Computer Algebra Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/29-1.1.3.8-P-x-c-x-^m-a+b-x^n-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [594]. This is test number [29].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (594)	0.00 (0)
Mathematica	100.00 (594)	0.00 (0)
Maple	97.14 (577)	2.86 (17)
Fricas	90.57 (538)	9.43 (56)
Mupad	75.59 (449)	24.41 (145)
Sympy	72.39 (430)	27.61 (164)
Giac	70.71 (420)	29.29 (174)
Maxima	69.87 (415)	30.13 (179)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

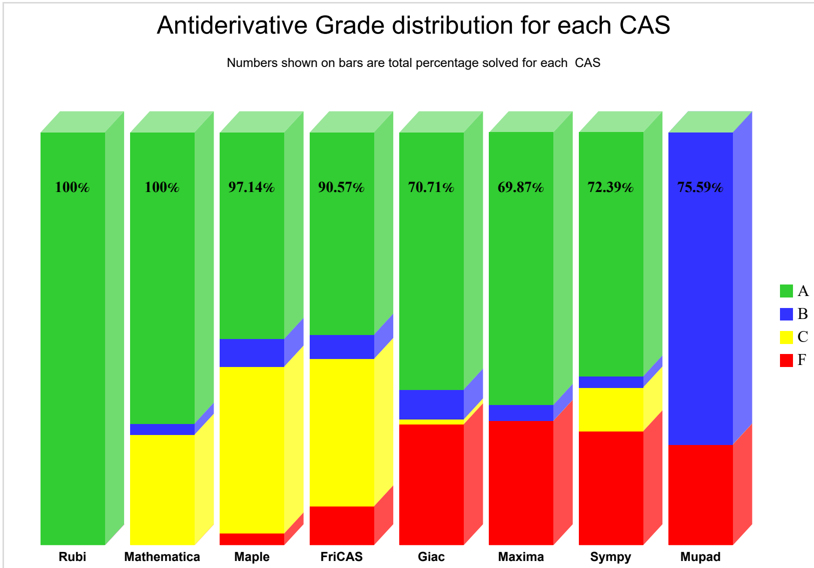
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

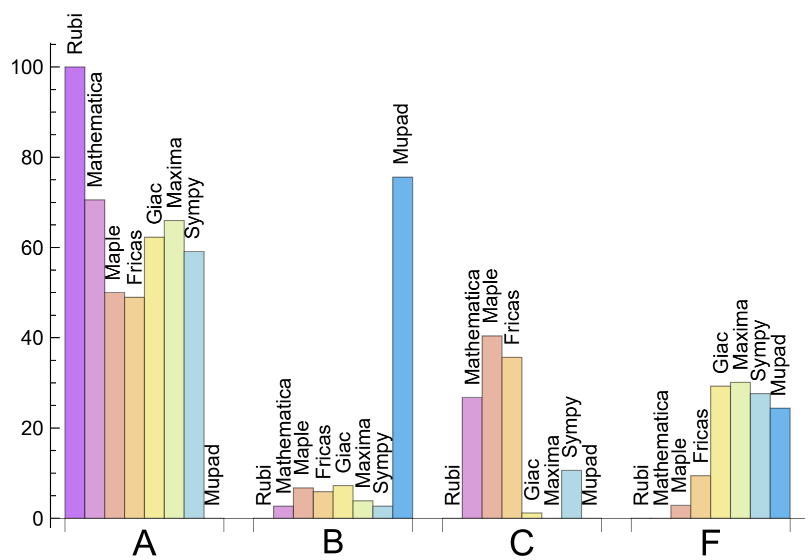
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	70.539	2.694	26.768	0.000
Maxima	65.993	3.872	0.000	30.135
Giac	62.290	7.239	1.178	29.293
Sympy	59.091	2.694	10.606	27.609
Maple	50.000	6.734	40.404	2.862
Fricas	48.990	5.892	35.690	9.428
Mupad	0.000	75.589	0.000	24.411

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	17	94.12	5.88	0.00
Fricas	56	55.36	41.07	3.57
Mupad	145	0.00	100.00	0.00
Sympy	164	1.22	96.95	1.83
Giac	174	92.53	2.30	5.17
Maxima	179	96.09	0.00	3.91

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.19
Maxima	0.26
Giac	0.28
Maple	1.67
Mathematica	2.80
Fricas	3.12
Sympy	4.78
Mupad	6.34

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	171.29	0.91	154.00	0.95
Sympy	185.59	1.15	128.00	0.93
Maxima	188.27	1.11	173.00	0.99
Giac	209.21	1.19	185.50	1.02
Maple	234.76	0.92	147.00	0.82
Rubi	245.10	1.00	221.50	1.00
Mupad	490.04	2.28	199.00	1.03
Fricas	15695.66	70.51	201.50	1.02

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

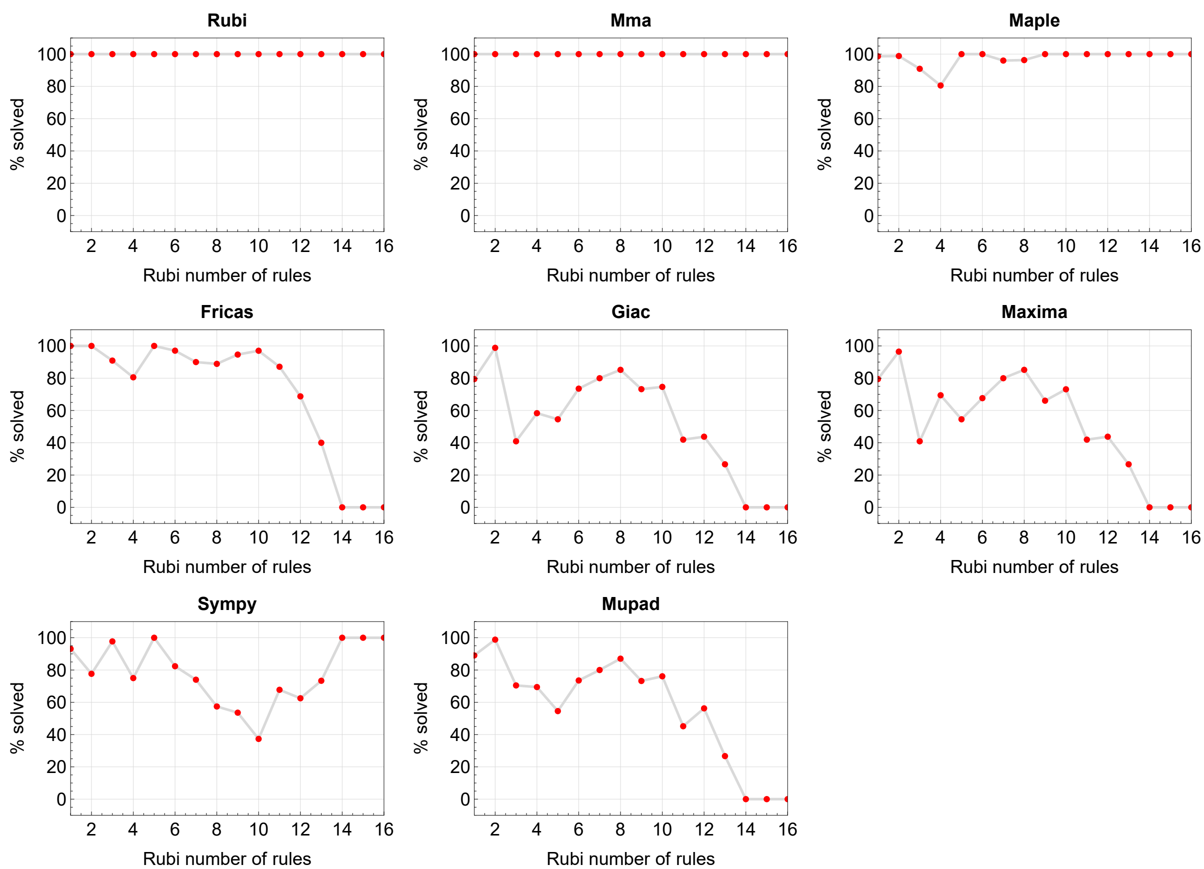


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

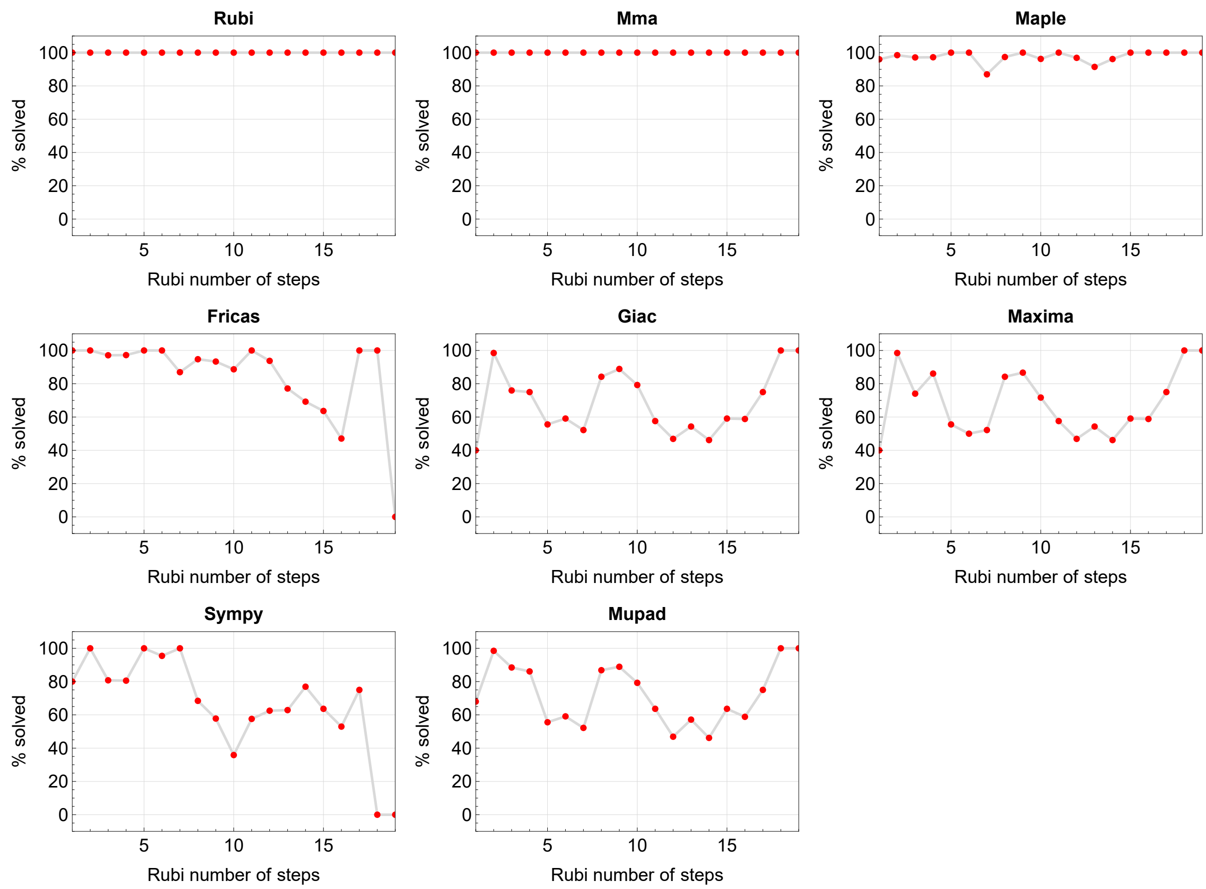


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

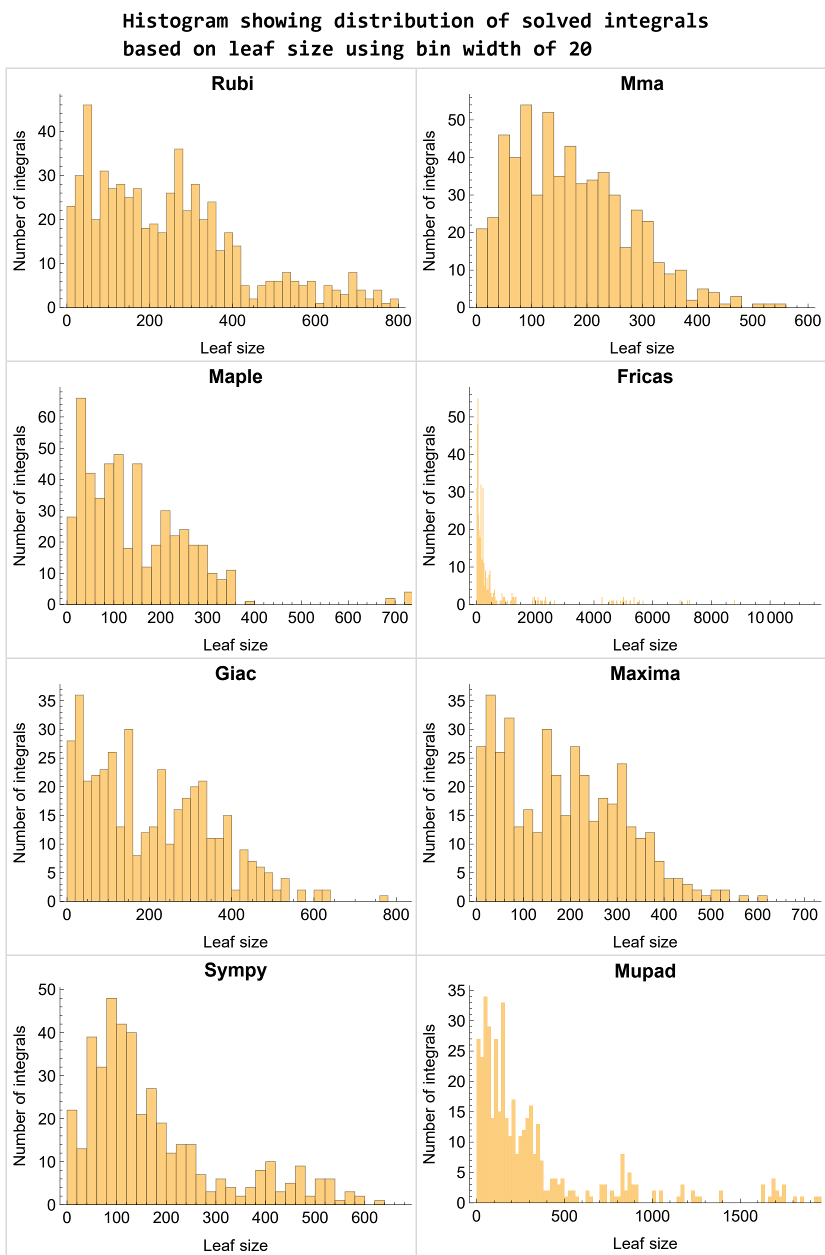


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

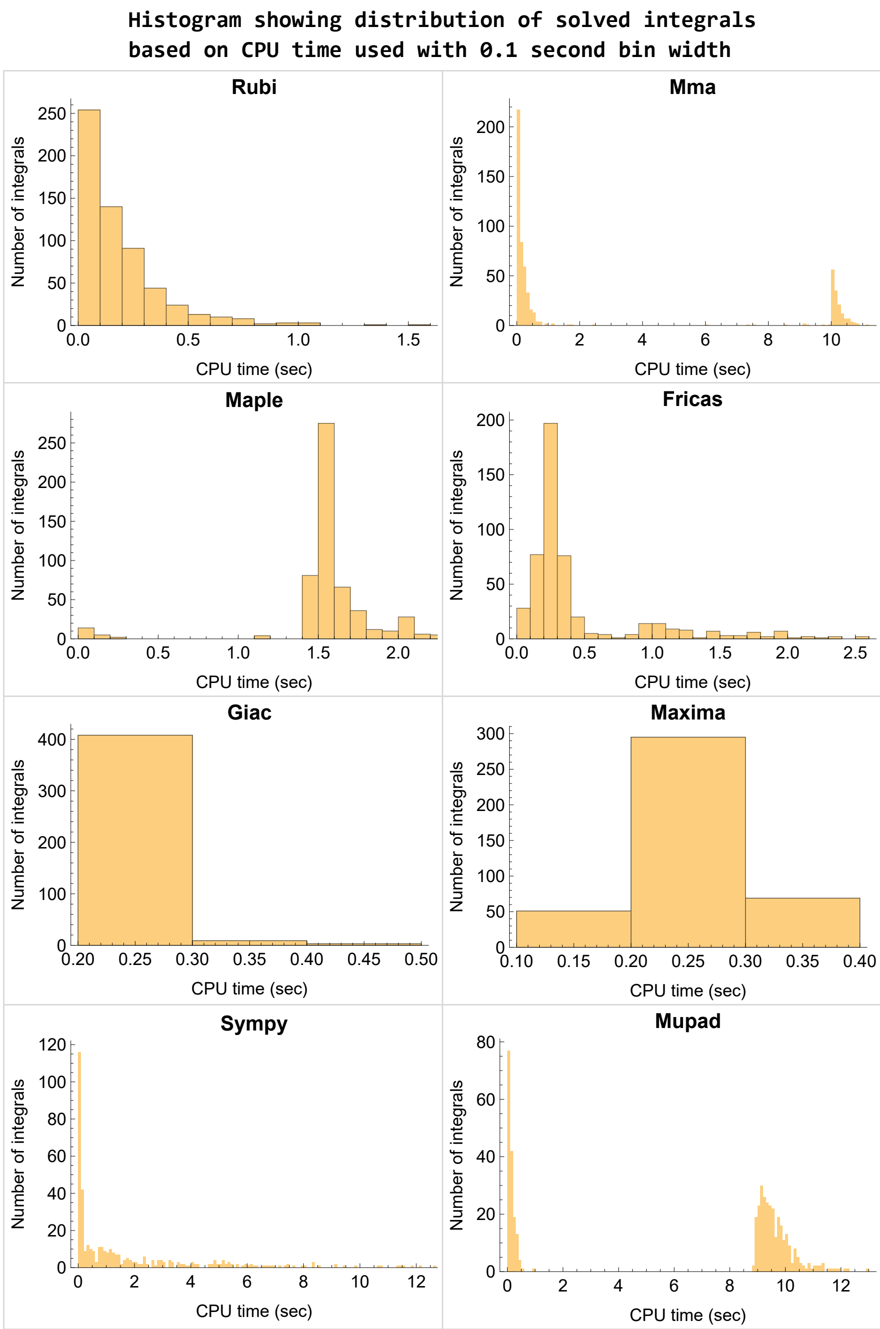


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

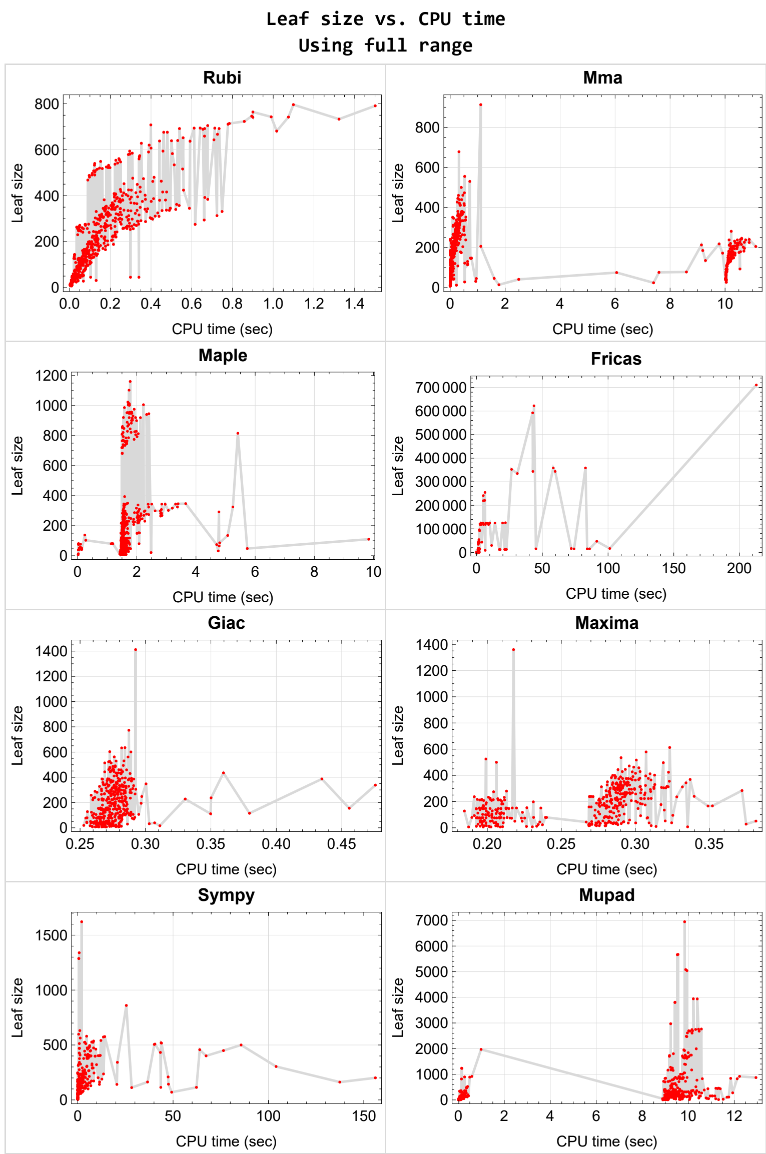


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple $\{81, 93, 97, 113, 592, 594\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
 # 1.7 is a fudge factor since it is low side from actual leaf count
 leafCount = round(1.7*count_ops(anti))

 except Exception as ee:
     leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

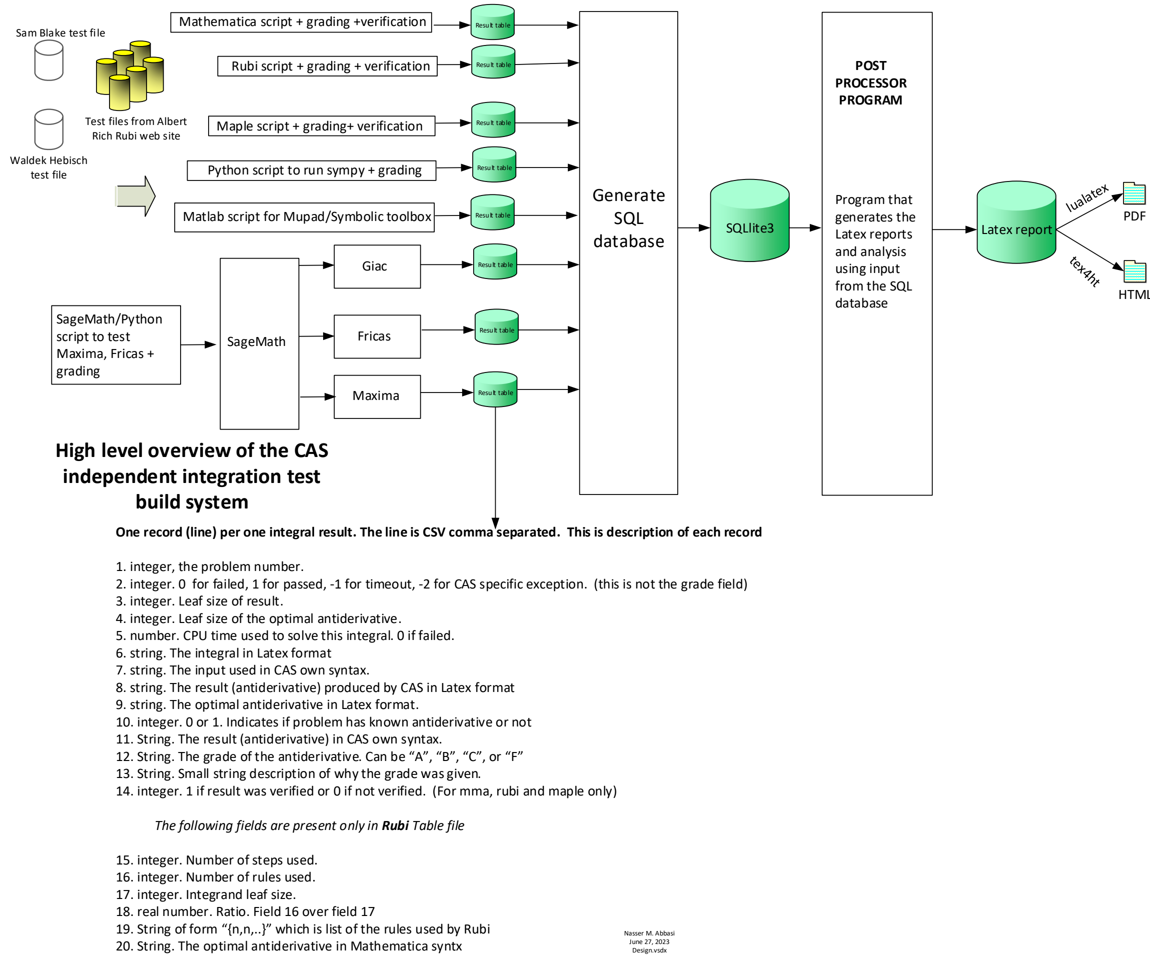
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 37, 38, 39, 40, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 76, 77, 78, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 551, 552, 553, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 591, 592, 593, 594 }

B grade { 21, 32, 33, 34, 35, 36, 41, 44, 45, 46, 47, 369, 370, 371, 372, 557 }

C grade { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 124, 161, 210, 211, 212, 213, 214, 220, 221, 222, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 567, 590 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 17, 18, 19, 28, 39, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 76, 77, 78, 107, 108, 109, 110, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 153, 159, 161, 167, 179, 180, 181, 182, 183, 184, 185, 211, 212, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 350, 357, 364, 365, 366, 367, 368, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 408, 409, 410, 411, 417, 418, 419, 420, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 477, 478, 479, 480, 481, 482, 483, 484, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 580, 589, 593 }

B grade { 20, 21, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 44, 45, 46, 47, 83, 84, 85, 86, 87, 88, 89, 90, 99, 100, 101, 102, 103, 104, 105, 106, 123, 369, 370, 371, 372, 557 }

C grade { 7, 8, 9, 10, 11, 12, 22, 23, 24, 25, 26, 27, 57, 58, 70, 71, 72, 73, 74, 75, 79, 80, 81, 82, 91, 92, 93, 94, 95, 96, 97, 98, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 149, 150, 151, 152, 154, 155, 156, 157, 158, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 213, 214, 220, 221, 222, 233, 234, 235, 236, 237, 238, 239, 260, 261, 262, 263, 264, 265, 266, 286, 287, 288, 289, 290, 291, 292, 293, 294, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 358, 359, 360, 361, 362, 363, 403, 404, 405, 406, 407, 412, 413, 414, 415, 416, 421, 422, 423, 424, 425, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 592, 594 }

F normal fail { 474, 475, 476, 551, 552, 553, 576, 581, 582, 583, 584, 585, 586, 587, 588, 591 }

F(-1) timedout fail { 590 }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 76, 77, 78, 123, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 153, 159, 161, 167, 180, 181, 182, 183, 184, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 477, 478, 479, 480, 481, 482, 483, 484, 495, 496, 497, 498, 499, 506, 507, 508, 509, 510, 511, 512, 513, 514, 525, 526, 527, 528, 529, 530, 531, 532, 533, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 571, 572, 573, 574, 575, 579, 580, 585, 589, 593, 594 }

B grade { 40, 41, 44, 45, 46, 47, 156, 160, 163, 164, 168, 179, 185, 221, 222, 283, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 557, 569, 570, 577, 578, 591, 592 }

C grade { 7, 8, 9, 10, 11, 12, 24, 25, 26, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 149, 150, 151, 152, 154, 155, 157, 158, 162, 165, 166, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 192, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494 }

F normal fail { 474, 475, 476, 500, 501, 502, 503, 504, 505, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 534, 535, 551, 552, 553, 576, 581, 582, 583, 586, 587, 588 }

F(-1) timedout fail { 186, 187, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209 }

F(-2) exception fail { 584, 590 }

Maxima

A grade { 1, 2, 4, 5, 7, 8, 9, 10, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 30, 33, 36, 39, 42, 44, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 76, 77, 78, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 580, 589, 592, 593 }

B grade { 3, 6, 20, 21, 31, 32, 34, 35, 37, 38, 40, 41, 43, 45, 46, 115, 161, 179, 185, 370, 371, 557, 594 }

C grade { }

F normal fail { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 210, 211, 212, 213, 214, 220, 221, 222, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 576, 581, 582, 583, 584, 585, 586, 587, 588, 590, 591 }

F(-1) timedout fail { }

F(-2) exception fail { 11, 12, 25, 26, 73, 74, 75 }

Giac

A grade { 1, 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 31, 32, 35, 36, 39, 42, 43, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 76, 77, 78, 116, 118, 120, 122, 124, 126, 128, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 170, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 189, 190, 191, 195, 196, 197, 201, 202, 203, 204, 207, 208, 209, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 477, 478, 479, 480, 481, 482, 483, 484, 487, 488, 489, 490, 491, 492, 493, 494, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 579, 580 }

B grade { 3, 6, 29, 115, 117, 119, 121, 123, 125, 127, 129, 131, 149, 150, 151, 161, 169, 171, 172, 173, 179, 186, 187, 188, 192, 193, 194, 198, 199, 200, 205, 206, 254, 371, 485, 486, 557, 577, 578, 591, 592, 593, 594 }

C grade { 30, 33, 34, 44, 45, 369, 370 }

F normal fail { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 86, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 107, 108, 109, 110, 111, 112, 113, 114, 210, 211, 212, 213, 214, 220, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 576, 581, 582, 583, 584, 585, 586, 588, 589, 590 }

F(-1) timedout fail { 37, 38, 40, 41 }

F(-2) exception fail { 87, 88, 89, 90, 103, 104, 105, 106, 587 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 91, 92, 93, 94, 95, 96, 97, 98, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 435, 444, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 536, 548, 549, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 580, 581, 582, 583, 589, 591, 592, 593, 594 }

C grade { }

F normal fail { }

F(-1) timedout fail { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 83, 84, 85, 86, 87, 88, 89, 90, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 210, 211, 212, 213, 214, 220, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 550, 551, 552, 553, 576, 584, 585, 586, 587, 588, 590 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 29, 31, 37, 38, 39, 42, 43, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 126, 128, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 159, 160, 162, 163, 164, 165, 167, 168, 169, 170, 180, 181, 182, 183, 184, 211, 212, 217, 218, 220, 223, 224, 225, 226, 227, 228, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 251, 252, 253, 254, 260, 262, 264, 266, 267, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 344, 345, 346, 351, 352, 353, 358, 359, 360, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 489, 495, 496, 497, 498, 499, 510, 511, 512, 513, 514, 515, 516, 517, 518, 529, 530, 531, 532, 533, 540, 541, 542, 543, 544, 545, 546, 552, 553, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 580, 581, 585 }

B grade { 3, 6, 125, 127, 140, 149, 158, 166, 179, 185, 221, 222, 557, 577, 578, 579 }

C grade { 18, 19, 20, 21, 27, 28, 30, 32, 33, 34, 35, 36, 49, 123, 161, 210, 213, 214, 215, 216, 219, 365, 366, 367, 368, 369, 370, 371, 372, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 534, 535, 536, 537, 538, 539, 547, 548, 549, 550, 576, 582, 584, 586 }

F normal fail { 589, 590 }

F(-1) timedout fail { 40, 41, 44, 45, 46, 47, 69, 72, 73, 74, 75, 129, 130, 131, 132, 150, 151, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 229, 230, 231, 232, 245, 246, 247, 248, 249, 250, 255, 256, 257, 258, 259, 261, 263, 265, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 341, 342, 343, 347, 348, 349, 350, 354, 355, 356, 357, 361, 362, 363, 364, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 485, 486, 487, 488, 490, 491, 492, 493, 494, 551, 583, 587, 588, 591 }

F(-2) exception fail { 592, 593, 594 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	53	48	77	53	102	77	58
N.S.	1	1.00	0.74	0.67	1.07	0.74	1.42	1.07	0.81
time (sec)	N/A	0.023	0.066	5.733	0.193	0.314	0.455	0.254	8.967

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	155	135	237	192	275	236	149
N.S.	1	1.00	0.96	0.84	1.47	1.19	1.71	1.47	0.93
time (sec)	N/A	0.069	0.148	5.071	0.191	0.270	0.799	0.260	0.069

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	355	325	525	457	631	526	299
N.S.	1	1.00	1.30	1.19	1.92	1.67	2.30	1.92	1.09
time (sec)	N/A	0.124	0.307	5.247	0.199	0.281	1.104	0.273	0.100

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	82	74	128	90	163	128	103
N.S.	1	1.00	0.72	0.65	1.12	0.79	1.43	1.12	0.90
time (sec)	N/A	0.050	0.106	4.699	0.185	0.309	0.634	0.256	8.997

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	320	292	500	417	563	514	316
N.S.	1	1.00	1.00	0.91	1.56	1.30	1.76	1.61	0.99
time (sec)	N/A	0.158	0.334	4.776	0.206	0.558	1.310	0.269	9.147

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	708	708	913	816	1360	1221	1622	1412	896
N.S.	1	1.00	1.29	1.15	1.92	1.72	2.29	1.99	1.27
time (sec)	N/A	0.399	1.114	5.415	0.218	0.499	2.069	0.292	0.274

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	124	32	135	1931	76	141	127
N.S.	1	1.00	0.77	0.20	0.84	11.99	0.47	0.88	0.79
time (sec)	N/A	0.079	0.059	4.752	0.273	1.919	0.339	0.270	9.239

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	180	65	169	2088	105	174	169
N.S.	1	1.00	0.95	0.34	0.89	11.05	0.56	0.92	0.89
time (sec)	N/A	0.094	0.157	4.784	0.281	0.931	0.449	0.273	9.056

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	205	86	203	2215	146	194	206
N.S.	1	1.00	0.95	0.40	0.94	10.30	0.68	0.90	0.96
time (sec)	N/A	0.129	0.161	4.801	0.283	0.931	0.597	0.274	0.283

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	229	110	238	2308	185	218	241
N.S.	1	1.00	0.95	0.46	0.99	9.62	0.77	0.91	1.00
time (sec)	N/A	0.147	0.201	9.837	0.269	0.940	0.709	0.268	9.336

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	125	32	0	1961	76	141	127
N.S.	1	1.00	0.78	0.20	0.00	12.18	0.47	0.88	0.79
time (sec)	N/A	0.081	0.059	1.504	0.000	1.003	0.337	0.285	9.199

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	125	34	0	1905	78	136	124
N.S.	1	1.00	0.78	0.21	0.00	11.83	0.48	0.84	0.77
time (sec)	N/A	0.063	0.055	1.680	0.000	0.985	0.343	0.268	0.229

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84
time (sec)	N/A	0.010	0.007	1.454	0.276	0.279	0.044	0.258	9.002

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84
time (sec)	N/A	0.009	0.006	1.469	0.280	0.274	0.045	0.268	0.031

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	17	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	0.77	0.73
time (sec)	N/A	0.008	0.007	1.514	0.269	0.322	0.041	0.264	9.011

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	17	19	18
N.S.	1	1.00	1.00	0.86	0.82	0.82	0.77	0.86	0.82
time (sec)	N/A	0.008	0.006	1.457	0.269	0.267	0.037	0.253	0.127

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	32	32	44	33	46
N.S.	1	1.00	1.00	0.80	0.78	0.78	1.07	0.80	1.12
time (sec)	N/A	0.017	0.011	1.486	0.287	0.277	0.064	0.271	0.152

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	34	28	54	28	28
N.S.	1	1.00	1.07	1.00	1.17	0.97	1.86	0.97	0.97
time (sec)	N/A	0.014	0.014	1.522	0.276	0.290	0.072	0.268	0.055

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	29	33	26	53	26	28
N.S.	1	1.00	1.00	1.00	1.14	0.90	1.83	0.90	0.97
time (sec)	N/A	0.013	0.010	1.500	0.281	0.411	0.080	0.267	0.051

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	195	163	107	88	48	49
N.S.	1	1.00	0.90	5.00	4.18	2.74	2.26	1.23	1.26
time (sec)	N/A	0.018	0.016	1.540	0.276	0.566	0.125	0.281	9.076

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	129	202	174	114	105	57	49
N.S.	1	1.00	3.15	4.93	4.24	2.78	2.56	1.39	1.20
time (sec)	N/A	0.033	0.057	1.523	0.275	0.457	0.153	0.282	0.241

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	90	47	159	310	26	103	98
N.S.	1	1.00	0.76	0.40	1.35	2.63	0.22	0.87	0.83
time (sec)	N/A	0.086	0.013	1.566	0.269	0.912	0.063	0.272	9.277

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	90	49	159	305	22	115	96
N.S.	1	1.00	0.76	0.42	1.35	2.58	0.19	0.97	0.81
time (sec)	N/A	0.070	0.013	1.620	0.270	0.285	0.073	0.274	9.406

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	124	52	188	1961	76	147	127
N.S.	1	1.00	0.77	0.32	1.17	12.18	0.47	0.91	0.79
time (sec)	N/A	0.114	0.041	1.716	0.274	1.024	0.340	0.275	9.278

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	122	36	0	1344	75	118	158
N.S.	1	1.00	0.91	0.27	0.00	10.03	0.56	0.88	1.18
time (sec)	N/A	0.081	0.033	1.530	0.000	1.425	0.151	0.279	0.193

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	123	36	0	1040	70	124	178
N.S.	1	1.00	0.92	0.27	0.00	7.76	0.52	0.93	1.33
time (sec)	N/A	0.061	0.032	1.523	0.000	1.533	0.197	0.269	9.330

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	72	42	42	36	60	37	84
N.S.	1	1.00	1.95	1.14	1.14	0.97	1.62	1.00	2.27
time (sec)	N/A	0.039	0.024	1.545	0.282	0.336	0.138	0.263	9.146

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	71	38	44	36	60	38	86
N.S.	1	1.00	1.82	0.97	1.13	0.92	1.54	0.97	2.21
time (sec)	N/A	0.028	0.021	1.508	0.267	0.266	0.148	0.272	9.026

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	76	115	47	134	58	115	147
N.S.	1	1.00	1.58	2.40	0.98	2.79	1.21	2.40	3.06
time (sec)	N/A	0.024	0.026	1.515	0.275	0.311	0.161	0.379	9.269

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	72	96	36	40	85	76	145
N.S.	1	1.00	1.53	2.04	0.77	0.85	1.81	1.62	3.09
time (sec)	N/A	0.023	0.033	1.526	0.274	0.283	0.143	0.280	9.593

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	99	97	122	182	58	56	176
N.S.	1	1.00	1.74	1.70	2.14	3.19	1.02	0.98	3.09
time (sec)	N/A	0.046	0.034	1.486	0.275	0.483	0.187	0.277	9.585

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	106	98	93	43	95	51	142
N.S.	1	1.00	2.26	2.09	1.98	0.91	2.02	1.09	3.02
time (sec)	N/A	0.040	0.045	1.524	0.273	0.480	0.164	0.265	0.371

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	146	116	51	52	100	143	172
N.S.	1	1.00	2.92	2.32	1.02	1.04	2.00	2.86	3.44
time (sec)	N/A	0.053	0.058	1.506	0.282	0.472	0.175	0.267	9.566

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	150	119	167	53	110	109	172
N.S.	1	1.00	2.83	2.25	3.15	1.00	2.08	2.06	3.25
time (sec)	N/A	0.054	0.066	1.485	0.280	0.782	0.190	0.283	9.635

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	149	117	168	56	109	91	173
N.S.	1	1.00	2.76	2.17	3.11	1.04	2.02	1.69	3.20
time (sec)	N/A	0.041	0.050	1.476	0.287	0.316	0.184	0.269	9.381

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	147	118	52	53	102	85	171
N.S.	1	1.00	2.77	2.23	0.98	1.00	1.92	1.60	3.23
time (sec)	N/A	0.040	0.046	1.496	0.278	0.291	0.187	0.273	9.544

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	95	112	162	160	70	0	193
N.S.	1	1.00	1.56	1.84	2.66	2.62	1.15	0.00	3.16
time (sec)	N/A	0.027	0.021	1.560	0.281	0.297	0.176	0.000	9.448

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	116	117	173	205	73	0	221
N.S.	1	1.00	1.66	1.67	2.47	2.93	1.04	0.00	3.16
time (sec)	N/A	0.050	0.033	1.571	0.278	0.357	0.199	0.000	9.428

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	50	32	31	31	42	32	46
N.S.	1	1.00	1.25	0.80	0.78	0.78	1.05	0.80	1.15
time (sec)	N/A	0.021	0.012	1.466	0.269	0.290	0.060	0.303	0.157

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	122	217	236	430	0	0	386
N.S.	1	1.00	1.74	3.10	3.37	6.14	0.00	0.00	5.51
time (sec)	N/A	0.046	0.048	1.593	0.281	1.992	0.000	0.000	10.698

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	238	227	252	470	0	0	444
N.S.	1	1.00	2.70	2.58	2.86	5.34	0.00	0.00	5.05
time (sec)	N/A	0.080	0.567	1.587	0.306	1.665	0.000	0.000	11.273

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	12	12	12	12	7	13	12
N.S.	1	1.00	1.09	1.09	1.09	1.09	0.64	1.18	1.09
time (sec)	N/A	0.007	0.001	1.492	0.201	0.384	0.025	0.278	0.038

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	210	17	20	16	15
N.S.	1	1.00	1.00	0.86	10.00	0.81	0.95	0.76	0.71
time (sec)	N/A	0.010	0.003	1.497	0.275	0.407	0.068	0.311	9.660

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F(-1)	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	247	219	78	429	0	194	436
N.S.	1	1.00	3.48	3.08	1.10	6.04	0.00	2.73	6.14
time (sec)	N/A	0.068	0.283	1.499	0.282	1.981	0.000	0.297	10.856

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	288	224	238	459	0	235	456
N.S.	1	1.00	3.79	2.95	3.13	6.04	0.00	3.09	6.00
time (sec)	N/A	0.070	0.194	1.524	0.271	1.021	0.000	0.288	11.331

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	253	222	239	450	0	133	453
N.S.	1	1.00	3.24	2.85	3.06	5.77	0.00	1.71	5.81
time (sec)	N/A	0.075	0.270	1.525	0.270	1.029	0.000	0.286	11.204

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	244	223	78	450	0	125	435
N.S.	1	1.00	3.25	2.97	1.04	6.00	0.00	1.67	5.80
time (sec)	N/A	0.074	0.278	1.521	0.279	1.780	0.000	0.286	11.394

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	28	26	26	24	27	35
N.S.	1	1.00	0.97	0.88	0.81	0.81	0.75	0.84	1.09
time (sec)	N/A	0.024	0.016	1.722	0.269	0.493	0.207	0.272	9.458

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	62	55	47	47	323	52	87
N.S.	1	1.00	1.13	1.00	0.85	0.85	5.87	0.95	1.58
time (sec)	N/A	0.041	0.039	1.496	0.271	0.369	0.462	0.275	10.083

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75
time (sec)	N/A	0.004	0.001	1.455	0.187	0.274	0.021	0.269	0.027

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	32	32	5	33	63
N.S.	1	1.00	1.00	1.10	1.07	1.07	0.17	1.10	2.10
time (sec)	N/A	0.019	0.010	1.481	0.291	0.288	0.059	0.274	0.133

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	15	17	16
N.S.	1	1.00	1.00	0.94	0.89	0.89	0.83	0.94	0.89
time (sec)	N/A	0.012	0.007	1.471	0.311	0.270	0.045	0.267	10.058

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	113	98	97	97	117	97	97
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.04	0.86	0.86
time (sec)	N/A	0.065	0.006	1.469	0.203	0.411	0.023	0.272	0.093

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	75	74	74	90	74	74
N.S.	1	1.00	1.00	0.85	0.84	0.84	1.02	0.84	0.84
time (sec)	N/A	0.041	0.005	1.482	0.223	0.378	0.022	0.268	0.039

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	58	50	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.97	0.83	0.83
time (sec)	N/A	0.024	0.003	0.138	0.224	0.343	0.018	0.270	0.030

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.009	0.001	1.425	0.226	0.652	0.022	0.280	0.020

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	124	32	135	1931	76	141	127
N.S.	1	1.00	0.77	0.20	0.84	11.99	0.47	0.88	0.79
time (sec)	N/A	0.072	0.046	1.464	0.320	0.900	0.344	0.270	0.213

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	180	65	169	2088	105	174	169
N.S.	1	1.00	0.95	0.34	0.89	11.05	0.56	0.92	0.89
time (sec)	N/A	0.087	0.148	1.468	0.279	0.923	0.448	0.285	10.158

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	585	585	78	786	0	117	265	0	0
N.S.	1	1.00	0.13	1.34	0.00	0.20	0.45	0.00	0.00
time (sec)	N/A	0.308	8.594	1.722	0.000	0.128	2.638	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	556	556	76	762	0	93	170	0	0
N.S.	1	1.00	0.14	1.37	0.00	0.17	0.31	0.00	0.00
time (sec)	N/A	0.245	7.596	1.591	0.000	0.107	1.827	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	525	525	75	744	0	69	163	0	0
N.S.	1	1.00	0.14	1.42	0.00	0.13	0.31	0.00	0.00
time (sec)	N/A	0.181	6.056	1.635	0.000	0.086	1.650	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	75	720	0	43	78	0	0
N.S.	1	1.00	0.15	1.47	0.00	0.09	0.16	0.00	0.00
time (sec)	N/A	0.124	10.041	1.515	0.000	0.089	1.615	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	96	765	0	94	163	0	0
N.S.	1	1.00	0.18	1.47	0.00	0.18	0.31	0.00	0.00
time (sec)	N/A	0.189	10.060	1.505	0.000	0.133	3.771	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	554	554	123	809	0	155	163	0	0
N.S.	1	1.00	0.22	1.46	0.00	0.28	0.29	0.00	0.00
time (sec)	N/A	0.227	10.095	1.500	0.000	0.112	11.466	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	138	853	0	214	163	0	0
N.S.	1	1.00	0.24	1.47	0.00	0.37	0.28	0.00	0.00
time (sec)	N/A	0.296	10.120	1.514	0.000	0.089	36.616	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	590	590	135	785	0	87	187	0	0
N.S.	1	1.00	0.23	1.33	0.00	0.15	0.32	0.00	0.00
time (sec)	N/A	0.388	10.146	1.632	0.000	0.082	1.738	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	594	594	130	821	0	153	189	0	0
N.S.	1	1.00	0.22	1.38	0.00	0.26	0.32	0.00	0.00
time (sec)	N/A	0.295	10.132	1.538	0.000	0.171	5.186	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	628	170	861	0	261	209	0	0
N.S.	1	1.00	0.27	1.37	0.00	0.42	0.33	0.00	0.00
time (sec)	N/A	0.353	10.193	1.517	0.000	0.151	47.416	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	676	676	196	921	0	373	0	0	0
N.S.	1	1.00	0.29	1.36	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.462	10.253	1.549	0.000	0.110	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	200	43	192	5014	156	175	357
N.S.	1	1.00	1.08	0.23	1.03	26.96	0.84	0.94	1.92
time (sec)	N/A	0.124	0.090	1.649	0.276	0.892	0.516	0.279	10.141

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	214	66	240	7245	245	214	370
N.S.	1	1.00	0.96	0.30	1.08	32.64	1.10	0.96	1.67
time (sec)	N/A	0.210	0.169	1.660	0.269	1.335	8.527	0.284	10.211

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	280	277	90	303	8787	0	294	513
N.S.	1	0.99	0.98	0.32	1.07	31.16	0.00	1.04	1.82
time (sec)	N/A	0.294	0.246	1.524	0.283	6.542	0.000	0.284	10.334

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	270	269	85	0	12827	0	282	769
N.S.	1	0.99	0.99	0.31	0.00	47.16	0.00	1.04	2.83
time (sec)	N/A	0.328	0.283	1.742	0.000	1.431	0.000	0.277	10.379

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	439	181	0	29479	0	462	1700
N.S.	1	1.00	1.06	0.44	0.00	70.86	0.00	1.11	4.09
time (sec)	N/A	0.457	0.419	1.690	0.000	11.406	0.000	0.272	9.180

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	645	643	678	350	0	47284	0	773	2971
N.S.	1	1.00	1.05	0.54	0.00	73.31	0.00	1.20	4.61
time (sec)	N/A	0.709	0.321	1.694	0.000	91.411	0.000	0.287	9.237

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	54	38	37	37	44	38	49
N.S.	1	1.00	1.26	0.88	0.86	0.86	1.02	0.88	1.14
time (sec)	N/A	0.052	0.016	1.594	0.309	0.432	0.062	0.261	0.107

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	54	36	37	37	46	38	51
N.S.	1	1.00	1.17	0.78	0.80	0.80	1.00	0.83	1.11
time (sec)	N/A	0.053	0.013	1.491	0.273	0.586	0.065	0.265	0.098

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	37	37	48	38	49
N.S.	1	1.00	1.00	0.86	0.84	0.84	1.09	0.86	1.11
time (sec)	N/A	0.027	0.010	1.450	0.300	0.321	0.061	0.267	9.379

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	47	47	0	21	92	0	312
N.S.	1	1.00	0.20	0.20	0.00	0.09	0.40	0.00	1.36
time (sec)	N/A	0.040	10.027	1.790	0.000	0.107	0.884	0.000	0.159

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	43	41	0	23	97	0	342
N.S.	1	1.00	0.17	0.16	0.00	0.09	0.38	0.00	1.33
time (sec)	N/A	0.045	10.023	1.614	0.000	0.152	1.207	0.000	9.597

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	144	144	63	95	0	21	82	0	326
N.S.	1	1.00	0.44	0.66	0.00	0.15	0.57	0.00	2.26
time (sec)	N/A	0.024	10.031	1.720	0.000	0.146	1.180	0.000	9.564

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	67	52	0	23	99	0	360
N.S.	1	1.00	0.50	0.39	0.00	0.17	0.73	0.00	2.67
time (sec)	N/A	0.023	10.030	1.748	0.000	0.188	0.984	0.000	0.274

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	468	468	90	1003	0	49	122	0	0
N.S.	1	1.00	0.19	2.14	0.00	0.10	0.26	0.00	0.00
time (sec)	N/A	0.089	10.069	1.749	0.000	0.098	1.712	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	481	481	91	949	0	55	128	0	0
N.S.	1	1.00	0.19	1.97	0.00	0.11	0.27	0.00	0.00
time (sec)	N/A	0.098	10.070	1.805	0.000	0.141	2.347	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	92	952	0	48	112	0	0
N.S.	1	1.00	0.34	3.51	0.00	0.18	0.41	0.00	0.00
time (sec)	N/A	0.050	10.048	1.758	0.000	0.105	2.378	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	93	1012	0	56	129	0	0
N.S.	1	1.00	0.35	3.80	0.00	0.21	0.48	0.00	0.00
time (sec)	N/A	0.046	10.054	1.723	0.000	0.088	1.948	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	520	520	89	1004	0	53	124	0	0
N.S.	1	1.00	0.17	1.93	0.00	0.10	0.24	0.00	0.00
time (sec)	N/A	0.147	10.057	1.742	0.000	0.100	1.256	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	533	533	89	950	0	56	129	0	0
N.S.	1	1.00	0.17	1.78	0.00	0.11	0.24	0.00	0.00
time (sec)	N/A	0.122	10.057	1.777	0.000	0.105	1.443	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	90	953	0	52	114	0	0
N.S.	1	1.00	0.35	3.72	0.00	0.20	0.45	0.00	0.00
time (sec)	N/A	0.068	10.042	1.667	0.000	0.121	1.395	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	92	1013	0	57	131	0	0
N.S.	1	1.00	0.37	4.04	0.00	0.23	0.52	0.00	0.00
time (sec)	N/A	0.049	10.041	1.727	0.000	0.197	1.342	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	49	48	0	21	92	0	313
N.S.	1	1.00	0.39	0.38	0.00	0.17	0.72	0.00	2.46
time (sec)	N/A	0.021	10.030	1.722	0.000	0.084	0.864	0.000	0.136

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	45	42	0	23	97	0	343
N.S.	1	1.00	0.32	0.30	0.00	0.16	0.68	0.00	2.42
time (sec)	N/A	0.025	10.020	1.613	0.000	0.120	1.220	0.000	9.148

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	264	264	63	96	0	21	82	0	327
N.S.	1	1.00	0.24	0.36	0.00	0.08	0.31	0.00	1.24
time (sec)	N/A	0.048	10.033	1.617	0.000	0.153	1.190	0.000	9.173

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	67	52	0	23	97	0	361
N.S.	1	1.00	0.27	0.21	0.00	0.09	0.39	0.00	1.46
time (sec)	N/A	0.045	10.031	1.641	0.000	0.254	0.971	0.000	0.118

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	47	48	0	21	92	0	312
N.S.	1	1.00	0.37	0.38	0.00	0.17	0.73	0.00	2.48
time (sec)	N/A	0.026	10.021	1.705	0.000	0.091	1.170	0.000	9.174

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	43	42	0	23	97	0	342
N.S.	1	1.00	0.30	0.29	0.00	0.16	0.68	0.00	2.39
time (sec)	N/A	0.025	10.016	1.646	0.000	0.161	1.003	0.000	0.056

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	263	263	63	96	0	21	82	0	326
N.S.	1	1.00	0.24	0.37	0.00	0.08	0.31	0.00	1.24
time (sec)	N/A	0.036	10.024	1.684	0.000	0.140	0.930	0.000	9.172

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	67	52	0	23	97	0	360
N.S.	1	1.00	0.27	0.21	0.00	0.09	0.39	0.00	1.45
time (sec)	N/A	0.044	10.025	1.615	0.000	0.253	1.189	0.000	9.153

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	90	1003	0	48	122	0	0
N.S.	1	1.00	0.35	3.92	0.00	0.19	0.48	0.00	0.00
time (sec)	N/A	0.040	10.074	1.757	0.000	0.101	1.762	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	90	949	0	56	128	0	0
N.S.	1	1.00	0.34	3.61	0.00	0.21	0.49	0.00	0.00
time (sec)	N/A	0.035	10.064	1.738	0.000	0.099	2.378	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	497	497	91	952	0	49	112	0	0
N.S.	1	1.00	0.18	1.92	0.00	0.10	0.23	0.00	0.00
time (sec)	N/A	0.111	10.045	1.733	0.000	0.154	2.402	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	488	488	93	1012	0	55	128	0	0
N.S.	1	1.00	0.19	2.07	0.00	0.11	0.26	0.00	0.00
time (sec)	N/A	0.096	10.052	1.739	0.000	0.236	1.985	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	89	1004	0	52	124	0	0
N.S.	1	1.00	0.37	4.17	0.00	0.22	0.51	0.00	0.00
time (sec)	N/A	0.065	10.054	1.715	0.000	0.095	1.233	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	89	950	0	57	129	0	0
N.S.	1	1.00	0.36	3.83	0.00	0.23	0.52	0.00	0.00
time (sec)	N/A	0.054	10.052	1.743	0.000	0.105	1.393	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	549	549	90	953	0	53	114	0	0
N.S.	1	1.00	0.16	1.74	0.00	0.10	0.21	0.00	0.00
time (sec)	N/A	0.152	10.040	1.694	0.000	0.119	1.384	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	540	540	92	1013	0	56	129	0	0
N.S.	1	1.00	0.17	1.88	0.00	0.10	0.24	0.00	0.00
time (sec)	N/A	0.123	10.043	1.748	0.000	0.283	1.378	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	75	720	0	43	78	0	0
N.S.	1	1.00	0.15	1.47	0.00	0.09	0.16	0.00	0.00
time (sec)	N/A	0.110	0.017	1.493	0.000	0.085	0.951	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	503	75	681	0	47	82	0	0
N.S.	1	1.00	0.15	1.35	0.00	0.09	0.16	0.00	0.00
time (sec)	N/A	0.124	10.037	1.510	0.000	0.093	1.052	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	76	683	0	43	73	0	0
N.S.	1	1.00	0.15	1.33	0.00	0.08	0.14	0.00	0.00
time (sec)	N/A	0.136	10.029	1.510	0.000	0.085	1.022	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	508	78	726	0	47	83	0	0
N.S.	1	1.00	0.15	1.43	0.00	0.09	0.16	0.00	0.00
time (sec)	N/A	0.118	10.032	1.526	0.000	0.132	1.017	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	42	33	0	18	61	0	373
N.S.	1	1.00	0.17	0.13	0.00	0.07	0.25	0.00	1.52
time (sec)	N/A	0.056	10.020	1.645	0.000	0.120	0.742	0.000	8.979

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	38	29	0	18	65	0	406
N.S.	1	1.00	0.14	0.11	0.00	0.07	0.24	0.00	1.50
time (sec)	N/A	0.078	10.019	1.661	0.000	0.301	0.848	0.000	9.001

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	275	275	58	65	0	18	56	0	374
N.S.	1	1.00	0.21	0.24	0.00	0.07	0.20	0.00	1.36
time (sec)	N/A	0.078	10.034	1.622	0.000	0.085	0.781	0.000	8.961

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	62	36	0	18	66	0	405
N.S.	1	1.00	0.24	0.14	0.00	0.07	0.25	0.00	1.55
time (sec)	N/A	0.061	10.032	1.582	0.000	0.084	0.820	0.000	8.965

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	134	34	126	39057	126	227	182
N.S.	1	1.00	1.54	0.39	1.45	448.93	1.45	2.61	2.09
time (sec)	N/A	0.046	0.035	1.473	0.289	1.803	0.451	0.285	9.250

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	184	32	207	41851	124	213	160
N.S.	1	1.00	0.84	0.15	0.95	191.10	0.57	0.97	0.73
time (sec)	N/A	0.111	0.088	1.471	0.297	1.214	0.434	0.283	9.063

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	168	69	157	40560	156	254	283
N.S.	1	1.00	1.53	0.63	1.43	368.73	1.42	2.31	2.57
time (sec)	N/A	0.057	0.154	1.498	0.281	1.878	0.594	0.274	9.190

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	224	66	238	43065	155	238	282
N.S.	1	1.00	0.93	0.27	0.99	178.69	0.64	0.99	1.17
time (sec)	N/A	0.131	0.198	1.479	0.293	1.492	0.585	0.279	9.464

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	193	89	186	40637	194	272	315
N.S.	1	1.00	1.42	0.65	1.37	298.80	1.43	2.00	2.32
time (sec)	N/A	0.075	0.141	1.518	0.284	1.211	1.151	0.283	9.566

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	249	86	269	43180	192	256	315
N.S.	1	1.00	0.94	0.32	1.01	162.33	0.72	0.96	1.18
time (sec)	N/A	0.154	0.198	1.480	0.294	2.189	1.137	0.277	9.415

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	217	113	223	40780	231	296	351
N.S.	1	1.00	1.34	0.70	1.38	251.73	1.43	1.83	2.17
time (sec)	N/A	0.097	0.179	1.484	0.286	1.265	0.890	0.283	9.441

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	274	110	304	43302	231	280	350
N.S.	1	1.00	0.94	0.38	1.04	148.80	0.79	0.96	1.20
time (sec)	N/A	0.183	0.290	1.508	0.288	4.058	0.864	0.288	0.317

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	42	40	35	35	313	37	100
N.S.	1	1.00	1.75	1.67	1.46	1.46	13.04	1.54	4.17
time (sec)	N/A	0.015	0.019	1.478	0.276	0.282	0.284	0.271	9.375

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	99	27	86	34376	83	86	71
N.S.	1	1.00	1.01	0.28	0.88	350.78	0.85	0.88	0.72
time (sec)	N/A	0.049	0.085	1.504	0.280	1.224	0.266	0.265	0.104

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	187	39	153	120560	471	259	725
N.S.	1	1.00	1.61	0.34	1.32	1039.31	4.06	2.23	6.25
time (sec)	N/A	0.061	0.050	1.508	0.286	2.582	5.327	0.276	9.881

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	229	37	257	121386	466	271	712
N.S.	1	1.00	0.83	0.13	0.93	438.22	1.68	0.98	2.57
time (sec)	N/A	0.129	0.100	1.517	0.283	2.586	5.313	0.273	9.597

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	211	83	191	116982	508	306	477
N.S.	1	1.00	1.45	0.57	1.31	801.25	3.48	2.10	3.27
time (sec)	N/A	0.082	0.193	1.620	0.290	3.010	40.489	0.280	9.538

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	305	80	294	124258	505	301	472
N.S.	1	1.00	0.99	0.26	0.95	403.44	1.64	0.98	1.53
time (sec)	N/A	0.167	0.324	1.655	0.297	3.375	40.164	0.280	0.357

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	244	114	230	118710	0	334	826
N.S.	1	1.00	1.36	0.64	1.28	663.18	0.00	1.87	4.61
time (sec)	N/A	0.111	0.202	1.489	0.287	5.142	0.000	0.291	9.735

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	337	111	336	124787	0	330	826
N.S.	1	1.00	0.99	0.33	0.99	365.94	0.00	0.97	2.42
time (sec)	N/A	0.212	0.336	1.489	0.287	7.020	0.000	0.285	9.634

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	276	150	279	118903	0	370	874
N.S.	1	1.00	1.31	0.71	1.32	563.52	0.00	1.75	4.14
time (sec)	N/A	0.140	0.264	1.505	0.282	9.179	0.000	0.289	9.951

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	369	147	383	124960	0	366	873
N.S.	1	1.00	0.99	0.40	1.03	335.91	0.00	0.98	2.35
time (sec)	N/A	0.250	0.415	1.520	0.284	13.972	0.000	0.275	9.763

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	24	25	24	27	25	25
N.S.	1	1.00	0.96	0.86	0.89	0.86	0.96	0.89	0.89
time (sec)	N/A	0.009	0.004	1.433	0.197	0.265	0.016	0.266	9.139

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	27	27	27	29	27	27
N.S.	1	1.00	0.97	0.82	0.82	0.82	0.88	0.82	0.82
time (sec)	N/A	0.009	0.003	1.472	0.207	0.245	0.018	0.268	0.037

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	58	50	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.97	0.83	0.83
time (sec)	N/A	0.041	0.003	1.486	0.197	0.245	0.024	0.266	0.026

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	27	27	31	27	27
N.S.	1	1.00	1.00	0.82	0.82	0.82	0.94	0.82	0.82
time (sec)	N/A	0.009	0.001	1.440	0.194	0.254	0.017	0.266	0.038

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	60	50	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.00	0.83	0.83
time (sec)	N/A	0.018	0.003	1.499	0.219	0.244	0.021	0.258	0.027

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	53	61	53	53
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.94	0.82	0.82
time (sec)	N/A	0.063	0.005	1.474	0.195	0.253	0.021	0.270	0.029

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	77	76	76	90	76	76
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.83
time (sec)	N/A	0.035	0.005	1.476	0.193	0.243	0.022	0.272	0.042

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	33	16	15	27	29	15	26
N.S.	1	1.00	1.94	0.94	0.88	1.59	1.71	0.88	1.53
time (sec)	N/A	0.004	0.001	1.473	0.198	0.242	0.021	0.276	0.035

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	60	51	50	50	58	50	50
N.S.	1	1.00	1.33	1.13	1.11	1.11	1.29	1.11	1.11
time (sec)	N/A	0.011	0.003	1.499	0.202	0.250	0.022	0.275	0.026

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	65	54	53	53	60	53	53
N.S.	1	1.00	1.30	1.08	1.06	1.06	1.20	1.06	1.06
time (sec)	N/A	0.012	0.004	1.500	0.203	0.241	0.029	0.271	0.028

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	76	76	88	76	76
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.14	0.99	0.99
time (sec)	N/A	0.035	0.004	1.508	0.195	0.260	0.022	0.282	0.039

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	65	54	53	53	61	53	53
N.S.	1	1.00	1.30	1.08	1.06	1.06	1.22	1.06	1.06
time (sec)	N/A	0.015	0.004	1.535	0.198	0.261	0.027	0.272	0.029

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	76	76	90	76	76
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.17	0.99	0.99
time (sec)	N/A	0.026	0.005	1.483	0.196	0.255	0.024	0.263	0.039

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	79	79	92	79	79
N.S.	1	1.00	1.18	0.98	0.96	0.96	1.12	0.96	0.96
time (sec)	N/A	0.054	0.005	1.503	0.190	0.263	0.023	0.273	0.042

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	124	103	102	102	121	102	102
N.S.	1	1.00	1.14	0.94	0.94	0.94	1.11	0.94	0.94
time (sec)	N/A	0.050	0.008	1.461	0.209	0.255	0.022	0.274	9.133

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	180	151	150	150	180	150	150
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.19	0.99	0.99
time (sec)	N/A	0.068	0.005	1.501	0.197	0.268	0.025	0.276	9.256

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	220	89	200	117016	520	315	483
N.S.	1	1.00	1.42	0.57	1.29	754.94	3.35	2.03	3.12
time (sec)	N/A	0.086	0.151	1.505	0.283	2.996	43.720	0.280	0.367

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	253	120	249	118761	0	352	832
N.S.	1	1.00	1.35	0.64	1.32	631.71	0.00	1.87	4.43
time (sec)	N/A	0.105	0.212	1.499	0.281	6.660	0.000	0.278	9.519

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	286	156	297	118945	0	388	880
N.S.	1	1.00	1.30	0.71	1.35	540.66	0.00	1.76	4.00
time (sec)	N/A	0.132	0.285	1.534	0.285	8.984	0.000	0.288	9.638

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	78	25	123	120	88	97	36
N.S.	1	1.00	0.77	0.25	1.22	1.19	0.87	0.96	0.36
time (sec)	N/A	0.065	0.038	1.452	0.283	0.270	0.176	0.289	0.125

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	16	15	15	19	15	15
N.S.	1	1.00	1.00	0.73	0.68	0.68	0.86	0.68	0.68
time (sec)	N/A	0.008	0.012	1.472	0.280	0.262	0.039	0.275	9.062

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	107	29	147	12348	88	115	119
N.S.	1	1.00	0.87	0.24	1.20	100.39	0.72	0.93	0.97
time (sec)	N/A	0.070	0.058	1.468	0.302	1.044	0.310	0.279	0.210

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	78	25	123	128	88	97	32
N.S.	1	1.00	0.77	0.25	1.22	1.27	0.87	0.96	0.32
time (sec)	N/A	0.053	0.019	1.458	0.284	0.257	0.181	0.277	0.143

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	113	31	167	493	68	131	315
N.S.	1	1.00	0.80	0.22	1.18	3.50	0.48	0.93	2.23
time (sec)	N/A	0.067	0.052	1.487	0.292	0.279	0.219	0.281	9.212

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	99	33	147	12741	85	114	162
N.S.	1	1.00	0.80	0.27	1.20	103.59	0.69	0.93	1.32
time (sec)	N/A	0.079	0.048	1.485	0.288	1.046	0.309	0.287	0.218

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	129	34	187	46651	292	143	270
N.S.	1	1.00	0.79	0.21	1.15	286.20	1.79	0.88	1.66
time (sec)	N/A	0.082	0.081	1.500	0.276	1.487	2.389	0.278	9.472

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	11	11	10	11	9
N.S.	1	1.00	1.00	0.77	0.85	0.85	0.77	0.85	0.69
time (sec)	N/A	0.003	0.003	1.488	0.193	0.267	0.030	0.273	0.032

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	108	31	149	191	51	109	117
N.S.	1	1.00	0.95	0.27	1.31	1.68	0.45	0.96	1.03
time (sec)	N/A	0.073	0.034	1.465	0.280	0.271	0.161	0.283	0.288

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	65	28	113	27	53	93	25
N.S.	1	1.00	1.81	0.78	3.14	0.75	1.47	2.58	0.69
time (sec)	N/A	0.021	0.036	1.467	0.285	0.274	0.185	0.281	0.056

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	128	34	171	17085	199	125	307
N.S.	1	1.00	0.94	0.25	1.26	125.62	1.46	0.92	2.26
time (sec)	N/A	0.080	0.063	1.490	0.286	1.108	0.844	0.281	9.482

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	108	35	152	239	70	109	117
N.S.	1	1.00	0.95	0.31	1.33	2.10	0.61	0.96	1.03
time (sec)	N/A	0.084	0.028	1.492	0.285	0.285	0.145	0.285	0.388

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	148	36	195	513	148	137	286
N.S.	1	1.00	0.96	0.23	1.27	3.33	0.96	0.89	1.86
time (sec)	N/A	0.086	0.099	1.482	0.301	0.294	0.672	0.284	10.292

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	125	38	174	18086	189	124	300
N.S.	1	1.00	0.92	0.28	1.28	132.99	1.39	0.91	2.21
time (sec)	N/A	0.095	0.065	1.485	0.289	1.151	0.833	0.289	10.470

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	164	39	207	54479	580	149	1168
N.S.	1	1.00	0.93	0.22	1.18	309.54	3.30	0.85	6.64
time (sec)	N/A	0.106	0.122	1.500	0.276	1.793	4.670	0.286	10.149

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75
time (sec)	N/A	0.005	0.001	1.475	0.197	0.258	0.022	0.267	0.023

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	50	31	76	151	73	70	156
N.S.	1	1.00	0.94	0.58	1.43	2.85	1.38	1.32	2.94
time (sec)	N/A	0.028	0.039	1.515	0.282	0.427	0.170	0.265	0.444

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	203	38	160	91748	187	290	312
N.S.	1	1.00	1.64	0.31	1.29	739.90	1.51	2.34	2.52
time (sec)	N/A	0.059	0.048	1.483	0.292	2.281	0.968	0.271	9.462

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	283	36	296	96349	187	270	305
N.S.	1	1.00	1.02	0.13	1.07	347.83	0.68	0.97	1.10
time (sec)	N/A	0.133	0.192	1.480	0.283	2.070	0.927	0.269	9.512

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	249	65	202	592528	0	299	5082
N.S.	1	1.00	1.68	0.44	1.36	4003.57	0.00	2.02	34.34
time (sec)	N/A	0.128	0.079	1.508	0.281	42.648	0.000	0.273	9.890

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	221	108	224	334837	0	339	1393
N.S.	1	1.00	1.28	0.63	1.30	1946.73	0.00	1.97	8.10
time (sec)	N/A	0.107	0.314	1.524	0.311	30.976	0.000	0.276	9.841

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	263	147	284	343626	0	387	1002
N.S.	1	1.00	1.19	0.67	1.29	1554.87	0.00	1.75	4.53
time (sec)	N/A	0.173	0.520	1.512	0.372	42.834	0.000	0.435	9.665

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	313	188	345	343822	0	435	1056
N.S.	1	1.00	1.18	0.71	1.30	1292.56	0.00	1.64	3.97
time (sec)	N/A	0.223	0.322	1.536	0.295	59.773	0.000	0.360	9.823

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	311	58	328	622377	0	338	5042
N.S.	1	1.00	0.97	0.18	1.03	1951.03	0.00	1.06	15.81
time (sec)	N/A	0.238	0.338	1.501	0.295	43.661	0.000	0.476	9.946

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	319	105	350	352423	0	360	1383
N.S.	1	1.00	0.94	0.31	1.03	1033.50	0.00	1.06	4.06
time (sec)	N/A	0.206	0.195	1.525	0.295	26.609	0.000	0.273	9.884

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	366	144	412	358509	0	410	1001
N.S.	1	1.00	0.93	0.37	1.05	909.92	0.00	1.04	2.54
time (sec)	N/A	0.303	0.287	1.530	0.310	58.341	0.000	0.276	9.754

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	411	185	472	358702	0	459	1053
N.S.	1	1.00	0.94	0.42	1.08	820.83	0.00	1.05	2.41
time (sec)	N/A	0.346	0.370	1.514	0.296	82.838	0.000	0.286	9.843

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	15	15	15	15	15
N.S.	1	1.00	0.82	0.73	1.36	1.36	1.36	1.36	1.36
time (sec)	N/A	0.009	0.002	1.455	0.206	0.267	0.031	0.277	0.032

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	14	8	12	12	8	12	11
N.S.	1	1.00	1.27	0.73	1.09	1.09	0.73	1.09	1.00
time (sec)	N/A	0.008	0.001	1.465	0.195	0.264	0.027	0.259	0.024

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	7	7	5	7	6
N.S.	1	1.00	1.00	0.78	0.78	0.78	0.56	0.78	0.67
time (sec)	N/A	0.006	0.001	0.020	0.335	0.270	0.020	0.262	0.018

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75
time (sec)	N/A	0.005	0.001	1.476	0.203	0.256	0.025	0.270	0.002

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	5	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	0.71	1.00	1.00
time (sec)	N/A	0.010	0.001	1.640	0.230	0.267	0.038	0.264	0.032

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	12	12	10	7	7
N.S.	1	1.00	0.82	0.73	1.09	1.09	0.91	0.64	0.64
time (sec)	N/A	0.012	0.002	1.633	0.207	0.275	0.050	0.264	9.102

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	17	17	17	7	7
N.S.	1	1.00	0.82	0.73	1.55	1.55	1.55	0.64	0.64
time (sec)	N/A	0.012	0.001	1.597	0.206	0.249	0.057	0.259	9.124

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	256	74	222	0	0	338	2478
N.S.	1	1.00	1.55	0.45	1.35	0.00	0.00	2.05	15.02
time (sec)	N/A	0.176	0.237	1.580	0.304	0.000	0.000	0.272	9.917

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	301	95	240	0	0	430	3810
N.S.	1	1.00	1.60	0.51	1.28	0.00	0.00	2.29	20.27
time (sec)	N/A	0.216	0.320	1.564	0.340	0.000	0.000	0.288	9.425

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	318	102	257	0	0	445	5673
N.S.	1	1.00	1.55	0.50	1.25	0.00	0.00	2.17	27.67
time (sec)	N/A	0.206	0.306	1.573	0.290	0.000	0.000	0.283	9.556

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	342	73	351	0	0	373	2469
N.S.	1	1.00	1.01	0.22	1.04	0.00	0.00	1.11	7.33
time (sec)	N/A	0.267	0.330	1.650	0.290	0.000	0.000	0.280	9.940

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	427	88	399	0	0	443	3798
N.S.	1	1.00	1.11	0.23	1.04	0.00	0.00	1.15	9.89
time (sec)	N/A	0.373	0.255	1.603	0.286	0.000	0.000	0.273	9.422

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	445	103	429	0	0	459	5664
N.S.	1	1.00	1.11	0.26	1.07	0.00	0.00	1.14	14.09
time (sec)	N/A	0.371	0.280	1.524	0.291	0.000	0.000	0.281	9.525

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	257	127	243	710521	0	375	1626
N.S.	1	1.00	1.40	0.69	1.32	3861.53	0.00	2.04	8.84
time (sec)	N/A	0.134	0.151	1.531	0.283	212.801	0.000	0.273	9.941

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	302	137	260	0	0	460	2611
N.S.	1	1.00	1.49	0.67	1.28	0.00	0.00	2.27	12.86
time (sec)	N/A	0.187	0.227	1.535	0.292	0.000	0.000	0.280	9.964

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	338	157	299	0	0	487	3943
N.S.	1	1.00	1.50	0.70	1.33	0.00	0.00	2.16	17.52
time (sec)	N/A	0.213	0.219	1.539	0.299	0.000	0.000	0.277	10.223

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	359	124	374	0	0	393	1623
N.S.	1	1.00	1.02	0.35	1.06	0.00	0.00	1.11	4.60
time (sec)	N/A	0.235	0.272	1.532	0.289	0.000	0.000	0.279	9.939

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	415	133	416	0	0	460	2605
N.S.	1	1.00	1.05	0.34	1.05	0.00	0.00	1.16	6.59
time (sec)	N/A	0.334	0.357	1.523	0.300	0.000	0.000	0.276	10.167

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	460	154	458	0	0	486	3939
N.S.	1	1.00	1.10	0.37	1.10	0.00	0.00	1.17	9.45
time (sec)	N/A	0.347	0.302	1.550	0.302	0.000	0.000	0.275	10.393

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	309	173	316	0	0	434	1687
N.S.	1	1.00	1.28	0.72	1.31	0.00	0.00	1.80	7.00
time (sec)	N/A	0.224	0.273	1.545	0.303	0.000	0.000	0.281	9.885

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	359	190	343	0	0	529	2680
N.S.	1	1.00	1.34	0.71	1.28	0.00	0.00	1.97	10.00
time (sec)	N/A	0.284	0.339	1.538	0.290	0.000	0.000	0.278	10.073

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	380	206	377	0	0	561	2696
N.S.	1	1.00	1.33	0.72	1.32	0.00	0.00	1.97	9.46
time (sec)	N/A	0.249	0.282	1.622	0.309	0.000	0.000	0.287	10.023

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	411	170	446	0	0	453	1686
N.S.	1	1.00	1.00	0.41	1.08	0.00	0.00	1.10	4.08
time (sec)	N/A	0.325	0.362	1.536	0.293	0.000	0.000	0.274	10.017

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	473	186	497	0	0	530	2680
N.S.	1	1.00	1.02	0.40	1.07	0.00	0.00	1.14	5.79
time (sec)	N/A	0.450	0.517	1.560	0.319	0.000	0.000	0.284	10.476

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	480	480	500	201	535	0	0	562	2695
N.S.	1	1.00	1.04	0.42	1.11	0.00	0.00	1.17	5.61
time (sec)	N/A	0.415	0.405	1.625	0.290	0.000	0.000	0.276	10.188

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	360	219	389	0	0	494	1747
N.S.	1	1.00	1.23	0.75	1.33	0.00	0.00	1.69	5.96
time (sec)	N/A	0.287	0.381	1.547	0.290	0.000	0.000	0.283	10.136

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	422	240	429	0	0	601	2747
N.S.	1	1.00	1.27	0.73	1.30	0.00	0.00	1.82	8.30
time (sec)	N/A	0.379	0.335	1.570	0.303	0.000	0.000	0.289	10.321

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	439	256	463	0	0	633	2764
N.S.	1	1.00	1.26	0.73	1.33	0.00	0.00	1.81	7.92
time (sec)	N/A	0.347	0.337	1.603	0.299	0.000	0.000	0.282	10.572

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	462	462	461	216	517	0	0	514	1743
N.S.	1	1.00	1.00	0.47	1.12	0.00	0.00	1.11	3.77
time (sec)	N/A	0.408	0.463	1.544	0.301	0.000	0.000	0.274	10.098

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	516	516	530	237	579	0	0	603	2741
N.S.	1	1.00	1.03	0.46	1.12	0.00	0.00	1.17	5.31
time (sec)	N/A	0.555	0.715	1.552	0.307	0.000	0.000	0.273	10.162

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	534	555	253	613	0	0	635	2757
N.S.	1	1.00	1.04	0.47	1.15	0.00	0.00	1.19	5.16
time (sec)	N/A	0.514	0.529	1.579	0.323	0.000	0.000	0.284	10.448

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	79	96	0	72	61	0	0
N.S.	1	1.00	0.65	0.79	0.00	0.60	0.50	0.00	0.00
time (sec)	N/A	0.041	10.065	1.565	0.000	0.143	1.035	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	90	0	79	95	0	0
N.S.	1	1.00	0.93	1.03	0.00	0.91	1.09	0.00	0.00
time (sec)	N/A	0.044	10.059	1.564	0.000	0.108	1.138	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	83	95	0	73	90	0	0
N.S.	1	1.00	0.93	1.07	0.00	0.82	1.01	0.00	0.00
time (sec)	N/A	0.040	10.049	1.548	0.000	0.122	1.134	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	85	101	0	82	66	0	0
N.S.	1	1.00	0.67	0.80	0.00	0.65	0.52	0.00	0.00
time (sec)	N/A	0.044	10.050	1.561	0.000	0.112	1.113	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	131	193	0	128	102	0	0
N.S.	1	1.00	0.51	0.75	0.00	0.50	0.40	0.00	0.00
time (sec)	N/A	0.087	10.117	1.577	0.000	0.113	1.260	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	80	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	5.71	0.86	0.86
time (sec)	N/A	0.004	1.775	1.529	0.233	0.306	2.844	0.282	9.034

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	24	25	34	104	22	23
N.S.	1	1.00	0.93	0.83	0.86	1.17	3.59	0.76	0.79
time (sec)	N/A	0.015	10.042	1.605	0.234	0.289	3.843	0.280	8.955

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	27	24	23	33	109	22	20
N.S.	1	1.00	1.08	0.96	0.92	1.32	4.36	0.88	0.80
time (sec)	N/A	0.018	10.046	1.609	0.237	0.291	4.165	0.277	8.953

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	44	44	133	30	29
N.S.	1	1.00	1.00	0.92	1.16	1.16	3.50	0.79	0.76
time (sec)	N/A	0.020	10.051	1.642	0.235	0.281	5.197	0.275	9.022

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	58	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	4.83	0.83	0.83
time (sec)	N/A	0.002	0.229	1.502	0.314	0.273	1.633	0.263	8.932

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	281	287	0	204	260	0	0
N.S.	1	1.00	0.73	0.75	0.00	0.53	0.68	0.00	0.00
time (sec)	N/A	0.296	10.225	2.011	0.000	0.174	3.294	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	51	45	0	835	1287	101	64
N.S.	1	1.00	0.47	0.41	0.00	7.66	11.81	0.93	0.59
time (sec)	N/A	0.057	0.013	1.492	0.000	0.989	0.577	0.284	9.139

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	47	41	0	799	1287	101	65
N.S.	1	1.00	0.43	0.38	0.00	7.33	11.81	0.93	0.60
time (sec)	N/A	0.030	0.012	1.483	0.000	1.024	0.554	0.280	0.201

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	187	195	209	210	216	240	237
N.S.	1	1.00	0.90	0.94	1.00	1.01	1.04	1.15	1.14
time (sec)	N/A	0.210	0.101	1.505	0.203	0.276	0.591	0.261	9.073

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	154	159	169	170	172	192	189
N.S.	1	1.00	0.91	0.94	0.99	1.00	1.01	1.13	1.11
time (sec)	N/A	0.164	0.072	1.500	0.197	0.295	0.546	0.274	9.081

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	119	123	129	130	128	144	141
N.S.	1	1.00	0.90	0.93	0.98	0.98	0.97	1.09	1.07
time (sec)	N/A	0.118	0.061	1.490	0.199	0.279	0.519	0.266	9.050

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	88	89	91	92	88	98	96
N.S.	1	1.00	0.92	0.93	0.95	0.96	0.92	1.02	1.00
time (sec)	N/A	0.090	0.044	1.535	0.201	0.266	0.479	0.259	8.967

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	75	75	77	80	70	77	76
N.S.	1	1.00	0.94	0.94	0.96	1.00	0.88	0.96	0.95
time (sec)	N/A	0.078	0.035	1.540	0.197	0.317	2.392	0.271	9.101

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	75	77	85	70	94	74
N.S.	1	1.00	0.95	0.93	0.95	1.05	0.86	1.16	0.91
time (sec)	N/A	0.077	0.048	1.510	0.208	0.312	49.265	0.269	9.139

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	88	90	93	101	0	123	92
N.S.	1	1.00	0.93	0.95	0.98	1.06	0.00	1.29	0.97
time (sec)	N/A	0.083	0.073	1.514	0.191	0.331	0.000	0.259	9.129

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	128	120	125	127	0	180	123
N.S.	1	1.00	1.00	0.94	0.98	0.99	0.00	1.41	0.96
time (sec)	N/A	0.106	0.067	1.507	0.207	0.320	0.000	0.267	9.229

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	164	156	166	168	0	230	161
N.S.	1	1.00	1.00	0.95	1.01	1.02	0.00	1.40	0.98
time (sec)	N/A	0.124	0.069	1.515	0.197	0.325	0.000	0.276	9.807

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	194	193	208	210	0	281	200
N.S.	1	1.00	0.95	0.94	1.01	1.02	0.00	1.37	0.98
time (sec)	N/A	0.138	0.152	1.527	0.198	0.339	0.000	0.263	0.262

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	351	242	351	342	469	446	358
N.S.	1	1.00	1.01	0.70	1.01	0.98	1.35	1.28	1.03
time (sec)	N/A	0.226	0.092	1.537	0.281	0.305	0.983	0.276	0.323

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	311	204	313	321	513	434	313
N.S.	1	1.00	0.98	0.65	0.99	1.02	1.62	1.37	0.99
time (sec)	N/A	0.200	0.110	1.540	0.277	0.304	0.847	0.277	9.855

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	306	194	311	304	423	394	311
N.S.	1	1.00	0.98	0.62	1.00	0.97	1.36	1.26	1.00
time (sec)	N/A	0.191	0.106	1.518	0.288	0.316	0.945	0.282	9.921

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	266	156	269	281	469	380	267
N.S.	1	1.00	0.95	0.56	0.96	1.01	1.68	1.36	0.96
time (sec)	N/A	0.174	0.109	1.532	0.293	0.298	0.770	0.287	10.583

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	264	146	267	249	376	340	264
N.S.	1	1.00	0.96	0.53	0.97	0.91	1.37	1.24	0.96
time (sec)	N/A	0.179	0.106	1.531	0.304	0.310	0.916	0.269	10.119

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	231	112	225	568	427	286	225
N.S.	1	1.00	0.94	0.46	0.92	2.32	1.74	1.17	0.92
time (sec)	N/A	0.146	0.148	1.525	0.304	0.307	0.775	0.275	9.760

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	229	104	223	600	342	248	222
N.S.	1	1.00	0.95	0.43	0.93	2.50	1.42	1.03	0.92
time (sec)	N/A	0.096	0.148	1.537	0.309	0.305	0.869	0.284	9.753

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	224	159	217	560	408	265	204
N.S.	1	1.00	0.99	0.70	0.96	2.47	1.80	1.17	0.90
time (sec)	N/A	0.131	0.114	1.542	0.288	0.307	1.222	0.266	9.502

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	218	155	214	565	326	228	201
N.S.	1	1.00	0.97	0.69	0.96	2.52	1.46	1.02	0.90
time (sec)	N/A	0.119	0.108	1.547	0.284	0.312	1.464	0.272	0.288

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	220	159	217	556	411	258	209
N.S.	1	1.00	0.97	0.70	0.96	2.45	1.81	1.14	0.92
time (sec)	N/A	0.133	0.109	1.553	0.284	0.318	3.954	0.258	9.299

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	220	155	214	584	328	217	207
N.S.	1	1.00	0.98	0.69	0.95	2.60	1.46	0.96	0.92
time (sec)	N/A	0.114	0.087	1.552	0.287	0.318	7.487	0.270	9.216

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	231	170	234	610	432	271	219
N.S.	1	1.00	0.95	0.70	0.97	2.52	1.79	1.12	0.90
time (sec)	N/A	0.127	0.123	1.534	0.279	0.309	43.346	0.281	9.259

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	231	170	234	595	0	293	220
N.S.	1	1.00	0.95	0.70	0.96	2.44	0.00	1.20	0.90
time (sec)	N/A	0.117	0.128	1.551	0.291	0.307	0.000	0.270	9.161

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	266	205	260	262	0	371	253
N.S.	1	1.00	0.96	0.74	0.94	0.95	0.00	1.34	0.91
time (sec)	N/A	0.142	0.140	1.528	0.290	0.289	0.000	0.275	9.250

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	266	205	260	295	0	333	253
N.S.	1	1.00	0.95	0.73	0.93	1.05	0.00	1.19	0.90
time (sec)	N/A	0.134	0.138	1.531	0.297	0.301	0.000	0.276	9.203

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	308	239	307	317	0	413	286
N.S.	1	1.00	0.98	0.76	0.98	1.01	0.00	1.32	0.91
time (sec)	N/A	0.158	0.110	1.563	0.290	0.295	0.000	0.278	9.570

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	311	240	307	335	0	387	287
N.S.	1	1.00	0.99	0.76	0.97	1.06	0.00	1.23	0.91
time (sec)	N/A	0.147	0.118	1.545	0.289	0.295	0.000	0.267	0.295

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	346	277	353	355	0	467	323
N.S.	1	1.00	0.99	0.79	1.01	1.01	0.00	1.33	0.92
time (sec)	N/A	0.176	0.124	1.657	0.292	0.304	0.000	0.275	10.004

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	205	217	222	303	236	293	356
N.S.	1	1.00	0.93	0.99	1.01	1.38	1.07	1.33	1.62
time (sec)	N/A	0.228	0.140	1.674	0.207	0.281	13.529	0.266	9.653

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	167	179	180	257	189	242	233
N.S.	1	1.00	0.93	0.99	1.00	1.43	1.05	1.34	1.29
time (sec)	N/A	0.176	0.120	1.520	0.199	0.293	12.664	0.271	9.730

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	129	138	138	202	141	212	155
N.S.	1	1.00	0.92	0.99	0.99	1.44	1.01	1.51	1.11
time (sec)	N/A	0.133	0.101	1.521	0.195	0.314	11.380	0.280	9.828

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	93	100	98	143	100	205	103
N.S.	1	1.00	0.90	0.97	0.95	1.39	0.97	1.99	1.00
time (sec)	N/A	0.100	0.065	1.542	0.200	0.259	5.434	0.273	0.090

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	95	97	100	145	0	123	100
N.S.	1	1.00	0.95	0.97	1.00	1.45	0.00	1.23	1.00
time (sec)	N/A	0.085	0.120	1.500	0.199	0.305	0.000	0.269	9.616

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	97	101	116	172	0	130	109
N.S.	1	1.00	0.89	0.93	1.06	1.58	0.00	1.19	1.00
time (sec)	N/A	0.100	0.079	1.495	0.201	0.305	0.000	0.269	9.794

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	118	123	138	208	0	196	130
N.S.	1	1.00	0.91	0.95	1.06	1.60	0.00	1.51	1.00
time (sec)	N/A	0.112	0.124	1.497	0.206	0.295	0.000	0.275	9.745

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	160	169	181	261	0	269	175
N.S.	1	1.00	0.91	0.97	1.03	1.49	0.00	1.54	1.00
time (sec)	N/A	0.144	0.112	1.501	0.210	0.311	0.000	0.278	9.651

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	198	209	226	310	0	324	216
N.S.	1	1.00	0.93	0.98	1.06	1.45	0.00	1.51	1.01
time (sec)	N/A	0.169	0.215	1.518	0.202	0.321	0.000	0.283	10.334

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	364	244	369	488	500	443	481
N.S.	1	1.00	0.99	0.66	1.00	1.32	1.36	1.20	1.30
time (sec)	N/A	0.301	0.428	1.528	0.337	0.295	85.665	0.269	0.354

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	319	204	325	455	0	435	362
N.S.	1	1.00	0.95	0.61	0.97	1.36	0.00	1.30	1.08
time (sec)	N/A	0.454	0.186	1.535	0.289	0.322	0.000	0.282	10.370

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	315	193	321	423	449	387	358
N.S.	1	1.00	0.96	0.59	0.98	1.29	1.37	1.18	1.09
time (sec)	N/A	0.249	0.279	1.543	0.291	0.321	76.438	0.272	10.180

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	282	157	277	920	0	338	287
N.S.	1	1.00	0.95	0.53	0.93	3.09	0.00	1.13	0.96
time (sec)	N/A	0.307	0.183	1.540	0.282	0.373	0.000	0.272	10.322

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	277	147	270	946	401	289	280
N.S.	1	1.00	0.96	0.51	0.94	3.28	1.39	1.00	0.97
time (sec)	N/A	0.225	0.182	1.557	0.287	0.323	67.315	0.261	0.344

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	255	130	259	874	0	313	246
N.S.	1	1.00	0.94	0.48	0.96	3.23	0.00	1.15	0.91
time (sec)	N/A	0.201	0.170	1.526	0.290	0.364	0.000	0.277	10.059

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	251	123	254	861	377	268	241
N.S.	1	1.00	0.95	0.47	0.96	3.26	1.43	1.02	0.91
time (sec)	N/A	0.181	0.169	1.526	0.293	0.322	3.097	0.269	9.768

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	255	187	258	860	457	301	244
N.S.	1	1.00	0.96	0.71	0.97	3.25	1.72	1.14	0.92
time (sec)	N/A	0.168	0.176	1.581	0.297	0.335	63.973	0.279	9.511

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	250	183	258	902	0	257	245
N.S.	1	1.00	0.96	0.70	0.99	3.47	0.00	0.99	0.94
time (sec)	N/A	0.174	0.173	1.539	0.300	0.300	0.000	0.274	9.472

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	255	195	267	902	0	306	247
N.S.	1	1.00	0.95	0.72	0.99	3.35	0.00	1.14	0.92
time (sec)	N/A	0.190	0.181	1.545	0.298	0.394	0.000	0.277	9.592

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	253	193	268	897	0	260	248
N.S.	1	1.00	0.94	0.71	0.99	3.32	0.00	0.96	0.92
time (sec)	N/A	0.182	0.181	1.544	0.296	0.421	0.000	0.273	9.439

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	281	215	292	982	0	328	274
N.S.	1	1.00	0.95	0.72	0.98	3.31	0.00	1.10	0.92
time (sec)	N/A	0.257	0.192	1.562	0.310	0.422	0.000	0.265	9.421

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	280	214	292	959	0	342	274
N.S.	1	1.00	0.94	0.72	0.98	3.23	0.00	1.15	0.92
time (sec)	N/A	0.241	0.191	1.579	0.304	0.399	0.000	0.268	9.519

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	319	251	323	442	0	431	310
N.S.	1	1.00	0.96	0.75	0.97	1.32	0.00	1.29	0.93
time (sec)	N/A	0.301	0.198	1.555	0.307	0.673	0.000	0.278	9.798

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	317	250	323	475	0	385	310
N.S.	1	1.00	0.95	0.75	0.96	1.42	0.00	1.15	0.93
time (sec)	N/A	0.287	0.208	1.554	0.294	0.270	0.000	0.267	9.523

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	370	287	374	507	0	475	348
N.S.	1	1.00	0.99	0.77	1.00	1.35	0.00	1.27	0.93
time (sec)	N/A	0.358	0.390	1.565	0.297	0.283	0.000	0.273	9.716

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	246	260	275	396	0	341	449
N.S.	1	1.00	0.92	0.98	1.03	1.49	0.00	1.28	1.69
time (sec)	N/A	0.293	0.152	1.526	0.204	0.250	0.000	0.282	9.236

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	208	218	233	353	0	291	293
N.S.	1	1.00	0.92	0.96	1.03	1.56	0.00	1.29	1.30
time (sec)	N/A	0.223	0.125	1.504	0.201	0.255	0.000	0.277	9.362

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	170	176	191	295	0	230	204
N.S.	1	1.00	0.91	0.95	1.03	1.59	0.00	1.24	1.10
time (sec)	N/A	0.188	0.103	1.505	0.199	0.259	0.000	0.278	9.567

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	145	137	147	225	0	142	152
N.S.	1	1.00	0.99	0.94	1.01	1.54	0.00	0.97	1.04
time (sec)	N/A	0.137	0.076	1.558	0.202	0.272	0.000	0.269	0.112

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	105	99	109	158	0	96	112
N.S.	1	1.00	0.96	0.91	1.00	1.45	0.00	0.88	1.03
time (sec)	N/A	0.106	0.061	1.520	0.201	0.252	0.000	0.273	9.161

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	104	113	129	187	0	126	123
N.S.	1	1.00	0.91	0.99	1.13	1.64	0.00	1.11	1.08
time (sec)	N/A	0.100	0.109	1.530	0.203	0.272	0.000	0.273	0.188

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	121	115	144	250	0	172	135
N.S.	1	1.00	0.90	0.86	1.07	1.87	0.00	1.28	1.01
time (sec)	N/A	0.119	0.093	1.533	0.207	0.263	0.000	0.273	9.154

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	149	153	182	316	0	185	167
N.S.	1	1.00	0.91	0.94	1.12	1.94	0.00	1.13	1.02
time (sec)	N/A	0.139	0.112	1.518	0.211	0.278	0.000	0.276	9.352

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	200	212	232	396	0	317	222
N.S.	1	1.00	0.92	0.97	1.06	1.82	0.00	1.45	1.02
time (sec)	N/A	0.173	0.149	1.528	0.211	0.285	0.000	0.275	9.496

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	238	253	280	448	0	372	265
N.S.	1	1.00	0.92	0.98	1.09	1.74	0.00	1.44	1.03
time (sec)	N/A	0.204	0.199	1.525	0.213	0.312	0.000	0.279	0.323

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	411	284	424	667	0	491	575
N.S.	1	1.00	0.99	0.68	1.02	1.60	0.00	1.18	1.38
time (sec)	N/A	0.477	0.514	1.623	0.322	0.275	0.000	0.277	9.483

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	380	244	380	634	0	483	425
N.S.	1	1.00	0.99	0.64	0.99	1.65	0.00	1.26	1.11
time (sec)	N/A	0.678	0.434	1.543	0.286	0.283	0.000	0.277	9.353

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	362	233	376	602	0	435	420
N.S.	1	1.00	0.97	0.62	1.00	1.61	0.00	1.16	1.12
time (sec)	N/A	0.401	0.384	1.543	0.291	0.281	0.000	0.279	9.627

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	329	195	330	1278	0	384	338
N.S.	1	1.00	0.95	0.57	0.96	3.70	0.00	1.11	0.98
time (sec)	N/A	0.495	0.247	1.544	0.284	0.294	0.000	0.280	9.661

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	323	186	326	1318	0	338	335
N.S.	1	1.00	0.96	0.55	0.97	3.92	0.00	1.01	1.00
time (sec)	N/A	0.331	0.352	1.543	0.296	0.287	0.000	0.272	9.328

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	300	166	311	1224	0	359	295
N.S.	1	1.00	0.95	0.53	0.98	3.87	0.00	1.14	0.93
time (sec)	N/A	0.341	0.224	1.527	0.313	0.294	0.000	0.267	9.263

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	294	159	305	1213	0	313	290
N.S.	1	1.00	0.96	0.52	0.99	3.95	0.00	1.02	0.94
time (sec)	N/A	0.280	0.220	1.531	0.291	0.311	0.000	0.277	9.303

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	284	151	296	1158	0	334	280
N.S.	1	1.00	0.94	0.50	0.98	3.85	0.00	1.11	0.93
time (sec)	N/A	0.248	0.221	1.565	0.299	0.285	0.000	0.272	9.207

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	279	146	291	1184	0	290	275
N.S.	1	1.00	0.96	0.50	1.00	4.05	0.00	0.99	0.94
time (sec)	N/A	0.213	0.212	1.555	0.286	0.279	0.000	0.271	9.150

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	286	218	300	1206	0	336	276
N.S.	1	1.00	0.94	0.72	0.99	3.98	0.00	1.11	0.91
time (sec)	N/A	0.225	0.233	1.556	0.288	0.308	0.000	0.271	11.933

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	283	216	302	1217	0	307	279
N.S.	1	1.00	0.94	0.72	1.00	4.04	0.00	1.02	0.93
time (sec)	N/A	0.226	0.224	1.583	0.286	0.294	0.000	0.266	10.052

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	303	230	317	1254	0	352	293
N.S.	1	1.00	0.96	0.73	1.00	3.96	0.00	1.11	0.92
time (sec)	N/A	0.255	0.233	1.657	0.292	0.295	0.000	0.278	9.328

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	299	228	318	1247	0	305	293
N.S.	1	1.00	0.95	0.72	1.01	3.95	0.00	0.97	0.93
time (sec)	N/A	0.242	0.240	1.556	0.291	0.303	0.000	0.279	0.325

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	328	253	343	1340	0	374	321
N.S.	1	1.00	0.96	0.74	1.00	3.91	0.00	1.09	0.94
time (sec)	N/A	0.383	0.245	1.545	0.287	0.296	0.000	0.271	9.339

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	324	252	343	1317	0	388	321
N.S.	1	1.00	0.95	0.74	1.01	3.86	0.00	1.14	0.94
time (sec)	N/A	0.370	0.248	1.554	0.294	0.285	0.000	0.271	9.286

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	366	289	376	621	0	479	359
N.S.	1	1.00	0.96	0.76	0.99	1.63	0.00	1.26	0.94
time (sec)	N/A	0.471	0.438	1.554	0.294	0.267	0.000	0.273	9.395

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	376	288	376	654	0	433	359
N.S.	1	1.00	0.99	0.76	0.99	1.72	0.00	1.14	0.94
time (sec)	N/A	0.442	0.483	1.577	0.295	0.277	0.000	0.279	9.345

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	419	325	427	686	0	523	397
N.S.	1	1.00	0.99	0.77	1.01	1.62	0.00	1.23	0.94
time (sec)	N/A	0.560	0.507	1.575	0.298	0.272	0.000	0.272	9.449

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	45	44	44	53	45	56
N.S.	1	1.00	1.09	0.83	0.81	0.81	0.98	0.83	1.04
time (sec)	N/A	0.049	0.016	1.499	0.292	0.251	0.062	0.261	0.099

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	25	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.83	0.80
time (sec)	N/A	0.026	0.005	1.505	0.288	0.264	0.038	0.269	0.034

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	53	36	37	37	44	38	49
N.S.	1	1.00	1.20	0.82	0.84	0.84	1.00	0.86	1.11
time (sec)	N/A	0.040	0.010	1.502	0.280	0.338	0.062	0.272	8.967

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	50	33	34	34	42	35	63
N.S.	1	1.00	1.22	0.80	0.83	0.83	1.02	0.85	1.54
time (sec)	N/A	0.027	0.009	1.512	0.274	0.329	0.060	0.261	0.081

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	53	35	36	36	46	38	48
N.S.	1	1.00	1.26	0.83	0.86	0.86	1.10	0.90	1.14
time (sec)	N/A	0.032	0.010	1.526	0.295	0.387	0.088	0.258	8.956

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	60	42	43	48	49	45	55
N.S.	1	1.00	1.22	0.86	0.88	0.98	1.00	0.92	1.12
time (sec)	N/A	0.037	0.018	1.518	0.283	0.341	0.101	0.255	0.079

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	26	28	33	27	29	25
N.S.	1	1.00	1.00	0.81	0.88	1.03	0.84	0.91	0.78
time (sec)	N/A	0.022	0.008	1.598	0.269	0.328	0.046	0.262	8.981

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	47	35	34	34	42	35	63
N.S.	1	1.00	1.15	0.85	0.83	0.83	1.02	0.85	1.54
time (sec)	N/A	0.026	0.011	1.679	0.285	0.248	0.062	0.263	9.024

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	53	33	32	32	41	33	63
N.S.	1	1.00	1.36	0.85	0.82	0.82	1.05	0.85	1.62
time (sec)	N/A	0.027	0.012	1.514	0.285	0.264	0.061	0.265	0.091

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	44	43	43	49	43	43
N.S.	1	1.00	1.00	0.80	0.78	0.78	0.89	0.78	0.78
time (sec)	N/A	0.038	0.005	0.143	0.195	0.242	0.016	0.260	0.028

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	44	43	43	49	43	43
N.S.	1	1.00	1.00	0.80	0.78	0.78	0.89	0.78	0.78
time (sec)	N/A	0.025	0.004	0.132	0.197	0.274	0.019	0.260	0.024

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	41	40	40	46	40	40
N.S.	1	1.00	1.00	0.82	0.80	0.80	0.92	0.80	0.80
time (sec)	N/A	0.016	0.003	0.137	0.206	0.283	0.017	0.255	0.024

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	44	39	38
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.96	0.85	0.83
time (sec)	N/A	0.017	0.005	0.026	0.199	0.261	0.045	0.265	0.029

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	38	45	41	39	38
N.S.	1	1.00	1.00	0.89	0.86	1.02	0.93	0.89	0.86
time (sec)	N/A	0.024	0.007	0.029	0.211	0.268	0.056	0.270	0.029

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	38	45	44	39	38
N.S.	1	1.00	1.00	0.89	0.86	1.02	1.00	0.89	0.86
time (sec)	N/A	0.026	0.006	0.032	0.204	0.263	0.100	0.307	0.030

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	79	79	92	79	79
N.S.	1	1.00	1.18	0.98	0.96	0.96	1.12	0.96	0.96
time (sec)	N/A	0.043	0.005	1.528	0.203	0.257	0.020	0.255	0.040

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	79	79	94	79	79
N.S.	1	1.00	1.18	0.98	0.96	0.96	1.15	0.96	0.96
time (sec)	N/A	0.035	0.004	1.554	0.208	0.265	0.019	0.264	0.038

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	76	76	88	76	76
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.14	0.99	0.99
time (sec)	N/A	0.042	0.004	1.539	0.199	0.260	0.021	0.263	0.037

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	75	74	74	88	75	74
N.S.	1	1.00	1.00	0.85	0.84	0.84	1.00	0.85	0.84
time (sec)	N/A	0.035	0.010	1.487	0.208	0.256	0.070	0.265	0.041

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	74	73	81	82	74	73
N.S.	1	1.00	1.00	0.89	0.88	0.98	0.99	0.89	0.88
time (sec)	N/A	0.043	0.010	1.509	0.216	0.268	0.069	0.266	0.042

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	75	74	81	87	75	74
N.S.	1	1.00	1.00	0.89	0.88	0.96	1.04	0.89	0.88
time (sec)	N/A	0.042	0.011	1.554	0.215	0.273	0.115	0.290	0.038

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	139	116	115	115	138	115	115
N.S.	1	1.00	1.26	1.05	1.05	1.05	1.25	1.05	1.05
time (sec)	N/A	0.050	0.005	1.529	0.200	0.267	0.022	0.267	0.095

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	139	116	115	115	138	115	115
N.S.	1	1.00	1.26	1.05	1.05	1.05	1.25	1.05	1.05
time (sec)	N/A	0.048	0.005	1.582	0.210	0.272	0.022	0.268	0.095

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	134	113	112	112	134	112	112
N.S.	1	1.00	1.28	1.08	1.07	1.07	1.28	1.07	1.07
time (sec)	N/A	0.067	0.005	1.532	0.194	0.284	0.025	0.261	0.092

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	110	109	109	131	110	109
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.03	0.87	0.86
time (sec)	N/A	0.051	0.011	1.494	0.206	0.271	0.073	0.275	0.103

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	125	110	109	117	128	110	109
N.S.	1	1.00	1.00	0.88	0.87	0.94	1.02	0.88	0.87
time (sec)	N/A	0.061	0.012	1.514	0.211	0.285	0.085	0.267	0.106

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	126	111	110	117	131	111	110
N.S.	1	1.00	1.00	0.88	0.87	0.93	1.04	0.88	0.87
time (sec)	N/A	0.062	0.011	1.544	0.208	0.264	0.127	0.277	8.985

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	181	152	151	151	184	151	151
N.S.	1	1.00	1.31	1.10	1.09	1.09	1.33	1.09	1.09
time (sec)	N/A	0.065	0.006	1.615	0.220	0.253	0.024	0.264	9.157

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	181	152	151	151	185	151	151
N.S.	1	1.00	1.31	1.10	1.09	1.09	1.34	1.09	1.09
time (sec)	N/A	0.066	0.005	1.569	0.217	0.268	0.024	0.267	0.163

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	173	148	147	147	178	147	147
N.S.	1	1.00	1.33	1.14	1.13	1.13	1.37	1.13	1.13
time (sec)	N/A	0.098	0.005	1.532	0.208	0.278	0.025	0.273	0.157

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	166	145	144	144	175	145	144
N.S.	1	1.00	1.00	0.87	0.87	0.87	1.05	0.87	0.87
time (sec)	N/A	0.071	0.011	1.494	0.213	0.266	0.088	0.268	0.162

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	162	145	144	153	168	145	144
N.S.	1	1.00	1.00	0.90	0.89	0.94	1.04	0.90	0.89
time (sec)	N/A	0.079	0.011	1.504	0.193	0.279	0.106	0.272	9.067

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	166	147	146	153	175	147	146
N.S.	1	1.00	1.00	0.89	0.88	0.92	1.05	0.89	0.88
time (sec)	N/A	0.080	0.013	1.608	0.209	0.263	0.171	0.270	9.048

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	191	67	190	4798	178	206	319
N.S.	1	1.00	0.93	0.33	0.93	23.40	0.87	1.00	1.56
time (sec)	N/A	0.175	0.099	1.539	0.289	0.995	0.743	0.277	8.974

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	184	59	181	4261	150	191	340
N.S.	1	1.00	0.95	0.31	0.94	22.08	0.78	0.99	1.76
time (sec)	N/A	0.169	0.084	1.541	0.305	0.963	0.732	0.277	9.079

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	200	49	173	4628	160	174	266
N.S.	1	1.00	1.09	0.27	0.95	25.29	0.87	0.95	1.45
time (sec)	N/A	0.152	0.063	1.543	0.278	1.216	0.752	0.277	9.114

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	176	37	159	4671	160	162	274
N.S.	1	1.00	0.99	0.21	0.90	26.39	0.90	0.92	1.55
time (sec)	N/A	0.090	0.089	1.521	0.282	1.212	0.709	0.280	0.254

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	176	190	176	4588	0	176	716
N.S.	1	1.00	0.96	1.03	0.96	24.93	0.00	0.96	3.89
time (sec)	N/A	0.138	0.085	1.530	0.294	1.269	0.000	0.276	9.125

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	184	207	186	4524	0	198	723
N.S.	1	1.00	0.96	1.08	0.97	23.56	0.00	1.03	3.77
time (sec)	N/A	0.140	0.226	1.536	0.279	1.167	0.000	0.277	8.929

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	192	209	177	4279	0	202	701
N.S.	1	1.00	0.95	1.03	0.87	21.08	0.00	1.00	3.45
time (sec)	N/A	0.136	0.164	1.677	0.317	1.070	0.000	0.282	8.993

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	174	67	163	2077	110	176	180
N.S.	1	1.00	0.92	0.35	0.86	10.93	0.58	0.93	0.95
time (sec)	N/A	0.115	0.138	1.583	0.288	0.932	1.084	0.283	0.211

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	186	73	185	2358	124	186	194
N.S.	1	1.00	0.93	0.36	0.92	11.79	0.62	0.93	0.97
time (sec)	N/A	0.109	0.183	1.582	0.303	0.947	0.845	0.286	8.977

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	189	71	179	2118	116	183	175
N.S.	1	1.00	0.95	0.36	0.90	10.64	0.58	0.92	0.88
time (sec)	N/A	0.093	0.183	1.563	0.290	0.933	0.672	0.283	8.999

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	199	235	203	5018	0	213	490
N.S.	1	1.00	0.90	1.06	0.91	22.60	0.00	0.96	2.21
time (sec)	N/A	0.213	0.152	1.562	0.292	1.081	0.000	0.280	0.374

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	213	245	222	4976	0	233	488
N.S.	1	1.00	0.92	1.06	0.96	21.54	0.00	1.01	2.11
time (sec)	N/A	0.224	0.266	1.555	0.278	1.130	0.000	0.276	9.279

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	221	249	220	4774	0	245	733
N.S.	1	1.00	0.91	1.03	0.91	19.73	0.00	1.01	3.03
time (sec)	N/A	0.233	0.171	1.614	0.276	1.087	0.000	0.282	9.200

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	225	271	236	5373	0	264	537
N.S.	1	1.00	0.86	1.03	0.90	20.51	0.00	1.01	2.05
time (sec)	N/A	0.277	0.174	1.560	0.328	1.187	0.000	0.279	9.289

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	198	87	203	2163	148	203	216
N.S.	1	1.00	0.92	0.40	0.94	10.06	0.69	0.94	1.00
time (sec)	N/A	0.137	0.189	1.535	0.300	1.084	4.671	0.285	0.235

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	214	96	223	2519	170	210	232
N.S.	1	1.00	0.90	0.40	0.93	10.54	0.71	0.88	0.97
time (sec)	N/A	0.138	0.274	1.525	0.306	1.055	1.710	0.282	9.028

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	213	92	219	2251	163	209	212
N.S.	1	1.00	0.95	0.41	0.97	10.00	0.72	0.93	0.94
time (sec)	N/A	0.134	0.286	1.519	0.290	0.924	1.066	0.280	0.264

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	229	269	246	5229	0	248	540
N.S.	1	1.00	0.89	1.05	0.96	20.35	0.00	0.96	2.10
time (sec)	N/A	0.282	0.201	1.551	0.293	1.107	0.000	0.297	9.238

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	248	277	266	5112	0	268	793
N.S.	1	1.00	0.93	1.04	1.00	19.15	0.00	1.00	2.97
time (sec)	N/A	0.305	0.222	1.553	0.307	1.143	0.000	0.292	9.352

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	253	283	265	4911	0	278	778
N.S.	1	1.00	0.92	1.03	0.96	17.79	0.00	1.01	2.82
time (sec)	N/A	0.339	0.224	1.515	0.288	1.211	0.000	0.289	9.297

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	255	301	283	5550	0	299	870
N.S.	1	1.00	0.86	1.01	0.95	18.62	0.00	1.00	2.92
time (sec)	N/A	0.408	0.386	1.536	0.289	1.768	0.000	0.282	9.794

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	230	110	248	2364	201	236	253
N.S.	1	1.00	0.93	0.44	1.00	9.53	0.81	0.95	1.02
time (sec)	N/A	0.176	0.250	1.523	0.281	1.984	156.077	0.280	0.289

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	241	119	260	2646	214	238	265
N.S.	1	1.00	0.89	0.44	0.96	9.80	0.79	0.88	0.98
time (sec)	N/A	0.177	0.363	1.549	0.294	1.127	6.081	0.285	0.267

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	239	116	254	2344	202	233	247
N.S.	1	1.00	0.96	0.46	1.02	9.38	0.81	0.93	0.99
time (sec)	N/A	0.157	0.242	1.513	0.283	0.905	1.803	0.282	0.305

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	259	305	293	5370	0	284	871
N.S.	1	1.00	0.89	1.05	1.01	18.45	0.00	0.98	2.99
time (sec)	N/A	0.350	0.256	1.543	0.300	1.063	0.000	0.279	12.945

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	279	313	313	5250	0	304	840
N.S.	1	1.00	0.93	1.04	1.04	17.44	0.00	1.01	2.79
time (sec)	N/A	0.403	0.292	1.553	0.322	1.102	0.000	0.279	11.846

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	284	319	312	5049	0	315	825
N.S.	1	1.00	0.92	1.03	1.01	16.29	0.00	1.02	2.66
time (sec)	N/A	0.439	0.283	1.562	0.332	1.070	0.000	0.285	12.130

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	284	336	330	5670	0	326	918
N.S.	1	1.00	0.84	0.99	0.97	16.68	0.00	0.96	2.70
time (sec)	N/A	0.521	0.501	1.561	0.313	1.440	0.000	0.278	12.233

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	57	29	26	26	54	27	26
N.S.	1	1.00	1.97	1.00	0.90	0.90	1.86	0.93	0.90
time (sec)	N/A	0.042	0.021	1.528	0.271	0.262	0.071	0.269	11.520

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	57	29	26	26	54	27	26
N.S.	1	1.00	1.97	1.00	0.90	0.90	1.86	0.93	0.90
time (sec)	N/A	0.024	0.009	1.508	0.269	0.252	0.081	0.262	0.029

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	58	29	28	28	54	29	27
N.S.	1	1.00	1.87	0.94	0.90	0.90	1.74	0.94	0.87
time (sec)	N/A	0.041	0.020	1.525	0.271	0.258	0.069	0.274	0.069

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	58	29	28	28	54	29	27
N.S.	1	1.00	1.87	0.94	0.90	0.90	1.74	0.94	0.87
time (sec)	N/A	0.025	0.010	1.615	0.375	0.267	0.078	0.286	0.031

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	146	116	51	52	100	149	154
N.S.	1	1.00	2.92	2.32	1.02	1.04	2.00	2.98	3.08
time (sec)	N/A	0.064	0.048	1.674	0.382	0.270	0.141	0.278	11.157

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	149	119	166	53	110	114	156
N.S.	1	1.00	2.81	2.25	3.13	1.00	2.08	2.15	2.94
time (sec)	N/A	0.069	0.079	1.498	0.349	0.265	0.160	0.284	11.037

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	148	117	167	56	109	97	155
N.S.	1	1.00	2.74	2.17	3.09	1.04	2.02	1.80	2.87
time (sec)	N/A	0.059	0.048	1.516	0.352	0.285	0.133	0.282	11.056

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	147	118	52	53	102	90	155
N.S.	1	1.00	2.77	2.23	0.98	1.00	1.92	1.70	2.92
time (sec)	N/A	0.058	0.059	1.525	0.320	0.279	0.149	0.268	10.956

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	79	79	90	85	82
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.93	0.88	0.85
time (sec)	N/A	0.089	0.030	1.185	0.231	0.271	0.019	0.264	0.051

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	79	79	90	85	82
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.93	0.88	0.85
time (sec)	N/A	0.067	0.029	1.161	0.240	0.281	0.019	0.273	0.043

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	79	79	90	85	82
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.93	0.88	0.85
time (sec)	N/A	0.065	0.027	1.136	0.239	0.273	0.019	0.266	0.042

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	79	79	90	85	82
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.93	0.88	0.85
time (sec)	N/A	0.060	0.019	1.191	0.209	0.283	0.019	0.268	0.043

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	77	76	76	87	82	79
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.95	0.89	0.86
time (sec)	N/A	0.051	0.015	0.101	0.205	0.289	0.021	0.292	0.042

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	78	74	74	85	81	77
N.S.	1	1.00	1.00	0.89	0.84	0.84	0.97	0.92	0.88
time (sec)	N/A	0.040	0.030	0.030	0.194	0.272	0.076	0.266	0.047

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	81	74	81	82	81	77
N.S.	1	1.00	1.00	0.94	0.86	0.94	0.95	0.94	0.90
time (sec)	N/A	0.047	0.041	0.033	0.211	0.276	0.083	0.260	0.047

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	78	78	74	81	83	78	76
N.S.	1	1.00	0.91	0.91	0.86	0.94	0.97	0.91	0.88
time (sec)	N/A	0.050	0.061	0.034	0.225	0.278	0.141	0.263	0.043

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	76	76	75	81	83	77	75
N.S.	1	1.00	0.88	0.88	0.87	0.94	0.97	0.90	0.87
time (sec)	N/A	0.050	0.060	0.034	0.206	0.272	0.324	0.264	0.043

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	77	74	75	81	83	75	74
N.S.	1	1.00	0.90	0.86	0.87	0.94	0.97	0.87	0.86
time (sec)	N/A	0.053	0.060	0.033	0.273	0.277	1.234	0.274	10.461

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	163	152	151	151	167	157	151
N.S.	1	1.00	1.00	0.93	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.143	0.046	2.062	0.213	0.343	0.028	0.266	0.118

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	163	152	151	151	167	157	151
N.S.	1	1.00	1.00	0.93	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.114	0.031	2.092	0.236	0.325	0.031	0.270	10.531

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	150	152	151	151	167	157	151
N.S.	1	1.00	0.95	0.96	0.96	0.96	1.06	0.99	0.96
time (sec)	N/A	0.091	0.076	2.076	0.196	0.379	0.041	0.271	0.112

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	163	152	151	151	167	157	151
N.S.	1	1.00	1.03	0.96	0.96	0.96	1.06	0.99	0.96
time (sec)	N/A	0.091	0.030	2.061	0.195	0.360	0.027	0.271	0.110

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	125	149	148	148	163	154	148
N.S.	1	1.00	0.82	0.97	0.97	0.97	1.07	1.01	0.97
time (sec)	N/A	0.093	0.093	2.006	0.199	0.436	0.030	0.264	0.108

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	154	147	146	146	162	153	146
N.S.	1	1.00	1.03	0.99	0.98	0.98	1.09	1.03	0.98
time (sec)	N/A	0.074	0.053	1.503	0.198	0.275	0.126	0.266	10.494

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	152	150	146	153	156	152	145
N.S.	1	1.00	1.03	1.02	0.99	1.04	1.06	1.03	0.99
time (sec)	N/A	0.086	0.073	1.518	0.193	0.285	0.137	0.281	0.118

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	127	148	146	153	158	150	145
N.S.	1	1.00	0.86	1.01	0.99	1.04	1.07	1.02	0.99
time (sec)	N/A	0.094	0.087	1.492	0.197	0.297	0.193	0.279	9.831

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	123	147	147	153	158	150	145
N.S.	1	1.00	0.81	0.97	0.97	1.01	1.04	0.99	0.95
time (sec)	N/A	0.085	0.077	1.483	0.209	0.281	0.401	0.272	0.096

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	125	144	147	153	156	149	145
N.S.	1	1.00	0.82	0.95	0.97	1.01	1.03	0.98	0.95
time (sec)	N/A	0.085	0.084	1.498	0.215	0.278	1.410	0.283	0.069

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	223	221	217	217	246	229	205
N.S.	1	1.00	1.00	0.99	0.97	0.97	1.10	1.03	0.92
time (sec)	N/A	0.216	0.058	2.058	0.193	0.272	0.035	0.267	0.198

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	223	221	217	217	246	229	205
N.S.	1	1.00	1.00	0.99	0.97	0.97	1.10	1.03	0.92
time (sec)	N/A	0.156	0.052	2.051	0.202	0.274	0.032	0.271	9.008

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	223	221	217	217	246	229	205
N.S.	1	1.00	1.05	1.04	1.02	1.02	1.16	1.08	0.97
time (sec)	N/A	0.126	0.055	2.029	0.196	0.277	0.033	0.267	0.181

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	223	221	217	217	246	229	205
N.S.	1	1.00	1.05	1.04	1.02	1.02	1.16	1.08	0.97
time (sec)	N/A	0.123	0.039	2.048	0.207	0.266	0.032	0.272	0.181

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	170	218	214	214	243	226	202
N.S.	1	1.00	0.82	1.05	1.03	1.03	1.17	1.09	0.98
time (sec)	N/A	0.124	0.094	2.037	0.204	0.391	0.037	0.268	0.180

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	214	215	212	212	240	224	199
N.S.	1	1.00	1.07	1.08	1.06	1.06	1.20	1.12	1.00
time (sec)	N/A	0.099	0.090	1.517	0.196	0.372	0.184	0.274	9.129

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	172	219	212	219	236	224	199
N.S.	1	1.00	0.87	1.11	1.07	1.11	1.19	1.13	1.01
time (sec)	N/A	0.129	0.159	1.503	0.212	0.376	0.192	0.264	9.554

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	174	217	212	219	238	222	199
N.S.	1	1.00	0.88	1.10	1.07	1.11	1.20	1.12	1.01
time (sec)	N/A	0.135	0.136	1.475	0.195	0.393	0.244	0.260	0.154

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	172	216	212	219	236	221	199
N.S.	1	1.00	0.82	1.03	1.01	1.05	1.13	1.06	0.95
time (sec)	N/A	0.134	0.122	1.491	0.209	0.354	0.459	0.275	0.147

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	170	215	212	219	235	220	199
N.S.	1	1.00	0.81	1.03	1.01	1.05	1.12	1.05	0.95
time (sec)	N/A	0.133	0.129	1.492	0.207	0.290	1.495	0.267	9.393

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	334	165	378	15635	0	375	1271
N.S.	1	1.00	1.01	0.50	1.14	47.24	0.00	1.13	3.84
time (sec)	N/A	0.749	0.470	1.545	0.303	1.775	0.000	0.289	9.232

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	299	138	332	15451	0	351	1236
N.S.	1	1.00	0.96	0.44	1.06	49.36	0.00	1.12	3.95
time (sec)	N/A	0.724	0.218	1.551	0.293	2.360	0.000	0.278	0.146

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	290	123	313	14746	0	329	1170
N.S.	1	1.00	0.99	0.42	1.06	50.16	0.00	1.12	3.98
time (sec)	N/A	0.661	0.186	1.553	0.303	2.301	0.000	0.271	9.253

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	272	104	300	14875	0	291	1161
N.S.	1	1.00	0.99	0.38	1.09	54.09	0.00	1.06	4.22
time (sec)	N/A	0.617	0.342	1.551	0.313	1.462	0.000	0.280	9.119

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	257	254	82	266	15235	0	271	1150
N.S.	1	0.99	0.98	0.32	1.03	58.82	0.00	1.05	4.44
time (sec)	N/A	0.262	0.268	1.562	0.295	1.782	0.000	0.275	9.344

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	256	258	259	290	15327	0	278	1731
N.S.	1	0.99	1.00	1.00	1.12	59.41	0.00	1.08	6.71
time (sec)	N/A	0.313	0.181	1.548	0.289	84.140	0.000	0.272	9.186

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	257	260	290	15238	0	274	1802
N.S.	1	1.00	1.02	1.03	1.15	60.23	0.00	1.08	7.12
time (sec)	N/A	0.298	0.270	1.546	0.290	85.937	0.000	0.279	9.355

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	258	257	251	271	15424	0	267	6948
N.S.	1	0.99	0.99	0.97	1.04	59.32	0.00	1.03	26.72
time (sec)	N/A	0.266	0.318	1.567	0.283	45.126	0.000	0.281	9.843

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	274	264	276	302	15204	0	287	1842
N.S.	1	0.99	0.96	1.00	1.09	55.09	0.00	1.04	6.67
time (sec)	N/A	0.317	0.443	1.566	0.291	74.149	0.000	0.274	10.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	334	162	364	16147	0	351	1241
N.S.	1	1.00	0.99	0.48	1.08	47.91	0.00	1.04	3.68
time (sec)	N/A	0.476	0.429	1.556	0.300	1.957	0.000	0.270	9.313

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	294	136	329	16285	0	329	1229
N.S.	1	1.00	0.95	0.44	1.06	52.36	0.00	1.06	3.95
time (sec)	N/A	0.420	0.224	1.546	0.288	1.991	0.000	0.281	0.158

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	288	280	115	283	12153	0	301	816
N.S.	1	0.99	0.97	0.40	0.98	41.91	0.00	1.04	2.81
time (sec)	N/A	0.337	0.187	1.532	0.280	1.604	0.000	0.288	0.138

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	285	113	311	12617	0	311	827
N.S.	1	1.00	0.99	0.39	1.08	43.66	0.00	1.08	2.86
time (sec)	N/A	0.334	0.185	1.543	0.285	1.660	0.000	0.290	10.613

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	268	120	292	12636	0	302	835
N.S.	1	1.00	0.97	0.43	1.06	45.78	0.00	1.09	3.03
time (sec)	N/A	0.253	0.164	1.525	0.288	1.521	0.000	0.274	9.775

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	287	269	293	302	12541	0	314	1660
N.S.	1	0.99	0.93	1.01	1.04	43.39	0.00	1.09	5.74
time (sec)	N/A	0.373	0.212	1.559	0.287	21.328	0.000	0.276	9.775

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	285	298	329	12556	0	324	1684
N.S.	1	1.00	0.95	0.99	1.09	41.71	0.00	1.08	5.59
time (sec)	N/A	0.400	0.312	1.604	0.290	22.128	0.000	0.276	10.014

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	304	292	293	316	12231	0	333	1632
N.S.	1	0.99	0.95	0.96	1.03	39.97	0.00	1.09	5.33
time (sec)	N/A	0.396	0.496	1.588	0.321	17.921	0.000	0.283	9.832

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	336	303	332	365	16568	0	359	1924
N.S.	1	0.99	0.90	0.98	1.08	49.02	0.00	1.06	5.69
time (sec)	N/A	0.490	0.504	1.560	0.311	71.983	0.000	0.284	9.839

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	342	168	391	12967	0	381	916
N.S.	1	1.00	0.99	0.49	1.13	37.59	0.00	1.10	2.66
time (sec)	N/A	0.589	0.305	1.544	0.298	2.149	0.000	0.287	0.593

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	315	173	366	12939	0	359	908
N.S.	1	1.00	0.97	0.53	1.13	39.81	0.00	1.10	2.79
time (sec)	N/A	0.433	0.259	1.553	0.298	1.961	0.000	0.278	9.593

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	287	151	308	6926	0	315	627
N.S.	1	1.00	0.97	0.51	1.04	23.32	0.00	1.06	2.11
time (sec)	N/A	0.290	0.249	1.552	0.318	1.467	0.000	0.270	9.371

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	297	158	344	7190	0	335	640
N.S.	1	1.00	0.92	0.49	1.07	22.26	0.00	1.04	1.98
time (sec)	N/A	0.323	0.331	1.667	0.285	1.740	0.000	0.280	9.433

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	295	149	327	6984	0	329	630
N.S.	1	1.00	0.94	0.48	1.04	22.31	0.00	1.05	2.01
time (sec)	N/A	0.291	0.250	1.710	0.281	1.524	0.000	0.287	9.293

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	345	311	339	368	12815	0	371	1716
N.S.	1	0.99	0.90	0.98	1.06	36.93	0.00	1.07	4.95
time (sec)	N/A	0.495	0.283	1.582	0.302	21.986	0.000	0.280	9.747

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	336	345	400	12951	0	386	1747
N.S.	1	1.00	0.93	0.95	1.10	35.78	0.00	1.07	4.83
time (sec)	N/A	0.526	0.543	1.563	0.285	22.720	0.000	0.291	9.869

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	357	337	340	390	12435	0	395	1697
N.S.	1	0.99	0.94	0.94	1.08	34.54	0.00	1.10	4.71
time (sec)	N/A	0.538	0.558	1.608	0.287	17.282	0.000	0.285	9.904

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	392	352	394	444	16697	0	427	1994
N.S.	1	0.99	0.89	1.00	1.12	42.27	0.00	1.08	5.05
time (sec)	N/A	0.664	0.572	1.589	0.288	101.305	0.000	0.281	10.349

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	583	583	132	765	0	81	129	0	0
N.S.	1	1.00	0.23	1.31	0.00	0.14	0.22	0.00	0.00
time (sec)	N/A	0.505	10.144	1.670	0.000	0.079	1.499	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	560	121	755	0	74	107	0	0
N.S.	1	1.00	0.22	1.35	0.00	0.13	0.19	0.00	0.00
time (sec)	N/A	0.341	10.088	1.631	0.000	0.082	1.410	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	537	537	114	746	0	66	107	0	0
N.S.	1	1.00	0.21	1.39	0.00	0.12	0.20	0.00	0.00
time (sec)	N/A	0.223	10.070	1.661	0.000	0.079	1.394	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	509	107	735	0	55	105	0	0
N.S.	1	1.00	0.21	1.44	0.00	0.11	0.21	0.00	0.00
time (sec)	N/A	0.126	10.060	1.601	0.000	0.088	1.094	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	518	518	128	740	0	190	105	0	0
N.S.	1	1.00	0.25	1.43	0.00	0.37	0.20	0.00	0.00
time (sec)	N/A	0.161	10.172	1.538	0.000	0.212	1.697	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	547	547	126	758	0	235	107	0	121
N.S.	1	1.00	0.23	1.39	0.00	0.43	0.20	0.00	0.22
time (sec)	N/A	0.250	10.120	1.661	0.000	0.114	1.432	0.000	9.724

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	569	569	131	763	0	246	112	0	0
N.S.	1	1.00	0.23	1.34	0.00	0.43	0.20	0.00	0.00
time (sec)	N/A	0.343	10.196	1.659	0.000	0.131	1.577	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	594	594	134	813	0	138	129	0	0
N.S.	1	1.00	0.23	1.37	0.00	0.23	0.22	0.00	0.00
time (sec)	N/A	0.458	10.107	2.049	0.000	0.096	6.884	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	574	574	127	793	0	130	129	0	0
N.S.	1	1.00	0.22	1.38	0.00	0.23	0.22	0.00	0.00
time (sec)	N/A	0.344	10.100	2.009	0.000	0.107	5.070	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	542	118	775	0	107	129	0	0
N.S.	1	1.00	0.22	1.43	0.00	0.20	0.24	0.00	0.00
time (sec)	N/A	0.236	10.098	1.910	0.000	0.095	4.003	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	107	759	0	99	109	0	0
N.S.	1	1.00	0.20	1.45	0.00	0.19	0.21	0.00	0.00
time (sec)	N/A	0.198	10.078	1.602	0.000	0.162	3.542	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	561	561	108	765	0	112	109	0	0
N.S.	1	1.00	0.19	1.36	0.00	0.20	0.19	0.00	0.00
time (sec)	N/A	0.242	10.067	1.557	0.000	0.212	3.309	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	532	109	771	0	98	107	0	0
N.S.	1	1.00	0.20	1.45	0.00	0.18	0.20	0.00	0.00
time (sec)	N/A	0.192	10.053	1.539	0.000	0.095	3.270	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	579	579	119	794	0	345	265	0	0
N.S.	1	1.00	0.21	1.37	0.00	0.60	0.46	0.00	0.00
time (sec)	N/A	0.289	10.104	1.542	0.000	0.123	5.697	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	607	607	121	806	0	377	267	0	136
N.S.	1	1.00	0.20	1.33	0.00	0.62	0.44	0.00	0.22
time (sec)	N/A	0.400	10.096	1.962	0.000	0.139	5.906	0.000	10.038

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	733	733	172	956	0	202	238	0	0
N.S.	1	1.00	0.23	1.30	0.00	0.28	0.32	0.00	0.00
time (sec)	N/A	1.323	9.910	1.758	0.000	0.093	2.328	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	681	681	158	920	0	177	223	0	0
N.S.	1	1.00	0.23	1.35	0.00	0.26	0.33	0.00	0.00
time (sec)	N/A	1.017	10.272	1.744	0.000	0.171	2.118	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	667	667	143	829	0	147	223	0	0
N.S.	1	1.00	0.21	1.24	0.00	0.22	0.33	0.00	0.00
time (sec)	N/A	0.726	10.144	1.685	0.000	0.126	2.018	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	639	639	135	863	0	141	194	0	0
N.S.	1	1.00	0.21	1.35	0.00	0.22	0.30	0.00	0.00
time (sec)	N/A	0.501	9.288	1.693	0.000	0.091	2.074	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	620	620	185	848	0	340	235	0	0
N.S.	1	1.00	0.30	1.37	0.00	0.55	0.38	0.00	0.00
time (sec)	N/A	0.391	9.188	1.565	0.000	0.232	4.294	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	638	638	213	829	0	307	236	0	0
N.S.	1	1.00	0.33	1.30	0.00	0.48	0.37	0.00	0.00
time (sec)	N/A	0.442	9.140	1.983	0.000	0.235	2.907	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	640	640	218	826	0	320	255	0	0
N.S.	1	1.00	0.34	1.29	0.00	0.50	0.40	0.00	0.00
time (sec)	N/A	0.531	9.784	1.929	0.000	0.244	2.976	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	637	637	238	822	0	348	265	0	0
N.S.	1	1.00	0.37	1.29	0.00	0.55	0.42	0.00	0.00
time (sec)	N/A	0.592	10.148	1.808	0.000	0.558	3.587	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	694	238	845	0	330	274	0	0
N.S.	1	1.00	0.34	1.22	0.00	0.48	0.39	0.00	0.00
time (sec)	N/A	0.713	10.448	1.776	0.000	0.642	3.607	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	652	652	180	874	0	346	240	0	0
N.S.	1	1.00	0.28	1.34	0.00	0.53	0.37	0.00	0.00
time (sec)	N/A	0.558	10.279	1.730	0.000	0.400	3.258	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	659	659	211	883	0	404	304	0	0
N.S.	1	1.00	0.32	1.34	0.00	0.61	0.46	0.00	0.00
time (sec)	N/A	0.663	10.431	1.816	0.000	0.167	4.756	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	711	711	213	901	0	439	308	0	0
N.S.	1	1.00	0.30	1.27	0.00	0.62	0.43	0.00	0.00
time (sec)	N/A	0.778	10.485	1.822	0.000	0.152	4.837	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	743	743	192	931	0	482	304	0	0
N.S.	1	1.00	0.26	1.25	0.00	0.65	0.41	0.00	0.00
time (sec)	N/A	0.990	10.195	1.872	0.000	0.139	4.713	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	791	791	179	1161	0	262	512	0	0
N.S.	1	1.00	0.23	1.47	0.00	0.33	0.65	0.00	0.00
time (sec)	N/A	1.501	10.563	1.778	0.000	0.091	4.225	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	742	742	162	1103	0	237	525	0	0
N.S.	1	1.00	0.22	1.49	0.00	0.32	0.71	0.00	0.00
time (sec)	N/A	1.075	10.361	1.739	0.000	0.134	3.752	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	723	723	148	889	0	207	525	0	0
N.S.	1	1.00	0.20	1.23	0.00	0.29	0.73	0.00	0.00
time (sec)	N/A	0.858	10.309	1.677	0.000	0.149	3.574	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	694	139	1024	0	201	444	0	0
N.S.	1	1.00	0.20	1.48	0.00	0.29	0.64	0.00	0.00
time (sec)	N/A	0.641	10.194	1.699	0.000	0.088	3.622	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	676	676	215	987	0	457	473	0	0
N.S.	1	1.00	0.32	1.46	0.00	0.68	0.70	0.00	0.00
time (sec)	N/A	0.480	10.448	1.583	0.000	0.249	8.305	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	692	692	224	946	0	424	474	0	0
N.S.	1	1.00	0.32	1.37	0.00	0.61	0.68	0.00	0.00
time (sec)	N/A	0.540	10.372	2.409	0.000	0.242	5.027	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	694	232	941	0	433	462	0	0
N.S.	1	1.00	0.33	1.36	0.00	0.62	0.67	0.00	0.00
time (sec)	N/A	0.612	10.345	2.332	0.000	0.263	5.169	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	692	692	243	920	0	434	484	0	0
N.S.	1	1.00	0.35	1.33	0.00	0.63	0.70	0.00	0.00
time (sec)	N/A	0.663	10.602	2.124	0.000	0.670	6.195	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	741	741	246	900	0	384	495	0	0
N.S.	1	1.00	0.33	1.21	0.00	0.52	0.67	0.00	0.00
time (sec)	N/A	0.900	10.605	1.993	0.000	0.820	6.221	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	689	689	191	920	0	382	476	0	0
N.S.	1	1.00	0.28	1.34	0.00	0.55	0.69	0.00	0.00
time (sec)	N/A	0.655	10.242	1.960	0.000	0.261	5.907	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	692	692	240	903	0	430	524	0	0
N.S.	1	1.00	0.35	1.30	0.00	0.62	0.76	0.00	0.00
time (sec)	N/A	0.734	10.560	1.855	0.000	0.394	7.999	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	746	240	916	0	446	536	0	0
N.S.	1	1.00	0.32	1.23	0.00	0.60	0.72	0.00	0.00
time (sec)	N/A	0.894	10.880	1.846	0.000	0.376	8.323	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	705	705	202	949	0	470	527	0	0
N.S.	1	1.00	0.29	1.35	0.00	0.67	0.75	0.00	0.00
time (sec)	N/A	0.678	10.444	1.834	0.000	0.262	7.320	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	714	714	226	958	0	525	573	0	0
N.S.	1	1.00	0.32	1.34	0.00	0.74	0.80	0.00	0.00
time (sec)	N/A	0.784	10.702	1.828	0.000	0.265	13.495	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	764	764	227	976	0	559	576	0	0
N.S.	1	1.00	0.30	1.28	0.00	0.73	0.75	0.00	0.00
time (sec)	N/A	0.900	10.543	1.913	0.000	0.227	14.172	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	796	796	194	1006	0	606	541	0	0
N.S.	1	1.00	0.24	1.26	0.00	0.76	0.68	0.00	0.00
time (sec)	N/A	1.099	10.429	2.226	0.000	0.170	12.155	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	120	114	0	0	0	112	0	0
N.S.	1	1.18	1.12	0.00	0.00	0.00	1.10	0.00	0.00
time (sec)	N/A	0.050	0.633	0.000	0.000	0.000	28.290	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	125	116	0	0	0	114	0	0
N.S.	1	1.17	1.08	0.00	0.00	0.00	1.07	0.00	0.00
time (sec)	N/A	0.061	0.631	0.000	0.000	0.000	43.564	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	125	116	0	0	0	114	0	0
N.S.	1	1.17	1.08	0.00	0.00	0.00	1.07	0.00	0.00
time (sec)	N/A	0.069	0.644	0.000	0.000	0.000	62.265	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	54	54	63	54	54
N.S.	1	1.00	1.00	0.81	0.79	0.79	0.93	0.79	0.79
time (sec)	N/A	0.029	0.011	0.092	0.196	0.306	0.018	0.263	0.036

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	58	57	57	66	57	57
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.90	0.78	0.78
time (sec)	N/A	0.043	0.004	0.096	0.198	0.282	0.017	0.272	0.034

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	124	103	102	102	121	102	102
N.S.	1	1.00	1.14	0.94	0.94	0.94	1.11	0.94	0.94
time (sec)	N/A	0.045	0.004	1.655	0.197	0.280	0.026	0.288	0.093

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	129	106	105	105	124	105	105
N.S.	1	1.00	1.13	0.93	0.92	0.92	1.09	0.92	0.92
time (sec)	N/A	0.054	0.006	1.512	0.203	0.285	0.020	0.262	0.088

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	180	151	150	150	180	150	150
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.19	0.99	0.99
time (sec)	N/A	0.066	0.006	1.462	0.213	0.278	0.027	0.271	0.175

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	185	154	153	153	184	153	153
N.S.	1	1.00	1.19	0.99	0.98	0.98	1.18	0.98	0.98
time (sec)	N/A	0.070	0.006	1.521	0.227	0.261	0.023	0.277	0.176

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	236	199	198	198	241	198	198
N.S.	1	1.00	1.22	1.03	1.03	1.03	1.25	1.03	1.03
time (sec)	N/A	0.102	0.007	1.546	0.231	0.288	0.027	0.271	9.304

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	241	202	201	201	245	201	201
N.S.	1	1.00	1.22	1.02	1.02	1.02	1.24	1.02	1.02
time (sec)	N/A	0.101	0.007	1.579	0.210	0.362	0.026	0.272	0.417

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	214	44	174	241149	0	276	1970
N.S.	1	1.00	1.61	0.33	1.31	1813.15	0.00	2.08	14.81
time (sec)	N/A	0.082	0.063	1.552	0.307	5.082	0.000	0.285	0.991

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	221	79	208	220680	0	325	846
N.S.	1	1.00	1.36	0.49	1.28	1362.22	0.00	2.01	5.22
time (sec)	N/A	0.144	0.082	1.550	0.288	6.173	0.000	0.276	9.018

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	296	42	277	254687	0	286	1952
N.S.	1	1.00	1.01	0.14	0.95	869.24	0.00	0.98	6.66
time (sec)	N/A	0.149	0.216	1.563	0.310	6.415	0.000	0.284	9.811

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	311	75	305	219615	0	305	838
N.S.	1	1.00	0.97	0.23	0.95	684.16	0.00	0.95	2.61
time (sec)	N/A	0.228	0.178	1.552	0.321	5.099	0.000	0.293	8.991

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	315	86	305	124301	517	311	478
N.S.	1	1.00	0.99	0.27	0.96	390.88	1.63	0.98	1.50
time (sec)	N/A	0.179	0.345	1.550	0.319	5.024	43.913	0.280	0.366

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	294	82	294	122993	0	300	559
N.S.	1	1.00	0.95	0.26	0.95	396.75	0.00	0.97	1.80
time (sec)	N/A	0.189	0.276	1.552	0.296	3.579	0.000	0.274	9.239

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	347	117	355	124838	0	348	832
N.S.	1	1.00	0.99	0.33	1.01	355.66	0.00	0.99	2.37
time (sec)	N/A	0.209	0.339	1.711	0.323	9.938	0.000	0.300	9.388

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	329	114	343	124542	0	334	521
N.S.	1	1.00	0.97	0.34	1.01	366.30	0.00	0.98	1.53
time (sec)	N/A	0.223	0.368	1.630	0.334	8.281	0.000	0.283	0.403

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	379	153	402	125011	0	384	879
N.S.	1	1.00	0.99	0.40	1.05	327.25	0.00	1.01	2.30
time (sec)	N/A	0.260	0.410	1.643	0.313	19.777	0.000	0.280	9.445

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	366	144	396	125996	0	375	888
N.S.	1	1.00	0.96	0.38	1.04	331.57	0.00	0.99	2.34
time (sec)	N/A	0.261	0.438	1.555	0.307	21.923	0.000	0.274	0.497

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	202	290	0	214	252	0	0
N.S.	1	1.00	0.48	0.69	0.00	0.51	0.60	0.00	0.00
time (sec)	N/A	0.252	10.662	2.043	0.000	0.223	3.383	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	215	269	0	205	212	0	0
N.S.	1	1.00	0.55	0.68	0.00	0.52	0.54	0.00	0.00
time (sec)	N/A	0.220	10.595	2.032	0.000	0.282	3.346	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	182	262	0	193	212	0	0
N.S.	1	1.00	0.49	0.71	0.00	0.52	0.57	0.00	0.00
time (sec)	N/A	0.210	10.685	2.080	0.000	0.125	3.233	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	211	254	0	170	158	0	0
N.S.	1	1.00	0.60	0.72	0.00	0.48	0.45	0.00	0.00
time (sec)	N/A	0.180	10.189	2.050	0.000	0.130	2.245	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	171	243	0	163	156	0	0
N.S.	1	1.00	0.52	0.73	0.00	0.49	0.47	0.00	0.00
time (sec)	N/A	0.131	10.133	1.894	0.000	0.125	2.100	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	208	274	0	0	204	0	0
N.S.	1	1.00	0.60	0.79	0.00	0.00	0.59	0.00	0.00
time (sec)	N/A	0.165	10.340	1.650	0.000	0.000	4.059	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	208	274	0	0	206	0	0
N.S.	1	1.00	0.61	0.80	0.00	0.00	0.60	0.00	0.00
time (sec)	N/A	0.168	10.361	2.404	0.000	0.000	2.919	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	204	269	0	0	230	0	0
N.S.	1	1.00	0.60	0.79	0.00	0.00	0.67	0.00	0.00
time (sec)	N/A	0.177	10.216	2.239	0.000	0.000	2.830	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	205	264	0	0	235	0	0
N.S.	1	1.00	0.57	0.74	0.00	0.00	0.66	0.00	0.00
time (sec)	N/A	0.203	10.281	2.075	0.000	0.000	2.901	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	175	261	0	0	211	0	0
N.S.	1	1.00	0.53	0.79	0.00	0.00	0.64	0.00	0.00
time (sec)	N/A	0.187	10.220	2.045	0.000	0.000	3.005	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	179	280	0	0	216	0	0
N.S.	1	1.00	0.50	0.78	0.00	0.00	0.60	0.00	0.00
time (sec)	N/A	0.209	10.238	2.140	0.000	0.000	3.048	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	145	262	0	166	189	0	0
N.S.	1	1.00	0.41	0.74	0.00	0.47	0.54	0.00	0.00
time (sec)	N/A	0.213	10.236	2.110	0.000	0.128	2.716	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	145	269	0	173	192	0	0
N.S.	1	1.00	0.39	0.72	0.00	0.46	0.51	0.00	0.00
time (sec)	N/A	0.242	10.243	2.145	0.000	0.266	2.854	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	146	277	0	196	246	0	0
N.S.	1	1.00	0.36	0.69	0.00	0.49	0.62	0.00	0.00
time (sec)	N/A	0.259	10.177	2.329	0.000	0.267	4.045	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	148	299	0	208	246	0	0
N.S.	1	1.00	0.35	0.70	0.00	0.49	0.58	0.00	0.00
time (sec)	N/A	0.290	10.180	2.619	0.000	0.173	4.198	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	225	338	0	264	462	0	0
N.S.	1	1.00	0.47	0.71	0.00	0.55	0.97	0.00	0.00
time (sec)	N/A	0.310	10.880	2.042	0.000	0.140	9.496	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	452	238	317	0	255	398	0	0
N.S.	1	1.00	0.53	0.70	0.00	0.56	0.88	0.00	0.00
time (sec)	N/A	0.267	10.748	1.988	0.000	0.162	9.142	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	427	205	310	0	247	398	0	0
N.S.	1	1.00	0.48	0.73	0.00	0.58	0.93	0.00	0.00
time (sec)	N/A	0.249	11.117	2.089	0.000	0.150	9.121	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	196	300	0	224	396	0	0
N.S.	1	1.00	0.48	0.73	0.00	0.55	0.97	0.00	0.00
time (sec)	N/A	0.219	10.705	2.063	0.000	0.131	5.195	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	175	287	0	214	394	0	0
N.S.	1	1.00	0.46	0.75	0.00	0.56	1.03	0.00	0.00
time (sec)	N/A	0.170	10.507	1.922	0.000	0.132	4.980	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	224	346	0	0	405	0	0
N.S.	1	1.00	0.56	0.86	0.00	0.00	1.00	0.00	0.00
time (sec)	N/A	0.221	10.517	1.653	0.000	0.000	11.830	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	222	346	0	0	406	0	0
N.S.	1	1.00	0.55	0.86	0.00	0.00	1.00	0.00	0.00
time (sec)	N/A	0.223	10.461	3.396	0.000	0.000	6.135	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	194	344	0	0	377	0	0
N.S.	1	1.00	0.48	0.85	0.00	0.00	0.93	0.00	0.00
time (sec)	N/A	0.227	10.340	3.311	0.000	0.000	4.896	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	194	343	0	0	381	0	0
N.S.	1	1.00	0.48	0.84	0.00	0.00	0.93	0.00	0.00
time (sec)	N/A	0.226	10.336	2.960	0.000	0.000	4.843	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	163	344	0	0	379	0	0
N.S.	1	1.00	0.42	0.89	0.00	0.00	0.98	0.00	0.00
time (sec)	N/A	0.246	10.274	2.845	0.000	0.000	5.484	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	165	344	0	0	386	0	0
N.S.	1	1.00	0.43	0.89	0.00	0.00	1.00	0.00	0.00
time (sec)	N/A	0.231	10.222	2.505	0.000	0.000	5.647	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	163	343	0	0	406	0	0
N.S.	1	1.00	0.42	0.88	0.00	0.00	1.04	0.00	0.00
time (sec)	N/A	0.234	10.219	2.385	0.000	0.000	5.253	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	164	299	0	0	415	0	0
N.S.	1	1.00	0.40	0.73	0.00	0.00	1.01	0.00	0.00
time (sec)	N/A	0.266	10.246	2.278	0.000	0.000	5.319	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	174	295	0	0	444	0	0
N.S.	1	1.00	0.46	0.78	0.00	0.00	1.18	0.00	0.00
time (sec)	N/A	0.251	10.309	2.253	0.000	0.000	6.502	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	174	324	0	0	449	0	0
N.S.	1	1.00	0.43	0.80	0.00	0.00	1.11	0.00	0.00
time (sec)	N/A	0.277	10.318	2.402	0.000	0.000	6.794	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	171	308	0	217	398	0	0
N.S.	1	1.00	0.43	0.77	0.00	0.54	1.00	0.00	0.00
time (sec)	N/A	0.287	10.338	2.833	0.000	0.210	6.667	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	172	317	0	227	401	0	0
N.S.	1	1.00	0.41	0.75	0.00	0.54	0.95	0.00	0.00
time (sec)	N/A	0.303	10.364	3.202	0.000	0.349	6.986	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	149	325	0	250	403	0	0
N.S.	1	1.00	0.33	0.72	0.00	0.56	0.90	0.00	0.00
time (sec)	N/A	0.332	10.217	3.374	0.000	0.133	10.769	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	151	347	0	258	403	0	0
N.S.	1	1.00	0.32	0.73	0.00	0.54	0.85	0.00	0.00
time (sec)	N/A	0.363	10.210	3.653	0.000	0.128	11.565	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	212	246	0	163	177	0	0
N.S.	1	1.00	0.59	0.68	0.00	0.45	0.49	0.00	0.00
time (sec)	N/A	0.217	10.152	2.182	0.000	0.124	2.825	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	212	235	0	156	156	0	0
N.S.	1	1.00	0.63	0.70	0.00	0.46	0.46	0.00	0.00
time (sec)	N/A	0.175	10.159	2.093	0.000	0.118	2.696	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	193	230	0	147	156	0	0
N.S.	1	1.00	0.63	0.75	0.00	0.48	0.51	0.00	0.00
time (sec)	N/A	0.159	10.189	2.271	0.000	0.122	2.618	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	160	222	0	133	129	0	0
N.S.	1	1.00	0.54	0.74	0.00	0.44	0.43	0.00	0.00
time (sec)	N/A	0.137	10.112	2.066	0.000	0.122	1.936	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	150	208	0	135	128	0	0
N.S.	1	1.00	0.54	0.75	0.00	0.49	0.46	0.00	0.00
time (sec)	N/A	0.093	10.102	1.895	0.000	0.114	1.507	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	159	216	0	0	126	0	0
N.S.	1	1.00	0.56	0.76	0.00	0.00	0.44	0.00	0.00
time (sec)	N/A	0.119	10.222	1.627	0.000	0.000	2.274	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	157	234	0	0	128	0	0
N.S.	1	1.00	0.51	0.76	0.00	0.00	0.41	0.00	0.00
time (sec)	N/A	0.150	10.227	2.029	0.000	0.000	1.810	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	148	225	0	139	126	0	118
N.S.	1	1.00	0.49	0.75	0.00	0.46	0.42	0.00	0.39
time (sec)	N/A	0.147	10.143	1.928	0.000	0.111	1.715	0.000	9.789

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	149	237	0	136	131	0	0
N.S.	1	1.00	0.46	0.73	0.00	0.42	0.41	0.00	0.00
time (sec)	N/A	0.182	10.165	2.051	0.000	0.114	1.884	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	147	243	0	150	158	0	0
N.S.	1	1.00	0.42	0.70	0.00	0.43	0.46	0.00	0.00
time (sec)	N/A	0.192	10.161	2.124	0.000	0.126	2.410	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	134	255	0	159	163	0	0
N.S.	1	1.00	0.36	0.68	0.00	0.42	0.43	0.00	0.00
time (sec)	N/A	0.223	10.206	2.009	0.000	0.122	2.641	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	220	302	0	242	202	0	0
N.S.	1	1.00	0.60	0.83	0.00	0.66	0.55	0.00	0.00
time (sec)	N/A	0.330	10.199	3.068	0.000	0.121	10.696	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	176	282	0	211	172	0	0
N.S.	1	1.00	0.51	0.82	0.00	0.62	0.50	0.00	0.00
time (sec)	N/A	0.248	10.169	2.807	0.000	0.116	8.390	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	166	264	0	223	172	0	0
N.S.	1	1.00	0.53	0.84	0.00	0.71	0.55	0.00	0.00
time (sec)	N/A	0.187	10.161	2.807	0.000	0.118	7.198	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	297	181	248	0	215	156	0	0
N.S.	1	0.98	0.60	0.82	0.00	0.71	0.52	0.00	0.00
time (sec)	N/A	0.140	10.135	1.799	0.000	0.126	6.388	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	165	251	0	201	156	0	0
N.S.	1	1.00	0.50	0.75	0.00	0.60	0.47	0.00	0.00
time (sec)	N/A	0.173	10.218	1.736	0.000	0.123	5.898	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	116	227	0	147	133	0	0
N.S.	1	1.00	0.38	0.75	0.00	0.49	0.44	0.00	0.00
time (sec)	N/A	0.126	10.083	1.876	0.000	0.092	5.279	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	116	230	0	129	131	0	0
N.S.	1	1.00	0.42	0.84	0.00	0.47	0.48	0.00	0.00
time (sec)	N/A	0.076	10.078	1.730	0.000	0.093	4.950	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	125	253	0	191	289	0	0
N.S.	1	1.00	0.39	0.78	0.00	0.59	0.89	0.00	0.00
time (sec)	N/A	0.193	10.127	1.732	0.000	0.123	7.639	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	123	268	0	215	291	0	133
N.S.	1	1.00	0.36	0.78	0.00	0.62	0.85	0.00	0.39
time (sec)	N/A	0.250	10.120	2.851	0.000	0.122	8.080	0.000	9.968

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	140	283	0	231	316	0	147
N.S.	1	1.00	0.38	0.77	0.00	0.63	0.86	0.00	0.40
time (sec)	N/A	0.318	10.110	2.836	0.000	0.123	7.472	0.000	10.027

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	136	299	0	216	321	0	0
N.S.	1	1.00	0.35	0.77	0.00	0.56	0.83	0.00	0.00
time (sec)	N/A	0.413	10.127	2.797	0.000	0.126	9.872	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	269	269	174	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.175	0.405	0.000	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	170	147	0	0	0	141	0	0
N.S.	1	1.19	1.03	0.00	0.00	0.00	0.99	0.00	0.00
time (sec)	N/A	0.085	0.735	0.000	0.000	0.000	20.537	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	175	175	145	0	0	0	143	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.82	0.00	0.00
time (sec)	N/A	0.112	0.757	0.000	0.000	0.000	47.592	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75
time (sec)	N/A	0.006	0.007	1.524	0.209	0.261	0.019	0.279	0.024

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	8	7	9	6
N.S.	1	1.00	1.00	0.70	0.80	0.80	0.70	0.90	0.60
time (sec)	N/A	0.009	0.001	1.531	0.210	0.272	0.023	0.268	0.056

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	8	8	9	6
N.S.	1	1.00	1.00	0.70	0.80	0.80	0.80	0.90	0.60
time (sec)	N/A	0.008	0.002	1.545	0.202	0.249	0.023	0.277	8.893

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	21	14	17	17	15	15	6
N.S.	1	1.00	2.10	1.40	1.70	1.70	1.50	1.50	0.60
time (sec)	N/A	0.006	0.022	1.528	0.199	0.259	0.041	0.276	0.106

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	17	16	16	24	16	16
N.S.	1	1.00	1.00	0.71	0.67	0.67	1.00	0.67	0.67
time (sec)	N/A	0.015	0.013	1.529	0.274	0.280	0.052	0.273	0.031

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	38	38	46	39	49
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.92	0.78	0.98
time (sec)	N/A	0.032	0.021	1.552	0.286	0.288	0.097	0.265	0.141

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	39	38	38	46	39	48
N.S.	1	1.00	0.92	0.78	0.76	0.76	0.92	0.78	0.96
time (sec)	N/A	0.034	0.016	1.593	0.287	0.265	0.089	0.272	8.885

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	56	47	46	46	56	48	52
N.S.	1	1.00	0.93	0.78	0.77	0.77	0.93	0.80	0.87
time (sec)	N/A	0.033	0.016	1.608	0.275	0.276	0.102	0.268	9.061

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	47	46	46	56	48	52
N.S.	1	1.00	0.87	0.78	0.77	0.77	0.93	0.80	0.87
time (sec)	N/A	0.032	0.017	1.527	0.292	0.275	0.103	0.261	9.024

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	38	38	48	35	46
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.96	0.70	0.92
time (sec)	N/A	0.017	0.008	1.522	0.284	0.438	0.061	0.280	0.099

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	38	38	48	39	46
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.96	0.78	0.92
time (sec)	N/A	0.028	0.013	1.518	0.325	0.447	0.073	0.275	9.065

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	100	83	84	115	105	86	100
N.S.	1	1.00	0.91	0.75	0.76	1.05	0.95	0.78	0.91
time (sec)	N/A	0.082	0.094	1.663	0.283	0.414	0.202	0.276	9.366

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	97	83	84	115	105	86	100
N.S.	1	1.00	0.88	0.75	0.76	1.05	0.95	0.78	0.91
time (sec)	N/A	0.082	0.084	1.613	0.281	0.436	0.200	0.279	0.198

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	122	61	61	91	70	63	52
N.S.	1	1.00	1.51	0.75	0.75	1.12	0.86	0.78	0.64
time (sec)	N/A	0.046	0.530	1.588	0.289	0.276	0.097	0.277	0.060

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	73	74	126	82	76	77
N.S.	1	1.00	0.91	0.79	0.80	1.37	0.89	0.83	0.84
time (sec)	N/A	0.079	0.034	1.568	0.308	0.280	0.153	0.280	0.128

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	119	104	105	256	124	111	120
N.S.	1	1.00	0.80	0.70	0.71	1.73	0.84	0.75	0.81
time (sec)	N/A	0.116	0.063	1.577	0.292	0.283	0.314	0.265	9.484

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	121	104	105	257	124	111	121
N.S.	1	1.00	0.83	0.71	0.72	1.76	0.85	0.76	0.83
time (sec)	N/A	0.125	0.081	1.563	0.276	0.270	0.330	0.289	9.929

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	111	94	95	187	116	106	110
N.S.	1	1.00	0.78	0.66	0.67	1.32	0.82	0.75	0.77
time (sec)	N/A	0.101	0.065	1.553	0.325	0.323	0.308	0.291	11.763

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	111	94	95	187	116	106	111
N.S.	1	1.00	0.78	0.66	0.67	1.32	0.82	0.75	0.78
time (sec)	N/A	0.108	0.080	1.543	0.289	0.389	0.294	0.295	0.195

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	103	86	87	131	110	89	102
N.S.	1	1.00	0.91	0.76	0.77	1.16	0.97	0.79	0.90
time (sec)	N/A	0.051	0.056	1.585	0.287	0.481	0.237	0.273	9.749

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	111	94	95	187	119	99	111
N.S.	1	1.00	0.85	0.72	0.73	1.43	0.91	0.76	0.85
time (sec)	N/A	0.102	0.068	1.538	0.289	0.461	0.289	0.282	0.194

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	91	76	75	75	102	69	91
N.S.	1	1.00	0.92	0.77	0.76	0.76	1.03	0.70	0.92
time (sec)	N/A	0.045	0.019	1.759	0.284	0.300	0.198	0.291	9.678

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	130	0	0	0	860	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	5.31	0.00	0.00
time (sec)	N/A	0.115	0.633	0.000	0.000	0.000	25.498	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	108	118	118	305	1340	392	115
N.S.	1	1.00	1.29	1.40	1.40	3.63	15.95	4.67	1.37
time (sec)	N/A	0.040	0.281	1.760	0.224	0.286	0.783	0.289	9.422

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	120	80	78	160	598	196	76
N.S.	1	1.00	1.97	1.31	1.28	2.62	9.80	3.21	1.25
time (sec)	N/A	0.027	0.176	2.077	0.213	0.328	0.484	0.277	9.179

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	42	43	39	56	180	65	38
N.S.	1	1.00	1.02	1.05	0.95	1.37	4.39	1.59	0.93
time (sec)	N/A	0.015	0.131	0.068	0.206	0.528	0.303	0.282	9.746

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	17	12	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.42	1.00	1.00	1.00
time (sec)	N/A	0.002	0.002	0.026	0.201	0.817	0.018	0.274	11.319

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	73	0	43
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.74	0.00	1.02
time (sec)	N/A	0.019	0.106	0.000	0.000	0.000	1.221	0.000	11.197

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	388	0	49
N.S.	1	1.00	1.00	0.00	0.00	0.00	8.82	0.00	1.11
time (sec)	N/A	0.019	0.138	0.000	0.000	0.000	4.866	0.000	10.885

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	63	0	0	0	0	0	59
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	1.28
time (sec)	N/A	0.019	0.126	0.000	0.000	0.000	0.000	0.000	10.811

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	305	305	206	0	0	0	342	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	1.12	0.00	0.00
time (sec)	N/A	0.165	1.115	0.000	0.000	0.000	20.808	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	66	162	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.47	3.60	0.00	0.00
time (sec)	N/A	0.299	0.952	0.000	0.000	0.397	137.421	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	273	273	178	0	0	0	304	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	1.11	0.00	0.00
time (sec)	N/A	0.130	0.375	0.000	0.000	0.000	104.019	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	204	0	0	0	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.151	0.461	0.000	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	147	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.085	0.774	0.000	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	20	0	0	20
N.S.	1	1.00	1.00	0.88	0.83	0.83	0.00	0.00	0.83
time (sec)	N/A	0.033	7.399	2.483	0.273	0.285	0.000	0.000	9.615

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	93	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	10.542	0.000	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	61	0	228	95
N.S.	1	1.00	1.00	0.00	0.00	2.18	0.00	8.14	3.39
time (sec)	N/A	0.068	0.528	0.000	0.000	0.269	0.000	0.330	9.356

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	45	45	46	138	77	119	0	237	124
N.S.	1	1.00	1.02	3.07	1.71	2.64	0.00	5.27	2.76
time (sec)	N/A	0.103	1.601	0.244	0.308	0.283	0.000	0.350	9.438

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	52	59	54	0	111	76
N.S.	1	1.00	1.00	1.68	1.90	1.74	0.00	3.58	2.45
time (sec)	N/A	0.131	0.931	0.056	0.296	0.286	0.000	0.350	10.776

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	45	45	41	103	92	88	0	155	106
N.S.	1	1.00	0.91	2.29	2.04	1.96	0.00	3.44	2.36
time (sec)	N/A	0.341	2.494	0.276	0.304	0.291	0.000	0.456	11.690

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [124] had the largest ratio of [.6923000000000000026]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	20	0.050
2	A	2	1	1.00	22	0.045
3	A	2	1	1.00	22	0.045
4	A	2	1	1.00	25	0.040
5	A	2	1	1.00	27	0.037
6	A	2	1	1.00	27	0.037
7	A	6	6	1.00	15	0.400
8	A	7	7	1.00	15	0.467
9	A	8	7	1.00	15	0.467
10	A	9	7	1.00	15	0.467
11	A	6	6	1.00	15	0.400
12	A	6	6	1.00	16	0.375
13	A	3	3	1.00	11	0.273
14	A	3	3	1.00	15	0.200
15	A	3	3	1.00	13	0.231
16	A	3	3	1.00	13	0.231
17	A	6	6	1.00	15	0.400
18	A	3	3	1.00	19	0.158
19	A	3	3	1.00	21	0.143
20	A	3	3	1.00	31	0.097
21	A	3	3	1.00	36	0.083
22	A	12	10	1.00	35	0.286
23	A	11	9	1.00	33	0.273
24	A	10	8	1.00	36	0.222

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	10	10	1.00	19	0.526
26	A	9	9	1.00	18	0.500
27	A	4	4	1.00	27	0.148
28	A	4	4	1.00	28	0.143
29	A	4	4	1.00	24	0.167
30	A	4	4	1.00	24	0.167
31	A	4	4	1.00	26	0.154
32	A	4	4	1.00	26	0.154
33	A	4	4	1.00	28	0.143
34	A	4	4	1.00	30	0.133
35	A	4	4	1.00	29	0.138
36	A	4	4	1.00	29	0.138
37	A	4	4	1.00	29	0.138
38	A	4	4	1.00	32	0.125
39	A	6	6	1.00	13	0.462
40	A	4	4	1.00	49	0.082
41	A	4	4	1.00	57	0.070
42	A	2	2	1.00	31	0.065
43	A	2	2	1.00	42	0.048
44	A	4	4	1.00	42	0.095
45	A	4	4	1.00	45	0.089
46	A	4	4	1.00	45	0.089
47	A	4	4	1.00	44	0.091
48	A	3	3	1.00	20	0.150
49	A	6	6	1.00	20	0.300
50	A	2	2	1.00	16	0.125
51	A	5	5	1.00	20	0.250
52	A	3	3	1.00	18	0.167
53	A	2	1	1.00	30	0.033
54	A	2	1	1.00	30	0.033
55	A	2	1	1.00	28	0.036
56	A	2	1	1.00	30	0.033
57	A	7	7	1.00	30	0.233
58	A	8	8	1.00	30	0.267
59	A	7	5	1.00	32	0.156

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	6	5	1.00	32	0.156
61	A	5	5	1.00	32	0.156
62	A	4	4	1.00	32	0.125
63	A	5	5	1.00	32	0.156
64	A	6	5	1.00	32	0.156
65	A	7	5	1.00	32	0.156
66	A	7	6	1.00	32	0.188
67	A	6	6	1.00	32	0.188
68	A	5	5	1.00	32	0.156
69	A	6	6	1.00	32	0.188
70	A	8	8	1.00	17	0.471
71	A	10	9	1.00	17	0.529
72	A	10	9	0.99	17	0.529
73	A	10	9	0.99	22	0.409
74	A	10	9	1.00	22	0.409
75	A	10	9	1.00	22	0.409
76	A	9	8	1.00	17	0.471
77	A	9	8	1.00	19	0.421
78	A	8	7	1.00	18	0.389
79	A	3	3	1.00	18	0.167
80	A	3	3	1.00	22	0.136
81	A	1	1	1.00	20	0.050
82	A	1	1	1.00	20	0.050
83	A	3	3	1.00	33	0.091
84	A	3	3	1.00	35	0.086
85	A	1	1	1.00	36	0.028
86	A	1	1	1.00	36	0.028
87	A	3	3	1.00	30	0.100
88	A	3	3	1.00	32	0.094
89	A	1	1	1.00	33	0.030
90	A	1	1	1.00	33	0.030
91	A	1	1	1.00	20	0.050
92	A	1	1	1.00	24	0.042
93	A	3	3	1.00	22	0.136
94	A	3	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	1	1	1.00	20	0.050
96	A	1	1	1.00	20	0.050
97	A	3	3	1.00	18	0.167
98	A	3	3	1.00	22	0.136
99	A	1	1	1.00	35	0.029
100	A	1	1	1.00	37	0.027
101	A	3	3	1.00	38	0.079
102	A	3	3	1.00	38	0.079
103	A	1	1	1.00	32	0.031
104	A	1	1	1.00	34	0.029
105	A	3	3	1.00	35	0.086
106	A	3	3	1.00	35	0.086
107	A	3	3	1.00	17	0.176
108	A	3	3	1.00	18	0.167
109	A	3	3	1.00	19	0.158
110	A	3	3	1.00	20	0.150
111	A	3	3	1.00	15	0.200
112	A	3	3	1.00	17	0.176
113	A	3	3	1.00	15	0.200
114	A	3	3	1.00	17	0.176
115	A	7	5	1.00	16	0.312
116	A	13	9	1.00	15	0.600
117	A	8	6	1.00	16	0.375
118	A	14	10	1.00	15	0.667
119	A	9	6	1.00	16	0.375
120	A	15	10	1.00	15	0.667
121	A	10	6	1.00	16	0.375
122	A	16	10	1.00	15	0.667
123	A	7	5	1.00	15	0.333
124	A	13	9	1.00	13	0.692
125	A	7	5	1.00	21	0.238
126	A	13	9	1.00	20	0.450
127	A	8	6	1.00	21	0.286
128	A	14	10	1.00	20	0.500
129	A	9	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	15	10	1.00	20	0.500
131	A	10	6	1.00	21	0.286
132	A	16	10	1.00	20	0.500
133	A	3	2	1.00	11	0.182
134	A	3	2	1.00	12	0.167
135	A	2	1	1.00	15	0.067
136	A	3	2	1.00	14	0.143
137	A	2	1	1.00	17	0.059
138	A	3	2	1.00	19	0.105
139	A	2	1	1.00	20	0.050
140	A	2	2	1.00	14	0.143
141	A	4	3	1.00	17	0.176
142	A	4	3	1.00	19	0.158
143	A	3	2	1.00	20	0.100
144	A	4	3	1.00	21	0.143
145	A	3	2	1.00	22	0.091
146	A	4	3	1.00	24	0.125
147	A	3	2	1.00	25	0.080
148	A	3	2	1.00	25	0.080
149	A	8	6	1.00	26	0.231
150	A	9	7	1.00	26	0.269
151	A	10	7	1.00	26	0.269
152	A	10	7	1.00	11	0.636
153	A	3	3	1.00	12	0.250
154	A	13	9	1.00	15	0.600
155	A	10	7	1.00	14	0.500
156	A	9	6	1.00	17	0.353
157	A	14	10	1.00	19	0.526
158	A	13	9	1.00	20	0.450
159	A	2	2	1.00	14	0.143
160	A	12	8	1.00	17	0.471
161	A	5	5	1.00	19	0.263
162	A	15	11	1.00	20	0.550
163	A	13	9	1.00	21	0.429
164	A	12	8	1.00	22	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	16	12	1.00	24	0.500
166	A	15	11	1.00	25	0.440
167	A	2	2	1.00	19	0.105
168	A	11	8	1.00	17	0.471
169	A	9	7	1.00	20	0.350
170	A	15	11	1.00	19	0.579
171	A	11	8	1.00	31	0.258
172	A	8	6	1.00	31	0.194
173	A	9	7	1.00	31	0.226
174	A	10	8	1.00	31	0.258
175	A	17	12	1.00	30	0.400
176	A	14	10	1.00	30	0.333
177	A	15	11	1.00	30	0.367
178	A	16	12	1.00	30	0.400
179	A	2	2	1.00	21	0.095
180	A	2	2	1.00	21	0.095
181	A	2	1	1.00	19	0.053
182	A	2	2	1.00	19	0.105
183	A	2	2	1.00	21	0.095
184	A	2	2	1.00	21	0.095
185	A	2	2	1.00	21	0.095
186	A	13	9	1.00	36	0.250
187	A	13	9	1.00	41	0.220
188	A	13	9	1.00	46	0.196
189	A	19	13	1.00	35	0.371
190	A	19	13	1.00	40	0.325
191	A	19	13	1.00	45	0.289
192	A	8	6	1.00	36	0.167
193	A	8	6	1.00	41	0.146
194	A	10	8	1.00	46	0.174
195	A	14	10	1.00	35	0.286
196	A	14	10	1.00	40	0.250
197	A	16	12	1.00	45	0.267
198	A	9	7	1.00	36	0.194
199	A	9	7	1.00	41	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	9	7	1.00	46	0.152
201	A	15	11	1.00	35	0.314
202	A	15	11	1.00	40	0.275
203	A	15	11	1.00	45	0.244
204	A	10	8	1.00	36	0.222
205	A	10	8	1.00	41	0.195
206	A	10	8	1.00	46	0.174
207	A	16	12	1.00	35	0.343
208	A	16	12	1.00	40	0.300
209	A	16	12	1.00	45	0.267
210	A	6	5	1.00	17	0.294
211	A	7	6	1.00	18	0.333
212	A	7	6	1.00	19	0.316
213	A	6	5	1.00	20	0.250
214	A	8	7	1.00	22	0.318
215	A	1	1	1.00	23	0.043
216	A	1	1	1.00	26	0.038
217	A	1	1	1.00	28	0.036
218	A	1	1	1.00	31	0.032
219	A	1	1	1.00	15	0.067
220	A	12	10	1.00	42	0.238
221	A	3	2	1.00	11	0.182
222	A	3	2	1.00	15	0.133
223	A	3	2	1.00	30	0.067
224	A	3	2	1.00	30	0.067
225	A	3	2	1.00	30	0.067
226	A	3	2	1.00	30	0.067
227	A	3	2	1.00	30	0.067
228	A	3	2	1.00	30	0.067
229	A	3	2	1.00	30	0.067
230	A	3	2	1.00	30	0.067
231	A	3	2	1.00	30	0.067
232	A	3	2	1.00	30	0.067
233	A	9	8	1.00	30	0.267
234	A	9	8	1.00	30	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	9	8	1.00	30	0.267
236	A	9	8	1.00	30	0.267
237	A	9	8	1.00	30	0.267
238	A	9	8	1.00	28	0.286
239	A	8	7	1.00	27	0.259
240	A	8	7	1.00	30	0.233
241	A	8	7	1.00	30	0.233
242	A	8	7	1.00	30	0.233
243	A	8	7	1.00	30	0.233
244	A	8	7	1.00	30	0.233
245	A	8	7	1.00	30	0.233
246	A	8	7	1.00	30	0.233
247	A	8	7	1.00	30	0.233
248	A	8	7	1.00	30	0.233
249	A	8	7	1.00	30	0.233
250	A	8	7	1.00	30	0.233
251	A	3	2	1.00	30	0.067
252	A	3	2	1.00	30	0.067
253	A	3	2	1.00	30	0.067
254	A	3	2	1.00	30	0.067
255	A	3	2	1.00	30	0.067
256	A	3	2	1.00	30	0.067
257	A	3	2	1.00	30	0.067
258	A	3	2	1.00	30	0.067
259	A	3	2	1.00	30	0.067
260	A	9	8	1.00	30	0.267
261	A	12	10	1.00	30	0.333
262	A	9	8	1.00	30	0.267
263	A	11	10	1.00	30	0.333
264	A	9	8	1.00	30	0.267
265	A	10	9	1.00	28	0.321
266	A	9	9	1.00	27	0.333
267	A	9	8	1.00	30	0.267
268	A	9	8	1.00	30	0.267
269	A	9	8	1.00	30	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	9	8	1.00	30	0.267
271	A	9	8	1.00	30	0.267
272	A	9	8	1.00	30	0.267
273	A	9	8	1.00	30	0.267
274	A	9	8	1.00	30	0.267
275	A	9	8	1.00	30	0.267
276	A	3	2	1.00	30	0.067
277	A	3	2	1.00	30	0.067
278	A	3	2	1.00	30	0.067
279	A	3	2	1.00	30	0.067
280	A	3	2	1.00	30	0.067
281	A	3	2	1.00	30	0.067
282	A	3	2	1.00	30	0.067
283	A	3	2	1.00	30	0.067
284	A	3	2	1.00	30	0.067
285	A	3	2	1.00	30	0.067
286	A	10	9	1.00	30	0.300
287	A	14	10	1.00	30	0.333
288	A	10	9	1.00	30	0.300
289	A	13	10	1.00	30	0.333
290	A	10	9	1.00	30	0.300
291	A	12	10	1.00	30	0.333
292	A	10	10	1.00	30	0.333
293	A	10	10	1.00	28	0.357
294	A	9	9	1.00	27	0.333
295	A	9	9	1.00	30	0.300
296	A	9	9	1.00	30	0.300
297	A	10	9	1.00	30	0.300
298	A	10	9	1.00	30	0.300
299	A	10	8	1.00	30	0.267
300	A	10	8	1.00	30	0.267
301	A	10	8	1.00	30	0.267
302	A	10	8	1.00	30	0.267
303	A	10	8	1.00	30	0.267
304	A	8	7	1.00	16	0.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	5	4	1.00	16	0.250
306	A	8	7	1.00	16	0.438
307	A	6	6	1.00	14	0.429
308	A	6	5	1.00	16	0.312
309	A	6	5	1.00	16	0.312
310	A	3	2	1.00	16	0.125
311	A	6	6	1.00	14	0.429
312	A	6	6	1.00	16	0.375
313	A	2	1	1.00	21	0.048
314	A	2	1	1.00	19	0.053
315	A	2	1	1.00	18	0.056
316	A	2	1	1.00	21	0.048
317	A	2	1	1.00	21	0.048
318	A	2	1	1.00	21	0.048
319	A	3	2	1.00	23	0.087
320	A	3	2	1.00	21	0.095
321	A	3	2	1.00	20	0.100
322	A	2	1	1.00	23	0.043
323	A	2	1	1.00	23	0.043
324	A	2	1	1.00	23	0.043
325	A	3	2	1.00	23	0.087
326	A	3	2	1.00	21	0.095
327	A	3	2	1.00	20	0.100
328	A	2	1	1.00	23	0.043
329	A	2	1	1.00	23	0.043
330	A	2	1	1.00	23	0.043
331	A	3	2	1.00	23	0.087
332	A	3	2	1.00	21	0.095
333	A	3	2	1.00	20	0.100
334	A	2	1	1.00	23	0.043
335	A	2	1	1.00	23	0.043
336	A	2	1	1.00	23	0.043
337	A	10	9	1.00	23	0.391
338	A	10	9	1.00	23	0.391
339	A	10	9	1.00	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	8	8	1.00	20	0.400
341	A	10	9	1.00	23	0.391
342	A	10	9	1.00	23	0.391
343	A	10	9	1.00	23	0.391
344	A	7	7	1.00	23	0.304
345	A	7	7	1.00	21	0.333
346	A	7	7	1.00	20	0.350
347	A	11	10	1.00	23	0.435
348	A	11	10	1.00	23	0.435
349	A	11	10	1.00	23	0.435
350	A	11	10	1.00	23	0.435
351	A	8	8	1.00	23	0.348
352	A	8	8	1.00	21	0.381
353	A	8	8	1.00	20	0.400
354	A	12	10	1.00	23	0.435
355	A	12	10	1.00	23	0.435
356	A	12	10	1.00	23	0.435
357	A	12	10	1.00	23	0.435
358	A	9	8	1.00	23	0.348
359	A	9	9	1.00	21	0.429
360	A	9	8	1.00	20	0.400
361	A	13	10	1.00	23	0.435
362	A	13	10	1.00	23	0.435
363	A	13	10	1.00	23	0.435
364	A	13	10	1.00	23	0.435
365	A	5	5	1.00	20	0.250
366	A	4	4	1.00	18	0.222
367	A	5	5	1.00	20	0.250
368	A	4	4	1.00	18	0.222
369	A	4	4	1.00	27	0.148
370	A	4	4	1.00	29	0.138
371	A	4	4	1.00	28	0.143
372	A	4	4	1.00	28	0.143
373	A	2	1	1.00	36	0.028
374	A	2	1	1.00	36	0.028

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	2	1	1.00	36	0.028
376	A	2	1	1.00	34	0.029
377	A	2	1	1.00	33	0.030
378	A	2	1	1.00	36	0.028
379	A	2	1	1.00	36	0.028
380	A	2	1	1.00	36	0.028
381	A	2	1	1.00	36	0.028
382	A	2	1	1.00	36	0.028
383	A	2	1	1.00	38	0.026
384	A	2	1	1.00	38	0.026
385	A	3	2	1.00	38	0.053
386	A	3	2	1.00	36	0.056
387	A	3	2	1.00	35	0.057
388	A	3	2	1.00	38	0.053
389	A	3	2	1.00	38	0.053
390	A	3	2	1.00	38	0.053
391	A	2	1	1.00	38	0.026
392	A	2	1	1.00	38	0.026
393	A	2	1	1.00	38	0.026
394	A	2	1	1.00	38	0.026
395	A	3	2	1.00	38	0.053
396	A	3	2	1.00	36	0.056
397	A	3	2	1.00	35	0.057
398	A	3	2	1.00	38	0.053
399	A	3	2	1.00	38	0.053
400	A	3	2	1.00	38	0.053
401	A	2	1	1.00	38	0.026
402	A	2	1	1.00	38	0.026
403	A	13	10	1.00	38	0.263
404	A	13	10	1.00	38	0.263
405	A	13	10	1.00	38	0.263
406	A	13	10	1.00	36	0.278
407	A	10	9	0.99	35	0.257
408	A	10	9	0.99	38	0.237
409	A	10	9	1.00	38	0.237

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	10	9	0.99	38	0.237
411	A	10	9	0.99	38	0.237
412	A	11	10	1.00	38	0.263
413	A	11	10	1.00	38	0.263
414	A	11	10	0.99	38	0.263
415	A	11	10	1.00	36	0.278
416	A	9	9	1.00	35	0.257
417	A	11	10	0.99	38	0.263
418	A	11	10	1.00	38	0.263
419	A	11	10	0.99	38	0.263
420	A	11	10	0.99	38	0.263
421	A	12	11	1.00	38	0.290
422	A	10	10	1.00	38	0.263
423	A	8	8	1.00	38	0.210
424	A	8	8	1.00	36	0.222
425	A	8	8	1.00	35	0.229
426	A	12	10	0.99	38	0.263
427	A	12	10	1.00	38	0.263
428	A	12	10	0.99	38	0.263
429	A	12	10	0.99	38	0.263
430	A	10	7	1.00	25	0.280
431	A	8	7	1.00	25	0.280
432	A	6	6	1.00	23	0.261
433	A	5	5	1.00	22	0.227
434	A	7	7	1.00	25	0.280
435	A	8	8	1.00	25	0.320
436	A	9	8	1.00	25	0.320
437	A	8	7	1.00	25	0.280
438	A	7	7	1.00	25	0.280
439	A	6	6	1.00	25	0.240
440	A	4	4	1.00	25	0.160
441	A	6	6	1.00	23	0.261
442	A	4	4	1.00	22	0.182
443	A	10	10	1.00	25	0.400
444	A	11	11	1.00	25	0.440

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
445	A	13	9	1.00	35	0.257
446	A	11	9	1.00	35	0.257
447	A	9	8	1.00	33	0.242
448	A	8	7	1.00	32	0.219
449	A	11	11	1.00	35	0.314
450	A	11	11	1.00	35	0.314
451	A	10	9	1.00	35	0.257
452	A	11	9	1.00	35	0.257
453	A	12	9	1.00	35	0.257
454	A	10	10	1.00	35	0.286
455	A	11	10	1.00	35	0.286
456	A	12	10	1.00	35	0.286
457	A	13	10	1.00	35	0.286
458	A	14	9	1.00	35	0.257
459	A	12	9	1.00	35	0.257
460	A	10	8	1.00	33	0.242
461	A	9	7	1.00	32	0.219
462	A	12	11	1.00	35	0.314
463	A	12	11	1.00	35	0.314
464	A	11	9	1.00	35	0.257
465	A	12	9	1.00	35	0.257
466	A	13	9	1.00	35	0.257
467	A	11	11	1.00	35	0.314
468	A	12	11	1.00	35	0.314
469	A	13	11	1.00	35	0.314
470	A	11	10	1.00	35	0.286
471	A	12	10	1.00	35	0.286
472	A	13	10	1.00	35	0.286
473	A	14	10	1.00	35	0.286
474	A	8	7	1.18	20	0.350
475	A	7	4	1.17	21	0.190
476	A	7	4	1.17	23	0.174
477	A	2	1	1.00	23	0.043
478	A	2	1	1.00	26	0.038
479	A	3	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
480	A	3	2	1.00	28	0.071
481	A	3	2	1.00	25	0.080
482	A	3	2	1.00	28	0.071
483	A	3	2	1.00	25	0.080
484	A	3	2	1.00	28	0.071
485	A	9	7	1.00	26	0.269
486	A	12	9	1.00	29	0.310
487	A	15	11	1.00	25	0.440
488	A	18	13	1.00	28	0.464
489	A	14	10	1.00	25	0.400
490	A	14	10	1.00	28	0.357
491	A	15	11	1.00	25	0.440
492	A	15	11	1.00	28	0.393
493	A	16	11	1.00	25	0.440
494	A	16	11	1.00	28	0.393
495	A	14	12	1.00	30	0.400
496	A	13	11	1.00	30	0.367
497	A	12	11	1.00	30	0.367
498	A	12	11	1.00	28	0.393
499	A	11	10	1.00	27	0.370
500	A	14	13	1.00	30	0.433
501	A	14	13	1.00	30	0.433
502	A	14	13	1.00	30	0.433
503	A	15	14	1.00	30	0.467
504	A	13	13	1.00	30	0.433
505	A	14	14	1.00	30	0.467
506	A	12	12	1.00	30	0.400
507	A	13	12	1.00	30	0.400
508	A	14	13	1.00	30	0.433
509	A	15	13	1.00	30	0.433
510	A	16	12	1.00	30	0.400
511	A	15	11	1.00	30	0.367
512	A	14	11	1.00	30	0.367
513	A	14	11	1.00	28	0.393
514	A	13	10	1.00	27	0.370

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
515	A	16	13	1.00	30	0.433
516	A	16	14	1.00	30	0.467
517	A	16	15	1.00	30	0.500
518	A	16	14	1.00	30	0.467
519	A	15	15	1.00	30	0.500
520	A	15	15	1.00	30	0.500
521	A	15	15	1.00	30	0.500
522	A	16	16	1.00	30	0.533
523	A	14	13	1.00	30	0.433
524	A	15	14	1.00	30	0.467
525	A	13	12	1.00	30	0.400
526	A	14	12	1.00	30	0.400
527	A	15	13	1.00	30	0.433
528	A	16	13	1.00	30	0.433
529	A	12	10	1.00	30	0.333
530	A	11	9	1.00	30	0.300
531	A	10	9	1.00	30	0.300
532	A	10	9	1.00	28	0.321
533	A	9	8	1.00	27	0.296
534	A	12	11	1.00	30	0.367
535	A	13	12	1.00	30	0.400
536	A	11	10	1.00	30	0.333
537	A	12	10	1.00	30	0.333
538	A	13	11	1.00	30	0.367
539	A	14	11	1.00	30	0.367
540	A	12	11	1.00	30	0.367
541	A	11	10	1.00	30	0.333
542	A	10	9	1.00	30	0.300
543	A	9	8	0.98	30	0.267
544	A	10	9	1.00	30	0.300
545	A	7	6	1.00	28	0.214
546	A	4	4	1.00	27	0.148
547	A	11	10	1.00	30	0.333
548	A	13	12	1.00	30	0.400
549	A	15	12	1.00	30	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
550	A	17	13	1.00	30	0.433
551	A	14	4	1.00	30	0.133
552	A	12	8	1.19	25	0.320
553	A	13	7	1.00	28	0.250
554	A	2	2	1.00	22	0.091
555	A	2	2	1.00	35	0.057
556	A	2	2	1.00	35	0.057
557	A	2	2	1.00	22	0.091
558	A	3	3	1.00	25	0.120
559	A	6	5	1.00	15	0.333
560	A	6	5	1.00	15	0.333
561	A	7	6	1.00	20	0.300
562	A	7	6	1.00	20	0.300
563	A	7	7	1.00	17	0.412
564	A	7	6	1.00	25	0.240
565	A	11	6	1.00	35	0.171
566	A	11	6	1.00	35	0.171
567	A	8	5	1.00	22	0.227
568	A	11	7	1.00	25	0.280
569	A	17	7	1.00	15	0.467
570	A	17	7	1.00	15	0.467
571	A	14	7	1.00	20	0.350
572	A	14	7	1.00	20	0.350
573	A	15	9	1.00	17	0.529
574	A	14	7	1.00	25	0.280
575	A	13	7	1.00	18	0.389
576	A	7	4	1.00	36	0.111
577	A	4	3	1.00	19	0.158
578	A	4	3	1.00	19	0.158
579	A	4	2	1.00	17	0.118
580	A	1	0	1.00	9	0.000
581	A	3	3	1.00	19	0.158
582	A	3	3	1.00	19	0.158
583	A	3	3	1.00	19	0.158
584	A	13	4	1.00	38	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
585	A	2	2	1.00	58	0.034
586	A	10	3	1.00	30	0.100
587	A	13	4	1.00	36	0.111
588	A	4	4	1.00	35	0.114
589	A	1	1	1.00	46	0.022
590	A	10	8	1.00	24	0.333
591	A	1	1	1.00	48	0.021
592	A	1	1	1.00	45	0.022
593	A	1	1	1.00	69	0.014
594	A	1	1	1.00	86	0.012

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx=\frac{2(b^2c-abd+a^2e)\sqrt{a+bx}}{b^3}+\frac{2(bd-2ae)(a+bx)^{3/2}}{3b^3}+\frac{2e(a+bx)^{5/2}}{5b^3}$$

```
[Out] 2/3*(-2*a*e+b*d)*(b*x+a)^(3/2)/b^3+2/5*e*(b*x+a)^(5/2)/b^3+2*(a^2*e-a*b*d+b
^2*c)*(b*x+a)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {712}

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx=\frac{2\sqrt{a+bx}(a^2e-abd+b^2c)}{b^3}+\frac{2(a+bx)^{3/2}(bd-2ae)}{3b^3}+\frac{2e(a+bx)^{5/2}}{5b^3}$$

```
[In] Int[(c + d*x + e*x^2)/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^2*c - a*b*d + a^2*e)*Sqrt[a + b*x])/b^3 + (2*(b*d - 2*a*e)*(a + b*x)^
(3/2))/(3*b^3) + (2*e*(a + b*x)^(5/2))/(5*b^3)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{b^2c - abd + a^2e}{b^2\sqrt{a+bx}} + \frac{(bd-2ae)\sqrt{a+bx}}{b^2} + \frac{e(a+bx)^{3/2}}{b^2} \right) dx$$
$$= \frac{2(b^2c - abd + a^2e)\sqrt{a+bx}}{b^3} + \frac{2(bd-2ae)(a+bx)^{3/2}}{3b^3} + \frac{2e(a+bx)^{5/2}}{5b^3}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}(8a^2e - 2ab(5d+2ex) + b^2(15c + x(5d+3ex)))}{15b^3}$$

```
[In] Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x],x]
[Out] (2*Sqrt[a + b*x]*(8*a^2*e - 2*a*b*(5*d + 2*e*x) + b^2*(15*c + x*(5*d + 3*e*
x))))/(15*b^3)
```

Maple [A] (verified)

Time = 5.73 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{16\left(\frac{\left(3e\,x^2+5dx+15c\right)b^2}{8}-\frac{5a\left(\frac{2ex}{5}+d\right)b}{4}+a^2e\right)\sqrt{bx+a}}{15b^3}$	48
gosper	$\frac{2\sqrt{bx+a}\,\left(3b^2e\,x^2-4abex+5b^2dx+8a^2e-10abd+15b^2c\right)}{15b^3}$	53
trager	$\frac{2\sqrt{bx+a}\,\left(3b^2e\,x^2-4abex+5b^2dx+8a^2e-10abd+15b^2c\right)}{15b^3}$	53
risch	$\frac{2\sqrt{bx+a}\,\left(3b^2e\,x^2-4abex+5b^2dx+8a^2e-10abd+15b^2c\right)}{15b^3}$	53
derivativedivides	$\frac{\frac{2e(bx+a)^{\frac{5}{2}}}{5}-\frac{4ae(bx+a)^{\frac{3}{2}}}{3}+\frac{2bd(bx+a)^{\frac{3}{2}}}{3}+2a^2e\sqrt{bx+a}-2abd\sqrt{bx+a}+2b^2c\sqrt{bx+a}}{b^3}$	75
default	$\frac{\frac{2e(bx+a)^{\frac{5}{2}}}{5}-\frac{4ae(bx+a)^{\frac{3}{2}}}{3}+\frac{2bd(bx+a)^{\frac{3}{2}}}{3}+2a^2e\sqrt{bx+a}-2abd\sqrt{bx+a}+2b^2c\sqrt{bx+a}}{b^3}$	75

```
[In] int((e*x^2+d*x+c)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 16/15*(1/8*(3*e*x^2+5*d*x+15*c)*b^2-5/4*a*(2/5*e*x+d)*b+a^2*e)*(b*x+a)^(1/2
)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx = \frac{2\,(3\,b^2ex^2+15\,b^2c-10\,abd+8\,a^2e+(5\,b^2d-4\,abe)x)\sqrt{bx+a}}{15\,b^3}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*e*x^2 + 15*b^2*c - 10*a*b*d + 8*a^2*e + (5*b^2*d - 4*a*b*e)*x)*
sqrt(b*x + a)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.42

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx = \begin{cases} \frac{2c\sqrt{a+bx}+\frac{2d\left(-a\sqrt{a+bx}+\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}+\frac{2e\left(a^2\sqrt{a+bx}-\frac{2a(a+bx)^{\frac{3}{2}}}{3}+\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}}{b} & \text{for } b\neq 0 \\ \frac{cx+\frac{dx^2}{2}+\frac{ex^3}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

```
[In] integrate((e*x**2+d*x+c)/(b*x+a)**(1/2),x)
[Out] Piecewise(((2*c*sqrt(a + b*x) + 2*d*(-a*sqrt(a + b*x) + (a + b*x)**(3/2)/3)
/b + 2*e*(a**2*sqrt(a + b*x) - 2*a*(a + b*x)**(3/2)/3 + (a + b*x)**(5/2)/5)
/b**2)/b, Ne(b, 0)), ((c*x + d*x**2/2 + e*x**3/3)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx = \frac{2\left(15\sqrt{bx+a}c+\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2}\right)}{15\,b}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/15*(15*sqrt(b*x + a)*c + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b + (3
*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2)/b
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx = \frac{2\left(15\sqrt{bx+a}c+\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}+\frac{\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2}\right)}{15\,b}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(b*x + a)*c + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b + (3
*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+b\,x}\left(3\,e\,(a+b\,x)^2+15\,b^2\,c+15\,a^2\,e-10\,a\,e\,(a+b\,x)+5\,b\,d\,(a+b\,x)-15\,a\,b\,d\right)}{15\,b^3}$$

```
[In] int((c + d*x + e*x^2)/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(1/2)*(3*e*(a + b*x)^2 + 15*b^2*c + 15*a^2*e - 10*a*e*(a + b*x
) + 5*b*d*(a + b*x) - 15*a*b*d))/(15*b^3)
```

3.2 $\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 161

$$\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx = \frac{2(b^2c-abd+a^2e)^2\sqrt{a+bx}}{b^5} + \frac{4(bd-2ae)\,(b^2c-abd+a^2e)\,(a+bx)^{3/2}}{3b^5} - \frac{2(6abde-6a^2e^2-b^2(d^2+2ce))\,(a+bx)^{5/2}}{5b^5} + \frac{4e(bd-2ae)(a+bx)^{7/2}}{7b^5} + \frac{2e^2(a+bx)^{9/2}}{9b^5}$$

```
[Out] 4/3*(-2*a*e+b*d)*(a^2*e-a*b*d+b^2*c)*(b*x+a)^(3/2)/b^5-2/5*(6*a*b*d*e-6*a^2
*e^2-b^2*(2*c*e+d^2))*(b*x+a)^(5/2)/b^5+4/7*e*(-2*a*e+b*d)*(b*x+a)^(7/2)/b^
5+2/9*e^2*(b*x+a)^(9/2)/b^5+2*(a^2*e-a*b*d+b^2*c)^2*(b*x+a)^(1/2)/b^5
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {712}

$$\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx = -\frac{2(a+bx)^{5/2}\,(-6a^2e^2+6abde-(b^2(2ce+d^2)))}{5b^5} + \frac{4(a+bx)^{3/2}(bd-2ae)\,(a^2e-abd+b^2c)}{3b^5} + \frac{2\sqrt{a+bx}(a^2e-abd+b^2c)^2}{b^5} + \frac{4e(a+bx)^{7/2}(bd-2ae)}{7b^5} + \frac{2e^2(a+bx)^{9/2}}{9b^5}$$

```
[In] Int[(c + d*x + e*x^2)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^2*c - a*b*d + a^2*e)^2*Sqrt[a + b*x])/b^5 + (4*(b*d - 2*a*e)*(b^2*c -
 a*b*d + a^2*e)*(a + b*x)^(3/2))/(3*b^5) - (2*(6*a*b*d*e - 6*a^2*e^2 - b^2*
(d^2 + 2*c*e))*(a + b*x)^(5/2))/(5*b^5) + (4*e*(b*d - 2*a*e)*(a + b*x)^(7/2
))/(7*b^5) + (2*e^2*(a + b*x)^(9/2))/(9*b^5)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{(b^2c-abd+a^2e)^2}{b^4\sqrt{a+bx}}+\frac{2(bd-2ae)\left(b^2c-abd+a^2e\right)\sqrt{a+bx}}{b^4}+\frac{\left(-6abde+6a^2e^2+b^2(d^2+2ce)\right)(a+bx)^{3/2}}{b^4}+\frac{2e(bd-2ae)(a+bx)^{5/2}}{b^4}+\frac{e^2(a+bx)^{7/2}}{b^4}\right) dx\\ &= \frac{2(b^2c-abd+a^2e)^2\sqrt{a+bx}}{b^5}+\frac{4(bd-2ae)\left(b^2c-abd+a^2e\right)(a+bx)^{3/2}}{3b^5}-\frac{2\left(6abde-6a^2e^2-b^2(d^2+2ce)\right)(a+bx)^{5/2}}{5b^5}+\frac{4e(bd-2ae)(a+bx)^{7/2}}{7b^5}+\frac{2e^2(a+bx)^{9/2}}{9b^5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}\left(128a^4e^2-32a^3be(9d+2ex)+24a^2b^2(7d^2+6dex+2e(7c+ex^2))-4ab^3(21c(5d+2ex)+x(2\right.}{315b^5}$$

```
[In] Integrate[(c + d*x + e*x^2)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(128*a^4*e^2 - 32*a^3*b*e*(9*d + 2*e*x) + 24*a^2*b^2*(7*d^
2 + 6*d*e*x + 2*e*(7*c + e*x^2)) - 4*a*b^3*(21*c*(5*d + 2*e*x) + x*(21*d^2
+ 27*d*e*x + 10*e^2*x^2)) + b^4*(315*c^2 + 42*c*x*(5*d + 3*e*x) + x^2*(63*d
^2 + 90*d*e*x + 35*e^2*x^2))))/(315*b^5)
```

Maple [A] (verified)

Time = 5.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{2e^2(bx+a)^{\frac{9}{2}}}{9}+\frac{4(-2ae+bd)e(bx+a)^{\frac{7}{2}}}{7}+\frac{2\left(2\left(a^2e-abd+b^2c\right)e+(-2ae+bd)^2\right)(bx+a)^{\frac{5}{2}}}{5}+\frac{4\left(a^2e-abd+b^2c\right)(-2ae+bd)(bx+a)^{\frac{3}{2}}}{3}+2(a^2}{b^5}$
default	$\frac{\frac{2e^2(bx+a)^{\frac{9}{2}}}{9}+\frac{4(-2ae+bd)e(bx+a)^{\frac{7}{2}}}{7}+\frac{2\left(2\left(a^2e-abd+b^2c\right)e+(-2ae+bd)^2\right)(bx+a)^{\frac{5}{2}}}{5}+\frac{4\left(a^2e-abd+b^2c\right)(-2ae+bd)(bx+a)^{\frac{3}{2}}}{3}+2(a^2}{b^5}$
pseudoelliptic	$\frac{256\sqrt{bx+a}\left(\frac{7\left(\frac{5e^2x^4}{2}+9\left(\frac{5dx}{7}+c\right)x^2e+\frac{9d^2x^2}{2}+15cdx+\frac{45c^2}{2}\right)b^4}{64}-\frac{105\left(\frac{2e^2x^3}{21}+\frac{2\left(\frac{9dx}{14}+c\right)xe}{5}+d\left(\frac{dx}{5}+c\right)\right)a\,b^3}{32}+\frac{21a^2\left(\frac{x^2e^2}{7}+(\right.}{}\right.}{315b^5}$
gosper	$\frac{2\sqrt{bx+a}\,(35e^2x^4b^4-40a\,b^3e^2x^3+90b^4de\,x^3+48a^2b^2e^2x^2-108a\,b^3de\,x^2+126b^4ce\,x^2+63b^4d^2x^2-64a^3b\,e^2x+144a^2b^2dex-}{315b^5}$
trager	$\frac{2\sqrt{bx+a}\,(35e^2x^4b^4-40a\,b^3e^2x^3+90b^4de\,x^3+48a^2b^2e^2x^2-108a\,b^3de\,x^2+126b^4ce\,x^2+63b^4d^2x^2-64a^3b\,e^2x+144a^2b^2dex-}{315b^5}$
risch	$\frac{2\sqrt{bx+a}\,(35e^2x^4b^4-40a\,b^3e^2x^3+90b^4de\,x^3+48a^2b^2e^2x^2-108a\,b^3de\,x^2+126b^4ce\,x^2+63b^4d^2x^2-64a^3b\,e^2x+144a^2b^2dex-}{315b^5}$

```
[In] int((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^5*(1/9*e^2*(b*x+a)^(9/2)+2/7*(-2*a*e+b*d)*e*(b*x+a)^(7/2)+1/5*(2*(a^2*e
-a*b*d+b^2*c)*e+(-2*a*e+b*d)^2)*(b*x+a)^(5/2)+2/3*(a^2*e-a*b*d+b^2*c)*(-2*a
*e+b*d)*(b*x+a)^(3/2)+(a^2*e-a*b*d+b^2*c)^2*(b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx = \frac{2\,(35\,b^4e^2x^4+315\,b^4c^2-420\,ab^3cd+168\,a^2b^2d^2+128\,a^4e^2+10\,(9\,b^4de-4\,ab^3e^2)x^3+3\,(21\,b^4d^2+16\,a^2b^2}{}$$

```
[In] integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*e^2*x^4 + 315*b^4*c^2 - 420*a*b^3*c*d + 168*a^2*b^2*d^2 + 128
*a^4*e^2 + 10*(9*b^4*d*e - 4*a*b^3*e^2)*x^3 + 3*(21*b^4*d^2 + 16*a^2*b^2*e^
2 + 6*(7*b^4*c - 6*a*b^3*d)*e)*x^2 + 48*(7*a^2*b^2*c - 6*a^3*b*d)*e + 2*(10
5*b^4*c*d - 42*a*b^3*d^2 - 32*a^3*b*e^2 - 12*(7*a*b^3*c - 6*a^2*b^2*d)*e)*x
)*sqrt(b*x + a)/b^5
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.71

$$\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{e^2(a+bx)^{\frac{9}{2}}}{9b^4}+\frac{(a+bx)^{\frac{7}{2}}(-4ae^2+2bde)}{7b^4}+\frac{(a+bx)^{\frac{5}{2}}\cdot(6a^2e^2-6abde+2b^2ce+b^2d^2)}{5b^4}+\frac{(a+bx)^{\frac{3}{2}}(-4a^3e^2+6a^2bde-4ab^2ce-2ab^2d^2+2b^3cd)}{3b^4}+\frac{\sqrt{a+bx}\,(a^4e^2-\cdots}{\cdots}\right.}{b} \\ \frac{c^2x+cdx^2+\frac{dex^4}{2}+\frac{e^2x^5}{5}+\frac{x^3\cdot(2ce+d^2)}{3}}{\sqrt{a}} \end{cases}$$

```
[In] integrate((e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((2*(e**2*(a + b*x)**(9/2)/(9*b**4) + (a + b*x)**(7/2)*(-4*a*e**2
+ 2*b*d*e)/(7*b**4) + (a + b*x)**(5/2)*(6*a**2*e**2 - 6*a*b*d*e + 2*b**2*c*
e + b**2*d**2)/(5*b**4) + (a + b*x)**(3/2)*(-4*a**3*e**2 + 6*a**2*b*d*e - 4
*a*b**2*c*e - 2*a*b**2*d**2 + 2*b**3*c*d)/(3*b**4) + sqrt(a + b*x)*(a**4*e*
*2 - 2*a**3*b*d*e + 2*a**2*b**2*c*e + a**2*b**2*d**2 - 2*a*b**3*c*d + b**4*
c**2)/b**4)/b, Ne(b, 0)), ((c**2*x + c*d*x**2 + d*e*x**4/2 + e**2*x**5/5 +
x**3*(2*c*e + d**2)/3)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.47

$$\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx$$
$$= \frac{2\left(315\sqrt{bx+a}c^2+42c\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}+\frac{\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2}\right)+\frac{21\left(3(bx+a)^{\frac{5}{2}}-10(bx+a\cdots\right.}{b^2}\cdots\right.}{\cdots}$$

```
[In] integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(b*x + a)*c^2 + 42*c*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a
)*d/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e
/b^2) + 21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2
)*d^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/
2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a
)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*
x + a)*a^4)*e^2/b^4)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.47

$$\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx = \frac{2\left(315\sqrt{bx+a}c^2 + \frac{210\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)cd}{b} + \frac{21\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)d^2}{b^2} + \frac{42\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+\cdots\right.}{b^2}\right.}{}$$

```
[In] integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(b*x + a)*c^2 + 210*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*c*
d/b + 21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*
d^2/b^2 + 42*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a
^2)*c*e/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(
3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x +
 a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(
b*x + a)*a^4)*e^2/b^4)/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx = \frac{2\,e^2\,(a+b\,x)^{9/2}}{9\,b^5} + \frac{(a+b\,x)^{5/2}\,(12\,a^2\,e^2-12\,a\,b\,d\,e+2\,b^2\,d^2+4\,c\,b^2\,e)}{5\,b^5} + \frac{2\,\sqrt{a+b\,x}\,(e\,a^2-d\,a\,b+c\,b^2)^2}{b^5} - \frac{(8\,a\,e^2-4\,b\,d\,e)\,(a+b\,x)^{7/2}}{7\,b^5} - \frac{4\,(2\,a\,e-b\,d)\,(a+b\,x)^{3/2}\,(e\,a^2-d\,a\,b+c\,b^2)}{3\,b^5}$$

```
[In] int((c + d*x + e*x^2)^2/(a + b*x)^(1/2),x)
```

```
[Out] (2*e^2*(a + b*x)^(9/2))/(9*b^5) + ((a + b*x)^(5/2)*(12*a^2*e^2 + 2*b^2*d^2
+ 4*b^2*c*e - 12*a*b*d*e))/(5*b^5) + (2*(a + b*x)^(1/2)*(b^2*c + a^2*e - a*
b*d)^2)/b^5 - ((8*a*e^2 - 4*b*d*e)*(a + b*x)^(7/2))/(7*b^5) - (4*(2*a*e - b
*d)*(a + b*x)^(3/2)*(b^2*c + a^2*e - a*b*d))/(3*b^5)
```

3.3 $\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 274

$$
\begin{aligned}
\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx &= \frac{2(b^2c-abd+a^2e)^3\sqrt{a+bx}}{b^7} \\
&+ \frac{2(bd-2ae)\left(b^2c-abd+a^2e\right)^2(a+bx)^{3/2}}{b^7} \\
&- \frac{6(b^2c-abd+a^2e)\left(5abde-5a^2e^2-b^2(d^2+ce)\right)(a+bx)^{5/2}}{5b^7} \\
&- \frac{2(bd-2ae)\left(10abde-10a^2e^2-b^2(d^2+6ce)\right)(a+bx)^{7/2}}{7b^7} \\
&- \frac{2e(5abde-5a^2e^2-b^2(d^2+ce))(a+bx)^{9/2}}{3b^7} \\
&+ \frac{6e^2(bd-2ae)(a+bx)^{11/2}}{11b^7} + \frac{2e^3(a+bx)^{13/2}}{13b^7}
\end{aligned}
$$

```
[Out] 2*(-2*a*e+b*d)*(a^2*e-a*b*d+b^2*c)^2*(b*x+a)^(3/2)/b^7-6/5*(a^2*e-a*b*d+b^2
*c)*(5*a*b*d*e-5*a^2*e^2-b^2*(c*e+d^2))*(b*x+a)^(5/2)/b^7-2/7*(-2*a*e+b*d)*
(10*a*b*d*e-10*a^2*e^2-b^2*(6*c*e+d^2))*(b*x+a)^(7/2)/b^7-2/3*e*(5*a*b*d*e-
5*a^2*e^2-b^2*(c*e+d^2))*(b*x+a)^(9/2)/b^7+6/11*e^2*(-2*a*e+b*d)*(b*x+a)^(1
1/2)/b^7+2/13*e^3*(b*x+a)^(13/2)/b^7+2*(a^2*e-a*b*d+b^2*c)^3*(b*x+a)^(1/2)/
b^7
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {712}

$$\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx = -\frac{2e(a+bx)^{9/2}\left(-5a^2e^2+5abde-\left(b^2(ce+d^2)\right)\right)}{3b^7} - \frac{2(a+bx)^{7/2}(bd-2ae)\left(-10a^2e^2+10abde-\left(b^2(6ce+d^2)\right)\right)}{7b^7} - \frac{6(a+bx)^{5/2}\left(a^2e-abd+b^2c\right)\left(-5a^2e^2+5abde-\left(b^2(ce+d^2)\right)\right)}{5b^7} + \frac{2(a+bx)^{3/2}(bd-2ae)\left(a^2e-abd+b^2c\right)^2}{b^7} + \frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)^3}{b^7} + \frac{6e^2(a+bx)^{11/2}(bd-2ae)}{11b^7} + \frac{2e^3(a+bx)^{13/2}}{13b^7}$$

```
[In] Int[(c + d*x + e*x^2)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^2*c - a*b*d + a^2*e)^3*Sqrt[a + b*x])/b^7 + (2*(b*d - 2*a*e)*(b^2*c -
 a*b*d + a^2*e)^2*(a + b*x)^(3/2))/b^7 - (6*(b^2*c - a*b*d + a^2*e)*(5*a*b*
d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^(5/2))/(5*b^7) - (2*(b*d - 2*a
*e)*(10*a*b*d*e - 10*a^2*e^2 - b^2*(d^2 + 6*c*e))*(a + b*x)^(7/2))/(7*b^7)
- (2*e*(5*a*b*d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^(9/2))/(3*b^7) +
 (6*e^2*(b*d - 2*a*e)*(a + b*x)^(11/2))/(11*b^7) + (2*e^3*(a + b*x)^(13/2))
/(13*b^7)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{(b^2c-abd+a^2e)^3}{b^6\sqrt{a+bx}} + \frac{3(bd-2ae)\,(b^2c-abd+a^2e)^2\,\sqrt{a+bx}}{b^6} + \frac{3(b^2c-abd+a^2e)\,(b^2d^2+b^2ce-5abde+5a^2e^2)\,(a+bx)^{3/2}}{b^6} + \frac{(bd-2ae)\,(-10abde+10a^2e^2+b^2(d^2+6ce))\,(a+bx)^{5/2}}{b^6} + \frac{3e(-5abde+5a^2e^2+b^2(d^2+ce))\,(a+bx)^{7/2}}{b^6} + \frac{3e^2(bd-2ae)(a+bx)^{9/2}}{b^6} + \frac{e^3(a+bx)^{11/2}}{b^6} \right) dx$$

$$= \frac{2(b^2c-abd+a^2e)^3\,\sqrt{a+bx}}{b^7} + \frac{2(bd-2ae)\,(b^2c-abd+a^2e)^2\,(a+bx)^{3/2}}{b^7} - \frac{6(b^2c-abd+a^2e)\,(5abde-5a^2e^2-b^2(d^2+ce))\,(a+bx)^{5/2}}{5b^7} - \frac{2(bd-2ae)\,(10abde-10a^2e^2-b^2(d^2+6ce))\,(a+bx)^{7/2}}{7b^7} - \frac{2e(5abde-5a^2e^2-b^2(d^2+ce))\,(a+bx)^{9/2}}{3b^7} + \frac{6e^2(bd-2ae)(a+bx)^{11/2}}{11b^7} + \frac{2e^3(a+bx)^{13/2}}{13b^7}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx$$
$$= \frac{2\sqrt{a+bx}(5120a^6e^3-1280a^5be^2(13d+2ex)+128a^4b^2e(143d^2+65dex+e(143c+15ex^2))-16a^3b^3(429\ldots}{}$$

```
[In] Integrate[(c + d*x + e*x^2)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(5120*a^6*e^3 - 1280*a^5*b*e^2*(13*d + 2*e*x) + 128*a^4*b^
2*e*(143*d^2 + 65*d*e*x + e*(143*c + 15*e*x^2)) - 16*a^3*b^3*(429*d^3 + 572
*d^2*e*x + 78*d*e*(33*c + 5*e*x^2) + 4*e^2*x*(143*c + 25*e*x^2)) + 8*a^2*b^
4*(3003*c^2*e + 429*c*(7*d^2 + 6*d*e*x + 2*e^2*x^2) + x*(429*d^3 + 858*d^2*
e*x + 650*d*e^2*x^2 + 175*e^3*x^3)) + b^6*(15015*c^3 + 3003*c^2*x*(5*d + 3*
e*x) + 143*c*x^2*(63*d^2 + 90*d*e*x + 35*e^2*x^2) + 5*x^3*(429*d^3 + 1001*d
^2*e*x + 819*d*e^2*x^2 + 231*e^3*x^3)) - 2*a*b^5*(3003*c^2*(5*d + 2*e*x) +
286*c*x*(21*d^2 + 27*d*e*x + 10*e^2*x^2) + x^2*(1287*d^3 + 2860*d^2*e*x + 2
275*d*e^2*x^2 + 630*e^3*x^3))))/(15015*b^7)
```

Maple [A] (verified)

Time = 5.25 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.19

method	result
pseudoelliptic	$\frac{2048\left(\left(\frac{231e^3x^6}{1024}+\frac{1001\left(\frac{9dx}{11}+c\right)x^4e^2}{1024}+\frac{9009x^2\left(\frac{5}{9}d^2x^2+\frac{10}{7}cdx+c^2\right)e}{5120}+\frac{3003c^3}{1024}+\frac{429d^3x^3}{1024}+\frac{3003c^2dx}{1024}+\frac{9009c\,d^2x^2}{5120}\right)b^6-\frac{3003\left(6\right.}{}\right.}{}$
derivativedivides	$\frac{2e^3(bx+a)^{\frac{13}{2}}}{13}+\frac{6(-2ae+bd)e^2(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(\left(a^2e-abd+b^2c\right)e^2+2(-2ae+bd)^2e+e\left(2\left(a^2e-abd+b^2c\right)e+(-2ae+bd)^2\right)\right)(bx+a)^{\frac{9}{2}}}{9}+$
default	$\frac{2e^3(bx+a)^{\frac{13}{2}}}{13}+\frac{6(-2ae+bd)e^2(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(\left(a^2e-abd+b^2c\right)e^2+2(-2ae+bd)^2e+e\left(2\left(a^2e-abd+b^2c\right)e+(-2ae+bd)^2\right)\right)(bx+a)^{\frac{9}{2}}}{9}+$
gosper	$2\sqrt{bx+a}\,(1155e^3x^6b^6-1260a\,b^5e^3x^5+4095b^6d\,e^2x^5+1400a^2b^4e^3x^4-4550a\,b^5d\,e^2x^4+5005b^6c\,e^2x^4+5005b^6d^2e\,x^4-1600$
trager	$2\sqrt{bx+a}\,(1155e^3x^6b^6-1260a\,b^5e^3x^5+4095b^6d\,e^2x^5+1400a^2b^4e^3x^4-4550a\,b^5d\,e^2x^4+5005b^6c\,e^2x^4+5005b^6d^2e\,x^4-1600$
risch	$2\sqrt{bx+a}\,(1155e^3x^6b^6-1260a\,b^5e^3x^5+4095b^6d\,e^2x^5+1400a^2b^4e^3x^4-4550a\,b^5d\,e^2x^4+5005b^6c\,e^2x^4+5005b^6d^2e\,x^4-1600$

```
[In] int((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2048/3003*((231/1024*e^3*x^6+1001/1024*(9/11*d*x+c)*x^4*e^2+9009/5120*x^2*(
5/9*d^2*x^2+10/7*c*d*x+c^2)*e+3003/1024*c^3+429/1024*d^3*x^3+3003/1024*c^2*
d*x+9009/5120*c*d^2*x^2)*b^6-3003/512*(6/143*e^3*x^5+(5/33*d*x^4+4/21*c*x^3
)*e^2+(4/21*d^2*x^3+18/35*c*d*x^2+2/5*c^2*x)*e+d*(3/35*d^2*x^2+2/5*c*d*x+c^
2))*a*b^5+3003/640*(25/429*e^3*x^4+2/7*x^2*(25/33*d*x+c)*e^2+(2/7*d^2*x^2+6
/7*c*d*x+c^2)*e+d^2*(1/7*d*x+c))*a^2*b^4-1287/160*(50/1287*e^3*x^3+(5/33*d*
x^2+2/9*c*x)*e^2+d*(2/9*d*x+c)*e+1/6*d^3)*a^3*b^3+143/40*e*a^4*(15/143*x^2*
e^2+(5/11*d*x+c)*e+d^2)*b^2-13/4*e^2*(2/13*e*x+d)*a^5*b+a^6*e^3)*(b*x+a)^(1
/2)/b^7
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx$$
$$=\frac{2\left(1155\,b^6e^3x^6+15015\,b^6c^3-30030\,ab^5c^2d+24024\,a^2b^4cd^2-6864\,a^3b^3d^3+5120\,a^6e^3+315\left(13\,b^6de^2-4\right.\right.}{}$$

```
[In] integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^6*e^3*x^6 + 15015*b^6*c^3 - 30030*a*b^5*c^2*d + 24024*a^2*b
^4*c*d^2 - 6864*a^3*b^3*d^3 + 5120*a^6*e^3 + 315*(13*b^6*d*e^2 - 4*a*b^5*e^
```

```
3)*x^5 + 35*(143*b^6*d^2*e + 40*a^2*b^4*e^3 + 13*(11*b^6*c - 10*a*b^5*d)*e^
2)*x^4 + 5*(429*b^6*d^3 - 320*a^3*b^3*e^3 - 104*(11*a*b^5*c - 10*a^2*b^4*d)
*e^2 + 286*(9*b^6*c*d - 4*a*b^5*d^2)*e)*x^3 + 1664*(11*a^4*b^2*c - 10*a^5*b
*d)*e^2 + 3*(3003*b^6*c*d^2 - 858*a*b^5*d^3 + 640*a^4*b^2*e^3 + 208*(11*a^2
*b^4*c - 10*a^3*b^3*d)*e^2 + 143*(21*b^6*c^2 - 36*a*b^5*c*d + 16*a^2*b^4*d^
2)*e)*x^2 + 1144*(21*a^2*b^4*c^2 - 36*a^3*b^3*c*d + 16*a^4*b^2*d^2)*e + (15
015*b^6*c^2*d - 12012*a*b^5*c*d^2 + 3432*a^2*b^4*d^3 - 2560*a^5*b*e^3 - 832
*(11*a^3*b^3*c - 10*a^4*b^2*d)*e^2 - 572*(21*a*b^5*c^2 - 36*a^2*b^4*c*d + 1
6*a^3*b^3*d^2)*e)*x)*sqrt(b*x + a)/b^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(277) = 554$.

Time = 1.10 (sec) , antiderivative size = 631, normalized size of antiderivative = 2.30

$$\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx = \begin{cases} \frac{2\left(\frac{e^3(a+bx)^{\frac{13}{2}}}{13b^6}+\frac{(a+bx)^{\frac{11}{2}}\left(-6ae^3+3bde^2\right)}{11b^6}+\frac{(a+bx)^{\frac{9}{2}}\cdot\left(15a^2e^3-15abde^2+3b^2ce^2+3b^2d^2e\right)}{9b^6}+\frac{(a+bx)^{\frac{7}{2}}\left(-20a^3e^3+30a^2bde^2-12ab^2ce^2-12ab^2d^2e+6b^3cc\right.}{7b^6} \\ \frac{c^3x+\frac{3c^2dx^2}{2}+\frac{de^2x^6}{2}+\frac{e^3x^7}{7}+\frac{x^5\cdot\left(3ce^2+3d^2e\right)}{5}+\frac{x^4\cdot\left(6cde+d^3\right)}{4}+\frac{x^3\cdot\left(3c^2e+3cd^2\right)}{3}}{\sqrt{a}} \end{cases}$$

```
[In] integrate((e*x**2+d*x+c)**3/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((2*(e**3*(a + b*x)**(13/2)/(13*b**6) + (a + b*x)**(11/2)*(-6*a*e*
*3 + 3*b*d*e**2)/(11*b**6) + (a + b*x)**(9/2)*(15*a**2*e**3 - 15*a*b*d*e**2
 + 3*b**2*c*e**2 + 3*b**2*d**2*e)/(9*b**6) + (a + b*x)**(7/2)*(-20*a**3*e**
3 + 30*a**2*b*d*e**2 - 12*a*b**2*c*e**2 - 12*a*b**2*d**2*e + 6*b**3*c*d*e +
 b**3*d**3)/(7*b**6) + (a + b*x)**(5/2)*(15*a**4*e**3 - 30*a**3*b*d*e**2 +
18*a**2*b**2*c*e**2 + 18*a**2*b**2*d**2*e - 18*a*b**3*c*d*e - 3*a*b**3*d**3
 + 3*b**4*c**2*e + 3*b**4*c*d**2)/(5*b**6) + (a + b*x)**(3/2)*(-6*a**5*e**3
 + 15*a**4*b*d*e**2 - 12*a**3*b**2*c*e**2 - 12*a**3*b**2*d**2*e + 18*a**2*b
**3*c*d*e + 3*a**2*b**3*d**3 - 6*a*b**4*c**2*e - 6*a*b**4*c*d**2 + 3*b**5*c
**2*d)/(3*b**6) + sqrt(a + b*x)*(a**6*e**3 - 3*a**5*b*d*e**2 + 3*a**4*b**2*
c*e**2 + 3*a**4*b**2*d**2*e - 6*a**3*b**3*c*d*e - a**3*b**3*d**3 + 3*a**2*b
**4*c**2*e + 3*a**2*b**4*c*d**2 - 3*a*b**5*c**2*d + b**6*c**3)/b**6)/b, Ne(
b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + d*e**2*x**6/2 + e**3*x**7/7 + x**5*(3*
c*e**2 + 3*d**2*e)/5 + x**4*(6*c*d*e + d**3)/4 + x**3*(3*c**2*e + 3*c*d**2)
/3)/sqrt(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(250) = 500$.

Time = 0.20 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.92

$$\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx = \frac{2\left(15015\sqrt{bx+a}c^3 + 3003c^2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b} + \frac{\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2}\right) + 143c\left(\frac{21\left(3(bx+\cdots\right.}{\cdots}\right.\right.}{\cdots}$$

```
[In] integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(15015*sqrt(b*x + a)*c^3 + 3003*c^2*(5*((b*x + a)^(3/2) - 3*sqrt(b*
x + a)*a)*d/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a
)*a^2)*e/b^2) + 143*c*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sq
rt(b*x + a)*a^2)*d^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 3
5*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2)
 - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^
3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4) + 429*(5*(b*x + a)^(7/2) - 21*(b*x + a)
^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 143*(35
*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b
*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d^2*e/b^4 + 65*(63*(b*x + a)^(11
/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2
)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2/b^5 + 5*(23
1*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8
580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a
^5 + 3003*sqrt(b*x + a)*a^6)*e^3/b^6)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(250) = 500$.

Time = 0.27 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.92

$$\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx = \frac{2\left(15015\sqrt{bx+a}c^3 + \frac{15015\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)c^2d}{b} + \frac{3003\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)cd^2}{b^2} + \frac{3003\left(3(bx+a)^{\frac{5}{2}}-1\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15015*(15015*sqrt(b*x + a)*c^3 + 15015*((b*x + a)^(3/2) - 3*sqrt(b*x + a)
*a)*c^2*d/b + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x
```

```
+ a)*a^2)*c*d^2/b^2 + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*s
qrt(b*x + a)*a^2)*c^2*e/b^2 + 429*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a
 + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 2574*(5*(b*x +
a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)
*a^3)*c*d*e/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*
x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d^2*e/b
^4 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*
a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*c*e^2/b^4 + 65*(63*(
b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b
*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2
/b^5 + 5*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(
9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x
+ a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*e^3/b^6)/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx = \frac{2\,e^3\,(a+b\,x)^{13/2}}{13\,b^7} - \frac{\left(12\,a\,e^3-6\,b\,d\,e^2\right)\,(a+b\,x)^{11/2}}{11\,b^7}$$
$$+ \frac{(a+b\,x)^{9/2}\,\left(30\,a^2\,e^3-30\,a\,b\,d\,e^2+6\,b^2\,d^2\,e+6\,c\,b^2\,e^2\right)}{9\,b^7} + \frac{2\,\sqrt{a+b\,x}\,\left(e\,a^2-d\,a\,b+c\,b^2\right)^3}{b^7}$$
$$+ \frac{(a+b\,x)^{5/2}\,\left(30\,a^4\,e^3-60\,a^3\,b\,d\,e^2+36\,a^2\,b^2\,c\,e^2+36\,a^2\,b^2\,d^2\,e-36\,a\,b^3\,c\,d\,e-6\,a\,b^3\,d^3+6\,b^4\,c^2\,e+6\,b^4\right.}{5\,b^7}$$
$$- \frac{2\,(2\,a\,e-b\,d)\,(a+b\,x)^{7/2}\,\left(10\,a^2\,e^2-10\,a\,b\,d\,e+b^2\,d^2+6\,c\,b^2\,e\right)}{7\,b^7}$$
$$- \frac{2\,(2\,a\,e-b\,d)\,(a+b\,x)^{3/2}\,\left(e\,a^2-d\,a\,b+c\,b^2\right)^2}{b^7}$$

```
[In] int((c + d*x + e*x^2)^3/(a + b*x)^(1/2),x)
```

```
[Out] (2*e^3*(a + b*x)^(13/2))/(13*b^7) - ((12*a*e^3 - 6*b*d*e^2)*(a + b*x)^(11/2
))/(11*b^7) + ((a + b*x)^(9/2)*(30*a^2*e^3 + 6*b^2*c*e^2 + 6*b^2*d^2*e - 30
*a*b*d*e^2))/(9*b^7) + (2*(a + b*x)^(1/2)*(b^2*c + a^2*e - a*b*d)^3)/b^7 +
((a + b*x)^(5/2)*(30*a^4*e^3 - 6*a*b^3*d^3 + 6*b^4*c*d^2 + 6*b^4*c^2*e + 36
*a^2*b^2*c*e^2 + 36*a^2*b^2*d^2*e - 60*a^3*b*d*e^2 - 36*a*b^3*c*d*e))/(5*b^
7) - (2*(2*a*e - b*d)*(a + b*x)^(7/2)*(10*a^2*e^2 + b^2*d^2 + 6*b^2*c*e - 1
0*a*b*d*e))/(7*b^7) - (2*(2*a*e - b*d)*(a + b*x)^(3/2)*(b^2*c + a^2*e - a*b
*d)^2)/b^7
```

3.4 $\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx = \frac{2(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx}}{b^4} + \frac{2(b^2d-2abe+3a^2f)(a+bx)^{3/2}}{3b^4} + \frac{2(be-3af)(a+bx)^{5/2}}{5b^4} + \frac{2f(a+bx)^{7/2}}{7b^4}$$

```
[Out] 2/3*(3*a^2*f-2*a*b*e+b^2*d)*(b*x+a)^(3/2)/b^4+2/5*(-3*a*f+b*e)*(b*x+a)^(5/2
)/b^4+2/7*f*(b*x+a)^(7/2)/b^4+2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x+a)^(1/2
)/b^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1864}

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx = \frac{2(a+bx)^{3/2}(3a^2f-2abe+b^2d)}{3b^4} + \frac{2\sqrt{a+bx}(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4} + \frac{2(a+bx)^{5/2}(be-3af)}{5b^4} + \frac{2f(a+bx)^{7/2}}{7b^4}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x])/b^4 + (2*(b^2*d - 2*a
*b*e + 3*a^2*f)*(a + b*x)^(3/2))/(3*b^4) + (2*(b*e - 3*a*f)*(a + b*x)^(5/2)
)/(5*b^4) + (2*f*(a + b*x)^(7/2))/(7*b^4)
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \int \left(\frac{b^3c - ab^2d + a^2be - a^3f}{b^3\sqrt{a+bx}} + \frac{(b^2d - 2abe + 3a^2f)\sqrt{a+bx}}{b^3} + \frac{(be-3af)(a+bx)^{3/2}}{b^3} + \frac{f(a+bx)^{5/2}}{b^3}\right) dx$$

$$= \frac{2(b^3c - ab^2d + a^2be - a^3f)\sqrt{a+bx}}{b^4} + \frac{2(b^2d - 2abe + 3a^2f)(a+bx)^{3/2}}{3b^4} + \frac{2(be-3af)(a+bx)^{5/2}}{5b^4} + \frac{2f(a+bx)^{7/2}}{7b^4}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.72

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}(-48a^3f + 8a^2b(7e+3fx) - 2ab^2(35d + x(14e+9fx)) + b^3(105c + x(35d + 3x(7e+5fx))))}{105b^4}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x) - 2*a*b^2*(35*d + x*(14
*e + 9*f*x)) + b^3*(105*c + x*(35*d + 3*x*(7*e + 5*f*x)))))/(105*b^4)
```

Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$-\frac{32\left(\frac{\left(-5 f \,x^{3}-7 e \,x^{2}-\frac{35}{3} d x -35 c \right) b^{3}}{16}+\frac{35 \left(\frac{9}{35} f \,x^{2}+\frac{2}{5} e x +d \right) a \,b^{2}}{24}-\frac{7 \left(\frac{3 f x}{7}+e \right) a^{2} b}{6}+f \,a^{3}\right) \sqrt{b x +a}}{35 b^{4}}$
gosper	$-\frac{2 \sqrt{b x +a}\, \left(-15 b^{3} f \,x^{3}+18 a \,b^{2} f \,x^{2}-21 b^{3} e \,x^{2}-24 x f \,a^{2} b +28 a \,b^{2} e x -35 b^{3} d x +48 f \,a^{3}-56 a^{2} b e +70 a \,b^{2} d -105 b^{3} c \right)}{105 b^{4}}$
trager	$-\frac{2 \sqrt{b x +a}\, \left(-15 b^{3} f \,x^{3}+18 a \,b^{2} f \,x^{2}-21 b^{3} e \,x^{2}-24 x f \,a^{2} b +28 a \,b^{2} e x -35 b^{3} d x +48 f \,a^{3}-56 a^{2} b e +70 a \,b^{2} d -105 b^{3} c \right)}{105 b^{4}}$
risch	$-\frac{2 \sqrt{b x +a}\, \left(-15 b^{3} f \,x^{3}+18 a \,b^{2} f \,x^{2}-21 b^{3} e \,x^{2}-24 x f \,a^{2} b +28 a \,b^{2} e x -35 b^{3} d x +48 f \,a^{3}-56 a^{2} b e +70 a \,b^{2} d -105 b^{3} c \right)}{105 b^{4}}$
derivativedivides	$\frac{\frac{2 f \left(b x +a \right)^{\frac{7}{2}}}{7}-\frac{6 a f \left(b x +a \right)^{\frac{5}{2}}}{5}+\frac{2 b e \left(b x +a \right)^{\frac{5}{2}}}{5}+2 a^{2} f \left(b x +a \right)^{\frac{3}{2}}-\frac{4 a b e \left(b x +a \right)^{\frac{3}{2}}}{3}+\frac{2 b^{2} d \left(b x +a \right)^{\frac{3}{2}}}{3}-2 a^{3} f \sqrt{b x +a}+2 a^{2} b e \sqrt{b x +a}-2 a \,b^{2} d}{b^{4}}$
default	$\frac{\frac{2 f \left(b x +a \right)^{\frac{7}{2}}}{7}-\frac{6 a f \left(b x +a \right)^{\frac{5}{2}}}{5}+\frac{2 b e \left(b x +a \right)^{\frac{5}{2}}}{5}+2 a^{2} f \left(b x +a \right)^{\frac{3}{2}}-\frac{4 a b e \left(b x +a \right)^{\frac{3}{2}}}{3}+\frac{2 b^{2} d \left(b x +a \right)^{\frac{3}{2}}}{3}-2 a^{3} f \sqrt{b x +a}+2 a^{2} b e \sqrt{b x +a}-2 a \,b^{2} d}{b^{4}}$

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -32/35*(1/16*(-5*f*x^3-7*e*x^2-35/3*d*x-35*c)*b^3+35/24*(9/35*f*x^2+2/5*e*x
+d)*a*b^2-7/6*(3/7*f*x+e)*a^2*b+f*a^3)*(b*x+a)^(1/2)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.79

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx = \frac{2\left(15\,b^3fx^3+105\,b^3c-70\,ab^2d+56\,a^2be-48\,a^3f+3\,(7\,b^3e-6\,ab^2f)x^2+(35\,b^3d-28\,ab^2e+24\,a^2bf)x\right)\sqrt{}}{105\,b^4}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/105*(15*b^3*f*x^3 + 105*b^3*c - 70*a*b^2*d + 56*a^2*b*e - 48*a^3*f + 3*(7
*b^3*e - 6*a*b^2*f)*x^2 + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x)*sqrt(b*x
+ a)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.43

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx = \begin{cases} \frac{2c\sqrt{a+bx}+\frac{2d\left(-a\sqrt{a+bx}+\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}+\frac{2e\left(a^2\sqrt{a+bx}-\frac{2a(a+bx)^{\frac{3}{2}}}{3}+\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}+\frac{2f\left(-a^3\sqrt{a+bx}+a^2(a+bx)^{\frac{3}{2}}-\frac{3a(a+bx)^{\frac{5}{2}}}{5}+\frac{(a+bx)^{\frac{7}{2}}}{7}\right)}{b^3}}{b} & \text{for } b \neq \\ \frac{cx+\frac{dx^2}{2}+\frac{ex^3}{3}+\frac{fx^4}{4}}{\sqrt{a}} & \text{otherwi} \end{cases}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise(((2*c*sqrt(a + b*x) + 2*d*(-a*sqrt(a + b*x) + (a + b*x)**(3/2)/3)
/b + 2*e*(a**2*sqrt(a + b*x) - 2*a*(a + b*x)**(3/2)/3 + (a + b*x)**(5/2)/5)
/b**2 + 2*f*(-a**3*sqrt(a + b*x) + a**2*(a + b*x)**(3/2) - 3*a*(a + b*x)**(
5/2)/5 + (a + b*x)**(7/2)/7)/b**3)/b, Ne(b, 0)), ((c*x + d*x**2/2 + e*x**3/
3 + f*x**4/4)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.12

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx = \frac{2\left(105\sqrt{bx+a}c + \frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b} + \frac{7\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2} + \frac{3\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(b\cdots\right.}{b^3}\right.}{105\,b}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(105*sqrt(b*x + a)*c + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b +
 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2
+ 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35
*sqrt(b*x + a)*a^3)*f/b^3)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.12

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx = \frac{2\left(105\sqrt{bx+a}c + \frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b} + \frac{7\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2} + \frac{3\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(b\cdots\right.}{b^3}\right.}{105\,b}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(b*x + a)*c + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b +
 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2
+ 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35
*sqrt(b*x + a)*a^3)*f/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx = \frac{(a+b\,x)^{3/2}\,(6\,f\,a^2-4\,e\,a\,b+2\,d\,b^2)}{3\,b^4} - \frac{(6\,a\,f-2\,b\,e)\,\left(a+b\,x\right)^{5/2}}{5\,b^4} + \frac{\sqrt{a+b\,x}\,(-2\,f\,a^3+2\,e\,a^2\,b-2\,d\,a\,b^2+2\,c\,b^3)}{b^4} + \frac{2\,f\,(a+b\,x)^{7/2}}{7\,b^4}$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x)^(1/2),x)
```

```
[Out] ((a + b*x)^(3/2)*(2*b^2*d + 6*a^2*f - 4*a*b*e))/(3*b^4) - ((6*a*f - 2*b*e)*
(a + b*x)^(5/2))/(5*b^4) + ((a + b*x)^(1/2)*(2*b^3*c - 2*a^3*f - 2*a*b^2*d
+ 2*a^2*b*e))/b^4 + (2*f*(a + b*x)^(7/2))/(7*b^4)
```

3.5 $\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 320

$$
\begin{aligned}
\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx
&= \frac{2(b^3c-ab^2d+a^2be-a^3f)^2\sqrt{a+bx}}{b^7} \\
&+ \frac{4(b^2d-2abe+3a^2f)(b^3c-ab^2d+a^2be-a^3f)(a+bx)^{3/2}}{3b^7} \\
&+ \frac{2(b^4(d^2+2ce)-20a^3bef+15a^4f^2-6ab^3(de+cf)+6a^2b^2(e^2+2df))(a+bx)^{5/2}}{5b^7} \\
&+ \frac{4(10a^2bef-10a^3f^2+b^3(de+cf)-2ab^2(e^2+2df))(a+bx)^{7/2}}{7b^7} \\
&- \frac{2(10abef-15a^2f^2-b^2(e^2+2df))(a+bx)^{9/2}}{9b^7} \\
&+ \frac{4f(be-3af)(a+bx)^{11/2}}{11b^7} + \frac{2f^2(a+bx)^{13/2}}{13b^7}
\end{aligned}
$$

```
[Out] 4/3*(3*a^2*f-2*a*b*e+b^2*d)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x+a)^(3/2)/b^
7+2/5*(b^4*(2*c*e+d^2)-20*a^3*b*e*f+15*a^4*f^2-6*a*b^3*(c*f+d*e)+6*a^2*b^2*
(2*d*f+e^2))*(b*x+a)^(5/2)/b^7+4/7*(10*a^2*b*e*f-10*a^3*f^2+b^3*(c*f+d*e)-2
*a*b^2*(2*d*f+e^2))*(b*x+a)^(7/2)/b^7-2/9*(10*a*b*e*f-15*a^2*f^2-b^2*(2*d*f
+e^2))*(b*x+a)^(9/2)/b^7+4/11*f*(-3*a*f+b*e)*(b*x+a)^(11/2)/b^7+2/13*f^2*(b
*x+a)^(13/2)/b^7+2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)^2*(b*x+a)^(1/2)/b^7
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1864}

$$\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx$$
$$= -\frac{2(a+bx)^{9/2}\left(-15a^2f^2+10abef-\left(b^2(2df+e^2)\right)\right)}{9b^7}$$
$$+\frac{4(a+bx)^{7/2}\left(-10a^3f^2+10a^2bef-2ab^2(2df+e^2)+b^3(cf+de)\right)}{7b^7}$$
$$+\frac{4(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^7}$$
$$+\frac{2\sqrt{a+bx}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)^2}{b^7}$$
$$+\frac{2(a+bx)^{5/2}\left(15a^4f^2-20a^3bef+6a^2b^2(2df+e^2)-6ab^3(cf+de)+b^4(2ce+d^2)\right)}{5b^7}$$
$$+\frac{4f(a+bx)^{11/2}(be-3af)}{11b^7}+\frac{2f^2(a+bx)^{13/2}}{13b^7}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*Sqrt[a + b*x])/b^7 + (4*(b^2*d - 2
*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a + b*x)^(3/2))/(3*b
^7) + (2*(b^4*(d^2 + 2*c*e) - 20*a^3*b*e*f + 15*a^4*f^2 - 6*a*b^3*(d*e + c*
f) + 6*a^2*b^2*(e^2 + 2*d*f))*(a + b*x)^(5/2))/(5*b^7) + (4*(10*a^2*b*e*f -
 10*a^3*f^2 + b^3*(d*e + c*f) - 2*a*b^2*(e^2 + 2*d*f))*(a + b*x)^(7/2))/(7*
b^7) - (2*(10*a*b*e*f - 15*a^2*f^2 - b^2*(e^2 + 2*d*f))*(a + b*x)^(9/2))/(9
*b^7) + (4*f*(b*e - 3*a*f)*(a + b*x)^(11/2))/(11*b^7) + (2*f^2*(a + b*x)^(1
3/2))/(13*b^7)
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \int \left(\frac{(b^3c-ab^2d+a^2be-a^3f)^2}{b^6\sqrt{a+bx}} + \frac{2(b^2d-2abe+3a^2f)(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx}}{b^6} + \frac{(b^4(d^2+2ce)-20a^3bef+15a^4f^2-6ab^3(de+cf)+6a^2b^2(e^2+2df))(a+bx)^{3/2}}{b^6} + \frac{2(10a^2bef-10a^3f^2+b^3(de+cf)-2ab^2(e^2+2df))(a+bx)^{5/2}}{b^6} + \frac{(-10abef+15a^2f^2+b^2(e^2+2df))(a+bx)^{7/2}}{b^6} + \frac{2f(be-3af)(a+bx)^{9/2}}{b^6} + \frac{f^2(a+bx)^{11/2}}{b^6}\right) dx$$

$$= \frac{2(b^3c-ab^2d+a^2be-a^3f)^2\sqrt{a+bx}}{b^7} + \frac{4(b^2d-2abe+3a^2f)(b^3c-ab^2d+a^2be-a^3f)(a+bx)^{3/2}}{3b^7} + \frac{2(b^4(d^2+2ce)-20a^3bef+15a^4f^2-6ab^3(de+cf)+6a^2b^2(e^2+2df))(a+bx)^{5/2}}{5b^7} + \frac{4(10a^2bef-10a^3f^2+b^3(de+cf)-2ab^2(e^2+2df))(a+bx)^{7/2}}{7b^7} - \frac{2(10abef-15a^2f^2-b^2(e^2+2df))(a+bx)^{9/2}}{9b^7} + \frac{4f(be-3af)(a+bx)^{11/2}}{11b^7} + \frac{2f^2(a+bx)^{13/2}}{13b^7}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}(15360a^6f^2-2560a^5bf(13e+3fx)+128a^4b^2(143e^2+130efx+f(286d+45fx^2))-32a^3b^3(1\cdots}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(15360*a^6*f^2 - 2560*a^5*b*f*(13*e + 3*f*x) + 128*a^4*b^2
*(143*e^2 + 130*e*f*x + f*(286*d + 45*f*x^2)) - 32*a^3*b^3*(1287*c*f + 143*
d*(9*e + 4*f*x) + 2*x*(143*e^2 + 195*e*f*x + 75*f^2*x^2)) + 8*a^2*b^4*(3003
```

```
*d^2 + 858*d*x*(3*e + 2*f*x) + 858*c*(7*e + 3*f*x) + x^2*(858*e^2 + 1300*e*
f*x + 525*f^2*x^2)) + b^6*(45045*c^2 + 858*c*x*(35*d + 3*x*(7*e + 5*f*x)) +
 x^2*(9009*d^2 + 1430*d*x*(9*e + 7*f*x) + 35*x^2*(143*e^2 + 234*e*f*x + 99*
f^2*x^2))) - 4*a*b^5*(429*c*(35*d + x*(14*e + 9*f*x)) + x*(3003*d^2 + 143*d
*x*(27*e + 20*f*x) + 5*x^2*(286*e^2 + 455*e*f*x + 189*f^2*x^2)))))/(45045*b
^7)
```

Maple [A] (verified)

Time = 4.78 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{2f^2(bx+a)^{\frac{13}{2}}}{13}+\frac{4(-3af+be)f(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(2\left(3a^2f-2aeb+b^2d\right)f+(-3af+be)^2\right)(bx+a)^{\frac{9}{2}}}{9}+\frac{2(2(-f\,a^3+a^2be-a\,b^2d+b^3c)f+2(3a^2}{7}}{}$
pseudoelliptic	$\frac{2048\left(\frac{\left(\frac{231f^2x^6}{2}+273ef\,x^5+\frac{1001\left(df+\frac{e^2}{2}\right)x^4}{3}+429(cf+de)x^3+\frac{3003\left(ce+\frac{d^2}{2}\right)x^2}{5}+1001cdx+\frac{3003c^2}{2}\right)b^6}{512}-\frac{1001(\frac{9f^2x^5}{143}+\frac{5ef\,x^4}{33}+}{}\right.}{}$
default	$\frac{\frac{2f^2(bx+a)^{\frac{13}{2}}}{13}-\frac{4(3af-be)f(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(-2\left(-3a^2f+2aeb-b^2d\right)f+(3af-be)^2\right)(bx+a)^{\frac{9}{2}}}{9}+\frac{2(-2(f\,a^3-a^2be+a\,b^2d-b^3c)f+2(-3a}{7}}{}$
gosper	$\frac{2\sqrt{bx+a}\,(3465f^2x^6b^6-3780a\,b^5f^2x^5+8190b^6ef\,x^5+4200a^2b^4f^2x^4-9100a\,b^5ef\,x^4+10010b^6df\,x^4+5005b^6e^2x^4-4800a^3b}{}$
trager	$\frac{2\sqrt{bx+a}\,(3465f^2x^6b^6-3780a\,b^5f^2x^5+8190b^6ef\,x^5+4200a^2b^4f^2x^4-9100a\,b^5ef\,x^4+10010b^6df\,x^4+5005b^6e^2x^4-4800a^3b}{}$
risch	$\frac{2\sqrt{bx+a}\,(3465f^2x^6b^6-3780a\,b^5f^2x^5+8190b^6ef\,x^5+4200a^2b^4f^2x^4-9100a\,b^5ef\,x^4+10010b^6df\,x^4+5005b^6e^2x^4-4800a^3b}{}$

```
[In] int((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^7*(1/13*f^2*(b*x+a)^(13/2)+2/11*(-3*a*f+b*e)*f*(b*x+a)^(11/2)+1/9*(2*(3
*a^2*f-2*a*b*e+b^2*d)*f+(-3*a*f+b*e)^2)*(b*x+a)^(9/2)+1/7*(2*(-a^3*f+a^2*b*
e-a*b^2*d+b^3*c)*f+2*(3*a^2*f-2*a*b*e+b^2*d)*(-3*a*f+b*e))*(b*x+a)^(7/2)+1/
5*(2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(-3*a*f+b*e)+(3*a^2*f-2*a*b*e+b^2*d)^2)
*(b*x+a)^(5/2)+2/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(3*a^2*f-2*a*b*e+b^2*d)*(
b*x+a)^(3/2)+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)^2*(b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx$$
$$= \frac{2\,(3465\,b^6f^2x^6+45045\,b^6c^2-60060\,ab^5cd+24024\,a^2b^4d^2+18304\,a^4b^2e^2+15360\,a^6f^2+630\,(13\,b^6ef-\cdots}{}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3465*b^6*f^2*x^6 + 45045*b^6*c^2 - 60060*a*b^5*c*d + 24024*a^2*b^4
*d^2 + 18304*a^4*b^2*e^2 + 15360*a^6*f^2 + 630*(13*b^6*e*f - 6*a*b^5*f^2)*x
^5 + 35*(143*b^6*e^2 + 120*a^2*b^4*f^2 + 26*(11*b^6*d - 10*a*b^5*e)*f)*x^4
+ 10*(1287*b^6*d*e - 572*a*b^5*e^2 - 480*a^3*b^3*f^2 + 13*(99*b^6*c - 88*a*
b^5*d + 80*a^2*b^4*e)*f)*x^3 + 3*(3003*b^6*d^2 + 2288*a^2*b^4*e^2 + 1920*a^
4*b^2*f^2 + 858*(7*b^6*c - 6*a*b^5*d)*e - 52*(99*a*b^5*c - 88*a^2*b^4*d + 8
0*a^3*b^3*e)*f)*x^2 + 6864*(7*a^2*b^4*c - 6*a^3*b^3*d)*e - 416*(99*a^3*b^3*
c - 88*a^4*b^2*d + 80*a^5*b*e)*f + 2*(15015*b^6*c*d - 6006*a*b^5*d^2 - 4576
*a^3*b^3*e^2 - 3840*a^5*b*f^2 - 1716*(7*a*b^5*c - 6*a^2*b^4*d)*e + 104*(99*
a^2*b^4*c - 88*a^3*b^3*d + 80*a^4*b^2*e)*f)*x)*sqrt(b*x + a)/b^7
```

Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.76

$$\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{f^2(a+bx)^{\frac{13}{2}}}{13b^6}+\frac{(a+bx)^{\frac{11}{2}}\left(-6af^2+2bef\right)}{11b^6}+\frac{(a+bx)^{\frac{9}{2}}\cdot\left(15a^2f^2-10abef+2b^2df+b^2e^2\right)}{9b^6}+\frac{(a+bx)^{\frac{7}{2}}\left(-20a^3f^2+20a^2bef-8ab^2df-4ab^2e^2+2b^3cf+2b^3de\right)}{7b^6}+\cdots\right.}{} \\ \frac{c^2x+cdx^2+\frac{efx^6}{3}+\frac{f^2x^7}{7}+\frac{x^5\cdot\left(2df+e^2\right)}{5}+\frac{x^4\cdot(2cf+2de)}{4}+\frac{x^3\cdot\left(2ce+d^2\right)}{3}}{\sqrt{a}} \end{cases}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((2*(f**2*(a + b*x)**(13/2)/(13*b**6) + (a + b*x)**(11/2)*(-6*a*f*
*2 + 2*b*e*f)/(11*b**6) + (a + b*x)**(9/2)*(15*a**2*f**2 - 10*a*b*e*f + 2*b
**2*d*f + b**2*e**2)/(9*b**6) + (a + b*x)**(7/2)*(-20*a**3*f**2 + 20*a**2*b
*e*f - 8*a*b**2*d*f - 4*a*b**2*e**2 + 2*b**3*c*f + 2*b**3*d*e)/(7*b**6) + (
a + b*x)**(5/2)*(15*a**4*f**2 - 20*a**3*b*e*f + 12*a**2*b**2*d*f + 6*a**2*b
**2*e**2 - 6*a*b**3*c*f - 6*a*b**3*d*e + 2*b**4*c*e + b**4*d**2)/(5*b**6) +
 (a + b*x)**(3/2)*(-6*a**5*f**2 + 10*a**4*b*e*f - 8*a**3*b**2*d*f - 4*a**3*
```

```
b**2*e**2 + 6*a**2*b**3*c*f + 6*a**2*b**3*d*e - 4*a*b**4*c*e - 2*a*b**4*d**
2 + 2*b**5*c*d)/(3*b**6) + sqrt(a + b*x)*(a**6*f**2 - 2*a**5*b*e*f + 2*a**4
*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*f - 2*a**3*b**3*d*e + 2*a**2*b**
4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**6*c**2)/b**6)/b, Ne(b, 0)), ((c*
*2*x + c*d*x**2 + e*f*x**6/3 + f**2*x**7/7 + x**5*(2*d*f + e**2)/5 + x**4*(
2*c*f + 2*d*e)/4 + x**3*(2*c*e + d**2)/3)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.56

$$\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx = \frac{2\left(45045\sqrt{bx+a}c^2+858\,c\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}+\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+\right.}{}\right.\right.}{}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(45045*sqrt(b*x + a)*c^2 + 858*c*(35*((b*x + a)^(3/2) - 3*sqrt(b*x
+ a)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a
)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(
3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*f/b^3) + 3003*(3*(b*x + a)^(5/2) - 10*(b*x
 + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d^2/b^2 + 143*(35*(b*x + a)^(9/2) - 1
80*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 +
315*sqrt(b*x + a)*a^4)*e^2/b^4 + 286*(35*(b*x + a)^(9/2)*f + 45*(b*e - 4*a*
f)*(b*x + a)^(7/2) - 189*(a*b*e - 2*a^2*f)*(b*x + a)^(5/2) + 105*(3*a^2*b*e
 - 4*a^3*f)*(b*x + a)^(3/2) - 315*(a^3*b*e - a^4*f)*sqrt(b*x + a))*d/b^4 +
130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2
- 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a
^5)*e*f/b^5 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*
x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 60
06*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx = \frac{2\left(45045\sqrt{bx+a}c^2+\frac{30030\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)cd}{b}+\frac{3003\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)d^2}{b^2}+\frac{6006\left(3\,(bx+a)^{\frac{5}{2}}-10\,(\right.}{}\right.}{}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45045*(45045*sqrt(b*x + a)*c^2 + 30030*((b*x + a)^(3/2) - 3*sqrt(b*x + a)
*a)*c*d/b + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x +
a)*a^2)*d^2/b^2 + 6006*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(
b*x + a)*a^2)*c*e/b^2 + 2574*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35
*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + 2574*(5*(b*x + a)^(7
/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)
*c*f/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^
(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4 + 286*
(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420
*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d*f/b^4 + 130*(63*(b*x + a)^(
11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5
/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*e*f/b^5 + 15*(2
31*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 -
8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*
a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6)/b
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.99

$$
\begin{aligned}
\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx = {} & \frac{2\sqrt{a+b\,x}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)^2}{b^7}+\frac{2\,f^2\,{\left(a+b\,x\right)}^{13/2}}{13\,b^7} \\
& -\frac{{\left(a+b\,x\right)}^{7/2}\,\left(40\,a^3\,f^2-40\,a^2\,b\,e\,f+8\,a\,b^2\,e^2+16\,d\,a\,b^2\,f-4\,d\,b^3\,e-4\,c\,b^3\,f\right)}{7\,b^7} \\
& +\frac{{\left(a+b\,x\right)}^{9/2}\,\left(30\,a^2\,f^2-20\,a\,b\,e\,f+2\,b^2\,e^2+4\,d\,b^2\,f\right)}{9\,b^7} \\
& +\frac{{\left(a+b\,x\right)}^{5/2}\,\left(30\,a^4\,f^2-40\,a^3\,b\,e\,f+24\,a^2\,b^2\,d\,f+12\,a^2\,b^2\,e^2-12\,a\,b^3\,d\,e-12\,c\,a\,b^3\,f+2\,b^4\,d^2+4\,c\,b^4\,e}{5\,b^7} \\
& -\frac{\left(12\,a\,f^2-4\,b\,e\,f\right)\,{\left(a+b\,x\right)}^{11/2}}{11\,b^7} \\
& +\frac{4\,{\left(a+b\,x\right)}^{3/2}\,\left(3\,f\,a^2-2\,e\,a\,b+d\,b^2\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^7}
\end{aligned}
$$

```
[In] int((c + d*x + e*x^2 + f*x^3)^2/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e)^2)/b^7 + (2*f^2*(a +
 b*x)^(13/2))/(13*b^7) - ((a + b*x)^(7/2)*(40*a^3*f^2 + 8*a*b^2*e^2 - 4*b^3
*c*f - 4*b^3*d*e + 16*a*b^2*d*f - 40*a^2*b*e*f))/(7*b^7) + ((a + b*x)^(9/2)
*(30*a^2*f^2 + 2*b^2*e^2 + 4*b^2*d*f - 20*a*b*e*f))/(9*b^7) + ((a + b*x)^(5
/2)*(2*b^4*d^2 + 30*a^4*f^2 + 12*a^2*b^2*e^2 + 4*b^4*c*e - 12*a*b^3*c*f - 1
2*a*b^3*d*e - 40*a^3*b*e*f + 24*a^2*b^2*d*f))/(5*b^7) - ((12*a*f^2 - 4*b*e*
f)*(a + b*x)^(11/2))/(11*b^7) + (4*(a + b*x)^(3/2)*(b^2*d + 3*a^2*f - 2*a*b
*e)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^7)
```

3.6 $\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 708

$$\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx = \frac{2(b^3c-ab^2d+a^2be-a^3f)^3\sqrt{a+bx}}{b^{10}}$$
$$+\frac{2(b^2d-2abe+3a^2f)\,(b^3c-ab^2d+a^2be-a^3f)^2\,(a+bx)^{3/2}}{b^{10}}$$
$$+\frac{6(b^3c-ab^2d+a^2be-a^3f)\,(b^4(d^2+ce)-16a^3bef+12a^4f^2-ab^3(5de+3cf)+a^2b^2(5e^2+9df))\,(a+b}{5b^{10}}$$
$$-\frac{2(168a^5bef^2-84a^6f^3-b^6(d^3+6cde+3c^2f)-105a^4b^2f(e^2+df)+12ab^5(d^2e+ce^2+2cdf)-30a^2b^4(}{7b^{10}}$$
$$+\frac{2(70a^4bef^2-42a^5f^3-35a^3b^2f(e^2+df)+b^5(d^2e+ce^2+2cdf)-5ab^4(de^2+d^2f+2cef)+5a^2b^3(e^3+}{3b^{10}}$$
$$-\frac{6(56a^3bef^2-42a^4f^3-21a^2b^2f(e^2+df)-b^4(de^2+d^2f+2cef)+2ab^3(e^3+6def+3cf^2))\,(a+bx)^{11/2}}{11b^{10}}$$
$$+\frac{2(84a^2bef^2-84a^3f^3-21ab^2f(e^2+df)+b^3(e^3+6def+3cf^2))\,(a+bx)^{13/2}}{13b^{10}}$$
$$-\frac{2f(8abef-12a^2f^2-b^2(e^2+df))\,(a+bx)^{15/2}}{5b^{10}}$$
$$+\frac{6f^2(be-3af)(a+bx)^{17/2}}{17b^{10}}+\frac{2f^3(a+bx)^{19/2}}{19b^{10}}$$

```
[Out] 2*(3*a^2*f-2*a*b*e+b^2*d)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)^2*(b*x+a)^(3/2)/b^
10+6/5*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b^4*(c*e+d^2)-16*a^3*b*e*f+12*a^4*f^
2-a*b^3*(3*c*f+5*d*e)+a^2*b^2*(9*d*f+5*e^2))*(b*x+a)^(5/2)/b^10-2/7*(168*a^
5*b*e*f^2-84*a^6*f^3-b^6*(3*c^2*f+6*c*d*e+d^3)-105*a^4*b^2*f*(d*f+e^2)+12*a
*b^5*(2*c*d*f+c*e^2+d^2*e)-30*a^2*b^4*(2*c*e*f+d^2*f+d*e^2)+20*a^3*b^3*(3*c
*f^2+6*d*e*f+e^3))*(b*x+a)^(7/2)/b^10+2/3*(70*a^4*b*e*f^2-42*a^5*f^3-35*a^3
*b^2*f*(d*f+e^2)+b^5*(2*c*d*f+c*e^2+d^2*e)-5*a*b^4*(2*c*e*f+d^2*f+d*e^2)+5*
```

```
a^2*b^3*(3*c*f^2+6*d*e*f+e^3))*(b*x+a)^(9/2)/b^10-6/11*(56*a^3*b*e*f^2-42*a
^4*f^3-21*a^2*b^2*f*(d*f+e^2)-b^4*(2*c*e*f+d^2*f+d*e^2)+2*a*b^3*(3*c*f^2+6*
d*e*f+e^3))*(b*x+a)^(11/2)/b^10+2/13*(84*a^2*b*e*f^2-84*a^3*f^3-21*a*b^2*f*
(d*f+e^2)+b^3*(3*c*f^2+6*d*e*f+e^3))*(b*x+a)^(13/2)/b^10-2/5*f*(8*a*b*e*f-1
2*a^2*f^2-b^2*(d*f+e^2))*(b*x+a)^(15/2)/b^10+6/17*f^2*(-3*a*f+b*e)*(b*x+a)^
(17/2)/b^10+2/19*f^3*(b*x+a)^(19/2)/b^10+2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)^3
*(b*x+a)^(1/2)/b^10
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1864}

$$\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx = -\frac{2f(a+bx)^{15/2}\left(-12a^2f^2+8abef-(b^2(df+e^2))\right)}{5b^{10}}$$
$$+\frac{2(a+bx)^{13/2}\left(-84a^3f^3+84a^2bef^2-21ab^2f(df+e^2)+b^3(3cf^2+6def+e^3)\right)}{13b^{10}}$$
$$+\frac{2(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)^2}{b^{10}}$$
$$+\frac{2\sqrt{a+bx}(a^3(-f)+a^2be-ab^2d+b^3c)^3}{b^{10}}$$
$$+\frac{6(a+bx)^{5/2}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)\left(12a^4f^2-16a^3bef+a^2b^2(9df+5e^2)-ab^3(3cf+5de)+b^4\right.}{5b^{10}}$$
$$-\frac{6(a+bx)^{11/2}\left(-42a^4f^3+56a^3bef^2-21a^2b^2f(df+e^2)+2ab^3(3cf^2+6def+e^3)-b^4(2cef+d^2f+d\right.}{11b^{10}}$$
$$+\frac{2(a+bx)^{9/2}\left(-42a^5f^3+70a^4bef^2-35a^3b^2f(df+e^2)+5a^2b^3(3cf^2+6def+e^3)-5ab^4(2cef+d^2f+\right.}{3b^{10}}$$
$$-\frac{2(a+bx)^{7/2}\left(-84a^6f^3+168a^5bef^2-105a^4b^2f(df+e^2)+20a^3b^3(3cf^2+6def+e^3)-30a^2b^4(2cef+\right.}{7b^{10}}$$
$$+\frac{6f^2(a+bx)^{17/2}(be-3af)}{17b^{10}}+\frac{2f^3(a+bx)^{19/2}}{19b^{10}}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^3*Sqrt[a + b*x])/b^10 + (2*(b^2*d -
2*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*(a + b*x)^(3/2))/b
^10 + (6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(b^4*(d^2 + c*e) - 16*a^3*b*e*
f + 12*a^4*f^2 - a*b^3*(5*d*e + 3*c*f) + a^2*b^2*(5*e^2 + 9*d*f))*(a + b*x)
^(5/2))/(5*b^10) - (2*(168*a^5*b*e*f^2 - 84*a^6*f^3 - b^6*(d^3 + 6*c*d*e +
3*c^2*f) - 105*a^4*b^2*f*(e^2 + d*f) + 12*a*b^5*(d^2*e + c*e^2 + 2*c*d*f) -
 30*a^2*b^4*(d*e^2 + d^2*f + 2*c*e*f) + 20*a^3*b^3*(e^3 + 6*d*e*f + 3*c*f^2
))*(a + b*x)^(7/2))/(7*b^10) + (2*(70*a^4*b*e*f^2 - 42*a^5*f^3 - 35*a^3*b^2
*f*(e^2 + d*f) + b^5*(d^2*e + c*e^2 + 2*c*d*f) - 5*a*b^4*(d*e^2 + d^2*f + 2
```

```
*c*e*f) + 5*a^2*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(9/2))/(3*b^10) -
(6*(56*a^3*b*e*f^2 - 42*a^4*f^3 - 21*a^2*b^2*f*(e^2 + d*f) - b^4*(d*e^2 + d
^2*f + 2*c*e*f) + 2*a*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(11/2))/(11*
b^10) + (2*(84*a^2*b*e*f^2 - 84*a^3*f^3 - 21*a*b^2*f*(e^2 + d*f) + b^3*(e^3
 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(13/2))/(13*b^10) - (2*f*(8*a*b*e*f - 12*a
^2*f^2 - b^2*(e^2 + d*f))*(a + b*x)^(15/2))/(5*b^10) + (6*f^2*(b*e - 3*a*f)
*(a + b*x)^(17/2))/(17*b^10) + (2*f^3*(a + b*x)^(19/2))/(19*b^10)
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \int \left(\frac{(b^3c - ab^2d + a^2be - a^3f)^3}{b^9\sqrt{a+bx}} + \frac{3(b^2d - 2abe + 3a^2f)\,(b^3c - ab^2d + a^2be - a^3f)^2\,\sqrt{a+bx}}{b^9} + \frac{3(b^3c - ab^2d + a^2be - a^3f)\,(b^4d^2 + b^4ce - 5ab^3de + 5a^2b^2e^2 - 3ab^3cf + 9a^2b^2df - 16a^3bef + 12a^4f^2)\,(a}{b^9} + \frac{(-168a^5bef^2 + 84a^6f^3 + b^6(d^3 + 6cde + 3c^2f) + 105a^4b^2f(e^2 + df) - 12ab^5(d^2e + ce^2 + 2cdf) + 30a^2b^4}{b^9} + \frac{3(70a^4bef^2 - 42a^5f^3 - 35a^3b^2f(e^2 + df) + b^5(d^2e + ce^2 + 2cdf) - 5ab^4(de^2 + d^2f + 2cef) + 5a^2b^3(e^3 +}{b^9} + \frac{3(-56a^3bef^2 + 42a^4f^3 + 21a^2b^2f(e^2 + df) + b^4(de^2 + d^2f + 2cef) - 2ab^3(e^3 + 6def + 3cf^2))\,(a+bx)^9}{b^9} + \frac{(84a^2bef^2 - 84a^3f^3 - 21ab^2f(e^2 + df) + b^3(e^3 + 6def + 3cf^2))\,(a+bx)^{11/2}}{b^9} + \frac{3f(-8abef + 12a^2f^2 + b^2(e^2 + df))\,(a+bx)^{13/2}}{b^9} + \frac{3f^2(be - 3af)(a+bx)^{15/2}}{b^9} + \frac{f^3(a+bx)^{17/2}}{b^9} \right) dx$$

$$
\begin{aligned}
&= \frac{2(b^3c - ab^2d + a^2be - a^3f)^3 \sqrt{a+bx}}{b^{10}} \\
&+ \frac{2(b^2d - 2abe + 3a^2f)(b^3c - ab^2d + a^2be - a^3f)^2 (a+bx)^{3/2}}{b^{10}} \\
&+ \frac{6(b^3c - ab^2d + a^2be - a^3f)(b^4(d^2+ce) - 16a^3bef + 12a^4f^2 - ab^3(5de+3cf) + a^2b^2(5e^2+9d}{5b^{10}} \\
&- \frac{2(168a^5bef^2 - 84a^6f^3 - b^6(d^3+6cde+3c^2f) - 105a^4b^2f(e^2+df) + 12ab^5(d^2e+ce^2+2cdf)}{7b^{10}} \\
&+ \frac{2(70a^4bef^2 - 42a^5f^3 - 35a^3b^2f(e^2+df) + b^5(d^2e+ce^2+2cdf) - 5ab^4(de^2+d^2f+2cef) + 5}{3b^{10}} \\
&- \frac{6(56a^3bef^2 - 42a^4f^3 - 21a^2b^2f(e^2+df) - b^4(de^2+d^2f+2cef) + 2ab^3(e^3+6def+3cf^2))(a}{11b^{10}} \\
&+ \frac{2(84a^2bef^2 - 84a^3f^3 - 21ab^2f(e^2+df) + b^3(e^3+6def+3cf^2))(a+bx)^{13/2}}{13b^{10}} \\
&- \frac{2f(8abef - 12a^2f^2 - b^2(e^2+df))(a+bx)^{15/2}}{5b^{10}} \\
&+ \frac{6f^2(be-3af)(a+bx)^{17/2}}{17b^{10}} + \frac{2f^3(a+bx)^{19/2}}{19b^{10}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 913, normalized size of antiderivative = 1.29

$$
\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx
= \frac{2\sqrt{a+bx}(-1376256a^9f^3 + 229376a^8bf^2(19e+3fx) - 14336a^7b^2f(323e^2+152efx+f(323d+36fx^2))}{}
$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-1376256*a^9*f^3 + 229376*a^8*b*f^2*(19*e + 3*f*x) - 1433
6*a^7*b^2*f*(323*e^2 + 152*e*f*x + f*(323*d + 36*f*x^2)) + 1024*a^6*b^3*(16
15*e^3 + 2261*e^2*f*x + 114*e*f*(85*d + 14*f*x^2) + f^2*(4845*c + 2261*d*x
+ 420*f*x^3)) - 256*a^5*b^4*(20995*d^2*f + 3230*c*f*(13*e + 3*f*x) + 323*d*
(65*e^2 + 60*e*f*x + 21*f^2*x^2) + x*(3230*e^3 + 6783*e^2*f*x + 5320*e*f^2*
x^2 + 1470*f^3*x^3)) + 128*a^4*b^5*(4199*d^2*(11*e + 5*f*x) + 323*c*(143*e^
2 + 130*e*f*x + 45*f^2*x^2) + x^2*(4845*e^3 + 11305*e^2*f*x + 9310*e*f^2*x^
2 + 2646*f^3*x^3) + 323*d*(286*c*f + 5*x*(13*e^2 + 18*e*f*x + 7*f^2*x^2)))
- 16*a^3*b^6*(138567*d^3 + 415701*c^2*f + 8398*d^2*x*(22*e + 15*f*x) + 1292
*c*x*(143*e^2 + 195*e*f*x + 75*f^2*x^2) + x^3*(32300*e^3 + 79135*e^2*f*x +
67032*e*f^2*x^2 + 19404*f^3*x^3) + 323*d*(286*c*(9*e + 4*f*x) + 5*x^2*(78*e
^2 + 120*e*f*x + 49*f^2*x^2))) + b^9*(4849845*c^3 + 138567*c^2*x*(35*d + 3*
x*(7*e + 5*f*x)) + 323*c*x^2*(9009*d^2 + 1430*d*x*(9*e + 7*f*x) + 35*x^2*(1
43*e^2 + 234*e*f*x + 99*f^2*x^2)) + x^3*(692835*d^3 + 146965*d^2*x*(11*e +
```

```
9*f*x) + 6783*d*x^2*(195*e^2 + 330*e*f*x + 143*f^2*x^2) + 231*x^3*(1615*e^3
 + 4199*e^2*f*x + 3705*e*f^2*x^2 + 1105*f^3*x^3))) + 8*a^2*b^7*(138567*c^2*
(7*e + 3*f*x) + 323*c*(3003*d^2 + 858*d*x*(3*e + 2*f*x) + x^2*(858*e^2 + 13
00*e*f*x + 525*f^2*x^2)) + x*(138567*d^3 + 8398*d^2*x*(33*e + 25*f*x) + 323
*d*x^2*(650*e^2 + 1050*e*f*x + 441*f^2*x^2) + 7*x^3*(8075*e^3 + 20349*e^2*f
*x + 17556*e*f^2*x^2 + 5148*f^3*x^3))) - 2*a*b^8*(138567*c^2*(35*d + x*(14*
e + 9*f*x)) + 646*c*x*(3003*d^2 + 143*d*x*(27*e + 20*f*x) + 5*x^2*(286*e^2
+ 455*e*f*x + 189*f^2*x^2)) + x^2*(415701*d^3 + 20995*d^2*x*(44*e + 35*f*x)
 + 2261*d*x^2*(325*e^2 + 540*e*f*x + 231*f^2*x^2) + 21*x^3*(9690*e^3 + 2487
1*e^2*f*x + 21736*e*f^2*x^2 + 6435*f^3*x^3)))))/(4849845*b^10)
```

Maple [A] (verified)

Time = 5.42 (sec) , antiderivative size = 816, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{131072\left(\frac{\left(-\frac{12155 f^{3} x^{9}}{2}-\frac{40755 e f^{2} x^{8}}{2}-\frac{46189 f \left(d f+e^{2}\right) x^{7}}{2}+17765\left(-\frac{1}{2} e^{3}-\frac{3}{2} c f^{2}-3 d e f\right) x^{6}+62985\left(\left(-\frac{d^{2}}{2}-c e\right) f-\frac{d e^{2}}{2}\right) x^{5}+ \cdots\right.}{32768 \cdots}\right.}{\cdots}$
derivativedivides	Expression too large to display
default	Expression too large to display
gosper	Expression too large to display
trager	Expression too large to display
risch	Expression too large to display

```
[In] int((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -131072/230945*(1/32768*(-12155/2*f^3*x^9-40755/2*e*f^2*x^8-46189/2*f*(d*f+
e^2)*x^7+17765*(-1/2*e^3-3/2*c*f^2-3*d*e*f)*x^6+62985*((-1/2*d^2-c*e)*f-1/2
*d*e^2)*x^5+230945/3*(-c*f*d-1/2*e*(c*e+d^2))*x^4+692835/7*(-1/2*c^2*f-(c*e
+1/6*d^2)*d)*x^3-138567/2*c*(c*e+d^2)*x^2-230945/2*c^2*d*x-230945/2*c^3)*b^
9+230945/32768*(9/323*f^3*x^8+8/85*e*f^2*x^7+7/65*f*(d*f+e^2)*x^6+6/143*(3*
c*f^2+6*d*e*f+e^3)*x^5+5/33*(f*(2*c*e+d^2)+d*e^2)*x^4+4/21*(2*c*f*d+e*(c*e+
d^2))*x^3+3/35*(3*c^2*f+6*c*d*e+d^3)*x^2+2/5*c*(c*e+d^2)*x+c^2*d)*a*b^8-461
89/8192*(12/323*f^3*x^7+28/221*e*f^2*x^6+21/143*f*(d*f+e^2)*x^5+25/143*(1/3
*e^3+c*f^2+2*d*e*f)*x^4+50/231*(f*(2*c*e+d^2)+d*e^2)*x^3+2/7*(2*c*f*d+e*(c*
e+d^2))*x^2+1/7*(3*c^2*f+6*c*d*e+d^3)*x+c^2*e+c*d^2)*a^2*b^7+138567/28672*(
196/4199*f^3*x^6+392/2431*e*f^2*x^5+245/1287*f*(d*f+e^2)*x^4+100/429*(1/3*e
^3+c*f^2+2*d*e*f)*x^3+10/33*(f*(2*c*e+d^2)+d*e^2)*x^2+4/9*(2*c*f*d+e*(c*e+d
^2))*x+1/3*d^3+c^2*f+2*c*d*e)*a^3*b^6-46189/5376*a^4*(1323/46189*f^3*x^5+24
```

```
5/2431*e*f^2*x^4+35/286*f*(d*f+e^2)*x^3+15/143*(3/2*c*f^2+3*d*e*f+1/2*e^3)*
x^2+5/11*((c*e+1/2*d^2)*f+1/2*d*e^2)*x+c*f*d+1/2*e*(c*e+d^2))*b^5+20995/268
8*(147/4199*f^3*x^4+28/221*e*f^2*x^3+21/130*f*(d*f+e^2)*x^2+1/13*(3*c*f^2+6
*d*e*f+e^3)*x+(c*e+1/2*d^2)*f+1/2*d*e^2)*a^5*b^4-1615/448*(28/323*f^3*x^3+2
8/85*e*f^2*x^2+7/15*f*(d*f+e^2)*x+1/3*e^3+c*f^2+2*d*e*f)*a^6*b^3+323/96*f*(
36/323*f^2*x^2+8/17*e*f*x+d*f+e^2)*a^7*b^2-19/6*(3/19*f*x+e)*f^2*a^8*b+a^9*
f^3)*(b*x+a)^(1/2)/b^10
```

Fricas [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 1221, normalized size of antiderivative = 1.72

$$\int \frac{(c + dx + ex^2 + fx^3)^3}{\sqrt{a + bx}} \, dx = \text{Too large to display}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/4849845*(255255*b^9*f^3*x^9 + 4849845*b^9*c^3 - 9699690*a*b^8*c^2*d + 775
9752*a^2*b^7*c*d^2 - 2217072*a^3*b^6*d^3 + 1653760*a^6*b^3*e^3 - 1376256*a^
9*f^3 + 45045*(19*b^9*e*f^2 - 6*a*b^8*f^3)*x^8 + 3003*(323*b^9*e^2*f + 96*a
^2*b^7*f^3 + 19*(17*b^9*d - 16*a*b^8*e)*f^2)*x^7 + 231*(1615*b^9*e^3 - 1344
*a^3*b^6*f^3 + 19*(255*b^9*c - 238*a*b^8*d + 224*a^2*b^7*e)*f^2 + 646*(15*b
^9*d*e - 7*a*b^8*e^2)*f)*x^6 + 63*(20995*b^9*d*e^2 - 6460*a*b^8*e^3 + 5376*
a^4*b^5*f^3 - 76*(255*a*b^8*c - 238*a^2*b^7*d + 224*a^3*b^6*e)*f^2 + 323*(6
5*b^9*d^2 + 56*a^2*b^7*e^2 + 10*(13*b^9*c - 12*a*b^8*d)*e)*f)*x^5 + 35*(461
89*b^9*d^2*e + 12920*a^2*b^7*e^3 - 10752*a^5*b^4*f^3 + 4199*(11*b^9*c - 10*
a*b^8*d)*e^2 + 152*(255*a^2*b^7*c - 238*a^3*b^6*d + 224*a^4*b^5*e)*f^2 + 64
6*(143*b^9*c*d - 65*a*b^8*d^2 - 56*a^3*b^6*e^2 - 10*(13*a*b^8*c - 12*a^2*b^
7*d)*e)*f)*x^4 + 5*(138567*b^9*d^3 - 103360*a^3*b^6*e^3 + 86016*a^6*b^3*f^3
 - 33592*(11*a*b^8*c - 10*a^2*b^7*d)*e^2 - 1216*(255*a^3*b^6*c - 238*a^4*b^
5*d + 224*a^5*b^4*e)*f^2 + 92378*(9*b^9*c*d - 4*a*b^8*d^2)*e + 323*(1287*b^
9*c^2 - 2288*a*b^8*c*d + 1040*a^2*b^7*d^2 + 896*a^4*b^5*e^2 + 160*(13*a^2*b
^7*c - 12*a^3*b^6*d)*e)*f)*x^3 + 537472*(11*a^4*b^5*c - 10*a^5*b^4*d)*e^2 +
 19456*(255*a^6*b^3*c - 238*a^7*b^2*d + 224*a^8*b*e)*f^2 + 3*(969969*b^9*c*
d^2 - 277134*a*b^8*d^3 + 206720*a^4*b^5*e^3 - 172032*a^7*b^2*f^3 + 67184*(1
1*a^2*b^7*c - 10*a^3*b^6*d)*e^2 + 2432*(255*a^4*b^5*c - 238*a^5*b^4*d + 224
*a^6*b^3*e)*f^2 + 46189*(21*b^9*c^2 - 36*a*b^8*c*d + 16*a^2*b^7*d^2)*e - 64
6*(1287*a*b^8*c^2 - 2288*a^2*b^7*c*d + 1040*a^3*b^6*d^2 + 896*a^5*b^4*e^2 +
 160*(13*a^3*b^6*c - 12*a^4*b^5*d)*e)*f)*x^2 + 369512*(21*a^2*b^7*c^2 - 36*
a^3*b^6*c*d + 16*a^4*b^5*d^2)*e - 5168*(1287*a^3*b^6*c^2 - 2288*a^4*b^5*c*d
 + 1040*a^5*b^4*d^2 + 896*a^7*b^2*e^2 + 160*(13*a^5*b^4*c - 12*a^6*b^3*d)*e
)*f + (4849845*b^9*c^2*d - 3879876*a*b^8*c*d^2 + 1108536*a^2*b^7*d^3 - 8268
80*a^5*b^4*e^3 + 688128*a^8*b*f^3 - 268736*(11*a^3*b^6*c - 10*a^4*b^5*d)*e^
```

```
2 - 9728*(255*a^5*b^4*c - 238*a^6*b^3*d + 224*a^7*b^2*e)*f^2 - 184756*(21*a
*b^8*c^2 - 36*a^2*b^7*c*d + 16*a^3*b^6*d^2)*e + 2584*(1287*a^2*b^7*c^2 - 22
88*a^3*b^6*c*d + 1040*a^4*b^5*d^2 + 896*a^6*b^3*e^2 + 160*(13*a^4*b^5*c - 1
2*a^5*b^4*d)*e)*f)*x)*sqrt(b*x + a)/b^10
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1622 vs. $2(758) = 1516$.

Time = 2.07 (sec) , antiderivative size = 1622, normalized size of antiderivative = 2.29

$$\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)**3/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((2*(f**3*(a + b*x)**(19/2)/(19*b**9) + (a + b*x)**(17/2)*(-9*a*f*
*3 + 3*b*e*f**2)/(17*b**9) + (a + b*x)**(15/2)*(36*a**2*f**3 - 24*a*b*e*f**
2 + 3*b**2*d*f**2 + 3*b**2*e**2*f)/(15*b**9) + (a + b*x)**(13/2)*(-84*a**3*
f**3 + 84*a**2*b*e*f**2 - 21*a*b**2*d*f**2 - 21*a*b**2*e**2*f + 3*b**3*c*f*
*2 + 6*b**3*d*e*f + b**3*e**3)/(13*b**9) + (a + b*x)**(11/2)*(126*a**4*f**3
 - 168*a**3*b*e*f**2 + 63*a**2*b**2*d*f**2 + 63*a**2*b**2*e**2*f - 18*a*b**
3*c*f**2 - 36*a*b**3*d*e*f - 6*a*b**3*e**3 + 6*b**4*c*e*f + 3*b**4*d**2*f +
 3*b**4*d*e**2)/(11*b**9) + (a + b*x)**(9/2)*(-126*a**5*f**3 + 210*a**4*b*e
*f**2 - 105*a**3*b**2*d*f**2 - 105*a**3*b**2*e**2*f + 45*a**2*b**3*c*f**2 +
 90*a**2*b**3*d*e*f + 15*a**2*b**3*e**3 - 30*a*b**4*c*e*f - 15*a*b**4*d**2*
f - 15*a*b**4*d*e**2 + 6*b**5*c*d*f + 3*b**5*c*e**2 + 3*b**5*d**2*e)/(9*b**
9) + (a + b*x)**(7/2)*(84*a**6*f**3 - 168*a**5*b*e*f**2 + 105*a**4*b**2*d*f
**2 + 105*a**4*b**2*e**2*f - 60*a**3*b**3*c*f**2 - 120*a**3*b**3*d*e*f - 20
*a**3*b**3*e**3 + 60*a**2*b**4*c*e*f + 30*a**2*b**4*d**2*f + 30*a**2*b**4*d
*e**2 - 24*a*b**5*c*d*f - 12*a*b**5*c*e**2 - 12*a*b**5*d**2*e + 3*b**6*c**2
*f + 6*b**6*c*d*e + b**6*d**3)/(7*b**9) + (a + b*x)**(5/2)*(-36*a**7*f**3 +
 84*a**6*b*e*f**2 - 63*a**5*b**2*d*f**2 - 63*a**5*b**2*e**2*f + 45*a**4*b**
3*c*f**2 + 90*a**4*b**3*d*e*f + 15*a**4*b**3*e**3 - 60*a**3*b**4*c*e*f - 30
*a**3*b**4*d**2*f - 30*a**3*b**4*d*e**2 + 36*a**2*b**5*c*d*f + 18*a**2*b**5
*c*e**2 + 18*a**2*b**5*d**2*e - 9*a*b**6*c**2*f - 18*a*b**6*c*d*e - 3*a*b**
6*d**3 + 3*b**7*c**2*e + 3*b**7*c*d**2)/(5*b**9) + (a + b*x)**(3/2)*(9*a**8
*f**3 - 24*a**7*b*e*f**2 + 21*a**6*b**2*d*f**2 + 21*a**6*b**2*e**2*f - 18*a
**5*b**3*c*f**2 - 36*a**5*b**3*d*e*f - 6*a**5*b**3*e**3 + 30*a**4*b**4*c*e*
f + 15*a**4*b**4*d**2*f + 15*a**4*b**4*d*e**2 - 24*a**3*b**5*c*d*f - 12*a**
3*b**5*c*e**2 - 12*a**3*b**5*d**2*e + 9*a**2*b**6*c**2*f + 18*a**2*b**6*c*d
*e + 3*a**2*b**6*d**3 - 6*a*b**7*c**2*e - 6*a*b**7*c*d**2 + 3*b**8*c**2*d)/
(3*b**9) + sqrt(a + b*x)*(-a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**2*d*f**2
 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a**6*b**3*
e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2 + 6*a**4
```

```
*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*f
- 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c*d
**2 - 3*a*b**8*c**2*d + b**9*c**3)/b**9)/b, Ne(b, 0)), ((c**3*x + 3*c**2*d*
x**2/2 + e*f**2*x**9/3 + f**3*x**10/10 + x**8*(3*d*f**2 + 3*e**2*f)/8 + x**
7*(3*c*f**2 + 6*d*e*f + e**3)/7 + x**6*(6*c*e*f + 3*d**2*f + 3*d*e**2)/6 +
x**5*(6*c*d*f + 3*c*e**2 + 3*d**2*e)/5 + x**4*(3*c**2*f + 6*c*d*e + d**3)/4
 + x**3*(3*c**2*e + 3*c*d**2)/3)/sqrt(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(673) = 1346$.

Time = 0.22 (sec) , antiderivative size = 1360, normalized size of antiderivative = 1.92

$$\int \frac{(c + dx + ex^2 + fx^3)^3}{\sqrt{a + bx}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/4849845*(4849845*sqrt(b*x + a)*c^3 + 138567*c^2*(35*((b*x + a)^(3/2) - 3*
sqrt(b*x + a)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqr
t(b*x + a)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b
*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*f/b^3) + 323*c*(3003*(3*(b*x + a)
^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d^2/b^2 + 143*(35*(b*
x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x +
 a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4 + 286*(35*(b*x + a)^(9/2)*f
+ 45*(b*e - 4*a*f)*(b*x + a)^(7/2) - 189*(a*b*e - 2*a^2*f)*(b*x + a)^(5/2)
+ 105*(3*a^2*b*e - 4*a^3*f)*(b*x + a)^(3/2) - 315*(a^3*b*e - a^4*f)*sqrt(b*
x + a))*d/b^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x
 + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693
*sqrt(b*x + a)*a^5)*e*f/b^5 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11
/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a
)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6) +
 138567*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2
- 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 4199*(315*(b*x + a)^(11/2)*f + 385*(b*e -
 5*a*f)*(b*x + a)^(9/2) - 990*(2*a*b*e - 5*a^2*f)*(b*x + a)^(7/2) + 1386*(3
*a^2*b*e - 5*a^3*f)*(b*x + a)^(5/2) - 1155*(4*a^3*b*e - 5*a^4*f)*(b*x + a)^
(3/2) + 3465*(a^4*b*e - a^5*f)*sqrt(b*x + a))*d^2/b^5 + 1615*(231*(b*x + a)
^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x +
a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*s
qrt(b*x + a)*a^6)*e^3/b^6 + 2261*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13
/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x
 + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6
435*sqrt(b*x + a)*a^7)*e^2*f/b^7 + 133*(6435*(b*x + a)^(17/2) - 58344*(b*x
+ a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 +
```

```
 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)
^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*e*f^2/b
^8 + 323*(3003*(b*x + a)^(15/2)*f^2 + 3465*(2*b*e*f - 7*a*f^2)*(b*x + a)^(1
3/2) + 4095*(b^2*e^2 - 12*a*b*e*f + 21*a^2*f^2)*(b*x + a)^(11/2) - 25025*(a
*b^2*e^2 - 6*a^2*b*e*f + 7*a^3*f^2)*(b*x + a)^(9/2) + 32175*(2*a^2*b^2*e^2
- 8*a^3*b*e*f + 7*a^4*f^2)*(b*x + a)^(7/2) - 9009*(10*a^3*b^2*e^2 - 30*a^4*
b*e*f + 21*a^5*f^2)*(b*x + a)^(5/2) + 15015*(5*a^4*b^2*e^2 - 12*a^5*b*e*f +
 7*a^6*f^2)*(b*x + a)^(3/2) - 45045*(a^5*b^2*e^2 - 2*a^6*b*e*f + a^7*f^2)*s
qrt(b*x + a))*d/b^7 + 21*(12155*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*
a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b
*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*
a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqr
t(b*x + a)*a^9)*f^3/b^9)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1412 vs. $2(673) = 1346$.

Time = 0.29 (sec) , antiderivative size = 1412, normalized size of antiderivative = 1.99

$$\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/4849845*(4849845*sqrt(b*x + a)*c^3 + 4849845*((b*x + a)^(3/2) - 3*sqrt(b*
x + a)*a)*c^2*d/b + 969969*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*s
qrt(b*x + a)*a^2)*c*d^2/b^2 + 969969*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2
)*a + 15*sqrt(b*x + a)*a^2)*c^2*e/b^2 + 138567*(5*(b*x + a)^(7/2) - 21*(b*x
 + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 83
1402*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 3
5*sqrt(b*x + a)*a^3)*c*d*e/b^3 + 415701*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(
5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*c^2*f/b^3 + 46189*(
35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*
(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d^2*e/b^4 + 46189*(35*(b*x + a
)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(
3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*c*e^2/b^4 + 92378*(35*(b*x + a)^(9/2) - 1
80*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 +
315*sqrt(b*x + a)*a^4)*c*d*f/b^4 + 20995*(63*(b*x + a)^(11/2) - 385*(b*x +
a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x
 + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2/b^5 + 20995*(63*(b*x + a)^(1
1/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/
2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d^2*f/b^5 + 4199
0*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 -
1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5
```

```
)*c*e*f/b^5 + 1615*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(
b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 -
6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*e^3/b^6 + 9690*(231*(b*x
 + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b
*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3
003*sqrt(b*x + a)*a^6)*d*e*f/b^6 + 4845*(231*(b*x + a)^(13/2) - 1638*(b*x +
 a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(
b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*c*f
^2/b^6 + 2261*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x
+ a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 2
7027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a
^7)*e^2*f/b^7 + 2261*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 1228
5*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*
a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x
 + a)*a^7)*d*f^2/b^7 + 133*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*
a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x
 + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 -
 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*e*f^2/b^8 + 21*(121
55*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a
^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*
(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)
*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*f^3/b^9)/b
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 896, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx$$

$$= \frac{(a+b\,x)^{11/2}\,(252\,a^4\,f^3-336\,a^3\,b\,e\,f^2+126\,a^2\,b^2\,d\,f^2+126\,a^2\,b^2\,e^2\,f-72\,a\,b^3\,d\,e\,f-12\,a\,b^3\,e^3-36\,c\,a\,b^3\,f}{11\,b^{10}}$$

$$+\frac{2\,\sqrt{a+b\,x}\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)^3}{b^{10}}$$

$$+\frac{(a+b\,x)^{9/2}\,(-252\,a^5\,f^3+420\,a^4\,b\,e\,f^2-210\,a^3\,b^2\,d\,f^2-210\,a^3\,b^2\,e^2\,f+180\,a^2\,b^3\,d\,e\,f+30\,a^2\,b^3\,e^3+90}{9\,b^{10}}$$

$$+\frac{2\,f^3\,(a+b\,x)^{19/2}}{19\,b^{10}}$$

$$+\frac{(a+b\,x)^{13/2}\,(-168\,a^3\,f^3+168\,a^2\,b\,e\,f^2-42\,a\,b^2\,e^2\,f-42\,d\,a\,b^2\,f^2+2\,b^3\,e^3+12\,d\,b^3\,e\,f+6\,c\,b^3\,f^2)}{13\,b^{10}}$$

$$-\frac{(18\,a\,f^3-6\,b\,e\,f^2)\,(a+b\,x)^{17/2}}{17\,b^{10}}$$

$$+\frac{(a+b\,x)^{15/2}\,(72\,a^2\,f^3-48\,a\,b\,e\,f^2+6\,b^2\,e^2\,f+6\,d\,b^2\,f^2)}{15\,b^{10}}$$

$$-\frac{(a+b\,x)^{5/2}\,(72\,a^7\,f^3-168\,a^6\,b\,e\,f^2+126\,a^5\,b^2\,d\,f^2+126\,a^5\,b^2\,e^2\,f-90\,a^4\,b^3\,c\,f^2-180\,a^4\,b^3\,d\,e\,f-30\,a}{}$$

$$+\frac{(a+b\,x)^{7/2}\,(168\,a^6\,f^3-336\,a^5\,b\,e\,f^2+210\,a^4\,b^2\,d\,f^2+210\,a^4\,b^2\,e^2\,f-120\,a^3\,b^3\,c\,f^2-240\,a^3\,b^3\,d\,e\,f-4}{}$$

$$+\frac{2\,(a+b\,x)^{3/2}\,(3\,f\,a^2-2\,e\,a\,b+d\,b^2)\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)^2}{b^{10}}$$

```
[In] int((c + d*x + e*x^2 + f*x^3)^3/(a + b*x)^(1/2),x)
```

```
[Out] ((a + b*x)^(11/2)*(252*a^4*f^3 - 12*a*b^3*e^3 + 6*b^4*d*e^2 + 6*b^4*d^2*f +
 126*a^2*b^2*d*f^2 + 126*a^2*b^2*e^2*f + 12*b^4*c*e*f - 36*a*b^3*c*f^2 - 33
6*a^3*b*e*f^2 - 72*a*b^3*d*e*f))/(11*b^10) + (2*(a + b*x)^(1/2)*(b^3*c - a^
3*f - a*b^2*d + a^2*b*e)^3)/b^10 + ((a + b*x)^(9/2)*(6*b^5*c*e^2 - 252*a^5*
f^3 + 6*b^5*d^2*e + 30*a^2*b^3*e^3 + 90*a^2*b^3*c*f^2 - 210*a^3*b^2*d*f^2 -
 210*a^3*b^2*e^2*f + 12*b^5*c*d*f - 30*a*b^4*d*e^2 - 30*a*b^4*d^2*f + 420*a
^4*b*e*f^2 + 180*a^2*b^3*d*e*f - 60*a*b^4*c*e*f))/(9*b^10) + (2*f^3*(a + b*
x)^(19/2))/(19*b^10) + ((a + b*x)^(13/2)*(2*b^3*e^3 - 168*a^3*f^3 + 6*b^3*c
*f^2 + 12*b^3*d*e*f - 42*a*b^2*d*f^2 - 42*a*b^2*e^2*f + 168*a^2*b*e*f^2))/(
13*b^10) - (((18*a*f^3 - 6*b*e*f^2)*(a + b*x)^(17/2))/(17*b^10) + ((a + b*x)
^(15/2)*(72*a^2*f^3 + 6*b^2*d*f^2 + 6*b^2*e^2*f - 48*a*b*e*f^2))/(15*b^10)
- ((a + b*x)^(5/2)*(72*a^7*f^3 + 6*a*b^6*d^3 - 6*b^7*c*d^2 - 6*b^7*c^2*e -
30*a^4*b^3*e^3 - 36*a^2*b^5*c*e^2 - 36*a^2*b^5*d^2*e + 60*a^3*b^4*d*e^2 - 9
0*a^4*b^3*c*f^2 + 60*a^3*b^4*d^2*f + 126*a^5*b^2*d*f^2 + 126*a^5*b^2*e^2*f
+ 18*a*b^6*c^2*f - 168*a^6*b*e*f^2 - 72*a^2*b^5*c*d*f + 120*a^3*b^4*c*e*f -
```

```
 180*a^4*b^3*d*e*f + 36*a*b^6*c*d*e))/(5*b^10) + ((a + b*x)^(7/2)*(2*b^6*d^
3 + 168*a^6*f^3 + 6*b^6*c^2*f - 40*a^3*b^3*e^3 + 60*a^2*b^4*d*e^2 - 120*a^3
*b^3*c*f^2 + 60*a^2*b^4*d^2*f + 210*a^4*b^2*d*f^2 + 210*a^4*b^2*e^2*f + 12*
b^6*c*d*e - 24*a*b^5*c*e^2 - 24*a*b^5*d^2*e - 336*a^5*b*e*f^2 + 120*a^2*b^4
*c*e*f - 240*a^3*b^3*d*e*f - 48*a*b^5*c*d*f))/(7*b^10) + (2*(a + b*x)^(3/2)
*(b^2*d + 3*a^2*f - 2*a*b*e)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e)^2)/b^10
```

3.7 $\int \frac{c+dx}{a+bx^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 161

$$\int \frac{c+dx}{a+bx^3} \, dx = -\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}} -\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/3*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(2/3)-1/6*(c-a^(1
/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)-1/
3*(b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a
^(2/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1874, 31, 648, 631, 210, 642}

$$\int \frac{c+dx}{a+bx^3} \, dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{\sqrt{3}a^{2/3}b^{2/3}} -\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} +\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}$$

```
[In] Int[(c + d*x)/(a + b*x^3),x]
```

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}c+\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}} + \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}} \\
&= \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{2/3}} \\
&\quad + \frac{1}{2}\left(\frac{c}{\sqrt[3]{a}}+\frac{d}{\sqrt[3]{b}}\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx \\
&= \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}} \\
&\quad + \frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}b^{2/3}} \\
&= -\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} \\
&\quad - \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.77

$$
\int \frac{c+dx}{a+bx^3}\,dx
$$

$$
= \frac{-2\sqrt{3}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right.\right.}{6a^{2/3}b^{2/3}}
$$

```
[In] Integrate[(c + d*x)/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + (b^(1/3)*c - a^(1/3)*d)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.75 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_Rd+c\right)\ln\left(x-\_R\right)}{\_R^2}}{3b}$
default	$c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x\right.}{6b\left(\frac{a}{b}\right)}\right.$

```
[In] int((d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b*sum((_R*d+c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 1931, normalized size of antiderivative = 11.99

$$\int \frac{c+dx}{a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d
^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*
d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(
2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)
/(a^2*b^2))^(1/3)))*a*b*c^2 + 2*a*c*d^2 + (b*c^3 + a*d^3)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2
))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^
2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
```

```
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^
(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2)
+ (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1
/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a
*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3 + a*d^3)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a^2*b*d + 2*a*b*c^2
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*
b))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^
3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c
^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^
3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d
^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^
2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3
+ a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*
a^2*b*d + 2*a*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^
2*a*b + 16*c*d)/(a*b)))
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$\int \frac{c+dx}{a+bx^3}\,dx$$
$$= \operatorname{RootSum}\left(27t^3a^2b^2+9tabcd+ad^3-bc^3,\left(t\mapsto t\log\left(x+\frac{9t^2a^2bd+3tabc^2+2acd^2}{ad^3+bc^3}\right)\right)\right)$$

```
[In] integrate((d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*a*b*c*d + a*d**3 - b*c**3, Lambda(_t, _t*
log(x + (9*_t**2*a**2*b*d + 3*_t*a*b*c**2 + 2*a*c*d**2)/(a*d**3 + b*c**3)))
)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

$$\int \frac{c+dx}{a+bx^3}\,dx = \frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(d*(a/b)^(1/3) + c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b*(a/b)^(2/3)) + 1/6*(d*(a/b)^(1/3) - c)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(d*(a/b)^(1/3) - c)*log(x + (a/b)^(1/3
))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int \frac{c+dx}{a+bx^3}\,dx = -\frac{\sqrt{3}\left(bc-\left(-ab^2\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(bc+\left(-ab^2\right)^{\frac{1}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}$$

```
[In] integrate((d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*c + (-a*b^2)^(1/3)*d)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*(d*(-a/b)^(1/3) + c)*(-a/
b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a
```

Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int \frac{c+dx}{a+bx^3}\,dx = \sum_{k=1}^{3} \ln\left(b\left(c\,d+d^2\,x+\operatorname{root}\left(27\,a^2\,b^2\,z^3+9\,a\,b\,c\,d\,z+a\,d^3-b\,c^3,z,k\right)^2 a\,b\,9 + \operatorname{root}\left(27\,a^2\,b^2\,z^3+9\,a\,b\,c\,d\,z+a\,d^3-b\,c^3,z,k\right)\,b\,c\,x\,3\right)\right)\operatorname{root}\left(27\,a^2\,b^2\,z^3 + 9\,a\,b\,c\,d\,z+a\,d^3-b\,c^3,z,k\right)$$

```
[In] int((c + d*x)/(a + b*x^3),x)
```

```
[Out] symsum(log(b*(c*d + d^2*x + 9*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b
*c^3, z, k)^2*a*b + 3*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3, z,
 k)*b*c*x))*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3, z, k), k, 1,
 3)
```

3.8 $\int \frac{c+dx}{\left(a+bx^3\right)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 189

$$\begin{aligned}\int \frac{c+dx}{\left(a+bx^3\right)^2} dx =& \frac{x(c+dx)}{3a\left(a+bx^3\right)} - \frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} \\ &+ \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} \\ &- \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}\end{aligned}$$

```
[Out] 1/3*x*(d*x+c)/a/(b*x^3+a)+1/9*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)
/a^(5/3)/b^(2/3)-1/18*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/a^(5/3)/b^(2/3)-1/9*(2*b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1869, 1874, 31, 648, 631, 210, 642}

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)}{3\sqrt{3}a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} + \frac{x(c+dx)}{3a\,(a+bx^3)}$$

```
[In] Int[(c + d*x)/(a + b*x^3)^2,x]
```

```
[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5
/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(c+dx)}{3a\,(a+bx^3)} - \frac{\int \frac{-2c-dx}{a+bx^3}\,dx}{3a} \\
&= \frac{x(c+dx)}{3a\,(a+bx^3)} - \frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}c-\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}\sqrt[3]{b}} + \frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}} \\
&= \frac{x(c+dx)}{3a\,(a+bx^3)} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} \\
&\quad - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{2/3}} \\
&\quad + \frac{\left(2c+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}}
\end{aligned}$$

$$= \frac{x(c+dx)}{3a(a+bx^3)} + \frac{\left(2\sqrt[3]{b}c - \sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c - \sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c + \sqrt[3]{a}d\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}b^{2/3}}$$

$$= \frac{x(c+dx)}{3a(a+bx^3)} - \frac{\left(2\sqrt[3]{b}c + \sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c - \sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c - \sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.95

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx$$

$$= \frac{\frac{6ax(c+dx)}{a+bx^3} - \frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{\left(-2\sqrt[3]{a}\sqrt[3]{b}c+a^{2/3}d\right)\log\left(\right.}{b^{2/}}}{18a^2}$$

```
[In] Integrate[(c + d*x)/(a + b*x^3)^2,x]
```

```
[Out] ((6*a*x*(c + d*x))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*c + a^(1/3)*
d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(2*a^(1/3)*b^(
1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-2*a^(1/3)*b^(1/3
)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(
18*a^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.78 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\frac{d x^2}{3a}+\frac{cx}{3a}}{b x^3+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_Rd+2c\right)\ln\left(x-\_R\right)}{\_R^2}}{9ba}$
default	$c\left(\frac{x}{3a(b x^3+a)}+\frac{\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}\right)+d\left(\frac{x^2}{3a(b x^3+a)}+\frac{-\frac{\ln\left(x+\left(\frac{a}{b}\right.\right.}{3b\left(\frac{a}{b}\right.}\right.$

```
[In] int((d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/3*d/a*x^2+1/3*c/a*x)/(b*x^3+a)+1/9/b/a*sum((_R*d+2*c)/_R^2*ln(x-_R),_R=R
ootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 2088, normalized size of antiderivative = 11.05

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*d*x^2 - 2*(a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
 a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*
(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^
5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^
5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3)
- 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/
3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2
) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/
(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*
```

```
a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) + 12*c*x + ((a*b*x^3 + a^2)*((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) + 3*sqrt(1/3)*(a*b*x^
3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) +
 (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^
3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a
^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(
I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3
)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqr
t(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2)
)^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d^3)*x + 3/4*sqrt(1/3)*(((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c
^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8
*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b +
32*c*d)/(a^3*b))) + ((a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(
a^5*b^2))^(1/3))) - 3*sqrt(1/3)*(a*b*x^3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt
(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)
+ 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (
8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/
(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*
(8*b*c^3 + a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(
a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1
)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/
2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3
 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b))))/(a*b*x^3 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.56

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx$$
$$=\operatorname{RootSum}\left(729t^3a^5b^2+54ta^2bcd+ad^3-8bc^3,\left(t\mapsto t\log\left(x+\frac{81t^2a^4bd+36ta^2bc^2+4acd^2}{ad^3+8bc^3}\right)\right)\right)$$
$$+\frac{cx+dx^2}{3a^2+3abx^3}$$

```
[In] integrate((d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda(
_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d**3
 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*a*b*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.89

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx=\frac{dx^2+cx}{3\,(abx^3+a^2)}+\frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(d*x^2 + c*x)/(a*b*x^3 + a^2) + 1/9*sqrt(3)*(d*(a/b)^(1/3) + 2*c)*arcta
n(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) + 1/18*(d*
(a/b)^(1/3) - 2*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/3))
 - 1/9*(d*(a/b)^(1/3) - 2*c)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.92

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(2\,bc-(-ab^2)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{\left(2\,bc+(-ab^2)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{dx^2+cx}{3\,(bx^3+a)a}$$

```
[In] integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + (-a*b^2)^(1/3)*d)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(d*(-a/b)^(1
/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(d*x^2 + c*x)/
((b*x^3 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.89

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx = \left(\sum_{k=1}^{3}\ln\left(\frac{b\left(2\,c\,d+d^2\,x+\mathrm{root}(729\,a^5\,b^2\,z^3+54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k)^2\,a^3\,b\,81+\mathrm{root}(729\,a^5\,b^2\,z^3+\dots}{a^2\,9}\right.\right.$$
$$\left.\left.+54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k\right)\right)+\frac{\frac{d\,x^2}{3\,a}+\frac{c\,x}{3\,a}}{b\,x^3+a}$$

```
[In] int((c + d*x)/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((b*(2*c*d + d^2*x + 81*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z - 8
*b*c^3 + a*d^3, z, k)^2*a^3*b + 18*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z -
8*b*c^3 + a*d^3, z, k)*a*b*c*x))/(9*a^2))*root(729*a^5*b^2*z^3 + 54*a^2*b*c
*d*z - 8*b*c^3 + a*d^3, z, k), k, 1, 3) + ((d*x^2)/(3*a) + (c*x)/(3*a))/(a
+ b*x^3)
```

3.9 $\int \frac{c+dx}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 215

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx = \frac{x(c+dx)}{6a(a+bx^3)^2}+\frac{x(5c+4dx)}{18a^2(a+bx^3)}-\frac{\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}$$

```
[Out] 1/6*x*(d*x+c)/a/(b*x^3+a)^2+1/18*x*(4*d*x+5*c)/a^2/(b*x^3+a)+1/27*(5*b^(1/3
)*c-2*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(2/3)-1/54*(5*b^(1/3)*c-2*
a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(2/3)-1/27*(
5*b^(1/3)*c+2*a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/
a^(8/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used

= {1869, 1874, 31, 648, 631, 210, 642}

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)}{9\sqrt{3}a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} + \frac{x(5c+4dx)}{18a^2(a+bx^3)} + \frac{x(c+dx)}{6a(a+bx^3)^2}$$

```
[In] Int[(c + d*x)/(a + b*x^3)^3,x]
```

```
[Out] (x*(c + d*x))/(6*a*(a + b*x^3)^2) + (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3))
- ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(1/
3) + b^(1/3)*x])/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(c+dx)}{6a\left(a+bx^3\right)^2}-\frac{\int \frac{-5c-4dx}{\left(a+bx^3\right)^2}\,dx}{6a}\\
&= \frac{x(c+dx)}{6a\left(a+bx^3\right)^2}+\frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}+\frac{\int \frac{10c+4dx}{a+bx^3}\,dx}{18a^2}\\
&= \frac{x(c+dx)}{6a\left(a+bx^3\right)^2}+\frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}+\frac{\int \frac{\sqrt[3]{a}\left(20\sqrt[3]{b}c+4\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-10\sqrt[3]{b}c+4\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}\sqrt[3]{b}}\\
&\quad+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{8/3}}\\
&= \frac{x(c+dx)}{6a\left(a+bx^3\right)^2}+\frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}\\
&\quad-\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}b^{2/3}}\\
&\quad+\frac{\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{7/3}\sqrt[3]{b}}
\end{aligned}$$

$$= \frac{x(c+dx)}{6a(a+bx^3)^2} + \frac{x(5c+4dx)}{18a^2(a+bx^3)} + \frac{\left(5\sqrt[3]{b}c - 2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}c - 2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}} + \frac{\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{8/3}b^{2/3}}$$

$$= \frac{x(c+dx)}{6a(a+bx^3)^2} + \frac{x(5c+4dx)}{18a^2(a+bx^3)} - \frac{\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}} + \frac{\left(5\sqrt[3]{b}c - 2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}c - 2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.95

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx$$

$$= \frac{\frac{9a^2x(c+dx)}{(a+bx^3)^2} + \frac{3ax(5c+4dx)}{a+bx^3} - \frac{2\sqrt{3}\sqrt[3]{a}\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{2\left(5\sqrt[3]{a}\sqrt[3]{b}c-2a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{\left(-5\sqrt[3]{a}\right.}{}}{54a^3}$$

```
[In] Integrate[(c + d*x)/(a + b*x^3)^3,x]
```

```
[Out] ((9*a^2*x*(c + d*x))/(a + b*x^3)^2 + (3*a*x*(5*c + 4*d*x))/(a + b*x^3) - (2
*Sqrt[3]*a^(1/3)*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(2/3) + (2*(5*a^(1/3)*b^(1/3)*c - 2*a^(2/3)*d)*Log[a^(1/3)
 + b^(1/3)*x])/b^(2/3) + ((-5*a^(1/3)*b^(1/3)*c + 2*a^(2/3)*d)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(54*a^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.80 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{2bd\,x^{5}}{9a^{2}}+\frac{5bc\,x^{4}}{18a^{2}}+\frac{7d\,x^{2}}{18a}+\frac{4cx}{9a}}{\left(b\,x^{3}+a\right)^{2}}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(2\_Rd+5c\right)\ln\left(x-\_R\right)}{\_R^{2}}}{27a^{2}b}$
default	$c\left(\frac{x}{6a\left(b\,x^{3}+a\right)^{2}}+\frac{\frac{5x}{18a\left(b\,x^{3}+a\right)}+\frac{5\left(\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{6a}}{a}\right)+d\left(\frac{x}{6a(b\,x}\right.$

[In] int((d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] (2/9*b*d/a^2*x^5+5/18*b*c/a^2*x^4+7/18*d/a*x^2+4/9*c/a*x)/(b*x^3+a)^2+1/27/
a^2/b*sum((2*_R*d+5*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 2215, normalized size of antiderivative = 10.30

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

[In] integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/108*(24*b*d*x^5 + 30*b*c*x^4 + 42*a*d*x^2 + 48*a*c*x - 2*(a^2*b^2*x^6 + 2
*a^3*b*x^3 + a^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*

```
b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt
(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a
^8*b^2))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3
)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(
-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*
d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d - 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125
*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(
1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (
125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 + 40*a*c*d^2 + (125*b*c^3
 + 8*a*d^3)*x) + ((a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(
1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a
^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3))) + 3*sqrt(1/3)*(a^2*b^2*x
^6 + 2*a^3*b*x^3 + a^4)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8
*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)
*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b)))*log(-1/2*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d
^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c
^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*
d + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (1
25*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/
(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^
(1/3)))*a^3*b*c^2 - 40*a*c*d^2 + 2*(125*b*c^3 + 8*a*d^3)*x + 3/2*sqrt(1/3)*
(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*(
(125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*
a^6*b*d + 25*a^3*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8
*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)
*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b))) + ((a^2*b^2*x^6
 + 2*a^3*b*x^3 + a^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a
^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*s
qrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)
/(a^8*b^2))^(1/3))) - 3*sqrt(1/3)*(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*sqrt(-(
((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((
125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2
*a^5*b + 160*c*d)/(a^5*b)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c
^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)
^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*
b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d + 25/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^
(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(
a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 - 40*a*c*d^2
```

```
+ 2*(125*b*c^3 + 8*a*d^3)*x - 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(
(125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) -
20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2)
 + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^6*b*d + 25*a^3*b*c^2)*sqrt(-(
((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
- 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((
125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2
*a^5*b + 160*c*d)/(a^5*b))))/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.68

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx = \text{RootSum}\left(19683t^3a^8b^2+810ta^3bcd+8ad^3-125bc^3, \left(t\mapsto t\log\left(x+\frac{1458t^2a^6bd+675ta^3bc^2+40acd^2}{8ad^3+125bc^3}\right)\right)\right) + \frac{8acx+7adx^2+5bcx^4+4bdx^5}{18a^4+36a^3bx^3+18a^2b^2x^6}$$

```
[In] integrate((d*x+c)/(b*x**3+a)**3,x)
[Out] RootSum(19683*_t**3*a**8*b**2 + 810*_t*a**3*b*c*d + 8*a*d**3 - 125*b*c**3,
Lambda(_t, _t*log(x + (1458*_t**2*a**6*b*d + 675*_t*a**3*b*c**2 + 40*a*c*d*
*2)/(8*a*d**3 + 125*b*c**3)))) + (8*a*c*x + 7*a*d*x**2 + 5*b*c*x**4 + 4*b*d
*x**5)/(18*a**4 + 36*a**3*b*x**3 + 18*a**2*b**2*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx = \frac{4\,bdx^5+5\,bcx^4+7\,adx^2+8\,acx}{18\,(a^2b^2x^6+2\,a^3bx^3+a^4)} + \frac{\sqrt{3}\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*b*d*x^5 + 5*b*c*x^4 + 7*a*d*x^2 + 8*a*c*x)/(a^2*b^2*x^6 + 2*a^3*b*x
^3 + a^4) + 1/27*sqrt(3)*(2*d*(a/b)^(1/3) + 5*c)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) + 1/54*(2*d*(a/b)^(1/3) - 5*c
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1/27*(2*d*(a
/b)^(1/3) - 5*c)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.90

$$\int \frac{c+dx}{\left(a+bx^3\right)^3}\,dx = -\frac{\sqrt{3}\left(5\,bc-2\left(-ab^2\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^2} - \frac{\left(5\,bc+2\left(-ab^2\right)^{\frac{1}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^2} - \frac{\left(2\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3} + \frac{4\,bdx^5+5\,bcx^4+7\,adx^2+8\,acx}{18\left(bx^3+a\right)^2a^2}$$

```
[In] integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(5*b*c - 2*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(5*b*c + 2*(-a*b^2)^(1/3)
*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(2
*d*(-a/b)^(1/3) + 5*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/18*(
4*b*d*x^5 + 5*b*c*x^4 + 7*a*d*x^2 + 8*a*c*x)/((b*x^3 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.96

$$\int \frac{c+dx}{\left(a+bx^3\right)^3}\,dx = \frac{\frac{7\,d\,x^2}{18\,a}+\frac{4\,c\,x}{9\,a}+\frac{5\,b\,c\,x^4}{18\,a^2}+\frac{2\,b\,d\,x^5}{9\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \left(\sum_{k=1}^{3}\ln\left(\frac{b\left(10\,c\,d+4\,d^2\,x+\mathrm{root}(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k)^2\,a^5\,b\,729+\mathrm{root}\right.}{a^4\,81}\right.\right.$$

$$\left.\left. + 810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k)\right)\right)$$

```
[In] int((c + d*x)/(a + b*x^3)^3,x)
```

```
[Out] ((7*d*x^2)/(18*a) + (4*c*x)/(9*a) + (5*b*c*x^4)/(18*a^2) + (2*b*d*x^5)/(9*a
^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((b*(10*c*d + 4*d^2*x + 729*ro
ot(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a*d^3, z, k)^2*a^5*b
 + 135*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a*d^3, z, k
)*a^2*b*c*x))/(81*a^4))*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^
3 + 8*a*d^3, z, k), k, 1, 3)
```

3.10 $\int \frac{c+dx}{(a+bx^3)^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 240

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx = \frac{x(c+dx)}{9a(a+bx^3)^3} + \frac{x(8c+7dx)}{54a^2(a+bx^3)^2} + \frac{2x(10c+7dx)}{81a^3(a+bx^3)} - \frac{2\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}} + \frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} - \frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}$$

```
[Out] 1/9*x*(d*x+c)/a/(b*x^3+a)^3+1/54*x*(7*d*x+8*c)/a^2/(b*x^3+a)^2+2/81*x*(7*d*
x+10*c)/a^3/(b*x^3+a)+2/243*(20*b^(1/3)*c-7*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x
)/a^(11/3)/b^(2/3)-1/243*(20*b^(1/3)*c-7*a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(2/3)-2/243*(20*b^(1/3)*c+7*a^(1/3)*d)*arctan
(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used

= {1869, 1874, 31, 648, 631, 210, 642}

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx = -\frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)}{81\sqrt{3}a^{11/3}b^{2/3}} - \frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}} + \frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} + \frac{2x(10c+7dx)}{81a^3(a+bx^3)} + \frac{x(8c+7dx)}{54a^2(a+bx^3)^2} + \frac{x(c+dx)}{9a(a+bx^3)^3}$$

```
[In] Int[(c + d*x)/(a + b*x^3)^4,x]
```

```
[Out] (x*(c + d*x))/(9*a*(a + b*x^3)^3) + (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2
) + (2*x*(10*c + 7*d*x))/(81*a^3*(a + b*x^3)) - (2*(20*b^(1/3)*c + 7*a^(1/3
)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(11/3
)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(243
*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3}-\frac{\int \frac{-8c-7dx}{\left(a+bx^3\right)^3}\,dx}{9a}\\
&= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3}+\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{\int \frac{40c+28dx}{\left(a+bx^3\right)^2}\,dx}{54a^2}\\
&= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3}+\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}-\frac{\int \frac{-80c-28dx}{a+bx^3}\,dx}{162a^3}\\
&= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3}+\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}\\
&\quad -\frac{\int \frac{\sqrt[3]{a}\left(-160\sqrt[3]{b}c-28\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(80\sqrt[3]{b}c-28\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{486a^{11/3}\sqrt[3]{b}}\\
&\quad +\frac{\left(2\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{243a^{11/3}}
\end{aligned}
$$

$$=\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}+\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{243a^{11/3}b^{2/3}}+\frac{\left(20c+\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{81a^{10/3}}$$

$$=\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}+\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{\left(2\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{81a^{11/3}b^{2/3}}$$

$$=\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}+\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}-\frac{2\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

$$\int\frac{c+dx}{\left(a+bx^3\right)^4}dx$$

$$=\frac{\frac{54a^3x(c+dx)}{(a+bx^3)^3}+\frac{9a^2x(8c+7dx)}{(a+bx^3)^2}+\frac{12ax(10c+7dx)}{a+bx^3}-\frac{4\sqrt{3}\sqrt[3]{a}\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{4\left(20\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(\right.}{b^{2/3}}}{486a^4}$$

```
[In] Integrate[(c + d*x)/(a + b*x^3)^4,x]
```

```
[Out] ((54*a^3*x*(c + d*x))/(a + b*x^3)^3 + (9*a^2*x*(8*c + 7*d*x))/(a + b*x^3)^2
 + (12*a*x*(10*c + 7*d*x))/(a + b*x^3) - (4*Sqrt[3]*a^(1/3)*(20*b^(1/3)*c +
 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (4*(20
*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (2*(-
20*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/b^(2/3))/(486*a^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 9.84 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.46

method	result
risch	$\frac{\frac{14d\,b^2x^8}{81a^3}+\frac{20c\,b^2x^7}{81a^3}+\frac{77bd\,x^5}{162a^2}+\frac{52bc\,x^4}{81a^2}+\frac{67d\,x^2}{162a}+\frac{41cx}{81a}}{(b\,x^3+a)^3}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(7\_Rd+20c\right)\ln\left(x-\_R\right)}{\_R^2}\right)}{243a^3b}$
default	$c\left(\frac{x}{9a(b\,x^3+a)^3}+\frac{\frac{4x}{27a\left(b\,x^3+a\right)^2}+\frac{8\left(\frac{5x}{18a\left(b\,x^3+a\right)}+\frac{5\left(\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{6a}\right)}{9a}}{a}\right.$

```
[In] int((d*x+c)/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (14/81*d/a^3*b^2*x^8+20/81*c/a^3*b^2*x^7+77/162*b*d/a^2*x^5+52/81*b*c/a^2*x
^4+67/162*d/a*x^2+41/81*c/a*x)/(b*x^3+a)^3+2/243/a^3/b*sum((7*_R*d+20*c)/_R
^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 2308, normalized size of antiderivative = 9.62

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(168*b^2*d*x^8 + 240*b^2*c*x^7 + 462*a*b*d*x^5 + 624*a*b*c*x^4 + 402*
a^2*d*x^2 + 492*a^2*c*x - 2*(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^
6)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*
c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*
b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2
))^(1/3)))*log(7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sq
rt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*
a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d - 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*
b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)
- 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^4*b*c^2 + 7840*a*c*d^2
 + 4*(8000*b*c^3 + 343*a*d^3)*x) + ((a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*
x^3 + a^6)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) +
(8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) +
1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(
a^11*b^2))^(1/3))) + 3*sqrt(1/3)*(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3
 + a^6)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2
) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3
) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^
3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b)))*log(-7/4*(4^(1/3)*(I*s
qrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)
/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 +
 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^8
*b*d + 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) +
(8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) +
1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(
a^11*b^2))^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2 + 8*(8000*b*c^3 + 343*a*d^3)*x
+ 3/4*sqrt(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11
```

```
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sq
rt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*
a*d^3)/(a^11*b^2))^(1/3)))*a^8*b*d + 1600*a^4*b*c^2)*sqrt(-((4^(1/3)*(I*sqr
t(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(
a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 3
43*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b
 + 8960*c*d)/(a^7*b))) + ((a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)
*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*
((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))
^(1/3))) - 3*sqrt(1/3)*(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)*sq
rt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*
b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^
7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b
^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b)))*log(-7/4*(4^(1/3)*(I*sqrt(3) + 1
)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2
))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3
)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d + 400
*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*
((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))
^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2 + 8*(8000*b*c^3 + 343*a*d^3)*x - 3/4*sqrt
(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8
000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)
/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^
11*b^2))^(1/3)))*a^8*b*d + 1600*a^4*b*c^2)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*
((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))
^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/
(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*
d)/(a^7*b))))/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.77

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx$$

$$=\operatorname{RootSum}\left(14348907t^3a^{11}b^2+408240ta^4bcd+2744ad^3-64000bc^3,\left(t\mapsto t\log\left(x+\frac{413343t^2a^8bd+1944}{1372ad^3+}\right.\right.\right.$$

$$+\frac{82a^2cx+67a^2dx^2+104abcx^4+77abdx^5+40b^2cx^7+28b^2dx^8}{162a^6+486a^5bx^3+486a^4b^2x^6+162a^3b^3x^9}$$

```
[In] integrate((d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**11*b**2 + 408240*_t*a**4*b*c*d + 2744*a*d**3 - 64
000*b*c**3, Lambda(_t, _t*log(x + (413343*_t**2*a**8*b*d + 194400*_t*a**4*b
*c**2 + 7840*a*c*d**2)/(1372*a*d**3 + 32000*b*c**3)))) + (82*a**2*c*x + 67*
a**2*d*x**2 + 104*a*b*c*x**4 + 77*a*b*d*x**5 + 40*b**2*c*x**7 + 28*b**2*d*x
**8)/(162*a**6 + 486*a**5*b*x**3 + 486*a**4*b**2*x**6 + 162*a**3*b**3*x**9)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.99

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx = \frac{28\,b^2dx^8+40\,b^2cx^7+77\,abdx^5+104\,abcx^4+67\,a^2dx^2+82\,a^2cx}{162\,(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6)} + \frac{2\,\sqrt{3}\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^3b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^3b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{2\,\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^3b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/162*(28*b^2*d*x^8 + 40*b^2*c*x^7 + 77*a*b*d*x^5 + 104*a*b*c*x^4 + 67*a^2*
d*x^2 + 82*a^2*c*x)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6) + 2/2
43*sqrt(3)*(7*d*(a/b)^(1/3) + 20*c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/
(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) + 1/243*(7*d*(a/b)^(1/3) - 20*c)*log(x^2 -
 x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) - 2/243*(7*d*(a/b)^(1/3)
- 20*c)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.91

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx = -\frac{2\sqrt{3}\left(20\,bc-7\,(-ab^2)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,(-ab^2)^{\frac{2}{3}}\,a^3} - \frac{\left(20\,bc+7\,(-ab^2)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,(-ab^2)^{\frac{2}{3}}\,a^3} - \frac{2\left(7\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^4} + \frac{28\,b^2dx^8+40\,b^2cx^7+77\,abdx^5+104\,abcx^4+67\,a^2dx^2+82\,a^2cx}{162\,(bx^3+a)^3a^3}$$

```
[In] integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -2/243*sqrt(3)*(20*b*c - 7*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/243*(20*b*c + 7*(-a*b^2)^(
1/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 2/2
43*(7*d*(-a/b)^(1/3) + 20*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 +
1/162*(28*b^2*d*x^8 + 40*b^2*c*x^7 + 77*a*b*d*x^5 + 104*a*b*c*x^4 + 67*a^2*
d*x^2 + 82*a^2*c*x)/((b*x^3 + a)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx = \left(\sum_{k=1}^{3}\ln\left(\frac{b\left(560\,c\,d+196\,d^2\,x+\mathrm{root}(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k)^2\right.}{a^6} + 408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k)\right)\right) + \frac{\frac{67\,d\,x^2}{162\,a}+\frac{41\,c\,x}{81\,a}+\frac{20\,b^2\,c\,x^7}{81\,a^3}+\frac{14\,b^2\,d\,x^8}{81\,a^3}+\frac{52\,b\,c\,x^4}{81\,a^2}+\frac{77\,b\,d\,x^5}{162\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9}$$

```
[In] int((c + d*x)/(a + b*x^3)^4,x)
```

```
[Out] symsum(log((b*(560*c*d + 196*d^2*x + 59049*root(14348907*a^11*b^2*z^3 + 408
240*a^4*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k)^2*a^7*b + 9720*root(14348
907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k)*a^3
```

```
*b*c*x))/(6561*a^6))*root(14348907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 6400
0*b*c^3 + 2744*a*d^3, z, k), k, 1, 3) + ((67*d*x^2)/(162*a) + (41*c*x)/(81*
a) + (20*b^2*c*x^7)/(81*a^3) + (14*b^2*d*x^8)/(81*a^3) + (52*b*c*x^4)/(81*a
^2) + (77*b*d*x^5)/(162*a^2))/(a^3 + b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2*x^6)
```

3.11 $\int \frac{a+bx}{d+ex^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 161

$$\int \frac{a+bx}{d+ex^3} \, dx = -\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}} - \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}} - \frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}$$

```
[Out] -1/3*(b*d^(1/3)-a*e^(1/3))*ln(d^(1/3)+e^(1/3)*x)/d^(2/3)/e^(2/3)-1/6*(a-b*d
^(1/3)/e^(1/3))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/e^(1/3)-1
/3*(b*d^(1/3)+a*e^(1/3))*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/
d^(2/3)/e^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1874, 31, 648, 631, 210, 642}

$$\int \frac{a+bx}{d+ex^3} \, dx = -\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}} - \frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}} - \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}$$

```
[In] Int[(a + b*x)/(d + e*x^3),x]
```

```
[Out] -(((b*d^(1/3) + a*e^(1/3))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))
])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) - a*e^(1/3))*Log[d^(1/3) + e^(1
/3)*x])/(3*d^(2/3)*e^(2/3)) - ((a - (b*d^(1/3))/e^(1/3))*Log[d^(2/3) - d^(1
/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\text{integral} = \frac{\left(a - \frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{3d^{2/3}} + \frac{\int \frac{\sqrt[3]{d}\left(b\sqrt[3]{d}+2a\sqrt[3]{e}\right)+\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{3d^{2/3}\sqrt[3]{e}}$$

$$= \frac{\left(a - \frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{1}{2}\left(\frac{a}{\sqrt[3]{d}} + \frac{b}{\sqrt[3]{e}}\right)\int \frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx$$
$$+ \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\int \frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{6d^{2/3}e^{2/3}}$$

$$= \frac{\left(a - \frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}}$$
$$+ \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{d^{2/3}e^{2/3}}$$

$$= -\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}} + \frac{\left(a - \frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$
$$+ \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

$$\int \frac{a+bx}{d+ex^3}\,dx$$
$$= \frac{-2\sqrt{3}\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right) - \left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\left(2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right) - \log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/}\right.\right.}{6d^{2/3}e^{2/3}}$$

```
[In] Integrate[(a + b*x)/(d + e*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b*d^(1/3) + a*e^(1/3))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt
[3]] - (b*d^(1/3) - a*e^(1/3))*(2*Log[d^(1/3) + e^(1/3)*x] - Log[d^(2/3) -
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)*e^(2/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

method	result
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(e\_Z^3+d\right)}\frac{\left(\_Rb+a\right)\ln\left(x-\_R\right)}{\_R^2}}{3e}$
default	$a\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)+b\left(-\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{d}{e}\right.\right.}{6e\left(\right.}\right.$

```
[In] int((b*x+a)/(e*x^3+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/e*sum((_R*b+a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e+d))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 1961, normalized size of antiderivative = 12.18

$$\int \frac{a+bx}{d+ex^3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3
*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^
3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)
 - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^
3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(
2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)
/(d^2*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d + (b^3*d + a^3*e)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2
))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^
2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
```

```
*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)
 - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^
3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e + 16*a*b)/(d*e)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^
(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2)
- (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1
/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a
^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e - 2*a*b^2*d + 2*(b^3*d + a^3*e)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d -
 a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d
+ a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*b*d^2*e + 2*a^2*d*e
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d -
 a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d
+ a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e + 16*a*b)/(d*
e))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*
d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3
*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3))) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*
e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3
*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e + 16*a*b)/(d*e)))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d -
 a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d
+ a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2
*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^
2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e - 2*a*b^2*d + 2*(b^3*d
+ a^3*e)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(
d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3)
 + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*
b*d^2*e + 2*a^2*d*e)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(
d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3)
 + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^
2*d*e + 16*a*b)/(d*e)))
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$\int \frac{a+bx}{d+ex^3}\,dx$$
$$=\mathrm{RootSum}\left(27t^3d^2e^2+9tabde-a^3e+b^3d,\left(t\mapsto t\log\left(x+\frac{9t^2bd^2e+3ta^2de+2ab^2d}{a^3e+b^3d}\right)\right)\right)$$

```
[In] integrate((b*x+a)/(e*x**3+d),x)
```

```
[Out] RootSum(27*_t**3*d**2*e**2 + 9*_t*a*b*d*e - a**3*e + b**3*d, Lambda(_t, _t*
log(x + (9*_t**2*b*d**2*e + 3*_t*a**2*d*e + 2*a*b**2*d)/(a**3*e + b**3*d)))
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx}{d+ex^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x+a)/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int \frac{a+bx}{d+ex^3}\,dx = -\frac{\sqrt{3}\left(ae-\left(-de^2\right)^{\frac{1}{3}}b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\left(-de^2\right)^{\frac{2}{3}}} - \frac{\left(ae+\left(-de^2\right)^{\frac{1}{3}}b\right)\log\left(x^2+x\left(-\frac{d}{e}\right)^{\frac{1}{3}}+\left(-\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(-de^2\right)^{\frac{2}{3}}} - \frac{\left(b\left(-\frac{d}{e}\right)^{\frac{1}{3}}+a\right)\left(-\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,d}$$

```
[In] integrate((b*x+a)/(e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(a*e - (-d*e^2)^(1/3)*b)*arctan(1/3*sqrt(3)*(2*x + (-d/e)^(1/3
))/(-d/e)^(1/3))/(-d*e^2)^(2/3) - 1/6*(a*e + (-d*e^2)^(1/3)*b)*log(x^2 + x*
(-d/e)^(1/3) + (-d/e)^(2/3))/(-d*e^2)^(2/3) - 1/3*(b*(-d/e)^(1/3) + a)*(-d/
e)^(1/3)*log(abs(x - (-d/e)^(1/3)))/d
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int \frac{a+bx}{d+ex^3}\,dx = \sum_{k=1}^{3} \ln\left(e\left(a\,b+b^2\,x+\mathrm{root}\left(27\,d^2\,e^2\,z^3+9\,a\,b\,d\,e\,z+b^3\,d-a^3\,e,z,k\right)^2\,d\,e\,9 + \mathrm{root}\left(27\,d^2\,e^2\,z^3+9\,a\,b\,d\,e\,z+b^3\,d-a^3\,e,z,k\right)\,a\,e\,x\,3\right)\right)\mathrm{root}\left(27\,d^2\,e^2\,z^3 + 9\,a\,b\,d\,e\,z+b^3\,d-a^3\,e,z,k\right)$$

```
[In] int((a + b*x)/(d + e*x^3),x)
```

```
[Out] symsum(log(e*(a*b + b^2*x + 9*root(27*d^2*e^2*z^3 + 9*a*b*d*e*z + b^3*d - a
^3*e, z, k)^2*d*e + 3*root(27*d^2*e^2*z^3 + 9*a*b*d*e*z + b^3*d - a^3*e, z,
 k)*a*e*x))*root(27*d^2*e^2*z^3 + 9*a*b*d*e*z + b^3*d - a^3*e, z, k), k, 1,
 3)
```

3.12 $\int \frac{a+bx}{d-ex^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 161

$$\int \frac{a+bx}{d-ex^3}\,dx = -\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}} - \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}} + \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}}$$

```
[Out] -1/3*(b*d^(1/3)+a*e^(1/3))*ln(d^(1/3)-e^(1/3)*x)/d^(2/3)/e^(2/3)+1/6*(b*d^(
1/3)+a*e^(1/3))*ln(d^(2/3)+d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/e^(2/3)-1
/3*(b*d^(1/3)-a*e^(1/3))*arctan(1/3*(d^(1/3)+2*e^(1/3)*x)/d^(1/3)*3^(1/2))/
d^(2/3)/e^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1875, 31, 648, 631, 210, 642}

$$\int \frac{a+bx}{d-ex^3}\,dx = -\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}} + \frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}} - \frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}$$

```
[In] Int[(a + b*x)/(d - e*x^3),x]
```

```
[Out] -(((b*d^(1/3) - a*e^(1/3))*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))
])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) + a*e^(1/3))*Log[d^(1/3) - e^(1
/3)*x])/(3*d^(2/3)*e^(2/3)) + ((b*d^(1/3) + a*e^(1/3))*Log[d^(2/3) + d^(1/3
)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1875

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Dist[r*((B*r + A*s)/(3*a*
s)), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s*(B*
r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && N
eQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rubi steps

$$\text{integral} = \frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int\frac{1}{\sqrt[3]{d}-\sqrt[3]{e}x}\,dx}{3d^{2/3}} - \frac{\int\frac{\sqrt[3]{d}\left(b\sqrt[3]{d}-2a\sqrt[3]{e}\right)-\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\sqrt[3]{e}x}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{3d^{2/3}\sqrt[3]{e}}$$

$$= -\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}} - \frac{1}{2}\left(-\frac{a}{\sqrt[3]{d}}+\frac{b}{\sqrt[3]{e}}\right)\int\frac{1}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx + \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\int\frac{\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{6d^{2/3}e^{2/3}}$$

$$= -\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}} + \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}} + \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{d^{2/3}e^{2/3}}$$

$$= -\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}} - \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}} + \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

$$\int\frac{a+bx}{d-ex^3}\,dx = \frac{-2\sqrt{3}\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\arctan\left(\frac{1+\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)-\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\left(2\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)-\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e\right.\right.}{6d^{2/3}e^{2/3}}$$

```
[In] Integrate[(a + b*x)/(d - e*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b*d^(1/3) - a*e^(1/3))*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/Sqrt
[3]] - (b*d^(1/3) + a*e^(1/3))*(2*Log[d^(1/3) - e^(1/3)*x] - Log[d^(2/3) +
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)*e^(2/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.68 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.21

method	result
risch	$-\frac{\sum_{\_R=\text{RootOf}\left(e\_Z^3-d\right)}\frac{\left(\_Rb+a\right)\ln\left(x-\_R\right)}{\_R^2}}{3e}$
default	$a\left(-\frac{\ln\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^2+\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)+b\left(-\frac{\ln\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2+\left(\frac{d}{e}\right.\right.}{6e}\right.$

[In] int((b*x+a)/(-e*x^3+d),x,method=_RETURNVERBOSE)

[Out] -1/3/e*sum((_R*b+a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e-d))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 1905, normalized size of antiderivative = 11.83

$$\int \frac{a+bx}{d-ex^3}\,dx = \text{Too large to display}$$

[In] integrate((b*x+a)/(-e*x^3+d),x, algorithm="fricas")

[Out] -1/18*(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a
^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(
d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(1/36*(9*(I*sqrt(3) +
 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3
) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*
d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e - 1/6*(9*(I*sqrt(3) + 1)*(-1/54*(b^
3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sq
rt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^
2*e^2))^(1/3)))*a^2*d*e - 2*a*b^2*d - (b^3*d - a^3*e)*x) + 1/36*(9*(I*sqrt(
3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^
(1/3) + 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e
^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1

```
/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d
*e - 144*a*b)/(d*e)) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^
2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(-1/36*(9*(I*sqrt(3) +
1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)
 + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d
 - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/6*(9*(I*sqrt(3) + 1)*(-1/54*(b^3
*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqr
t(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2
*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d - 2*(b^3*d - a^3*e)*x + 1/12*sqrt(1/3)*(
(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/
(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^
2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*b*d^2*e + 6*a^2*d*e)*sqrt(-((9
*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d
^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2)
 - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e - 144*a*b)/(d*e))) + 1/36*
(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/
(d^2*e^2))^(1/3) - 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^
3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) +
1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^
(1/3)))^2*d*e - 144*a*b)/(d*e)) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d +
 a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(-1/36*(9*(I
*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*
e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) -
1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/6*(9*(I*sqrt(3) + 1)*
(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) +
a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d -
a^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d - 2*(b^3*d - a^3*e)*x - 1/12*
sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*
d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3
*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*b*d^2*e + 6*a^2*d*e
)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d
- a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e
)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e - 144*a*b)/(d*e
)))
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.48

$$\int \frac{a+bx}{d-ex^3}\,dx = -\operatorname{RootSum}\left(27t^3d^2e^2-9tabde-a^3e-b^3d, \left(t\mapsto t\log\left(x+\frac{9t^2bd^2e-3ta^2de-2ab^2d}{a^3e-b^3d}\right)\right)\right)$$

```
[In] integrate((b*x+a)/(-e*x**3+d),x)
```

```
[Out] -RootSum(27*_t**3*d**2*e**2 - 9*_t*a*b*d*e - a**3*e - b**3*d, Lambda(_t, _t
*log(x + (9*_t**2*b*d**2*e - 3*_t*a**2*d*e - 2*a*b**2*d)/(a**3*e - b**3*d))
))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx}{d-ex^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x+a)/(-e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int \frac{a+bx}{d-ex^3}\,dx = \frac{\left(ae+\left(de^2\right)^{\frac{1}{3}}b\right)\log\left(x^2+x\left(\frac{d}{e}\right)^{\frac{1}{3}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(de^2\right)^{\frac{2}{3}}} - \frac{\left(b\left(\frac{d}{e}\right)^{\frac{1}{3}}+a\right)\left(\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,d} + \frac{\sqrt{3}\left(\left(de^2\right)^{\frac{1}{3}}ae-\left(de^2\right)^{\frac{2}{3}}b\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\,de^2}$$

```
[In] integrate((b*x+a)/(-e*x^3+d),x, algorithm="giac")
```

```
[Out] 1/6*(a*e + (d*e^2)^(1/3)*b)*log(x^2 + x*(d/e)^(1/3) + (d/e)^(2/3))/(d*e^2)^
(2/3) - 1/3*(b*(d/e)^(1/3) + a)*(d/e)^(1/3)*log(abs(x - (d/e)^(1/3)))/d + 1
/3*sqrt(3)*((d*e^2)^(1/3)*a*e - (d*e^2)^(2/3)*b)*arctan(1/3*sqrt(3)*(2*x +
(d/e)^(1/3))/(d/e)^(1/3))/(d*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.77

$$\int \frac{a+bx}{d-ex^3}\,dx=\sum_{k=1}^{3}\ln\left(e\left(a\,b+b^2\,x-\mathrm{root}\left(27\,d^2\,e^2\,z^3-9\,a\,b\,d\,e\,z+b^3\,d+a^3\,e,z,k\right)^2\,d\,e\,9-\mathrm{root}\left(27\,d^2\,e^2\,z^3-9\,a\,b\,d\,e\,z+b^3\,d+a^3\,e,z,k\right)\,a\,e\,x\,3\right)\right)\,\mathrm{root}\left(27\,d^2\,e^2\,z^3-9\,a\,b\,d\,e\,z+b^3\,d+a^3\,e,z,k\right)$$

```
[In] int((a + b*x)/(d - e*x^3),x)
```

```
[Out] symsum(log(e*(a*b + b^2*x - 9*root(27*d^2*e^2*z^3 - 9*a*b*d*e*z + b^3*d + a
^3*e, z, k)^2*d*e - 3*root(27*d^2*e^2*z^3 - 9*a*b*d*e*z + b^3*d + a^3*e, z,
 k)*a*e*x))*root(27*d^2*e^2*z^3 - 9*a*b*d*e*z + b^3*d + a^3*e, z, k), k, 1,
 3)
```

3.13 $\int \frac{1+x}{1+x^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{1+x}{1+x^3}\,dx = -\frac{2\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -2/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1600, 632, 210}

$$\int \frac{1+x}{1+x^3}\,dx = -\frac{2\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[(1 + x)/(1 + x^3),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{1-x+x^2}\,dx\\ &= -\left(2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right)\right)\\ &= -\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{1+x^3}\,dx = \frac{2\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Integrate[(1 + x)/(1 + x^3),x]
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$
meijerg	$\frac{x\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{1}{3}}}-\frac{x\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{1}{3}}}+\frac{x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{1}{3}}}-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}$

```
[In] int((1+x)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1+x}{1+x^3} \, dx = \frac{2}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2 x - 1)\right)$$

```
[In] integrate((1+x)/(x^3+1),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1+x}{1+x^3} \, dx = \frac{2\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((1+x)/(x**3+1),x)
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1+x}{1+x^3} \, dx = \frac{2}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2 x - 1)\right)$$

```
[In] integrate((1+x)/(x^3+1),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1+x}{1+x^3} \, dx = \frac{2}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right)$$

```
[In] integrate((1+x)/(x^3+1),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1+x}{1+x^3} \, dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2x-1)}{3}\right)}{3}$$

```
[In] int((x + 1)/(x^3 + 1),x)
[Out] (2*3^(1/2)*atan((3^(1/2)*(2*x - 1))/3))/3
```

3.14 $\int \frac{1-x}{1-x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1600, 632, 210}

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[(1 - x)/(1 - x^3),x]
```

```
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{1+x+x^2}\,dx \\ &= -\left(2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right)\right) \\ &= \frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Integrate[(1 - x)/(1 - x^3),x]
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result
default	$\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
risch	$\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
meijerg	$-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\text{arct}\right.}{3(x^3)^{\frac{2}{3}}}$

```
[In] int((1-x)/(-x^3+1),x,method=_RETURNVERBOSE)
[Out] 2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)$$

```
[In] integrate((1-x)/(-x^3+1),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((1-x)/(-x**3+1),x)
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)$$

```
[In] integrate((1-x)/(-x^3+1),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)$$

```
[In] integrate((1-x)/(-x^3+1),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\,x+1)}{3}\right)}{3}$$

```
[In] int((x - 1)/(x^3 - 1),x)
[Out] (2*3^(1/2)*atan((3^(1/2)*(2*x + 1))/3))/3
```

3.15 $\int \frac{1+x}{1-x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1+x}{1-x^3} \, dx = -\frac{2}{3}\log(1-x) + \frac{1}{3}\log\left(1+x+x^2\right)$$

```
[Out] -2/3*ln(1-x)+1/3*ln(x^2+x+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1875, 31, 642}

$$\int \frac{1+x}{1-x^3} \, dx = \frac{1}{3}\log\left(x^2+x+1\right) - \frac{2}{3}\log(1-x)$$

```
[In] Int[(1 + x)/(1 - x^3),x]
```

```
[Out] (-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1875

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Dist[r*((B*r + A*s)/(3*a*
s)), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s*(B*
r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && N
eQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{3}\int \frac{-1-2x}{1+x+x^2}\,dx\right)+\frac{2}{3}\int \frac{1}{1-x}\,dx\\ &= -\frac{2}{3}\log(1-x)+\frac{1}{3}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{1-x^3}\,dx=-\frac{2}{3}\log(1-x)+\frac{1}{3}\log\left(1+x+x^2\right)$$

```
[In] Integrate[(1 + x)/(1 - x^3),x]
[Out] (-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result
default	$-\frac{2\ln(-1+x)}{3}+\frac{\ln(x^2+x+1)}{3}$
norman	$-\frac{2\ln(-1+x)}{3}+\frac{\ln(x^2+x+1)}{3}$
risch	$-\frac{2\ln(-1+x)}{3}+\frac{\ln(x^2+x+1)}{3}$
parallelrisch	$-\frac{2\ln(-1+x)}{3}+\frac{\ln(x^2+x+1)}{3}$
meijerg	$-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{1}{3}}}-\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{\right.}}{3(x^3)^{\frac{2}{3}}}$

```
[In] int((1+x)/(-x^3+1),x,method=_RETURNVERBOSE)
[Out] -2/3*ln(-1+x)+1/3*ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{1-x^3}\,dx = \frac{1}{3}\,\log\left(x^2+x+1\right) - \frac{2}{3}\,\log\left(x-1\right)$$

```
[In] integrate((1+x)/(-x^3+1),x, algorithm="fricas")
[Out] 1/3*log(x^2 + x + 1) - 2/3*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1+x}{1-x^3}\,dx = -\frac{2\log\left(x-1\right)}{3} + \frac{\log\left(x^2+x+1\right)}{3}$$

```
[In] integrate((1+x)/(-x**3+1),x)
[Out] -2*log(x - 1)/3 + log(x**2 + x + 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{1-x^3}\,dx = \frac{1}{3}\,\log\left(x^2+x+1\right) - \frac{2}{3}\,\log\left(x-1\right)$$

```
[In] integrate((1+x)/(-x^3+1),x, algorithm="maxima")
[Out] 1/3*log(x^2 + x + 1) - 2/3*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1+x}{1-x^3}\,dx = \frac{1}{3}\,\log\left(x^2+x+1\right) - \frac{2}{3}\,\log\left(|x-1|\right)$$

```
[In] integrate((1+x)/(-x^3+1),x, algorithm="giac")
[Out] 1/3*log(x^2 + x + 1) - 2/3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{1-x^3} dx = \frac{\ln(x^2+x+1)}{3} - \frac{2 \ln(x-1)}{3}$$

```
[In] int(-(x + 1)/(x^3 - 1),x)
[Out] log(x + x^2 + 1)/3 - (2*log(x - 1))/3
```

3.16 $\int \frac{1-x}{1+x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1-x}{1+x^3} \, dx = \frac{2}{3} \log(1+x) - \frac{1}{3} \log \left(1-x+x^2\right)$$

```
[Out] 2/3*ln(1+x)-1/3*ln(x^2-x+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1874, 31, 642}

$$\int \frac{1-x}{1+x^3} \, dx = \frac{2}{3} \log(x+1) - \frac{1}{3} \log \left(x^2-x+1\right)$$

```
[In] Int[(1 - x)/(1 + x^3),x]
```

```
[Out] (2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\int \frac{1-2x}{1-x+x^2}\,dx+\frac{2}{3}\int \frac{1}{1+x}\,dx\\ &= \frac{2}{3}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1-x}{1+x^3}\,dx=\frac{2}{3}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)$$

```
[In] Integrate[(1 - x)/(1 + x^3),x]
[Out] (2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result
default	$\frac{2\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{3}$
norman	$\frac{2\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{3}$
risch	$\frac{2\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{3}$
parallelrisch	$\frac{2\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{3}$
meijerg	$\frac{x\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{1}{3}}}-\frac{x\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{1}{3}}}+\frac{x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}-\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3\right.}{6(x^3)^{\frac{2}{3}}}$

```
[In] int((1-x)/(x^3+1),x,method=_RETURNVERBOSE)
[Out] 2/3*ln(1+x)-1/3*ln(x^2-x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1-x}{1+x^3} \, dx = -\frac{1}{3} \log \left(x^2 - x + 1\right) + \frac{2}{3} \log \left(x + 1\right)$$

```
[In] integrate((1-x)/(x^3+1),x, algorithm="fricas")
[Out] -1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1-x}{1+x^3} \, dx = \frac{2 \log \left(x + 1\right)}{3} - \frac{\log \left(x^2 - x + 1\right)}{3}$$

```
[In] integrate((1-x)/(x**3+1),x)
[Out] 2*log(x + 1)/3 - log(x**2 - x + 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1-x}{1+x^3} \, dx = -\frac{1}{3} \log \left(x^2 - x + 1\right) + \frac{2}{3} \log \left(x + 1\right)$$

```
[In] integrate((1-x)/(x^3+1),x, algorithm="maxima")
[Out] -1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1-x}{1+x^3} \, dx = -\frac{1}{3} \log \left(x^2 - x + 1\right) + \frac{2}{3} \log \left(|x + 1|\right)$$

```
[In] integrate((1-x)/(x^3+1),x, algorithm="giac")
[Out] -1/3*log(x^2 - x + 1) + 2/3*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1-x}{1+x^3}\,dx = \frac{2\,\ln\left(x+1\right)}{3} - \frac{\ln\left(x^2-x+1\right)}{3}$$

```
[In] int(-(x - 1)/(x^3 + 1),x)
[Out] (2*log(x + 1))/3 - log(x^2 - x + 1)/3
```

3.17 $\int \frac{3-x}{1-x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{3-x}{1-x^3} dx = \frac{4 \arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\log(1-x) + \frac{1}{3}\log\left(1+x+x^2\right)$$

```
[Out] -2/3*ln(1-x)+1/3*ln(x^2+x+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1875, 31, 648, 632, 210, 642}

$$\int \frac{3-x}{1-x^3} dx = \frac{4 \arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(x^2+x+1\right) - \frac{2}{3}\log(1-x)$$

```
[In] Int[(3 - x)/(1 - x^3),x]
```

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1 + x + x^2]
/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1875

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Dist[r*((B*r + A*s)/(3*a*
s)), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s*(B*
r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && N
eQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\frac{1}{3}\int \frac{-7-2x}{1+x+x^2}\,dx\right)+\frac{2}{3}\int \frac{1}{1-x}\,dx\\
&= -\frac{2}{3}\log(1-x)+\frac{1}{3}\int \frac{1+2x}{1+x+x^2}\,dx+2\int \frac{1}{1+x+x^2}\,dx\\
&= -\frac{2}{3}\log(1-x)+\frac{1}{3}\log\left(1+x+x^2\right)-4\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= \frac{4\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1-x)+\frac{1}{3}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{3-x}{1-x^3}\,dx = \frac{4\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\log(1-x) + \frac{1}{3}\log\left(1+x+x^2\right)$$

```
[In] Integrate[(3 - x)/(1 - x^3),x]
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1 + x + x^2]
/3
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result
default	$-\frac{2\ln(-1+x)}{3}+\frac{\ln(x^2+x+1)}{3}+\frac{4\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
risch	$-\frac{2\ln(-1+x)}{3}+\frac{\ln(16x^2+16x+16)}{3}+\frac{4\sqrt{3}\,\arctan\left(\frac{(2+4x)\sqrt{3}}{6}\right)}{3}$
meijerg	$-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{(x^3)^{\frac{1}{3}}}+\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\cdots\right.}{3(x^3)^{\frac{2}{3}}}$

```
[In] int((3-x)/(-x^3+1),x,method=_RETURNVERBOSE)
[Out] -2/3*ln(-1+x)+1/3*ln(x^2+x+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{3-x}{1-x^3}\,dx = \frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{3}\log\left(x^2+x+1\right) - \frac{2}{3}\log\left(x-1\right)$$

```
[In] integrate((3-x)/(-x^3+1),x, algorithm="fricas")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log(
x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{3-x}{1-x^3}\,dx = -\frac{2\log(x-1)}{3} + \frac{\log(x^2+x+1)}{3} + \frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((3-x)/(-x**3+1),x)
[Out] -2*log(x - 1)/3 + log(x**2 + x + 1)/3 + 4*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt
(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{3-x}{1-x^3}\,dx = \frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{3}\log\left(x^2+x+1\right) - \frac{2}{3}\log(x-1)$$

```
[In] integrate((3-x)/(-x^3+1),x, algorithm="maxima")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log(
x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{3-x}{1-x^3}\,dx = \frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{3}\log\left(x^2+x+1\right) - \frac{2}{3}\log(|x-1|)$$

```
[In] integrate((3-x)/(-x^3+1),x, algorithm="giac")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log(
abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{3-x}{1-x^3}\,dx = -\frac{2\,\ln\left(x-1\right)}{3} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)$$

```
[In] int((x - 3)/(x^3 - 1),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*2i)/3 + 1/3) - log(x - (3^(1/2)*1i)
/2 + 1/2)*((3^(1/2)*2i)/3 - 1/3) - (2*log(x - 1))/3
```

3.18 $\int \frac{c+dx}{c^3+d^3x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 29

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx=-\frac{2\arctan\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

```
[Out] -2/3*arctan(1/3*(-2*d*x+c)/c*3^(1/2))/c/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1600, 631, 210}

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx=-\frac{2\arctan\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

```
[In] Int[(c + d*x)/(c^3 + d^3*x^3),x]
```

```
[Out] (-2*ArcTan[(c - 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{c^2 - cdx + d^2x^2}\,dx\\ &= \frac{2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2dx}{c}\right)}{cd}\\ &= -\frac{2\tan^{-1}\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = \frac{2\arctan\left(\frac{-c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

```
[In] Integrate[(c + d*x)/(c^3 + d^3*x^3),x]
[Out] (2*ArcTan[(-c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{2d\sqrt{3}\,x}{3c}-\frac{\sqrt{3}}{3}\right)}{3dc}$	29
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x\,d^2-cd\right)\sqrt{3}}{3cd}\right)}{3cd}$	35

```
[In] int((d*x+c)/(d^3*x^3+c^3),x,method=_RETURNVERBOSE)
[Out] 2/3*3^(1/2)/d/c*arctan(2/3*d*3^(1/2)/c*x-1/3*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = \frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx-c)}{3\,c}\right)}{3\,cd}$$

```
[In] integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = \frac{-\frac{\sqrt{3}i\log\left(x+\frac{-c-\sqrt{3}ic}{2d}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{-c+\sqrt{3}ic}{2d}\right)}{3}}{cd}$$

```
[In] integrate((d*x+c)/(d**3*x**3+c**3),x)
[Out] (-sqrt(3)*I*log(x + (-c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x + (-c + s
qrt(3)*I*c)/(2*d))/3)/(c*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = \frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,d^2x-cd)}{3\,cd}\right)}{3\,cd}$$

```
[In] integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x - c*d)/(c*d))/(c*d)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx-c)}{3\,c}\right)}{3\,cd}$$

```
[In] integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = -\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,d\,x}{3\,c}\right)}{3\,c\,d}$$

```
[In] int((c + d*x)/(c^3 + d^3*x^3),x)
[Out] -(2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*d*x)/(3*c)))/(3*c*d)
```

3.19 $\int \frac{c-dx}{c^3-d^3x^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{c-dx}{c^3-d^3x^3} dx = \frac{2\arctan\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

```
[Out] 2/3*arctan(1/3*(2*d*x+c)/c*3^(1/2))/c/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1600, 631, 210}

$$\int \frac{c-dx}{c^3-d^3x^3} dx = \frac{2\arctan\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

```
[In] Int[(c - d*x)/(c^3 - d^3*x^3),x]
```

```
[Out] (2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{c^2 + cdx + d^2x^2}\,dx \\
&= -\frac{2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{2dx}{c}\right)}{cd} \\
&= \frac{2\tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{c - dx}{c^3 - d^3x^3}\,dx = \frac{2\arctan\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

```
[In] Integrate[(c - d*x)/(c^3 - d^3*x^3),x]
[Out] (2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{2d\sqrt{3}\,x}{3c}+\frac{\sqrt{3}}{3}\right)}{3dc}$	29
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x\,d^2+cd\right)\sqrt{3}}{3cd}\right)}{3cd}$	34

```
[In] int((-d*x+c)/(-d^3*x^3+c^3),x,method=_RETURNVERBOSE)
[Out] 2/3*3^(1/2)/d/c*arctan(2/3*d*3^(1/2)/c*x+1/3*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx+c)}{3\,c}\right)}{3\,cd}$$

[In] integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="fricas")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx=\frac{-\frac{\sqrt{3}i\log\left(x+\frac{c-\sqrt{3}ic}{2d}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{c+\sqrt{3}ic}{2d}\right)}{3}}{cd}$$

[In] integrate((-d*x+c)/(-d**3*x**3+c**3),x)

[Out] (-sqrt(3)*I*log(x + (c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x + (c + sqrt(3)*I*c)/(2*d))/3)/(c*d)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,d^2x+cd)}{3\,cd}\right)}{3\,cd}$$

[In] integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="maxima")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x + c*d)/(c*d))/(c*d)

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx = \frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx+c)}{3\,c}\right)}{3\,cd}$$

```
[In] integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx = \frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\,\sqrt{3}\,d\,x}{3\,c}\right)}{3\,c\,d}$$

```
[In] int((c - d*x)/(c^3 - d^3*x^3),x)
[Out] (2*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*d*x)/(3*c)))/(3*c*d)
```

3.20 $\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 39

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3}\,dx=-\frac{2B\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

```
[Out] -2/3*B*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1600, 631, 210}

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3}\,dx=-\frac{2B\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

```
[In] Int[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] (-2*B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\frac{a^{2/3}}{\sqrt[3]{b}B} - \frac{\sqrt[3]{a}x}{B} + \frac{\sqrt[3]{b}x^2}{B}}\,dx\\ &= \frac{(2B)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}\\ &= -\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a+bx^3}\,dx = -\frac{2B\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

```
[In] Integrate[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3),x]
[Out] (-2*B*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(1/3))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(28) = 56.

Time = 1.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 5.00

method	result
default	$B\,b^{\frac{1}{3}}\left(a^{\frac{1}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+b^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+-\ln\right.\right.$

```
[In] int((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] B*b^(1/3)*(a^(1/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1)))+b^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/
b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.74

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3}\,dx=\left[\sqrt{\frac{1}{3}}B\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3-3\,a^{\frac{2}{3}}b^{\frac{1}{3}}x+3\,\sqrt{\frac{1}{3}}\left(2\,a^{\frac{2}{3}}b^{\frac{2}{3}}x^2+ab^{\frac{1}{3}}x-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-a}{bx^3+a}\right.\right.$$

```
[In] integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [sqrt(1/3)*B*sqrt(-1/a^(2/3))*log((2*b*x^3 - 3*a^(2/3)*b^(1/3)*x + 3*sqrt(1
/3)*(2*a^(2/3)*b^(2/3)*x^2 + a*b^(1/3)*x - a^(4/3))*sqrt(-1/a^(2/3)) - a)/(
b*x^3 + a)), 2*sqrt(1/3)*B*arctan(sqrt(1/3)*(2*b^(1/3)*x - a^(1/3))/a^(1/3)
)/a^(1/3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.26

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a + bx^3}\, dx = \frac{B\left(-\frac{\sqrt{3}i\log\left(x + \frac{-B\sqrt[3]{a} - \sqrt{3}iB\sqrt[3]{a}}{2B\sqrt[3]{b}}\right)}{3} + \frac{\sqrt{3}i\log\left(x + \frac{-B\sqrt[3]{a} + \sqrt{3}iB\sqrt[3]{a}}{2B\sqrt[3]{b}}\right)}{3}\right)}{\sqrt[3]{a}}$$

```
[In] integrate((a**(1/3)*b**(1/3)*B+b**(2/3)*B*x)/(b*x**3+a),x)
```

```
[Out] B*(-sqrt(3)*I*log(x + (-B*a**(1/3) - sqrt(3)*I*B*a**(1/3))/(2*B*b**(1/3)))/
3 + sqrt(3)*I*log(x + (-B*a**(1/3) + sqrt(3)*I*B*a**(1/3))/(2*B*b**(1/3)))/
3)/a**(1/3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(30) = 60.

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 4.18

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a + bx^3}\, dx = \frac{\sqrt{3}\left(Bb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + Ba^{\frac{1}{3}}b^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(Bb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} - Ba^{\frac{1}{3}}b^{\frac{1}{3}}\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(Bb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} - Ba^{\frac{1}{3}}b^{\frac{1}{3}}\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*b^(2/3)*(a/b)^(1/3) + B*a^(1/3)*b^(1/3))*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2/3)) + 1/6*(B*b^(2/3)*(a/b)^(1/
3) - B*a^(1/3)*b^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/
3)) - 1/3*(B*b^(2/3)*(a/b)^(1/3) - B*a^(1/3)*b^(1/3))*log(x + (a/b)^(1/3))/
(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3}\,dx=\frac{2\sqrt{3}Bb^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,b^{\frac{2}{3}}x-a^{\frac{1}{3}}b^{\frac{1}{3}}\right)}{3\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}}\right)}{3\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}}$$

```
[In] integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="giac")
[Out] 2/3*sqrt(3)*B*b^(1/3)*arctan(1/3*sqrt(3)*(2*b^(2/3)*x - a^(1/3)*b^(1/3))/sq
rt(a^(2/3)*b^(2/3)))/sqrt(a^(2/3)*b^(2/3))
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3}\,dx=\frac{2\sqrt{3}\,B\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\sqrt{b}}{3\,\sqrt{-b}}-\frac{2\,\sqrt{3}\,b^{5/6}\,x}{3\,a^{1/3}\,\sqrt{-b}}\right)}{3\,a^{1/3}\,\sqrt{-b}}$$

```
[In] int((B*a^(1/3)*b^(1/3) + B*b^(2/3)*x)/(a + b*x^3),x)
[Out] (2*3^(1/2)*B*b^(1/2)*atanh((3^(1/2)*b^(1/2))/(3*(-b)^(1/2)) - (2*3^(1/2)*b^
(5/6)*x)/(3*a^(1/3)*(-b)^(1/2))))/(3*a^(1/3)*(-b)^(1/2))
```

3.21 $\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 41

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx=\frac{2B\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

```
[Out] 2/3*B*arctan(1/3*(a^(1/3)+2*(-b)^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1600, 631, 210}

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx=\frac{2B\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

```
[In] Int[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] (2*B*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\text{integral} = \int \frac{1}{-\frac{a^{2/3}(-b)^{2/3}}{bB} + \frac{\sqrt[3]{a}x}{B} + \frac{\sqrt[3]{-b}x^2}{B}}\,dx$$

$$= -\frac{(2B)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$= \frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 129 vs. $2(41) = 82$.

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.15

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - (-b)^{2/3}Bx}{a+bx^3}\,dx = \frac{\sqrt[3]{-b}B\left(2\sqrt{3}\left(\sqrt[3]{-b}-\sqrt[3]{b}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(\sqrt[3]{-b}+\sqrt[3]{b}\right)\left(2\log\left(\sqrt[3]{a}\right.\right.\right.}{6\sqrt[3]{a}b^{2/3}}$$

```
[In] Integrate[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] ((-b)^(1/3)*B*(2*Sqrt[3]*((-b)^(1/3) - b^(1/3))*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + ((-b)^(1/3) + b^(1/3))*(2*Log[a^(1/3) + b^(1/3)*x] - Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])))/(6*a^(1/3)*b^(2/3))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(30) = 60$.

Time = 1.52 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.93

method	result
default	$-B\,b^{\frac{1}{3}}\left(-a^{\frac{1}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(-b)^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.\right.$

```
[In] int((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x,method=_RETURNVERBOSE
)
```

```
[Out] -B*b^(1/3)*(-a^(1/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1)))+(-b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))
+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))*(-1)^(1/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.78

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx = \left[\sqrt{\frac{1}{3}}B\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3+3\,a^{\frac{2}{3}}(-b)^{\frac{1}{3}}\,x-3\,\sqrt{\frac{1}{3}}\left(2\,a^{\frac{2}{3}}(-b)^{\frac{2}{3}}\,x^2-a(-b)^{\frac{1}{3}}\,x\right.}{bx^3+a}\right.\right.$$

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="fri
cas")
```

```
[Out] [sqrt(1/3)*B*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*a^(2/3)*(-b)^(1/3)*x - 3*sqr
t(1/3)*(2*a^(2/3)*(-b)^(2/3)*x^2 - a*(-b)^(1/3)*x - a^(4/3))*sqrt(-1/a^(2/3
)) - a)/(b*x^3 + a)), 2*sqrt(1/3)*B*arctan(sqrt(1/3)*(2*(-b)^(1/3)*x + a^(1
/3))/a^(1/3))/a^(1/3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - (-b)^{2/3}Bx}{a + bx^3}\,dx = -\frac{B\left(-\frac{\sqrt{3}i\log\left(-\frac{\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b} - \frac{\sqrt{3}i\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b} + x\right)}{3} + \frac{\sqrt{3}i\log\left(-\frac{\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b} + \frac{\sqrt{3}i\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b} + x\right)}{3}\right)}{\sqrt[3]{a}}$$

```
[In] integrate((a**(1/3)*(-b)**(1/3)*B-(-b)**(2/3)*B*x)/(b*x**3+a),x)
[Out] -B*(-sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)/(2*b) - sqrt(3)*I*a**(1/3)*(-b)**(
2/3)/(2*b) + x)/3 + sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)/(2*b) + sqrt(3)*I*a
**(1/3)*(-b)**(2/3)/(2*b) + x)/3)/a**(1/3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 4.24

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - (-b)^{2/3}Bx}{a + bx^3}\,dx = -\frac{\sqrt{3}\left(B(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} - Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(B(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(B(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="max
ima")
[Out] -1/3*sqrt(3)*(B*(-b)^(2/3)*(a/b)^(1/3) - B*a^(1/3)*(-b)^(1/3))*arctan(1/3*s
qrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2/3)) - 1/6*(B*(-b)^(2/3)
*(a/b)^(1/3) + B*a^(1/3)*(-b)^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(b*(a/b)^(2/3)) + 1/3*(B*(-b)^(2/3)*(a/b)^(1/3) + B*a^(1/3)*(-b)^(1/3))*lo
g(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx=-\frac{2\sqrt{3}B(-b)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\left(2(-b)^{\frac{2}{3}}x+a^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)}{3\sqrt{a^{\frac{2}{3}}(-b)^{\frac{2}{3}}}}\right)}{3\sqrt{a^{\frac{2}{3}}(-b)^{\frac{2}{3}}}}$$

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="gia
c")
```

```
[Out] -2/3*sqrt(3)*B*(-b)^(1/3)*arctan(-1/3*sqrt(3)*(2*(-b)^(2/3)*x + a^(1/3)*(-b
)^(1/3))/sqrt(a^(2/3)*(-b)^(2/3)))/sqrt(a^(2/3)*(-b)^(2/3))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx=-\frac{2\sqrt{3}\,B\,\sqrt{-b}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\sqrt{-b}}{3\,\sqrt{b}}-\frac{2\,\sqrt{3}\,\sqrt{b}\,x}{3\,a^{1/3}\,(-b)^{1/6}}\right)}{3\,a^{1/3}\,\sqrt{b}}$$

```
[In] int(-(B*(-b)^(2/3)*x - B*a^(1/3)*(-b)^(1/3))/(a + b*x^3),x)
```

```
[Out] -(2*3^(1/2)*B*(-b)^(1/2)*atanh((3^(1/2)*(-b)^(1/2))/(3*b^(1/2)) - (2*3^(1/2
)*b^(1/2)*x)/(3*a^(1/3)*(-b)^(1/6))))/(3*a^(1/3)*b^(1/2))
```

3.22 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right) dx$

Optimal result

Integrand size = 35, antiderivative size = 118

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right) dx = -\frac{B\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

```
[Out] -1/3*B*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(2/3)+1/6*B*ln(a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/a^(1/3)/b^(2/3)-1/3*B*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/a^(1/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {266, 1607, 1885, 12, 298, 31, 648, 631, 210, 642}

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right) dx = \frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}-\frac{B\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

```
[In] Int[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] -((B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^
(2/3))) - (B*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)) + (B*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(C\int \frac{x^2}{a+bx^3}\,dx\right)+\int \frac{Bx+Cx^2}{a+bx^3}\,dx\\
&= -\frac{C\log\left(a+bx^3\right)}{3b}+\int \frac{x(B+Cx)}{a+bx^3}\,dx\\
&= -\frac{C\log\left(a+bx^3\right)}{3b}+C\int \frac{x^2}{a+bx^3}\,dx+\int \frac{Bx}{a+bx^3}\,dx\\
&= B\int \frac{x}{a+bx^3}\,dx\\
&= -\frac{B\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}+\frac{B\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}\\
&= -\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{B\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}b^{2/3}}+\frac{B\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}}\\
&= -\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}\\
&\quad+\frac{B\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}
\end{aligned}$$

$$= -\frac{B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}} - \frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.76

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)dx$$
$$= \frac{B\left(-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6\sqrt[3]{a}b^{2/3}}$$

```
[In] Integrate[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3),x]
[Out] (B*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3)
+ b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(1/3)*
b^(2/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.57 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

method	result	size
risch	$-\frac{C\ln\left(b\,x^3+a\right)}{3b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(C\_R+B\right)\ln\left(x-\_R\right)}{\_R}}{3b}$	47
default	$-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{B\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{B\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	94

```
[In] int(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] -1/3*C/b*ln(b*x^3+a)+1/3/b*sum(1/_R*(C*_R+B)*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.91 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.63

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx = \left[\frac{3\sqrt{\frac{1}{3}}Bab\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\,(-ab^2)^{\frac{2}{3}}x^2+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}x}{bx^3+a}\right)+(-ab^2)^{\frac{2}{3}}B\log\left(b\right.}{6\,ab^2}\right.$$

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*B*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqr
t(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3
)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*B*log(b^2*x^2 + (-
a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*B*log(b*x - (-a*b^2)^
(1/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*B*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt
(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*
B*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*B*l
og(b*x - (-a*b^2)^(1/3)))/(a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.22

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx = B\operatorname{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)$$

```
[In] integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x)/(b*x**3+a),x)
```

```
[Out] B*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.35

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx = -\frac{C\log\left(bx^3+a\right)}{3\,b}+\frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{1}{3}}+B\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}-B\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\left(2\,Ca-\left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}+\frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*C*log(b*x^3 + a)/b + 1/6*(2*C*(a/b)^(1/3) + B)*log(x^2 - x*(a/b)^(1/3)
 + (a/b)^(2/3))/(b*(a/b)^(1/3)) + 1/3*(C*(a/b)^(1/3) - B)*log(x + (a/b)^(1/
3))/(b*(a/b)^(1/3)) - 1/9*sqrt(3)*(2*C*a - (3*B*(a/b)^(2/3) + 2*C*a/b)*b)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx = \frac{\sqrt{3}B\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{1}{3}}}-\frac{B\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{1}{3}}}-\frac{B\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}$$

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*B*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(-a*b^2
)^(1/3) - 1/6*B*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(1/3) - 1
/3*B*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx = -\frac{B\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{3\,a^{1/3}\,b^{2/3}} + \frac{\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(B-\sqrt{3}\,B\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}} + \frac{\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}+\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(B+\sqrt{3}\,B\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}$$

```
[In] int((B*x + C*x^2)/(a + b*x^3) - (C*x^2)/(a + b*x^3),x)
```

```
[Out] (log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i - 2*a^(1/3))*(B - 3^(1/2)*B*1i))/(6*a
^(1/3)*b^(2/3)) - (B*log(b^(1/3)*x + a^(1/3)))/(3*a^(1/3)*b^(2/3)) + (log(3
^(1/2)*a^(1/3)*2i + 4*b^(1/3)*x - 2*a^(1/3))*(B + 3^(1/2)*B*1i))/(6*a^(1/3)
*b^(2/3))
```

3.23 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 118

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx = -\frac{A\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/3*A*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(1/3)-1/6*A*ln(a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)-1/3*A*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/a^(2/3)/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {266, 1885, 12, 206, 31, 648, 631, 210, 642}

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx = -\frac{A\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

```
[In] Int[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3),x]
```

```
[Out] -((A*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^
(1/3))) + (A*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) - (A*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = -\left(C\int \frac{x^2}{a+bx^3}\,dx\right) + \int \frac{A+Cx^2}{a+bx^3}\,dx$$

$$= -\frac{C\log(a+bx^3)}{3b} + C\int \frac{x^2}{a+bx^3}\,dx + \int \frac{A}{a+bx^3}\,dx$$

$$= A\int \frac{1}{a+bx^3}\,dx$$

$$= \frac{A\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}} + \frac{A\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}}$$

$$= \frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{A\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}} - \frac{A\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}\sqrt[3]{b}}$$

$$= \frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{A\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

$$= -\frac{A\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}} + \frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.76

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)dx$$
$$=-\frac{A\left(2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6a^{2/3}\sqrt[3]{b}}$$

```
[In] Integrate[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3),x]
```

```
[Out] -1/6*(A*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1
/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(2/3
)*b^(1/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

method	result	size
risch	$-\frac{C\ln(bx^3+a)}{3b}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(C\_R^2+A)\ln(x-\_R)}{\_R^2}}{3b}$	49
default	$\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{A\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	94

```
[In] int(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*C/b*ln(b*x^3+a)+1/3/b*sum((C*_R^2+A)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.58

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)\,dx = \left[\frac{3\sqrt{\frac{1}{3}}Aab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-(a^2b)^{\frac{2}{3}}A\log\left(abx^2-\right.}{6\,a^2b}\right.$$

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*A*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(
1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*A*log(a*b*x^2 - (a^2
*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*A*log(a*b*x + (a^2*b)^(2/3
)))/(a^2*b), 1/6*(6*sqrt(1/3)*A*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*
(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^
(2/3)*A*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*
A*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b)]
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.19

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)\,dx = A\,\mathrm{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

```
[In] integrate(-C*x**2/(b*x**3+a)+(C*x**2+A)/(b*x**3+a),x)
```

```
[Out] A*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.35

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)\,dx = -\frac{C\log\left(bx^3+a\right)}{3\,b}-\frac{\sqrt{3}\left(2\,Ca-\left(3\,A\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}+\frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}-A\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+A\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*C*log(b*x^3 + a)/b - 1/9*sqrt(3)*(2*C*a - (3*A*(a/b)^(1/3) + 2*C*a/b)*
b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*C*(a/
b)^(2/3) - A)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*
(C*(a/b)^(2/3) + A)*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)\,dx = -\frac{A\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}A\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{\left(-ab^2\right)^{\frac{1}{3}}A\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*A*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/3*sqrt(3)*(-a*b^2)^(1/
3)*A*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/6*(-a*
b^2)^(1/3)*A*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)\,dx = \frac{A\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{3\,a^{2/3}\,b^{1/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(A-\sqrt{3}\,A\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(A+\sqrt{3}\,A\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}}$$

```
[In] int((A + C*x^2)/(a + b*x^3) - (C*x^2)/(a + b*x^3),x)
```

```
[Out] (A*log(b^(1/3)*x + a^(1/3)))/(3*a^(2/3)*b^(1/3)) - (log(a^(1/3) - 2*b^(1/3)
*x - 3^(1/2)*a^(1/3)*1i)*(A - 3^(1/2)*A*1i))/(6*a^(2/3)*b^(1/3)) - (log(2*b
^(1/3)*x - 3^(1/2)*a^(1/3)*1i - a^(1/3))*(A + 3^(1/2)*A*1i))/(6*a^(2/3)*b^(
1/3))
```

3.24 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right) \, dx$

Optimal result

Integrand size = 36, antiderivative size = 161

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right) \, dx = -\frac{\left(A\sqrt[3]{b}+\sqrt[3]{a}B\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}} - \frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/3*(A*b^(1/3)-a^(1/3)*B)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(2/3)-1/6*(A-a^(1
/3)*B/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)-1/
3*(A*b^(1/3)+a^(1/3)*B)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a
^(2/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used

$= \{266, 1885, 1874, 31, 648, 631, 210, 642\}$

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)dx = -\frac{\left(\sqrt[3]{a}B+A\sqrt[3]{b}\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} - \frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}$$

```
[In] Int[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] -(((A*b^(1/3) + a^(1/3)*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((A*b^(1/3) - a^(1/3)*B)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((A - (a^(1/3)*B)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(C\int \frac{x^2}{a+bx^3}\,dx\right)+\int \frac{A+Bx+Cx^2}{a+bx^3}\,dx\\
&= -\frac{C\log\left(a+bx^3\right)}{3b}+C\int \frac{x^2}{a+bx^3}\,dx+\int \frac{A+Bx}{a+bx^3}\,dx\\
&= \frac{\int \frac{\sqrt[3]{a}\left(2A\sqrt[3]{b}+\sqrt[3]{a}B\right)+\sqrt[3]{b}\left(-A\sqrt[3]{b}+\sqrt[3]{a}B\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}}\\
&= \frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{2/3}}\\
&\quad+\frac{1}{2}\left(\frac{A}{\sqrt[3]{a}}+\frac{B}{\sqrt[3]{b}}\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx\\
&= \frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}}\\
&\quad+\frac{\left(A\sqrt[3]{b}+\sqrt[3]{a}B\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}b^{2/3}}
\end{aligned}
$$

$$= -\frac{\left(A\sqrt[3]{b}+\sqrt[3]{a}B\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+\frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.77

$$\int\left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)dx = \frac{-2\sqrt{3}\left(A\sqrt[3]{b}+\sqrt[3]{a}B\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.\right.}{6a^{2/3}b^{2/3}}$$

```
[In] Integrate[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(A*b^(1/3) + a^(1/3)*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + (A*b^(1/3) - a^(1/3)*B)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.72 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.32

method	result
risch	$-\frac{C\ln(b\,x^3+a)}{3b}+\frac{\sum_{\_R=\operatorname{RootOf}(b\_Z^3+a)}\frac{\left(C\_R^2+B\_R+A\right)\ln\left(x-\_R\right)}{\_R^2}}{3b}$
default	$\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{A\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{B\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right.\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$

```
[In] int(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*C/b*ln(b*x^3+a)+1/3/b*sum((C*_R^2+B*_R+A)/_R^2*ln(x-_R),_R=RootOf(_Z^3
*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 1961, normalized size of antiderivative = 12.18

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx = \text{Too large to display}$$

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3
*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^
3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^
3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(
2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)
/(a^2*b^2))^(1/3)))*A^2*a*b + 2*A*B^2*a + (B^3*a + A^3*b)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2
))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^
2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^
3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*a*b + 16*A*B)/(a*b)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^
(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2)
- (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b + 1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1
/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A
^3*b)/(a^2*b^2))^(1/3)))*A^2*a*b - 2*A*B^2*a + 2*(B^3*a + A^3*b)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a -
 A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a
+ A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*B*a^2*b + 2*A^2*a*b
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a -
 A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a
+ A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*a*b + 16*A*B)/(a*
b))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*
a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3
*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*
b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3
*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*a*b + 16*A*B)/(a*b)))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a -
 A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a
```

```
+ A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^
2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*A^2*a*b - 2*A*B^2*a + 2*(B^3*a
+ A^3*b)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(
a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3)
 + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*
B*a^2*b + 2*A^2*a*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(
a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3)
 + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^
2*a*b + 16*A*B)/(a*b)))
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$\int\left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)dx$$
$$=\operatorname{RootSum}\left(27t^3a^2b^2+9tABab-A^3b+B^3a,\left(t\mapsto t\log\left(x+\frac{9t^2Ba^2b+3tA^2ab+2AB^2a}{A^3b+B^3a}\right)\right)\right)$$

```
[In] integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x+A)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*A*B*a*b - A**3*b + B**3*a, Lambda(_t, _t*
log(x + (9*_t**2*B*a**2*b + 3*_t*A**2*a*b + 2*A*B**2*a)/(A**3*b + B**3*a)))
)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.17

$$\int\left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)dx$$
$$=-\frac{C\log\left(bx^3+a\right)}{3\,b}-\frac{\sqrt{3}\left(2\,Ca-\left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\,A\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$
$$+\frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}+B\left(\frac{a}{b}\right)^{\frac{1}{3}}-A\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-B\left(\frac{a}{b}\right)^{\frac{1}{3}}+A\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*C*log(b*x^3 + a)/b - 1/9*sqrt(3)*(2*C*a - (3*B*(a/b)^(2/3) + 3*A*(a/b)
^(1/3) + 2*C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a
*b) + 1/6*(2*C*(a/b)^(2/3) + B*(a/b)^(1/3) - A)*log(x^2 - x*(a/b)^(1/3) + (
a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(C*(a/b)^(2/3) - B*(a/b)^(1/3) + A)*log(x
 + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx=-\frac{\sqrt{3}\left(Ab-(-ab^2)^{\frac{1}{3}}\,B\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{2}{3}}}-\frac{\left(Ab+(-ab^2)^{\frac{1}{3}}\,B\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{2}{3}}}-\frac{\left(Bb\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(A*b - (-a*b^2)^(1/3)*B)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(A*b + (-a*b^2)^(1/3)*B)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*(B*b*(-a/b)^(1/3) + A*b)*
(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx=\sum_{k=1}^{3}\ln\left(b\left(B^2\,x+A\,B+\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,A\,B\,a\,b\,z+B^3\,a-A^3\,b,z,k\right)^2\,a\,b\,9+A\,\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,A\,B\,a\,b\,z+B^3\,a-A^3\,b,z,k\right)\,b\,x\,3\right)\right)\,\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,A\,B\,a\,b\,z+B^3\,a-A^3\,b,z,k\right)$$

```
[In] int((A + B*x + C*x^2)/(a + b*x^3) - (C*x^2)/(a + b*x^3),x)
```

```
[Out] symsum(log(b*(B^2*x + A*B + 9*root(27*a^2*b^2*z^3 + 9*A*B*a*b*z + B^3*a - A
^3*b, z, k)^2*a*b + 3*A*root(27*a^2*b^2*z^3 + 9*A*B*a*b*z + B^3*a - A^3*b,
z, k)*b*x))*root(27*a^2*b^2*z^3 + 9*A*B*a*b*z + B^3*a - A^3*b, z, k), k, 1,
 3)
```

3.25 $\int \frac{bx+cx^2}{d+ex^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{bx+cx^2}{d+ex^3}\,dx = -\frac{b\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{d}e^{2/3}} - \frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}} + \frac{c\log\left(d+ex^3\right)}{3e}$$

```
[Out] -1/3*b*ln(d^(1/3)+e^(1/3)*x)/d^(1/3)/e^(2/3)+1/6*b*ln(d^(2/3)-d^(1/3)*e^(1/
3)*x+e^(2/3)*x^2)/d^(1/3)/e^(2/3)+1/3*c*ln(e*x^3+d)/e-1/3*b*arctan(1/3*(d^(
1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(1/3)/e^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1607, 1885, 12, 298, 31, 648, 631, 210, 642, 266}

$$\int \frac{bx+cx^2}{d+ex^3}\,dx = -\frac{b\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{d}e^{2/3}} + \frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}} - \frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{c\log\left(d+ex^3\right)}{3e}$$

```
[In] Int[(b*x + c*x^2)/(d + e*x^3),x]
```

```
[Out] -((b*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(1/3)*e^
(2/3))) - (b*Log[d^(1/3) + e^(1/3)*x])/(3*d^(1/3)*e^(2/3)) + (b*Log[d^(2/3)
```

```
 - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(1/3)*e^(2/3)) + (c*Log[d + e*x^3
])/(3*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x(b+cx)}{d+ex^3}\,dx \\
&= c\int \frac{x^2}{d+ex^3}\,dx + \int \frac{bx}{d+ex^3}\,dx \\
&= \frac{c\log\left(d+ex^3\right)}{3e} + b\int \frac{x}{d+ex^3}\,dx \\
&= \frac{c\log\left(d+ex^3\right)}{3e} - \frac{b\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{3\sqrt[3]{d}\sqrt[3]{e}} + \frac{b\int \frac{\sqrt[3]{d}+\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{3\sqrt[3]{d}\sqrt[3]{e}} \\
&= -\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{c\log\left(d+ex^3\right)}{3e} \\
&\quad + \frac{b\int \frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{6\sqrt[3]{d}e^{2/3}} + \frac{b\int \frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{2\sqrt[3]{e}} \\
&= -\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}} \\
&\quad + \frac{c\log\left(d+ex^3\right)}{3e} + \frac{b\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\sqrt[3]{d}e^{2/3}}
\end{aligned}
$$

$$= -\frac{b\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{d}e^{2/3}} - \frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}} + \frac{c\log\left(d+ex^3\right)}{3e}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int \frac{bx+cx^2}{d+ex^3}\,dx = \frac{-2\sqrt{3}b\sqrt[3]{e}\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right) - 2b\sqrt[3]{e}\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right) + b\sqrt[3]{e}\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right) + 2c\sqrt[3]{d}\log(}{6\sqrt[3]{d}e}$$

```
[In] Integrate[(b*x + c*x^2)/(d + e*x^3),x]
```

```
[Out] (-2*Sqrt[3]*b*e^(1/3)*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - 2*b*e^(
1/3)*Log[d^(1/3) + e^(1/3)*x] + b*e^(1/3)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x +
 e^(2/3)*x^2] + 2*c*d^(1/3)*Log[d + e*x^3])/(6*d^(1/3)*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^3 e+d\right)} \frac{\left(\_R^2 c+\_R b\right)\ln\left(x-\_R\right)}{\_R^2}}{3e}$	36
default	$b\left(-\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)+\frac{c\ln\left(e\,x^3+d\right)}{3e}$	108

```
[In] int((c*x^2+b*x)/(e*x^3+d),x,method=_RETURNVERBOSE)
[Out] 1/3/e*sum((_R^2*c+_R*b)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e+d))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 1344, normalized size of antiderivative = 10.03

$$\int \frac{bx + cx^2}{d + ex^3} \, dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="fricas")
[Out] -1/12*(2*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d
 - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*e*log(1/4*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e
^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*d*e^
2 + (3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^
3*e)/(d*e^3))^(1/3) - 2*c/e)*c*d*e + b^2*e*x + c^2*d) - ((3*(I*sqrt(3) + 1)
*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) -
2*c/e)*e + 3*sqrt(1/3)*e*sqrt(-((3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^
3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*e^2 + 4*(3*(I*sq
rt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3)
)^(1/3) - 2*c/e)*c*e + 4*c^2)/e^2) + 6*c)*log(-1/4*(3*(I*sqrt(3) + 1)*(-1/5
4*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)
^2*d*e^2 - (3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3
*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*d*e + 2*b^2*e*x - c^2*d + 3/4*sqrt(1/
3)*((3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^
3*e)/(d*e^3))^(1/3) - 2*c/e)*d*e^2 + 2*c*d*e)*sqrt(-((3*(I*sqrt(3) + 1)*(-1
/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/
e)^2*e^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c
^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*e + 4*c^2)/e^2)) - ((3*(I*sqrt(3) +
 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3)
 - 2*c/e)*e - 3*sqrt(1/3)*e*sqrt(-((3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54
*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*e^2 + 4*(3*(I
*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e
^3))^(1/3) - 2*c/e)*c*e + 4*c^2)/e^2) + 6*c)*log(-1/4*(3*(I*sqrt(3) + 1)*(-
1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c
/e)^2*d*e^2 - (3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(
c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*d*e + 2*b^2*e*x - c^2*d - 3/4*sqrt
(1/3)*((3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d -
 b^3*e)/(d*e^3))^(1/3) - 2*c/e)*d*e^2 + 2*c*d*e)*sqrt(-((3*(I*sqrt(3) + 1)*
(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2
*c/e)^2*e^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54
*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*e + 4*c^2)/e^2)))/e
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.56

$$\int \frac{bx+cx^2}{d+ex^3}\,dx = \operatorname{RootSum}\left(27t^3de^3-27t^2cde^2+9tc^2de+b^3e-c^3d, \left(t\mapsto t\log\left(x+\frac{9t^2de^2-6tcde+c^2d}{b^2e}\right)\right)\right)$$

```
[In] integrate((c*x**2+b*x)/(e*x**3+d),x)
[Out] RootSum(27*_t**3*d*e**3 - 27*_t**2*c*d*e**2 + 9*_t*c**2*d*e + b**3*e - c**3
*d, Lambda(_t, _t*log(x + (9*_t**2*d*e**2 - 6*_t*c*d*e + c**2*d)/(b**2*e)))
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{bx+cx^2}{d+ex^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88

$$\int \frac{bx+cx^2}{d+ex^3}\,dx = \frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\left(-de^2\right)^{\frac{1}{3}}} - \frac{b\log\left(x^2+x\left(-\frac{d}{e}\right)^{\frac{1}{3}}+\left(-\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(-de^2\right)^{\frac{1}{3}}} - \frac{b\left(-\frac{d}{e}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3d} + \frac{c\log\left(\left|ex^3+d\right|\right)}{3e}$$

```
[In] integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="giac")
[Out] 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x + (-d/e)^(1/3))/(-d/e)^(1/3))/(-d*e^2
)^(1/3) - 1/6*b*log(x^2 + x*(-d/e)^(1/3) + (-d/e)^(2/3))/(-d*e^2)^(1/3) - 1
/3*b*(-d/e)^(2/3)*log(abs(x - (-d/e)^(1/3)))/d + 1/3*c*log(abs(e*x^3 + d))/
e
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.18

$$\int \frac{bx+cx^2}{d+ex^3}\,dx = \sum_{k=1}^{3}\ln\left(-\mathrm{root}\left(27\,d\,e^3\,z^3-27\,c\,d\,e^2\,z^2+9\,c^2\,d\,e\,z+b^3\,e-c^3\,d,z,k\right)\,\left(6\,c\,d\,e-\mathrm{root}\left(27\,d\,e^3\,z^3-27\,c\,d\,e^2\,z^2+9\,c^2\,d\,e\,z+b^3\,e-c^3\,d,z,k\right)\,d\,e^2\,9\right)+c^2\,d+b^2\,e\,x\right)\,\mathrm{root}\left(27\,d\,e^3\,z^3-27\,c\,d\,e^2\,z^2+9\,c^2\,d\,e\,z+b^3\,e-c^3\,d,z,k\right)$$

```
[In] int((b*x + c*x^2)/(d + e*x^3),x)
```

```
[Out] symsum(log(c^2*d - root(27*d*e^3*z^3 - 27*c*d*e^2*z^2 + 9*c^2*d*e*z + b^3*e
 - c^3*d, z, k)*(6*c*d*e - 9*root(27*d*e^3*z^3 - 27*c*d*e^2*z^2 + 9*c^2*d*e
*z + b^3*e - c^3*d, z, k)*d*e^2) + b^2*e*x)*root(27*d*e^3*z^3 - 27*c*d*e^2*
z^2 + 9*c^2*d*e*z + b^3*e - c^3*d, z, k), k, 1, 3)
```

3.26 $\int \frac{a+cx^2}{d-ex^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 134

$$\int \frac{a+cx^2}{d-ex^3}\,dx=\frac{a\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{e}}-\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}+\frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}$$

```
[Out] -1/3*a*ln(d^(1/3)-e^(1/3)*x)/d^(2/3)/e^(1/3)+1/6*a*ln(d^(2/3)+d^(1/3)*e^(1/
3)*x+e^(2/3)*x^2)/d^(2/3)/e^(1/3)-1/3*c*ln(-e*x^3+d)/e+1/3*a*arctan(1/3*(d^
(1/3)+2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(2/3)/e^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1885, 12, 206, 31, 648, 631, 210, 642, 266}

$$\int \frac{a+cx^2}{d-ex^3}\,dx=\frac{a\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{e}}+\frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}$$

```
[In] Int[(a + c*x^2)/(d - e*x^3),x]
```

```
[Out] (a*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(1
/3)) - (a*Log[d^(1/3) - e^(1/3)*x])/(3*d^(2/3)*e^(1/3)) + (a*Log[d^(2/3) +
```

```
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(1/3)) - (c*Log[d - e*x^3])/
(3*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= c\int \frac{x^2}{d-ex^3}\,dx+\int \frac{a}{d-ex^3}\,dx\\
&= -\frac{c\log\left(d-ex^3\right)}{3e}+a\int \frac{1}{d-ex^3}\,dx\\
&= -\frac{c\log\left(d-ex^3\right)}{3e}+\frac{a\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e}x}\,dx}{3d^{2/3}}+\frac{a\int \frac{2\sqrt[3]{d}+\sqrt[3]{e}x}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{3d^{2/3}}\\
&= -\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}\\
&\quad+\frac{a\int \frac{1}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{2\sqrt[3]{d}}+\frac{a\int \frac{\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{6d^{2/3}\sqrt[3]{e}}\\
&= -\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}+\frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}\\
&\quad-\frac{c\log\left(d-ex^3\right)}{3e}-\frac{a\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{d^{2/3}\sqrt[3]{e}}\\
&= \frac{a\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{e}}-\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\\
&\quad+\frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int \frac{a+cx^2}{d-ex^3}\,dx$$
$$= \frac{2\sqrt{3}ae^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)-2ae^{2/3}\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)+ae^{2/3}\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)-2cd^{2/3}\log}{6d^{2/3}e}$$

[In] Integrate[(a + c*x^2)/(d - e*x^3),x]

[Out] (2*Sqrt[3]*a*e^(2/3)*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - 2*a*e^(2/3)*Log[d^(1/3) - e^(1/3)*x] + a*e^(2/3)*Log[d^(2/3) + d^(1/3)*e^(1/3)*x + e^(2/3)*x^2] - 2*c*d^(2/3)*Log[d - e*x^3])/(6*d^(2/3)*e)

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

method	result	size
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3e-d\right)}\frac{\left(\_R^2c+a\right)\ln\left(x-\_R\right)}{\_R^2}}{3e}$	36
default	$a\left(-\frac{\ln\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^2+\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)-\frac{c\ln\left(-e\,x^3+d\right)}{3e}$	110

[In] int((c*x^2+a)/(-e*x^3+d),x,method=_RETURNVERBOSE)

[Out] -1/3/e*sum((_R^2*c+a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e-d))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 1040, normalized size of antiderivative = 7.76

$$\int \frac{a+cx^2}{d-ex^3}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="fricas")
```

```
[Out] -1/12*(2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a
^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*e*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(
c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*d*e +
 a*e*x + c*d) - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3
*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*e + 3*sqrt(1/3)*e*sqrt(-(((1/2)^(
1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3)
)^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2
*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*c*e + 4*c^2)/e^2) - 6*c
)*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 +
a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*d*e + 2*a*e*x + 3/2*sqrt(1/3)*d*e*sqrt(-
(((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/
(d^2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 +
 a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*c*e + 4*c^2)/e
^2) - c*d) - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^
2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*e - 3*sqrt(1/3)*e*sqrt(-(((1/2)^(1/3
)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(
1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e)
 - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*c*e + 4*c^2)/e^2) - 6*c)*l
og(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3
*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*d*e + 2*a*e*x - 3/2*sqrt(1/3)*d*e*sqrt(-(((
1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^
2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^
3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*c*e + 4*c^2)/e^2)
 - c*d))/e
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.52

$$\int \frac{a+cx^2}{d-ex^3}\,dx = -\operatorname{RootSum}\left(27t^3d^2e^3 - 27t^2cd^2e^2 + 9tc^2d^2e - a^3e^2 - c^3d^2, \left(t \mapsto t\log\left(x + \frac{-3tde + cd}{ae}\right)\right)\right)$$

```
[In] integrate((c*x**2+a)/(-e*x**3+d),x)
```

```
[Out] -RootSum(27*_t**3*d**2*e**3 - 27*_t**2*c*d**2*e**2 + 9*_t*c**2*d**2*e - a**
3*e**2 - c**3*d**2, Lambda(_t, _t*log(x + (-3*_t*d*e + c*d)/(a*e))))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{d-ex^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

$$\int \frac{a+cx^2}{d-ex^3}\,dx = -\frac{a\left(\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,d} - \frac{c\log\left(\left|ex^3-d\right|\right)}{3\,e} + \frac{\sqrt{3}(de^2)^{\frac{1}{3}}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\,de} + \frac{(de^2)^{\frac{1}{3}}\,a\log\left(x^2+x\left(\frac{d}{e}\right)^{\frac{1}{3}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,de}$$

```
[In] integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*a*(d/e)^(1/3)*log(abs(x - (d/e)^(1/3)))/d - 1/3*c*log(abs(e*x^3 - d))/
e + 1/3*sqrt(3)*(d*e^2)^(1/3)*a*arctan(1/3*sqrt(3)*(2*x + (d/e)^(1/3))/(d/e
)^(1/3))/(d*e) + 1/6*(d*e^2)^(1/3)*a*log(x^2 + x*(d/e)^(1/3) + (d/e)^(2/3))
/(d*e)
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.33

$$\int \frac{a+cx^2}{d-ex^3}\,dx = \sum_{k=1}^{3} \ln\Big(-\big(c + \mathrm{root}\big(27\,d^2\,e^3\,z^3 + 27\,c\,d^2\,e^2\,z^2 + 9\,c^2\,d^2\,e\,z + c^3\,d^2 + a^3\,e^2, z, k\big)\,e\,3\big)\,\big(c\,d + \mathrm{root}\big(27\,d^2\,e^3\,z^3 + 27\,c\,d^2\,e^2\,z^2 + 9\,c^2\,d^2\,e\,z + c^3\,d^2 + a^3\,e^2, z, k\big)\,d\,e\,3 + a\,e\,x\big)\Big)\,\mathrm{root}\big(27\,d^2\,e^3\,z^3 + 27\,c\,d^2\,e^2\,z^2 + 9\,c^2\,d^2\,e\,z + c^3\,d^2 + a^3\,e^2, z, k\big)$$

```
[In] int((a + c*x^2)/(d - e*x^3),x)
```

```
[Out] symsum(log(-(c + 3*root(27*d^2*e^3*z^3 + 27*c*d^2*e^2*z^2 + 9*c^2*d^2*e*z +
 c^3*d^2 + a^3*e^2, z, k)*e)*(c*d + 3*root(27*d^2*e^3*z^3 + 27*c*d^2*e^2*z^
2 + 9*c^2*d^2*e*z + c^3*d^2 + a^3*e^2, z, k)*d*e + a*e*x))*root(27*d^2*e^3*
z^3 + 27*c*d^2*e^2*z^2 + 9*c^2*d^2*e*z + c^3*d^2 + a^3*e^2, z, k), k, 1, 3)
```

3.27 $\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 37

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx=-\frac{2\arctan\left(\frac{a-2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}+\frac{\log(a+bx)}{b}$$

```
[Out] ln(b*x+a)/b-2/3*arctan(1/3*(-2*b*x+a)/a*3^(1/2))/b*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1882, 31, 631, 210}

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx=\frac{\log(a+bx)}{b}-\frac{2\arctan\left(\frac{a-2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}$$

```
[In] Int[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x]
```

```
[Out] (-2*ArcTan[(a - 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) + Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, 
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1882

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{a\int \frac{1}{\frac{a^2}{b^2}-\frac{ax}{b}+x^2}\,dx}{b^2}+\frac{\int \frac{1}{\frac{a}{b}+x}\,dx}{b}\\ &= \frac{\log(a+bx)}{b}+\frac{2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2bx}{a}\right)}{b}\\ &= -\frac{2\tan^{-1}\left(\frac{a-2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}+\frac{\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.95

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{-a+2bx}{\sqrt{3}a}\right)+2\log(a+bx)-\log\left(a^2-abx+b^2x^2\right)+\log\left(a^3+b^3x^3\right)}{3b}$$

```
[In] Integrate[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*b*x)/(Sqrt[3]*a)] + 2*Log[a + b*x] - Log[a^2 - a*
b*x + b^2*x^2] + Log[a^3 + b^3*x^3])/(3*b)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

method	result	size
risch	$\frac{\ln(bx+a)}{b}+\left(\sum_{\_R=\mathrm{RootOf}(3b^2\_Z^2+1)}\_R\ln\left(3ab\_R+2bx-a\right)\right)$	42
default	$\frac{\ln(bx+a)}{b}+\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2b^2x-ab\right)\sqrt{3}}{3ab}\right)}{3b}$	43

```
[In] int((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x,method=_RETURNVERBOSE)
[Out] ln(b*x+a)/b+sum(_R*ln(3*_R*a*b+2*b*x-a),_R=RootOf(3*_Z^2*b^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,bx-a)}{3\,a}\right)+3\,\log\left(bx+a\right)}{3\,b}$$

```
[In] integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="fricas")
[Out] 1/3*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x - a)/a) + 3*log(b*x + a))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx=\frac{-\frac{\sqrt{3}i\log\left(x+\frac{-a-\sqrt{3}ia}{2b}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{-a+\sqrt{3}ia}{2b}\right)}{3}+\log\left(\frac{a}{b}+x\right)}{b}$$

```
[In] integrate((b**2*x**2+2*a**2)/(b**3*x**3+a**3),x)
[Out] (-sqrt(3)*I*log(x + (-a - sqrt(3)*I*a)/(2*b))/3 + sqrt(3)*I*log(x + (-a + s
qrt(3)*I*a)/(2*b))/3 + log(a/b + x))/b
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,b^2x-ab)}{3\,ab}\right)}{3\,b}+\frac{\log\left(bx+a\right)}{b}$$

```
[In] integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b^2*x - a*b)/(a*b))/b + log(b*x + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,bx-a)}{3\,a}\right)}{3\,b}+\frac{\log\left(|bx+a|\right)}{b}$$

```
[In] integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x - a)/a)/b + log(abs(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.27

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx=\frac{\ln\left(a+b\,x\right)}{b}-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}\,a^3\,b^4}{4\,a^3\,b^4+4\,x\,a^2\,b^5}-\frac{4\sqrt{3}\,a^2\,b^5\,x}{4\,a^3\,b^4+4\,x\,a^2\,b^5}\right)}{3\,b}$$

```
[In] int((2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x)
[Out] log(a + b*x)/b - (2*3^(1/2)*atan((4*3^(1/2)*a^3*b^4)/(4*a^3*b^4 + 4*a^2*b^5
*x) - (4*3^(1/2)*a^2*b^5*x)/(4*a^3*b^4 + 4*a^2*b^5*x))))/(3*b)
```

3.28 $\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 39

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx=\frac{2\arctan\left(\frac{a+2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}-\frac{\log(a-bx)}{b}$$

```
[Out] -ln(-b*x+a)/b+2/3*arctan(1/3*(2*b*x+a)/a*3^(1/2))/b*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1882, 31, 631, 210}

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx=\frac{2\arctan\left(\frac{a+2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}-\frac{\log(a-bx)}{b}$$

```
[In] Int[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x]
```

```
[Out] (2*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) - Log[a - b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1882

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a \int \frac{1}{\frac{a^2}{b^2}+\frac{ax}{b}+x^2}\,dx}{b^2} - \frac{\int \frac{1}{-\frac{a}{b}+x}\,dx}{b} \\
&= -\frac{\log(a-bx)}{b} - \frac{2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2bx}{a}\right)}{b} \\
&= \frac{2\tan^{-1}\left(\frac{a+2bx}{\sqrt{3}a}\right)}{\sqrt{3}b} - \frac{\log(a-bx)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.82

$$
\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{a+2bx}{\sqrt{3}a}\right) - 2\log(a-bx) + \log\left(a^2+abx+b^2x^2\right) - \log\left(a^3-b^3x^3\right)}{3b}
$$

```
[In] Integrate[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)] - 2*Log[a - b*x] + Log[a^2 + a*b
*x + b^2*x^2] - Log[a^3 - b^3*x^3])/(3*b)
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{2 \arctan\left(\frac{(2bx+a)\sqrt{3}}{3a}\right)\sqrt{3}}{3b}-\frac{\ln(bx-a)}{b}$	38
default	$-\frac{\ln(-bx+a)}{b}+\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2b^2x+ab\right)\sqrt{3}}{3ab}\right)}{3b}$	44

```
[In] int((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x,method=_RETURNVERBOSE)
[Out] 2/3*arctan(1/3*(2*b*x+a)/a*3^(1/2))/b*3^(1/2)-1/b*ln(b*x-a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,bx+a)}{3\,a}\right)-3\,\log\left(bx-a\right)}{3\,b}$$

```
[In] integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="fricas")
[Out] 1/3*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x + a)/a) - 3*log(b*x - a))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx=-\frac{\frac{\sqrt{3}i\log\left(x+\frac{a-\sqrt{3}ia}{2b}\right)}{3}-\frac{\sqrt{3}i\log\left(x+\frac{a+\sqrt{3}ia}{2b}\right)}{3}+\log\left(-\frac{a}{b}+x\right)}{b}$$

```
[In] integrate((b**2*x**2+2*a**2)/(-b**3*x**3+a**3),x)
[Out] -(sqrt(3)*I*log(x + (a - sqrt(3)*I*a)/(2*b))/3 - sqrt(3)*I*log(x + (a + sqr
t(3)*I*a)/(2*b))/3 + log(-a/b + x))/b
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,b^2x+ab)}{3\,ab}\right)}{3\,b} - \frac{\log\left(bx-a\right)}{b}$$

[In] integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="maxima")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b^2*x + a*b)/(a*b))/b - log(b*x - a)/b

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,bx+a)}{3\,a}\right)}{3\,b} - \frac{\log\left(|bx-a|\right)}{b}$$

[In] integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="giac")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x + a)/a)/b - log(abs(b*x - a))/b

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.21

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}\,a^3\,b^4}{4\,a^3\,b^4-4\,a^2\,b^5\,x}+\frac{4\sqrt{3}\,a^2\,b^5\,x}{4\,a^3\,b^4-4\,a^2\,b^5\,x}\right)}{3\,b} - \frac{\ln\left(a-b\,x\right)}{b}$$

[In] int((2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x)

[Out] (2*3^(1/2)*atan((4*3^(1/2)*a^3*b^4)/(4*a^3*b^4 - 4*a^2*b^5*x) + (4*3^(1/2)*a^2*b^5*x)/(4*a^3*b^4 - 4*a^2*b^5*x)))/(3*b) - log(a - b*x)/b

3.29 $\int \frac{8C+b^{2/3}Cx^2}{8+bx^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx = -\frac{2C\arctan\left(\frac{1-\sqrt[3]{b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{C\log\left(2+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

```
[Out] C*ln(2+b^(1/3)*x)/b^(1/3)-2/3*C*arctan(1/3*(1-b^(1/3)*x)*3^(1/2))/b^(1/3)*3
^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1877, 31, 631, 210}

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx = \frac{C\log\left(\sqrt[3]{b}x+2\right)}{\sqrt[3]{b}}-\frac{2C\arctan\left(\frac{1-\sqrt[3]{b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

```
[In] Int[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 - b^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (C*Log[2 + b^(1/
3)*x])/b^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1877

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(2C)\int \frac{1}{\frac{4}{b^{2/3}}-\frac{2x}{\sqrt[3]{b}}+x^2}\,dx}{b^{2/3}}+\frac{C\int \frac{1}{\frac{2}{\sqrt[3]{b}}+x}\,dx}{\sqrt[3]{b}}\\ &= \frac{C\log\left(2+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\sqrt[3]{b}x\right)}{\sqrt[3]{b}}\\ &= -\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{C\log\left(2+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx = \frac{C\left(2\sqrt{3}\arctan\left(\frac{-1+\sqrt[3]{b}x}{\sqrt{3}}\right)+2\log\left(2+\sqrt[3]{b}x\right)-\log\left(4-2\sqrt[3]{b}x+b^{2/3}x^2\right)+\log\left(8+\right.\right.}{3\sqrt[3]{b}}$$

```
[In] Integrate[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*ArcTan[(-1 + b^(1/3)*x)/Sqrt[3]] + 2*Log[2 + b^(1/3)*x] - Log
[4 - 2*b^(1/3)*x + b^(2/3)*x^2] + Log[8 + b*x^3]))/(3*b^(1/3))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(37) = 74$.

Time = 1.52 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.40

method	result	size
default	$C\left(\frac{8^{\frac{1}{3}}\ln\left(x+8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{1}{b}\right)^{\frac{2}{3}}}-\frac{8^{\frac{1}{3}}\ln\left(x^{2}-8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}x+8^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{1}{b}\right)^{\frac{2}{3}}}+\frac{8^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{8^{\frac{2}{3}}x}{4\left(\frac{1}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{1}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(b\,x^{3}+8\right)}{3b^{\frac{1}{3}}}\right)$	115
meijerg	$\frac{2C\left(\frac{b^{\frac{1}{3}}x\ln\left(1+\frac{\left(b\,x^{3}\right)^{\frac{1}{3}}}{2}\right)}{\left(b\,x^{3}\right)^{\frac{1}{3}}}-\frac{b^{\frac{1}{3}}x\ln\left(1-\frac{\left(b\,x^{3}\right)^{\frac{1}{3}}}{2}+\frac{\left(b\,x^{3}\right)^{\frac{2}{3}}}{4}\right)}{2\left(b\,x^{3}\right)^{\frac{1}{3}}}+\frac{b^{\frac{1}{3}}x\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b\,x^{3}\right)^{\frac{1}{3}}}{4-\left(b\,x^{3}\right)^{\frac{1}{3}}}\right)}{\left(b\,x^{3}\right)^{\frac{1}{3}}}\right)}{3b^{\frac{1}{3}}}+\frac{C\ln\left(1+\frac{b\,x^{3}}{8}\right)}{3b^{\frac{1}{3}}}$	123

```
[In] int((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x,method=_RETURNVERBOSE)
```

```
[Out] C*(1/3/b*8^(1/3)/(1/b)^(2/3)*ln(x+8^(1/3)*(1/b)^(1/3))-1/6/b*8^(1/3)/(1/b)^
(2/3)*ln(x^2-8^(1/3)*(1/b)^(1/3)*x+8^(2/3)*(1/b)^(2/3))+1/3/b*8^(1/3)/(1/b)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/4*8^(2/3)/(1/b)^(1/3)*x-1))+1/3/b^(1/3
)*ln(b*x^3+8))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.79

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx=\left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{bx^3+6\sqrt{\frac{1}{3}}\left(bx^2+b^{\frac{2}{3}}x-2\,b^{\frac{1}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-6\,b^{\frac{1}{3}}x-4}{bx^3+8}\right)+Cb^{\frac{2}{3}}\log\left(bx+2\,b^{\frac{2}{3}}\right)}{b},\ 2\ldots\right.$$

```
[In] integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt(-1/b^(2/3))*log((b*x^3 + 6*sqrt(1/3)*(b*x^2 + b^(2/3)*
x - 2*b^(1/3))*sqrt(-1/b^(2/3)) - 6*b^(1/3)*x - 4)/(b*x^3 + 8)) + C*b^(2/3)
*log(b*x + 2*b^(2/3)))/b, (2*sqrt(1/3)*C*b^(2/3)*arctan(sqrt(1/3)*(b^(2/3)*
x - b^(1/3))/b^(1/3)) + C*b^(2/3)*log(b*x + 2*b^(2/3)))/b]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx = \operatorname{RootSum}\left(3t^3b^{\frac{5}{3}}-3t^2Cb^{\frac{4}{3}}+tC^2b-C^3b^{\frac{2}{3}},\left(t\mapsto t\log\left(x+\frac{3t\sqrt[3]{b}-C}{C\sqrt[3]{b}}\right)\right)\right)$$

```
[In] integrate((8*C+b**(2/3)*C*x**2)/(b*x**3+8),x)
```

```
[Out] RootSum(3*_t**3*b**(5/3) - 3*_t**2*C*b**(4/3) + _t*C**2*b - C**3*b**(2/3),
Lambda(_t, _t*log(x + (3*_t*b**(1/3) - C)/(C*b**(1/3)))))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx = \frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(b^{\frac{2}{3}}x-b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{3b^{\frac{1}{3}}}+\frac{C\log\left(\frac{b^{\frac{1}{3}}x+2}{b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}$$

```
[In] integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(b^(2/3)*x - b^(1/3))/b^(1/3))/b^(1/3) + C
*log((b^(1/3)*x + 2)/b^(1/3))/b^(1/3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(36) = 72.

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.40

$$\begin{aligned}\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx &= \frac{2}{3}\sqrt{3}C\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(x+\left(-\frac{1}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{b}\right)^{\frac{1}{3}}}\right)\\ &\quad-\frac{1}{3}\left(Cb^{\frac{2}{3}}\left(-\frac{1}{b}\right)^{\frac{2}{3}}+2C\right)\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(\left|x-2\left(-\frac{1}{b}\right)^{\frac{1}{3}}\right|\right)\\ &\quad+\frac{1}{3}\left(C\left(-\frac{1}{b}\right)^{\frac{1}{3}}+\frac{C}{b^{\frac{1}{3}}}\right)\log\left(x^2+2x\left(-\frac{1}{b}\right)^{\frac{1}{3}}+4\left(-\frac{1}{b}\right)^{\frac{2}{3}}\right)\end{aligned}$$

```
[In] integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*C*(-1/b)^(1/3)*arctan(1/3*sqrt(3)*(x + (-1/b)^(1/3))/(-1/b)^(1/
3)) - 1/3*(C*b^(2/3)*(-1/b)^(2/3) + 2*C)*(-1/b)^(1/3)*log(abs(x - 2*(-1/b)^
(1/3))) + 1/3*(C*(-1/b)^(1/3) + C/b^(1/3))*log(x^2 + 2*x*(-1/b)^(1/3) + 4*(
-1/b)^(2/3))
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.06

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx=\sum_{k=1}^{3}\ln\left(-\frac{\left(C-\mathrm{root}\left(27\,b^3\,z^3-27\,C\,b^{8/3}\,z^2+9\,C^2\,b^{7/3}\,z-9\,C^3\,b^2,z,k\right)\,b^{1/3}\,3\right)\,\left(-C\right.}{b}\right.$$

```
[In] int((8*C + C*b^(2/3)*x^2)/(b*x^3 + 8),x)
```

```
[Out] symsum(log(-(8*(C - 3*root(27*b^3*z^3 - 27*C*b^(8/3)*z^2 + 9*C^2*b^(7/3)*z
- 9*C^3*b^2, z, k)*b^(1/3))*(3*root(27*b^3*z^3 - 27*C*b^(8/3)*z^2 + 9*C^2*b
^(7/3)*z - 9*C^3*b^2, z, k)*b^(1/3) - C + C*b^(1/3)*x))/b^(5/3))*root(27*b^
3*z^3 - 27*C*b^(8/3)*z^2 + 9*C^2*b^(7/3)*z - 9*C^3*b^2, z, k), k, 1, 3)
```

3.30 $\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx=-\frac{C\arctan\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}}+\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)$$

```
[Out] 1/4*C*ln(a^(1/3)+2*x)-1/6*C*arctan(1/3*(a^(1/3)-4*x)/a^(1/3)*3^(1/2))*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1877, 31, 631, 210}

$$\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx=\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)-\frac{C\arctan\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}}$$

```
[In] Int[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3),x]
```

```
[Out] -1/2*(C*ArcTan[(a^(1/3) - 4*x)/(Sqrt[3]*a^(1/3))])/Sqrt[3] + (C*Log[a^(1/3)
 + 2*x])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1877

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}C\int\frac{1}{\frac{\sqrt[3]{a}}{2}+x}\,dx+\frac{1}{8}\left(\sqrt[3]{a}C\right)\int\frac{1}{\frac{a^{2/3}}{4}-\frac{\sqrt[3]{a}x}{2}+x^2}\,dx\\
&= \frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)+\frac{1}{2}C\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{4x}{\sqrt[3]{a}}\right)\\
&= -\frac{C\tan^{-1}\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}}+\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53

$$
\int\frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx=\frac{1}{12}C\left(-2\sqrt{3}\arctan\left(\frac{1-\frac{4x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{a}+2x\right)-\log\left(a^{2/3}-2\sqrt[3]{a}x+4x^2\right)+\log\left(a+8x^3\right)\right)
$$

```
[In] Integrate[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3),x]
[Out] (C*(-2*Sqrt[3]*ArcTan[(1 - (4*x)/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3) + 2*x] -
 Log[a^(2/3) - 2*a^(1/3)*x + 4*x^2] + Log[a + 8*x^3]))/12
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(34) = 68$.

Time = 1.53 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.04

method	result	size
default	$C\left(a^{\frac{2}{3}}\left(\frac{8^{\frac{2}{3}}\ln\left(x+\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}}{8}\right)}{24a^{\frac{2}{3}}}-\frac{8^{\frac{2}{3}}\ln\left(x^{2}-\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}x}{8}+\frac{8^{\frac{1}{3}}a^{\frac{2}{3}}}{8}\right)}{48a^{\frac{2}{3}}}+\frac{8^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\,8^{\frac{1}{3}}x}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{24a^{\frac{2}{3}}}\right)+\frac{\ln(8x^{3}+a)}{12}\right)$	96

[In] int((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x,method=_RETURNVERBOSE)

[Out] C*(a^(2/3)*(1/24*8^(2/3)/a^(2/3)*ln(x+1/8*8^(2/3)*a^(1/3))-1/48*8^(2/3)/a^(2/3)*ln(x^2-1/8*8^(2/3)*a^(1/3)*x+1/8*8^(1/3)*a^(2/3))+1/24*8^(2/3)/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*8^(1/3)/a^(1/3)*x-1)))+1/12*ln(8*x^3+a))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx = \frac{1}{6}\sqrt{3}C\arctan\left(\frac{4\sqrt{3}a^{\frac{2}{3}}x-\sqrt{3}a}{3\,a}\right)+\frac{1}{4}C\log\left(2\,x+a^{\frac{1}{3}}\right)$$

[In] integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*C*arctan(1/3*(4*sqrt(3)*a^(2/3)*x - sqrt(3)*a)/a) + 1/4*C*log(2*x + a^(1/3))

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.81

$$\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx = C\left(\frac{\log\left(\frac{\sqrt[3]{a}}{2}+x\right)}{4}-\frac{\sqrt{3}i\log\left(x+\frac{-C\sqrt[3]{a}-\sqrt{3}iC\sqrt[3]{a}}{4C}\right)}{12}+\frac{\sqrt{3}i\log\left(x+\frac{-C\sqrt[3]{a}+\sqrt{3}iC\sqrt[3]{a}}{4C}\right)}{12}\right)$$

```
[In] integrate((a**(2/3)*C+2*C*x**2)/(8*x**3+a),x)
```

```
[Out] C*(log(a**(1/3)/2 + x)/4 - sqrt(3)*I*log(x + (-C*a**(1/3) - sqrt(3)*I*C*a**
(1/3))/(4*C))/12 + sqrt(3)*I*log(x + (-C*a**(1/3) + sqrt(3)*I*C*a**(1/3))/(
4*C))/12)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3}\,dx = \frac{1}{6}\,\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(4\,x - a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) + \frac{1}{4}\,C\log\left(x + \frac{1}{2}\,a^{\frac{1}{3}}\right)$$

```
[In] integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*C*arctan(1/3*sqrt(3)*(4*x - a^(1/3))/a^(1/3)) + 1/4*C*log(x + 1
/2*a^(1/3))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3}\,dx = -\frac{\sqrt{3}\left(-i\,\sqrt{3}|a| - a\right)C\arctan\left(\frac{\sqrt{3}\left(4\,x+(-a)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)}{12\,a} - \frac{\left(C(-a)^{\frac{2}{3}} + 2\,Ca^{\frac{2}{3}}\right)(-a)^{\frac{1}{3}}\log\left(\left|x - \frac{1}{2}\,(-a)^{\frac{1}{3}}\right|\right)}{12\,a}$$

```
[In] integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*(-I*sqrt(3)*abs(a) - a)*C*arctan(1/3*sqrt(3)*(4*x + (-a)^(1/3
))/(-a)^(1/3))/a - 1/12*(C*(-a)^(2/3) + 2*C*a^(2/3))*(-a)^(1/3)*log(abs(x -
 1/2*(-a)^(1/3)))/a
```

Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.09

$$\int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3}\,dx = \sum_{k=1}^{3} \ln\left(-\frac{a^{2/3}\left(C - 12\,\mathrm{root}(1728\,a^2\,z^3 - 432\,C\,a^2\,z^2 + 36\,C^2\,a^2\,z - 9\,C^3\,a^2, z, k)\right)\,(4\,C$$

```
[In] int((C*a^(2/3) + 2*C*x^2)/(a + 8*x^3),x)
```

```
[Out] symsum(log(-(a^(2/3)*(C - 12*root(1728*a^2*z^3 - 432*C*a^2*z^2 + 36*C^2*a^2
*z - 9*C^3*a^2, z, k))*(4*C*x - C*a^(1/3) + 12*root(1728*a^2*z^3 - 432*C*a^
2*z^2 + 36*C^2*a^2*z - 9*C^3*a^2, z, k)*a^(1/3)))/128)*root(1728*a^2*z^3 -
432*C*a^2*z^2 + 36*C^2*a^2*z - 9*C^3*a^2, z, k), k, 1, 3)
```

3.31 $\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx=\frac{2C\arctan\left(\frac{1-\sqrt[3]{-b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-b}}-\frac{C\log\left(2+\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

```
[Out] -C*ln(2+(-b)^(1/3)*x)/(-b)^(1/3)+2/3*C*arctan(1/3*(1-(-b)^(1/3)*x)*3^(1/2))
/(-b)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1878, 31, 631, 210}

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx=\frac{2C\arctan\left(\frac{1-\sqrt[3]{-b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-b}}-\frac{C\log\left(\sqrt[3]{-b}x+2\right)}{\sqrt[3]{-b}}$$

```
[In] Int[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 - (-b)^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*(-b)^(1/3)) - (C*Log[2 +
(-b)^(1/3)*x])/(-b)^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1878

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a)^(1/3)/(-b)^(1/3)},
Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2)
, x], x]] /; EqQ[A*(-b)^(2/3) - (-a)^(1/3)*(-b)^(1/3)*B - 2*(-a)^(2/3)*C, 0
]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(2C)\int \frac{1}{\frac{4}{(-b)^{2/3}}-\frac{2x}{\sqrt[3]{-b}}+x^2}\,dx}{(-b)^{2/3}} - \frac{C\int \frac{1}{\frac{2}{\sqrt[3]{-b}}+x}\,dx}{\sqrt[3]{-b}}\\
&= -\frac{C\log\left(2+\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}} - \frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}\\
&= \frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{-b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-b}} - \frac{C\log\left(2+\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.74

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx = \frac{C\left(-2\sqrt{3}b^{2/3}\arctan\left(\frac{1+\sqrt[3]{b}x}{\sqrt{3}}\right)+2b^{2/3}\log\left(2-\sqrt[3]{b}x\right)-b^{2/3}\log\left(4+2\sqrt[3]{b}x+b^{2/3}\right.\right.}{3b}$$

```
[In] Integrate[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*b^(2/3)*ArcTan[(1 + b^(1/3)*x)/Sqrt[3]] + 2*b^(2/3)*Log[2 -
b^(1/3)*x] - b^(2/3)*Log[4 + 2*b^(1/3)*x + b^(2/3)*x^2] + (-b)^(2/3)*Log[8
- b*x^3]))/(3*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(46) = 92$.

Time = 1.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.70

method	result
meijerg	$\frac{2Cx\left(\ln\left(1-\frac{\left(b\,x^{3}\right)^{\frac{1}{3}}}{2}\right)-\frac{\ln\left(1+\frac{\left(b\,x^{3}\right)^{\frac{1}{3}}}{2}+\frac{\left(b\,x^{3}\right)^{\frac{2}{3}}}{4}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(b\,x^{3}\right)^{\frac{1}{3}}}{4+\left(b\,x^{3}\right)^{\frac{1}{3}}}\right)\right)}{3\left(b\,x^{3}\right)^{\frac{1}{3}}}-\frac{C\ln\left(1-\frac{b\,x^{3}}{8}\right)}{3(-b)^{\frac{1}{3}}}$
default	$C\left(\frac{8^{\frac{1}{3}}\ln\left(x-8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{1}{b}\right)^{\frac{2}{3}}}-\frac{8^{\frac{1}{3}}\ln\left(x^{2}+8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}x+8^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{1}{b}\right)^{\frac{2}{3}}}-\frac{8^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{8^{\frac{2}{3}}x}{4\left(\frac{1}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3b\left(\frac{1}{b}\right)^{\frac{2}{3}}}+\frac{(-b)^{\frac{2}{3}}\ln\left(b\,x^{3}-8\right)}{3b}\right)$

```
[In] int((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*C*x/(b*x^3)^(1/3)*(ln(1-1/2*(b*x^3)^(1/3))-1/2*ln(1+1/2*(b*x^3)^(1/3)+1
/4*(b*x^3)^(2/3))-3^(1/2)*arctan(1/4*3^(1/2)*(b*x^3)^(1/3)/(1+1/4*(b*x^3)^(
1/3))))-1/3*C/(-b)^(1/3)*ln(1-1/8*b*x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.19

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx = \left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(\frac{bx^3-6\sqrt{\frac{1}{3}}\left(bx^2-(-b)^{\frac{2}{3}}x+2\,(-b)^{\frac{1}{3}}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+6\,(-b)^{\frac{1}{3}}x+4}{bx^3-8}\right)+C(-b)^{\frac{2}{3}}\text{lo}}{b}\right.$$

$$\left.-\frac{2\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(\sqrt{\frac{1}{3}}\left((-b)^{\frac{2}{3}}x-(-b)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\right)-C(-b)^{\frac{2}{3}}\log\left(bx-2\,(-b)^{\frac{2}{3}}\right)}{b}\right]$$

```
[In] integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt((-b)^(1/3)/b)*log((b*x^3 - 6*sqrt(1/3)*(b*x^2 - (-b)^(
2/3)*x + 2*(-b)^(1/3))*sqrt((-b)^(1/3)/b) + 6*(-b)^(1/3)*x + 4)/(b*x^3 - 8)
) + C*(-b)^(2/3)*log(b*x - 2*(-b)^(2/3)))/b, -(2*sqrt(1/3)*C*b*sqrt(-(-b)^(
1/3)/b)*arctan(sqrt(1/3)*((-b)^(2/3)*x - (-b)^(1/3))*sqrt(-(-b)^(1/3)/b)) -
 C*(-b)^(2/3)*log(b*x - 2*(-b)^(2/3)))/b]
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx = \text{RootSum}\left(3t^3b^2-3t^2Cb(-b)^{\frac{2}{3}}+tC^2(-b)^{\frac{4}{3}}-C^3b,\left(t\mapsto t\log\left(-\frac{3t}{C}+x+\frac{(-b)^{\frac{2}{3}}}{b}\right.\right.\right.$$

```
[In] integrate((8*C+(-b)**(2/3)*C*x**2)/(b*x**3-8),x)
```

```
[Out] RootSum(3*_t**3*b**2 - 3*_t**2*C*b*(-b)**(2/3) + _t*C**2*(-b)**(4/3) - C**3
*b, Lambda(_t, _t*log(-3*_t/C + x + (-b)**(2/3)/b)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(45) = 90.

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.14

$$\begin{aligned}\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx &= \frac{\left(C(-b)^{\frac{2}{3}}-Cb^{\frac{2}{3}}\right)\log\left(b^{\frac{2}{3}}x^2+2\,b^{\frac{1}{3}}x+4\right)}{3\,b} \\ &+ \frac{\left(C(-b)^{\frac{2}{3}}+2\,Cb^{\frac{2}{3}}\right)\log\left(\frac{b^{\frac{1}{3}}x-2}{b^{\frac{1}{3}}}\right)}{3\,b} \\ &+ \frac{2\sqrt{3}\left(C(-b)^{\frac{2}{3}}\,b^{\frac{4}{3}}-\left(C(-b)^{\frac{2}{3}}\,b^{\frac{1}{3}}+3\,Cb\right)b\right)\arctan\left(\frac{\sqrt{3}\left(b^{\frac{2}{3}}x+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{9\,b^{\frac{7}{3}}}\end{aligned}$$

```
[In] integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="maxima")
```

```
[Out] 1/3*(C*(-b)^(2/3) - C*b^(2/3))*log(b^(2/3)*x^2 + 2*b^(1/3)*x + 4)/b + 1/3*(
C*(-b)^(2/3) + 2*C*b^(2/3))*log((b^(1/3)*x - 2)/b^(1/3))/b + 2/9*sqrt(3)*(C
*(-b)^(2/3)*b^(4/3) - (C*(-b)^(2/3)*b^(1/3) + 3*C*b)*b)*arctan(1/3*sqrt(3)*
(b^(2/3)*x + b^(1/3))/b^(1/3))/b^(7/3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx = -\frac{2\sqrt{3}C|b|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}b^{\frac{1}{3}}\left(x+\frac{1}{b^{\frac{1}{3}}}\right)\right)}{3\,b} + \frac{\left(2\,C+\frac{C(-b)^{\frac{2}{3}}}{b^{\frac{2}{3}}}\right)\log\left(\left|x-\frac{2}{b^{\frac{1}{3}}}\right|\right)}{3\,b^{\frac{1}{3}}}$$

```
[In] integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*C*abs(b)^(2/3)*arctan(1/3*sqrt(3)*b^(1/3)*(x + 1/b^(1/3)))/b +
 1/3*(2*C + C*(-b)^(2/3)/b^(2/3))*log(abs(x - 2/b^(1/3)))/b^(1/3)
```

Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.09

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx = \sum_{k=1}^{3}\ln\left(\frac{8\,C^2}{(-b)^{5/3}} + \mathrm{root}\left(27\,b^3\,z^3-27\,C\,(-b)^{8/3}\,z^2-9\,C^2\,(-b)^{7/3}\,z-9\,C^3\,b^2,z,k\right)\left(-\frac{\mathrm{root}\left(27\,b^3\,z^3-27\,C\,(-b)^{8/3}\,z^2-9\,C^2\,(-\right.}{b}\right.\right.$$

```
[In] int((8*C + C*(-b)^(2/3)*x^2)/(b*x^3 - 8),x)
```

```
[Out] symsum(log((8*C^2)/(-b)^(5/3) + root(27*b^3*z^3 - 27*C*(-b)^(8/3)*z^2 - 9*C
^2*(-b)^(7/3)*z - 9*C^3*b^2, z, k)*((48*C)/(-b)^(4/3) - (72*root(27*b^3*z^3
 - 27*C*(-b)^(8/3)*z^2 - 9*C^2*(-b)^(7/3)*z - 9*C^3*b^2, z, k))/b + (24*C*x
)/b) - (8*C^2*x)/(-b)^(4/3))*root(27*b^3*z^3 - 27*C*(-b)^(8/3)*z^2 - 9*C^2*
(-b)^(7/3)*z - 9*C^3*b^2, z, k), k, 1, 3)
```

3.32 $\int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 47

$$\int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3}\,dx=\frac{C\arctan\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)$$

```
[Out] -1/4*C*ln((-a)^(1/3)+2*x)+1/6*C*arctan(1/3*(1-4*x/(-a)^(1/3))*3^(1/2))*3^(1
/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1878, 31, 631, 210}

$$\int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3}\,dx=\frac{C\arctan\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)$$

```
[In] Int[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3),x]
```

```
[Out] (C*ArcTan[(1 - (4*x)/(-a)^(1/3))/Sqrt[3]])/(2*Sqrt[3]) - (C*Log[(-a)^(1/3)
+ 2*x])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1878

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a)^(1/3)/(-b)^(1/3)},
Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2)
, x], x]] /; EqQ[A*(-b)^(2/3) - (-a)^(1/3)*(-b)^(1/3)*B - 2*(-a)^(2/3)*C, 0
]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{4}C\int \frac{1}{\frac{\sqrt[3]{-a}}{2}+x}\,dx\right)-\frac{1}{8}\left(\sqrt[3]{-a}C\right)\int \frac{1}{\frac{1}{4}(-a)^{2/3}-\frac{1}{2}\sqrt[3]{-a}x+x^2}\,dx\\
&= -\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)-\frac{1}{2}C\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{4x}{\sqrt[3]{-a}}\right)\\
&= \frac{C\tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 106 vs. $2(47) = 94$.

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.26

$$\int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3}\,dx = \frac{C\left(2\sqrt{3}(-a)^{2/3}\arctan\left(\frac{1+\frac{4x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2(-a)^{2/3}\log\left(\sqrt[3]{a}-2x\right)+(-a)^{2/3}\log\left(a^{2/3}\right.\right.}{12a^{2/3}}$$

```
[In] Integrate[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3),x]
[Out] (C*(2*Sqrt[3]*(-a)^(2/3)*ArcTan[(1 + (4*x)/a^(1/3))/Sqrt[3]] - 2*(-a)^(2/3)
*Log[a^(1/3) - 2*x] + (-a)^(2/3)*Log[a^(2/3) + 2*a^(1/3)*x + 4*x^2] - a^(2/
3)*Log[-a + 8*x^3]))/(12*a^(2/3))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(36) = 72$.

Time = 1.52 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.09

method	result
default	$C\left((-a)^{\frac{2}{3}}\left(-\frac{8^{\frac{2}{3}}\ln\left(x-\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}}{8}\right)}{24a^{\frac{2}{3}}}+\frac{8^{\frac{2}{3}}\ln\left(x^{2}+\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}x}{8}+\frac{8^{\frac{1}{3}}a^{\frac{2}{3}}}{8}\right)}{48a^{\frac{2}{3}}}+\frac{8^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,8^{\frac{1}{3}}x}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{24a^{\frac{2}{3}}}\right)-\frac{\ln(-8x^{3}+a)}{12}\right)$

```
[In] int(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*((-a)^(2/3)*(-1/24*8^(2/3)/a^(2/3)*ln(x-1/8*8^(2/3)*a^(1/3))+1/48*8^(2/3)
/a^(2/3)*ln(x^2+1/8*8^(2/3)*a^(1/3)*x+1/8*8^(1/3)*a^(2/3))+1/24*8^(2/3)/a^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*8^(1/3)/a^(1/3)*x+1)))-1/12*ln(-8*x^3+a)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{(-a)^{2/3}C + 2Cx^2}{a-8x^3}\,dx = \frac{1}{6}\sqrt{3}C\arctan\left(\frac{4\sqrt{3}(-a)^{\frac{2}{3}}x+\sqrt{3}a}{3a}\right) - \frac{1}{4}C\log\left(2x+(-a)^{\frac{1}{3}}\right)$$

```
[In] integrate(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*C*arctan(1/3*(4*sqrt(3)*(-a)^(2/3)*x + sqrt(3)*a)/a) - 1/4*C*lo
g(2*x + (-a)^(1/3))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.02

$$\int \frac{(-a)^{2/3}C + 2Cx^2}{a - 8x^3}\,dx = -C\left(\frac{\log\left(-\frac{a}{2(-a)^{\frac{2}{3}}} + x\right)}{4} + \frac{\sqrt{3}i\log\left(\frac{a}{4(-a)^{\frac{2}{3}}} - \frac{\sqrt{3}ia}{4(-a)^{\frac{2}{3}}} + x\right)}{12} - \frac{\sqrt{3}i\log\left(\frac{a}{4(-a)^{\frac{2}{3}}} + \frac{\sqrt{3}ia}{4(-a)^{\frac{2}{3}}} + x\right)}{12}\right)$$

```
[In] integrate(((-a)**(2/3)*C+2*C*x**2)/(-8*x**3+a),x)
[Out] -C*(log(-a/(2*(-a)**(2/3)) + x)/4 + sqrt(3)*I*log(a/(4*(-a)**(2/3)) - sqrt(
3)*I*a/(4*(-a)**(2/3)) + x)/12 - sqrt(3)*I*log(a/(4*(-a)**(2/3)) + sqrt(3)*
I*a/(4*(-a)**(2/3)) + x)/12)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(36) = 72$.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.98

$$\int \frac{(-a)^{2/3}C + 2Cx^2}{a - 8x^3}\,dx = \frac{\sqrt{3}C(-a)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(4x + a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{6a^{\frac{2}{3}}} + \frac{\left(C(-a)^{\frac{2}{3}} - Ca^{\frac{2}{3}}\right)\log\left(4x^2 + 2a^{\frac{1}{3}}x + a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}} - \frac{\left(2C(-a)^{\frac{2}{3}} + Ca^{\frac{2}{3}}\right)\log\left(x - \frac{1}{2}a^{\frac{1}{3}}\right)}{12a^{\frac{2}{3}}}$$

```
[In] integrate(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*C*(-a)^(2/3)*arctan(1/3*sqrt(3)*(4*x + a^(1/3))/a^(1/3))/a^(2/3
) + 1/12*(C*(-a)^(2/3) - C*a^(2/3))*log(4*x^2 + 2*a^(1/3)*x + a^(2/3))/a^(2
/3) - 1/12*(2*C*(-a)^(2/3) + C*a^(2/3))*log(x - 1/2*a^(1/3))/a^(2/3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{(-a)^{2/3}C + 2Cx^2}{a - 8x^3}\,dx = \frac{1}{6}\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(4x + a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right) - \frac{\left(2C(-a)^{\frac{2}{3}} + Ca^{\frac{2}{3}}\right)\log\left(\left|x - \frac{1}{2}a^{\frac{1}{3}}\right|\right)}{12a^{\frac{2}{3}}}$$

```
[In] integrate(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*C*arctan(1/3*sqrt(3)*(4*x + a^(1/3))/a^(1/3)) - 1/12*(2*C*(-a)^
(2/3) + C*a^(2/3))*log(abs(x - 1/2*a^(1/3)))/a^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.02

$$\int \frac{(-a)^{2/3}C + 2Cx^2}{a - 8x^3}\,dx = \sum_{k=1}^{3} \ln\left(-\frac{(C + 12\,\mathrm{root}(1728\,a^2\,z^3 + 432\,C\,a^2\,z^2 + 36\,C^2\,a^2\,z + 9\,C^3\,a^2, z, k))\,\left(C\,a\right.}{}\right.$$

```
[In] int((2*C*x^2 + C*(-a)^(2/3))/(a - 8*x^3),x)
```

```
[Out] symsum(log(-((C + 12*root(1728*a^2*z^3 + 432*C*a^2*z^2 + 36*C^2*a^2*z + 9*C
^3*a^2, z, k))*(C*a + 12*root(1728*a^2*z^3 + 432*C*a^2*z^2 + 36*C^2*a^2*z +
 9*C^3*a^2, z, k)*a + 4*C*(-a)^(2/3)*x))/128)*root(1728*a^2*z^3 + 432*C*a^2
*z^2 + 36*C^2*a^2*z + 9*C^3*a^2, z, k), k, 1, 3)
```

3.33 $\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 50

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=-\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

```
[Out] C*ln((a/b)^(1/3)+x)/b-2/3*C*arctan(1/3*(1-2*x/(a/b)^(1/3))*3^(1/2))/b*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1881, 31, 631, 210}

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

```
[In] Int[(2*(a/b)^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^(
1/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1881

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\text{integral} = \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}+x}\,dx}{b} + \frac{\left(\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}$$

$$= \frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b} + \frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}$$

$$= -\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} + \frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 146 vs. 2(50) = 100.

Time = 0.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.92

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=\frac{C\left(-2\sqrt{3}\left(\frac{a}{b}\right)^{2/3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(\frac{a}{b}\right)^{2/3}b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\left(\frac{a}{b}\right)^{2/3}\right.}{3a^{2/3}b}$$

```
[In] Integrate[(2*(a/b)^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*(a/b)^(2/3)*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] + 2*(a/b)^(2/3)*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - (a/b)^(2/3)*b^(2/3)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*Log[a + b*x^3]))/(
3*a^(2/3)*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(43) = 86.

Time = 1.51 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.32

method	result	size
default	$C\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\frac{\ln(bx^3+a)}{3b}\right)$	116

```
[In] int((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(2*(a/b)^(2/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(
x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1)))+1/3*ln(b*x^3+a)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3} C + Cx^2}{a + bx^3} \, dx = \frac{2\sqrt{3}C \arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}a}{3a}\right) + 3C \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 3*
C*log(x + (a/b)^(1/3)))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.00

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3} C + Cx^2}{a + bx^3} \, dx = \frac{C\left(\log\left(\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + x\right) - \frac{\sqrt{3}i \log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + x\right)}{3} + \frac{\sqrt{3}i \log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + x\right)}{3}\right)}{b}$$

```
[In] integrate((2*(a/b)**(2/3)*C+C*x**2)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(a/b)**(2/3)) + x) - sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) - sqrt
(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3} C + Cx^2}{a + bx^3} \, dx = \frac{2\sqrt{3}C \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b} + \frac{C \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/b + C*log
(x + (a/b)^(1/3))/b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.86

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=-\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+2\left(ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}+\frac{\left(3ab^2+i\sqrt{3}\sqrt{a^2b^4}\right)C\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^3}$$

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1/
3*(C*b^2*(-a/b)^(2/3) + 2*(a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)^
(1/3)))/(a*b^2) + 1/6*(3*a*b^2 + I*sqrt(3)*sqrt(a^2*b^4))*C*log(x^2 + x*(-a
/b)^(1/3) + (-a/b)^(2/3))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.44

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=\sum_{k=1}^{3}\ln\left(-\frac{\left(C-\mathrm{root}(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+9\,C^2\,a^2\,b\,z-9\,C^3\,a^2,z,k)\;b\,3\right)\,\left(\cdots\right.}{\cdots}\right.$$

```
[In] int((C*x^2 + 2*C*(a/b)^(2/3))/(a + b*x^3),x)
```

```
[Out] symsum(log(-((C - 3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
- 9*C^3*a^2, z, k)*b)*(3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2
*b*z - 9*C^3*a^2, z, k)*a*b - C*a + 2*C*b*x*(a/b)^(2/3)))/b^3)*root(27*a^2*
b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z - 9*C^3*a^2, z, k), k, 1, 3)
```

3.34 $\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 53

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx=\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

```
[Out] -C*ln((-a/b)^(1/3)+x)/b+2/3*C*arctan(1/3*(1-2*x/(-a/b)^(1/3))*3^(1/2))/b*3^
(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1881, 31, 631, 210}

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx=\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

```
[In] Int[(2*(-(a/b))^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(-(a/
b))^(1/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1881

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\text{integral} = -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}+x}\,dx}{b} - \frac{\left(\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}-\sqrt[3]{-\frac{a}{b}}x+x^2}\,dx}{b}$$

$$= -\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b} - \frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$

$$= \frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 150 vs. 2(53) = 106.

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.83

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx = \frac{C\left(2\sqrt{3}\left(-\frac{a}{b}\right)^{2/3}b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\left(-\frac{a}{b}\right)^{2/3}b^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)+\left(-\frac{a}{b}\right.\right.}{3a^{2/3}b}$$

```
[In] Integrate[(2*(-(a/b))^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(-(a/b))^(2/3)*b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] - 2*(-(a/b))^(2/3)*b^(2/3)*Log[a^(1/3) - b^(1/3)*x] + (-(a/b))^(2/3)*
b^(2/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*Log[a - b*
x^3]))/(3*a^(2/3)*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(46) = 92.

Time = 1.48 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.25

method	result	size
default	$C\left(2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-\frac{\ln(-bx^3+a)}{3b}\right)$	119

```
[In] int((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(2*(-a/b)^(2/3)*(-1/3/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/6/b/(a/b)^(2/3)*l
n(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*(1+2/
(a/b)^(1/3)*x)*3^(1/2)))-1/3*ln(-b*x^3+a)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.78 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + Cx^2}{a - bx^3}\, dx = \frac{2\sqrt{3}C \arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}} + \sqrt{3}a}{3a}\right) - 3C\log\left(x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="fricas")
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) + sqrt(3)*a)/a) - 3
*C*log(x + (-a/b)^(1/3)))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.08

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + Cx^2}{a - bx^3}\, dx = -\frac{C\left(\log\left(-\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + x\right) + \frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + x\right)}{3} - \frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + x\right)}{3}\right)}{b}$$

```
[In] integrate((2*(-a/b)**(2/3)*C+C*x**2)/(-b*x**3+a),x)
[Out] -C*(log(-a/(b*(-a/b)**(2/3)) + x) + sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3)) - s
qrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(46) = 92$.

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.15

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx = -\frac{2\sqrt{3}\left(Ca-\left(3\,C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab} - \frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*(C*a - (3*C*(a/b)^(1/3)*(-a/b)^(2/3) + C*a/b)*b)*arctan(1/3*sq
rt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b) - 1/3*(C*(a/b)^(2/3) - C*(-a/b
)^(2/3))*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(C*(a
/b)^(2/3) + 2*C*(-a/b)^(2/3))*log(x - (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.06

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx = -\frac{\sqrt{3}\left(ab^2-i\sqrt{3}\sqrt{a^2b^4}\right)C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3} - \frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}$$

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(a*b^2 - I*sqrt(3)*sqrt(a^2*b^4))*C*arctan(1/3*sqrt(3)*(2*x +
(a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) - 1/3*(C*b^2*(a/b)^(2/3) + 2*(-a*b^2)^(2/
3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.25

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + Cx^2}{a - bx^3} \, dx = \sum_{k=1}^{3} \ln\left(-\frac{(C + \mathrm{root}(27\, a^2\, b^3\, z^3 + 27\, C\, a^2\, b^2\, z^2 + 9\, C^2\, a^2\, b\, z + 9\, C^3\, a^2, z, k)\; b\, 3)}{}\right.$$

```
[In] int((C*x^2 + 2*C*(-a/b)^(2/3))/(a - b*x^3),x)
```

```
[Out] symsum(log(-((C + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
+ 9*C^3*a^2, z, k)*b)*(C*a + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C
^2*a^2*b*z + 9*C^3*a^2, z, k)*a*b + 2*C*b*x*(-a/b)^(2/3)))/b^3)*root(27*a^2
*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z + 9*C^3*a^2, z, k), k, 1, 3)
```

3.35 $\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 54

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=-\frac{2C\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

```
[Out] C*ln((-a/b)^(1/3)-x)/b-2/3*C*arctan(1/3*(1+2*x/(-a/b)^(1/3))*3^(1/2))/b*3^(
1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1883, 31, 631, 210}

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{2C\arctan\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

```
[In] Int[(2*(-(a/b))^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(-(a
/b))^(1/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1883

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Dist[-C/b
, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]
] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &&
PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x}\,dx}{b} - \frac{\left(\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}+\sqrt[3]{-\frac{a}{b}}x+x^2}\,dx}{b}$$

$$= \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b} + \frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$

$$= -\frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} + \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 149 vs. $2(54) = 108$.

Time = 0.05 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.76

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx = \frac{C\left(-2\sqrt{3}\left(-\frac{a}{b}\right)^{2/3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(-\frac{a}{b}\right)^{2/3}b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-(-\cdots}{3a^{2/3}b}$$

```
[In] Integrate[(2*(-(a/b))^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*(-(a/b))^(2/3)*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]] + 2*(-(a/b))^(2/3)*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - (-(a/b))^(2/3)
*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*Log[a + b
*x^3]))/(3*a^(2/3)*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(47) = 94$.

Time = 1.48 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.17

method	result	size
default	$C\left(2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\frac{\ln\left(b\,x^3+a\right)}{3b}\right)$	117

```
[In] int((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(2*(-a/b)^(2/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1)))+1/3*ln(b*x^3+a)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + Cx^2}{a + bx^3}\, dx = \frac{2\sqrt{3}C \arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}a}{3\,a}\right) + 3\,C \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 3
*C*log(x - (-a/b)^(1/3)))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.02

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + Cx^2}{a + bx^3}\, dx = \frac{C\left(\log\left(\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + x\right) - \frac{\sqrt{3}i \log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + x\right)}{3} + \frac{\sqrt{3}i \log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + x\right)}{3}\right)}{b}$$

```
[In] integrate((2*(-a/b)**(2/3)*C+C*x**2)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(-a/b)**(2/3)) + x) - sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3)) - sq
rt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(47) = 94$.

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.11

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx = -\frac{2\sqrt{3}\left(Ca-\left(3C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab} + \frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+2C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*(C*a - (3*C*(a/b)^(1/3)*(-a/b)^(2/3) + C*a/b)*b)*arctan(1/3*sq
rt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/3*(C*(a/b)^(2/3) - C*(-a/b
)^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(C*(a
/b)^(2/3) + 2*C*(-a/b)^(2/3))*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.69

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx = -\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b} - \frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1/
3*(C*b^2*(-a/b)^(2/3) + 2*(-a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.20

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + Cx^2}{a + bx^3} \, dx = \sum_{k=1}^{3} \ln \left(-\frac{(C - \mathrm{root}(27\, a^2\, b^3\, z^3 - 27\, C\, a^2\, b^2\, z^2 + 9\, C^2\, a^2\, b\, z - 9\, C^3\, a^2, z, k)\ b\, 3)}{} \right.$$

```
[In] int((C*x^2 + 2*C*(-a/b)^(2/3))/(a + b*x^3),x)
```

```
[Out] symsum(log(-((C - 3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
- 9*C^3*a^2, z, k)*b)*(3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2
*b*z - 9*C^3*a^2, z, k)*a*b - C*a + 2*C*b*x*(-a/b)^(2/3)))/b^3)*root(27*a^2
*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z - 9*C^3*a^2, z, k), k, 1, 3)
```

3.36 $\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 53

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx=\frac{2C\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

```
[Out] -C*ln((a/b)^(1/3)-x)/b+2/3*C*arctan(1/3*(1+2*x/(a/b)^(1/3))*3^(1/2))/b*3^(1
/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1883, 31, 631, 210}

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx=\frac{2C\arctan\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

```
[In] Int[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^(1
/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1883

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Dist[-C/b
, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]
] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &&
PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}-x}\,dx}{b} + \frac{\left(\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}+\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}$$

$$= -\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b} - \frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}$$

$$= \frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 147 vs. $2(53) = 106$.

Time = 0.05 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.77

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3} C + Cx^2}{a - bx^3} \, dx = \frac{C\left(2\sqrt{3}\left(\frac{a}{b}\right)^{2/3} b^{2/3} \arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\left(\frac{a}{b}\right)^{2/3} b^{2/3} \log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right) + \left(\frac{a}{b}\right)^{2/3} b^{2/3}\right.}{3a^{2/3}b}$$

```
[In] Integrate[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(a/b)^(2/3)*b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] - 2*(a/b)^(2/3)*b^(2/3)*Log[a^(1/3) - b^(1/3)*x] + (a/b)^(2/3)*b^(2/3)*L
og[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*Log[a - b*x^3]))/(3
*a^(2/3)*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(46) = 92$.

Time = 1.50 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.23

method	result	size
default	$C\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-\frac{\ln\left(-b\,x^{3}+a\right)}{3b}\right)$	118

```
[In] int((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(2*(a/b)^(2/3)*(-1/3/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/6/b/(a/b)^(2/3)*ln
(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*(1+2/(
a/b)^(1/3)*x)*3^(1/2)))-1/3*ln(-b*x^3+a)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx=\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)-3C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) + sqrt(3)*a)/a) - 3*
C*log(x - (a/b)^(1/3)))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.92

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx=-\frac{C\left(\log\left(-\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

```
[In] integrate((2*(a/b)**(2/3)*C+C*x**2)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(a/b)**(2/3)) + x) + sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) - sqr
t(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx=\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - C*log
(x - (a/b)^(1/3))/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.60

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C + Cx^2}{a - bx^3}\,dx = \frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b} - \frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}} + 2\,(ab^2)^{\frac{2}{3}}\,C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}$$

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - 1/3*(
C*b^2*(a/b)^(2/3) + 2*(a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3))
)/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.54 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.23

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C + Cx^2}{a - bx^3}\,dx = \sum_{k=1}^{3}\ln\left(-\frac{\left(C + \mathrm{root}(27\,a^2\,b^3\,z^3 + 27\,C\,a^2\,b^2\,z^2 + 9\,C^2\,a^2\,b\,z + 9\,C^3\,a^2, z, k)\ b\,3\right)\left(C\right.}{}\right.$$

```
[In] int((C*x^2 + 2*C*(a/b)^(2/3))/(a - b*x^3),x)
```

```
[Out] symsum(log(-((C + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
+ 9*C^3*a^2, z, k)*b)*(C*a + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C
^2*a^2*b*z + 9*C^3*a^2, z, k)*a*b + 2*C*b*x*(a/b)^(2/3)))/b^3)*root(27*a^2*
b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z + 9*C^3*a^2, z, k), k, 1, 3)
```

3.37 $\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 61

$$\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx=-\frac{2C\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

```
[Out] C*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)-2/3*C*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1
/3)*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1877, 31, 631, 210}

$$\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx=\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2C\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

```
[In] Int[(2*a^(2/3)*C + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3))
+ (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1877

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt[3]{a}C\right)\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx}{b^{2/3}}+\frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx}{\sqrt[3]{b}}\\
&= \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{(2C)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}\\
&= -\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.56

$$\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx=\frac{C\left(-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right.}{3\sqrt[3]{b}}$$

```
[In] Integrate[(2*a^(2/3)*C + b^(2/3)*C*x^2)/(a + b*x^3),x]
[Out] (C*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3)
+ b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + Log[a + b*x
^3]))/(3*b^(1/3))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(44) = 88.

Time = 1.56 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.84

method	result	size
default	$C\left(2a^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\frac{\ln\left(b\,x^{3}+a\right)}{3b^{\frac{1}{3}}}\right)$	112

```
[In] int((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(2*a^(2/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1)))+1/3/b^(1/3)*ln(b*x^3+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.62

$$\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx = \left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2\,bx^{3}-3\,a^{\frac{2}{3}}b^{\frac{1}{3}}x+3\,\sqrt{\frac{1}{3}}\left(2\,a^{\frac{1}{3}}bx^{2}+a^{\frac{2}{3}}b^{\frac{2}{3}}x-ab^{\frac{1}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-a}{bx^{3}+a}\right)+Cb^{\frac{2}{3}}\log\left(b\right.}{b}\right.$$

```
[In] integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt(-1/b^(2/3))*log((2*b*x^3 - 3*a^(2/3)*b^(1/3)*x + 3*sqr
t(1/3)*(2*a^(1/3)*b*x^2 + a^(2/3)*b^(2/3)*x - a*b^(1/3))*sqrt(-1/b^(2/3)) -
 a)/(b*x^3 + a)) + C*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b, (2*sqrt(1/3)*C*
b^(2/3)*arctan(sqrt(1/3)*(2*a^(2/3)*b^(2/3)*x - a*b^(1/3))/(a*b^(1/3))) + C
*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b]
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{2a^{2/3}C + b^{2/3}Cx^2}{a + bx^3}\,dx = \operatorname{RootSum}\left(3t^3b^{\frac{5}{3}} - 3t^2Cb^{\frac{4}{3}} + tC^2b - C^3b^{\frac{2}{3}}, \left(t \mapsto t\log\left(x + \frac{3t\sqrt[3]{a}\sqrt[3]{b} - C\sqrt[3]{a}}{2C\sqrt[3]{b}}\right)\right)\right)$$

```
[In] integrate((2*a**(2/3)*C+b**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] RootSum(3*_t**3*b**(5/3) - 3*_t**2*C*b**(4/3) + _t*C**2*b - C**3*b**(2/3),
Lambda(_t, _t*log(x + (3*_t*a**(1/3)*b**(1/3) - C*a**(1/3))/(2*C*b**(1/3)))
))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(46) = 92.

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.66

$$\int \frac{2a^{2/3}C + b^{2/3}Cx^2}{a + bx^3}\,dx = -\frac{2\sqrt{3}\left(Cab^{\frac{2}{3}} - \left(3Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + \frac{Ca}{b^{\frac{1}{3}}}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab} + \frac{\left(Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}} - Ca^{\frac{2}{3}}\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}} + 2Ca^{\frac{2}{3}}\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*(C*a*b^(2/3) - (3*C*a^(2/3)*(a/b)^(1/3) + C*a/b^(1/3))*b)*arct
an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/3*(C*b^(2/3)*(a/b
)^(2/3) - C*a^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3))
 + 1/3*(C*b^(2/3)*(a/b)^(2/3) + 2*C*a^(2/3))*log(x + (a/b)^(1/3))/(b*(a/b)^
(2/3))
```

Giac [F(-1)]

Timed out.

$$\int \frac{2a^{2/3}C + b^{2/3}Cx^2}{a + bx^3}\, dx = \text{Timed out}$$

```
[In] integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.16

$$\int \frac{2a^{2/3}C + b^{2/3}Cx^2}{a + bx^3}\, dx = \sum_{k=1}^{3} \ln\left(-\frac{a^{2/3}\left(C - \operatorname{root}\left(27\,a^2\,b^3\,z^3 - 27\,C\,a^2\,b^{8/3}\,z^2 + 9\,C^2\,a^2\,b^{7/3}\,z - 9\,C^3\,a^2\,b^2,\right.\right.}{}\right.$$

```
[In] int((2*C*a^(2/3) + C*b^(2/3)*x^2)/(a + b*x^3),x)
```

```
[Out] symsum(log(-(a^(2/3)*(C - 3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^(8/3)*z^2 + 9*
C^2*a^2*b^(7/3)*z - 9*C^3*a^2*b^2, z, k)*b^(1/3))*(3*root(27*a^2*b^3*z^3 -
27*C*a^2*b^(8/3)*z^2 + 9*C^2*a^2*b^(7/3)*z - 9*C^3*a^2*b^2, z, k)*a^(1/3)*b
^(1/3) - C*a^(1/3) + 2*C*b^(1/3)*x))/b^(5/3))*root(27*a^2*b^3*z^3 - 27*C*a^
2*b^(8/3)*z^2 + 9*C^2*a^2*b^(7/3)*z - 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

3.38 $\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 70

$$\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx = -\frac{2C\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

```
[Out] C*ln(a^(1/3)-(-b)^(1/3)*x)/(-b)^(1/3)-2/3*C*arctan(1/3*(a^(1/3)+2*(-b)^(1/3
)*x)/a^(1/3)*3^(1/2))/(-b)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1880, 31, 631, 210}

$$\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx = \frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}-\frac{2C\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}}$$

```
[In] Int[(-2*a^(2/3)*C - (-b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(-b)^(
1/3)) + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1880

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/(-b)^(1/3)}, Dis
t[-C/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2),
x], x]] /; EqQ[A*(-b)^(2/3) + a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C, 0]] /; Fr
eeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(\sqrt[3]{a}C\right)\int \frac{1}{\frac{a^{2/3}}{(-b)^{2/3}}+\frac{\sqrt[3]{a}x}{\sqrt[3]{-b}}+x^2}\,dx}{(-b)^{2/3}}-\frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{-b}}-x}\,dx}{\sqrt[3]{-b}}\\
&= \frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}+\frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{-b}}\\
&= -\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.66

$$\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx =$$

$$-\frac{C\left(-2\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+(-b)^{2/3}\right.}{3b}$$

```
[In] Integrate[(-2*a^(2/3)*C - (-b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] -1/3*(C*(-2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2
*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2] + (-b)^(2/3)*Log[a + b*x^3]))/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(53) = 106$.

Time = 1.57 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.67

method	result	size
default	$C\left(-2a^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-\frac{(-b)^{\frac{2}{3}}\ln\left(b\,x^3+a\right)}{3b}\right)$	117

```
[In] int((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-2*a^(2/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1)))-1/3*(-b)^(2/3)*ln(b*x^3+a)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.93

$$\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx = \left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,bx^3+3\,a^{\frac{2}{3}}(-b)^{\frac{1}{3}}x-3\,\sqrt{\frac{1}{3}}\left(2\,a^{\frac{1}{3}}bx^2+a^{\frac{2}{3}}(-b)^{\frac{2}{3}}x+a(-b)^{\frac{1}{3}}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}-a}{bx^3+a}\right)}{b}\right.$$

```
[In] integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt((-b)^(1/3)/b)*log((2*b*x^3 + 3*a^(2/3)*(-b)^(1/3)*x -
3*sqrt(1/3)*(2*a^(1/3)*b*x^2 + a^(2/3)*(-b)^(2/3)*x + a*(-b)^(1/3))*sqrt((-
b)^(1/3)/b) - a)/(b*x^3 + a)) - C*(-b)^(2/3)*log(b*x + a^(1/3)*(-b)^(2/3)))
```

```
/b, -(2*sqrt(1/3)*C*b*sqrt(-(-b)^(1/3)/b)*arctan(sqrt(1/3)*(2*a^(2/3)*(-b)^
(2/3)*x + a*(-b)^(1/3))*sqrt(-(-b)^(1/3)/b)/a) + C*(-b)^(2/3)*log(b*x + a^(
1/3)*(-b)^(2/3)))/b]
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx = -\operatorname{RootSum}\left(3t^3b^2-3t^2Cb(-b)^{\frac{2}{3}}+tC^2(-b)^{\frac{4}{3}}-C^3b,\left(t\mapsto t\log\left(\frac{3t\sqrt[3]{a}}{2C}-\frac{\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b}+x\right)\right)\right)$$

```
[In] integrate((-2*a**(2/3)*C-(-b)**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] -RootSum(3*_t**3*b**2 - 3*_t**2*C*b*(-b)**(2/3) + _t*C**2*(-b)**(4/3) - C**
3*b, Lambda(_t, _t*log(3*_t*a**(1/3)/(2*C) - a**(1/3)*(-b)**(2/3)/(2*b) + x
)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(53) = 106.

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.47

$$\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx = \frac{2\sqrt{3}\left(Ca(-b)^{\frac{2}{3}}-\left(3\,Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{Ca(-b)^{\frac{2}{3}}}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab} - \frac{\left(C(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}-Ca^{\frac{2}{3}}\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(C(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,Ca^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*(C*a*(-b)^(2/3) - (3*C*a^(2/3)*(a/b)^(1/3) + C*a*(-b)^(2/3)/b)*
b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) - 1/3*(C*(-b)^
(2/3)*(a/b)^(2/3) - C*a^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a
/b)^(2/3)) - 1/3*(C*(-b)^(2/3)*(a/b)^(2/3) + 2*C*a^(2/3))*log(x + (a/b)^(1/
3))/(b*(a/b)^(2/3))
```

Giac [F(-1)]

Timed out.

$$\int \frac{-2a^{2/3}C - (-b)^{2/3}Cx^2}{a + bx^3} \, dx = \text{Timed out}$$

```
[In] integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 221, normalized size of antiderivative = 3.16

$$\int \frac{-2a^{2/3}C - (-b)^{2/3}Cx^2}{a + bx^3} \, dx = \sum_{k=1}^{3} \ln \left(\text{root}\left(27\,a^2\,b^3\,z^3 + 27\,C\,a^2\,(-b)^{8/3}\,z^2 - 9\,C^2\,a^2\,(-b)^{7/3}\,z + 9\,C^3\,a^2\,b^2, z, \right.\right.$$

```
[In] int(-(2*C*a^(2/3) + C*(-b)^(2/3)*x^2)/(a + b*x^3),x)
```

```
[Out] symsum(log(root(27*a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 - 9*C^2*a^2*(-b)^(
7/3)*z + 9*C^3*a^2*b^2, z, k)*((6*C*a)/(-b)^(4/3) + (9*root(27*a^2*b^3*z^3
+ 27*C*a^2*(-b)^(8/3)*z^2 - 9*C^2*a^2*(-b)^(7/3)*z + 9*C^3*a^2*b^2, z, k)*a
)/b - (6*C*a^(2/3)*x)/b) - (C^2*a)/(-b)^(5/3) - (2*C^2*a^(2/3)*x)/(-b)^(4/3
))*root(27*a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 - 9*C^2*a^2*(-b)^(7/3)*z +
 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

3.39 $\int \frac{-3+x^2}{-1+x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{-3+x^2}{-1+x^3}\,dx=\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-\frac{2}{3}\log(1-x)+\frac{5}{6}\log\left(1+x+x^2\right)$$

```
[Out] -2/3*ln(1-x)+5/6*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1889, 31, 648, 632, 210, 642}

$$\int \frac{-3+x^2}{-1+x^3}\,dx=\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)$$

```
[In] Int[(-3 + x^2)/(-1 + x^3),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - (2*Log[1 - x])/3 + (5*Log[1 + x + x^2])
/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1889

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Dist[q*((A + B*q
 + C*q^2)/(3*a)), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q -
 C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b
*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
 && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{3}\int \frac{-7-5x}{1+x+x^2}\,dx\right)+\frac{2}{3}\int \frac{1}{1-x}\,dx\\
&= -\frac{2}{3}\log(1-x)+\frac{5}{6}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{3}{2}\int \frac{1}{1+x+x^2}\,dx\\
&= -\frac{2}{3}\log(1-x)+\frac{5}{6}\log\left(1+x+x^2\right)-3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-\frac{2}{3}\log(1-x)+\frac{5}{6}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \frac{-3+x^2}{-1+x^3}\,dx=\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-\log(1-x)+\frac{1}{2}\log\left(1+x+x^2\right)+\frac{1}{3}\log\left(1-x^3\right)$$

```
[In] Integrate[(-3 + x^2)/(-1 + x^3),x]
[Out] Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[1 - x] + Log[1 + x + x^2]/2 + Log[1
 - x^3]/3
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{2\ln(-1+x)}{3}+\frac{5\ln(x^2+x+1)}{6}+\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}$	32
risch	$-\frac{2\ln(-1+x)}{3}+\frac{5\ln(9x^2+9x+9)}{6}+\sqrt{3}\,\arctan\left(\frac{2(\frac{3}{2}+3x)\sqrt{3}}{9}\right)$	36
meijerg	$-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{(x^3)^{\frac{1}{3}}}+\frac{\ln(-x^3+1)}{3}$	73

```
[In] int((x^2-3)/(x^3-1),x,method=_RETURNVERBOSE)
[Out] -2/3*ln(-1+x)+5/6*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{-3+x^2}{-1+x^3}\,dx=\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(x-1\right)$$

```
[In] integrate((x^2-3)/(x^3-1),x, algorithm="fricas")
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(x -
1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{-3+x^2}{-1+x^3}\,dx = -\frac{2\log\left(x-1\right)}{3} + \frac{5\log\left(x^2+x+1\right)}{6} + \sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)$$

```
[In] integrate((x**2-3)/(x**3-1),x)
[Out] -2*log(x - 1)/3 + 5*log(x**2 + x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt
(3)/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{-3+x^2}{-1+x^3}\,dx = \sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{5}{6}\log\left(x^2+x+1\right) - \frac{2}{3}\log\left(x-1\right)$$

```
[In] integrate((x^2-3)/(x^3-1),x, algorithm="maxima")
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(x -
1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{-3+x^2}{-1+x^3}\,dx = \sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{5}{6}\log\left(x^2+x+1\right) - \frac{2}{3}\log\left(|x-1|\right)$$

```
[In] integrate((x^2-3)/(x^3-1),x, algorithm="giac")
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(abs(
x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{-3+x^2}{-1+x^3}\,dx = -\frac{2\ln(x-1)}{3} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{5}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{5}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)$$

```
[In] int((x^2 - 3)/(x^3 - 1),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/2 + 5/6) - log(x - (3^(1/2)*1i)
/2 + 1/2)*((3^(1/2)*1i)/2 - 5/6) - (2*log(x - 1))/3
```

3.40 $\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 70

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx = -\frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}+\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

```
[Out] C*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)-2/3*(B/a^(1/3)+C/b^(1/3))*arctan(1/3*(a^(1/
3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.082, Rules used = {1877, 31, 631, 210}

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx = \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)}{\sqrt{3}}$$

```
[In] Int[(a^(1/3)*b^(1/3)*B + 2*a^(2/3)*C + b^(2/3)*B*x + b^(2/3)*C*x^2)/(a + b*
x^3),x]
```

```
[Out] (-2*(B/a^(1/3) + C/b^(1/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/Sqrt[3] + (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1877

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = \frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx}{\sqrt[3]{b}} + \frac{\left(\sqrt[3]{b}B+\sqrt[3]{a}C\right)\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx}{b^{2/3}}$$

$$= \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \left(2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)$$

$$= -\frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}} + \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx = \frac{-2\sqrt{3}\left(\sqrt[3]{b}B+\sqrt[3]{a}C\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\sqrt[3]{a}C\left(2\log\left(\sqrt[3]{a}\right.\right.}{3\sqrt[3]{a}}$$

```
[In] Integrate[(a^(1/3)*b^(1/3)*B + 2*a^(2/3)*C + b^(2/3)*B*x + b^(2/3)*C*x^2)/(
a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b^(1/3)*B + a^(1/3)*C)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + a^(1/3)*C*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2] + Log[a + b*x^3]))/(3*a^(1/3)*b^(1/3))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(51) = 102$.

Time = 1.59 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.10

method	result
default	$\left(a^{\frac{1}{3}}b^{\frac{1}{3}}B+2a^{\frac{2}{3}}C\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+b^{\frac{2}{3}}B\left(-\frac{\ln\left(x+\right.}{3b\left(\right.}\right.$

```
[In] int((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+a),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] (a^(1/3)*b^(1/3)*B+2*a^(2/3)*C)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/
(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+b^(2/3)*B*(-1/3/b/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/
b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*C/b^(1/3)*ln(b*x
^3+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(53) = 106.

Time = 1.99 (sec) , antiderivative size = 430, normalized size of antiderivative = 6.14

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + 2a^{2/3}C + b^{2/3}Bx + b^{2/3}Cx^2}{a + bx^3}\,dx = \left[\frac{\sqrt{\frac{1}{3}}b\sqrt{-\frac{C^2ab^{\frac{1}{3}}+2\,BCa^{\frac{2}{3}}b^{\frac{2}{3}}+B^2a^{\frac{1}{3}}b}{ab}}\log\left(-\frac{C^3a^2+B^3ab-2\,(C^3ab+B^3}{}\right.}{}\right.$$

[In] integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")

[Out] [(sqrt(1/3)*b*sqrt(-(C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b)/(a*b))*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 + 3*(C^3*a + B^3*b)*a^(2/3)*b^(1/3)*x - 3*sqrt(1/3)*((2*B^2*b*x^2 + C^2*a*x + B*C*a)*a^(2/3)*b^(2/3) + (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*a^(1/3) - (2*B*C*a*b*x^2 - B^2*a*b*x + C^2*a^2)*b^(1/3))*sqrt(-(C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b)/(a*b)))/(b*x^3 + a)) + C*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b, (2*sqrt(1/3)*b*sqrt((C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b)/(a*b))*arctan(sqrt(1/3)*((2*C^2*x + B*C)*a^(2/3)*b^(2/3) - (2*B*C*b*x + B^2*b)*a^(1/3) + (2*B^2*b*x - C^2*a)*b^(1/3))*sqrt((C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b)/(a*b))/(C^3*a + B^3*b)) + C*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b]

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + 2a^{2/3}C + b^{2/3}Bx + b^{2/3}Cx^2}{a + bx^3}\,dx = \text{Timed out}$$

[In] integrate((a**(1/3)*b**(1/3)*B+2*a**(2/3)*C+b**(2/3)*B*x+b**(2/3)*C*x**2)/(b*x**3+a),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(53) = 106.

Time = 0.28 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.37

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + 2a^{2/3}C + b^{2/3}Bx + b^{2/3}Cx^2}{a + bx^3}\,dx = -\frac{\sqrt{3}\left(2\,Cab^{\frac{2}{3}} - \left(6\,Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + 3\,Ba^{\frac{1}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}} + \frac{2\,Ca}{b}\right)b^{\frac{2}{3}}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab} - \frac{\left(2\,Ca^{\frac{2}{3}} + Ba^{\frac{1}{3}}b^{\frac{1}{3}} - \left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}} + B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^{\frac{2}{3}}\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(2\,Ca^{\frac{2}{3}} + Ba^{\frac{1}{3}}b^{\frac{1}{3}} + \left(C\left(\frac{a}{b}\right)^{\frac{2}{3}} - B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^{\frac{2}{3}}\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+
a),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*(2*C*a*b^(2/3) - (6*C*a^(2/3)*(a/b)^(1/3) + 3*B*a^(1/3)*b^(1/3
)*(a/b)^(1/3) + (3*B*(a/b)^(2/3) + 2*C*a/b)*b^(2/3))*b)*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) - 1/6*(2*C*a^(2/3) + B*a^(1/3)*b^(1/
3) - (2*C*(a/b)^(2/3) + B*(a/b)^(1/3))*b^(2/3))*log(x^2 - x*(a/b)^(1/3) + (
a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(2*C*a^(2/3) + B*a^(1/3)*b^(1/3) + (C*(a/
b)^(2/3) - B*(a/b)^(1/3))*b^(2/3))*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + 2a^{2/3}C + b^{2/3}Bx + b^{2/3}Cx^2}{a + bx^3}\,dx = \text{Timed out}$$

```
[In] integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+
a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 386, normalized size of antiderivative = 5.51

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + 2a^{2/3}C + b^{2/3}Bx + b^{2/3}Cx^2}{a + bx^3}\,dx = \sum_{k=1}^{3} \ln\left(-\frac{x\left(2\,C^2\,a^{2/3}\,b^{2/3} - B^2\,b^{4/3} + B\,C\,a^{1/3}\,b\right)}{b^2} + \frac{a^{1/3}\,(B$$

```
[In] int((2*C*a^(2/3) + B*a^(1/3)*b^(1/3) + C*b^(2/3)*x^2 + B*b^(2/3)*x)/(a + b*
x^3),x)
```

```
[Out] symsum(log((a^(1/3)*(B*b^(1/3) + C*a^(1/3))^2)/b^(5/3) - (x*(2*C^2*a^(2/3)*
b^(2/3) - B^2*b^(4/3) + B*C*a^(1/3)*b))/b^2 + (root(27*a^2*b^3*z^3 - 27*C*a
^2*b^(8/3)*z^2 + 18*B*C*a^(5/3)*b^(8/3)*z + 9*C^2*a^2*b^(7/3)*z + 9*B^2*a^(
4/3)*b^3*z - 18*B*C^2*a^(5/3)*b^(7/3) - 9*B^2*C*a^(4/3)*b^(8/3) - 9*C^3*a^2
*b^2, z, k)*(9*root(27*a^2*b^3*z^3 - 27*C*a^2*b^(8/3)*z^2 + 18*B*C*a^(5/3)*
b^(8/3)*z + 9*C^2*a^2*b^(7/3)*z + 9*B^2*a^(4/3)*b^3*z - 18*B*C^2*a^(5/3)*b^
(7/3) - 9*B^2*C*a^(4/3)*b^(8/3) - 9*C^3*a^2*b^2, z, k)*a*b^(1/3) - 6*C*a +
3*B*a^(1/3)*b^(2/3)*x + 6*C*a^(2/3)*b^(1/3)*x))/b^(4/3))*root(27*a^2*b^3*z^
3 - 27*C*a^2*b^(8/3)*z^2 + 18*B*C*a^(5/3)*b^(8/3)*z + 9*C^2*a^2*b^(7/3)*z +
 9*B^2*a^(4/3)*b^3*z - 18*B*C^2*a^(5/3)*b^(7/3) - 9*B^2*C*a^(4/3)*b^(8/3) -
 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

3.41 $\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal result

Integrand size = 57, antiderivative size = 88

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx=\frac{2\left(bB+\sqrt[3]{a}(-b)^{2/3}C\right)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

```
[Out] C*ln(a^(1/3)-(-b)^(1/3)*x)/(-b)^(1/3)+2/3*(b*B+a^(1/3)*(-b)^(2/3)*C)*arctan
(1/3*(a^(1/3)+2*(-b)^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {1880, 31, 631, 210}

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx=\frac{2\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}(-b)^{2/3}C+bB\right)}{\sqrt{3}\sqrt[3]{a}b}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

```
[In] Int[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (-b)^(2/3)*C*x^2
)/(a + b*x^3),x]
```

```
[Out] (2*(b*B + a^(1/3)*(-b)^(2/3)*C)*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(1/3)*b) + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1880

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/(-b)^(1/3)}, Dis
t[-C/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2),
x], x]] /; EqQ[A*(-b)^(2/3) + a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C, 0]] /; Fr
eeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{-b}}-x}\,dx}{\sqrt[3]{-b}}+\frac{\left(\sqrt[3]{-b}B-\sqrt[3]{a}C\right)\int \frac{1}{\frac{a^{2/3}}{(-b)^{2/3}}+\frac{\sqrt[3]{a}x}{\sqrt[3]{-b}}+x^2}\,dx}{(-b)^{2/3}}\\
&= \frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}-\left(2\left(\frac{B}{\sqrt[3]{a}}+\frac{bC}{(-b)^{4/3}}\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}\right)\\
&= \frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{bC}{(-b)^{4/3}}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 238 vs. 2(88) = 176.

Time = 0.57 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.70

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - 2a^{2/3}C - (-b)^{2/3}Bx - (-b)^{2/3}Cx^2}{a + bx^3}\,dx = \frac{2\sqrt{3}\sqrt[3]{b}\left(\left((-b)^{2/3} - \sqrt[3]{-b^2}\right)B + 2\sqrt[3]{a}\sqrt[3]{b}C\right)\arctan}{}$$

```
[In] Integrate[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (-b)^(2/3)
*C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[3]*b^(1/3)*(((-b)^(2/3) - (-b^2)^(1/3))*B + 2*a^(1/3)*b^(1/3)*C)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (-2*b*((-(-b)^(2/3) + b^(2/3))*
B + 2*a^(1/3)*(-b)^(1/3)*C)*Log[a^(1/3) + b^(1/3)*x] + ((-b)^(5/3)*B + b^(5
/3)*B + 2*a^(1/3)*(-b)^(1/3)*b*C)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2] - 2*a^(1/3)*(-b)^(2/3)*(-b^2)^(1/3)*C*Log[a + b*x^3])/(-b^2)^(1/3))/(
6*a^(1/3)*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(67) = 134.

Time = 1.59 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.58

method	result
default	$\left(a^{\frac{1}{3}}(-b)^{\frac{1}{3}}B - 2a^{\frac{2}{3}}C\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right) - (-b)^{\frac{2}{3}}B$

```
[In] int((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2)/(b*x
^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] (a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6
/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-(-b)^(2/3)*B*(-1/3/b/(a/b)^(1/3)*l
n(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^
```

```
(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-1/3*C*(-b)^(2/
3)*ln(b*x^3+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(67) = 134$.

Time = 1.66 (sec) , antiderivative size = 470, normalized size of antiderivative = 5.34

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - 2a^{2/3}C - (-b)^{2/3}Bx - (-b)^{2/3}Cx^2}{a + bx^3}\,dx = \left[\frac{\sqrt{\frac{1}{3}}b\sqrt{\frac{C^2a(-b)^{\frac{1}{3}} - 2\,BCa^{\frac{2}{3}}(-b)^{\frac{2}{3}} - B^2a^{\frac{1}{3}}b}{ab}}\log\left(-\frac{C^3a^2+B}{}\right.}{}\right.$$

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2
)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*b*sqrt((C^2*a*(-b)^(1/3) - 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/
3)*b)/(a*b))*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 - 3*(C^3*a
 + B^3*b)*a^(2/3)*(-b)^(1/3)*x + 3*sqrt(1/3)*((2*B^2*b*x^2 + C^2*a*x + B*C*
a)*a^(2/3)*(-b)^(2/3) + (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*a^(1/3) + (2*
B*C*a*b*x^2 - B^2*a*b*x + C^2*a^2)*(-b)^(1/3))*sqrt((C^2*a*(-b)^(1/3) - 2*B
*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/3)*b)/(a*b)))/(b*x^3 + a)) - C*(-b)^(2/3)*
log(b*x + a^(1/3)*(-b)^(2/3)))/b, -(2*sqrt(1/3)*b*sqrt(-(C^2*a*(-b)^(1/3) -
 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/3)*b)/(a*b))*arctan(sqrt(1/3)*((2*C^2*
x + B*C)*a^(2/3)*(-b)^(2/3) - (2*B*C*b*x + B^2*b)*a^(1/3) - (2*B^2*b*x - C^
2*a)*(-b)^(1/3))*sqrt(-(C^2*a*(-b)^(1/3) - 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a
^(1/3)*b)/(a*b))/(C^3*a + B^3*b)) + C*(-b)^(2/3)*log(b*x + a^(1/3)*(-b)^(2/
3)))/b]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - 2a^{2/3}C - (-b)^{2/3}Bx - (-b)^{2/3}Cx^2}{a + bx^3}\,dx = \text{Timed out}$$

```
[In] integrate((a**(1/3)*(-b)**(1/3)*B-2*a**(2/3)*C-(-b)**(2/3)*B*x-(-b)**(2/3)*
C*x**2)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(67) = 134.

Time = 0.31 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx = \frac{\sqrt{3}\left(2\,Ca(-b)^{\frac{2}{3}}-\left(6\,Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}-3\,Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.\right.}{9} + \frac{\left(2\,Ca^{\frac{2}{3}}-Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}-\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}+B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)(-b)^{\frac{2}{3}}\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(2\,Ca^{\frac{2}{3}}-Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}+\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)(-b)^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2
)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*(2*C*a*(-b)^(2/3) - (6*C*a^(2/3)*(a/b)^(1/3) - 3*B*a^(1/3)*(-b)
^(1/3)*(a/b)^(1/3) + (3*B*(a/b)^(2/3) + 2*C*a/b)*(-b)^(2/3))*b)*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*C*a^(2/3) - B*a^(1/
3)*(-b)^(1/3) - (2*C*(a/b)^(2/3) + B*(a/b)^(1/3))*(-b)^(2/3))*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(2*C*a^(2/3) - B*a^(1/3)*(-
b)^(1/3) + (C*(a/b)^(2/3) - B*(a/b)^(1/3))*(-b)^(2/3))*log(x + (a/b)^(1/3))
/(b*(a/b)^(2/3))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2
)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 11.27 (sec) , antiderivative size = 444, normalized size of antiderivative = 5.05

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - 2a^{2/3}C - (-b)^{2/3}Bx - (-b)^{2/3}Cx^2}{a + bx^3}\, dx = \sum_{k=1}^{3} \ln\left(\operatorname{root}\left(27\, a^2\, b^3\, z^3 + 27\, C\, a^2\, (-b)^{8/3}\, z^2 + 18\, B\right.\right.$$

```
[In] int(-(2*C*a^(2/3) + B*(-b)^(2/3)*x - B*a^(1/3)*(-b)^(1/3) + C*(-b)^(2/3)*x^
2)/(a + b*x^3),x)
```

```
[Out] symsum(log(root(27*a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 + 18*B*C*a^(5/3)*(
-b)^(8/3)*z + 9*B^2*a^(4/3)*b^3*z - 9*C^2*a^2*(-b)^(7/3)*z - 18*B*C^2*a^(5/
3)*(-b)^(7/3) + 9*B^2*C*a^(4/3)*(-b)^(8/3) + 9*C^3*a^2*b^2, z, k)*((6*C*a)/
(-b)^(4/3) - (x*(3*B*a^(1/3)*(-b)^(4/3) + 6*C*a^(2/3)*b))/b^2 + (9*root(27*
a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 + 18*B*C*a^(5/3)*(-b)^(8/3)*z + 9*B^2
*a^(4/3)*b^3*z - 9*C^2*a^2*(-b)^(7/3)*z - 18*B*C^2*a^(5/3)*(-b)^(7/3) + 9*B
^2*C*a^(4/3)*(-b)^(8/3) + 9*C^3*a^2*b^2, z, k)*a)/b) + (B^2*a^(1/3)*b^2 + C
^2*a*(-b)^(4/3) - 2*B*C*a^(2/3)*(-b)^(5/3))/b^3 - (x*(2*C^2*a^(2/3)*(-b)^(2
/3) - B^2*(-b)^(4/3) + B*C*a^(1/3)*b))/b^2)*root(27*a^2*b^3*z^3 + 27*C*a^2*
(-b)^(8/3)*z^2 + 18*B*C*a^(5/3)*(-b)^(8/3)*z + 9*B^2*a^(4/3)*b^3*z - 9*C^2*
a^2*(-b)^(7/3)*z - 18*B*C^2*a^(5/3)*(-b)^(7/3) + 9*B^2*C*a^(4/3)*(-b)^(8/3)
 + 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

3.42 $\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 11

$$\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3} \, dx = \frac{\log(B-Cx)}{C}$$

```
[Out] ln(-C*x+B)/C
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1600, 31}

$$\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3} \, dx = \frac{\log(B-Cx)}{C}$$

```
[In] Int[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3),x]
```

```
[Out] Log[B - C*x]/C
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\text{integral} = \int \frac{1}{-B+Cx}\,dx$$
$$= \frac{\log(B-Cx)}{C}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3}\,dx = \frac{\log(-B+Cx)}{C}$$

```
[In] Integrate[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3),x]
[Out] Log[-B + C*x]/C
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\ln(-Cx+B)}{C}$	12
norman	$\frac{\ln(-Cx+B)}{C}$	12
risch	$\frac{\ln(-Cx+B)}{C}$	12
parallelrisch	$\frac{\ln(-Cx+B)}{C}$	12

```
[In] int((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x,method=_RETURNVERBOSE)
[Out] ln(-C*x+B)/C
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3}\,dx = \frac{\log\left(Cx-B\right)}{C}$$

```
[In] integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="fricas")
[Out] log(C*x - B)/C
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3}\,dx = \frac{\log(-B + Cx)}{C}$$

```
[In] integrate((C**2*x**2+B*C*x+B**2)/(C**3*x**3-B**3),x)
[Out] log(-B + C*x)/C
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3}\,dx = \frac{\log(Cx - B)}{C}$$

```
[In] integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="maxima")
[Out] log(C*x - B)/C
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3}\,dx = \frac{\log(|Cx - B|)}{C}$$

```
[In] integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="giac")
[Out] log(abs(C*x - B))/C
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3}\,dx = \frac{\ln(C\,x - B)}{C}$$

```
[In] int(-(B^2 + C^2*x^2 + B*C*x)/(B^3 - C^3*x^3),x)
[Out] log(C*x - B)/C
```

3.43 $\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 21

$$\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx=\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

```
[Out] C*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1600, 31}

$$\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx=\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

```
[In] Int[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\frac{\sqrt[3]{a}}{C} + \frac{\sqrt[3]{b}x}{C}}\,dx$$
$$= \frac{C\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a + bx^3}\,dx = \frac{C\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

```
[In] Integrate[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3),x]
[Out] (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result
risch	$\frac{C\ln\left(a^{\frac{1}{3}}b^{\frac{2}{3}}+bx\right)}{b^{\frac{1}{3}}}$
default	$C\left(a^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-b^{\frac{1}{3}}a^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(\right.}{}\right.\right.$

```
[In] int((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x,method=_RETUR
NVERBOSE)
[Out] C/b^(1/3)*ln(a^(1/3)*b^(2/3)+b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a + bx^3}\,dx = \frac{C\log\left(bx + a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}$$

```
[In] integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algori
thm="fricas")
[Out] C*log(b*x + a^(1/3)*b^(2/3))/b^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a + bx^3}\,dx = \frac{C\log\left(\sqrt[3]{a}b^{\frac{2}{3}} + bx\right)}{\sqrt[3]{b}}$$

```
[In] integrate((a**(2/3)*C-a**(1/3)*b**(1/3)*C*x+b**(2/3)*C*x**2)/(b*x**3+a),x)
[Out] C*log(a**(1/3)*b**(2/3) + b*x)/b**(1/3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(15) = 30.

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 10.00

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a + bx^3}\,dx = -\frac{\sqrt{3}\left(2\,Cab^{\frac{2}{3}} + \left(3\,Ca^{\frac{1}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}} - 3\,Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} - \frac{2\,Ca}{b^{\frac{1}{3}}}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab} + \frac{\left(2\,Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}} - Ca^{\frac{1}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} - Ca^{\frac{2}{3}}\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}} + Ca^{\frac{1}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + Ca^{\frac{2}{3}}\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algori
thm="maxima")
```

```
[Out] -1/9*sqrt(3)*(2*C*a*b^(2/3) + (3*C*a^(1/3)*b^(1/3)*(a/b)^(2/3) - 3*C*a^(2/3
)*(a/b)^(1/3) - 2*C*a/b^(1/3))*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a
/b)^(1/3))/(a*b) + 1/6*(2*C*b^(2/3)*(a/b)^(2/3) - C*a^(1/3)*b^(1/3)*(a/b)^(
1/3) - C*a^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) +
1/3*(C*b^(2/3)*(a/b)^(2/3) + C*a^(1/3)*b^(1/3)*(a/b)^(1/3) + C*a^(2/3))*log
(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a + bx^3}\,dx = \frac{C\log\left(\left|b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right|\right)}{b^{\frac{1}{3}}}$$

```
[In] integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algori
thm="giac")
```

```
[Out] C*log(abs(b^(1/3)*x + a^(1/3)))/b^(1/3)
```

Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a + bx^3}\,dx = \frac{C\,\ln\left(x + \frac{a^{1/3}}{b^{1/3}}\right)}{b^{1/3}}$$

```
[In] int((C*a^(2/3) + C*b^(2/3)*x^2 - C*a^(1/3)*b^(1/3)*x)/(a + b*x^3),x)
```

```
[Out] (C*log(x + a^(1/3)/b^(1/3)))/b^(1/3)
```

3.44 $\int \frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 71

$$\int \frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx = -\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B+\sqrt[3]{\frac{a}{b}}C\right)\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}a}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

```
[Out] C*ln((a/b)^(1/3)+x)/b-2/3*(a/b)^(2/3)*(B+(a/b)^(1/3)*C)*arctan(1/3*(1-2*x/(
a/b)^(1/3))*3^(1/2))/a*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1881, 31, 631, 210}

$$\int \frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx=\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2\left(\frac{a}{b}\right)^{2/3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)\left(C\sqrt[3]{\frac{a}{b}}+B\right)}{\sqrt{3}a}$$

```
[In] Int[((a/b)^(1/3)*B + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*(a/b)^(2/3)*(B + (a/b)^(1/3)*C)*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]]
)/(Sqrt[3]*a) + (C*Log[(a/b)^(1/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1881

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}+x}\,dx}{b}+\frac{\left(B+\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}\\
&= \frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}+\left(2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a}+\frac{C}{b}\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)\\
&= -\frac{2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a}+\frac{C}{b}\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 247 vs. $2(71)=142$.

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.48

$$
\int\frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx=\frac{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}B+\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\left(B+2\sqrt[3]{\frac{a}{b}}C\right)\right)\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right.}{}
$$

```
[In] Integrate[((a/b)^(1/3)*B + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (a/b)^(1/3)*b^(1/3)*(B + 2*(a/b)^(1
/3)*C))*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 2*b^(1/3)*(-(a
^(2/3)*B) + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(B + 2*(a/b)^(1/3)*C))*Log[a^(1/3)
+ b^(1/3)*x] + b^(1/3)*(a^(2/3)*B - a^(1/3)*(a/b)^(1/3)*b^(1/3)*(B + 2*(a/b
)^(1/3)*C))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*C*Log[a +
b*x^3])/(6*a*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(60) = 120.

Time = 1.50 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.08

method	result
default	$\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}} C+\left(\frac{a}{b}\right)^{\frac{1}{3}} B\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+B\left(-\frac{\ln\left(x+\right.}{3b\left(\right.}\right.$

[In] int(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] (2*(a/b)^(2/3)*C+(a/b)^(1/3)*B)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+B*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*C/b*ln(b*x^3+a)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(60) = 120.

Time = 1.98 (sec) , antiderivative size = 429, normalized size of antiderivative = 6.04

$$\int \frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx=\left[\frac{C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+\sqrt{\frac{1}{3}}\sqrt{-\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}+B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}+C^2a}{a}}\log\left(-\frac{C^3a^2+B^3\cdots}{\cdots}\right.}{\cdots}\right.$$

[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm="fricas")

[Out] [(C*log(x + (a/b)^(1/3)) + sqrt(1/3)*sqrt(-(2*B*C*b*(a/b)^(2/3) + B^2*b*(a/b)^(1/3) + C^2*a)/a)*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 + 3*(C^3*a*b + B^3*b^2)*x*(a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*a*b*

```
x + C^2*a^2 - (2*B^2*b^2*x^2 + C^2*a*b*x + B*C*a*b)*(a/b)^(2/3) - (2*C^2*a*
b*x^2 - B*C*a*b*x - B^2*a*b)*(a/b)^(1/3))*sqrt(-(2*B*C*b*(a/b)^(2/3) + B^2*
b*(a/b)^(1/3) + C^2*a)/a))/(b*x^3 + a)))/b, (2*sqrt(1/3)*sqrt((2*B*C*b*(a/b
)^(2/3) + B^2*b*(a/b)^(1/3) + C^2*a)/a)*arctan(sqrt(1/3)*(2*B^2*b*x - C^2*a
 + (2*C^2*b*x + B*C*b)*(a/b)^(2/3) - (2*B*C*b*x + B^2*b)*(a/b)^(1/3))*sqrt(
(2*B*C*b*(a/b)^(2/3) + B^2*b*(a/b)^(1/3) + C^2*a)/a)/(C^3*a + B^3*b)) + C*l
og(x + (a/b)^(1/3)))/b]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a + bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(((a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a + bx^3}\,dx = \frac{C\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b} - \frac{2\sqrt{3}\left(Ca - \left(3B\left(\frac{a}{b}\right)^{\frac{2}{3}} + \frac{4Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab}$$

```
[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm=
"maxima")
```

```
[Out] C*log(x + (a/b)^(1/3))/b - 2/9*sqrt(3)*(C*a - (3*B*(a/b)^(2/3) + 4*C*a/b)*b
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx=\frac{\left(2\,Cab+\left(-a^2b^4\right)^{\frac{1}{3}}\,B\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,ab^2-i\,\sqrt{3}\sqrt{a^2b^4}}-\frac{2\,\sqrt{3}\left(Cab+\left(ab^2\right)^{\frac{2}{3}}\,B\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}-\frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+Bb^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(ab^2\right)^{\frac{1}{3}}\,Bb+2\,\left(ab^2\right)^{\frac{2}{3}}\,C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}$$

```
[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm=
"giac")
```

```
[Out] (2*C*a*b + (-a^2*b^4)^(1/3)*B)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(3*
a*b^2 - I*sqrt(3)*sqrt(a^2*b^4)) - 2/3*sqrt(3)*(C*a*b + (a*b^2)^(2/3)*B)*ar
ctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(-
a/b)^(2/3) + B*b^2*(-a/b)^(1/3) + (a*b^2)^(1/3)*B*b + 2*(a*b^2)^(2/3)*C)*(-
a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 10.86 (sec) , antiderivative size = 436, normalized size of antiderivative = 6.14

$$\int \frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx=\sum_{k=1}^{3}\ln\left(\frac{C^2\,a+B^2\,b\,\left(\frac{a}{b}\right)^{1/3}+2\,B\,C\,b\,\left(\frac{a}{b}\right)^{2/3}}{b^3}+\frac{\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+18\,B\,C\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{2/3}+9\,B^2\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,\left(\frac{a}{b}\right)^{2/3}\right.}{}-\frac{x\left(2\,C^2\,\left(\frac{a}{b}\right)^{2/3}-B^2+B\,C\,\left(\frac{a}{b}\right)^{1/3}\right)}{b^2}\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+18\,B\,C\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{2/3}+9\,B^2\,a\,b^2\,z\,\left(\frac{a}{b}\right.\right.$$

```
[In] int((B*x + C*x^2 + B*(a/b)^(1/3) + 2*C*(a/b)^(2/3))/(a + b*x^3),x)
```

```
[Out] symsum(log((C^2*a + B^2*b*(a/b)^(1/3) + 2*B*C*b*(a/b)^(2/3))/b^3 + (root(27
*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 18*B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*
z*(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(a/b)^(2/3) - 9*B^2*C*a*b*(a/b
)^(1/3) - 9*C^3*a^2, z, k)*(9*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 18*B
```

```
*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C
^2*a*b*(a/b)^(2/3) - 9*B^2*C*a*b*(a/b)^(1/3) - 9*C^3*a^2, z, k)*a*b - 6*C*a
 + 3*B*b*x*(a/b)^(1/3) + 6*C*b*x*(a/b)^(2/3)))/b^2 - (x*(2*C^2*(a/b)^(2/3)
- B^2 + B*C*(a/b)^(1/3)))/b^2)*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 18*
B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*
C^2*a*b*(a/b)^(2/3) - 9*B^2*C*a*b*(a/b)^(1/3) - 9*C^3*a^2, z, k), k, 1, 3)
```

3.45 $\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 76

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx=\frac{2\left(B+\sqrt[3]{-\frac{a}{b}}C\right)\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-\frac{a}{b}}b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

```
[Out] -C*ln((-a/b)^(1/3)+x)/b+2/3*(B+(-a/b)^(1/3)*C)*arctan(1/3*(1-2*x/(-a/b)^(1/
3))*3^(1/2))/(-a/b)^(1/3)/b*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {1881, 31, 631, 210}

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx=\frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)\left(C\sqrt[3]{-\frac{a}{b}}+B\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

```
[In] Int[((-(a/b))^(1/3)*B + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]

[Out] (2*(B + (-(a/b))^(1/3)*C)*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt
[3]*(-(a/b))^(1/3)*b) - (C*Log[(-(a/b))^(1/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1881

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
```

```
Q[P2, x, 2]
```

Rubi steps

$$\text{integral} = -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}+x}\,dx}{b} - \frac{\left(B+\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}-\sqrt[3]{-\frac{a}{b}}x+x^2}\,dx}{b}$$

$$= -\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b} - \frac{\left(2\left(\frac{B}{\sqrt[3]{-\frac{a}{b}}}+C\right)\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$

$$= \frac{2\left(\frac{B}{\sqrt[3]{-\frac{a}{b}}}+C\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 288 vs. $2(76) = 152$.

Time = 0.19 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.79

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx = -\frac{\left(a^{2/3}B-\sqrt[3]{a}\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}B-2\sqrt[3]{a}\left(-\frac{a}{b}\right)^{2/3}\sqrt[3]{b}C\right)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}ab^{2/3}} - \frac{\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}B+2\sqrt[3]{a}\left(-\frac{a}{b}\right)^{2/3}\sqrt[3]{b}C\right)\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3ab^{2/3}} - \frac{\left(-a^{2/3}B-\sqrt[3]{a}\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}B-2\sqrt[3]{a}\left(-\frac{a}{b}\right)^{2/3}\sqrt[3]{b}C\right)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6ab^{2/3}} - \frac{C\log\left(a-bx^3\right)}{3b}$$

```
[In] Integrate[((-(a/b))^(1/3)*B + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a - b*x^3)
,x]
```

```
[Out] -(((a^(2/3)*B - a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B - 2*a^(1/3)*(-(a/b))^(2/3)
*b^(1/3)*C)*ArcTan[(a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a*b
^(2/3))) - ((a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B + 2*a^(1/3)*(-(a/
b))^(2/3)*b^(1/3)*C)*Log[a^(1/3) - b^(1/3)*x])/(3*a*b^(2/3)) - ((-(a^(2/3)*
B) - a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B - 2*a^(1/3)*(-(a/b))^(2/3)*b^(1/3)*C)
*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a*b^(2/3)) - (C*Log[a -
 b*x^3])/(3*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(65) = 130$.

Time = 1.52 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.95

method	result
default	$\left(2\left(-\frac{a}{b}\right)^{\frac{2}{3}} C+\left(-\frac{a}{b}\right)^{\frac{1}{3}} B\right)\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+B\left(\right.$

```
[In] int(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x,method=_RETURN
VERBOSE)
```

```
[Out] (2*(-a/b)^(2/3)*C+(-a/b)^(1/3)*B)*(-1/3/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/6
/b/(a/b)^(2/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2)))+B*(-1/3/b/(a/b)^(1/3)*ln(x-(a/b)^(
1/3))+1/6/b/(a/b)^(1/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*3^(1/2)/b/(a/
b)^(1/3)*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2)))-1/3*C*ln(-b*x^3+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(65) = 130$.

Time = 1.02 (sec) , antiderivative size = 459, normalized size of antiderivative = 6.04

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a - bx^3}\,dx = \left[-\frac{C\log\left(x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) - \sqrt{\frac{1}{3}}\sqrt{\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}} + B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}} - C^2a}{a}}\log\left(-\right.}{}\right.$$

$$-\frac{2\sqrt{\frac{1}{3}}\sqrt{-\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}} + B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}} - C^2a}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2\,B^2bx + C^2a + \left(2\,C^2bx + BCb\right)\left(-\frac{a}{b}\right)^{\frac{2}{3}} - \left(2\,BCbx + B^2b\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{2\,BCb\left(-\right.}{}}}{C^3a - B^3b}\right.}{b}$$

```
[In] integrate(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorit
hm="fricas")
```

```
[Out] [-(C*log(x + (-a/b)^(1/3)) - sqrt(1/3)*sqrt((2*B*C*b*(-a/b)^(2/3) + B^2*b*(
-a/b)^(1/3) - C^2*a)/a)*log(-(C^3*a^2 - B^3*a*b + 2*(C^3*a*b - B^3*b^2)*x^3
 - 3*(C^3*a*b - B^3*b^2)*x*(-a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*
a*b*x - C^2*a^2 + (2*B^2*b^2*x^2 - C^2*a*b*x - B*C*a*b)*(-a/b)^(2/3) - (2*C
^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(-a/b)^(1/3))*sqrt((2*B*C*b*(-a/b)^(2/3)
+ B^2*b*(-a/b)^(1/3) - C^2*a)/a))/(b*x^3 - a)))/b, -(2*sqrt(1/3)*sqrt(-(2*B
*C*b*(-a/b)^(2/3) + B^2*b*(-a/b)^(1/3) - C^2*a)/a)*arctan(-sqrt(1/3)*(2*B^2
*b*x + C^2*a + (2*C^2*b*x + B*C*b)*(-a/b)^(2/3) - (2*B*C*b*x + B^2*b)*(-a/b
)^(1/3))*sqrt(-(2*B*C*b*(-a/b)^(2/3) + B^2*b*(-a/b)^(1/3) - C^2*a)/a)/(C^3*
a - B^3*b)) + C*log(x + (-a/b)^(1/3)))/b]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a - bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(((-a/b)**(1/3)*B+2*(-a/b)**(2/3)*C+B*x+C*x**2)/(-b*x**3+a),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(65) = 130.

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.13

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a - bx^3}\,dx =$$
$$-\frac{\sqrt{3}\left(2\,Ca - \left(6\,C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}} - 3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}} + 3\,B\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab} - \frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}} - B\left(\frac{a}{b}\right)^{\frac{1}{3}} - B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}} + 2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}} + B\left(\frac{a}{b}\right)^{\frac{1}{3}} + B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorit
hm="maxima")
[Out] -1/9*sqrt(3)*(2*C*a - (6*C*(a/b)^(1/3)*(-a/b)^(2/3) - 3*B*(a/b)^(2/3) + 3*B
*(a/b)^(1/3)*(-a/b)^(1/3) + 2*C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/
3))/(a/b)^(1/3))/(a*b) - 1/6*(2*C*(a/b)^(2/3) - 2*C*(-a/b)^(2/3) - B*(a/b)^
(1/3) - B*(-a/b)^(1/3))*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/
3)) - 1/3*(C*(a/b)^(2/3) + 2*C*(-a/b)^(2/3) + B*(a/b)^(1/3) + B*(-a/b)^(1/3
))*log(x - (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.09

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx = -\frac{\left(2\,Cab-\left(-a^2b^4\right)^{\frac{1}{3}}B\right)\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,ab^2+i\,\sqrt{3}\sqrt{a^2b^4}} - \frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+Bb^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}Bb+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2} + \frac{\sqrt{3}\left(\left(9\left(-a^2b^4\right)^{\frac{1}{3}}ab^2+27^{\frac{5}{6}}\left(-a^2b^4\right)^{\frac{5}{6}}\right)B-18\left(a^2b^3+i\,\sqrt{3}\sqrt{a^4b^6}\right)C\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{54\,a^2b^4}$$

```
[In] integrate(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorit
hm="giac")
```

```
[Out] -(2*C*a*b - (-a^2*b^4)^(1/3)*B)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(3*a
*b^2 + I*sqrt(3)*sqrt(a^2*b^4)) - 1/3*(C*b^2*(a/b)^(2/3) + B*b^2*(a/b)^(1/3
) + (-a*b^2)^(1/3)*B*b + 2*(-a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^
(1/3)))/(a*b^2) + 1/54*sqrt(3)*((9*(-a^2*b^4)^(1/3)*a*b^2 + 27^(5/6)*(-a^2*
b^4)^(5/6))*B - 18*(a^2*b^3 + I*sqrt(3)*sqrt(a^4*b^6))*C)*arctan(1/3*sqrt(3
)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 11.33 (sec) , antiderivative size = 456, normalized size of antiderivative = 6.00

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx = \sum_{k=1}^{3}\ln\left(\frac{B^2\,b\,\left(-\frac{a}{b}\right)^{1/3}-C^2\,a+2\,B\,C\,b\,\left(-\frac{a}{b}\right)^{2/3}}{b^3} - \frac{\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2-18\,B\,C\,a\,b^2\,z\left(-\frac{a}{b}\right)^{2/3}-9\,B^2\,a\,b^2\,z\left(-\frac{a}{b}\right)^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,(-\right.}{} - \frac{x\left(2\,C^2\left(-\frac{a}{b}\right)^{2/3}-B^2+B\,C\left(-\frac{a}{b}\right)^{1/3}\right)}{b^2}\right)\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2-18\,B\,C\,a\,b^2\,z\left(-\frac{a}{b}\right)^{2/3}-9\,B^2\,a\,b^2\right.$$

```
[In] int((B*x + C*x^2 + B*(-a/b)^(1/3) + 2*C*(-a/b)^(2/3))/(a - b*x^3),x)
```

```
[Out] symsum(log((B^2*b*(-a/b)^(1/3) - C^2*a + 2*B*C*b*(-a/b)^(2/3))/b^3 - (root(
27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 - 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b
^2*z*(-a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) - 9*B^2*C*a*b
*(-a/b)^(1/3) + 9*C^3*a^2, z, k)*(6*C*a + 9*root(27*a^2*b^3*z^3 + 27*C*a^2*
b^2*z^2 - 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3) + 9*C^2*
a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) - 9*B^2*C*a*b*(-a/b)^(1/3) + 9*C^3*a^2,
 z, k)*a*b + 3*B*b*x*(-a/b)^(1/3) + 6*C*b*x*(-a/b)^(2/3)))/b^2 - (x*(2*C^2*
(-a/b)^(2/3) - B^2 + B*C*(-a/b)^(1/3)))/b^2)*root(27*a^2*b^3*z^3 + 27*C*a^2
*b^2*z^2 - 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3) + 9*C^2
*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) - 9*B^2*C*a*b*(-a/b)^(1/3) + 9*C^3*a^2
, z, k), k, 1, 3)
```

3.46 $\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 78

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx=\frac{2\left(B-\sqrt[3]{-\frac{a}{b}}C\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-\frac{a}{b}}b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

```
[Out] C*ln((-a/b)^(1/3)-x)/b+2/3*(B-(-a/b)^(1/3)*C)*arctan(1/3*(1+2*x/(-a/b)^(1/3
))*3^(1/2))/(-a/b)^(1/3)/b*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {1883, 31, 631, 210}

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx=\frac{2\arctan\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)\left(B-C\sqrt[3]{-\frac{a}{b}}\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

```
[In] Int[(-((-(a/b))^(1/3)*B) + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*(B - (-(a/b))^(1/3)*C)*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt
[3]*(-(a/b))^(1/3)*b) + (C*Log[(-(a/b))^(1/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1883

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Dist[-C/b
, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]
] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &&
```

```
PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x}\,dx}{b} + \frac{\left(B-\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}+\sqrt[3]{-\frac{a}{b}}x+x^2}\,dx}{b}$$

$$= \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b} - \frac{\left(2\left(B-\sqrt[3]{-\frac{a}{b}}C\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{\sqrt[3]{-\frac{a}{b}}b}$$

$$= \frac{2\left(B-\sqrt[3]{-\frac{a}{b}}C\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-\frac{a}{b}}b} + \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 253 vs. $2(78) = 156$.

Time = 0.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.24

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx = \frac{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}B+\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\left(-B+2\sqrt[3]{-\frac{a}{b}}C\right)\right)\arctan\left(\frac{-\cdots}{\cdots}\right)\cdots}{\cdots}$$

```
[In] Integrate[(-((-(a/b))^(1/3)*B) + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a + b*x
^3),x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (-(a/b))^(1/3)*b^(1/3)*(-B + 2*(-(a
/b))^(1/3)*C))*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] - 2*b^(1/
3)*(a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*(B - 2*(-(a/b))^(1/3)*C))*Lo
g[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3
)*(B - 2*(-(a/b))^(1/3)*C))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
+ 2*a*C*Log[a + b*x^3])/(6*a*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(67) = 134.

Time = 1.52 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.85

method	result
default	$\left(2\left(-\frac{a}{b}\right)^{\frac{2}{3}} C-\left(-\frac{a}{b}\right)^{\frac{1}{3}} B\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+B\left(-\right.$

```
[In] int((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x,method=_RETURN
VERBOSE)
```

```
[Out] (2*(-a/b)^(2/3)*C-(-a/b)^(1/3)*B)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/
b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+B*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1
/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b
)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*C/b*ln(b*x^3+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(68) = 136.

Time = 1.03 (sec) , antiderivative size = 450, normalized size of antiderivative = 5.77

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx=\left[\frac{C\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+\sqrt{\frac{1}{3}}\sqrt{-\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+C^2a}{a}}\log\right.}{}\right.$$

```
[In] integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorit
hm="fricas")
```

```
[Out] [(C*log(x - (-a/b)^(1/3)) + sqrt(1/3)*sqrt(-(2*B*C*b*(-a/b)^(2/3) - B^2*b*(
-a/b)^(1/3) + C^2*a)/a)*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3
 + 3*(C^3*a*b + B^3*b^2)*x*(-a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*
```

```
a*b*x + C^2*a^2 - (2*B^2*b^2*x^2 + C^2*a*b*x + B*C*a*b)*(-a/b)^(2/3) + (2*C
^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(-a/b)^(1/3))*sqrt(-(2*B*C*b*(-a/b)^(2/3)
 - B^2*b*(-a/b)^(1/3) + C^2*a)/a))/(b*x^3 + a)))/b, (2*sqrt(1/3)*sqrt((2*B*
C*b*(-a/b)^(2/3) - B^2*b*(-a/b)^(1/3) + C^2*a)/a)*arctan(sqrt(1/3)*(2*B^2*b
*x - C^2*a + (2*C^2*b*x + B*C*b)*(-a/b)^(2/3) + (2*B*C*b*x + B^2*b)*(-a/b)^
(1/3))*sqrt((2*B*C*b*(-a/b)^(2/3) - B^2*b*(-a/b)^(1/3) + C^2*a)/a)/(C^3*a +
 B^3*b)) + C*log(x - (-a/b)^(1/3)))/b]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate((-(-a/b)**(1/3)*B+2*(-a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(68) = 136.

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.06

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a+bx^3}\,dx =$$
$$-\frac{\sqrt{3}\left(2\,Ca - \left(6\,C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}} + 3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}} - 3\,B\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$
$$+\frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}} + B\left(\frac{a}{b}\right)^{\frac{1}{3}} + B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}} + 2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}} - B\left(\frac{a}{b}\right)^{\frac{1}{3}} - B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorit
hm="maxima")
[Out] -1/9*sqrt(3)*(2*C*a - (6*C*(a/b)^(1/3)*(-a/b)^(2/3) + 3*B*(a/b)^(2/3) - 3*B
*(a/b)^(1/3)*(-a/b)^(1/3) + 2*C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/
3))/(a/b)^(1/3))/(a*b) + 1/6*(2*C*(a/b)^(2/3) - 2*C*(-a/b)^(2/3) + B*(a/b)^
```

```
(1/3) + B*(-a/b)^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/
3)) + 1/3*(C*(a/b)^(2/3) + 2*C*(-a/b)^(2/3) - B*(a/b)^(1/3) - B*(-a/b)^(1/3
))*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.71

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx = -\frac{2\sqrt{3}\left(Cab+\left(-ab^2\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2} - \frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+Bb^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}Bb+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

```
[In] integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorit
hm="giac")
```

```
[Out] -2/3*sqrt(3)*(C*a*b + (-a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(-a/b)^(2/3) + B*b^2*(-a/b)^(1/3) -
 (-a*b^2)^(1/3)*B*b + 2*(-a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(
1/3)))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 11.20 (sec) , antiderivative size = 453, normalized size of antiderivative = 5.81

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx = \sum_{k=1}^{3}\ln\left(\frac{C^2\,a-B^2\,b\,\left(-\frac{a}{b}\right)^{1/3}+2\,B\,C\,b\,\left(-\frac{a}{b}\right)^{2/3}}{b^3} - \frac{\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+18\,B\,C\,a\,b^2\,z\,\left(-\frac{a}{b}\right)^{2/3}-9\,B^2\,a\,b^2\,z\,\left(-\frac{a}{b}\right)^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,\left(\right.\right.}{} + \frac{x\left(B^2-2\,C^2\,\left(-\frac{a}{b}\right)^{2/3}+B\,C\,\left(-\frac{a}{b}\right)^{1/3}\right)}{b^2}\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+18\,B\,C\,a\,b^2\,z\,\left(-\frac{a}{b}\right)^{2/3}-9\,B^2\,a\,b\right.$$

```
[In] int((B*x + C*x^2 - B*(-a/b)^(1/3) + 2*C*(-a/b)^(2/3))/(a + b*x^3),x)
```

```
[Out] symsum(log((C^2*a - B^2*b*(-a/b)^(1/3) + 2*B*C*b*(-a/b)^(2/3))/b^3 - (root(
27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b
^2*z*(-a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) + 9*B^2*C*a*b
*(-a/b)^(1/3) - 9*C^3*a^2, z, k)*(6*C*a - 9*root(27*a^2*b^3*z^3 - 27*C*a^2*
b^2*z^2 + 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3) + 9*C^2*
a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) + 9*B^2*C*a*b*(-a/b)^(1/3) - 9*C^3*a^2,
 z, k)*a*b + 3*B*b*x*(-a/b)^(1/3) - 6*C*b*x*(-a/b)^(2/3)))/b^2 + (x*(B^2 -
2*C^2*(-a/b)^(2/3) + B*C*(-a/b)^(1/3)))/b^2)*root(27*a^2*b^3*z^3 - 27*C*a^2
*b^2*z^2 + 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3) + 9*C^2
*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) + 9*B^2*C*a*b*(-a/b)^(1/3) - 9*C^3*a^2
, z, k), k, 1, 3)
```

3.47 $\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 75

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx = -\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B-\sqrt[3]{\frac{a}{b}}C\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}a}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

```
[Out] -C*ln((a/b)^(1/3)-x)/b-2/3*(a/b)^(2/3)*(B-(a/b)^(1/3)*C)*arctan(1/3*(1+2*x/
(a/b)^(1/3))*3^(1/2))/a*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1883, 31, 631, 210}

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx = -\frac{2\left(\frac{a}{b}\right)^{2/3}\arctan\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)\left(B-C\sqrt[3]{\frac{a}{b}}\right)}{\sqrt{3}a}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

```
[In] Int[(-((a/b)^(1/3)*B) + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

```
[Out] (-2*(a/b)^(2/3)*(B - (a/b)^(1/3)*C)*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]]
)/(Sqrt[3]*a) - (C*Log[(a/b)^(1/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1883

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Dist[-C/b
, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]
] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &&
PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}-x}\,dx}{b} - \frac{\left(B-\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}+\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}$$

$$= -\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b} + \left(2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a}-\frac{C}{b}\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)$$

$$= -\frac{2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a}-\frac{C}{b}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 244 vs. $2(75) = 150$.

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.25

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx = \frac{-2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}B+\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\left(B-2\sqrt[3]{\frac{a}{b}}C\right)\right)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{}$$

```
[In] Integrate[(-((a/b)^(1/3)*B) + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (a/b)^(1/3)*b^(1/3)*(B - 2*(a/b)^(
1/3)*C))*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*b^(1/3)*(a^(2/3)*B
 + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(-B + 2*(a/b)^(1/3)*C))*Log[a^(1/3) - b^(1/3
)*x] + b^(1/3)*(a^(2/3)*B + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(-B + 2*(a/b)^(1/3)
*C))*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*C*Log[a - b*x^3])
/(6*a*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(64) = 128.

Time = 1.52 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.97

method	result
default	$\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}} C-\left(\frac{a}{b}\right)^{\frac{1}{3}} B\right)\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan\left(\frac{\left(1+\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+B\left(-\frac{\ln\left(\right.}{3}\right.$

```
[In] int((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x,method=_RETURNV
ERBOSE)
```

```
[Out] (2*(a/b)^(2/3)*C-(a/b)^(1/3)*B)*(-1/3/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/6/b
/(a/b)^(2/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2)))+B*(-1/3/b/(a/b)^(1/3)*ln(x-(a/b)^(1/
3))+1/6/b/(a/b)^(1/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*3^(1/2)/b/(a/b)
^(1/3)*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2)))-1/3*C*ln(-b*x^3+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(65) = 130.

Time = 1.78 (sec) , antiderivative size = 450, normalized size of antiderivative = 6.00

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx = \left[-\frac{C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-\sqrt{\frac{1}{3}}\sqrt{\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}\log\left(-\frac{C^3a^2-}{}\right.}{}\right.$$

$$-\frac{2\sqrt{\frac{1}{3}}\sqrt{-\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2\,B^2bx+C^2a+\left(2\,C^2bx+BCb\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}+\left(2\,BCbx+B^2b\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}-B^2}{a}}}{C^3a-B^3b}\right.}{b}$$

```
[In] integrate((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorith
m="fricas")
```

```
[Out] [-(C*log(x - (a/b)^(1/3)) - sqrt(1/3)*sqrt((2*B*C*b*(a/b)^(2/3) - B^2*b*(a/
b)^(1/3) - C^2*a)/a)*log(-(C^3*a^2 - B^3*a*b + 2*(C^3*a*b - B^3*b^2)*x^3 -
3*(C^3*a*b - B^3*b^2)*x*(a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*a*b*
x - C^2*a^2 + (2*B^2*b^2*x^2 - C^2*a*b*x - B*C*a*b)*(a/b)^(2/3) + (2*C^2*a*
b*x^2 - B*C*a*b*x - B^2*a*b)*(a/b)^(1/3))*sqrt((2*B*C*b*(a/b)^(2/3) - B^2*b
*(a/b)^(1/3) - C^2*a)/a))/(b*x^3 - a)))/b, -(2*sqrt(1/3)*sqrt(-(2*B*C*b*(a/
b)^(2/3) - B^2*b*(a/b)^(1/3) - C^2*a)/a)*arctan(-sqrt(1/3)*(2*B^2*b*x + C^2
*a + (2*C^2*b*x + B*C*b)*(a/b)^(2/3) + (2*B*C*b*x + B^2*b)*(a/b)^(1/3))*sqr
t(-(2*B*C*b*(a/b)^(2/3) - B^2*b*(a/b)^(1/3) - C^2*a)/a)/(C^3*a - B^3*b)) +
C*log(x - (a/b)^(1/3)))/b]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3} C + Bx + Cx^2}{a - bx^3}\, dx = \text{Timed out}$$

```
[In] integrate((-(a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(-b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3} C + Bx + Cx^2}{a - bx^3}\, dx = -\frac{C \log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b} - \frac{2\sqrt{3}\left(Ca + \left(3B\left(\frac{a}{b}\right)^{\frac{2}{3}} - \frac{4Ca}{b}\right)b\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab}$$

```
[In] integrate((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorith
m="maxima")
```

```
[Out] -C*log(x - (a/b)^(1/3))/b - 2/9*sqrt(3)*(C*a + (3*B*(a/b)^(2/3) - 4*C*a/b)*
b)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.67

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a - bx^3}\,dx = \frac{2\sqrt{3}\left(Cab - (ab^2)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} - \frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}} + Bb^2\left(\frac{a}{b}\right)^{\frac{1}{3}} - (ab^2)^{\frac{1}{3}}Bb + 2\,(ab^2)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}$$

```
[In] integrate((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorith
m="giac")
```

```
[Out] 2/3*sqrt(3)*(C*a*b - (a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3)
)/(a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(a/b)^(2/3) + B*b^2*(a/b)^(1/3) - (a*b^
2)^(1/3)*B*b + 2*(a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/(a*
b^2)
```

Mupad [B] (verification not implemented)

Time = 11.39 (sec) , antiderivative size = 435, normalized size of antiderivative = 5.80

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a - bx^3}\,dx = \sum_{k=1}^{3}\ln\left(-\frac{C^2\,a + B^2\,b\,\left(\frac{a}{b}\right)^{1/3} - 2\,B\,C\,b\,\left(\frac{a}{b}\right)^{2/3}}{b^3} - \frac{\mathrm{root}\left(27\,a^2\,b^3\,z^3 + 27\,C\,a^2\,b^2\,z^2 - 18\,B\,C\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{2/3} + 9\,B^2\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{1/3} + 9\,C^2\,a^2\,b\,z - 18\,B\,C^2\,a\,b\,\left(\frac{a}{b}\right)^{2/3} + \right.}{} + \frac{x\left(B^2 - 2\,C^2\,\left(\frac{a}{b}\right)^{2/3} + B\,C\,\left(\frac{a}{b}\right)^{1/3}\right)}{b^2}\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3 + 27\,C\,a^2\,b^2\,z^2 - 18\,B\,C\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{2/3} + 9\,B^2\,a\,b^2\,z\,\left(\frac{a}{b}\right.\right.$$

```
[In] int((B*x + C*x^2 - B*(a/b)^(1/3) + 2*C*(a/b)^(2/3))/(a - b*x^3),x)
```

```
[Out] symsum(log((x*(B^2 - 2*C^2*(a/b)^(2/3) + B*C*(a/b)^(1/3)))/b^2 - (root(27*a
^2*b^3*z^3 + 27*C*a^2*b^2*z^2 - 18*B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*
(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(a/b)^(2/3) + 9*B^2*C*a*b*(a/b)^
(1/3) + 9*C^3*a^2, z, k)*(6*C*a + 9*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2
- 18*B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b*z -
18*B*C^2*a*b*(a/b)^(2/3) + 9*B^2*C*a*b*(a/b)^(1/3) + 9*C^3*a^2, z, k)*a*b -
 3*B*b*x*(a/b)^(1/3) + 6*C*b*x*(a/b)^(2/3)))/b^2 - (C^2*a + B^2*b*(a/b)^(1/
3) - 2*B*C*b*(a/b)^(2/3))/b^3)*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 - 18*
B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*
C^2*a*b*(a/b)^(2/3) + 9*B^2*C*a*b*(a/b)^(1/3) + 9*C^3*a^2, z, k), k, 1, 3)
```

3.48 $\int \frac{a+ax+cx^2}{1-x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 32

$$\int \frac{a+ax+cx^2}{1-x^3} \, dx = -\frac{1}{3}(2a+c)\log(1-x)+\frac{1}{3}(a-c)\log\left(1+x+x^2\right)$$

```
[Out] -1/3*(2*a+c)*ln(1-x)+1/3*(a-c)*ln(x^2+x+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1889, 31, 642}

$$\int \frac{a+ax+cx^2}{1-x^3} \, dx = \frac{1}{3}(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}(2a+c)\log(1-x)$$

```
[In] Int[(a + a*x + c*x^2)/(1 - x^3),x]
```

```
[Out] -1/3*((2*a + c)*Log[1 - x]) + ((a - c)*Log[1 + x + x^2])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1889

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Dist[q*((A + B*q
 + C*q^2)/(3*a)), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q -
 C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b
*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
 && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\int \frac{a-c+(2a-2c)x}{1+x+x^2}\,dx+\frac{1}{3}(2a+c)\int \frac{1}{1-x}\,dx\\ &= -\frac{1}{3}(2a+c)\log(1-x)+\frac{1}{3}(a-c)\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{a+ax+cx^2}{1-x^3}\,dx=\frac{1}{3}\left(-((2a+c)\log(1-x))+(a-c)\log\left(1+x+x^2\right)\right)$$

```
[In] Integrate[(a + a*x + c*x^2)/(1 - x^3),x]
[Out] (-((2*a + c)*Log[1 - x]) + (a - c)*Log[1 + x + x^2])/3
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

method	result
default	$\left(-\frac{2a}{3}-\frac{c}{3}\right)\ln\left(-1+x\right)+\frac{(a-c)\ln(x^2+x+1)}{3}$
norman	$\left(-\frac{2a}{3}-\frac{c}{3}\right)\ln\left(-1+x\right)+\left(\frac{a}{3}-\frac{c}{3}\right)\ln\left(x^2+x+1\right)$
parallelrisch	$-\frac{2\ln(-1+x)a}{3}-\frac{\ln(-1+x)c}{3}+\frac{\ln(x^2+x+1)a}{3}-\frac{\ln(x^2+x+1)c}{3}$
risch	$-\frac{2\ln(-1+x)a}{3}-\frac{\ln(-1+x)c}{3}+\frac{\ln(-x^2-x-1)a}{3}-\frac{\ln(-x^2-x-1)c}{3}$
meijerg	$-\frac{c\ln(-x^3+1)}{3}-\frac{a\,x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}-\frac{ax\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(\right.}{}\right.}{}$

```
[In] int((c*x^2+a*x+a)/(-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/3*a-1/3*c)*ln(-1+x)+1/3*(a-c)*ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a+ax+cx^2}{1-x^3}\,dx = \frac{1}{3}\,(a-c)\log\left(x^2+x+1\right) - \frac{1}{3}\,(2\,a+c)\log\left(x-1\right)$$

```
[In] integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="fricas")
[Out] 1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{a+ax+cx^2}{1-x^3}\,dx = \frac{(a-c)\log\left(x^2+x+1\right)}{3} - \frac{(2a+c)\log\left(x-1\right)}{3}$$

```
[In] integrate((c*x**2+a*x+a)/(-x**3+1),x)
[Out] (a - c)*log(x**2 + x + 1)/3 - (2*a + c)*log(x - 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a+ax+cx^2}{1-x^3}\,dx = \frac{1}{3}\,(a-c)\log\left(x^2+x+1\right) - \frac{1}{3}\,(2\,a+c)\log\left(x-1\right)$$

```
[In] integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="maxima")
[Out] 1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{a+ax+cx^2}{1-x^3}\,dx = \frac{1}{3}\,(a-c)\log\left(x^2+x+1\right) - \frac{1}{3}\,(2\,a+c)\log\left(|x-1|\right)$$

```
[In] integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="giac")
[Out] 1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{a+ax+cx^2}{1-x^3}\,dx = \frac{a\,\ln\left(x^2+x+1\right)}{3} - \frac{c\,\ln\left(x-1\right)}{3} - \frac{2\,a\,\ln\left(x-1\right)}{3} - \frac{c\,\ln\left(x^2+x+1\right)}{3}$$

```
[In] int(-(a + a*x + c*x^2)/(x^3 - 1),x)
[Out] (a*log(x + x^2 + 1))/3 - (c*log(x - 1))/3 - (2*a*log(x - 1))/3 - (c*log(x +
 x^2 + 1))/3
```

3.49 $\int \frac{a+bx+cx^2}{1-x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{a+bx+cx^2}{1-x^3} \, dx = \frac{(a-b)\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}(a+b+c)\log(1-x) + \frac{1}{6}(a+b-2c)\log\left(1+x+x^2\right)$$

```
[Out] -1/3*(a+b+c)*ln(1-x)+1/6*(a+b-2*c)*ln(x^2+x+1)+1/3*(a-b)*arctan(1/3*(1+2*x)
*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1889, 31, 648, 632, 210, 642}

$$\int \frac{a+bx+cx^2}{1-x^3} \, dx = \frac{(a-b)\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{6}\log\left(x^2+x+1\right)(a+b-2c) - \frac{1}{3}\log(1-x)(a+b+c)$$

```
[In] Int[(a + b*x + c*x^2)/(1 - x^3),x]
```

```
[Out] ((a - b)*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - ((a + b + c)*Log[1 - x])/3 +
((a + b - 2*c)*Log[1 + x + x^2])/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1889

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Dist[q*((A + B*q
 + C*q^2)/(3*a)), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q -
 C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b
*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
 && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\int \frac{2a-b-c+(a+b-2c)x}{1+x+x^2}\,dx+\frac{1}{3}(a+b+c)\int \frac{1}{1-x}\,dx\\
&= -\frac{1}{3}(a+b+c)\log(1-x)+\frac{1}{2}(a-b)\int \frac{1}{1+x+x^2}\,dx+\frac{1}{6}(a+b-2c)\int \frac{1+2x}{1+x+x^2}\,dx\\
&= -\frac{1}{3}(a+b+c)\log(1-x)+\frac{1}{6}(a+b-2c)\log\left(1+x+x^2\right)\\
&\quad+(-a+b)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= \frac{(a-b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}(a+b+c)\log(1-x)+\frac{1}{6}(a+b-2c)\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx=\frac{1}{6}\left(2\sqrt{3}(a-b)\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-2(a+b)\log(1-x)+(a+b)\log\left(1+x+x^2\right)-2c\log\left(1-x^3\right)\right)$$

```
[In] Integrate[(a + b*x + c*x^2)/(1 - x^3),x]
[Out] (2*Sqrt[3]*(a - b)*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*(a + b)*Log[1 - x] + (a +
b)*Log[1 + x + x^2] - 2*c*Log[1 - x^3])/6
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

method	result
default	$\left(-\frac{c}{3}-\frac{b}{3}-\frac{a}{3}\right)\ln\left(-1+x\right)+\frac{(a+b-2c)\ln\left(x^2+x+1\right)}{6}+\frac{2\left(\frac{3a}{2}-\frac{3b}{2}\right)\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}$
risch	$-\frac{\ln(-1+x)c}{3}-\frac{\ln(-1+x)b}{3}-\frac{\ln(-1+x)a}{3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^2+(-a-b+2c)\_Z+a^2-ab-ac+b^2-bc+c^2\right)}\_R\ln\left(\left(-\_Ra-ac+b\right.\right.\right.}{3}$
meijerg	$-\frac{c\ln\left(-x^3+1\right)}{3}-\frac{bx^2\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2+\left(x^3\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^3\right)^{\frac{2}{3}}}-\frac{ax\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}\right.}{2}\right.}{3}$

```
[In] int((c*x^2+b*x+a)/(-x^3+1),x,method=_RETURNVERBOSE)
[Out] (-1/3*c-1/3*b-1/3*a)*ln(-1+x)+1/6*(a+b-2*c)*ln(x^2+x+1)+2/9*(3/2*a-3/2*b)*a
rctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx=\frac{1}{3}\sqrt{3}(a-b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}(a+b-2c)\log\left(x^2+x+1\right)-\frac{1}{3}(a+b+c)\log\left(x-1\right)$$

```
[In] integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(a - b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*c)*log(x
^2 + x + 1) - 1/3*(a + b + c)*log(x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 323, normalized size of antiderivative = 5.87

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx$$
$$= -\frac{(a+b+c)\log\left(x+\frac{a^2c-a^2(a+b+c)-2ab^2+bc^2-2bc(a+b+c)+b(a+b+c)^2}{a^3-b^3}\right)}{3} - \left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}\right.$$
$$\left.-\frac{\sqrt{3}i(a-b)}{6}\right)\log\left(x+\frac{a^2c-3a^2\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}i(a-b)}{6}\right)-2ab^2+bc^2-6bc\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}i(a-b)}{6}\right)}{a^3-b^3}\right.$$
$$-\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}\right.$$
$$\left.+\frac{\sqrt{3}i(a-b)}{6}\right)\log\left(x+\frac{a^2c-3a^2\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}i(a-b)}{6}\right)-2ab^2+bc^2-6bc\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}i(a-b)}{6}\right)}{a^3-b^3}\right.$$

```
[In] integrate((c*x**2+b*x+a)/(-x**3+1),x)
```

```
[Out] -(a + b + c)*log(x + (a**2*c - a**2*(a + b + c) - 2*a*b**2 + b*c**2 - 2*b*c
*(a + b + c) + b*(a + b + c)**2)/(a**3 - b**3))/3 - (-a/6 - b/6 + c/3 - sqr
t(3)*I*(a - b)/6)*log(x + (a**2*c - 3*a**2*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a
 - b)/6) - 2*a*b**2 + b*c**2 - 6*b*c*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a - b)/
6) + 9*b*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a - b)/6)**2)/(a**3 - b**3)) - (-a/
6 - b/6 + c/3 + sqrt(3)*I*(a - b)/6)*log(x + (a**2*c - 3*a**2*(-a/6 - b/6 +
 c/3 + sqrt(3)*I*(a - b)/6) - 2*a*b**2 + b*c**2 - 6*b*c*(-a/6 - b/6 + c/3 +
 sqrt(3)*I*(a - b)/6) + 9*b*(-a/6 - b/6 + c/3 + sqrt(3)*I*(a - b)/6)**2)/(a
**3 - b**3))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx = \frac{1}{3}\sqrt{3}(a-b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}(a+b-2c)\log\left(x^2+x+1\right) - \frac{1}{3}(a+b+c)\log(x-1)$$

```
[In] integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(a - b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*c)*log(x
^2 + x + 1) - 1/3*(a + b + c)*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx = \frac{1}{3}\left(\sqrt{3}a-\sqrt{3}b\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}(a+b-2c)\log\left(x^2+x+1\right) - \frac{1}{3}(a+b+c)\log(|x-1|)$$

```
[In] integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(3)*a - sqrt(3)*b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*
c)*log(x^2 + x + 1) - 1/3*(a + b + c)*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 10.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.58

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx = \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{a}{6}+\frac{b}{6}-\frac{c}{3}-\frac{\sqrt{3}\,a\,1\mathrm{i}}{6}+\frac{\sqrt{3}\,b\,1\mathrm{i}}{6}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{a}{6}+\frac{b}{6}-\frac{c}{3}+\frac{\sqrt{3}\,a\,1\mathrm{i}}{6}-\frac{\sqrt{3}\,b\,1\mathrm{i}}{6}\right) - \ln(x-1)\left(\frac{a}{3}+\frac{b}{3}+\frac{c}{3}\right)$$

```
[In] int(-(a + b*x + c*x^2)/(x^3 - 1),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*(a/6 + b/6 - c/3 - (3^(1/2)*a*1i)/6 + (3^(1/2
)*b*1i)/6) + log(x + (3^(1/2)*1i)/2 + 1/2)*(a/6 + b/6 - c/3 + (3^(1/2)*a*1i
)/6 - (3^(1/2)*b*1i)/6) - log(x - 1)*(a/3 + b/3 + c/3)
```

3.50 $\int \frac{1+x+x^2}{1-x^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 8

$$\int \frac{1+x+x^2}{1-x^3} \, dx = -\log(1-x)$$

```
[Out] -ln(1-x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1600, 31}

$$\int \frac{1+x+x^2}{1-x^3} \, dx = -\log(1-x)$$

```
[In] Int[(1 + x + x^2)/(1 - x^3),x]
```

```
[Out] -Log[1 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{1-x}\,dx \\ &= -\log(1-x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{1-x^3}\,dx = -\log(1-x)$$

```
[In] Integrate[(1 + x + x^2)/(1 - x^3),x]
[Out] -Log[1 - x]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result
default	$-\ln(-1+x)$
norman	$-\ln(-1+x)$
risch	$-\ln(-1+x)$
parallelrisch	$-\ln(-1+x)$
meijerg	$-\frac{\ln(-x^3+1)}{3}-\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3\right.}{}\right.}{}$

```
[In] int((x^2+x+1)/(-x^3+1),x,method=_RETURNVERBOSE)
[Out] -ln(-1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2}{1-x^3}\,dx = -\log(x-1)$$

```
[In] integrate((x^2+x+1)/(-x^3+1),x, algorithm="fricas")
[Out] -log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1+x+x^2}{1-x^3}\,dx = -\log(x-1)$$

```
[In] integrate((x**2+x+1)/(-x**3+1),x)
[Out] -log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2}{1-x^3}\,dx = -\log(x-1)$$

```
[In] integrate((x^2+x+1)/(-x^3+1),x, algorithm="maxima")
[Out] -log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^2}{1-x^3}\,dx = -\log(|x-1|)$$

```
[In] integrate((x^2+x+1)/(-x^3+1),x, algorithm="giac")
[Out] -log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2}{1-x^3}\,dx = -\ln(x-1)$$

```
[In] int(-(x + x^2 + 1)/(x^3 - 1),x)
[Out] -log(x - 1)
```

3.51 $\int \frac{1-x+3x^2}{1-x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 30

$$\int \frac{1-x+3x^2}{1-x^3} \, dx = \frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

```
[Out] -ln(-x^3+1)+2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1885, 1600, 632, 210, 266}

$$\int \frac{1-x+3x^2}{1-x^3} \, dx = \frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

```
[In] Int[(1 - x + 3*x^2)/(1 - x^3),x]
```

```
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\int \frac{x^2}{1-x^3}\,dx + \int \frac{1-x}{1-x^3}\,dx \\
&= -\log\left(1-x^3\right) + \int \frac{1}{1+x+x^2}\,dx \\
&= -\log\left(1-x^3\right) - 2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) \\
&= \frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = \frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

```
[In] Integrate[(1 - x + 3*x^2)/(1 - x^3),x]
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result
default	$-\ln\left(-1+x\right)-\ln\left(x^{2}+x+1\right)+\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
risch	$\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\ln\left(4x^{2}+4x+4\right)-\ln\left(-1+x\right)$
meijerg	$-\ln\left(-x^{3}+1\right)+\frac{x^{2}\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{3}\right)^{\frac{1}{3}}}{2+\left(x^{3}\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^{3}\right)^{\frac{2}{3}}}-\frac{x\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}\right.}{2}\right.}{3}$

```
[In] int((3*x^2-x+1)/(-x^3+1),x,method=_RETURNVERBOSE)
[Out] -ln(-1+x)-ln(x^2+x+1)+2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\log\left(x^2+x+1\right)-\log\left(x-1\right)$$

```
[In] integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.17

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = -\log\left(x-1\right)$$

```
[In] integrate((3*x**2-x+1)/(-x**3+1),x)
[Out] -log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \log\left(x^2+x+1\right) - \log\left(x-1\right)$$

```
[In] integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \log\left(x^2+x+1\right) - \log\left(|x-1|\right)$$

```
[In] integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(abs(x -
1))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.10

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = -\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) - \ln\left(x-1\right) - \frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{3} + \frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{3}$$

```
[In] int(-(3*x^2 - x + 1)/(x^3 - 1),x)
```

```
[Out] (3^(1/2)*log(x + (3^(1/2)*1i)/2 + 1/2)*1i)/3 - log(x + (3^(1/2)*1i)/2 + 1/2
) - log(x - 1) - (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/3 - log(x - (3^
(1/2)*1i)/2 + 1/2)
```

3.52 $\int \frac{1+x+4x^2}{1-x^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1+x+4x^2}{1-x^3} \, dx = -2\log(1-x) - \log\left(1+x+x^2\right)$$

```
[Out] -2*ln(1-x)-ln(x^2+x+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1889, 31, 642}

$$\int \frac{1+x+4x^2}{1-x^3} \, dx = -\log\left(x^2+x+1\right) - 2\log(1-x)$$

```
[In] Int[(1 + x + 4*x^2)/(1 - x^3),x]
```

```
[Out] -2*Log[1 - x] - Log[1 + x + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1889

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Dist[q*((A + B*q
 + C*q^2)/(3*a)), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q -
 C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b
*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
 && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\int \frac{-3-6x}{1+x+x^2}\,dx + 2\int \frac{1}{1-x}\,dx\\ &= -2\log(1-x) - \log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -2\log(1-x) - \log\left(1+x+x^2\right)$$

```
[In] Integrate[(1 + x + 4*x^2)/(1 - x^3),x]
[Out] -2*Log[1 - x] - Log[1 + x + x^2]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result
default	$-2\ln(-1+x) - \ln(x^2+x+1)$
norman	$-2\ln(-1+x) - \ln(x^2+x+1)$
risch	$-2\ln(-1+x) - \ln(x^2+x+1)$
parallelrisch	$-2\ln(-1+x) - \ln(x^2+x+1)$
meijerg	$-\frac{4\ln(-x^3+1)}{3} - \frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right) - \frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2} + \sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}} - \frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right) - \frac{\ln\left(1+(x \cdots\right.}{}\right.}{}$

```
[In] int((4*x^2+x+1)/(-x^3+1),x,method=_RETURNVERBOSE)
[Out] -2*ln(-1+x)-ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -\log\left(x^2+x+1\right) - 2\,\log\left(x-1\right)$$

```
[In] integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="fricas")
[Out] -log(x^2 + x + 1) - 2*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -2\log\left(x-1\right) - \log\left(x^2+x+1\right)$$

```
[In] integrate((4*x**2+x+1)/(-x**3+1),x)
[Out] -2*log(x - 1) - log(x**2 + x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -\log\left(x^2+x+1\right) - 2\,\log\left(x-1\right)$$

```
[In] integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="maxima")
[Out] -log(x^2 + x + 1) - 2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -\log\left(x^2+x+1\right) - 2\,\log\left(|x-1|\right)$$

```
[In] integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="giac")
[Out] -log(x^2 + x + 1) - 2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -\ln\left(x^2+x+1\right)-2\,\ln\left(x-1\right)$$

```
[In] int(-(x + 4*x^2 + 1)/(x^3 - 1),x)
[Out] - log(x + x^2 + 1) - 2*log(x - 1)
```

3.53 $\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 113

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4)\,dx = a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5 +\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10} +\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

```
[Out] a^4*c*x+1/2*a^4*d*x^2+a^3*b*c*x^4+4/5*a^3*b*d*x^5+6/7*a^2*b^2*c*x^7+3/4*a^2
*b^2*d*x^8+2/5*a*b^3*c*x^10+4/11*a*b^3*d*x^11+1/13*b^4*c*x^13+1/14*b^4*d*x^
14
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1864}

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4)\,dx = a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5 +\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10} +\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

```
[In] Int[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^
7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (b^
4*c*x^13)/13 + (b^4*d*x^14)/14
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^4c + a^4dx + 4a^3bcx^3 + 4a^3bdx^4 + 6a^2b^2cx^6 + 6a^2b^2dx^7 + 4ab^3cx^9 + 4ab^3dx^{10} + b^4cx^{12} + b^4dx^{13}\right) dx \\ &= a^4cx + \frac{1}{2}a^4dx^2 + a^3bcx^4 + \frac{4}{5}a^3bdx^5 + \frac{6}{7}a^2b^2cx^7 + \frac{3}{4}a^2b^2dx^8 + \frac{2}{5}ab^3cx^{10} + \frac{4}{11}ab^3dx^{11} + \frac{1}{13}b^4cx^{13} + \frac{1}{14}b^4dx^{14}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)^3 \left(ac+adx+bcx^3+bdx^4\right) dx = a^4cx + \frac{1}{2}a^4dx^2 + a^3bcx^4 + \frac{4}{5}a^3bdx^5 + \frac{6}{7}a^2b^2cx^7 + \frac{3}{4}a^2b^2dx^8 + \frac{2}{5}ab^3cx^{10} + \frac{4}{11}ab^3dx^{11} + \frac{1}{13}b^4cx^{13} + \frac{1}{14}b^4dx^{14}$$

```
[In] Integrate[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^
7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (b^
4*c*x^13)/13 + (b^4*d*x^14)/14
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

method	result
default	$a^4cx+\frac{1}{2}a^4d\,x^2+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{5}a\,b^3c\,x^{10}+\frac{4}{11}x^{11}d\,b^3a+\frac{1}{13}b$
norman	$a^4cx+\frac{1}{2}a^4d\,x^2+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{5}a\,b^3c\,x^{10}+\frac{4}{11}x^{11}d\,b^3a+\frac{1}{13}b$
risch	$a^4cx+\frac{1}{2}a^4d\,x^2+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{5}a\,b^3c\,x^{10}+\frac{4}{11}x^{11}d\,b^3a+\frac{1}{13}b$
parallelrisch	$a^4cx+\frac{1}{2}a^4d\,x^2+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{5}a\,b^3c\,x^{10}+\frac{4}{11}x^{11}d\,b^3a+\frac{1}{13}b$
gosper	$\frac{x(1430b^4d\,x^{13}+1540b^4c\,x^{12}+7280a\,b^3d\,x^{10}+8008a\,b^3c\,x^9+15015a^2b^2d\,x^7+17160a^2b^2c\,x^6+16016a^3bd\,x^4+20020a^3bc\,x^3+10010}{20020}$

```
[In] int((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] a^4*c*x+1/2*a^4*d*x^2+a^3*b*c*x^4+4/5*d*x^5*b*a^3+6/7*a^2*b^2*c*x^7+3/4*x^8
*b^2*d*a^2+2/5*a*b^3*c*x^10+4/11*x^11*d*b^3*a+1/13*b^4*c*x^13+1/14*b^4*d*x^
14
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int (a+bx^3)^3(ac+adx+bcx^3+bdx^4)\,dx=\frac{1}{14}b^4dx^{14}+\frac{1}{13}b^4cx^{13}+\frac{4}{11}ab^3dx^{11}+\frac{2}{5}ab^3cx^{10}+\frac{3}{4}a^2b^2dx^8+\frac{6}{7}a^2b^2cx^7+\frac{4}{5}a^3bdx^5+a^3bcx^4+\frac{1}{2}a^4dx^2+a^4cx$$

```
[In] integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/2
*a^4*d*x^2 + a^4*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int (a+bx^3)^3(ac+adx+bcx^3+bdx^4)\,dx=a^4cx+\frac{a^4dx^2}{2}+a^3bcx^4+\frac{4a^3bdx^5}{5}+\frac{6a^2b^2cx^7}{7}+\frac{3a^2b^2dx^8}{4}+\frac{2ab^3cx^{10}}{5}+\frac{4ab^3dx^{11}}{11}+\frac{b^4cx^{13}}{13}+\frac{b^4dx^{14}}{14}$$

```
[In] integrate((b*x**3+a)**3*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**3*b*c*x**4 + 4*a**3*b*d*x**5/5 + 6*a**2*b**2*
c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*a*b**3*c*x**10/5 + 4*a*b**3*d*x**11/11
+ b**4*c*x**13/13 + b**4*d*x**14/14
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int \left(a+bx^3\right)^3 \left(ac+adx+bcx^3+bdx^4\right)\,dx = \frac{1}{14}\,b^4dx^{14}+\frac{1}{13}\,b^4cx^{13}+\frac{4}{11}\,ab^3dx^{11}+\frac{2}{5}\,ab^3cx^{10}+\frac{3}{4}\,a^2b^2dx^8+\frac{6}{7}\,a^2b^2cx^7+\frac{4}{5}\,a^3bdx^5+a^3bcx^4+\frac{1}{2}\,a^4dx^2+a^4cx$$

```
[In] integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/2
*a^4*d*x^2 + a^4*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int \left(a+bx^3\right)^3 \left(ac+adx+bcx^3+bdx^4\right)\,dx = \frac{1}{14}\,b^4dx^{14}+\frac{1}{13}\,b^4cx^{13}+\frac{4}{11}\,ab^3dx^{11}+\frac{2}{5}\,ab^3cx^{10}+\frac{3}{4}\,a^2b^2dx^8+\frac{6}{7}\,a^2b^2cx^7+\frac{4}{5}\,a^3bdx^5+a^3bcx^4+\frac{1}{2}\,a^4dx^2+a^4cx$$

```
[In] integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/2
*a^4*d*x^2 + a^4*c*x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4)\ dx = \frac{d\,a^4\,x^2}{2}+c\,a^4\,x+\frac{4\,d\,a^3\,b\,x^5}{5}+c\,a^3\,b\,x^4+\frac{3\,d\,a^2\,b^2\,x^8}{4}+\frac{6\,c\,a^2\,b^2\,x^7}{7}+\frac{4\,d\,a\,b^3\,x^{11}}{11}+\frac{2\,c\,a\,b^3\,x^{10}}{5}+\frac{d\,b^4\,x^{14}}{14}+\frac{c\,b^4\,x^{13}}{13}$$

```
[In] int((a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

```
[Out] (a^4*d*x^2)/2 + (b^4*c*x^13)/13 + (b^4*d*x^14)/14 + a^4*c*x + (6*a^2*b^2*c*
x^7)/7 + (3*a^2*b^2*d*x^8)/4 + a^3*b*c*x^4 + (2*a*b^3*c*x^10)/5 + (4*a^3*b*
d*x^5)/5 + (4*a*b^3*d*x^11)/11
```

3.54 $\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 88

$$\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4)\,dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

```
[Out] a^3*c*x+1/2*a^3*d*x^2+3/4*a^2*b*c*x^4+3/5*a^2*b*d*x^5+3/7*a*b^2*c*x^7+3/8*a
*b^2*d*x^8+1/10*b^3*c*x^10+1/11*b^3*d*x^11
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1864}

$$\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4)\,dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

```
[In] Int[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*
c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \int \left(a^3c + a^3dx + 3a^2bcx^3 + 3a^2bdx^4 + 3ab^2cx^6 + 3ab^2dx^7 + b^3cx^9 + b^3dx^{10}\right) dx$$
$$= a^3cx + \frac{1}{2}a^3dx^2 + \frac{3}{4}a^2bcx^4 + \frac{3}{5}a^2bdx^5 + \frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right) dx = a^3cx + \frac{1}{2}a^3dx^2 + \frac{3}{4}a^2bcx^4 + \frac{3}{5}a^2bdx^5 + \frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11}$$

```
[In] Integrate[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*
c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

method	result	size
default	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{3}{4}a^2bc\,x^4+\frac{3}{5}x^5bd\,a^2+\frac{3}{7}a\,b^2c\,x^7+\frac{3}{8}x^8b^2da+\frac{1}{10}b^3c\,x^{10}+\frac{1}{11}b^3d\,x^{11}$	75
norman	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{3}{4}a^2bc\,x^4+\frac{3}{5}x^5bd\,a^2+\frac{3}{7}a\,b^2c\,x^7+\frac{3}{8}x^8b^2da+\frac{1}{10}b^3c\,x^{10}+\frac{1}{11}b^3d\,x^{11}$	75
risch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{3}{4}a^2bc\,x^4+\frac{3}{5}x^5bd\,a^2+\frac{3}{7}a\,b^2c\,x^7+\frac{3}{8}x^8b^2da+\frac{1}{10}b^3c\,x^{10}+\frac{1}{11}b^3d\,x^{11}$	75
parallelrisch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{3}{4}a^2bc\,x^4+\frac{3}{5}x^5bd\,a^2+\frac{3}{7}a\,b^2c\,x^7+\frac{3}{8}x^8b^2da+\frac{1}{10}b^3c\,x^{10}+\frac{1}{11}b^3d\,x^{11}$	75
gosper	$\frac{x\left(280b^3d\,x^{10}+308b^3c\,x^9+1155a\,b^2d\,x^7+1320a\,b^2c\,x^6+1848a^2bd\,x^4+2310a^2x^3bc+1540a^3dx+3080c\,a^3\right)}{3080}$	76

```
[In] int((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] a^3*c*x+1/2*a^3*d*x^2+3/4*a^2*b*c*x^4+3/5*x^5*b*d*a^2+3/7*a*b^2*c*x^7+3/8*x
^8*b^2*d*a+1/10*b^3*c*x^10+1/11*b^3*d*x^11
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{2}\,a^3dx^2+a^3cx$$

```
[In] integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/5
*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right)\,dx=a^3cx+\frac{a^3dx^2}{2}+\frac{3a^2bcx^4}{4}+\frac{3a^2bdx^5}{5}+\frac{3ab^2cx^7}{7}+\frac{3ab^2dx^8}{8}+\frac{b^3cx^{10}}{10}+\frac{b^3dx^{11}}{11}$$

```
[In] integrate((b*x**3+a)**2*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + 3*a**2*b*c*x**4/4 + 3*a**2*b*d*x**5/5 + 3*a*b**2
*c*x**7/7 + 3*a*b**2*d*x**8/8 + b**3*c*x**10/10 + b**3*d*x**11/11
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{2}\,a^3dx^2+a^3cx$$

```
[In] integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/5
*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4) \, dx = \frac{1}{11} b^3dx^{11} + \frac{1}{10} b^3cx^{10} + \frac{3}{8} ab^2dx^8 + \frac{3}{7} ab^2cx^7 + \frac{3}{5} a^2bdx^5 + \frac{3}{4} a^2bcx^4 + \frac{1}{2} a^3dx^2 + a^3cx$$

```
[In] integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
[Out] 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/5
*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4) \, dx = \frac{d\,a^3\,x^2}{2} + c\,a^3\,x + \frac{3\,d\,a^2\,b\,x^5}{5} + \frac{3\,c\,a^2\,b\,x^4}{4} + \frac{3\,d\,a\,b^2\,x^8}{8} + \frac{3\,c\,a\,b^2\,x^7}{7} + \frac{d\,b^3\,x^{11}}{11} + \frac{c\,b^3\,x^{10}}{10}$$

```
[In] int((a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
[Out] (a^3*d*x^2)/2 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + a^3*c*x + (3*a^2*b*c*x^
4)/4 + (3*a*b^2*c*x^7)/7 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*d*x^8)/8
```

3.55 $\int (a+bx^3) (ac+adx+bcx^3+bdx^4) \, dx$

Optimal result

Integrand size = 28, antiderivative size = 60

$$\int (a+bx^3) (ac+adx+bcx^3+bdx^4) \, dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/2*a*b*c*x^4+2/5*a*b*d*x^5+1/7*b^2*c*x^7+1/8*b^2*d*x
^8
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1864}

$$\int (a+bx^3) (ac+adx+bcx^3+bdx^4) \, dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

```
[In] Int[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 +
 (b^2*d*x^8)/8
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(a^2c+a^2dx+2abcx^3+2abdx^4+b^2cx^6+b^2dx^7\right) \, dx \\ &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)\left(ac+adx+bcx^3+bdx^4\right)\,dx=a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

```
[In] Integrate[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 +
 (b^2*d*x^8)/8
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
default	$a^2cx+\frac{1}{2}a^2d\,x^2+\frac{1}{2}abc\,x^4+\frac{2}{5}x^5dba+\frac{1}{7}b^2c\,x^7+\frac{1}{8}b^2d\,x^8$	51
norman	$a^2cx+\frac{1}{2}a^2d\,x^2+\frac{1}{2}abc\,x^4+\frac{2}{5}x^5dba+\frac{1}{7}b^2c\,x^7+\frac{1}{8}b^2d\,x^8$	51
risch	$a^2cx+\frac{1}{2}a^2d\,x^2+\frac{1}{2}abc\,x^4+\frac{2}{5}x^5dba+\frac{1}{7}b^2c\,x^7+\frac{1}{8}b^2d\,x^8$	51
parallelrisch	$a^2cx+\frac{1}{2}a^2d\,x^2+\frac{1}{2}abc\,x^4+\frac{2}{5}x^5dba+\frac{1}{7}b^2c\,x^7+\frac{1}{8}b^2d\,x^8$	51
gosper	$\frac{x\left(35b^2d\,x^7+40b^2c\,x^6+112abd\,x^4+140abc\,x^3+140a^2dx+280a^2c\right)}{280}$	52

```
[In] int((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/2*a*b*c*x^4+2/5*x^5*d*b*a+1/7*b^2*c*x^7+1/8*b^2*d*x
^8
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+bx^3\right)\left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{2}\,a^2dx^2+a^2cx$$

```
[In] integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*x
^2 + a^2*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \left(a+bx^3\right) \left(ac+adx+bcx^3+bdx^4\right) \, dx = a^2cx+\frac{a^2dx^2}{2}+\frac{abcx^4}{2}+\frac{2abdx^5}{5}+\frac{b^2cx^7}{7}+\frac{b^2dx^8}{8}$$

```
[In] integrate((b*x**3+a)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a*b*c*x**4/2 + 2*a*b*d*x**5/5 + b**2*c*x**7/7 +
b**2*d*x**8/8
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+bx^3\right) \left(ac+adx+bcx^3+bdx^4\right) \, dx = \frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{2}{5}\,abdx^5 + \frac{1}{2}\,abcx^4+\frac{1}{2}\,a^2dx^2+a^2cx$$

```
[In] integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*x
^2 + a^2*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+bx^3\right) \left(ac+adx+bcx^3+bdx^4\right) \, dx = \frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{2}{5}\,abdx^5 + \frac{1}{2}\,abcx^4+\frac{1}{2}\,a^2dx^2+a^2cx$$

```
[In] integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*x
^2 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+bx^3\right)\left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{2\,d\,a\,b\,x^5}{5}+\frac{c\,a\,b\,x^4}{2}+\frac{d\,b^2\,x^8}{8}+\frac{c\,b^2\,x^7}{7}$$

```
[In] int((a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

```
[Out] (a^2*d*x^2)/2 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8 + a^2*c*x + (a*b*c*x^4)/2 + (
2*a*b*d*x^5)/5
```

3.56 $\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 12

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3} \, dx = cx+\frac{dx^2}{2}$$

```
[Out] c*x+1/2*d*x^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1600}

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3} \, dx = cx+\frac{dx^2}{2}$$

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]
```

```
[Out] c*x + (d*x^2)/2
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\text{integral} = \int (c+dx) \, dx$$

$$= cx+\frac{dx^2}{2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx=cx+\frac{dx^2}{2}$$

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]
[Out] c*x + (d*x^2)/2
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$\frac{x(dx+2c)}{2}$	11
default	$cx+\frac{1}{2}d\,x^2$	11
norman	$cx+\frac{1}{2}d\,x^2$	11
risch	$cx+\frac{1}{2}d\,x^2$	11
parallelrisch	$cx+\frac{1}{2}d\,x^2$	11
parts	$cx+\frac{1}{2}d\,x^2$	11

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(d*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx=\frac{1}{2}\,dx^2+cx$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="fricas")
[Out] 1/2*d*x^2 + c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{ac + adx + bcx^3 + bdx^4}{a + bx^3} \, dx = cx + \frac{dx^2}{2}$$

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a),x)
[Out] c*x + d*x**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ac + adx + bcx^3 + bdx^4}{a + bx^3} \, dx = \frac{1}{2} \, dx^2 + cx$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="maxima")
[Out] 1/2*d*x^2 + c*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ac + adx + bcx^3 + bdx^4}{a + bx^3} \, dx = \frac{1}{2} \, dx^2 + cx$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="giac")
[Out] 1/2*d*x^2 + c*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ac + adx + bcx^3 + bdx^4}{a + bx^3} \, dx = \frac{d\,x^2}{2} + c\,x$$

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x)
[Out] c*x + (d*x^2)/2
```

3.57 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 161

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2} dx = -\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}} - \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/3*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(2/3)-1/6*(c-a^(1
/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)-1/
3*(b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a
^(2/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used

= {1600, 1874, 31, 648, 631, 210, 642}

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{\sqrt{3}a^{2/3}b^{2/3}} - \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}$$

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]
```

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{c+dx}{a+bx^3}\,dx\\
&= \frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}c+\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}}\\
&= \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{2/3}}\\
&\quad+\frac{1}{2}\left(\frac{c}{\sqrt[3]{a}}+\frac{d}{\sqrt[3]{b}}\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx\\
&= \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}}\\
&\quad+\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}b^{2/3}}\\
&= -\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\\
&\quad-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.77

$$\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-2\sqrt{3}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2}\right.\right.}{6a^{2/3}b^{2/3}}$$

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]
```

```
[Out] (-2*Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + (b^(1/3)*c - a^(1/3)*d)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_Rd+c\right)\ln\left(x-\_R\right)}{\_R^2}}{3b}$
default	$c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x\right.}{6b\left(\frac{a}{b}\right)}\right.$

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b*sum((_R*d+c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 1931, normalized size of antiderivative = 11.99

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^2} dx = \text{Too large to display}$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d
^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*
d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(
2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)
/(a^2*b^2))^(1/3)))*a*b*c^2 + 2*a*c*d^2 + (b*c^3 + a*d^3)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2
))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^
2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^
(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2)
+ (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1
/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a
*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3 + a*d^3)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a^2*b*d + 2*a*b*c^2
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*
b))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^
3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c
^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^
3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d
^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
```

```
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^
2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3
+ a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*
a^2*b*d + 2*a*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^
2*a*b + 16*c*d)/(a*b)))
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^2}\,dx$$
$$= \operatorname{RootSum}\left(27t^3a^2b^2 + 9tabcd + ad^3 - bc^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2bd + 3tabc^2 + 2acd^2}{ad^3 + bc^3}\right)\right)\right)$$

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*a*b*c*d + a*d**3 - b*c**3, Lambda(_t, _t*
log(x + (9*_t**2*a**2*b*d + 3*_t*a*b*c**2 + 2*a*c*d**2)/(a*d**3 + b*c**3)))
)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^2}\,dx = \frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}} + c\right)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}} - c\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}} - c\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(d*(a/b)^(1/3) + c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b*(a/b)^(2/3)) + 1/6*(d*(a/b)^(1/3) - c)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(d*(a/b)^(1/3) - c)*log(x + (a/b)^(1/3
))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(bc-(-ab^2)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{2}{3}}} - \frac{\left(bc+(-ab^2)^{\frac{1}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{2}{3}}} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*c + (-a*b^2)^(1/3)*d)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*(d*(-a/b)^(1/3) + c)*(-a/
b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx = \sum_{k=1}^{3}\ln\left(b\left(c\,d+d^2\,x+\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,a\,b\,c\,d\,z+a\,d^3-b\,c^3,z,k\right)^2\,a\,b\,9+\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,a\,b\,c\,d\,z+a\,d^3-b\,c^3,z,k\right)\,b\,c\,x\,3\right)\right)\,\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,a\,b\,c\,d\,z+a\,d^3-b\,c^3,z,k\right)$$

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x)
```

```
[Out] symsum(log(b*(c*d + d^2*x + 9*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b
*c^3, z, k)^2*a*b + 3*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3, z,
 k)*b*c*x))*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3, z, k), k, 1,
 3)
```

3.58 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 189

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3} dx = \frac{x(c+dx)}{3a(a+bx^3)} - \frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}$$

```
[Out] 1/3*x*(d*x+c)/a/(b*x^3+a)+1/9*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)
/a^(5/3)/b^(2/3)-1/18*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/a^(5/3)/b^(2/3)-1/9*(2*b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used

$= \{1600, 1869, 1874, 31, 648, 631, 210, 642\}$

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)}{3\sqrt{3}a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} + \frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]
```

```
[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5
/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\text{integral} = \int \frac{c+dx}{\left(a+bx^3\right)^2}\,dx$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} - \frac{\int \frac{-2c-dx}{a+bx^3}\,dx}{3a}$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} - \frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}c-\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}\sqrt[3]{b}} + \frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}$$
$$- \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{2/3}}$$
$$+ \frac{\left(2c+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}}$$

$$= \frac{x(c+dx)}{3a(a+bx^3)} + \frac{\left(2\sqrt[3]{b}c - \sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c - \sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c + \sqrt[3]{a}d\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}b^{2/3}}$$

$$= \frac{x(c+dx)}{3a(a+bx^3)} - \frac{\left(2\sqrt[3]{b}c + \sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c - \sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c - \sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.95

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx$$

$$= \frac{\frac{6ax(c+dx)}{a+bx^3} - \frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{\left(-2\sqrt[3]{a}\sqrt[3]{b}c+a^{2/3}d\right)\log\left(\ldots\right.}{b^{2/\ldots}}}{18a^2}$$

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]
```

```
[Out] ((6*a*x*(c + d*x))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*c + a^(1/3)*
d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(2*a^(1/3)*b^(
1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-2*a^(1/3)*b^(1/3
)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(
18*a^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.47 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\frac{d x^2}{3a}+\frac{cx}{3a}}{b x^3+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_Rd+2c\right)\ln\left(x-\_R\right)}{\_R^2}}{9ba}$
default	$c\left(\frac{x}{3a(b x^3+a)}+\frac{\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}\right)+d\left(\frac{x^2}{3a(b x^3+a)}+\frac{-\frac{\ln\left(x+\left(\frac{a}{b}\right.\right.}{3b\left(\frac{a}{b}\right)}\right.$

[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] (1/3*d/a*x^2+1/3*c/a*x)/(b*x^3+a)+1/9/b/a*sum((_R*d+2*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 2088, normalized size of antiderivative = 11.05

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx=\text{Too large to display}$$

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/36*(12*d*x^2 - 2*(a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*

```
a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) + 12*c*x + ((a*b*x^3 + a^2)*((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) + 3*sqrt(1/3)*(a*b*x^
3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) +
 (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^
3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a
^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(
I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3
)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqr
t(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2)
)^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d^3)*x + 3/4*sqrt(1/3)*(((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c
^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8
*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b +
32*c*d)/(a^3*b))) + ((a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(
a^5*b^2))^(1/3))) - 3*sqrt(1/3)*(a*b*x^3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt
(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)
+ 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (
8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/
(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*
(8*b*c^3 + a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(
a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1
)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/
2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3
 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b))))/(a*b*x^3 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.56

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx$$
$$=\operatorname{RootSum}\left(729t^3a^5b^2+54ta^2bcd+ad^3-8bc^3,\left(t\mapsto t\log\left(x+\frac{81t^2a^4bd+36ta^2bc^2+4acd^2}{ad^3+8bc^3}\right)\right)\right)$$
$$+\frac{cx+dx^2}{3a^2+3abx^3}$$

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda(
_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d**3
 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*a*b*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.89

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx=\frac{dx^2+cx}{3\,(abx^3+a^2)}+\frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(d*x^2 + c*x)/(a*b*x^3 + a^2) + 1/9*sqrt(3)*(d*(a/b)^(1/3) + 2*c)*arcta
n(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) + 1/18*(d*
(a/b)^(1/3) - 2*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/3))
 - 1/9*(d*(a/b)^(1/3) - 2*c)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.92

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}\left(2\,bc-(-ab^2)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{\left(2\,bc+(-ab^2)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{dx^2+cx}{3\,(bx^3+a)a}$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + (-a*b^2)^(1/3)*d)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(d*(-a/b)^(1
/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(d*x^2 + c*x)/
((b*x^3 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 10.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.89

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx = \left(\sum_{k=1}^{3}\ln\left(\frac{b\left(2\,c\,d+d^2\,x+\mathrm{root}(729\,a^5\,b^2\,z^3+54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k)^2\,a^3\,b\,81+\mathrm{root}(729\,a^5\,b^2\,z^3+\dots\right.}{a^2\,9}\right.\right. + 54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k)\Bigg) + \frac{\frac{d\,x^2}{3\,a}+\frac{c\,x}{3\,a}}{b\,x^3+a}$$

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x)
```

```
[Out] symsum(log((b*(2*c*d + d^2*x + 81*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z - 8
*b*c^3 + a*d^3, z, k)^2*a^3*b + 18*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z -
8*b*c^3 + a*d^3, z, k)*a*b*c*x))/(9*a^2))*root(729*a^5*b^2*z^3 + 54*a^2*b*c
*d*z - 8*b*c^3 + a*d^3, z, k), k, 1, 3) + ((d*x^2)/(3*a) + (c*x)/(3*a))/(a
+ b*x^3)
```

3.59 $\int (a+bx^3)^{3/2} (ac+adx+bcx^3+bdx^4) \, dx$

Optimal result

Integrand size = 32, antiderivative size = 585

$$\int (a+bx^3)^{3/2} (ac+adx+bcx^3+bdx^4) \, dx = \frac{810a^3d\sqrt{a+bx^3}}{1729b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{54a^2(1729cx+935dx^2)\sqrt{a+bx^3}}{323323} + \frac{30a(247cx+187dx^2)(a+bx^3)^{3/2}}{46189} + \frac{2}{323}(19cx+17dx^2)(a+bx^3)^{5/2} - \frac{405\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{(1\ldots}{(1\ldots}\right.\right.}{1729b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 30/46189*a*(187*d*x^2+247*c*x)*(b*x^3+a)^(3/2)+2/323*(17*d*x^2+19*c*x)*(b*x
^3+a)^(5/2)+54/323323*a^2*(935*d*x^2+1729*c*x)*(b*x^3+a)^(1/2)+810/1729*a^3
*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-405/1729*3^(1/4)
*a^(10/3)*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)+54/323323*3^(3/4)*a^3*(a^(1/3)+b^(1/3)*x)*EllipticF(
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1729*b^(1/3)*c-935*a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1866, 1867, 1892, 224, 1891}

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right) \, dx = -\frac{405\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{810a^3d\sqrt{a+bx^3}}{1729b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{54a^2\sqrt{a+bx^3}(1729cx+935dx^2)}{323323} + \frac{54\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(1729\sqrt[3]{b}c-935\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\arcsin\left(\right.\right.}{323323b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{30a(a+bx^3)^{3/2}\left(247cx+187dx^2\right)}{46189} + \frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

```
[In] Int[(a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] (810*a^3*d*Sqrt[a + b*x^3])/(1729*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)) + (54*a^2*(1729*c*x + 935*d*x^2)*Sqrt[a + b*x^3])/323323 + (30*a*(247*c
*x + 187*d*x^2)*(a + b*x^3)^(3/2))/46189 + (2*(19*c*x + 17*d*x^2)*(a + b*x^
3)^(5/2))/323 - (405*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*d*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(2/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) + (54*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(1729*b^(1/3)*c - 935*(1 - Sqrt
[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(323323*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1866

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[PolynomialQuoti
ent[Pq, a + b*x^n, x]*(a + b*x^n)^(p + 1), x] /; FreeQ[{a, b, p}, x] && Pol
yQ[Pq, x] && IGtQ[n, 0] && GeQ[Expon[Pq, x], n] && EqQ[PolynomialRemainder[
Pq, a + b*x^n, x], 0]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \int (c+dx)\left(a+bx^3\right)^{5/2}\,dx$$

$$= \frac{2}{323}\left(19cx+17dx^2\right)\left(a+bx^3\right)^{5/2}+\frac{1}{2}(15a)\int\left(\frac{2c}{17}+\frac{2dx}{19}\right)\left(a+bx^3\right)^{3/2}\,dx$$

$$= \frac{30a(247cx+187dx^2)\left(a+bx^3\right)^{3/2}}{46189} + \frac{2}{323}\left(19cx+17dx^2\right)\left(a+bx^3\right)^{5/2}+\frac{1}{4}\left(135a^2\right)\int\left(\frac{4c}{187}+\frac{4dx}{247}\right)\sqrt{a+bx^3}\,dx$$

$$= \frac{54a^2(1729cx+935dx^2)\sqrt{a+bx^3}}{323323}+\frac{30a(247cx+187dx^2)\left(a+bx^3\right)^{3/2}}{46189} + \frac{2}{323}\left(19cx+17dx^2\right)\left(a+bx^3\right)^{5/2}+\frac{1}{8}\left(405a^3\right)\int\frac{\frac{8c}{935}+\frac{8dx}{1729}}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{54a^2(1729cx+935dx^2)\sqrt{a+bx^3}}{323323}+\frac{30a(247cx+187dx^2)\left(a+bx^3\right)^{3/2}}{46189} + \frac{2}{323}\left(19cx+17dx^2\right)\left(a+bx^3\right)^{5/2}+\frac{\left(405a^3d\right)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{1729\sqrt[3]{b}} + \frac{\left(81a^3\left(1729c-\frac{935\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{323323}$$

$$= \frac{810a^3d\sqrt{a+bx^3}}{1729b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{54a^2(1729cx+935dx^2)\sqrt{a+bx^3}}{323323} + \frac{30a(247cx+187dx^2)\left(a+bx^3\right)^{3/2}}{46189}+\frac{2}{323}\left(19cx+17dx^2\right)\left(a+bx^3\right)^{5/2} - \frac{405\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\cdots}{1729b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{54\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(1729\sqrt[3]{b}c-935\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\cdots\right.}{323323b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.59 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.13

$$\int (a+bx^3)^{3/2} (ac+adx+bcx^3+bdx^4)\ dx = \frac{a^2x\sqrt{a+bx^3}\left(2c\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2}{3},\frac{5}{3},-\frac{b}{}\right.\right.}{2\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] (a^2*x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-5/2, 1/3, 4/3, -((b*x^3)/a)]
 + d*x*Hypergeometric2F1[-5/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x^3
)/a])
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.34

method	result	size
risch	Expression too large to display	786
elliptic	Expression too large to display	830
default	Expression too large to display	1618

```
[In] int((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/323323*x*(17017*b^2*d*x^7+19019*b^2*c*x^6+53669*a*b*d*x^4+63973*a*b*c*x^3
+61897*a^2*d*x+91637*a^2*c)*(b*x^3+a)^(1/2)+81/323323*a^3*(-3458/3*I*c*3^(1
/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-1870/3*I*d*3^(1
/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^
(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
```

```
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.20

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right) dx = \frac{2 \left(140049\, a^3 \sqrt{b} c \text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right) - 75735\, a^3 \sqrt{b} d \text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstr}\right.\right.}{}$$

```
[In] integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas"
)
```

```
[Out] 2/323323*(140049*a^3*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 75735*a^
3*sqrt(b)*d*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) +
 (17017*b^3*d*x^8 + 19019*b^3*c*x^7 + 53669*a*b^2*d*x^5 + 63973*a*b^2*c*x^4
 + 61897*a^2*b*d*x^2 + 91637*a^2*b*c*x)*sqrt(b*x^3 + a))/b
```

Sympy [A] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.45

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right) dx = \frac{a^{\frac{5}{2}} cx \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{5}{2}} dx^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{2a^{\frac{3}{2}} bcx^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2a^{\frac{3}{2}} bdx^5 \Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a} b^2 cx^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{a} b^2 dx^8 \Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(3/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**(5/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(5/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a**(3/2)*b*c*x**4*gamma(4/3)*hyp
er((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(3/
2)*b*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(8/3)) + sqrt(a)*b**2*c*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b**2*d*x**8*gamma(8/3
)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3))
```

Maxima [F]

$$\int (a+bx^3)^{3/2}\left(ac+adx+bcx^3+bdx^4\right)\,dx=\int \left(bdx^4+bcx^3+adx+ac\right)\left(bx^3+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int (a+bx^3)^{3/2}\left(ac+adx+bcx^3+bdx^4\right)\,dx=\int \left(bdx^4+bcx^3+adx+ac\right)\left(bx^3+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^3)^{3/2}\left(ac+adx+bcx^3+bdx^4\right)\,dx=\int \left(b\,x^3+a\right)^{3/2}\,\left(b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c\right)\,dx$$

```
[In] int((a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

```
[Out] int((a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x)
```

3.60 $\int \sqrt{a+bx^3}(ac+adx+bcx^3+bdx^4)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 556

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{54a^2d\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{18a(91cx+55dx^2)\sqrt{a+bx^3}}{5005}+\frac{2}{143}\left(13cx+11dx^2\right)\left(a+bx^3\right)^{3/2}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(91\sqrt[3]{b}c-55\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{5005b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/143*(11*d*x^2+13*c*x)*(b*x^3+a)^(3/2)+18/5005*a*(55*d*x^2+91*c*x)*(b*x^3+
a)^(1/2)+54/91*a^2*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)-27/91*3^(1/4)*a^(7/3)*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/
2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+18/5005*3^(3/4)*a^2*(a^(1/3)+b^(1/3)*x
)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(91*b^(1/3)*c-55*a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^
```

```
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1866, 1867, 1892, 224, 1891}

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx = -\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{54a^2d\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(91\sqrt[3]{b}c-55\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\arcsin\left(\ldots\right.\right.}{5005b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{18a\sqrt{a+bx^3}\left(91cx+55dx^2\right)}{5005} + \frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

```
[In] Int[Sqrt[a + b*x^3]*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] (54*a^2*d*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 + (18*a*(91*c*x + 55*d*x^2)*Sqrt[a + b*x^3])/5005 + (2*(13*c*x + 11*d*x^2)
*(a + b*x^3)^(3/2))/143 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*d*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(91*b^(1/3)*c - 55*(1 - S
qrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(5005*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1866

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[PolynomialQuoti
ent[Pq, a + b*x^n, x]*(a + b*x^n)^(p + 1), x] /; FreeQ[{a, b, p}, x] && Pol
yQ[Pq, x] && IGtQ[n, 0] && GeQ[Expon[Pq, x], n] && EqQ[PolynomialRemainder[
Pq, a + b*x^n, x], 0]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \int (c+dx)\left(a+bx^3\right)^{3/2}\,dx$$

$$= \frac{2}{143}\left(13cx+11dx^2\right)\left(a+bx^3\right)^{3/2}+\frac{1}{2}(9a)\int\left(\frac{2c}{11}+\frac{2dx}{13}\right)\sqrt{a+bx^3}\,dx$$

$$= \frac{18a(91cx+55dx^2)\sqrt{a+bx^3}}{5005}+\frac{2}{143}\left(13cx+11dx^2\right)\left(a+bx^3\right)^{3/2}+\frac{1}{4}\left(27a^2\right)\int\frac{\frac{4c}{55}+\frac{4dx}{91}}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{18a(91cx+55dx^2)\sqrt{a+bx^3}}{5005}+\frac{2}{143}\left(13cx+11dx^2\right)\left(a+bx^3\right)^{3/2} + \frac{(27a^2d)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{91\sqrt[3]{b}}+\frac{\left(27a^2\left(91c-\frac{55\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{5005}$$

$$= \frac{54a^2d\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{18a(91cx+55dx^2)\sqrt{a+bx^3}}{5005} + \frac{2}{143}\left(13cx+11dx^2\right)\left(a+bx^3\right)^{3/2} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(91\sqrt[3]{b}c-55\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{5005b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.60 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.14

$$\int\sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx = \frac{ax\sqrt{a+bx^3}\left(2c\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[Sqrt[a + b*x^3]*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
[Out] (a*x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] +
 d*x*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x^3)/
a])
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 762, normalized size of antiderivative = 1.37

method	result
risch	$\frac{2x(385bd\,x^4+455bc\,x^3+880adx+1274ac)\sqrt{b\,x^3+a}}{5005}+\frac{27a^2\left(-\frac{182ic\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\cdots}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\cdots}}\cdots}{\cdots}\cdots\right)}{\cdots}$
elliptic	Expression too large to display
default	Expression too large to display

```
[In] int((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5005*x*(385*b*d*x^4+455*b*c*x^3+880*a*d*x+1274*a*c)*(b*x^3+a)^(1/2)+27/50
05*a^2*(-182/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2
)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
```

```
^(1/2))-110/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)
^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.17

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx$$
$$=\frac{2\left(2457\,a^2\sqrt{b}c\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-1485\,a^2\sqrt{b}d\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\right.\right.}{5005\,b}$$

```
[In] integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas"
)
```

```
[Out] 2/5005*(2457*a^2*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 1485*a^2*sqr
t(b)*d*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (385
*b^2*d*x^5 + 455*b^2*c*x^4 + 880*a*b*d*x^2 + 1274*a*b*c*x)*sqrt(b*x^3 + a))
/b
```

Sympy [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.31

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx = \frac{a^{\frac{3}{2}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(1/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**(3/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(3/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x**4*gamma(4/3)*hyper(
(-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*d
*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*ga
mma(8/3))
```

Maxima [F]

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx = \int \left(bdx^4+bcx^3+adx+ac\right)\sqrt{bx^3+a}\,dx$$

```
[In] integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx=\int \left(bdx^4+bcx^3+adx+ac\right)\sqrt{bx^3+a}\,dx$$

```
[In] integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx=\int \sqrt{b\,x^3+a}\,\left(b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c\right)\,dx$$

```
[In] int((a + b*x^3)^(1/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
[Out] int((a + b*x^3)^(1/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x)
```

3.61 $\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 525

$$\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx = \frac{6ad\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2}{35}\left(7cx+5dx^2\right)\sqrt{a+bx^3}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(7\sqrt[3]{b}c-5\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.\right.}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/35*(5*d*x^2+7*c*x)*(b*x^3+a)^(1/2)+6/7*a*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2)))-3/7*3^(1/4)*a^(4/3)*d*(a^(1/3)+b^(1/3)*x)*Ellipti
cE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/35*3^(3/4)*a*(
a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*c-5*a^(1/3)*d*(1-3^(1/2)))*(1/2
*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1600, 1867, 1892, 224, 1891}

$$\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx = \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}c-5\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{6ad\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2}{35}\sqrt{a+bx^3}\left(7cx+5dx^2\right)$$

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/Sqrt[a + b*x^3],x]
```

```
[Out] (6*a*d*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (
2*(7*c*x + 5*d*x^2)*Sqrt[a + b*x^3])/35 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4
/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
7*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(7*b^(1/3)*c -
 5*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(35*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (c+dx)\sqrt{a+bx^3}\,dx\\ &= \frac{2}{35}\left(7cx+5dx^2\right)\sqrt{a+bx^3}+\frac{1}{2}(3a)\int \frac{\frac{2c}{5}+\frac{2dx}{7}}{\sqrt{a+bx^3}}\,dx\end{aligned}$$

$$= \frac{2}{35}\left(7cx+5dx^2\right)\sqrt{a+bx^3}+\frac{(3ad)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{7\sqrt[3]{b}}$$

$$+\frac{1}{35}\left(3a\left(7c-\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{6ad\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2}{35}\left(7cx+5dx^2\right)\sqrt{a+bx^3}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{\cdots}\right.}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(7\sqrt[3]{b}c-5\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\cdots\right.}{\left(1+\cdots\right.}\right.\right.}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.14

$$\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{x\sqrt{a+bx^3}\left(2c\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)] + d
*x*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x^3)/a]
)
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 744, normalized size of antiderivative = 1.42

method	result
risch	$\frac{2x(5dx+7c)\sqrt{b\,x^{3}+a}}{35}+\frac{3a\left(-\frac{14ic\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\cdots\right.}{2\cdots}\right.}{\cdots}}\cdots}{\cdots}\cdots\right)}{\cdots}$ $2iac\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x\cdots\right.}{\cdots}}\cdots$
elliptic	$\frac{2d\,x^{2}\sqrt{b\,x^{3}+a}}{7}+\frac{2cx\sqrt{b\,x^{3}+a}}{5}-\frac{\cdots}{\cdots}$
default	Expression too large to display

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35*x*(5*d*x+7*c)*(b*x^3+a)^(1/2)+3/35*a*(-14/3*I*c*3^(1/2)/b*(-a*b^2)^(1/
3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-10/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.13

$$\int \frac{ac + adx + bcx^3 + bdx^4}{\sqrt{a + bx^3}}\,dx$$
$$= \frac{2\left(21\,a\sqrt{b}c\text{weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right) - 15\,a\sqrt{b}d\text{weierstrassZeta}\left(0, -\frac{4\,a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4\,a}{b},\right.\right.\right.}{35\,b}$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/35*(21*a*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 15*a*sqrt(b)*d*wei
erstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (5*b*d*x^2 + 7
*b*c*x)*sqrt(b*x^3 + a))/b
```

Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.31

$$\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) + sqrt(a)*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((1/2, 4/3),
(7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + b*d*x**5*gamma(5
/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/
3))
```

Maxima [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx = \int \frac{bdx^4+bcx^3+adx+ac}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx = \int \frac{bdx^4+bcx^3+adx+ac}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ac + adx + bcx^3 + bdx^4}{\sqrt{a + bx^3}}\,dx = \int \frac{b\,d\,x^4 + b\,c\,x^3 + a\,d\,x + a\,c}{\sqrt{b\,x^3 + a}}\,dx$$

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(1/2),x)
[Out] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(1/2), x)
```

3.62 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 490

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx = \frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)
*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)+2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/
2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c-a^(1/3)*d*(1
-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)
/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1600, 1892, 224, 1891}

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{Ellip}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+}}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*
(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \int \frac{c+dx}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} + \left(c - \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.15

$$\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\right.}{2\sqrt{a+bx^3}}$$

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x]
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
+ d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.47

method	result
elliptic	$-\frac{2ic\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,F}{3b\sqrt{b\,x^{3}+a}}$
default	Expression too large to display

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/
3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.09

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(\sqrt{b}c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{b}d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weiers}\right.\right.}{b}$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="fricas"
)
[Out] 2*(sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - sqrt(b)*d*weierstrassZeta(
0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.16

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(3/2),x)
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Maxima [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx = \int \frac{bdx^4+bcx^3+adx+ac}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx=\int \frac{bdx^4+bcx^3+adx+ac}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx=\int \frac{b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c}{(b\,x^3+a)^{3/2}}\,dx$$

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x)
[Out] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2), x)
```

3.63 $\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^{5/2}} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 522

$$\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^{5/2}} \, dx = \frac{2x(c+dx)}{3a\sqrt{a+bx^3}} - \frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+ \frac{\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c+\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/3*x*(d*x+c)/a/(b*x^3+a)^(1/2)-2/3*d*(b*x^3+a)^(1/2)/a/b^(2/3)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))+1/3*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3
)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/9*(a^(1/3)+b^(1/3)*
x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*(b^(1/3)*c+a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2)
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)*3^(3/4)/a/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1600, 1869, 1892, 224, 1891}

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)\text{Elli}}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a}}$$

$$+\frac{\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x(c+dx)}{3a\sqrt{a+bx^3}}$$

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x*(c + d*x))/(3*a*Sqrt[a + b*x^3]) - (2*d*Sqrt[a + b*x^3])/(3*a*b^(2/3)*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c + (1 - Sqrt[3])*a^(1
/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \int \frac{c+dx}{(a+bx^3)^{3/2}}\,dx$$

$$= \frac{2x(c+dx)}{3a\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{c}{2}+\frac{dx}{2}}{\sqrt{a+bx^3}}\,dx}{3a}$$

$$= \frac{2x(c+dx)}{3a\sqrt{a+bx^3}} - \frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{3a\sqrt[3]{b}} + \frac{\left(c+\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3a}$$

$$= \frac{2x(c+dx)}{3a\sqrt{a+bx^3}} - \frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+ \frac{\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{2\sqrt{2+\sqrt{3}}\left(c+\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.18

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx = \frac{x\left(4c+2c\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+3dx\sqrt{1+\frac{bx^3}{a}}\,\text{Hy}\right.}{6a\sqrt{a+bx^3}}$$

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x]
```

```
[Out] (x*(4*c + 2*c*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3
)/a)] + 3*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3
)/a)]))/(6*a*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.47

method	result
elliptic	$-\frac{2b\left(-\frac{d\,x^{2}}{3ab}-\frac{cx}{3ba}\right)}{\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2ic\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i}{}\right.}{\left(-a\,b\right.}}}{9ab\sqrt{b\,x^{3}}}$
default	Expression too large to display

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(-1/3/a/b*d*x^2-1/3/b/a*c*x)/((x^3+a/b)*b)^(1/2)-2/9*I*c/a*3^(1/2)/b*(
-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/9*I*d/a*3^(1/2)/b*(-
a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.18

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx = \frac{2\left((bcx^3+ac)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+(bdx^3+ad)\sqrt{b}\text{weierstra}\right.}{3\left(ab^2x^3\right.}$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*((b*c*x^3 + a*c)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (b*d*x^3 +
 a*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))
 + (b*d*x^2 + b*c*x)*sqrt(b*x^3 + a))/(a*b^2*x^3 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.31

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}+\frac{bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{5}{2}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(5/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(3/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 5/2)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(7/3)) + b*d*x**5*gamm
a(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamm
a(8/3))
```

Maxima [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{5/2}} \, dx = \int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{5}{2}}} \, dx$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="maxima"
)
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{5/2}} \, dx = \int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{5}{2}}} \, dx$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{5/2}} \, dx = \int \frac{b\,d\,x^4 + b\,c\,x^3 + a\,d\,x + a\,c}{(b\,x^3 + a)^{5/2}} \, dx$$

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x)
[Out] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2), x)
```

3.64 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 554

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx = \frac{2x(c+dx)}{9a(a+bx^3)^{3/2}} + \frac{2x(7c+5dx)}{27a^2\sqrt{a+bx^3}} - \frac{10d\sqrt{a+bx^3}}{27a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{5\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}c+5\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{27\sqrt[4]{3}a^2b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/9*x*(d*x+c)/a/(b*x^3+a)^(3/2)+2/27*x*(5*d*x+7*c)/a^2/(b*x^3+a)^(1/2)-10/2
7*d*(b*x^3+a)^(1/2)/a^2/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+5/27*d*(a^(
1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5
/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)+2/81*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3
```

```
)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*c+
5*a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^2/b^(2/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1600, 1869, 1892, 224, 1891}

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx = \frac{5\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{9\ 3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{10d\sqrt{a+bx^3}}{27a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x(7c+5dx)}{27a^2\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)\sqrt[3]{a}d+7\sqrt[3]{b}c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{27\sqrt[4]{3}a^2b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x]
```

```
[Out] (2*x*(c + d*x))/(9*a*(a + b*x^3)^(3/2)) + (2*x*(7*c + 5*d*x))/(27*a^2*Sqrt[
a + b*x^3]) - (10*d*Sqrt[a + b*x^3])/(27*a^2*b^(2/3)*((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)) + (5*Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*a^(5/3)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*c + 5*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^
```

```
2*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{c+dx}{\left(a+bx^3\right)^{5/2}}\,dx \\
&= \frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}} - \frac{2\int \frac{-\frac{7c}{2}-\frac{5dx}{2}}{\left(a+bx^3\right)^{3/2}}\,dx}{9a} \\
&= \frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}} + \frac{2x(7c+5dx)}{27a^2\sqrt{a+bx^3}} + \frac{4\int \frac{\frac{7c}{4}-\frac{5dx}{4}}{\sqrt{a+bx^3}}\,dx}{27a^2} \\
&= \frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}} + \frac{2x(7c+5dx)}{27a^2\sqrt{a+bx^3}} - \frac{(5d)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{27a^2\sqrt[3]{b}} \\
&\quad + \frac{\left(7c+\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{27a^2} \\
&= \frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}} + \frac{2x(7c+5dx)}{27a^2\sqrt{a+bx^3}} - \frac{10d\sqrt{a+bx^3}}{27a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} \\
&\quad + \frac{5\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} \\
&\quad + \frac{2\sqrt{2+\sqrt{3}}\left(7c+\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{27\sqrt[4]{3}a^2\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.22

$$
\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^{7/2}}\,dx = \frac{4cx\left(10a+7bx^3\right)+14cx\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right.}{54a^2\left(a+bx^3\right.}
$$

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x]
```

```
[Out] (4*c*x*(10*a + 7*b*x^3) + 14*c*x*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeome
tric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 27*d*x^2*(a + b*x^3)*Sqrt[1 + (b*x^3
)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(54*a^2*(a + b*x^3)^(3
/2))
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.46

method	result	size
elliptic	Expression too large to display	809
default	Expression too large to display	1782

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/9*d/a/b^2*x^2+2/9*c/a/b^2*x)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2*b*(-5/27/a^2/
b*d*x^2-7/27/a^2/b*c*x)/((x^3+a/b)*b)^(1/2)-14/81*I/a^2*c*3^(1/2)/b*(-a*b^2
)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+10/81*I*d/a^2*3^(1/2)/b*(-a*
b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.28

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx = \frac{2\left(7\left(b^2cx^6+2\,abcx^3+a^2c\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+5\left(b^2dx^6+2\right.\right.}{}$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="fricas"
)
```

```
[Out] 2/27*(7*(b^2*c*x^6 + 2*a*b*c*x^3 + a^2*c)*sqrt(b)*weierstrassPInverse(0, -4
*a/b, x) + 5*(b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d)*sqrt(b)*weierstrassZeta(0, -
4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (5*b^2*d*x^5 + 7*b^2*c*x^4 + 8*
a*b*d*x^2 + 10*a*b*c*x)*sqrt(b*x^3 + a))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4
*b)
```

Sympy [A] (verification not implemented)

Time = 11.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.29

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{7}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{7}{2}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{7}{2}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{5}{3},\frac{7}{2}\\\frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(7/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 7/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5
/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 7/2), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(5/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 7/2)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7/2)*gamma(7/3)) + b*d*x**5*gamm
a(5/3)*hyper((5/3, 7/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7/2)*gamm
a(8/3))
```

Maxima [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{7/2}} \, dx = \int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{7}{2}}} \, dx$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="maxima"
)
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{7/2}} \, dx = \int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{7}{2}}} \, dx$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="giac")
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{7/2}} \, dx = \int \frac{b\,d\,x^4 + b\,c\,x^3 + a\,d\,x + a\,c}{(b\,x^3 + a)^{7/2}} \, dx$$

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x)
[Out] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2), x)
```

3.65 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 581

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx = \frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}+\frac{2x(13c+11dx)}{135a^2\left(a+bx^3\right)^{3/2}}$$
$$+\frac{2x(91c+55dx)}{405a^3\sqrt{a+bx^3}}-\frac{22d\sqrt{a+bx^3}}{81a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{11\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{27\ 3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(91\sqrt[3]{b}c+55\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{405\sqrt[4]{3}a^3b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/15*x*(d*x+c)/a/(b*x^3+a)^(5/2)+2/135*x*(11*d*x+13*c)/a^2/(b*x^3+a)^(3/2)+
2/405*x*(55*d*x+91*c)/a^3/(b*x^3+a)^(1/2)-22/81*d*(b*x^3+a)^(1/2)/a^3/b^(2/
3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+11/81*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b
^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I
)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)/b^(2/3)/(b*x^3+a)^(1/2)/
(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/121
```

```
5*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(91*b^(1/3)*c+55*a^(1/3)*d*(1-3^(1/2)))
*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^3/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1600, 1869, 1892, 224, 1891}

$$\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^{9/2}}dx = \frac{11\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{27\ 3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{22d\sqrt{a+bx^3}}{81a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x(91c+55dx)}{405a^3\sqrt{a+bx^3}}+\frac{2x(13c+11dx)}{135a^2\left(a+bx^3\right)^{3/2}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(55\left(1-\sqrt{3}\right)\sqrt[3]{a}d+91\sqrt[3]{b}c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\right.}{\sqrt[3]{b}x+\left(1+\right.}\right.\right.}{405\sqrt[4]{3}a^3b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x]
```

```
[Out] (2*x*(c + d*x))/(15*a*(a + b*x^3)^(5/2)) + (2*x*(13*c + 11*d*x))/(135*a^2*(
a + b*x^3)^(3/2)) + (2*x*(91*c + 55*d*x))/(405*a^3*Sqrt[a + b*x^3]) - (22*d
*Sqrt[a + b*x^3])/(81*a^3*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (1
1*Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(27*3^(3/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 +
Sqrt[3]]*(91*b^(1/3)*c + 55*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*a^3*b^(2/3)*Sqrt
```

```
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \int \frac{c+dx}{\left(a+bx^3\right)^{7/2}}\,dx$$

$$= \frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}} - \frac{2\int \frac{-\frac{13c}{2}-\frac{11dx}{2}}{\left(a+bx^3\right)^{5/2}}\,dx}{15a}$$

$$= \frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}} + \frac{2x(13c+11dx)}{135a^2\left(a+bx^3\right)^{3/2}} + \frac{4\int \frac{\frac{91c}{4}+\frac{55dx}{4}}{\left(a+bx^3\right)^{3/2}}\,dx}{135a^2}$$

$$= \frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}} + \frac{2x(13c+11dx)}{135a^2\left(a+bx^3\right)^{3/2}} + \frac{2x(91c+55dx)}{405a^3\sqrt{a+bx^3}} - \frac{8\int \frac{-\frac{91c}{8}+\frac{55dx}{8}}{\sqrt{a+bx^3}}\,dx}{405a^3}$$

$$= \frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}} + \frac{2x(13c+11dx)}{135a^2\left(a+bx^3\right)^{3/2}} + \frac{2x(91c+55dx)}{405a^3\sqrt{a+bx^3}} - \frac{(11d)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{81a^3\sqrt[3]{b}} + \frac{\left(91c+\frac{55\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{405a^3}$$

$$= \frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}} + \frac{2x(13c+11dx)}{135a^2\left(a+bx^3\right)^{3/2}} + \frac{2x(91c+55dx)}{405a^3\sqrt{a+bx^3}} - \frac{22d\sqrt{a+bx^3}}{81a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{11\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{27\ 3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(91c+\frac{55\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}\right.\right.}{405\sqrt[4]{3}a^3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.24

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx = \frac{4cx(157a^2+221abx^3+91b^2x^6)+182cx(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometri}}{810}$$

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x]
```

```
[Out] (4*c*x*(157*a^2 + 221*a*b*x^3 + 91*b^2*x^6) + 182*c*x*(a + b*x^3)^2*Sqrt[1
+ (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 405*d*x^2*(a
+ b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 7/2, 5/3, -((b*x^3)/a
)])/(810*a^3*(a + b*x^3)^(5/2))
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 853, normalized size of antiderivative = 1.47

method	result	size
elliptic	Expression too large to display	853
default	Expression too large to display	1902

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/15*d/a/b^3*x^2+2/15*c/a/b^3*x)*(b*x^3+a)^(1/2)/(x^3+a/b)^3+(22/135/b^2/a
^2*d*x^2+26/135/b^2/a^2*c*x)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2*b*(-11/81/a^3/b*
d*x^2-91/405/a^3/b*c*x)/((x^3+a/b)*b)^(1/2)-182/1215*I*c/a^3*3^(1/2)/b*(-a*
b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+22/243*I*d/a^3*3^(1/2)/b*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
```

```
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.37

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx = \frac{2\left(91\left(b^3cx^9+3\,ab^2cx^6+3\,a^2bcx^3+a^3c\right)\sqrt{b}\,\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right.}{}$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="fricas"
)
```

```
[Out] 2/405*(91*(b^3*c*x^9 + 3*a*b^2*c*x^6 + 3*a^2*b*c*x^3 + a^3*c)*sqrt(b)*weier
strassPInverse(0, -4*a/b, x) + 55*(b^3*d*x^9 + 3*a*b^2*d*x^6 + 3*a^2*b*d*x^
3 + a^3*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b
, x)) + (55*b^3*d*x^8 + 91*b^3*c*x^7 + 143*a*b^2*d*x^5 + 221*a*b^2*c*x^4 +
115*a^2*b*d*x^2 + 157*a^2*b*c*x)*sqrt(b*x^3 + a))/(a^3*b^4*x^9 + 3*a^4*b^3*
x^6 + 3*a^5*b^2*x^3 + a^6*b)
```

Sympy [A] (verification not implemented)

Time = 36.62 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.28

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{9}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{9}{2}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{9}{2}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{9}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{9}{2}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{9}{2}}\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(9/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 9/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7
/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 9/2), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(7/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 9/2)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(9/2)*gamma(7/3)) + b*d*x**5*gamm
a(5/3)*hyper((5/3, 9/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(9/2)*gamm
a(8/3))
```

Maxima [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{9/2}}\,dx = \int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2), x)
```

Giac [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{9/2}}\,dx = \int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{9/2}}\,dx = \int \frac{b\,d\,x^4 + b\,c\,x^3 + a\,d\,x + a\,c}{(b\,x^3 + a)^{9/2}}\,dx$$

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x)
```

```
[Out] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2), x)
```

3.66 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 590

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{2e\sqrt{a+bx^3}}{3b}+\frac{2fx\sqrt{a+bx^3}}{5b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}+\frac{2(7bd-4ag)\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}(5bc-2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{Elliptic}}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/3*e*(b*x^3+a)^(1/2)/b+2/5*f*x*(b*x^3+a)^(1/2)/b+2/7*g*x^2*(b*x^3+a)^(1/2)
/b+2/7*(-4*a*g+7*b*d)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))-1/7*3^(1/4)*a^(1/3)*(-4*a*g+7*b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/105*(a^(1/3)+b^(1/3)*x)*
EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I
```

```
*3^(1/2)+2*I)*(7*b^(1/3)*(-2*a*f+5*b*c)-5*a^(1/3)*(-4*a*g+7*b*d)*(1-3^(1/2)
))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3
)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1902, 1900, 267, 1892, 224, 1891}

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(5\right.}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7bd-4ag)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{}\right.}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{a+bx^3}(7bd-4ag)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2e\sqrt{a+bx^3}}{3b}+\frac{2fx\sqrt{a+bx^3}}{5b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*f*x*Sqrt[a + b*x^3])/(5*b) + (2*g*x^2*Sqrt
[a + b*x^3])/(7*b) + (2*(7*b*d - 4*a*g)*Sqrt[a + b*x^3])/(7*b^(5/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*b*d -
 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*(5*b*c - 2*
a*f) - 5*(1 - Sqrt[3])*a^(1/3)*(7*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
```

```
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{2\int \frac{\frac{7bc}{2}+\frac{1}{2}(7bd-4ag)x+\frac{7}{2}bex^2+\frac{7}{2}bfx^3}{\sqrt{a+bx^3}}\,dx}{7b}$$

$$= \frac{2fx\sqrt{a+bx^3}}{5b} + \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{4\int \frac{\frac{7}{4}b(5bc-2af)+\frac{5}{4}b(7bd-4ag)x+\frac{35}{4}b^2ex^2}{\sqrt{a+bx^3}}\,dx}{35b^2}$$

$$= \frac{2fx\sqrt{a+bx^3}}{5b} + \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{4\int \frac{\frac{7}{4}b(5bc-2af)+\frac{5}{4}b(7bd-4ag)x}{\sqrt{a+bx^3}}\,dx}{35b^2} + e\int \frac{x^2}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2e\sqrt{a+bx^3}}{3b} + \frac{2fx\sqrt{a+bx^3}}{5b} + \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{(7bd-4ag)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}} + \frac{\left(7\sqrt[3]{b}(5bc-2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd-4ag)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{35b^{4/3}}$$

$$= \frac{2e\sqrt{a+bx^3}}{3b} + \frac{2fx\sqrt{a+bx^3}}{5b} + \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{2(7bd-4ag)\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-\right.}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}(5bc-2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.23

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{4(a+bx^3)(35e+3x(7f+5gx))+42(5bc-2af)x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+15(7b}{210b\sqrt{a+bx^3}}$$

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/Sqrt[a + b*x^3],x]

[Out] (4*(a + b*x^3)*(35*e + 3*x*(7*f + 5*g*x)) + 42*(5*b*c - 2*a*f)*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 15*(7*b*d - 4*a*g)*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(210*b*Sqrt[a + b*x^3])

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.33

method	result
elliptic	$\frac{2g\,x^2\sqrt{b\,x^3+a}}{7b}+\frac{2fx\sqrt{b\,x^3+a}}{5b}+\frac{2e\sqrt{b\,x^3+a}}{3b}-\frac{2i\left(c-\frac{2af}{5b}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\left(\right.}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}}{}$
risch	Expression too large to display
default	Expression too large to display

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/7*g*x^2*(b*x^3+a)^(1/2)/b+2/5*f*x*(b*x^3+a)^(1/2)/b+2/3*e*(b*x^3+a)^(1/2)/b-2/3*I*(c-2/5*a/b*f)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a

```
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))-2/3*I*(d-4/7*a/b*g)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.15

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{2\left(21\,(5\,bc-2\,af)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-15\,(7\,bd-4\,ag)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstr}\right.\right.}{105\,b^2}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(21*(5*b*c - 2*a*f)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - 15*(7
*b*d - 4*a*g)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*
a/b, x)) + (15*b*g*x^2 + 21*b*f*x + 35*b*e)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.32

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx = e\left(\begin{cases} \frac{x^3}{3\sqrt{a}} & \text{for } b=0 \\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
[Out] e*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True))
 + c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + f*x**4*gamma(4/3)*hyper((1/2, 4/3),
 (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + g*x**5*gamma(5/
3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3
))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx = \int \frac{gx^4+fx^3+ex^2+dx+c}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx = \int \frac{gx^4+fx^3+ex^2+dx+c}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx = \int \frac{g\,x^4+f\,x^3+e\,x^2+d\,x+c}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(1/2),x)
[Out] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(1/2), x)
```

3.67 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 594

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = \frac{2x(bc-af+(bd-ag)x+bex^2)}{3ab\sqrt{a+bx^3}} - \frac{2e\sqrt{a+bx^3}}{3ab} - \frac{2(bd-4ag)\sqrt{a+bx^3}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{\sqrt{2-\sqrt{3}}(bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}(bc+2af)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\text{arcs}\right.}{3\sqrt[4]{3}ab^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/3*x*(b*c-a*f+(-a*g+b*d)*x+b*e*x^2)/a/b/(b*x^3+a)^(1/2)-2/3*e*(b*x^3+a)^(1
/2)/a/b-2/3*(-4*a*g+b*d)*(b*x^3+a)^(1/2)/a/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))+1/3*(-4*a*g+b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(
1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2
*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/9*(a^(1/3)+b^(1/3)*x)
```

```
*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*(b^(1/3)*(2*a*f+b*c)+a^(1/3)*(-4*a*g+b*d)*(1-3^(1/2)))*(1/2*
6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1872, 1900, 267, 1892, 224, 1891}

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(\cdots}{\sqrt[3]{b}x+(\cdots}\right.\right.}{3\sqrt[4]{3}ab^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}}$$

$$+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(bd-4ag)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{a+bx^3}(bd-4ag)}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}-\frac{2e\sqrt{a+bx^3}}{3ab}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(3*a*b*Sqrt[a + b*x^3]) - (2*e*
Sqrt[a + b*x^3])/(3*a*b) - (2*(b*d - 4*a*g)*Sqrt[a + b*x^3])/(3*a*b^(5/3)*(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*(b*d - 4*a*g)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^
(2/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*(b*c + 2*a*
f) + (1 - Sqrt[3])*a^(1/3)*(b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(5/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
```

```
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$\text{integral} = \frac{2x(bc-af+(bd-ag)x+bex^2)}{3ab\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{1}{2}b(bc+2af)+\frac{1}{2}b(bd-4ag)x+\frac{3}{2}b^2ex^2}{\sqrt{a+bx^3}}\,dx}{3ab^2}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{3ab\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{1}{2}b(bc+2af)+\frac{1}{2}b(bd-4ag)x}{\sqrt{a+bx^3}}\,dx}{3ab^2} - \frac{e\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{a}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{3ab\sqrt{a+bx^3}} - \frac{2e\sqrt{a+bx^3}}{3ab} - \frac{(bd-4ag)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{3ab^{4/3}} + \frac{\left(\sqrt[3]{b}(bc+2af)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3ab^{4/3}}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{3ab\sqrt{a+bx^3}} - \frac{2e\sqrt{a+bx^3}}{3ab} - \frac{2(bd-4ag)\sqrt{a+bx^3}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+ \frac{\sqrt{2-\sqrt{3}}(bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{\ldots}\right.}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}(bc+2af)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin\ldots\right.}{3\sqrt[4]{3}ab^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.22

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = \frac{4bcx-4a(e+x(f-3gx))+2(bc+2af)x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\ldots}{6ab\sqrt{\ldots}}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x]
```

```
[Out] (4*b*c*x - 4*a*(e + x*(f - 3*g*x)) + 2*(b*c + 2*a*f)*x*Sqrt[1 + (b*x^3)/a]*
Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 3*(b*d - 4*a*g)*x^2*Sqrt[1
 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)])/(6*a*b*Sqrt[a
 + b*x^3])
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.38

method	result	size
elliptic	Expression too large to display	821
default	Expression too large to display	1547

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(1/3*(a*g-b*d)/a/b^2*x^2+1/3*(a*f-b*c)/b^2/a*x+1/3*e/b^2)/((x^3+a/b)*b
)^(1/2)-2/3*I*(f/b-1/3*(a*f-b*c)/a/b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(g/b+1/3*(a*g-b*d)/a/b)*3^(1/2)/b*(-a*b^2)
^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.26

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(((b^2c+2\,abf)x^3+abc+2\,a^2f)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+((b$$

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] 2/3*(((b^2*c + 2*a*b*f)*x^3 + a*b*c + 2*a^2*f)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + ((b^2*d - 4*a*b*g)*x^3 + a*b*d - 4*a^2*g)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - sqrt(b*x^3 + a)*(a*b*e - (b^2*d - a*b*g)*x^2 - (b^2*c - a*b*f)*x))/(a*b^3*x^3 + a^2*b^2)

Sympy [A] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.32

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = e\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)

[Out] e*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True)) + c*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3)) + f*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3)) + g*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3))

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx=\int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx=\int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx=\int \frac{g\,x^4+f\,x^3+e\,x^2+d\,x+c}{(b\,x^3+a)^{3/2}}\,dx$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x)
[Out] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2), x)
```

3.68 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 628

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx = \frac{2x(bc-af+(bd-ag)x+bex^2)}{9ab\,(a+bx^3)^{3/2}} - \frac{2(5bd+4ag)\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2(3ae-x(7bc+2af+(5bd+4ag)x))}{27a^2b\sqrt{a+bx^3}} + \frac{\sqrt{2-\sqrt{3}}(5bd+4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{9\;3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}(7bc+2af)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd+4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\text{arc}\right.}{27\sqrt[4]{3}a^2b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/9*x*(b*c-a*f+(-a*g+b*d)*x+b*e*x^2)/a/b/(b*x^3+a)^(3/2)-2/27*(3*a*e-x*(7*b
*c+2*a*f+(4*a*g+5*b*d)*x))/a^2/b/(b*x^3+a)^(1/2)-2/27*(4*a*g+5*b*d)*(b*x^3+
a)^(1/2)/a^2/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/27*(4*a*g+5*b*d)*(a^
(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(
5/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3
```

```
)*(1+3^(1/2)))^2)^(1/2)+2/81*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/
3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*(2*
a*f+7*b*c)+a^(1/3)*(4*a*g+5*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)*3^(3/4)/a^2/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1872, 1868, 1892, 224, 1891}

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4ag+5bd)E\left(\arcsin\left(\frac{\sqrt[3]{b}\ldots}{\sqrt[3]{b}\ldots}\right.\right.}{9\ 3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}\ldots}$$

$$-\frac{2\sqrt{a+bx^3}(4ag+5bd)}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2(3ae-x(x(4ag+5bd)+2af+7bc))}{27a^2b\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{b}(2\ldots\right.}{27\sqrt[4]{3}a^2b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2x(x(bd-ag)-af+bc+bex^2)}{9ab\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(9*a*b*(a + b*x^3)^(3/2)) - (2*
(5*b*d + 4*a*g)*Sqrt[a + b*x^3])/(27*a^2*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)) - (2*(3*a*e - x*(7*b*c + 2*a*f + (5*b*d + 4*a*g)*x)))/(27*a^2*b*
Sqrt[a + b*x^3]) + (Sqrt[2 - Sqrt[3]]*(5*b*d + 4*a*g)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*a^(5/3)*b^(5/3)*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*(7*b*c + 2*a*f) + (1 - Sqr
t[3])*a^(1/3)*(5*b*d + 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF
```

```
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^2*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2x(bc-af+(bd-ag)x+bex^2)}{9ab\left(a+bx^3\right)^{3/2}} - \frac{2\int \frac{-\frac{1}{2}b(7bc+2af)-\frac{1}{2}b(5bd+4ag)x-\frac{3}{2}b^2ex^2}{(a+bx^3)^{3/2}}\,dx}{9ab^2}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{9ab\left(a+bx^3\right)^{3/2}} - \frac{2(3ae-x(7bc+2af+(5bd+4ag)x))}{27a^2b\sqrt{a+bx^3}} + \frac{4\int \frac{\frac{1}{4}b(7bc+2af)-\frac{1}{4}b(5bd+4ag)x}{\sqrt{a+bx^3}}\,dx}{27a^2b^2}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{9ab\left(a+bx^3\right)^{3/2}} - \frac{2(3ae-x(7bc+2af+(5bd+4ag)x))}{27a^2b\sqrt{a+bx^3}} - \frac{(5bd+4ag)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{27a^2b^{4/3}} + \frac{\left(\sqrt[3]{b}(7bc+2af)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd+4ag)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{27a^2b^{4/3}}$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{9ab\left(a+bx^3\right)^{3/2}} - \frac{2(5bd+4ag)\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2(3ae-x(7bc+2af+(5bd+4ag)x))}{27a^2b\sqrt{a+bx^3}} + \frac{\sqrt{2-\sqrt{3}}(5bd+4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{9\,3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}(7bc+2af)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd+4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F}{27\sqrt[4]{3}a^2b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.27

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx = \frac{140b^2cx^4+40abx(5c+fx^3)-4a^2(15e+x(5f+27gx))+10(7bc+2af)x}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x]
```

```
[Out] (140*b^2*c*x^4 + 40*a*b*x*(5*c + f*x^3) - 4*a^2*(15*e + x*(5*f + 27*g*x)) +
 10*(7*b*c + 2*a*f)*x*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3
, 1/2, 4/3, -((b*x^3)/a)] + 27*(5*b*d + 4*a*g)*x^2*(a + b*x^3)*Sqrt[1 + (b*
x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(270*a^2*b*(a + b*x
^3)^(3/2))
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 861, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	861
default	Expression too large to display	1673

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/9/a/b^3*(a*g-b*d)*x^2-2/9/a/b^3*(a*f-b*c)*x-2/9/b^3*e)*(b*x^3+a)^(1/2)/
(x^3+a/b)^2-2*b*(-1/27/a^2/b^2*(4*a*g+5*b*d)*x^2-1/27/a^2/b^2*(2*a*f+7*b*c)
*x)/((x^3+a/b)*b)^(1/2)-2/81*I/a^2/b^2*(2*a*f+7*b*c)*3^(1/2)*(-a*b^2)^(1/3)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/81*I/a^2/b^2*(4*a*g+5*b*d)*3^(1/2
)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2
)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/
```

```
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.42

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx = \frac{2\left(\left((7\,b^3c+2\,ab^2f)x^6+7\,a^2bc+2\,a^3f+2\,(7\,ab^2c+2\,a^2bf)x^3\right)\sqrt{b}\text{weierst}\right.}{}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/27*(((7*b^3*c + 2*a*b^2*f)*x^6 + 7*a^2*b*c + 2*a^3*f + 2*(7*a*b^2*c + 2*a
^2*b*f)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + ((5*b^3*d + 4*a*b^
2*g)*x^6 + 5*a^2*b*d + 4*a^3*g + 2*(5*a*b^2*d + 4*a^2*b*g)*x^3)*sqrt(b)*wei
erstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + ((5*b^3*d + 4*
a*b^2*g)*x^5 + (7*b^3*c + 2*a*b^2*f)*x^4 - 3*a^2*b*e + (8*a*b^2*d + a^2*b*g
)*x^2 + (10*a*b^2*c - a^2*b*f)*x)*sqrt(b*x^3 + a))/(a^2*b^4*x^6 + 2*a^3*b^3
*x^3 + a^4*b^2)
```

Sympy [A] (verification not implemented)

Time = 47.42 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx = e\left(\begin{cases} -\frac{2}{9ab\sqrt{a+bx^3}+9b^2x^3\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^3}{3a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{3},\frac{5}{2} \\ \frac{4}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix} \frac{2}{3},\frac{5}{2} \\ \frac{5}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix} \frac{4}{3},\frac{5}{2} \\ \frac{7}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix} \frac{5}{3},\frac{5}{2} \\ \frac{8}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(5/2),x)
```

```
[Out] e*Piecewise((-2/(9*a*b*sqrt(a + b*x**3) + 9*b**2*x**3*sqrt(a + b*x**3)), Ne
(b, 0)), (x**3/(3*a**(5/2)), True)) + c*x*gamma(1/3)*hyper((1/3, 5/2), (4/3
```

```
,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(4/3)) + d*x**2*gamma(2/3)*h
yper((2/3, 5/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(5/3))
+ f*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(5/2)*gamma(7/3)) + g*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(5/2)*gamma(8/3))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx=\int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="maxima")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx=\int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx=\int \frac{g\,x^4+f\,x^3+e\,x^2+d\,x+c}{{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x)
[Out] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2), x)
```

3.69 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 676

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx &= \frac{2x(bc-af+(bd-ag)x+bex^2)}{15ab\,(a+bx^3)^{5/2}} \\
&+ \frac{2x(7(13bc+2af)+5(11bd+4ag)x)}{405a^3b\sqrt{a+bx^3}} - \frac{2(11bd+4ag)\sqrt{a+bx^3}}{81a^3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} \\
&- \frac{2(9ae-x(13bc+2af+(11bd+4ag)x))}{135a^2b\,(a+bx^3)^{3/2}} \\
&+ \frac{\sqrt{2-\sqrt{3}}(11bd+4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{27\ 3^{3/4}a^{8/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} \\
&+ \frac{2\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}(13bc+2af)+5\left(1-\sqrt{3}\right)\sqrt[3]{a}(11bd+4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{Elliptic}}{405\sqrt[4]{3}a^3b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}
$$

```
[Out] 2/15*x*(b*c-a*f+(-a*g+b*d)*x+b*e*x^2)/a/b/(b*x^3+a)^(5/2)-2/135*(9*a*e-x*(1
3*b*c+2*a*f+(4*a*g+11*b*d)*x))/a^2/b/(b*x^3+a)^(3/2)+2/405*x*(14*a*f+91*b*c
+5*(4*a*g+11*b*d)*x)/a^3/b/(b*x^3+a)^(1/2)-2/81*(4*a*g+11*b*d)*(b*x^3+a)^(1
/2)/a^3/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/81*(4*a*g+11*b*d)*(a^(1/3
)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(
```

```
1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)
/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)+2/1215*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*(2*
a*f+13*b*c)+5*a^(1/3)*(4*a*g+11*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)*3^(3/4)/a^3/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1872, 1868, 1869, 1892, 224, 1891}

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4ag+11bd)E\left(\arcsin\left(\frac{\sqrt[3]{b}\ldots}{\sqrt[3]{b}\ldots}\right.\right.}{27\ 3^{3/4}a^{8/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}\ldots}$$

$$-\frac{2\sqrt{a+bx^3}(4ag+11bd)}{81a^3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x(7(2af+13bc)+5x(4ag+11bd))}{405a^3b\sqrt{a+bx^3}}$$

$$-\frac{2(9ae-x(x(4ag+11bd)+2af+13bc))}{135a^2b\left(a+bx^3\right)^{3/2}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(2a\ldots\right.}{405\sqrt[4]{3}a^3b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x]
```

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(15*a*b*(a + b*x^3)^(5/2)) + (2
*x*(7*(13*b*c + 2*a*f) + 5*(11*b*d + 4*a*g)*x))/(405*a^3*b*Sqrt[a + b*x^3])
 - (2*(11*b*d + 4*a*g)*Sqrt[a + b*x^3])/(81*a^3*b^(5/3)*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) - (2*(9*a*e - x*(13*b*c + 2*a*f + (11*b*d + 4*a*g)*x)))/(
135*a^2*b*(a + b*x^3)^(3/2)) + (Sqrt[2 - Sqrt[3]]*(11*b*d + 4*a*g)*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
```

```
])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(3/4)*a^(
8/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*(13*b*c +
2*a*f) + 5*(1 - Sqrt[3])*a^(1/3)*(11*b*d + 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*a^3*b^(5/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x(bc-af+(bd-ag)x+bex^2)}{15ab\left(a+bx^3\right)^{5/2}}-\frac{2\int\frac{-\frac{1}{2}b(13bc+2af)-\frac{1}{2}b(11bd+4ag)x-\frac{9}{2}b^2ex^2}{(a+bx^3)^{5/2}}\,dx}{15ab^2}\\
&= \frac{2x(bc-af+(bd-ag)x+bex^2)}{15ab\left(a+bx^3\right)^{5/2}}-\frac{2(9ae-x(13bc+2af+(11bd+4ag)x))}{135a^2b\left(a+bx^3\right)^{3/2}}\\
&\quad+\frac{4\int\frac{\frac{7}{4}b(13bc+2af)+\frac{5}{4}b(11bd+4ag)x}{(a+bx^3)^{3/2}}\,dx}{135a^2b^2}\\
&= \frac{2x(bc-af+(bd-ag)x+bex^2)}{15ab\left(a+bx^3\right)^{5/2}}+\frac{2x(7(13bc+2af)+5(11bd+4ag)x)}{405a^3b\sqrt{a+bx^3}}\\
&\quad-\frac{2(9ae-x(13bc+2af+(11bd+4ag)x))}{135a^2b\left(a+bx^3\right)^{3/2}}-\frac{8\int\frac{-\frac{7}{8}b(13bc+2af)+\frac{5}{8}b(11bd+4ag)x}{\sqrt{a+bx^3}}\,dx}{405a^3b^2}\\
&= \frac{2x(bc-af+(bd-ag)x+bex^2)}{15ab\left(a+bx^3\right)^{5/2}}+\frac{2x(7(13bc+2af)+5(11bd+4ag)x)}{405a^3b\sqrt{a+bx^3}}\\
&\quad-\frac{2(9ae-x(13bc+2af+(11bd+4ag)x))}{135a^2b\left(a+bx^3\right)^{3/2}}-\frac{(11bd+4ag)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{81a^3b^{4/3}}\\
&\quad+\frac{\left(7\sqrt[3]{b}(13bc+2af)+5\left(1-\sqrt{3}\right)\sqrt[3]{a}(11bd+4ag)\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{405a^3b^{4/3}}
\end{aligned}
$$

$$= \frac{2x(bc-af+(bd-ag)x+bex^2)}{15ab\,(a+bx^3)^{5/2}} + \frac{2x(7(13bc+2af)+5(11bd+4ag)x)}{405a^3b\sqrt{a+bx^3}} - \frac{2(11bd+4ag)\sqrt{a+bx^3}}{81a^3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2(9ae-x(13bc+2af+(11bd+4ag)x))}{135a^2b\,(a+bx^3)^{3/2}}$$

$$+ \frac{\sqrt{2-\sqrt{3}}(11bd+4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-}{27\,3^{3/4}a^{8/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{2\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}(13bc+2af)+5\left(1-\sqrt{3}\right)\sqrt[3]{a}(11bd+4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right.}}}{405\sqrt[4]{3}a^3b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.25 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.29

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \frac{4004b^3cx^7+44ab^2x^4(221c+14fx^3)+44a^2bx(157c+34fx^3)-4a^3(297e}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x]
```

```
[Out] (4004*b^3*c*x^7 + 44*a*b^2*x^4*(221*c + 14*f*x^3) + 44*a^2*b*x*(157*c + 34*
f*x^3) - 4*a^3*(297*e + x*(77*f + 405*g*x)) + 154*(13*b*c + 2*a*f)*x*(a + b
*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
+ 405*(11*b*d + 4*a*g)*x^2*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[2/3, 7/2, 5/3, -((b*x^3)/a)])/(8910*a^3*b*(a + b*x^3)^(5/2))
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 921, normalized size of antiderivative = 1.36

method	result	size
elliptic	Expression too large to display	921
default	Expression too large to display	1793

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/15/a/b^4*(a*g-b*d)*x^2-2/15/a/b^4*(a*f-b*c)*x-2/15/b^4*e)*(b*x^3+a)^(1/
2)/(x^3+a/b)^3+(2/135/a^2/b^3*(4*a*g+11*b*d)*x^2+2/135/a^2/b^3*(2*a*f+13*b*
c)*x)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2*b*(-1/81*(4*a*g+11*b*d)/a^3/b^2*x^2-7/4
05*(2*a*f+13*b*c)/a^3/b^2*x)/((x^3+a/b)*b)^(1/2)-14/1215*I*(2*a*f+13*b*c)/a
^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(
b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/243*
I*(4*a*g+11*b*d)/a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a
*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.55

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \frac{2\left(7\left((13\,b^4c+2\,ab^3f)x^9+3\,(13\,ab^3c+2\,a^2b^2f)x^6+13\,a^3bc+2\,a^4f+3\,(\right.\right.}{}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/405*(7*((13*b^4*c + 2*a*b^3*f)*x^9 + 3*(13*a*b^3*c + 2*a^2*b^2*f)*x^6 + 1
3*a^3*b*c + 2*a^4*f + 3*(13*a^2*b^2*c + 2*a^3*b*f)*x^3)*sqrt(b)*weierstrass
PInverse(0, -4*a/b, x) + 5*((11*b^4*d + 4*a*b^3*g)*x^9 + 3*(11*a*b^3*d + 4*
a^2*b^2*g)*x^6 + 11*a^3*b*d + 4*a^4*g + 3*(11*a^2*b^2*d + 4*a^3*b*g)*x^3)*s
qrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (5*(
11*b^4*d + 4*a*b^3*g)*x^8 + 7*(13*b^4*c + 2*a*b^3*f)*x^7 + 13*(11*a*b^3*d +
 4*a^2*b^2*g)*x^5 - 27*a^3*b*e + 17*(13*a*b^3*c + 2*a^2*b^2*f)*x^4 + 5*(23*
a^2*b^2*d + a^3*b*g)*x^2 + (157*a^2*b^2*c - 7*a^3*b*f)*x)*sqrt(b*x^3 + a))/
(a^3*b^5*x^9 + 3*a^4*b^4*x^6 + 3*a^5*b^3*x^3 + a^6*b^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="maxima")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="giac")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \int \frac{g\,x^4+f\,x^3+e\,x^2+d\,x+c}{(b\,x^3+a)^{7/2}}\,dx$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x)
[Out] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2), x)
```

3.70 $\int \frac{(a+bx)^2}{c+dx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 186

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = -\frac{a\left(2b\sqrt[3]{c}+a\sqrt[3]{d}\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{2/3}} - \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}} + \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{2/3}} + \frac{b^2\log\left(c+dx^3\right)}{3d}$$

```
[Out] -1/3*a*(2*b*c^(1/3)-a*d^(1/3))*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/d^(2/3)+1/6*a*
(2*b*c^(1/3)-a*d^(1/3))*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/d
^(2/3)+1/3*b^2*ln(d*x^3+c)/d-1/3*a*(2*b*c^(1/3)+a*d^(1/3))*arctan(1/3*(c^(1
/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/d^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = -\frac{a\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{2/3}} + \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{2/3}} - \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}} + \frac{b^2\log\left(c+dx^3\right)}{3d}$$

```
[In] Int[(a + b*x)^2/(c + d*x^3),x]
```

```
[Out] -((a*(2*b*c^(1/3) + a*d^(1/3))*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1
/3))])/(Sqrt[3]*c^(2/3)*d^(2/3))) - (a*(2*b*c^(1/3) - a*d^(1/3))*Log[c^(1/3
) + d^(1/3)*x])/(3*c^(2/3)*d^(2/3)) + (a*(2*b*c^(1/3) - a*d^(1/3))*Log[c^(2
/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(2/3)) + (b^2*Log[c +
d*x^3])/(3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = b^2\int \frac{x^2}{c+dx^3}\,dx + \int \frac{a^2+2abx}{c+dx^3}\,dx$$

$$= \frac{b^2\log\left(c+dx^3\right)}{3d} + \frac{\int \frac{\sqrt[3]{c}\left(2ab\sqrt[3]{c}+2a^2\sqrt[3]{d}\right)+\left(2ab\sqrt[3]{c}-a^2\sqrt[3]{d}\right)\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{2/3}\sqrt[3]{d}} - \frac{\left(2ab\sqrt[3]{c}-a^2\sqrt[3]{d}\right)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}\sqrt[3]{d}}$$

$$= -\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}} + \frac{b^2\log\left(c+dx^3\right)}{3d} + \frac{1}{2}\left(a\left(\frac{a}{\sqrt[3]{c}}+\frac{2b}{\sqrt[3]{d}}\right)\right)\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx + \frac{\left(a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\right)\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{2/3}d^{2/3}}$$

$$= -\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}} + \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{2/3}} + \frac{b^2\log\left(c+dx^3\right)}{3d} + \frac{\left(a\left(2b\sqrt[3]{c}+a\sqrt[3]{d}\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{2/3}d^{2/3}}$$

$$= -\frac{a\left(2b\sqrt[3]{c}+a\sqrt[3]{d}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{2/3}} - \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}} + \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{2/3}} + \frac{b^2\log\left(c+dx^3\right)}{3d}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08

$$\int\frac{(a+bx)^2}{c+dx^3}\,dx = \frac{\left(2abc^{2/3}+a^2\sqrt[3]{c}\sqrt[3]{d}\right)\arctan\left(\frac{-\sqrt[3]{c}+2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}cd^{2/3}} + \frac{\left(-2abc^{2/3}+a^2\sqrt[3]{c}\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3cd^{2/3}} - \frac{\left(-2abc^{2/3}+a^2\sqrt[3]{c}\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6cd^{2/3}} + \frac{b^2\log\left(c+dx^3\right)}{3d}$$

```
[In] Integrate[(a + b*x)^2/(c + d*x^3),x]
```

```
[Out] ((2*a*b*c^(2/3) + a^2*c^(1/3)*d^(1/3))*ArcTan[(-c^(1/3) + 2*d^(1/3)*x)/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*c*d^(2/3)) + ((-2*a*b*c^(2/3) + a^2*c^(1/3)*d^(1/3
))*Log[c^(1/3) + d^(1/3)*x])/(3*c*d^(2/3)) - ((-2*a*b*c^(2/3) + a^2*c^(1/3)
*d^(1/3))*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c*d^(2/3)) + (
b^2*Log[c + d*x^3])/(3*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.65 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3d+c\right)}\frac{\left(\_R^2b^2+2\_Rab+a^2\right)\ln\left(x-\_R\right)}{\_R^2}}{3d}$
default	$a^2\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)+2ab\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right.}{6d\left(\frac{c}{d}\right.}\right.$

```
[In] int((b*x+a)^2/(d*x^3+c),x,method=_RETURNVERBOSE)
[Out] 1/3/d*sum((_R^2*b^2+2*_R*a*b+a^2)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 5014, normalized size of antiderivative = 26.96

$$\int\frac{(a+bx)^2}{c+dx^3}\,dx=\text{Too large to display}$$

```
[In] integrate((b*x+a)^2/(d*x^3+c),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.84

$$\int\frac{(a+bx)^2}{c+dx^3}\,dx$$
$$=\mathrm{RootSum}\left(27t^3c^2d^3-27t^2b^2c^2d^2+t\left(18a^3bcd^2+9b^4c^2d\right)-a^6d^2+2a^3b^3cd-b^6c^2,\left(t\mapsto t\log\left(x+\frac{18t^2bc}{}\right.\right.\right.$$

```
[In] integrate((b*x+a)**2/(d*x**3+c),x)
[Out] RootSum(27*_t**3*c**2*d**3 - 27*_t**2*b**2*c**2*d**2 + _t*(18*a**3*b*c*d**2
 + 9*b**4*c**2*d) - a**6*d**2 + 2*a**3*b**3*c*d - b**6*c**2, Lambda(_t, _t*
log(x + (18*_t**2*b*c**2*d**2 + 3*_t*a**3*c*d**2 - 12*_t*b**3*c**2*d + 7*a*
*3*b**2*c*d + 2*b**5*c**2)/(a**5*d**2 + 8*a**2*b**3*c*d))))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = -\frac{\sqrt{3}\left(2\,b^2c-\left(6\,ab\left(\frac{c}{d}\right)^{\frac{2}{3}}+3\,a^2\left(\frac{c}{d}\right)^{\frac{1}{3}}+\frac{2\,b^2c}{d}\right)d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd} + \frac{\left(2\,b^2\left(\frac{c}{d}\right)^{\frac{2}{3}}+2\,ab\left(\frac{c}{d}\right)^{\frac{1}{3}}-a^2\right)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{\left(b^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,ab\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^2\right)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

```
[In] integrate((b*x+a)^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*(2*b^2*c - (6*a*b*(c/d)^(2/3) + 3*a^2*(c/d)^(1/3) + 2*b^2*c/d)
*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d) + 1/6*(2*b^2*
(c/d)^(2/3) + 2*a*b*(c/d)^(1/3) - a^2)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3
))/(d*(c/d)^(2/3)) + 1/3*(b^2*(c/d)^(2/3) - 2*a*b*(c/d)^(1/3) + a^2)*log(x
+ (c/d)^(1/3))/(d*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = \frac{b^2\log\left(|dx^3+c|\right)}{3\,d} - \frac{\sqrt{3}\left(a^2d-2\left(-cd^2\right)^{\frac{1}{3}}ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}} - \frac{\left(a^2d+2\left(-cd^2\right)^{\frac{1}{3}}ab\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}} - \frac{\left(2\,abd\left(-\frac{c}{d}\right)^{\frac{1}{3}}+a^2d\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}$$

```
[In] integrate((b*x+a)^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^2*log(abs(d*x^3 + c))/d - 1/3*sqrt(3)*(a^2*d - 2*(-c*d^2)^(1/3)*a*b)*
arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(-c*d^2)^(2/3) - 1/6*
(a^2*d + 2*(-c*d^2)^(1/3)*a*b)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(-c
*d^2)^(2/3) - 1/3*(2*a*b*d*(-c/d)^(1/3) + a^2*d)*(-c/d)^(1/3)*log(abs(x - (
-c/d)^(1/3)))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.92

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = \sum_{k=1}^{3} \ln\Big(b^4\,c + \mathrm{root}\big(27\,c^2\,d^3\,z^3 - 27\,b^2\,c^2\,d^2\,z^2 + 18\,a^3\,b\,c\,d^2\,z + 9\,b^4\,c^2\,d\,z + 2\,a^3\,b^3\,c\,d - b^6\,c^2 - a^6\,d^2, z, k\big)^2\,c\,d^2\,9 + 2\,a^3\,b\,d - \mathrm{root}\big(27\,c^2\,d^3\,z^3 - 27\,b^2\,c^2\,d^2\,z^2 + 18\,a^3\,b\,c\,d^2\,z + 9\,b^4\,c^2\,d\,z + 2\,a^3\,b^3\,c\,d - b^6\,c^2 - a^6\,d^2, z, k\big)\,b^2\,c\,d\,6 + \mathrm{root}\big(27\,c^2\,d^3\,z^3 - 27\,b^2\,c^2\,d^2\,z^2 + 18\,a^3\,b\,c\,d^2\,z + 9\,b^4\,c^2\,d\,z + 2\,a^3\,b^3\,c\,d - b^6\,c^2 - a^6\,d^2, z, k\big)\,a^2\,d^2\,x\,3 + 3\,a^2\,b^2\,d\,x\Big)\,\mathrm{root}\big(27\,c^2\,d^3\,z^3 - 27\,b^2\,c^2\,d^2\,z^2 + 18\,a^3\,b\,c\,d^2\,z + 9\,b^4\,c^2\,d\,z + 2\,a^3\,b^3\,c\,d - b^6\,c^2 - a^6\,d^2, z, k\big)$$

```
[In] int((a + b*x)^2/(c + d*x^3),x)
```

```
[Out] symsum(log(b^4*c + 9*root(27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*
d^2*z + 9*b^4*c^2*d*z + 2*a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k)^2*c*d^2 +
2*a^3*b*d - 6*root(27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*d^2*z +
 9*b^4*c^2*d*z + 2*a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k)*b^2*c*d + 3*root(
27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*d^2*z + 9*b^4*c^2*d*z + 2*
a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k)*a^2*d^2*x + 3*a^2*b^2*d*x)*root(27*c
^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*d^2*z + 9*b^4*c^2*d*z + 2*a^3*
b^3*c*d - b^6*c^2 - a^6*d^2, z, k), k, 1, 3)
```

3.71 $\int \frac{(a+bx)^3}{c+dx^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 222

$$\begin{aligned}\int \frac{(a+bx)^3}{c+dx^3} dx = {} & \frac{b^3x}{d} + \frac{\left(b^3c - 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} \\ & - \frac{\left(b^3c + 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} \\ & + \frac{\left(b^3c + 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} \\ & + \frac{ab^2\log\left(c+dx^3\right)}{d}\end{aligned}$$

```
[Out] b^3*x/d-1/3*(b^3*c+3*a^2*b*c^(1/3)*d^(2/3)-a^3*d)*ln(c^(1/3)+d^(1/3)*x)/c^(
2/3)/d^(4/3)+1/6*(b^3*c+3*a^2*b*c^(1/3)*d^(2/3)-a^3*d)*ln(c^(2/3)-c^(1/3)*d
^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/d^(4/3)+a*b^2*ln(d*x^3+c)/d+1/3*(b^3*c-3*a^2*
b*c^(1/3)*d^(2/3)-a^3*d)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/
c^(2/3)/d^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used

$= \{1901, 1885, 1874, 31, 648, 631, 210, 642, 266\}$

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \frac{\left(a^3(-d)-3a^2b\sqrt[3]{c}d^{2/3}+b^3c\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} + \frac{\left(a^3(-d)+3a^2b\sqrt[3]{c}d^{2/3}+b^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} - \frac{\left(a^3(-d)+3a^2b\sqrt[3]{c}d^{2/3}+b^3c\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{ab^2\log\left(c+dx^3\right)}{d} + \frac{b^3x}{d}$$

```
[In] Int[(a + b*x)^3/(c + d*x^3),x]
```

```
[Out] (b^3*x)/d + ((b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*ArcTan[(c^(1/3) - 2*
d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(4/3)) - ((b^3*c + 3*a^2*
b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(4/3)) +
((b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x
+ d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3)) + (a*b^2*Log[c + d*x^3])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \left(\frac{b^3}{d} - \frac{b^3c - a^3d - 3a^2bdx - 3ab^2dx^2}{d\,(c+dx^3)}\right) dx$$

$$= \frac{b^3x}{d} - \frac{\int \frac{b^3c-a^3d-3a^2bdx-3ab^2dx^2}{c+dx^3}\,dx}{d}$$

$$= \frac{b^3x}{d} + \left(3ab^2\right)\int \frac{x^2}{c+dx^3}\,dx - \frac{\int \frac{b^3c-a^3d-3a^2bdx}{c+dx^3}\,dx}{d}$$

$$= \frac{b^3x}{d} + \frac{ab^2\log\left(c+dx^3\right)}{d} - \frac{\int \frac{\sqrt[3]{c}\left(-3a^2b\sqrt[3]{c}d+2\sqrt[3]{d}(b^3c-a^3d)\right)+\sqrt[3]{d}\left(-3a^2b\sqrt[3]{c}d-\sqrt[3]{d}(b^3c-a^3d)\right)x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{2/3}d^{4/3}} - \frac{\left(b^3c+3a^2b\sqrt[3]{c}d^{2/3}-a^3d\right)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}d}$$

$$= \frac{b^3x}{d} - \frac{\left(b^3c + 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\log\left(\sqrt[3]{c} + \sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{ab^2\log\left(c + dx^3\right)}{d} - \frac{\left(b^3c - 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\int \frac{1}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2}\,dx}{2\sqrt[3]{c}d} + \frac{\left(b^3c + 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\int \frac{-\sqrt[3]{c}\sqrt[3]{d} + 2d^{2/3}x}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2}\,dx}{6c^{2/3}d^{4/3}}$$

$$= \frac{b^3x}{d} - \frac{\left(b^3c + 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\log\left(\sqrt[3]{c} + \sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{\left(b^3c + 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} + \frac{ab^2\log\left(c + dx^3\right)}{d} - \frac{\left(b^3c - 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{2/3}d^{4/3}}$$

$$= \frac{b^3x}{d} + \frac{\left(b^3c - 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} - \frac{\left(b^3c + 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\log\left(\sqrt[3]{c} + \sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{\left(b^3c + 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} + \frac{ab^2\log\left(c + dx^3\right)}{d}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx$$

$$= \frac{6b^3c^{2/3}\sqrt[3]{d}x + 2\sqrt{3}\left(b^3c - 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\arctan\left(\frac{1 - \frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right) - 2\left(b^3c + 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\log\left(\sqrt[3]{c} + \right.}{6c^{2/3}d^{4/3}}$$

```
[In] Integrate[(a + b*x)^3/(c + d*x^3),x]
```

```
[Out] (6*b^3*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3
*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] - 2*(b^3*c + 3*a^2*b*c^(1/3
)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x] + (b^3*c + 3*a^2*b*c^(1/3)*d^(2
/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2] + 6*a*b^2*c^(2/
3)*d^(1/3)*Log[c + d*x^3])/(6*c^(2/3)*d^(4/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.66 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.30

method	result
risch	$\frac{b^{3} x}{d}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{3} d+c\right)} \frac{\left(3 \_R^{2} a \,b^{2} d+3 a^{2} b d \_R+a^{3} d-b^{3} c\right) \ln \left(x-\_R\right)}{\_R^{2}}}{3 d^{2}}$
default	$\frac{b^{3} x}{d}+\frac{\left(a^{3} d-b^{3} c\right)\left(\frac{\ln \left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3 d \left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln \left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}} x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6 d \left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 d \left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)+3 d \,a^{2} b \left(-\frac{\ln \left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3 d \left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln \left(x^{2}-\left(\frac{c}{d}\right.\right.}{6 d (}\right.}{d}$

[In] int((b*x+a)^3/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] b^3*x/d+1/3/d^2*sum((3*_R^2*a*b^2*d+3*_R*a^2*b*d+a^3*d-b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 7245, normalized size of antiderivative = 32.64

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \text{Too large to display}$$

[In] integrate((b*x+a)^3/(d*x^3+c),x, algorithm="fricas")

[Out] Too large to include

Sympy [A] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \frac{b^3x}{d} + \text{RootSum}\left(27t^3c^2d^4 - 81t^2ab^2c^2d^3 + t\left(27a^5bcd^3 + 54a^2b^4c^2d^2\right) - a^9d^3 + 3a^6b^3cd^2 - 3a^3b^6c^2d + b^9c^3, \left(t\right.\right.$$

```
[In] integrate((b*x+a)**3/(d*x**3+c),x)
```

```
[Out] b**3*x/d + RootSum(27*_t**3*c**2*d**4 - 81*_t**2*a*b**2*c**2*d**3 + _t*(27*
a**5*b*c*d**3 + 54*a**2*b**4*c**2*d**2) - a**9*d**3 + 3*a**6*b**3*c*d**2 -
3*a**3*b**6*c**2*d + b**9*c**3, Lambda(_t, _t*log(x + (27*_t**2*a**2*b*c**2
*d**3 + 3*_t*a**6*c*d**3 - 60*_t*a**3*b**3*c**2*d**2 + 3*_t*b**6*c**3*d + 1
5*a**7*b**2*c*d**2 + 15*a**4*b**5*c**2*d - 3*a*b**8*c**3)/(a**9*d**3 + 24*a
**6*b**3*c*d**2 + 3*a**3*b**6*c**2*d - b**9*c**3))))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{(a+bx)^3}{c+dx^3}\,dx &= \frac{b^3x}{d} - \frac{\sqrt{3}\left(\left(b^3\left(\frac{c}{d}\right)^{\frac{1}{3}} + 2\,ab^2\right)c - \left(3\,a^2b\left(\frac{c}{d}\right)^{\frac{2}{3}} + a^3\left(\frac{c}{d}\right)^{\frac{1}{3}} + \frac{2\,ab^2c}{d}\right)d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd} \\ &\quad + \frac{\left(b^3c + \left(6\,ab^2\left(\frac{c}{d}\right)^{\frac{2}{3}} + 3\,a^2b\left(\frac{c}{d}\right)^{\frac{1}{3}} - a^3\right)d\right)\log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} \\ &\quad - \frac{\left(b^3c - \left(3\,ab^2\left(\frac{c}{d}\right)^{\frac{2}{3}} - 3\,a^2b\left(\frac{c}{d}\right)^{\frac{1}{3}} + a^3\right)d\right)\log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate((b*x+a)^3/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] b^3*x/d - 1/3*sqrt(3)*((b^3*(c/d)^(1/3) + 2*a*b^2)*c - (3*a^2*b*(c/d)^(2/3)
 + a^3*(c/d)^(1/3) + 2*a*b^2*c/d)*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))
/(c/d)^(1/3))/(c*d) + 1/6*(b^3*c + (6*a*b^2*(c/d)^(2/3) + 3*a^2*b*(c/d)^(1/
3) - a^3)*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(d^2*(c/d)^(2/3)) - 1/3
*(b^3*c - (3*a*b^2*(c/d)^(2/3) - 3*a^2*b*(c/d)^(1/3) + a^3)*d)*log(x + (c/d
)^(1/3))/(d^2*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \frac{b^3x}{d} + \frac{ab^2\log\left(|dx^3+c|\right)}{d} + \frac{\sqrt{3}\left(b^3c - a^3d + 3\left(-cd^2\right)^{\frac{1}{3}}a^2b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}} + \frac{\left(b^3c - a^3d - 3\left(-cd^2\right)^{\frac{1}{3}}a^2b\right)\log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}} - \frac{\left(3\,a^2bd^3\left(-\frac{c}{d}\right)^{\frac{1}{3}} - b^3cd^2 + a^3d^3\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd^3}$$

```
[In] integrate((b*x+a)^3/(d*x^3+c),x, algorithm="giac")
```

```
[Out] b^3*x/d + a*b^2*log(abs(d*x^3 + c))/d + 1/3*sqrt(3)*(b^3*c - a^3*d + 3*(-c*
d^2)^(1/3)*a^2*b)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(-c
*d^2)^(2/3) + 1/6*(b^3*c - a^3*d - 3*(-c*d^2)^(1/3)*a^2*b)*log(x^2 + x*(-c/
d)^(1/3) + (-c/d)^(2/3))/(-c*d^2)^(2/3) - 1/3*(3*a^2*b*d^3*(-c/d)^(1/3) - b
^3*c*d^2 + a^3*d^3)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d^3)
```

Mupad [B] (verification not implemented)

Time = 10.21 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \left(\sum_{k=1}^{3} \ln\left(\mathrm{root}\left(27\,c^2\,d^4\,z^3 - 81\,a\,b^2\,c^2\,d^3\,z^2 + 54\,a^2\,b^4\,c^2\,d^2\,z + 27\,a^5\,b\,c\,d^3\,z + 3\,a^6\,b^3\,c\,d^2 - 3\,a^3\,b^6\,c^2\,d + b^9\,c^3 - a^9 \ldots + x\left(6\,d\,a^4\,b^2 + 3\,c\,a\,b^5\right) + 6\,a^2\,b^4\,c + 3\,a^5\,b\,d\right)\,\mathrm{root}\left(27\,c^2\,d^4\,z^3 - 81\,a\,b^2\,c^2\,d^3\,z^2 + 54\,a^2\,b^4\,c^2\,d^2\,z + 27\,a^5\,b\,c\,d^3\,z + 3\,a^6\,b^3\,c\,d^2 - 3\,a^3\,b^6\,c^2\,d + b^9\,c^3 - a^9\,d^3, z, k\right)\right) + \frac{b^3\,x}{d}$$

```
[In] int((a + b*x)^3/(c + d*x^3),x)
```

```
[Out] symsum(log(root(27*c^2*d^4*z^3 - 81*a*b^2*c^2*d^3*z^2 + 54*a^2*b^4*c^2*d^2*
z + 27*a^5*b*c*d^3*z + 3*a^6*b^3*c*d^2 - 3*a^3*b^6*c^2*d + b^9*c^3 - a^9*d^
3, z, k)*(x*(3*a^3*d^2 - 3*b^3*c*d) + 9*root(27*c^2*d^4*z^3 - 81*a*b^2*c^2*
```

```
d^3*z^2 + 54*a^2*b^4*c^2*d^2*z + 27*a^5*b*c*d^3*z + 3*a^6*b^3*c*d^2 - 3*a^3
*b^6*c^2*d + b^9*c^3 - a^9*d^3, z, k)*c*d^2 - 18*a*b^2*c*d) + x*(6*a^4*b^2*
d + 3*a*b^5*c) + 6*a^2*b^4*c + 3*a^5*b*d)*root(27*c^2*d^4*z^3 - 81*a*b^2*c^
2*d^3*z^2 + 54*a^2*b^4*c^2*d^2*z + 27*a^5*b*c*d^3*z + 3*a^6*b^3*c*d^2 - 3*a
^3*b^6*c^2*d + b^9*c^3 - a^9*d^3, z, k), k, 1, 3) + (b^3*x)/d
```

3.72 $\int \frac{(a+bx)^4}{c+dx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 282

$$\begin{aligned}\int \frac{(a+bx)^4}{c+dx^3}\,dx =\ & \frac{4ab^3x}{d}+\frac{b^4x^2}{2d} \\ & +\frac{\left(b^4c^{4/3}+4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}d-a^4d^{4/3}\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{5/3}} \\ & +\frac{\left(b\sqrt[3]{c}(b^3c-4a^3d)-\sqrt[3]{d}(4ab^3c-a^4d)\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{5/3}} \\ & -\frac{\left(b\sqrt[3]{c}(b^3c-4a^3d)-\sqrt[3]{d}(4ab^3c-a^4d)\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{5/3}} \\ & +\frac{2a^2b^2\log(c+dx^3)}{d}\end{aligned}$$

```
[Out] 4*a*b^3*x/d+1/2*b^4*x^2/d+1/3*(b*c^(1/3)*(-4*a^3*d+b^3*c)-d^(1/3)*(-a^4*d+4
*a*b^3*c))*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/d^(5/3)-1/6*(b*c^(1/3)*(-4*a^3*d+b
^3*c)-d^(1/3)*(-a^4*d+4*a*b^3*c))*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)
/c^(2/3)/d^(5/3)+2*a^2*b^2*ln(d*x^3+c)/d+1/3*(b^4*c^(4/3)+4*a*b^3*c*d^(1/3)
-4*a^3*b*c^(1/3)*d-a^4*d^(4/3))*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^
(1/2))/c^(2/3)/d^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx = \frac{2a^2b^2\log(c+dx^3)}{d} + \frac{\left(a^4\left(-d^{4/3}\right)-4a^3b\sqrt[3]{c}d+4ab^3c\sqrt[3]{d}+b^4c^{4/3}\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{5/3}} + \frac{\left(a^4(-d)-\frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}}+4ab^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} + \frac{\left(b\sqrt[3]{c}(b^3c-4a^3d)-\sqrt[3]{d}(4ab^3c-a^4d)\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{5/3}} + \frac{4ab^3x}{d} + \frac{b^4x^2}{2d}$$

```
[In] Int[(a + b*x)^4/(c + d*x^3),x]
```

```
[Out] (4*a*b^3*x)/d + (b^4*x^2)/(2*d) + ((b^4*c^(4/3) + 4*a*b^3*c*d^(1/3) - 4*a^3
*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3)
)])/(Sqrt[3]*c^(2/3)*d^(5/3)) + ((b*c^(1/3)*(b^3*c - 4*a^3*d) - d^(1/3)*(4*
a*b^3*c - a^4*d))*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(5/3)) + ((4*a*b^3
*c - a^4*d - (b*c^(1/3)*(b^3*c - 4*a^3*d))/d^(1/3))*Log[c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3)) + (2*a^2*b^2*Log[c + d*x^3])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{4ab^3}{d}+\frac{b^4x}{d}-\frac{4ab^3c-a^4d+b\left(b^3c-4a^3d\right)x-6a^2b^2dx^2}{d\left(c+dx^3\right)}\right)dx\\
&= \frac{4ab^3x}{d}+\frac{b^4x^2}{2d}-\frac{\int \frac{4ab^3c-a^4d+b\left(b^3c-4a^3d\right)x-6a^2b^2dx^2}{c+dx^3}\,dx}{d}\\
&= \frac{4ab^3x}{d}+\frac{b^4x^2}{2d}+\left(6a^2b^2\right)\int \frac{x^2}{c+dx^3}\,dx-\frac{\int \frac{4ab^3c-a^4d+b\left(b^3c-4a^3d\right)x}{c+dx^3}\,dx}{d}
\end{aligned}$$

$$= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} + \frac{2a^2b^2\log(c+dx^3)}{d} - \frac{\int \frac{\sqrt[3]{c}\left(b\sqrt[3]{c}(b^3c-4a^3d)+2\sqrt[3]{d}(4ab^3c-a^4d)\right)+\sqrt[3]{d}\left(b\sqrt[3]{c}(b^3c-4a^3d)-\sqrt[3]{d}(4ab^3c-a^4d)\right)x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}dx}{3c^{2/3}d^{4/3}} - \frac{\left(4ab^3c-a^4d-\frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}}\right)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}dx}{3c^{2/3}d}$$

$$= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} - \frac{\left(4ab^3c-a^4d-\frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{2a^2b^2\log(c+dx^3)}{d} - \frac{\left(b^4c^{4/3}+4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}d-a^4d^{4/3}\right)\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}dx}{2\sqrt[3]{c}d^{4/3}} + \frac{\left(4ab^3c-a^4d-\frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}}\right)\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}dx}{6c^{2/3}d^{4/3}}$$

$$= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} - \frac{\left(4ab^3c-a^4d-\frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{\left(4ab^3c-a^4d-\frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} + \frac{2a^2b^2\log(c+dx^3)}{d} - \frac{\left(b^4c^{4/3}+4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}d-a^4d^{4/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{2/3}d^{5/3}}$$

$$= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} + \frac{\left(b^4c^{4/3}+4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}d-a^4d^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{5/3}} - \frac{\left(4ab^3c-a^4d-\frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{\left(4ab^3c-a^4d-\frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} + \frac{2a^2b^2\log(c+dx^3)}{d}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx$$

$$= \frac{24ab^3d^{2/3}x+3b^4d^{2/3}x^2+\frac{2\sqrt{3}\left(b^4c^{4/3}+4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}d-a^4d^{4/3}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{2/3}}+\frac{2\left(b^4c^{4/3}-4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}d+\right.}{c^{2/3}}}{6d}$$

```
[In] Integrate[(a + b*x)^4/(c + d*x^3),x]
```

```
[Out] (24*a*b^3*d^(2/3)*x + 3*b^4*d^(2/3)*x^2 + (2*Sqrt[3]*(b^4*c^(4/3) + 4*a*b^3
*c*d^(1/3) - 4*a^3*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(1 - (2*d^(1/3)*x)/c^(
1/3))/Sqrt[3]])/c^(2/3) + (2*(b^4*c^(4/3) - 4*a*b^3*c*d^(1/3) - 4*a^3*b*c^(
1/3)*d + a^4*d^(4/3))*Log[c^(1/3) + d^(1/3)*x])/c^(2/3) - ((b^4*c^(4/3) - 4
*a*b^3*c*d^(1/3) - 4*a^3*b*c^(1/3)*d + a^4*d^(4/3))*Log[c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2])/c^(2/3) + 12*a^2*b^2*d^(2/3)*Log[c + d*x^3])/(6*d^
(5/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.32

method	result
risch	$\frac{b^4x^2}{2d}+\frac{4ab^3x}{d}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^3d+c)}\frac{\left(6a^2b^2d\_R^2+b\left(4a^3d-b^3c\right)\_R+a^4d-4ab^3c\right)\ln\left(x-\_R\right)}{\_R^2}}{3d^2}$
default	$\frac{b^3\left(\frac{1}{2}bx^2+4ax\right)}{d}+\frac{(a^4d-4ab^3c)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)+(4da^3b-b^4c)\left(-\frac{\ln\left(x\right.}{3}\right.}{d}$

```
[In] int((b*x+a)^4/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^4*x^2/d+4*a*b^3*x/d+1/3/d^2*sum((6*a^2*b^2*d*_R^2+b*(4*a^3*d-b^3*c)*_
R+a^4*d-4*a*b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.54 (sec) , antiderivative size = 8787, normalized size of antiderivative = 31.16

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^4/(d*x^3+c),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**4/(d*x**3+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{(a+bx)^4}{c+dx^3}\,dx =& -\frac{\sqrt{3}\left(\left(b^4\left(\frac{c}{d}\right)^{\frac{2}{3}}+4\,ab^3\left(\frac{c}{d}\right)^{\frac{1}{3}}+4\,a^2b^2\right)c-\left(4\,a^3b\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^4\left(\frac{c}{d}\right)^{\frac{1}{3}}+\frac{4\,a^2b^2c}{d}\right)d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd}\\&+\frac{b^4x^2+8\,ab^3x}{2\,d}\\&-\frac{\left(\left(b^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-4\,ab^3\right)c-\left(12\,a^2b^2\left(\frac{c}{d}\right)^{\frac{2}{3}}+4\,a^3b\left(\frac{c}{d}\right)^{\frac{1}{3}}-a^4\right)d\right)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}\\&+\frac{\left(\left(b^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-4\,ab^3\right)c+\left(6\,a^2b^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-4\,a^3b\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^4\right)d\right)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate((b*x+a)^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*((b^4*(c/d)^(2/3) + 4*a*b^3*(c/d)^(1/3) + 4*a^2*b^2)*c - (4*a^
3*b*(c/d)^(2/3) + a^4*(c/d)^(1/3) + 4*a^2*b^2*c/d)*d)*arctan(1/3*sqrt(3)*(2
*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d) + 1/2*(b^4*x^2 + 8*a*b^3*x)/d - 1/6*((
b^4*(c/d)^(1/3) - 4*a*b^3)*c - (12*a^2*b^2*(c/d)^(2/3) + 4*a^3*b*(c/d)^(1/3
) - a^4)*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(d^2*(c/d)^(2/3)) + 1/3*
((b^4*(c/d)^(1/3) - 4*a*b^3)*c + (6*a^2*b^2*(c/d)^(2/3) - 4*a^3*b*(c/d)^(1/
3) + a^4)*d)*log(x + (c/d)^(1/3))/(d^2*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx = \frac{2\,a^2b^2\log\left(|dx^3+c|\right)}{d} + \frac{\sqrt{3}\left(4\,ab^3cd - a^4d^2 - \left(-cd^2\right)^{\frac{1}{3}}b^4c + 4\left(-cd^2\right)^{\frac{1}{3}}a^3bd\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}d} + \frac{\left(4\,ab^3cd - a^4d^2 + \left(-cd^2\right)^{\frac{1}{3}}b^4c - 4\left(-cd^2\right)^{\frac{1}{3}}a^3bd\right)\log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}d} + \frac{b^4dx^2 + 8\,ab^3dx}{2\,d^2} + \frac{\left(b^4cd^4\left(-\frac{c}{d}\right)^{\frac{1}{3}} - 4\,a^3bd^5\left(-\frac{c}{d}\right)^{\frac{1}{3}} + 4\,ab^3cd^4 - a^4d^5\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd^5}$$

```
[In] integrate((b*x+a)^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 2*a^2*b^2*log(abs(d*x^3 + c))/d + 1/3*sqrt(3)*(4*a*b^3*c*d - a^4*d^2 - (-c*
d^2)^(1/3)*b^4*c + 4*(-c*d^2)^(1/3)*a^3*b*d)*arctan(1/3*sqrt(3)*(2*x + (-c/
d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*d) + 1/6*(4*a*b^3*c*d - a^4*d^2 + (
-c*d^2)^(1/3)*b^4*c - 4*(-c*d^2)^(1/3)*a^3*b*d)*log(x^2 + x*(-c/d)^(1/3) +
(-c/d)^(2/3))/((-c*d^2)^(2/3)*d) + 1/2*(b^4*d*x^2 + 8*a*b^3*d*x)/d^2 + 1/3*
(b^4*c*d^4*(-c/d)^(1/3) - 4*a^3*b*d^5*(-c/d)^(1/3) + 4*a*b^3*c*d^4 - a^4*d^
5)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d^5)
```

Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx$$

$$= \left(\sum_{k=1}^{3} \ln\left(\mathrm{root}\left(27\,c^2\,d^5\,z^3 - 162\,a^2\,b^2\,c^2\,d^4\,z^2 + 171\,a^4\,b^4\,c^2\,d^3\,z + 36\,a\,b^7\,c^3\,d^2\,z + 36\,a^7\,b\,c\,d^4\,z - 6\,a^6\,b^6\,c^2\,d^2 + 4\,a\right.\right.\right.$$

$$+ \frac{4\,a^7\,b\,d^2 + 19\,a^4\,b^4\,c\,d + 4\,a\,b^7\,c^2}{d} + \frac{x\,(10\,a^6\,b^2\,d^2 + 16\,a^3\,b^5\,c\,d + b^8\,c^2)}{d}\Big)\,\mathrm{root}\big(27\,c^2\,d^5\,z^3$$

$$- 162\,a^2\,b^2\,c^2\,d^4\,z^2 + 171\,a^4\,b^4\,c^2\,d^3\,z + 36\,a\,b^7\,c^3\,d^2\,z + 36\,a^7\,b\,c\,d^4\,z - 6\,a^6\,b^6\,c^2\,d^2$$

$$\left. + 4\,a^9\,b^3\,c\,d^3 + 4\,a^3\,b^9\,c^3\,d - b^{12}\,c^4 - a^{12}\,d^4, z, k\big)\right) + \frac{b^4\,x^2}{2\,d} + \frac{4\,a\,b^3\,x}{d}$$

```
[In] int((a + b*x)^4/(c + d*x^3),x)
```

```
[Out] symsum(log(root(27*c^2*d^5*z^3 - 162*a^2*b^2*c^2*d^4*z^2 + 171*a^4*b^4*c^2*
d^3*z + 36*a*b^7*c^3*d^2*z + 36*a^7*b*c*d^4*z - 6*a^6*b^6*c^2*d^2 + 4*a^9*b
^3*c*d^3 + 4*a^3*b^9*c^3*d - b^12*c^4 - a^12*d^4, z, k)*((x*(3*a^4*d^3 - 12
*a*b^3*c*d^2))/d + 9*root(27*c^2*d^5*z^3 - 162*a^2*b^2*c^2*d^4*z^2 + 171*a^
4*b^4*c^2*d^3*z + 36*a*b^7*c^3*d^2*z + 36*a^7*b*c*d^4*z - 6*a^6*b^6*c^2*d^2
 + 4*a^9*b^3*c*d^3 + 4*a^3*b^9*c^3*d - b^12*c^4 - a^12*d^4, z, k)*c*d^2 - 3
6*a^2*b^2*c*d) + (4*a*b^7*c^2 + 4*a^7*b*d^2 + 19*a^4*b^4*c*d)/d + (x*(b^8*c
^2 + 10*a^6*b^2*d^2 + 16*a^3*b^5*c*d))/d)*root(27*c^2*d^5*z^3 - 162*a^2*b^2
*c^2*d^4*z^2 + 171*a^4*b^4*c^2*d^3*z + 36*a*b^7*c^3*d^2*z + 36*a^7*b*c*d^4*
z - 6*a^6*b^6*c^2*d^2 + 4*a^9*b^3*c*d^3 + 4*a^3*b^9*c^3*d - b^12*c^4 - a^12
*d^4, z, k), k, 1, 3) + (b^4*x^2)/(2*d) + (4*a*b^3*x)/d
```

3.73 $\int \frac{(a+bx+cx^2)^2}{d+ex^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 272

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{d+ex^3} dx &= \frac{2bcx}{e}+\frac{c^2x^2}{2e}+\frac{\left(c^2d^{4/3}+2bcd\sqrt[3]{e}-a\left(2b\sqrt[3]{d}+a\sqrt[3]{e}\right)e\right)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{5/3}} \\ &\quad -\frac{\left(\sqrt[3]{e}(2bcd-a^2e)-\sqrt[3]{d}(c^2d-2abe)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{5/3}} \\ &\quad +\frac{\left(\sqrt[3]{e}(2bcd-a^2e)-\sqrt[3]{d}(c^2d-2abe)\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{5/3}} \\ &\quad +\frac{(b^2+2ac)\log(d+ex^3)}{3e}\end{aligned}$$

```
[Out] 2*b*c*x/e+1/2*c^2*x^2/e-1/3*(e^(1/3)*(-a^2*e+2*b*c*d)-d^(1/3)*(-2*a*b*e+c^2
*d))*ln(d^(1/3)+e^(1/3)*x)/d^(2/3)/e^(5/3)+1/6*(e^(1/3)*(-a^2*e+2*b*c*d)-d^
(1/3)*(-2*a*b*e+c^2*d))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/e
^(5/3)+1/3*(2*a*c+b^2)*ln(e*x^3+d)/e+1/3*(c^2*d^(4/3)+2*b*c*d*e^(1/3)-a*(2*
b*d^(1/3)+a*e^(1/3))*e)*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d
^(2/3)/e^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(a^2(-e)-\frac{\sqrt[3]{d}(c^2d-2abe)}{\sqrt[3]{e}}+2bcd\right)}{6d^{2/3}e^{4/3}} - \frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(\sqrt[3]{e}(2bcd-a^2e)-\sqrt[3]{d}(c^2d-2abe)\right)}{3d^{2/3}e^{5/3}} + \frac{\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(-ae\left(a\sqrt[3]{e}+2b\sqrt[3]{d}\right)+2bcd\sqrt[3]{e}+c^2d^{4/3}\right)}{\sqrt{3}d^{2/3}e^{5/3}} + \frac{(2ac+b^2)\log\left(d+ex^3\right)}{3e} + \frac{2bcx}{e} + \frac{c^2x^2}{2e}$$

```
[In] Int[(a + b*x + c*x^2)^2/(d + e*x^3),x]
```

```
[Out] (2*b*c*x)/e + (c^2*x^2)/(2*e) + ((c^2*d^(4/3) + 2*b*c*d*e^(1/3) - a*(2*b*d^
(1/3) + a*e^(1/3))*e)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(S
qrt[3]*d^(2/3)*e^(5/3)) - ((e^(1/3)*(2*b*c*d - a^2*e) - d^(1/3)*(c^2*d - 2*
a*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(5/3)) + ((2*b*c*d - a^2*e -
 (d^(1/3)*(c^2*d - 2*a*b*e))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(
2/3)*x^2])/(6*d^(2/3)*e^(4/3)) + ((b^2 + 2*a*c)*Log[d + e*x^3])/(3*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2bc}{e}+\frac{c^2x}{e}-\frac{2bcd-a^2e+(c^2d-2abe)\,x-(b^2+2ac)\,ex^2}{e\,(d+ex^3)}\right)dx\\
&= \frac{2bcx}{e}+\frac{c^2x^2}{2e}-\frac{\int \frac{2bcd-a^2e+(c^2d-2abe)x-(b^2+2ac)ex^2}{d+ex^3}\,dx}{e}\\
&= \frac{2bcx}{e}+\frac{c^2x^2}{2e}-\left(-b^2-2ac\right)\int \frac{x^2}{d+ex^3}\,dx-\frac{\int \frac{2bcd-a^2e+(c^2d-2abe)x}{d+ex^3}\,dx}{e}
\end{aligned}$$

$$= \frac{2bcx}{e} + \frac{c^2x^2}{2e} + \frac{(b^2+2ac)\log(d+ex^3)}{3e} - \frac{\int \frac{\sqrt[3]{d}\left(2\sqrt[3]{e}(2bcd-a^2e)+\sqrt[3]{d}(c^2d-2abe)\right)+\sqrt[3]{e}\left(-\sqrt[3]{e}(2bcd-a^2e)+\sqrt[3]{d}(c^2d-2abe)\right)x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}dx}{3d^{2/3}e^{4/3}} - \frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}(c^2d-2abe)}{\sqrt[3]{e}}\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}dx}{3d^{2/3}e}$$

$$= \frac{2bcx}{e} + \frac{c^2x^2}{2e} - \frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}(c^2d-2abe)}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{4/3}} + \frac{(b^2+2ac)\log(d+ex^3)}{3e} - \frac{\left(c^2d^{4/3}+2bcd\sqrt[3]{e}-a\left(2b\sqrt[3]{d}+a\sqrt[3]{e}\right)e\right)\int \frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}dx}{2\sqrt[3]{d}e^{4/3}} + \frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}(c^2d-2abe)}{\sqrt[3]{e}}\right)\int \frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}dx}{6d^{2/3}e^{4/3}}$$

$$= \frac{2bcx}{e} + \frac{c^2x^2}{2e} - \frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}(c^2d-2abe)}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{4/3}} + \frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}(c^2d-2abe)}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{4/3}} + \frac{(b^2+2ac)\log(d+ex^3)}{3e} - \frac{\left(c^2d^{4/3}+2bcd\sqrt[3]{e}-a\left(2b\sqrt[3]{d}+a\sqrt[3]{e}\right)e\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{d^{2/3}e^{5/3}}$$

$$= \frac{2bcx}{e} + \frac{c^2x^2}{2e} + \frac{\left(c^2d^{4/3}+2bcd\sqrt[3]{e}-a\left(2b\sqrt[3]{d}+a\sqrt[3]{e}\right)e\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{5/3}} - \frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}(c^2d-2abe)}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{4/3}} + \frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}(c^2d-2abe)}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{4/3}} + \frac{(b^2+2ac)\log(d+ex^3)}{3e}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx$$

$$= \frac{12bce^{2/3}x+3c^2e^{2/3}x^2+\frac{2\sqrt{3}(cd^{2/3}-ae^{2/3})\left(cd^{2/3}+2b\sqrt[3]{d}\sqrt[3]{e}+ae^{2/3}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{2/3}}+\frac{2\left(c^2d^{4/3}-2bcd\sqrt[3]{e}+a\left(-2b\sqrt[3]{d}+\right.\right.}{d^{2/3}}}{}$$

[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x^3),x]

[Out] (12*b*c*e^(2/3)*x + 3*c^2*e^(2/3)*x^2 + (2*Sqrt[3]*(c*d^(2/3) - a*e^(2/3))*(c*d^(2/3) + 2*b*d^(1/3)*e^(1/3) + a*e^(2/3))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (2*(c^2*d^(4/3) - 2*b*c*d*e^(1/3) + a*(-2*b*d^(1/3) + a*e^(1/3))*e)*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - ((c^2*d^(4/3) - 2*b*c*d*e^(1/3) + a*(-2*b*d^(1/3) + a*e^(1/3))*e)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3) + 2*(b^2 + 2*a*c)*e^(2/3)*Log[d + e*x^3])/(6*e^(5/3))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.74 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.31

method	result
risch	$\frac{c^2x^2}{2e}+\frac{2bcx}{e}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3e+d\right)}\frac{\left(e\left(2ac+b^2\right)\_R^2+\left(2aeb-c^2d\right)\_R+a^2e-2bcd\right)\ln\left(x-\_R\right)}{\_R^2}}{3e^2}$
default	$\frac{c\left(\frac{1}{2}cx^2+2bx\right)}{e}+\frac{(a^2e-2bcd)\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)+(2aeb-c^2d)\left(-\frac{\ln\left(x+\left(\frac{d}{e}\right.\right.}{3e\left(\frac{d}{e}\right)}\right.}{e}$

[In] int((c*x^2+b*x+a)^2/(e*x^3+d),x,method=_RETURNVERBOSE)

```
[Out] 1/2*c^2*x^2/e+2*b*c*x/e+1/3/e^2*sum((e*(2*a*c+b^2)*_R^2+(2*a*b*e-c^2*d)*_R+
a^2*e-2*b*c*d)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e+d))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.43 (sec) , antiderivative size = 12827, normalized size of antiderivative = 47.16

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**2/(e*x**3+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \frac{(b^2+2\,ac)\log\left(|ex^3+d|\right)}{3\,e} + \frac{\sqrt{3}\left(2\,bcde-a^2e^2-(-de^2)^{\frac{1}{3}}\,c^2d+2\,(-de^2)^{\frac{1}{3}}\,abe\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\,(-de^2)^{\frac{2}{3}}\,e} + \frac{\left(2\,bcde-a^2e^2+(-de^2)^{\frac{1}{3}}\,c^2d-2\,(-de^2)^{\frac{1}{3}}\,abe\right)\log\left(x^2+x\left(-\frac{d}{e}\right)^{\frac{1}{3}}+\left(-\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,(-de^2)^{\frac{2}{3}}\,e} + \frac{c^2ex^2+4\,bcex}{2\,e^2} + \frac{\left(c^2de^4\left(-\frac{d}{e}\right)^{\frac{1}{3}}-2\,abe^5\left(-\frac{d}{e}\right)^{\frac{1}{3}}+2\,bcde^4-a^2e^5\right)\left(-\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,de^5}$$

```
[In] integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="giac")
```

```
[Out] 1/3*(b^2 + 2*a*c)*log(abs(e*x^3 + d))/e + 1/3*sqrt(3)*(2*b*c*d*e - a^2*e^2
- (-d*e^2)^(1/3)*c^2*d + 2*(-d*e^2)^(1/3)*a*b*e)*arctan(1/3*sqrt(3)*(2*x +
(-d/e)^(1/3))/(-d/e)^(1/3))/((-d*e^2)^(2/3)*e) + 1/6*(2*b*c*d*e - a^2*e^2 +
 (-d*e^2)^(1/3)*c^2*d - 2*(-d*e^2)^(1/3)*a*b*e)*log(x^2 + x*(-d/e)^(1/3) +
(-d/e)^(2/3))/((-d*e^2)^(2/3)*e) + 1/2*(c^2*e*x^2 + 4*b*c*e*x)/e^2 + 1/3*(c
^2*d*e^4*(-d/e)^(1/3) - 2*a*b*e^5*(-d/e)^(1/3) + 2*b*c*d*e^4 - a^2*e^5)*(-d
/e)^(1/3)*log(abs(x - (-d/e)^(1/3)))/(d*e^5)
```

Mupad [B] (verification not implemented)

Time = 10.38 (sec) , antiderivative size = 769, normalized size of antiderivative = 2.83

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \left(\sum_{k=1}^{3}\ln\left(\frac{2\,a^3\,b\,e^2+3\,a^2\,c^2\,d\,e+b^4\,d\,e+2\,b\,c^3\,d^2}{e} + \frac{x\,(-2\,a^3\,c\,e^2+3\,a^2\,b^2\,e^2+2\,b^3\,c\,d\,e+c^4\,d^2)}{e} - \mathrm{root}\big(27\,d^2\,e^5\,z^3-54\,a\,c\,d^2\,e^4\,z^2-27\,b^2\,d^2\,e^4\,z^2+27\,a^2\,c^2\,d^2\,e^3\,z+18\,b\,c^3\,d^3\,e^2\,z+18\,a^3\,b\,d\,e^4\,z+9\,b^4\,d^2\,e^3\,z+ \right.\right.$$

$$- 54\,a\,c\,d^2\,e^4\,z^2 - 27\,b^2\,d^2\,e^4\,z^2 + 27\,a^2\,c^2\,d^2\,e^3\,z + 18\,b\,c^3\,d^3\,e^2\,z + 18\,a^3\,b\,d\,e^4\,z + 9\,b^4\,d^2\,e^3\,z + 6\,a\,b^4\,c\,d^2\,e^2 - 9\,a^2\,b^2\,c^2\,d^2\,e^2 - 6\,a^4\,b\,c\,d\,e^3 - 6\,a\,b\,c^4\,d^3\,e - 2\,a^3\,c^3\,d^2\,e^2 \left.\left. + 2\,b^3\,c^3\,d^3\,e + 2\,a^3\,b^3\,d\,e^3 - b^6\,d^2\,e^2 - c^6\,d^4 - a^6\,e^4, z, k\big)\right)\right) + \frac{c^2\,x^2}{2\,e} + \frac{2\,b\,c\,x}{e}$$

```
[In] int((a + b*x + c*x^2)^2/(d + e*x^3),x)
```

```
[Out] symsum(log((2*a^3*b*e^2 + 2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e)/e + (x*(c^
4*d^2 - 2*a^3*c*e^2 + 3*a^2*b^2*e^2 + 2*b^3*c*d*e))/e - 3*root(27*d^2*e^5*z
^3 - 54*a*c*d^2*e^4*z^2 - 27*b^2*d^2*e^4*z^2 + 27*a^2*c^2*d^2*e^3*z + 18*b*
c^3*d^3*e^2*z + 18*a^3*b*d*e^4*z + 9*b^4*d^2*e^3*z + 6*a*b^4*c*d^2*e^2 - 9*
a^2*b^2*c^2*d^2*e^2 - 6*a^4*b*c*d*e^3 - 6*a*b*c^4*d^3*e - 2*a^3*c^3*d^2*e^2
 + 2*b^3*c^3*d^3*e + 2*a^3*b^3*d*e^3 - b^6*d^2*e^2 - c^6*d^4 - a^6*e^4, z,
k)*e*(2*b^2*d - 3*root(27*d^2*e^5*z^3 - 54*a*c*d^2*e^4*z^2 - 27*b^2*d^2*e^4
*z^2 + 27*a^2*c^2*d^2*e^3*z + 18*b*c^3*d^3*e^2*z + 18*a^3*b*d*e^4*z + 9*b^4
*d^2*e^3*z + 6*a*b^4*c*d^2*e^2 - 9*a^2*b^2*c^2*d^2*e^2 - 6*a^4*b*c*d*e^3 -
6*a*b*c^4*d^3*e - 2*a^3*c^3*d^2*e^2 + 2*b^3*c^3*d^3*e + 2*a^3*b^3*d*e^3 - b
^6*d^2*e^2 - c^6*d^4 - a^6*e^4, z, k)*d*e + 4*a*c*d - a^2*e*x + 2*b*c*d*x))
*root(27*d^2*e^5*z^3 - 54*a*c*d^2*e^4*z^2 - 27*b^2*d^2*e^4*z^2 + 27*a^2*c^2
*d^2*e^3*z + 18*b*c^3*d^3*e^2*z + 18*a^3*b*d*e^4*z + 9*b^4*d^2*e^3*z + 6*a*
b^4*c*d^2*e^2 - 9*a^2*b^2*c^2*d^2*e^2 - 6*a^4*b*c*d*e^3 - 6*a*b*c^4*d^3*e -
 2*a^3*c^3*d^2*e^2 + 2*b^3*c^3*d^3*e + 2*a^3*b^3*d*e^3 - b^6*d^2*e^2 - c^6*
d^4 - a^6*e^4, z, k), k, 1, 3) + (c^2*x^2)/(2*e) + (2*b*c*x)/e
```

3.74 $\int \frac{(a+bx+cx^2)^3}{d+ex^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 416

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3} \, dx = -\frac{(c^3d-b^3e-6abce)\,x}{e^2}+\frac{3c(b^2+ac)\,x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e} - \frac{\left(c^3d^2-3b^2cd^{4/3}e^{2/3}-3ac^2d^{4/3}e^{2/3}-b^3de-6abcde+3a^2b\sqrt[3]{d}e^{5/3}+a^3e^2\right)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{7/3}} + \frac{\left(c^3d^2-6abcde-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}(b^2cd+ac^2d-a^2be)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}} - \frac{\left(c^3d^2-6abcde-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}(b^2cd+ac^2d-a^2be)\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{7/3}} - \frac{(bc^2d-ab^2e-a^2ce)\log(d+ex^3)}{e^2}$$

```
[Out] -(-6*a*b*c*e-b^3*e+c^3*d)*x/e^2+3/2*c*(a*c+b^2)*x^2/e+b*c^2*x^3/e+1/4*c^3*x
^4/e+1/3*(c^3*d^2-6*a*b*c*d*e-e*(-a^3*e+b^3*d)+3*d^(1/3)*e^(2/3)*(-a^2*b*e+
a*c^2*d+b^2*c*d))*ln(d^(1/3)+e^(1/3)*x)/d^(2/3)/e^(7/3)-1/6*(c^3*d^2-6*a*b*
c*d*e-e*(-a^3*e+b^3*d)+3*d^(1/3)*e^(2/3)*(-a^2*b*e+a*c^2*d+b^2*c*d))*ln(d^(
2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/e^(7/3)-(-a^2*c*e-a*b^2*e+b*c^2
*d)*ln(e*x^3+d)/e^2-1/3*(c^3*d^2-3*b^2*c*d^(4/3)*e^(2/3)-3*a*c^2*d^(4/3)*e^
(2/3)-b^3*d*e-6*a*b*c*d*e+3*a^2*b*d^(1/3)*e^(5/3)+a^3*e^2)*arctan(1/3*(d^(1
/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(2/3)/e^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx = -\frac{\log(d+ex^3)\left(a^2(-c)e-ab^2e+bc^2d\right)}{e^2} - \frac{\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(a^3e^2+3a^2b\sqrt[3]{d}e^{5/3}-6abcde-3ac^2d^{4/3}e^{2/3}-b^3de-3b^2cd^{4/3}e^{2/3}+c^3d^2\right)}{\sqrt{3}d^{2/3}e^{7/3}} - \frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}(a^2(-b)e+ac^2d+b^2cd)-6abcde+c^3d^2\right)}{6d^{2/3}e^{7/3}} + \frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}(a^2(-b)e+ac^2d+b^2cd)-6abcde+c^3d^2\right)}{3d^{2/3}e^{7/3}} - \frac{x(-6abce+b^3(-e)+c^3d)}{e^2} + \frac{3cx^2(ac+b^2)}{2e} + \frac{bc^2x^3}{e} + \frac{c^3x^4}{4e}$$

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x^3),x]
```

```
[Out] -(((c^3*d - b^3*e - 6*a*b*c*e)*x)/e^2) + (3*c*(b^2 + a*c)*x^2)/(2*e) + (b*c
^2*x^3)/e + (c^3*x^4)/(4*e) - ((c^3*d^2 - 3*b^2*c*d^(4/3)*e^(2/3) - 3*a*c^2
*d^(4/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e + 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^
2)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(7
/3)) + ((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2
*c*d + a*c^2*d - a^2*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(7/3)) -
((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2*c*d +
a*c^2*d - a^2*b*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2
/3)*e^(7/3)) - ((b*c^2*d - a*b^2*e - a^2*c*e)*Log[d + e*x^3])/e^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{c^3d-b^3e-6abce}{e^2}+\frac{3c(b^2+ac)\,x}{e}+\frac{3bc^2x^2}{e}+\frac{c^3x^3}{e} + \frac{c^3d^2-6abcde-e(b^3d-a^3e)-3e(b^2cd+ac^2d-a^2be)\,x-3e(bc^2d-ab^2e-a^2ce)\,x^2}{e^2\,(d+ex^3)}\right)dx$$

$$= -\frac{(c^3d-b^3e-6abce)\,x}{e^2}+\frac{3c(b^2+ac)\,x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\frac{\int \frac{c^3d^2-6abcde-e(b^3d-a^3e)-3e(b^2cd+ac^2d-a^2be)x-3e(bc^2d-ab^2e-a^2ce)x^2}{d+ex^3}\,dx}{e^2}$$

$$= -\frac{(c^3d-b^3e-6abce)\,x}{e^2}+\frac{3c(b^2+ac)\,x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\frac{\int \frac{c^3d^2-6abcde-e(b^3d-a^3e)-3e(b^2cd+ac^2d-a^2be)x}{d+ex^3}\,dx}{e^2}-\frac{(3(bc^2d-ab^2e-a^2ce))\int \frac{x^2}{d+ex^3}\,dx}{e}$$

$$= -\frac{(c^3d-b^3e-6abce)\,x}{e^2}+\frac{3c(b^2+ac)\,x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}-\frac{(bc^2d-ab^2e-a^2ce)\log\left(d+ex^3\right)}{e^2}+\frac{\int \frac{\sqrt[3]{d}\left(-3\sqrt[3]{d}e\left(b^2cd+ac^2d-a^2be\right)+2\sqrt[3]{e}\left(c^3d^2-6abcde-e\left(b^3d-a^3e\right)\right)\right)+\sqrt[3]{e}\left(-3\sqrt[3]{d}e\left(b^2cd+ac^2d-a^2be\right)-\sqrt[3]{e}\left(c^3d^2-6abcd\right.\right.}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}}{3d^{2/3}e^{7/3}}+\frac{\left(c^3d^2-6abcde-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}(b^2cd+ac^2d-a^2be)\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{3d^{2/3}e^2}$$

$$= -\frac{(c^3d-b^3e-6abce)\,x}{e^2}+\frac{3c(b^2+ac)\,x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\frac{\left(c^3d^2-6abcde-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}(b^2cd+ac^2d-a^2be)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}}-\frac{(bc^2d-ab^2e-a^2ce)\log\left(d+ex^3\right)}{e^2}+\frac{\left(c^3d^2-3b^2cd^{4/3}e^{2/3}-3ac^2d^{4/3}e^{2/3}-b^3de-6abcde+3a^2b\sqrt[3]{d}e^{5/3}+a^3e^2\right)\int \frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}}{2\sqrt[3]{d}e^2}-\frac{\left(c^3d^2-6abcde-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}(b^2cd+ac^2d-a^2be)\right)\int \frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{6d^{2/3}e^{7/3}}$$

$$= -\frac{(c^3d-b^3e-6abce)\,x}{e^2}+\frac{3c(b^2+ac)\,x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\frac{\left(c^3d^2-6abcde-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}(b^2cd+ac^2d-a^2be)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}}-\frac{\left(c^3d^2-6abcde-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}(b^2cd+ac^2d-a^2be)\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{7/3}}-\frac{(bc^2d-ab^2e-a^2ce)\log\left(d+ex^3\right)}{e^2}+\frac{\left(c^3d^2-3b^2cd^{4/3}e^{2/3}-3ac^2d^{4/3}e^{2/3}-b^3de-6abcde+3a^2b\sqrt[3]{d}e^{5/3}+a^3e^2\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x\right.}{d^{2/3}e^{7/3}}$$

$$= -\frac{(c^3d - b^3e - 6abce)\,x}{e^2} + \frac{3c(b^2 + ac)\,x^2}{2e} + \frac{bc^2x^3}{e} + \frac{c^3x^4}{4e}$$
$$- \frac{\left(c^3d^2 - 3b^2cd^{4/3}e^{2/3} - 3ac^2d^{4/3}e^{2/3} - b^3de - 6abcde + 3a^2b\sqrt[3]{d}e^{5/3} + a^3e^2\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{7/3}}$$
$$+ \frac{\left(c^3d^2 - 6abcde - e(b^3d - a^3e) + 3\sqrt[3]{d}e^{2/3}(b^2cd + ac^2d - a^2be)\right)\log\left(\sqrt[3]{d} + \sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}}$$
$$- \frac{\left(c^3d^2 - 6abcde - e(b^3d - a^3e) + 3\sqrt[3]{d}e^{2/3}(b^2cd + ac^2d - a^2be)\right)\log\left(d^{2/3} - \sqrt[3]{d}\sqrt[3]{e}x + e^{2/3}x^2\right)}{6d^{2/3}e^{7/3}}$$
$$- \frac{(bc^2d - ab^2e - a^2ce)\log\left(d + ex^3\right)}{e^2}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.06

$$\int \frac{(a + bx + cx^2)^3}{d + ex^3}\,dx$$

$$= \frac{12\sqrt[3]{e}(-c^3d + b^3e + 6abce)\,x + 18c(b^2 + ac)\,e^{4/3}x^2 + 12bc^2e^{4/3}x^3 + 3c^3e^{4/3}x^4 - \frac{4\sqrt{3}\left(c^3d^2 - 3ac^2d^{4/3}e^{2/3} + e\left(-b^3\right.\right.}{}}{}$$

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x^3),x]
```

```
[Out] (12*e^(1/3)*(-(c^3*d) + b^3*e + 6*a*b*c*e)*x + 18*c*(b^2 + a*c)*e^(4/3)*x^2
 + 12*b*c^2*e^(4/3)*x^3 + 3*c^3*e^(4/3)*x^4 - (4*Sqrt[3]*(c^3*d^2 - 3*a*c^2
*d^(4/3)*e^(2/3) + e*(-(b^3*d) + 3*a^2*b*d^(1/3)*e^(2/3) + a^3*e) - 3*c*(b^
2*d^(4/3)*e^(2/3) + 2*a*b*d*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]]
)/d^(2/3) + (4*(c^3*d^2 + 3*b^2*c*d^(4/3)*e^(2/3) + 3*a*c^2*d^(4/3)*e^(2/3)
 - b^3*d*e - 6*a*b*c*d*e - 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^2)*Log[d^(1/3) +
 e^(1/3)*x])/d^(2/3) - (2*(c^3*d^2 + 3*b^2*c*d^(4/3)*e^(2/3) + 3*a*c^2*d^(4
/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e - 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^2)*Lo
g[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3) + 12*e^(1/3)*(-(b*c^2
*d) + a*b^2*e + a^2*c*e)*Log[d + e*x^3])/(12*e^(7/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.69 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.44

method	result
risch	$\frac{c^3x^4}{4e}+\frac{b\,c^2x^3}{e}+\frac{3a\,c^2x^2}{2e}+\frac{3b^2c\,x^2}{2e}+\frac{6abcx}{e}+\frac{b^3x}{e}-\frac{c^3dx}{e^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3e+d\right)}\frac{\left(3e\left(a^2ce+a\,b^2e-b\,c^2d\right)\_R^2+3e\left(a^2be-\right.\right.}{}}{3e^3}$
default	$\frac{\frac{1}{4}c^3x^4e+b\,c^2x^3e+\frac{3}{2}a\,c^2e\,x^2+\frac{3}{2}b^2ce\,x^2+6abcex+b^3ex-c^3dx}{e^2}+\frac{\left(a^3e^2-6abcde-b^3de+c^3d^2\right)\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right.}{}$

[In] int((c*x^2+b*x+a)^3/(e*x^3+d),x,method=_RETURNVERBOSE)

[Out] 1/4*c^3*x^4/e+b*c^2*x^3/e+3/2/e*a*c^2*x^2+3/2/e*b^2*c*x^2+6/e*a*b*c*x+1/e*b^3*x-1/e^2*c^3*d*x+1/3/e^3*sum((3*e*(a^2*c*e+a*b^2*e-b*c^2*d)*_R^2+3*e*(a^2*b*e-a*c^2*d-b^2*c*d)*_R+a^3*e^2-6*a*b*c*d*e-b^3*d*e+c^3*d^2)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e+d))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.41 (sec) , antiderivative size = 29479, normalized size of antiderivative = 70.86

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx = \text{Too large to display}$$

[In] integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**3/(e*x**3+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx =$$

$$-\frac{\sqrt{3}\left(c^3d^2 - b^3de - 6\,abcde + a^3e^2 + 3\left(-de^2\right)^{\frac{1}{3}}b^2cd + 3\left(-de^2\right)^{\frac{1}{3}}ac^2d - 3\left(-de^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x\right.}{3\left(\right.}\right.}{3\left(-de^2\right)^{\frac{2}{3}}e}$$

$$-\frac{\left(c^3d^2 - b^3de - 6\,abcde + a^3e^2 - 3\left(-de^2\right)^{\frac{1}{3}}b^2cd - 3\left(-de^2\right)^{\frac{1}{3}}ac^2d + 3\left(-de^2\right)^{\frac{1}{3}}a^2be\right)\log\left(x^2 + x\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right.}{6\left(-de^2\right)^{\frac{2}{3}}e}$$

$$-\frac{(bc^2d - ab^2e - a^2ce)\log\left(|ex^3+d|\right)}{e^2}$$

$$+\frac{c^3e^3x^4 + 4\,bc^2e^3x^3 + 6\,b^2ce^3x^2 + 6\,ac^2e^3x^2 - 4\,c^3de^2x + 4\,b^3e^3x + 24\,abce^3x}{4\,e^4}$$

$$+\frac{\left(3\,b^2cde^8\left(-\frac{d}{e}\right)^{\frac{1}{3}} + 3\,ac^2de^8\left(-\frac{d}{e}\right)^{\frac{1}{3}} - 3\,a^2be^9\left(-\frac{d}{e}\right)^{\frac{1}{3}} - c^3d^2e^7 + b^3de^8 + 6\,abcde^8 - a^3e^9\right)\left(-\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x\right.\right.}{3\,de^9}$$

```
[In] integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(c^3*d^2 - b^3*d*e - 6*a*b*c*d*e + a^3*e^2 + 3*(-d*e^2)^(1/3)*
b^2*c*d + 3*(-d*e^2)^(1/3)*a*c^2*d - 3*(-d*e^2)^(1/3)*a^2*b*e)*arctan(1/3*s
qrt(3)*(2*x + (-d/e)^(1/3))/(-d/e)^(1/3))/((-d*e^2)^(2/3)*e) - 1/6*(c^3*d^2
 - b^3*d*e - 6*a*b*c*d*e + a^3*e^2 - 3*(-d*e^2)^(1/3)*b^2*c*d - 3*(-d*e^2)^
(1/3)*a*c^2*d + 3*(-d*e^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-d/e)^(1/3) + (-d/e)
^(2/3))/((-d*e^2)^(2/3)*e) - (b*c^2*d - a*b^2*e - a^2*c*e)*log(abs(e*x^3 +
d))/e^2 + 1/4*(c^3*e^3*x^4 + 4*b*c^2*e^3*x^3 + 6*b^2*c*e^3*x^2 + 6*a*c^2*e^
3*x^2 - 4*c^3*d*e^2*x + 4*b^3*e^3*x + 24*a*b*c*e^3*x)/e^4 + 1/3*(3*b^2*c*d*
e^8*(-d/e)^(1/3) + 3*a*c^2*d*e^8*(-d/e)^(1/3) - 3*a^2*b*e^9*(-d/e)^(1/3) -
c^3*d^2*e^7 + b^3*d*e^8 + 6*a*b*c*d*e^8 - a^3*e^9)*(-d/e)^(1/3)*log(abs(x -
 (-d/e)^(1/3)))/(d*e^9)
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 1700, normalized size of antiderivative = 4.09

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x + c*x^2)^3/(d + e*x^3),x)
```

```
[Out] x*((b^3 + 6*a*b*c)/e - (c^3*d)/e^2) + symsum(log(root(27*d^2*e^7*z^3 + 81*b
*c^2*d^3*e^5*z^2 - 81*a^2*c*d^2*e^6*z^2 - 81*a*b^2*d^2*e^6*z^2 - 27*a^3*b^2
*c*d^2*e^5*z + 27*a^2*b*c^3*d^3*e^4*z + 27*a*b^3*c^2*d^3*e^4*z + 54*b^2*c^4
*d^4*e^3*z + 54*a^4*c^2*d^2*e^5*z + 54*a^2*b^4*d^2*e^5*z + 27*b^5*c*d^3*e^4
*z - 27*a*c^5*d^4*e^3*z + 27*a^5*b*d*e^6*z + 18*a^4*b^4*c*d^2*e^4 - 18*a^4*
b*c^4*d^3*e^3 + 18*a*b^4*c^4*d^4*e^2 - 9*a*b^7*c*d^3*e^3 - 27*a^5*b^2*c^2*d
^2*e^4 + 27*a^2*b^5*c^2*d^3*e^3 - 27*a^2*b^2*c^5*d^4*e^2 - 21*a^3*b^3*c^3*d
^3*e^3 - 9*a^7*b*c*d*e^5 - 9*a*b*c^7*d^5*e - 3*b^6*c^3*d^4*e^2 - 3*a^6*c^3*
d^2*e^4 - 3*a^3*c^6*d^4*e^2 - 3*a^3*b^6*d^2*e^4 + 3*b^3*c^6*d^5*e + 3*a^6*b
^3*d*e^5 + b^9*d^3*e^3 - c^9*d^6 - a^9*e^6, z, k)*((3*x*(a^3*e^4 - b^3*d*e^
3 + c^3*d^2*e^2 - 6*a*b*c*d*e^3))/e^2 - (3*(6*a*b^2*d*e^3 - 6*b*c^2*d^2*e^2
 + 6*a^2*c*d*e^3))/e^2 + 9*root(27*d^2*e^7*z^3 + 81*b*c^2*d^3*e^5*z^2 - 81*
a^2*c*d^2*e^6*z^2 - 81*a*b^2*d^2*e^6*z^2 - 27*a^3*b^2*c*d^2*e^5*z + 27*a^2*
b*c^3*d^3*e^4*z + 27*a*b^3*c^2*d^3*e^4*z + 54*b^2*c^4*d^4*e^3*z + 54*a^4*c^
2*d^2*e^5*z + 54*a^2*b^4*d^2*e^5*z + 27*b^5*c*d^3*e^4*z - 27*a*c^5*d^4*e^3*
z + 27*a^5*b*d*e^6*z + 18*a^4*b^4*c*d^2*e^4 - 18*a^4*b*c^4*d^3*e^3 + 18*a*b
^4*c^4*d^4*e^2 - 9*a*b^7*c*d^3*e^3 - 27*a^5*b^2*c^2*d^2*e^4 + 27*a^2*b^5*c^
2*d^3*e^3 - 27*a^2*b^2*c^5*d^4*e^2 - 21*a^3*b^3*c^3*d^3*e^3 - 9*a^7*b*c*d*e
^5 - 9*a*b*c^7*d^5*e - 3*b^6*c^3*d^4*e^2 - 3*a^6*c^3*d^2*e^4 - 3*a^3*c^6*d^
4*e^2 - 3*a^3*b^6*d^2*e^4 + 3*b^3*c^6*d^5*e + 3*a^6*b^3*d*e^5 + b^9*d^3*e^3
 - c^9*d^6 - a^9*e^6, z, k)*d*e^2) + (3*(a^5*b*e^3 - a*c^5*d^3 + 2*b^2*c^4*
d^3 + 2*a^2*b^4*d*e^2 + 2*a^4*c^2*d*e^2 + b^5*c*d^2*e + a*b^3*c^2*d^2*e + a
```

```
^2*b*c^3*d^2*e - a^3*b^2*c*d*e^2))/e^2 + (3*x*(b*c^5*d^3 - a^5*c*e^3 + 2*a^
4*b^2*e^3 + 2*a^2*c^4*d^2*e + 2*b^4*c^2*d^2*e + a*b^5*d*e^2 - a*b^2*c^3*d^2
*e + a^2*b^3*c*d*e^2 + a^3*b*c^2*d*e^2))/e^2)*root(27*d^2*e^7*z^3 + 81*b*c^
2*d^3*e^5*z^2 - 81*a^2*c*d^2*e^6*z^2 - 81*a*b^2*d^2*e^6*z^2 - 27*a^3*b^2*c*
d^2*e^5*z + 27*a^2*b*c^3*d^3*e^4*z + 27*a*b^3*c^2*d^3*e^4*z + 54*b^2*c^4*d^
4*e^3*z + 54*a^4*c^2*d^2*e^5*z + 54*a^2*b^4*d^2*e^5*z + 27*b^5*c*d^3*e^4*z
- 27*a*c^5*d^4*e^3*z + 27*a^5*b*d*e^6*z + 18*a^4*b^4*c*d^2*e^4 - 18*a^4*b*c
^4*d^3*e^3 + 18*a*b^4*c^4*d^4*e^2 - 9*a*b^7*c*d^3*e^3 - 27*a^5*b^2*c^2*d^2*
e^4 + 27*a^2*b^5*c^2*d^3*e^3 - 27*a^2*b^2*c^5*d^4*e^2 - 21*a^3*b^3*c^3*d^3*
e^3 - 9*a^7*b*c*d*e^5 - 9*a*b*c^7*d^5*e - 3*b^6*c^3*d^4*e^2 - 3*a^6*c^3*d^2
*e^4 - 3*a^3*c^6*d^4*e^2 - 3*a^3*b^6*d^2*e^4 + 3*b^3*c^6*d^5*e + 3*a^6*b^3*
d*e^5 + b^9*d^3*e^3 - c^9*d^6 - a^9*e^6, z, k), k, 1, 3) + (c^3*x^4)/(4*e)
+ (b*c^2*x^3)/e + (3*c*x^2*(a*c + b^2))/(2*e)
```

3.75 $\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 645

$$
\begin{aligned}
&\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx \\
&= -\frac{2(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce)\,x}{e^2} - \frac{(4bc^3d-b^4e-12ab^2ce-6a^2c^2e)\,x^2}{2e^2} \\
&\quad - \frac{c(c^3d-4b^3e-12abce)\,x^3}{3e^2} + \frac{c^2(3b^2+2ac)\,x^4}{2e} + \frac{4bc^3x^5}{5e} + \frac{c^4x^6}{6e} \\
&\quad - \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\left(4c^3d^2+6c^2\left(bd^{5/3}\sqrt[3]{e}-ad^{4/3}e^{2/3}\right)-12abcde-e\left(b^3d+3ab^2d^{2/3}\sqrt[3]{e}-3a^2b\sqrt[3]{d}e^{2/3}-a^3e\right.\right.}{\sqrt{3}d^{2/3}e^{8/3}} \\
&\quad + \frac{\left(\sqrt[3]{e}(6b^2c^2d^2+4ac^3d^2-4ab^3de-12a^2bcde+a^4e^2)+\sqrt[3]{d}(b^4de+12ab^2cde+6a^2c^2de-4b(c^3d^2+a^3e^2)\right)}{3d^{2/3}e^{8/3}} \\
&\quad - \frac{\left(\sqrt[3]{e}(6b^2c^2d^2+4ac^3d^2-4ab^3de-12a^2bcde+a^4e^2)+\sqrt[3]{d}(b^4de+12ab^2cde+6a^2c^2de-4b(c^3d^2+a^3e^2)\right)}{6d^{2/3}e^{8/3}} \\
&\quad + \frac{(c^4d^2-12abc^2de+6a^2b^2e^2-4ce(b^3d-a^3e))\log(d+ex^3)}{3e^3}
\end{aligned}
$$

```
[Out] -2*(-6*a^2*b*c*e-2*a*b^3*e+2*a*c^3*d+3*b^2*c^2*d)*x/e^2-1/2*(-6*a^2*c^2*e-1
2*a*b^2*c*e-b^4*e+4*b*c^3*d)*x^2/e^2-1/3*c*(-12*a*b*c*e-4*b^3*e+c^3*d)*x^3/
e^2+1/2*c^2*(2*a*c+3*b^2)*x^4/e+4/5*b*c^3*x^5/e+1/6*c^4*x^6/e+1/3*(e^(1/3)*
(a^4*e^2-12*a^2*b*c*d*e-4*a*b^3*d*e+4*a*c^3*d^2+6*b^2*c^2*d^2)+d^(1/3)*(b^4
*d*e+12*a*b^2*c*d*e+6*a^2*c^2*d*e-4*b*(a^3*e^2+c^3*d^2)))*ln(d^(1/3)+e^(1/3
)*x)/d^(2/3)/e^(8/3)-1/6*(e^(1/3)*(a^4*e^2-12*a^2*b*c*d*e-4*a*b^3*d*e+4*a*c
^3*d^2+6*b^2*c^2*d^2)+d^(1/3)*(b^4*d*e+12*a*b^2*c*d*e+6*a^2*c^2*d*e-4*b*(a^
3*e^2+c^3*d^2)))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/e^(8/3)+
1/3*(c^4*d^2-12*a*b*c^2*d*e+6*a^2*b^2*e^2-4*c*e*(-a^3*e+b^3*d))*ln(e*x^3+d)
```

```
/e^3-1/3*(b*d^(1/3)+a*e^(1/3))*(4*c^3*d^2+6*c^2*(b*d^(5/3)*e^(1/3)-a*d^(4/3
)*e^(2/3))-12*a*b*c*d*e-e*(b^3*d+3*a*b^2*d^(2/3)*e^(1/3)-3*a^2*b*d^(1/3)*e^
(2/3)-a^3*e))*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(2/3)/e^(
8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx$$

$$= -\frac{x^2(-6a^2c^2e-12ab^2ce+b^4(-e)+4bc^3d)}{2e^2} - \frac{2x(-6a^2bce-2ab^3e+2ac^3d+3b^2c^2d)}{e^2}$$

$$-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(-e\left(a^3(-e)-3a^2b\sqrt[3]{d}e^{2/3}+3ab^2d^{2/3}\sqrt[3]{e}+b^3d\right)+6c^2\left(bd^{5/3}\sqrt[3]{e}-\cdots\right.\right.}{\sqrt{3}d^{2/3}e^{8/3}}$$

$$+\frac{\log(d+ex^3)\left(-4ce(b^3d-a^3e)+6a^2b^2e^2-12abc^2de+c^4d^2\right)}{3e^3}$$

$$-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(a^4e^2-12a^2bcde+\frac{\sqrt[3]{d}(-4b(a^3e^2+c^3d^2)+6a^2c^2de+12ab^2cde+b^4de)}{\sqrt[3]{e}}-4ab^3de+4\cdots\right.}{6d^{2/3}e^{7/3}}$$

$$+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(\sqrt[3]{e}(a^4e^2-12a^2bcde-4ab^3de+4ac^3d^2+6b^2c^2d^2)+\sqrt[3]{d}(-4b(a^3e^2+c^3d^2)+6a^2c^2\cdots\right.}{3d^{2/3}e^{8/3}}$$

$$-\frac{cx^3(-12abce-4b^3e+c^3d)}{3e^2}+\frac{c^2x^4(2ac+3b^2)}{2e}+\frac{4bc^3x^5}{5e}+\frac{c^4x^6}{6e}$$

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x^3),x]
```

```
[Out] (-2*(3*b^2*c^2*d + 2*a*c^3*d - 2*a*b^3*e - 6*a^2*b*c*e)*x)/e^2 - ((4*b*c^3*
d - b^4*e - 12*a*b^2*c*e - 6*a^2*c^2*e)*x^2)/(2*e^2) - (c*(c^3*d - 4*b^3*e
- 12*a*b*c*e)*x^3)/(3*e^2) + (c^2*(3*b^2 + 2*a*c)*x^4)/(2*e) + (4*b*c^3*x^5
)/(5*e) + (c^4*x^6)/(6*e) - ((b*d^(1/3) + a*e^(1/3))*(4*c^3*d^2 + 6*c^2*(b*
d^(5/3)*e^(1/3) - a*d^(4/3)*e^(2/3)) - 12*a*b*c*d*e - e*(b^3*d + 3*a*b^2*d^
(2/3)*e^(1/3) - 3*a^2*b*d^(1/3)*e^(2/3) - a^3*e))*ArcTan[(d^(1/3) - 2*e^(1/
3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(8/3)) + ((e^(1/3)*(6*b^2*c^2*
d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2) + d^(1/3)*(b^4*
d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))*Log[d^(1/3
) + e^(1/3)*x])/(3*d^(2/3)*e^(8/3)) - ((6*b^2*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b
^3*d*e - 12*a^2*b*c*d*e + a^4*e^2 + (d^(1/3)*(b^4*d*e + 12*a*b^2*c*d*e + 6*
a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1
/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(7/3)) + ((c^4*d^2 - 12*a*b*c^2*d*e + 6*
a^2*b^2*e^2 - 4*c*e*(b^3*d - a^3*e))*Log[d + e*x^3])/(3*e^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
```

```
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \Bigg(-\frac{2(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce)}{e^2} - \frac{(4bc^3d-b^4e-12ab^2ce-6a^2c^2e)\,x}{e^2} - \frac{c(c^3d-4b^3e-12abce)\,x^2}{e^2} + \frac{2c^2(3b^2+2ac)\,x^3}{e} + \frac{4bc^3x^4}{e} + \frac{c^4x^5}{e} + \frac{6b^2c^2d^2+4ac^3d^2-4ab^3de-12a^2bcde+a^4e^2-(b^4de+12ab^2cde+6a^2c^2de-4b(c^3d^2+a^3e^2))\,x+(c^4}{e^2\,(d+ex^3)}$$

$$= -\frac{2(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce)\,x}{e^2} - \frac{(4bc^3d-b^4e-12ab^2ce-6a^2c^2e)\,x^2}{2e^2} - \frac{c(c^3d-4b^3e-12abce)\,x^3}{3e^2} + \frac{c^2(3b^2+2ac)\,x^4}{2e} + \frac{4bc^3x^5}{5e} + \frac{c^4x^6}{6e} + \frac{\int \frac{6b^2c^2d^2+4ac^3d^2-4ab^3de-12a^2bcde+a^4e^2-(b^4de+12ab^2cde+6a^2c^2de-4b(c^3d^2+a^3e^2))x+(c^4d^2-12abc^2de+6a^2b^2e^2-4ce(b^3c}{d+ex^3}}{e^2}$$

$$= -\frac{2(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce)\,x}{e^2} - \frac{(4bc^3d-b^4e-12ab^2ce-6a^2c^2e)\,x^2}{2e^2} - \frac{c(c^3d-4b^3e-12abce)\,x^3}{3e^2} + \frac{c^2(3b^2+2ac)\,x^4}{2e} + \frac{4bc^3x^5}{5e} + \frac{c^4x^6}{6e} + \frac{\int \frac{6b^2c^2d^2+4ac^3d^2-4ab^3de-12a^2bcde+a^4e^2+(-b^4de-12ab^2cde-6a^2c^2de+4b(c^3d^2+a^3e^2))x}{d+ex^3}\,dx}{e^2} + \frac{(c^4d^2-12abc^2de+6a^2b^2e^2-4ce(b^3d-a^3e))\int \frac{x^2}{d+ex^3}\,dx}{e^2}$$

$$= -\frac{2(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce)\,x}{e^2} - \frac{(4bc^3d-b^4e-12ab^2ce-6a^2c^2e)\,x^2}{2e^2} - \frac{c(c^3d-4b^3e-12abce)\,x^3}{3e^2} + \frac{c^2(3b^2+2ac)\,x^4}{2e} + \frac{4bc^3x^5}{5e} + \frac{c^4x^6}{6e} + \frac{(c^4d^2-12abc^2de+6a^2b^2e^2-4ce(b^3d-a^3e))\log(d+ex^3)}{3e^3} + \frac{\int \frac{\sqrt[3]{d}\left(2\sqrt[3]{e}(6b^2c^2d^2+4ac^3d^2-4ab^3de-12a^2bcde+a^4e^2)+\sqrt[3]{d}(-b^4de-12ab^2cde-6a^2c^2de+4b(c^3d^2+a^3e^2))\right)+\sqrt[3]{e}\left(-\sqrt[3]{e}(\right.}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}}{3d^{2/3}e^{7/3}} + \frac{\left(6b^2c^2d^2+4ac^3d^2-4ab^3de-12a^2bcde+a^4e^2+\frac{\sqrt[3]{d}(b^4de+12ab^2cde+6a^2c^2de-4b(c^3d^2+a^3e^2))}{\sqrt[3]{e}}\right)\int \frac{}{\sqrt[3]{d}-}}{3d^{2/3}e^2}$$

$$= -\frac{2(3b^2c^2d + 2ac^3d - 2ab^3e - 6a^2bce)\,x}{e^2} - \frac{(4bc^3d - b^4e - 12ab^2ce - 6a^2c^2e)\,x^2}{2e^2} - \frac{c(c^3d - 4b^3e - 12abce)\,x^3}{3e^2} + \frac{c^2(3b^2 + 2ac)\,x^4}{2e} + \frac{4bc^3x^5}{5e} + \frac{c^4x^6}{6e} + \frac{\left(6b^2c^2d^2 + 4ac^3d^2 - 4ab^3de - 12a^2bcde + a^4e^2 + \frac{\sqrt[3]{d}(b^4de+12ab^2cde+6a^2c^2de-4b(c^3d^2+a^3e^2))}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}\right.}{3d^{2/3}e^{7/3}} + \frac{(c^4d^2 - 12abc^2de + 6a^2b^2e^2 - 4ce(b^3d - a^3e))\log(d + ex^3)}{3e^3} + \frac{\left(\left(b\sqrt[3]{d} + a\sqrt[3]{e}\right)\left(4c^3d^2 + 6c^2\left(bd^{5/3}\sqrt[3]{e} - ad^{4/3}e^{2/3}\right) - 12abcde - e\left(b^3d + 3ab^2d^{2/3}\sqrt[3]{e} - 3a^2b\sqrt[3]{d}e\right.\right.\right.}{2\sqrt[3]{d}e^{7/3}} - \frac{\left(6b^2c^2d^2 + 4ac^3d^2 - 4ab^3de - 12a^2bcde + a^4e^2 + \frac{\sqrt[3]{d}(b^4de+12ab^2cde+6a^2c^2de-4b(c^3d^2+a^3e^2))}{\sqrt[3]{e}}\right)\int \frac{-\sqrt[3]{}}{d^{2/3}-\sqrt[3]{}}}{6d^{2/3}e^{7/3}}$$

$$= -\frac{2(3b^2c^2d + 2ac^3d - 2ab^3e - 6a^2bce)\,x}{e^2} - \frac{(4bc^3d - b^4e - 12ab^2ce - 6a^2c^2e)\,x^2}{2e^2} - \frac{c(c^3d - 4b^3e - 12abce)\,x^3}{3e^2} + \frac{c^2(3b^2 + 2ac)\,x^4}{2e} + \frac{4bc^3x^5}{5e} + \frac{c^4x^6}{6e} + \frac{\left(6b^2c^2d^2 + 4ac^3d^2 - 4ab^3de - 12a^2bcde + a^4e^2 + \frac{\sqrt[3]{d}(b^4de+12ab^2cde+6a^2c^2de-4b(c^3d^2+a^3e^2))}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}\right.}{3d^{2/3}e^{7/3}} - \frac{\left(6b^2c^2d^2 + 4ac^3d^2 - 4ab^3de - 12a^2bcde + a^4e^2 + \frac{\sqrt[3]{d}(b^4de+12ab^2cde+6a^2c^2de-4b(c^3d^2+a^3e^2))}{\sqrt[3]{e}}\right)\log\left(d^{2/}\right.}{6d^{2/3}e^{7/3}} + \frac{(c^4d^2 - 12abc^2de + 6a^2b^2e^2 - 4ce(b^3d - a^3e))\log(d + ex^3)}{3e^3} + \frac{\left(\left(b\sqrt[3]{d} + a\sqrt[3]{e}\right)\left(4c^3d^2 + 6c^2\left(bd^{5/3}\sqrt[3]{e} - ad^{4/3}e^{2/3}\right) - 12abcde - e\left(b^3d + 3ab^2d^{2/3}\sqrt[3]{e} - 3a^2b\sqrt[3]{d}e\right.\right.\right.}{d^{2/3}e^{8/3}}$$

$$= -\frac{2(3b^2c^2d + 2ac^3d - 2ab^3e - 6a^2bce)\,x}{e^2} - \frac{(4bc^3d - b^4e - 12ab^2ce - 6a^2c^2e)\,x^2}{2e^2}$$
$$- \frac{c(c^3d - 4b^3e - 12abce)\,x^3}{3e^2} + \frac{c^2(3b^2 + 2ac)\,x^4}{2e} + \frac{4bc^3x^5}{5e} + \frac{c^4x^6}{6e}$$
$$- \frac{\left(b\sqrt[3]{d} + a\sqrt[3]{e}\right)\left(4c^3d^2 + 6c^2\left(bd^{5/3}\sqrt[3]{e} - ad^{4/3}e^{2/3}\right) - 12abcde - e\left(b^3d + 3ab^2d^{2/3}\sqrt[3]{e} - 3a^2b\sqrt[3]{de}\right.\right.}{\sqrt{3}d^{2/3}e^{8/3}}$$
$$+ \frac{\left(6b^2c^2d^2 + 4ac^3d^2 - 4ab^3de - 12a^2bcde + a^4e^2 + \frac{\sqrt[3]{d}(b^4de+12ab^2cde+6a^2c^2de-4b(c^3d^2+a^3e^2))}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{\right.}{3d^{2/3}e^{7/3}}$$
$$- \frac{\left(6b^2c^2d^2 + 4ac^3d^2 - 4ab^3de - 12a^2bcde + a^4e^2 + \frac{\sqrt[3]{d}(b^4de+12ab^2cde+6a^2c^2de-4b(c^3d^2+a^3e^2))}{\sqrt[3]{e}}\right)\log\left(d\right.}{6d^{2/3}e^{7/3}}$$
$$+ \frac{(c^4d^2 - 12abc^2de + 6a^2b^2e^2 - 4ce(b^3d - a^3e))\log\left(d + ex^3\right)}{3e^3}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.05

$$\int \frac{(a + bx + cx^2)^4}{d + ex^3}\,dx$$

$$= \frac{60e^{2/3}(-3b^2c^2d - 2ac^3d + 2ab^3e + 6a^2bce)\,x + 15e^{2/3}(-4bc^3d + b^4e + 12ab^2ce + 6a^2c^2e)\,x^2 + 10ce^{2/3}(-c}{}$$

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x^3),x]
```

```
[Out] (60*e^(2/3)*(-3*b^2*c^2*d - 2*a*c^3*d + 2*a*b^3*e + 6*a^2*b*c*e)*x + 15*e^(
2/3)*(-4*b*c^3*d + b^4*e + 12*a*b^2*c*e + 6*a^2*c^2*e)*x^2 + 10*c*e^(2/3)*(
-(c^3*d) + 4*b^3*e + 12*a*b*c*e)*x^3 + 15*c^2*(3*b^2 + 2*a*c)*e^(5/3)*x^4 +
 24*b*c^3*e^(5/3)*x^5 + 5*c^4*e^(5/3)*x^6 + (10*Sqrt[3]*(b*d^(1/3) + a*e^(1
/3))*(-4*c^3*d^2 + c^2*(-6*b*d^(5/3)*e^(1/3) + 6*a*d^(4/3)*e^(2/3)) + 12*a*
b*c*d*e + e*(b^3*d + 3*a*b^2*d^(2/3)*e^(1/3) - 3*a^2*b*d^(1/3)*e^(2/3) - a^
3*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (10*(4*a*c^3*d
^2*e^(1/3) + b^4*d^(4/3)*e + 6*a^2*c^2*d^(4/3)*e - 4*a*b^3*d*e^(4/3) + a^4*
e^(7/3) + 6*b^2*(c^2*d^2*e^(1/3) + 2*a*c*d^(4/3)*e) - 4*b*(c^3*d^(7/3) + 3*
a^2*c*d*e^(4/3) + a^3*d^(1/3)*e^2))*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - (5*
(4*a*c^3*d^2*e^(1/3) + b^4*d^(4/3)*e + 6*a^2*c^2*d^(4/3)*e - 4*a*b^3*d*e^(4
/3) + a^4*e^(7/3) + 6*b^2*(c^2*d^2*e^(1/3) + 2*a*c*d^(4/3)*e) - 4*b*(c^3*d^
(7/3) + 3*a^2*c*d*e^(4/3) + a^3*d^(1/3)*e^2))*Log[d^(2/3) - d^(1/3)*e^(1/3)
*x + e^(2/3)*x^2])/d^(2/3) + (10*(c^4*d^2 - 12*a*b*c^2*d*e + 6*a^2*b^2*e^2
+ 4*c*e*(-(b^3*d) + a^3*e))*Log[d + e*x^3])/e^(1/3))/(30*e^(8/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.69 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.54

method	result
risch	$\frac{c^4x^6}{6e}+\frac{4b\,c^3x^5}{5e}+\frac{a\,c^3x^4}{e}+\frac{3b^2c^2x^4}{2e}+\frac{4ab\,c^2x^3}{e}+\frac{4b^3c\,x^3}{3e}-\frac{c^4d\,x^3}{3e^2}+\frac{3a^2c^2x^2}{e}+\frac{6a\,b^2c\,x^2}{e}+\frac{b^4x^2}{2e}-\frac{2b\,c^3d\,x^2}{e^2}+12a$
default	$\frac{\frac{1}{6}c^4x^6e+\frac{4}{5}b\,c^3x^5e+a\,c^3e\,x^4+\frac{3}{2}b^2c^2e\,x^4+4ab\,c^2e\,x^3+\frac{4}{3}b^3ce\,x^3-\frac{1}{3}c^4d\,x^3+3a^2c^2e\,x^2+6a\,b^2ce\,x^2+\frac{1}{2}b^4e\,x^2-2b\,c^3d\,x^2+12a^2bcex+4a\,b^3e}{e^2}$

```
[In] int((c*x^2+b*x+a)^4/(e*x^3+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c^4*x^6/e+4/5*b*c^3*x^5/e+1/e*a*c^3*x^4+3/2/e*b^2*c^2*x^4+4/e*a*b*c^2*x
^3+4/3/e*b^3*c*x^3-1/3/e^2*c^4*d*x^3+3/e*a^2*c^2*x^2+6/e*a*b^2*c*x^2+1/2/e*
b^4*x^2-2/e^2*b*c^3*d*x^2+12/e*a^2*b*c*x+4/e*a*b^3*x-4/e^2*a*c^3*d*x-6/e^2*
x*b^2*c^2*d+1/3/e^3*sum(((4*a^3*c*e^2+6*a^2*b^2*e^2-12*a*b*c^2*d*e-4*b^3*c*
d*e+c^4*d^2)*_R^2+_R*(4*a^3*b*e^2-6*a^2*c^2*d*e-12*a*b^2*c*d*e-b^4*d*e+4*b*
c^3*d^2)+a^4*e^2-12*a^2*b*c*d*e-4*a*b^3*d*e+4*a*c^3*d^2+6*b^2*c^2*d^2)/_R^2
*ln(x-_R),_R=RootOf(_Z^3*e+d))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 91.41 (sec) , antiderivative size = 47284, normalized size of antiderivative = 73.31

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**4/(e*x**3+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 773, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx =$$

$$-\frac{\sqrt{3}\Big(6\,b^2c^2d^2e+4\,ac^3d^2e-4\,ab^3de^2-12\,a^2bcde^2+a^4e^3-4\,(-de^2)^{\frac{1}{3}}\,bc^3d^2+(-de^2)^{\frac{1}{3}}\,b^4de+12\,(-de^2)}{3\,(-de^2)^{\frac{2}{3}}\,e^2}$$

$$-\frac{\Big(6\,b^2c^2d^2e+4\,ac^3d^2e-4\,ab^3de^2-12\,a^2bcde^2+a^4e^3+4\,(-de^2)^{\frac{1}{3}}\,bc^3d^2-(-de^2)^{\frac{1}{3}}\,b^4de-12\,(-de^2)^{\frac{1}{3}}\,a}{6\,(-de^2)^{\frac{2}{3}}\,e^2}$$

$$+\frac{(c^4d^2-4\,b^3cde-12\,abc^2de+6\,a^2b^2e^2+4\,a^3ce^2)\log\left(|ex^3+d|\right)}{3\,e^3}$$

$$+\frac{5\,c^4e^5x^6+24\,bc^3e^5x^5+45\,b^2c^2e^5x^4+30\,ac^3e^5x^4-10\,c^4de^4x^3+40\,b^3ce^5x^3+120\,abc^2e^5x^3-60\,bc^3de^4x}{30\,e}$$

$$-\frac{\Big(4\,bc^3d^2e^{11}\left(-\frac{d}{e}\right)^{\frac{1}{3}}-b^4de^{12}\left(-\frac{d}{e}\right)^{\frac{1}{3}}-12\,ab^2cde^{12}\left(-\frac{d}{e}\right)^{\frac{1}{3}}-6\,a^2c^2de^{12}\left(-\frac{d}{e}\right)^{\frac{1}{3}}+4\,a^3be^{13}\left(-\frac{d}{e}\right)^{\frac{1}{3}}+6\,b^2c^2d^2e}{3\,de^{13}}$$

```
[In] integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(6*b^2*c^2*d^2*e + 4*a*c^3*d^2*e - 4*a*b^3*d*e^2 - 12*a^2*b*c*
d*e^2 + a^4*e^3 - 4*(-d*e^2)^(1/3)*b*c^3*d^2 + (-d*e^2)^(1/3)*b^4*d*e + 12*
(-d*e^2)^(1/3)*a*b^2*c*d*e + 6*(-d*e^2)^(1/3)*a^2*c^2*d*e - 4*(-d*e^2)^(1/3
)*a^3*b*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d/e)^(1/3))/(-d/e)^(1/3))/((-d*e^2
)^(2/3)*e^2) - 1/6*(6*b^2*c^2*d^2*e + 4*a*c^3*d^2*e - 4*a*b^3*d*e^2 - 12*a^
2*b*c*d*e^2 + a^4*e^3 + 4*(-d*e^2)^(1/3)*b*c^3*d^2 - (-d*e^2)^(1/3)*b^4*d*e
 - 12*(-d*e^2)^(1/3)*a*b^2*c*d*e - 6*(-d*e^2)^(1/3)*a^2*c^2*d*e + 4*(-d*e^2
)^(1/3)*a^3*b*e^2)*log(x^2 + x*(-d/e)^(1/3) + (-d/e)^(2/3))/((-d*e^2)^(2/3)
*e^2) + 1/3*(c^4*d^2 - 4*b^3*c*d*e - 12*a*b*c^2*d*e + 6*a^2*b^2*e^2 + 4*a^3
*c*e^2)*log(abs(e*x^3 + d))/e^3 + 1/30*(5*c^4*e^5*x^6 + 24*b*c^3*e^5*x^5 +
45*b^2*c^2*e^5*x^4 + 30*a*c^3*e^5*x^4 - 10*c^4*d*e^4*x^3 + 40*b^3*c*e^5*x^3
 + 120*a*b*c^2*e^5*x^3 - 60*b*c^3*d*e^4*x^2 + 15*b^4*e^5*x^2 + 180*a*b^2*c*
e^5*x^2 + 90*a^2*c^2*e^5*x^2 - 180*b^2*c^2*d*e^4*x - 120*a*c^3*d*e^4*x + 12
0*a*b^3*e^5*x + 360*a^2*b*c*e^5*x)/e^6 - 1/3*(4*b*c^3*d^2*e^11*(-d/e)^(1/3)
 - b^4*d*e^12*(-d/e)^(1/3) - 12*a*b^2*c*d*e^12*(-d/e)^(1/3) - 6*a^2*c^2*d*e
^12*(-d/e)^(1/3) + 4*a^3*b*e^13*(-d/e)^(1/3) + 6*b^2*c^2*d^2*e^11 + 4*a*c^3
*d^2*e^11 - 4*a*b^3*d*e^12 - 12*a^2*b*c*d*e^12 + a^4*e^13)*(-d/e)^(1/3)*log
(abs(x - (-d/e)^(1/3)))/(d*e^13)
```

Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 2971, normalized size of antiderivative = 4.61

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3} dx = \text{Too large to display}$$

```
[In] int((a + b*x + c*x^2)^4/(d + e*x^3),x)
```

```
[Out] x^2*((b^4 + 6*a^2*c^2 + 12*a*b^2*c)/(2*e) - (2*b*c^3*d)/e^2) - x^3*((c^4*d)
/(3*e^2) - (4*b*c*(3*a*c + b^2))/(3*e)) + symsum(log(root(27*d^2*e^9*z^3 +
324*a*b*c^2*d^3*e^7*z^2 + 108*b^3*c*d^3*e^7*z^2 - 108*a^3*c*d^2*e^8*z^2 - 1
62*a^2*b^2*d^2*e^8*z^2 - 27*c^4*d^4*e^6*z^2 - 72*a*b*c^6*d^5*e^4*z + 216*a^
2*b^2*c^4*d^4*e^5*z + 144*a^3*b^3*c^2*d^3*e^6*z - 108*a^5*b^2*c*d^2*e^7*z +
 108*a^2*b^5*c*d^3*e^6*z - 36*a^4*b*c^3*d^3*e^6*z + 36*a*b^4*c^3*d^4*e^5*z
+ 144*b^3*c^5*d^5*e^4*z + 90*b^6*c^2*d^4*e^5*z - 144*a^3*c^5*d^4*e^5*z + 90
*a^6*c^2*d^2*e^7*z + 171*a^4*b^4*d^2*e^7*z + 36*a*b^7*d^3*e^6*z + 36*a^7*b*
d*e^8*z + 9*c^8*d^6*e^3*z + 36*a^7*b^4*c*d^2*e^6 - 36*a^7*b*c^4*d^3*e^5 - 3
6*a^4*b^7*c*d^3*e^5 - 36*a^4*b*c^7*d^5*e^3 - 36*a*b^7*c^4*d^5*e^3 + 36*a*b^
4*c^7*d^6*e^2 + 12*a*b^10*c*d^4*e^4 + 108*a^5*b^5*c^2*d^3*e^5 - 108*a^5*b^2
*c^5*d^4*e^4 + 108*a^2*b^5*c^5*d^5*e^3 - 96*a^6*b^3*c^3*d^3*e^5 + 96*a^3*b^
6*c^3*d^4*e^4 - 96*a^3*b^3*c^6*d^5*e^3 - 54*a^8*b^2*c^2*d^2*e^6 - 54*a^2*b^
8*c^2*d^4*e^4 - 54*a^2*b^2*c^8*d^6*e^2 - 9*a^4*b^4*c^4*d^4*e^4 - 12*a^10*b*
c*d*e^7 - 12*a*b*c^10*d^7*e - 6*b^6*c^6*d^6*e^2 + 4*b^9*c^3*d^5*e^3 - 6*a^6
```

*c^6*d^4*e^4 - 4*a^9*c^3*d^2*e^6 - 4*a^3*c^9*d^6*e^2 - 6*a^6*b^6*d^2*e^6 +
4*a^3*b^9*d^3*e^5 + 4*b^3*c^9*d^7*e + 4*a^9*b^3*d*e^7 - b^12*d^4*e^4 - c^12
*d^8 - a^12*e^8, z, k)*((x*(3*a^4*e^5 + 12*a*c^3*d^2*e^3 + 18*b^2*c^2*d^2*e
^3 - 12*a*b^3*d*e^4 - 36*a^2*b*c*d*e^4))/e^3 - (6*c^4*d^3*e^3 + 36*a^2*b^2*
d*e^5 - 24*b^3*c*d^2*e^4 + 24*a^3*c*d*e^5 - 72*a*b*c^2*d^2*e^4)/e^4 + 9*roo
t(27*d^2*e^9*z^3 + 324*a*b*c^2*d^3*e^7*z^2 + 108*b^3*c*d^3*e^7*z^2 - 108*a^
3*c*d^2*e^8*z^2 - 162*a^2*b^2*d^2*e^8*z^2 - 27*c^4*d^4*e^6*z^2 - 72*a*b*c^6
*d^5*e^4*z + 216*a^2*b^2*c^4*d^4*e^5*z + 144*a^3*b^3*c^2*d^3*e^6*z - 108*a^
5*b^2*c*d^2*e^7*z + 108*a^2*b^5*c*d^3*e^6*z - 36*a^4*b*c^3*d^3*e^6*z + 36*a
*b^4*c^3*d^4*e^5*z + 144*b^3*c^5*d^5*e^4*z + 90*b^6*c^2*d^4*e^5*z - 144*a^3
*c^5*d^4*e^5*z + 90*a^6*c^2*d^2*e^7*z + 171*a^4*b^4*d^2*e^7*z + 36*a*b^7*d^
3*e^6*z + 36*a^7*b*d*e^8*z + 9*c^8*d^6*e^3*z + 36*a^7*b^4*c*d^2*e^6 - 36*a^
7*b*c^4*d^3*e^5 - 36*a^4*b^7*c*d^3*e^5 - 36*a^4*b*c^7*d^5*e^3 - 36*a*b^7*c^
4*d^5*e^3 + 36*a*b^4*c^7*d^6*e^2 + 12*a*b^10*c*d^4*e^4 + 108*a^5*b^5*c^2*d^
3*e^5 - 108*a^5*b^2*c^5*d^4*e^4 + 108*a^2*b^5*c^5*d^5*e^3 - 96*a^6*b^3*c^3*
d^3*e^5 + 96*a^3*b^6*c^3*d^4*e^4 - 96*a^3*b^3*c^6*d^5*e^3 - 54*a^8*b^2*c^2*
d^2*e^6 - 54*a^2*b^8*c^2*d^4*e^4 - 54*a^2*b^2*c^8*d^6*e^2 - 9*a^4*b^4*c^4*d
^4*e^4 - 12*a^10*b*c*d*e^7 - 12*a*b*c^10*d^7*e - 6*b^6*c^6*d^6*e^2 + 4*b^9*
c^3*d^5*e^3 - 6*a^6*c^6*d^4*e^4 - 4*a^9*c^3*d^2*e^6 - 4*a^3*c^9*d^6*e^2 - 6
*a^6*b^6*d^2*e^6 + 4*a^3*b^9*d^3*e^5 + 4*b^3*c^9*d^7*e + 4*a^9*b^3*d*e^7 -
b^12*d^4*e^4 - c^12*d^8 - a^12*e^8, z, k)*d*e^2) + (c^8*d^5 + 4*a^7*b*e^5 +
4*a*b^7*d^2*e^3 + 19*a^4*b^4*d*e^4 + 10*a^6*c^2*d*e^4 + 16*b^3*c^5*d^4*e -
16*a^3*c^5*d^3*e^2 + 10*b^6*c^2*d^3*e^2 - 8*a*b*c^6*d^4*e + 24*a^2*b^2*c^4
*d^3*e^2 + 16*a^3*b^3*c^2*d^2*e^3 - 12*a^5*b^2*c*d*e^4 + 4*a*b^4*c^3*d^3*e^
2 + 12*a^2*b^5*c*d^2*e^3 - 4*a^4*b*c^3*d^2*e^3)/e^4 + (x*(10*a^6*b^2*e^4 -
4*a^7*c*e^4 - 4*a*c^7*d^4 + 10*b^2*c^6*d^4 + b^8*d^2*e^2 + 16*a^3*b^5*d*e^3
+ 16*b^5*c^3*d^3*e + 19*a^4*c^4*d^2*e^2 + 24*a^2*b^4*c^2*d^2*e^2 - 16*a^3*
b^2*c^3*d^2*e^2 - 4*a*b^3*c^4*d^3*e + 8*a*b^6*c*d^2*e^2 + 12*a^2*b*c^5*d^3*
e - 4*a^4*b^3*c*d*e^3 + 12*a^5*b*c^2*d*e^3))/e^3)*root(27*d^2*e^9*z^3 + 324
*a*b*c^2*d^3*e^7*z^2 + 108*b^3*c*d^3*e^7*z^2 - 108*a^3*c*d^2*e^8*z^2 - 162*
a^2*b^2*d^2*e^8*z^2 - 27*c^4*d^4*e^6*z^2 - 72*a*b*c^6*d^5*e^4*z + 216*a^2*b
^2*c^4*d^4*e^5*z + 144*a^3*b^3*c^2*d^3*e^6*z - 108*a^5*b^2*c*d^2*e^7*z + 10
8*a^2*b^5*c*d^3*e^6*z - 36*a^4*b*c^3*d^3*e^6*z + 36*a*b^4*c^3*d^4*e^5*z + 1
44*b^3*c^5*d^5*e^4*z + 90*b^6*c^2*d^4*e^5*z - 144*a^3*c^5*d^4*e^5*z + 90*a^
6*c^2*d^2*e^7*z + 171*a^4*b^4*d^2*e^7*z + 36*a*b^7*d^3*e^6*z + 36*a^7*b*d*e
^8*z + 9*c^8*d^6*e^3*z + 36*a^7*b^4*c*d^2*e^6 - 36*a^7*b*c^4*d^3*e^5 - 36*a
^4*b^7*c*d^3*e^5 - 36*a^4*b*c^7*d^5*e^3 - 36*a*b^7*c^4*d^5*e^3 + 36*a*b^4*c
^7*d^6*e^2 + 12*a*b^10*c*d^4*e^4 + 108*a^5*b^5*c^2*d^3*e^5 - 108*a^5*b^2*c^
5*d^4*e^4 + 108*a^2*b^5*c^5*d^5*e^3 - 96*a^6*b^3*c^3*d^3*e^5 + 96*a^3*b^6*c
^3*d^4*e^4 - 96*a^3*b^3*c^6*d^5*e^3 - 54*a^8*b^2*c^2*d^2*e^6 - 54*a^2*b^8*c
^2*d^4*e^4 - 54*a^2*b^2*c^8*d^6*e^2 - 9*a^4*b^4*c^4*d^4*e^4 - 12*a^10*b*c*d
*e^7 - 12*a*b*c^10*d^7*e - 6*b^6*c^6*d^6*e^2 + 4*b^9*c^3*d^5*e^3 - 6*a^6*c^
6*d^4*e^4 - 4*a^9*c^3*d^2*e^6 - 4*a^3*c^9*d^6*e^2 - 6*a^6*b^6*d^2*e^6 + 4*a
^3*b^9*d^3*e^5 + 4*b^3*c^9*d^7*e + 4*a^9*b^3*d*e^7 - b^12*d^4*e^4 - c^12*d^
8 - a^12*e^8, z, k), k, 1, 3) - x*((d*(4*a*c^3 + 6*b^2*c^2))/e^2 - (4*a*b*(

```
3*a*c + b^2))/e) + (c^4*x^6)/(6*e) + (x^4*(4*a*c^3 + 6*b^2*c^2))/(4*e) + (4
*b*c^3*x^5)/(5*e)
```

3.76 $\int \frac{2x^2+x^4}{1+x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{2x^2+x^4}{1+x^3}\,dx=\frac{x^2}{2}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\frac{1}{2}\log\left(1-x+x^2\right)$$

```
[Out] 1/2*x^2+ln(1+x)+1/2*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1607, 1901, 1888, 31, 648, 632, 210, 642}

$$\int \frac{2x^2+x^4}{1+x^3}\,dx=\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^2}{2}+\frac{1}{2}\log\left(x^2-x+1\right)+\log(x+1)$$

```
[In] Int[(2*x^2 + x^4)/(1 + x^3),x]
```

```
[Out] x^2/2 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x] + Log[1 - x + x^2]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1888

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[q*((A - B*q
+ C*q^2)/(3*a)), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(2+x^2)}{1+x^3}\,dx$$

$$= \int \left(x + \frac{x(-1+2x)}{1+x^3}\right) dx$$

$$= \frac{x^2}{2} + \int \frac{x(-1+2x)}{1+x^3} dx$$

$$= \frac{x^2}{2} + \frac{1}{3}\int \frac{-3+3x}{1-x+x^2} dx + \int \frac{1}{1+x} dx$$

$$= \frac{x^2}{2} + \log(1+x) - \frac{1}{2}\int \frac{1}{1-x+x^2} dx + \frac{1}{2}\int \frac{-1+2x}{1-x+x^2} dx$$

$$= \frac{x^2}{2} + \log(1+x) + \frac{1}{2}\log\left(1-x+x^2\right) + \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right)$$

$$= \frac{x^2}{2} - \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log(1+x) + \frac{1}{2}\log\left(1-x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{2x^2+x^4}{1+x^3} dx = \frac{1}{6}\left(3x^2 - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 2\log(1+x) - \log\left(1-x+x^2\right) + 4\log\left(1+x^3\right)\right)$$

```
[In] Integrate[(2*x^2 + x^4)/(1 + x^3),x]
[Out] (3*x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - Log[1 - x +
x^2] + 4*Log[1 + x^3])/6
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x^2}{2} + \ln(1+x) + \frac{\ln(x^2-x+1)}{2} - \frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	38
risch	$\frac{x^2}{2} + \frac{\ln(4x^2-4x+4)}{2} - \frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3} + \ln(1+x)$	40
meijerg	$\frac{x^2}{2} - \frac{x^2\left(-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{(x^3)^{\frac{2}{3}}} + \frac{\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2(x^3)^{\frac{2}{3}}} + \frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{(x^3)^{\frac{2}{3}}}\right)}{3} + \frac{2\ln(x^3+1)}{3}$	89

```
[In] int((x^4+2*x^2)/(x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+ln(1+x)+1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{2x^2+x^4}{1+x^3}\,dx=\frac{1}{2}\,x^2-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)+\log\left(x+1\right)$$

```
[In] integrate((x^4+2*x^2)/(x^3+1),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
+ log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{2x^2+x^4}{1+x^3}\,dx=\frac{x^2}{2}+\log\left(x+1\right)+\frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((x**4+2*x**2)/(x**3+1),x)
[Out] x**2/2 + log(x + 1) + log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sq
rt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{2x^2+x^4}{1+x^3}\,dx=\frac{1}{2}\,x^2-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)+\log\left(x+1\right)$$

```
[In] integrate((x^4+2*x^2)/(x^3+1),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
+ log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{2x^2+x^4}{1+x^3}\,dx = \frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{2}\,\log\left(x^2-x+1\right) + \log\left(|x+1|\right)$$

```
[In] integrate((x^4+2*x^2)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
+ log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{2x^2+x^4}{1+x^3}\,dx = \ln(x+1) + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \frac{x^2}{2}$$

```
[In] int((2*x^2 + x^4)/(x^3 + 1),x)
```

```
[Out] log(x + 1) + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/2) - log(x +
 (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/2) + x^2/2
```

3.77 $\int \frac{2x^2+x^4}{1-x^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{2x^2+x^4}{1-x^3} dx = -\frac{x^2}{2} - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log(1-x) - \frac{1}{2}\log\left(1+x+x^2\right)$$

```
[Out] -1/2*x^2-ln(1-x)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1607, 1901, 1889, 31, 648, 632, 210, 642}

$$\int \frac{2x^2+x^4}{1-x^3} dx = -\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{x^2}{2} - \frac{1}{2}\log\left(x^2+x+1\right) - \log(1-x)$$

```
[In] Int[(2*x^2 + x^4)/(1 - x^3),x]
```

```
[Out] -1/2*x^2 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x] - Log[1 + x + x^2
]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1889

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Dist[q*((A + B*q
 + C*q^2)/(3*a)), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q -
 C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b
*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
 && LtQ[a/b, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(2+x^2)}{1-x^3}\,dx$$

$$= \int \left(-x + \frac{x(1+2x)}{1-x^3}\right) dx$$

$$= -\frac{x^2}{2} + \int \frac{x(1+2x)}{1-x^3} \, dx$$

$$= -\frac{x^2}{2} + \frac{1}{3} \int \frac{-3-3x}{1+x+x^2} \, dx + \int \frac{1}{1-x} \, dx$$

$$= -\frac{x^2}{2} - \log(1-x) - \frac{1}{2} \int \frac{1}{1+x+x^2} \, dx - \frac{1}{2} \int \frac{1+2x}{1+x+x^2} \, dx$$

$$= -\frac{x^2}{2} - \log(1-x) - \frac{1}{2} \log\left(1+x+x^2\right) + \text{Subst}\left(\int \frac{1}{-3-x^2} \, dx, x, 1+2x\right)$$

$$= -\frac{x^2}{2} - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log(1-x) - \frac{1}{2} \log\left(1+x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{2x^2+x^4}{1-x^3} \, dx = \frac{1}{6}\left(-3x^2 - 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 2\log(1-x) + \log\left(1+x+x^2\right) - 4\log\left(1-x^3\right)\right)$$

```
[In] Integrate[(2*x^2 + x^4)/(1 - x^3),x]
```

```
[Out] (-3*x^2 - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[1 - x] + Log[1 + x +
x^2] - 4*Log[1 - x^3])/6
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{x^2}{2}-\ln\left(-1+x\right)-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}-\frac{\ln\left(x^2+x+1\right)}{2}$	36
default	$-\frac{x^2}{2}-\ln\left(-1+x\right)-\frac{\ln\left(x^2+x+1\right)}{2}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	38
meijerg	$\frac{(-1)^{\frac{1}{3}}\left(\frac{3x^2(-1)^{\frac{2}{3}}}{2}+\frac{x^2(-1)^{\frac{2}{3}}\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2+\left(x^3\right)^{\frac{1}{3}}}\right)\right)}{\left(x^3\right)^{\frac{2}{3}}}\right)}{3}-\frac{2\ln\left(-x^3+1\right)}{3}$	90

```
[In] int((x^4+2*x^2)/(-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2-ln(-1+x)-1/3*3^(1/2)*arctan(2/3*(x+1/2)*3^(1/2))-1/2*ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{2x^2+x^4}{1-x^3}\,dx=-\frac{1}{2}\,x^2-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{1}{2}\,\log\left(x^2+x+1\right)-\log\left(x-1\right)$$

```
[In] integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1)
 - log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{2x^2+x^4}{1-x^3}\,dx=-\frac{x^2}{2}-\log\left(x-1\right)-\frac{\log\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((x**4+2*x**2)/(-x**3+1),x)
```

```
[Out] -x**2/2 - log(x - 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + s
qrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{2x^2+x^4}{1-x^3}\,dx = -\frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{2}\,\log\left(x^2+x+1\right) - \log\left(x-1\right)$$

```
[In] integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="maxima")
[Out] -1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1)
 - log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{2x^2+x^4}{1-x^3}\,dx = -\frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{2}\,\log\left(x^2+x+1\right) - \log\left(|x-1|\right)$$

```
[In] integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="giac")
[Out] -1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1)
 - log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{2x^2+x^4}{1-x^3}\,dx = -\ln\left(x-1\right) + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \frac{x^2}{2}$$

```
[In] int(-(2*x^2 + x^4)/(x^3 - 1),x)
[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/2) - log(x - 1) - log(x +
 (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/2) - x^2/2
```

3.78 $\int \frac{1-x+4x^3}{1+x^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 44

$$\int \frac{1-x+4x^3}{1+x^3} \, dx = 4x + \frac{4\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\log(1+x) + \frac{1}{3}\log\left(1-x+x^2\right)$$

```
[Out] 4*x-2/3*ln(1+x)+1/3*ln(x^2-x+1)+4/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1901, 1874, 31, 648, 632, 210, 642}

$$\int \frac{1-x+4x^3}{1+x^3} \, dx = \frac{4\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(x^2-x+1\right) + 4x - \frac{2}{3}\log(x+1)$$

```
[In] Int[(1 - x + 4*x^3)/(1 + x^3),x]
```

```
[Out] 4*x + (4*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 - x
+ x^2]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(4-\frac{3+x}{1+x^3}\right) dx \\
&= 4x-\int \frac{3+x}{1+x^3}\,dx \\
&= 4x-\frac{1}{3}\int \frac{7-2x}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1}{1+x}\,dx \\
&= 4x-\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-1+2x}{1-x+x^2}\,dx-2\int \frac{1}{1-x+x^2}\,dx
\end{aligned}
$$

$$= 4x - \frac{2}{3}\log(1+x) + \frac{1}{3}\log\left(1 - x + x^2\right) + 4\text{Subst}\left(\int \frac{1}{-3 - x^2}\,dx, x, -1 + 2x\right)$$

$$= 4x + \frac{4\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\log(1+x) + \frac{1}{3}\log\left(1 - x + x^2\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1 - x + 4x^3}{1 + x^3}\,dx = 4x - \frac{4\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\log(1+x) + \frac{1}{3}\log\left(1 - x + x^2\right)$$

```
[In] Integrate[(1 - x + 4*x^3)/(1 + x^3),x]
[Out] 4*x - (4*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 - x
 + x^2]/3
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result
default	$4x - \frac{2\ln(1+x)}{3} + \frac{\ln(x^2-x+1)}{3} - \frac{4\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$
risch	$4x - \frac{2\ln(1+x)}{3} + \frac{\ln(16x^2-16x+16)}{3} - \frac{4\sqrt{3}\,\arctan\left(\frac{(-2+4x)\sqrt{3}}{6}\right)}{3}$
meijerg	$4x - \frac{4x\left(\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}\right)}{\left(x^3\right)^{\frac{1}{3}}} - \frac{\ln\left(1-\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2\left(x^3\right)^{\frac{1}{3}}} + \frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2-\left(x^3\right)^{\frac{1}{3}}}\right)}{\left(x^3\right)^{\frac{1}{3}}}\right)}{3} + \frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}} - \frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}$

```
[In] int((4*x^3-x+1)/(x^3+1),x,method=_RETURNVERBOSE)
[Out] 4*x-2/3*ln(1+x)+1/3*ln(x^2-x+1)-4/3*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = -\frac{4}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+4\,x+\frac{1}{3}\,\log\left(x^2-x+1\right)-\frac{2}{3}\,\log\left(x+1\right)$$

```
[In] integrate((4*x^3-x+1)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) - 2
/3*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = 4x-\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{3}-\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((4*x**3-x+1)/(x**3+1),x)
```

```
[Out] 4*x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3 - 4*sqrt(3)*atan(2*sqrt(3)*x/3 -
 sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = -\frac{4}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+4\,x+\frac{1}{3}\,\log\left(x^2-x+1\right)-\frac{2}{3}\,\log\left(x+1\right)$$

```
[In] integrate((4*x^3-x+1)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) - 2
/3*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = -\frac{4}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+4\,x+\frac{1}{3}\,\log\left(x^2-x+1\right)-\frac{2}{3}\,\log\left(|x+1|\right)$$

```
[In] integrate((4*x^3-x+1)/(x^3+1),x, algorithm="giac")
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) - 2
/3*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = 4\,x-\frac{2\,\ln\left(x+1\right)}{3}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)$$

```
[In] int((4*x^3 - x + 1)/(x^3 + 1),x)
[Out] 4*x - (2*log(x + 1))/3 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*2i)/3 + 1/
3) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*2i)/3 - 1/3)
```

3.79 $\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 230

$$\begin{aligned}\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx &= \frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} \\ &\quad + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\end{aligned}$$

```
[Out] 2*(x^3+1)^(1/2)/(1+x+3^(1/2))-3^(1/4)*(1+x)*EllipticE((1+x-3^(1/2))/(1+x+3^
(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)
^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1+x)*Elliptic
F((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^
2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1892, 224, 1891}

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} + \frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

```
[In] Int[(1 + Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sq
rt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4
*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

<u>Rule 224</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

<u>Rule 1891</u>

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx + \int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx\\ &= \frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\\ &\quad + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.20

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \left(1+\sqrt{3}\right)x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right) + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

```
[In] Integrate[(1 + Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

```
[Out] (1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (x^2*Hypergeometri
c2F1[1/2, 2/3, 5/3, -x^3])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.79 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.20

method	result
meijerg	$x {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -x^3\right) + \frac{x^2 {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -x^3\right)}{2} + \sqrt{3}\, x {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -x^3\right)$
elliptic	$\frac{2\left(1+\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} + 2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} + 2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}$

```
[In] int((1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*hypergeom([1/3,1/2],[4/3],-x^3)+1/2*x^2*hypergeom([1/2,2/3],[5/3],-x^3)+3
^(1/2)*x*hypergeom([1/3,1/2],[4/3],-x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.09

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = 2\left(\sqrt{3}+1\right)\text{weierstrassPInverse}(0,-4,x) - 2\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

```
[In] integrate((1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(3) + 1)*weierstrassPInverse(0, -4, x) - 2*weierstrassZeta(0, -4, we
ierstrassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.40

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3}, \frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3}, \frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((1+x+3**(1/2))/(x**3+1)**(1/2),x)
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/
3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma
(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi
))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.36

$$\begin{aligned}
&\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx \\
&= \sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right) \\
&\quad - \frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} \\
&\quad + \frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}
\end{aligned}$$

```
[In] int((x + 3^(1/2) + 1)/(x^3 + 1)^(1/2),x)
```

```
[Out] 3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, -x^3) - (6*((x + (3^(1/2)*1i)/2 - 1/2)
/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^
(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x + 1
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) + (6*((x + (3^(1/2)*1i)/2
- 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((
(x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1
i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1)
- ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.80 $\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 257

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2*(-x^3+1)^(1/2)/(1-x+3^(1/2))+3^(1/4)*(1-x)*EllipticE((1-x-3^(1/2))/(1-x+
3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^
2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-4*3^(1/4)*(1-x)*Ellip
ticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*(
(x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1892, 224, 1891}

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

```
[In] Int[(1 + Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(
1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt
[1 - x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + S
qrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 -
4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}\,dx + \int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx\\ &= -\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}\\ &\quad - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.17

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \left(1+\sqrt{3}\right)x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right) - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)$$

```
[In] Integrate[(1 + Sqrt[3] - x)/Sqrt[1 - x^3],x]
[Out] (1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] - (x^2*Hypergeometric
2F1[1/2, 2/3, 5/3, x^3])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.61 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.16

method	result
meijerg	$x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)-\frac{x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)}{2}+\sqrt{3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)$
elliptic	$-\frac{2i\left(1+\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}\right.}}{}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{}$

```
[In] int((1-x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*hypergeom([1/3,1/2],[4/3],x^3)-1/2*x^2*hypergeom([1/2,2/3],[5/3],x^3)+3^(
1/2)*x*hypergeom([1/3,1/2],[4/3],x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.09

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -2\left(i\sqrt{3}+i\right)\text{weierstrassPInverse}(0,4,x) - 2i\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

```
[In] integrate((1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(I*sqrt(3) + I)*weierstrassPInverse(0, 4, x) - 2*I*weierstrassZeta(0, 4,
 weierstrassPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.38

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -\frac{x^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((1-x+3**(1/2))/(-x**3+1)**(1/2),x)
[Out] -x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*
gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar
(2*I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

```
[In] integrate((1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x - sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

```
[In] integrate((1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x - sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.33

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$$
$$=\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)$$
$$+\frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$-\frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int((3^(1/2) - x + 1)/(1 - x^3)^(1/2),x)
```

```
[Out] 3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, x^3) + (6*(x^3 - 1)^(1/2)*(-(x - (3^(1
/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellip
ticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)
/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^
3)^(1/2)) - (6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-
(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 -
x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.81 $\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 144

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx=\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] 2*(x^3-1)^(1/2)/(1-x-3^(1/2))-3^(1/4)*(1-x)*EllipticE((1-x+3^(1/2))/(1-x-3^
(1/2)),2*I-I*3^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^
(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1893}

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx=\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[In] Int[(1 + Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*S
qrt[-1 + x^3])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.44

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = -\frac{x\sqrt{1-x^3}\left(-2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)}{2\sqrt{-1+x^3}}$$

```
[In] Integrate[(1 + Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 - x^3]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^
3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/Sqrt[-1 + x^3]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.72 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.66

method	result
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x\, {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}} - \frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x^2\, {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)}{2\sqrt{\operatorname{signum}(x^3-1)}} + \frac{\sqrt{3}\,\sqrt{-\operatorname{signum}(x^3-1)}\, x\, {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}$
elliptic	$\frac{2\left(1+\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\cdots}{\frac{3}{2}+\cdots}}\cdots}{\cdots}$

```
[In] int((1-x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/2],[4/3],x^
3)-1/2/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^2*hypergeom([1/2,2/3],[
5/3],x^3)+3^(1/2)/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1
/3,1/2],[4/3],x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.15

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = 2\left(\sqrt{3}+1\right)\text{weierstrassPInverse}(0,4,x) + 2\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

```
[In] integrate((1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(3) + 1)*weierstrassPInverse(0, 4, x) + 2*weierstrassZeta(0, 4, weie
rstrassPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.57

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \frac{ix^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - sqrt(3)*
I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - I*x*gamma(1
/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

```
[In] integrate((1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

```
[In] integrate((1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.26

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$$
$$=\frac{\sqrt{3}\,x\,\sqrt{1-x^3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)}{\sqrt{x^3-1}}$$
$$+\frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$
$$-\frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

```
[In] int((3^(1/2) - x + 1)/(x^3 - 1)^(1/2),x)
```

```
[Out] (3^(1/2)*x*(1 - x^3)^(1/2)*hypergeom([1/3, 1/2], 4/3, x^3))/(x^3 - 1)^(1/2)
 + (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(
1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x
^3)^(1/2) - (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*(
(x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 + 1) + x^3)^(1/2)
```

3.82 $\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 135

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx=-\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] -2*(-x^3-1)^(1/2)/(1+x-3^(1/2))+3^(1/4)*(1+x)*EllipticE((1+x+3^(1/2))/(1+x-
3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*
2^(1/2))/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1893}

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx=\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

```
[In] Int[(1 + Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/
(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*
Sqrt[-1 - x^3])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \frac{x\sqrt{1+x^3}\left(2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

```
[In] Integrate[(1 + Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

```
[Out] (x*Sqrt[1 + x^3]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] +
x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.75 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.39

method	result
meijerg	$-ix\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)-\frac{ix^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)}{2}-i\sqrt{3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)$
elliptic	$-\frac{2i\left(1+\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\cdots\right.}}{\cdots}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\cdots}{\cdots}$

```
[In] int((1+x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*x*hypergeom([1/3,1/2],[4/3],-x^3)-1/2*I*x^2*hypergeom([1/2,2/3],[5/3],-x
^3)-I*3^(1/2)*x*hypergeom([1/3,1/2],[4/3],-x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.17

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -2\left(i\sqrt{3}+i\right)\text{weierstrassPInverse}(0,-4,x) + 2i\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

```
[In] integrate((1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(I*sqrt(3) + I)*weierstrassPInverse(0, -4, x) + 2*I*weierstrassZeta(0, -
4, weierstrassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.73

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -\frac{ix^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((1+x+3**(1/2))/(-x**3-1)**(1/2),x)
[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma
(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*
pi))/(3*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_pol
ar(I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

```
[In] integrate((1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

```
[In] integrate((1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.67

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \frac{\sqrt{3}\,x\,\sqrt{x^3+1}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)}{\sqrt{-x^3-1}} - \frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} + \frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int((x + 3^(1/2) + 1)/(- x^3 - 1)^(1/2),x)
```

```
[Out] (3^(1/2)*x*(x^3 + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3, -x^3))/(- x^3 - 1)^(1
/2) - (6*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2)
)^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1
/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) + (6*(x^3 + 1)^(1/2)*((x + (3
^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ell
ipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2
)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2))^(1/2))
```

3.83 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 468

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2*(b*x^3+a)^(1/2)/b^(1/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)*(
a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(
b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)+4*3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3
)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1892, 224, 1891}

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[
3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[
3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \left(2\sqrt{3}\sqrt[3]{a}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx + \int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.19

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2(1+\sqrt{3})\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2,
 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/
a)]))/(2*Sqrt[a + b*x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. 2(346) = 692.

Time = 1.75 (sec) , antiderivative size = 1003, normalized size of antiderivative = 2.14

method	result	size
default	Expression too large to display	1003

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))-2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(
```

```
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2*I*a^(1/3)/b*(
-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.10

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \frac{2\left(a^{\frac{1}{3}}\sqrt{b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-b^{\frac{5}{6}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b}\right.\right.\right.}{b}$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2*(a^(1/3)*sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - b^(5/6
)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.26

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*ex
```

```
p_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3,
 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Maxima [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}+1\right)}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(a + b*x^3)^(1/2),x)
```

```
[Out] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(a + b*x^3)^(1/2), x)
```

3.84 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 481

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] -2*(-b*x^3+a)^(1/2)/b^(1/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+3^(1/4)*a^(1/3
)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(
1/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)-4*3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)
*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b
^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1892, 224, 1891}

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx =$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt
[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
- b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \left(2\sqrt{3}\sqrt[3]{a}\right)\int\frac{1}{\sqrt{a-bx^3}}\,dx+\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$$

$$= -\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.19

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = \frac{x\sqrt{1-\frac{bx^3}{a}}\left(2(1+\sqrt{3})\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)-\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2,
 4/3, (b*x^3)/a] - b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/
(2*Sqrt[a - b*x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 948 vs. 2(359) = 718.

Time = 1.80 (sec) , antiderivative size = 949, normalized size of antiderivative = 1.97

method	result	size
default	Expression too large to display	949

```
[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 2/3*I*a^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/
(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-
b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1
/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2*I*a^(1/3
)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))
*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*
3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I/b^(2/3)*3^(1/2)*(a*b^2)^
(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(
```

```
a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^
2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-
I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/
3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/
2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1
/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -\frac{2\left(a^{\frac{1}{3}}\sqrt{-b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+\sqrt{-b}b^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0\right.\right.\right.}{b}$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] -2*(a^(1/3)*sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + sqrt(
-b)*b^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.27

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -\frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*I*pi
)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3
```

```
*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper(
(1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Maxima [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}+1)}{\sqrt{-bx^3+a}}\,dx$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}+1)}{\sqrt{-bx^3+a}}\,dx$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -\int \frac{b^{1/3}\,x-a^{1/3}\,(\sqrt{3}+1)}{\sqrt{a-b\,x^3}}\,dx$$

```
[In] int(-(b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(a - b*x^3)^(1/2),x)
```

```
[Out] -int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(a - b*x^3)^(1/2), x)
```

3.85 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 271

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

```
[Out] 2*(b*x^3-a)^(1/2)/b^(1/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))-3^(1/4)*a^(1/3)*
(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+
a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(1/
3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used

$= \{1893\}$

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.34

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{x\sqrt{1-\frac{bx^3}{a}}\left(2(1+\sqrt{3})\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)-\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2,
 4/3, (b*x^3)/a] - b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/
(2*Sqrt[-a + b*x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 951 vs. 2(206) = 412.

Time = 1.76 (sec) , antiderivative size = 952, normalized size of antiderivative = 3.51

method	result	size
default	Expression too large to display	952

```
[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(
-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*
x^3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*Elliptic
E(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(
1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3
^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/
(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*a^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(
-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^
(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3
))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I
```

```
*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1
/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(a*b^2)^(1/3)))^(1/2))+2*I*a^(1/3)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*
(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*
(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^
(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2
)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/
2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.18

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\left(a^{\frac{1}{3}}\sqrt{b}(\sqrt{3}+1)\operatorname{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+b^{\frac{5}{6}}\operatorname{weierstrassZeta}\left(0,\frac{4a}{b},\operatorname{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2*(a^(1/3)*sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + b^(5/6)
*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.41

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*g
amma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*
```

```
a**(1/6)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(
3*a**(1/6)*gamma(4/3))
```

Maxima [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3-a}}\,dx$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="ma
xima")
[Out] -integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x)
```

Giac [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3-a}}\,dx$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="gi
ac")
[Out] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx=-\int \frac{b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}+1\right)}{\sqrt{b\,x^3-a}}\,dx$$

```
[In] int(-(b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(b*x^3 - a)^(1/2),x)
[Out] -int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(b*x^3 - a)^(1/2), x)
```

3.86 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 266

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx=-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] -2*(-b*x^3-a)^(1/2)/b^(1/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))+3^(1/4)*a^(1/3)
*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a
^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(1/3)
/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/
2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used

$= \{1893\}$

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$$
$$= \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1
/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[
ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.35

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2,
 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/
a)]))/(2*Sqrt[-a - b*x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1011 vs. 2(201) = 402.

Time = 1.72 (sec) , antiderivative size = 1012, normalized size of antiderivative = 3.80

method	result	size
default	Expression too large to display	1012

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))-2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)
```

```
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2*I*a^(1/3)/b
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.21

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\left(a^{\frac{1}{3}}\sqrt{-b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{-b}b^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInvers}\right.\right.}{b}$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -2*(a^(1/3)*sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - sqrt
(-b)*b^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))
/b
```

Sympy [A] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.48

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

[Out] -I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))

Maxima [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3-a}}\,dx$$

[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxima")

[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a), x)

Giac [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3-a}}\,dx$$

[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac")

[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}+1\right)}{\sqrt{-b\,x^3-a}}\,dx$$

[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(- a - b*x^3)^(1/2),x)

[Out] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(- a - b*x^3)^(1/2), x)

3.87 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 520

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2*(b/a)^(1/3)*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/
4)*a^(1/3)*(b/a)^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(-a
^(1/3)*(b/a)^(1/3)*(1-3^(1/2))+b^(1/3)*(1+3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2
```

```
))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1892, 224, 1891}

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(b/a)^(1/3)*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (
2*Sqrt[2 + Sqrt[3]]*((1 + Sqrt[3])*b^(1/3) - (1 - Sqrt[3])*a^(1/3)*(b/a)^(1
/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3
^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} + \left(1+\sqrt{3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\right)\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.17

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -(
(b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])
)/(2*Sqrt[a + b*x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1003 vs. 2(386) = 772.

Time = 1.74 (sec) , antiderivative size = 1004, normalized size of antiderivative = 1.93

method	result	size
default	Expression too large to display	1004

```
[In] int((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2*I/
b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3
^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt{b}(\sqrt{3}+1)\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0\right.\right.\right.}{b}$$

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - sqrt(b)*(b/a)^
(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.24

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{x^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3+a)**(1/2),x)
```

```
[Out] x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*p
i)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**
3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1
/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx=\int \frac{\sqrt{3}+x\,{\left(\frac{b}{a}\right)}^{1/3}+1}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((3^(1/2) + x*(b/a)^(1/3) + 1)/(a + b*x^3)^(1/2),x)
[Out] int((3^(1/2) + x*(b/a)^(1/3) + 1)/(a + b*x^3)^(1/2), x)
```

3.88 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 533

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] -2*(b/a)^(1/3)*(-b*x^3+a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+3^
(1/4)*a^(1/3)*(b/a)^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-
1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/
(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-2/3*(a^(1/3)-b^(1/3)*x)*EllipticF
((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(-a^(1/3)*(b/a)^(1/3)*(1-3^(1/2))+b^(1/3)*(1+3^(1/2)))*(1/2*6^(1/2)+
```

```
1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(
1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1892, 224, 1891}

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx =$$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$-\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (-2*(b/a)^(1/3)*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) -
(2*Sqrt[2 + Sqrt[3]]*((1 + Sqrt[3])*b^(1/3) - (1 - Sqrt[3])*a^(1/3)*(b/a)^(
1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^
(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{\frac{b}{a}}\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx}{\sqrt[3]{b}} - \left(-1-\sqrt{3}+\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{a-bx^3}}\,dx$$

$$= -\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right.}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.17

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(-2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 - (b*x^3)/a]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4
/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])
)/Sqrt[a - b*x^3]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 949 vs. 2(399) = 798.

Time = 1.78 (sec) , antiderivative size = 950, normalized size of antiderivative = 1.78

method	result	size
default	Expression too large to display	950

```
[In] int((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)
^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*
b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/
2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2*I/b*(a*b^2)^(1/3
)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^
2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(
1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)
*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*
(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/
3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b
^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*
3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/
b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)
)+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^
2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.11

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{2\left(\sqrt{-b}(\sqrt{3}+1)\mathrm{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\mathrm{weierstrassZeta}\left(0,\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0\right.\right.\right.}{b}$$

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + sqrt(-b)*(b/a
)^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.24

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{x^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*
I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*
x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyp
er((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{-bx^3+a}}\,dx$$

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] -integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \int \frac{\sqrt{3}-x\,{\left(\frac{b}{a}\right)}^{1/3}+1}{\sqrt{a-b\,x^3}}\,dx$$

```
[In] int((3^(1/2) - x*(b/a)^(1/3) + 1)/(a - b*x^3)^(1/2),x)
[Out] int((3^(1/2) - x*(b/a)^(1/3) + 1)/(a - b*x^3)^(1/2), x)
```

3.89 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 256

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a+bx^3}}{b\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}\,E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{-a+bx^3}}$$

```
[Out] 2*(b/a)^(2/3)*(b*x^3-a)^(1/2)/b/(1-(b/a)^(1/3)*x-3^(1/2))-3^(1/4)*(1-(b/a)^
(1/3)*x)*EllipticE((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2)),2*I-
I*3^(1/2))*((1+(b/a)^(1/3)*x+(b/a)^(2/3)*x^2)/(1-(b/a)^(1/3)*x-3^(1/2))^2)^
(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(b/a)^(1/3)/(b*x^3-a)^(1/2)/((-1+(b/a)^(1/3
)*x)/(1-(b/a)^(1/3)*x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {1893}

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{bx^3-a}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}\sqrt{bx^3-a}}$$

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*(b/a)^(2/3)*Sqrt[-a + b*x^3])/(b*(1 - Sqrt[3] - (b/a)^(1/3)*x)) - (3^(1/
4)*Sqrt[2 + Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a)^(1/3)*x + (b/a)^(2
/3)*x^2)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (
b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(1/3)
*Sqrt[-((1 - (b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2)]*Sqrt[-a + b*x
^3])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a+bx^3}}{b\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)} \\
&\quad - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{-a+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.35

$$
\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(-2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}
$$

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 - (b*x^3)/a]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4
/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])
)/Sqrt[-a + b*x^3]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 952 vs. 2(211) = 422.

Time = 1.67 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.72

method	result	size
default	Expression too large to display	953

```
[In] int((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^
(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b
^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2
/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2*I/b*(a*b^2)^(1/3)
*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1
/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(
-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^
(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x
+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3)
)^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2
)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^
(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(
a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1
/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^
(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.20

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\left(\sqrt{b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right.\right.\right.}{b}$$

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="fricas")
[Out] 2*(sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + sqrt(b)*(b/a)^(
1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.45

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3-a)**(1/2),x)
[Out] I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(
a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)
/(3*sqrt(a)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a
)/(3*sqrt(a)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{bx^3-a}}\,dx$$

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="maxima")
[Out] -integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \int \frac{\sqrt{3}-x\,{\left(\frac{b}{a}\right)}^{1/3}+1}{\sqrt{b\,x^3-a}}\,dx$$

```
[In] int((3^(1/2) - x*(b/a)^(1/3) + 1)/(b*x^3 - a)^(1/2),x)
[Out] int((3^(1/2) - x*(b/a)^(1/3) + 1)/(b*x^3 - a)^(1/2), x)
```

3.90 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 251

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-bx^3}}{b\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}\,E\left(\arcsin\left(\frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] -2*(b/a)^(2/3)*(-b*x^3-a)^(1/2)/b/(1+(b/a)^(1/3)*x-3^(1/2))+3^(1/4)*(1+(b/a
)^(1/3)*x)*EllipticE((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2)),2*
I-I*3^(1/2))*((1-(b/a)^(1/3)*x+(b/a)^(2/3)*x^2)/(1+(b/a)^(1/3)*x-3^(1/2))^2
)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(b/a)^(1/3)/(-b*x^3-a)^(1/2)/((-1-(b/a)^(
1/3)*x)/(1+(b/a)^(1/3)*x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {1893}

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$$
$$= \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}\sqrt{-a-bx^3}}$$
$$-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}$$

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*(b/a)^(2/3)*Sqrt[-a - b*x^3])/(b*(1 - Sqrt[3] + (b/a)^(1/3)*x)) + (3^(1
/4)*Sqrt[2 + Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)^(1/3)*x + (b/a)^(
2/3)*x^2)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] +
(b/a)^(1/3)*x)/(1 - Sqrt[3] + (b/a)^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(1/3
)*Sqrt[-((1 + (b/a)^(1/3)*x)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2)]*Sqrt[-a - b*
x^3])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-bx^3}}{b\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.37

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2(1+\sqrt{3})\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -(
(b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])
)/(2*Sqrt[-a - b*x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1012 vs. 2(208) = 416.

Time = 1.73 (sec) , antiderivative size = 1013, normalized size of antiderivative = 4.04

method	result	size
default	Expression too large to display	1013

```
[In] int((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2*I
/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)
*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x
^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\left(\sqrt{-b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInver}\right.\right.}{b}$$

[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] -2*(sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - sqrt(-b)*(b/a)^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b

Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.52

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(-b*x**3-a)**(1/2),x)

[Out] -I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))

Maxima [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{-bx^3-a}}\,dx$$

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="maxima")
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{\sqrt{3}+x\,{\left(\frac{b}{a}\right)}^{1/3}+1}{\sqrt{-b\,x^3-a}}\,dx$$

```
[In] int((3^(1/2) + x*(b/a)^(1/3) + 1)/(- a - b*x^3)^(1/2),x)
[Out] int((3^(1/2) + x*(b/a)^(1/3) + 1)/(- a - b*x^3)^(1/2), x)
```

3.91 $\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}} dx$

Optimal result

Integrand size = 20, antiderivative size = 127

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}} dx = \frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 2*(x^3+1)^(1/2)/(1+x+3^(1/2))-3^(1/4)*(1+x)*EllipticE((1+x-3^(1/2))/(1+x+3^
(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)
^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1891}

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}} dx = \frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[In] Int[(1 - Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.39

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \left(1-\sqrt{3}\right)x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right) + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

```
[In] Integrate[(1 - Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

```
[Out] (1 - Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (x^2*Hypergeometri
c2F1[1/2, 2/3, 5/3, -x^3])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.72 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.38

method	result
meijerg	$x{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+\frac{x^2{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)}{2}-\sqrt{3}\,x{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)$
elliptic	$\frac{2\left(1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}}{-\frac{3}{2}}}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\right.$

```
[In] int((1+x-3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] x*hypergeom([1/3,1/2],[4/3],-x^3)+1/2*x^2*hypergeom([1/2,2/3],[5/3],-x^3)-3
^(1/2)*x*hypergeom([1/3,1/2],[4/3],-x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.17

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = -2\left(\sqrt{3}-1\right)\text{weierstrassPInverse}(0,-4,x) - 2\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

```
[In] integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] -2*(sqrt(3) - 1)*weierstrassPInverse(0, -4, x) - 2*weierstrassZeta(0, -4, w
eierstrassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/
3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/
(3*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi
))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.46

$$\begin{aligned}
&\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx\\
&=-\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)\\
&\quad-\frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\\
&\quad+\frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}
\end{aligned}$$

```
[In] int((x - 3^(1/2) + 1)/(x^3 + 1)^(1/2),x)
```

```
[Out] (6*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*
1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1
/2) - (6*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/
2))^(1/2) - 3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, -x^3)
```

3.92 $\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2*(-x^3+1)^(1/2)/(1-x+3^(1/2))+3^(1/4)*(1-x)*EllipticE((1-x-3^(1/2))/(1-x+
3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^
2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1891}

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

```
[In] Int[(1 - Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(
1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt
[1 - x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.32

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \left(1-\sqrt{3}\right)x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right) - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)$$

```
[In] Integrate[(1 - Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

```
[Out] (1 - Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] - (x^2*Hypergeometric
2F1[1/2, 2/3, 5/3, x^3])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.61 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.30

method	result
meijerg	$x {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)-\frac{x^2 {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)}{2}-\sqrt{3}\, x {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)$
elliptic	$-\frac{2i\left(1-\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}\right.}}{}$
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{}$

```
[In] int((1-x-3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*hypergeom([1/3,1/2],[4/3],x^3)-1/2*x^2*hypergeom([1/2,2/3],[5/3],x^3)-3^(
1/2)*x*hypergeom([1/3,1/2],[4/3],x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.16

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -2\left(-i\sqrt{3}+i\right)\text{weierstrassPInverse}(0,4,x) - 2i\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

```
[In] integrate((1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(-I*sqrt(3) + I)*weierstrassPInverse(0, 4, x) - 2*I*weierstrassZeta(0, 4
, weierstrassPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.68

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((1-x-3**(1/2))/(-x**3+1)**(1/2),x)
[Out] -x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*
pi))/(3*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar
(2*I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

```
[In] integrate((1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

```
[In] integrate((1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.42

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$$
$$= -\sqrt{3}\,x\,{}_2\mathrm{F}_1\!\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)$$
$$+\frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$-\frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(-(x + 3^(1/2) - 1)/(1 - x^3)^(1/2),x)
```

```
[Out] (6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/2)*(
((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*(
(3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - 3^(1/2)*x*hypergeom([1/3, 1/2],
4/3, x^3) - (6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-
(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 -
x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.93 $\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 264

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] 2*(x^3-1)^(1/2)/(1-x-3^(1/2))+4*3^(1/4)*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-
3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^
2)^(1/2)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)-3^(1/4)*(1-x)*Ellipti
cE((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(
1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1894, 225, 1893}

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$$
$$= \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}$$

```
[In] Int[(1 - Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*S
qrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -
7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx\right)+\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx\\ &= \frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\\ &\quad+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.24

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = -\frac{x\sqrt{1-x^3}\left(2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)}{2\sqrt{-1+x^3}}$$

```
[In] Integrate[(1 - Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 - x^3]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^
3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/Sqrt[-1 + x^3]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.62 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.36

method	result
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x\, {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}} - \frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x^2\, {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)}{2\sqrt{\operatorname{signum}(x^3-1)}} - \frac{\sqrt{3}\,\sqrt{-\operatorname{signum}(x^3-1)}\, x\, {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}$
elliptic	$\frac{2\left(1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{}$

```
[In] int((1-x-3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/2],[4/3],x^
3)-1/2/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^2*hypergeom([1/2,2/3],[
5/3],x^3)-3^(1/2)/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1
/3,1/2],[4/3],x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.08

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = -2\left(\sqrt{3}-1\right)\text{weierstrassPInverse}(0,4,x) + 2\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

```
[In] integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(3) - 1)*weierstrassPInverse(0, 4, x) + 2*weierstrassZeta(0, 4, wei
erstrassPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.31

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((1-x-3**(1/2))/(x**3-1)**(1/2),x)
[Out] I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - I*x*gamm
a(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + sqrt(3)*I*x*gamma(1
/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

```
[In] integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

```
[In] integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.24

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$$
$$= -\frac{\sqrt{3}\,x\,\sqrt{1-x^3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)}{\sqrt{x^3-1}} + \frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(-(x + 3^(1/2) - 1)/(x^3 - 1)^(1/2),x)
```

```
[Out] (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2
)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/
2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)
^(1/2) - (3^(1/2)*x*(1 - x^3)^(1/2)*hypergeom([1/3, 1/2], 4/3, x^3))/(x^3 -
 1)^(1/2) - (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*(
(x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 + 1) + x^3)^(1/2)
```

3.94 $\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 247

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] -2*(-x^3-1)^(1/2)/(1+x-3^(1/2))-4*3^(1/4)*(1+x)*EllipticF((1+x+3^(1/2))/(1+
x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2)
)^2)^(1/2)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)+3^(1/4)*(1+x)*Elli
pticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2
)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1894, 225, 1893}

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,E\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} - \frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

```
[In] Int[(1 - Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/
(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*
Sqrt[-1 - x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(
1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)],
-7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\left(\left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx\right) + \int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$$

$$= -\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$- \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.27

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{x\sqrt{1+x^3}\left(-2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

```
[In] Integrate[(1 - Sqrt[3] + x)/Sqrt[-1 - x^3],x]
[Out] (x*Sqrt[1 + x^3]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3]
+ x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.64 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.21

method	result
meijerg	$-ix\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)-\frac{ix^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)}{2}+i\sqrt{3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)$
elliptic	$-\frac{2i\left(1-\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i}{}\right.}}{}$
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{}}{}$

```
[In] int((1+x-3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*x*hypergeom([1/3,1/2],[4/3],-x^3)-1/2*I*x^2*hypergeom([1/2,2/3],[5/3],-x
^3)+I*3^(1/2)*x*hypergeom([1/3,1/2],[4/3],-x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.09

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -2\left(-i\sqrt{3}+i\right)\text{weierstrassPInverse}(0,-4,x) + 2i\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

```
[In] integrate((1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(-I*sqrt(3) + I)*weierstrassPInverse(0, -4, x) + 2*I*weierstrassZeta(0,
-4, weierstrassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((1+x-3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma
(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*
gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_pol
ar(I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

```
[In] integrate((1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

```
[In] integrate((1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.46

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$$
$$= -\frac{\sqrt{3}\,x\,\sqrt{x^3+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)}{\sqrt{-x^3-1}}$$
$$-\frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$+\frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int((x - 3^(1/2) + 1)/(- x^3 - 1)^(1/2),x)
```

```
[Out] (6*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x
^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (6*(x^3 + 1)^(1/2)*((x + (3^(1/2)
*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE
(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^
(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^
(1/2)) - (3^(1/2)*x*(x^3 + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3, -x^3))/(- x^
3 - 1)^(1/2)
```

3.95 $\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 126

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = -\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -2*(x^3+1)^(1/2)/(1+x+3^(1/2))+3^(1/4)*(1+x)*EllipticE((1+x-3^(1/2))/(1+x+3
^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2
)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1891}

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

```
[In] Int[(-1 + Sqrt[3] - x)/Sqrt[1 + x^3],x]
```

```
[Out] (-2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt
[1 + x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.37

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = \left(-1+\sqrt{3}\right)x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right) - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

```
[In] Integrate[(-1 + Sqrt[3] - x)/Sqrt[1 + x^3],x]
```

```
[Out] (-1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] - (x^2*Hypergeometr
ic2F1[1/2, 2/3, 5/3, -x^3])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.70 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.38

method	result
meijerg	$-x{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)-\frac{x^2{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)}{2}+\sqrt{3}\,x{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)$
elliptic	$\frac{2\left(\sqrt{3}-1\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\cdots}}{\cdots}$
default	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$

```
[In] int((-1-x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x*hypergeom([1/3,1/2],[4/3],-x^3)-1/2*x^2*hypergeom([1/2,2/3],[5/3],-x^3)+
3^(1/2)*x*hypergeom([1/3,1/2],[4/3],-x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.17

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = 2\left(\sqrt{3}-1\right)\text{weierstrassPInverse}(0,-4,x) + 2\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

```
[In] integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(3) - 1)*weierstrassPInverse(0, -4, x) + 2*weierstrassZeta(0, -4, we
ierstrassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.73

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = -\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((-1-x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] -x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5
/3)) - x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamm
a(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*p
i))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = \int -\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = \int -\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.48

$$\begin{aligned}
&\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx \\
&= \sqrt{3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right) \\
&\quad + \frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}} \\
&\quad - \frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}
\end{aligned}$$

```
[In] int(-(x - 3^(1/2) + 1)/(x^3 + 1)^(1/2),x)
```

```
[Out] 3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, -x^3) + (6*((x + (3^(1/2)*1i)/2 - 1/2)
/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^
(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x + 1
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (6*((x + (3^(1/2)*1i)/2
- 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((
(x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1
i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1)
- ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.96 $\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = \frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] 2*(-x^3+1)^(1/2)/(1-x+3^(1/2))-3^(1/4)*(1-x)*EllipticE((1-x-3^(1/2))/(1-x+3
^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2
)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1891}

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = \frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[In] Int[(-1 + Sqrt[3] + x)/Sqrt[1 - x^3],x]
```

```
[Out] (2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sq
rt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(1
 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.30

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = \frac{1}{2}x\left(2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right) + x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)$$

```
[In] Integrate[(-1 + Sqrt[3] + x)/Sqrt[1 - x^3],x]
```

```
[Out] (x*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + x*Hypergeometr
ic2F1[1/2, 2/3, 5/3, x^3]))/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.65 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result
meijerg	$-x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+\frac{x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)}{2}+\sqrt{3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)$
elliptic	$-\frac{2i\left(\sqrt{3}-1\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}\right.}}{}$
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$

```
[In] int((-1+x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x*hypergeom([1/3,1/2],[4/3],x^3)+1/2*x^2*hypergeom([1/2,2/3],[5/3],x^3)+3^
(1/2)*x*hypergeom([1/3,1/2],[4/3],x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.16

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = -2\left(i\sqrt{3}-i\right)\text{weierstrassPInverse}(0,4,x) + 2i\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

```
[In] integrate((-1+x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(I*sqrt(3) - I)*weierstrassPInverse(0, 4, x) + 2*I*weierstrassZeta(0, 4,
 weierstrassPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.68

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((-1+x+3**(1/2))/(-x**3+1)**(1/2),x)
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma(
5/3)) - x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*g
amma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(
2*I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

```
[In] integrate((-1+x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Giac [F]

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

```
[In] integrate((-1+x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.39

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx$$
$$=\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)$$
$$-\frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$+\frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int((x + 3^(1/2) - 1)/(1 - x^3)^(1/2),x)
```

```
[Out] 3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, x^3) - (6*(x^3 - 1)^(1/2)*(-(x - (3^(1
/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellip
ticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)
/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^
3)^(1/2)) + (6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-
(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 -
x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.97 $\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 263

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = -\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -2*(x^3-1)^(1/2)/(1-x-3^(1/2))-4*3^(1/4)*(1-x)*EllipticF((1-x+3^(1/2))/(1-x
-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))
^2)^(1/2)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)+3^(1/4)*(1-x)*Ellipt
icE((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^
(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1894, 225, 1893}

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,E\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}$$

```
[In] Int[(-1 + Sqrt[3] + x)/Sqrt[-1 + x^3],x]
```

```
[Out] (-2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/
(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*
Sqrt[-1 + x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
-7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx-\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx\\ &= -\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\\ &\quad-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.24

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = \frac{x\sqrt{1-x^3}\left(2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)}{2\sqrt{-1+x^3}}$$

```
[In] Integrate[(-1 + Sqrt[3] + x)/Sqrt[-1 + x^3],x]
```

```
[Out] (x*Sqrt[1 - x^3]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] +
x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/(2*Sqrt[-1 + x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.68 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.37

method	result
meijerg	$-\frac{\sqrt{-\operatorname{signum}\left(x^{3}-1\right)}\, x\, {}_{2}F_{1}\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^{3}\right)}{\sqrt{\operatorname{signum}\left(x^{3}-1\right)}}+\frac{\sqrt{-\operatorname{signum}\left(x^{3}-1\right)}\, x^{2}\, {}_{2}F_{1}\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^{3}\right)}{2 \sqrt{\operatorname{signum}\left(x^{3}-1\right)}}+\frac{\sqrt{3}\, \sqrt{-\operatorname{signum}\left(x^{3}-1\right)}\, x\, {}_{2}F_{1}\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^{3}\right)}{\sqrt{\operatorname{signum}\left(x^{3}-1\right)}}$
elliptic	$\frac{2\left(\sqrt{3}-1\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}+\ldots$
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}+\ldots$

```
[In] int((-1+x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/2],[4/3],x
^3)+1/2/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^2*hypergeom([1/2,2/3],
[5/3],x^3)+3^(1/2)/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([
1/3,1/2],[4/3],x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.08

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\, dx = 2\left(\sqrt{3}-1\right)\text{weierstrassPInverse}(0,4,x) - 2\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

```
[In] integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(3) - 1)*weierstrassPInverse(0, 4, x) - 2*weierstrassZeta(0, 4, weie
rstrassPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.31

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = -\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((-1+x+3**(1/2))/(x**3-1)**(1/2),x)
[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - sqrt(3)
*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + I*x*gamma(
1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

```
[In] integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Giac [F]

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

```
[In] integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.24

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = \frac{\sqrt{3}\,x\,\sqrt{1-x^3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)}{\sqrt{x^3-1}} - \frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}} + \frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

```
[In] int((x + 3^(1/2) - 1)/(x^3 - 1)^(1/2),x)
```

```
[Out] (3^(1/2)*x*(1 - x^3)^(1/2)*hypergeom([1/3, 1/2], 4/3, x^3))/(x^3 - 1)^(1/2)
 - (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(
1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x
^3)^(1/2) + (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*(
(x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 + 1) + x^3)^(1/2)
```

3.98 $\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 248

$$\begin{aligned}
&\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}} \, dx \\
&= \frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} \\
&\quad + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}
\end{aligned}$$

```
[Out] 2*(-x^3-1)^(1/2)/(1+x-3^(1/2))+4*3^(1/4)*(1+x)*EllipticF((1+x+3^(1/2))/(1+x
-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))
^2)^(1/2)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)-3^(1/4)*(1+x)*Ellip
ticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)
^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1894, 225, 1893}

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx$$
$$=\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

```
[In] Int[(-1 + Sqrt[3] - x)/Sqrt[-1 - x^3],x]
```

```
[Out] (2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*S
qrt[-1 - x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -
7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx - \int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx \\ &= \frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} \\ &\quad + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.27

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = -\frac{x\sqrt{1+x^3}\left(-2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

```
[In] Integrate[(-1 + Sqrt[3] - x)/Sqrt[-1 - x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 + x^3]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -
x^3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/Sqrt[-1 - x^3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.62 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.21

method	result
meijerg	$ix{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+\frac{ix^2{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)}{2}-i\sqrt{3}\,x{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)$
elliptic	$-\frac{2i\left(\sqrt{3}-1\right)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\cdots\right.}}{\cdots}$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\cdots}}{\cdots}$

```
[In] int((-1-x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*x*hypergeom([1/3,1/2],[4/3],-x^3)+1/2*I*x^2*hypergeom([1/2,2/3],[5/3],-x^
3)-I*3^(1/2)*x*hypergeom([1/3,1/2],[4/3],-x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.09

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = -2\left(i\sqrt{3}-i\right)\text{weierstrassPInverse}(0,-4,x) - 2i\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

```
[In] integrate((-1-x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(I*sqrt(3) - I)*weierstrassPInverse(0, -4, x) - 2*I*weierstrassZeta(0, -
4, weierstrassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = \frac{ix^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((-1-x+3**(1/2))/(-x**3-1)**(1/2),x)
[Out] I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(
5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*p
i))/(3*gamma(4/3)) + I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_pola
r(I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = \int -\frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

```
[In] integrate((-1-x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Giac [F]

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = \int -\frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

```
[In] integrate((-1-x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.45

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = \frac{\sqrt{3}\,x\,\sqrt{x^3+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)}{\sqrt{-x^3-1}} + \frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}} - \frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

```
[In] int(-(x - 3^(1/2) + 1)/(- x^3 - 1)^(1/2),x)
```

```
[Out] (3^(1/2)*x*(x^3 + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3, -x^3))/(- x^3 - 1)^(1
/2) + (6*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2)
)^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1
/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (6*(x^3 + 1)^(1/2)*((x + (3
^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ell
ipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2
)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2))^(1/2))
```

3.99 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 256

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2*(b*x^3+a)^(1/2)/b^(1/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)*(
a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(
b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used

$= \{1891\}$

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.35

$$\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x}{\sqrt{a+b x^3}} \, dx$$
$$= \frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{b}{}\right.\right.}{2\sqrt{a+bx^3}}$$

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/
2, 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3
)/a)]))/(2*Sqrt[a + b*x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. 2(189) = 378.

Time = 1.76 (sec) , antiderivative size = 1003, normalized size of antiderivative = 3.92

method	result	size
default	Expression too large to display	1003

```
[In] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))-2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(
```

```
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+2*I*a^(1/3)/b*(
-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.19

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = -\frac{2\left(a^{\frac{1}{3}}\sqrt{b}(\sqrt{3}-1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+b^{\frac{5}{6}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,\right.\right.\right.}{b}$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -2*(a^(1/3)*sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + b^(5/
6)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.48

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b
```

```
*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + x*gamma(1/3)*hyper((1/3,
 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Maxima [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="max
ima")
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="gia
c")
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}-1\right)}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(a + b*x^3)^(1/2),x)
[Out] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(a + b*x^3)^(1/2), x)
```

3.100 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 263

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] -2*(-b*x^3+a)^(1/2)/b^(1/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+3^(1/4)*a^(1/3
)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(
1/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used

$= \{1891\}$

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$$

$$= \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$- \frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

$$+ \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.34

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3
, 1/2, 4/3, (b*x^3)/a] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)
/a]))/Sqrt[a - b*x^3]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 948 vs. 2(196) = 392.

Time = 1.74 (sec) , antiderivative size = 949, normalized size of antiderivative = 3.61

method	result	size
default	Expression too large to display	949

```
[In] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(
-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b
*x^3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^
(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b
/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*a^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*
(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)
^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/
3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(
```

```
-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^
(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(a*b^2)^(1/3)))^(1/2))-2*I*a^(1/3)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/
b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2
)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(
1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^
(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.21

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = \frac{2\left(a^{\frac{1}{3}}\sqrt{-b}(\sqrt{3}-1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{-b}b^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2*(a^(1/3)*sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - sqrt(-
b)*b^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.49

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -\frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*I*pi
)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,)
```

```
, b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + x*gamma(1/3)*hyper(
(1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Maxima [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3+a}}\,dx$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a), x)
```

Giac [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3+a}}\,dx$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=\int -\frac{b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}-1\right)}{\sqrt{a-b\,x^3}}\,dx$$

```
[In] int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(a - b*x^3)^(1/2),x)
```

```
[Out] int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(a - b*x^3)^(1/2), x)
```

3.101 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 497

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

```
[Out] 2*(b*x^3-a)^(1/2)/b^(1/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))+4*3^(1/4)*a^(1/3
)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*
x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^(
1/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3
^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(
2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/
2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(1/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(
1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {1894, 225, 1893}

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$$
$$=\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$
$$+\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) + (4*3^(1/4)*Sqrt[2 -
Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*
Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^
(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\left(\left(2\sqrt{3}\sqrt[3]{a}\right)\int \frac{1}{\sqrt{-a+bx^3}}\,dx\right) + \int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$$

$$= \frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.18

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right.\right.}{2\sqrt{-a+bx^3}}$$

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3
, 1/2, 4/3, (b*x^3)/a] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)
/a]))/Sqrt[-a + b*x^3]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 951 vs. 2(375) = 750.

Time = 1.73 (sec) , antiderivative size = 952, normalized size of antiderivative = 1.92

method	result	size
default	Expression too large to display	952

```
[In] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(
-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*
x^3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*Elliptic
E(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(
1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3
^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/
(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2/3*I*a^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(
-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^
(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3
))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I
```

```
*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1
/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(a*b^2)^(1/3)))^(1/2))-2*I*a^(1/3)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*
(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*
(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^
(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2
)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/
2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.10

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = -\frac{2\left(a^{\frac{1}{3}}\sqrt{b}(\sqrt{3}-1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-b^{\frac{5}{6}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -2*(a^(1/3)*sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - b^(5/6
)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.23

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*g
amma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*a**(1/6)
```

```
*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(
3*a**(1/6)*gamma(4/3))
```

Maxima [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3-a}}\,dx$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="ma
xima")
[Out] -integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x)
```

Giac [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3-a}}\,dx$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="gi
ac")
[Out] integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx=\int -\frac{b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}-1\right)}{\sqrt{b\,x^3-a}}\,dx$$

```
[In] int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(b*x^3 - a)^(1/2),x)
[Out] int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(b*x^3 - a)^(1/2), x)
```

3.102 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 488

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] -2*(-b*x^3-a)^(1/2)/b^(1/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))-4*3^(1/4)*a^(1/
3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^(1/
3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1
/2*6^(1/2)+1/2*2^(1/2))/b^(1/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)
*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {1894, 225, 1893}

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} - \frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1
/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[
ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) - (4*3^(1/4)*Sqrt[2 -
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4
*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\left(\left(2\sqrt{3}\sqrt[3]{a}\right)\int \frac{1}{\sqrt{-a-bx^3}}\,dx\right) + \int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$$

$$= -\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.19

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\right.\right.}{2\sqrt{-a-bx^3}}$$

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/
2, 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3
)/a)]))/(2*Sqrt[-a - b*x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1011 vs. 2(366) = 732.

Time = 1.74 (sec) , antiderivative size = 1012, normalized size of antiderivative = 2.07

method	result	size
default	Expression too large to display	1012

```
[In] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+2*I*a^(1/3)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2
/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
```

```
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = \frac{2\left(a^{\frac{1}{3}}\sqrt{-b}\left(\sqrt{3}-1\right)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{-b}b^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(\right.\right.\right.}{b}$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2*(a^(1/3)*sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + sqrt(
-b)*b^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/
b
```

Sympy [A] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.26

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hype
r((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Maxima [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3-a}}\,dx$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a), x)
```

Giac [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3-a}}\,dx$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}-1\right)}{\sqrt{-b\,x^3-a}}\,dx$$

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(- a - b*x^3)^(1/2),x)
```

```
[Out] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(- a - b*x^3)^(1/2), x)
```

3.103 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 241

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+bx^3}}{b\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2*(b/a)^(2/3)*(b*x^3+a)^(1/2)/b/(1+(b/a)^(1/3)*x+3^(1/2))-3^(1/4)*(1+(b/a)^
(1/3)*x)*EllipticE((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2)),I*3^
(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((1-(b/a)^(1/3)*x+(b/a)^(2/3)*x^2)/(1+
(b/a)^(1/3)*x+3^(1/2))^2)^(1/2)/(b/a)^(1/3)/(b*x^3+a)^(1/2)/((1+(b/a)^(1/3)
*x)/(1+(b/a)^(1/3)*x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1891}

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}\sqrt{a+bx^3}}$$

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(b/a)^(2/3)*Sqrt[a + b*x^3])/(b*(1 + Sqrt[3] + (b/a)^(1/3)*x)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)^(1/3)*x + (b/a)^(2/
3)*x^2)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + (b
/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(1/3)*
Sqrt[(1 + (b/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

integral

$$= \frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+bx^3}}{b\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.37

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
-((b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)
]))/(2*Sqrt[a + b*x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1003 vs. 2(196) = 392.

Time = 1.72 (sec) , antiderivative size = 1004, normalized size of antiderivative = 4.17

method	result	size
default	Expression too large to display	1004

```
[In] int((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2*I/
b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3
^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.22

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = -\frac{2\left(\sqrt{b}(\sqrt{3}-1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] -2*(sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + sqrt(b)*(b/a)
^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.51

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{x^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3+a)**(1/2),x)
[Out] x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*p
i)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + x*gamma(1/3)*hyper((1
/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \int \frac{x\,\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(a + b*x^3)^(1/2),x)
[Out] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(a + b*x^3)^(1/2), x)
```

3.104 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 248

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-bx^3}}{b\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}\,E\left(\arcsin\left(\frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] -2*(b/a)^(2/3)*(-b*x^3+a)^(1/2)/b/(1-(b/a)^(1/3)*x+3^(1/2))+3^(1/4)*(1-(b/a
)^(1/3)*x)*EllipticE((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2)),I*
3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(b/a)^(1/3)*x+(b/a)^(2/3)*x^2)/(
1-(b/a)^(1/3)*x+3^(1/2))^2)^(1/2)/(b/a)^(1/3)/(-b*x^3+a)^(1/2)/((1-(b/a)^(1
/3)*x)/(1-(b/a)^(1/3)*x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1891}

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx$$

$$= \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}\sqrt{a-bx^3}}$$

$$-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-bx^3}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)}$$

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (-2*(b/a)^(2/3)*Sqrt[a - b*x^3])/(b*(1 + Sqrt[3] - (b/a)^(1/3)*x)) + (3^(1/
4)*Sqrt[2 - Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a)^(1/3)*x + (b/a)^(2
/3)*x^2)/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - (
b/a)^(1/3)*x)/(1 + Sqrt[3] - (b/a)^(1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(1/3)
*Sqrt[(1 - (b/a)^(1/3)*x)/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

integral

$$= -\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-bx^3}}{b\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.36

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4
/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])
)/Sqrt[a - b*x^3]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 949 vs. 2(203) = 406.

Time = 1.74 (sec) , antiderivative size = 950, normalized size of antiderivative = 3.83

method	result	size
default	Expression too large to display	950

```
[In] int((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)
^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*
b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/
2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2*I/b*(a*b^2)^(1/3
)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^
2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(
1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)
*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*
(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/
3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b
^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*
3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/
b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)
)+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^
2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \frac{2\left(\sqrt{-b}(\sqrt{3}-1)\,\mathrm{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\mathrm{weierstrassZeta}\left(0,\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,\right.\right.\right.}{b}$$

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] 2*(sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - sqrt(-b)*(b/a)
^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.52

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{x^2\sqrt[3]{\frac{b}{a}}\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(-b*x**3+a)**(1/2),x)
[Out] -x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*
I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4
/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + x*gamma(1/3)*hyp
er((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{-bx^3+a}}\,dx$$

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] -integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \int -\frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{a-b\,x^3}}\,dx$$

```
[In] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(a - b*x^3)^(1/2),x)
[Out] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(a - b*x^3)^(1/2), x)
```

3.105 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 549

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$
$$-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$
$$-\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

```
[Out] 2*(b/a)^(1/3)*(b*x^3-a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))-2/3*
(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+
a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*(1-3^(1/2))-a^(1/3)*(b/a)^(1/3
)*(1+3^(1/2)))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(b*x^3-a)^
(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/
2)-3^(1/4)*a^(1/3)*(b/a)^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3
```

```
)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*
(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3
)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1894, 225, 1893}

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = -\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}} + \frac{2\sqrt[3]{\frac{b}{a}}\sqrt{bx^3-a}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*(b/a)^(1/3)*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqr
t[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(
1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]
) - (2*Sqrt[2 - Sqrt[3]]*((1 - Sqrt[3])*b^(1/3) - (1 + Sqrt[3])*a^(1/3)*(b/
a)^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3]
)*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]
])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx}{\sqrt[3]{b}} - \left(-1+\sqrt{3}+\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a+bx^3}}\,dx$$

$$= \frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{\ }\right.}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}} - \frac{2\sqrt{2-\sqrt{3}}\left(1-\sqrt{3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)}{\left(1-\sqrt{3}\right)}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.16

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] -1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4
/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])
)/Sqrt[-a + b*x^3]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(415) = 830$.

Time = 1.69 (sec) , antiderivative size = 953, normalized size of antiderivative = 1.74

method	result	size
default	Expression too large to display	953

```
[In] int((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^
(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b
^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2
/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2*I/b*(a*b^2)^(1/3)
*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1
/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(
-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^
(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x
+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3)
)^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2
)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^
(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(
a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1
/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^
(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = -\frac{2\left(\sqrt{b}(\sqrt{3}-1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\right.\right.\right.}{b}$$

[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] -2*(sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - sqrt(b)*(b/a)^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b

Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.21

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3-a)**(1/2),x)

[Out] I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sqrt(a)*gamma(4/3))

Maxima [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{bx^3-a}}\,dx$$

[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="maxima")

[Out] -integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen &

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \int -\frac{\sqrt{3}+x\,\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{b\,x^3-a}}\,dx$$

[In] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(b*x^3 - a)^(1/2),x)

[Out] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(b*x^3 - a)^(1/2), x)

3.106 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 540

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$+\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] -2*(b/a)^(1/3)*(-b*x^3-a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))+2/3
*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a
^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*(1-3^(1/2))-a^(1/3)*(b/a)^(1/3)
*(1+3^(1/2)))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(
1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(-b*x^3-a)^(
1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
+3^(1/4)*a^(1/3)*(b/a)^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^
```

```
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*
6^(1/2)+1/2*2^(1/2))/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)
/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1894, 225, 1893}

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$-\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*(b/a)^(1/3)*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3
]) + (2*Sqrt[2 - Sqrt[3]]*((1 - Sqrt[3])*b^(1/3) - (1 + Sqrt[3])*a^(1/3)*(b
/a)^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3
]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx}{\sqrt[3]{b}} + \left(1-\sqrt{3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a-bx^3}}\,dx$$

$$= -\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right.}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.17

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right.}{2\sqrt{-a-bx^3}}$$

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
-((b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)
]))/(2*Sqrt[-a - b*x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1012 vs. 2(406) = 812.

Time = 1.75 (sec) , antiderivative size = 1013, normalized size of antiderivative = 1.88

method	result	size
default	Expression too large to display	1013

```
[In] int((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2*I
/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)
*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x
^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.10

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \frac{2\left(\sqrt{-b}(\sqrt{3}-1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\right.\right.}{b}$$

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + sqrt(-b)*(b/a
)^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.24

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,),
b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*h
yper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{-bx^3-a}}\,dx$$

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{x\,{\left(\frac{b}{a}\right)}^{1/3}-\sqrt{3}+1}{\sqrt{-b\,x^3-a}}\,dx$$

```
[In] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(- a - b*x^3)^(1/2),x)
```

```
[Out] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(- a - b*x^3)^(1/2), x)
```

3.107 $\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 490

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)
*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)+2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/
2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c-a^(1/3)*d*(1
-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)
/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1892, 224, 1891}

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[(c + d*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*
(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} + \left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.15

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

```
[In] Integrate[(c + d*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
+ d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.47

method	result
default	$-\frac{2 i c \sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}} \sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}} \sqrt{-\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\, F\left(\right.}{3b\sqrt{b\,x^{3}+a}}$
elliptic	$-\frac{2 i c \sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}} \sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}} \sqrt{-\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\, F\left(\right.}{3b\sqrt{b\,x^{3}+a}}$

```
[In] int((d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/
3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
```

```
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.09

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt{b}c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{b}d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - sqrt(b)*d*weierstrassZeta(
0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.16

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x + c)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \int \frac{c+d\,x}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((c + d*x)/(a + b*x^3)^(1/2),x)
[Out] int((c + d*x)/(a + b*x^3)^(1/2), x)
```

3.108 $\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 503

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \frac{2d\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$
$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c+\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] 2*d*(-b*x^3+a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/
3)*d*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/
3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3
)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/
b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)-2/3*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c+a^(
1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-b*x
^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1892, 224, 1891}

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} + \frac{2d\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[In] Int[(c + d*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (2*d*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (2*Sqrt[2 + Sqrt[3]]*
(b^(1/3)*c + (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) +
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) -
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx}{\sqrt[3]{b}} - \left(-c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{a-bx^3}}\,dx$$

$$= \frac{2d\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c+\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\cdots}\cdots\right)\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.15

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \frac{x\sqrt{1-\frac{bx^3}{a}}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

```
[In] Integrate[(c + d*x)/Sqrt[a - b*x^3],x]
[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] + d
*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[a - b*x^3])
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.35

method	result
default	$\frac{2ic\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,F\left(\frac{\sqrt{3}\,\sqrt{-\frac{i\left(x\cdots\right.}{\cdots}}}{\cdots}\right.}{3b\sqrt{-b\,x^{3}+a}}$
elliptic	$\frac{2ic\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,F\left(\frac{\sqrt{3}\,\sqrt{-\frac{i\left(x\cdots\right.}{\cdots}}}{\cdots}\right.}{3b\sqrt{-b\,x^{3}+a}}$

```
[In] int((d*x+c)/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*c*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+
a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-
3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2/3*I*d*3^(1/2)/
b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3
^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x
+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3)
```

```
)^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*
(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)
,(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^
(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.09

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = -\frac{2\left(\sqrt{-b}c\,\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{-b}d\,\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

```
[In] integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(-b)*c*weierstrassPInverse(0, 4*a/b, x) - sqrt(-b)*d*weierstrassZet
a(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.16

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((d*x+c)/(-b*x**3+a)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqr
t(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_p
olar(2*I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \int \frac{dx+c}{\sqrt{-bx^3+a}}\,dx$$

```
[In] integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x + c)/sqrt(-b*x^3 + a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \int \frac{dx+c}{\sqrt{-bx^3+a}}\,dx$$

```
[In] integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)/sqrt(-b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \int \frac{c+d\,x}{\sqrt{a-b\,x^3}}\,dx$$

```
[In] int((c + d*x)/(a - b*x^3)^(1/2),x)
[Out] int((c + d*x)/(a - b*x^3)^(1/2), x)
```

3.109 $\int \frac{c+dx}{\sqrt{-a+bx^3}} dx$

Optimal result

Integrand size = 19, antiderivative size = 515

$$\int \frac{c+dx}{\sqrt{-a+bx^3}} dx = -\frac{2d\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}c+\left(1+\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

```
[Out] -2*d*(b*x^3-a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))-2/3*(a^(1/3)-
b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(
1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*c+a^(1/3)*d*(1+3^(1/2)))*((a^(2/3)+a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6
^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1
/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*d*(a^(1/3)
-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*
(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1
/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(b*x^
3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1894, 225, 1893}

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = -\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}} - \frac{2d\sqrt{bx^3-a}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[In] Int[(c + d*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (-2*d*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^
(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*d*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/
((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) - (2*Sqrt[2 - Sqr
t[3]]*(b^(1/3)*c + (1 + Sqrt[3])*a^(1/3)*d)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(
2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

<u>Rule 225</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{d\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx}{\sqrt[3]{b}} - \left(-c-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{-a+bx^3}}\,dx$$

$$= -\frac{2d\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$- \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}c+\left(1+\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\ }}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\ }}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.15

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = \frac{x\sqrt{1-\frac{bx^3}{a}}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

```
[In] Integrate[(c + d*x)/Sqrt[-a + b*x^3],x]
[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] + d
*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[-a + b*x^3])
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.33

method	result
default	$\frac{2ic\sqrt{3}\,(a\,b^2)^{\frac{1}{3}}\sqrt{-\frac{i\left(x+\frac{(a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{\frac{i\left(x+\frac{(a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(a\,b^2)^{\frac{1}{3}}}}\,F\left(\frac{\sqrt{3}\sqrt{-\frac{i\left(x+\cdots\right.}{\cdots}}}{\cdots}\right.\cdots}{3b\sqrt{b\,x^3-a}}$
elliptic	$\frac{2ic\sqrt{3}\,(a\,b^2)^{\frac{1}{3}}\sqrt{-\frac{i\left(x+\frac{(a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{\frac{i\left(x+\frac{(a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(a\,b^2)^{\frac{1}{3}}}}\,F\left(\frac{\sqrt{3}\sqrt{-\frac{i\left(x+\cdots\right.}{\cdots}}}{\cdots}\right.\cdots}{3b\sqrt{b\,x^3-a}}$

```
[In] int((d*x+c)/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*c*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a
)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3
/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2/3*I*d*3^(1/2)/b
*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^
(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1
/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^
```

```
(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*
b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(
-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1
/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.08

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = \frac{2\left(\sqrt{b}c\mathrm{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{b}d\mathrm{weierstrassZeta}\left(0,\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

```
[In] integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(b)*c*weierstrassPInverse(0, 4*a/b, x) - sqrt(b)*d*weierstrassZeta(0
, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.14

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = -\frac{icx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}-\frac{idx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((d*x+c)/(b*x**3-a)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sqrt(a)*gamma(4/3)
) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*gamm
a(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx=\int \frac{dx+c}{\sqrt{bx^3-a}}\,dx$$

```
[In] integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x + c)/sqrt(b*x^3 - a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx=\int \frac{dx+c}{\sqrt{bx^3-a}}\,dx$$

```
[In] integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)/sqrt(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx=\int \frac{c+d\,x}{\sqrt{b\,x^3-a}}\,dx$$

```
[In] int((c + d*x)/(b*x^3 - a)^(1/2),x)
[Out] int((c + d*x)/(b*x^3 - a)^(1/2), x)
```

3.110 $\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 508

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = -\frac{2d\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}c-\left(1+\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\dots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\dots}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] -2*d*(-b*x^3-a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))+2/3*(a^(1/3)+
b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-
3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*c-a^(1/3)*d*(1+3^(1/2)))*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1
/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*d*(a^(1/3)+b^
(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^
(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(-b*x^3-a)^
(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1894, 225, 1893}

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1+\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)\ldots\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$-\frac{2d\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[(c + d*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*d*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^
(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) + (2*Sqrt[2 - Sqr
t[3]]*(b^(1/3)*c - (1 + Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

<u>Rule 225</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{d\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx}{\sqrt[3]{b}} + \left(c-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a-bx^3}}\,dx$$

$$= -\frac{2d\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} + \frac{2\sqrt{2-\sqrt{3}}\left(c-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.15

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

```
[In] Integrate[(c + d*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
+ d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[-a - b*x^3])
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 726, normalized size of antiderivative = 1.43

method	result
default	$-\frac{2ic\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,F}{3b\sqrt{-b\,x^{3}-a}}$
elliptic	$-\frac{2ic\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,F}{3b\sqrt{-b\,x^{3}-a}}$

```
[In] int((d*x+c)/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2
/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
```

```
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.09

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = -\frac{2\left(\sqrt{-b}c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{-b}d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

```
[In] integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(-b)*c*weierstrassPInverse(0, -4*a/b, x) - sqrt(-b)*d*weierstrassZe
ta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.16

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = -\frac{icx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}-\frac{idx^2\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((d*x+c)/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = \int \frac{dx+c}{\sqrt{-bx^3-a}}\,dx$$

```
[In] integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x + c)/sqrt(-b*x^3 - a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = \int \frac{dx+c}{\sqrt{-bx^3-a}}\,dx$$

```
[In] integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)/sqrt(-b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = \int \frac{c+d\,x}{\sqrt{-b\,x^3-a}}\,dx$$

```
[In] int((c + d*x)/(- a - b*x^3)^(1/2),x)
[Out] int((c + d*x)/(- a - b*x^3)^(1/2), x)
```

3.111 $\int \frac{c+dx}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 246

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = \frac{2d\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 2*d*(x^3+1)^(1/2)/(1+x+3^(1/2))-3^(1/4)*d*(1+x)*EllipticE((1+x-3^(1/2))/(1+
x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2)
)^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+2/3*(1+x)*EllipticF(
(1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(c-d*(1-3^(1/2)))*(1/2*6^(1/2)+1
/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/
(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1892, 224, 1891}

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} + \frac{2d\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

```
[In] Int[(c + d*x)/Sqrt[1 + x^3],x]
```

```
[Out] (2*d*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d*(1 + x
)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x
)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*S
qrt[1 + x^3]) + (2*Sqrt[2 + Sqrt[3]]*(c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1
- x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqr
t[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqr
t[1 + x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx + \left(c-\left(1-\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{1+x^3}}\,dx \\
&= \frac{2d\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} \\
&\quad + \frac{2\sqrt{2+\sqrt{3}}\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.17

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = cx\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right) + \frac{1}{2}dx^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

```
[In] Integrate[(c + d*x)/Sqrt[1 + x^3],x]
[Out] c*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (d*x^2*Hypergeometric2F1[1/2,
2/3, 5/3, -x^3])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.64 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.13

method	result
meijerg	$\frac{d x^{2} {}_{2}F_{1}\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^{3}\right)}{2}+c x {}_{2}F_{1}\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^{3}\right)$
default	$\frac{2c\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}+\frac{2d\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}$
elliptic	$\frac{2c\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}+\frac{2d\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}$

```
[In] int((d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*d*x^2*hypergeom([1/2,2/3],[5/3],-x^3)+c*x*hypergeom([1/3,1/2],[4/3],-x^
3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.07

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = 2\,c\text{weierstrassPInverse}(0,-4,x) - 2\,d\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

```
[In] integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 2*c*weierstrassPInverse(0, -4, x) - 2*d*weierstrassZeta(0, -4, weierstrassP
Inverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.25

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{dx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((d*x+c)/(x**3+1)**(1/2),x)
```

[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3
)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*g
amma(5/3))

Maxima [F]

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = \int \frac{dx+c}{\sqrt{x^3+1}}\,dx$$

[In] integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate((d*x + c)/sqrt(x^3 + 1), x)

Giac [F]

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = \int \frac{dx+c}{\sqrt{x^3+1}}\,dx$$

[In] integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate((d*x + c)/sqrt(x^3 + 1), x)

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.52

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = -\frac{2\,d\,\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\right.}} + \frac{2\,c\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

[In] int((c + d*x)/(x^3 + 1)^(1/2),x)

[Out] (2*c*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 +

```
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*(
(3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*d*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(
((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*
1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1
/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*
((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2
)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.112 $\int \frac{c+dx}{\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 271

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = \frac{2d\sqrt{1-x^3}}{1+\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2+\sqrt{3}}(c+d-\sqrt{3}d)\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] 2*d*(-x^3+1)^(1/2)/(1-x+3^(1/2))-3^(1/4)*d*(1-x)*EllipticE((1-x-3^(1/2))/(1
-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2
))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-2/3*(1-x)*Elliptic
F((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(c+d-d*3^(1/2))*(1/2*6^(1/2)+1
/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)
/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1892, 224, 1891}

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(c-\sqrt{3}d+d)\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} + \frac{2d\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

```
[In] Int[(c + d*x)/Sqrt[1 - x^3],x]
```

```
[Out] (2*d*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d*(1 - x
)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*S
qrt[1 - x^3]) - (2*Sqrt[2 + Sqrt[3]]*(c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 +
x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[
3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(d\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx\right)+\left(c+d-\sqrt{3}d\right)\int \frac{1}{\sqrt{1-x^3}}\,dx \\ &= \frac{2d\sqrt{1-x^3}}{1+\sqrt{3}-x}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} \\ &\quad -\frac{2\sqrt{2+\sqrt{3}}\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.14

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = cx\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right) + \frac{1}{2}dx^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)$$

```
[In] Integrate[(c + d*x)/Sqrt[1 - x^3],x]
[Out] c*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + (d*x^2*Hypergeometric2F1[1/2, 2
/3, 5/3, x^3])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.66 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.11

method	result
meijerg	$\frac{d x^{2} {}_{2}F_{1}\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^{3}\right)}{2}+c x {}_{2}F_{1}\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^{3}\right)$
default	$-\frac{2 i c \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}}-\frac{2 i d \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right)}}{}$
elliptic	$-\frac{2 i c \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}}-\frac{2 i d \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right)}}{}$

```
[In] int((d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*d*x^2*hypergeom([1/2,2/3],[5/3],x^3)+c*x*hypergeom([1/3,1/2],[4/3],x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.07

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = -2i\,c\text{weierstrassPInverse}(0,4,x) + 2i\,d\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

```
[In] integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] -2*I*c*weierstrassPInverse(0, 4, x) + 2*I*d*weierstrassZeta(0, 4, weierstra
ssPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.24

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4
/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/
(3*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = \int \frac{dx+c}{\sqrt{-x^3+1}}\,dx$$

```
[In] integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x + c)/sqrt(-x^3 + 1), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = \int \frac{dx+c}{\sqrt{-x^3+1}}\,dx$$

```
[In] integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)/sqrt(-x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.50

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = -\frac{2\,c\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\,d\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}}$$

```
[In] int((c + d*x)/(1 - x^3)^(1/2),x)
[Out] - (2*c*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/
((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x -
```

```
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) -
x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2*d*
(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3
/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)
/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(
-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i
)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*
(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.113 $\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 275

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = -\frac{2d\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{2-\sqrt{3}}\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -2*d*(x^3-1)^(1/2)/(1-x-3^(1/2))-2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(
1/2)),2*I-I*3^(1/2))*(c+d+d*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(
1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)+
3^(1/4)*d*(1-x)*EllipticE((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*((x^2+
x+1)/(1-x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(x^3-1)^(1/2)/((-1+x)
/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1894, 225, 1893}

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = -\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(c+\sqrt{3}d+d)\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{2d\sqrt{x^3-1}}{-x-\sqrt{3}+1}$$

```
[In] Int[(c + d*x)/Sqrt[-1 + x^3],x]
```

```
[Out] (-2*d*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*d*(1 -
 x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
 x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^
2)]*Sqrt[-1 + x^3]) - (2*Sqrt[2 - Sqrt[3]]*(c + d + Sqrt[3]*d)*(1 - x)*Sqrt
[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 -
 Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^
2)]*Sqrt[-1 + x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
```

```
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\left(d\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx\right)+\left(c+d+\sqrt{3}d\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2d\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$-\frac{2\sqrt{2-\sqrt{3}}\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.21

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = \frac{x\sqrt{1-x^3}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)}{2\sqrt{-1+x^3}}$$

```
[In] Integrate[(c + d*x)/Sqrt[-1 + x^3],x]
```

```
[Out] (x*Sqrt[1 - x^3]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + d*x*Hypergeom
etric2F1[1/2, 2/3, 5/3, x^3]))/(2*Sqrt[-1 + x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.62 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.24

method	result
meijerg	$\frac{d\sqrt{-\operatorname{signum}(x^3-1)}\,x^2{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)}{2\sqrt{\operatorname{signum}(x^3-1)}}+\frac{c\sqrt{-\operatorname{signum}(x^3-1)}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}$
default	$\frac{2c\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+2d\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots$
elliptic	$\frac{2c\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+2d\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots$

```
[In] int((d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^2*hypergeom([1/2,2/3],[5
/3],x^3)+c/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/2]
,[4/3],x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.07

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = 2\,c\text{weierstrassPInverse}(0,4,x) - 2\,d\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

```
[In] integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*c*weierstrassPInverse(0, 4, x) - 2*d*weierstrassZeta(0, 4, weierstrassPIn
verse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.20

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = -\frac{icx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - I*d*x**2
*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = \int \frac{dx+c}{\sqrt{x^3-1}}\,dx$$

```
[In] integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x + c)/sqrt(x^3 - 1), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = \int \frac{dx+c}{\sqrt{x^3-1}}\,dx$$

```
[In] integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)/sqrt(x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.36

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = -\frac{2\,c\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\,d\,\Big(\Big(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)\,\mathrm{F}\Big(\mathrm{asin}\Big(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\Big)\Big|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\Big)-\Big(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)\,\mathrm{E}\Big(\mathrm{asin}\Big(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\Big)\Big|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}{\sqrt{x^3+\Big(-\Big(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)\,\Big(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)-1\Big)\,x+\Big(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)}}$$

```
[In] int((c + d*x)/(x^3 - 1)^(1/2),x)
[Out] - (2*c*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
- 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1
```

```
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*d*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(
asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^
(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin((-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))
*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2)
)^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

3.114 $\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 261

$$\begin{aligned}
&\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx \\
&= -\frac{2d\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} \\
&\quad+\frac{2\sqrt{2-\sqrt{3}}\left(c-\left(1+\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}
\end{aligned}$$

```
[Out] -2*d*(-x^3-1)^(1/2)/(1+x-3^(1/2))+2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^
(1/2)),2*I-I*3^(1/2))*(c-d*(1+3^(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1
)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1
/2)+3^(1/4)*d*(1+x)*EllipticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((
x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(-x^3-1)^(1/2)/((
-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1894, 225, 1893}

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} - \frac{2d\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

```
[In] Int[(c + d*x)/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*d*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*d*(1 +
 x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] +
 x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^
2)]*Sqrt[-1 - x^3]) + (2*Sqrt[2 - Sqrt[3]]*(c - (1 + Sqrt[3])*d)*(1 + x)*Sq
rt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1
 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x
)^2)]*Sqrt[-1 - x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
```

```
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 1894

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = d\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx + \left(c-\left(1+\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= -\frac{2d\sqrt{-1-x^3}}{1-\sqrt{3}+x} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,d(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+ \frac{2\sqrt{2-\sqrt{3}}\left(c-\left(1+\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.24

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = \frac{x\sqrt{1+x^3}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

```
[In] Integrate[(c + d*x)/Sqrt[-1 - x^3],x]
```

```
[Out] (x*Sqrt[1 + x^3]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + d*x*Hypergeo
metric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.58 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.14

method	result
meijerg	$-\frac{i d x^2 {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)}{2}-icx {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)$
default	$-\frac{2ic\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2id\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$
elliptic	$-\frac{2ic\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2id\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$

```
[In] int((d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*d*x^2*hypergeom([1/2,2/3],[5/3],-x^3)-I*c*x*hypergeom([1/3,1/2],[4/3
],-x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.07

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = -2i\,c\text{weierstrassPInverse}(0,-4,x) + 2i\,d\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

```
[In] integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*I*c*weierstrassPInverse(0, -4, x) + 2*I*d*weierstrassZeta(0, -4, weierst
rassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.25

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = -\frac{icx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((d*x+c)/(-x**3-1)**(1/2),x)
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(
4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))
/(3*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = \int \frac{dx+c}{\sqrt{-x^3-1}}\,dx$$

```
[In] integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x + c)/sqrt(-x^3 - 1), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = \int \frac{dx+c}{\sqrt{-x^3-1}}\,dx$$

```
[In] integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)/sqrt(-x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.55

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx$$
$$=\frac{2c\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$-\frac{2d\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right.}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\right.}}$$

```
[In] int((c + d*x)/(- x^3 - 1)^(1/2),x)
```

```
[Out] (2*c*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2
)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*d*(((
3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))
, -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*
ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x +
(3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/
((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
+ 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.115 $\int \frac{c+dx}{a-bx^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{c+dx}{a-bx^4} dx = \frac{c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{c\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] 1/2*c*arctan(b^(1/4)*x/a^(1/4))/a^(3/4)/b^(1/4)+1/2*c*arctanh(b^(1/4)*x/a^(
1/4))/a^(3/4)/b^(1/4)+1/2*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1890, 218, 214, 211, 281}

$$\int \frac{c+dx}{a-bx^4} dx = \frac{c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{c\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[In] Int[(c + d*x)/(a - b*x^4),x]
```

```
[Out] (c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (c*ArcTanh[(b^(1/4)*x
)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqr
t[a]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{c}{a-bx^4}+\frac{dx}{a-bx^4}\right) dx \\
&= c\int \frac{1}{a-bx^4}\,dx + d\int \frac{x}{a-bx^4}\,dx \\
&= \frac{c\int \frac{1}{\sqrt{a}-\sqrt{b}x^2}\,dx}{2\sqrt{a}} + \frac{c\int \frac{1}{\sqrt{a}+\sqrt{b}x^2}\,dx}{2\sqrt{a}} + \frac{1}{2}d\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right) \\
&= \frac{c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.54

$$\int \frac{c+dx}{a-bx^4}\,dx$$
$$=\frac{2\sqrt[4]{b}c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt[4]{b}c+\sqrt[4]{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)+\sqrt[4]{b}c\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)-\sqrt[4]{a}d\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)+}{4a^{3/4}\sqrt{b}}$$

```
[In] Integrate[(c + d*x)/(a - b*x^4),x]
```

```
[Out] (2*b^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)] - (b^(1/4)*c + a^(1/4)*d)*Log[a^(1
/4) - b^(1/4)*x] + b^(1/4)*c*Log[a^(1/4) + b^(1/4)*x] - a^(1/4)*d*Log[a^(1/
4) + b^(1/4)*x] + a^(1/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*a^(3/4)*Sqrt[b])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.39

method	result	size
risch	$-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(\_R d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	34
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}$	87

```
[In] int((d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*sum((_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 39057, normalized size of antiderivative = 448.93

$$\int \frac{c+dx}{a-bx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \frac{c+dx}{a-bx^4}\,dx = -\operatorname{RootSum}\left(256t^4a^3b^2-32t^2a^2bd^2-16tabc^2d+ad^4-bc^4,\left(t\mapsto t\log\left(x+\frac{-128t^3a^3bd^2+16t^2a^2bc^2d}{4acd^4}\right.\right.\right.$$

```
[In] integrate((d*x+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**2 - 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c**2*d + a*
d**4 - b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b*d**2 + 16*_t**2*a**
2*b*c**2*d + 8*_t*a**2*d**4 - 4*_t*a*b*c**4 + 5*a*c**2*d**3)/(4*a*c*d**4 +
b*c**5))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(57) = 114.

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \frac{c+dx}{a-bx^4}\,dx = \frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}} - \frac{d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}} - \frac{c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}$$

```
[In] integrate((d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + 1/4*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 1/4*d*log(sqrt(b)
*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 1/4*c*log((sqrt(b)*x - sqrt(sqrt(a)*sqr
t(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(57) = 114$.

Time = 0.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.61

$$\int \frac{c+dx}{a-bx^4}\,dx = \frac{\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}bd-(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}bd-(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2}$$

```
[In] integrate((d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))
/(a*b) - 1/8*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sq
rt(-a/b))/(a*b) - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b*d - (-a*b^3)^(1/4)*b*c)
*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2) - 1/
4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b*d - (-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.09

$$\int \frac{c+dx}{a-bx^4}\,dx = \begin{cases} \frac{2\,c+3\,d\,x}{6\,b\,x^3} \\ \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,(-b)^{1/4}\,x}{a^{1/4}}-1\right)\left(2\,a^{1/4}\,d+\sqrt{2}\,(-b)^{1/4}\,c\right)}{4\,a^{3/4}\,\sqrt{-b}} - \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,(-b)^{1/4}\,x}{a^{1/4}}+1\right)\left(4\,a^{1/4}\,d-2\,\sqrt{2}\,(-b)^{1/4}\,c\right)}{8\,a^{3/4}\,\sqrt{-b}} + \frac{\sqrt{2}\,c\,\ln\left(\frac{\sqrt{-b}\,x^2+\sqrt{a}+\sqrt{2}\,a^{1/4}\,(-b}{\sqrt{-b}\,x^2+\sqrt{a}-\sqrt{2}\,a^{1/4}\,(-b}\right.}{8\,a^{3/4}\,(-b)^{1/4}} \end{cases}$$

```
[In] int((c + d*x)/(a - b*x^4),x)
```

```
[Out] piecewise(a == 0, (2*c + 3*d*x)/(6*b*x^3), a ~= 0, (atan((2^(1/2)*(-b)^(1/4
)*x)/a^(1/4) - 1)*(2*a^(1/4)*d + 2^(1/2)*(-b)^(1/4)*c))/(4*a^(3/4)*(-b)^(1/
2)) - (atan((2^(1/2)*(-b)^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*d - 2*2^(1/2)*(-
b)^(1/4)*c))/(8*a^(3/4)*(-b)^(1/2)) + (2^(1/2)*c*log(((-b)^(1/2)*x^2 + a^(1
/2) + 2^(1/2)*a^(1/4)*(-b)^(1/4)*x)/((-b)^(1/2)*x^2 + a^(1/2) - 2^(1/2)*a^(
1/4)*(-b)^(1/4)*x)))/(8*a^(3/4)*(-b)^(1/4)))
```

3.116 $\int \frac{c+dx}{a+bx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 219

$$\int \frac{c+dx}{a+bx^4}\,dx=\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

```
[Out] 1/4*c*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)*2^(1/2)+1/4*c*ar
ctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)*2^(1/2)-1/8*c*ln(-a^(1/4)
*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)+1/8*c*ln(a^
(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)+1/2*d*
arctan(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$\int \frac{c+dx}{a+bx^4}\,dx=-\frac{c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[In] Int[(c + d*x)/(a + b*x^4),x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (c*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*ArcTan[1 + (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - (c*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1
/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{c}{a+bx^4}+\frac{dx}{a+bx^4}\right)dx\\
&= c\int \frac{1}{a+bx^4}\,dx + d\int \frac{x}{a+bx^4}\,dx\\
&= \frac{c\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}} + \frac{c\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}} + \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)\\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} + \frac{c\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\sqrt{b}} + \frac{c\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\sqrt{b}}\\
&\quad - \frac{c\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{c\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}\\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}\\
&\quad + \frac{c\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{c\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}
\end{aligned}
$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.84

$$\int \frac{c+dx}{a+bx^4}\,dx$$

$$= \frac{-2\left(\sqrt{2}\sqrt[4]{b}c+2\sqrt[4]{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt[4]{b}c-2\sqrt[4]{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+\sqrt{2}\sqrt[4]{b}c\left(-\log\right.}{8a^{3/4}\sqrt{b}}$$

```
[In] Integrate[(c + d*x)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*b^(1/4)*c + 2*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)] + 2*(Sqrt[2]*b^(1/4)*c - 2*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(
1/4)] + Sqrt[2]*b^(1/4)*c*(-Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(8*a^(3/
4)*Sqrt[b])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.47 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.15

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b+a\right)} \frac{\left(\_Rd+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	32
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{d\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}$	124

```
[In] int((d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*sum((_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 41851, normalized size of antiderivative = 191.10

$$\int \frac{c+dx}{a+bx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)/(b*x^4+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.57

$$\int \frac{c+dx}{a+bx^4}\,dx = \operatorname{RootSum}\left(256t^4a^3b^2+32t^2a^2bd^2-16tabc^2d+ad^4+bc^4, \left(t\mapsto t\log\left(x+\frac{-128t^3a^3bd^2-16t^2a^2bc^2d-}{4acd^4-}\right.\right.\right.$$

```
[In] integrate((d*x+c)/(b*x**4+a),x)
[Out] RootSum(256*_t**4*a**3*b**2 + 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c**2*d + a*d
**4 + b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b*d**2 - 16*_t**2*a**2
*b*c**2*d - 8*_t*a**2*d**4 - 4*_t*a*b*c**4 + 5*a*c**2*d**3)/(4*a*c*d**4 - b
*c**5))))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.95

$$\int \frac{c+dx}{a+bx^4}\,dx = \frac{\sqrt{2}c\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}c\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-2\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{1}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c+2\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{1}{4}}}$$

```
[In] integrate((d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*c*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(1/4)) - 1/8*sqrt(2)*c*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + s
qrt(a))/(a^(3/4)*b^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*b^(1/4)*c - 2*sqrt(a)*d)*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt
(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*b^(1/4
)*c + 2*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4
))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.97

$$\int \frac{c+dx}{a+bx^4}\,dx = \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}bd-(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}bd-(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2}$$

```
[In] integrate((d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
*b) - 1/8*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(a*b) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b*d - (a*b^3)^(1/4)*b*c)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) - 1/4*sqrt(2)*
(sqrt(2)*sqrt(a*b)*b*d - (a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(
2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73

$$\int \frac{c+dx}{a+bx^4}\,dx$$
$$= \begin{cases} -\frac{2c+3dx}{6bx^3} & \text{if } a \\ \frac{\operatorname{atan}\left(\frac{\sqrt{2}\,b^{1/4}\,x}{a^{1/4}}-1\right)\left(2\,a^{1/4}\,d+\sqrt{2}\,b^{1/4}\,c\right)}{4\,a^{3/4}\,\sqrt{b}}-\frac{\operatorname{atan}\left(\frac{\sqrt{2}\,b^{1/4}\,x}{a^{1/4}}+1\right)\left(4\,a^{1/4}\,d-2\,\sqrt{2}\,b^{1/4}\,c\right)}{8\,a^{3/4}\,\sqrt{b}}+\frac{\sqrt{2}\,c\,\ln\left(\frac{\sqrt{a}+\sqrt{b}\,x^2+\sqrt{2}\,a^{1/4}\,b^{1/4}\,x}{\sqrt{a}+\sqrt{b}\,x^2-\sqrt{2}\,a^{1/4}\,b^{1/4}\,x}\right)}{8\,a^{3/4}\,b^{1/4}} & \text{if } a \end{cases}$$

```
[In] int((c + d*x)/(a + b*x^4),x)
```

```
[Out] piecewise(a == 0, -(2*c + 3*d*x)/(6*b*x^3), a ~= 0, (atan((2^(1/2)*b^(1/4)*
x)/a^(1/4) - 1)*(2*a^(1/4)*d + 2^(1/2)*b^(1/4)*c))/(4*a^(3/4)*b^(1/2)) - (a
tan((2^(1/2)*b^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*d - 2*2^(1/2)*b^(1/4)*c))/(
8*a^(3/4)*b^(1/2)) + (2^(1/2)*c*log((a^(1/2) + b^(1/2)*x^2 + 2^(1/2)*a^(1/4
)*b^(1/4)*x)/(a^(1/2) + b^(1/2)*x^2 - 2^(1/2)*a^(1/4)*b^(1/4)*x)))/(8*a^(3/
4)*b^(1/4)))
```

3.117 $\int \frac{c+dx}{(a-bx^4)^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 110

$$\int \frac{c+dx}{(a-bx^4)^2} \, dx = \frac{x(c+dx)}{4a(a-bx^4)} + \frac{3c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}} + \frac{3c\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}} + \frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

```
[Out] 1/4*x*(d*x+c)/a/(-b*x^4+a)+3/8*c*arctan(b^(1/4)*x/a^(1/4))/a^(7/4)/b^(1/4)+
3/8*c*arctanh(b^(1/4)*x/a^(1/4))/a^(7/4)/b^(1/4)+1/4*d*arctanh(x^2*b^(1/2)/
a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1869, 1890, 218, 214, 211, 281}

$$\int \frac{c+dx}{(a-bx^4)^2} \, dx = \frac{3c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}} + \frac{3c\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}} + \frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{x(c+dx)}{4a(a-bx^4)}$$

```
[In] Int[(c + d*x)/(a - b*x^4)^2,x]
```

```
[Out] (x*(c + d*x))/(4*a*(a - b*x^4)) + (3*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7
/4)*b^(1/4)) + (3*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(1/4)) + (d*
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(c+dx)}{4a\left(a-bx^4\right)}-\frac{\int \frac{-3c-2dx}{a-bx^4}\,dx}{4a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)}-\frac{\int \left(-\frac{3c}{a-bx^4}-\frac{2dx}{a-bx^4}\right)\,dx}{4a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)}+\frac{(3c)\int \frac{1}{a-bx^4}\,dx}{4a}+\frac{d\int \frac{x}{a-bx^4}\,dx}{2a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)}+\frac{(3c)\int \frac{1}{\sqrt{a}-\sqrt{b}x^2}\,dx}{8a^{3/2}}+\frac{(3c)\int \frac{1}{\sqrt{a}+\sqrt{b}x^2}\,dx}{8a^{3/2}}+\frac{d\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{4a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)}+\frac{3c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{3c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.53

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx$$
$$= \frac{\frac{4ax(c+dx)}{a-bx^4} + \frac{6\sqrt[4]{a}c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} - \frac{\left(3\sqrt[4]{a}\sqrt[4]{b}c+2\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}} + \frac{\left(3\sqrt[4]{a}\sqrt[4]{b}c-2\sqrt{a}d\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{\sqrt{b}} + \frac{2\sqrt{a}d\log}{}}{16a^2}$$

```
[In] Integrate[(c + d*x)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a*x*(c + d*x))/(a - b*x^4) + (6*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/
b^(1/4) - ((3*a^(1/4)*b^(1/4)*c + 2*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/Sq
rt[b] + ((3*a^(1/4)*b^(1/4)*c - 2*Sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/Sqrt
[b] + (2*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(16*a^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

method	result	size
risch	$\frac{\frac{d x^2}{4a}+\frac{cx}{4a}}{-b x^4+a} - \frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b-a\right)} \frac{\left(2\_R d+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$	69
default	$c\left(\frac{x}{4a(-b x^4+a)} + \frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{16a^2}\right) + d\left(\frac{x^2}{4a(-b x^4+a)} + \frac{\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)$	128

```
[In] int((d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4*d/a*x^2+1/4*c/a*x)/(-b*x^4+a)-1/16/b/a*sum((2*_R*d+3*c)/_R^3*ln(x-_R),
_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 40560, normalized size of antiderivative = 368.73

$$\int \frac{c+dx}{(a-bx^4)^2} \, dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.42

$$\int \frac{c+dx}{(a-bx^4)^2} \, dx$$
$$= \text{RootSum}\left(65536t^4a^7b^2 - 2048t^2a^4bd^2 + 1152ta^2bc^2d + 16ad^4 - 81bc^4, \left(t \mapsto t\log\left(x + \frac{32768t^3a^6bd^2 + \cdots}{\cdots}\right.\right.\right.$$
$$+ \frac{-cx - dx^2}{-4a^2 + 4abx^4}$$

```
[In] integrate((d*x+c)/(-b*x**4+a)**2,x)
[Out] RootSum(65536*_t**4*a**7*b**2 - 2048*_t**2*a**4*b*d**2 + 1152*_t*a**2*b*c**
2*d + 16*a*d**4 - 81*b*c**4, Lambda(_t, _t*log(x + (32768*_t**3*a**6*b*d**2
 + 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d**4 + 1296*_t*a**2*b*c**4 + 360*
a*c**2*d**3)/(192*a*c*d**4 + 243*b*c**5)))) + (-c*x - d*x**2)/(-4*a**2 + 4*
a*b*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.43

$$\int \frac{c+dx}{(a-bx^4)^2} \, dx = -\frac{dx^2 + cx}{4\,(abx^4 - a^2)} + \frac{\frac{6\,c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{2\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{3\,c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{16\,a}$$

```
[In] integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(d*x^2 + c*x)/(a*b*x^4 - a^2) + 1/16*(6*c*arctan(sqrt(b)*x/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*d*log(sqrt(b)*x^2 + sqrt(a)
)/(sqrt(a)*sqrt(b)) - 2*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 3*
c*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b)
)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(79) = 158$.

Time = 0.27 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.31

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx = \frac{3\sqrt{2}(-ab^3)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,a^2b} - \frac{3\sqrt{2}(-ab^3)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,a^2b} - \frac{dx^2+cx}{4\,(bx^4-a)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-ab}bd+3\,(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-ab}bd+3\,(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2}$$

```
[In] integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b)
)/(a^2*b) - 3/32*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4)
+ sqrt(-a/b))/(a^2*b) - 1/4*(d*x^2 + c*x)/((b*x^4 - a)*a) + 1/16*sqrt(2)*(2
*sqrt(2)*sqrt(-a*b)*b*d + 3*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + s
qrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt
(-a*b)*b*d + 3*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)
^(1/4))/(-a/b)^(1/4))/(a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.57

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx$$
$$= \left(\sum_{k=1}^{4}\ln\left(-\frac{b^2\left(3\,c\,d^2+2\,d^3\,x+\mathrm{root}(65536\,a^7\,b^2\,z^4-2048\,a^4\,b\,d^2\,z^2+1152\,a^2\,b\,c^2\,d\,z-81\,b\,c^4+16\,a\,d^4,\right.}{}\right.\right.$$
$$\left.\left. -2048\,a^4\,b\,d^2\,z^2+1152\,a^2\,b\,c^2\,d\,z-81\,b\,c^4+16\,a\,d^4,z,k)\right)\right)+\frac{\frac{d\,x^2}{4\,a}+\frac{c\,x}{4\,a}}{a-b\,x^4}$$

```
[In] int((c + d*x)/(a - b*x^4)^2,x)
```

```
[Out] symsum(log(-(b^2*(3*c*d^2 + 2*d^3*x + 192*root(65536*a^7*b^2*z^4 - 2048*a^4
*b*d^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b*c^4 + 16*a*d^4, z, k)^2*a^3*b*c - 12
8*root(65536*a^7*b^2*z^4 - 2048*a^4*b*d^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b*c
^4 + 16*a*d^4, z, k)^2*a^3*b*d*x + 36*root(65536*a^7*b^2*z^4 - 2048*a^4*b*d
^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b*c^4 + 16*a*d^4, z, k)*a*b*c^2*x))/(16*a^
3))*root(65536*a^7*b^2*z^4 - 2048*a^4*b*d^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b
*c^4 + 16*a*d^4, z, k), k, 1, 4) + ((d*x^2)/(4*a) + (c*x)/(4*a))/(a - b*x^4
)
```

3.118 $\int \frac{c+dx}{(a+bx^4)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 241

$$\int \frac{c+dx}{(a+bx^4)^2} dx = \frac{x(c+dx)}{4a(a+bx^4)} + \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{3c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3c\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} - \frac{3c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

```
[Out] 1/4*x*(d*x+c)/a/(b*x^4+a)+3/16*c*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/
4)/b^(1/4)*2^(1/2)+3/16*c*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(1/
4)*2^(1/2)-3/32*c*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4
)/b^(1/4)*2^(1/2)+3/32*c*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/
a^(7/4)/b^(1/4)*2^(1/2)+1/4*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules

used = {1869, 1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx = -\frac{3c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{3c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{x(c+dx)}{4a\left(a+bx^4\right)}$$

```
[In] Int[(c + d*x)/(a + b*x^4)^2,x]
```

```
[Out] (x*(c + d*x))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3
/2)*Sqrt[b]) - (3*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*b^(1/4)) + (3*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*b^(1/4)) - (3*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
])/(16*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(c+dx)}{4a\left(a+bx^4\right)} - \frac{\int \frac{-3c-2dx}{a+bx^4}\,dx}{4a}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} - \frac{\int \left(-\frac{3c}{a+bx^4} - \frac{2dx}{a+bx^4}\right)\,dx}{4a}$$

$$= \frac{x(c+dx)}{4a(a+bx^4)} + \frac{(3c)\int \frac{1}{a+bx^4}\,dx}{4a} + \frac{d\int \frac{x}{a+bx^4}\,dx}{2a}$$

$$= \frac{x(c+dx)}{4a(a+bx^4)} + \frac{(3c)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}} + \frac{(3c)\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}} + \frac{d\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{4a}$$

$$= \frac{x(c+dx)}{4a(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{(3c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}\sqrt{b}} + \frac{(3c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}\sqrt{b}} - \frac{(3c)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\sqrt[4]{b}} - \frac{(3c)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

$$= \frac{x(c+dx)}{4a(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{3c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{(3c)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} - \frac{(3c)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

$$= \frac{x(c+dx)}{4a(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{3c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3c\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} - \frac{3c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.93

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx$$

$$= \frac{\frac{8a^{3/4}x(c+dx)}{a+bx^4} - \frac{2\left(3\sqrt{2}\sqrt[4]{b}c+4\sqrt[4]{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{2\left(3\sqrt{2}\sqrt[4]{b}c-4\sqrt[4]{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}} - \frac{3\sqrt{2}c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{\sqrt[4]{b}}}{32a^{7/4}}$$

```
[In] Integrate[(c + d*x)/(a + b*x^4)^2,x]
```

```
[Out] ((8*a^(3/4)*x*(c + d*x))/(a + b*x^4) - (2*(3*Sqrt[2]*b^(1/4)*c + 4*a^(1/4)*
d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] + (2*(3*Sqrt[2]*b^(1/4)
*c - 4*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] - (3*Sqr
t[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4) + (3
*Sqrt[2]*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))
/(32*a^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.27

method	result
risch	$\frac{\frac{d\,x^2}{4a}+\frac{cx}{4a}}{b\,x^4+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(2\_Rd+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(b\,x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+d\left(\frac{x^2}{4a(b\,x^4+a)}+\right.$

```
[In] int((d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4*d/a*x^2+1/4*c/a*x)/(b*x^4+a)+1/16/b/a*sum((2*_R*d+3*c)/_R^3*ln(x-_R),_
R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 43065, normalized size of antiderivative = 178.69

$$\int \frac{c+dx}{\left(a+bx^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.64

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx$$
$$=\text{RootSum}\left(65536t^4a^7b^2+2048t^2a^4bd^2-1152ta^2bc^2d+16ad^4+81bc^4,\left(t\mapsto t\log\left(x+\frac{-32768t^3a^6bd^2-\right.\right.\right.$$
$$+\frac{cx+dx^2}{4a^2+4abx^4}$$

```
[In] integrate((d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**2 + 2048*_t**2*a**4*b*d**2 - 1152*_t*a**2*b*c**
2*d + 16*a*d**4 + 81*b*c**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*b*d**
2 - 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d**4 - 1296*_t*a**2*b*c**4 + 360
*a*c**2*d**3)/(192*a*c*d**4 - 243*b*c**5)))) + (c*x + d*x**2)/(4*a**2 + 4*a
*b*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.99

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx=\frac{dx^2+cx}{4\left(abx^4+a^2\right)}$$
$$+\frac{\frac{3\sqrt{2}c\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{3\sqrt{2}c\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-4\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{1}{4}}}+\frac{2}{-}}{32\,a}$$

```
[In] integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(d*x^2 + c*x)/(a*b*x^4 + a^2) + 1/32*(3*sqrt(2)*c*log(sqrt(b)*x^2 + sqr
t(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - 3*sqrt(2)*c*log(sqrt(
b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) + 2*(3*sqrt
(2)*a^(1/4)*b^(1/4)*c - 4*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt
(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*
b^(1/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(1/4)*c + 4*sqrt(a)*d)*arctan(1/2*sqrt(2)
*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sq
rt(sqrt(a)*sqrt(b))*b^(1/4)))/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.99

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx = \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b} - \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b} + \frac{dx^2+cx}{4\,(bx^4+a)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}bd+3\,(ab^3)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}bd+3\,(ab^3)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2}$$

```
[In] integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(
a^2*b) - 3/32*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(a^2*b) + 1/4*(d*x^2 + c*x)/((b*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(
2)*sqrt(a*b)*b*d + 3*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b*d
+ 3*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)
^(1/4))/(a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.17

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx = \left(\sum_{k=1}^{4}\ln\left(\frac{b^2\left(3\,c\,d^2+2\,d^3\,x-\mathrm{root}(65536\,a^7\,b^2\,z^4+2048\,a^4\,b\,d^2\,z^2-1152\,a^2\,b\,c^2\,d\,z+81\,b\,c^4+16\,a\,d^4,z,k\right.}{\cdots}\right.\right.$$
$$\left.\left.+2048\,a^4\,b\,d^2\,z^2-1152\,a^2\,b\,c^2\,d\,z+81\,b\,c^4+16\,a\,d^4,z,k)\right)\right)+\frac{\frac{d\,x^2}{4\,a}+\frac{c\,x}{4\,a}}{b\,x^4+a}$$

```
[In] int((c + d*x)/(a + b*x^4)^2,x)
```

```
[Out] symsum(log((b^2*(3*c*d^2 + 2*d^3*x - 192*root(65536*a^7*b^2*z^4 + 2048*a^4*
b*d^2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*c^4 + 16*a*d^4, z, k)^2*a^3*b*c + 128
*root(65536*a^7*b^2*z^4 + 2048*a^4*b*d^2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*c^
4 + 16*a*d^4, z, k)^2*a^3*b*d*x - 36*root(65536*a^7*b^2*z^4 + 2048*a^4*b*d^
2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*c^4 + 16*a*d^4, z, k)*a*b*c^2*x))/(16*a^3
))*root(65536*a^7*b^2*z^4 + 2048*a^4*b*d^2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*
c^4 + 16*a*d^4, z, k), k, 1, 4) + ((d*x^2)/(4*a) + (c*x)/(4*a))/(a + b*x^4)
```

3.119 $\int \frac{c+dx}{(a-bx^4)^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 136

$$\int \frac{c+dx}{(a-bx^4)^3} dx = \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{21c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{21c\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{3d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

```
[Out] 1/8*x*(d*x+c)/a/(-b*x^4+a)^2+1/32*x*(6*d*x+7*c)/a^2/(-b*x^4+a)+21/64*c*arct
an(b^(1/4)*x/a^(1/4))/a^(11/4)/b^(1/4)+21/64*c*arctanh(b^(1/4)*x/a^(1/4))/a
^(11/4)/b^(1/4)+3/16*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1869, 1890, 218, 214, 211, 281}

$$\int \frac{c+dx}{(a-bx^4)^3} dx = \frac{21c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{21c\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{3d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{x(c+dx)}{8a(a-bx^4)^2}$$

```
[In] Int[(c + d*x)/(a - b*x^4)^3,x]
```

```
[Out] (x*(c + d*x))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a - b*x^4))
+ (21*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) + (21*c*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sq
rt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(c+dx)}{8a\left(a-bx^4\right)^2} - \frac{\int \frac{-7c-6dx}{\left(a-bx^4\right)^2}\,dx}{8a}$$

$$= \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{\int \frac{21c+12dx}{a-bx^4}\,dx}{32a^2}$$

$$= \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{\int \left(\frac{21c}{a-bx^4} + \frac{12dx}{a-bx^4}\right)\,dx}{32a^2}$$

$$= \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{(21c)\int \frac{1}{a-bx^4}\,dx}{32a^2} + \frac{(3d)\int \frac{x}{a-bx^4}\,dx}{8a^2}$$

$$= \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{(21c)\int \frac{1}{\sqrt{a}-\sqrt{b}x^2}\,dx}{64a^{5/2}} + \frac{(21c)\int \frac{1}{\sqrt{a}+\sqrt{b}x^2}\,dx}{64a^{5/2}} + \frac{(3d)\mathrm{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{16a^2}$$

$$= \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{21c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{21c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.42

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx$$

$$= \frac{\frac{16a^2x(c+dx)}{(a-bx^4)^2} + \frac{4ax(7c+6dx)}{a-bx^4} + \frac{42\sqrt[4]{a}c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} - \frac{3\left(7\sqrt[4]{a}\sqrt[4]{b}c+4\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}} + \frac{3\left(7\sqrt[4]{a}\sqrt[4]{b}c-4\sqrt{a}d\right)\log\left(\sqrt[4]{a}\right.}{\sqrt{b}}}{128a^3}$$

```
[In] Integrate[(c + d*x)/(a - b*x^4)^3,x]
```

```
[Out] ((16*a^2*x*(c + d*x))/(a - b*x^4)^2 + (4*a*x*(7*c + 6*d*x))/(a - b*x^4) + (
42*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(1/4) - (3*(7*a^(1/4)*b^(1/4)*c
 + 4*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/Sqrt[b] + (3*(7*a^(1/4)*b^(1/4)*c
 - 4*Sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/Sqrt[b] + (12*Sqrt[a]*d*Log[Sqrt[
a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.65

method	result
risch	$\frac{-\frac{3bd\,x^{6}}{16a^{2}}-\frac{7bc\,x^{5}}{32a^{2}}+\frac{5d\,x^{2}}{16a}+\frac{11cx}{32a}}{\left(-b\,x^{4}+a\right)^{2}}-\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b-a\right)}\frac{\left(4\_Rd+7c\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{128a^{2}b}$
default	$c\left(\frac{x}{8a\left(-b\,x^{4}+a\right)^{2}}+\frac{\frac{7x}{32a\left(-b\,x^{4}+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{128a^{2}}}{a}\right)+d\left(\frac{x^{2}}{8a\left(-b\,x^{4}+a\right)^{2}}+\frac{\frac{3x^{2}}{16a\left(-b\,x^{4}+a\right)}+\cdots}{a}\right.$

```
[In] int((d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/16*b*d/a^2*x^6-7/32*b*c/a^2*x^5+5/16*d/a*x^2+11/32*c/a*x)/(-b*x^4+a)^2-
3/128/a^2/b*sum((4*_R*d+7*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 40637, normalized size of antiderivative = 298.80

$$\int \frac{c+dx}{\left(a-bx^4\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.43

$$\int \frac{c+dx}{\left(a-bx^4\right)^3}\,dx =$$
$$-\operatorname{RootSum}\left(268435456t^4a^{11}b^2-4718592t^2a^6bd^2-2709504ta^3bc^2d+20736ad^4-194481bc^4,\left(t\mapsto t\log\right.\right.$$
$$-\frac{-11acx-10adx^2+7bcx^5+6bdx^6}{32a^4-64a^3bx^4+32a^2b^2x^8}$$

```
[In] integrate((d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] -RootSum(268435456*_t**4*a**11*b**2 - 4718592*_t**2*a**6*b*d**2 - 2709504*_
t*a**3*b*c**2*d + 20736*a*d**4 - 194481*b*c**4, Lambda(_t, _t*log(x + (-671
08864*_t**3*a**9*b*d**2 + 9633792*_t**2*a**6*b*c**2*d + 589824*_t*a**4*d**4
 - 2765952*_t*a**3*b*c**4 + 423360*a*c**2*d**3)/(193536*a*c*d**4 + 453789*b
*c**5)))) - (-11*a*c*x - 10*a*d*x**2 + 7*b*c*x**5 + 6*b*d*x**6)/(32*a**4 -
64*a**3*b*x**4 + 32*a**2*b**2*x**8)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.37

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx = -\frac{6\,bdx^6+7\,bcx^5-10\,adx^2-11\,acx}{32\,(a^2b^2x^8-2\,a^3bx^4+a^4)} + \frac{3\left(\frac{14\,c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{4\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{4\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{7\,c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\right)}{128\,a^2}$$

```
[In] integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(6*b*d*x^6 + 7*b*c*x^5 - 10*a*d*x^2 - 11*a*c*x)/(a^2*b^2*x^8 - 2*a^3*
b*x^4 + a^4) + 3/128*(14*c*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)
*sqrt(sqrt(a)*sqrt(b))) + 4*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b))
- 4*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 7*c*log((sqrt(b)*x - s
qrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sq
rt(a)*sqrt(b))))/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(104) = 208$.

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.00

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx = \frac{21\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^3b} - \frac{21\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^3b} - \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{-ab}bd-7\,(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2} - \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{-ab}bd-7\,(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2} - \frac{6\,bdx^6+7\,bcx^5-10\,adx^2-11\,acx}{32\,(bx^4-a)^2a^2}$$

```
[In] integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/
b))/(a^3*b) - 21/256*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1
/4) + sqrt(-a/b))/(a^3*b) - 3/128*sqrt(2)*(4*sqrt(2)*sqrt(-a*b)*b*d - 7*(-a
*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/(a^3*b^2) - 3/128*sqrt(2)*(4*sqrt(2)*sqrt(-a*b)*b*d - 7*(-a*b^3)^(1/4)*
b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^2
) - 1/32*(6*b*d*x^6 + 7*b*c*x^5 - 10*a*d*x^2 - 11*a*c*x)/((b*x^4 - a)^2*a^2
)
```

Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.32

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx = \frac{\frac{5\,d\,x^2}{16\,a}+\frac{11\,c\,x}{32\,a}-\frac{7\,b\,c\,x^5}{32\,a^2}-\frac{3\,b\,d\,x^6}{16\,a^2}}{a^2-2\,a\,b\,x^4+b^2\,x^8} + \left(\sum_{k=1}^{4}\ln\left(-\frac{b^2\left(63\,c\,d^2+36\,d^3\,x+\mathrm{root}(268435456\,a^{11}\,b^2\,z^4-4718592\,a^6\,b\,d^2\,z^2+2709504\,a^3\,b\,c^2\,d\,z-\right.}{}\right.\right.$$

$$\left.\left.-4718592\,a^6\,b\,d^2\,z^2+2709504\,a^3\,b\,c^2\,d\,z-194481\,b\,c^4+20736\,a\,d^4,z,k\right)\right)$$

```
[In] int((c + d*x)/(a - b*x^4)^3,x)
```

```
[Out] ((5*d*x^2)/(16*a) + (11*c*x)/(32*a) - (7*b*c*x^5)/(32*a^2) - (3*b*d*x^6)/(1
6*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4) + symsum(log(-(3*b^2*(63*c*d^2 + 36*d^3
*x + 7168*root(268435456*a^11*b^2*z^4 - 4718592*a^6*b*d^2*z^2 + 2709504*a^3
*b*c^2*d*z - 194481*b*c^4 + 20736*a*d^4, z, k)^2*a^5*b*c + 1176*root(268435
456*a^11*b^2*z^4 - 4718592*a^6*b*d^2*z^2 + 2709504*a^3*b*c^2*d*z - 194481*b
*c^4 + 20736*a*d^4, z, k)*a^2*b*c^2*x - 4096*root(268435456*a^11*b^2*z^4 -
4718592*a^6*b*d^2*z^2 + 2709504*a^3*b*c^2*d*z - 194481*b*c^4 + 20736*a*d^4,
 z, k)^2*a^5*b*d*x))/(2048*a^6))*root(268435456*a^11*b^2*z^4 - 4718592*a^6*
b*d^2*z^2 + 2709504*a^3*b*c^2*d*z - 194481*b*c^4 + 20736*a*d^4, z, k), k, 1
, 4)
```

3.120 $\int \frac{c+dx}{(a+bx^4)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 266

$$\int \frac{c+dx}{(a+bx^4)^3} \, dx = \frac{x(c+dx)}{8a(a+bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a+bx^4)} + \frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{21c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21c\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} - \frac{21c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

```
[Out] 1/8*x*(d*x+c)/a/(b*x^4+a)^2+1/32*x*(6*d*x+7*c)/a^2/(b*x^4+a)+21/128*c*arcta
n(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/b^(1/4)*2^(1/2)+21/128*c*arctan(1+
b^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/b^(1/4)*2^(1/2)-21/256*c*ln(-a^(1/4)*b^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)+21/256*c*ln(a
^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)+3/16
*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1869, 1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx = -\frac{21c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{21c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{21c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{21c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{x(7c+6dx)}{32a^2(a+bx^4)}+\frac{x(c+dx)}{8a(a+bx^4)^2}$$

```
[In] Int[(c + d*x)/(a + b*x^4)^3,x]
```

```
[Out] (x*(c + d*x))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a + b*x^4))
+ (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - (21*c*ArcTan[1
 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(1/4)) + (21*c*ArcT
an[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(1/4)) - (21*c*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/
4)*b^(1/4)) + (21*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(128*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} - \frac{\int \frac{-7c-6dx}{\left(a+bx^4\right)^2}\,dx}{8a}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{\int \frac{21c+12dx}{a+bx^4}\,dx}{32a^2}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{\int \left(\frac{21c}{a+bx^4} + \frac{12dx}{a+bx^4}\right)\,dx}{32a^2}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{(21c)\int \frac{1}{a+bx^4}\,dx}{32a^2} + \frac{(3d)\int \frac{x}{a+bx^4}\,dx}{8a^2}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{(21c)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{64a^{5/2}} + \frac{(21c)\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{64a^{5/2}} + \frac{(3d)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{16a^2}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} + \frac{(21c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{128a^{5/2}\sqrt{b}} + \frac{(21c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{128a^{5/2}\sqrt{b}} - \frac{(21c)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{128\sqrt{2}a^{11/4}\sqrt[4]{b}} - \frac{(21c)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{128\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{21c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{(21c)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} - \frac{(21c)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{21c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21c\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} - \frac{21c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.94

$$\int \frac{c+dx}{\left(a+bx^4\right)^3}\,dx$$

$$= \frac{\frac{32a^{7/4}x(c+dx)}{\left(a+bx^4\right)^2} + \frac{8a^{3/4}x(7c+6dx)}{a+bx^4} - \frac{6\left(7\sqrt{2}\sqrt[4]{b}c+8\sqrt[4]{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{6\left(7\sqrt{2}\sqrt[4]{b}c-8\sqrt[4]{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}} - \cdots}{256a^{11/4}}$$

```
[In] Integrate[(c + d*x)/(a + b*x^4)^3,x]
```

```
[Out] ((32*a^(7/4)*x*(c + d*x))/(a + b*x^4)^2 + (8*a^(3/4)*x*(7*c + 6*d*x))/(a +
b*x^4) - (6*(7*Sqrt[2]*b^(1/4)*c + 8*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/Sqrt[b] + (6*(7*Sqrt[2]*b^(1/4)*c - 8*a^(1/4)*d)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] - (21*Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4) + (21*Sqrt[2]*c*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))/(256*a^(11/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\frac{3bd\,x^6}{16a^2}+\frac{7bc\,x^5}{32a^2}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}}{(b\,x^4+a)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{(4\_Rd+7c)\ln(x-\_R)}{\_R^3}\right)}{128a^2b}$
default	$c\left(\frac{x}{8a(b\,x^4+a)^2}+\frac{\frac{7x}{32a(b\,x^4+a)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)+d\left(\frac{}{8a(b}\right.$

[In] int((d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)

[Out] (3/16*b*d/a^2*x^6+7/32*b*c/a^2*x^5+5/16*d/a*x^2+11/32*c/a*x)/(b*x^4+a)^2+3/128/a^2/b*sum((4*_R*d+7*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 43180, normalized size of antiderivative = 162.33

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

[In] integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")

[Out] Too large to include

Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.72

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx$$
$$= \operatorname{RootSum}\left(268435456t^4a^{11}b^2+4718592t^2a^6bd^2-2709504ta^3bc^2d+20736ad^4+194481bc^4,\left(t\mapsto t\log\left(x\right.\right.\right.$$
$$+\frac{11acx+10adx^2+7bcx^5+6bdx^6}{32a^4+64a^3bx^4+32a^2b^2x^8}$$

```
[In] integrate((d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*b**2 + 4718592*_t**2*a**6*b*d**2 - 2709504*_t
*a**3*b*c**2*d + 20736*a*d**4 + 194481*b*c**4, Lambda(_t, _t*log(x + (-6710
8864*_t**3*a**9*b*d**2 - 9633792*_t**2*a**6*b*c**2*d - 589824*_t*a**4*d**4
- 2765952*_t*a**3*b*c**4 + 423360*a*c**2*d**3)/(193536*a*c*d**4 - 453789*b*
c**5)))) + (11*a*c*x + 10*a*d*x**2 + 7*b*c*x**5 + 6*b*d*x**6)/(32*a**4 + 64
*a**3*b*x**4 + 32*a**2*b**2*x**8)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx = \frac{6\,bdx^6+7\,bcx^5+10\,adx^2+11\,acx}{32\,(a^2b^2x^8+2\,a^3bx^4+a^4)} + \frac{3\left(\frac{7\,\sqrt{2}c\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{7\,\sqrt{2}c\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{2\left(7\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-8\,\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{1}{4}}}\right.}{256\,a^2}$$

```
[In] integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(6*b*d*x^6 + 7*b*c*x^5 + 10*a*d*x^2 + 11*a*c*x)/(a^2*b^2*x^8 + 2*a^3*b
*x^4 + a^4) + 3/256*(7*sqrt(2)*c*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*
x + sqrt(a))/(a^(3/4)*b^(1/4)) - 7*sqrt(2)*c*log(sqrt(b)*x^2 - sqrt(2)*a^(1
/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) + 2*(7*sqrt(2)*a^(1/4)*b^(1/4)*c
 - 8*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/
sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)) + 2*(7*sqrt(
2)*a^(1/4)*b^(1/4)*c + 8*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b
^(1/4)))/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.96

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx = \frac{21\sqrt{2}(ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b} - \frac{21\sqrt{2}(ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ab}bd+7\,(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ab}bd+7\,(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2} + \frac{6\,bdx^6+7\,bcx^5+10\,adx^2+11\,acx}{32\,(bx^4+a)^2a^2}$$

```
[In] integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(a^3*b) - 21/256*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(a^3*b) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b*d + 7*(a*b^3)^(1
/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b
^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b*d + 7*(a*b^3)^(1/4)*b*c)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^2) + 1/32*(6*b*
d*x^6 + 7*b*c*x^5 + 10*a*d*x^2 + 11*a*c*x)/((b*x^4 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.18

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx = \frac{\frac{5\,d\,x^2}{16\,a}+\frac{11\,c\,x}{32\,a}+\frac{7\,b\,c\,x^5}{32\,a^2}+\frac{3\,b\,d\,x^6}{16\,a^2}}{a^2+2\,a\,b\,x^4+b^2\,x^8} + \left(\sum_{k=1}^{4}\ln\left(\frac{b^2\left(63\,c\,d^2+36\,d^3\,x-\mathrm{root}(268435456\,a^{11}\,b^2\,z^4+4718592\,a^6\,b\,d^2\,z^2-2709504\,a^3\,b\,c^2\,d\,z+194\right.}{} + 4718592\,a^6\,b\,d^2\,z^2-2709504\,a^3\,b\,c^2\,d\,z+194481\,b\,c^4+20736\,a\,d^4,z,k)\right)$$

```
[In] int((c + d*x)/(a + b*x^4)^3,x)
```

```
[Out] ((5*d*x^2)/(16*a) + (11*c*x)/(32*a) + (7*b*c*x^5)/(32*a^2) + (3*b*d*x^6)/(1
6*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4) + symsum(log((3*b^2*(63*c*d^2 + 36*d^3*
```

```
x - 7168*root(268435456*a^11*b^2*z^4 + 4718592*a^6*b*d^2*z^2 - 2709504*a^3*
b*c^2*d*z + 194481*b*c^4 + 20736*a*d^4, z, k)^2*a^5*b*c - 1176*root(2684354
56*a^11*b^2*z^4 + 4718592*a^6*b*d^2*z^2 - 2709504*a^3*b*c^2*d*z + 194481*b*
c^4 + 20736*a*d^4, z, k)*a^2*b*c^2*x + 4096*root(268435456*a^11*b^2*z^4 + 4
718592*a^6*b*d^2*z^2 - 2709504*a^3*b*c^2*d*z + 194481*b*c^4 + 20736*a*d^4,
z, k)^2*a^5*b*d*x))/(2048*a^6))*root(268435456*a^11*b^2*z^4 + 4718592*a^6*b
*d^2*z^2 - 2709504*a^3*b*c^2*d*z + 194481*b*c^4 + 20736*a*d^4, z, k), k, 1,
 4)
```

3.121 $\int \frac{c+dx}{(a-bx^4)^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 162

$$\int \frac{c+dx}{(a-bx^4)^4} dx = \frac{x(c+dx)}{12a(a-bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a-bx^4)} + \frac{77c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{77c\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{5d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

```
[Out] 1/12*x*(d*x+c)/a/(-b*x^4+a)^3+1/96*x*(10*d*x+11*c)/a^2/(-b*x^4+a)^2+1/384*x
*(60*d*x+77*c)/a^3/(-b*x^4+a)+77/256*c*arctan(b^(1/4)*x/a^(1/4))/a^(15/4)/b
^(1/4)+77/256*c*arctanh(b^(1/4)*x/a^(1/4))/a^(15/4)/b^(1/4)+5/32*d*arctanh(
x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1869, 1890, 218, 214, 211, 281}

$$\int \frac{c+dx}{(a-bx^4)^4} dx = \frac{77c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{77c\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{5d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} + \frac{x(77c+60dx)}{384a^3(a-bx^4)} + \frac{x(11c+10dx)}{96a^2(a-bx^4)^2} + \frac{x(c+dx)}{12a(a-bx^4)^3}$$

```
[In] Int[(c + d*x)/(a - b*x^4)^4,x]
```

```
[Out] (x*(c + d*x))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*(a - b*x^4
)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a - b*x^4)) + (77*c*ArcTan[(b^(1/4)*x)
```

```
/a^(1/4)])/(256*a^(15/4)*b^(1/4)) + (77*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(25
6*a^(15/4)*b^(1/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt
[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(c+dx)}{12a\left(a-bx^4\right)^3} - \frac{\int \frac{-11c-10dx}{\left(a-bx^4\right)^3}\,dx}{12a}$$

$$= \frac{x(c+dx)}{12a(a-bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a-bx^4)^2} + \frac{\int \frac{77c+60dx}{(a-bx^4)^2}\,dx}{96a^2}$$

$$= \frac{x(c+dx)}{12a(a-bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a-bx^4)} - \frac{\int \frac{-231c-120dx}{a-bx^4}\,dx}{384a^3}$$

$$= \frac{x(c+dx)}{12a(a-bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a-bx^4)} - \frac{\int \left(-\frac{231c}{a-bx^4} - \frac{120dx}{a-bx^4}\right)dx}{384a^3}$$

$$= \frac{x(c+dx)}{12a(a-bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a-bx^4)} + \frac{(77c)\int \frac{1}{a-bx^4}\,dx}{128a^3} + \frac{(5d)\int \frac{x}{a-bx^4}\,dx}{16a^3}$$

$$= \frac{x(c+dx)}{12a(a-bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a-bx^4)} + \frac{(77c)\int \frac{1}{\sqrt{a}-\sqrt{b}x^2}\,dx}{256a^{7/2}} + \frac{(77c)\int \frac{1}{\sqrt{a}+\sqrt{b}x^2}\,dx}{256a^{7/2}} + \frac{(5d)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{32a^3}$$

$$= \frac{x(c+dx)}{12a(a-bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a-bx^4)} + \frac{77c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{77c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.34

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx$$

$$= \frac{\frac{128a^3x(c+dx)}{(a-bx^4)^3} + \frac{16a^2x(11c+10dx)}{(a-bx^4)^2} + \frac{4ax(77c+60dx)}{a-bx^4} + \frac{462\sqrt[4]{a}c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} - \frac{3\left(77\sqrt[4]{a}\sqrt[4]{b}c+40\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}} + \frac{3(}{}}{1536a^4}$$

```
[In] Integrate[(c + d*x)/(a - b*x^4)^4,x]
```

```
[Out] ((128*a^3*x*(c + d*x))/(a - b*x^4)^3 + (16*a^2*x*(11*c + 10*d*x))/(a - b*x^
4)^2 + (4*a*x*(77*c + 60*d*x))/(a - b*x^4) + (462*a^(1/4)*c*ArcTan[(b^(1/4)
*x)/a^(1/4)])/b^(1/4) - (3*(77*a^(1/4)*b^(1/4)*c + 40*Sqrt[a]*d)*Log[a^(1/4
) - b^(1/4)*x]])/Sqrt[b] + (3*(77*a^(1/4)*b^(1/4)*c - 40*Sqrt[a]*d)*Log[a^(1
/4) + b^(1/4)*x]])/Sqrt[b] + (120*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt
[b])/(1536*a^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{\frac{5d\,b^{2}x^{10}}{32a^{3}}+\frac{77c\,b^{2}x^{9}}{384a^{3}}-\frac{5bd\,x^{6}}{12a^{2}}-\frac{33bc\,x^{5}}{64a^{2}}+\frac{11d\,x^{2}}{32a}+\frac{51cx}{128a}}{\left(-b\,x^{4}+a\right)^{3}}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b-a\right)}\frac{\left(40\_Rd+77c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{512a^{3}b}$	113
default	$\frac{\frac{5d\,b^{2}x^{10}}{32a^{3}}+\frac{77c\,b^{2}x^{9}}{384a^{3}}-\frac{5bd\,x^{6}}{12a^{2}}-\frac{33bc\,x^{5}}{64a^{2}}+\frac{11d\,x^{2}}{32a}+\frac{51cx}{128a}}{\left(-b\,x^{4}+a\right)^{3}}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{10d\ln\left(\frac{a+x^{2}\sqrt{ab}}{a-x^{2}\sqrt{ab}}\right)}{\sqrt{ab}}}{128a^{3}}$	165

```
[In] int((d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9-5/12*b*d/a^2*x^6-33/64*b*c/a^2*x^
5+11/32*d/a*x^2+51/128*c/a*x)/(-b*x^4+a)^3-1/512/a^3/b*sum((40*_R*d+77*c)/_
R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 40780, normalized size of antiderivative = 251.73

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.43

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx$$
$$= \mathrm{RootSum}\left(68719476736t^4a^{15}b^2 - 838860800t^2a^8bd^2 + 485703680ta^4bc^2d + 2560000ad^4 - 35153041bc^4, \left(\right.\right.$$
$$+ \frac{-153a^2cx - 132a^2dx^2 + 198abcx^5 + 160abdx^6 - 77b^2cx^9 - 60b^2dx^{10}}{-384a^6 + 1152a^5bx^4 - 1152a^4b^2x^8 + 384a^3b^3x^{12}}$$

```
[In] integrate((d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**2 - 838860800*_t**2*a**8*b*d**2 + 485703
680*_t*a**4*b*c**2*d + 2560000*a*d**4 - 35153041*b*c**4, Lambda(_t, _t*log(
x + (429496729600*_t**3*a**12*b*d**2 + 62170071040*_t**2*a**8*b*c**2*d - 26
21440000*_t*a**5*d**4 + 17998356992*_t*a**4*b*c**4 + 1897280000*a*c**2*d**3
)/(788480000*a*c*d**4 + 2706784157*b*c**5)))) + (-153*a**2*c*x - 132*a**2*d
*x**2 + 198*a*b*c*x**5 + 160*a*b*d*x**6 - 77*b**2*c*x**9 - 60*b**2*d*x**10)
/(-384*a**6 + 1152*a**5*b*x**4 - 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.38

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx = -\frac{60\,b^2dx^{10}+77\,b^2cx^9-160\,abdx^6-198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\,(a^3b^3x^{12}-3\,a^4b^2x^8+3\,a^5bx^4-a^6)} + \frac{\frac{154\,c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{40\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{40\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{77\,c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{512\,a^3}$$

```
[In] integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 - 160*a*b*d*x^6 - 198*a*b*c*x^5 + 132*
a^2*d*x^2 + 153*a^2*c*x)/(a^3*b^3*x^12 - 3*a^4*b^2*x^8 + 3*a^5*b*x^4 - a^6)
 + 1/512*(154*c*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(
a)*sqrt(b))) + 40*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 40*d*log
(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 77*c*log((sqrt(b)*x - sqrt(sqrt
(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sq
rt(b))))/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(129) = 258$.

Time = 0.28 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.83

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx = \frac{77\sqrt{2}(-ab^3)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,a^4b} - \frac{77\sqrt{2}(-ab^3)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,a^4b} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}bd+77\left(-ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}bd+77\left(-ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2} - \frac{60\,b^2dx^{10}+77\,b^2cx^9-160\,abdx^6-198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\left(bx^4-a\right)^3a^3}$$

```
[In] integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
[Out] 77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a
/b))/(a^4*b) - 77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^
(1/4) + sqrt(-a/b))/(a^4*b) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b*d + 77
*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)
^(1/4))/(a^4*b^2) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b*d + 77*(-a*b^3)^
(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a
^4*b^2) - 1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 - 160*a*b*d*x^6 - 198*a*b*c*x
^5 + 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 - a)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.17

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx = \left(\sum_{k=1}^{4}\ln\left(-\frac{b^2\left(1925\,c\,d^2+1000\,d^3\,x+\mathrm{root}(68719476736\,a^{15}\,b^2\,z^4-838860800\,a^8\,b\,d^2\,z^2+485703680\,a^4\right.}{\cdots}\right.\right.$$

$$\left.-838860800\,a^8\,b\,d^2\,z^2+485703680\,a^4\,b\,c^2\,d\,z-35153041\,b\,c^4+2560000\,a\,d^4,z,k)\right)$$

$$+\frac{\frac{11\,d\,x^2}{32\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}-\frac{33\,b\,c\,x^5}{64\,a^2}-\frac{5\,b\,d\,x^6}{12\,a^2}}{a^3-3\,a^2\,b\,x^4+3\,a\,b^2\,x^8-b^3\,x^{12}}$$

```
[In] int((c + d*x)/(a - b*x^4)^4,x)
```

```
[Out] symsum(log(-(b^2*(1925*c*d^2 + 1000*d^3*x + 315392*root(68719476736*a^15*b^
2*z^4 - 838860800*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4
+ 2560000*a*d^4, z, k)^2*a^7*b*c + 47432*root(68719476736*a^15*b^2*z^4 - 83
8860800*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4 + 2560000*
a*d^4, z, k)*a^3*b*c^2*x - 163840*root(68719476736*a^15*b^2*z^4 - 838860800
*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4 + 2560000*a*d^4,
z, k)^2*a^7*b*d*x))/(32768*a^9))*root(68719476736*a^15*b^2*z^4 - 838860800*
a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4 + 2560000*a*d^4, z
, k), k, 1, 4) + ((11*d*x^2)/(32*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9)/(38
4*a^3) + (5*b^2*d*x^10)/(32*a^3) - (33*b*c*x^5)/(64*a^2) - (5*b*d*x^6)/(12*
a^2))/(a^3 - b^3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8)
```

3.122 $\int \frac{c+dx}{(a+bx^4)^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 291

$$\int \frac{c+dx}{(a+bx^4)^4} dx = \frac{x(c+dx)}{12a(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a+bx^4)} + \frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{77c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}} + \frac{77c\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}} - \frac{77c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}} + \frac{77c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

```
[Out] 1/12*x*(d*x+c)/a/(b*x^4+a)^3+1/96*x*(10*d*x+11*c)/a^2/(b*x^4+a)^2+1/384*x*(
60*d*x+77*c)/a^3/(b*x^4+a)+77/512*c*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^
(15/4)/b^(1/4)*2^(1/2)+77/512*c*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(15/4
)/b^(1/4)*2^(1/2)-77/1024*c*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/
2))/a^(15/4)/b^(1/4)*2^(1/2)+77/1024*c*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*b^(1/2))/a^(15/4)/b^(1/4)*2^(1/2)+5/32*d*arctan(x^2*b^(1/2)/a^(1/2))/a
^(7/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1869, 1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$\begin{aligned}\int \frac{c+dx}{(a+bx^4)^4}\,dx = &-\frac{77c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{77c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}}\\&+\frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}-\frac{77c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}\\&+\frac{77c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}\\&+\frac{x(77c+60dx)}{384a^3(a+bx^4)}+\frac{x(11c+10dx)}{96a^2(a+bx^4)^2}+\frac{x(c+dx)}{12a(a+bx^4)^3}\end{aligned}$$

```
[In] Int[(c + d*x)/(a + b*x^4)^4,x]
```

```
[Out] (x*(c + d*x))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*(a + b*x^4
)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a + b*x^4)) + (5*d*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - (77*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4)) - (77*c*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b
^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
```

```
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} - \frac{\int \frac{-11c-10dx}{\left(a+bx^4\right)^3}\,dx}{12a}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{\int \frac{77c+60dx}{\left(a+bx^4\right)^2}\,dx}{96a^2}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} - \frac{\int \frac{-231c-120dx}{a+bx^4}\,dx}{384a^3}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} - \frac{\int \left(-\frac{231c}{a+bx^4} - \frac{120dx}{a+bx^4}\right)\,dx}{384a^3}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} + \frac{(77c)\int \frac{1}{a+bx^4}\,dx}{128a^3} + \frac{(5d)\int \frac{x}{a+bx^4}\,dx}{16a^3}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} + \frac{(77c)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{256a^{7/2}} + \frac{(77c)\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{256a^{7/2}} + \frac{(5d)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{32a^3}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} + \frac{(77c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{512a^{7/2}\sqrt{b}} + \frac{(77c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{512a^{7/2}\sqrt{b}} - \frac{(77c)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{512\sqrt{2}a^{15/4}\sqrt[4]{b}} - \frac{(77c)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

$$= \frac{x(c+dx)}{12a(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a+bx^4)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{77c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}} + \frac{77c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}} + \frac{(77c)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}} - \frac{(77c)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

$$= \frac{x(c+dx)}{12a(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a+bx^4)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{77c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}} + \frac{77c\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}} - \frac{77c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}} + \frac{77c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.94

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx$$

$$= \frac{\frac{256a^{11/4}x(c+dx)}{(a+bx^4)^3} + \frac{32a^{7/4}x(11c+10dx)}{(a+bx^4)^2} + \frac{8a^{3/4}x(77c+60dx)}{a+bx^4} - \frac{6\left(77\sqrt{2}\sqrt[4]{b}c+80\sqrt[4]{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{6\left(77\sqrt{2}\sqrt[4]{b}c-80\sqrt[4]{\ }\right.}{}}{3072a^{15/4}}$$

```
[In] Integrate[(c + d*x)/(a + b*x^4)^4,x]
```

```
[Out] ((256*a^(11/4)*x*(c + d*x))/(a + b*x^4)^3 + (32*a^(7/4)*x*(11*c + 10*d*x))/
(a + b*x^4)^2 + (8*a^(3/4)*x*(77*c + 60*d*x))/(a + b*x^4) - (6*(77*Sqrt[2]*
b^(1/4)*c + 80*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b]
+ (6*(77*Sqrt[2]*b^(1/4)*c - 80*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/Sqrt[b] - (231*Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
 Sqrt[b]*x^2])/b^(1/4) + (231*Sqrt[2]*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/b^(1/4))/(3072*a^(15/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.51 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.38

method	result
risch	$\frac{\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{(b\,x^4+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{(40\_Rd+77c)\ln(x-\_R)}{\_R^3}}{512a^3b}$
default	$\frac{\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{(b\,x^4+a)^3}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\right.\right.}{8a}}{128a^3}$

[In] int((d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)

[Out] (5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9+5/12*b*d/a^2*x^6+33/64*b*c/a^2*x^5+11/32*d/a*x^2+51/128*c/a*x)/(b*x^4+a)^3+1/512/a^3/b*sum((40*_R*d+77*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.06 (sec) , antiderivative size = 43302, normalized size of antiderivative = 148.80

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

[In] integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")

[Out] Too large to include

Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.79

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx$$
$$=\mathrm{RootSum}\left(68719476736t^4a^{15}b^2+838860800t^2a^8bd^2-485703680ta^4bc^2d+2560000ad^4+35153041bc^4,\left(t\right.\right.$$
$$+\frac{153a^2cx+132a^2dx^2+198abcx^5+160abdx^6+77b^2cx^9+60b^2dx^{10}}{384a^6+1152a^5bx^4+1152a^4b^2x^8+384a^3b^3x^{12}}$$

[In] integrate((d*x+c)/(b*x**4+a)**4,x)

```
[Out] RootSum(68719476736*_t**4*a**15*b**2 + 838860800*_t**2*a**8*b*d**2 - 485703
680*_t*a**4*b*c**2*d + 2560000*a*d**4 + 35153041*b*c**4, Lambda(_t, _t*log(
x + (-429496729600*_t**3*a**12*b*d**2 - 62170071040*_t**2*a**8*b*c**2*d - 2
621440000*_t*a**5*d**4 - 17998356992*_t*a**4*b*c**4 + 1897280000*a*c**2*d**
3)/(788480000*a*c*d**4 - 2706784157*b*c**5)))) + (153*a**2*c*x + 132*a**2*d
*x**2 + 198*a*b*c*x**5 + 160*a*b*d*x**6 + 77*b**2*c*x**9 + 60*b**2*d*x**10)
/(384*a**6 + 1152*a**5*b*x**4 + 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.04

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx = \frac{60\,b^2dx^{10}+77\,b^2cx^9+160\,abdx^6+198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\,(a^3b^3x^{12}+3\,a^4b^2x^8+3\,a^5bx^4+a^6)} + \frac{\frac{77\,\sqrt{2}c\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{77\,\sqrt{2}c\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-80\,\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{1}{4}}}}{1024\,a^3}$$

```
[In] integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 132*a
^2*d*x^2 + 153*a^2*c*x)/(a^3*b^3*x^12 + 3*a^4*b^2*x^8 + 3*a^5*b*x^4 + a^6)
+ 1/1024*(77*sqrt(2)*c*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a
))/(a^(3/4)*b^(1/4)) - 77*sqrt(2)*c*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/
4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(1/4)*c - 80*sq
rt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sq
rt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)) + 2*(77*sqrt(2)*a^(
1/4)*b^(1/4)*c + 80*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^
(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4
)))/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.96

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx = \frac{77\sqrt{2}(ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b} - \frac{77\sqrt{2}(ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}bd+77\,(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}bd+77\,(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2} + \frac{60\,b^2dx^{10}+77\,b^2cx^9+160\,abdx^6+198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\,(bx^4+a)^3a^3}$$

```
[In] integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 77/1024*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b)
)/(a^4*b) - 77/1024*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4)
 + sqrt(a/b))/(a^4*b) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b*d + 77*(a*b^3
)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
^4*b^2) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b*d + 77*(a*b^3)^(1/4)*b*c)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^2) + 1/38
4*(60*b^2*d*x^10 + 77*b^2*c*x^9 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 132*a^2*d
*x^2 + 153*a^2*c*x)/((b*x^4 + a)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.20

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx$$

$$= \left(\sum_{k=1}^{4}\ln\left(\frac{b^2\left(1925\,c\,d^2+1000\,d^3\,x-\mathrm{root}(68719476736\,a^{15}\,b^2\,z^4+838860800\,a^8\,b\,d^2\,z^2-485703680\,a^4\,b\,c^2\right.}{}\right.\right.$$

$$\left.+838860800\,a^8\,b\,d^2\,z^2-485703680\,a^4\,b\,c^2\,d\,z+35153041\,b\,c^4+2560000\,a\,d^4,z,k)\right)$$

$$+\frac{\frac{11\,d\,x^2}{32\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}+\frac{33\,b\,c\,x^5}{64\,a^2}+\frac{5\,b\,d\,x^6}{12\,a^2}}{a^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^8+b^3\,x^{12}}$$

```
[In] int((c + d*x)/(a + b*x^4)^4,x)
```

```
[Out] symsum(log((b^2*(1925*c*d^2 + 1000*d^3*x - 315392*root(68719476736*a^15*b^2
*z^4 + 838860800*a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 +
 2560000*a*d^4, z, k)^2*a^7*b*c - 47432*root(68719476736*a^15*b^2*z^4 + 838
860800*a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 + 2560000*a
*d^4, z, k)*a^3*b*c^2*x + 163840*root(68719476736*a^15*b^2*z^4 + 838860800*
a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 + 2560000*a*d^4, z
, k)^2*a^7*b*d*x))/(32768*a^9))*root(68719476736*a^15*b^2*z^4 + 838860800*a
^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 + 2560000*a*d^4, z,
 k), k, 1, 4) + ((11*d*x^2)/(32*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9)/(384
*a^3) + (5*b^2*d*x^10)/(32*a^3) + (33*b*c*x^5)/(64*a^2) + (5*b*d*x^6)/(12*a
^2))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8)
```

3.123 $\int \frac{c+dx}{1-x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{c+dx}{1-x^4} dx = \frac{1}{2}c\arctan(x) + \frac{1}{2}c\text{arctanh}(x) + \frac{1}{2}d\text{arctanh}\left(x^2\right)$$

```
[Out] 1/2*c*arctan(x)+1/2*c*arctanh(x)+1/2*d*arctanh(x^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1890, 218, 212, 209, 281}

$$\int \frac{c+dx}{1-x^4} dx = \frac{1}{2}c\arctan(x) + \frac{1}{2}c\text{arctanh}(x) + \frac{1}{2}d\text{arctanh}\left(x^2\right)$$

```
[In] Int[(c + d*x)/(1 - x^4),x]
```

```
[Out] (c*ArcTan[x])/2 + (c*ArcTanh[x])/2 + (d*ArcTanh[x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{1-x^4}+\frac{dx}{1-x^4}\right) dx \\
&= c\int \frac{1}{1-x^4}\,dx+d\int \frac{x}{1-x^4}\,dx \\
&= \frac{1}{2}c\int \frac{1}{1-x^2}\,dx+\frac{1}{2}c\int \frac{1}{1+x^2}\,dx+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}c\tan^{-1}(x)+\frac{1}{2}c\tanh^{-1}(x)+\frac{1}{2}d\tanh^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{c+dx}{1-x^4}\,dx=\frac{1}{4}\left(2c\arctan(x)-(c+d)\log(1-x)+c\log(1+x)-d\log(1+x)+d\log\left(1+x^2\right)\right)$$

```
[In] Integrate[(c + d*x)/(1 - x^4),x]
[Out] (2*c*ArcTan[x] - (c + d)*Log[1 - x] + c*Log[1 + x] - d*Log[1 + x] + d*Log[1
 + x^2])/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(18) = 36.

Time = 1.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

method	result
default	$\frac{(-c-d)\ln(-1+x)}{4}-\frac{(-c+d)\ln(1+x)}{4}+\frac{d\ln(x^2+1)}{4}+\frac{c\arctan(x)}{2}$
meijerg	$\frac{d\,\operatorname{arctanh}(x^2)}{2}-\frac{cx\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}$
parallelrisch	$\frac{\ln(1+x)c}{4}-\frac{\ln(1+x)d}{4}-\frac{\ln(-1+x)c}{4}-\frac{\ln(-1+x)d}{4}+\frac{\ln(x-i)d}{4}-\frac{i\ln(x-i)c}{4}+\frac{\ln(x+i)d}{4}+\frac{i\ln(x+i)c}{4}$
risch	$\frac{d\ln(c^4x^2+4d^4x^2+c^4+4d^4)}{4}+\frac{c\arctan\left(\frac{c^4x}{c^4+4d^4}+\frac{4d^4x}{c^4+4d^4}\right)}{2}+\frac{c\arctan\left(\frac{2cd}{c^2-2d^2}\right)}{2}+\frac{\ln(-x-1)c}{4}-\frac{\ln(-x-1)d}{4}-\frac{\ln(-1+}{4}$

```
[In] int((d*x+c)/(-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*(-c-d)*ln(-1+x)-1/4*(-c+d)*ln(1+x)+1/4*d*ln(x^2+1)+1/2*c*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{c+dx}{1-x^4}\,dx = \frac{1}{2}\,c\arctan(x) + \frac{1}{4}\,d\log\left(x^2+1\right) + \frac{1}{4}\,(c-d)\log(x+1) - \frac{1}{4}\,(c+d)\log(x-1)$$

```
[In] integrate((d*x+c)/(-x^4+1),x, algorithm="fricas")
[Out] 1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(x + 1) - 1/4*(c + d)
*log(x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 13.04

$$\int \frac{c+dx}{1-x^4}\,dx = \frac{(c-d)\log\left(x+\frac{c^4(c-d)+5c^2d^3+c^2d(c-d)^2-2d^4(c-d)+2d^2(c-d)^3}{c^5+4cd^4}\right)}{4} - \frac{(c+d)\log\left(x+\frac{-c^4(c+d)+5c^2d^3+c^2d(c+d)^2+2d^4(c+d)-2d^2(c+d)^3}{c^5+4cd^4}\right)}{4} - \left(-\frac{ic}{4}-\frac{d}{4}\right)\log\left(x+\frac{-4c^4\left(-\frac{ic}{4}-\frac{d}{4}\right)+5c^2d^3+16c^2d\left(-\frac{ic}{4}-\frac{d}{4}\right)^2+8d^4\left(-\frac{ic}{4}-\frac{d}{4}\right)-128d^2\left(-\frac{ic}{4}-\frac{d}{4}\right)^3}{c^5+4cd^4}\right) - \left(\frac{ic}{4}-\frac{d}{4}\right)\log\left(x+\frac{-4c^4\left(\frac{ic}{4}-\frac{d}{4}\right)+5c^2d^3+16c^2d\left(\frac{ic}{4}-\frac{d}{4}\right)^2+8d^4\left(\frac{ic}{4}-\frac{d}{4}\right)-128d^2\left(\frac{ic}{4}-\frac{d}{4}\right)^3}{c^5+4cd^4}\right)$$

```
[In] integrate((d*x+c)/(-x**4+1),x)
```

```
[Out] (c - d)*log(x + (c**4*(c - d) + 5*c**2*d**3 + c**2*d*(c - d)**2 - 2*d**4*(c
 - d) + 2*d**2*(c - d)**3)/(c**5 + 4*c*d**4))/4 - (c + d)*log(x + (-c**4*(c
 + d) + 5*c**2*d**3 + c**2*d*(c + d)**2 + 2*d**4*(c + d) - 2*d**2*(c + d)**
3)/(c**5 + 4*c*d**4))/4 - (-I*c/4 - d/4)*log(x + (-4*c**4*(-I*c/4 - d/4) +
5*c**2*d**3 + 16*c**2*d*(-I*c/4 - d/4)**2 + 8*d**4*(-I*c/4 - d/4) - 128*d**
2*(-I*c/4 - d/4)**3)/(c**5 + 4*c*d**4)) - (I*c/4 - d/4)*log(x + (-4*c**4*(I
*c/4 - d/4) + 5*c**2*d**3 + 16*c**2*d*(I*c/4 - d/4)**2 + 8*d**4*(I*c/4 - d/
4) - 128*d**2*(I*c/4 - d/4)**3)/(c**5 + 4*c*d**4))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{c+dx}{1-x^4}\,dx = \frac{1}{2}c\arctan(x)+\frac{1}{4}d\log\left(x^2+1\right)+\frac{1}{4}(c-d)\log(x+1)-\frac{1}{4}(c+d)\log(x-1)$$

```
[In] integrate((d*x+c)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(x + 1) - 1/4*(c + d)
*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{c+dx}{1-x^4}\,dx = \frac{1}{2}\,c\arctan\left(x\right) + \frac{1}{4}\,d\log\left(x^2+1\right) + \frac{1}{4}\,(c-d)\log\left(|x+1|\right) - \frac{1}{4}\,(c+d)\log\left(|x-1|\right)$$

```
[In] integrate((d*x+c)/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(abs(x + 1)) - 1/4*(c
 + d)*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.17

$$\int \frac{c+dx}{1-x^4}\,dx = -\frac{(-1)^{1/4}\operatorname{atan}\left((-1)^{3/4}\sqrt{2}\,x+1\right)\left(\sqrt{2}\,c+2\,(-1)^{1/4}\,d\right)}{4} - \frac{(-1)^{1/4}\operatorname{atan}\left((-1)^{3/4}\sqrt{2}\,x-1\right)\left(2\sqrt{2}\,c-4\,(-1)^{1/4}\,d\right)}{8} + \frac{(-1)^{1/4}\sqrt{2}\,c\,\ln\left(\frac{x^2+(-1)^{1/4}\sqrt{2}\,x+1\mathrm{i}}{x^2-(-1)^{1/4}\sqrt{2}\,x+1\mathrm{i}}\right)}{8}$$

```
[In] int(-(c + d*x)/(x^4 - 1),x)
```

```
[Out] ((-1)^(1/4)*2^(1/2)*c*log((x^2 + (-1)^(1/4)*2^(1/2)*x + 1i)/(x^2 - (-1)^(1/
4)*2^(1/2)*x + 1i)))/8 - ((-1)^(1/4)*atan((-1)^(3/4)*2^(1/2)*x - 1)*(2*2^(1
/2)*c - 4*(-1)^(1/4)*d))/8 - ((-1)^(1/4)*atan((-1)^(3/4)*2^(1/2)*x + 1)*(2^
(1/2)*c + 2*(-1)^(1/4)*d))/4
```

3.124 $\int \frac{c+dx}{1+x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \frac{c+dx}{1+x^4} dx = \frac{1}{2}d\arctan\left(x^2\right) - \frac{c\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{c\arctan\left(1+\sqrt{2}x\right)}{2\sqrt{2}} - \frac{c\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{c\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

```
[Out] 1/2*d*arctan(x^2)+1/4*c*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*c*arctan(1+x*2^(1/
2))*2^(1/2)-1/8*c*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/8*c*ln(1+x^2+x*2^(1/2))*2^(
1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {1890, 217, 1179, 642, 1176, 631, 210, 281, 209}

$$\int \frac{c+dx}{1+x^4} dx = -\frac{c\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{c\arctan\left(\sqrt{2}x+1\right)}{2\sqrt{2}} + \frac{1}{2}d\arctan\left(x^2\right) - \frac{c\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}} + \frac{c\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}$$

```
[In] Int[(c + d*x)/(1 + x^4),x]
```

```
[Out] (d*ArcTan[x^2])/2 - (c*ArcTan[1 - Sqrt[2]*x])/(2*Sqrt[2]) + (c*ArcTan[1 + S
qrt[2]*x])/(2*Sqrt[2]) - (c*Log[1 - Sqrt[2]*x + x^2])/(4*Sqrt[2]) + (c*Log[
1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{1+x^4}+\frac{dx}{1+x^4}\right) dx \\
&= c\int \frac{1}{1+x^4}\,dx+d\int \frac{x}{1+x^4}\,dx \\
&= \frac{1}{2}c\int \frac{1-x^2}{1+x^4}\,dx+\frac{1}{2}c\int \frac{1+x^2}{1+x^4}\,dx+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}d\tan^{-1}\left(x^2\right)+\frac{1}{4}c\int \frac{1}{1-\sqrt{2}x+x^2}\,dx+\frac{1}{4}c\int \frac{1}{1+\sqrt{2}x+x^2}\,dx \\
&\quad -\frac{c\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}-\frac{c\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{4\sqrt{2}} \\
&= \frac{1}{2}d\tan^{-1}\left(x^2\right)-\frac{c\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{c\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}} \\
&\quad +\frac{c\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{2\sqrt{2}}-\frac{c\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}x\right)}{2\sqrt{2}} \\
&= \frac{1}{2}d\tan^{-1}\left(x^2\right)-\frac{c\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{c\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}} \\
&\quad -\frac{c\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{c\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{1+x^4}\,dx = \frac{1}{4}\Big(-\big((\sqrt[4]{-1}c+id)\log\big(\sqrt[4]{-1}-x\big)\big)+\big(-(-1)^{3/4}c+id\big)\log\big((-1)^{3/4}-x\big)+\big(\sqrt[4]{-1}c-id\big)\log\big(\sqrt[4]{-1}+x\big)+\big((-1)^{3/4}c+id\big)\log\big((-1)^{3/4}+x\big)\Big)$$

```
[In] Integrate[(c + d*x)/(1 + x^4),x]
```

```
[Out] (-(((-1)^(1/4)*c + I*d)*Log[(-1)^(1/4) - x]) + (-((-1)^(3/4)*c) + I*d)*Log[
(-1)^(3/4) - x] + ((-1)^(1/4)*c - I*d)*Log[(-1)^(1/4) + x] + ((-1)^(3/4)*c
+ I*d)*Log[(-1)^(3/4) + x])/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.28

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+1)}\frac{(\_Rd+c)\ln(x-\_R)}{\_R^3}\right)}{4}$
default	$\frac{c\sqrt{2}\left(\ln\left(\frac{1+x^2+\sqrt{2}\,x}{1+x^2-\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{8}+\frac{d\arctan(x^2)}{2}$
meijerg	$\frac{d\arctan(x^2)}{2}+\frac{c\left(-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{2(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^4)^{\frac{1}{4}}}\right)}{(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{2(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2+\sqrt{2}\,(x^4)^{\frac{1}{4}}}\right)}{(x^4)^{\frac{1}{4}}}\right)}{4}$

```
[In] int((d*x+c)/(x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 34376, normalized size of antiderivative = 350.78

$$\int \frac{c+dx}{1+x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)/(x^4+1),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \frac{c+dx}{1+x^4}\,dx = \operatorname{RootSum}\left(256t^4+32t^2d^2-16tc^2d+c^4+d^4,\ \left(t\mapsto t\log\left(x+\frac{128t^3d^2+16t^2c^2d+4tc^4+8td^4-5c^2d^3}{c^5-4cd^4}\right)\right)\right)$$

```
[In] integrate((d*x+c)/(x**4+1),x)
[Out] RootSum(256*_t**4 + 32*_t**2*d**2 - 16*_t*c**2*d + c**4 + d**4, Lambda(_t,
_t*log(x + (128*_t**3*d**2 + 16*_t**2*c**2*d + 4*_t*c**4 + 8*_t*d**4 - 5*c*
*2*d**3)/(c**5 - 4*c*d**4))))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{c+dx}{1+x^4}\,dx =& \frac{1}{8}\sqrt{2}c\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}c\log\left(x^2-\sqrt{2}x+1\right)\\&+\frac{1}{4}\left(\sqrt{2}c-2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)\\&+\frac{1}{4}\left(\sqrt{2}c+2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)\end{aligned}$$

```
[In] integrate((d*x+c)/(x^4+1),x, algorithm="maxima")
[Out] 1/8*sqrt(2)*c*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*c*log(x^2 - sqrt(2)*x
+ 1) + 1/4*(sqrt(2)*c - 2*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqr
t(2)*c + 2*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \frac{c+dx}{1+x^4}\,dx = \frac{1}{8}\sqrt{2}c\log\left(x^2+\sqrt{2}x+1\right) - \frac{1}{8}\sqrt{2}c\log\left(x^2-\sqrt{2}x+1\right) + \frac{1}{4}\left(\sqrt{2}c-2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{4}\left(\sqrt{2}c+2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)$$

```
[In] integrate((d*x+c)/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*c*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*c*log(x^2 - sqrt(2)*x
+ 1) + 1/4*(sqrt(2)*c - 2*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqr
t(2)*c + 2*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{c+dx}{1+x^4}\,dx = \mathrm{atan}\left(\sqrt{2}\,x-1\right)\,\left(\frac{d}{2}+\frac{\sqrt{2}\,c}{4}\right) - \mathrm{atan}\left(\sqrt{2}\,x+1\right)\,\left(\frac{d}{2}-\frac{\sqrt{2}\,c}{4}\right) + \frac{\sqrt{2}\,c\,\ln\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)}{8}$$

```
[In] int((c + d*x)/(x^4 + 1),x)
```

```
[Out] atan(2^(1/2)*x - 1)*(d/2 + (2^(1/2)*c)/4) - atan(2^(1/2)*x + 1)*(d/2 - (2^(
1/2)*c)/4) + (2^(1/2)*c*log((2^(1/2)*x + x^2 + 1)/(x^2 - 2^(1/2)*x + 1)))/8
```

3.125 $\int \frac{c+dx+ex^2}{a-bx^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{c+dx+ex^2}{a-bx^4} dx = \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] 1/2*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)+1/2*arctan(b^(1/4)*x/a^(
1/4))*(-e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)+1/2*arctanh(b^(1/4)*x/a^(1/4))
*(e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1890, 281, 214, 1181, 211}

$$\int \frac{c+dx+ex^2}{a-bx^4} dx = \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}c-\sqrt{a}e\right)}{2a^{3/4}b^{3/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2a^{3/4}b^{3/4}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
 + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{dx}{a-bx^4}+\frac{c+ex^2}{a-bx^4}\right) dx \\
&= d\int \frac{x}{a-bx^4}\,dx+\int \frac{c+ex^2}{a-bx^4}\,dx \\
&= \frac{1}{2}d\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}\left(-\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx \\
&\quad +\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx
\end{aligned}
$$

$$= \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.61

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx$$
$$= \frac{2\left(\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}c+\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{a}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)+\sqrt{b}c\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)-\sqrt[4]{\ }}{4a^{3/4}b^{3/4}}$$

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4),x]
```

```
[Out] (2*(Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c + a^(1/
4)*b^(1/4)*d + Sqrt[a]*e)*Log[a^(1/4) - b^(1/4)*x] + Sqrt[b]*c*Log[a^(1/4)
+ b^(1/4)*x] - a^(1/4)*b^(1/4)*d*Log[a^(1/4) + b^(1/4)*x] + Sqrt[a]*e*Log[a
^(1/4) + b^(1/4)*x] + a^(1/4)*b^(1/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*a^(3
/4)*b^(3/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.34

method	result	size
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4b-a\right)}\frac{\left(\_R^2e+\_Rd+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	39
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	139

```
[In] int((e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*sum((_R^2*e+_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.58 (sec) , antiderivative size = 120560, normalized size of antiderivative = 1039.31

$$\int \frac{c + dx + ex^2}{a - bx^4} \, dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(105) = 210.

Time = 5.33 (sec) , antiderivative size = 471, normalized size of antiderivative = 4.06

$$\int \frac{c + dx + ex^2}{a - bx^4} \, dx = -\operatorname{RootSum}\left(256t^4a^3b^3 + t^2(-64a^2b^2ce - 32a^2b^2d^2) + t(-16a^2bde^2 - 16ab^2c^2d) - a^2e^4 + 2abc^2e^2 - 4abc\right.$$

```
[In] integrate((e*x**2+d*x+c)/(-b*x**4+a),x)
[Out] -RootSum(256*_t**4*a**3*b**3 + _t**2*(-64*a**2*b**2*c*e - 32*a**2*b**2*d**2
) + _t*(-16*a**2*b*d*e**2 - 16*a*b**2*c**2*d) - a**2*e**4 + 2*a*b*c**2*e**2
 - 4*a*b*c*d**2*e + a*b*d**4 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*
a**4*b**2*e**3 - 64*_t**3*a**3*b**3*c**2*e + 128*_t**3*a**3*b**3*c*d**2 + 4
8*_t**2*a**3*b**2*c*d*e**2 - 32*_t**2*a**3*b**2*d**3*e - 16*_t**2*a**2*b**3
*c**3*d + 12*_t*a**3*b*c*e**4 + 12*_t*a**3*b*d**2*e**3 + 16*_t*a**2*b**2*c*
*3*e**2 - 36*_t*a**2*b**2*c**2*d**2*e - 8*_t*a**2*b**2*c*d**4 + 4*_t*a*b**3
*c**5 + 3*a**3*d*e**5 - 5*a**2*b*c*d**3*e**2 + 2*a**2*b*d**5*e + 5*a*b**2*c
**4*d*e - 5*a*b**2*c**3*d**3)/(a**3*e**6 + a**2*b*c**2*e**4 - 8*a**2*b*c*d*
*2*e**3 + 4*a**2*b*d**4*e**2 - a*b**2*c**4*e**2 + 8*a*b**2*c**3*d**2*e - 4*
a*b**2*c**2*d**4 - b**3*c**6))))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.32

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx = \frac{d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}} - \frac{d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}} + \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 1/4*d*log(sqrt(b)*x^2
- sqrt(a))/(sqrt(a)*sqrt(b)) + 1/2*(sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt
(b)*c + sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqr
t(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(78) = 156.

Time = 0.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.23

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx = -\frac{\sqrt{2}\left(b^2c-\sqrt{2}(-ab^3)^{\frac{1}{4}}bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(b^2c+\sqrt{2}(-ab^3)^{\frac{1}{4}}bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(b^2c-\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}} + \frac{\sqrt{2}\left(b^2c-\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}$$

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c - sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*
```

```
sqrt(2)*(b^2*c + sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*sqrt(
2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/
(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
```

Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 725, normalized size of antiderivative = 6.25

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx = \sum_{k=1}^{4} \ln\Big(-b^2\,c\,d^2+b^2\,c^2\,e-b^2\,d^3\,x-a\,b\,e^3$$

$$-\mathrm{root}\big(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z+16\,a\,b^2\,c^2\,d\,z-4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2+$$

$$-\mathrm{root}\big(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z+16\,a\,b^2\,c^2\,d\,z$$

$$-4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2+a\,b\,d^4-a^2\,e^4-b^2\,c^4,z,k\big)\;b^3\,c^2\,x\,4$$

$$+\mathrm{root}\big(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z+16\,a\,b^2\,c^2\,d\,z-4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2+$$

$$-\mathrm{root}\big(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z+16\,a\,b^2\,c^2\,d\,z$$

$$-4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2+a\,b\,d^4-a^2\,e^4-b^2\,c^4,z,k\big)\;a\,b^2\,e^2\,x\,4$$

$$+\mathrm{root}\big(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z+16\,a\,b^2\,c^2\,d\,z$$

$$-4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2+a\,b\,d^4-a^2\,e^4-b^2\,c^4,z,k\big)\;a\,b^2\,d\,e\,8$$

$$+2\,b^2\,c\,d\,e\,x\Big)\;\mathrm{root}\big(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z$$

$$+16\,a\,b^2\,c^2\,d\,z-4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2+a\,b\,d^4-a^2\,e^4-b^2\,c^4,z,k\big)$$

```
[In] int((c + d*x + e*x^2)/(a - b*x^4),x)
```

```
[Out] symsum(log(b^2*c^2*e - b^2*c*d^2 - b^2*d^3*x - a*b*e^3 - 16*root(256*a^3*b^
3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b
^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z
, k)^2*a*b^3*c - 4*root(256*a^3*b^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d
^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^
2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)*b^3*c^2*x + 16*root(256*a^3*b^3*z^4
- 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2
*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)^2
*a*b^3*d*x - 4*root(256*a^3*b^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z
^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 +
a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)*a*b^2*e^2*x + 8*root(256*a^3*b^3*z^4 - 6
4*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*
z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)*a*b^
2*d*e + 2*b^2*c*d*e*x)*root(256*a^3*b^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b
^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^
2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k), k, 1, 4)
```

3.126 $\int \frac{c+dx+ex^2}{a+bx^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 277

$$\int \frac{c+dx+ex^2}{a+bx^4} \, dx = \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}$$

```
[Out] 1/2*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)-1/8*ln(-a^(1/4)*b^(1/4)*x
*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2
)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-e*a^(1/2)+c*b^(1/
2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(
1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1
/4))*(e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4),x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4))
+ ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sq
rt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{dx}{a+bx^4}+\frac{c+ex^2}{a+bx^4}\right) dx\\ &= d\int \frac{x}{a+bx^4}\,dx+\int \frac{c+ex^2}{a+bx^4}\,dx\end{aligned}$$

$$=\frac{1}{2}d\text{Subst}\left(\int\frac{1}{a+bx^2}\,dx,x,x^2\right)+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-e\right)\int\frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int\frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b}$$

$$=\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b}$$

$$-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{3/4}}$$

$$=\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}$$

$$+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}$$

$$+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

$$-\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

$$=\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

$$+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

$$-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}$$

$$+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.83

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx$$
$$= \frac{-2\left(\sqrt{2}\sqrt{b}c+2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt{b}c-2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1+\dots\right.}{8a^{3/4}b^{3\dots}}$$

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*Sqrt[b]*c + 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1
- (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[b]*c - 2*a^(1/4)*b^(1/4)*d
 + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*(Sq
rt[b]*c - Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(8*a^(3/4)*b^(
3/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.13

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(\_R^2 e+\_R d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{d\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\dots\right.}{x^2+\left(\dots\right.}\right.\right.}{\dots}$

```
[In] int((e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*sum((_R^2*e+_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.59 (sec) , antiderivative size = 121386, normalized size of antiderivative = 438.22

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.68

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx$$
$$= \text{RootSum}\left(256t^4a^3b^3 + t^2\cdot\left(64a^2b^2ce + 32a^2b^2d^2\right) + t\left(16a^2bde^2 - 16ab^2c^2d\right) + a^2e^4 + 2abc^2e^2 - 4abcd^2e +\right.$$

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a),x)
[Out] RootSum(256*_t**4*a**3*b**3 + _t**2*(64*a**2*b**2*c*e + 32*a**2*b**2*d**2)
+ _t*(16*a**2*b*d*e**2 - 16*a*b**2*c**2*d) + a**2*e**4 + 2*a*b*c**2*e**2 -
4*a*b*c*d**2*e + a*b*d**4 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**4
*b**2*e**3 - 64*_t**3*a**3*b**3*c**2*e + 128*_t**3*a**3*b**3*c*d**2 + 48*_t
**2*a**3*b**2*c*d*e**2 - 32*_t**2*a**3*b**2*d**3*e + 16*_t**2*a**2*b**3*c**
3*d + 12*_t*a**3*b*c*e**4 + 12*_t*a**3*b*d**2*e**3 - 16*_t*a**2*b**2*c**3*e
**2 + 36*_t*a**2*b**2*c**2*d**2*e + 8*_t*a**2*b**2*c*d**4 + 4*_t*a*b**3*c**
5 + 3*a**3*d*e**5 + 5*a**2*b*c*d**3*e**2 - 2*a**2*b*d**5*e + 5*a*b**2*c**4*
d*e - 5*a*b**2*c**3*d**3)/(a**3*e**6 - a**2*b*c**2*e**4 + 8*a**2*b*c*d**2*e
**3 - 4*a**2*b*d**4*e**2 - a*b**2*c**4*e**2 + 8*a*b**2*c**3*d**2*e - 4*a*b*
*2*c**2*d**4 + b**3*c**6))))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.93

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-2\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{3}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e+2\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{3}{4}}}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
[Out] 1/8*sqrt(2)*(sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - 1/8*sqrt(2)*(sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 1/4*(sqrt(2)*a^(1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e - 2*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 1/4*(sqrt(2)*a^(1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e + 2*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.98

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx = -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-(ab^3)^{\frac{1}{4}}b^2c-(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-(ab^3)^{\frac{1}{4}}b^2c-(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c-(ab^3)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c-(ab^3)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)
*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) - 1
/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8
*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/
4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 712, normalized size of antiderivative = 2.57

$$
\begin{aligned}
\int \frac{c+dx+ex^2}{a+bx^4}\,dx = \sum_{k=1}^{4} \ln\Big(& b^2\,c\,d^2 - b^2\,c^2\,e + b^2\,d^3\,x - a\,b\,e^3 \\
&-\mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z - 16\,a\,b^2\,c^2\,d\,z - 4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2 + \\
&- \mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z - 16\,a\,b^2\,c^2\,d\,z \\
&\quad - 4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2 + a\,b\,d^4 + a^2\,e^4 + b^2\,c^4, z, k\big)\; b^3\,c^2\,x\,4 \\
&+\mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z - 16\,a\,b^2\,c^2\,d\,z - 4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2 + \\
&+ \mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z - 16\,a\,b^2\,c^2\,d\,z \\
&\quad - 4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2 + a\,b\,d^4 + a^2\,e^4 + b^2\,c^4, z, k\big)\; a\,b^2\,e^2\,x\,4 \\
&- \mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z - 16\,a\,b^2\,c^2\,d\,z \\
&\quad - 4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2 + a\,b\,d^4 + a^2\,e^4 + b^2\,c^4, z, k\big)\; a\,b^2\,d\,e\,8 \\
&- 2\,b^2\,c\,d\,e\,x\Big)\; \mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z \\
&\quad - 16\,a\,b^2\,c^2\,d\,z - 4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2 + a\,b\,d^4 + a^2\,e^4 + b^2\,c^4, z, k\big)
\end{aligned}
$$

```
[In] int((c + d*x + e*x^2)/(a + b*x^4),x)
```

```
[Out] symsum(log(b^2*c*d^2 - b^2*c^2*e + b^2*d^3*x - a*b*e^3 - 16*root(256*a^3*b^
3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b
^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z
, k)^2*a*b^3*c - 4*root(256*a^3*b^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d
^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^
2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)*b^3*c^2*x + 16*root(256*a^3*b^3*z^4
+ 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2
*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)^2
*a*b^3*d*x + 4*root(256*a^3*b^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z
^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 +
a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)*a*b^2*e^2*x - 8*root(256*a^3*b^3*z^4 + 6
4*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*
z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)*a*b^
```

```
2*d*e - 2*b^2*c*d*e*x)*root(256*a^3*b^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b
^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^
2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k), k, 1, 4)
```

3.127 $\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx = \frac{x(c+dx+ex^2)}{4a(a-bx^4)} + \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

```
[Out] 1/4*x*(e*x^2+d*x+c)/a/(-b*x^4+a)+1/4*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(3/2)
/b^(1/2)+1/8*arctan(b^(1/4)*x/a^(1/4))*(-e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(
3/4)+1/8*arctanh(b^(1/4)*x/a^(1/4))*(e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(3/4)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1869, 1890, 281, 214, 1181, 211}

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}c-\sqrt{a}e\right)}{8a^{7/4}b^{3/4}} + \frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{8a^{7/4}b^{3/4}} + \frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{x(c+dx+ex^2)}{4a(a-bx^4)}$$

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4)^2,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(4*a*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*ArcTan
[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*Arc
Tanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/S
qrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(c+dx+ex^2)}{4a\,(a-bx^4)} - \frac{\int \frac{-3c-2dx-ex^2}{a-bx^4}\,dx}{4a}$$

$$=\frac{x(c+dx+ex^2)}{4a\left(a-bx^4\right)}-\frac{\int\left(-\frac{2dx}{a-bx^4}+\frac{-3c-ex^2}{a-bx^4}\right)dx}{4a}$$

$$=\frac{x(c+dx+ex^2)}{4a\left(a-bx^4\right)}-\frac{\int\frac{-3c-ex^2}{a-bx^4}dx}{4a}+\frac{d\int\frac{x}{a-bx^4}dx}{2a}$$

$$=\frac{x(c+dx+ex^2)}{4a\left(a-bx^4\right)}+\frac{d\text{Subst}\left(\int\frac{1}{a-bx^2}dx,x,x^2\right)}{4a}$$
$$-\frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}-e\right)\int\frac{1}{-\sqrt{a}\sqrt{b}-bx^2}dx}{8a}+\frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\int\frac{1}{\sqrt{a}\sqrt{b}-bx^2}dx}{8a^{3/2}}$$

$$=\frac{x(c+dx+ex^2)}{4a\left(a-bx^4\right)}+\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}$$
$$+\frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.45

$$\int\frac{c+dx+ex^2}{\left(a-bx^4\right)^2}dx$$
$$=\frac{\frac{4ax(c+x(d+ex))}{a-bx^4}-\frac{2\sqrt[4]{a}\left(-3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}-\frac{\left(3\sqrt[4]{a}\sqrt{b}c+2\sqrt{a}\sqrt[4]{b}d+a^{3/4}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{b^{3/4}}+\frac{\left(3\sqrt[4]{a}\sqrt{b}c-2\sqrt{a}\sqrt[4]{b}d}{}}{16a^2}$$

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a*x*(c + x*(d + e*x)))/(a - b*x^4) - (2*a^(1/4)*(-3*Sqrt[b]*c + Sqrt[a]
*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((3*a^(1/4)*Sqrt[b]*c + 2*Sqrt[a
]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((3*a^(1/4)*Sq
rt[b]*c - 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4
) + (2*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(16*a^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}+\frac{cx}{4a}}{-b\,x^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4b-a\right)}\frac{\left(\_R^2e+2\_Rd+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(-b\,x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{16a^2}\right)+d\left(\frac{x^2}{4a(-b\,x^4+a)}+\frac{\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)+e\left(\frac{x^3}{4a(-b\,x\right.$

```
[In] int((e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4/a*e*x^3+1/4*d/a*x^2+1/4*c/a*x)/(-b*x^4+a)-1/16/b/a*sum((_R^2*e+2*_R*d+
3*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.01 (sec) , antiderivative size = 116982, normalized size of antiderivative = 801.25

$$\int\frac{c+dx+ex^2}{(a-bx^4)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(131) = 262.

Time = 40.49 (sec) , antiderivative size = 508, normalized size of antiderivative = 3.48

$$\int\frac{c+dx+ex^2}{(a-bx^4)^2}\,dx$$
$$=\mathrm{RootSum}\left(65536t^4a^7b^3+t^2\left(-3072a^4b^2ce-2048a^4b^2d^2\right)+t\left(128a^3bde^2+1152a^2b^2c^2d\right)-a^2e^4+18abc^2\right.$$
$$+\frac{-cx-dx^2-ex^3}{-4a^2+4abx^4}$$

```
[In] integrate((e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048*a**4*b**2
*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d) - a**2*e**4 + 18*a*
b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 - 81*b**2*c**4, Lambda(_t, _t*l
og(x + (4096*_t**3*a**7*b**2*e**3 + 36864*_t**3*a**6*b**3*c**2*e - 98304*_t
**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2
*d**3*e - 13824*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_t*a**4
*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e
+ 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 - 120*a
**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c*
*3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4 - 96*a**2*b*c*d**2*e**3 + 64*a**2*
b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**
2*d**4 - 729*b**3*c**6)))) + (-c*x - d*x**2 - e*x**3)/(-4*a**2 + 4*a*b*x**4
)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.31

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx = -\frac{ex^3+dx^2+cx}{4\,(abx^4-a^2)} + \frac{\frac{2\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{2\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\left(3\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{16\,a}$$

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(e*x^3 + d*x^2 + c*x)/(a*b*x^4 - a^2) + 1/16*(2*d*log(sqrt(b)*x^2 + sq
rt(a))/(sqrt(a)*sqrt(b)) - 2*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b))
 + 2*(3*sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqr
t(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*sqrt(b)*c + sqrt(a)*e)*log((sqrt(b
)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*
sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(107) = 214$.

Time = 0.28 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.10

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx = -\frac{\sqrt{2}\left(3\,b^2c-2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{\sqrt{2}\left(3\,b^2c+2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{\sqrt{2}\left(3\,b^2c-\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a} + \frac{\sqrt{2}\left(3\,b^2c-\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{ex^3+dx^2+cx}{4\,(bx^4-a)a}$$

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*b^2*c - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arc
tan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*
a) - 1/16*sqrt(2)*(3*b^2*c + 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3
/4)*a) - 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)
^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b
)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) -
1/4*(e*x^3 + d*x^2 + c*x)/((b*x^4 - a)*a)
```

Mupad [B] (verification not implemented)

Time = 9.54 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.27

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx = \frac{\frac{d\,x^2}{4\,a}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{a-b\,x^4}$$

$$+\left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\big(65536\,a^7\,b^3\,z^4-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\right.\right.$$

$$\left.-\frac{-9\,b^2\,c^2\,e+12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3}-\frac{x\,(2\,b^2\,d^3-3\,b^2\,c\,d\,e)}{16\,a^3}\right)\mathrm{root}\big(65536\,a^7\,b^3\,z^4$$

$$-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e$$

$$\left.+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4-81\,b^2\,c^4-a^2\,e^4,z,k\big)\right)$$

```
[In] int((c + d*x + e*x^2)/(a - b*x^4)^2,x)
```

```
[Out] ((d*x^2)/(4*a) + (e*x^3)/(4*a) + (c*x)/(4*a))/(a - b*x^4) + symsum(log(- ro
ot(65536*a^7*b^3*z^4 - 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a
^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a
*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k)*(root(65536*a^7*b^3*z^4 - 3072*a^4*b^2
*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z
- 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k
)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 + 4*a^2*b^2*e^2))/(16*a^3) - (b
^2*d*e)/a) - (12*b^2*c*d^2 - 9*b^2*c^2*e + a*b*e^3)/(64*a^3) - (x*(2*b^2*d^
3 - 3*b^2*c*d*e))/(16*a^3))*root(65536*a^7*b^3*z^4 - 3072*a^4*b^2*c*e*z^2 -
 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c
*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k), k, 1, 4
)
```

3.128 $\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 308

$$\begin{aligned}\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx &= \frac{x(c+dx+ex^2)}{4a(a+bx^4)}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} \\ &\quad-\frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} \\ &\quad+\frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} \\ &\quad-\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} \\ &\quad+\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}\end{aligned}$$

```
[Out] 1/4*x*(e*x^2+d*x+c)/a/(b*x^4+a)+1/4*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b
^(1/2)-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-e*a^(1/2)+
3*c*b^(1/2))/a^(7/4)/b^(3/4)*2^(1/2)+1/32*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1
/2)+x^2*b^(1/2))*(-e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(3/4)*2^(1/2)+1/16*arct
an(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(3/4)*2^
(1/2)+1/16*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+3*c*b^(1/2))/a^(7
/4)/b^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{8\sqrt{2}a^{7/4}b^{3/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{8\sqrt{2}a^{7/4}b^{3/4}} + \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{x(c+dx+ex^2)}{4a\left(a+bx^4\right)}$$

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4)^2,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])
/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) - ((3*Sq
rt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(16*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(c+dx+ex^2)}{4a\,(a+bx^4)} - \frac{\int \frac{-3c-2dx-ex^2}{a+bx^4}\,dx}{4a}$$

$$= \frac{x(c+dx+ex^2)}{4a\,(a+bx^4)} - \frac{\int \left(-\frac{2dx}{a+bx^4} + \frac{-3c-ex^2}{a+bx^4}\right)dx}{4a}$$

$$= \frac{x(c+dx+ex^2)}{4a\,(a+bx^4)} - \frac{\int \frac{-3c-ex^2}{a+bx^4}\,dx}{4a} + \frac{d\int \frac{x}{a+bx^4}\,dx}{2a}$$

$$= \frac{x(c+dx+ex^2)}{4a\,(a+bx^4)} + \frac{d\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{4a} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}} - e\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{8ab} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}} + e\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{8ab}$$

$$= \frac{x(c+dx+ex^2)}{4a\,(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}} + e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx}{16ab} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}} + e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx}{16ab} - \frac{\left(3\sqrt{b}c - \sqrt{a}e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} - x^2}\,dx}{16\sqrt{2}a^{7/4}b^{3/4}} - \frac{\left(3\sqrt{b}c - \sqrt{a}e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} - 2x}{-\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} - x^2}\,dx}{16\sqrt{2}a^{7/4}b^{3/4}}$$

$$= \frac{x(c+dx+ex^2)}{4a\,(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}c - \sqrt{a}e\right)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c - \sqrt{a}e\right)\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c + \sqrt{a}e\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} - \frac{\left(3\sqrt{b}c + \sqrt{a}e\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}}$$

$$= \frac{x(c+dx+ex^2)}{4a(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} - \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx$$

$$= \frac{\frac{8ax(c+x(d+ex))}{a+bx^4} - \frac{2\sqrt[4]{a}\left(3\sqrt{2}\sqrt{b}c+4\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{2\sqrt[4]{a}\left(3\sqrt{2}\sqrt{b}c-4\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1+\sqrt{\ldots}\right)}{b^{3/4}}}{32a^2}$$

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4)^2,x]
```

```
[Out] ((8*a*x*(c + x*(d + e*x)))/(a + b*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[b]*c +
4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/b^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[b]*c - 4*a^(1/4)*b^(1/4)*d + Sq
rt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2
]*(-3*a^(1/4)*Sqrt[b]*c + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(32*a^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.26

method	result
risch	$\frac{\frac{e\,x^{3}}{4a}+\frac{d\,x^{2}}{4a}+\frac{cx}{4a}}{b\,x^{4}+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}b+a\right)}\frac{\left(\_R^{2}e+2\_Rd+3c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{16ba}$
default	$c\left(\frac{x}{4a(b\,x^{4}+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^{2}}\right)+d\left(\frac{x^{2}}{4a(b\,x^{4}+a)}+\right.$

```
[In] int((e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4/a*e*x^3+1/4*d/a*x^2+1/4*c/a*x)/(b*x^4+a)+1/16/b/a*sum((_R^2*e+2*_R*d+3
*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.38 (sec) , antiderivative size = 124258, normalized size of antiderivative = 403.44

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 40.16 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.64

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx$$
$$=\operatorname{RootSum}\left(65536t^4a^7b^3+t^2\cdot\left(3072a^4b^2ce+2048a^4b^2d^2\right)+t\left(128a^3bde^2-1152a^2b^2c^2d\right)+a^2e^4+18abc^2e^2\right.$$
$$+\frac{cx+dx^2+ex^3}{4a^2+4abx^4}$$

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*a**4*b**2*
d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d) + a**2*e**4 + 18*a*b
*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 + 81*b**2*c**4, Lambda(_t, _t*lo
```

```
g(x + (4096*_t**3*a**7*b**2*e**3 - 36864*_t**3*a**6*b**3*c**2*e + 98304*_t*
*3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2*
d**3*e + 13824*_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t*a**4*
b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e +
 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 + 120*a*
*2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c**
3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4 + 96*a**2*b*c*d**2*e**3 - 64*a**2*b
*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**2
*d**4 + 729*b**3*c**6)))) + (c*x + d*x**2 + e*x**3)/(4*a**2 + 4*a*b*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.95

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^2}\,dx = \frac{ex^3+dx^2+cx}{4\left(abx^4+a^2\right)} + \frac{\frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-4\sqrt{a}\sqrt{b}d\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{}}}}{32\,a}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(e*x^3 + d*x^2 + c*x)/(a*b*x^4 + a^2) + 1/32*(sqrt(2)*(3*sqrt(b)*c - sq
rt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^
(3/4)) - sqrt(2)*(3*sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4
)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*c +
 sqrt(2)*a^(3/4)*b^(1/4)*e - 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqr
t(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt
(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^
(1/4)*e + 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^
(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4
)))/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.98

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx$$
$$= \frac{ex^3+dx^2+cx}{4\,(bx^4+a)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$
$$- \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(e*x^3 + d*x^2 + c*x)/((b*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*
b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sq
rt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^
3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(
x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.53

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx = \frac{\frac{d\,x^2}{4\,a}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{b\,x^4+a}$$
$$+\left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(65536\,a^7\,b^3\,z^4+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\right.\right.\right.$$
$$-\frac{9\,b^2\,c^2\,e-12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3}+\frac{x\,(2\,b^2\,d^3-3\,b^2\,c\,d\,e)}{16\,a^3}\Bigg)\,\mathrm{root}\big(65536\,a^7\,b^3\,z^4$$
$$+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e$$
$$\left.\left.+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4+81\,b^2\,c^4+a^2\,e^4,z,k\right)\right)$$

```
[In] int((c + d*x + e*x^2)/(a + b*x^4)^2,x)
```

```
[Out] ((d*x^2)/(4*a) + (e*x^3)/(4*a) + (c*x)/(4*a))/(a + b*x^4) + symsum(log((x*(
2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3) - (9*b^2*c^2*e - 12*b^2*c*d^2 + a*b*e^3)
/(64*a^3) - root(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2*d^
2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*
c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k)*(root(65536*a^7*b^3*z^4
+ 3072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*
a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 +
 a^2*e^4, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 - 4*a^2*b^2*e^2))
/(16*a^3) + (b^2*d*e)/a))*root(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 + 2
048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d
^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k), k, 1, 4)
```

3.129 $\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx = \frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}+\frac{x(7c+6dx+5ex^2)}{32a^2(a-bx^4)} + \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{3d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

```
[Out] 1/8*x*(e*x^2+d*x+c)/a/(-b*x^4+a)^2+1/32*x*(5*e*x^2+6*d*x+7*c)/a^2/(-b*x^4+a
)+3/16*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)+1/64*arctan(b^(1/4)*x
/a^(1/4))*(-5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)+1/64*arctanh(b^(1/4)
*x/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1869, 1890, 281, 214, 1181, 211}

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(21\sqrt{b}c-5\sqrt{a}e\right)}{64a^{11/4}b^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}e+21\sqrt{b}c\right)}{64a^{11/4}b^{3/4}} + \frac{3d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x(7c+6dx+5ex^2)}{32a^2(a-bx^4)}+\frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}$$

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4)^3,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x + 5*e*x^2))/(32
*a^2*(a - b*x^4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4
)])/(64*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh[(b^(1/4)*
x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(
16*a^(5/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(c+dx+ex^2)}{8a\left(a-bx^4\right)^2} - \frac{\int \frac{-7c-6dx-5ex^2}{\left(a-bx^4\right)^2}\,dx}{8a}$$

$$= \frac{x(c+dx+ex^2)}{8a\left(a-bx^4\right)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2\left(a-bx^4\right)} + \frac{\int \frac{21c+12dx+5ex^2}{a-bx^4}\,dx}{32a^2}$$

$$= \frac{x(c+dx+ex^2)}{8a\left(a-bx^4\right)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2\left(a-bx^4\right)} + \frac{\int \left(\frac{12dx}{a-bx^4} + \frac{21c+5ex^2}{a-bx^4}\right)dx}{32a^2}$$

$$= \frac{x(c+dx+ex^2)}{8a\left(a-bx^4\right)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2\left(a-bx^4\right)} + \frac{\int \frac{21c+5ex^2}{a-bx^4}\,dx}{32a^2} + \frac{(3d)\int \frac{x}{a-bx^4}\,dx}{8a^2}$$

$$= \frac{x(c+dx+ex^2)}{8a\left(a-bx^4\right)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2\left(a-bx^4\right)} + \frac{(3d)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{16a^2} - \frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{64a^2} + \frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}+5e\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx}{64a^2}$$

$$= \frac{x(c+dx+ex^2)}{8a\left(a-bx^4\right)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2\left(a-bx^4\right)} + \frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{3/4}} + \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.36

$$\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^3}\,dx$$

$$= \frac{\frac{16a^2x(c+x(d+ex))}{\left(a-bx^4\right)^2} + \frac{4ax(7c+x(6d+5ex))}{a-bx^4} + \frac{2\sqrt[4]{a}\left(21\sqrt{b}c-5\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}} - \frac{\left(21\sqrt[4]{a}\sqrt{b}c+12\sqrt{a}\sqrt[4]{b}d+5a^{3/4}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\right.}{b^{3/4}}}{128a^3}$$

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4)^3,x]
```

```
[Out] ((16*a^2*x*(c + x*(d + e*x)))/(a - b*x^4)^2 + (4*a*x*(7*c + x*(6*d + 5*e*x)
))/(a - b*x^4) + (2*a^(1/4)*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)
/a^(1/4)])/b^(3/4) - ((21*a^(1/4)*Sqrt[b]*c + 12*Sqrt[a]*b^(1/4)*d + 5*a^(3
/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((21*a^(1/4)*Sqrt[b]*c - 12*Sqrt
[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (12*Sqrt[a
]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.49 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.64

method	result
risch	$\frac{-\frac{5be\,x^{7}}{32a^{2}}-\frac{3bd\,x^{6}}{16a^{2}}-\frac{7bc\,x^{5}}{32a^{2}}+\frac{9e\,x^{3}}{32a}+\frac{5d\,x^{2}}{16a}+\frac{11cx}{32a}}{\left(-b\,x^{4}+a\right)^{2}}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b-a\right)}\frac{\left(5\_R^{2}e+12\_Rd+21c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{128a^{2}b}$
default	$c\left(\frac{x}{8a\left(-b\,x^{4}+a\right)^{2}}+\frac{\frac{7x}{32a\left(-b\,x^{4}+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{128a^{2}}}{a}\right)+d\left(\frac{x^{2}}{8a\left(-b\,x^{4}+a\right)^{2}}+\frac{\frac{3x^{2}}{16a\left(-b\,x^{4}+a\right)}+ \cdots}{a}\right.$

[In] int((e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)

[Out] (-5/32*b*e/a^2*x^7-3/16*b*d/a^2*x^6-7/32*b*c/a^2*x^5+9/32/a*e*x^3+5/16*d/a*x^2+11/32*c/a*x)/(-b*x^4+a)^2-1/128/a^2/b*sum((5*_R^2*e+12*_R*d+21*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.14 (sec) , antiderivative size = 118710, normalized size of antiderivative = 663.18

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx = \text{Too large to display}$$

[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx = \text{Timed out}$$

[In] integrate((e*x**2+d*x+c)/(-b*x**4+a)**3,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.28

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx = -\frac{5\,bex^7+6\,bdx^6+7\,bcx^5-9\,aex^3-10\,adx^2-11\,acx}{32\,(a^2b^2x^8-2\,a^3bx^4+a^4)} + \frac{\frac{12\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{12\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(21\,\sqrt{b}c+5\,\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{128\,a^2}$$

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(5*b*e*x^7 + 6*b*d*x^6 + 7*b*c*x^5 - 9*a*e*x^3 - 10*a*d*x^2 - 11*a*c*
x)/(a^2*b^2*x^8 - 2*a^3*b*x^4 + a^4) + 1/128*(12*d*log(sqrt(b)*x^2 + sqrt(a
))/(sqrt(a)*sqrt(b)) - 12*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) +
2*(21*sqrt(b)*c - 5*sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqr
t(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (21*sqrt(b)*c + 5*sqrt(a)*e)*log((sqr
t(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(139) = 278.

Time = 0.29 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.87

$$\begin{aligned}\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx &= -\frac{\sqrt{2}\left(21\,b^2c-12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2} \\ &\quad - \frac{\sqrt{2}\left(21\,b^2c+12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2} \\ &\quad - \frac{\sqrt{2}(21\,b^2c-5\,\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2} \\ &\quad + \frac{\sqrt{2}(21\,b^2c-5\,\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2} \\ &\quad - \frac{5\,bex^7+6\,bdx^6+7\,bcx^5-9\,aex^3-10\,adx^2-11\,acx}{32\,(bx^4-a)^2a^2}\end{aligned}$$

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*(21*b^2*c - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 5*sqrt(-a*b)*b*e
)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(
3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*c + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 5*sqr
t(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/
((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2
+ sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) + 1/256*sqrt(2)
*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b
))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b*e*x^7 + 6*b*d*x^6 + 7*b*c*x^5 - 9*a*e*x
^3 - 10*a*d*x^2 - 11*a*c*x)/((b*x^4 - a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 826, normalized size of antiderivative = 4.61

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx = \frac{\frac{5\,d\,x^2}{16\,a}+\frac{9\,e\,x^3}{32\,a}+\frac{11\,c\,x}{32\,a}-\frac{7\,b\,c\,x^5}{32\,a^2}-\frac{3\,b\,d\,x^6}{16\,a^2}-\frac{5\,b\,e\,x^7}{32\,a^2}}{a^2-2\,a\,b\,x^4+b^2\,x^8} + \left(\sum_{k=1}^{4}\ln\left(-\frac{b\left(125\,a\,e^3+3024\,b\,c\,d^2-2205\,b\,c^2\,e+1728\,b\,d^3\,x+\mathrm{root}(268435456\,a^{11}\,b^3\,z^4-6881280\,a^6\right.}{}\right.\right.$$
$$-6881280\,a^6\,b^2\,c\,e\,z^2-4718592\,a^6\,b^2\,d^2\,z^2+2709504\,a^3\,b^2\,c^2\,d\,z+153600\,a^4\,b\,d\,e^2\,z$$
$$\left.-60480\,a\,b\,c\,d^2\,e+22050\,a\,b\,c^2\,e^2+20736\,a\,b\,d^4-625\,a^2\,e^4-194481\,b^2\,c^4,z,k)\right)$$

```
[In] int((c + d*x + e*x^2)/(a - b*x^4)^3,x)
```

```
[Out] ((5*d*x^2)/(16*a) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) - (7*b*c*x^5)/(32*a^
2) - (3*b*d*x^6)/(16*a^2) - (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^
4) + symsum(log(-(b*(125*a*e^3 + 3024*b*c*d^2 - 2205*b*c^2*e + 1728*b*d^3*x
 + 344064*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a
^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b
*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4
, z, k)^2*a^5*b^2*c + 3200*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*
e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*
e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4
 - 194481*b^2*c^4, z, k)*a^3*b*e^2*x - 2520*b*c*d*e*x + 56448*root(26843545
6*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 270950
4*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^
2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k)*a^2*b^2*c^2*x -
 196608*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6
*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c
*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4,
z, k)^2*a^5*b^2*d*x - 15360*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c
```

```
*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d
*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^
4 - 194481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435456*a^11*b^3*
z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c
^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 207
36*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k), k, 1, 4)
```

3.130 $\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 341

$$\begin{aligned}\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx &= \frac{x(c+dx+ex^2)}{8a(a+bx^4)^2}+\frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)}+\frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}\\ &-\frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}}\\ &+\frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}}\\ &-\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}\\ &+\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}\end{aligned}$$

```
[Out] 1/8*x*(e*x^2+d*x+c)/a/(b*x^4+a)^2+1/32*x*(5*e*x^2+6*d*x+7*c)/a^2/(b*x^4+a)+
3/16*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)-1/256*ln(-a^(1/4)*b^(1/4
)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/
4)*2^(1/2)+1/256*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*e*a^
(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)*2^(1/2)+1/128*arctan(-1+b^(1/4)*x*2^(1
/2)/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)*2^(1/2)+1/128*arct
an(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)
*2^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}e+21\sqrt{b}c\right)}{64\sqrt{2}a^{11/4}b^{3/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}e+21\sqrt{b}c\right)}{64\sqrt{2}a^{11/4}b^{3/4}} + \frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}} + \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}} + \frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)} + \frac{x(c+dx+ex^2)}{8a(a+bx^4)^2}$$

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4)^3,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x + 5*e*x^2))/(32
*a^2*(a + b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]
) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/
(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 + (S
qrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*Sqrt[b]*c
- 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128
*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral}=\frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2}-\frac{\int\frac{-7c-6dx-5ex^2}{\left(a+bx^4\right)^2}\,dx}{8a}$$

$$=\frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2}+\frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)}+\frac{\int\frac{21c+12dx+5ex^2}{a+bx^4}\,dx}{32a^2}$$

$$=\frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2}+\frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)}+\frac{\int\left(\frac{12dx}{a+bx^4}+\frac{21c+5ex^2}{a+bx^4}\right)dx}{32a^2}$$

$$=\frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2}+\frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)}+\frac{\int\frac{21c+5ex^2}{a+bx^4}\,dx}{32a^2}+\frac{(3d)\int\frac{x}{a+bx^4}\,dx}{8a^2}$$

$$=\frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2}+\frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)}+\frac{(3d)\text{Subst}\left(\int\frac{1}{a+bx^2}\,dx,x,x^2\right)}{16a^2}+\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\int\frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{64a^2b}+\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}+5e\right)\int\frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{64a^2b}$$

$$=\frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2}+\frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{128\sqrt{2}a^{9/4}b^{3/4}}-\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{128\sqrt{2}a^{9/4}b^{3/4}}+\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}+5e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{128a^2b}+\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}+5e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{128a^2b}$$

$$= \frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$
$$- \frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{3/4}}$$
$$+ \frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{3/4}}$$
$$+ \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}}$$
$$- \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}}$$

$$= \frac{x(c+dx+ex^2)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$
$$- \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}} + \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}}$$
$$- \frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{3/4}}$$
$$+ \frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{3/4}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^3}\,dx$$

$$= \frac{\frac{32a^2x(c+x(d+ex))}{(a+bx^4)^2} + \frac{8ax(7c+x(6d+5ex))}{a+bx^4} - \frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{b}c+24\sqrt[4]{a}\sqrt[4]{b}d+5\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{b}c-24\sqrt[4]{\ldots}\right.}{\ldots}}{\ldots}$$

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4)^3,x]
```

```
[Out] ((32*a^2*x*(c + x*(d + e*x)))/(a + b*x^4)^2 + (8*a*x*(7*c + x*(6*d + 5*e*x)
))/(a + b*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c + 24*a^(1/4)*b^(1/4)*d +
5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (2*
a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c - 24*a^(1/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)
```

```
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2]*(-21*a^(1/4)*S
qrt[b]*c + 5*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2])/b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3/4)*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(256*a^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.33

method	result
risch	$\frac{\frac{5be\,x^7}{32a^2}+\frac{3bd\,x^6}{16a^2}+\frac{7bc\,x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{\left(5\_R^2e+12\_Rd+21c\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2b}$
default	$c\left(\frac{x}{8a(b\,x^4+a)^2}+\frac{\frac{7x}{32a\left(b\,x^4+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)+d\left(\frac{}{8a(b}\right.$

```
[In] int((e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (5/32*b*e/a^2*x^7+3/16*b*d/a^2*x^6+7/32*b*c/a^2*x^5+9/32/a*e*x^3+5/16*d/a*x
^2+11/32*c/a*x)/(b*x^4+a)^2+1/128/a^2/b*sum((5*_R^2*e+12*_R*d+21*c)/_R^3*ln
(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.02 (sec) , antiderivative size = 124787, normalized size of antiderivative = 365.94

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx = \frac{5\,bex^7+6\,bdx^6+7\,bcx^5+9\,aex^3+10\,adx^2+11\,acx}{32\,(a^2b^2x^8+2\,a^3bx^4+a^4)} + \frac{\frac{\sqrt{2}\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(21\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+5\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-24\right.}{a^{\frac{3}{4}}}}{256\,a^2}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(5*b*e*x^7 + 6*b*d*x^6 + 7*b*c*x^5 + 9*a*e*x^3 + 10*a*d*x^2 + 11*a*c*x
)/(a^2*b^2*x^8 + 2*a^3*b*x^4 + a^4) + 1/256*(sqrt(2)*(21*sqrt(b)*c - 5*sqrt
(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3
/4)) - sqrt(2)*(21*sqrt(b)*c - 5*sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(3/4)*c
 + 5*sqrt(2)*a^(3/4)*b^(1/4)*e - 24*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(
2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt
(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(3/4)*c + 5*sqrt(2)*a^
(3/4)*b^(1/4)*e + 24*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - s
qrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b
))*b^(3/4)))/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.97

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx = \frac{5\,bex^7+6\,bdx^6+7\,bcx^5+9\,aex^3+10\,adx^2+11\,acx}{32\,(bx^4+a)^2a^2} + \frac{\sqrt{2}\left(12\,\sqrt{2}\sqrt{ab}b^2d+21\,(ab^3)^{\frac{1}{4}}\,b^2c+5\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3} + \frac{\sqrt{2}\left(12\,\sqrt{2}\sqrt{ab}b^2d+21\,(ab^3)^{\frac{1}{4}}\,b^2c+5\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3} + \frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^2c-5\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3} - \frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^2c-5\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*b*e*x^7 + 6*b*d*x^6 + 7*b*c*x^5 + 9*a*e*x^3 + 10*a*d*x^2 + 11*a*c*x
)/((b*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b
^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d
 + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)
*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^
2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.42

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx = \frac{\frac{5\,d\,x^2}{16\,a}+\frac{9\,e\,x^3}{32\,a}+\frac{11\,c\,x}{32\,a}+\frac{7\,b\,c\,x^5}{32\,a^2}+\frac{3\,b\,d\,x^6}{16\,a^2}+\frac{5\,b\,e\,x^7}{32\,a^2}}{a^2+2\,a\,b\,x^4+b^2\,x^8} + \left(\sum_{k=1}^{4}\ln\left(-\frac{b\left(125\,a\,e^3-3024\,b\,c\,d^2+2205\,b\,c^2\,e-1728\,b\,d^3\,x+\mathrm{root}(268435456\,a^{11}\,b^3\,z^4+6881280\,a^6\right.}{}\right.\right.$$
$$+6881280\,a^6\,b^2\,c\,e\,z^2+4718592\,a^6\,b^2\,d^2\,z^2-2709504\,a^3\,b^2\,c^2\,d\,z+153600\,a^4\,b\,d\,e^2\,z$$
$$\left.-60480\,a\,b\,c\,d^2\,e+22050\,a\,b\,c^2\,e^2+20736\,a\,b\,d^4+625\,a^2\,e^4+194481\,b^2\,c^4,z,k)\right)$$

```
[In] int((c + d*x + e*x^2)/(a + b*x^4)^3,x)
```

```
[Out] ((5*d*x^2)/(16*a) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) + (7*b*c*x^5)/(32*a^
2) + (3*b*d*x^6)/(16*a^2) + (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^
4) + symsum(log(-(b*(125*a*e^3 - 3024*b*c*d^2 + 2205*b*c^2*e - 1728*b*d^3*x
 + 344064*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a
^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b
*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4
, z, k)^2*a^5*b^2*c - 3200*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*
e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*
e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4
 + 194481*b^2*c^4, z, k)*a^3*b*e^2*x + 2520*b*c*d*e*x + 56448*root(26843545
6*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 270950
4*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^
2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k)*a^2*b^2*c^2*x -
 196608*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6
*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c
*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4,
z, k)^2*a^5*b^2*d*x + 15360*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c
*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d
*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^
4 + 194481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435456*a^11*b^3*
z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c
^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 207
36*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k), k, 1, 4)
```

3.131 $\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 211

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx=\frac{x(c+dx+ex^2)}{12a(a-bx^4)^3}+\frac{x(11c+10dx+9ex^2)}{96a^2(a-bx^4)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3(a-bx^4)}+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{5d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

```
[Out] 1/12*x*(e*x^2+d*x+c)/a/(-b*x^4+a)^3+1/96*x*(9*e*x^2+10*d*x+11*c)/a^2/(-b*x^
4+a)^2+1/384*x*(45*e*x^2+60*d*x+77*c)/a^3/(-b*x^4+a)+5/32*d*arctanh(x^2*b^(
1/2)/a^(1/2))/a^(7/2)/b^(1/2)+1/256*arctan(b^(1/4)*x/a^(1/4))*(-15*e*a^(1/2
)+77*c*b^(1/2))/a^(15/4)/b^(3/4)+1/256*arctanh(b^(1/4)*x/a^(1/4))*(15*e*a^(
1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

= {1869, 1890, 281, 214, 1181, 211}

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(77\sqrt{b}c-15\sqrt{a}e\right)}{256a^{15/4}b^{3/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}e+77\sqrt{b}c\right)}{256a^{15/4}b^{3/4}} + \frac{5d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} + \frac{x(77c+60dx+45ex^2)}{384a^3(a-bx^4)} + \frac{x(11c+10dx+9ex^2)}{96a^2(a-bx^4)^2} + \frac{x(c+dx+ex^2)}{12a(a-bx^4)^3}$$

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4)^4,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x + 9*e*x^2))/
(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a - b*x^4
)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15
/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(256*a^(15/4)*b^(3/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*
Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
```

```
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(c+dx+ex^2)}{12a\,(a-bx^4)^3} - \frac{\int \frac{-11c-10dx-9ex^2}{(a-bx^4)^3}\,dx}{12a} \\
&= \frac{x(c+dx+ex^2)}{12a\,(a-bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2\,(a-bx^4)^2} + \frac{\int \frac{77c+60dx+45ex^2}{(a-bx^4)^2}\,dx}{96a^2} \\
&= \frac{x(c+dx+ex^2)}{12a\,(a-bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2\,(a-bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a-bx^4)} - \frac{\int \frac{-231c-120dx-45ex^2}{a-bx^4}\,dx}{384a^3} \\
&= \frac{x(c+dx+ex^2)}{12a\,(a-bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2\,(a-bx^4)^2} \\
&\quad + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a-bx^4)} - \frac{\int \left(-\frac{120dx}{a-bx^4} + \frac{-231c-45ex^2}{a-bx^4}\right)dx}{384a^3} \\
&= \frac{x(c+dx+ex^2)}{12a\,(a-bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2\,(a-bx^4)^2} \\
&\quad + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a-bx^4)} - \frac{\int \frac{-231c-45ex^2}{a-bx^4}\,dx}{384a^3} + \frac{(5d)\int \frac{x}{a-bx^4}\,dx}{16a^3} \\
&= \frac{x(c+dx+ex^2)}{12a\,(a-bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2\,(a-bx^4)^2} \\
&\quad + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a-bx^4)} + \frac{(5d)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{32a^3} \\
&\quad - \frac{\left(\frac{77\sqrt{b}c}{\sqrt{a}} - 15e\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{256a^3} + \frac{\left(\frac{77\sqrt{b}c}{\sqrt{a}} + 15e\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx}{256a^3}
\end{aligned}$$

$$= \frac{x(c+dx+ex^2)}{12a\left(a-bx^4\right)^3}+\frac{x(11c+10dx+9ex^2)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a-bx^4\right)}+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.31

$$\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^4}\,dx$$

$$= \frac{\frac{128a^3x(c+x(d+ex))}{(a-bx^4)^3}+\frac{4ax(77c+15x(4d+3ex))}{a-bx^4}+\frac{16a^2x(11c+x(10d+9ex))}{(a-bx^4)^2}+\frac{6\sqrt[4]{a}\left(77\sqrt{b}c-15\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}-\frac{3\left(77\sqrt[4]{a}\sqrt{b}c+\cdots\right.}{}}{1536a^4}$$

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4)^4,x]
```

```
[Out] ((128*a^3*x*(c + x*(d + e*x)))/(a - b*x^4)^3 + (4*a*x*(77*c + 15*x*(4*d + 3
*e*x)))/(a - b*x^4) + (16*a^2*x*(11*c + x*(10*d + 9*e*x)))/(a - b*x^4)^2 +
(6*a^(1/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(3/
4) - (3*(77*a^(1/4)*Sqrt[b]*c + 40*Sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Log[a^
(1/4) - b^(1/4)*x])/b^(3/4) + (3*(77*a^(1/4)*Sqrt[b]*c - 40*Sqrt[a]*b^(1/4)
*d + 15*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (120*Sqrt[a]*d*Log[S
qrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(1536*a^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\frac{15 e\,b^{2} x^{11}}{128 a^{3}}+\frac{5 d\,b^{2} x^{10}}{32 a^{3}}+\frac{77 c\,b^{2} x^{9}}{384 a^{3}}-\frac{21 b e\,x^{7}}{64 a^{2}}-\frac{5 b d\,x^{6}}{12 a^{2}}-\frac{33 b c\,x^{5}}{64 a^{2}}+\frac{113 e\,x^{3}}{384 a}+\frac{11 d\,x^{2}}{32 a}+\frac{51 c x}{128 a}}{\left(-b\,x^{4}+a\right)^{3}}-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4} b-a\right)}\frac{\left(15\_R^{2} e+40\_R d+77 c\right)}{\_R^{3}}}{512 a^{3} b}$
default	$\frac{\frac{15 e\,b^{2} x^{11}}{128 a^{3}}+\frac{5 d\,b^{2} x^{10}}{32 a^{3}}+\frac{77 c\,b^{2} x^{9}}{384 a^{3}}-\frac{21 b e\,x^{7}}{64 a^{2}}-\frac{5 b d\,x^{6}}{12 a^{2}}-\frac{33 b c\,x^{5}}{64 a^{2}}+\frac{113 e\,x^{3}}{384 a}+\frac{11 d\,x^{2}}{32 a}+\frac{51 c x}{128 a}}{\left(-b\,x^{4}+a\right)^{3}}+\frac{\frac{77 c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+}{}$

```
[In] int((e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9-21/64*b*e/a
^2*x^7-5/12*b*d/a^2*x^6-33/64*b*c/a^2*x^5+113/384/a*e*x^3+11/32*d/a*x^2+51/
128*c/a*x)/(-b*x^4+a)^3-1/512/a^3/b*sum((15*_R^2*e+40*_R*d+77*c)/_R^3*ln(x-
_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.18 (sec) , antiderivative size = 118903, normalized size of antiderivative = 563.52

$$\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.32

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx = -\frac{45\,b^2ex^{11}+60\,b^2dx^{10}+77\,b^2cx^9-126\,abex^7-160\,abdx^6-198\,abcx^5+113\,a^2ex^3+132\,a^2dx^2+153\,a}{384\,(a^3b^3x^{12}-3\,a^4b^2x^8+3\,a^5bx^4-a^6)} + \frac{\frac{40\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{40\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\,\sqrt{b}c+15\,\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{512\,a^3}$$

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(45*b^2*e*x^11 + 60*b^2*d*x^10 + 77*b^2*c*x^9 - 126*a*b*e*x^7 - 160*
a*b*d*x^6 - 198*a*b*c*x^5 + 113*a^2*e*x^3 + 132*a^2*d*x^2 + 153*a^2*c*x)/(a
^3*b^3*x^12 - 3*a^4*b^2*x^8 + 3*a^5*b*x^4 - a^6) + 1/512*(40*d*log(sqrt(b)*
x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 40*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)
*sqrt(b)) + 2*(77*sqrt(b)*c - 15*sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (77*sqrt(b)*c + 15*sqrt(
a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqr
t(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(170) = 340.

Time = 0.29 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.75

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx$$

$$= -\frac{\sqrt{2}\left(77\,b^2c-40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}$$

$$-\frac{\sqrt{2}\left(77\,b^2c+40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}$$

$$-\frac{\sqrt{2}(77\,b^2c-15\,\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}$$

$$+\frac{\sqrt{2}(77\,b^2c-15\,\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}$$

$$-\frac{45\,b^2ex^{11}+60\,b^2dx^{10}+77\,b^2cx^9-126\,abex^7-160\,abdx^6-198\,abcx^5+113\,a^2ex^3+132\,a^2dx^2+153\,a^2c}{384\,(bx^4-a)^3a^3}$$

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^2*c - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 15*sqrt(-a*b)*b*
e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^
(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2*c + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 15*s
qrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4)
)/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(
x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) + 1/1024*sq
rt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqr
t(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/384*(45*b^2*e*x^11 + 60*b^2*d*x^10 + 77*b
^2*c*x^9 - 126*a*b*e*x^7 - 160*a*b*d*x^6 - 198*a*b*c*x^5 + 113*a^2*e*x^3 +
132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 - a)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 9.95 (sec) , antiderivative size = 874, normalized size of antiderivative = 4.14

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx = \frac{\frac{11\,d\,x^2}{32\,a}+\frac{113\,e\,x^3}{384\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}+\frac{15\,b^2\,e\,x^{11}}{128\,a^3}-\frac{33\,b\,c\,x^5}{64\,a^2}-\frac{5\,b\,d\,x^6}{12\,a^2}-\frac{21\,b\,e\,x^7}{64\,a^2}}{a^3-3\,a^2\,b\,x^4+3\,a\,b^2\,x^8-b^3\,x^{12}} + \left(\sum_{k=1}^{4}\ln\left(-\frac{b\left(3375\,a\,e^3+123200\,b\,c\,d^2-88935\,b\,c^2\,e+64000\,b\,d^3\,x+\mathrm{root}(68719476736\,a^{15}\,b^3\,z^4-12\right.}{}\right.\right.$$

$$-1211105280\,a^8\,b^2\,c\,e\,z^2-838860800\,a^8\,b^2\,d^2\,z^2+485703680\,a^4\,b^2\,c^2\,d\,z+18432000\,a^5\,b\,d\,e^2\,z-7392000\,a\,b\,c\,d^2\,e+2668050\,a\,b\,c^2\,e^2+2560000\,a\,b\,d^4-35153041\,b^2\,c^4-50625\,a^2\,e^4,z,k)\Bigg)$$

```
[In] int((c + d*x + e*x^2)/(a - b*x^4)^4,x)
```

```
[Out] ((11*d*x^2)/(32*a) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9
)/(384*a^3) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) - (33*b*c
*x^5)/(64*a^2) - (5*b*d*x^6)/(12*a^2) - (21*b*e*x^7)/(64*a^2))/(a^3 - b^3*x
^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log(-(b*(3375*a*e^3 + 123200*b*c*
d^2 - 88935*b*c^2*e + 64000*b*d^3*x + 20185088*root(68719476736*a^15*b^3*z^
4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*
b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)^2*a^7*b^2
*c + 115200*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 83
8860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*
z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*
b^2*c^4 - 50625*a^2*e^4, z, k)*a^4*b*e^2*x - 92400*b*c*d*e*x + 3035648*root
(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*
d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*
c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*
a^2*e^4, z, k)*a^3*b^2*c^2*x - 10485760*root(68719476736*a^15*b^3*z^4 - 121
1105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2
*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 +
 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x -
614400*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 8388608
00*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7
392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c
^4 - 50625*a^2*e^4, z, k)*a^4*b*d*e))/(2097152*a^9))*root(68719476736*a^15*
b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 48570368
0*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*
```

```
a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k), k,
 1, 4)
```

3.132 $\int \frac{c+dx+ex^2}{(a+bx^4)^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 372

$$\begin{aligned}\int \frac{c+dx+ex^2}{(a+bx^4)^4} dx = {} & \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3}+\frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} \\ & +\frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}-\frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}} \\ & +\frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}} \\ & -\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}} \\ & +\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}\end{aligned}$$

```
[Out] 1/12*x*(e*x^2+d*x+c)/a/(b*x^4+a)^3+1/96*x*(9*e*x^2+10*d*x+11*c)/a^2/(b*x^4+
a)^2+1/384*x*(45*e*x^2+60*d*x+77*c)/a^3/(b*x^4+a)+5/32*d*arctan(x^2*b^(1/2)
/a^(1/2))/a^(7/2)/b^(1/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
b^(1/2))*(-15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)+1/1024*ln(a^
(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-15*e*a^(1/2)+77*c*b^(1/2))/a
^(15/4)/b^(3/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(15*e*a^
(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)+1/512*arctan(1+b^(1/4)*x*2^(1/
2)/a^(1/4))*(15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}e+77\sqrt{b}c\right)}{256\sqrt{2}a^{15/4}b^{3/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}e+77\sqrt{b}c\right)}{256\sqrt{2}a^{15/4}b^{3/4}} + \frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}} + \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3}$$

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4)^4,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x + 9*e*x^2))/
(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a + b*x^4
)) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*Sqrt[b
]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a
^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c - 15*Sqrt[a
]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a
^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(c+dx+ex^2)}{12a\left(a+bx^4\right)^3}-\frac{\int \frac{-11c-10dx-9ex^2}{\left(a+bx^4\right)^3}\,dx}{12a}\\
&= \frac{x(c+dx+ex^2)}{12a\left(a+bx^4\right)^3}+\frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2}+\frac{\int \frac{77c+60dx+45ex^2}{\left(a+bx^4\right)^2}\,dx}{96a^2}\\
&= \frac{x(c+dx+ex^2)}{12a\left(a+bx^4\right)^3}+\frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a+bx^4\right)}-\frac{\int \frac{-231c-120dx-45ex^2}{a+bx^4}\,dx}{384a^3}\\
&= \frac{x(c+dx+ex^2)}{12a\left(a+bx^4\right)^3}+\frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2}\\
&\quad+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a+bx^4\right)}-\frac{\int \left(-\frac{120dx}{a+bx^4}+\frac{-231c-45ex^2}{a+bx^4}\right)dx}{384a^3}\\
&= \frac{x(c+dx+ex^2)}{12a\left(a+bx^4\right)^3}+\frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2}\\
&\quad+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a+bx^4\right)}-\frac{\int \frac{-231c-45ex^2}{a+bx^4}\,dx}{384a^3}+\frac{(5d)\int \frac{x}{a+bx^4}\,dx}{16a^3}\\
&= \frac{x(c+dx+ex^2)}{12a\left(a+bx^4\right)^3}+\frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2}\\
&\quad+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a+bx^4\right)}+\frac{(5d)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{32a^3}\\
&\quad+\frac{\left(\frac{77\sqrt{b}c}{\sqrt{a}}-15e\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{256a^3b}+\frac{\left(\frac{77\sqrt{b}c}{\sqrt{a}}+15e\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{256a^3b}
\end{aligned}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} + \frac{\left(\frac{77\sqrt{b}c}{\sqrt{a}}+15e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{512a^3b} + \frac{\left(\frac{77\sqrt{b}c}{\sqrt{a}}+15e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{512a^3b} - \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{512\sqrt{2}a^{15/4}b^{3/4}} - \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{512\sqrt{2}a^{15/4}b^{3/4}}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}} + \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}} + \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}} - \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}}$$

$$
\begin{aligned}
&= \frac{x(c+dx+ex^2)}{12a\,(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2\,(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\,(a+bx^4)} \\
&\quad + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}} \\
&\quad + \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}} \\
&\quad - \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}} \\
&\quad + \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$$

$$= \frac{\frac{256a^3x(c+x(d+ex))}{(a+bx^4)^3} + \frac{8ax(77c+15x(4d+3ex))}{a+bx^4} + \frac{32a^2x(11c+x(10d+9ex))}{(a+bx^4)^2} - \frac{6\sqrt[4]{a}\left(77\sqrt{2}\sqrt{b}c+80\sqrt[4]{a}\sqrt[4]{b}d+15\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.}{b^{3/4}}}{}$$

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4)^4,x]
```

```
[Out] ((256*a^3*x*(c + x*(d + e*x)))/(a + b*x^4)^3 + (8*a*x*(77*c + 15*x*(4*d + 3
*e*x)))/(a + b*x^4) + (32*a^2*x*(11*c + x*(10*d + 9*e*x)))/(a + b*x^4)^2 -
(6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c + 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a
]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (6*a^(1/4)*(77*Sqrt
[2]*Sqrt[b]*c - 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (3*Sqrt[2]*(-77*a^(1/4)*Sqrt[b]*c + 15
*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4)
 + (3*Sqrt[2]*(77*a^(1/4)*Sqrt[b]*c - 15*a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(3072*a^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{15e\,b^{2}x^{11}}{128a^{3}}+\frac{5d\,b^{2}x^{10}}{32a^{3}}+\frac{77c\,b^{2}x^{9}}{384a^{3}}+\frac{21be\,x^{7}}{64a^{2}}+\frac{5bd\,x^{6}}{12a^{2}}+\frac{33bc\,x^{5}}{64a^{2}}+\frac{113e\,x^{3}}{384a}+\frac{11d\,x^{2}}{32a}+\frac{51cx}{128a}}{\left(b\,x^{4}+a\right)^{3}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}\frac{\left(15\_R^{2}e+40\_Rd+77c\right.}{\_R^{3}}}{512a^{3}b}$
default	$\frac{\frac{15e\,b^{2}x^{11}}{128a^{3}}+\frac{5d\,b^{2}x^{10}}{32a^{3}}+\frac{77c\,b^{2}x^{9}}{384a^{3}}+\frac{21be\,x^{7}}{64a^{2}}+\frac{5bd\,x^{6}}{12a^{2}}+\frac{33bc\,x^{5}}{64a^{2}}+\frac{113e\,x^{3}}{384a}+\frac{11d\,x^{2}}{32a}+\frac{51cx}{128a}}{\left(b\,x^{4}+a\right)^{3}}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\right.}{8a}}{}$

[In] int((e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)

[Out] (15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9+21/64*b*e/a^2*x^7+5/12*b*d/a^2*x^6+33/64*b*c/a^2*x^5+113/384/a*e*x^3+11/32*d/a*x^2+51/128*c/a*x)/(b*x^4+a)^3+1/512/a^3/b*sum((15*_R^2*e+40*_R*d+77*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.97 (sec) , antiderivative size = 124960, normalized size of antiderivative = 335.91

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^4}\,dx = \text{Too large to display}$$

[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^4}\,dx = \text{Timed out}$$

[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**4,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.03

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$$
$$= \frac{45\,b^2ex^{11}+60\,b^2dx^{10}+77\,b^2cx^9+126\,abex^7+160\,abdx^6+198\,abcx^5+113\,a^2ex^3+132\,a^2dx^2+153\,a^2cx}{384\,(a^3b^3x^{12}+3\,a^4b^2x^8+3\,a^5bx^4+a^6)}$$
$$+\frac{\frac{\sqrt{2}\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+15\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-8\right.}{a^{\frac{3}{4}}}}{1024\,a^3}$$

[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")

[Out] 1/384*(45*b^2*e*x^11 + 60*b^2*d*x^10 + 77*b^2*c*x^9 + 126*a*b*e*x^7 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 113*a^2*e*x^3 + 132*a^2*d*x^2 + 153*a^2*c*x)/(a^3*b^3*x^12 + 3*a^4*b^2*x^8 + 3*a^5*b*x^4 + a^6) + 1/1024*(sqrt(2)*(77*sqrt(b)*c - 15*sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*sqrt(b)*c - 15*sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e - 80*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e + 80*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/a^3

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.98

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$$

$$= \frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{ab}b^2d+77\,(ab^3)^{\frac{1}{4}}\,b^2c+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$

$$+ \frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{ab}b^2d+77\,(ab^3)^{\frac{1}{4}}\,b^2c+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$

$$+ \frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$

$$- \frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$

$$+ \frac{45\,b^2ex^{11}+60\,b^2dx^{10}+77\,b^2cx^9+126\,abex^7+160\,abdx^6+198\,abcx^5+113\,a^2ex^3+132\,a^2dx^2+153\,a^2}{384\,(bx^4+a)^3a^3}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 15*(a*
b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2
*c + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/
b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(
3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqr
t(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b
)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^2*e*x^11 + 60*b^2*d*x^10 + 77*
b^2*c*x^9 + 126*a*b*e*x^7 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 113*a^2*e*x^3 +
 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 + a)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 873, normalized size of antiderivative = 2.35

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$$
$$= \frac{\frac{11\,d\,x^2}{32\,a}+\frac{113\,e\,x^3}{384\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}+\frac{15\,b^2\,e\,x^{11}}{128\,a^3}+\frac{33\,b\,c\,x^5}{64\,a^2}+\frac{5\,b\,d\,x^6}{12\,a^2}+\frac{21\,b\,e\,x^7}{64\,a^2}}{a^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^8+b^3\,x^{12}}$$
$$+\left(\sum_{k=1}^{4}\ln\left(-\frac{b\left(3375\,a\,e^3-123200\,b\,c\,d^2+88935\,b\,c^2\,e-64000\,b\,d^3\,x+\mathrm{root}(68719476736\,a^{15}\,b^3\,z^4+121\right.}{}\right.\right.$$
$$+1211105280\,a^8\,b^2\,c\,e\,z^2+838860800\,a^8\,b^2\,d^2\,z^2-485703680\,a^4\,b^2\,c^2\,d\,z$$
$$+18432000\,a^5\,b\,d\,e^2\,z-7392000\,a\,b\,c\,d^2\,e+2668050\,a\,b\,c^2\,e^2+2560000\,a\,b\,d^4$$
$$\left.\left.+35153041\,b^2\,c^4+50625\,a^2\,e^4,z,k)\right)\right)$$

```
[In] int((c + d*x + e*x^2)/(a + b*x^4)^4,x)
```

```
[Out] ((11*d*x^2)/(32*a) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9
)/(384*a^3) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) + (33*b*c
*x^5)/(64*a^2) + (5*b*d*x^6)/(12*a^2) + (21*b*e*x^7)/(64*a^2))/(a^3 + b^3*x
^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log(-(b*(3375*a*e^3 - 123200*b*c*
d^2 + 88935*b*c^2*e - 64000*b*d^3*x + 20185088*root(68719476736*a^15*b^3*z^
4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*
b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)^2*a^7*b^2
*c - 115200*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 83
8860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*
z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*
b^2*c^4 + 50625*a^2*e^4, z, k)*a^4*b*e^2*x + 92400*b*c*d*e*x + 3035648*root
(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*
d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*
c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*
a^2*e^4, z, k)*a^3*b^2*c^2*x - 10485760*root(68719476736*a^15*b^3*z^4 + 121
1105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2
*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 +
 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x +
614400*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 8388608
00*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7
392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c
^4 + 50625*a^2*e^4, z, k)*a^4*b*d*e))/(2097152*a^9))*root(68719476736*a^15*
b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 48570368
0*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*
```

```
a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k), k,
 1, 4)
```

3.133 $\int a(e+fx^4)^2\,dx$

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int a(e+fx^4)^2\,dx = ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9$$

```
[Out] a*e^2*x+2/5*a*e*f*x^5+1/9*a*f^2*x^9
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 200}

$$\int a(e+fx^4)^2\,dx = ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9$$

```
[In] Int[a*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= a\int \left(e+fx^4\right)^2\,dx\\ &= a\int \left(e^2+2efx^4+f^2x^8\right)\,dx\\ &= ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int a\left(e+fx^4\right)^2\,dx=a\left(e^2x+\frac{2}{5}efx^5+\frac{f^2x^9}{9}\right)$$

```
[In] Integrate[a*(e + f*x^4)^2,x]
[Out] a*(e^2*x + (2*e*f*x^5)/5 + (f^2*x^9)/9)
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
default	$\left(\frac{1}{9}f^2x^9+\frac{2}{5}ef\,x^5+e^2x\right)a$	24
parallelrisch	$\left(\frac{1}{9}f^2x^9+\frac{2}{5}ef\,x^5+e^2x\right)a$	24
norman	$a\,e^2x+\frac{2}{5}aef\,x^5+\frac{1}{9}a\,f^2x^9$	25
risch	$a\,e^2x+\frac{2}{5}aef\,x^5+\frac{1}{9}a\,f^2x^9$	25
gosper	$\frac{x\left(5f^2x^8+18ef\,x^4+45e^2\right)a}{45}$	26

```
[In] int(a*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
[Out] (1/9*f^2*x^9+2/5*e*f*x^5+e^2*x)*a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int a(e+fx^4)^2\,dx=\frac{1}{9}af^2x^9+\frac{2}{5}aefx^5+ae^2x$$

```
[In] integrate(a*(f*x^4+e)^2,x, algorithm="fricas")
[Out] 1/9*a*f^2*x^9 + 2/5*a*e*f*x^5 + a*e^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int a(e+fx^4)^2\,dx=ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}$$

```
[In] integrate(a*(f*x**4+e)**2,x)
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int a(e+fx^4)^2\,dx=\frac{1}{45}\left(5f^2x^9+18efx^5+45e^2x\right)a$$

```
[In] integrate(a*(f*x^4+e)^2,x, algorithm="maxima")
[Out] 1/45*(5*f^2*x^9 + 18*e*f*x^5 + 45*e^2*x)*a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int a(e+fx^4)^2\,dx=\frac{1}{45}\left(5f^2x^9+18efx^5+45e^2x\right)a$$

```
[In] integrate(a*(f*x^4+e)^2,x, algorithm="giac")
[Out] 1/45*(5*f^2*x^9 + 18*e*f*x^5 + 45*e^2*x)*a
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int a(e+fx^4)^2\,dx=\frac{a\,x\,\left(45\,e^2+18\,e\,f\,x^4+5\,f^2\,x^8\right)}{45}$$

```
[In] int(a*(e + f*x^4)^2,x)
[Out] (a*x*(45*e^2 + 5*f^2*x^8 + 18*e*f*x^4))/45
```

3.134 $\int bx(e+fx^4)^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int bx(e+fx^4)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

```
[Out] 1/2*b*e^2*x^2+1/3*b*e*f*x^6+1/10*b*f^2*x^10
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {12, 276}

$$\int bx(e+fx^4)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

```
[In] Int[b*x*(e + f*x^4)^2,x]
[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= b\int x(e+fx^4)^2\,dx \\ &= b\int (e^2x+2efx^5+f^2x^9)\,dx \\ &= \frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int bx(e+fx^4)^2\,dx = b\left(\frac{e^2x^2}{2}+\frac{1}{3}efx^6+\frac{f^2x^{10}}{10}\right)$$

```
[In] Integrate[b*x*(e + f*x^4)^2,x]
[Out] b*((e^2*x^2)/2 + (e*f*x^6)/3 + (f^2*x^10)/10)
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
default	$\left(\frac{1}{10}f^2x^{10}+\frac{1}{3}ef\,x^6+\frac{1}{2}x^2e^2\right)b$	27
parallelrisch	$\left(\frac{1}{10}f^2x^{10}+\frac{1}{3}ef\,x^6+\frac{1}{2}x^2e^2\right)b$	27
gosper	$\frac{x^2(3f^2x^8+10ef\,x^4+15e^2)b}{30}$	28
norman	$\frac{1}{2}b\,e^2x^2+\frac{1}{3}bef\,x^6+\frac{1}{10}b\,f^2x^{10}$	28
risch	$\frac{1}{2}b\,e^2x^2+\frac{1}{3}bef\,x^6+\frac{1}{10}b\,f^2x^{10}$	28

```
[In] int(b*x*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
[Out] (1/10*f^2*x^10+1/3*e*f*x^6+1/2*x^2*e^2)*b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int bx(e+fx^4)^2\,dx = \frac{1}{10}\,bf^2x^{10} + \frac{1}{3}\,befx^6 + \frac{1}{2}\,be^2x^2$$

```
[In] integrate(b*x*(f*x^4+e)^2,x, algorithm="fricas")
[Out] 1/10*b*f^2*x^10 + 1/3*b*e*f*x^6 + 1/2*b*e^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int bx(e+fx^4)^2\,dx = \frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10}$$

```
[In] integrate(b*x*(f*x**4+e)**2,x)
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int bx(e+fx^4)^2\,dx = \frac{1}{30}\,(3\,f^2x^{10} + 10\,efx^6 + 15\,e^2x^2)b$$

```
[In] integrate(b*x*(f*x^4+e)^2,x, algorithm="maxima")
[Out] 1/30*(3*f^2*x^10 + 10*e*f*x^6 + 15*e^2*x^2)*b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int bx(e+fx^4)^2\,dx = \frac{1}{30}\,(3\,f^2x^{10} + 10\,efx^6 + 15\,e^2x^2)b$$

```
[In] integrate(b*x*(f*x^4+e)^2,x, algorithm="giac")
[Out] 1/30*(3*f^2*x^10 + 10*e*f*x^6 + 15*e^2*x^2)*b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int bx(e+fx^4)^2 \, dx = \frac{b\,x^2\,(15\,e^2+10\,e\,f\,x^4+3\,f^2\,x^8)}{30}$$

```
[In] int(b*x*(e + f*x^4)^2,x)
[Out] (b*x^2*(15*e^2 + 3*f^2*x^8 + 10*e*f*x^4))/30
```

3.135 $\int (a+bx)\left(e+fx^4\right)^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int (a+bx)\left(e+fx^4\right)^2\,dx=ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}$$

```
[Out] a*e^2*x+1/2*b*e^2*x^2+2/5*a*e*f*x^5+1/3*b*e*f*x^6+1/9*a*f^2*x^9+1/10*b*f^2*
x^10
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1864}

$$\int (a+bx)\left(e+fx^4\right)^2\,dx=ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

```
[In] Int[(a + b*x)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 +
 (b*f^2*x^10)/10
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(ae^2+be^2x+2aefx^4+2befx^5+af^2x^8+bf^2x^9\right)\,dx\\&=ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (a+bx)\left(e+fx^4\right)^2\,dx=ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}$$

```
[In] Integrate[(a + b*x)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 +
 (b*f^2*x^10)/10
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
gosper	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}$	51
default	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}$	51
norman	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}$	51
risch	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}$	51
parallelrisch	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}$	51

```
[In] int((b*x+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*e^2*x+1/2*b*e^2*x^2+2/5*a*e*f*x^5+1/3*b*e*f*x^6+1/9*a*f^2*x^9+1/10*b*f^2*
x^10
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx)\left(e+fx^4\right)^2\,dx=\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{2}\,be^2x^2+ae^2x$$

```
[In] integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/2*b*e^2
*x^2 + a*e^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int (a+bx) \left(e+fx^4\right)^2 \, dx = ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10}$$

```
[In] integrate((b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
b*f**2*x**10/10
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx) \left(e+fx^4\right)^2 \, dx = \frac{1}{10} bf^2x^{10} + \frac{1}{9} af^2x^9 + \frac{1}{3} befx^6 + \frac{2}{5} aefx^5 + \frac{1}{2} be^2x^2 + ae^2x$$

```
[In] integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/2*b*e^2
*x^2 + a*e^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx) \left(e+fx^4\right)^2 \, dx = \frac{1}{10} bf^2x^{10} + \frac{1}{9} af^2x^9 + \frac{1}{3} befx^6 + \frac{2}{5} aefx^5 + \frac{1}{2} be^2x^2 + ae^2x$$

```
[In] integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/2*b*e^2
*x^2 + a*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx)\left(e+fx^4\right)^2\,dx=\frac{b\,e^2\,x^2}{2}+a\,e^2\,x+\frac{b\,e\,f\,x^6}{3}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{b\,f^2\,x^{10}}{10}+\frac{a\,f^2\,x^9}{9}$$

```
[In] int((e + f*x^4)^2*(a + b*x),x)
[Out] (b*e^2*x^2)/2 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + a*e^2*x + (2*a*e*f*x^5)/5
 + (b*e*f*x^6)/3
```

3.136 $\int cx^2(e+fx^4)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int cx^2\left(e+fx^4\right)^2\,dx=\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

```
[Out] 1/3*c*e^2*x^3+2/7*c*e*f*x^7+1/11*c*f^2*x^11
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 276}

$$\int cx^2\left(e+fx^4\right)^2\,dx=\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

```
[In] Int[c*x^2*(e + f*x^4)^2,x]
```

```
[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= c\int x^2\left(e+fx^4\right)^2\,dx\\ &= c\int \left(e^2x^2+2efx^6+f^2x^{10}\right)\,dx\\ &= \frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int cx^2\left(e+fx^4\right)^2\,dx=\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

```
[In] Integrate[c*x^2*(e + f*x^4)^2,x]
[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
default	$\left(\frac{1}{11}f^2x^{11}+\frac{2}{7}ef\,x^7+\frac{1}{3}e^2x^3\right)c$	27
parallelrisch	$\left(\frac{1}{11}f^2x^{11}+\frac{2}{7}ef\,x^7+\frac{1}{3}e^2x^3\right)c$	27
gosper	$\frac{x^3\left(21f^2x^8+66ef\,x^4+77e^2\right)c}{231}$	28
norman	$\frac{1}{3}c\,e^2x^3+\frac{2}{7}cef\,x^7+\frac{1}{11}c\,f^2x^{11}$	28
risch	$\frac{1}{3}c\,e^2x^3+\frac{2}{7}cef\,x^7+\frac{1}{11}c\,f^2x^{11}$	28

```
[In] int(c*x^2*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
[Out] (1/11*f^2*x^11+2/7*e*f*x^7+1/3*e^2*x^3)*c
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int cx^2(e+fx^4)^2\,dx = \frac{1}{11}\,cf^2x^{11} + \frac{2}{7}\,cefx^7 + \frac{1}{3}\,ce^2x^3$$

```
[In] integrate(c*x^2*(f*x^4+e)^2,x, algorithm="fricas")
[Out] 1/11*c*f^2*x^11 + 2/7*c*e*f*x^7 + 1/3*c*e^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int cx^2(e+fx^4)^2\,dx = \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11}$$

```
[In] integrate(c*x**2*(f*x**4+e)**2,x)
[Out] c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int cx^2(e+fx^4)^2\,dx = \frac{1}{231}\,\left(21\,f^2x^{11} + 66\,efx^7 + 77\,e^2x^3\right)c$$

```
[In] integrate(c*x^2*(f*x^4+e)^2,x, algorithm="maxima")
[Out] 1/231*(21*f^2*x^11 + 66*e*f*x^7 + 77*e^2*x^3)*c
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int cx^2(e+fx^4)^2\,dx = \frac{1}{231}\,\left(21\,f^2x^{11} + 66\,efx^7 + 77\,e^2x^3\right)c$$

```
[In] integrate(c*x^2*(f*x^4+e)^2,x, algorithm="giac")
[Out] 1/231*(21*f^2*x^11 + 66*e*f*x^7 + 77*e^2*x^3)*c
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int cx^2 \left(e+fx^4\right)^2 \, dx = \frac{c\, x^3\, \left(77\, e^2+66\, e\, f\, x^4+21\, f^2\, x^8\right)}{231}$$

```
[In] int(c*x^2*(e + f*x^4)^2,x)
[Out] (c*x^3*(77*e^2 + 21*f^2*x^8 + 66*e*f*x^4))/231
```

3.137 $\int (a+cx^2)(e+fx^4)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int (a+cx^2)(e+fx^4)^2\,dx=ae^2x+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}$$

```
[Out] a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1/11*c*f^2*
x^11
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1168}

$$\int (a+cx^2)(e+fx^4)^2\,dx=ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

```
[In] Int[(a + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9
 + (c*f^2*x^11)/11
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(ae^2+ce^2x^2+2aefx^4+2cefx^6+af^2x^8+cf^2x^{10}\right)dx\\&=ae^2x+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (a+cx^2)(e+fx^4)^2\,dx=ae^2x+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}$$

```
[In] Integrate[(a + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9
 + (c*f^2*x^11)/11
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
gosper	$a\,e^2x+\frac{1}{3}c\,e^2x^3+\frac{2}{5}aef\,x^5+\frac{2}{7}cef\,x^7+\frac{1}{9}a\,f^2x^9+\frac{1}{11}c\,f^2x^{11}$	51
default	$a\,e^2x+\frac{1}{3}c\,e^2x^3+\frac{2}{5}aef\,x^5+\frac{2}{7}cef\,x^7+\frac{1}{9}a\,f^2x^9+\frac{1}{11}c\,f^2x^{11}$	51
norman	$a\,e^2x+\frac{1}{3}c\,e^2x^3+\frac{2}{5}aef\,x^5+\frac{2}{7}cef\,x^7+\frac{1}{9}a\,f^2x^9+\frac{1}{11}c\,f^2x^{11}$	51
risch	$a\,e^2x+\frac{1}{3}c\,e^2x^3+\frac{2}{5}aef\,x^5+\frac{2}{7}cef\,x^7+\frac{1}{9}a\,f^2x^9+\frac{1}{11}c\,f^2x^{11}$	51
parallelrisch	$a\,e^2x+\frac{1}{3}c\,e^2x^3+\frac{2}{5}aef\,x^5+\frac{2}{7}cef\,x^7+\frac{1}{9}a\,f^2x^9+\frac{1}{11}c\,f^2x^{11}$	51

```
[In] int((c*x^2+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1/11*c*f^2*
x^11
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+cx^2)(e+fx^4)^2\,dx=\frac{1}{11}cf^2x^{11}+\frac{1}{9}af^2x^9+\frac{2}{7}cefx^7+\frac{2}{5}aefx^5+\frac{1}{3}ce^2x^3+ae^2x$$

```
[In] integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 2/5*a*e*f*x^5 + 1/3*c*e^2
*x^3 + a*e^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \left(a+cx^2\right)\left(e+fx^4\right)^2\,dx = ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11}$$

```
[In] integrate((c*x**2+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e*f*x**7/7
+ c*f**2*x**11/11
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+cx^2\right)\left(e+fx^4\right)^2\,dx = \frac{1}{11}\,cf^2x^{11} + \frac{1}{9}\,af^2x^9 + \frac{2}{7}\,cefx^7 + \frac{2}{5}\,aefx^5 + \frac{1}{3}\,ce^2x^3 + ae^2x$$

```
[In] integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 2/5*a*e*f*x^5 + 1/3*c*e^2
*x^3 + a*e^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+cx^2\right)\left(e+fx^4\right)^2\,dx = \frac{1}{11}\,cf^2x^{11} + \frac{1}{9}\,af^2x^9 + \frac{2}{7}\,cefx^7 + \frac{2}{5}\,aefx^5 + \frac{1}{3}\,ce^2x^3 + ae^2x$$

```
[In] integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 2/5*a*e*f*x^5 + 1/3*c*e^2
*x^3 + a*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+cx^2\right)\left(e+fx^4\right)^2\,dx=\frac{c\,e^2\,x^3}{3}+a\,e^2\,x+\frac{2\,c\,e\,f\,x^7}{7}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{c\,f^2\,x^{11}}{11}+\frac{a\,f^2\,x^9}{9}$$

```
[In] int((a + c*x^2)*(e + f*x^4)^2,x)
[Out] (a*f^2*x^9)/9 + (c*e^2*x^3)/3 + (c*f^2*x^11)/11 + a*e^2*x + (2*a*e*f*x^5)/5
 + (2*c*e*f*x^7)/7
```

3.138 $\int (bx+cx^2)(e+fx^4)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int (bx+cx^2)(e+fx^4)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$$

```
[Out] 1/2*b*e^2*x^2+1/3*c*e^2*x^3+1/3*b*e*f*x^6+2/7*c*e*f*x^7+1/10*b*f^2*x^10+1/1
1*c*f^2*x^11
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1607, 1634}

$$\int (bx+cx^2)(e+fx^4)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

```
[In] Int[(b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^
10)/10 + (c*f^2*x^11)/11
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
```

```
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int x(b+cx)\left(e+fx^4\right)^2 \, dx \\ &= \int \left(be^2x+ce^2x^2+2befx^5+2cefx^6+bf^2x^9+cf^2x^{10}\right) dx \\ &= \frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \left(bx+cx^2\right)\left(e+fx^4\right)^2 \, dx = \frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$$

```
[In] Integrate[(b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^
10)/10 + (c*f^2*x^11)/11
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^2\left(210cf^2x^9+231bf^2x^8+660cefx^5+770befx^4+770ce^2x+1155e^2b\right)}{2310}$	54
default	$\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$	54
norman	$\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$	54
risch	$\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$	54
parallelrisch	$\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$	54

```
[In] int((c*x^2+b*x)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2310*x^2*(210*c*f^2*x^9+231*b*f^2*x^8+660*c*e*f*x^5+770*b*e*f*x^4+770*c*e
^2*x+1155*b*e^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \left(bx+cx^2\right)\left(e+fx^4\right)^2\,dx=\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{2}{7}\,cefx^7+\frac{1}{3}\,befx^6+\frac{1}{3}\,ce^2x^3+\frac{1}{2}\,be^2x^2$$

```
[In] integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/3*c*e
^2*x^3 + 1/2*b*e^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \left(bx+cx^2\right)\left(e+fx^4\right)^2\,dx=\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}$$

```
[In] integrate((c*x**2+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x*
*7/7 + c*f**2*x**11/11
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \left(bx+cx^2\right)\left(e+fx^4\right)^2\,dx=\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{2}{7}\,cefx^7+\frac{1}{3}\,befx^6+\frac{1}{3}\,ce^2x^3+\frac{1}{2}\,be^2x^2$$

```
[In] integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/3*c*e
^2*x^3 + 1/2*b*e^2*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (bx+cx^2) (e+fx^4)^2 \, dx = \frac{1}{11} cf^2x^{11} + \frac{1}{10} bf^2x^{10} + \frac{2}{7} cefx^7 + \frac{1}{3} befx^6 + \frac{1}{3} ce^2x^3 + \frac{1}{2} be^2x^2$$

```
[In] integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/3*c*e
^2*x^3 + 1/2*b*e^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (bx+cx^2) (e+fx^4)^2 \, dx = \frac{c\,e^2\,x^3}{3} + \frac{b\,e^2\,x^2}{2} + \frac{2\,c\,e\,f\,x^7}{7} + \frac{b\,e\,f\,x^6}{3} + \frac{c\,f^2\,x^{11}}{11} + \frac{b\,f^2\,x^{10}}{10}$$

```
[In] int((b*x + c*x^2)*(e + f*x^4)^2,x)
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11 + (b*e*f*
x^6)/3 + (2*c*e*f*x^7)/7
```

3.139 $\int (a+bx+cx^2)(e+fx^4)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int (a+bx+cx^2)(e+fx^4)^2 \, dx = ae^2x+\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$$

```
[Out] a*e^2*x+1/2*b*e^2*x^2+1/3*c*e^2*x^3+2/5*a*e*f*x^5+1/3*b*e*f*x^6+2/7*c*e*f*x
^7+1/9*a*f^2*x^9+1/10*b*f^2*x^10+1/11*c*f^2*x^11
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1671}

$$\int (a+bx+cx^2)(e+fx^4)^2 \, dx = ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

```
[In] Int[(a + b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 +
 (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(ae^2+be^2x+ce^2x^2+2aefx^4+2befx^5+2cefx^6+af^2x^8+bf^2x^9+cf^2x^{10}\right)\,dx$$
$$=ae^2x+\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \left(a+bx+cx^2\right)\left(e+fx^4\right)^2\,dx = ae^2x+\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{1}{3}befx^6 +\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$$

```
[In] Integrate[(a + b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 +
 (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

method	result
gosper	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{1}{3}c\,e^2x^3+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{2}{7}cef\,x^7+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}+\frac{1}{11}c\,f^2x^{11}$
default	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{1}{3}c\,e^2x^3+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{2}{7}cef\,x^7+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}+\frac{1}{11}c\,f^2x^{11}$
norman	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{1}{3}c\,e^2x^3+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{2}{7}cef\,x^7+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}+\frac{1}{11}c\,f^2x^{11}$
risch	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{1}{3}c\,e^2x^3+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{2}{7}cef\,x^7+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}+\frac{1}{11}c\,f^2x^{11}$
parallelrisch	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{1}{3}c\,e^2x^3+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{2}{7}cef\,x^7+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}+\frac{1}{11}c\,f^2x^{11}$

```
[In] int((c*x^2+b*x+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*e^2*x+1/2*b*e^2*x^2+1/3*c*e^2*x^3+2/5*a*e*f*x^5+1/3*b*e*f*x^6+2/7*c*e*f*x
^7+1/9*a*f^2*x^9+1/10*b*f^2*x^10+1/11*c*f^2*x^11
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int (a+bx+cx^2)\left(e+fx^4\right)^2\,dx = \frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cefx^7+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{3}\,ce^2x^3+\frac{1}{2}\,be^2x^2+ae^2x$$

[In] integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")

[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2 + a*e^2*x

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

$$\int (a+bx+cx^2)\left(e+fx^4\right)^2\,dx = ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}$$

[In] integrate((c*x**2+b*x+a)*(f*x**4+e)**2,x)

[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int (a+bx+cx^2)\left(e+fx^4\right)^2\,dx = \frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cefx^7+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{3}\,ce^2x^3+\frac{1}{2}\,be^2x^2+ae^2x$$

[In] integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")

[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2 + a*e^2*x

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int (a + bx + cx^2) (e + fx^4)^2 \, dx = \frac{1}{11} cf^2x^{11} + \frac{1}{10} bf^2x^{10} + \frac{1}{9} af^2x^9 + \frac{2}{7} cefx^7 + \frac{1}{3} befx^6 + \frac{2}{5} aefx^5 + \frac{1}{3} ce^2x^3 + \frac{1}{2} be^2x^2 + ae^2x$$

```
[In] integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 1/3*b*e
*f*x^6 + 2/5*a*e*f*x^5 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2 + a*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int (a + bx + cx^2) (e + fx^4)^2 \, dx = \frac{c\,e^2\,x^3}{3} + \frac{b\,e^2\,x^2}{2} + a\,e^2\,x + \frac{2\,c\,e\,f\,x^7}{7} + \frac{b\,e\,f\,x^6}{3} + \frac{2\,a\,e\,f\,x^5}{5} + \frac{c\,f^2\,x^{11}}{11} + \frac{b\,f^2\,x^{10}}{10} + \frac{a\,f^2\,x^9}{9}$$

```
[In] int((e + f*x^4)^2*(a + b*x + c*x^2),x)
```

```
[Out] (b*e^2*x^2)/2 + (a*f^2*x^9)/9 + (c*e^2*x^3)/3 + (b*f^2*x^10)/10 + (c*f^2*x^
11)/11 + a*e^2*x + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7
```

3.140 $\int dx^3(e+fx^4)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 17

$$\int dx^3\left(e+fx^4\right)^2\,dx=\frac{d(e+fx^4)^3}{12f}$$

```
[Out] 1/12*d*(f*x^4+e)^3/f
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 267}

$$\int dx^3\left(e+fx^4\right)^2\,dx=\frac{d(e+fx^4)^3}{12f}$$

```
[In] Int[d*x^3*(e + f*x^4)^2,x]
```

```
[Out] (d*(e + f*x^4)^3)/(12*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = d\int x^3\left(e+fx^4\right)^2\,dx$$
$$= \frac{d(e+fx^4)^3}{12f}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int dx^3\left(e+fx^4\right)^2\,dx = \frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

```
[In] Integrate[d*x^3*(e + f*x^4)^2,x]
[Out] (d*e^2*x^4)/4 + (d*e*f*x^8)/4 + (d*f^2*x^12)/12
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{d\left(f\,x^{4}+e\right)^{3}}{12f}$	16
gosper	$\frac{x^{4}\left(f^{2}x^{8}+3ef\,x^{4}+3e^{2}\right)d}{12}$	27
parallelrisch	$\left(\frac{1}{12}f^{2}x^{12}+\frac{1}{4}ef\,x^{8}+\frac{1}{4}e^{2}x^{4}\right)d$	27
norman	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{4}d\,x^{4}e^{2}+\frac{1}{4}def\,x^{8}$	28
risch	$\frac{d\,f^{2}x^{12}}{12}+\frac{def\,x^{8}}{4}+\frac{d\,x^{4}e^{2}}{4}+\frac{d\,e^{3}}{12f}$	37

```
[In] int(d*x^3*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
[Out] 1/12*d*(f*x^4+e)^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int dx^3 (e + fx^4)^2 \, dx = \frac{1}{12} df^2 x^{12} + \frac{1}{4} defx^8 + \frac{1}{4} de^2 x^4$$

```
[In] integrate(d*x^3*(f*x^4+e)^2,x, algorithm="fricas")
[Out] 1/12*d*f^2*x^12 + 1/4*d*e*f*x^8 + 1/4*d*e^2*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int dx^3 (e + fx^4)^2 \, dx = \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

```
[In] integrate(d*x**3*(f*x**4+e)**2,x)
[Out] d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int dx^3 (e + fx^4)^2 \, dx = \frac{(fx^4 + e)^3 d}{12 f}$$

```
[In] integrate(d*x^3*(f*x^4+e)^2,x, algorithm="maxima")
[Out] 1/12*(f*x^4 + e)^3*d/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int dx^3 \left(e+fx^4\right)^2 \, dx = \frac{\left(fx^4+e\right)^3 d}{12 \, f}$$

```
[In] integrate(d*x^3*(f*x^4+e)^2,x, algorithm="giac")
[Out] 1/12*(f*x^4 + e)^3*d/f
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int dx^3 \left(e+fx^4\right)^2 \, dx = \frac{d \, x^4 \, \left(3 \, e^2 + 3 \, e \, f \, x^4 + f^2 \, x^8\right)}{12}$$

```
[In] int(d*x^3*(e + f*x^4)^2,x)
[Out] (d*x^4*(3*e^2 + f^2*x^8 + 3*e*f*x^4))/12
```

3.141 $\int (a+dx^3)(e+fx^4)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int (a+dx^3)(e+fx^4)^2 \, dx = ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{d(e+fx^4)^3}{12f}$$

```
[Out] a*e^2*x+2/5*a*e*f*x^5+1/9*a*f^2*x^9+1/12*d*(f*x^4+e)^3/f
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1596, 12, 200}

$$\int (a+dx^3)(e+fx^4)^2 \, dx = ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{d(e+fx^4)^3}{12f}$$

```
[In] Int[(a + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{d(e+fx^4)^3}{12f} + \int a\left(e+fx^4\right)^2\,dx \\ &= \frac{d(e+fx^4)^3}{12f} + a\int \left(e+fx^4\right)^2\,dx \\ &= \frac{d(e+fx^4)^3}{12f} + a\int \left(e^2+2efx^4+f^2x^8\right)\,dx \\ &= ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{d(e+fx^4)^3}{12f} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \left(a+dx^3\right)\left(e+fx^4\right)^2\,dx = ae^2x + \frac{1}{4}de^2x^4 + \frac{2}{5}aefx^5 + \frac{1}{4}defx^8 + \frac{1}{9}af^2x^9 + \frac{1}{12}df^2x^{12}$$

```
[In] Integrate[(a + d*x^3)*(e + f*x^4)^2,x]
[Out] a*e^2*x + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9 +
 (d*f^2*x^12)/12
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

method	result	size
gosper	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{9}a\,f^{2}x^{9}+\frac{1}{4}def\,x^{8}+\frac{2}{5}aef\,x^{5}+\frac{1}{4}d\,x^{4}e^{2}+a\,e^{2}x$	51
default	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{9}a\,f^{2}x^{9}+\frac{1}{4}def\,x^{8}+\frac{2}{5}aef\,x^{5}+\frac{1}{4}d\,x^{4}e^{2}+a\,e^{2}x$	51
norman	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{9}a\,f^{2}x^{9}+\frac{1}{4}def\,x^{8}+\frac{2}{5}aef\,x^{5}+\frac{1}{4}d\,x^{4}e^{2}+a\,e^{2}x$	51
risch	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{9}a\,f^{2}x^{9}+\frac{1}{4}def\,x^{8}+\frac{2}{5}aef\,x^{5}+\frac{1}{4}d\,x^{4}e^{2}+a\,e^{2}x$	51
parallelrisch	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{9}a\,f^{2}x^{9}+\frac{1}{4}def\,x^{8}+\frac{2}{5}aef\,x^{5}+\frac{1}{4}d\,x^{4}e^{2}+a\,e^{2}x$	51

```
[In] int((d*x^3+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*d*f^2*x^12+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/5*a*e*f*x^5+1/4*d*x^4*e^2+a*e
^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \left(a+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+ae^2x$$

```
[In] integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/5*a*e*f*x^5 + 1/4*d*e^2
*x^4 + a*e^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \left(a+dx^3\right)\left(e+fx^4\right)^2\,dx=ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

```
[In] integrate((d*x**3+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + d*e**2*x**4/4 + d*e*f*x**8/4 +
d*f**2*x**12/12
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (a+dx^3)(e+fx^4)^2 \, dx = \frac{1}{12}df^2x^{12} + \frac{1}{9}af^2x^9 + \frac{1}{4}defx^8 + \frac{2}{5}aefx^5 + \frac{1}{4}de^2x^4 + ae^2x$$

```
[In] integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/5*a*e*f*x^5 + 1/4*d*e^2
*x^4 + a*e^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (a+dx^3)(e+fx^4)^2 \, dx = \frac{1}{12}df^2x^{12} + \frac{1}{9}af^2x^9 + \frac{1}{4}defx^8 + \frac{2}{5}aefx^5 + \frac{1}{4}de^2x^4 + ae^2x$$

```
[In] integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/5*a*e*f*x^5 + 1/4*d*e^2
*x^4 + a*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (a+dx^3)(e+fx^4)^2 \, dx = \frac{d\,e^2\,x^4}{4} + a\,e^2\,x + \frac{d\,e\,f\,x^8}{4} + \frac{2\,a\,e\,f\,x^5}{5} + \frac{d\,f^2\,x^{12}}{12} + \frac{a\,f^2\,x^9}{9}$$

```
[In] int((a + d*x^3)*(e + f*x^4)^2,x)
```

```
[Out] (a*f^2*x^9)/9 + (d*e^2*x^4)/4 + (d*f^2*x^12)/12 + a*e^2*x + (2*a*e*f*x^5)/5
 + (d*e*f*x^8)/4
```

3.142 $\int (bx+dx^3)(e+fx^4)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int (bx+dx^3)\left(e+fx^4\right)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)^3}{12f}$$

```
[Out] 1/2*b*e^2*x^2+1/3*b*e*f*x^6+1/10*b*f^2*x^10+1/12*d*(f*x^4+e)^3/f
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1596, 12, 276}

$$\int (bx+dx^3)\left(e+fx^4\right)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)^3}{12f}$$

```
[In] Int[(b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d(e+fx^4)^3}{12f} + \int bx\left(e+fx^4\right)^2\,dx\\ &= \frac{d(e+fx^4)^3}{12f} + b\int x\left(e+fx^4\right)^2\,dx\\ &= \frac{d(e+fx^4)^3}{12f} + b\int \left(e^2x+2efx^5+f^2x^9\right)\,dx\\ &= \frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)^3}{12f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \left(bx+dx^3\right)\left(e+fx^4\right)^2\,dx = \frac{1}{2}be^2x^2+\frac{1}{4}de^2x^4+\frac{1}{3}befx^6+\frac{1}{4}defx^8+\frac{1}{10}bf^2x^{10}+\frac{1}{12}df^2x^{12}$$

```
[In] Integrate[(b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (d*e*f*x^8)/4 + (b*f^2*x^10
)/10 + (d*f^2*x^12)/12
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2$	54
norman	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2$	54
risch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2$	54
parallelrisch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2$	54
gosper	$\frac{x^2(5d\,f^2x^{10}+6b\,f^2x^8+15def\,x^6+20bef\,x^4+15d\,e^2x^2+30e^2b)}{60}$	56

```
[In] int((d*x^3+b*x)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/4*d*e*f*x^8+1/3*b*e*f*x^6+1/4*d*x^4*e^2+1
/2*b*e^2*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (bx+dx^3)(e+fx^4)^2\,dx=\frac{1}{12}df^2x^{12}+\frac{1}{10}bf^2x^{10}+\frac{1}{4}defx^8+\frac{1}{3}befx^6+\frac{1}{4}de^2x^4+\frac{1}{2}be^2x^2$$

```
[In] integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 1/3*b*e*f*x^6 + 1/4*d*e
^2*x^4 + 1/2*b*e^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int (bx+dx^3)(e+fx^4)^2\,dx=\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

```
[In] integrate((d*x**3+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + d*e**2*x**4/4 + d*e*f*x**8
/4 + d*f**2*x**12/12
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (bx+dx^3)\left(e+fx^4\right)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{4}\,defx^8+\frac{1}{3}\,befx^6+\frac{1}{4}\,de^2x^4+\frac{1}{2}\,be^2x^2$$

```
[In] integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 1/3*b*e*f*x^6 + 1/4*d*e
^2*x^4 + 1/2*b*e^2*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (bx+dx^3)\left(e+fx^4\right)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{4}\,defx^8+\frac{1}{3}\,befx^6+\frac{1}{4}\,de^2x^4+\frac{1}{2}\,be^2x^2$$

```
[In] integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 1/3*b*e*f*x^6 + 1/4*d*e
^2*x^4 + 1/2*b*e^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (bx+dx^3)\left(e+fx^4\right)^2\,dx=\frac{d\,e^2\,x^4}{4}+\frac{b\,e^2\,x^2}{2}+\frac{d\,e\,f\,x^8}{4}+\frac{b\,e\,f\,x^6}{3}+\frac{d\,f^2\,x^{12}}{12}+\frac{b\,f^2\,x^{10}}{10}$$

```
[In] int((b*x + d*x^3)*(e + f*x^4)^2,x)
```

```
[Out] (b*e^2*x^2)/2 + (b*f^2*x^10)/10 + (d*e^2*x^4)/4 + (d*f^2*x^12)/12 + (b*e*f*
x^6)/3 + (d*e*f*x^8)/4
```

3.143 $\int (a+bx+dx^3)(e+fx^4)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 77

$$\int (a+bx+dx^3)(e+fx^4)^2\,dx = ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)^3}{12f}$$

```
[Out] a*e^2*x+1/2*b*e^2*x^2+2/5*a*e*f*x^5+1/3*b*e*f*x^6+1/9*a*f^2*x^9+1/10*b*f^2*
x^10+1/12*d*(f*x^4+e)^3/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1596, 1864}

$$\int (a+bx+dx^3)(e+fx^4)^2\,dx = ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)^3}{12f}$$

```
[In] Int[(a + b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 +
 (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
```

```
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(e+fx^4)^3}{12f} + \int (a+bx)\left(e+fx^4\right)^2\,dx \\
&= \frac{d(e+fx^4)^3}{12f} + \int \left(ae^2+be^2x+2aefx^4+2befx^5+af^2x^8+bf^2x^9\right)\,dx \\
&= ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)^3}{12f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\begin{aligned}
\int \left(a+bx+dx^3\right)\left(e+fx^4\right)^2\,dx &= ae^2x+\frac{1}{2}be^2x^2+\frac{1}{4}de^2x^4+\frac{2}{5}aefx^5+\frac{1}{3}befx^6 \\
&\quad +\frac{1}{4}defx^8+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{1}{12}df^2x^{12}
\end{aligned}$$

```
[In] Integrate[(a + b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 +
 (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (d*f^2*x^12)/12
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result
gosper	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2+a\,e^2x$
default	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2+a\,e^2x$
norman	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2+a\,e^2x$
risch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2+a\,e^2x$
parallelrisch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2+a\,e^2x$

```
[In] int((d*x^3+b*x+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/9*a*f^2*x^9+1/4*d*e*f*x^8+1/3*b*e*f*x^6+2
/5*a*e*f*x^5+1/4*d*x^4*e^2+1/2*b*e^2*x^2+a*e^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a+bx+dx^3)\,(e+fx^4)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+\frac{1}{2}\,be^2x^2+ae^2x$$

```
[In] integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 1/3*b*e
*f*x^6 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/2*b*e^2*x^2 + a*e^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int (a+bx+dx^3)\,(e+fx^4)^2\,dx=ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

```
[In] integrate((d*x**3+b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
b*f**2*x**10/10 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a+bx+dx^3)(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12} + \frac{1}{10}bf^2x^{10} + \frac{1}{9}af^2x^9 + \frac{1}{4}defx^8 + \frac{1}{3}befx^6 + \frac{2}{5}aefx^5 + \frac{1}{4}de^2x^4 + \frac{1}{2}be^2x^2 + ae^2x$$

```
[In] integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 1/3*b*e
*f*x^6 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/2*b*e^2*x^2 + a*e^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a+bx+dx^3)(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12} + \frac{1}{10}bf^2x^{10} + \frac{1}{9}af^2x^9 + \frac{1}{4}defx^8 + \frac{1}{3}befx^6 + \frac{2}{5}aefx^5 + \frac{1}{4}de^2x^4 + \frac{1}{2}be^2x^2 + ae^2x$$

```
[In] integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 1/3*b*e
*f*x^6 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/2*b*e^2*x^2 + a*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a+bx+dx^3)(e+fx^4)^2\,dx = \frac{d\,e^2\,x^4}{4} + \frac{b\,e^2\,x^2}{2} + a\,e^2\,x + \frac{d\,e\,f\,x^8}{4} + \frac{b\,e\,f\,x^6}{3} + \frac{2\,a\,e\,f\,x^5}{5} + \frac{d\,f^2\,x^{12}}{12} + \frac{b\,f^2\,x^{10}}{10} + \frac{a\,f^2\,x^9}{9}$$

```
[In] int((e + f*x^4)^2*(a + b*x + d*x^3),x)
```

```
[Out] (b*e^2*x^2)/2 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (d*e^2*x^4)/4 + (d*f^2*x^
12)/12 + a*e^2*x + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (d*e*f*x^8)/4
```

3.144 $\int (cx^2+dx^3)(e+fx^4)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int (cx^2+dx^3)(e+fx^4)^2\,dx=\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

```
[Out] 1/3*c*e^2*x^3+2/7*c*e*f*x^7+1/11*c*f^2*x^11+1/12*d*(f*x^4+e)^3/f
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1596, 12, 276}

$$\int (cx^2+dx^3)(e+fx^4)^2\,dx=\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

```
[In] Int[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(e+fx^4)^3}{12f} + \int cx^2\left(e+fx^4\right)^2\,dx \\
&= \frac{d(e+fx^4)^3}{12f} + c\int x^2\left(e+fx^4\right)^2\,dx \\
&= \frac{d(e+fx^4)^3}{12f} + c\int \left(e^2x^2+2efx^6+f^2x^{10}\right)\,dx \\
&= \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11} + \frac{d(e+fx^4)^3}{12f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \left(cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx = \frac{1}{3}ce^2x^3 + \frac{1}{4}de^2x^4 + \frac{2}{7}cefx^7 + \frac{1}{4}defx^8 + \frac{1}{11}cf^2x^{11} + \frac{1}{12}df^2x^{12}$$

```
[In] Integrate[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
[Out] (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4 + (c*f^2*x^
11)/11 + (d*f^2*x^12)/12
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{x^{3}\left(77d\,f^{2}x^{9}+84c\,f^{2}x^{8}+231def\,x^{5}+264cef\,x^{4}+231d\,e^{2}x+308c\,e^{2}\right)}{924}$	54
default	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{11}c\,f^{2}x^{11}+\frac{1}{4}def\,x^{8}+\frac{2}{7}cef\,x^{7}+\frac{1}{4}d\,x^{4}e^{2}+\frac{1}{3}c\,e^{2}x^{3}$	54
norman	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{11}c\,f^{2}x^{11}+\frac{1}{4}def\,x^{8}+\frac{2}{7}cef\,x^{7}+\frac{1}{4}d\,x^{4}e^{2}+\frac{1}{3}c\,e^{2}x^{3}$	54
risch	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{11}c\,f^{2}x^{11}+\frac{1}{4}def\,x^{8}+\frac{2}{7}cef\,x^{7}+\frac{1}{4}d\,x^{4}e^{2}+\frac{1}{3}c\,e^{2}x^{3}$	54
parallelrisch	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{11}c\,f^{2}x^{11}+\frac{1}{4}def\,x^{8}+\frac{2}{7}cef\,x^{7}+\frac{1}{4}d\,x^{4}e^{2}+\frac{1}{3}c\,e^{2}x^{3}$	54

```
[In] int((d*x^3+c*x^2)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/924*x^3*(77*d*f^2*x^9+84*c*f^2*x^8+231*d*e*f*x^5+264*c*e*f*x^4+231*d*e^2*
x+308*c*e^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (cx^2+dx^3)\left(e+fx^4\right)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3$$

```
[In] integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*e*f*x^8 + 2/7*c*e*f*x^7 + 1/4*d*e
^2*x^4 + 1/3*c*e^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int (cx^2+dx^3)\left(e+fx^4\right)^2\,dx=\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

```
[In] integrate((d*x**3+c*x**2)*(f*x**4+e)**2,x)
```

```
[Out] c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x*
*8/4 + d*f**2*x**12/12
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (cx^2+dx^3)(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12}+\frac{1}{11}cf^2x^{11}+\frac{1}{4}defx^8+\frac{2}{7}cefx^7+\frac{1}{4}de^2x^4+\frac{1}{3}ce^2x^3$$

```
[In] integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*e*f*x^8 + 2/7*c*e*f*x^7 + 1/4*d*e
^2*x^4 + 1/3*c*e^2*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (cx^2+dx^3)(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12}+\frac{1}{11}cf^2x^{11}+\frac{1}{4}defx^8+\frac{2}{7}cefx^7+\frac{1}{4}de^2x^4+\frac{1}{3}ce^2x^3$$

```
[In] integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*e*f*x^8 + 2/7*c*e*f*x^7 + 1/4*d*e
^2*x^4 + 1/3*c*e^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (cx^2+dx^3)(e+fx^4)^2\,dx = \frac{d\,e^2\,x^4}{4}+\frac{c\,e^2\,x^3}{3}+\frac{d\,e\,f\,x^8}{4}+\frac{2\,c\,e\,f\,x^7}{7}+\frac{d\,f^2\,x^{12}}{12}+\frac{c\,f^2\,x^{11}}{11}$$

```
[In] int((e + f*x^4)^2*(c*x^2 + d*x^3),x)
```

```
[Out] (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12 + (2*c*e*
f*x^7)/7 + (d*e*f*x^8)/4
```

3.145 $\int (a+cx^2+dx^3)(e+fx^4)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int (a+cx^2+dx^3)(e+fx^4)^2\,dx = ae^2x+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

```
[Out] a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1/11*c*f^2*
x^11+1/12*d*(f*x^4+e)^3/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1596, 1168}

$$\int (a+cx^2+dx^3)(e+fx^4)^2\,dx = ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

```
[In] Int[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9
 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
```

```
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(e+fx^4)^3}{12f} + \int (a+cx^2)\,(e+fx^4)^2\,dx \\
&= \frac{d(e+fx^4)^3}{12f} + \int \left(ae^2+ce^2x^2+2aefx^4+2cefx^6+af^2x^8+cf^2x^{10}\right)dx \\
&= ae^2x+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int (a+cx^2+dx^3)\,(e+fx^4)^2\,dx = ae^2x+\frac{1}{3}ce^2x^3+\frac{1}{4}de^2x^4+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7 + \frac{1}{4}defx^8+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}+\frac{1}{12}df^2x^{12}$$

```
[In] Integrate[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7
 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result
gosper	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{11}c\,f^{2}x^{11}+\frac{1}{9}a\,f^{2}x^{9}+\frac{1}{4}def\,x^{8}+\frac{2}{7}cef\,x^{7}+\frac{2}{5}aef\,x^{5}+\frac{1}{4}d\,x^{4}e^{2}+\frac{1}{3}c\,e^{2}x^{3}+a\,e^{2}x$
default	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{11}c\,f^{2}x^{11}+\frac{1}{9}a\,f^{2}x^{9}+\frac{1}{4}def\,x^{8}+\frac{2}{7}cef\,x^{7}+\frac{2}{5}aef\,x^{5}+\frac{1}{4}d\,x^{4}e^{2}+\frac{1}{3}c\,e^{2}x^{3}+a\,e^{2}x$
norman	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{11}c\,f^{2}x^{11}+\frac{1}{9}a\,f^{2}x^{9}+\frac{1}{4}def\,x^{8}+\frac{2}{7}cef\,x^{7}+\frac{2}{5}aef\,x^{5}+\frac{1}{4}d\,x^{4}e^{2}+\frac{1}{3}c\,e^{2}x^{3}+a\,e^{2}x$
risch	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{11}c\,f^{2}x^{11}+\frac{1}{9}a\,f^{2}x^{9}+\frac{1}{4}def\,x^{8}+\frac{2}{7}cef\,x^{7}+\frac{2}{5}aef\,x^{5}+\frac{1}{4}d\,x^{4}e^{2}+\frac{1}{3}c\,e^{2}x^{3}+a\,e^{2}x$
parallelrisch	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{11}c\,f^{2}x^{11}+\frac{1}{9}a\,f^{2}x^{9}+\frac{1}{4}def\,x^{8}+\frac{2}{7}cef\,x^{7}+\frac{2}{5}aef\,x^{5}+\frac{1}{4}d\,x^{4}e^{2}+\frac{1}{3}c\,e^{2}x^{3}+a\,e^{2}x$

```
[In] int((d*x^3+c*x^2+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*d*f^2*x^12+1/11*c*f^2*x^11+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/7*c*e*f*x^7+2
/5*a*e*f*x^5+1/4*d*x^4*e^2+1/3*c*e^2*x^3+a*e^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \left(a+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3+ae^2x$$

```
[In] integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/7*c*e
*f*x^7 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + a*e^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int \left(a+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx=ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

```
[In] integrate((d*x**3+c*x**2+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e*f*x**7/7
+ c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a + cx^2 + dx^3) (e + fx^4)^2 \, dx = \frac{1}{12} df^2x^{12} + \frac{1}{11} cf^2x^{11} + \frac{1}{9} af^2x^9 + \frac{1}{4} defx^8 + \frac{2}{7} cefx^7 + \frac{2}{5} aefx^5 + \frac{1}{4} de^2x^4 + \frac{1}{3} ce^2x^3 + ae^2x$$

```
[In] integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/7*c*e
*f*x^7 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + a*e^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a + cx^2 + dx^3) (e + fx^4)^2 \, dx = \frac{1}{12} df^2x^{12} + \frac{1}{11} cf^2x^{11} + \frac{1}{9} af^2x^9 + \frac{1}{4} defx^8 + \frac{2}{7} cefx^7 + \frac{2}{5} aefx^5 + \frac{1}{4} de^2x^4 + \frac{1}{3} ce^2x^3 + ae^2x$$

```
[In] integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/7*c*e
*f*x^7 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + a*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a + cx^2 + dx^3) (e + fx^4)^2 \, dx = \frac{d\,e^2\,x^4}{4} + \frac{c\,e^2\,x^3}{3} + a\,e^2\,x + \frac{d\,e\,f\,x^8}{4} + \frac{2\,c\,e\,f\,x^7}{7} + \frac{2\,a\,e\,f\,x^5}{5} + \frac{d\,f^2\,x^{12}}{12} + \frac{c\,f^2\,x^{11}}{11} + \frac{a\,f^2\,x^9}{9}$$

```
[In] int((e + f*x^4)^2*(a + c*x^2 + d*x^3),x)
```

```
[Out] (a*f^2*x^9)/9 + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (c*f^2*x^11)/11 + (d*f^2*x^
12)/12 + a*e^2*x + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4
```

3.146 $\int (bx+cx^2+dx^3)(e+fx^4)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int (bx+cx^2+dx^3)(e+fx^4)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

```
[Out] 1/2*b*e^2*x^2+1/3*c*e^2*x^3+1/3*b*e*f*x^6+2/7*c*e*f*x^7+1/10*b*f^2*x^10+1/1
1*c*f^2*x^11+1/12*d*(f*x^4+e)^3/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1596, 1607, 1634}

$$\int (bx+cx^2+dx^3)(e+fx^4)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

```
[In] Int[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^
10)/10 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
```

```
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(e+fx^4)^3}{12f} + \int (bx+cx^2)(e+fx^4)^2 \, dx \\
&= \frac{d(e+fx^4)^3}{12f} + \int x(b+cx)(e+fx^4)^2 \, dx \\
&= \frac{d(e+fx^4)^3}{12f} + \int \left(be^2x+ce^2x^2+2befx^5+2cefx^6+bf^2x^9+cf^2x^{10}\right) dx \\
&= \frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int (bx+cx^2+dx^3)(e+fx^4)^2 \, dx = \frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{4}de^2x^4+\frac{1}{3}befx^6+\frac{2}{7}cefx^7 + \frac{1}{4}defx^8+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}+\frac{1}{12}df^2x^{12}$$

```
[In] Integrate[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (2*c*e*f*x^
7)/7 + (d*e*f*x^8)/4 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

method	result
gosper	$\frac{x^{2}\left(385 d f^{2} x^{10}+420 c f^{2} x^{9}+462 b f^{2} x^{8}+1155 d e f x^{6}+1320 c e f x^{5}+1540 b e f x^{4}+1155 d e^{2} x^{2}+1540 c e^{2} x+2310 e^{2} b\right)}{4620}$
default	$\frac{1}{12} d f^{2} x^{12}+\frac{1}{11} c f^{2} x^{11}+\frac{1}{10} b f^{2} x^{10}+\frac{1}{4} d e f x^{8}+\frac{2}{7} c e f x^{7}+\frac{1}{3} b e f x^{6}+\frac{1}{4} d x^{4} e^{2}+\frac{1}{3} c e^{2} x^{3}+\frac{1}{2} b e^{2}$
norman	$\frac{1}{12} d f^{2} x^{12}+\frac{1}{11} c f^{2} x^{11}+\frac{1}{10} b f^{2} x^{10}+\frac{1}{4} d e f x^{8}+\frac{2}{7} c e f x^{7}+\frac{1}{3} b e f x^{6}+\frac{1}{4} d x^{4} e^{2}+\frac{1}{3} c e^{2} x^{3}+\frac{1}{2} b e^{2}$
risch	$\frac{1}{12} d f^{2} x^{12}+\frac{1}{11} c f^{2} x^{11}+\frac{1}{10} b f^{2} x^{10}+\frac{1}{4} d e f x^{8}+\frac{2}{7} c e f x^{7}+\frac{1}{3} b e f x^{6}+\frac{1}{4} d x^{4} e^{2}+\frac{1}{3} c e^{2} x^{3}+\frac{1}{2} b e^{2}$
parallelrisch	$\frac{1}{12} d f^{2} x^{12}+\frac{1}{11} c f^{2} x^{11}+\frac{1}{10} b f^{2} x^{10}+\frac{1}{4} d e f x^{8}+\frac{2}{7} c e f x^{7}+\frac{1}{3} b e f x^{6}+\frac{1}{4} d x^{4} e^{2}+\frac{1}{3} c e^{2} x^{3}+\frac{1}{2} b e^{2}$

```
[In] int((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
[Out] 1/4620*x^2*(385*d*f^2*x^10+420*c*f^2*x^9+462*b*f^2*x^8+1155*d*e*f*x^6+1320*
c*e*f*x^5+1540*b*e*f*x^4+1155*d*e^2*x^2+1540*c*e^2*x+2310*b*e^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \left(bx+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{1}{12}df^2x^{12}+\frac{1}{11}cf^2x^{11}+\frac{1}{10}bf^2x^{10}+\frac{1}{4}defx^8+\frac{2}{7}cefx^7+\frac{1}{3}befx^6+\frac{1}{4}de^2x^4+\frac{1}{3}ce^2x^3+\frac{1}{2}be^2x^2$$

```
[In] integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 2/7*c
*e*f*x^7 + 1/3*b*e*f*x^6 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \left(bx+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

```
[In] integrate((d*x**3+c*x**2+b*x)*(f*x**4+e)**2,x)
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x*
*7/7 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int (bx+cx^2+dx^3)(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12}+\frac{1}{11}cf^2x^{11}+\frac{1}{10}bf^2x^{10}+\frac{1}{4}defx^8+\frac{2}{7}cefx^7+\frac{1}{3}befx^6+\frac{1}{4}de^2x^4+\frac{1}{3}ce^2x^3+\frac{1}{2}be^2x^2$$

```
[In] integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 2/7*c
*e*f*x^7 + 1/3*b*e*f*x^6 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int (bx+cx^2+dx^3)(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12}+\frac{1}{11}cf^2x^{11}+\frac{1}{10}bf^2x^{10}+\frac{1}{4}defx^8+\frac{2}{7}cefx^7+\frac{1}{3}befx^6+\frac{1}{4}de^2x^4+\frac{1}{3}ce^2x^3+\frac{1}{2}be^2x^2$$

```
[In] integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 2/7*c
*e*f*x^7 + 1/3*b*e*f*x^6 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int (bx+cx^2+dx^3)(e+fx^4)^2\,dx = \frac{d\,e^2\,x^4}{4}+\frac{c\,e^2\,x^3}{3}+\frac{b\,e^2\,x^2}{2}+\frac{d\,e\,f\,x^8}{4}+\frac{2\,c\,e\,f\,x^7}{7}+\frac{b\,e\,f\,x^6}{3}+\frac{d\,f^2\,x^{12}}{12}+\frac{c\,f^2\,x^{11}}{11}+\frac{b\,f^2\,x^{10}}{10}$$

```
[In] int((e + f*x^4)^2*(b*x + c*x^2 + d*x^3),x)
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*f^2*x^10)/10 + (d*e^2*x^4)/4 + (c*f^2*x^
11)/11 + (d*f^2*x^12)/12 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4
```

3.147 $\int (c+dx+ex^2+fx^3)(a+bx^4)^2 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 109

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2 \, dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7 + \frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{f(a+bx^4)^3}{12b}$$

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/3*a^2*e*x^3+2/5*a*b*c*x^5+1/3*a*b*d*x^6+2/7*a*b*e*x
^7+1/9*b^2*c*x^9+1/10*b^2*d*x^10+1/11*b^2*e*x^11+1/12*f*(b*x^4+a)^3/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1596, 1671}

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2 \, dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7 + \frac{f(a+bx^4)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 +
 (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f*(
a + b*x^4)^3)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
```

```
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{f(a+bx^4)^3}{12b} + \int (c+dx+ex^2)\left(a+bx^4\right)^2\,dx \\ &= \frac{f(a+bx^4)^3}{12b} + \int \left(a^2c+a^2dx+a^2ex^2+2abcx^4+2abdx^5+2abex^6+b^2cx^8+b^2dx^9 + b^2ex^{10}\right)dx \\ &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7 + \frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{f(a+bx^4)^3}{12b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{4}a^2fx^4+\frac{2}{5}abcx^5 + \frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{4}abfx^8+\frac{1}{9}b^2cx^9 + \frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{12}b^2fx^{12}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*c*x^5)/5 +
 (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x
^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2 f$
default	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2 f$
norman	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2 f$
risch	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2 f$
parallelrisch	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2 f$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*a*b*f*x^8
+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*a*b*c*x^5+1/4*a^2*f*x^4+1/3*a^2*e*x^3+1/2*
a^2*d*x^2+a^2*c*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int (c+dx+ex^2+fx^3)\,(a+bx^4)^2\,dx=\frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{a^2fx^4}{4}+\frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{abfx^8}{4}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}+\frac{b^2fx^{12}}{12}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*c*x**5/5 +
 a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*x**9/9 + b**2*d*x**1
0/10 + b**2*e*x**11/11 + b**2*f*x**12/12
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = \frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = \frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx=\frac{f\,a^2\,x^4}{4}+\frac{e\,a^2\,x^3}{3}+\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{f\,a\,b\,x^8}{4}+\frac{2\,e\,a\,b\,x^7}{7}+\frac{d\,a\,b\,x^6}{3}+\frac{2\,c\,a\,b\,x^5}{5}+\frac{f\,b^2\,x^{12}}{12}+\frac{e\,b^2\,x^{11}}{11}+\frac{d\,b^2\,x^{10}}{10}+\frac{c\,b^2\,x^9}{9}$$

```
[In] int((a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^2*d*x^2)/2 + (b^2*c*x^9)/9 + (a^2*e*x^3)/3 + (b^2*d*x^10)/10 + (a^2*f*x^
4)/4 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12 + a^2*c*x + (2*a*b*c*x^5)/5 + (a*b
*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4
```

3.148 $\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 151

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{f(a+bx^4)^4}{16b}$$

```
[Out] a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+3/5*a^2*b*c*x^5+1/2*a^2*b*d*x^6+3/7*a^2
*b*e*x^7+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/11*a*b^2*e*x^11+1/13*b^3*c*x^1
3+1/14*b^3*d*x^14+1/15*b^3*e*x^15+1/16*f*(b*x^4+a)^4/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1596, 1671}

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f(a+bx^4)^4}{16b}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)
/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e
```

```
*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a + b
*x^4)^4)/(16*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f(a+bx^4)^4}{16b} + \int (c+dx+ex^2)\left(a+bx^4\right)^3\,dx \\
&= \frac{f(a+bx^4)^4}{16b} + \int \left(a^3c+a^3dx+a^3ex^2+3a^2bcx^4+3a^2bdx^5+3a^2bex^6+3ab^2cx^8\right. \\
&\qquad \left.+3ab^2dx^9+3ab^2ex^{10}+b^3cx^{12}+b^3dx^{13}+b^3ex^{14}\right)\,dx \\
&= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9 \\
&\quad +\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{f(a+bx^4)^4}{16b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\begin{aligned}
\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx &= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{4}a^3fx^4+\frac{3}{5}a^2bcx^5 \\
&\quad +\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}a^2bfx^8+\frac{1}{3}ab^2cx^9 \\
&\quad +\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{4}ab^2fx^{12} \\
&\quad +\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{1}{16}b^3fx^{16}
\end{aligned}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*b*c*x^5)/5
 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8)/8 + (a*b^2*c*x^9)/
3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (a*b^2*f*x^12)/4 + (b^3*c*x
^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

method	result
gosper	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x$
default	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x$
norman	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x$
risch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x$
parallelrisch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+1/4*f*a^3*x^4+3/5*a^2*b*c*x^5+1/2*a^2*b
*d*x^6+3/7*a^2*b*e*x^7+3/8*f*a^2*b*x^8+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/
11*a*b^2*e*x^11+1/4*a*b^2*f*x^12+1/13*b^3*c*x^13+1/14*b^3*d*x^14+1/15*b^3*e
*x^15+1/16*b^3*f*x^16
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx=\frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\,b^3ex^{15}+\frac{1}{14}\,b^3dx^{14}+\frac{1}{13}\,b^3cx^{13}+\frac{1}{4}\,ab^2fx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bfx^8+\frac{3}{7}\,a^2bex^7+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3fx^4+\frac{1}{3}\,a^3ex^3+\frac{1}{2}\,a^3dx^2+a^3cx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx = a^3cx + \frac{a^3dx^2}{2} + \frac{a^3ex^3}{3} + \frac{a^3fx^4}{4} + \frac{3a^2bcx^5}{5} + \frac{a^2bdx^6}{2} + \frac{3a^2bex^7}{7} + \frac{3a^2bfx^8}{8} + \frac{ab^2cx^9}{3} + \frac{3ab^2dx^{10}}{10} + \frac{3ab^2ex^{11}}{11} + \frac{ab^2fx^{12}}{4} + \frac{b^3cx^{13}}{13} + \frac{b^3dx^{14}}{14} + \frac{b^3ex^{15}}{15} + \frac{b^3fx^{16}}{16}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2*b*c*x**5/
5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x**8/8 + a*b**2*c*x**9
/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + a*b**2*f*x**12/4 + b**3*c*
x**13/13 + b**3*d*x**14/14 + b**3*e*x**15/15 + b**3*f*x**16/16
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx = \frac{1}{16}\,b^3fx^{16} + \frac{1}{15}\,b^3ex^{15} + \frac{1}{14}\,b^3dx^{14} + \frac{1}{13}\,b^3cx^{13} + \frac{1}{4}\,ab^2fx^{12} + \frac{3}{11}\,ab^2ex^{11} + \frac{3}{10}\,ab^2dx^{10} + \frac{1}{3}\,ab^2cx^9 + \frac{3}{8}\,a^2bfx^8 + \frac{3}{7}\,a^2bex^7 + \frac{1}{2}\,a^2bdx^6 + \frac{3}{5}\,a^2bcx^5 + \frac{1}{4}\,a^3fx^4 + \frac{1}{3}\,a^3ex^3 + \frac{1}{2}\,a^3dx^2 + a^3cx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx=\frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\,b^3ex^{15}+\frac{1}{14}\,b^3dx^{14}+\frac{1}{13}\,b^3cx^{13}+\frac{1}{4}\,ab^2fx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bfx^8+\frac{3}{7}\,a^2bex^7+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3fx^4+\frac{1}{3}\,a^3ex^3+\frac{1}{2}\,a^3dx^2+a^3cx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx=\frac{f\,a^3\,x^4}{4}+\frac{e\,a^3\,x^3}{3}+\frac{d\,a^3\,x^2}{2}+c\,a^3\,x+\frac{3\,f\,a^2\,b\,x^8}{8}+\frac{3\,e\,a^2\,b\,x^7}{7}+\frac{d\,a^2\,b\,x^6}{2}+\frac{3\,c\,a^2\,b\,x^5}{5}+\frac{f\,a\,b^2\,x^{12}}{4}+\frac{3\,e\,a\,b^2\,x^{11}}{11}+\frac{3\,d\,a\,b^2\,x^{10}}{10}+\frac{c\,a\,b^2\,x^9}{3}+\frac{f\,b^3\,x^{16}}{16}+\frac{e\,b^3\,x^{15}}{15}+\frac{d\,b^3\,x^{14}}{14}+\frac{c\,b^3\,x^{13}}{13}$$

```
[In] int((a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^3*d*x^2)/2 + (b^3*c*x^13)/13 + (a^3*e*x^3)/3 + (b^3*d*x^14)/14 + (a^3*f*
x^4)/4 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16 + a^3*c*x + (3*a^2*b*c*x^5)/5 +
(a*b^2*c*x^9)/3 + (a^2*b*d*x^6)/2 + (3*a*b^2*d*x^10)/10 + (3*a^2*b*e*x^7)/7
 + (3*a*b^2*e*x^11)/11 + (3*a^2*b*f*x^8)/8 + (a*b^2*f*x^12)/4
```

3.149 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 155

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx=\frac{af+bx(c+dx+ex^2)}{4ab\,(a-bx^4)}+\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

```
[Out] 1/4*(a*f+b*x*(e*x^2+d*x+c))/a/b/(-b*x^4+a)+1/4*d*arctanh(x^2*b^(1/2)/a^(1/2
))/a^(3/2)/b^(1/2)+1/8*arctan(b^(1/4)*x/a^(1/4))*(-e*a^(1/2)+3*c*b^(1/2))/a
^(7/4)/b^(3/4)+1/8*arctanh(b^(1/4)*x/a^(1/4))*(e*a^(1/2)+3*c*b^(1/2))/a^(7/
4)/b^(3/4)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1868, 1890, 281, 214, 1181, 211}

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}c-\sqrt{a}e\right)}{8a^{7/4}b^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{8a^{7/4}b^{3/4}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{af+bx(c+dx+ex^2)}{4ab\,(a-bx^4)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]
```

```
[Out] (a*f + b*x*(c + d*x + e*x^2))/(4*a*b*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]
*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt
[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + (d*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{af+bx(c+dx+ex^2)}{4ab\,(a-bx^4)} - \frac{\int \frac{-3c-2dx-ex^2}{a-bx^4}\,dx}{4a}$$

$$= \frac{af+bx(c+dx+ex^2)}{4ab\,(a-bx^4)} - \frac{\int \left(-\frac{2dx}{a-bx^4} + \frac{-3c-ex^2}{a-bx^4}\right)dx}{4a}$$

$$= \frac{af+bx(c+dx+ex^2)}{4ab\,(a-bx^4)} - \frac{\int \frac{-3c-ex^2}{a-bx^4}\,dx}{4a} + \frac{d\int \frac{x}{a-bx^4}\,dx}{2a}$$

$$= \frac{af+bx(c+dx+ex^2)}{4ab\,(a-bx^4)} + \frac{d\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{4a} - \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}-e\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{8a} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx}{8a^{3/2}}$$

$$= \frac{af+bx(c+dx+ex^2)}{4ab\,(a-bx^4)} + \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.42

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx = \frac{\frac{4a(af+bx(c+x(d+ex)))}{a-bx^4} - 2\sqrt[4]{a}\sqrt[4]{b}\left(-3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \sqrt[4]{b}\left(3\sqrt[4]{a}\sqrt{b}c+2\sqrt{a}\sqrt[4]{b}d+a^{3/4}e\right)\log\left(\sqrt[4]{a}\right.}{16a^2b}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a*(a*f + b*x*(c + x*(d + e*x))))/(a - b*x^4) - 2*a^(1/4)*b^(1/4)*(-3*Sq
rt[b]*c + Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - b^(1/4)*(3*a^(1/4)*Sqrt[
b]*c + 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x] + b^(1/4)*
(3*a^(1/4)*Sqrt[b]*c - 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) + b^(1/
4)*x] + 2*Sqrt[a]*Sqrt[b]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^2*b)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{e x^3}{4a}+\frac{d x^2}{4a}+\frac{cx}{4a}+\frac{f}{4b}}{-b x^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 b-a)}\frac{(\_R^2 e+2\_R d+3c)\ln(x-\_R)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(-b x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{16a^2}\right)+d\left(\frac{x^2}{4a(-b x^4+a)}+\frac{\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)+e\left(\frac{x^3}{4a(-b x\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
[Out] (1/4/a*e*x^3+1/4*d/a*x^2+1/4*c/a*x+1/4*f/b)/(-b*x^4+a)-1/16/b/a*sum((_R^2*e
+2*_R*d+3*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.00 (sec) , antiderivative size = 117016, normalized size of antiderivative = 754.94

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(139) = 278.

Time = 43.72 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.35

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx$$
$$=\mathrm{RootSum}\left(65536t^4a^7b^3+t^2\left(-3072a^4b^2ce-2048a^4b^2d^2\right)+t\left(128a^3bde^2+1152a^2b^2c^2d\right)-a^2e^4+18abc^2\right.$$
$$+\frac{-af-bcx-bdx^2-bex^3}{-4a^2b+4ab^2x^4}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048*a**4*b**2
*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d) - a**2*e**4 + 18*a*
b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 - 81*b**2*c**4, Lambda(_t, _t*l
og(x + (4096*_t**3*a**7*b**2*e**3 + 36864*_t**3*a**6*b**3*c**2*e - 98304*_t
**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2
*d**3*e - 13824*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_t*a**4
*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e
+ 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 - 120*a
**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c*
*3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4 - 96*a**2*b*c*d**2*e**3 + 64*a**2*
b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**
2*d**4 - 729*b**3*c**6))))) + (-a*f - b*c*x - b*d*x**2 - b*e*x**3)/(-4*a**2*
b + 4*a*b**2*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.29

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx = -\frac{bex^3+bdx^2+bcx+af}{4\,(ab^2x^4-a^2b)} + \frac{\frac{2\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{2\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\left(3\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{16\,a}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b*e*x^3 + b*d*x^2 + b*c*x + a*f)/(a*b^2*x^4 - a^2*b) + 1/16*(2*d*log(
sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 2*d*log(sqrt(b)*x^2 - sqrt(a))/(
sqrt(a)*sqrt(b)) + 2*(3*sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*sqrt(b)*c + sqrt(a
)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt
(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(116) = 232$.

Time = 0.28 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.03

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx$$
$$= -\frac{\sqrt{2}\left(3\,b^2c-2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a}$$
$$-\frac{\sqrt{2}\left(3\,b^2c+2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a}$$
$$-\frac{\sqrt{2}(3\,b^2c-\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a}$$
$$+\frac{\sqrt{2}(3\,b^2c-\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a}-\frac{bex^3+bdx^2+bcx+af}{4\,(bx^4-a)ab}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*b^2*c - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arc
tan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*
a) - 1/16*sqrt(2)*(3*b^2*c + 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3
/4)*a) - 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)
^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b
)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) -
1/4*(b*e*x^3 + b*d*x^2 + b*c*x + a*f)/((b*x^4 - a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 483, normalized size of antiderivative = 3.12

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(-\operatorname{root}\left(65536\,a^7\,b^3\,z^4-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\right.\right.\right.$$
$$-\frac{-9\,b^2\,c^2\,e+12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3}-\frac{x\,(2\,b^2\,d^3-3\,b^2\,c\,d\,e)}{16\,a^3}\Bigg)\operatorname{root}\big(65536\,a^7\,b^3\,z^4$$
$$-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e$$
$$+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4-81\,b^2\,c^4-a^2\,e^4,z,k\big)\Bigg)+\frac{\frac{f}{4\,b}+\frac{d\,x^2}{4\,a}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{a-b\,x^4}$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x)
```

```
[Out] symsum(log(- root(65536*a^7*b^3*z^4 - 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*d
^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b
*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k)*(root(65536*a^7*b^3*z^4
 - 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128
*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4
- a^2*e^4, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 + 4*a^2*b^2*e^2)
)/(16*a^3) - (b^2*d*e)/a) - (12*b^2*c*d^2 - 9*b^2*c^2*e + a*b*e^3)/(64*a^3)
 - (x*(2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3))*root(65536*a^7*b^3*z^4 - 3072*a^
4*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e
^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4,
 z, k), k, 1, 4) + (f/(4*b) + (d*x^2)/(4*a) + (e*x^3)/(4*a) + (c*x)/(4*a))/
(a - b*x^4)
```

3.150 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 188

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx = \frac{x(7c+6dx+5ex^2)}{32a^2(a-bx^4)}+\frac{af+bx(c+dx+ex^2)}{8ab(a-bx^4)^2}+\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{3d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

```
[Out] 1/32*x*(5*e*x^2+6*d*x+7*c)/a^2/(-b*x^4+a)+1/8*(a*f+b*x*(e*x^2+d*x+c))/a/b/(
-b*x^4+a)^2+3/16*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)+1/64*arctan
(b^(1/4)*x/a^(1/4))*(-5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)+1/64*arcta
nh(b^(1/4)*x/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used

$= \{1868, 1869, 1890, 281, 214, 1181, 211\}$

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(21\sqrt{b}c-5\sqrt{a}e\right)}{64a^{11/4}b^{3/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}e+21\sqrt{b}c\right)}{64a^{11/4}b^{3/4}} + \frac{3d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} + \frac{x(7c+6dx+5ex^2)}{32a^2(a-bx^4)} + \frac{af+bx(c+dx+ex^2)}{8ab(a-bx^4)^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x]
```

```
[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a - b*x^4)) + (a*f + b*x*(c + d*x + e*
x^2))/(8*a*b*(a - b*x^4)^2) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)
*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh
[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/S
qrt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
```

```
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2}-\frac{\int \frac{-7c-6dx-5ex^2}{(a-bx^4)^2}\,dx}{8a}\\
&= \frac{x(7c+6dx+5ex^2)}{32a^2\,(a-bx^4)}+\frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2}+\frac{\int \frac{21c+12dx+5ex^2}{a-bx^4}\,dx}{32a^2}\\
&= \frac{x(7c+6dx+5ex^2)}{32a^2\,(a-bx^4)}+\frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2}+\frac{\int \left(\frac{12dx}{a-bx^4}+\frac{21c+5ex^2}{a-bx^4}\right)\,dx}{32a^2}\\
&= \frac{x(7c+6dx+5ex^2)}{32a^2\,(a-bx^4)}+\frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2}+\frac{\int \frac{21c+5ex^2}{a-bx^4}\,dx}{32a^2}+\frac{(3d)\int \frac{x}{a-bx^4}\,dx}{8a^2}\\
&= \frac{x(7c+6dx+5ex^2)}{32a^2\,(a-bx^4)}+\frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2}+\frac{(3d)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{16a^2}\\
&\quad -\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{64a^2}+\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}+5e\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx}{64a^2}\\
&= \frac{x(7c+6dx+5ex^2)}{32a^2\,(a-bx^4)}+\frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2}+\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{3/4}}\\
&\quad +\frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.35

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$$
$$= \frac{\frac{4ax(7c+x(6d+5ex))}{a-bx^4}+\frac{16a^2(af+bx(c+x(d+ex)))}{b(a-bx^4)^2}+\frac{2\sqrt[4]{a}(21\sqrt{b}c-5\sqrt{a}e)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}-\frac{\left(21\sqrt[4]{a}\sqrt{b}c+12\sqrt{a}\sqrt[4]{b}d+5a^{3/4}e\right)\log\left(\sqrt[4]{a}\right.}{b^{3/4}}}{128a^3}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x]
```

```
[Out] ((4*a*x*(7*c + x*(6*d + 5*e*x)))/(a - b*x^4) + (16*a^2*(a*f + b*x*(c + x*(d
 + e*x))))/(b*(a - b*x^4)^2) + (2*a^(1/4)*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcT
an[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((21*a^(1/4)*Sqrt[b]*c + 12*Sqrt[a]*b^(1
/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((21*a^(1/4)*Sqrt[
b]*c - 12*Sqrt[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4
) + (12*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.64

method	result
risch	$\frac{-\frac{5be\,x^7}{32a^2}-\frac{3bd\,x^6}{16a^2}-\frac{7bc\,x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}+\frac{f}{8b}}{(-b\,x^4+a)^2}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)}\frac{(5\_R^2e+12\_Rd+21c)\ln(x-\_R)}{\_R^3}}{128a^2b}$
default	$c\left(\frac{x}{8a(-b\,x^4+a)^2}+\frac{\frac{7x}{32a(-b\,x^4+a)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{128a^2}}{a}\right)+d\left(\frac{x^2}{8a(-b\,x^4+a)^2}+\frac{\frac{3x^2}{16a(-b\,x^4+a)}+\frac{3\ln}{}}{a}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-5/32*b*e/a^2*x^7-3/16*b*d/a^2*x^6-7/32*b*c/a^2*x^5+9/32/a*e*x^3+5/16*d/a*
x^2+11/32*c/a*x+1/8*f/b)/(-b*x^4+a)^2-1/128/a^2/b*sum((5*_R^2*e+12*_R*d+21*
c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.66 (sec) , antiderivative size = 118761, normalized size of antiderivative = 631.71

$$\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.32

$$\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^3}\,dx = -\frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5-9\,abex^3-10\,abdx^2-11\,abcx-4\,a^2f}{32\,\left(a^2b^3x^8-2\,a^3b^2x^4+a^4b\right)} + \frac{\frac{12\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{12\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\,\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\,\sqrt{b}c+5\,\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{128\,a^2}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
[Out] -1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*c*x^5 - 9*a*b*e*x^3 - 10*a*b*d*x^2
 - 11*a*b*c*x - 4*a^2*f)/(a^2*b^3*x^8 - 2*a^3*b^2*x^4 + a^4*b) + 1/128*(12*
d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 12*d*log(sqrt(b)*x^2 - sqr
t(a))/(sqrt(a)*sqrt(b)) + 2*(21*sqrt(b)*c - 5*sqrt(a)*e)*arctan(sqrt(b)*x/s
qrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (21*sqrt(b)
*c + 5*sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt
(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(148) = 296.

Time = 0.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.87

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$$
$$= -\frac{\sqrt{2}\left(21\,b^2c-12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,\left(-ab^3\right)^{\frac{3}{4}}\,a^2}$$
$$-\frac{\sqrt{2}\left(21\,b^2c+12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,\left(-ab^3\right)^{\frac{3}{4}}\,a^2}$$
$$-\frac{\sqrt{2}\left(21\,b^2c-5\,\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,\left(-ab^3\right)^{\frac{3}{4}}\,a^2}$$
$$+\frac{\sqrt{2}\left(21\,b^2c-5\,\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,\left(-ab^3\right)^{\frac{3}{4}}\,a^2}$$
$$-\frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5-9\,abex^3-10\,abdx^2-11\,abcx-4\,a^2f}{32\,(bx^4-a)^2a^2b}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*(21*b^2*c - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 5*sqrt(-a*b)*b*e
)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(
3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*c + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 5*sqr
t(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/
((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2
+ sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) + 1/256*sqrt(2)
*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b
))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*c*x^5 - 9
*a*b*e*x^3 - 10*a*b*d*x^2 - 11*a*b*c*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 832, normalized size of antiderivative = 4.43

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(-\frac{b\left(125\,a\,e^3+3024\,b\,c\,d^2-2205\,b\,c^2\,e+1728\,b\,d^3\,x+\mathrm{root}(268435456\,a^{11}\,b^3\,z^4-6881280\,a^6\,b^2\right.}{}\right.\right.$$
$$-6881280\,a^6\,b^2\,c\,e\,z^2-4718592\,a^6\,b^2\,d^2\,z^2+2709504\,a^3\,b^2\,c^2\,d\,z+153600\,a^4\,b\,d\,e^2\,z$$
$$\left.\left.-60480\,a\,b\,c\,d^2\,e+22050\,a\,b\,c^2\,e^2+20736\,a\,b\,d^4-625\,a^2\,e^4-194481\,b^2\,c^4,z,k\right)\right)$$
$$+\frac{\frac{f}{8\,b}+\frac{5\,d\,x^2}{16\,a}+\frac{9\,e\,x^3}{32\,a}+\frac{11\,c\,x}{32\,a}-\frac{7\,b\,c\,x^5}{32\,a^2}-\frac{3\,b\,d\,x^6}{16\,a^2}-\frac{5\,b\,e\,x^7}{32\,a^2}}{a^2-2\,a\,b\,x^4+b^2\,x^8}$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x)
```

```
[Out] symsum(log(-(b*(125*a*e^3 + 3024*b*c*d^2 - 2205*b*c^2*e + 1728*b*d^3*x + 34
4064*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^
2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^
2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z,
k)^2*a^5*b^2*c + 3200*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2
 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z
 - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 19
4481*b^2*c^4, z, k)*a^3*b*e^2*x - 2520*b*c*d*e*x + 56448*root(268435456*a^1
1*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3
*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2
 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k)*a^2*b^2*c^2*x - 1966
08*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*
d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*
e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k)
^2*a^5*b^2*d*x - 15360*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^
2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*
z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 1
94481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435456*a^11*b^3*z^4 -
 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*
z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*
b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k), k, 1, 4) + (f/(8*b) + (5*d*x^2
)/(16*a) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) - (7*b*c*x^5)/(32*a^2) - (3*b
*d*x^6)/(16*a^2) - (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4)
```

3.151 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 220

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx = \frac{x(11c+10dx+9ex^2)}{96a^2(a-bx^4)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3(a-bx^4)}+\frac{af+bx(c+dx+ex^2)}{12ab(a-bx^4)^3}+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{5d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

```
[Out] 1/96*x*(9*e*x^2+10*d*x+11*c)/a^2/(-b*x^4+a)^2+1/384*x*(45*e*x^2+60*d*x+77*c
)/a^3/(-b*x^4+a)+1/12*(a*f+b*x*(e*x^2+d*x+c))/a/b/(-b*x^4+a)^3+5/32*d*arcta
nh(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)+1/256*arctan(b^(1/4)*x/a^(1/4))*(-1
5*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)+1/256*arctanh(b^(1/4)*x/a^(1/4))
*(15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used

$= \{1868, 1869, 1890, 281, 214, 1181, 211\}$

$$\int \frac{c + dx + ex^2 + fx^3}{(a - bx^4)^4}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(77\sqrt{b}c - 15\sqrt{a}e\right)}{256a^{15/4}b^{3/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}e + 77\sqrt{b}c\right)}{256a^{15/4}b^{3/4}} + \frac{5d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a - bx^4)} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} + \frac{af + bx(c + dx + ex^2)}{12ab(a - bx^4)^3}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x]
```

```
[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x + 
45*e*x^2))/(384*a^3*(a - b*x^4)) + (a*f + b*x*(c + d*x + e*x^2))/(12*a*b*(a
 - b*x^4)^3) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/
(256*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)
/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(3
2*a^(7/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{af+bx(c+dx+ex^2)}{12ab\left(a-bx^4\right)^3}-\frac{\int \frac{-11c-10dx-9ex^2}{\left(a-bx^4\right)^3}\,dx}{12a}\\
&= \frac{x(11c+10dx+9ex^2)}{96a^2\left(a-bx^4\right)^2}+\frac{af+bx(c+dx+ex^2)}{12ab\left(a-bx^4\right)^3}+\frac{\int \frac{77c+60dx+45ex^2}{\left(a-bx^4\right)^2}\,dx}{96a^2}\\
&= \frac{x(11c+10dx+9ex^2)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a-bx^4\right)}\\
&\quad+\frac{af+bx(c+dx+ex^2)}{12ab\left(a-bx^4\right)^3}-\frac{\int \frac{-231c-120dx-45ex^2}{a-bx^4}\,dx}{384a^3}\\
&= \frac{x(11c+10dx+9ex^2)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a-bx^4\right)}\\
&\quad+\frac{af+bx(c+dx+ex^2)}{12ab\left(a-bx^4\right)^3}-\frac{\int \left(-\frac{120dx}{a-bx^4}+\frac{-231c-45ex^2}{a-bx^4}\right)dx}{384a^3}\\
&= \frac{x(11c+10dx+9ex^2)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a-bx^4\right)}\\
&\quad+\frac{af+bx(c+dx+ex^2)}{12ab\left(a-bx^4\right)^3}-\frac{\int \frac{-231c-45ex^2}{a-bx^4}\,dx}{384a^3}+\frac{(5d)\int \frac{x}{a-bx^4}\,dx}{16a^3}
\end{aligned}$$

$$= \frac{x(11c+10dx+9ex^2)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a-bx^4)} + \frac{af+bx(c+dx+ex^2)}{12ab(a-bx^4)^3} + \frac{(5d)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{32a^3} - \frac{\left(\frac{77\sqrt{b}c}{\sqrt{a}}-15e\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{256a^3} + \frac{\left(\frac{77\sqrt{b}c}{\sqrt{a}}+15e\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx}{256a^3}$$

$$= \frac{x(11c+10dx+9ex^2)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a-bx^4)} + \frac{af+bx(c+dx+ex^2)}{12ab(a-bx^4)^3} + \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.30

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$$

$$= \frac{\frac{4ax(77c+15x(4d+3ex))}{a-bx^4} + \frac{16a^2x(11c+x(10d+9ex))}{(a-bx^4)^2} - \frac{128a^3(af+bx(c+x(d+ex)))}{b(-a+bx^4)^3} + \frac{6\sqrt[4]{a}\left(77\sqrt{b}c-15\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}} - \frac{3\left(77\sqrt[4]{\ldots}\right.}{\ldots}}{1536\ldots}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x]
```

```
[Out] ((4*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a - b*x^4) + (16*a^2*x*(11*c + x*(10*
d + 9*e*x)))/(a - b*x^4)^2 - (128*a^3*(a*f + b*x*(c + x*(d + e*x))))/(b*(-a
 + b*x^4)^3) + (6*a^(1/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/
a^(1/4)])/b^(3/4) - (3*(77*a^(1/4)*Sqrt[b]*c + 40*Sqrt[a]*b^(1/4)*d + 15*a^
(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + (3*(77*a^(1/4)*Sqrt[b]*c - 40*
Sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (120*
Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(1536*a^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}-\frac{21be\,x^7}{64a^2}-\frac{5bd\,x^6}{12a^2}-\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}+\frac{f}{12b}}{(-b\,x^4+a)^3}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)}\frac{\left(15\_R^2e+40\_Rd\cdots\right.}{\_R\cdots}}{512a^3b}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}-\frac{21be\,x^7}{64a^2}-\frac{5bd\,x^6}{12a^2}-\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}+\frac{f}{12b}}{(-b\,x^4+a)^3}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\cdots\right.}{4a}}{\cdots}$

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9-21/64*b*e/a
^2*x^7-5/12*b*d/a^2*x^6-33/64*b*c/a^2*x^5+113/384/a*e*x^3+11/32*d/a*x^2+51/
128*c/a*x+1/12*f/b)/(-b*x^4+a)^3-1/512/a^3/b*sum((15*_R^2*e+40*_R*d+77*c)/_
R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.98 (sec) , antiderivative size = 118945, normalized size of antiderivative = 540.66

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.35

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx = -\frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9-126\,ab^2ex^7-160\,ab^2dx^6-198\,ab^2cx^5+113\,a^2bex^3+132\,a^2bdx^2+}{384\,(a^3b^4x^{12}-3\,a^4b^3x^8+3\,a^5b^2x^4-a^6b)} + \frac{\frac{40\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{40\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\,\sqrt{b}c+15\,\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{512\,a^3}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 77*b^3*c*x^9 - 126*a*b^2*e*x^7 - 16
0*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 153*a
^2*b*c*x + 32*a^3*f)/(a^3*b^4*x^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6*b)
 + 1/512*(40*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 40*d*log(sqrt
(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(77*sqrt(b)*c - 15*sqrt(a)*e)*arct
an(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
 - (77*sqrt(b)*c + 15*sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(s
qrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))
/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(179) = 358.

Time = 0.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.76

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$$

$$= -\frac{\sqrt{2}\left(77\,b^2c-40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,\left(-ab^3\right)^{\frac{3}{4}}\,a^3}$$

$$-\frac{\sqrt{2}\left(77\,b^2c+40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,\left(-ab^3\right)^{\frac{3}{4}}\,a^3}$$

$$-\frac{\sqrt{2}\left(77\,b^2c-15\,\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,\left(-ab^3\right)^{\frac{3}{4}}\,a^3}$$

$$+\frac{\sqrt{2}\left(77\,b^2c-15\,\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,\left(-ab^3\right)^{\frac{3}{4}}\,a^3}$$

$$-\frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9-126\,ab^2ex^7-160\,ab^2dx^6-198\,ab^2cx^5+113\,a^2bex^3+132\,a^2bdx^2+15}{384\,(bx^4-a)^3a^3b}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^2*c - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 15*sqrt(-a*b)*b*
e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^
(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2*c + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 15*s
qrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4)
)/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(
x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) + 1/1024*sq
rt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqr
t(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 77*b
^3*c*x^9 - 126*a*b^2*e*x^7 - 160*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 113*a^2*b*
e*x^3 + 132*a^2*b*d*x^2 + 153*a^2*b*c*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 880, normalized size of antiderivative = 4.00

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$$

$$= \left(\sum_{k=1}^{4} \ln\left(-\frac{b\left(3375\,a\,e^3+123200\,b\,c\,d^2-88935\,b\,c^2\,e+64000\,b\,d^3\,x+\mathrm{root}(68719476736\,a^{15}\,b^3\,z^4-1211\right.}{}\right.\right.$$

$$-1211105280\,a^8\,b^2\,c\,e\,z^2-838860800\,a^8\,b^2\,d^2\,z^2+485703680\,a^4\,b^2\,c^2\,d\,z+18432000\,a^5\,b\,d\,e^2\,z-7392000\,a\,b\,c\,d^2\,e+2668050\,a\,b\,c^2\,e^2+2560000\,a\,b\,d^4-35153041\,b^2\,c^4-50625\,a^2\,e^4,z,k)\Big)\Big)$$

$$+\frac{\frac{f}{12\,b}+\frac{11\,d\,x^2}{32\,a}+\frac{113\,e\,x^3}{384\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}+\frac{15\,b^2\,e\,x^{11}}{128\,a^3}-\frac{33\,b\,c\,x^5}{64\,a^2}-\frac{5\,b\,d\,x^6}{12\,a^2}-\frac{21\,b\,e\,x^7}{64\,a^2}}{a^3-3\,a^2\,b\,x^4+3\,a\,b^2\,x^8-b^3\,x^{12}}$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x)
```

```
[Out] symsum(log(-(b*(3375*a*e^3 + 123200*b*c*d^2 - 88935*b*c^2*e + 64000*b*d^3*x
 + 20185088*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 83
8860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*
z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*
b^2*c^4 - 50625*a^2*e^4, z, k)^2*a^7*b^2*c + 115200*root(68719476736*a^15*b
^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680
*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a
*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)*a^4*
b*e^2*x - 92400*b*c*d*e*x + 3035648*root(68719476736*a^15*b^3*z^4 - 1211105
280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z
 + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 256
0000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)*a^3*b^2*c^2*x - 1048
5760*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800
*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 739
2000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4
 - 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x - 614400*root(68719476736*a^15*b^3*z^
4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*
b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)*a^4*b*d*e
))/(2097152*a^9))*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^
2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*
d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 351
53041*b^2*c^4 - 50625*a^2*e^4, z, k), k, 1, 4) + (f/(12*b) + (11*d*x^2)/(32
*a) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9)/(384*a^3) + (
5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) - (33*b*c*x^5)/(64*a^2)
```

```
- (5*b*d*x^6)/(12*a^2) - (21*b*e*x^7)/(64*a^2))/(a^3 - b^3*x^12 - 3*a^2*b*x
^4 + 3*a*b^2*x^8)
```

3.152 $\int \frac{a}{2+3x^4} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 101

$$\int \frac{a}{2+3x^4} \, dx = -\frac{a \arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a \arctan\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a \log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a \log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}$$

```
[Out] 1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)-1/48
*a*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)+1/48*a*ln(6^(3/4)*x+3*x^2+6^(1/2))*
6^(3/4)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {12, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{a}{2+3x^4} \, dx = -\frac{a \arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a \arctan\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}} - \frac{a \log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}} + \frac{a \log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}$$

```
[In] Int[a/(2 + 3*x^4),x]
```

```
[Out] -1/4*(a*ArcTan[1 - 6^(1/4)*x])/6^(1/4) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/
4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(8*6^(1/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= a\int \frac{1}{2+3x^4}\,dx\\
&= \frac{a\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}} + \frac{a\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}}\\
&= \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} + \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}}\\
&= -\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}\\
&\quad + \frac{a\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}\\
&= -\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}\\
&\quad - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.77

$$
\int \frac{a}{2+3x^4}\,dx = \frac{a\left(-2\arctan\left(1-\sqrt[4]{6}x\right)+2\arctan\left(1+\sqrt[4]{6}x\right)-\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)+\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)\right)}{8\sqrt[4]{6}}
$$

```
[In] Integrate[a/(2 + 3*x^4),x]
```

```
[Out] (a*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] - Log[2 - 2*6^(1/4)*
x + Sqrt[6]*x^2] + Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(1/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.25

method	result
risch	$\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^{4}+2\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}$
meijerg	$\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}(}{8+3^{\frac{1}{4}}8^{\frac{3}{4}}}\right)}{\left(x^{4}\right)^{\frac{1}{4}}}\right.}{96}$

```
[In] int(a/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*a*sum(1/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{a}{2+3x^4}\,dx =& \frac{1}{96}\cdot 24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\log\left(12\,ax+24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\right)\\&+\frac{1}{96}i\cdot 24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\log\left(12\,ax+i\cdot 24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\right)\\&-\frac{1}{96}i\cdot 24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\log\left(12\,ax-i\cdot 24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\right)\\&-\frac{1}{96}\cdot 24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\log\left(12\,ax-24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\right)\end{aligned}$$

```
[In] integrate(a/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/96*24^(3/4)*(-a^4)^(1/4)*log(12*a*x + 24^(3/4)*(-a^4)^(1/4)) + 1/96*I*24^
(3/4)*(-a^4)^(1/4)*log(12*a*x + I*24^(3/4)*(-a^4)^(1/4)) - 1/96*I*24^(3/4)*
(-a^4)^(1/4)*log(12*a*x - I*24^(3/4)*(-a^4)^(1/4)) - 1/96*24^(3/4)*(-a^4)^(
1/4)*log(12*a*x - 24^(3/4)*(-a^4)^(1/4))
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{a}{2+3x^4}\,dx = a\left(-\frac{6^{\frac{3}{4}}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{48}+\frac{6^{\frac{3}{4}}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{48}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}x-1\right)}{24}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}x+1\right)}{24}\right)$$

```
[In] integrate(a/(3*x**4+2),x)
```

```
[Out] a*(-6**(3/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*log(x**2 +
6**(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*atan(6**(1/4)*x - 1)/24 + 6**(3/4)*
atan(6**(1/4)*x + 1)/24)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.22

$$\int \frac{a}{2+3x^4}\,dx = \frac{1}{48}\left(2\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+2\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+3^{\frac{3}{4}}2^{\frac{3}{4}}\,\mathrm{l}\right.$$

```
[In] integrate(a/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/48*(2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1
/4)*2^(3/4))) + 3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(
2)) - 3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)))*a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{a}{2+3x^4}\,dx = \frac{1}{48}\left(2\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+2\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\right.$$

```
[In] integrate(a/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/48*(2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) +
6^(3/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 6^(3/4)*log(x^2 - sq
rt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*a
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.36

$$\int \frac{a}{2+3x^4}\,dx = -\frac{(-1)^{1/4}\,6144^{3/4}\,a\left(\mathrm{atan}\left(\frac{(-1)^{1/4}\,6144^{1/4}\,x}{8}\right)\,1\mathrm{i}+\mathrm{atanh}\left(\frac{(-1)^{1/4}\,6144^{1/4}\,x}{8}\right)\,1\mathrm{i}\right)}{3072}$$

```
[In] int(a/(3*x^4 + 2),x)
```

```
[Out] -((-1)^(1/4)*6144^(3/4)*a*(atan(((-1)^(1/4)*6144^(1/4)*x)/8)*1i + atanh(((-
1)^(1/4)*6144^(1/4)*x)/8)*1i))/3072
```

3.153 $\int \frac{bx}{2+3x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{bx}{2+3x^4}\,dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

```
[Out] 1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {12, 281, 209}

$$\int \frac{bx}{2+3x^4}\,dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

```
[In] Int[(b*x)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b\int \frac{x}{2+3x^4}\,dx\\ &= \frac{1}{2}b\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right)\\ &= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{bx}{2+3x^4}\,dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

```
[In] Integrate[(b*x)/(2 + 3*x^4),x]
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}$	16
risch	$\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}$	16
meijerg	$\frac{\sqrt{6}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,x^2}{2}\right)}{12}$	19

```
[In] int(b*x/(3*x^4+2),x,method=_RETURNVERBOSE)
[Out] 1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{bx}{2+3x^4}\,dx = \frac{1}{12}\,\sqrt{6}b\arctan\left(\frac{1}{2}\,\sqrt{6}x^2\right)$$

```
[In] integrate(b*x/(3*x^4+2),x, algorithm="fricas")
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{bx}{2+3x^4}\,dx = \frac{\sqrt{6}b\operatorname{atan}\left(\frac{\sqrt{6}x^2}{2}\right)}{12}$$

```
[In] integrate(b*x/(3*x**4+2),x)
[Out] sqrt(6)*b*atan(sqrt(6)*x**2/2)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{bx}{2+3x^4}\,dx = \frac{1}{12}\,\sqrt{6}b\arctan\left(\frac{1}{2}\,\sqrt{6}x^2\right)$$

```
[In] integrate(b*x/(3*x^4+2),x, algorithm="maxima")
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{bx}{2+3x^4}\,dx = \frac{1}{12}\,\sqrt{6}b\arctan\left(\frac{1}{2}\,\sqrt{6}x^2\right)$$

```
[In] integrate(b*x/(3*x^4+2),x, algorithm="giac")
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{bx}{2+3x^4}\,dx=\frac{\sqrt{6}\,b\,\mathrm{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{12}$$

```
[In] int((b*x)/(3*x^4 + 2),x)
[Out] (6^(1/2)*b*atan((6^(1/2)*x^2)/2))/12
```

3.154 $\int \frac{a+bx}{2+3x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 123

$$\int \frac{a+bx}{2+3x^4}\,dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{a\arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a\arctan\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}$$

```
[Out] 1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)-1/48
*a*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)+1/48*a*ln(6^(3/4)*x+3*x^2+6^(1/2))*
6^(3/4)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1890, 217, 1179, 642, 1176, 631, 210, 281, 209}

$$\int \frac{a+bx}{2+3x^4}\,dx = -\frac{a\arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a\arctan\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}} - \frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}} + \frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}} + \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

```
[In] Int[(a + b*x)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4
)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a}{2+3x^4}+\frac{bx}{2+3x^4}\right) dx \\
&= a\int \frac{1}{2+3x^4}\,dx+b\int \frac{x}{2+3x^4}\,dx \\
&= \frac{a\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}}+\frac{a\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}}+\frac{1}{2}b\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx,x,x^2\right) \\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{a\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}}+\frac{a\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} \\
&\quad -\frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}}-\frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}} \\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} \\
&\quad +\frac{a\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}-\frac{a\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} \\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} \\
&\quad -\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.87

$$\int \frac{a+bx}{2+3x^4}\,dx$$
$$= \frac{-2\left(\sqrt[4]{6}a+2b\right)\arctan\left(1-\sqrt[4]{6}x\right)+2\left(\sqrt[4]{6}a-2b\right)\arctan\left(1+\sqrt[4]{6}x\right)+\sqrt[4]{6}a\left(-\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)\right.}{8\sqrt{6}}$$

```
[In] Integrate[(a + b*x)/(2 + 3*x^4),x]
```

```
[Out] (-2*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*(6^(1/4)*a - 2*b)*ArcTan[1
+ 6^(1/4)*x] + 6^(1/4)*a*(-Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + Log[2 + 2*6
^(1/4)*x + Sqrt[6]*x^2]))/(8*Sqrt[6])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.24

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_Rb+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}$
meijerg	$\frac{\sqrt{6}\,b\arctan\left(\frac{\sqrt{2}\sqrt{3}\,x^2}{2}\right)}{12}+\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{\ldots}}{2}\right.}{2\left(x^4\right)^{\frac{1}{4}}}\right.}{96}$

```
[In] int((b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*sum((_R*b+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 12348, normalized size of antiderivative = 100.39

$$\int \frac{a+bx}{2+3x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)/(3*x^4+2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.72

$$\int \frac{a+bx}{2+3x^4}\,dx = \text{RootSum}\left(18432t^4+384t^2b^2-96ta^2b+3a^4+2b^4, \left(t\mapsto t\log\left(x+\frac{3072t^3b^2+192t^2a^2b+24ta^4+32tb}{3a^5-8ab^4}\right.\right.\right.$$

```
[In] integrate((b*x+a)/(3*x**4+2),x)
[Out] RootSum(18432*_t**4 + 384*_t**2*b**2 - 96*_t*a**2*b + 3*a**4 + 2*b**4, Lamb
da(_t, _t*log(x + (3072*_t**3*b**2 + 192*_t**2*a**2*b + 24*_t*a**4 + 32*_t*
b**4 - 10*a**2*b**3)/(3*a**5 - 8*a*b**4))))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{a+bx}{2+3x^4}\,dx &= \frac{1}{48}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)-\frac{1}{48}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\\ &\quad+\frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a-2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\\ &\quad+\frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\end{aligned}$$

```
[In] integrate((b*x+a)/(3*x^4+2),x, algorithm="maxima")
[Out] 1/48*3^(3/4)*2^(3/4)*a*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) - 1/4
8*3^(3/4)*2^(3/4)*a*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*s
qrt(3)*(3^(1/4)*2^(3/4)*a - 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt
(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a + 2*sqrt(2)*b)*
arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int \frac{a+bx}{2+3x^4}\,dx = \frac{1}{48}\cdot 6^{\frac{3}{4}}a\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) - \frac{1}{48}\cdot 6^{\frac{3}{4}}a\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a-2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a+2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$

```
[In] integrate((b*x+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/48*6^(3/4)*a*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*6^(3/4)*
a*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(3/4)*a - 2*sqrt(6
)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^
(3/4)*a + 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^
(1/4)))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.97

$$\int \frac{a+bx}{2+3x^4}\,dx = \frac{2^{3/4}\,3^{3/4}\,a\,\ln\left(x^2+\frac{6^{3/4}\,x}{3}+\frac{\sqrt{6}}{3}\right)}{48} - \frac{2^{3/4}\,3^{3/4}\,a\,\ln\left(x^2-\frac{6^{3/4}\,x}{3}+\frac{\sqrt{6}}{3}\right)}{48} + \frac{2^{3/4}\,3^{3/4}\,a\,\mathrm{atan}\left(6^{1/4}\,x-1\right)}{24} + \frac{2^{3/4}\,3^{3/4}\,a\,\mathrm{atan}\left(6^{1/4}\,x+1\right)}{24} + \frac{\sqrt{2}\,\sqrt{3}\,b\,\mathrm{atan}\left(6^{1/4}\,x-1\right)}{12} - \frac{\sqrt{2}\,\sqrt{3}\,b\,\mathrm{atan}\left(6^{1/4}\,x+1\right)}{12}$$

```
[In] int((a + b*x)/(3*x^4 + 2),x)
```

```
[Out] (2^(3/4)*3^(3/4)*a*log((6^(3/4)*x)/3 + 6^(1/2)/3 + x^2))/48 - (2^(3/4)*3^(3
/4)*a*log(6^(1/2)/3 - (6^(3/4)*x)/3 + x^2))/48 + (2^(3/4)*3^(3/4)*a*atan(6^
(1/4)*x - 1))/24 + (2^(3/4)*3^(3/4)*a*atan(6^(1/4)*x + 1))/24 + (2^(1/2)*3^
(1/2)*b*atan(6^(1/4)*x - 1))/12 - (2^(1/2)*3^(1/2)*b*atan(6^(1/4)*x + 1))/1
2
```

3.155 $\int \frac{cx^2}{2+3x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 101

$$\int \frac{cx^2}{2+3x^4}\,dx = -\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\arctan\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}-\frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}$$

```
[Out] 1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)+1/24
*c*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)-1/24*c*ln(6^(3/4)*x+3*x^2+6^(1/2))*
6^(1/4)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {12, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{cx^2}{2+3x^4}\,dx = -\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\arctan\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}} + \frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}$$

```
[In] Int[(c*x^2)/(2 + 3*x^4),x]
```

```
[Out] -1/2*(c*ArcTan[1 - 6^(1/4)*x])/6^(3/4) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/
4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(4*6^(3/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c\int \frac{x^2}{2+3x^4}\,dx \\
&= -\frac{c\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}}+\frac{c\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}} \\
&= \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx+\frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx \\
&\quad +\frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\ 6^{3/4}}+\frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\ 6^{3/4}} \\
&= \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}-\frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} \\
&\quad +\frac{c\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}-\frac{c\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}} \\
&= -\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}} \\
&\quad +\frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}-\frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.77

$$\int \frac{cx^2}{2+3x^4}\,dx = \frac{c\left(-2\arctan\left(1-\sqrt[4]{6}x\right)+2\arctan\left(1+\sqrt[4]{6}x\right)+\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)-\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)\right)}{4\ 6^{3/4}}$$

```
[In] Integrate[(c*x^2)/(2 + 3*x^4),x]
[Out] (c*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] + Log[2 - 2*6^(1/4)*
x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(4*6^(3/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.25

method	result
risch	$\frac{c\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^{4}+2\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{12}$
default	$\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{144}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^{3}\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}-\frac{x^{3}\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\ldots}{8+3^{\frac{1}{4}}8\ldots}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}\ldots\right.}{216}$

```
[In] int(c*x^2/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*c*sum(1/_R*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \frac{cx^2}{2+3x^4}\,dx = {} & \frac{1}{216}\cdot 54^{\frac{3}{4}}\left(-c^4\right)^{\frac{1}{4}}\log\left(3\,c^3x+54^{\frac{1}{4}}\left(-c^4\right)^{\frac{3}{4}}\right) \\ & -\frac{1}{216}i\cdot 54^{\frac{3}{4}}\left(-c^4\right)^{\frac{1}{4}}\log\left(3\,c^3x+i\cdot 54^{\frac{1}{4}}\left(-c^4\right)^{\frac{3}{4}}\right) \\ & +\frac{1}{216}i\cdot 54^{\frac{3}{4}}\left(-c^4\right)^{\frac{1}{4}}\log\left(3\,c^3x-i\cdot 54^{\frac{1}{4}}\left(-c^4\right)^{\frac{3}{4}}\right) \\ & -\frac{1}{216}\cdot 54^{\frac{3}{4}}\left(-c^4\right)^{\frac{1}{4}}\log\left(3\,c^3x-54^{\frac{1}{4}}\left(-c^4\right)^{\frac{3}{4}}\right)\end{aligned}$$

```
[In] integrate(c*x^2/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/216*54^(3/4)*(-c^4)^(1/4)*log(3*c^3*x + 54^(1/4)*(-c^4)^(3/4)) - 1/216*I*
54^(3/4)*(-c^4)^(1/4)*log(3*c^3*x + I*54^(1/4)*(-c^4)^(3/4)) + 1/216*I*54^(
3/4)*(-c^4)^(1/4)*log(3*c^3*x - I*54^(1/4)*(-c^4)^(3/4)) - 1/216*54^(3/4)*(
-c^4)^(1/4)*log(3*c^3*x - 54^(1/4)*(-c^4)^(3/4))
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{cx^2}{2+3x^4}\,dx = c\left(\frac{\sqrt[4]{6}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}-\frac{\sqrt[4]{6}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}+\frac{\sqrt[4]{6}\,\mathrm{atan}\left(\sqrt[4]{6}x-1\right)}{12}+\frac{\sqrt[4]{6}\,\mathrm{atan}\left(\sqrt[4]{6}x+1\right)}{12}\right)$$

```
[In] integrate(c*x**2/(3*x**4+2),x)
```

```
[Out] c*(6**(1/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/24 - 6**(1/4)*log(x**2 + 6
**(3/4)*x/3 + sqrt(6)/3)/24 + 6**(1/4)*atan(6**(1/4)*x - 1)/12 + 6**(1/4)*a
tan(6**(1/4)*x + 1)/12)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.22

$$\int \frac{cx^2}{2+3x^4}\,dx = \frac{1}{24}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-3^{\frac{1}{4}}2^{\frac{1}{4}}\,\mathrm{l}\right.$$

```
[In] integrate(c*x^2/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*(2*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 2*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1
/4)*2^(3/4))) - 3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(
2)) + 3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)))*c
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{cx^2}{2+3x^4}\,dx = \frac{1}{24}\left(2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\right.$$

```
[In] integrate(c*x^2/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) -
6^(1/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 6^(1/4)*log(x^2 - sq
rt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*c
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.32

$$\int \frac{cx^2}{2+3x^4}\,dx = \frac{(-1)^{1/4}\,24^{1/4}\,c\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\,24^{1/4}\,x}{2}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\,24^{1/4}\,x}{2}\right)\right)}{12}$$

```
[In] int((c*x^2)/(3*x^4 + 2),x)
```

```
[Out] ((-1)^(1/4)*24^(1/4)*c*(atan(((-1)^(1/4)*24^(1/4)*x)/2) - atanh(((-1)^(1/4)
*24^(1/4)*x)/2)))/12
```

3.156 $\int \frac{a+cx^2}{2+3x^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 141

$$\int \frac{a+cx^2}{2+3x^4} dx = -\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}} - \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}$$

```
[Out] -1/48*ln(-6^(3/4)*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1/4)+1/48*ln(6^(3/4)
*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(-1+6^(1/4)*x)*(2*c+a
*6^(1/2))*6^(1/4)+1/24*arctan(1+6^(1/4)*x)*(2*c+a*6^(1/2))*6^(1/4)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1182, 1176, 631, 210, 1179, 642}

$$\int \frac{a+cx^2}{2+3x^4} dx = -\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\arctan\left(\sqrt[4]{6}x+1\right)}{4\ 6^{3/4}} - \frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}$$

```
[In] Int[(a + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] -1/4*((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/6^(3/4) + ((Sqrt[6]*a + 2*c)
*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3
/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x +
3*x^2])/(8*6^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rubi steps

$$\text{integral} = \frac{1}{12}\left(\sqrt{6}a-2c\right)\int\frac{\sqrt{6}-3x^2}{2+3x^4}\,dx+\frac{1}{12}\left(\sqrt{6}a+2c\right)\int\frac{\sqrt{6}+3x^2}{2+3x^4}\,dx$$

$$= -\frac{\left(\sqrt{6}a-2c\right)\int\frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\int\frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}}+\frac{1}{24}\left(\sqrt{6}a+2c\right)\int\frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx+\frac{1}{24}\left(\sqrt{6}a+2c\right)\int\frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx$$

$$= -\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}}$$

$$= -\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

$$\int\frac{a+cx^2}{2+3x^4}\,dx$$

$$= \frac{-2\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)+2\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)-\left(\sqrt{6}a-2c\right)\left(\log\left(2-2\sqrt[4]{6}x+\sqrt{\ }\right.\right.}{8\ 6^{3/4}}$$

```
[In] Integrate[(a + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (-2*(Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x] + 2*(Sqrt[6]*a + 2*c)*ArcTan[1
+ 6^(1/4)*x] - (Sqrt[6]*a - 2*c)*(Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - Log[
2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(3/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.22

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)} \frac{\left(\_R^2 c+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\cdots}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\cdots}\right.\cdots\right.}{\cdots}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\cdots}}{8+3^{\frac{1}{4}}8^{\cdots}}\right)}{\left(x^4\right)^{\frac{3}{4}}}\cdots\right)}{216}$

```
[In] int((c*x^2+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*sum((_R^2*c+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(104) = 208.

Time = 0.28 (sec) , antiderivative size = 493, normalized size of antiderivative = 3.50

$$\int \frac{a+cx^2}{2+3x^4}\,dx$$
$$= -\frac{1}{24}\sqrt{-12\,ac+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\log\left(-3\left(9\,a^4-4\,c^4\right)x+\left(9\,a^3-6\,ac^2-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}c\right)\sqrt{-12\,ac+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\right)$$
$$+\frac{1}{24}\sqrt{-12\,ac+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\log\left(-3\left(9\,a^4-4\,c^4\right)x-\left(9\,a^3-6\,ac^2-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}c\right)\sqrt{-12\,ac+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\right)$$
$$-\frac{1}{24}\sqrt{-12\,ac-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\log\left(-3\left(9\,a^4-4\,c^4\right)x+\left(9\,a^3-6\,ac^2+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}c\right)\sqrt{-12\,ac-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\right)$$
$$+\frac{1}{24}\sqrt{-12\,ac-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\log\left(-3\left(9\,a^4-4\,c^4\right)x-\left(9\,a^3-6\,ac^2+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}c\right)\sqrt{-12\,ac-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\right)$$

```
[In] integrate((c*x^2+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))*log(-3*(9*a
^4 - 4*c^4)*x + (9*a^3 - 6*a*c^2 - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4
)*c)*sqrt(-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))) + 1/24*sqrt
(-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))*log(-3*(9*a^4 - 4*c^4
)*x - (9*a^3 - 6*a*c^2 - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)*c)*sqrt(
-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))) - 1/24*sqrt(-12*a*c -
 sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))*log(-3*(9*a^4 - 4*c^4)*x + (9*a
^3 - 6*a*c^2 + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)*c)*sqrt(-12*a*c -
sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))) + 1/24*sqrt(-12*a*c - sqrt(6)*s
qrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))*log(-3*(9*a^4 - 4*c^4)*x - (9*a^3 - 6*a*c
^2 + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)*c)*sqrt(-12*a*c - sqrt(6)*sq
rt(-9*a^4 + 12*a^2*c^2 - 4*c^4)))
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.48

$$\int \frac{a+cx^2}{2+3x^4}\,dx$$
$$=\operatorname{RootSum}\left(55296t^4+2304t^2ac+9a^4+12a^2c^2+4c^4,\left(t\mapsto t\log\left(x+\frac{-4608t^3c+72ta^3-144tac^2}{9a^4-4c^4}\right)\right)\right)$$

```
[In] integrate((c*x**2+a)/(3*x**4+2),x)
```

```
[Out] RootSum(55296*_t**4 + 2304*_t**2*a*c + 9*a**4 + 12*a**2*c**2 + 4*c**4, Lamb
da(_t, _t*log(x + (-4608*_t**3*c + 72*_t*a**3 - 144*_t*a*c**2)/(9*a**4 - 4*
c**4))))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{a+cx^2}{2+3x^4}\,dx =& \frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a+\sqrt{2}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\\ &+\frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a+\sqrt{2}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\\ &+\frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\\ &-\frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\end{aligned}$$

```
[In] integrate((c*x^2+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*3^(1/4)*2^(3/4)*(sqrt(3)*a + sqrt(2)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*
sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*3^(1/4)*2^(3/4)*(sqrt(3)*a + sqrt(2)*c
)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 1/48*3^(1/4
)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqr
t(2)) - 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 - 3^(1
/4)*2^(3/4)*x + sqrt(2))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int \frac{a+cx^2}{2+3x^4}\,dx = \frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

```
[In] integrate((c*x^2+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2
)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(
3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*log(x^2
+ sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*log(x
^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.23

$$\int \frac{a+cx^2}{2+3x^4}\,dx = -2\,\mathrm{atanh}\left(\frac{216\,a^2\,x\,\sqrt{-\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3+18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2-12\,c^3} - \frac{144\,c^2\,x\,\sqrt{-\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3+18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2-12\,c^3}\right)\sqrt{-\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}} + 2\,\mathrm{atanh}\left(\frac{216\,a^2\,x\,\sqrt{\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3-18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2+12\,c^3} - \frac{144\,c^2\,x\,\sqrt{\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3-18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2+12\,c^3}\right)\sqrt{\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}$$

```
[In] int((a + c*x^2)/(3*x^4 + 2),x)
```

```
[Out] 2*atanh((216*a^2*x*((6^(1/2)*a^2*1i)/192 - (a*c)/48 - (6^(1/2)*c^2*1i)/288)
^(1/2))/(6^(1/2)*a^3*9i - 18*a^2*c + 12*c^3 - 6^(1/2)*a*c^2*6i) - (144*c^2*
x*((6^(1/2)*a^2*1i)/192 - (a*c)/48 - (6^(1/2)*c^2*1i)/288)^(1/2))/(6^(1/2)*
a^3*9i - 18*a^2*c + 12*c^3 - 6^(1/2)*a*c^2*6i))*((6^(1/2)*a^2*1i)/192 - (a*
c)/48 - (6^(1/2)*c^2*1i)/288)^(1/2) - 2*atanh((216*a^2*x*((6^(1/2)*c^2*1i)/
288 - (6^(1/2)*a^2*1i)/192 - (a*c)/48)^(1/2))/(6^(1/2)*a^3*9i + 18*a^2*c -
12*c^3 - 6^(1/2)*a*c^2*6i) - (144*c^2*x*((6^(1/2)*c^2*1i)/288 - (6^(1/2)*a^
2*1i)/192 - (a*c)/48)^(1/2))/(6^(1/2)*a^3*9i + 18*a^2*c - 12*c^3 - 6^(1/2)*
a*c^2*6i))*((6^(1/2)*c^2*1i)/288 - (6^(1/2)*a^2*1i)/192 - (a*c)/48)^(1/2)
```

3.157 $\int \frac{bx+cx^2}{2+3x^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{bx+cx^2}{2+3x^4}\,dx=\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\arctan\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}-\frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}$$

```
[Out] 1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)+1/24
*c*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)-1/24*c*ln(6^(3/4)*x+3*x^2+6^(1/2))*
6^(1/4)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1607, 1845, 281, 209, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{bx+cx^2}{2+3x^4}\,dx=\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\arctan\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}$$

```
[In] Int[(b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4
)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(b+cx)}{2+3x^4}\,dx \\
&= \int \left(\frac{bx}{2+3x^4}+\frac{cx^2}{2+3x^4}\right)dx \\
&= b\int \frac{x}{2+3x^4}\,dx + c\int \frac{x^2}{2+3x^4}\,dx \\
&= \frac{1}{2}b\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right) - \frac{c\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}} + \frac{c\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}} \\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx \\
&\quad + \frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\ 6^{3/4}} + \frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\ 6^{3/4}} \\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} \\
&\quad + \frac{c\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}} - \frac{c\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}}
\end{aligned}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

$$\int \frac{bx+cx^2}{2+3x^4}\,dx$$
$$= \frac{-2\left(\sqrt[4]{6}b+c\right)\arctan\left(1-\sqrt[4]{6}x\right)+2\left(-\sqrt[4]{6}b+c\right)\arctan\left(1+\sqrt[4]{6}x\right)+c\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)-c\log\left(\right.}{4\ 6^{3/4}}$$

```
[In] Integrate[(b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (-2*(6^(1/4)*b + c)*ArcTan[1 - 6^(1/4)*x] + 2*(-(6^(1/4)*b) + c)*ArcTan[1 +
 6^(1/4)*x] + c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - c*Log[2 + 2*6^(1/4)*x
+ Sqrt[6]*x^2])/(4*6^(3/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.27

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^2c+\_Rb\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{144}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\ \ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\ \arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ \ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\ \arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}}{8+3^{\frac{1}{4}}8}\right)}{\left(x^4\right)^{\frac{3}{4}}}\right.}{216}$

```
[In] int((c*x^2+b*x)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*sum((_R^2*c+_R*b)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 12741, normalized size of antiderivative = 103.59

$$\int \frac{bx + cx^2}{2 + 3x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

$$\int \frac{bx + cx^2}{2 + 3x^4}\,dx = \text{RootSum}\left(27648t^4 + 576t^2b^2 + 96tbc^2 + 3b^4 + 2c^4, \left(t \mapsto t\log\left(x + \frac{-1152t^3c^2 + 288t^2b^3 - 36tb^2c^2 + 3b\right.\right.\right.}{6b^4c - c^5}$$

```
[In] integrate((c*x**2+b*x)/(3*x**4+2),x)
[Out] RootSum(27648*_t**4 + 576*_t**2*b**2 + 96*_t*b*c**2 + 3*b**4 + 2*c**4, Lamb
da(_t, _t*log(x + (-1152*_t**3*c**2 + 288*_t**2*b**3 - 36*_t*b**2*c**2 + 3*
b**5 - 3*b*c**4)/(6*b**4*c - c**5))))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{bx + cx^2}{2 + 3x^4}\,dx &= \frac{1}{24}\sqrt{2}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}c - 2\sqrt{3}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x + 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) \\ &\quad + \frac{1}{24}\sqrt{2}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}c + 2\sqrt{3}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x - 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) - \frac{1}{24} \\ &\quad \cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\log\left(\sqrt{3}x^2 + 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right) + \frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\log\left(\sqrt{3}x^2 - 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right)\end{aligned}$$

```
[In] integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="maxima")
[Out] 1/24*sqrt(2)*(3^(1/4)*2^(3/4)*c - 2*sqrt(3)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(
2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sqrt(2)*(3^(1/4)*2^(3/4)*c + 2*sqrt(
3)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) - 1/24*3^
(1/4)*2^(1/4)*c*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*3^(1/
4)*2^(1/4)*c*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = -\frac{1}{24}\cdot 6^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) + \frac{1}{24}\cdot 6^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) - \frac{1}{12}\left(\sqrt{6}b-6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\left(\sqrt{6}b+6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$

```
[In] integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/24*6^(1/4)*c*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*6^(1/4)
*c*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/12*(sqrt(6)*b - 6^(1/4)
*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/12*(sqr
t(6)*b + 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/
4)))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.32

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = \sum_{k=1}^{4}\ln\left(9\,b^3\,x-6\,c^3-\mathrm{root}\left(z^4+\frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)\,b\,c\,144 + \mathrm{root}\left(z^4+\frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)^2\,b\,x\,864 + \mathrm{root}\left(z^4+\frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)\,c^2\,x\,72\right)\,\mathrm{root}\left(z^4 + \frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)$$

```
[In] int((b*x + c*x^2)/(3*x^4 + 2),x)
```

```
[Out] symsum(log(9*b^3*x - 6*c^3 - 144*root(z^4 + (b^2*z^2)/48 + (b*c^2*z)/288 +
c^4/13824 + b^4/9216, z, k)*b*c + 864*root(z^4 + (b^2*z^2)/48 + (b*c^2*z)/2
88 + c^4/13824 + b^4/9216, z, k)^2*b*x + 72*root(z^4 + (b^2*z^2)/48 + (b*c^
2*z)/288 + c^4/13824 + b^4/9216, z, k)*c^2*x)*root(z^4 + (b^2*z^2)/48 + (b*
c^2*z)/288 + c^4/13824 + b^4/9216, z, k), k, 1, 4)
```

3.158 $\int \frac{a+bx+cx^2}{2+3x^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 163

$$\int \frac{a+bx+cx^2}{2+3x^4} \, dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}} + \frac{\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}} - \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}$$

```
[Out] 1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)-1/48*ln(-6^(3/4)*x+3*x^2+6^(1/2))*(-
2*c+a*6^(1/2))*6^(1/4)+1/48*ln(6^(3/4)*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^
(1/4)+1/24*arctan(-1+6^(1/4)*x)*(2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(1+6^(1/
4)*x)*(2*c+a*6^(1/2))*6^(1/4)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used

= {1890, 281, 209, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx = -\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}} + \frac{\left(\sqrt{6}a+2c\right)\arctan\left(\sqrt[4]{6}x+1\right)}{4\ 6^{3/4}} - \frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}} + \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

```
[In] Int[(a + b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/
4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4))
- ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt
[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{bx}{2+3x^4}+\frac{a+cx^2}{2+3x^4}\right) dx\\ &= b\int \frac{x}{2+3x^4}\,dx+\int \frac{a+cx^2}{2+3x^4}\,dx\\ &= \frac{1}{2}b\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx,x,x^2\right)+\frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{2+3x^4}\,dx\\ &\quad+\frac{1}{12}\left(\sqrt{6}a+2c\right)\int \frac{\sqrt{6}+3x^2}{2+3x^4}\,dx\end{aligned}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}a-2c\right)\int\frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}} - \frac{\left(\sqrt{6}a-2c\right)\int\frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}} + \frac{1}{24}\left(\sqrt{6}a+2c\right)\int\frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{1}{24}\left(\sqrt{6}a+2c\right)\int\frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{\left(\sqrt{6}a+2c\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}} - \frac{\left(\sqrt{6}a+2c\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}} + \frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}} - \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.79

$$\int\frac{a+bx+cx^2}{2+3x^4}\,dx = \frac{-2\left(\sqrt{6}a+2\left(\sqrt[4]{6}b+c\right)\right)\arctan\left(1-\sqrt[4]{6}x\right)+2\left(\sqrt{6}a-2\sqrt[4]{6}b+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)-\left(\sqrt{6}a-2c\right)\left(\text{lo}\right.}{8\ 6^{3/4}}$$

```
[In] Integrate[(a + b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (-2*(Sqrt[6]*a + 2*(6^(1/4)*b + c))*ArcTan[1 - 6^(1/4)*x] + 2*(Sqrt[6]*a -
2*6^(1/4)*b + 2*c)*ArcTan[1 + 6^(1/4)*x] - (Sqrt[6]*a - 2*c)*(Log[2 - 2*6^(
1/4)*x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(3/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.21

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)} \frac{\left(\_R^2 c+\_R b+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\cdots}{\cdots}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}\cdots}{8+3\cdots}\right)}{\left(x^4\right)^{\cdots}}\cdots\right)}{216}$

[In] int((c*x^2+b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)

[Out] 1/12*sum((_R^2*c+_R*b+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 46651, normalized size of antiderivative = 286.20

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx = \text{Too large to display}$$

[In] integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")

[Out] Too large to include

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(144) = 288.

Time = 2.39 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.79

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx$$

$$=\operatorname{RootSum}\Bigl(55296t^4+t^2\cdot\left(2304ac+1152b^2\right)+t\left(-288a^2b+192bc^2\right)+9a^4+12a^2c^2-24ab^2c+6b^4+4c\cdots$$

```
[In] integrate((c*x**2+b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(55296*_t**4 + _t**2*(2304*a*c + 1152*b**2) + _t*(-288*a**2*b + 192*
b*c**2) + 9*a**4 + 12*a**2*c**2 - 24*a*b**2*c + 6*b**4 + 4*c**4, Lambda(_t,
 _t*log(x + (-13824*_t**3*a**2*c + 27648*_t**3*a*b**2 + 9216*_t**3*c**3 + 1
728*_t**2*a**3*b + 3456*_t**2*a*b*c**2 - 2304*_t**2*b**3*c + 216*_t*a**5 -
576*_t*a**3*c**2 + 1296*_t*a**2*b**2*c + 288*_t*a*b**4 + 288*_t*a*c**4 + 28
8*_t*b**2*c**3 + 90*a**4*b*c - 90*a**3*b**3 + 60*a*b**3*c**2 - 24*b**5*c +
24*b*c**5)/(27*a**6 - 18*a**4*c**2 + 144*a**3*b**2*c - 72*a**2*b**4 - 12*a*
*2*c**4 + 96*a*b**2*c**3 - 48*b**4*c**2 + 8*c**6))))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.15

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx = \frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) - \frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{24}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}a-2\sqrt{3}\sqrt{2}b+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{24}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}a+2\sqrt{3}\sqrt{2}b+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

```
[In] integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3
/4)*x + sqrt(2)) - 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)
*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*(3^(3/4)*2^(3/4)*a - 2*sqrt(3)*s
qrt(2)*b + 2*3^(1/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3
^(1/4)*2^(3/4))) + 1/24*(3^(3/4)*2^(3/4)*a + 2*sqrt(3)*sqrt(2)*b + 2*3^(1/4
)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{a+bx+cx^2}{2+3x^4}\,dx &= \frac{1}{24}\left(6^{\frac{3}{4}}a-2\sqrt{6}b+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\\ &\quad+\frac{1}{24}\left(6^{\frac{3}{4}}a+2\sqrt{6}b+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\\ &\quad+\frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)\\ &\quad-\frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)\end{aligned}$$

```
[In] integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)
*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b + 2*6^(1/4)*c
)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/
4)*a - 2*6^(1/4)*c)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^
(3/4)*a - 2*6^(1/4)*c)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.66

$$\begin{aligned}\int \frac{a+bx+cx^2}{2+3x^4}\,dx = \sum_{k=1}^{4}\ln\Bigg(9\,a\,b^2-9\,a^2\,c\\ -\mathrm{root}\left(z^4+\frac{z^2\,(2304\,a\,c+1152\,b^2)}{55296}-\frac{z\,(288\,a^2\,b-192\,b\,c^2)}{55296}-\frac{a\,b^2\,c}{2304}+\frac{a^2\,c^2}{4608}+\frac{c^4}{13824}+\frac{b^4}{9216}\right.\\ \left.+\frac{a^4}{6144},z,k\right)\left(\mathrm{root}\left(z^4+\frac{z^2\,(2304\,a\,c+1152\,b^2)}{55296}-\frac{z\,(288\,a^2\,b-192\,b\,c^2)}{55296}-\frac{a\,b^2\,c}{2304}+\frac{a^2\,c^2}{4608}+\frac{c^4}{13824}+\frac{b^4}{9216}\right.\right.\\ \left.\left.+144\,b\,c+x\,(108\,a^2-72\,c^2)\right)-6\,c^3+x\,(9\,b^3-18\,a\,b\,c)\right)\,\mathrm{root}\Bigg(z^4+\frac{z^2\,(2304\,a\,c+1152\,b^2)}{55296}\\ -\frac{z\,(288\,a^2\,b-192\,b\,c^2)}{55296}-\frac{a\,b^2\,c}{2304}+\frac{a^2\,c^2}{4608}+\frac{c^4}{13824}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\Bigg)\end{aligned}$$

```
[In] int((a + b*x + c*x^2)/(3*x^4 + 2),x)
```

```
[Out] symsum(log(9*a*b^2 - 9*a^2*c - root(z^4 + (z^2*(2304*a*c + 1152*b^2))/55296
 - (z*(288*a^2*b - 192*b*c^2))/55296 - (a*b^2*c)/2304 + (a^2*c^2)/4608 + c^
4/13824 + b^4/9216 + a^4/6144, z, k)*(root(z^4 + (z^2*(2304*a*c + 1152*b^2)
)/55296 - (z*(288*a^2*b - 192*b*c^2))/55296 - (a*b^2*c)/2304 + (a^2*c^2)/46
08 + c^4/13824 + b^4/9216 + a^4/6144, z, k)*(864*a - 864*b*x) + 144*b*c + x
*(108*a^2 - 72*c^2)) - 6*c^3 + x*(9*b^3 - 18*a*b*c))*root(z^4 + (z^2*(2304*
a*c + 1152*b^2))/55296 - (z*(288*a^2*b - 192*b*c^2))/55296 - (a*b^2*c)/2304
 + (a^2*c^2)/4608 + c^4/13824 + b^4/9216 + a^4/6144, z, k), k, 1, 4)
```

3.159 $\int \frac{dx^3}{2+3x^4} dx$

Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{dx^3}{2+3x^4} dx = \frac{1}{12} d \log \left(2+3x^4\right)$$

```
[Out] 1/12*d*ln(3*x^4+2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 266}

$$\int \frac{dx^3}{2+3x^4} dx = \frac{1}{12} d \log \left(3x^4+2\right)$$

```
[In] Int[(d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (d*Log[2 + 3*x^4])/12
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\text{integral} = d\int \frac{x^3}{2+3x^4}\,dx$$
$$= \frac{1}{12}d\log\left(2+3x^4\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{1}{12}d\log\left(2+3x^4\right)$$

```
[In] Integrate[(d*x^3)/(2 + 3*x^4),x]
[Out] (d*Log[2 + 3*x^4])/12
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{d\ln\left(x^4+\frac{2}{3}\right)}{12}$	10
derivativedivides	$\frac{d\ln\left(3x^4+2\right)}{12}$	12
default	$\frac{d\ln\left(3x^4+2\right)}{12}$	12
norman	$\frac{d\ln\left(3x^4+2\right)}{12}$	12
meijerg	$\frac{d\ln\left(\frac{3x^4}{2}+1\right)}{12}$	12
risch	$\frac{d\ln\left(3x^4+2\right)}{12}$	12

```
[In] int(d*x^3/(3*x^4+2),x,method=_RETURNVERBOSE)
[Out] 1/12*d*ln(x^4+2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{1}{12}\,d\log\left(3\,x^4+2\right)$$

```
[In] integrate(d*x^3/(3*x^4+2),x, algorithm="fricas")
[Out] 1/12*d*log(3*x^4 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{d\log\left(3x^4+2\right)}{12}$$

```
[In] integrate(d*x**3/(3*x**4+2),x)
[Out] d*log(3*x**4 + 2)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{1}{12}\,d\log\left(3\,x^4+2\right)$$

```
[In] integrate(d*x^3/(3*x^4+2),x, algorithm="maxima")
[Out] 1/12*d*log(3*x^4 + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{1}{12}\,d\log\left(3\,x^4+2\right)$$

```
[In] integrate(d*x^3/(3*x^4+2),x, algorithm="giac")
[Out] 1/12*d*log(3*x^4 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{d\,\ln\left(x^4+\frac{2}{3}\right)}{12}$$

```
[In] int((d*x^3)/(3*x^4 + 2),x)
```

```
[Out] (d*log(x^4 + 2/3))/12
```

3.160 $\int \frac{a+dx^3}{2+3x^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 114

$$\int \frac{a+dx^3}{2+3x^4} dx = -\frac{a \arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a \arctan\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a \log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a \log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{1}{12} d \log\left(2+3x^4\right)$$

```
[Out] 1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)+1/12
*d*ln(3*x^4+2)-1/48*a*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)+1/48*a*ln(6^(3/4
)*x+3*x^2+6^(1/2))*6^(3/4)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1890, 217, 1179, 642, 1176, 631, 210, 266}

$$\int \frac{a+dx^3}{2+3x^4} dx = -\frac{a \arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a \arctan\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}} - \frac{a \log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}} + \frac{a \log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}} + \frac{1}{12} d \log\left(3x^4+2\right)$$

```
[In] Int[(a + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] -1/4*(a*ArcTan[1 - 6^(1/4)*x])/6^(1/4) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/
4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (d*Log[2 + 3*x^4])/12
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a}{2+3x^4}+\frac{dx^3}{2+3x^4}\right) dx \\
&= a\int \frac{1}{2+3x^4}\,dx + d\int \frac{x^3}{2+3x^4}\,dx \\
&= \frac{1}{12}d\log\left(2+3x^4\right)+\frac{a\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}}+\frac{a\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}} \\
&= \frac{1}{12}d\log\left(2+3x^4\right)+\frac{a\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}}+\frac{a\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} \\
&\quad -\frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}}-\frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}} \\
&= -\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{1}{12}d\log\left(2+3x^4\right) \\
&\quad +\frac{a\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}-\frac{a\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} \\
&= -\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}-\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} \\
&\quad +\frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{1}{12}d\log\left(2+3x^4\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.95

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \frac{1}{48}\Big(-2 6^{3/4} a \arctan\left(1-\sqrt[4]{6}x\right) +2\, 6^{3/4} a \arctan\left(1+\sqrt[4]{6}x\right) - 6^{3/4} a \log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right) + 6^{3/4} a \log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right) + 4d\log\left(2+3x^4\right)\Big)$$

```
[In] Integrate[(a + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(3/4)*a*ArcTan[1 - 6^(1/4)*x] + 2*6^(3/4)*a*ArcTan[1 + 6^(1/4)*x] - 6
^(3/4)*a*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(3/4)*a*Log[2 + 2*6^(1/4)*x
 + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.46 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.27

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}(3\_Z^4+2)} \frac{(\_R^3 d+a)\ln(x-\_R)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{d\ln(3x^4+2)}{12}$
meijerg	$\frac{d\ln\left(\frac{3x^4}{2}+1\right)}{12}+\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\ln\left(1-6^{\frac{1}{4}}(x^4)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}(x^4)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}(x^4)^{\frac{1}{4}}}\right)}{(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\ln\left(1+6^{\frac{1}{4}}(x^4)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{}}{}\right.}{96}$

```
[In] int((d*x^3+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*sum((_R^3*d+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(83) = 166.

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.68

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \frac{1}{24}\left(\sqrt{3}\sqrt{\sqrt{6}\sqrt{-a^4}}+2\,d\right)\log\left(3\,ax+\sqrt{3}\sqrt{\sqrt{6}\sqrt{-a^4}}\right) - \frac{1}{24}\left(\sqrt{3}\sqrt{\sqrt{6}\sqrt{-a^4}}-2\,d\right)\log\left(3\,ax-\sqrt{3}\sqrt{\sqrt{6}\sqrt{-a^4}}\right) + \frac{1}{24}\left(\sqrt{3}\sqrt{-\sqrt{6}\sqrt{-a^4}}+2\,d\right)\log\left(3\,ax+\sqrt{3}\sqrt{-\sqrt{6}\sqrt{-a^4}}\right) - \frac{1}{24}\left(\sqrt{3}\sqrt{-\sqrt{6}\sqrt{-a^4}}-2\,d\right)\log\left(3\,ax-\sqrt{3}\sqrt{-\sqrt{6}\sqrt{-a^4}}\right)$$

```
[In] integrate((d*x^3+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(3)*sqrt(sqrt(6)*sqrt(-a^4)) + 2*d)*log(3*a*x + sqrt(3)*sqrt(sqrt
(6)*sqrt(-a^4))) - 1/24*(sqrt(3)*sqrt(sqrt(6)*sqrt(-a^4)) - 2*d)*log(3*a*x
- sqrt(3)*sqrt(sqrt(6)*sqrt(-a^4))) + 1/24*(sqrt(3)*sqrt(-sqrt(6)*sqrt(-a^4
)) + 2*d)*log(3*a*x + sqrt(3)*sqrt(-sqrt(6)*sqrt(-a^4))) - 1/24*(sqrt(3)*sq
rt(-sqrt(6)*sqrt(-a^4)) - 2*d)*log(3*a*x - sqrt(3)*sqrt(-sqrt(6)*sqrt(-a^4)
))
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.45

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \operatorname{RootSum}\left(165888t^4-55296t^3d+6912t^2d^2-384td^3+27a^4+8d^4,\left(t\mapsto t\log\left(x+\frac{24t-2d}{3a}\right)\right)\right)$$

```
[In] integrate((d*x**3+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + 6912*_t**2*d**2 - 384*_t*d**3 + 27*a
**4 + 8*d**4, Lambda(_t, _t*log(x + (24*_t - 2*d)/(3*a))))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.31

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d+3a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

```
[In] integrate((d*x^3+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*3^(3/4)*2^(3/4)*a*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^
(3/4))) + 1/24*3^(3/4)*2^(3/4)*a*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x -
3^(1/4)*2^(3/4))) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d + 3*a)*log(s
qrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4
)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{48}\left(6^{\frac{3}{4}}a+4d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)-\frac{1}{48}\left(6^{\frac{3}{4}}a-4d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

```
[In] integrate((d*x^3+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
+ 1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))
) + 1/48*(6^(3/4)*a + 4*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1
/48*(6^(3/4)*a - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \ln\left(x - \frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12} - \frac{6^{1/4}\sqrt{\frac{3}{4}\mathrm{i}}\,a}{12}\right) + \ln\left(x + \frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12} + \frac{6^{1/4}\sqrt{\frac{3}{4}\mathrm{i}}\,a}{12}\right) + \ln\left(x - \frac{(-1)\right.$$

```
[In] int((a + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] log(x - ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(3i/4)^(1/2)*a)/12
) + log(x + ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(3i/4)^(1/2)*a
)/12) + log(x - ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(-3i/4)^(1
/2)*a)/12) + log(x + ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(-3i/
4)^(1/2)*a)/12)
```

3.161 $\int \frac{bx+dx^3}{2+3x^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \frac{bx+dx^3}{2+3x^4}\,dx=\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(2+3x^4\right)$$

```
[Out] 1/12*d*ln(3*x^4+2)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1607, 1262, 649, 209, 266}

$$\int \frac{bx+dx^3}{2+3x^4}\,dx=\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

```
[In] Int[(b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) + (d*Log[2 + 3*x^4])/12
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(b+dx^2)}{2+3x^4}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}b\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right)+\frac{1}{2}d\text{Subst}\left(\int \frac{x}{2+3x^2}\,dx, x, x^2\right)\\ &= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(2+3x^4\right)\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.81

$$\int \frac{bx+dx^3}{2+3x^4}\,dx = \frac{1}{24}\left(i\sqrt{6}b+2d\right)\log\left(\sqrt{6}-3ix^2\right)+\frac{1}{24}\left(-i\sqrt{6}b+2d\right)\log\left(\sqrt{6}+3ix^2\right)$$

```
[In] Integrate[(b*x + d*x^3)/(2 + 3*x^4),x]
[Out] ((I*Sqrt[6]*b + 2*d)*Log[Sqrt[6] - (3*I)*x^2])/24 + (((-I)*Sqrt[6]*b + 2*d)
*Log[Sqrt[6] + (3*I)*x^2])/24
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{d \ln(3x^4+2)}{12}+\frac{b \arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}$	28
risch	$\frac{d \ln(9x^4+6)}{12}+\frac{b \arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}$	28
meijerg	$\frac{d \ln\left(\frac{3x^4}{2}+1\right)}{12}+\frac{\sqrt{6}\, b \arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,x^2}{2}\right)}{12}$	31

```
[In] int((d*x^3+b*x)/(3*x^4+2),x,method=_RETURNVERBOSE)
[Out] 1/12*d*ln(3*x^4+2)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{bx+dx^3}{2+3x^4}\,dx=\frac{1}{12}\,\sqrt{6}b\arctan\left(\frac{1}{2}\,\sqrt{6}x^2\right)+\frac{1}{12}\,d\log\left(3\,x^4+2\right)$$

```
[In] integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="fricas")
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2) + 1/12*d*log(3*x^4 + 2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{bx+dx^3}{2+3x^4}\,dx=\left(-\frac{\sqrt{6}ib}{24}+\frac{d}{12}\right)\log\left(x^2-\frac{\sqrt{6}i}{3}\right)+\left(\frac{\sqrt{6}ib}{24}+\frac{d}{12}\right)\log\left(x^2+\frac{\sqrt{6}i}{3}\right)$$

```
[In] integrate((d*x**3+b*x)/(3*x**4+2),x)
[Out] (-sqrt(6)*I*b/24 + d/12)*log(x**2 - sqrt(6)*I/3) + (sqrt(6)*I*b/24 + d/12)*
log(x**2 + sqrt(6)*I/3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(27) = 54$.

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.14

$$\int \frac{bx + dx^3}{2 + 3x^4}\, dx = -\frac{1}{12}\sqrt{3}\sqrt{2}b \arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x + 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{12}\sqrt{3}\sqrt{2}b \arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x - 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{12}d\log\left(\sqrt{3}x^2 + 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right) + \frac{1}{12}d\log\left(\sqrt{3}x^2 - 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right)$$

```
[In] integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*sqrt(2)*b*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 1/12*sqrt(3)*sqrt(2)*b*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x -
 3^(1/4)*2^(3/4))) + 1/12*d*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2))
+ 1/12*d*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(27) = 54$.

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.58

$$\int \frac{bx + dx^3}{2 + 3x^4}\, dx = -\frac{1}{12}\sqrt{6}b \arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\sqrt{6}b \arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}d\log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) + \frac{1}{12}d\log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

```
[In] integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4)
)) + 1/12*d*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/12*d*log(x^2 -
 sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{bx+dx^3}{2+3x^4}\,dx = \frac{d\,\ln\left(x^4+\frac{2}{3}\right)}{12}+\frac{\sqrt{6}\,b\,\mathrm{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{12}$$

```
[In] int((b*x + d*x^3)/(3*x^4 + 2),x)
[Out] (d*log(x^4 + 2/3))/12 + (6^(1/2)*b*atan((6^(1/2)*x^2)/2))/12
```

3.162 $\int \frac{a+bx+dx^3}{2+3x^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 136

$$\int \frac{a+bx+dx^3}{2+3x^4} \, dx = \frac{b \arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{a \arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a \arctan\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a \log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a \log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{1}{12} d \log\left(2+3x^4\right)$$

```
[Out] 1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)+1/12
*d*ln(3*x^4+2)-1/48*a*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)+1/48*a*ln(6^(3/4
)*x+3*x^2+6^(1/2))*6^(3/4)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {1890, 217, 1179, 642, 1176, 631, 210, 1262, 649, 209, 266}

$$\int \frac{a+bx+dx^3}{2+3x^4} \, dx = -\frac{a \arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a \arctan\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}} - \frac{a \log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}} + \frac{a \log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}} + \frac{b \arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{1}{12} d \log\left(3x^4+2\right)$$

```
[In] Int[(a + b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4
)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (d*
Log[2 + 3*x^4])/12
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a}{2+3x^4}+\frac{x(b+dx^2)}{2+3x^4}\right)dx\\
&= a\int \frac{1}{2+3x^4}\,dx+\int \frac{x(b+dx^2)}{2+3x^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx,x,x^2\right)+\frac{a\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}}+\frac{a\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}}
\end{aligned}$$

$$= \frac{a\int \frac{1}{\sqrt{\frac{2}{3}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}}dx}{4\sqrt{6}} + \frac{a\int \frac{1}{\sqrt{\frac{2}{3}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}}dx}{4\sqrt{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}dx}{8\sqrt[4]{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}dx}{8\sqrt[4]{6}} + \frac{1}{2}b\text{Subst}\left(\int \frac{1}{2+3x^2}dx,x,x^2\right) + \frac{1}{2}d\text{Subst}\left(\int \frac{x}{2+3x^2}dx,x,x^2\right)$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{1}{12}d\log\left(2+3x^4\right) + \frac{a\text{Subst}\left(\int \frac{1}{-1-x^2}dx,x,1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a\text{Subst}\left(\int \frac{1}{-1-x^2}dx,x,1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{1}{12}d\log\left(2+3x^4\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int \frac{a+bx+dx^3}{2+3x^4}dx = \frac{1}{48}\Big(-2\sqrt{6}\left(\sqrt[4]{6}a+2b\right)\arctan\left(1-\sqrt[4]{6}x\right) + 2\sqrt{6}\left(\sqrt[4]{6}a-2b\right)\arctan\left(1+\sqrt[4]{6}x\right) - 6^{3/4}a\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right) + 6^{3/4}a\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right) + 4d\log\left(2+3x^4\right)\Big)$$

```
[In] Integrate[(a + b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*Sqrt[6]*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*Sqrt[6]*(6^(1/4)*a
- 2*b)*ArcTan[1 + 6^(1/4)*x] - 6^(3/4)*a*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2]
 + 6^(3/4)*a*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.25

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^3 d+\_R b+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}+\frac{d\ln(3x}{12}$
meijerg	$\frac{d\ln\left(\frac{3x^4}{2}+1\right)}{12}+\frac{\sqrt{6}\,b\arctan\left(\frac{\sqrt{2}\sqrt{3}\,x^2}{2}\right)}{12}+\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\ln\left(1\right.}{}\right.}{96}$

```
[In] int((d*x^3+b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
[Out] 1/12*sum((_R^3*d+_R*b+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 17085, normalized size of antiderivative = 125.62

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.46

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx$$
$$= \text{RootSum}\left(165888t^4-55296t^3d+t^2\cdot\left(3456b^2+6912d^2\right)+t\left(-864a^2b-576b^2d-384d^3\right)+27a^4+72a^2\right.$$

```
[In] integrate((d*x**3+b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(3456*b**2 + 6912*d**2) + _t*(
-864*a**2*b - 576*b**2*d - 384*d**3) + 27*a**4 + 72*a**2*b*d + 18*b**4 + 24
*b**2*d**2 + 8*d**4, Lambda(_t, _t*log(x + (27648*_t**3*b**2 + 1728*_t**2*a
**2*b - 6912*_t**2*b**2*d + 216*_t*a**4 - 288*_t*a**2*b*d + 288*_t*b**4 + 5
76*_t*b**2*d**2 - 18*a**4*d - 90*a**2*b**3 + 12*a**2*b*d**2 - 24*b**4*d - 1
6*b**2*d**3)/(27*a**5 - 72*a*b**4)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.26

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx = \frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d+3\,a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a-2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

```
[In] integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d + 3*a)*log(sqrt(3)*x^2 + 3^(1/4)
*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d - 3*a)*l
og(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*sqrt(3)*(3^(1/4)*2^(3/
4)*a - 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/
4))) + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a + 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^
(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx = \frac{1}{24}\left(6^{\frac{3}{4}}a-2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a+2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a+4\,d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a-4\,d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

```
[In] integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2
)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(
3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a + 4*d)*log(x^2 + sqrt(2
)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 4*d)*log(x^2 - sqrt(2)*(2/
3)^(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.26

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx=\sum_{k=1}^{4}\ln\left(x\left(9\,a^2\,d+9\,b^3+6\,b\,d^2\right)+9\,a\,b^2-6\,a\,d^2-\mathrm{root}\left(z^4-\frac{d\,z^3}{3}+\frac{z^2\,\left(3456\,b^2+6912\,d^2\right)}{165888}-\frac{z\,\left(864\,a^2\,b+576\,b^2\,d+384\,d^3\right)}{165888}+\frac{a^2\,b\,d}{2304}+\frac{b^2\,d^2}{6912}+\frac{d^4}{20736}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\right)\left(\mathrm{root}\left(z^4-\frac{d\,z^3}{3}+\frac{z^2\,\left(3456\,b^2+6912\,d^2\right)}{165888}-\frac{z\,\left(864\,a^2\,b+576\,b^2\,d+384\,d^3\right)}{165888}+\frac{a^2\,b\,d}{2304}+\frac{b^2\,d^2}{6912}+\right.\right.\right.$$
$$\left.\left.\left.-144\,a\,d+x\left(108\,a^2+144\,b\,d\right)\right)\right)\mathrm{root}\left(z^4-\frac{d\,z^3}{3}+\frac{z^2\,\left(3456\,b^2+6912\,d^2\right)}{165888}-\frac{z\,\left(864\,a^2\,b+576\,b^2\,d+384\,d^3\right)}{165888}+\frac{a^2\,b\,d}{2304}+\frac{b^2\,d^2}{6912}+\frac{d^4}{20736}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\right)$$

```
[In] int((a + b*x + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] symsum(log(x*(9*a^2*d + 6*b*d^2 + 9*b^3) + 9*a*b^2 - 6*a*d^2 - root(z^4 - (
d*z^3)/3 + (z^2*(3456*b^2 + 6912*d^2))/165888 - (z*(864*a^2*b + 576*b^2*d +
 384*d^3))/165888 + (a^2*b*d)/2304 + (b^2*d^2)/6912 + d^4/20736 + b^4/9216
+ a^4/6144, z, k)*(root(z^4 - (d*z^3)/3 + (z^2*(3456*b^2 + 6912*d^2))/16588
8 - (z*(864*a^2*b + 576*b^2*d + 384*d^3))/165888 + (a^2*b*d)/2304 + (b^2*d^
2)/6912 + d^4/20736 + b^4/9216 + a^4/6144, z, k)*(864*a - 864*b*x) - 144*a*
d + x*(144*b*d + 108*a^2)))*root(z^4 - (d*z^3)/3 + (z^2*(3456*b^2 + 6912*d^
2))/165888 - (z*(864*a^2*b + 576*b^2*d + 384*d^3))/165888 + (a^2*b*d)/2304
+ (b^2*d^2)/6912 + d^4/20736 + b^4/9216 + a^4/6144, z, k), k, 1, 4)
```

3.163 $\int \frac{cx^2+dx^3}{2+3x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx=-\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}}+\frac{c\arctan\left(1+\sqrt[4]{6}x\right)}{2\,6^{3/4}}+\frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\,6^{3/4}}-\frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\,6^{3/4}}+\frac{1}{12}d\log\left(2+3x^4\right)$$

```
[Out] 1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)+1/12
*d*ln(3*x^4+2)+1/24*c*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)-1/24*c*ln(6^(3/4
)*x+3*x^2+6^(1/2))*6^(1/4)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1607, 1845, 303, 1176, 631, 210, 1179, 642, 266}

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx=-\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}}+\frac{c\arctan\left(\sqrt[4]{6}x+1\right)}{2\,6^{3/4}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

```
[In] Int[(c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] -1/2*(c*ArcTan[1 - 6^(1/4)*x])/6^(3/4) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/
4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(4*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2(c+dx)}{2+3x^4}\,dx\\
&= \int \left(\frac{cx^2}{2+3x^4}+\frac{dx^3}{2+3x^4}\right)dx\\
&= c\int \frac{x^2}{2+3x^4}\,dx+d\int \frac{x^3}{2+3x^4}\,dx\\
&= \frac{1}{12}d\log\left(2+3x^4\right)-\frac{c\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}}+\frac{c\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}}\\
&= \frac{1}{12}d\log\left(2+3x^4\right)+\frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx\\
&\quad+\frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx+\frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\ 6^{3/4}}+\frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\ 6^{3/4}}\\
&= \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}-\frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}\\
&\quad+\frac{1}{12}d\log\left(2+3x^4\right)+\frac{c\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}-\frac{c\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}}\\
&= -\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}\\
&\quad-\frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}+\frac{1}{12}d\log\left(2+3x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.95

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{24}\Big(-2\sqrt[4]{6}c\arctan\left(1-\sqrt[4]{6}x\right)+2\sqrt[4]{6}c\arctan\left(1+\sqrt[4]{6}x\right) + \sqrt[4]{6}c\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)-\sqrt[4]{6}c\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right) + 2d\log\left(2+3x^4\right)\Big)$$

```
[In] Integrate[(c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*c*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*c*ArcTan[1 + 6^(1/4)*x] + 6
^(1/4)*c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - 6^(1/4)*c*Log[2 + 2*6^(1/4)*x
 + Sqrt[6]*x^2] + 2*d*Log[2 + 3*x^4])/24
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.31

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^3 d+\_R^2 c\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{144}+\frac{d\ln\left(3x^4+2\right)}{12}$
meijerg	$\frac{d\ln\left(\frac{3x^4}{2}+1\right)}{12}+\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\right.}{216}$

```
[In] int((d*x^3+c*x^2)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*sum((_R^3*d+_R^2*c)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(83) = 166.

Time = 0.28 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.10

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{24}\left(\sqrt{2}\sqrt{\sqrt{6}\sqrt{-c^4}}+2\,d\right)\log\left(6\,c^3x+\sqrt{6}\sqrt{2}\sqrt{-c^4}\sqrt{\sqrt{6}\sqrt{-c^4}}\right) - \frac{1}{24}\left(\sqrt{2}\sqrt{\sqrt{6}\sqrt{-c^4}}-2\,d\right)\log\left(6\,c^3x-\sqrt{6}\sqrt{2}\sqrt{-c^4}\sqrt{\sqrt{6}\sqrt{-c^4}}\right) - \frac{1}{24}\left(\sqrt{2}\sqrt{-\sqrt{6}\sqrt{-c^4}}-2\,d\right)\log\left(6\,c^3x+\sqrt{6}\sqrt{2}\sqrt{-c^4}\sqrt{-\sqrt{6}\sqrt{-c^4}}\right) + \frac{1}{24}\left(\sqrt{2}\sqrt{-\sqrt{6}\sqrt{-c^4}}+2\,d\right)\log\left(6\,c^3x-\sqrt{6}\sqrt{2}\sqrt{-c^4}\sqrt{-\sqrt{6}\sqrt{-c^4}}\right)$$

```
[In] integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(2)*sqrt(sqrt(6)*sqrt(-c^4)) + 2*d)*log(6*c^3*x + sqrt(6)*sqrt(2)
*sqrt(-c^4)*sqrt(sqrt(6)*sqrt(-c^4))) - 1/24*(sqrt(2)*sqrt(sqrt(6)*sqrt(-c^
4)) - 2*d)*log(6*c^3*x - sqrt(6)*sqrt(2)*sqrt(-c^4)*sqrt(sqrt(6)*sqrt(-c^4)
)) - 1/24*(sqrt(2)*sqrt(-sqrt(6)*sqrt(-c^4)) - 2*d)*log(6*c^3*x + sqrt(6)*s
qrt(2)*sqrt(-c^4)*sqrt(-sqrt(6)*sqrt(-c^4))) + 1/24*(sqrt(2)*sqrt(-sqrt(6)*
sqrt(-c^4)) + 2*d)*log(6*c^3*x - sqrt(6)*sqrt(2)*sqrt(-c^4)*sqrt(-sqrt(6)*s
qrt(-c^4)))
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx = \operatorname{RootSum}\left(41472t^4-13824t^3d+1728t^2d^2-96td^3+3c^4+2d^4,\left(t\mapsto t\log\left(x+\frac{3456t^3-864t^2d+72td^2}{3c^3}\right.\right.\right.$$

```
[In] integrate((d*x**3+c*x**2)/(3*x**4+2),x)
```

```
[Out] RootSum(41472*_t**4 - 13824*_t**3*d + 1728*_t**2*d**2 - 96*_t*d**3 + 3*c**4
 + 2*d**4, Lambda(_t, _t*log(x + (3456*_t**3 - 864*_t**2*d + 72*_t*d**2 - 2
*d**3)/(3*c**3))))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.33

$$\int \frac{cx^2 + dx^3}{2 + 3x^4} \, dx = \frac{1}{72} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(3^{\frac{1}{4}} 2^{\frac{3}{4}} d - \sqrt{3}c\right) \log\left(\sqrt{3}x^2 + 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right) + \frac{1}{72} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(3^{\frac{1}{4}} 2^{\frac{3}{4}} d + \sqrt{3}c\right) \log\left(\sqrt{3}x^2 - 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right) + \frac{1}{12} \cdot 3^{\frac{1}{4}} 2^{\frac{1}{4}} c \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2\sqrt{3}x + 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right) + \frac{1}{12} \cdot 3^{\frac{1}{4}} 2^{\frac{1}{4}} c \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2\sqrt{3}x - 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right)$$

```
[In] integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d - sqrt(3)*c)*log(sqrt(3)*x^2 + 3^(1
/4)*2^(3/4)*x + sqrt(2)) + 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d + sqrt(3
)*c)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/12*3^(1/4)*2^(1/4)*
c*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/12*3^(1/4
)*2^(1/4)*c*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{cx^2 + dx^3}{2 + 3x^4} \, dx = \frac{1}{12} \cdot 6^{\frac{1}{4}} c \arctan\left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3}\right)^{\frac{3}{4}} \left(2x + \sqrt{2} \left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12} \cdot 6^{\frac{1}{4}} c \arctan\left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3}\right)^{\frac{3}{4}} \left(2x - \sqrt{2} \left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) - \frac{1}{24} \left(6^{\frac{1}{4}} c - 2d\right) \log\left(x^2 + \sqrt{2} \left(\frac{2}{3}\right)^{\frac{1}{4}} x + \sqrt{\frac{2}{3}}\right) + \frac{1}{24} \left(6^{\frac{1}{4}} c + 2d\right) \log\left(x^2 - \sqrt{2} \left(\frac{2}{3}\right)^{\frac{1}{4}} x + \sqrt{\frac{2}{3}}\right)$$

```
[In] integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
+ 1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))
) - 1/24*(6^(1/4)*c - 2*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1
/24*(6^(1/4)*c + 2*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx = \ln\left(x - \frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}+\frac{6^{1/4}\sqrt{-\frac{1}{2}\mathrm{i}}\,c}{12}\right)+\ln\left(x+\frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}-\frac{6^{1/4}\sqrt{-\frac{1}{2}\mathrm{i}}\,c}{12}\right)+\ln\left(x-\frac{(-\right.$$

```
[In] int((c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] log(x - ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(-1i/2)^(1/2)*c)/1
2) + log(x + ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(-1i/2)^(1/2)
*c)/12) + log(x - ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(1i/2)^(
1/2)*c)/12) + log(x + ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(1i/
2)^(1/2)*c)/12)
```

3.164 $\int \frac{a+cx^2+dx^3}{2+3x^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{a+cx^2+dx^3}{2+3x^4} dx = -\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{1}{12}d\log\left(2+3x^4\right)$$

```
[Out] 1/12*d*ln(3*x^4+2)-1/48*ln(-6^(3/4)*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1/
4)+1/48*ln(6^(3/4)*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(-1
+6^(1/4)*x)*(2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(1+6^(1/4)*x)*(2*c+a*6^(1/2)
)*6^(1/4)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1890, 266, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{a+cx^2+dx^3}{2+3x^4} dx = -\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\arctan\left(\sqrt[4]{6}x+1\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

```
[In] Int[(a + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] -1/4*((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/6^(3/4) + ((Sqrt[6]*a + 2*c)
*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3
/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x +
3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \int \left(\frac{dx^3}{2+3x^4} + \frac{a+cx^2}{2+3x^4}\right) dx$$

$$= d\int \frac{x^3}{2+3x^4}\,dx + \int \frac{a+cx^2}{2+3x^4}\,dx$$

$$= \frac{1}{12}d\log\left(2+3x^4\right) + \frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{2+3x^4}\,dx + \frac{1}{12}\left(\sqrt{6}a+2c\right)\int \frac{\sqrt{6}+3x^2}{2+3x^4}\,dx$$

$$= \frac{1}{12}d\log\left(2+3x^4\right) - \frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}} - \frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}}$$
$$+ \frac{1}{24}\left(\sqrt{6}a+2c\right)\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{1}{24}\left(\sqrt{6}a+2c\right)\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx$$

$$= -\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}$$
$$+ \frac{1}{12}d\log\left(2+3x^4\right) + \frac{\left(\sqrt{6}a+2c\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}} - \frac{\left(\sqrt{6}a+2c\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx\right.}{4\ 6^{3/4}}$$

$$= -\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}} + \frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}}$$
$$- \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}$$
$$+ \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{1}{12}d\log\left(2+3x^4\right)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.96

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{48}\Big(-2\sqrt[4]{6}\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right) + 2\sqrt[4]{6}\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right) - \sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right) + \sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)+4d\log\left(2+3x^4\right)\Big)$$

```
[In] Integrate[(a + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*(Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*(Sqrt[6]*a
+ 2*c)*ArcTan[1 + 6^(1/4)*x] - 6^(1/4)*(Sqrt[6]*a - 2*c)*Log[2 - 2*6^(1/4)*
x + Sqrt[6]*x^2] + 6^(1/4)*(Sqrt[6]*a - 2*c)*Log[2 + 2*6^(1/4)*x + Sqrt[6]*
x^2] + 4*d*Log[2 + 3*x^4])/48
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^3 d+\_R^2 c+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\cdots}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\cdots}\right)\cdots\right.}{\cdots}$
meijerg	$\frac{d\ln\left(\frac{3x^4}{2}+1\right)}{12}+\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\cdots\right.}{216}$

```
[In] int((d*x^3+c*x^2+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*sum((_R^3*d+_R^2*c+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(115) = 230$.

Time = 0.29 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.33

$$\int \frac{a + cx^2 + dx^3}{2 + 3x^4}\,dx$$
$$= \frac{1}{24}\left(2\,d - \sqrt{-12\,ac + \sqrt{6}\sqrt{-9\,a^4 + 12\,a^2c^2 - 4\,c^4}}\right)\log\left(-3\left(9\,a^4 - 4\,c^4\right)x + \left(9\,a^3 - 6\,ac^2 - \sqrt{6}\sqrt{-9\,a^4 + 12\,a^2c^2 - 4\,c^4}c\right)\sqrt{-12\,ac + \sqrt{6}\sqrt{-9\,a^4 + 12\,a^2c^2 - 4\,c^4}}\right)$$
$$+ \frac{1}{24}\left(2\,d + \sqrt{-12\,ac + \sqrt{6}\sqrt{-9\,a^4 + 12\,a^2c^2 - 4\,c^4}}\right)\log\left(-3\left(9\,a^4 - 4\,c^4\right)x - \left(9\,a^3 - 6\,ac^2 - \sqrt{6}\sqrt{-9\,a^4 + 12\,a^2c^2 - 4\,c^4}c\right)\sqrt{-12\,ac + \sqrt{6}\sqrt{-9\,a^4 + 12\,a^2c^2 - 4\,c^4}}\right)$$
$$+ \frac{1}{24}\left(2\,d - \sqrt{-12\,ac - \sqrt{6}\sqrt{-9\,a^4 + 12\,a^2c^2 - 4\,c^4}}\right)\log\left(-3\left(9\,a^4 - 4\,c^4\right)x + \left(9\,a^3 - 6\,ac^2 + \sqrt{6}\sqrt{-9\,a^4 + 12\,a^2c^2 - 4\,c^4}c\right)\sqrt{-12\,ac - \sqrt{6}\sqrt{-9\,a^4 + 12\,a^2c^2 - 4\,c^4}}\right)$$
$$+ \frac{1}{24}\left(2\,d + \sqrt{-12\,ac - \sqrt{6}\sqrt{-9\,a^4 + 12\,a^2c^2 - 4\,c^4}}\right)\log\left(-3\left(9\,a^4 - 4\,c^4\right)x - \left(9\,a^3 - 6\,ac^2 + \sqrt{6}\sqrt{-9\,a^4 + 12\,a^2c^2 - 4\,c^4}c\right)\sqrt{-12\,ac - \sqrt{6}\sqrt{-9\,a^4 + 12\,a^2c^2 - 4\,c^4}}\right)$$

```
[In] integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/24*(2*d - sqrt(-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)))*log(
-3*(9*a^4 - 4*c^4)*x + (9*a^3 - 6*a*c^2 - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2
- 4*c^4)*c)*sqrt(-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))) + 1/
24*(2*d + sqrt(-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)))*log(-3
*(9*a^4 - 4*c^4)*x - (9*a^3 - 6*a*c^2 - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 -
4*c^4)*c)*sqrt(-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))) + 1/24
*(2*d - sqrt(-12*a*c - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)))*log(-3*(
9*a^4 - 4*c^4)*x + (9*a^3 - 6*a*c^2 + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*
c^4)*c)*sqrt(-12*a*c - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))) + 1/24*(
2*d + sqrt(-12*a*c - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)))*log(-3*(9*
a^4 - 4*c^4)*x - (9*a^3 - 6*a*c^2 + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^
4)*c)*sqrt(-12*a*c - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)))
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.96

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx$$
$$= \text{RootSum}\left(165888t^4-55296t^3d+t^2\cdot\left(6912ac+6912d^2\right)+t\left(-1152acd-384d^3\right)+27a^4+36a^2c^2+48ac\right.$$

```
[In] integrate((d*x**3+c*x**2+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 6912*d**2) + _t*(-
1152*a*c*d - 384*d**3) + 27*a**4 + 36*a**2*c**2 + 48*a*c*d**2 + 12*c**4 + 8
*d**4, Lambda(_t, _t*log(x + (-13824*_t**3*c + 3456*_t**2*c*d + 216*_t*a**3
 - 432*_t*a*c**2 - 288*_t*c*d**2 - 18*a**3*d + 36*a*c**2*d + 8*c*d**3)/(27*
a**4 - 12*c**4))))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx = &-\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c-2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\\&+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\\&+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\\&+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\end{aligned}$$

```
[In] integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/144*3^(3/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c - 2*3^(1/4)*2^(1/4)*d - 3*a)*log(
sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(sqrt(3)
*sqrt(2)*c + 2*3^(1/4)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x
 + sqrt(2)) + 1/72*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c)*arct
an(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/72*sqrt(3)*(3*3
^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(
3)*x - 3^(1/4)*2^(3/4)))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.89

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c+4\,d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c-4\,d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

```
[In] integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2
)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(
3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c + 4*d)*lo
g(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*c
- 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 10.29 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.86

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx = \ln\left(-2\,c+\sqrt{6}\,a\,1\mathrm{i}+x\,\sqrt{3\mathrm{i}\,\sqrt{6}\,a^2-12\,a\,c-2\mathrm{i}\,\sqrt{6}\,c^2}\right)\,\left(\frac{d}{12}+\frac{\sqrt{\frac{3\mathrm{i}\,\sqrt{6}\,a^2}{4}-3\,a\,c-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{2}}}{12}\right)+\ln\left(2\,c-\sqrt{6}\,a\,1\mathrm{i}+x\,\sqrt{3\mathrm{i}\,\sqrt{6}\,a^2-12\,a\,c-2\mathrm{i}\,\sqrt{6}\,c^2}\right)\,\left(\frac{d}{12}-\frac{\sqrt{\frac{3\mathrm{i}\,\sqrt{6}\,a^2}{4}-3\,a\,c-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{2}}}{12}\right)+\ln\left(2\,c+\sqrt{6}\,a\,1\mathrm{i}+x\,\sqrt{-3\mathrm{i}\,\sqrt{6}\,a^2-12\,a\,c+2\mathrm{i}\,\sqrt{6}\,c^2}\right)\,\left(\frac{d}{12}-\frac{\sqrt{-\frac{3\mathrm{i}\,\sqrt{6}\,a^2}{4}-3\,a\,c+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{2}}}{12}\right)+\ln\left(2\,c+\sqrt{6}\,a\,1\mathrm{i}-x\,\sqrt{-3\mathrm{i}\,\sqrt{6}\,a^2-12\,a\,c+2\mathrm{i}\,\sqrt{6}\,c^2}\right)\,\left(\frac{d}{12}+\frac{\sqrt{-\frac{3\mathrm{i}\,\sqrt{6}\,a^2}{4}-3\,a\,c+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{2}}}{12}\right)$$

```
[In] int((a + c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] log(6^(1/2)*a*1i - 2*c + x*(6^(1/2)*a^2*3i - 12*a*c - 6^(1/2)*c^2*2i)^(1/2)
)*(d/12 + ((6^(1/2)*a^2*3i)/4 - 3*a*c - (6^(1/2)*c^2*1i)/2)^(1/2)/12) + log
(2*c - 6^(1/2)*a*1i + x*(6^(1/2)*a^2*3i - 12*a*c - 6^(1/2)*c^2*2i)^(1/2))*(
d/12 - ((6^(1/2)*a^2*3i)/4 - 3*a*c - (6^(1/2)*c^2*1i)/2)^(1/2)/12) + log(2*
c + 6^(1/2)*a*1i + x*(6^(1/2)*c^2*2i - 6^(1/2)*a^2*3i - 12*a*c)^(1/2))*(d/1
2 - ((6^(1/2)*c^2*1i)/2 - (6^(1/2)*a^2*3i)/4 - 3*a*c)^(1/2)/12) + log(2*c +
 6^(1/2)*a*1i - x*(6^(1/2)*c^2*2i - 6^(1/2)*a^2*3i - 12*a*c)^(1/2))*(d/12 +
 ((6^(1/2)*c^2*1i)/2 - (6^(1/2)*a^2*3i)/4 - 3*a*c)^(1/2)/12)
```

3.165 $\int \frac{bx+cx^2+dx^3}{2+3x^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 136

$$\int \frac{bx+cx^2+dx^3}{2+3x^4} dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\arctan\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} + \frac{1}{12}d\log\left(2+3x^4\right)$$

```
[Out] 1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)+1/12
*d*ln(3*x^4+2)+1/24*c*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)-1/24*c*ln(6^(3/4
)*x+3*x^2+6^(1/2))*6^(1/4)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1608, 1845, 303, 1176, 631, 210, 1179, 642, 1262, 649, 209, 266}

$$\int \frac{bx+cx^2+dx^3}{2+3x^4} dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\arctan\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}} + \frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}} - \frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}} + \frac{1}{12}d\log\left(3x^4+2\right)$$

```
[In] Int[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4
)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) + (d*
Log[2 + 3*x^4])/12
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(b+cx+dx^2)}{2+3x^4}\,dx \\ &= \int \left(\frac{cx^2}{2+3x^4}+\frac{x(b+dx^2)}{2+3x^4}\right)\,dx \\ &= c\int \frac{x^2}{2+3x^4}\,dx+\int \frac{x(b+dx^2)}{2+3x^4}\,dx\end{aligned}$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx,x,x^2\right) - \frac{c\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}} + \frac{c\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{1}{2}b\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx,x,x^2\right) + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx$$

$$+ \frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\ 6^{3/4}} + \frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\ 6^{3/4}} + \frac{1}{2}d\text{Subst}\left(\int \frac{x}{2+3x^2}\,dx,x,x^2\right)$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}$$

$$+ \frac{1}{12}d\log\left(2+3x^4\right) + \frac{c\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}} - \frac{c\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}}$$

$$+ \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} + \frac{1}{12}d\log\left(2+3x^4\right)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92

$$\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{24}\left(-2\sqrt[4]{6}\left(\sqrt[4]{6}b+c\right)\arctan\left(1-\sqrt[4]{6}x\right) + 2\sqrt[4]{6}\left(-\sqrt[4]{6}b+c\right)\arctan\left(1+\sqrt[4]{6}x\right) + \sqrt[4]{6}c\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right) - \sqrt[4]{6}c\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right) + 2d\log\left(2+3x^4\right)\right)$$

```
[In] Integrate[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*(6^(1/4)*b + c)*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*(-(6^(1/4)*b)
 + c)*ArcTan[1 + 6^(1/4)*x] + 6^(1/4)*c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2]
- 6^(1/4)*c*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 2*d*Log[2 + 3*x^4])/24
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.28

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^3 d+\_R^2 c+\_R b\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{144}+\frac{d\ln(3x}{12}$
meijerg	$\frac{d\ln\left(\frac{3x^4}{2}+1\right)}{12}+\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\right.}{216}$

```
[In] int((d*x^3+c*x^2+b*x)/(3*x^4+2),x,method=_RETURNVERBOSE)
[Out] 1/12*sum((_R^3*d+_R^2*c+_R*b)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 18086, normalized size of antiderivative = 132.99

$$\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.39

$$\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx$$
$$=\mathrm{RootSum}\left(82944t^4-27648t^3d+t^2\cdot\left(1728b^2+3456d^2\right)+t\left(-288b^2d+288bc^2-192d^3\right)+9b^4+12b^2d^2\right.$$

```
[In] integrate((d*x**3+c*x**2+b*x)/(3*x**4+2),x)
```

```
[Out] RootSum(82944*_t**4 - 27648*_t**3*d + _t**2*(1728*b**2 + 3456*d**2) + _t*(-
288*b**2*d + 288*b*c**2 - 192*d**3) + 9*b**4 + 12*b**2*d**2 - 24*b*c**2*d +
 6*c**4 + 4*d**4, Lambda(_t, _t*log(x + (-3456*_t**3*c**2 + 864*_t**2*b**3
+ 864*_t**2*c**2*d - 144*_t*b**3*d - 108*_t*b**2*c**2 - 72*_t*c**2*d**2 + 9
*b**5 + 6*b**3*d**2 + 9*b**2*c**2*d - 9*b*c**4 + 2*c**2*d**3)/(18*b**4*c -
3*c**5))))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx =& \frac{1}{72}\sqrt{3}\sqrt{2}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}c-6\,b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\\ &+\frac{1}{72}\sqrt{3}\sqrt{2}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}c+6\,b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\\ &+\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d-\sqrt{3}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\\ &+\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d+\sqrt{3}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\end{aligned}$$

```
[In] integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/72*sqrt(3)*sqrt(2)*(3^(3/4)*2^(3/4)*c - 6*b)*arctan(1/6*3^(3/4)*2^(1/4)*(
2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/72*sqrt(3)*sqrt(2)*(3^(3/4)*2^(3/4)*c +
 6*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 1/72*3^
(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d - sqrt(3)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(
3/4)*x + sqrt(2)) + 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d + sqrt(3)*c)*lo
g(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx =& -\frac{1}{12}\left(\sqrt{6}b-6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\\ &+\frac{1}{12}\left(\sqrt{6}b+6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\\ &-\frac{1}{24}\left(6^{\frac{1}{4}}c-2\,d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)\\ &+\frac{1}{24}\left(6^{\frac{1}{4}}c+2\,d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)\end{aligned}$$

```
[In] integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/12*(sqrt(6)*b - 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)
*(2/3)^(1/4))) + 1/12*(sqrt(6)*b + 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4
)*(2*x - sqrt(2)*(2/3)^(1/4))) - 1/24*(6^(1/4)*c - 2*d)*log(x^2 + sqrt(2)*(
2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(1/4)*c + 2*d)*log(x^2 - sqrt(2)*(2/3)^
(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 10.47 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.21

$$\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx = \sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(z^4-\frac{d\,z^3}{3}+\frac{z^2\,(1728\,b^2+3456\,d^2)}{82944}-\frac{z\,(-288\,b\,c^2+288\,b^2\,d+192\,d^3)}{82944}-\frac{b\,c^2\,d}{3456}+\frac{b^2\,d^2}{6912}+\frac{d^4}{20736}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)\left(144\,b\,c+x\,(144\,b\,d-72\,c^2)-\mathrm{root}\left(z^4-\frac{d\,z^3}{3}+\frac{z^2\,(1728\,b^2+3456\,d^2)}{82944}-\frac{z\,(-288\,b\,c^2+288\,b^2\,d+192\,d^3)}{82944}-\frac{b\,c^2\,d}{3456}+\frac{b^2\,d^2}{6912}+\frac{d^4}{20736}+\frac{c}{138\ldots}\right.\right.\right.$$
$$\left.+x\,(9\,b^3+6\,b\,d^2-6\,c^2\,d)-6\,c^3+12\,b\,c\,d\right)\mathrm{root}\left(z^4-\frac{d\,z^3}{3}+\frac{z^2\,(1728\,b^2+3456\,d^2)}{82944}-\frac{z\,(-288\,b\,c^2+288\,b^2\,d+192\,d^3)}{82944}-\frac{b\,c^2\,d}{3456}+\frac{b^2\,d^2}{6912}+\frac{d^4}{20736}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)$$

```
[In] int((b*x + c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] symsum(log(x*(6*b*d^2 - 6*c^2*d + 9*b^3) - root(z^4 - (d*z^3)/3 + (z^2*(172
8*b^2 + 3456*d^2))/82944 - (z*(- 288*b*c^2 + 288*b^2*d + 192*d^3))/82944 -
(b*c^2*d)/3456 + (b^2*d^2)/6912 + d^4/20736 + c^4/13824 + b^4/9216, z, k)*(
144*b*c + x*(144*b*d - 72*c^2) - 864*root(z^4 - (d*z^3)/3 + (z^2*(1728*b^2
+ 3456*d^2))/82944 - (z*(- 288*b*c^2 + 288*b^2*d + 192*d^3))/82944 - (b*c^2
*d)/3456 + (b^2*d^2)/6912 + d^4/20736 + c^4/13824 + b^4/9216, z, k)*b*x) -
6*c^3 + 12*b*c*d)*root(z^4 - (d*z^3)/3 + (z^2*(1728*b^2 + 3456*d^2))/82944
- (z*(- 288*b*c^2 + 288*b^2*d + 192*d^3))/82944 - (b*c^2*d)/3456 + (b^2*d^2
)/6912 + d^4/20736 + c^4/13824 + b^4/9216, z, k), k, 1, 4)
```

3.166 $\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 176

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx=\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{1}{12}d\log\left(2+3x^4\right)$$

```
[Out] 1/12*d*ln(3*x^4+2)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)-1/48*ln(-6^(3/4)*
x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1/4)+1/48*ln(6^(3/4)*x+3*x^2+6^(1/2))*
(-2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(-1+6^(1/4)*x)*(2*c+a*6^(1/2))*6^(1/4)+
1/24*arctan(1+6^(1/4)*x)*(2*c+a*6^(1/2))*6^(1/4)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules

used = {1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 209, 266}

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = -\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}} + \frac{\left(\sqrt{6}a+2c\right)\arctan\left(\sqrt[4]{6}x+1\right)}{4\ 6^{3/4}} - \frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}} + \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{1}{12}d\log\left(3x^4+2\right)$$

```
[In] Int[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/
4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4))
- ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt
[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x
^4])/12
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a+cx^2}{2+3x^4}+\frac{x(b+dx^2)}{2+3x^4}\right) dx\\
&= \int \frac{a+cx^2}{2+3x^4}\,dx+\int \frac{x(b+dx^2)}{2+3x^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx,x,x^2\right)+\frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{2+3x^4}\,dx\\
&\quad+\frac{1}{12}\left(\sqrt{6}a+2c\right)\int \frac{\sqrt{6}+3x^2}{2+3x^4}\,dx\\
&= \frac{1}{2}b\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx,x,x^2\right)\\
&\quad-\frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}}\\
&\quad+\frac{1}{24}\left(\sqrt{6}a+2c\right)\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx+\frac{1}{24}\left(\sqrt{6}a+2c\right)\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx+\frac{1}{2}d\text{Subst}\left(\int \frac{}{2}\right.\\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}\\
&\quad+\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}\\
&\quad+\frac{1}{12}d\log\left(2+3x^4\right)+\frac{\left(\sqrt{6}a+2c\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx\right.}{4\ 6^{3/4}}\\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}\\
&\quad+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}\\
&\quad+\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{1}{12}d\log\left(2+3x^4\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.93

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{48}\Big(-2\sqrt[4]{6}\left(\sqrt{6}a+2\left(\sqrt[4]{6}b+c\right)\right)\arctan\left(1-\sqrt[4]{6}x\right) + 2\sqrt[4]{6}\left(\sqrt{6}a-2\sqrt[4]{6}b+2c\right)\arctan\left(1+\sqrt[4]{6}x\right) - \sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right) + \sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)+4d\log\left(2+3x^4\right)\Big)$$

```
[In] Integrate[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*(Sqrt[6]*a + 2*(6^(1/4)*b + c))*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/
4)*(Sqrt[6]*a - 2*6^(1/4)*b + 2*c)*ArcTan[1 + 6^(1/4)*x] - 6^(1/4)*(Sqrt[6]
*a - 2*c)*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(1/4)*(Sqrt[6]*a - 2*c)*Lo
g[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.22

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^3 d+\_R^2 c+\_R b+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\ldots}{}$
meijerg	$\frac{d\ln\left(\frac{3x^4}{2}+1\right)}{12}+\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\ldots\right)}{216}$

```
[In] int((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*sum((_R^3*d+_R^2*c+_R*b+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 54479, normalized size of antiderivative = 309.54

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(156) = 312.

Time = 4.67 (sec) , antiderivative size = 580, normalized size of antiderivative = 3.30

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx$$
$$=\text{RootSum}\left(165888t^4-55296t^3d+t^2\cdot\left(6912ac+3456b^2+6912d^2\right)+t\left(-864a^2b-1152acd-576b^2d+\right.\right.$$

```
[In] integrate((d*x**3+c*x**2+b*x+a)/(3*x**4+2),x)
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 3456*b**2 + 6912*d
**2) + _t*(-864*a**2*b - 1152*a*c*d - 576*b**2*d + 576*b*c**2 - 384*d**3) +
 27*a**4 + 72*a**2*b*d + 36*a**2*c**2 - 72*a*b**2*c + 48*a*c*d**2 + 18*b**4
 + 24*b**2*d**2 - 48*b*c**2*d + 12*c**4 + 8*d**4, Lambda(_t, _t*log(x + (-4
1472*_t**3*a**2*c + 82944*_t**3*a*b**2 + 27648*_t**3*c**3 + 5184*_t**2*a**3
*b + 10368*_t**2*a**2*c*d - 20736*_t**2*a*b**2*d + 10368*_t**2*a*b*c**2 - 6
912*_t**2*b**3*c - 6912*_t**2*c**3*d + 648*_t*a**5 - 864*_t*a**3*b*d - 1728
*_t*a**3*c**2 + 3888*_t*a**2*b**2*c - 864*_t*a**2*c*d**2 + 864*_t*a*b**4 +
1728*_t*a*b**2*d**2 - 1728*_t*a*b*c**2*d + 864*_t*a*c**4 + 1152*_t*b**3*c*d
 + 864*_t*b**2*c**3 + 576*_t*c**3*d**2 - 54*a**5*d + 270*a**4*b*c - 270*a**
3*b**3 + 36*a**3*b*d**2 + 144*a**3*c**2*d - 324*a**2*b**2*c*d + 24*a**2*c*d
**3 - 72*a*b**4*d + 180*a*b**3*c**2 - 48*a*b**2*d**3 + 72*a*b*c**2*d**2 - 7
2*a*c**4*d - 72*b**5*c - 48*b**3*c*d**2 - 72*b**2*c**3*d + 72*b*c**5 - 16*c
**3*d**3)/(81*a**6 - 54*a**4*c**2 + 432*a**3*b**2*c - 216*a**2*b**4 - 36*a*
*2*c**4 + 288*a*b**2*c**3 - 144*b**4*c**2 + 24*c**6)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.18

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = -\frac{1}{144} \cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c - 2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d - 3\,a\right)\log\left(\sqrt{3}x^2 + 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right) + \frac{1}{144} \cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c + 2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d - 3\,a\right)\log\left(\sqrt{3}x^2 - 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right) + \frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a + 2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c - 6\sqrt{2}b\right)\arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x + 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a + 2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c + 6\sqrt{2}b\right)\arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x - 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/144*3^(3/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c - 2*3^(1/4)*2^(1/4)*d - 3*a)*log(
sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(sqrt(3)
*sqrt(2)*c + 2*3^(1/4)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x
 + sqrt(2)) + 1/72*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c - 6*s
qrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/7
2*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c + 6*sqrt(2)*b)*arctan(
1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.85

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx$$
$$=\frac{1}{24}\left(6^{\frac{3}{4}}a-2\sqrt{6}b+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$
$$+\frac{1}{24}\left(6^{\frac{3}{4}}a+2\sqrt{6}b+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$
$$+\frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c+4d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$
$$-\frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c-4d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)
*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b + 2*6^(1/4)*c
)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/
4)*a - 2*6^(1/4)*c + 4*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/
48*(6^(3/4)*a - 2*6^(1/4)*c - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2
/3))
```

Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 1168, normalized size of antiderivative = 6.64

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x + c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] symsum(log(9*a*b^2 - 864*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24
 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)
/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*
b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4
/9216 + a^4/6144, z, k)^2*a - 9*a^2*c - 6*a*d^2 + 9*b^3*x - 6*c^3 + 144*roo
t(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/
144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*
d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912
 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216 + a^4/6144, z, k)*a*d
- 144*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (
a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432
```

```
- (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*
d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216 + a^4/6144, z
, k)*b*c + 12*b*c*d - 108*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/2
4 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z
)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a
*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^
4/9216 + a^4/6144, z, k)*a^2*x + 864*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 +
(d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288
 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d
)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4
/13824 + b^4/9216 + a^4/6144, z, k)^2*b*x + 72*root(z^4 - (d*z^3)/3 + (a*c*
z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*
c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456
+ (a^2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/2
0736 + c^4/13824 + b^4/9216 + a^4/6144, z, k)*c^2*x + 9*a^2*d*x + 6*b*d^2*x
 - 6*c^2*d*x - 144*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^
2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 -
 (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)
/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216
+ a^4/6144, z, k)*b*d*x - 18*a*b*c*x)*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 +
 (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/28
8 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*
d)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^
4/13824 + b^4/9216 + a^4/6144, z, k), k, 1, 4)
```

3.167 $\int \frac{1+x+x^2+x^3}{1-x^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 8

$$\int \frac{1+x+x^2+x^3}{1-x^4} \, dx = -\log(1-x)$$

```
[Out] -ln(1-x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1600, 31}

$$\int \frac{1+x+x^2+x^3}{1-x^4} \, dx = -\log(1-x)$$

```
[In] Int[(1 + x + x^2 + x^3)/(1 - x^4),x]
```

```
[Out] -Log[1 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{1-x}\,dx \\
&= -\log(1-x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log(1-x)$$

```
[In] Integrate[(1 + x + x^2 + x^3)/(1 - x^4),x]
[Out] -Log[1 - x]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result
default	$-\ln(-1+x)$
norman	$-\ln(-1+x)$
risch	$-\ln(-1+x)$
parallelrisch	$-\ln(-1+x)$
meijerg	$-\frac{\ln(-x^4+1)}{4}-\frac{x^3\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}+\frac{\operatorname{arctanh}(x^2)}{2}-\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4 \ldots\right.\right.}{4(x^4 \ldots}$

```
[In] int((x^3+x^2+x+1)/(-x^4+1),x,method=_RETURNVERBOSE)
[Out] -ln(-1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log\left(x-1\right)$$

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="fricas")
[Out] -log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log\left(x-1\right)$$

```
[In] integrate((x**3+x**2+x+1)/(-x**4+1),x)
[Out] -log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log\left(x-1\right)$$

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="maxima")
[Out] -log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log\left(|x-1|\right)$$

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="giac")
[Out] -log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx=-\ln\left(x-1\right)$$

```
[In] int(-(x + x^2 + x^3 + 1)/(x^4 - 1),x)
[Out] -log(x - 1)
```

3.168 $\int \frac{1+x+x^2+x^3}{1+x^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx=\frac{\arctan(x^2)}{2}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}x\right)}{\sqrt{2}}+\frac{1}{4}\log\left(1+x^4\right)$$

```
[Out] 1/2*arctan(x^2)+1/4*ln(x^4+1)+1/2*arctan(-1+x*2^(1/2))*2^(1/2)+1/2*arctan(1
+x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1890, 1176, 631, 210, 1262, 649, 209, 266}

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx=\frac{\arctan(x^2)}{2}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}+\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}+\frac{1}{4}\log\left(x^4+1\right)$$

```
[In] Int[(1 + x + x^2 + x^3)/(1 + x^4),x]
```

```
[Out] ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/Sqrt[
2] + Log[1 + x^4]/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \int \left(\frac{1+x^2}{1+x^4} + \frac{x(1+x^2)}{1+x^4}\right) dx$$

$$
\begin{aligned}
&= \int \frac{1+x^2}{1+x^4} \, dx + \int \frac{x(1+x^2)}{1+x^4} \, dx \\
&= \frac{1}{2} \int \frac{1}{1-\sqrt{2}x+x^2} \, dx + \frac{1}{2} \int \frac{1}{1+\sqrt{2}x+x^2} \, dx + \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{1+x^2} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{x}{1+x^2} \, dx, x, x^2\right) \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} \, dx, x, 1-\sqrt{2}x\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} \, dx, x, 1+\sqrt{2}x\right)}{\sqrt{2}} \\
&= \frac{1}{2} \tan^{-1}\left(x^2\right) - \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{\sqrt{2}} + \frac{1}{4} \log\left(1+x^4\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$
\int \frac{1+x+x^2+x^3}{1+x^4} \, dx = \frac{1}{4}\left(-2\left(1+\sqrt{2}\right) \arctan\left(1-\sqrt{2}x\right) + 2\left(-1+\sqrt{2}\right) \arctan\left(1+\sqrt{2}x\right) + \log\left(1+x^4\right)\right)
$$

```
[In] Integrate[(1 + x + x^2 + x^3)/(1 + x^4),x]
[Out] (-2*(1 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(-1 + Sqrt[2])*ArcTan[1 + Sqrt[
2]*x] + Log[1 + x^4])/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+1\right)} \frac{\left(\_R^3+\_R^2+\_R+1\right) \ln\left(x-\_R\right)}{\_R^3}\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+\sqrt{2}\,x}{1+x^2-\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{8} + \frac{\arctan\left(x^2\right)}{2} + \frac{\sqrt{2}\left(\ln\left(\frac{1+x^2-\sqrt{2}\,x}{1+x^2+\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\,\text{arc}\right.}{8}$
meijerg	$\frac{\ln\left(x^4+1\right)}{4} + \frac{x^3\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{3}{4}}} + \frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}\right)}{4\left(x^4\right)^{\frac{3}{4}}} - \frac{x^3\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{3}{4}}} + \frac{x^3\sqrt{2}\,\text{arcta}}{}$

```
[In] int((x^3+x^2+x+1)/(x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R^3+_R^2+_R+1)/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(41) = 82$.

Time = 0.43 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.85

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = \frac{1}{4}\left(\sqrt{2\sqrt{2}-3}+1\right)\log\left(\sqrt{2\sqrt{2}-3}\left(\sqrt{2}+2\right)+2x+\sqrt{2}\right) - \frac{1}{4}\left(\sqrt{2\sqrt{2}-3}-1\right)\log\left(-\sqrt{2\sqrt{2}-3}\left(\sqrt{2}+2\right)+2x+\sqrt{2}\right) - \frac{1}{4}\left(\sqrt{-2\sqrt{2}-3}-1\right)\log\left(\left(\sqrt{2}-2\right)\sqrt{-2\sqrt{2}-3}+2x-\sqrt{2}\right) + \frac{1}{4}\left(\sqrt{-2\sqrt{2}-3}+1\right)\log\left(-\left(\sqrt{2}-2\right)\sqrt{-2\sqrt{2}-3}+2x-\sqrt{2}\right)$$

```
[In] integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2*sqrt(2) - 3) + 1)*log(sqrt(2*sqrt(2) - 3)*(sqrt(2) + 2) + 2*x +
 sqrt(2)) - 1/4*(sqrt(2*sqrt(2) - 3) - 1)*log(-sqrt(2*sqrt(2) - 3)*(sqrt(2)
 + 2) + 2*x + sqrt(2)) - 1/4*(sqrt(-2*sqrt(2) - 3) - 1)*log((sqrt(2) - 2)*s
qrt(-2*sqrt(2) - 3) + 2*x - sqrt(2)) + 1/4*(sqrt(-2*sqrt(2) - 3) + 1)*log(-
(sqrt(2) - 2)*sqrt(-2*sqrt(2) - 3) + 2*x - sqrt(2))
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = \frac{\log\left(x^2-\sqrt{2}x+1\right)}{4} + \frac{\log\left(x^2+\sqrt{2}x+1\right)}{4} + 2\cdot\left(\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x-1\right) + 2\left(-\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x+1\right)$$

```
[In] integrate((x**3+x**2+x+1)/(x**4+1),x)
```

```
[Out] log(x**2 - sqrt(2)*x + 1)/4 + log(x**2 + sqrt(2)*x + 1)/4 + 2*(1/4 + sqrt(2
)/4)*atan(sqrt(2)*x - 1) + 2*(-1/4 + sqrt(2)/4)*atan(sqrt(2)*x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = -\frac{1}{4}\sqrt{2}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{4}\sqrt{2}\left(\sqrt{2}+2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{1}{4}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{4}\log\left(x^2-\sqrt{2}x+1\right)$$

```
[In] integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2
)*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/4*log(x^2 + sqrt(2)
*x + 1) + 1/4*log(x^2 - sqrt(2)*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = \frac{1}{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{2}\left(\sqrt{2}+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{1}{4}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{4}\log\left(x^2-\sqrt{2}x+1\right)$$

```
[In] integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*(sqrt(2) + 1)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/4*log(x^2 + sqrt(2)*x + 1) + 1/4*log
(x^2 - sqrt(2)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.94

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = \ln\left((16\,x-16)\,\left(\frac{\sqrt{-2\,\sqrt{2}-3}}{4}+\frac{1}{4}\right)-8\,x\right)\,\left(\frac{\sqrt{-2\,\sqrt{2}-3}}{4}+\frac{1}{4}\right) - \ln\left(8\,x+(16\,x-16)\,\left(\frac{\sqrt{-2\,\sqrt{2}-3}}{4}-\frac{1}{4}\right)\right)\,\left(\frac{\sqrt{-2\,\sqrt{2}-3}}{4}-\frac{1}{4}\right) - \ln\left(8\,x+(16\,x-16)\,\left(\frac{\sqrt{2\,\sqrt{2}-3}}{4}-\frac{1}{4}\right)\right)\,\left(\frac{\sqrt{2\,\sqrt{2}-3}}{4}-\frac{1}{4}\right) + \ln\left(8\,x-(16\,x-16)\,\left(\frac{\sqrt{2\,\sqrt{2}-3}}{4}+\frac{1}{4}\right)\right)\,\left(\frac{\sqrt{2\,\sqrt{2}-3}}{4}+\frac{1}{4}\right)$$

```
[In] int((x + x^2 + x^3 + 1)/(x^4 + 1),x)
```

```
[Out] log((16*x - 16)*((- 2*2^(1/2) - 3)^(1/2)/4 + 1/4) - 8*x)*((- 2*2^(1/2) - 3)
^(1/2)/4 + 1/4) - log(8*x + (16*x - 16)*((- 2*2^(1/2) - 3)^(1/2)/4 - 1/4))*
((- 2*2^(1/2) - 3)^(1/2)/4 - 1/4) - log(8*x + (16*x - 16)*((2*2^(1/2) - 3)^
(1/2)/4 - 1/4))*((2*2^(1/2) - 3)^(1/2)/4 - 1/4) + log(8*x - (16*x - 16)*((2
*2^(1/2) - 3)^(1/2)/4 + 1/4))*((2*2^(1/2) - 3)^(1/2)/4 + 1/4)
```

3.169 $\int \frac{1+x+x^2+x^3}{a-bx^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 124

$$\int \frac{1+x+x^2+x^3}{a-bx^4} dx = -\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{\left(\sqrt{a}+\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\log\left(a-bx^4\right)}{4b}$$

```
[Out] -1/4*ln(-b*x^4+a)/b-1/2*arctan(b^(1/4)*x/a^(1/4))*(a^(1/2)-b^(1/2))/a^(3/4)
/b^(3/4)+1/2*arctanh(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)+1/2*arctanh(b^(1/
4)*x/a^(1/4))*(a^(1/2)+b^(1/2))/a^(3/4)/b^(3/4)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1890, 1181, 211, 214, 1262, 649, 266}

$$\int \frac{1+x+x^2+x^3}{a-bx^4} dx = -\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{\left(\sqrt{a}+\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\log\left(a-bx^4\right)}{4b}$$

```
[In] Int[(1 + x + x^2 + x^3)/(a - b*x^4),x]
```

```
[Out] -1/2*((Sqrt[a] - Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(a^(3/4)*b^(3/4)) +
((Sqrt[a] + Sqrt[b])*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + Ar
cTanh[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - Log[a - b*x^4]/(4*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \int \left(\frac{1+x^2}{a-bx^4}+\frac{x(1+x^2)}{a-bx^4}\right)dx$$
$$= \int \frac{1+x^2}{a-bx^4}\,dx+\int \frac{x(1+x^2)}{a-bx^4}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx$$
$$+\frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx$$
$$= -\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$
$$+\frac{1}{2}\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{x}{a-bx^2}\,dx,x,x^2\right)$$
$$= -\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\log\left(a-bx^4\right)}{4b}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.64

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx = \frac{\left(-a^{3/4}+\sqrt[4]{a}\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2ab^{3/4}} - \frac{\left(a^{3/4}+\sqrt{a}\sqrt[4]{b}+\sqrt[4]{a}\sqrt{b}\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{4ab^{3/4}} - \frac{\left(-a^{3/4}+\sqrt{a}\sqrt[4]{b}-\sqrt[4]{a}\sqrt{b}\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{4ab^{3/4}} + \frac{\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{a}\sqrt{b}} - \frac{\log\left(a-bx^4\right)}{4b}$$

```
[In] Integrate[(1 + x + x^2 + x^3)/(a - b*x^4),x]
```

```
[Out] ((-a^(3/4) + a^(1/4)*Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a*b^(3/4)) -
((a^(3/4) + Sqrt[a]*b^(1/4) + a^(1/4)*Sqrt[b])*Log[a^(1/4) - b^(1/4)*x])/(4
*a*b^(3/4)) - ((-a^(3/4) + Sqrt[a]*b^(1/4) - a^(1/4)*Sqrt[b])*Log[a^(1/4) +
 b^(1/4)*x])/(4*a*b^(3/4)) + Log[Sqrt[a] + Sqrt[b]*x^2]/(4*Sqrt[a]*Sqrt[b])
 - Log[a - b*x^4]/(4*b)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.31

method	result	size
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b-a\right)}\frac{\left(\_R^{3}+\_R^{2}+\_R+1\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b}$	38
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{\ln\left(\frac{a+x^{2}\sqrt{ab}}{a-x^{2}\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{\ln\left(-b\,x^{4}+a\right)}{4b}$	150

```
[In] int((x^3+x^2+x+1)/(-b*x^4+a),x,method=_RETURNVERBOSE)
[Out] -1/4/b*sum((_R^3+_R^2+_R+1)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.28 (sec) , antiderivative size = 91748, normalized size of antiderivative = 739.90

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.51

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx = -\operatorname{RootSum}\left(256t^4a^3b^4-256t^3a^3b^3+t^2\cdot\left(96a^3b^2-96a^2b^3\right)+t\left(-16a^3b+32a^2b^2-16ab^3\right)+a^3-3a^2b+\right.$$

```
[In] integrate((x**3+x**2+x+1)/(-b*x**4+a),x)
[Out] -RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3 + _t**2*(96*a**3*b**2 -
96*a**2*b**3) + _t*(-16*a**3*b + 32*a**2*b**2 - 16*a*b**3) + a**3 - 3*a**2*
b + 3*a*b**2 - b**3, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**3 + 48*_t**2*
a**3*b**2 + 16*_t**2*a**2*b**3 - 12*_t*a**3*b + 16*_t*a**2*b**2 - 4*_t*a*b*
*3 + a**3 - 2*a**2*b + a*b**2)/(a**2*b - 2*a*b**2 + b**3))))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.29

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx = -\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}b} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}b} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

```
[In] integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="maxima")
[Out] -1/2*(sqrt(a) - sqrt(b))*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt(a) - sqrt(b))*log(sqrt(b)*x^2 + s
qrt(a))/(sqrt(a)*b) - 1/4*(sqrt(a) + sqrt(b))*log(sqrt(b)*x^2 - sqrt(a))/(s
qrt(a)*b) - 1/4*(sqrt(a) + sqrt(b))*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))
/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b
))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(84) = 168.

Time = 0.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.34

$$\begin{aligned}\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx &= -\frac{\log\left(|bx^4-a|\right)}{4\,b} + \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\sqrt{2}\sqrt{-ab^3}b+\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}\\
&+ \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2+\sqrt{2}\sqrt{-ab^3}b+\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}\\
&+ \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\left(-ab^3\right)^{\frac{3}{4}}\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3}\\
&- \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\left(-ab^3\right)^{\frac{3}{4}}\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3}\end{aligned}$$

```
[In] integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(b*x^4 - a))/b + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2 - sqrt(2)*sqrt
(-a*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4)
)/(-a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2 + sqrt(2)*sqrt(-a
*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(
-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*lo
g(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3
)^(1/4)*b^2 - (-a*b^3)^(3/4))*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b)
)/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.52

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx = \sum_{k=1}^{4} \ln\Big(-\mathrm{root}\big(256\,a^3\,b^4\,z^4+256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2-96\,a^2\,b^3\,z^2 + 16\,a^3\,b\,z+16\,a\,b^3\,z-32\,a^2\,b^2\,z-3\,a^2\,b+3\,a\,b^2-b^3 + a^3, z, k\big)\ \big(\mathrm{root}\big(256\,a^3\,b^4\,z^4+256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2-96\,a^2\,b^3\,z^2+16\,a^3\,b\,z+16\,a\,b^3\,z-32\,a^2\,b^2\,z-3\,a^2\,b+ \ldots - x\,\big(4\,a\,b^2-4\,b^3\big)\big)\Big)\ \mathrm{root}\big(256\,a^3\,b^4\,z^4+256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2-96\,a^2\,b^3\,z^2+16\,a^3\,b\,z + 16\,a\,b^3\,z-32\,a^2\,b^2\,z-3\,a^2\,b+3\,a\,b^2-b^3+a^3, z, k\big)$$

```
[In] int((x + x^2 + x^3 + 1)/(a - b*x^4),x)
```

```
[Out] symsum(log(-root(256*a^3*b^4*z^4 + 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 - 96*a^
2*b^3*z^2 + 16*a^3*b*z + 16*a*b^3*z - 32*a^2*b^2*z - 3*a^2*b + 3*a*b^2 - b^
3 + a^3, z, k)*(root(256*a^3*b^4*z^4 + 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 - 9
6*a^2*b^3*z^2 + 16*a^3*b*z + 16*a*b^3*z - 32*a^2*b^2*z - 3*a^2*b + 3*a*b^2
- b^3 + a^3, z, k)*(16*a*b^3 - 16*a*b^3*x) - x*(4*a*b^2 - 4*b^3)))*root(256
*a^3*b^4*z^4 + 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 - 96*a^2*b^3*z^2 + 16*a^3*b
*z + 16*a*b^3*z - 32*a^2*b^2*z - 3*a^2*b + 3*a*b^2 - b^3 + a^3, z, k), k, 1
, 4)
```

3.170 $\int \frac{1+x+x^2+x^3}{a+bx^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 277

$$\begin{aligned}\int \frac{1+x+x^2+x^3}{a+bx^4} dx =& \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} \\ &+ \frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} \\ &+ \frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} \\ &- \frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\log\left(a+bx^4\right)}{4b}\end{aligned}$$

```
[Out] 1/4*ln(b*x^4+a)/b+1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(a
^(1/2)-b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)-1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^
(1/2)+x^2*b^(1/2))*(a^(1/2)-b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/2*arctan(x^2
*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)+1/4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*
(a^(1/2)+b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^
(1/4))*(a^(1/2)+b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 211, 266}

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx = -\frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} + \frac{\log\left(a+bx^4\right)}{4b}$$

```
[In] Int[(1 + x + x^2 + x^3)/(a + b*x^4),x]
```

```
[Out] ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt
[a] + Sqrt[b])*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
b^(3/4)) + ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) +
Log[a + b*x^4]/(4*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1+x^2}{a+bx^4}+\frac{x(1+x^2)}{a+bx^4}\right)dx\\
&= \int \frac{1+x^2}{a+bx^4}\,dx+\int \frac{x(1+x^2)}{a+bx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{a+bx^2}\,dx,x,x^2\right)-\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}+\frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b}\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx,x,x^2\right)\\
&\quad+\frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b}+\frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b}\\
&\quad+\frac{\left(\sqrt{a}-\sqrt{b}\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}-\sqrt{b}\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{3/4}}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}\\
&\quad-\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\log\left(a+bx^4\right)}{4b}\\
&\quad+\frac{\left(\sqrt{a}+\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}\\
&\quad-\frac{\left(\sqrt{a}+\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}
\end{aligned}
$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\log\left(a+bx^4\right)}{4b}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.02

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx = \frac{-2\sqrt[4]{a}\left(\sqrt{2}\sqrt{a}+2\sqrt[4]{a}\sqrt[4]{b}+\sqrt{2}\sqrt{b}\right)\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\left(\sqrt{2}\sqrt{a}-2\sqrt[4]{a}\sqrt[4]{b}+\sqrt{2}\sqrt{b}\right)\sqrt[4]{b}\,\mathrm{arct}}{}$$

```
[In] Integrate[(1 + x + x^2 + x^3)/(a + b*x^4),x]
```

```
[Out] (-2*a^(1/4)*(Sqrt[2]*Sqrt[a] + 2*a^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b])*b^(1/4)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*(Sqrt[2]*Sqrt[a] - 2*a
^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^
(1/4)] + Sqrt[2]*(a^(3/4) - a^(1/4)*Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(-a^(3/4) + a^(1/4)*Sqrt[b])*b^(
1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a*Log[a + b
*x^4])/(8*a*b)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.13

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 b+a)}\frac{\left(\_R^3+\_R^2+\_R+1\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{}$

```
[In] int((x^3+x^2+x+1)/(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4/b*sum((_R^3+_R^2+_R+1)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 96349, normalized size of antiderivative = 347.83

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.68

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx$$
$$= \text{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3 + t^2\cdot\left(96a^3b^2 + 96a^2b^3\right) + t\left(-16a^3b - 32a^2b^2 - 16ab^3\right) + a^3 + 3a^2b + 3a\right.$$

```
[In] integrate((x**3+x**2+x+1)/(b*x**4+a),x)
[Out] RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3 + _t**2*(96*a**3*b**2 + 9
6*a**2*b**3) + _t*(-16*a**3*b - 32*a**2*b**2 - 16*a*b**3) + a**3 + 3*a**2*b
 + 3*a*b**2 + b**3, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**3 - 48*_t**2*a*
*3*b**2 + 16*_t**2*a**2*b**3 + 12*_t*a**3*b + 16*_t*a**2*b**2 + 4*_t*a*b**3
 - a**3 - 2*a**2*b - a*b**2)/(a**2*b + 2*a*b**2 + b**3))))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.07

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}-\sqrt{a}\sqrt{b}+b\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}+\sqrt{a}\sqrt{b}-b\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\left(\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{a}\right)b+\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}+2\,a\right)\sqrt{b}-2\,a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{5}{4}}} + \frac{\left(\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{a}\right)b+\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}-2\,a\right)\sqrt{b}+2\,a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{5}{4}}}$$

```
[In] integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(sqrt(2)*a^(3/4)*b^(1/4) - sqrt(a)*sqrt(b) + b)*log(sqrt(b)*x^2
 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/8*sqrt(2)*(sq
rt(2)*a^(3/4)*b^(1/4) + sqrt(a)*sqrt(b) - b)*log(sqrt(b)*x^2 - sqrt(2)*a^(1
/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/4*((sqrt(2)*a^(1/4)*b^(1/4)
- 2*sqrt(a))*b + (sqrt(2)*a^(3/4)*b^(1/4) + 2*a)*sqrt(b) - 2*a*sqrt(b))*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 1/4*((sqrt(2)*a^(1/4)*b^(1/4)
 + 2*sqrt(a))*b + (sqrt(2)*a^(3/4)*b^(1/4) - 2*a)*sqrt(b) + 2*a*sqrt(b))*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.97

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx = \frac{\log(|bx^4+a|)}{4\,b} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2-\sqrt{2}\sqrt{ab^3}b+(ab^3)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2+\sqrt{2}\sqrt{ab^3}b+(ab^3)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2-(ab^3)^{\frac{3}{4}}\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2-(ab^3)^{\frac{3}{4}}\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

```
[In] integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2 - sqrt(2)*sqrt(a
*b^3)*b + (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/
b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2 + sqrt(2)*sqrt(a*b^3)*b
+ (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*log(x^2 + sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2 - (a
*b^3)^(3/4))*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.10

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx = \sum_{k=1}^{4}\ln\Big(-\mathrm{root}\big(256\,a^3\,b^4\,z^4-256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2+96\,a^2\,b^3\,z^2-16\,a^3\,b\,z-16\,a\,b^3\,z-32\,a^2\,b^2\,z+3\,a^2\,b+3\,a\,b^2+b^3+a^3,z,k\big)\,\big(\mathrm{root}\big(256\,a^3\,b^4\,z^4-256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2+96\,a^2\,b^3\,z^2-16\,a^3\,b\,z-16\,a\,b^3\,z-32\,a^2\,b^2\,z+3\,a^2\,b+ +x\left(4\,b^3+4\,a\,b^2\right)\big)\big)\Big)\,\mathrm{root}\big(256\,a^3\,b^4\,z^4-256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2+96\,a^2\,b^3\,z^2-16\,a^3\,b\,z-16\,a\,b^3\,z-32\,a^2\,b^2\,z+3\,a^2\,b+3\,a\,b^2+b^3+a^3,z,k\big)$$

```
[In] int((x + x^2 + x^3 + 1)/(a + b*x^4),x)
```

```
[Out] symsum(log(-root(256*a^3*b^4*z^4 - 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 + 96*a^
2*b^3*z^2 - 16*a^3*b*z - 16*a*b^3*z - 32*a^2*b^2*z + 3*a^2*b + 3*a*b^2 + b^
3 + a^3, z, k)*(root(256*a^3*b^4*z^4 - 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 + 9
6*a^2*b^3*z^2 - 16*a^3*b*z - 16*a*b^3*z - 32*a^2*b^2*z + 3*a^2*b + 3*a*b^2
+ b^3 + a^3, z, k)*(16*a*b^3 - 16*a*b^3*x) + x*(4*a*b^2 + 4*b^3)))*root(256
*a^3*b^4*z^4 - 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 + 96*a^2*b^3*z^2 - 16*a^3*b
*z - 16*a*b^3*z - 32*a^2*b^2*z + 3*a^2*b + 3*a*b^2 + b^3 + a^3, z, k), k, 1
, 4)
```

3.171 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4} dx$

Optimal result

Integrand size = 31, antiderivative size = 148

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4} dx = -\frac{gx}{b}+\frac{\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}} + \frac{\left(bc+\sqrt{a}\sqrt{b}e+ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{f\log\left(a-bx^4\right)}{4b}$$

```
[Out] -g*x/b-1/4*f*ln(-b*x^4+a)/b+1/2*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1
/2)+1/2*arctan(b^(1/4)*x/a^(1/4))*(b*c+a*g-e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/
4)+1/2*arctanh(b^(1/4)*x/a^(1/4))*(b*c+a*g+e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/
4)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used

$= \{1899, 1262, 649, 214, 266, 1901, 1181, 211\}$

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{f\log\left(a-bx^4\right)}{4b} - \frac{gx}{b}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]
```

```
[Out] -((g*x)/b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/
(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTanh[(b^(1/4)*x)/
a^(1/4)])/(2*a^(3/4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[
a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x(d+fx^2)}{a-bx^4}+\frac{c+ex^2+gx^4}{a-bx^4}\right)dx\\
&= \int \frac{x(d+fx^2)}{a-bx^4}\,dx+\int \frac{c+ex^2+gx^4}{a-bx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{a-bx^2}\,dx,x,x^2\right)+\int \left(-\frac{g}{b}+\frac{bc+ag+bex^2}{b\left(a-bx^4\right)}\right)dx\\
&= -\frac{gx}{b}+\frac{\int \frac{bc+ag+bex^2}{a-bx^4}\,dx}{b}+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}f\text{Subst}\left(\int \frac{x}{a-bx^2}\,dx,x,x^2\right)\\
&= -\frac{gx}{b}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}\\
&\quad+\frac{1}{2}\left(e-\frac{bc+ag}{\sqrt{a}\sqrt{b}}\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(e+\frac{bc+ag}{\sqrt{a}\sqrt{b}}\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx\\
&= -\frac{gx}{b}+\frac{\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}\\
&\quad+\frac{\left(bc+\sqrt{a}\sqrt{b}e+ag\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.68

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx$$
$$= \frac{-4a^{3/4}\sqrt[4]{b}gx+2\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(bc+\sqrt[4]{a}b^{3/4}d+\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]
```

```
[Out] (-4*a^(3/4)*b^(1/4)*g*x + 2*(b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)
*x)/a^(1/4)] - (b*c + a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e + a*g)*Log[a^(1
/4) - b^(1/4)*x] + b*c*Log[a^(1/4) + b^(1/4)*x] - a^(1/4)*b^(3/4)*d*Log[a^(
1/4) + b^(1/4)*x] + Sqrt[a]*Sqrt[b]*e*Log[a^(1/4) + b^(1/4)*x] + a*g*Log[a^
(1/4) + b^(1/4)*x] + a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2] - a^(3/4)
*b^(1/4)*f*Log[a - b*x^4])/(4*a^(3/4)*b^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{gx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(-\_R^3 bf-\_R^2 be-\_R bd-ag-bc\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$-\frac{gx}{b}+\frac{\frac{(ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{bd\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{f\ln\left(-b\,x^4+\right.}{4}}{b}$

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -g*x/b+1/4/b^2*sum((-_R^3*b*f-_R^2*b*e-_R*b*d-a*g-b*c)/_R^3*ln(x-_R),_R=Roo
tOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 42.65 (sec) , antiderivative size = 592528, normalized size of antiderivative = 4003.57

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx = \text{Timed out}$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.36

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx = -\frac{gx}{b} + \frac{\frac{2\left(b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\left(b^{\frac{3}{2}}d-\sqrt{a}bf\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(b^{\frac{3}{2}}d+\sqrt{a}bf\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(b^{\frac{3}{2}}c+\sqrt{a}be+a\sqrt{b}g\right)\log\left(\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{4\,b}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] -g*x/b + 1/4*(2*(b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g)*arctan(sqrt(b)*x/sq
rt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b^(3/2)*d -
 sqrt(a)*b*f)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (b^(3/2)*d + sqrt(a)
*b*f)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (b^(3/2)*c + sqrt(a)*b*e + a
*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(
a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(108) = 216$.

Time = 0.27 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.02

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx$$
$$= -\frac{\sqrt{2}\left(b^2c+abg-\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,(-ab^3)^{\frac{3}{4}}}$$
$$-\frac{\sqrt{2}\left(b^2c+abg+\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,(-ab^3)^{\frac{3}{4}}}$$
$$-\frac{\sqrt{2}(b^2c+abg-\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,(-ab^3)^{\frac{3}{4}}}$$
$$+\frac{\sqrt{2}(b^2c+abg-\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,(-ab^3)^{\frac{3}{4}}}-\frac{gx}{b}-\frac{f\log\left(|bx^4-a|\right)}{4\,b}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c + a*b*g - sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*
arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4
) - 1/4*sqrt(2)*(b^2*c + a*b*g + sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*
e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(
3/4) - 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a
/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(
-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) -
g*x/b - 1/4*f*log(abs(b*x^4 - a))/b
```

Mupad [B] (verification not implemented)

Time = 9.89 (sec) , antiderivative size = 5082, normalized size of antiderivative = 34.34

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x)
```

```
[Out] symsum(log(b^2*c^2*e - b^2*c*d^2 + a^2*e*g^2 - a^2*f^2*g - b^2*d^3*x - a*b*
e^3 - a*b*c*f^2 - a*b*d^2*g - 16*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 -
```

64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)^2*a*b^3*c - 4*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*b^3*c^2*x - b^2*c^2*f*x - a^2*f*g^2*x - 16*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)^2*a^2*b^2*g + 16*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)^2*a*b^3*d*x - 4*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*a*b^2*e^2*x - 4*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f

*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^
2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z +
8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2
*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*
f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2
*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^
4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*a^2*b*g^2*x + 2*a*b*c*e*g + 2*a*
b*d*e*f - 8*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 -
 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*
e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16
*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z
 + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*
e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2
*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2
 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3
*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*a*b^2*c*f + 8*root(256*a^3*b^
5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^
3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*
z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b
^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^
2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 -
 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*
a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3
*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^
4 - b^4*c^4, z, k)*a*b^2*d*e - 8*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 -
 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*
d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16
*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d
*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d
*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*
d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g
 - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^
2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*a^2*b*f*
g + a*b*d*f^2*x - a*b*e^2*f*x + 2*b^2*c*d*e*x - 8*root(256*a^3*b^5*z^4 + 25
6*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*
z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2
*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z
 + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*
e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e
*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^
3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2
 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^
4, z, k)*a*b^2*c*g*x + 8*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*
b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2
 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2

*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*a*b^2*d*f*x - 2*a*b*c*f*g*x + 2*a*b*d*e*g*x)*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k), k, 1, 4) - (g*x)/b

3.172 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 172

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2} dx = \frac{x(bc+ag+bdx+bex^2+bfx^3)}{4ab(a-bx^4)} + \frac{\left(3bc-\sqrt{a}\sqrt{b}e-ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}} + \frac{\left(3bc+\sqrt{a}\sqrt{b}e-ag\right)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}} + \frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

```
[Out] 1/4*x*(b*f*x^3+b*e*x^2+b*d*x+a*g+b*c)/a/b/(-b*x^4+a)+1/4*d*arctanh(x^2*b^(1
/2)/a^(1/2))/a^(3/2)/b^(1/2)+1/8*arctan(b^(1/4)*x/a^(1/4))*(3*b*c-a*g-e*a^(
1/2)*b^(1/2))/a^(7/4)/b^(5/4)+1/8*arctanh(b^(1/4)*x/a^(1/4))*(3*b*c-a*g+e*a
^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used

$= \{1872, 1890, 281, 214, 1181, 211\}$

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{x(ag+bc+bdx+bex^2+bfx^3)}{4ab\,(a-bx^4)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]
```

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + ((3*b*c -
 Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4))
+ ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/
4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
```

```
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{4ab\,(a-bx^4)}+\frac{\int \frac{3bc-ag+2bdx+bex^2}{a-bx^4}\,dx}{4ab}\\
&= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{4ab\,(a-bx^4)}+\frac{\int \left(\frac{2bdx}{a-bx^4}+\frac{3bc-ag+bex^2}{a-bx^4}\right)dx}{4ab}\\
&= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{4ab\,(a-bx^4)}+\frac{\int \frac{3bc-ag+bex^2}{a-bx^4}\,dx}{4ab}+\frac{d\int \frac{x}{a-bx^4}\,dx}{2a}\\
&= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{4ab\,(a-bx^4)}+\frac{d\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{4a}\\
&\quad -\frac{\left(3bc-\sqrt{a}\sqrt{b}e-ag\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{8a^{3/2}\sqrt{b}}+\frac{\left(3bc+\sqrt{a}\sqrt{b}e-ag\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx}{8a^{3/2}\sqrt{b}}\\
&= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{4ab\,(a-bx^4)}+\frac{\left(3bc-\sqrt{a}\sqrt{b}e-ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}}\\
&\quad +\frac{\left(3bc+\sqrt{a}\sqrt{b}e-ag\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.28

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx$$
$$= \frac{\frac{4a^{3/4}\sqrt[4]{b}(a(f+gx)+bx(c+x(d+ex)))}{a-bx^4}-2\left(-3bc+\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(3bc+2\sqrt[4]{a}b^{3/4}d+\sqrt{a}\sqrt{b}e-ag\right.}{16a^{7/4}}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*(a*(f + g*x) + b*x*(c + x*(d + e*x))))/(a - b*x^4) - 2*
(-3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (3*b*c + 2
*a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e - a*g)*Log[a^(1/4) - b^(1/4)*x] + (3
*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e - a*g)*Log[a^(1/4) + b^(1/4)
*x] + 2*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.63

method	result
risch	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^4+a}-\frac{\sum_{\_R=\operatorname{RootOf}(\_Z^4 b-a)}\frac{\left(\_R^2 e+2\_R d-\frac{ag-3bc}{b}\right)\ln(x-\_R)}{\_R^3}}{16ba}$
default	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^4+a}+\frac{\frac{(-ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{bd\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{2\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right.}{x-\left(\frac{a}{b}\right.}\right.\right.}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{4ba}$

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4/a*e*x^3+1/4*d/a*x^2+1/4*(a*g+b*c)/a/b*x+1/4*f/b)/(-b*x^4+a)-1/16/b/a*s
um((_R^2*e+2*_R*d-1/b*(a*g-3*b*c))/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 30.98 (sec) , antiderivative size = 334837, normalized size of antiderivative = 1946.73

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.30

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx = -\frac{bex^3+bdx^2+af+(bc+ag)x}{4\,(ab^2x^4-a^2b)} + \frac{\frac{2\,\sqrt{b}d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}} - \frac{2\,\sqrt{b}d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}} + \frac{2\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be-a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(3\,b^{\frac{3}{2}}c+\sqrt{a}be-a\sqrt{b}g\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{16\,ab}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b*e*x^3 + b*d*x^2 + a*f + (b*c + a*g)*x)/(a*b^2*x^4 - a^2*b) + 1/16*(
2*sqrt(b)*d*log(sqrt(b)*x^2 + sqrt(a))/sqrt(a) - 2*sqrt(b)*d*log(sqrt(b)*x^
2 - sqrt(a))/sqrt(a) + 2*(3*b^(3/2)*c - sqrt(a)*b*e - a*sqrt(b)*g)*arctan(s
qrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (
3*b^(3/2)*c + sqrt(a)*b*e - a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt
(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)))/(a*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(133) = 266.

Time = 0.28 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.97

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx$$
$$= -\frac{\sqrt{2}\left(3\,b^2c-abg-2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{\sqrt{2}\left(3\,b^2c-abg+2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{\sqrt{2}(3\,b^2c-abg-\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a} + \frac{\sqrt{2}(3\,b^2c-abg-\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{bex^3+bdx^2+bcx+agx+af}{4\,(bx^4-a)ab}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*b^2*c - a*b*g - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*
b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3
)^(3/4)*a) - 1/16*sqrt(2)*(3*b^2*c - a*b*g + 2*sqrt(2)*(-a*b^3)^(1/4)*b*d -
 sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/((-a*b^3)^(3/4)*a) - 1/32*sqrt(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*lo
g(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt
(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + s
qrt(-a/b))/((-a*b^3)^(3/4)*a) - 1/4*(b*e*x^3 + b*d*x^2 + b*c*x + a*g*x + a*
f)/((b*x^4 - a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 1393, normalized size of antiderivative = 8.10

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx$$
$$= \left(\sum_{k=1}^{4}\ln\left(-\frac{-a^2\,e\,g^2+6\,a\,b\,c\,e\,g-4\,a\,b\,d^2\,g+a\,b\,e^3-9\,b^2\,c^2\,e+12\,b^2\,c\,d^2}{64\,a^3} - \frac{\mathrm{root}(65536\,a^7\,b^5\,z^4+1024\,a^5\,b^3\,e\,g\,z^2-3072\,a^4\,b^4\,c\,e\,z^2-2048\,a^4\,b^4\,d^2\,z^2-768\,a^3\,b^3\,c\,d\,g\,z+128\,a^4\,b^2}{} - \frac{b\,d\,x\,(2\,b\,d^2-3\,b\,c\,e+a\,e\,g)}{16\,a^3}\right)\mathrm{root}\big(65536\,a^7\,b^5\,z^4+1024\,a^5\,b^3\,e\,g\,z^2-3072\,a^4\,b^4\,c\,e\,z^2 - 2048\,a^4\,b^4\,d^2\,z^2-768\,a^3\,b^3\,c\,d\,g\,z+128\,a^4\,b^2\,d\,g^2\,z+128\,a^3\,b^3\,d\,e^2\,z+1152\,a^2\,b^4\,c^2\,d\,z + 16\,a^2\,b^2\,d^2\,e\,g-12\,a^2\,b^2\,c\,e^2\,g-48\,a\,b^3\,c\,d^2\,e+108\,a\,b^3\,c^3\,g+12\,a^3\,b\,c\,g^3-54\,a^2\,b^2\,c^2\,g^2 + 2\,a^3\,b\,e^2\,g^2+18\,a\,b^3\,c^2\,e^2+16\,a\,b^3\,d^4-81\,b^4\,c^4-a^2\,b^2\,e^4-a^4\,g^4,z,k\big)\right) + \frac{\frac{f}{4\,b}+\frac{d\,x^2}{4\,a}+\frac{e\,x^3}{4\,a}+\frac{x\,(b\,c+a\,g)}{4\,a\,b}}{a-b\,x^4}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x)
```

```
[Out] symsum(log(- (12*b^2*c*d^2 - 9*b^2*c^2*e - a^2*e*g^2 + a*b*e^3 - 4*a*b*d^2*
g + 6*a*b*c*e*g)/(64*a^3) - (root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2
- 3072*a^4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z + 128*a
^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2*d*z + 16*a^2*b^2*d^
2*e*g - 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*
c*g^3 - 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*
d^4 - 81*b^4*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k)*b*(9*b^2*c^2*x + a^2*g^2*x
- 16*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 - 3072*a^4*b^4*c*e*z^2 -
 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z + 128*a^4*b^2*d*g^2*z + 128*a^3
*b^3*d*e^2*z + 1152*a^2*b^4*c^2*d*z + 16*a^2*b^2*d^2*e*g - 12*a^2*b^2*c*e^2
*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 - 54*a^2*b^2*c^2*g
^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 - 81*b^4*c^4 - a^2*b
^2*e^4 - a^4*g^4, z, k)*a^3*b*g + a*b*e^2*x + 48*root(65536*a^7*b^5*z^4 + 1
024*a^5*b^3*e*g*z^2 - 3072*a^4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3
*b^3*c*d*g*z + 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2
*d*z + 16*a^2*b^2*d^2*e*g - 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b
^3*c^3*g + 12*a^3*b*c*g^3 - 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3
*c^2*e^2 + 16*a*b^3*d^4 - 81*b^4*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k)*a^2*b^2
*c - 4*a*b*d*e - 32*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 - 3072*a^
4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z + 128*a^4*b^2*d*
g^2*z + 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2*d*z + 16*a^2*b^2*d^2*e*g - 1
2*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 - 5
```

```
4*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 - 81*
b^4*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k)*a^2*b^2*d*x - 6*a*b*c*g*x))/(4*a^2)
- (b*d*x*(2*b*d^2 - 3*b*c*e + a*e*g))/(16*a^3))*root(65536*a^7*b^5*z^4 + 10
24*a^5*b^3*e*g*z^2 - 3072*a^4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3*
b^3*c*d*g*z + 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2*
d*z + 16*a^2*b^2*d^2*e*g - 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^
3*c^3*g + 12*a^3*b*c*g^3 - 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*
c^2*e^2 + 16*a*b^3*d^4 - 81*b^4*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k), k, 1, 4
) + (f/(4*b) + (d*x^2)/(4*a) + (e*x^3)/(4*a) + (x*(b*c + a*g))/(4*a*b))/(a
- b*x^4)
```

3.173 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 221

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx &= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab(a-bx^4)^2} \\ &+ \frac{4af+x(7bc-ag+6bdx+5bex^2)}{32a^2b(a-bx^4)} \\ &+ \frac{\left(21bc-5\sqrt{a}\sqrt{b}e-3ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{5/4}} \\ &+ \frac{\left(21bc+5\sqrt{a}\sqrt{b}e-3ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{5/4}} \\ &+ \frac{3d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}\end{aligned}$$

```
[Out] 1/8*x*(b*f*x^3+b*e*x^2+b*d*x+a*g+b*c)/a/b/(-b*x^4+a)^2+1/32*(4*a*f+x*(5*b*e
*x^2+6*b*d*x-a*g+7*b*c))/a^2/b/(-b*x^4+a)+3/16*d*arctanh(x^2*b^(1/2)/a^(1/2
))/a^(5/2)/b^(1/2)+1/64*arctan(b^(1/4)*x/a^(1/4))*(21*b*c-3*a*g-5*e*a^(1/2)
*b^(1/2))/a^(11/4)/b^(5/4)+1/64*arctanh(b^(1/4)*x/a^(1/4))*(21*b*c-3*a*g+5*
e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1872, 1868, 1890, 281, 214, 1181, 211}

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}} + \frac{3d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} + \frac{x(-ag+7bc+6bdx+5bex^2)+4af}{32a^2b\,(a-bx^4)} + \frac{x(ag+bc+bdx+bex^2+bfx^3)}{8ab\,(a-bx^4)^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]
```

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) + (4*a*f
+ x*(7*b*c - a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a - b*x^4)) + ((21*b*c
- 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^
(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4
)])/(64*a^(11/4)*b^(5/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2
)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab\left(a-bx^4\right)^2}+\frac{\int \frac{7bc-ag+6bdx+5bex^2+4bfx^3}{\left(a-bx^4\right)^2}\,dx}{8ab}\\
&= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab\left(a-bx^4\right)^2}\\
&\quad+\frac{4af+x(7bc-ag+6bdx+5bex^2)}{32a^2b\left(a-bx^4\right)}-\frac{\int \frac{-3(7bc-ag)-12bdx-5bex^2}{a-bx^4}\,dx}{32a^2b}\\
&= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab\left(a-bx^4\right)^2}+\frac{4af+x(7bc-ag+6bdx+5bex^2)}{32a^2b\left(a-bx^4\right)}\\
&\quad-\frac{\int \left(-\frac{12bdx}{a-bx^4}+\frac{-3(7bc-ag)-5bex^2}{a-bx^4}\right)dx}{32a^2b}
\end{aligned}$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab(a-bx^4)^2} + \frac{4af+x(7bc-ag+6bdx+5bex^2)}{32a^2b(a-bx^4)} - \frac{\int \frac{-3(7bc-ag)-5bex^2}{a-bx^4}\,dx}{32a^2b} + \frac{(3d)\int \frac{x}{a-bx^4}\,dx}{8a^2}$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab(a-bx^4)^2} + \frac{4af+x(7bc-ag+6bdx+5bex^2)}{32a^2b(a-bx^4)} + \frac{(3d)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{16a^2} - \frac{\left(21bc-5\sqrt{a}\sqrt{b}e-3ag\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{64a^{5/2}\sqrt{b}} + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e-3ag\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx}{64a^{5/2}\sqrt{b}}$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab(a-bx^4)^2} + \frac{4af+x(7bc-ag+6bdx+5bex^2)}{32a^2b(a-bx^4)} + \frac{\left(21bc-5\sqrt{a}\sqrt{b}e-3ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{5/4}} + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e-3ag\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{5/4}} + \frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.19

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx = \frac{\frac{4a^{3/4}\sqrt[4]{b}(a^2(4f+3gx)-b^2x^5(7c+x(6d+5ex))+abx(11c+x(10d+9ex+gx^3)))}{(a-bx^4)^2} + 2\left(21bc-5\sqrt{a}\sqrt{b}e-3ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \left(\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*(a^2*(4*f + 3*g*x) - b^2*x^5*(7*c + x*(6*d + 5*e*x)) +
a*b*x*(11*c + x*(10*d + 9*e*x + g*x^3))))/(a - b*x^4)^2 + 2*(21*b*c - 5*Sqr
t[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (21*b*c + 12*a^(1/4)*
b^(3/4)*d + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*Log[a^(1/4) - b^(1/4)*x] + (21*b*c
 - 12*a^(1/4)*b^(3/4)*d + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*Log[a^(1/4) + b^(1/4
)*x] + 12*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a^(11/4)*b^(5/
4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.51 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.67

method	result
risch	$\frac{-\frac{5be\,x^7}{32a^2}-\frac{3bd\,x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-b\,x^4+a)^2}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)}\frac{\left(5\_R^2e+12\_Rd-\frac{3(ag-7bc)}{b}\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2b}$
default	$\frac{-\frac{5be\,x^7}{32a^2}-\frac{3bd\,x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-b\,x^4+a)^2}+\frac{\frac{(-3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{3bd\ln\left(\frac{a-}{a-}\right.}{\sqrt{}}}{32a^2b}$

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)

[Out] (-5/32*b*e/a^2*x^7-3/16*b*d/a^2*x^6+1/32*(a*g-7*b*c)/a^2*x^5+9/32/a*e*x^3+5/16*d/a*x^2+1/32*(3*a*g+11*b*c)/a/b*x+1/8*f/b)/(-b*x^4+a)^2-1/128/a^2/b*sum((5*_R^2*e+12*_R*d-3/b*(a*g-7*b*c))/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 42.83 (sec) , antiderivative size = 343626, normalized size of antiderivative = 1554.87

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx = \text{Too large to display}$$

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx = \text{Timed out}$$

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.29

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx$$
$$= -\frac{5\,b^2ex^7+6\,b^2dx^6-9\,abex^3+(7\,b^2c-abg)x^5-10\,abdx^2-4\,a^2f-(11\,abc+3\,a^2g)x}{32\,(a^2b^3x^8-2\,a^3b^2x^4+a^4b)}$$
$$+\frac{\frac{12\,\sqrt{b}d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}}-\frac{12\,\sqrt{b}d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}}+\frac{2\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be-3\,a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\,b^{\frac{3}{2}}c+5\,\sqrt{a}be-3\,a\sqrt{b}g\right)\log\left(\frac{\sqrt{b}\cdots}{\sqrt{b}\cdots}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{128\,a^2b}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 - 9*a*b*e*x^3 + (7*b^2*c - a*b*g)*x^5 - 10
*a*b*d*x^2 - 4*a^2*f - (11*a*b*c + 3*a^2*g)*x)/(a^2*b^3*x^8 - 2*a^3*b^2*x^4
 + a^4*b) + 1/128*(12*sqrt(b)*d*log(sqrt(b)*x^2 + sqrt(a))/sqrt(a) - 12*sqr
t(b)*d*log(sqrt(b)*x^2 - sqrt(a))/sqrt(a) + 2*(21*b^(3/2)*c - 5*sqrt(a)*b*e
 - 3*a*sqrt(b)*g)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))*sqrt(b)) - (21*b^(3/2)*c + 5*sqrt(a)*b*e - 3*a*sqrt(b)*g)*log
((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(
sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^2*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(181) = 362$.

Time = 0.43 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.75

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx$$

$$= -\frac{\sqrt{2}\left(21\,b^2c-3\,abg-12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2} - \frac{\sqrt{2}\left(21\,b^2c-3\,abg+12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2} - \frac{\sqrt{2}(21\,b^2c-3\,abg-5\,\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2} + \frac{\sqrt{2}(21\,b^2c-3\,abg-5\,\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2} - \frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5-abgx^5-9\,abex^3-10\,abdx^2-11\,abcx-3\,a^2gx-4\,a^2f}{32\,(bx^4-a)^2a^2b}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*(21*b^2*c - 3*a*b*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 5*sqrt
(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(
(-a*b^3)^(3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*c - 3*a*b*g + 12*sqrt(2)*(-a*b^
3)^(1/4)*b*d - 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(
1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 3*a*b*
g - 5*sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b
^3)^(3/4)*a^2) + 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a*b)*b*e)*log(
x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b
^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*c*x^5 - a*b*g*x^5 - 9*a*b*e*x^3 - 10*a*b*d*x
^2 - 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 1002, normalized size of antiderivative = 4.53

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx$$
$$=\frac{\frac{f}{8\,b}+\frac{5\,d\,x^2}{16\,a}+\frac{9\,e\,x^3}{32\,a}-\frac{x^5\,(7\,b\,c-a\,g)}{32\,a^2}+\frac{x\,(11\,b\,c+3\,a\,g)}{32\,a\,b}-\frac{3\,b\,d\,x^6}{16\,a^2}-\frac{5\,b\,e\,x^7}{32\,a^2}}{a^2-2\,a\,b\,x^4+b^2\,x^8}$$
$$+\left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\big(268435456\,a^{11}\,b^5\,z^4+983040\,a^7\,b^3\,e\,g\,z^2-6881280\,a^6\,b^4\,c\,e\,z^2-4718592\,a^6\,b^4\,d^2\,z^2-77414\right.\right.$$
$$-\frac{-45\,a^2\,e\,g^2+630\,a\,b\,c\,e\,g-432\,a\,b\,d^2\,g+125\,a\,b\,e^3-2205\,b^2\,c^2\,e+3024\,b^2\,c\,d^2}{32768\,a^6}$$
$$\left.-\frac{x\,(216\,b^2\,d^3-315\,c\,e\,b^2\,d+45\,a\,e\,g\,b\,d)}{4096\,a^6}\right)\mathrm{root}\big(268435456\,a^{11}\,b^5\,z^4+983040\,a^7\,b^3\,e\,g\,z^2$$
$$-6881280\,a^6\,b^4\,c\,e\,z^2-4718592\,a^6\,b^4\,d^2\,z^2-774144\,a^4\,b^3\,c\,d\,g\,z+55296\,a^5\,b^2\,d\,g^2\,z$$
$$+153600\,a^4\,b^3\,d\,e^2\,z+2709504\,a^3\,b^4\,c^2\,d\,z+8640\,a^2\,b^2\,d^2\,e\,g-6300\,a^2\,b^2\,c\,e^2\,g$$
$$-60480\,a\,b^3\,c\,d^2\,e+111132\,a\,b^3\,c^3\,g+2268\,a^3\,b\,c\,g^3-23814\,a^2\,b^2\,c^2\,g^2+450\,a^3\,b\,e^2\,g^2$$
$$\left.+22050\,a\,b^3\,c^2\,e^2-625\,a^2\,b^2\,e^4+20736\,a\,b^3\,d^4-81\,a^4\,g^4-194481\,b^4\,c^4,z,k\big)\right)$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x)
```

```
[Out] (f/(8*b) + (5*d*x^2)/(16*a) + (9*e*x^3)/(32*a) - (x^5*(7*b*c - a*g))/(32*a^
2) + (x*(11*b*c + 3*a*g))/(32*a*b) - (3*b*d*x^6)/(16*a^2) - (5*b*e*x^7)/(32
*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4) + symsum(log(- root(268435456*a^11*b^5*z
^4 + 983040*a^7*b^3*e*g*z^2 - 6881280*a^6*b^4*c*e*z^2 - 4718592*a^6*b^4*d^2
*z^2 - 774144*a^4*b^3*c*d*g*z + 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d*e^
2*z + 2709504*a^3*b^4*c^2*d*z + 8640*a^2*b^2*d^2*e*g - 6300*a^2*b^2*c*e^2*g
 - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 - 23814*a^2*
b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 - 625*a^2*b^2*e^4 + 2
0736*a*b^3*d^4 - 81*a^4*g^4 - 194481*b^4*c^4, z, k)*(root(268435456*a^11*b^
5*z^4 + 983040*a^7*b^3*e*g*z^2 - 6881280*a^6*b^4*c*e*z^2 - 4718592*a^6*b^4*
d^2*z^2 - 774144*a^4*b^3*c*d*g*z + 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d
*e^2*z + 2709504*a^3*b^4*c^2*d*z + 8640*a^2*b^2*d^2*e*g - 6300*a^2*b^2*c*e^
2*g - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 - 23814*a
^2*b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 - 625*a^2*b^2*e^4
+ 20736*a*b^3*d^4 - 81*a^4*g^4 - 194481*b^4*c^4, z, k)*((344064*a^5*b^3*c -
 49152*a^6*b^2*g)/(32768*a^6) - (6*b^3*d*x)/a) + (x*(144*a^4*b*g^2 + 7056*a
^2*b^3*c^2 + 400*a^3*b^2*e^2 - 2016*a^3*b^2*c*g))/(4096*a^6) - (15*b^2*d*e)
/(32*a^3)) - (3024*b^2*c*d^2 - 2205*b^2*c^2*e - 45*a^2*e*g^2 + 125*a*b*e^3
- 432*a*b*d^2*g + 630*a*b*c*e*g)/(32768*a^6) - (x*(216*b^2*d^3 - 315*b^2*c*
d*e + 45*a*b*d*e*g))/(4096*a^6))*root(268435456*a^11*b^5*z^4 + 983040*a^7*b
^3*e*g*z^2 - 6881280*a^6*b^4*c*e*z^2 - 4718592*a^6*b^4*d^2*z^2 - 774144*a^4
```

```
*b^3*c*d*g*z + 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d*e^2*z + 2709504*a^3
*b^4*c^2*d*z + 8640*a^2*b^2*d^2*e*g - 6300*a^2*b^2*c*e^2*g - 60480*a*b^3*c*
d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 - 23814*a^2*b^2*c^2*g^2 + 450
*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 - 625*a^2*b^2*e^4 + 20736*a*b^3*d^4 -
81*a^4*g^4 - 194481*b^4*c^4, z, k), k, 1, 4)
```

3.174 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 266

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab(a-bx^4)^3} + \frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b(a-bx^4)} + \frac{8af+x(11bc-ag+10bdx+9bex^2)}{96a^2b(a-bx^4)^2} + \frac{\left(77bc-15\sqrt{a}\sqrt{b}e-7ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{5/4}} + \frac{\left(77bc+15\sqrt{a}\sqrt{b}e-7ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{5/4}} + \frac{5d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

```
[Out] 1/12*x*(b*f*x^3+b*e*x^2+b*d*x+a*g+b*c)/a/b/(-b*x^4+a)^3+1/384*x*(45*b*e*x^2
+60*b*d*x-7*a*g+77*b*c)/a^3/b/(-b*x^4+a)+1/96*(8*a*f+x*(9*b*e*x^2+10*b*d*x-
a*g+11*b*c))/a^2/b/(-b*x^4+a)^2+5/32*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(7/2)
/b^(1/2)+1/256*arctan(b^(1/4)*x/a^(1/4))*(77*b*c-7*a*g-15*e*a^(1/2)*b^(1/2)
)/a^(15/4)/b^(5/4)+1/256*arctanh(b^(1/4)*x/a^(1/4))*(77*b*c-7*a*g+15*e*a^(1
/2)*b^(1/2))/a^(15/4)/b^(5/4)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {1872, 1868, 1869, 1890, 281, 214, 1181, 211}

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}} + \frac{5d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} + \frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b(a-bx^4)} + \frac{x(-ag+11bc+10bdx+9bex^2)+8af}{96a^2b(a-bx^4)^2} + \frac{x(ag+bc+bdx+bex^2+bfx^3)}{12ab(a-bx^4)^3}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x]
```

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3) + (x*(7*
(11*b*c - a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a - b*x^4)) + (8*a*f +
 x*(11*b*c - a*g + 10*b*d*x + 9*b*e*x^2))/(96*a^2*b*(a - b*x^4)^2) + ((77*b
*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/
4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTanh[(b^(1/4)*x)/
a^(1/4)])/(256*a^(15/4)*b^(5/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32
*a^(7/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{\int \frac{11bc-ag+10bdx+9bex^2+8bfx^3}{(a-bx^4)^3}\,dx}{12ab}$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{8af+x(11bc-ag+10bdx+9bex^2)}{96a^2b\,(a-bx^4)^2} - \frac{\int \frac{-7(11bc-ag)-60bdx-45bex^2}{(a-bx^4)^2}\,dx}{96a^2b}$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b\,(a-bx^4)} + \frac{8af+x(11bc-ag+10bdx+9bex^2)}{96a^2b\,(a-bx^4)^2} + \frac{\int \frac{21(11bc-ag)+120bdx+45bex^2}{a-bx^4}\,dx}{384a^3b}$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b\,(a-bx^4)} + \frac{8af+x(11bc-ag+10bdx+9bex^2)}{96a^2b\,(a-bx^4)^2} + \frac{\int \left(\frac{120bdx}{a-bx^4} + \frac{21(11bc-ag)+45bex^2}{a-bx^4}\right)dx}{384a^3b}$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b\,(a-bx^4)} + \frac{8af+x(11bc-ag+10bdx+9bex^2)}{96a^2b\,(a-bx^4)^2} + \frac{\int \frac{21(11bc-ag)+45bex^2}{a-bx^4}\,dx}{384a^3b} + \frac{(5d)\int \frac{x}{a-bx^4}\,dx}{16a^3}$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b\,(a-bx^4)} + \frac{8af+x(11bc-ag+10bdx+9bex^2)}{96a^2b\,(a-bx^4)^2} + \frac{(5d)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{32a^3} - \frac{\left(77bc-15\sqrt{a}\sqrt{b}e-7ag\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{256a^{7/2}\sqrt{b}} + \frac{\left(77bc+15\sqrt{a}\sqrt{b}e-7ag\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx}{256a^{7/2}\sqrt{b}}$$

$$= \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} + \frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b\,(a-bx^4)} + \frac{8af+x(11bc-ag+10bdx+9bex^2)}{96a^2b\,(a-bx^4)^2} + \frac{\left(77bc-15\sqrt{a}\sqrt{b}e-7ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{5/4}} + \frac{\left(77bc+15\sqrt{a}\sqrt{b}e-7ag\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{5/4}} + \frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.18

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$$
$$= \frac{\frac{4a^{3/4}\sqrt[4]{b}x(77bc-7ag+15bx(4d+3ex))}{a-bx^4}+\frac{16a^{7/4}\sqrt[4]{b}x(11bc-ag+bx(10d+9ex))}{(a-bx^4)^2}+\frac{128a^{11/4}\sqrt[4]{b}(a(f+gx)+bx(c+x(d+ex)))}{(a-bx^4)^3}+6\Big(77bc-}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*x*(77*b*c - 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a - b*x^4)
+ (16*a^(7/4)*b^(1/4)*x*(11*b*c - a*g + b*x*(10*d + 9*e*x)))/(a - b*x^4)^2
+ (128*a^(11/4)*b^(1/4)*(a*(f + g*x) + b*x*(c + x*(d + e*x))))/(a - b*x^4)^
3 + 6*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] -
 3*(77*b*c + 40*a^(1/4)*b^(3/4)*d + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*Log[a^(1/
4) - b^(1/4)*x] + 3*(77*b*c - 40*a^(1/4)*b^(3/4)*d + 15*Sqrt[a]*Sqrt[b]*e -
 7*a*g)*Log[a^(1/4) + b^(1/4)*x] + 120*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt
[b]*x^2])/(1536*a^(15/4)*b^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\frac{15eb^2x^{11}}{128a^3}+\frac{5db^2x^{10}}{32a^3}-\frac{7(ag-11bc)bx^9}{384a^3}-\frac{21bex^7}{64a^2}-\frac{5bdx^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{113ex^3}{384a}+\frac{11dx^2}{32a}+\frac{(7ag+51bc)x}{128ab}+\frac{f}{12b}}{(-bx^4+a)^3}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)}}{}$
default	$\frac{\frac{15eb^2x^{11}}{128a^3}+\frac{5db^2x^{10}}{32a^3}-\frac{7(ag-11bc)bx^9}{384a^3}-\frac{21bex^7}{64a^2}-\frac{5bdx^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{113ex^3}{384a}+\frac{11dx^2}{32a}+\frac{(7ag+51bc)x}{128ab}+\frac{f}{12b}}{(-bx^4+a)^3}+\frac{(-7ag+77bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\Big(\ln\Big(}{}$

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10-7/384*(a*g-11*b*c)/a^3*b*x^9-21/
64*b*e/a^2*x^7-5/12*b*d/a^2*x^6+3/64/a^2*(a*g-11*b*c)*x^5+113/384/a*e*x^3+1
1/32*d/a*x^2+1/128*(7*a*g+51*b*c)/a/b*x+1/12*f/b)/(-b*x^4+a)^3-1/512/a^3/b*
sum((15*_R^2*e+40*_R*d-7*(a*g-11*b*c)/b)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 59.77 (sec) , antiderivative size = 343822, normalized size of antiderivative = 1292.56

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = \text{Timed out}$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.30

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = -\frac{45\,b^3ex^{11}+60\,b^3dx^{10}-126\,ab^2ex^7-160\,ab^2dx^6+7\,(11\,b^3c-ab^2g)x^9+113\,a^2bex^3+132\,a^2bdx^2-18}{384\,(a^3b^4x^{12}-3\,a^4b^3x^8+3\,a^5b^2x^4-a^6b)} + \frac{\frac{40\,\sqrt{b}d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}}-\frac{40\,\sqrt{b}d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}}+\frac{2\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be-7\,a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\,b^{\frac{3}{2}}c+15\,\sqrt{a}be-7\,a\sqrt{b}g\right)\log}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{512\,a^3b}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 - 126*a*b^2*e*x^7 - 160*a*b^2*d*x^6 +
 7*(11*b^3*c - a*b^2*g)*x^9 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 - 18*(11*a*
b^2*c - a^2*b*g)*x^5 + 32*a^3*f + 3*(51*a^2*b*c + 7*a^3*g)*x)/(a^3*b^4*x^12
 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6*b) + 1/512*(40*sqrt(b)*d*log(sqrt(b)
*x^2 + sqrt(a))/sqrt(a) - 40*sqrt(b)*d*log(sqrt(b)*x^2 - sqrt(a))/sqrt(a) +
 2*(77*b^(3/2)*c - 15*sqrt(a)*b*e - 7*a*sqrt(b)*g)*arctan(sqrt(b)*x/sqrt(sq
```

```
rt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (77*b^(3/2)*c + 1
5*sqrt(a)*b*e - 7*a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqr
t(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(
a^3*b)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.64

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$$

$$= -\frac{\sqrt{2}\left(77\,b^2c-7\,abg-40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3} - \frac{\sqrt{2}\left(77\,b^2c-7\,abg+40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3} - \frac{\sqrt{2}(77\,b^2c-7\,abg-15\,\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3} + \frac{\sqrt{2}(77\,b^2c-7\,abg-15\,\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3} - \frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9-7\,ab^2gx^9-126\,ab^2ex^7-160\,ab^2dx^6-198\,ab^2cx^5+18\,a^2bgx^5+113\,a}{384\,(bx^4-a)^3a^3b}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^2*c - 7*a*b*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 15*sqr
t(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/
((-a*b^3)^(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2*c - 7*a*b*g + 40*sqrt(2)*(-a*b
^3)^(1/4)*b*d - 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)
^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 7*a
*b*g - 15*sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((
-a*b^3)^(3/4)*a^3) + 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqrt(-a*b)*b*e
)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/3
84*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 77*b^3*c*x^9 - 7*a*b^2*g*x^9 - 126*a*b^
2*e*x^7 - 160*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*e*
x^3 + 132*a^2*b*d*x^2 + 153*a^2*b*c*x + 21*a^3*g*x + 32*a^3*f)/((b*x^4 - a)
^3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 9.82 (sec) , antiderivative size = 1056, normalized size of antiderivative = 3.97

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$$

$$= \left(\sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(68719476736\,a^{15}\,b^5\,z^4-1211105280\,a^8\,b^4\,c\,e\,z^2+110100480\,a^9\,b^3\,e\,g\,z^2-838860800\,a^8\,b^4\,d\right.\right.\right.$$

$$-\frac{-735\,a^2\,e\,g^2+16170\,a\,b\,c\,e\,g-11200\,a\,b\,d^2\,g+3375\,a\,b\,e^3-88935\,b^2\,c^2\,e+123200\,b^2\,c\,d^2}{2097152\,a^9}$$

$$\left.-\frac{x\,(4000\,b^2\,d^3-5775\,c\,e\,b^2\,d+525\,a\,e\,g\,b\,d)}{131072\,a^9}\right)\mathrm{root}\big(68719476736\,a^{15}\,b^5\,z^4$$

$$-1211105280\,a^8\,b^4\,c\,e\,z^2+110100480\,a^9\,b^3\,e\,g\,z^2-838860800\,a^8\,b^4\,d^2\,z^2$$

$$-88309760\,a^5\,b^3\,c\,d\,g\,z+485703680\,a^4\,b^4\,c^2\,d\,z+4014080\,a^6\,b^2\,d\,g^2\,z$$

$$+18432000\,a^5\,b^3\,d\,e^2\,z+672000\,a^2\,b^2\,d^2\,e\,g-485100\,a^2\,b^2\,c\,e^2\,g-7392000\,a\,b^3\,c\,d^2\,e$$

$$+12782924\,a\,b^3\,c^3\,g+105644\,a^3\,b\,c\,g^3-1743126\,a^2\,b^2\,c^2\,g^2+22050\,a^3\,b\,e^2\,g^2$$

$$\left.+2668050\,a\,b^3\,c^2\,e^2-50625\,a^2\,b^2\,e^4+2560000\,a\,b^3\,d^4-2401\,a^4\,g^4-35153041\,b^4\,c^4,z,k\big)\right)$$

$$+\frac{\frac{f}{12\,b}+\frac{11\,d\,x^2}{32\,a}+\frac{113\,e\,x^3}{384\,a}-\frac{3\,x^5\,(11\,b\,c-a\,g)}{64\,a^2}+\frac{7\,b\,x^9\,(11\,b\,c-a\,g)}{384\,a^3}+\frac{x\,(51\,b\,c+7\,a\,g)}{128\,a\,b}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}+\frac{15\,b^2\,e\,x^{11}}{128\,a^3}-\frac{5\,b\,d\,x^6}{12\,a^2}-\frac{21\,b\,e}{64\,a}}{a^3-3\,a^2\,b\,x^4+3\,a\,b^2\,x^8-b^3\,x^{12}}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x)
```

```
[Out] symsum(log(- root(68719476736*a^15*b^5*z^4 - 1211105280*a^8*b^4*c*e*z^2 + 1
10100480*a^9*b^3*e*g*z^2 - 838860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d
*g*z + 485703680*a^4*b^4*c^2*d*z + 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b
^3*d*e^2*z + 672000*a^2*b^2*d^2*e*g - 485100*a^2*b^2*c*e^2*g - 7392000*a*b^
3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g^3 - 1743126*a^2*b^2*c^2
*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2 - 50625*a^2*b^2*e^4 + 25
60000*a*b^3*d^4 - 2401*a^4*g^4 - 35153041*b^4*c^4, z, k)*(root(68719476736*
a^15*b^5*z^4 - 1211105280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 - 838
860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d*g*z + 485703680*a^4*b^4*c^2*d
*z + 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b^3*d*e^2*z + 672000*a^2*b^2*d^
2*e*g - 485100*a^2*b^2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 12782924*a*b^3*c^3
*g + 105644*a^3*b*c*g^3 - 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2
668050*a*b^3*c^2*e^2 - 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 - 2401*a^4*g^4
 - 35153041*b^4*c^4, z, k)*((20185088*a^7*b^3*c - 1835008*a^8*b^2*g)/(20971
52*a^9) - (5*b^3*d*x)/a^2) + (x*(1568*a^5*b*g^2 + 189728*a^3*b^3*c^2 + 7200
*a^4*b^2*e^2 - 34496*a^4*b^2*c*g))/(131072*a^9) - (75*b^2*d*e)/(256*a^5)) -
 (123200*b^2*c*d^2 - 88935*b^2*c^2*e - 735*a^2*e*g^2 + 3375*a*b*e^3 - 11200
*a*b*d^2*g + 16170*a*b*c*e*g)/(2097152*a^9) - (x*(4000*b^2*d^3 - 5775*b^2*c
*d*e + 525*a*b*d*e*g))/(131072*a^9))*root(68719476736*a^15*b^5*z^4 - 121110
5280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 - 838860800*a^8*b^4*d^2*z^
```

2 - 88309760*a^5*b^3*c*d*g*z + 485703680*a^4*b^4*c^2*d*z + 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b^3*d*e^2*z + 672000*a^2*b^2*d^2*e*g - 485100*a^2*b^2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g^3 - 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2 - 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 - 2401*a^4*g^4 - 35153041*b^4*c^4, z, k), k, 1, 4) + (f/(12*b) + (11*d*x^2)/(32*a) + (113*e*x^3)/(384*a) - (3*x^5*(11*b*c - a*g))/(64*a^2) + (7*b*x^9*(11*b*c - a*g))/(384*a^3) + (x*(51*b*c + 7*a*g))/(128*a*b) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) - (5*b*d*x^6)/(12*a^2) - (21*b*e*x^7)/(64*a^2))/(a^3 - b^3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8)

3.175 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4} dx$

Optimal result

Integrand size = 30, antiderivative size = 319

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx &= \frac{gx}{b}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} \\ &\quad-\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} \\ &\quad+\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} \\ &\quad-\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} \\ &\quad+\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} \\ &\quad+\frac{f\log\left(a+bx^4\right)}{4b}\end{aligned}$$

```
[Out] g*x/b+1/4*f*ln(b*x^4+a)/b+1/2*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
-1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(b*c-a*g-e*a^(1/2)*
b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x
^2*b^(1/2))*(b*c-a*g-e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+1/4*arctan(
-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(b*c-a*g+e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/4)*2
^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(b*c-a*g+e*a^(1/2)*b^(1/2))/
a^(3/4)/b^(5/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1899, 1262, 649, 211, 266, 1901, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx$$
$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} + \frac{f\log\left(a+bx^4\right)}{4b} + \frac{gx}{b}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]
```

```
[Out] (g*x)/b + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((b*c + S
qrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - Sqrt[a]*Sqrt[b]*e
- a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (f*Log[a + b*
x^4])/(4*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x(d+fx^2)}{a+bx^4}+\frac{c+ex^2+gx^4}{a+bx^4}\right)dx\\
&= \int \frac{x(d+fx^2)}{a+bx^4}\,dx+\int \frac{c+ex^2+gx^4}{a+bx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{a+bx^2}\,dx,x,x^2\right)+\int \left(\frac{g}{b}+\frac{bc-ag+bex^2}{b\,(a+bx^4)}\right)dx\\
&= \frac{gx}{b}+\frac{\int \frac{bc-ag+bex^2}{a+bx^4}\,dx}{b}+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)+\frac{1}{2}f\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx,x,x^2\right)\\
&= \frac{gx}{b}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}\\
&\quad+\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2\sqrt{a}b^{3/2}}+\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2\sqrt{a}b^{3/2}}
\end{aligned}$$

$$= \frac{gx}{b} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} + \frac{f\log(a+bx^4)}{4b} - \frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{5/4}}$$

$$- \frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{5/4}}$$

$$+ \frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}b^{3/2}}$$

$$+ \frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}b^{3/2}}$$

$$= \frac{gx}{b} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}$$

$$+ \frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{f\log(a+bx^4)}{4b}$$

$$+ \frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

$$- \frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

$$= \frac{gx}{b} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

$$+ \frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

$$- \frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}$$

$$+ \frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{f\log(a+bx^4)}{4b}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.97

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx$$
$$= \frac{8a^{3/4}\sqrt[4]{b}gx-2\left(\sqrt{2}bc+2\sqrt[4]{a}b^{3/4}d+\sqrt{2}\sqrt{a}\sqrt{b}e-\sqrt{2}ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}bc-2\sqrt[4]{a}b^{3/4}d+\cdots\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]
```

```
[Out] (8*a^(3/4)*b^(1/4)*g*x - 2*(Sqrt[2]*b*c + 2*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqr
t[a]*Sqrt[b]*e - Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(
Sqrt[2]*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e - Sqrt[2]*a*g
)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-(b*c) + Sqrt[a]*Sqrt[
b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2
]*(b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x +
 Sqrt[b]*x^2] + 2*a^(3/4)*b^(1/4)*f*Log[a + b*x^4])/(8*a^(3/4)*b^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.18

method	result
risch	$\frac{gx}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(\_R^3 bf+\_R^2 be+\_R bd-ag+bc\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$\frac{gx}{b}+\frac{\frac{(-ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{bd\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)\cdots}{x^2+\left(\frac{a}{b}\right)\cdots}\right.\right.}{}}{b}$

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] g*x/b+1/4/b^2*sum((_R^3*b*f+_R^2*b*e+_R*b*d-a*g+b*c)/_R^3*ln(x-_R),_R=RootO
f(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 43.66 (sec) , antiderivative size = 622377, normalized size of antiderivative = 1951.03

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.03

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \frac{gx}{b} + \frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f+b^2c-\sqrt{a}b^{\frac{3}{2}}e-abg\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f-b^2c+\sqrt{a}b^{\frac{3}{2}}e+abg\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{2(}{}}{}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
[Out] g*x/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f + b^2*c - sqrt(a)*b^(3/2)*e
 - a*b*g)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b
^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f - b^2*c + sqrt(a)*b^(3/2)*e +
a*b*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5
/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a
^(5/4)*b^(5/4)*g - 2*sqrt(a)*b^2*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b
^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt(2
)*a^(5/4)*b^(5/4)*g + 2*sqrt(a)*b^2*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sq
rt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b)
)*b^(5/4)))/b
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.06

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \frac{gx}{b}+\frac{f\log\left(|bx^4+a|\right)}{4\,b} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] g*x/b + 1/4*f*log(abs(b*x^4 + a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d
- (a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*arctan(1/2*s
qrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(sqrt
(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(
3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3)
 + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e
)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^
3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.95 (sec) , antiderivative size = 5042, normalized size of antiderivative = 15.81

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x)
```

```
[Out] symsum(log(b^2*c*d^2 - b^2*c^2*e - a^2*e*g^2 + a^2*f^2*g + b^2*d^3*x - a*b*
e^3 - a*b*c*f^2 - a*b*d^2*g - 16*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 -
```

64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*
d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16
*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d
*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d
*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*
d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g
 + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^
2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)^2*a*b^3*
c - 4*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^
2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*
z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b
^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a
^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g
+ 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f -
 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^
2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4
 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*b^3*c^2*x + b^2*c^2*f*x + a^2*f*g^2
*x + 16*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*
a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*
g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2
*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8
*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*
g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f
 - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*
a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f
^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)^2*a^2*b^2*g + 16*root(256*a^3*b^5
*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3
*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z
 + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^
3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2
*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 -
4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a
^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*
b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4
 + b^4*c^4, z, k)^2*a*b^3*d*x + 4*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3
- 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4
*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 1
6*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*
d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*
d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b
*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*
g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e
^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a*b^2*e
^2*x - 4*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64
*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f

*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a^2*b*g^2*x + 2*a*b*c*e*g + 2*a*b*d*e*f + 8*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a*b^2*c*f - 8*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a*b^2*d*e - 8*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a^2*b*f*g + a*b*d*f^2*x - a*b*e^2*f*x - 2*b^2*c*d*e*x + 8*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a*b^2*c*g*x - 8*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2

```
*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*
a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f -
 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2
+ 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2
*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*
b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a*b^2*d*f*x - 2*
a*b*c*f*g*x + 2*a*b*d*e*g*x)*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*
a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*
z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3
*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z -
 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2
*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*
g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6
*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 +
a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k), k, 1, 4) +
(g*x)/b
```

3.176 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 341

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx &= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab\,(a+bx^4)}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}\\&-\frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}\\&+\frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}\\&-\frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}\\&+\frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}\end{aligned}$$

```
[Out] 1/4*x*(b*f*x^3+b*e*x^2+b*d*x-a*g+b*c)/a/b/(b*x^4+a)+1/4*d*arctan(x^2*b^(1/2
)/a^(1/2))/a^(3/2)/b^(1/2)-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b
^(1/2))*(3*b*c+a*g-e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/32*ln(a^(1/
4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(3*b*c+a*g-e*a^(1/2)*b^(1/2))/a^(
7/4)/b^(5/4)*2^(1/2)+1/16*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(3*b*c+a*g+e
*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/16*arctan(1+b^(1/4)*x*2^(1/2)/a
^(1/4))*(3*b*c+a*g+e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1872, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx\\
&=-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{8\sqrt{2}a^{7/4}b^{5/4}}\\
&\quad+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}\\
&\quad-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}\\
&\quad+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}\\
&\quad+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{4ab\,(a+bx^4)}
\end{aligned}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x]
```

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + (d*ArcTan
[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*b*c + Sqrt[a]*Sqrt[b]*e
+ a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4))
 + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/
4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
```

```
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\int \frac{-3bc-ag-2bdx-bex^2}{a+bx^4}\,dx}{4ab}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\int \left(-\frac{2bdx}{a+bx^4} + \frac{-3bc-ag-bex^2}{a+bx^4}\right)dx}{4ab}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\int \frac{-3bc-ag-bex^2}{a+bx^4}\,dx}{4ab} + \frac{d\int \frac{x}{a+bx^4}\,dx}{2a}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab\,(a+bx^4)} + \frac{d\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{4a} + \frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{8a^{3/2}b^{3/2}} + \frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{8a^{3/2}b^{3/2}}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab\left(a+bx^4\right)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

$$-\frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}b^{5/4}}$$

$$-\frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}b^{5/4}}$$

$$+\frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{3/2}}$$

$$+\frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{3/2}}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab\left(a+bx^4\right)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

$$-\frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}$$

$$+\frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}$$

$$+\frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}$$

$$-\frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}$$

$$= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{4ab\,(a + bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3bc + \sqrt{a}\sqrt{b}e + ag\right)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{\left(3bc + \sqrt{a}\sqrt{b}e + ag\right)\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} - \frac{\left(3bc - \sqrt{a}\sqrt{b}e + ag\right)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{\left(3bc - \sqrt{a}\sqrt{b}e + ag\right)\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.94

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^4)^2}\,dx$$

$$= \frac{-\frac{8a^{3/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{a+bx^4} - 2\left(3\sqrt{2}bc + 4\sqrt[4]{a}b^{3/4}d + \sqrt{2}\sqrt{a}\sqrt{b}e + \sqrt{2}ag\right)\arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 2\left(\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4) - 2
*(3*Sqrt[2]*b*c + 4*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e + Sqrt[2]
*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b*c - 4*a^(1/4
)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e + Sqrt[2]*a*g)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b*c - Sqrt[a]*Sqr
t[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a
^(7/4)*b^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.31

method	result
risch	$\frac{\frac{e x^3}{4a}+\frac{d x^2}{4a}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(\_R^2 e+2\_R d+\frac{ag+3bc}{b}\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$\frac{\frac{e x^3}{4a}+\frac{d x^2}{4a}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b x^4+a}+\frac{\frac{(ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{bd\arctan\left(x^2\sqrt{\right.}}{\sqrt{ab}}}{4ba}$

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4/a*e*x^3+1/4*d/a*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/(b*x^4+a)+1/16/b/a*su
m((_R^2*e+2*_R*d+1/b*(a*g+3*b*c))/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 26.61 (sec) , antiderivative size = 352423, normalized size of antiderivative = 1033.50

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.03

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx = \frac{bex^3+bdx^2-af+(bc-ag)x}{4\,(ab^2x^4+a^2b)}$$

$$+\frac{\frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}a^{\frac{3}{4}}\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*e*x^3 + b*d*x^2 - a*f + (b*c - a*g)*x)/(a*b^2*x^4 + a^2*b) + 1/32*(s
qrt(2)*(3*b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g)*log(sqrt(b)*x^2 + sqrt(2)*
a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(3*b^(3/2)*c - sqr
t(a)*b*e + a*sqrt(b)*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(
a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a^(3/4)*b^
(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g - 4*sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(
2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*
sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a
^(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g + 4*sqrt(a)*b^(3/2)*d)*arctan(
1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.06

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx$$
$$= \frac{bex^3+bdx^2+bcx-agx-af}{4\,(bx^4+a)ab} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3} - \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*e*x^3 + b*d*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sqrt
(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*
g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)
*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - s
qrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*
b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1
/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^
(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 1383, normalized size of antiderivative = 4.06

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx$$
$$= \left(\sum_{k=1}^{4}\ln\left(-\frac{a^2\,e\,g^2+6\,a\,b\,c\,e\,g-4\,a\,b\,d^2\,g+a\,b\,e^3+9\,b^2\,c^2\,e-12\,b^2\,c\,d^2}{64\,a^3} - \frac{\mathrm{root}(65536\,a^7\,b^5\,z^4+1024\,a^5\,b^3\,e\,g\,z^2+3072\,a^4\,b^4\,c\,e\,z^2+2048\,a^4\,b^4\,d^2\,z^2-768\,a^3\,b^3\,c\,d\,g\,z-128\,a^4\,b^2}{} - \frac{b\,d\,x\,(-2\,b\,d^2+3\,b\,c\,e+a\,e\,g)}{16\,a^3}\right)\mathrm{root}\big(65536\,a^7\,b^5\,z^4+1024\,a^5\,b^3\,e\,g\,z^2+3072\,a^4\,b^4\,c\,e\,z^2+2048\,a^4\,b^4\,d^2\,z^2-768\,a^3\,b^3\,c\,d\,g\,z-128\,a^4\,b^2\,d\,g^2\,z+128\,a^3\,b^3\,d\,e^2\,z-1152\,a^2\,b^4\,c^2\,d\,z-16\,a^2\,b^2\,d^2\,e\,g+12\,a^2\,b^2\,c\,e^2\,g-48\,a\,b^3\,c\,d^2\,e+108\,a\,b^3\,c^3\,g+12\,a^3\,b\,c\,g^3+54\,a^2\,b^2\,c^2\,g^2+2\,a^3\,b\,e^2\,g^2+18\,a\,b^3\,c^2\,e^2+16\,a\,b^3\,d^4+81\,b^4\,c^4+a^2\,b^2\,e^4+a^4\,g^4,z,k\big)\right) + \frac{\frac{d\,x^2}{4\,a}-\frac{f}{4\,b}+\frac{e\,x^3}{4\,a}+\frac{x\,(b\,c-a\,g)}{4\,a\,b}}{b\,x^4+a}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x)
```

```
[Out] symsum(log(- (9*b^2*c^2*e - 12*b^2*c*d^2 + a^2*e*g^2 + a*b*e^3 - 4*a*b*d^2*
g + 6*a*b*c*e*g)/(64*a^3) - (root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2
+ 3072*a^4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z - 128*a
^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2*d*z - 16*a^2*b^2*d^
2*e*g + 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*
c*g^3 + 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*
d^4 + 81*b^4*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k)*b*(9*b^2*c^2*x + a^2*g^2*x
+ 16*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 + 3072*a^4*b^4*c*e*z^2 +
 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z - 128*a^4*b^2*d*g^2*z + 128*a^3
*b^3*d*e^2*z - 1152*a^2*b^4*c^2*d*z - 16*a^2*b^2*d^2*e*g + 12*a^2*b^2*c*e^2
*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 + 54*a^2*b^2*c^2*g
^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 + 81*b^4*c^4 + a^2*b
^2*e^4 + a^4*g^4, z, k)*a^3*b*g - a*b*e^2*x + 48*root(65536*a^7*b^5*z^4 + 1
024*a^5*b^3*e*g*z^2 + 3072*a^4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3
*b^3*c*d*g*z - 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2
*d*z - 16*a^2*b^2*d^2*e*g + 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b
^3*c^3*g + 12*a^3*b*c*g^3 + 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3
*c^2*e^2 + 16*a*b^3*d^4 + 81*b^4*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k)*a^2*b^2
*c + 4*a*b*d*e - 32*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 + 3072*a^
4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z - 128*a^4*b^2*d*
g^2*z + 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2*d*z - 16*a^2*b^2*d^2*e*g + 1
2*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 + 5
```

4*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 + 81*
b^4*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k)*a^2*b^2*d*x + 6*a*b*c*g*x))/(4*a^2)
- (b*d*x*(3*b*c*e - 2*b*d^2 + a*e*g))/(16*a^3))*root(65536*a^7*b^5*z^4 + 10
24*a^5*b^3*e*g*z^2 + 3072*a^4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3*
b^3*c*d*g*z - 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2*
d*z - 16*a^2*b^2*d^2*e*g + 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^
3*c^3*g + 12*a^3*b*c*g^3 + 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*
c^2*e^2 + 16*a*b^3*d^4 + 81*b^4*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k), k, 1, 4
) + ((d*x^2)/(4*a) - f/(4*b) + (e*x^3)/(4*a) + (x*(b*c - a*g))/(4*a*b))/(a
+ b*x^4)

3.177 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 394

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3} dx \\
&= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab(a+bx^4)^2} - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b(a+bx^4)} \\
&+ \frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} \\
&+ \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} \\
&- \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}} \\
&+ \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}}
\end{aligned}$$

```
[Out] 1/8*x*(b*f*x^3+b*e*x^2+b*d*x-a*g+b*c)/a/b/(b*x^4+a)^2+1/32*(-4*a*f+x*(5*b*e
*x^2+6*b*d*x+a*g+7*b*c))/a^2/b/(b*x^4+a)+3/16*d*arctan(x^2*b^(1/2)/a^(1/2))
/a^(5/2)/b^(1/2)-1/256*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(
21*b*c+3*a*g-5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+1/256*ln(a^(1/4)
*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(21*b*c+3*a*g-5*e*a^(1/2)*b^(1/2))/
a^(11/4)/b^(5/4)*2^(1/2)+1/128*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(21*b*c
+3*a*g+5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+1/128*arctan(1+b^(1/4)
*x*2^(1/2)/a^(1/4))*(21*b*c+3*a*g+5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)*2^(
1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1872, 1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx\\
&=-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}\\
&\quad+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}\\
&\quad-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}\\
&\quad+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}\\
&\quad-\frac{4af-x(ag+7bc+6bdx+5bex^2)}{32a^2b\,(a+bx^4)}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2}
\end{aligned}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]
```

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) - (4*a*f
- x*(7*b*c + a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a + b*x^4)) + (3*d*ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - ((21*b*c + 5*Sqrt[a]*Sqrt
[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4
)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) - ((21*b*c - 5*Sqrt[a]*Sqr
t[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(12
8*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^
(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
```

```
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{\int \frac{-7bc-ag-6bdx-5bex^2-4bfx^3}{(a+bx^4)^2}\,dx}{8ab} \\
&= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} \\
&\quad - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b\,(a+bx^4)} + \frac{\int \frac{-3(-7bc-ag)+12bdx+5bex^2}{a+bx^4}\,dx}{32a^2b} \\
&= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b\,(a+bx^4)} \\
&\quad + \frac{\int \left(\frac{12bdx}{a+bx^4}+\frac{-3(-7bc-ag)+5bex^2}{a+bx^4}\right)dx}{32a^2b} \\
&= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b\,(a+bx^4)} \\
&\quad + \frac{\int \frac{-3(-7bc-ag)+5bex^2}{a+bx^4}\,dx}{32a^2b} + \frac{(3d)\int \frac{x}{a+bx^4}\,dx}{8a^2}
\end{aligned}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b\left(a+bx^4\right)} + \frac{(3d)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{16a^2} + \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{64a^{5/2}b^{3/2}} + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{64a^{5/2}b^{3/2}}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{128\sqrt{2}a^{11/4}b^{5/4}} - \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{128\sqrt{2}a^{11/4}b^{5/4}} + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{128a^{5/2}b^{3/2}} + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{128a^{5/2}b^{3/2}}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}} + \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}} + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} - \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} - \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}} + \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.93

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^4\right)^3}\,dx$$

$$= \frac{\frac{8a^{3/4}\sqrt[4]{b}x(7bc+ag+bx(6d+5ex))}{a+bx^4} - \frac{32a^{7/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{\left(a+bx^4\right)^2} - 2\left(21\sqrt{2}bc+24\sqrt[4]{a}b^{3/4}d+5\sqrt{2}\sqrt{a}\sqrt{b}e+3\sqrt{2}\cdots\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*b^(1/4)*x*(7*b*c + a*g + b*x*(6*d + 5*e*x)))/(a + b*x^4) - (32*
a^(7/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4)^2 - 2*(2
1*Sqrt[2]*b*c + 24*a^(1/4)*b^(3/4)*d + 5*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 3*Sqrt
[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(21*Sqrt[2]*b*c - 24*a
^(1/4)*b^(3/4)*d + 5*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 3*Sqrt[2]*a*g)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a
*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(21*b*
c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/(256*a^(11/4)*b^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.37

method	result
risch	$\frac{\frac{5be\,x^7}{32a^2}+\frac{3bd\,x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{\left(5\_R^2e+12\_Rd+\frac{3ag+21bc}{b}\right)\ln(x-\_R)}{\_R^3}}{128a^2b}$
default	$\frac{\frac{5be\,x^7}{32a^2}+\frac{3bd\,x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\frac{(3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)\right.}{8a}}{}$

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)

[Out] (5/32*b*e/a^2*x^7+3/16*b*d/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x^5+9/32/a*e*x^3+5/16*d/a*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8*f/b)/(b*x^4+a)^2+1/128/a^2/b*sum((5*_R^2*e+12*_R*d+3/b*(a*g+7*b*c))/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 58.34 (sec) , antiderivative size = 358509, normalized size of antiderivative = 909.92

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx = \text{Timed out}$$

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.05

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx = \frac{5\,b^2ex^7+6\,b^2dx^6+9\,abex^3+(7\,b^2c+abg)x^5+10\,abdx^2-4\,a^2f+(11\,abc-3\,a^2g)x}{32\,(a^2b^3x^8+2\,a^3b^2x^4+a^4b)} + \frac{\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be+3\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be+3\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(21\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c\right.}{}}{}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 9*a*b*e*x^3 + (7*b^2*c + a*b*g)*x^5 + 10*
a*b*d*x^2 - 4*a^2*f + (11*a*b*c - 3*a^2*g)*x)/(a^2*b^3*x^8 + 2*a^3*b^2*x^4
+ a^4*b) + 1/256*(sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g)*lo
g(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sq
rt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g)*log(sqrt(b)*x^2 - sqrt
(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*
b^(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g - 24*
sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4
))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*s
qrt(2)*a^(1/4)*b^(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*
b^(3/4)*g + 24*sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)
*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(
3/4)))/(a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$$
$$= \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+21\,(ab^3)^{\frac{1}{4}}\,b^2c+3\,(ab^3)^{\frac{1}{4}}\,abg+5\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$
$$+ \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+21\,(ab^3)^{\frac{1}{4}}\,b^2c+3\,(ab^3)^{\frac{1}{4}}\,abg+5\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$
$$+ \frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^2c+3\,(ab^3)^{\frac{1}{4}}\,abg-5\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$
$$- \frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^2c+3\,(ab^3)^{\frac{1}{4}}\,abg-5\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$
$$+ \frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5+abgx^5+9\,abex^3+10\,abdx^2+11\,abcx-3\,a^2gx-4\,a^2f}{32\,(bx^4+a)^2a^2b}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b
^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d
 + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arct
an(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*s
qrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)
*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21
*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 -
 sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*e*x^7 + 6*b^2*d
*x^6 + 7*b^2*c*x^5 + a*b*g*x^5 + 9*a*b*e*x^3 + 10*a*b*d*x^2 + 11*a*b*c*x -
3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 1001, normalized size of antiderivative = 2.54

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$$
$$= \frac{\frac{5\,d\,x^2}{16\,a}-\frac{f}{8\,b}+\frac{9\,e\,x^3}{32\,a}+\frac{x^5\,(7\,b\,c+a\,g)}{32\,a^2}+\frac{x\,(11\,b\,c-3\,a\,g)}{32\,a\,b}+\frac{3\,b\,d\,x^6}{16\,a^2}+\frac{5\,b\,e\,x^7}{32\,a^2}}{a^2+2\,a\,b\,x^4+b^2\,x^8}$$
$$+\left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(268435456\,a^{11}\,b^5\,z^4+983040\,a^7\,b^3\,e\,g\,z^2+6881280\,a^6\,b^4\,c\,e\,z^2+4718592\,a^6\,b^4\,d^2\,z^2-77414\right.\right.\right.$$
$$-\frac{45\,a^2\,e\,g^2+630\,a\,b\,c\,e\,g-432\,a\,b\,d^2\,g+125\,a\,b\,e^3+2205\,b^2\,c^2\,e-3024\,b^2\,c\,d^2}{32768\,a^6}$$
$$-\frac{x\,(-216\,b^2\,d^3+315\,c\,e\,b^2\,d+45\,a\,e\,g\,b\,d)}{4096\,a^6}\Bigg)\,\mathrm{root}\big(268435456\,a^{11}\,b^5\,z^4+983040\,a^7\,b^3\,e\,g\,z^2$$
$$+6881280\,a^6\,b^4\,c\,e\,z^2+4718592\,a^6\,b^4\,d^2\,z^2-774144\,a^4\,b^3\,c\,d\,g\,z-55296\,a^5\,b^2\,d\,g^2\,z$$
$$+153600\,a^4\,b^3\,d\,e^2\,z-2709504\,a^3\,b^4\,c^2\,d\,z-8640\,a^2\,b^2\,d^2\,e\,g+6300\,a^2\,b^2\,c\,e^2\,g$$
$$-60480\,a\,b^3\,c\,d^2\,e+111132\,a\,b^3\,c^3\,g+2268\,a^3\,b\,c\,g^3+23814\,a^2\,b^2\,c^2\,g^2+450\,a^3\,b\,e^2\,g^2$$
$$+22050\,a\,b^3\,c^2\,e^2+625\,a^2\,b^2\,e^4+20736\,a\,b^3\,d^4+81\,a^4\,g^4+194481\,b^4\,c^4,z,k\big)\Bigg)$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x)
```

```
[Out] ((5*d*x^2)/(16*a) - f/(8*b) + (9*e*x^3)/(32*a) + (x^5*(7*b*c + a*g))/(32*a^
2) + (x*(11*b*c - 3*a*g))/(32*a*b) + (3*b*d*x^6)/(16*a^2) + (5*b*e*x^7)/(32
*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4) + symsum(log(- root(268435456*a^11*b^5*z
^4 + 983040*a^7*b^3*e*g*z^2 + 6881280*a^6*b^4*c*e*z^2 + 4718592*a^6*b^4*d^2
*z^2 - 774144*a^4*b^3*c*d*g*z - 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d*e^
2*z - 2709504*a^3*b^4*c^2*d*z - 8640*a^2*b^2*d^2*e*g + 6300*a^2*b^2*c*e^2*g
 - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 + 23814*a^2*
b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 + 625*a^2*b^2*e^4 + 2
0736*a*b^3*d^4 + 81*a^4*g^4 + 194481*b^4*c^4, z, k)*(root(268435456*a^11*b^
5*z^4 + 983040*a^7*b^3*e*g*z^2 + 6881280*a^6*b^4*c*e*z^2 + 4718592*a^6*b^4*
d^2*z^2 - 774144*a^4*b^3*c*d*g*z - 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d
*e^2*z - 2709504*a^3*b^4*c^2*d*z - 8640*a^2*b^2*d^2*e*g + 6300*a^2*b^2*c*e^
2*g - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 + 23814*a
^2*b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 + 625*a^2*b^2*e^4
+ 20736*a*b^3*d^4 + 81*a^4*g^4 + 194481*b^4*c^4, z, k)*((344064*a^5*b^3*c +
 49152*a^6*b^2*g)/(32768*a^6) - (6*b^3*d*x)/a) + (x*(144*a^4*b*g^2 + 7056*a
^2*b^3*c^2 - 400*a^3*b^2*e^2 + 2016*a^3*b^2*c*g))/(4096*a^6) + (15*b^2*d*e)
/(32*a^3)) - (2205*b^2*c^2*e - 3024*b^2*c*d^2 + 45*a^2*e*g^2 + 125*a*b*e^3
- 432*a*b*d^2*g + 630*a*b*c*e*g)/(32768*a^6) - (x*(315*b^2*c*d*e - 216*b^2*
d^3 + 45*a*b*d*e*g))/(4096*a^6))*root(268435456*a^11*b^5*z^4 + 983040*a^7*b
^3*e*g*z^2 + 6881280*a^6*b^4*c*e*z^2 + 4718592*a^6*b^4*d^2*z^2 - 774144*a^4
```

```
*b^3*c*d*g*z - 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d*e^2*z - 2709504*a^3
*b^4*c^2*d*z - 8640*a^2*b^2*d^2*e*g + 6300*a^2*b^2*c*e^2*g - 60480*a*b^3*c*
d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 + 23814*a^2*b^2*c^2*g^2 + 450
*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 + 625*a^2*b^2*e^4 + 20736*a*b^3*d^4 +
81*a^4*g^4 + 194481*b^4*c^4, z, k), k, 1, 4)
```

3.178 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 437

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx &= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\,(a+bx^4)^3}+\frac{x(7(11bc+ag)+60bdx+45bex^2)}{384a^3b\,(a+bx^4)}\\
&-\frac{8af-x(11bc+ag+10bdx+9bex^2)}{96a^2b\,(a+bx^4)^2}+\frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}\\
&-\frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}}\\
&+\frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}}\\
&-\frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}}\\
&+\frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}}\end{aligned}$$

```
[Out] 1/12*x*(b*f*x^3+b*e*x^2+b*d*x-a*g+b*c)/a/b/(b*x^4+a)^3+1/384*x*(45*b*e*x^2+
60*b*d*x+7*a*g+77*b*c)/a^3/b/(b*x^4+a)+1/96*(-8*a*f+x*(9*b*e*x^2+10*b*d*x+a
*g+11*b*c))/a^2/b/(b*x^4+a)^2+5/32*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^
(1/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(77*b*c+7*a
*g-15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)+1/1024*ln(a^(1/4)*b^(1/4)
*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(77*b*c+7*a*g-15*e*a^(1/2)*b^(1/2))/a^(15/4
```

```
)/b^(5/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(77*b*c+7*a*g+
15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)+1/512*arctan(1+b^(1/4)*x*2^(
1/2)/a^(1/4))*(77*b*c+7*a*g+15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1872, 1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx \\
&= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}} \\
&+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}}+\frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} \\
&-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}} \\
&+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}} \\
&+\frac{x(7(ag+11bc)+60bdx+45bex^2)}{384a^3b\,(a+bx^4)} \\
&-\frac{8af-x(ag+11bc+10bdx+9bex^2)}{96a^2b\,(a+bx^4)^2}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{12ab\,(a+bx^4)^3}
\end{aligned}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x]
```

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3) + (x*(7*
(11*b*c + a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a + b*x^4)) - (8*a*f -
 x*(11*b*c + a*g + 10*b*d*x + 9*b*e*x^2))/(96*a^2*b*(a + b*x^4)^2) + (5*d*A
rcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*b*c + 15*Sqrt[a]*
Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^
(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqr
t[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a
*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^
(15/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\,(a+bx^4)^3} - \frac{\int \frac{-11bc-ag-10bdx-9bex^2-8bfx^3}{(a+bx^4)^3}\,dx}{12ab}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\,(a+bx^4)^3} - \frac{8af-x(11bc+ag+10bdx+9bex^2)}{96a^2b\,(a+bx^4)^2} + \frac{\int \frac{-7(-11bc-ag)+60bdx+45bex^2}{(a+bx^4)^2}\,dx}{96a^2b}$$

$$= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{12ab\left(a + bx^4\right)^3} + \frac{x(7(11bc + ag) + 60bdx + 45bex^2)}{384a^3b\left(a + bx^4\right)} - \frac{8af - x(11bc + ag + 10bdx + 9bex^2)}{96a^2b\left(a + bx^4\right)^2} - \frac{\int \frac{-21(11bc+ag)-120bdx-45bex^2}{a+bx^4}\,dx}{384a^3b}$$

$$= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{12ab\left(a + bx^4\right)^3} + \frac{x(7(11bc + ag) + 60bdx + 45bex^2)}{384a^3b\left(a + bx^4\right)} - \frac{8af - x(11bc + ag + 10bdx + 9bex^2)}{96a^2b\left(a + bx^4\right)^2} - \frac{\int \left(-\frac{120bdx}{a+bx^4} + \frac{-21(11bc+ag)-45bex^2}{a+bx^4}\right)dx}{384a^3b}$$

$$= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{12ab\left(a + bx^4\right)^3} + \frac{x(7(11bc + ag) + 60bdx + 45bex^2)}{384a^3b\left(a + bx^4\right)} - \frac{8af - x(11bc + ag + 10bdx + 9bex^2)}{96a^2b\left(a + bx^4\right)^2} - \frac{\int \frac{-21(11bc+ag)-45bex^2}{a+bx^4}\,dx}{384a^3b} + \frac{(5d)\int \frac{x}{a+bx^4}\,dx}{16a^3}$$

$$= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{12ab\left(a + bx^4\right)^3} + \frac{x(7(11bc + ag) + 60bdx + 45bex^2)}{384a^3b\left(a + bx^4\right)} - \frac{8af - x(11bc + ag + 10bdx + 9bex^2)}{96a^2b\left(a + bx^4\right)^2} + \frac{(5d)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{32a^3} + \frac{\left(77bc + 15\sqrt{a}\sqrt{b}e + 7ag\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{256a^{7/2}b^{3/2}} - \frac{\left(15e - \frac{7(11bc+ag)}{\sqrt{a}\sqrt{b}}\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{256a^3b}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3}+\frac{x(7(11bc+ag)+60bdx+45bex^2)}{384a^3b\left(a+bx^4\right)} - \frac{8af-x(11bc+ag+10bdx+9bex^2)}{96a^2b\left(a+bx^4\right)^2}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{512\sqrt{2}a^{15/4}b^{5/4}} - \frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{512\sqrt{2}a^{15/4}b^{5/4}} + \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{512a^{7/2}b^{3/2}} + \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{512a^{7/2}b^{3/2}}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3}+\frac{x(7(11bc+ag)+60bdx+45bex^2)}{384a^3b\left(a+bx^4\right)} - \frac{8af-x(11bc+ag+10bdx+9bex^2)}{96a^2b\left(a+bx^4\right)^2}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}} + \frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}} + \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}} - \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+60bdx+45bex^2)}{384a^3b\,(a+bx^4)}$$
$$- \frac{8af-x(11bc+ag+10bdx+9bex^2)}{96a^2b\,(a+bx^4)^2} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$
$$- \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}}$$
$$+ \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}}$$
$$- \frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}}$$
$$+ \frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.94

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$$
$$= \frac{\frac{8a^{3/4}\sqrt[4]{b}x(77bc+7ag+15bx(4d+3ex))}{a+bx^4} + \frac{32a^{7/4}\sqrt[4]{b}x(11bc+ag+bx(10d+9ex))}{(a+bx^4)^2} - \frac{256a^{11/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{(a+bx^4)^3} - 6\Big(77\sqrt{2}bc}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x]
```

```
[Out] ((8*a^(3/4)*b^(1/4)*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a + b*x^4)
+ (32*a^(7/4)*b^(1/4)*x*(11*b*c + a*g + b*x*(10*d + 9*e*x)))/(a + b*x^4)^2
- (256*a^(11/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4)^
3 - 6*(77*Sqrt[2]*b*c + 80*a^(1/4)*b^(3/4)*d + 15*Sqrt[2]*Sqrt[a]*Sqrt[b]*e
 + 7*Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b
*c - 80*a^(1/4)*b^(3/4)*d + 15*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 7*Sqrt[2]*a*g)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(77*b*c - 15*Sqrt[a]*Sqr
t[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*
Sqrt[2]*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.51 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.42

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}-\frac{(7ag-51bc)x}{128ab}-\frac{f}{12b}}{(b\,x^4+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}}{}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}-\frac{(7ag-51bc)x}{128ab}-\frac{f}{12b}}{(b\,x^4+a)^3}+\frac{\frac{(7ag+77bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,(\ln}{}}{}$

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)

[Out] (15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+7/384*(a*g+11*b*c)/a^3*b*x^9+21/64*b*e/a^2*x^7+5/12*b*d/a^2*x^6+3/64/a^2*(a*g+11*b*c)*x^5+113/384/a*e*x^3+11/32*d/a*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12*f/b)/(b*x^4+a)^3+1/512/a^3/b*sum((15*_R^2*e+40*_R*d+7*(a*g+11*b*c)/b)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 82.84 (sec) , antiderivative size = 358702, normalized size of antiderivative = 820.83

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx = \text{Timed out}$$

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.08

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$$
$$= \frac{45\,b^3ex^{11}+60\,b^3dx^{10}+126\,ab^2ex^7+160\,ab^2dx^6+7\,(11\,b^3c+ab^2g)x^9+113\,a^2bex^3+132\,a^2bdx^2+18\,(11\,\ldots}{384\,(a^3b^4x^{12}+3\,a^4b^3x^8+3\,a^5b^2x^4+a^6b)}$$
$$+\frac{\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be+7\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be+7\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b\ldots\right.}{\ldots}}{\ldots}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 126*a*b^2*e*x^7 + 160*a*b^2*d*x^6 +
7*(11*b^3*c + a*b^2*g)*x^9 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 18*(11*a*b
^2*c + a^2*b*g)*x^5 - 32*a^3*f + 3*(51*a^2*b*c - 7*a^3*g)*x)/(a^3*b^4*x^12
+ 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*b) + 1/1024*(sqrt(2)*(77*b^(3/2)*c -
15*sqrt(a)*b*e + 7*a*sqrt(b)*g)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x
 + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*e + 7*
a*sqrt(b)*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4
)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*sqrt(2)*a^(3/4)*b^(5/4)*e
 + 7*sqrt(2)*a^(5/4)*b^(3/4)*g - 80*sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(2)*(
2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt
(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*sqrt(2)*a
^(3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 80*sqrt(a)*b^(3/2)*d)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^3*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.05

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$$
$$= \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$
$$+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$
$$+\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$
$$-\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$
$$+\frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9+7\,ab^2gx^9+126\,ab^2ex^7+160\,ab^2dx^6+198\,ab^2cx^5+18\,a^2bgx^5+113}{384\,(bx^4+a)^3a^3b}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b
^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/
b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*
d + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/102
4*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4
)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2
)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log
(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*e*x^11
+ 60*b^3*d*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 126*a*b^2*e*x^7 + 160*a*b^
2*d*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*e*x^3 + 132*a^2*b*d*
x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 1053, normalized size of antiderivative = 2.41

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$$

$$= \left(\sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(68719476736\,a^{15}\,b^5\,z^4+1211105280\,a^8\,b^4\,c\,e\,z^2+110100480\,a^9\,b^3\,e\,g\,z^2+838860800\,a^8\,b^4\,d^2\right.\right.\right.$$

$$-\frac{735\,a^2\,e\,g^2+16170\,a\,b\,c\,e\,g-11200\,a\,b\,d^2\,g+3375\,a\,b\,e^3+88935\,b^2\,c^2\,e-123200\,b^2\,c\,d^2}{2097152\,a^9}$$

$$-\frac{x\,(-4000\,b^2\,d^3+5775\,c\,e\,b^2\,d+525\,a\,e\,g\,b\,d)}{131072\,a^9}\Bigg)\,\mathrm{root}\big(68719476736\,a^{15}\,b^5\,z^4$$

$$+1211105280\,a^8\,b^4\,c\,e\,z^2+110100480\,a^9\,b^3\,e\,g\,z^2+838860800\,a^8\,b^4\,d^2\,z^2$$

$$-88309760\,a^5\,b^3\,c\,d\,g\,z-485703680\,a^4\,b^4\,c^2\,d\,z-4014080\,a^6\,b^2\,d\,g^2\,z$$

$$+18432000\,a^5\,b^3\,d\,e^2\,z-672000\,a^2\,b^2\,d^2\,e\,g+485100\,a^2\,b^2\,c\,e^2\,g-7392000\,a\,b^3\,c\,d^2\,e$$

$$+12782924\,a\,b^3\,c^3\,g+105644\,a^3\,b\,c\,g^3+1743126\,a^2\,b^2\,c^2\,g^2+22050\,a^3\,b\,e^2\,g^2$$

$$+2668050\,a\,b^3\,c^2\,e^2+50625\,a^2\,b^2\,e^4+2560000\,a\,b^3\,d^4+2401\,a^4\,g^4+35153041\,b^4\,c^4,z,k\big)\Bigg)$$

$$+\frac{\frac{11\,d\,x^2}{32\,a}-\frac{f}{12\,b}+\frac{113\,e\,x^3}{384\,a}+\frac{3\,x^5\,(11\,b\,c+a\,g)}{64\,a^2}+\frac{7\,b\,x^9\,(11\,b\,c+a\,g)}{384\,a^3}+\frac{x\,(51\,b\,c-7\,a\,g)}{128\,a\,b}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}+\frac{15\,b^2\,e\,x^{11}}{128\,a^3}+\frac{5\,b\,d\,x^6}{12\,a^2}+\frac{21\,b\,e\,x^7}{64\,a^2}}{a^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^8+b^3\,x^{12}}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x)
```

```
[Out] symsum(log(- root(68719476736*a^15*b^5*z^4 + 1211105280*a^8*b^4*c*e*z^2 + 1
10100480*a^9*b^3*e*g*z^2 + 838860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d
*g*z - 485703680*a^4*b^4*c^2*d*z - 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b
^3*d*e^2*z - 672000*a^2*b^2*d^2*e*g + 485100*a^2*b^2*c*e^2*g - 7392000*a*b^
3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g^3 + 1743126*a^2*b^2*c^2
*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2 + 50625*a^2*b^2*e^4 + 25
60000*a*b^3*d^4 + 2401*a^4*g^4 + 35153041*b^4*c^4, z, k)*(root(68719476736*
a^15*b^5*z^4 + 1211105280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 + 838
860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d*g*z - 485703680*a^4*b^4*c^2*d
*z - 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b^3*d*e^2*z - 672000*a^2*b^2*d^
2*e*g + 485100*a^2*b^2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 12782924*a*b^3*c^3
*g + 105644*a^3*b*c*g^3 + 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2
668050*a*b^3*c^2*e^2 + 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 + 2401*a^4*g^4
 + 35153041*b^4*c^4, z, k)*((20185088*a^7*b^3*c + 1835008*a^8*b^2*g)/(20971
52*a^9) - (5*b^3*d*x)/a^2) + (x*(1568*a^5*b*g^2 + 189728*a^3*b^3*c^2 - 7200
*a^4*b^2*e^2 + 34496*a^4*b^2*c*g))/(131072*a^9) + (75*b^2*d*e)/(256*a^5)) -
 (88935*b^2*c^2*e - 123200*b^2*c*d^2 + 735*a^2*e*g^2 + 3375*a*b*e^3 - 11200
*a*b*d^2*g + 16170*a*b*c*e*g)/(2097152*a^9) - (x*(5775*b^2*c*d*e - 4000*b^2
*d^3 + 525*a*b*d*e*g))/(131072*a^9))*root(68719476736*a^15*b^5*z^4 + 121110
5280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 + 838860800*a^8*b^4*d^2*z^
```

```
2 - 88309760*a^5*b^3*c*d*g*z - 485703680*a^4*b^4*c^2*d*z - 4014080*a^6*b^2*
d*g^2*z + 18432000*a^5*b^3*d*e^2*z - 672000*a^2*b^2*d^2*e*g + 485100*a^2*b^
2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g
^3 + 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2
+ 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 + 2401*a^4*g^4 + 35153041*b^4*c^4,
z, k), k, 1, 4) + ((11*d*x^2)/(32*a) - f/(12*b) + (113*e*x^3)/(384*a) + (3*
x^5*(11*b*c + a*g))/(64*a^2) + (7*b*x^9*(11*b*c + a*g))/(384*a^3) + (x*(51*
b*c - 7*a*g))/(128*a*b) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^
3) + (5*b*d*x^6)/(12*a^2) + (21*b*e*x^7)/(64*a^2))/(a^3 + b^3*x^12 + 3*a^2*
b*x^4 + 3*a*b^2*x^8)
```

3.179 $\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{1}{4}(1-x)^4$$

```
[Out] -1/4*(1-x)^4
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1600, 32}

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{1}{4}(1-x)^4$$

```
[In] Int[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]
```

```
[Out] -1/4*(1 - x)^4
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\text{integral} = \int (1-x)^3\,dx$$
$$= -\frac{1}{4}(1-x)^4$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{1}{4}(-1+x)^4$$

```
[In] Integrate[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]
[Out] -1/4*(-1 + x)^4
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{(-1+x)^4}{4}$	8
parallelrisch	$-\frac{1}{4}x^4+x^3-\frac{3}{2}x^2+x$	16
gosper	$-\frac{x(x^3-4x^2+6x-4)}{4}$	17
risch	$-\frac{1}{4}x^4+x^3-\frac{3}{2}x^2+x-\frac{1}{4}$	17
norman	$\frac{-2x^5-2x^3-x^4-\frac{7}{4}x^2-\frac{1}{2}x-\frac{1}{4}x^8+\frac{1}{2}x^9-\frac{1}{4}x^{10}-\frac{3}{4}}{(x^3+x^2+x+1)^2}$	53

```
[In] int((-x^4+1)^3/(x^3+x^2+x+1)^3,x,method=_RETURNVERBOSE)
[Out] -1/4*(-1+x)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{1}{4}\,x^4 + x^3 - \frac{3}{2}\,x^2 + x$$

```
[In] integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="fricas")
[Out] -1/4*x^4 + x^3 - 3/2*x^2 + x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{x^4}{4} + x^3 - \frac{3x^2}{2} + x$$

```
[In] integrate((-x**4+1)**3/(x**3+x**2+x+1)**3,x)
[Out] -x**4/4 + x**3 - 3*x**2/2 + x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{1}{4}\,x^4 + x^3 - \frac{3}{2}\,x^2 + x$$

```
[In] integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="maxima")
[Out] -1/4*x^4 + x^3 - 3/2*x^2 + x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{1}{4}\,x^4+x^3-\frac{3}{2}\,x^2+x$$

```
[In] integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="giac")
[Out] -1/4*x^4 + x^3 - 3/2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{x^4}{4}+x^3-\frac{3\,x^2}{2}+x$$

```
[In] int(-(x^4 - 1)^3/(x + x^2 + x^3 + 1)^3,x)
[Out] x - (3*x^2)/2 + x^3 - x^4/4
```

3.180 $\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} \, dx = -\frac{1}{3}(1-x)^3$$

```
[Out] -1/3*(1-x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1600, 32}

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} \, dx = -\frac{1}{3}(1-x)^3$$

```
[In] Int[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]
```

```
[Out] -1/3*(1 - x)^3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\text{integral} = \int (1-x)^2 \, dx$$
$$= -\frac{1}{3}(1-x)^3$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} \, dx = x - x^2 + \frac{x^3}{3}$$

```
[In] Integrate[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]
[Out] x - x^2 + x^3/3
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{(-1+x)^3}{3}$	8
gosper	$\frac{x(x^2-3x+3)}{3}$	12
parallelrisch	$\frac{1}{3}x^3 - x^2 + x$	13
risch	$\frac{1}{3}x^3 - x^2 + x - \frac{1}{3}$	14
norman	$\frac{-\frac{1}{3}x^2+\frac{2}{3}x+\frac{1}{3}x^4-\frac{2}{3}x^5+\frac{1}{3}x^6-\frac{1}{3}}{x^3+x^2+x+1}$	38

```
[In] int((-x^4+1)^2/(x^3+x^2+x+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*(-1+x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx = \frac{1}{3}\,x^3 - x^2 + x$$

```
[In] integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="fricas")
[Out] 1/3*x^3 - x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx = \frac{x^3}{3} - x^2 + x$$

```
[In] integrate((-x**4+1)**2/(x**3+x**2+x+1)**2,x)
[Out] x**3/3 - x**2 + x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx = \frac{1}{3}\,x^3 - x^2 + x$$

```
[In] integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="maxima")
[Out] 1/3*x^3 - x^2 + x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx = \frac{1}{3}\,x^3 - x^2 + x$$

```
[In] integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="giac")
[Out] 1/3*x^3 - x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx=\frac{x\,(x^2-3\,x+3)}{3}$$

```
[In] int((x^4 - 1)^2/(x + x^2 + x^3 + 1)^2,x)
[Out] (x*(x^2 - 3*x + 3))/3
```

3.181 $\int \frac{1-x^4}{1+x+x^2+x^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 9

$$\int \frac{1-x^4}{1+x+x^2+x^3} \, dx = x - \frac{x^2}{2}$$

```
[Out] x-1/2*x^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1600}

$$\int \frac{1-x^4}{1+x+x^2+x^3} \, dx = x - \frac{x^2}{2}$$

```
[In] Int[(1 - x^4)/(1 + x + x^2 + x^3),x]
```

```
[Out] x - x^2/2
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int (1-x) \, dx \\ &= x - \frac{x^2}{2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-x^4}{1+x+x^2+x^3}\,dx = x-\frac{x^2}{2}$$

```
[In] Integrate[(1 - x^4)/(1 + x + x^2 + x^3),x]
[Out] x - x^2/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

method	result	size
gosper	$-\frac{x(-2+x)}{2}$	7
default	$x-\frac{1}{2}x^2$	8
norman	$x-\frac{1}{2}x^2$	8
risch	$x-\frac{1}{2}x^2$	8
parallelrisch	$x-\frac{1}{2}x^2$	8
parts	$x-\frac{1}{2}x^2$	8

```
[In] int((-x^4+1)/(x^3+x^2+x+1),x,method=_RETURNVERBOSE)
[Out] -1/2*x*(-2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1-x^4}{1+x+x^2+x^3}\,dx = -\frac{1}{2}\,x^2+x$$

```
[In] integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="fricas")
[Out] -1/2*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int \frac{1-x^4}{1+x+x^2+x^3} \, dx = -\frac{x^2}{2} + x$$

```
[In] integrate((-x**4+1)/(x**3+x**2+x+1),x)
[Out] -x**2/2 + x
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1-x^4}{1+x+x^2+x^3} \, dx = -\frac{1}{2} \, x^2 + x$$

```
[In] integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="maxima")
[Out] -1/2*x^2 + x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1-x^4}{1+x+x^2+x^3} \, dx = -\frac{1}{2} \, x^2 + x$$

```
[In] integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="giac")
[Out] -1/2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{1-x^4}{1+x+x^2+x^3} \, dx = -\frac{x\,(x-2)}{2}$$

```
[In] int(-(x^4 - 1)/(x + x^2 + x^3 + 1),x)
[Out] -(x*(x - 2))/2
```

3.182 $\int \frac{1+x+x^2+x^3}{1-x^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 8

$$\int \frac{1+x+x^2+x^3}{1-x^4} dx = -\log(1-x)$$

```
[Out] -ln(1-x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1600, 31}

$$\int \frac{1+x+x^2+x^3}{1-x^4} dx = -\log(1-x)$$

```
[In] Int[(1 + x + x^2 + x^3)/(1 - x^4),x]
```

```
[Out] -Log[1 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{1-x}\,dx \\ &= -\log(1-x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log(1-x)$$

```
[In] Integrate[(1 + x + x^2 + x^3)/(1 - x^4),x]
[Out] -Log[1 - x]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result
default	$-\ln(-1+x)$
norman	$-\ln(-1+x)$
risch	$-\ln(-1+x)$
parallelrisch	$-\ln(-1+x)$
meijerg	$-\frac{\ln(-x^4+1)}{4}-\frac{x^3\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}+\frac{\operatorname{arctanh}(x^2)}{2}-\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4\right.\right.}{4(x^4)}$

```
[In] int((x^3+x^2+x+1)/(-x^4+1),x,method=_RETURNVERBOSE)
[Out] -ln(-1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log\left(x-1\right)$$

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="fricas")
[Out] -log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log\left(x-1\right)$$

```
[In] integrate((x**3+x**2+x+1)/(-x**4+1),x)
[Out] -log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log\left(x-1\right)$$

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="maxima")
[Out] -log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log\left(|x-1|\right)$$

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="giac")
[Out] -log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\ln{(x-1)}$$

```
[In] int(-(x + x^2 + x^3 + 1)/(x^4 - 1),x)
[Out] -log(x - 1)
```

3.183 $\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 7

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} dx = \frac{1}{1-x}$$

```
[Out] 1/(1-x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1600, 32}

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} dx = \frac{1}{1-x}$$

```
[In] Int[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]
```

```
[Out] (1 - x)^(-1)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{(1-x)^2}\,dx \\ &= \frac{1}{1-x} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2}\,dx = -\frac{1}{-1+x}$$

```
[In] Integrate[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]
[Out] -(-1 + x)^(-1)
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result
gosper	$-\frac{1}{-1+x}$
default	$-\frac{1}{-1+x}$
risch	$-\frac{1}{-1+x}$
parallelrisch	$-\frac{1}{-1+x}$
norman	$\frac{-x^3-x^2-x-1}{x^4-1}$
meijerg	$\frac{(-1)^{\frac{1}{4}}\left(-\frac{x^3(-1)^{\frac{3}{4}}}{-x^4+1}-\frac{3x^3(-1)^{\frac{3}{4}}\left(\ln\left(1-\left(x^4\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^4\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(x^4\right)^{\frac{1}{4}}\right)\right)}{4\left(x^4\right)^{\frac{3}{4}}}\right)}{4}+\frac{i\left(-\frac{ix^2}{-x^4+1}+i\,\text{arctanh}\left(x^2\right)\right)}{2}+\frac{3(-1)}{}$

```
[In] int((x^3+x^2+x+1)^2/(-x^4+1)^2,x,method=_RETURNVERBOSE)
[Out] -1/(-1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} \, dx = -\frac{1}{x-1}$$

```
[In] integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="fricas")
[Out] -1/(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} \, dx = -\frac{1}{x-1}$$

```
[In] integrate((x**3+x**2+x+1)**2/(-x**4+1)**2,x)
[Out] -1/(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} \, dx = -\frac{1}{x-1}$$

```
[In] integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="maxima")
[Out] -1/(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} \, dx = -\frac{1}{x-1}$$

```
[In] integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="giac")
[Out] -1/(x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2}\,dx=-\frac{1}{x-1}$$

```
[In] int((x + x^2 + x^3 + 1)^2/(x^4 - 1)^2,x)
[Out] -1/(x - 1)
```

3.184 $\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3} dx = \frac{1}{2(1-x)^2}$$

```
[Out] 1/2/(1-x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1600, 32}

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3} dx = \frac{1}{2(1-x)^2}$$

```
[In] Int[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]
```

```
[Out] 1/(2*(1 - x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{(1-x)^3}\,dx \\ &= \frac{1}{2(1-x)^2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2(-1+x)^2}$$

```
[In] Integrate[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]
[Out] 1/(2*(-1 + x)^2)
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result
gosper	$\frac{1}{2(-1+x)^2}$
default	$\frac{1}{2(-1+x)^2}$
risch	$\frac{1}{2(-1+x)^2}$
parallelrisch	$\frac{1}{2(-1+x)^2}$
norman	$\frac{x+x^5+\frac{3}{2}x^4+\frac{3}{2}x^2+2x^3+\frac{1}{2}x^6+\frac{1}{2}}{(x^4-1)^2}$
meijerg	$-\frac{(-1)^{\frac{3}{4}}\left(\frac{(-1)^{\frac{1}{4}}x(-7x^4+11)}{4(-x^4+1)^2}-\frac{21x(-1)^{\frac{1}{4}}\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{16(x^4)^{\frac{1}{4}}}\right)}{8}+\frac{5(-1)^{\frac{1}{4}}\left(-\frac{x^3(-1)^{\frac{3}{4}}(21x^4+7}{28(-x^4+1)^2}\right.}{}$

```
[In] int((x^3+x^2+x+1)^3/(-x^4+1)^3,x,method=_RETURNVERBOSE)
[Out] 1/2/(-1+x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2\,(x^2-2\,x+1)}$$

```
[In] integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="fricas")
[Out] 1/2/(x^2 - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2x^2-4x+2}$$

```
[In] integrate((x**3+x**2+x+1)**3/(-x**4+1)**3,x)
[Out] 1/(2*x**2 - 4*x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2\,(x^2-2\,x+1)}$$

```
[In] integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="maxima")
[Out] 1/2/(x^2 - 2*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2\,(x-1)^2}$$

```
[In] integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="giac")
[Out] 1/2/(x - 1)^2
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2\,(x-1)^2}$$

```
[In] int(-(x + x^2 + x^3 + 1)^3/(x^4 - 1)^3,x)
[Out] 1/(2*(x - 1)^2)
```

3.185 $\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4} dx = \frac{1}{3(1-x)^3}$$

```
[Out] 1/3/(1-x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1600, 32}

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4} dx = \frac{1}{3(1-x)^3}$$

```
[In] Int[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]
```

```
[Out] 1/(3*(1 - x)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(1-x)^4}\,dx$$
$$= \frac{1}{3(1-x)^3}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3(-1+x)^3}$$

```
[In] Integrate[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]
[Out] -1/3*1/(-1 + x)^3
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{1}{3(-1+x)^3}$	8
default	$-\frac{1}{3(-1+x)^3}$	8
risch	$-\frac{1}{3(-1+x)^3}$	8
parallelrisch	$-\frac{1}{3(-1+x)^3}$	8
norman	$\frac{-4x^4-x^8-x-2x^2-\frac{10}{3}x^3-4x^5-\frac{10}{3}x^6-2x^7-\frac{1}{3}x^9-\frac{1}{3}}{(x^4-1)^3}$	54
meijerg	Expression too large to display	698

```
[In] int((x^3+x^2+x+1)^4/(-x^4+1)^4,x,method=_RETURNVERBOSE)
[Out] -1/3/(-1+x)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3\,(x^3-3\,x^2+3\,x-1)}$$

```
[In] integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="fricas")
[Out] -1/3/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3x^3-9x^2+9x-3}$$

```
[In] integrate((x**3+x**2+x+1)**4/(-x**4+1)**4,x)
[Out] -1/(3*x**3 - 9*x**2 + 9*x - 3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3\,(x^3-3\,x^2+3\,x-1)}$$

```
[In] integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="maxima")
[Out] -1/3/(x^3 - 3*x^2 + 3*x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3\,(x-1)^3}$$

```
[In] integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="giac")
[Out] -1/3/(x - 1)^3
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3\,(x-1)^3}$$

```
[In] int((x + x^2 + x^3 + 1)^4/(x^4 - 1)^4,x)
[Out] -1/(3*(x - 1)^3)
```

3.186 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 165

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4} \, dx = -\frac{gx}{b}-\frac{hx^2}{2b}+\frac{\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\sqrt{b}e+ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{(bd+ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

```
[Out] -g*x/b-1/2*h*x^2/b-1/4*f*ln(-b*x^4+a)/b+1/2*(a*h+b*d)*arctanh(x^2*b^(1/2)/a
^(1/2))/b^(3/2)/a^(1/2)+1/2*arctan(b^(1/4)*x/a^(1/4))*(b*c+a*g-e*a^(1/2)*b^
(1/2))/a^(3/4)/b^(5/4)+1/2*arctanh(b^(1/4)*x/a^(1/4))*(b*c+a*g+e*a^(1/2)*b^
(1/2))/a^(3/4)/b^(5/4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{1899, 1901, 1181, 211, 214, 1833, 1824, 649, 266\}$

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+bd)}{2\sqrt{a}b^{3/2}} - \frac{f\log\left(a-bx^4\right)}{4b} - \frac{gx}{b} - \frac{hx^2}{2b}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x]
```

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/
4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcT
anh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - (f*Log[a - b*x^4])/(4*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
```

```
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{c+ex^2+gx^4}{a-bx^4}+\frac{x(d+fx^2+hx^4)}{a-bx^4}\right)\,dx\\
&= \int \frac{c+ex^2+gx^4}{a-bx^4}\,dx+\int \frac{x(d+fx^2+hx^4)}{a-bx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{a-bx^2}\,dx,x,x^2\right)+\int \left(-\frac{g}{b}+\frac{bc+ag+bex^2}{b\,(a-bx^4)}\right)\,dx\\
&= -\frac{gx}{b}+\frac{1}{2}\text{Subst}\left(\int \left(-\frac{h}{b}+\frac{bd+ah+bfx}{b\,(a-bx^2)}\right)\,dx,x,x^2\right)+\frac{\int \frac{bc+ag+bex^2}{a-bx^4}\,dx}{b}\\
&= -\frac{gx}{b}-\frac{hx^2}{2b}+\frac{\text{Subst}\left(\int \frac{bd+ah+bfx}{a-bx^2}\,dx,x,x^2\right)}{2b}\\
&\quad+\frac{1}{2}\left(e-\frac{bc+ag}{\sqrt{a}\sqrt{b}}\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(e+\frac{bc+ag}{\sqrt{a}\sqrt{b}}\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx
\end{aligned}
$$

$$= -\frac{gx}{b} - \frac{hx^2}{2b} + \frac{\left(bc - \sqrt{a}\sqrt{b}e + ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}} + \frac{\left(bc + \sqrt{a}\sqrt{b}e + ag\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}} + \frac{1}{2}f\text{Subst}\left(\int \frac{x}{a-bx^2}\,dx, x, x^2\right) + \frac{(bd+ah)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{2b}$$

$$= -\frac{gx}{b} - \frac{hx^2}{2b} + \frac{\left(bc - \sqrt{a}\sqrt{b}e + ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}} + \frac{\left(bc + \sqrt{a}\sqrt{b}e + ag\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}} + \frac{(bd+ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}} - \frac{f\log\left(a-bx^4\right)}{4b}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.55

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx = \frac{-4a^{3/4}\sqrt{b}gx - 2a^{3/4}\sqrt{b}hx^2 + 2\sqrt[4]{b}\left(bc - \sqrt{a}\sqrt{b}e + ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \left(b^{5/4}c + \sqrt[4]{a}bd + \sqrt{a}b^{3/4}e + a\sqrt[4]{b}\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x]
```

```
[Out] (-4*a^(3/4)*Sqrt[b]*g*x - 2*a^(3/4)*Sqrt[b]*h*x^2 + 2*b^(1/4)*(b*c - Sqrt[a
]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (b^(5/4)*c + a^(1/4)*b*d +
 Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g + a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (b
^(5/4)*c - a^(1/4)*b*d + Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g - a^(5/4)*h)*Log[a
^(1/4) + b^(1/4)*x] + a^(1/4)*(b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2] - a^(3
/4)*Sqrt[b]*f*Log[a - b*x^4])/(4*a^(3/4)*b^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.58 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.45

method	result
risch	$-\frac{h\,x^{2}}{2b}-\frac{gx}{b}-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}b-a\right)}\frac{\left(bc+ag+(ah+bd)\_R+\_R^{2}be+\_R^{3}bf\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b^{2}}$
default	$-\frac{\frac{1}{2}h\,x^{2}+gx}{b}+\frac{\frac{(ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{(ah+bd)\ln\left(\frac{a+x^{2}\sqrt{ab}}{a-x^{2}\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
[Out] -1/2*h*x^2/b-g*x/b-1/4/b^2*sum((b*c+a*g+(a*h+b*d)*_R+_R^2*b*e+_R^3*b*f)/_R^
3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx=\text{Timed out}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx=\text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.35

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx = -\frac{hx^2+2\,gx}{2\,b} + \frac{\frac{2\left(b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\left(b^{\frac{3}{2}}d-\sqrt{a}bf+a\sqrt{b}h\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(b^{\frac{3}{2}}d+\sqrt{a}bf+a\sqrt{b}h\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(b^{\frac{3}{2}}c+\sqrt{a}b\right.}{}}{4\,b}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/2*(h*x^2 + 2*g*x)/b + 1/4*(2*(b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g)*arc
tan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)
) + (b^(3/2)*d - sqrt(a)*b*f + a*sqrt(b)*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqr
t(a)*b) - (b^(3/2)*d + sqrt(a)*b*f + a*sqrt(b)*h)*log(sqrt(b)*x^2 - sqrt(a)
)/(sqrt(a)*b) - (b^(3/2)*c + sqrt(a)*b*e + a*sqrt(b)*g)*log((sqrt(b)*x - sq
rt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))*sqrt(b)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(123) = 246.

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.05

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx = -\frac{\sqrt{2}\left(b^2c+abg-\sqrt{2}(-ab^3)^{\frac{1}{4}}bd-\sqrt{2}(-ab^3)^{\frac{1}{4}}ah+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,(-ab^3)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(b^2c+abg+\sqrt{2}(-ab^3)^{\frac{1}{4}}bd+\sqrt{2}(-ab^3)^{\frac{1}{4}}ah-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,(-ab^3)^{\frac{3}{4}}} - \frac{\sqrt{2}(b^2c+abg-\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,(-ab^3)^{\frac{3}{4}}} + \frac{\sqrt{2}(b^2c+abg-\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,(-ab^3)^{\frac{3}{4}}} - \frac{f\log\left(|bx^4-a|\right)}{4\,b} - \frac{bhx^2+2\,bgx}{2\,b^2}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c + a*b*g - sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(2)*(-a*b^3)
^(1/4)*a*h + sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4)
)/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c + a*b*g + sqrt(2)*(-a*b
^3)^(1/4)*b*d + sqrt(2)*(-a*b^3)^(1/4)*a*h - sqrt(-a*b)*b*e)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*sqrt(2
)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(
-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^
2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) - 1/4*f*log(abs(b*x
^4 - a))/b - 1/2*(b*h*x^2 + 2*b*g*x)/b^2
```

Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 2478, normalized size of antiderivative = 15.02

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x)
```

```
[Out] symsum(log(- root(256*a^3*b^6*z^4 + 256*a^3*b^5*f*z^3 - 64*a^3*b^4*e*g*z^2
- 64*a^3*b^4*d*h*z^2 - 64*a^2*b^5*c*e*z^2 - 32*a^4*b^3*h^2*z^2 + 96*a^3*b^4
*f^2*z^2 - 32*a^2*b^5*d^2*z^2 - 32*a^3*b^3*e*f*g*z - 32*a^3*b^3*d*f*h*z + 3
2*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*a^4*b^2*g^
2*h*z - 16*a^4*b^2*f*h^2*z + 16*a^3*b^3*e^2*h*z + 16*a^3*b^3*d*g^2*z + 16*a
^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z + 16*a*b^5*c^2*d*z
 + 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h + 8*a^2*b^3*c*d
*f*g - 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g - 4*a^3*b^
2*d*f^2*h + 4*a^3*b^2*d*f*g^2 + 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e*h^2 - 4*a
^2*b^3*d^2*e*g + 4*a^2*b^3*d*e^2*f + 4*a^2*b^3*c*e^2*g - 4*a^2*b^3*c*e*f^2
+ 4*a^4*b*f*g^2*h - 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c*d^2*e + 4
*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 - 6*a^
2*b^3*c^2*g^2 - 2*a^2*b^3*d^2*f^2 - 2*a^4*b*f^2*h^2 + 4*a^2*b^3*d^3*h - 4*a
^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a*b^4*d^4 + a^5*h^4 - a^2*b^
3*e^4 - a^4*b*g^4 - b^5*c^4, z, k)*((8*a*b^3*c*f - 8*a*b^3*d*e - 8*a^2*b^2*
e*h + 8*a^2*b^2*f*g)/b + root(256*a^3*b^6*z^4 + 256*a^3*b^5*f*z^3 - 64*a^3*
b^4*e*g*z^2 - 64*a^3*b^4*d*h*z^2 - 64*a^2*b^5*c*e*z^2 - 32*a^4*b^3*h^2*z^2
+ 96*a^3*b^4*f^2*z^2 - 32*a^2*b^5*d^2*z^2 - 32*a^3*b^3*e*f*g*z - 32*a^3*b^3
*d*f*h*z + 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 1
6*a^4*b^2*g^2*h*z - 16*a^4*b^2*f*h^2*z + 16*a^3*b^3*e^2*h*z + 16*a^3*b^3*d*
g^2*z + 16*a^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z + 16*a
*b^5*c^2*d*z + 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h + 8
*a^2*b^3*c*d*f*g - 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*
g - 4*a^3*b^2*d*f^2*h + 4*a^3*b^2*d*f*g^2 + 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c
*e*h^2 - 4*a^2*b^3*d^2*e*g + 4*a^2*b^3*d*e^2*f + 4*a^2*b^3*c*e^2*g - 4*a^2*
```

b^3*c*e*f^2 + 4*a^4*b*f*g^2*h - 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4
*c*d^2*e + 4*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^
2*g^2 - 6*a^2*b^3*c^2*g^2 - 2*a^2*b^3*d^2*f^2 - 2*a^4*b*f^2*h^2 + 4*a^2*b^3
*d^3*h - 4*a^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a*b^4*d^4 + a^5*
h^4 - a^2*b^3*e^4 - a^4*b*g^4 - b^5*c^4, z, k)*((16*a^2*b^3*g + 16*a*b^4*c)
/b - (x*(16*a^2*b^3*h + 16*a*b^4*d))/b) + (x*(4*b^4*c^2 + 4*a*b^3*e^2 + 4*a
^2*b^2*g^2 + 8*a*b^3*c*g - 8*a*b^3*d*f - 8*a^2*b^2*f*h))/b) - (a*b^2*e^3 +
b^3*c*d^2 - b^3*c^2*e + a^3*g*h^2 + a*b^2*c*f^2 + a*b^2*d^2*g + a^2*b*c*h^2
 - a^2*b*e*g^2 + a^2*b*f^2*g + 2*a*b^2*c*d*h - 2*a*b^2*c*e*g - 2*a*b^2*d*e*
f + 2*a^2*b*d*g*h - 2*a^2*b*e*f*h)/b - (x*(b^3*d^3 + a^3*h^3 + b^3*c^2*f -
2*b^3*c*d*e - a*b^2*d*f^2 + a*b^2*e^2*f + 3*a*b^2*d^2*h + 3*a^2*b*d*h^2 + a
^2*b*f*g^2 - a^2*b*f^2*h - 2*a*b^2*c*e*h + 2*a*b^2*c*f*g - 2*a*b^2*d*e*g -
2*a^2*b*e*g*h))/b)*root(256*a^3*b^6*z^4 + 256*a^3*b^5*f*z^3 - 64*a^3*b^4*e*
g*z^2 - 64*a^3*b^4*d*h*z^2 - 64*a^2*b^5*c*e*z^2 - 32*a^4*b^3*h^2*z^2 + 96*a
^3*b^4*f^2*z^2 - 32*a^2*b^5*d^2*z^2 - 32*a^3*b^3*e*f*g*z - 32*a^3*b^3*d*f*h
*z + 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*a^4*
b^2*g^2*h*z - 16*a^4*b^2*f*h^2*z + 16*a^3*b^3*e^2*h*z + 16*a^3*b^3*d*g^2*z
+ 16*a^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z + 16*a*b^5*c
^2*d*z + 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h + 8*a^2*b
^3*c*d*f*g - 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g - 4*
a^3*b^2*d*f^2*h + 4*a^3*b^2*d*f*g^2 + 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e*h^2
 - 4*a^2*b^3*d^2*e*g + 4*a^2*b^3*d*e^2*f + 4*a^2*b^3*c*e^2*g - 4*a^2*b^3*c*
e*f^2 + 4*a^4*b*f*g^2*h - 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c*d^2
*e + 4*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2
- 6*a^2*b^3*c^2*g^2 - 2*a^2*b^3*d^2*f^2 - 2*a^4*b*f^2*h^2 + 4*a^2*b^3*d^3*h
 - 4*a^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a*b^4*d^4 + a^5*h^4 -
a^2*b^3*e^4 - a^4*b*g^4 - b^5*c^4, z, k), k, 1, 4) - (h*x^2)/(2*b) - (g*x)/
b

3.187 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4} dx$

Optimal result

Integrand size = 41, antiderivative size = 188

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4} dx = -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b} - \frac{\left(be-\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}+ai\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}} + \frac{\left(be+\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}+ai\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}} + \frac{(bd+ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}} - \frac{f\log(a-bx^4)}{4b}$$

```
[Out] -g*x/b-1/2*h*x^2/b-1/3*i*x^3/b-1/4*f*ln(-b*x^4+a)/b+1/2*(a*h+b*d)*arctanh(x
^2*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)-1/2*arctan(b^(1/4)*x/a^(1/4))*(b*e+a*i-
(a*g+b*c)*b^(1/2)/a^(1/2))/a^(1/4)/b^(7/4)+1/2*arctanh(b^(1/4)*x/a^(1/4))*(
b*e+a*i+(a*g+b*c)*b^(1/2)/a^(1/2))/a^(1/4)/b^(7/4)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.220, Rules used

$= \{1899, 1833, 1824, 649, 214, 266, 1901, 1181, 211\}$

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx = -\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+bd)}{2\sqrt{a}b^{3/2}} - \frac{f\log\left(a-bx^4\right)}{4b} - \frac{gx}{b} - \frac{hx^2}{2b} - \frac{ix^3}{3b}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x]
```

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - ((b*e - (Sqrt[b]*(b*c + a*g))/
Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*e + (
Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4
)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2
)) - (f*Log[a - b*x^4])/(4*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
```

```
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x(d+fx^2+hx^4)}{a-bx^4}+\frac{c+ex^2+gx^4+ix^6}{a-bx^4}\right) dx \\
&= \int \frac{x(d+fx^2+hx^4)}{a-bx^4}\, dx+\int \frac{c+ex^2+gx^4+ix^6}{a-bx^4}\, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{a-bx^2}\, dx,x,x^2\right)+\int \left(-\frac{g}{b}-\frac{ix^2}{b}+\frac{bc+ag+(be+ai)x^2}{b\,(a-bx^4)}\right) dx \\
&= -\frac{gx}{b}-\frac{ix^3}{3b}+\frac{1}{2}\text{Subst}\left(\int \left(-\frac{h}{b}+\frac{bd+ah+bfx}{b\,(a-bx^2)}\right) dx,x,x^2\right)+\frac{\int \frac{bc+ag+(be+ai)x^2}{a-bx^4}\, dx}{b} \\
&= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}+\frac{\text{Subst}\left(\int \frac{bd+ah+bfx}{a-bx^2}\, dx,x,x^2\right)}{2b} \\
&\quad +\frac{\left(be-\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}+ai\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\, dx}{2b}+\frac{\left(be+\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}+ai\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\, dx}{2b}
\end{aligned}$$

$$= -\frac{gx}{b} - \frac{hx^2}{2b} - \frac{ix^3}{3b} - \frac{\left(be - \frac{\sqrt{b}(bc+ag)}{\sqrt{a}} + ai\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}} + \frac{\left(be + \frac{\sqrt{b}(bc+ag)}{\sqrt{a}} + ai\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}} + \frac{1}{2}f\text{Subst}\left(\int \frac{x}{a-bx^2}\,dx, x, x^2\right) + \frac{(bd+ah)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{2b}$$

$$= -\frac{gx}{b} - \frac{hx^2}{2b} - \frac{ix^3}{3b} - \frac{\left(be - \frac{\sqrt{b}(bc+ag)}{\sqrt{a}} + ai\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}} + \frac{\left(be + \frac{\sqrt{b}(bc+ag)}{\sqrt{a}} + ai\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}} + \frac{(bd+ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}} - \frac{f\log(a-bx^4)}{4b}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.60

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx$$

$$= \frac{-12b^{3/4}gx - 6b^{3/4}hx^2 - 4b^{3/4}ix^3 + \frac{6\left(b^{3/2}c-\sqrt{a}be+a\sqrt{b}g-a^{3/2}i\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}} - \frac{3\left(b^{3/2}c+\sqrt[4]{a}b^{5/4}d+\sqrt{a}be+a\sqrt{b}g+a^{5/4}\sqrt[4]{b}\right.}{a^{3/4}}}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x]
```

```
[Out] (-12*b^(3/4)*g*x - 6*b^(3/4)*h*x^2 - 4*b^(3/4)*i*x^3 + (6*(b^(3/2)*c - Sqrt
[a]*b*e + a*Sqrt[b]*g - a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/a^(3/4) - (
3*(b^(3/2)*c + a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g + a^(5/4)*b^(1
/4)*h + a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x])/a^(3/4) + (3*(b^(3/2)*c - a^(1
/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g - a^(5/4)*b^(1/4)*h + a^(3/2)*i)*
Log[a^(1/4) + b^(1/4)*x])/a^(3/4) + (3*b^(1/4)*(b*d + a*h)*Log[Sqrt[a] + Sq
rt[b]*x^2])/Sqrt[a] - 3*b^(3/4)*f*Log[a - b*x^4])/(12*b^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.51

method	result
risch	$-\frac{i\,x^{3}}{3b}-\frac{h\,x^{2}}{2b}-\frac{gx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b-a\right)}\frac{\left(-\_R^{3}bf+\left(-ai-be\right)\_R^{2}+\left(-ah-bd\right)\_R-ag-bc\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b^{2}}$
default	$-\frac{\frac{1}{3}i\,x^{3}+\frac{1}{2}h\,x^{2}+gx}{b}+\frac{\frac{\left(ag+bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{\left(ah+bd\right)\ln\left(\frac{a+x^{2}\sqrt{ab}}{a-x^{2}\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{\left(ai+be\right)\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\right.\right.}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b}$

[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)

[Out] -1/3*i*x^3/b-1/2*h*x^2/b-g*x/b+1/4/b^2*sum((-_R^3*b*f+(-a*i-b*e)*_R^2+(-a*h-b*d)*_R-a*g-b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx=\text{Timed out}$$

[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx=\text{Timed out}$$

[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.28

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx = -\frac{2\,ix^3+3\,hx^2+6\,gx}{6\,b} + \frac{\frac{2\left(b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g-a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b^{\frac{3}{2}}d-\sqrt{a}bf+a\sqrt{b}h\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}b}-\frac{\left(b^{\frac{3}{2}}d+\sqrt{a}bf+a\sqrt{b}h\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}b}-\frac{\left(b^{\frac{3}{2}}c\cdots\right.}{\cdots}}{4\,b}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="ma
xima")
```

```
[Out] -1/6*(2*i*x^3 + 3*h*x^2 + 6*g*x)/b + 1/4*(2*(b^(3/2)*c - sqrt(a)*b*e + a*sq
rt(b)*g - a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(b))*sqrt(b)) + (b^(3/2)*d - sqrt(a)*b*f + a*sqrt(b)*h)*log(sqr
t(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (b^(3/2)*d + sqrt(a)*b*f + a*sqrt(b)*h)*l
og(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (b^(3/2)*c + sqrt(a)*b*e + a*sqrt(b
)*g + a^(3/2)*i)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(
sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(144) = 288.

Time = 0.29 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.29

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx = -\frac{f\log\left(|bx^4-a|\right)}{4\,b}$$
$$-\frac{\sqrt{2}\left(b^3c+ab^2g-\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d-\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-\sqrt{-ab}b^2e-\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,(-ab^3)^{\frac{3}{4}}\,b}$$
$$-\frac{\sqrt{2}\left(b^3c+ab^2g+\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d+\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-\sqrt{-ab}b^2e+\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,(-ab^3)^{\frac{3}{4}}\,b}$$
$$-\frac{\sqrt{2}\left(b^3c+ab^2g-\sqrt{-ab}b^2e-\sqrt{-ab}abi\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,(-ab^3)^{\frac{3}{4}}\,b}$$
$$+\frac{\sqrt{2}\left(b^3c+ab^2g-\sqrt{-ab}b^2e-\sqrt{-ab}abi\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,(-ab^3)^{\frac{3}{4}}\,b}$$
$$-\frac{2\,b^2ix^3+3\,b^2hx^2+6\,b^2gx}{6\,b^3}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="gi
ac")
```

```
[Out] -1/4*f*log(abs(b*x^4 - a))/b - 1/4*sqrt(2)*(b^3*c + a*b^2*g - sqrt(2)*(-a*b
^3)^(1/4)*b^2*d - sqrt(2)*(-a*b^3)^(1/4)*a*b*h - sqrt(-a*b)*b^2*e - sqrt(-a
*b)*a*b*i)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((
-a*b^3)^(3/4)*b) - 1/4*sqrt(2)*(b^3*c + a*b^2*g + sqrt(2)*(-a*b^3)^(1/4)*b^
2*d + sqrt(2)*(-a*b^3)^(1/4)*a*b*h - sqrt(-a*b)*b^2*e + sqrt(-a*b)*a*b*i)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4
)*b) - 1/8*sqrt(2)*(b^3*c + a*b^2*g - sqrt(-a*b)*b^2*e - sqrt(-a*b)*a*b*i)*
log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*b) + 1/8*sqr
t(2)*(b^3*c + a*b^2*g - sqrt(-a*b)*b^2*e - sqrt(-a*b)*a*b*i)*log(x^2 - sqrt
(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*b) - 1/6*(2*b^2*i*x^3 + 3*
b^2*h*x^2 + 6*b^2*g*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 3810, normalized size of antiderivative = 20.27

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x)
```

```
[Out] symsum(log(- (a^4*i^3 + a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^2*b^2*c*h^2 -
 a^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i + a*b^3*c*f^2 + a*b^3*d^2*
g - a*b^3*c^2*i + 3*a^3*b*e*i^2 + a^3*b*g*h^2 - a^3*b*g^2*i - 2*a^2*b^2*c*g
*i - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h + 2*a*b^3*c*d*h -
2*a*b^3*c*e*g - 2*a*b^3*d*e*f - 2*a^3*b*f*h*i)/b^2 - root(256*a^3*b^7*z^4 +
 256*a^3*b^6*f*z^3 - 64*a^4*b^4*g*i*z^2 - 64*a^3*b^5*e*g*z^2 - 64*a^3*b^5*d
*h*z^2 - 64*a^3*b^5*c*i*z^2 - 64*a^2*b^6*c*e*z^2 - 32*a^4*b^4*h^2*z^2 + 96*
a^3*b^5*f^2*z^2 - 32*a^2*b^6*d^2*z^2 - 32*a^4*b^3*f*g*i*z + 32*a^4*b^3*e*h*
i*z - 32*a^3*b^4*e*f*g*z - 32*a^3*b^4*d*f*h*z + 32*a^3*b^4*d*e*i*z + 32*a^3
*b^4*c*g*h*z - 32*a^3*b^4*c*f*i*z - 32*a^2*b^5*c*e*f*z + 32*a^2*b^5*c*d*g*z
 + 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 16*a^4*b^3*f*h^2*z + 16*a^4*b^
3*d*i^2*z + 16*a^3*b^4*e^2*h*z + 16*a^3*b^4*d*g^2*z + 16*a^2*b^5*c^2*h*z -
16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z + 16*a*b^6*c^2*d*z + 16*a^3*b^4*f^3
*z + 8*a^4*b^2*e*f*h*i - 8*a^4*b^2*d*g*h*i - 8*a^3*b^3*d*e*g*h + 8*a^3*b^3*
d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*g*i - 8*a^3*b^3*c*d*h*i + 8*a^2
*b^4*c*d*f*g - 8*a^2*b^4*c*d*e*h - 4*a^4*b^2*f^2*g*i + 4*a^4*b^2*f*g^2*h +
4*a^4*b^2*e*g^2*i - 4*a^4*b^2*e*g*h^2 - 4*a^4*b^2*c*h^2*i - 4*a^3*b^3*d^2*g
*i + 4*a^4*b^2*d*f*i^2 + 4*a^4*b^2*c*g*i^2 + 4*a^3*b^3*e^2*f*h - 4*a^3*b^3*
e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^2*i + 4*a^3*b^3*d*f*g^2 + 4*a^2
*b^4*c^2*f*h + 4*a^2*b^4*c^2*e*i - 4*a^3*b^3*c*e*h^2 - 4*a^2*b^4*d^2*e*g -
4*a^2*b^4*c*d^2*i + 4*a^2*b^4*d*e^2*f + 4*a^2*b^4*c*e^2*g - 4*a^2*b^4*c*e*f
^2 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*h*i^2 + 4*a*b^5*c^2*d*f - 4*a*b^5*c*d^2*e
```

- 4*a^5*b*e*i^3 - 4*a*b^5*c^3*g - 6*a^4*b^2*e^2*i^2 - 2*a^4*b^2*f^2*h^2 + 6
*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 2*a^3*b^3*c^2*i^2 - 6*a^2*b^4*c^2*g^
2 - 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g^2*i^2 - 4*a^3*b^3*e^3*i + 4*a^4*b^2*d*h^3
 + 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*g^3 + 2*a*b^5*c^2*e^2 + a^3*b^3*f^4 + a^5*
b*h^4 + a*b^5*d^4 - a^4*b^2*g^4 - a^2*b^4*e^4 - a^6*i^4 - b^6*c^4, z, l)*(r
oot(256*a^3*b^7*z^4 + 256*a^3*b^6*f*z^3 - 64*a^4*b^4*g*i*z^2 - 64*a^3*b^5*e
*g*z^2 - 64*a^3*b^5*d*h*z^2 - 64*a^3*b^5*c*i*z^2 - 64*a^2*b^6*c*e*z^2 - 32*
a^4*b^4*h^2*z^2 + 96*a^3*b^5*f^2*z^2 - 32*a^2*b^6*d^2*z^2 - 32*a^4*b^3*f*g*
i*z + 32*a^4*b^3*e*h*i*z - 32*a^3*b^4*e*f*g*z - 32*a^3*b^4*d*f*h*z + 32*a^3
*b^4*d*e*i*z + 32*a^3*b^4*c*g*h*z - 32*a^3*b^4*c*f*i*z - 32*a^2*b^5*c*e*f*z
 + 32*a^2*b^5*c*d*g*z + 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 16*a^4*b^
3*f*h^2*z + 16*a^4*b^3*d*i^2*z + 16*a^3*b^4*e^2*h*z + 16*a^3*b^4*d*g^2*z +
16*a^2*b^5*c^2*h*z - 16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z + 16*a*b^6*c^2
*d*z + 16*a^3*b^4*f^3*z + 8*a^4*b^2*e*f*h*i - 8*a^4*b^2*d*g*h*i - 8*a^3*b^3
*d*e*g*h + 8*a^3*b^3*d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*g*i - 8*a^
3*b^3*c*d*h*i + 8*a^2*b^4*c*d*f*g - 8*a^2*b^4*c*d*e*h - 4*a^4*b^2*f^2*g*i +
 4*a^4*b^2*f*g^2*h + 4*a^4*b^2*e*g^2*i - 4*a^4*b^2*e*g*h^2 - 4*a^4*b^2*c*h^
2*i - 4*a^3*b^3*d^2*g*i + 4*a^4*b^2*d*f*i^2 + 4*a^4*b^2*c*g*i^2 + 4*a^3*b^3
*e^2*f*h - 4*a^3*b^3*e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^2*i + 4*a^
3*b^3*d*f*g^2 + 4*a^2*b^4*c^2*f*h + 4*a^2*b^4*c^2*e*i - 4*a^3*b^3*c*e*h^2 -
 4*a^2*b^4*d^2*e*g - 4*a^2*b^4*c*d^2*i + 4*a^2*b^4*d*e^2*f + 4*a^2*b^4*c*e^
2*g - 4*a^2*b^4*c*e*f^2 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*h*i^2 + 4*a*b^5*c^2*d
*f - 4*a*b^5*c*d^2*e - 4*a^5*b*e*i^3 - 4*a*b^5*c^3*g - 6*a^4*b^2*e^2*i^2 -
2*a^4*b^2*f^2*h^2 + 6*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 2*a^3*b^3*c^2*i
^2 - 6*a^2*b^4*c^2*g^2 - 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g^2*i^2 - 4*a^3*b^3*e^
3*i + 4*a^4*b^2*d*h^3 + 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*g^3 + 2*a*b^5*c^2*e^2
 + a^3*b^3*f^4 + a^5*b*h^4 + a*b^5*d^4 - a^4*b^2*g^4 - a^2*b^4*e^4 - a^6*i^
4 - b^6*c^4, z, l)*((16*a^2*b^4*g + 16*a*b^5*c)/b^2 - (x*(16*a^2*b^3*h + 16
*a*b^4*d))/b) - (8*a*b^4*d*e - 8*a*b^4*c*f + 8*a^2*b^3*d*i + 8*a^2*b^3*e*h
- 8*a^2*b^3*f*g + 8*a^3*b^2*h*i)/b^2 + (x*(4*b^4*c^2 + 4*a*b^3*e^2 + 4*a^3*
b*i^2 + 4*a^2*b^2*g^2 + 8*a*b^3*c*g - 8*a*b^3*d*f + 8*a^2*b^2*e*i - 8*a^2*b
^2*f*h))/b) - (x*(b^3*d^3 + a^3*h^3 + b^3*c^2*f + a^3*f*i^2 - 2*b^3*c*d*e -
 2*a^3*g*h*i - a*b^2*d*f^2 + a*b^2*e^2*f + 3*a*b^2*d^2*h + 3*a^2*b*d*h^2 +
a^2*b*f*g^2 - a^2*b*f^2*h - 2*a*b^2*c*d*i - 2*a*b^2*c*e*h + 2*a*b^2*c*f*g -
 2*a*b^2*d*e*g - 2*a^2*b*c*h*i - 2*a^2*b*d*g*i + 2*a^2*b*e*f*i - 2*a^2*b*e*
g*h))/b)*root(256*a^3*b^7*z^4 + 256*a^3*b^6*f*z^3 - 64*a^4*b^4*g*i*z^2 - 64
*a^3*b^5*e*g*z^2 - 64*a^3*b^5*d*h*z^2 - 64*a^3*b^5*c*i*z^2 - 64*a^2*b^6*c*e
*z^2 - 32*a^4*b^4*h^2*z^2 + 96*a^3*b^5*f^2*z^2 - 32*a^2*b^6*d^2*z^2 - 32*a^
4*b^3*f*g*i*z + 32*a^4*b^3*e*h*i*z - 32*a^3*b^4*e*f*g*z - 32*a^3*b^4*d*f*h*
z + 32*a^3*b^4*d*e*i*z + 32*a^3*b^4*c*g*h*z - 32*a^3*b^4*c*f*i*z - 32*a^2*b
^5*c*e*f*z + 32*a^2*b^5*c*d*g*z + 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z -
 16*a^4*b^3*f*h^2*z + 16*a^4*b^3*d*i^2*z + 16*a^3*b^4*e^2*h*z + 16*a^3*b^4*
d*g^2*z + 16*a^2*b^5*c^2*h*z - 16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z + 16
*a*b^6*c^2*d*z + 16*a^3*b^4*f^3*z + 8*a^4*b^2*e*f*h*i - 8*a^4*b^2*d*g*h*i -
 8*a^3*b^3*d*e*g*h + 8*a^3*b^3*d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*

```
g*i - 8*a^3*b^3*c*d*h*i + 8*a^2*b^4*c*d*f*g - 8*a^2*b^4*c*d*e*h - 4*a^4*b^2
*f^2*g*i + 4*a^4*b^2*f*g^2*h + 4*a^4*b^2*e*g^2*i - 4*a^4*b^2*e*g*h^2 - 4*a^
4*b^2*c*h^2*i - 4*a^3*b^3*d^2*g*i + 4*a^4*b^2*d*f*i^2 + 4*a^4*b^2*c*g*i^2 +
 4*a^3*b^3*e^2*f*h - 4*a^3*b^3*e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^
2*i + 4*a^3*b^3*d*f*g^2 + 4*a^2*b^4*c^2*f*h + 4*a^2*b^4*c^2*e*i - 4*a^3*b^3
*c*e*h^2 - 4*a^2*b^4*d^2*e*g - 4*a^2*b^4*c*d^2*i + 4*a^2*b^4*d*e^2*f + 4*a^
2*b^4*c*e^2*g - 4*a^2*b^4*c*e*f^2 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*h*i^2 + 4*a
*b^5*c^2*d*f - 4*a*b^5*c*d^2*e - 4*a^5*b*e*i^3 - 4*a*b^5*c^3*g - 6*a^4*b^2*
e^2*i^2 - 2*a^4*b^2*f^2*h^2 + 6*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 2*a^3
*b^3*c^2*i^2 - 6*a^2*b^4*c^2*g^2 - 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g^2*i^2 - 4*
a^3*b^3*e^3*i + 4*a^4*b^2*d*h^3 + 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*g^3 + 2*a*b
^5*c^2*e^2 + a^3*b^3*f^4 + a^5*b*h^4 + a*b^5*d^4 - a^4*b^2*g^4 - a^2*b^4*e^
4 - a^6*i^4 - b^6*c^4, z, l), l, 1, 4) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - (g
*x)/b
```

3.188 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 205

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx = -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}-\frac{jx^4}{4b}-\frac{\left(be-\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}+ai\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\left(be+\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}+ai\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{(bd+ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{(bf+aj)\log(a-bx^4)}{4b^2}$$

```
[Out] -g*x/b-1/2*h*x^2/b-1/3*i*x^3/b-1/4*j*x^4/b-1/4*(a*j+b*f)*ln(-b*x^4+a)/b^2+1
/2*(a*h+b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)-1/2*arctan(b^(1/4
)*x/a^(1/4))*(b*e+a*i-(a*g+b*c)*b^(1/2)/a^(1/2))/a^(1/4)/b^(7/4)+1/2*arctan
h(b^(1/4)*x/a^(1/4))*(b*e+a*i+(a*g+b*c)*b^(1/2)/a^(1/2))/a^(1/4)/b^(7/4)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.196, Rules used

$= \{1899, 1901, 1181, 211, 214, 1833, 1824, 649, 266\}$

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}$$
$$+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+bd)}{2\sqrt{a}b^{3/2}}-\frac{(aj+bf)\log(a-bx^4)}{4b^2}-\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}-\frac{jx^4}{4b}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4),x
]
```

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - (j*x^4)/(4*b) - ((b*e - (Sqrt[
b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7
/4)) + ((b*e + (Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(b^(1/4)*x)/a^(
1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(
2*Sqrt[a]*b^(3/2)) - ((b*f + a*j)*Log[a - b*x^4])/(4*b^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c+ex^2+gx^4+ix^6}{a-bx^4}+\frac{x(d+fx^2+hx^4+jx^6)}{a-bx^4}\right)\,dx\\
&= \int \frac{c+ex^2+gx^4+ix^6}{a-bx^4}\,dx+\int \frac{x(d+fx^2+hx^4+jx^6)}{a-bx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2+jx^3}{a-bx^2}\,dx,x,x^2\right)+\int \left(-\frac{g}{b}-\frac{ix^2}{b}+\frac{bc+ag+(be+ai)x^2}{b\,(a-bx^4)}\right)\,dx\\
&= -\frac{gx}{b}-\frac{ix^3}{3b}+\frac{1}{2}\text{Subst}\left(\int \left(-\frac{h}{b}-\frac{jx}{b}+\frac{bd+ah+(bf+aj)x}{b\,(a-bx^2)}\right)\,dx,x,x^2\right)\\
&\quad+\frac{\int \frac{bc+ag+(be+ai)x^2}{a-bx^4}\,dx}{b}\\
&= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}-\frac{jx^4}{4b}+\frac{\text{Subst}\left(\int \frac{bd+ah+(bf+aj)x}{a-bx^2}\,dx,x,x^2\right)}{2b}\\
&\quad+\frac{\left(be-\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}+ai\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{2b}+\frac{\left(be+\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}+ai\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx}{2b}
\end{aligned}$$

$$= -\frac{gx}{b} - \frac{hx^2}{2b} - \frac{ix^3}{3b} - \frac{jx^4}{4b} - \frac{\left(be - \frac{\sqrt{b}(bc+ag)}{\sqrt{a}} + ai\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}} + \frac{\left(be + \frac{\sqrt{b}(bc+ag)}{\sqrt{a}} + ai\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}} + \frac{(bd+ah)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{2b} + \frac{(bf+aj)\text{Subst}\left(\int \frac{x}{a-bx^2}\,dx, x, x^2\right)}{2b}$$

$$= -\frac{gx}{b} - \frac{hx^2}{2b} - \frac{ix^3}{3b} - \frac{jx^4}{4b} - \frac{\left(be - \frac{\sqrt{b}(bc+ag)}{\sqrt{a}} + ai\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}} + \frac{\left(be + \frac{\sqrt{b}(bc+ag)}{\sqrt{a}} + ai\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}} + \frac{(bd+ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}} - \frac{(bf+aj)\log\left(a-bx^4\right)}{4b^2}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.55

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx$$

$$= \frac{-12b^{3/4}gx - 6b^{3/4}hx^2 - 4b^{3/4}ix^3 - 3b^{3/4}jx^4 + \frac{6\left(b^{3/2}c-\sqrt{a}be+a\sqrt{b}g-a^{3/2}i\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}} - \frac{3\left(b^{3/2}c+\sqrt[4]{a}b^{5/4}d+\sqrt{a}be\right.}{}}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4),x]
```

```
[Out] (-12*b^(3/4)*g*x - 6*b^(3/4)*h*x^2 - 4*b^(3/4)*i*x^3 - 3*b^(3/4)*j*x^4 + (6
*(b^(3/2)*c - Sqrt[a]*b*e + a*Sqrt[b]*g - a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(
1/4)])/a^(3/4) - (3*(b^(3/2)*c + a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b
]*g + a^(5/4)*b^(1/4)*h + a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x]])/a^(3/4) + (3
*(b^(3/2)*c - a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g - a^(5/4)*b^(1/
4)*h + a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x]])/a^(3/4) + (3*b^(1/4)*(b*d + a*h
)*Log[Sqrt[a] + Sqrt[b]*x^2]])/Sqrt[a] - (3*(b*f + a*j)*Log[a - b*x^4]])/b^(1
/4))/(12*b^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.50

method	result
risch	$-\frac{j\,x^{4}}{4b}-\frac{i\,x^{3}}{3b}-\frac{h\,x^{2}}{2b}-\frac{gx}{b}-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}b-a\right)}\frac{\left(bc+ag+(ah+bd)\_R+(ai+be)\_R^{2}+(aj+bf)\_R^{3}\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b^{2}}$
default	$-\frac{\frac{1}{4}j\,x^{4}+\frac{1}{3}i\,x^{3}+\frac{1}{2}h\,x^{2}+gx}{b}+\frac{\frac{(ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{(ah+bd)\ln\left(\frac{a+x^{2}\sqrt{ab}}{a-x^{2}\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{(ai+be)\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right.}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b}$

[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)

[Out] -1/4*j*x^4/b-1/3*i*x^3/b-1/2*h*x^2/b-g*x/b-1/4/b^2*sum((b*c+a*g+(a*h+b*d)*_R+(a*i+b*e)*_R^2+(a*j+b*f)*_R^3)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx=\text{Timed out}$$

[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx=\text{Timed out}$$

[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.25

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx = -\frac{3\,jx^4+4\,ix^3+6\,hx^2+12\,gx}{12\,b} + \frac{\frac{2\left(b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g-a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b^{\frac{3}{2}}d-\sqrt{a}bf+a\sqrt{b}h-a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}b}-\frac{\left(b^{\frac{3}{2}}d+\sqrt{a}bf+a\sqrt{b}h+a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}b}-\cdots}{4\,b}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorit
hm="maxima")
```

```
[Out] -1/12*(3*j*x^4 + 4*i*x^3 + 6*h*x^2 + 12*g*x)/b + 1/4*(2*(b^(3/2)*c - sqrt(a
)*b*e + a*sqrt(b)*g - a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b^(3/2)*d - sqrt(a)*b*f + a*sqrt(b
)*h - a^(3/2)*j)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (b^(3/2)*d + sqrt
(a)*b*f + a*sqrt(b)*h + a^(3/2)*j)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) -
 (b^(3/2)*c + sqrt(a)*b*e + a*sqrt(b)*g + a^(3/2)*i)*log((sqrt(b)*x - sqrt(
sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a
)*sqrt(b))*sqrt(b)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(159) = 318.

Time = 0.28 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.17

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx =$$

$$-\frac{\sqrt{2}\left(b^3c+ab^2g-\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d-\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-\sqrt{-ab}b^2e-\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,(-ab^3)^{\frac{3}{4}}\,b}$$
$$-\frac{\sqrt{2}\left(b^3c+ab^2g+\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d+\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-\sqrt{-ab}b^2e+\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,(-ab^3)^{\frac{3}{4}}\,b}$$
$$-\frac{\sqrt{2}\left(b^3c+ab^2g-\sqrt{-ab}b^2e-\sqrt{-ab}abi\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,(-ab^3)^{\frac{3}{4}}\,b}$$
$$+\frac{\sqrt{2}\left(b^3c+ab^2g-\sqrt{-ab}b^2e-\sqrt{-ab}abi\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,(-ab^3)^{\frac{3}{4}}\,b}$$
$$-\frac{(bf+aj)\log\left(|bx^4-a|\right)}{4\,b^2}-\frac{3\,b^3jx^4+4\,b^3ix^3+6\,b^3hx^2+12\,b^3gx}{12\,b^4}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorit
hm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^3*c + a*b^2*g - sqrt(2)*(-a*b^3)^(1/4)*b^2*d - sqrt(2)*(-a*
b^3)^(1/4)*a*b*h - sqrt(-a*b)*b^2*e - sqrt(-a*b)*a*b*i)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*b) - 1/4*sqrt(2)
*(b^3*c + a*b^2*g + sqrt(2)*(-a*b^3)^(1/4)*b^2*d + sqrt(2)*(-a*b^3)^(1/4)*a
*b*h - sqrt(-a*b)*b^2*e + sqrt(-a*b)*a*b*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(
2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*b) - 1/8*sqrt(2)*(b^3*c + a*
b^2*g - sqrt(-a*b)*b^2*e - sqrt(-a*b)*a*b*i)*log(x^2 + sqrt(2)*x*(-a/b)^(1/
4) + sqrt(-a/b))/((-a*b^3)^(3/4)*b) + 1/8*sqrt(2)*(b^3*c + a*b^2*g - sqrt(-
a*b)*b^2*e - sqrt(-a*b)*a*b*i)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b
))/((-a*b^3)^(3/4)*b) - 1/4*(b*f + a*j)*log(abs(b*x^4 - a))/b^2 - 1/12*(3*b
^3*j*x^4 + 4*b^3*i*x^3 + 6*b^3*h*x^2 + 12*b^3*g*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 5673, normalized size of antiderivative = 27.67

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4),x
)
```

```
[Out] symsum(log(- (a^4*i^3 + a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^4*g*j^2 + a^2
*b^2*c*h^2 - a^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i - 2*a^4*h*i*j
+ a*b^3*c*f^2 + a*b^3*d^2*g - a*b^3*c^2*i + a^3*b*c*j^2 + 3*a^3*b*e*i^2 + a
^3*b*g*h^2 - a^3*b*g^2*i + 2*a^2*b^2*c*f*j - 2*a^2*b^2*c*g*i - 2*a^2*b^2*d*
e*j - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h + 2*a*b^3*c*d*h -
 2*a*b^3*c*e*g - 2*a*b^3*d*e*f - 2*a^3*b*d*i*j - 2*a^3*b*e*h*j + 2*a^3*b*f*
g*j - 2*a^3*b*f*h*i)/b^2 - root(256*a^3*b^8*z^4 + 256*a^4*b^6*j*z^3 + 256*a
^3*b^7*f*z^3 + 192*a^4*b^5*f*j*z^2 - 64*a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^
2 - 64*a^3*b^6*d*h*z^2 - 64*a^3*b^6*c*i*z^2 - 64*a^2*b^7*c*e*z^2 + 96*a^5*b
^4*j^2*z^2 - 32*a^4*b^5*h^2*z^2 + 96*a^3*b^6*f^2*z^2 - 32*a^2*b^7*d^2*z^2 -
 32*a^5*b^3*g*i*j*z - 32*a^4*b^4*f*g*i*z + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*
e*g*j*z - 32*a^4*b^4*d*h*j*z - 32*a^4*b^4*c*i*j*z - 32*a^3*b^5*e*f*g*z - 32
*a^3*b^5*d*f*h*z + 32*a^3*b^5*d*e*i*z + 32*a^3*b^5*c*g*h*z - 32*a^3*b^5*c*f
*i*z - 32*a^3*b^5*c*e*j*z - 32*a^2*b^6*c*e*f*z + 32*a^2*b^6*c*d*g*z - 16*a^
5*b^3*h^2*j*z + 16*a^5*b^3*h*i^2*z + 48*a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*
z + 16*a^4*b^4*g^2*h*z - 16*a^4*b^4*f*h^2*z - 16*a^3*b^5*d^2*j*z + 16*a^4*b
^4*d*i^2*z + 16*a^3*b^5*e^2*h*z + 16*a^3*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z -
 16*a^2*b^6*d^2*f*z + 16*a^2*b^6*d*e^2*z + 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^
3*z + 16*a^3*b^5*f^3*z - 8*a^5*b^2*f*g*i*j + 8*a^5*b^2*e*h*i*j + 8*a^4*b^3*
e*f*h*i - 8*a^4*b^3*e*f*g*j - 8*a^4*b^3*d*g*h*i - 8*a^4*b^3*d*f*h*j + 8*a^4
*b^3*d*e*i*j + 8*a^4*b^3*c*g*h*j - 8*a^4*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h +
8*a^3*b^4*d*e*f*i + 8*a^3*b^4*c*f*g*h + 8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f
*j - 8*a^3*b^4*c*d*h*i + 8*a^3*b^4*c*d*g*j + 8*a^2*b^5*c*d*f*g - 8*a^2*b^5*
c*d*e*h + 4*a^5*b^2*g^2*h*j - 4*a^5*b^2*g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5
*b^2*f*h*i^2 + 4*a^5*b^2*d*i^2*j + 4*a^4*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 -
4*a^5*b^2*d*h*j^2 - 4*a^5*b^2*c*i*j^2 - 4*a^4*b^3*f^2*g*i + 4*a^4*b^3*f*g^2
*h + 4*a^4*b^3*e*g^2*i + 4*a^4*b^3*d*g^2*j + 4*a^3*b^4*c^2*h*j - 4*a^4*b^3*
e*g*h^2 - 4*a^4*b^3*c*h^2*i - 4*a^3*b^4*d^2*g*i - 4*a^3*b^4*d^2*f*j + 4*a^4
*b^3*d*f*i^2 + 4*a^4*b^3*c*g*i^2 + 4*a^3*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j -
4*a^4*b^3*c*e*j^2 - 4*a^3*b^4*e*f^2*g - 4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2
*i + 4*a^3*b^4*d*f*g^2 + 4*a^2*b^5*c^2*f*h + 4*a^2*b^5*c^2*e*i + 4*a^2*b^5*
c^2*d*j - 4*a^3*b^4*c*e*h^2 - 4*a^2*b^5*d^2*e*g - 4*a^2*b^5*c*d^2*i + 4*a^2
*b^5*d*e^2*f + 4*a^2*b^5*c*e^2*g - 4*a^2*b^5*c*e*f^2 + 4*a^6*b*h*i^2*j - 4*
a^6*b*g*i*j^2 + 4*a*b^6*c^2*d*f - 4*a*b^6*c*d^2*e + 4*a^6*b*f*j^3 - 4*a*b^6
*c^3*g + 6*a^5*b^2*f^2*j^2 + 2*a^5*b^2*g^2*i^2 - 6*a^4*b^3*e^2*i^2 - 2*a^4*
```

b^3*f^2*h^2 - 2*a^4*b^3*d^2*j^2 + 6*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 2 *a^3*b^4*c^2*i^2 - 6*a^2*b^5*c^2*g^2 - 2*a^2*b^5*d^2*f^2 - 2*a^6*b*h^2*j^2 + 4*a^4*b^3*f^3*j - 4*a^5*b^2*e*i^3 - 4*a^3*b^4*e^3*i + 4*a^4*b^3*d*h^3 + 4 *a^2*b^5*d^3*h - 4*a^3*b^4*c*g^3 + 2*a*b^6*c^2*e^2 + a^5*b^2*h^4 + a^3*b^4* f^4 + a*b^6*d^4 + a^7*j^4 - a^4*b^3*g^4 - a^2*b^5*e^4 - a^6*b*i^4 - b^7*c^4 , z, m)*((8*a*b^4*c*f - 8*a*b^4*d*e + 8*a^2*b^3*c*j - 8*a^2*b^3*d*i - 8*a^2 *b^3*e*h + 8*a^2*b^3*f*g + 8*a^3*b^2*g*j - 8*a^3*b^2*h*i)/b^2 + root(256*a^ 3*b^8*z^4 + 256*a^4*b^6*j*z^3 + 256*a^3*b^7*f*z^3 + 192*a^4*b^5*f*j*z^2 - 6 4*a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^2 - 64*a^3*b^6*d*h*z^2 - 64*a^3*b^6*c* i*z^2 - 64*a^2*b^7*c*e*z^2 + 96*a^5*b^4*j^2*z^2 - 32*a^4*b^5*h^2*z^2 + 96*a ^3*b^6*f^2*z^2 - 32*a^2*b^7*d^2*z^2 - 32*a^5*b^3*g*i*j*z - 32*a^4*b^4*f*g*i *z + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*e*g*j*z - 32*a^4*b^4*d*h*j*z - 32*a^4* b^4*c*i*j*z - 32*a^3*b^5*e*f*g*z - 32*a^3*b^5*d*f*h*z + 32*a^3*b^5*d*e*i*z + 32*a^3*b^5*c*g*h*z - 32*a^3*b^5*c*f*i*z - 32*a^3*b^5*c*e*j*z - 32*a^2*b^6 *c*e*f*z + 32*a^2*b^6*c*d*g*z - 16*a^5*b^3*h^2*j*z + 16*a^5*b^3*h*i^2*z + 4 8*a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*z + 16*a^4*b^4*g^2*h*z - 16*a^4*b^4*f* h^2*z - 16*a^3*b^5*d^2*j*z + 16*a^4*b^4*d*i^2*z + 16*a^3*b^5*e^2*h*z + 16*a ^3*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z - 16*a^2*b^6*d^2*f*z + 16*a^2*b^6*d*e^2 *z + 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^3*z + 16*a^3*b^5*f^3*z - 8*a^5*b^2*f*g *i*j + 8*a^5*b^2*e*h*i*j + 8*a^4*b^3*e*f*h*i - 8*a^4*b^3*e*f*g*j - 8*a^4*b^ 3*d*g*h*i - 8*a^4*b^3*d*f*h*j + 8*a^4*b^3*d*e*i*j + 8*a^4*b^3*c*g*h*j - 8*a ^4*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h + 8*a^3*b^4*d*e*f*i + 8*a^3*b^4*c*f*g*h + 8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f*j - 8*a^3*b^4*c*d*h*i + 8*a^3*b^4*c*d *g*j + 8*a^2*b^5*c*d*f*g - 8*a^2*b^5*c*d*e*h + 4*a^5*b^2*g^2*h*j - 4*a^5*b^ 2*g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5*b^2*f*h*i^2 + 4*a^5*b^2*d*i^2*j + 4*a ^4*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 - 4*a^5*b^2*d*h*j^2 - 4*a^5*b^2*c*i*j^2 - 4*a^4*b^3*f^2*g*i + 4*a^4*b^3*f*g^2*h + 4*a^4*b^3*e*g^2*i + 4*a^4*b^3*d*g ^2*j + 4*a^3*b^4*c^2*h*j - 4*a^4*b^3*e*g*h^2 - 4*a^4*b^3*c*h^2*i - 4*a^3*b^ 4*d^2*g*i - 4*a^3*b^4*d^2*f*j + 4*a^4*b^3*d*f*i^2 + 4*a^4*b^3*c*g*i^2 + 4*a ^3*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j - 4*a^4*b^3*c*e*j^2 - 4*a^3*b^4*e*f^2*g - 4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2*i + 4*a^3*b^4*d*f*g^2 + 4*a^2*b^5*c^2 *f*h + 4*a^2*b^5*c^2*e*i + 4*a^2*b^5*c^2*d*j - 4*a^3*b^4*c*e*h^2 - 4*a^2*b^ 5*d^2*e*g - 4*a^2*b^5*c*d^2*i + 4*a^2*b^5*d*e^2*f + 4*a^2*b^5*c*e^2*g - 4*a ^2*b^5*c*e*f^2 + 4*a^6*b*h*i^2*j - 4*a^6*b*g*i*j^2 + 4*a*b^6*c^2*d*f - 4*a* b^6*c*d^2*e + 4*a^6*b*f*j^3 - 4*a*b^6*c^3*g + 6*a^5*b^2*f^2*j^2 + 2*a^5*b^2 *g^2*i^2 - 6*a^4*b^3*e^2*i^2 - 2*a^4*b^3*f^2*h^2 - 2*a^4*b^3*d^2*j^2 + 6*a^ 3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 2*a^3*b^4*c^2*i^2 - 6*a^2*b^5*c^2*g^2 - 2*a^2*b^5*d^2*f^2 - 2*a^6*b*h^2*j^2 + 4*a^4*b^3*f^3*j - 4*a^5*b^2*e*i^3 - 4*a^3*b^4*e^3*i + 4*a^4*b^3*d*h^3 + 4*a^2*b^5*d^3*h - 4*a^3*b^4*c*g^3 + 2*a *b^6*c^2*e^2 + a^5*b^2*h^4 + a^3*b^4*f^4 + a*b^6*d^4 + a^7*j^4 - a^4*b^3*g^ 4 - a^2*b^5*e^4 - a^6*b*i^4 - b^7*c^4, z, m)*((16*a^2*b^4*g + 16*a*b^5*c)/b ^2 - (x*(16*a^2*b^4*h + 16*a*b^5*d))/b^2) + (x*(4*b^5*c^2 + 4*a*b^4*e^2 + 4 *a^2*b^3*g^2 + 4*a^3*b^2*i^2 + 8*a*b^4*c*g - 8*a*b^4*d*f - 8*a^2*b^3*d*j + 8*a^2*b^3*e*i - 8*a^2*b^3*f*h - 8*a^3*b^2*h*j))/b^2) - (x*(b^4*d^3 + a^3*b* h^3 + b^4*c^2*f - a^4*h*j^2 + a^4*i^2*j + 3*a^2*b^2*d*h^2 + a^2*b^2*f*g^2 -

a^2*b^2*f^2*h + a^2*b^2*e^2*j - 2*b^4*c*d*e - a*b^3*d*f^2 + a*b^3*e^2*f + 3*a*b^3*d^2*h + a*b^3*c^2*j - a^3*b*d*j^2 + a^3*b*f*i^2 + a^3*b*g^2*j + 2*a^2*b^2*c*g*j - 2*a^2*b^2*c*h*i - 2*a^2*b^2*d*f*j - 2*a^2*b^2*d*g*i + 2*a^2*b^2*e*f*i - 2*a^2*b^2*e*g*h - 2*a*b^3*c*d*i - 2*a*b^3*c*e*h + 2*a*b^3*c*f*g - 2*a*b^3*d*e*g + 2*a^3*b*e*i*j - 2*a^3*b*f*h*j - 2*a^3*b*g*h*i))/b^2)*root(256*a^3*b^8*z^4 + 256*a^4*b^6*j*z^3 + 256*a^3*b^7*f*z^3 + 192*a^4*b^5*f*j*z^2 - 64*a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^2 - 64*a^3*b^6*d*h*z^2 - 64*a^3*b^6*c*i*z^2 - 64*a^2*b^7*c*e*z^2 + 96*a^5*b^4*j^2*z^2 - 32*a^4*b^5*h^2*z^2 + 96*a^3*b^6*f^2*z^2 - 32*a^2*b^7*d^2*z^2 - 32*a^5*b^3*g*i*j*z - 32*a^4*b^4*f*g*i*z + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*e*g*j*z - 32*a^4*b^4*d*h*j*z - 32*a^4*b^4*c*i*j*z - 32*a^3*b^5*e*f*g*z - 32*a^3*b^5*d*f*h*z + 32*a^3*b^5*d*e*i*z + 32*a^3*b^5*c*g*h*z - 32*a^3*b^5*c*f*i*z - 32*a^3*b^5*c*e*j*z - 32*a^2*b^6*c*e*f*z + 32*a^2*b^6*c*d*g*z - 16*a^5*b^3*h^2*j*z + 16*a^5*b^3*h*i^2*z + 48*a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*z + 16*a^4*b^4*g^2*h*z - 16*a^4*b^4*f*h^2*z - 16*a^3*b^5*d^2*j*z + 16*a^4*b^4*d*i^2*z + 16*a^3*b^5*e^2*h*z + 16*a^3*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z - 16*a^2*b^6*d^2*f*z + 16*a^2*b^6*d*e^2*z + 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^3*z + 16*a^3*b^5*f^3*z - 8*a^5*b^2*f*g*i*j + 8*a^5*b^2*e*h*i*j + 8*a^4*b^3*e*f*h*i - 8*a^4*b^3*e*f*g*j - 8*a^4*b^3*d*g*h*i - 8*a^4*b^3*d*f*h*j + 8*a^4*b^3*d*e*i*j + 8*a^4*b^3*c*g*h*j - 8*a^4*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h + 8*a^3*b^4*d*e*f*i + 8*a^3*b^4*c*f*g*h + 8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f*j - 8*a^3*b^4*c*d*h*i + 8*a^3*b^4*c*d*g*j + 8*a^2*b^5*c*d*f*g - 8*a^2*b^5*c*d*e*h + 4*a^5*b^2*g^2*h*j - 4*a^5*b^2*g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5*b^2*f*h*i^2 + 4*a^5*b^2*d*i^2*j + 4*a^4*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 - 4*a^5*b^2*d*h*j^2 - 4*a^5*b^2*c*i*j^2 - 4*a^4*b^3*f^2*g*i + 4*a^4*b^3*f*g^2*h + 4*a^4*b^3*e*g^2*i + 4*a^4*b^3*d*g^2*j + 4*a^3*b^4*c^2*h*j - 4*a^4*b^3*e*g*h^2 - 4*a^4*b^3*c*h^2*i - 4*a^3*b^4*d^2*g*i - 4*a^3*b^4*d^2*f*j + 4*a^4*b^3*d*f*i^2 + 4*a^4*b^3*c*g*i^2 + 4*a^3*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j - 4*a^4*b^3*c*e*j^2 - 4*a^3*b^4*e*f^2*g - 4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2*i + 4*a^3*b^4*d*f*g^2 + 4*a^2*b^5*c^2*f*h + 4*a^2*b^5*c^2*e*i + 4*a^2*b^5*c^2*d*j - 4*a^3*b^4*c*e*h^2 - 4*a^2*b^5*d^2*e*g - 4*a^2*b^5*c*d^2*i + 4*a^2*b^5*d*e^2*f + 4*a^2*b^5*c*e^2*g - 4*a^2*b^5*c*e*f^2 + 4*a^6*b*h*i^2*j - 4*a^6*b*g*i*j^2 + 4*a*b^6*c^2*d*f - 4*a*b^6*c*d^2*e + 4*a^6*b*f*j^3 - 4*a*b^6*c^3*g + 6*a^5*b^2*f^2*j^2 + 2*a^5*b^2*g^2*i^2 - 6*a^4*b^3*e^2*i^2 - 2*a^4*b^3*f^2*h^2 - 2*a^4*b^3*d^2*j^2 + 6*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 2*a^3*b^4*c^2*i^2 - 6*a^2*b^5*c^2*g^2 - 2*a^2*b^5*d^2*f^2 - 2*a^6*b*h^2*j^2 + 4*a^4*b^3*f^3*j - 4*a^5*b^2*e*i^3 - 4*a^3*b^4*e^3*i + 4*a^4*b^3*d*h^3 + 4*a^2*b^5*d^3*h - 4*a^3*b^4*c*g^3 + 2*a*b^6*c^2*e^2 + a^5*b^2*h^4 + a^3*b^4*f^4 + a*b^6*d^4 + a^7*j^4 - a^4*b^3*g^4 - a^2*b^5*e^4 - a^6*b*i^4 - b^7*c^4, z, m), m, 1, 4) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - (j*x^4)/(4*b) - (g*x)/b

3.189 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 337

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx &= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{(bd-ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}\\&+\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}\\&-\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}\\&+\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{f\log\left(a+bx^4\right)}{4b}\end{aligned}$$

```
[Out] g*x/b+1/2*h*x^2/b+1/4*f*ln(b*x^4+a)/b+1/2*(-a*h+b*d)*arctan(x^2*b^(1/2)/a^(
1/2))/b^(3/2)/a^(1/2)-1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2)
)*(b*c-a*g-e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4
)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(b*c-a*g-e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/4
)*2^(1/2)+1/4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(b*c-a*g+e*a^(1/2)*b^(1/
2))/a^(3/4)/b^(5/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(b*c-a*
g+e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {1899, 1901, 1182, 1176, 631, 210, 1179, 642, 1833, 1824, 649, 211, 266}

$$\begin{aligned}&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}dx\\&=-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}\\&+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}\\&-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}\\&+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}\\&+\frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(bd-ah)}{2\sqrt{a}b^{3/2}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}+\frac{hx^2}{2b}\end{aligned}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sq
rt[a]*b^(3/2)) - ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a
*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) -
((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4
)*b^(5/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{c+ex^2+gx^4}{a+bx^4}+\frac{x(d+fx^2+hx^4)}{a+bx^4}\right)\,dx\\
&= \int \frac{c+ex^2+gx^4}{a+bx^4}\,dx+\int \frac{x(d+fx^2+hx^4)}{a+bx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{a+bx^2}\,dx,x,x^2\right)+\int \left(\frac{g}{b}+\frac{bc-ag+bex^2}{b\,(a+bx^4)}\right)\,dx\\
&= \frac{gx}{b}+\frac{1}{2}\text{Subst}\left(\int \left(\frac{h}{b}+\frac{bd-ah+bfx}{b\,(a+bx^2)}\right)\,dx,x,x^2\right)+\frac{\int \frac{bc-ag+bex^2}{a+bx^4}\,dx}{b}\\
&= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{\text{Subst}\left(\int \frac{bd-ah+bfx}{a+bx^2}\,dx,x,x^2\right)}{2b}\\
&\quad+\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2\sqrt{a}b^{3/2}}+\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2\sqrt{a}b^{3/2}}
\end{aligned}
$$

$$=\frac{gx}{b}+\frac{hx^2}{2b}+\frac{1}{2}f\text{Subst}\left(\int\frac{x}{a+bx^2}\,dx,x,x^2\right)-\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{5/4}}$$

$$-\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}b^{3/2}}$$

$$+\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}b^{3/2}}+\frac{(bd-ah)\text{Subst}\left(\int\frac{1}{a+bx^2}\,dx,x,x^2\right)}{2b}$$

$$=\frac{gx}{b}+\frac{hx^2}{2b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$

$$-\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}$$

$$+\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{f\log\left(a+bx^4\right)}{4b}$$

$$+\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

$$-\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

$$=\frac{gx}{b}+\frac{hx^2}{2b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

$$+\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

$$-\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}$$

$$+\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{f\log\left(a+bx^4\right)}{4b}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx = \frac{-2\left(\sqrt{2}b^{5/4}c+2\sqrt[4]{a}bd+\sqrt{2}\sqrt{a}b^{3/4}e-\sqrt{2}a\sqrt[4]{b}g-2a^{5/4}h\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}b^{5/4}c-2\sqrt[4]{a}bd\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*b^(5/4)*c + 2*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e - Sqrt[2
]*a*b^(1/4)*g - 2*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(S
qrt[2]*b^(5/4)*c - 2*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e - Sqrt[2]*a*b^
(1/4)*g + 2*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + b^(1/4)*(S
qrt[2]*(-(b*c) + Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a^(3/4)*b^(1/4)*(2*x*(2*g +
 h*x) + f*Log[a + b*x^4])))/(8*a^(3/4)*b^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.65 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.22

method	result
risch	$\frac{h\,x^{2}}{2b}+\frac{gx}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}b+a\right)}\frac{\left(bc-ag+\left(-ah+bd\right)\_R+\_R^{2}be+\_R^{3}bf\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b^{2}}$
default	$\frac{\frac{1}{2}h\,x^{2}+gx}{b}+\frac{\frac{\left(-ag+bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{\left(-ah+bd\right)\arctan\left(x^{2}\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+}{b}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*h*x^2/b+g*x/b+1/4/b^2*sum((b*c-a*g+(-a*h+b*d)*_R+_R^2*b*e+_R^3*b*f)/_R^
3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx = \frac{hx^2+2\,gx}{2\,b} + \frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f+b^2c-\sqrt{a}b^{\frac{3}{2}}e-abg\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f-b^2c+\sqrt{a}b^{\frac{3}{2}}e+abg\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{2\left(\sqrt{\cdots}\right.}{\cdots}}{\cdots}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
[Out] 1/2*(h*x^2 + 2*g*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f + b^2*c - s
qrt(a)*b^(3/2)*e - a*b*g)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqr
t(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f - b^2*c + sqrt
(a)*b^(3/2)*e + a*b*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a
))/(a^(3/4)*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/
4)*e - sqrt(2)*a^(5/4)*b^(5/4)*g - 2*sqrt(a)*b^2*d + 2*a^(3/2)*b*h)*arctan(
1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sq
rt(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a^(5/4)*b^(5/4)*g + 2*sqrt(a)*b^2*d - 2*a
^(3/2)*b*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt
(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.11

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx = \frac{f\log\left(|bx^4+a|\right)}{4\,b}+\frac{bhx^2+2\,bgx}{2\,b^2} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d+\sqrt{2}\sqrt{ab}abh-(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d+\sqrt{2}\sqrt{ab}abh-(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*f*log(abs(b*x^4 + a))/b + 1/2*(b*h*x^2 + 2*b*g*x)/b^2 - 1/4*sqrt(2)*(sq
rt(2)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h - (a*b^3)^(1/4)*b^2*c + (a*
b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)
^(1/4))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + sqrt(
2)*sqrt(a*b)*a*b*h - (a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3
/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3)
+ 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)
*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3
)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*
(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.94 (sec) , antiderivative size = 2469, normalized size of antiderivative = 7.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x)
```

```
[Out] symsum(log(root(256*a^3*b^6*z^4 - 256*a^3*b^5*f*z^3 - 64*a^3*b^4*e*g*z^2 -
64*a^3*b^4*d*h*z^2 + 64*a^2*b^5*c*e*z^2 + 32*a^4*b^3*h^2*z^2 + 96*a^3*b^4*f
```

^2*z^2 + 32*a^2*b^5*d^2*z^2 + 32*a^3*b^3*e*f*g*z + 32*a^3*b^3*d*f*h*z - 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*a^4*b^2*g^2*h*z - 16*a^4*b^2*f*h^2*z - 16*a^3*b^3*e^2*h*z - 16*a^3*b^3*d*g^2*z + 16*a^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z - 16*a*b^5*c^2*d*z - 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h - 8*a^2*b^3*c*d*f*g + 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g - 4*a^3*b^2*d*f^2*h + 4*a^3*b^2*d*f*g^2 - 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e*h^2 + 4*a^2*b^3*d^2*e*g - 4*a^2*b^3*d*e^2*f - 4*a^2*b^3*c*e^2*g + 4*a^2*b^3*c*e*f^2 - 4*a^4*b*f*g^2*h + 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c*d^2*e - 4*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 + 6*a^2*b^3*c^2*g^2 + 2*a^2*b^3*d^2*f^2 + 2*a^4*b*f^2*h^2 - 4*a^2*b^3*d^3*h - 4*a^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a^2*b^3*e^4 + a^4*b*g^4 + a*b^4*d^4 + a^5*h^4 + b^5*c^4, z, k)*((8*a*b^3*c*f - 8*a*b^3*d*e + 8*a^2*b^2*e*h - 8*a^2*b^2*f*g)/b + root(256*a^3*b^6*z^4 - 256*a^3*b^5*f*z^3 - 64*a^3*b^4*e*g*z^2 - 64*a^3*b^4*d*h*z^2 + 64*a^2*b^5*c*e*z^2 + 32*a^4*b^3*h^2*z^2 + 96*a^3*b^4*f^2*z^2 + 32*a^2*b^5*d^2*z^2 + 32*a^3*b^3*e*f*g*z + 32*a^3*b^3*d*f*h*z - 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*a^4*b^2*g^2*h*z - 16*a^4*b^2*f*h^2*z - 16*a^3*b^3*e^2*h*z - 16*a^3*b^3*d*g^2*z + 16*a^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z - 16*a*b^5*c^2*d*z - 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h - 8*a^2*b^3*c*d*f*g + 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g - 4*a^3*b^2*d*f^2*h + 4*a^3*b^2*d*f*g^2 - 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e*h^2 + 4*a^2*b^3*d^2*e*g - 4*a^2*b^3*d*e^2*f - 4*a^2*b^3*c*e^2*g + 4*a^2*b^3*c*e*f^2 - 4*a^4*b*f*g^2*h + 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c*d^2*e - 4*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 + 6*a^2*b^3*c^2*g^2 + 2*a^2*b^3*d^2*f^2 + 2*a^4*b*f^2*h^2 - 4*a^2*b^3*d^3*h - 4*a^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a^2*b^3*e^4 + a^4*b*g^4 + a*b^4*d^4 + a^5*h^4 + b^5*c^4, z, k)*((16*a^2*b^3*g - 16*a*b^4*c)/b - (x*(16*a^2*b^3*h - 16*a*b^4*d))/b) - (x*(4*b^4*c^2 - 4*a*b^3*e^2 + 4*a^2*b^2*g^2 - 8*a*b^3*c*g + 8*a*b^3*d*f - 8*a^2*b^2*f*h))/b) - (a*b^2*e^3 - b^3*c*d^2 + b^3*c^2*e + a^3*g*h^2 + a*b^2*c*f^2 + a*b^2*d^2*g - a^2*b*c*h^2 + a^2*b*e*g^2 - a^2*b*f^2*g + 2*a*b^2*c*d*h - 2*a*b^2*c*e*g - 2*a*b^2*d*e*f - 2*a^2*b*d*g*h + 2*a^2*b*e*f*h)/b + (x*(b^3*d^3 - a^3*h^3 + b^3*c^2*f - 2*b^3*c*d*e + a*b^2*d*f^2 - a*b^2*e^2*f - 3*a*b^2*d^2*h + 3*a^2*b*d*h^2 + a^2*b*f*g^2 - a^2*b*f^2*h + 2*a*b^2*c*e*h - 2*a*b^2*c*f*g + 2*a*b^2*d*e*g - 2*a^2*b*e*g*h))/b)*root(256*a^3*b^6*z^4 - 256*a^3*b^5*f*z^3 - 64*a^3*b^4*e*g*z^2 - 64*a^3*b^4*d*h*z^2 + 64*a^2*b^5*c*e*z^2 + 32*a^4*b^3*h^2*z^2 + 96*a^3*b^4*f^2*z^2 + 32*a^2*b^5*d^2*z^2 + 32*a^3*b^3*e*f*g*z + 32*a^3*b^3*d*f*h*z - 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*a^4*b^2*g^2*h*z - 16*a^4*b^2*f*h^2*z - 16*a^3*b^3*e^2*h*z - 16*a^3*b^3*d*g^2*z + 16*a^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z - 16*a*b^5*c^2*d*z - 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h - 8*a^2*b^3*c*d*f*g + 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g - 4*a^3*b^2*d*f^2*h + 4*a^3*b^2*d*f*g^2 - 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e*h^2 + 4*a^2*b^3*d^2*e*g - 4*a^2*b^3*d*e^2*f - 4*a^2*b^3*c*e^2*g + 4*a^2*b^3*c*e*

```
f^2 - 4*a^4*b*f*g^2*h + 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c*d^2*e
 - 4*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 +
6*a^2*b^3*c^2*g^2 + 2*a^2*b^3*d^2*f^2 + 2*a^4*b*f^2*h^2 - 4*a^2*b^3*d^3*h -
 4*a^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a^2*b^3*e^4 + a^4*b*g^4
+ a*b^4*d^4 + a^5*h^4 + b^5*c^4, z, k), k, 1, 4) + (h*x^2)/(2*b) + (g*x)/b
```

3.190 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4} dx$

Optimal result

Integrand size = 40, antiderivative size = 384

$$\begin{aligned}&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx\\
&=\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{(bd-ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}\\
&\quad-\frac{\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{7/4}}\\
&\quad+\frac{\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{7/4}}\\
&\quad-\frac{\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{7/4}}\\
&\quad+\frac{\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{f\log\left(a+bx^4\right)}{4b}\end{aligned}$$

```
[Out] g*x/b+1/2*h*x^2/b+1/3*i*x^3/b+1/4*f*ln(b*x^4+a)/b+1/2*(-a*h+b*d)*arctan(x^2
*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)-1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*b^(1/2))*(-(-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b^(1/2))/a^(3/4)/b^(7/4)*2^(1
/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(-a*i+b*e)*a^(1
/2)+(-a*g+b*c)*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)+1/4*arctan(-1+b^(1/4)*x*2^(
1/2)/a^(1/4))*((-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/
2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*((-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b
^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {1899, 1833, 1824, 649, 211, 266, 1901, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx \\
&= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}} \\
&+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}} \\
&-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}} \\
&+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}} \\
&+\frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(bd-ah)}{2\sqrt{a}b^{3/2}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}
\end{aligned}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)
/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i
))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) + (
(Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*
e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2
]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4))
+ (f*Log[a + b*x^4])/(4*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x(d+fx^2+hx^4)}{a+bx^4}+\frac{c+ex^2+gx^4+ix^6}{a+bx^4}\right)dx\\
&= \int \frac{x(d+fx^2+hx^4)}{a+bx^4}\,dx+\int \frac{c+ex^2+gx^4+ix^6}{a+bx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{a+bx^2}\,dx,x,x^2\right)+\int \left(\frac{g}{b}+\frac{ix^2}{b}+\frac{bc-ag+(be-ai)x^2}{b\,(a+bx^4)}\right)dx\\
&= \frac{gx}{b}+\frac{ix^3}{3b}+\frac{1}{2}\text{Subst}\left(\int \left(\frac{h}{b}+\frac{bd-ah+bfx}{b\,(a+bx^2)}\right)dx,x,x^2\right)+\frac{\int \frac{bc-ag+(be-ai)x^2}{a+bx^4}\,dx}{b}\\
&= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{\text{Subst}\left(\int \frac{bd-ah+bfx}{a+bx^2}\,dx,x,x^2\right)}{2b}\\
&\quad -\frac{\left(be-\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b^2}+\frac{\left(be+\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b^2}
\end{aligned}$$

$$=\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{1}{2}f\text{Subst}\left(\int\frac{x}{a+bx^2}\,dx,x,x^2\right)+\frac{(bd-ah)\text{Subst}\left(\int\frac{1}{a+bx^2}\,dx,x,x^2\right)}{2b}$$
$$+\frac{\left(be-\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{\left(be-\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}$$
$$+\frac{\left(be+\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b^2}+\frac{\left(be+\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b^2}$$

$$=\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$
$$+\frac{\left(be-\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}$$
$$-\frac{\left(be-\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{f\log\left(a+bx^4\right)}{4b}$$
$$+\frac{\left(be+\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}$$
$$-\frac{\left(be+\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

$$=\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$
$$-\frac{\left(be+\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}$$
$$+\frac{\left(be+\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}$$
$$+\frac{\left(be-\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}$$
$$-\frac{\left(be-\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{f\log\left(a+bx^4\right)}{4b}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.11

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx$$
$$= \frac{24b^{3/4}gx+12b^{3/4}hx^2+8b^{3/4}ix^3+\frac{6\left(-\sqrt{2}b^{3/2}c-2\sqrt[4]{a}b^{5/4}d-\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+2a^{5/4}\sqrt[4]{b}h+\sqrt{2}a^{3/2}i\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}}}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x]
```

```
[Out] (24*b^(3/4)*g*x + 12*b^(3/4)*h*x^2 + 8*b^(3/4)*i*x^3 + (6*(-(Sqrt[2]*b^(3/2
)*c) - 2*a^(1/4)*b^(5/4)*d - Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 2*
a^(5/4)*b^(1/4)*h + Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/a^(3/4) + (6*(Sqrt[2]*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a
]*b*e - Sqrt[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - Sqrt[2]*a^(3/2)*i)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (3*Sqrt[2]*(b^(3/2)*c - Sqrt
[a]*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2])/a^(3/4) + (3*Sqrt[2]*(b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g
 + a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/
4) + 6*b^(3/4)*f*Log[a + b*x^4])/(24*b^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.60 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.23

method	result
risch	$\frac{i\,x^3}{3b}+\frac{h\,x^2}{2b}+\frac{gx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 b+a)}\frac{\left(bc-ag+(-ah+bd)\_R+(-ai+be)\_R^2+\_R^3 bf\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$\frac{\frac{1}{3}i\,x^3+\frac{1}{2}h\,x^2+gx}{b}+\frac{\frac{(-ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{(-ah+bd)\arctan\left(x^2\right.}{2\sqrt{ab}}}{b}$

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/3*i*x^3/b+1/2*h*x^2/b+g*x/b+1/4/b^2*sum((b*c-a*g+(-a*h+b*d)*_R+(-a*i+b*e)
*_R^2+_R^3*b*f)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx = \frac{2\,ix^3+3\,hx^2+6\,gx}{6\,b} + \frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f+b^2c-\sqrt{a}b^{\frac{3}{2}}e-abg+a^{\frac{3}{2}}\sqrt{b}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f-b^2c+\sqrt{a}b^{\frac{3}{2}}e+abg-a^{\frac{3}{2}}\sqrt{b}i\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right.}{a^{\frac{3}{4}}b^{\frac{5}{4}}}}{}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="max
ima")
```

```
[Out] 1/6*(2*i*x^3 + 3*h*x^2 + 6*g*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f
 + b^2*c - sqrt(a)*b^(3/2)*e - a*b*g + a^(3/2)*sqrt(b)*i)*log(sqrt(b)*x^2 +
 sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*
a^(3/4)*b^(5/4)*f - b^2*c + sqrt(a)*b^(3/2)*e + a*b*g - a^(3/2)*sqrt(b)*i)*
log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) +
2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a^(5/4)*
b^(5/4)*g - sqrt(2)*a^(7/4)*b^(3/4)*i - 2*sqrt(a)*b^2*d + 2*a^(3/2)*b*h)*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
```

```
b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c
 + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a^(5/4)*b^(5/4)*g - sqrt(2)*a^(7/4)*
b^(3/4)*i + 2*sqrt(a)*b^2*d - 2*a^(3/2)*b*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*
x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*s
qrt(b))*b^(5/4)))/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.15

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx = \frac{f\log\left(|bx^4+a|\right)}{4\,b} + \frac{2\,b^2ix^3+3\,b^2hx^2+6\,b^2gx}{6\,b^3}$$
$$- \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^3d+\sqrt{2}\sqrt{ab}ab^2h-(ab^3)^{\frac{1}{4}}b^3c+(ab^3)^{\frac{1}{4}}ab^2g-(ab^3)^{\frac{3}{4}}be+(ab^3)^{\frac{3}{4}}ai\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right.\right.}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{4\,ab^4}$$
$$- \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^3d+\sqrt{2}\sqrt{ab}ab^2h-(ab^3)^{\frac{1}{4}}b^3c+(ab^3)^{\frac{1}{4}}ab^2g-(ab^3)^{\frac{3}{4}}be+(ab^3)^{\frac{3}{4}}ai\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right.\right.}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{4\,ab^4}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c-(ab^3)^{\frac{1}{4}}ab^2g-(ab^3)^{\frac{3}{4}}be+(ab^3)^{\frac{3}{4}}ai\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^4}$$
$$- \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c-(ab^3)^{\frac{1}{4}}ab^2g-(ab^3)^{\frac{3}{4}}be+(ab^3)^{\frac{3}{4}}ai\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^4}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="gia
c")
```

```
[Out] 1/4*f*log(abs(b*x^4 + a))/b + 1/6*(2*b^2*i*x^3 + 3*b^2*h*x^2 + 6*b^2*g*x)/b
^3 - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^3*d + sqrt(2)*sqrt(a*b)*a*b^2*h - (a*
b^3)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b^2*g - (a*b^3)^(3/4)*b*e + (a*b^3)^(3/4
)*a*i)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4)
- 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^3*d + sqrt(2)*sqrt(a*b)*a*b^2*h - (a*b^3
)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b^2*g - (a*b^3)^(3/4)*b*e + (a*b^3)^(3/4)*a
*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1
/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c - (a*b^3)^(1/4)*a*b^2*g - (a*b^3)^(3/4)*b*e
 + (a*b^3)^(3/4)*a*i)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)
- 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c - (a*b^3)^(1/4)*a*b^2*g - (a*b^3)^(3/4)*
b*e + (a*b^3)^(3/4)*a*i)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^
4)
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 3798, normalized size of antiderivative = 9.89

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{a + bx^4} dx = \text{Too large to display}$$

[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x)

[Out] symsum(log((a^4*i^3 - a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^2*b^2*c*h^2 - a^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i - a*b^3*c*f^2 - a*b^3*d^2*g + a*b^3*c^2*i - 3*a^3*b*e*i^2 - a^3*b*g*h^2 + a^3*b*g^2*i - 2*a^2*b^2*c*g*i - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h - 2*a*b^3*c*d*h + 2*a*b^3*c*e*g + 2*a*b^3*d*e*f + 2*a^3*b*f*h*i)/b^2 + root(256*a^3*b^7*z^4 - 256*a^3*b^6*f*z^3 + 64*a^4*b^4*g*i*z^2 - 64*a^3*b^5*e*g*z^2 - 64*a^3*b^5*d*h*z^2 - 64*a^3*b^5*c*i*z^2 + 64*a^2*b^6*c*e*z^2 + 32*a^4*b^4*h^2*z^2 + 96*a^3*b^5*f^2*z^2 + 32*a^2*b^6*d^2*z^2 - 32*a^4*b^3*f*g*i*z + 32*a^4*b^3*e*h*i*z + 32*a^3*b^4*e*f*g*z + 32*a^3*b^4*d*f*h*z - 32*a^3*b^4*d*e*i*z - 32*a^3*b^4*c*g*h*z + 32*a^3*b^4*c*f*i*z - 32*a^2*b^5*c*e*f*z + 32*a^2*b^5*c*d*g*z - 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 16*a^4*b^3*f*h^2*z + 16*a^4*b^3*d*i^2*z - 16*a^3*b^4*e^2*h*z - 16*a^3*b^4*d*g^2*z + 16*a^2*b^5*c^2*h*z - 16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z - 16*a*b^6*c^2*d*z - 16*a^3*b^4*f^3*z - 8*a^4*b^2*e*f*h*i + 8*a^4*b^2*d*g*h*i - 8*a^3*b^3*d*e*g*h + 8*a^3*b^3*d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*g*i - 8*a^3*b^3*c*d*h*i - 8*a^2*b^4*c*d*f*g + 8*a^2*b^4*c*d*e*h + 4*a^4*b^2*f^2*g*i - 4*a^4*b^2*f*g^2*h - 4*a^4*b^2*e*g^2*i + 4*a^4*b^2*e*g*h^2 + 4*a^4*b^2*c*h^2*i - 4*a^3*b^3*d^2*g*i - 4*a^4*b^2*d*f*i^2 - 4*a^4*b^2*c*g*i^2 + 4*a^3*b^3*e^2*f*h - 4*a^3*b^3*e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^2*i + 4*a^3*b^3*d*f*g^2 - 4*a^2*b^4*c^2*f*h - 4*a^2*b^4*c^2*e*i - 4*a^3*b^3*c*e*h^2 + 4*a^2*b^4*d^2*e*g + 4*a^2*b^4*c*d^2*i - 4*a^2*b^4*d*e^2*f - 4*a^2*b^4*c*e^2*g + 4*a^2*b^4*c*e*f^2 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*h*i^2 + 4*a*b^5*c^2*d*f - 4*a*b^5*c*d^2*e - 4*a^5*b*e*i^3 - 4*a*b^5*c^3*g + 6*a^4*b^2*e^2*i^2 + 2*a^4*b^2*f^2*h^2 + 6*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 2*a^3*b^3*c^2*i^2 + 6*a^2*b^4*c^2*g^2 + 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g^2*i^2 - 4*a^3*b^3*e^3*i - 4*a^4*b^2*d*h^3 - 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*g^3 + 2*a*b^5*c^2*e^2 + a^4*b^2*g^4 + a^3*b^3*f^4 + a^2*b^4*e^4 + a^5*b*h^4 + a*b^5*d^4 + a^6*i^4 + b^6*c^4, z, l)*((8*a*b^4*c*f - 8*a*b^4*d*e + 8*a^2*b^3*d*i + 8*a^2*b^3*e*h - 8*a^2*b^3*f*g - 8*a^3*b^2*h*i)/b^2 + root(256*a^3*b^7*z^4 - 256*a^3*b^6*f*z^3 + 64*a^4*b^4*g*i*z^2 - 64*a^3*b^5*e*g*z^2 - 64*a^3*b^5*d*h*z^2 - 64*a^3*b^5*c*i*z^2 + 64*a^2*b^6*c*e*z^2 + 32*a^4*b^4*h^2*z^2 + 96*a^3*b^5*f^2*z^2 + 32*a^2*b^6*d^2*z^2 - 32*a^4*b^3*f*g*i*z + 32*a^4*b^3*e*h*i*z + 32*a^3*b^4*e*f*g*z + 32*a^3*b^4*d*f*h*z - 32*a^3*b^4*d*e*i*z - 32*a^3*b^4*c*g*h*z + 32*a^3*b^4*c*f*i*z - 32*a^2*b^5*c*e*f*z + 32*a^2*b^5*c*d*g*z - 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 16*a^4*b^3*f*h^2*z + 16*a^4*b^3*d*i^2*z - 16*a^3*b^4*e^2*h*z - 16*a^3*b^4*d*g^2*z + 16*a^2*b^5*c^2*h*z - 16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d *e^2*z - 16*a*b^6*c^2*d*z - 16*a^3*b^4*f^3*z - 8*a^4*b^2*e*f*h*i + 8*a^4*b^2*d*g*h*i - 8*a^3*b^3*d*e*g*h + 8*a^3*b^3*d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*g*i - 8*a^3*b^3*c*d*h*i - 8*a^2*b^4*c*d*f*g + 8*a^2*b^4*c*d*e*h + 4*a^4*b^2*f^2*g*i - 4*a^4*b^2*f*g^2*h - 4*a^4*b^2*e*g^2*i + 4*a^4*b^2*e*g*h^2 + 4*a^4*b^2*c*h^2*i - 4*a^3*b^3*d^2*g*i - 4*a^4*b^2*d*f*i^2 - 4*a^4*b^2*c*g*i^2 + 4*a^3*b^3*e^2*f*h - 4*a^3*b^3*e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^2*i + 4*a^3*b^3*d*f*g^2 - 4*a^2*b^4*c^2*f*h - 4*a^2*b^4*c^2*e*i - 4*a^3*b^3*c*e*h^2 + 4*a^2*b^4*d^2*e*g + 4*a^2*b^4*c*d^2*i - 4*a^2*b^4*d*e^2*f - 4*a^2*b^4*c*e^2*g + 4*a^2*b^4*c*e*f^2 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*h*i^2 + 4*a*b^5*c^2*d*f - 4*a*b^5*c*d^2*e - 4*a^5*b*e*i^3 - 4*a*b^5*c^3*g + 6*a^4*b^2*e^2*i^2 + 2*a^4*b^2*f^2*h^2 + 6*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 2*a^3*b^3*c^2*i^2 + 6*a^2*b^4*c^2*g^2 + 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g^2*i^2 - 4*a^3*b^3*e^3*i - 4*a^4*b^2*d*h^3 - 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*g^3 + 2*a*b^5*c^2*e^2 + a^4*b^2*g^4 + a^3*b^3*f^4 + a^2*b^4*e^4 + a^5*b*h^4 + a*b^5*d^4 + a^6*i^4 + b^6*c^4, z, l)*((16*a^2*b^4*g - 16*a*b^5*c)/b^2 - (x*(16*a^2*b^3*h - 16*a*b^4*d))/b) - (x*(4*b^4*c^2 - 4*a*b^3*e^2 - 4*a^3*b*i^2 + 4*a^2*b^2*g^2 - 8*a*b^3*c*g + 8*a*b^3*d*f + 8*a^2*b^2*e*i - 8*a^2*b^2*f*h))/b) + (x*(b^3*d^3 - a^3*h^3 + b^3*c^2*f - a^3*f*i^2 - 2*b^3*c*d*e + 2*a^3*g*h*i + a*b^2*d*f^2 - a*b^2*e^2*f - 3*a*b^2*d^2*h + 3*a^2*b*d*h^2 + a^2*b*f*g^2 - a^2*b*f^2*h + 2*a*b^2*c*d*i + 2*a*b^2*c*e*h - 2*a*b^2*c*f*g + 2*a*b^2*d*e*g - 2*a^2*b*c*h*i - 2*a^2*b*d*g*i + 2*a^2*b*e*f*i - 2*a^2*b*e*g*h))/b)*root(256*a^3*b^7*z^4 - 256*a^3*b^6*f*z^3 + 64*a^4*b^4*g*i*z^2 - 64*a^3*b^5*e*g*z^2 - 64*a^3*b^5*d*h*z^2 - 64*a^3*b^5*c*i*z^2 + 64*a^2*b^6*c*e*z^2 + 32*a^4*b^4*h^2*z^2 + 96*a^3*b^5*f^2*z^2 + 32*a^2*b^6*d^2*z^2 - 32*a^4*b^3*f*g*i*z + 32*a^4*b^3*e*h*i*z + 32*a^3*b^4*e*f*g*z + 32*a^3*b^4*d*f*h*z - 32*a^3*b^4*d*e*i*z - 32*a^3*b^4*c*g*h*z + 32*a^3*b^4*c*f*i*z - 32*a^2*b^5*c*e*f*z + 32*a^2*b^5*c*d*g*z - 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 16*a^4*b^3*f*h^2*z + 16*a^4*b^3*d*i^2*z - 16*a^3*b^4*e^2*h*z - 16*a^3*b^4*d*g^2*z + 16*a^2*b^5*c^2*h*z - 16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z - 16*a*b^6*c^2*d*z - 16*a^3*b^4*f^3*z - 8*a^4*b^2*e*f*h*i + 8*a^4*b^2*d*g*h*i - 8*a^3*b^3*d*e*g*h + 8*a^3*b^3*d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*g*i - 8*a^3*b^3*c*d*h*i - 8*a^2*b^4*c*d*f*g + 8*a^2*b^4*c*d*e*h + 4*a^4*b^2*f^2*g*i - 4*a^4*b^2*f*g^2*h - 4*a^4*b^2*e*g^2*i + 4*a^4*b^2*e*g*h^2 + 4*a^4*b^2*c*h^2*i - 4*a^3*b^3*d^2*g*i - 4*a^4*b^2*d*f*i^2 - 4*a^4*b^2*c*g*i^2 + 4*a^3*b^3*e^2*f*h - 4*a^3*b^3*e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^2*i + 4*a^3*b^3*d*f*g^2 - 4*a^2*b^4*c^2*f*h - 4*a^2*b^4*c^2*e*i - 4*a^3*b^3*c*e*h^2 + 4*a^2*b^4*d^2*e*g + 4*a^2*b^4*c*d^2*i - 4*a^2*b^4*d*e^2*f - 4*a^2*b^4*c*e^2*g + 4*a^2*b^4*c*e*f^2 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*h*i^2 + 4*a*b^5*c^2*d*f - 4*a*b^5*c*d^2*e - 4*a^5*b*e*i^3 - 4*a*b^5*c^3*g + 6*a^4*b^2*e^2*i^2 + 2*a^4*b^2*f^2*h^2 + 6*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 2*a^3*b^3*c^2*i^2 + 6*a^2*b^4*c^2*g^2 + 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g^2*i^2 - 4*a^3*b^3*e^3*i - 4*a^4*b^2*d*h^3 - 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*g^3 + 2*a*b^5*c^2*e^2 + a^4*b^2*g^4 + a^3*b^3*f^4 + a^2*b^4*e^4 + a^5*b*h^4 + a*b^5*d^4 + a^6*i^4 + b^6*c^4, z, l), l, 1, 4) + (h*x^2)/(2*b) + (i*x^3)/(3*b) + (g*x)/b

3.191 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4} dx$

Optimal result

Integrand size = 45, antiderivative size = 402

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4} dx \\
&= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{jx^4}{4b}+\frac{(bd-ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}} \\
&-\frac{\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{7/4}} \\
&+\frac{\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{7/4}} \\
&-\frac{\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{7/4}} \\
&+\frac{\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{7/4}} \\
&+\frac{(bf-aj)\log\left(a+bx^4\right)}{4b^2}
\end{aligned}$$

```
[Out] g*x/b+1/2*h*x^2/b+1/3*i*x^3/b+1/4*j*x^4/b+1/4*(-a*j+b*f)*ln(b*x^4+a)/b^2+1/
2*(-a*h+b*d)*arctan(x^2*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)-1/8*ln(-a^(1/4)*b^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b^(1/2
))/a^(3/4)/b^(7/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(
1/2))*(-(-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)+1/4*
arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*((-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b^(1/2)
)/a^(3/4)/b^(7/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*((-a*i+b*
e)*a^(1/2)+(-a*g+b*c)*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {1899, 1901, 1182, 1176, 631, 210, 1179, 642, 1833, 1824, 649, 211, 266}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx$$
$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}$$
$$+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}$$
$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}$$
$$+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}$$
$$+\frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(bd-ah)}{2\sqrt{a}b^{3/2}}+\frac{(bf-aj)\log\left(a+bx^4\right)}{4b^2}+\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{jx^4}{4b}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4),x
]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + (j*x^4)/(4*b) + ((b*d - a*h)*ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) + Sq
rt[a]*(b*e - a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*
g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e -
 a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(3/4)*b^(7/4)) + ((b*f - a*j)*Log[a + b*x^4])/(4*b^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c+ex^2+gx^4+ix^6}{a+bx^4}+\frac{x(d+fx^2+hx^4+jx^6)}{a+bx^4}\right)dx\\
&= \int \frac{c+ex^2+gx^4+ix^6}{a+bx^4}\,dx+\int \frac{x(d+fx^2+hx^4+jx^6)}{a+bx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2+jx^3}{a+bx^2}\,dx,x,x^2\right)+\int \left(\frac{g}{b}+\frac{ix^2}{b}+\frac{bc-ag+(be-ai)x^2}{b\,(a+bx^4)}\right)dx\\
&= \frac{gx}{b}+\frac{ix^3}{3b}+\frac{1}{2}\text{Subst}\left(\int \left(\frac{h}{b}+\frac{jx}{b}+\frac{bd-ah+(bf-aj)x}{b\,(a+bx^2)}\right)dx,x,x^2\right)+\frac{\int \frac{bc-ag+(be-ai)x^2}{a+bx^4}\,dx}{b}\\
&= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{jx^4}{4b}+\frac{\text{Subst}\left(\int \frac{bd-ah+(bf-aj)x}{a+bx^2}\,dx,x,x^2\right)}{2b}\\
&\quad-\frac{\left(be-\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b^2}+\frac{\left(be+\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b^2}
\end{aligned}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{ix^3}{3b} + \frac{jx^4}{4b} + \frac{(bd-ah)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2b}$$
$$+ \frac{\left(be - \frac{\sqrt{b}(bc-ag)}{\sqrt{a}} - ai\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}$$
$$+ \frac{\left(be - \frac{\sqrt{b}(bc-ag)}{\sqrt{a}} - ai\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}$$
$$+ \frac{\left(be + \frac{\sqrt{b}(bc-ag)}{\sqrt{a}} - ai\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b^2}$$
$$+ \frac{\left(be + \frac{\sqrt{b}(bc-ag)}{\sqrt{a}} - ai\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b^2} + \frac{(bf-aj)\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, x^2\right)}{2b}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{ix^3}{3b} + \frac{jx^4}{4b} + \frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$
$$+ \frac{\left(be - \frac{\sqrt{b}(bc-ag)}{\sqrt{a}} - ai\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}$$
$$- \frac{\left(be - \frac{\sqrt{b}(bc-ag)}{\sqrt{a}} - ai\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}} + \frac{(bf-aj)\log\left(a+bx^4\right)}{4b^2}$$
$$+ \frac{\left(be + \frac{\sqrt{b}(bc-ag)}{\sqrt{a}} - ai\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}$$
$$- \frac{\left(be + \frac{\sqrt{b}(bc-ag)}{\sqrt{a}} - ai\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

$$=\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{jx^4}{4b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$
$$-\frac{\left(be+\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}$$
$$+\frac{\left(be+\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}$$
$$+\frac{\left(be-\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}$$
$$-\frac{\left(be-\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}-ai\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{(bf-aj)\log(a+bx^4)}{4b^2}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.11

$$\int\frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx$$
$$=\frac{24b^{3/4}gx+12b^{3/4}hx^2+8b^{3/4}ix^3+6b^{3/4}jx^4+\frac{6\left(-\sqrt{2}b^{3/2}c-2\sqrt[4]{a}b^{5/4}d-\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+2a^{5/4}\sqrt[4]{b}h+\sqrt{2}a^{3/2}i\right)\arctan\left(\right.}{a^{3/4}}}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4),x]
```

```
[Out] (24*b^(3/4)*g*x + 12*b^(3/4)*h*x^2 + 8*b^(3/4)*i*x^3 + 6*b^(3/4)*j*x^4 + (6
*(-(Sqrt[2]*b^(3/2)*c) - 2*a^(1/4)*b^(5/4)*d - Sqrt[2]*Sqrt[a]*b*e + Sqrt[2
]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (6*(Sqrt[2]*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*
d + Sqrt[2]*Sqrt[a]*b*e - Sqrt[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - Sqrt[
2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (3*Sqrt[2]
*(b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (3*Sqrt[2]*(b^(3/2)*c - Sqrt[a]
*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/a^(3/4) + (6*(b*f - a*j)*Log[a + b*x^4])/b^(1/4))/(24*b^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.26

method	result
risch	$\frac{j\,x^{4}}{4b}+\frac{i\,x^{3}}{3b}+\frac{h\,x^{2}}{2b}+\frac{gx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}\frac{\left(bc-ag+\left(-ah+bd\right)\_R+\left(-ai+be\right)\_R^{2}+\left(-aj+bf\right)\_R^{3}\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b^{2}}$
default	$\frac{\frac{1}{4}j\,x^{4}+\frac{1}{3}i\,x^{3}+\frac{1}{2}h\,x^{2}+gx}{b}+\frac{\frac{\left(-ag+bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{\left(-ah+bd\right)\arctan}{2\sqrt{ab}}}{}$

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/4*j*x^4/b+1/3*i*x^3/b+1/2*h*x^2/b+g*x/b+1/4/b^2*sum((b*c-a*g+(-a*h+b*d)*_
R+(-a*i+b*e)*_R^2+(-a*j+b*f)*_R^3)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx=\text{Timed out}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx=\text{Timed out}$$

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.07

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx = \frac{3\,jx^4+4\,ix^3+6\,hx^2+12\,gx}{12\,b} + \frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f-\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}j+b^2c-\sqrt{a}b^{\frac{3}{2}}e-abg+a^{\frac{3}{2}}\sqrt{b}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f-\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}j-b^2c+\sqrt{a}b^{\frac{3}{2}}e+abg-a^{\frac{3}{2}}\cdots}{a^{\frac{3}{4}}b^{\frac{5}{4}}}}{}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorith
m="maxima")
```

```
[Out] 1/12*(3*j*x^4 + 4*i*x^3 + 6*h*x^2 + 12*g*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/
4)*b^(5/4)*f - sqrt(2)*a^(7/4)*b^(1/4)*j + b^2*c - sqrt(a)*b^(3/2)*e - a*b*
g + a^(3/2)*sqrt(b)*i)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a
))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f - sqrt(2)*a^(7/4)
*b^(1/4)*j - b^2*c + sqrt(a)*b^(3/2)*e + a*b*g - a^(3/2)*sqrt(b)*i)*log(sqr
t(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 2*(sqrt
(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a^(5/4)*b^(5/4)
*g - sqrt(2)*a^(7/4)*b^(3/4)*i - 2*sqrt(a)*b^2*d + 2*a^(3/2)*b*h)*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt
(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a^(5/4)*b^(5/4)*g - sqrt(2)*a^(7/4)*b^(3/4)
*i + 2*sqrt(a)*b^2*d - 2*a^(3/2)*b*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqr
t(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))
*b^(5/4)))/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.14

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx = \frac{(bf-aj)\log\left(|bx^4+a|\right)}{4\,b^2} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^3d+\sqrt{2}\sqrt{ab}ab^2h-(ab^3)^{\frac{1}{4}}b^3c+(ab^3)^{\frac{1}{4}}ab^2g-(ab^3)^{\frac{3}{4}}be+(ab^3)^{\frac{3}{4}}ai\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^4} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^3d+\sqrt{2}\sqrt{ab}ab^2h-(ab^3)^{\frac{1}{4}}b^3c+(ab^3)^{\frac{1}{4}}ab^2g-(ab^3)^{\frac{3}{4}}be+(ab^3)^{\frac{3}{4}}ai\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^4} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c-(ab^3)^{\frac{1}{4}}ab^2g-(ab^3)^{\frac{3}{4}}be+(ab^3)^{\frac{3}{4}}ai\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^4} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c-(ab^3)^{\frac{1}{4}}ab^2g-(ab^3)^{\frac{3}{4}}be+(ab^3)^{\frac{3}{4}}ai\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^4} + \frac{3\,b^3jx^4+4\,b^3ix^3+6\,b^3hx^2+12\,b^3gx}{12\,b^4}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorith
m="giac")
```

```
[Out] 1/4*(b*f - a*j)*log(abs(b*x^4 + a))/b^2 - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^
3*d + sqrt(2)*sqrt(a*b)*a*b^2*h - (a*b^3)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b^2
*g - (a*b^3)^(3/4)*b*e + (a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^3*d
 + sqrt(2)*sqrt(a*b)*a*b^2*h - (a*b^3)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b^2*g
- (a*b^3)^(3/4)*b*e + (a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c - (a*b
^3)^(1/4)*a*b^2*g - (a*b^3)^(3/4)*b*e + (a*b^3)^(3/4)*a*i)*log(x^2 + sqrt(2
)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c - (
a*b^3)^(1/4)*a*b^2*g - (a*b^3)^(3/4)*b*e + (a*b^3)^(3/4)*a*i)*log(x^2 - sqr
t(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) + 1/12*(3*b^3*j*x^4 + 4*b^3*i*x^3 +
 6*b^3*h*x^2 + 12*b^3*g*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 5664, normalized size of antiderivative = 14.09

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4),x
)
```

```
[Out] symsum(log((a^4*i^3 - a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^4*g*j^2 + a^2*b
^2*c*h^2 - a^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i - 2*a^4*h*i*j -
a*b^3*c*f^2 - a*b^3*d^2*g + a*b^3*c^2*i - a^3*b*c*j^2 - 3*a^3*b*e*i^2 - a^3
*b*g*h^2 + a^3*b*g^2*i + 2*a^2*b^2*c*f*j - 2*a^2*b^2*c*g*i - 2*a^2*b^2*d*e*
j - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h - 2*a*b^3*c*d*h + 2
*a*b^3*c*e*g + 2*a*b^3*d*e*f + 2*a^3*b*d*i*j + 2*a^3*b*e*h*j - 2*a^3*b*f*g*
j + 2*a^3*b*f*h*i)/b^2 + root(256*a^3*b^8*z^4 + 256*a^4*b^6*j*z^3 - 256*a^3
*b^7*f*z^3 - 192*a^4*b^5*f*j*z^2 + 64*a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^2
- 64*a^3*b^6*d*h*z^2 - 64*a^3*b^6*c*i*z^2 + 64*a^2*b^7*c*e*z^2 + 96*a^5*b^4
*j^2*z^2 + 32*a^4*b^5*h^2*z^2 + 96*a^3*b^6*f^2*z^2 + 32*a^2*b^7*d^2*z^2 + 3
2*a^5*b^3*g*i*j*z - 32*a^4*b^4*f*g*i*z + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*e*
g*j*z - 32*a^4*b^4*d*h*j*z - 32*a^4*b^4*c*i*j*z + 32*a^3*b^5*e*f*g*z + 32*a
^3*b^5*d*f*h*z - 32*a^3*b^5*d*e*i*z - 32*a^3*b^5*c*g*h*z + 32*a^3*b^5*c*f*i
*z + 32*a^3*b^5*c*e*j*z - 32*a^2*b^6*c*e*f*z + 32*a^2*b^6*c*d*g*z + 16*a^5*
b^3*h^2*j*z - 16*a^5*b^3*h*i^2*z - 48*a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*z
+ 16*a^4*b^4*g^2*h*z - 16*a^4*b^4*f*h^2*z + 16*a^3*b^5*d^2*j*z + 16*a^4*b^4
*d*i^2*z - 16*a^3*b^5*e^2*h*z - 16*a^3*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z - 1
6*a^2*b^6*d^2*f*z + 16*a^2*b^6*d*e^2*z - 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^3*
z - 16*a^3*b^5*f^3*z - 8*a^5*b^2*f*g*i*j + 8*a^5*b^2*e*h*i*j - 8*a^4*b^3*e*
f*h*i + 8*a^4*b^3*e*f*g*j + 8*a^4*b^3*d*g*h*i + 8*a^4*b^3*d*f*h*j - 8*a^4*b
^3*d*e*i*j - 8*a^4*b^3*c*g*h*j + 8*a^4*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h + 8*
a^3*b^4*d*e*f*i + 8*a^3*b^4*c*f*g*h + 8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f*j
 - 8*a^3*b^4*c*d*h*i + 8*a^3*b^4*c*d*g*j - 8*a^2*b^5*c*d*f*g + 8*a^2*b^5*c*
d*e*h + 4*a^5*b^2*g^2*h*j - 4*a^5*b^2*g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5*b
^2*f*h*i^2 + 4*a^5*b^2*d*i^2*j - 4*a^4*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 - 4*
a^5*b^2*d*h*j^2 - 4*a^5*b^2*c*i*j^2 + 4*a^4*b^3*f^2*g*i - 4*a^4*b^3*f*g^2*h
 - 4*a^4*b^3*e*g^2*i - 4*a^4*b^3*d*g^2*j + 4*a^3*b^4*c^2*h*j + 4*a^4*b^3*e*
g*h^2 + 4*a^4*b^3*c*h^2*i - 4*a^3*b^4*d^2*g*i - 4*a^3*b^4*d^2*f*j - 4*a^4*b
^3*d*f*i^2 - 4*a^4*b^3*c*g*i^2 + 4*a^3*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j + 4*
a^4*b^3*c*e*j^2 - 4*a^3*b^4*e*f^2*g - 4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2*i
 + 4*a^3*b^4*d*f*g^2 - 4*a^2*b^5*c^2*f*h - 4*a^2*b^5*c^2*e*i - 4*a^2*b^5*c^
2*d*j - 4*a^3*b^4*c*e*h^2 + 4*a^2*b^5*d^2*e*g + 4*a^2*b^5*c*d^2*i - 4*a^2*b
^5*d*e^2*f - 4*a^2*b^5*c*e^2*g + 4*a^2*b^5*c*e*f^2 - 4*a^6*b*h*i^2*j + 4*a^
6*b*g*i*j^2 + 4*a*b^6*c^2*d*f - 4*a*b^6*c*d^2*e - 4*a^6*b*f*j^3 - 4*a*b^6*c
^3*g + 6*a^5*b^2*f^2*j^2 + 2*a^5*b^2*g^2*i^2 + 6*a^4*b^3*e^2*i^2 + 2*a^4*b^
```

3*f^2*h^2 + 2*a^4*b^3*d^2*j^2 + 6*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 2*a
^3*b^4*c^2*i^2 + 6*a^2*b^5*c^2*g^2 + 2*a^2*b^5*d^2*f^2 + 2*a^6*b*h^2*j^2 -
4*a^4*b^3*f^3*j - 4*a^5*b^2*e*i^3 - 4*a^3*b^4*e^3*i - 4*a^4*b^3*d*h^3 - 4*a
^2*b^5*d^3*h - 4*a^3*b^4*c*g^3 + 2*a*b^6*c^2*e^2 + a^5*b^2*h^4 + a^4*b^3*g^
4 + a^3*b^4*f^4 + a^2*b^5*e^4 + a^6*b*i^4 + a*b^6*d^4 + a^7*j^4 + b^7*c^4,
z, m)*((8*a*b^4*c*f - 8*a*b^4*d*e - 8*a^2*b^3*c*j + 8*a^2*b^3*d*i + 8*a^2*b
^3*e*h - 8*a^2*b^3*f*g + 8*a^3*b^2*g*j - 8*a^3*b^2*h*i)/b^2 + root(256*a^3*
b^8*z^4 + 256*a^4*b^6*j*z^3 - 256*a^3*b^7*f*z^3 - 192*a^4*b^5*f*j*z^2 + 64*
a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^2 - 64*a^3*b^6*d*h*z^2 - 64*a^3*b^6*c*i*
z^2 + 64*a^2*b^7*c*e*z^2 + 96*a^5*b^4*j^2*z^2 + 32*a^4*b^5*h^2*z^2 + 96*a^3
*b^6*f^2*z^2 + 32*a^2*b^7*d^2*z^2 + 32*a^5*b^3*g*i*j*z - 32*a^4*b^4*f*g*i*z
 + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*e*g*j*z - 32*a^4*b^4*d*h*j*z - 32*a^4*b^
4*c*i*j*z + 32*a^3*b^5*e*f*g*z + 32*a^3*b^5*d*f*h*z - 32*a^3*b^5*d*e*i*z -
32*a^3*b^5*c*g*h*z + 32*a^3*b^5*c*f*i*z + 32*a^3*b^5*c*e*j*z - 32*a^2*b^6*c
*e*f*z + 32*a^2*b^6*c*d*g*z + 16*a^5*b^3*h^2*j*z - 16*a^5*b^3*h*i^2*z - 48*
a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*z + 16*a^4*b^4*g^2*h*z - 16*a^4*b^4*f*h^
2*z + 16*a^3*b^5*d^2*j*z + 16*a^4*b^4*d*i^2*z - 16*a^3*b^5*e^2*h*z - 16*a^3
*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z - 16*a^2*b^6*d^2*f*z + 16*a^2*b^6*d*e^2*z
 - 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^3*z - 16*a^3*b^5*f^3*z - 8*a^5*b^2*f*g*i
*j + 8*a^5*b^2*e*h*i*j - 8*a^4*b^3*e*f*h*i + 8*a^4*b^3*e*f*g*j + 8*a^4*b^3*
d*g*h*i + 8*a^4*b^3*d*f*h*j - 8*a^4*b^3*d*e*i*j - 8*a^4*b^3*c*g*h*j + 8*a^4
*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h + 8*a^3*b^4*d*e*f*i + 8*a^3*b^4*c*f*g*h +
8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f*j - 8*a^3*b^4*c*d*h*i + 8*a^3*b^4*c*d*g
*j - 8*a^2*b^5*c*d*f*g + 8*a^2*b^5*c*d*e*h + 4*a^5*b^2*g^2*h*j - 4*a^5*b^2*
g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5*b^2*f*h*i^2 + 4*a^5*b^2*d*i^2*j - 4*a^4
*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 - 4*a^5*b^2*d*h*j^2 - 4*a^5*b^2*c*i*j^2 +
4*a^4*b^3*f^2*g*i - 4*a^4*b^3*f*g^2*h - 4*a^4*b^3*e*g^2*i - 4*a^4*b^3*d*g^2
*j + 4*a^3*b^4*c^2*h*j + 4*a^4*b^3*e*g*h^2 + 4*a^4*b^3*c*h^2*i - 4*a^3*b^4*
d^2*g*i - 4*a^3*b^4*d^2*f*j - 4*a^4*b^3*d*f*i^2 - 4*a^4*b^3*c*g*i^2 + 4*a^3
*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j + 4*a^4*b^3*c*e*j^2 - 4*a^3*b^4*e*f^2*g -
4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2*i + 4*a^3*b^4*d*f*g^2 - 4*a^2*b^5*c^2*f
*h - 4*a^2*b^5*c^2*e*i - 4*a^2*b^5*c^2*d*j - 4*a^3*b^4*c*e*h^2 + 4*a^2*b^5*
d^2*e*g + 4*a^2*b^5*c*d^2*i - 4*a^2*b^5*d*e^2*f - 4*a^2*b^5*c*e^2*g + 4*a^2
*b^5*c*e*f^2 - 4*a^6*b*h*i^2*j + 4*a^6*b*g*i*j^2 + 4*a*b^6*c^2*d*f - 4*a*b^
6*c*d^2*e - 4*a^6*b*f*j^3 - 4*a*b^6*c^3*g + 6*a^5*b^2*f^2*j^2 + 2*a^5*b^2*g
^2*i^2 + 6*a^4*b^3*e^2*i^2 + 2*a^4*b^3*f^2*h^2 + 2*a^4*b^3*d^2*j^2 + 6*a^3*
b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 2*a^3*b^4*c^2*i^2 + 6*a^2*b^5*c^2*g^2 + 2
*a^2*b^5*d^2*f^2 + 2*a^6*b*h^2*j^2 - 4*a^4*b^3*f^3*j - 4*a^5*b^2*e*i^3 - 4*
a^3*b^4*e^3*i - 4*a^4*b^3*d*h^3 - 4*a^2*b^5*d^3*h - 4*a^3*b^4*c*g^3 + 2*a*b
^6*c^2*e^2 + a^5*b^2*h^4 + a^4*b^3*g^4 + a^3*b^4*f^4 + a^2*b^5*e^4 + a^6*b*
i^4 + a*b^6*d^4 + a^7*j^4 + b^7*c^4, z, m)*((16*a^2*b^4*g - 16*a*b^5*c)/b^2
 - (x*(16*a^2*b^4*h - 16*a*b^5*d))/b^2) - (x*(4*b^5*c^2 - 4*a*b^4*e^2 + 4*a
^2*b^3*g^2 - 4*a^3*b^2*i^2 - 8*a*b^4*c*g + 8*a*b^4*d*f - 8*a^2*b^3*d*j + 8*
a^2*b^3*e*i - 8*a^2*b^3*f*h + 8*a^3*b^2*h*j))/b^2) + (x*(b^4*d^3 - a^3*b*h^
3 + b^4*c^2*f - a^4*h*j^2 + a^4*i^2*j + 3*a^2*b^2*d*h^2 + a^2*b^2*f*g^2 - a

```
^2*b^2*f^2*h + a^2*b^2*e^2*j - 2*b^4*c*d*e + a*b^3*d*f^2 - a*b^3*e^2*f - 3*
a*b^3*d^2*h - a*b^3*c^2*j + a^3*b*d*j^2 - a^3*b*f*i^2 - a^3*b*g^2*j + 2*a^2
*b^2*c*g*j - 2*a^2*b^2*c*h*i - 2*a^2*b^2*d*f*j - 2*a^2*b^2*d*g*i + 2*a^2*b^
2*e*f*i - 2*a^2*b^2*e*g*h + 2*a*b^3*c*d*i + 2*a*b^3*c*e*h - 2*a*b^3*c*f*g +
 2*a*b^3*d*e*g - 2*a^3*b*e*i*j + 2*a^3*b*f*h*j + 2*a^3*b*g*h*i))/b^2)*root(
256*a^3*b^8*z^4 + 256*a^4*b^6*j*z^3 - 256*a^3*b^7*f*z^3 - 192*a^4*b^5*f*j*z
^2 + 64*a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^2 - 64*a^3*b^6*d*h*z^2 - 64*a^3*
b^6*c*i*z^2 + 64*a^2*b^7*c*e*z^2 + 96*a^5*b^4*j^2*z^2 + 32*a^4*b^5*h^2*z^2
+ 96*a^3*b^6*f^2*z^2 + 32*a^2*b^7*d^2*z^2 + 32*a^5*b^3*g*i*j*z - 32*a^4*b^4
*f*g*i*z + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*e*g*j*z - 32*a^4*b^4*d*h*j*z - 3
2*a^4*b^4*c*i*j*z + 32*a^3*b^5*e*f*g*z + 32*a^3*b^5*d*f*h*z - 32*a^3*b^5*d*
e*i*z - 32*a^3*b^5*c*g*h*z + 32*a^3*b^5*c*f*i*z + 32*a^3*b^5*c*e*j*z - 32*a
^2*b^6*c*e*f*z + 32*a^2*b^6*c*d*g*z + 16*a^5*b^3*h^2*j*z - 16*a^5*b^3*h*i^2
*z - 48*a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*z + 16*a^4*b^4*g^2*h*z - 16*a^4*
b^4*f*h^2*z + 16*a^3*b^5*d^2*j*z + 16*a^4*b^4*d*i^2*z - 16*a^3*b^5*e^2*h*z
- 16*a^3*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z - 16*a^2*b^6*d^2*f*z + 16*a^2*b^6
*d*e^2*z - 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^3*z - 16*a^3*b^5*f^3*z - 8*a^5*b
^2*f*g*i*j + 8*a^5*b^2*e*h*i*j - 8*a^4*b^3*e*f*h*i + 8*a^4*b^3*e*f*g*j + 8*
a^4*b^3*d*g*h*i + 8*a^4*b^3*d*f*h*j - 8*a^4*b^3*d*e*i*j - 8*a^4*b^3*c*g*h*j
 + 8*a^4*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h + 8*a^3*b^4*d*e*f*i + 8*a^3*b^4*c*
f*g*h + 8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f*j - 8*a^3*b^4*c*d*h*i + 8*a^3*b
^4*c*d*g*j - 8*a^2*b^5*c*d*f*g + 8*a^2*b^5*c*d*e*h + 4*a^5*b^2*g^2*h*j - 4*
a^5*b^2*g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5*b^2*f*h*i^2 + 4*a^5*b^2*d*i^2*j
 - 4*a^4*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 - 4*a^5*b^2*d*h*j^2 - 4*a^5*b^2*c*
i*j^2 + 4*a^4*b^3*f^2*g*i - 4*a^4*b^3*f*g^2*h - 4*a^4*b^3*e*g^2*i - 4*a^4*b
^3*d*g^2*j + 4*a^3*b^4*c^2*h*j + 4*a^4*b^3*e*g*h^2 + 4*a^4*b^3*c*h^2*i - 4*
a^3*b^4*d^2*g*i - 4*a^3*b^4*d^2*f*j - 4*a^4*b^3*d*f*i^2 - 4*a^4*b^3*c*g*i^2
 + 4*a^3*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j + 4*a^4*b^3*c*e*j^2 - 4*a^3*b^4*e*
f^2*g - 4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2*i + 4*a^3*b^4*d*f*g^2 - 4*a^2*b
^5*c^2*f*h - 4*a^2*b^5*c^2*e*i - 4*a^2*b^5*c^2*d*j - 4*a^3*b^4*c*e*h^2 + 4*
a^2*b^5*d^2*e*g + 4*a^2*b^5*c*d^2*i - 4*a^2*b^5*d*e^2*f - 4*a^2*b^5*c*e^2*g
 + 4*a^2*b^5*c*e*f^2 - 4*a^6*b*h*i^2*j + 4*a^6*b*g*i*j^2 + 4*a*b^6*c^2*d*f
- 4*a*b^6*c*d^2*e - 4*a^6*b*f*j^3 - 4*a*b^6*c^3*g + 6*a^5*b^2*f^2*j^2 + 2*a
^5*b^2*g^2*i^2 + 6*a^4*b^3*e^2*i^2 + 2*a^4*b^3*f^2*h^2 + 2*a^4*b^3*d^2*j^2
+ 6*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 2*a^3*b^4*c^2*i^2 + 6*a^2*b^5*c^2
*g^2 + 2*a^2*b^5*d^2*f^2 + 2*a^6*b*h^2*j^2 - 4*a^4*b^3*f^3*j - 4*a^5*b^2*e*
i^3 - 4*a^3*b^4*e^3*i - 4*a^4*b^3*d*h^3 - 4*a^2*b^5*d^3*h - 4*a^3*b^4*c*g^3
 + 2*a*b^6*c^2*e^2 + a^5*b^2*h^4 + a^4*b^3*g^4 + a^3*b^4*f^4 + a^2*b^5*e^4
+ a^6*b*i^4 + a*b^6*d^4 + a^7*j^4 + b^7*c^4, z, m), m, 1, 4) + (h*x^2)/(2*b
) + (i*x^3)/(3*b) + (j*x^4)/(4*b) + (g*x)/b
```

3.192 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 184

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx = \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{4ab\,(a-bx^4)} + \frac{\left(3bc-\sqrt{a}\sqrt{b}e-ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}} + \frac{\left(3bc+\sqrt{a}\sqrt{b}e-ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}} + \frac{(bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

```
[Out] 1/4*x*(b*c+a*g+(a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(-b*x^4+a)+1/4*(-a*h+b*d)*a
rctanh(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)+1/8*arctan(b^(1/4)*x/a^(1/4))*(
3*b*c-a*g-e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)+1/8*arctanh(b^(1/4)*x/a^(1/4))
*(3*b*c-a*g+e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used

$= \{1872, 1890, 281, 214, 1181, 211\}$

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(bd-ah)}{4a^{3/2}b^{3/2}} + \frac{x(x(ah+bd)+ag+bc+bex^2+bfx^3)}{4ab\,(a-bx^4)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + (
(3*b*c - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b
^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/
(8*a^(7/4)*b^(5/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/
2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{4ab\left(a-bx^4\right)}-\frac{\int \frac{-b(3bc-ag)-2b(bd-ah)x-b^2ex^2}{a-bx^4}\,dx}{4ab^2}\\
&= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{4ab\left(a-bx^4\right)}-\frac{\int \left(-\frac{2b(bd-ah)x}{a-bx^4}+\frac{-b(3bc-ag)-b^2ex^2}{a-bx^4}\right)dx}{4ab^2}\\
&= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{4ab\left(a-bx^4\right)}-\frac{\int \frac{-b(3bc-ag)-b^2ex^2}{a-bx^4}\,dx}{4ab^2}+\frac{(bd-ah)\int \frac{x}{a-bx^4}\,dx}{2ab}\\
&= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{4ab\left(a-bx^4\right)}-\frac{\left(3bc-\sqrt{a}\sqrt{b}e-ag\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{8a^{3/2}\sqrt{b}}\\
&\quad+\frac{\left(3bc+\sqrt{a}\sqrt{b}e-ag\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx}{8a^{3/2}\sqrt{b}}+\frac{(bd-ah)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{4ab}\\
&= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{4ab\left(a-bx^4\right)}+\frac{\left(3bc-\sqrt{a}\sqrt{b}e-ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}}\\
&\quad+\frac{\left(3bc+\sqrt{a}\sqrt{b}e-ag\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.40

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx$$
$$=\frac{\frac{4a^{3/4}\sqrt{b}(bx(c+x(d+ex))+a(f+x(g+hx)))}{a-bx^4}-2\sqrt[4]{b}\left(-3bc+\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+\left(-3b^{5/4}c-2\sqrt[4]{a}bd-\sqrt{\ }\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a^(3/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + h*x))))/(a - b*x
^4) - 2*b^(1/4)*(-3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/
4)] + (-3*b^(5/4)*c - 2*a^(1/4)*b*d - Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g + 2*a
^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (3*b^(5/4)*c - 2*a^(1/4)*b*d + Sqrt[a]
*b^(3/4)*e - a*b^(1/4)*g + 2*a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] - 2*a^(1/4
)*(-(b*d) + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

method	result
risch	$\frac{\frac{e\,x^3}{4a}+\frac{(ah+bd)x^2}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^4+a}-\frac{\sum_{\_R=\text{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(\_R^2 e-\frac{2(ah-bd)\_R}{b}-\frac{ag-3bc}{b}\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$\frac{\frac{e\,x^3}{4a}+\frac{(ah+bd)x^2}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^4+a}+\frac{\frac{(-ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{(-2ah+2bd)\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\right.\right.}{}}{4ba}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4/a*e*x^3+1/4*(a*h+b*d)/a/b*x^2+1/4*(a*g+b*c)/a/b*x+1/4*f/b)/(-b*x^4+a)-
1/16/b/a*sum((_R^2*e-2/b*(a*h-b*d)*_R-1/b*(a*g-3*b*c))/_R^3*ln(x-_R),_R=Roo
tOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 212.80 (sec) , antiderivative size = 710521, normalized size of antiderivative = 3861.53

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas
")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.32

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx = -\frac{bex^3+(bd+ah)x^2+af+(bc+ag)x}{4\,(ab^2x^4-a^2b)} + \frac{\frac{2\,(bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{2\,(bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be-a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(3\,b^{\frac{3}{2}}c+\sqrt{a}be-a\sqrt{b}g\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{16\,ab}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima
")
```

```
[Out] -1/4*(b*e*x^3 + (b*d + a*h)*x^2 + a*f + (b*c + a*g)*x)/(a*b^2*x^4 - a^2*b)
+ 1/16*(2*(b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 2*(b*d
 - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(3*b^(3/2)*c - sqr
t(a)*b*e - a*sqrt(b)*g)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sq
rt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*b^(3/2)*c + sqrt(a)*b*e - a*sqrt(b)*g)*lo
g((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(145) = 290$.

Time = 0.27 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx =$$

$$-\frac{\sqrt{2}\left(3\,b^2c-abg-2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,ah+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a}$$
$$-\frac{\sqrt{2}\left(3\,b^2c-abg+2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,ah-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a}$$
$$-\frac{\sqrt{2}\left(3\,b^2c-abg-\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a}$$
$$+\frac{\sqrt{2}\left(3\,b^2c-abg-\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a}$$
$$-\frac{bex^3+bdx^2+ahx^2+bcx+agx+af}{4\,(bx^4-a)ab}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*b^2*c - a*b*g - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + 2*sqrt(2)*(
-a*b^3)^(1/4)*a*h + sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b
)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a) - 1/16*sqrt(2)*(3*b^2*c - a*b*g +
 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - 2*sqrt(2)*(-a*b^3)^(1/4)*a*h - sqrt(-a*b)*b
*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)
^(3/4)*a) - 1/32*sqrt(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(
2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c
- a*b*g - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((
-a*b^3)^(3/4)*a) - 1/4*(b*e*x^3 + b*d*x^2 + a*h*x^2 + b*c*x + a*g*x + a*f)/
((b*x^4 - a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 9.94 (sec) , antiderivative size = 1626, normalized size of antiderivative = 8.84

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x)
```

```
[Out] symsum(log(- root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z^2 + 1024*a^5*b^4*e
*g*z^2 - 3072*a^4*b^5*c*e*z^2 - 2048*a^6*b^3*h^2*z^2 - 2048*a^4*b^5*d^2*z^2
 + 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b^2*g^2*h*z - 128*a^
4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z + 128*a^4*b^3*d*g^2*z + 128*a^3*b^4*d*
e^2*z + 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h + 96*a^2*b^3*c*d*e*h - 48
*a^3*b^2*c*e*h^2 + 16*a^2*b^3*d^2*e*g - 12*a^2*b^3*c*e^2*g + 16*a^4*b*e*g*h
^2 - 48*a*b^4*c*d^2*e - 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g + 96*a^3*b^2*d^2*h
^2 + 2*a^3*b^2*e^2*g^2 - 54*a^2*b^3*c^2*g^2 - 64*a^2*b^3*d^3*h + 12*a^3*b^2
*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 - 81*b^5*c^4 - a^2*b^
3*e^4 - a^4*b*g^4, z, k)*(root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z^2 + 1
024*a^5*b^4*e*g*z^2 - 3072*a^4*b^5*c*e*z^2 - 2048*a^6*b^3*h^2*z^2 - 2048*a^
4*b^5*d^2*z^2 + 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b^2*g^2
*h*z - 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z + 128*a^4*b^3*d*g^2*z + 1
28*a^3*b^4*d*e^2*z + 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h + 96*a^2*b^3
*c*d*e*h - 48*a^3*b^2*c*e*h^2 + 16*a^2*b^3*d^2*e*g - 12*a^2*b^3*c*e^2*g + 1
6*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e - 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g + 96*
a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 - 54*a^2*b^3*c^2*g^2 - 64*a^2*b^3*d^3*h
 + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 - 81*b^5
*c^4 - a^2*b^3*e^4 - a^4*b*g^4, z, k)*((768*a^3*b^4*c - 256*a^4*b^3*g)/(64*
a^3*b) - (x*(128*a^3*b^4*d - 128*a^4*b^3*h))/(16*a^3*b)) - (64*a^2*b^3*d*e
- 64*a^3*b^2*e*h)/(64*a^3*b) + (x*(36*a*b^4*c^2 + 4*a^2*b^3*e^2 + 4*a^3*b^2
*g^2 - 24*a^2*b^3*c*g))/(16*a^3*b)) - (a*b^2*e^3 + 12*b^3*c*d^2 - 9*b^3*c^2
*e - 4*a^3*g*h^2 - 4*a*b^2*d^2*g + 12*a^2*b*c*h^2 - a^2*b*e*g^2 - 24*a*b^2*
c*d*h + 6*a*b^2*c*e*g + 8*a^2*b*d*g*h)/(64*a^3*b) - (x*(2*b^3*d^3 - 2*a^3*h
^3 - 3*b^3*c*d*e - 6*a*b^2*d^2*h + 6*a^2*b*d*h^2 + 3*a*b^2*c*e*h + a*b^2*d*
e*g - a^2*b*e*g*h))/(16*a^3*b))*root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z
^2 + 1024*a^5*b^4*e*g*z^2 - 3072*a^4*b^5*c*e*z^2 - 2048*a^6*b^3*h^2*z^2 - 2
048*a^4*b^5*d^2*z^2 + 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b
^2*g^2*h*z - 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z + 128*a^4*b^3*d*g^2
*z + 128*a^3*b^4*d*e^2*z + 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h + 96*a
^2*b^3*c*d*e*h - 48*a^3*b^2*c*e*h^2 + 16*a^2*b^3*d^2*e*g - 12*a^2*b^3*c*e^2
*g + 16*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e - 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g
 + 96*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 - 54*a^2*b^3*c^2*g^2 - 64*a^2*b^3
*d^3*h + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 -
81*b^5*c^4 - a^2*b^3*e^4 - a^4*b*g^4, z, k), k, 1, 4) + (f/(4*b) + (e*x^3)/
(4*a) + (x*(b*c + a*g))/(4*a*b) + (x^2*(b*d + a*h))/(4*a*b))/(a - b*x^4)
```

3.193 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2} dx$

Optimal result

Integrand size = 41, antiderivative size = 203

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2} dx = \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{4ab(a-bx^4)} - \frac{\left(be-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}} + \frac{\left(be+\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}} + \frac{(bd-ah)\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

```
[Out] 1/4*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+b*f*x^3)/a/b/(-b*x^4+a)+1/4*(-a*h+
b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-1/8*arctan(b^(1/4)*x/a^(1
/4))*(b*e-3*a*i-(-a*g+3*b*c)*b^(1/2)/a^(1/2))/a^(5/4)/b^(7/4)+1/8*arctanh(b
^(1/4)*x/a^(1/4))*(b*e-3*a*i+(-a*g+3*b*c)*b^(1/2)/a^(1/2))/a^(5/4)/b^(7/4)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {1872, 1890, 281, 214, 1181, 211}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}$$
$$+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(bd-ah)}{4a^{3/2}b^{3/2}}+\frac{x(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3)}{4ab\,(a-bx^4)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(4*a*b*(a - b*x
^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/
a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3
*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a*h)*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

integral

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{4ab\,(a-bx^4)} - \frac{\int \frac{-b(3bc-ag)-2b(bd-ah)x-b(be-3ai)x^2}{a-bx^4}\,dx}{4ab^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{4ab\,(a-bx^4)} - \frac{\int \left(-\frac{2b(bd-ah)x}{a-bx^4} + \frac{-b(3bc-ag)-b(be-3ai)x^2}{a-bx^4}\right) dx}{4ab^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{4ab\,(a-bx^4)} - \frac{\int \frac{-b(3bc-ag)-b(be-3ai)x^2}{a-bx^4}\,dx}{4ab^2} + \frac{(bd-ah)\int \frac{x}{a-bx^4}\,dx}{2ab}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{4ab\,(a-bx^4)} + \frac{(bd-ah)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{4ab} + \frac{\left(be-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{8ab} + \frac{\left(be+\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx}{8ab}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{4ab\,(a-bx^4)} - \frac{\left(be-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}} + \frac{\left(be+\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}} + \frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.49

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx$$
$$= \frac{\frac{4a^{3/4}b^{3/4}(bx(c+x(d+ex))+a(f+x(g+x(h+ix))))}{a-bx^4}+2\left(3b^{3/2}c-\sqrt{a}be-a\sqrt{b}g+3a^{3/2}i\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+\left(-3b^{3/2}c-2\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x
]
```

```
[Out] ((4*a^(3/4)*b^(3/4)*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + x*(h + i*x)))))/
(a - b*x^4) + 2*(3*b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/2)*i)*Arc
Tan[(b^(1/4)*x)/a^(1/4)] + (-3*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d - Sqrt[a]*b*
e + a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*
x] + (3*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e - a*Sqrt[b]*g + 2*a^(
5/4)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 2*a^(1/4)*b^(1/4)*
(-(b*d) + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.67

method	result
risch	$\frac{\frac{(ai+be)x^3}{4ab}+\frac{(ah+bd)x^2}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-bx^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)}\frac{\left(-(3ai-be)\_R^2-2(ah-bd)\_R-ag+3bc\right)\ln(x-\_R)}{\_R^3}}{16ab^2}$
default	$\frac{\frac{(ai+be)x^3}{4ab}+\frac{(ah+bd)x^2}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-bx^4+a}+\frac{\frac{(-ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{(-2ah+2bd)\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{(-3ai+be)}{}}{4ba}$

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERB
OSE)
```

```
[Out] (1/4*(a*i+b*e)/a/b*x^3+1/4*(a*h+b*d)/a/b*x^2+1/4*(a*g+b*c)/a/b*x+1/4*f/b)/(
-b*x^4+a)-1/16/a/b^2*sum((-(3*a*i-b*e)*_R^2-2*(a*h-b*d)*_R-a*g+3*b*c)/_R^3*
ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.28

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx$$
$$=-\frac{(be+ai)x^3+(bd+ah)x^2+af+(bc+ag)x}{4\,(ab^2x^4-a^2b)}$$
$$+\frac{\frac{2\,(bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{2\,(bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be-a\sqrt{b}g+3\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(3\,b^{\frac{3}{2}}c+\sqrt{a}be-a\sqrt{b}g-3\cdots\right.}{\sqrt{a}\sqrt{\cdots}}}{16\,ab}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="
maxima")
```

```
[Out] -1/4*((b*e + a*i)*x^3 + (b*d + a*h)*x^2 + a*f + (b*c + a*g)*x)/(a*b^2*x^4 -
 a^2*b) + 1/16*(2*(b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b))
- 2*(b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(3*b^(3/2)
*c - sqrt(a)*b*e - a*sqrt(b)*g + 3*a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)
*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*b^(3/2)*c + sqrt(a)
*b*e - a*sqrt(b)*g - 3*a^(3/2)*i)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(
sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
)/(a*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(164) = 328$.

Time = 0.28 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.27

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{(a - bx^4)^2}\,dx = -\frac{\sqrt{2}\left(3\,b^3c - ab^2g - 2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d + 2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh - \sqrt{-ab}b^2e + 3\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(-\right.}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right.}{16\,(-ab^3)^{\frac{3}{4}}\,ab} - \frac{\sqrt{2}\left(3\,b^3c - ab^2g + 2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d - 2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh - \sqrt{-ab}b^2e - 3\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(-\right.}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right.}{16\,(-ab^3)^{\frac{3}{4}}\,ab} - \frac{\sqrt{2}\left(3\,b^3c - ab^2g - \sqrt{-ab}b^2e + 3\,\sqrt{-ab}abi\right)\log\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,ab} + \frac{\sqrt{2}\left(3\,b^3c - ab^2g - \sqrt{-ab}b^2e + 3\,\sqrt{-ab}abi\right)\log\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,ab} - \frac{bex^3 + aix^3 + bdx^2 + ahx^2 + bcx + agx + af}{4\,(bx^4 - a)ab}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="
giac")
```

```
[Out] -1/16*sqrt(2)*(3*b^3*c - a*b^2*g - 2*sqrt(2)*(-a*b^3)^(1/4)*b^2*d + 2*sqrt(
2)*(-a*b^3)^(1/4)*a*b*h - sqrt(-a*b)*b^2*e + 3*sqrt(-a*b)*a*b*i)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a*b) -
1/16*sqrt(2)*(3*b^3*c - a*b^2*g + 2*sqrt(2)*(-a*b^3)^(1/4)*b^2*d - 2*sqrt(2
)*(-a*b^3)^(1/4)*a*b*h - sqrt(-a*b)*b^2*e - 3*sqrt(-a*b)*a*b*i)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a*b) - 1
/32*sqrt(2)*(3*b^3*c - a*b^2*g - sqrt(-a*b)*b^2*e + 3*sqrt(-a*b)*a*b*i)*log
(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a*b) + 1/32*sqr
t(2)*(3*b^3*c - a*b^2*g - sqrt(-a*b)*b^2*e + 3*sqrt(-a*b)*a*b*i)*log(x^2 -
sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a*b) - 1/4*(b*e*x^3 +
a*i*x^3 + b*d*x^2 + a*h*x^2 + b*c*x + a*g*x + a*f)/((b*x^4 - a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 9.96 (sec) , antiderivative size = 2611, normalized size of antiderivative = 12.86

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x)
```

```
[Out] symsum(log((27*a^4*i^3 - a*b^3*e^3 - 12*b^4*c*d^2 + 9*b^4*c^2*e - 12*a^2*b^
2*c*h^2 + a^2*b^2*e*g^2 + 9*a^2*b^2*e^2*i + 4*a*b^3*d^2*g - 27*a*b^3*c^2*i
- 27*a^3*b*e*i^2 + 4*a^3*b*g*h^2 - 3*a^3*b*g^2*i + 18*a^2*b^2*c*g*i - 8*a^2
*b^2*d*g*h + 24*a*b^3*c*d*h - 6*a*b^3*c*e*g)/(64*a^3*b^2) - root(65536*a^7*
b^7*z^4 - 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b^5*c*i*z^2 + 4096*a^5*b^5*d*h*z^
2 + 1024*a^5*b^5*e*g*z^2 - 3072*a^4*b^6*c*e*z^2 - 2048*a^6*b^4*h^2*z^2 - 20
48*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z - 768*a^4*b^4*d*e*i*z + 768*a^4*b^
4*c*g*h*z - 768*a^3*b^5*c*d*g*z - 1152*a^6*b^2*h*i^2*z - 128*a^5*b^3*g^2*h*
z + 1152*a^5*b^3*d*i^2*z - 128*a^4*b^4*e^2*h*z - 1152*a^3*b^5*c^2*h*z + 128
*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z + 1152*a^2*b^6*c^2*d*z + 96*a^4*b^2*
d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*c*e*g*i - 32*a^3*b^3*d*e*g*h + 9
6*a^2*b^4*c*d*e*h - 12*a^4*b^2*e*g^2*i + 144*a^4*b^2*c*h^2*i - 48*a^3*b^3*d
^2*g*i + 16*a^4*b^2*e*g*h^2 - 108*a^4*b^2*c*g*i^2 - 108*a^2*b^4*c^2*e*i + 1
44*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 + 16*a^2*b^4*d^2*e*g - 12*a^2*b^4*c
*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*e + 108*a^5*b*e*i^3 + 108*a*b^5*
c^3*g - 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^2*i^2 + 96*a^3*b^3*d^2*h^2 + 2*a
^3*b^3*e^2*g^2 - 54*a^2*b^4*c^2*g^2 + 18*a^5*b*g^2*i^2 + 12*a^3*b^3*e^3*i -
 64*a^4*b^2*d*h^3 - 64*a^2*b^4*d^3*h + 12*a^3*b^3*c*g^3 + 18*a*b^5*c^2*e^2
+ 16*a^5*b*h^4 + 16*a*b^5*d^4 - 81*a^6*i^4 - 81*b^6*c^4 - a^4*b^2*g^4 - a^2
*b^4*e^4, z, l)*(root(65536*a^7*b^7*z^4 - 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b
^5*c*i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^5*e*g*z^2 - 3072*a^4*b^6*c*e
*z^2 - 2048*a^6*b^4*h^2*z^2 - 2048*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z -
768*a^4*b^4*d*e*i*z + 768*a^4*b^4*c*g*h*z - 768*a^3*b^5*c*d*g*z - 1152*a^6*
b^2*h*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^3*d*i^2*z - 128*a^4*b^4*e^2*
h*z - 1152*a^3*b^5*c^2*h*z + 128*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z + 11
52*a^2*b^6*c^2*d*z + 96*a^4*b^2*d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*
c*e*g*i - 32*a^3*b^3*d*e*g*h + 96*a^2*b^4*c*d*e*h - 12*a^4*b^2*e*g^2*i + 14
4*a^4*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i + 16*a^4*b^2*e*g*h^2 - 108*a^4*b^2*c
*g*i^2 - 108*a^2*b^4*c^2*e*i + 144*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 + 1
6*a^2*b^4*d^2*e*g - 12*a^2*b^4*c*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*
e + 108*a^5*b*e*i^3 + 108*a*b^5*c^3*g - 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^
2*i^2 + 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 - 54*a^2*b^4*c^2*g^2 + 18*a^
5*b*g^2*i^2 + 12*a^3*b^3*e^3*i - 64*a^4*b^2*d*h^3 - 64*a^2*b^4*d^3*h + 12*a
^3*b^3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4 + 16*a*b^5*d^4 - 81*a^6*i^4
- 81*b^6*c^4 - a^4*b^2*g^4 - a^2*b^4*e^4, z, l)*((768*a^3*b^5*c - 256*a^4*b
^4*g)/(64*a^3*b^2) - (x*(128*a^3*b^4*d - 128*a^4*b^3*h))/(16*a^3*b)) - (64*
```

```
a^2*b^4*d*e - 192*a^3*b^3*d*i - 64*a^3*b^3*e*h + 192*a^4*b^2*h*i)/(64*a^3*b
^2) + (x*(36*a*b^4*c^2 + 36*a^4*b*i^2 + 4*a^2*b^3*e^2 + 4*a^3*b^2*g^2 - 24*
a^2*b^3*c*g - 24*a^3*b^2*e*i))/(16*a^3*b)) - (x*(2*b^3*d^3 - 2*a^3*h^3 - 3*
b^3*c*d*e + 3*a^3*g*h*i - 6*a*b^2*d^2*h + 6*a^2*b*d*h^2 + 9*a*b^2*c*d*i + 3
*a*b^2*c*e*h + a*b^2*d*e*g - 9*a^2*b*c*h*i - 3*a^2*b*d*g*i - a^2*b*e*g*h))/
(16*a^3*b))*root(65536*a^7*b^7*z^4 - 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b^5*c*
i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^5*e*g*z^2 - 3072*a^4*b^6*c*e*z^2
- 2048*a^6*b^4*h^2*z^2 - 2048*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z - 768*a
^4*b^4*d*e*i*z + 768*a^4*b^4*c*g*h*z - 768*a^3*b^5*c*d*g*z - 1152*a^6*b^2*h
*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^3*d*i^2*z - 128*a^4*b^4*e^2*h*z -
 1152*a^3*b^5*c^2*h*z + 128*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z + 1152*a^
2*b^6*c^2*d*z + 96*a^4*b^2*d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*c*e*g
*i - 32*a^3*b^3*d*e*g*h + 96*a^2*b^4*c*d*e*h - 12*a^4*b^2*e*g^2*i + 144*a^4
*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i + 16*a^4*b^2*e*g*h^2 - 108*a^4*b^2*c*g*i^
2 - 108*a^2*b^4*c^2*e*i + 144*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 + 16*a^2
*b^4*d^2*e*g - 12*a^2*b^4*c*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*e + 1
08*a^5*b*e*i^3 + 108*a*b^5*c^3*g - 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^2*i^2
 + 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 - 54*a^2*b^4*c^2*g^2 + 18*a^5*b*g
^2*i^2 + 12*a^3*b^3*e^3*i - 64*a^4*b^2*d*h^3 - 64*a^2*b^4*d^3*h + 12*a^3*b^
3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4 + 16*a*b^5*d^4 - 81*a^6*i^4 - 81*
b^6*c^4 - a^4*b^2*g^4 - a^2*b^4*e^4, z, l), l, 1, 4) + (f/(4*b) + (x*(b*c +
 a*g))/(4*a*b) + (x^2*(b*d + a*h))/(4*a*b) + (x^3*(b*e + a*i))/(4*a*b))/(a
- b*x^4)
```

3.194 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2} dx$

Optimal result

Integrand size = 46, antiderivative size = 225

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2} dx = \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{4ab(a-bx^4)} - \frac{\left(be-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}} + \frac{\left(be+\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}} + \frac{(bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} + \frac{j\log(a-bx^4)}{4b^2}$$

```
[Out] 1/4*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+(a*j+b*f)*x^3)/a/b/(-b*x^4+a)+1/4*
(-a*h+b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)+1/4*j*ln(-b*x^4+a)/
b^2-1/8*arctan(b^(1/4)*x/a^(1/4))*(b*e-3*a*i-(-a*g+3*b*c)*b^(1/2)/a^(1/2))/
a^(5/4)/b^(7/4)+1/8*arctanh(b^(1/4)*x/a^(1/4))*(b*e-3*a*i+(-a*g+3*b*c)*b^(1
/2)/a^(1/2))/a^(5/4)/b^(7/4)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used

$= \{1872, 1890, 1181, 211, 214, 1262, 649, 266\}$

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(bd-ah)}{4a^{3/2}b^{3/2}} + \frac{j\log(a-bx^4)}{4b^2} + \frac{x(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc)}{4ab(a-bx^4)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^2
,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(4*a*b*
(a - b*x^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(
1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(3*b*c - a*g))/Sqr
t[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a
*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) + (j*Log[a - b*x^4]
)/(4*b^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{4ab\,(a-bx^4)} - \frac{\int \frac{-b(3bc-ag)-2b(bd-ah)x-b(be-3ai)x^2+4abjx^3}{a-bx^4}\,dx}{4ab^2} \\
&= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{4ab\,(a-bx^4)} - \frac{\int \left(\frac{-b(3bc-ag)-b(be-3ai)x^2}{a-bx^4}+\frac{x(-2b(bd-ah)+4abjx^2)}{a-bx^4}\right)dx}{4ab^2} \\
&= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{4ab\,(a-bx^4)} - \frac{\int \frac{-b(3bc-ag)-b(be-3ai)x^2}{a-bx^4}\,dx}{4ab^2} - \frac{\int \frac{x(-2b(bd-ah)+4abjx^2)}{a-bx^4}\,dx}{4ab^2}
\end{aligned}
$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{4ab\,(a-bx^4)} - \frac{\text{Subst}\left(\int \frac{-2b(bd-ah)+4abjx}{a-bx^2}\,dx, x, x^2\right)}{8ab^2} + \frac{\left(be-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{8ab} + \frac{\left(be+\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx}{8ab}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{4ab\,(a-bx^4)} - \frac{\left(be-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}} + \frac{\left(be+\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}} + \frac{(bd-ah)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{4ab} - \frac{j\text{Subst}\left(\int \frac{x}{a-bx^2}\,dx, x, x^2\right)}{2b}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{4ab\,(a-bx^4)} - \frac{\left(be-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}} + \frac{\left(be+\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}} + \frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} + \frac{j\log\left(a-bx^4\right)}{4b^2}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.50

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx$$

$$= \frac{\frac{4(a^2j+b^2x(c+x(d+ex))+ab(f+x(g+x(h+ix))))}{a(a-bx^4)} + \frac{2\sqrt[4]{b}\left(3b^{3/2}c-\sqrt{a}be-a\sqrt{b}g+3a^{3/2}i\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{\sqrt[4]{b}\left(-3b^{3/2}c-2\sqrt[4]{a}b^{5/4}d-\sqrt{a}\right.}{}}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4)^2,x]
```

```
[Out] ((4*(a^2*j + b^2*x*(c + x*(d + e*x)) + a*b*(f + x*(g + x*(h + i*x)))))/(a*(
a - b*x^4)) + (2*b^(1/4)*(3*b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/
2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/a^(7/4) + (b^(1/4)*(-3*b^(3/2)*c - 2*a^(
1/4)*b^(5/4)*d - Sqrt[a]*b*e + a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + 3*a^(3/2
)*i)*Log[a^(1/4) - b^(1/4)*x])/a^(7/4) + (b^(1/4)*(3*b^(3/2)*c - 2*a^(1/4)*
b^(5/4)*d + Sqrt[a]*b*e - a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*
Log[a^(1/4) + b^(1/4)*x])/a^(7/4) + (2*Sqrt[b]*(b*d - a*h)*Log[Sqrt[a] + Sq
rt[b]*x^2])/a^(3/2) + 4*j*Log[a - b*x^4])/(16*b^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.70

method	result
risch	$\frac{\frac{(ai+be)x^3}{4ab}+\frac{(ah+bd)x^2}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{aj+bf}{4b^2}}{-b\,x^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 b-a)}\frac{\left(-4j\_R^3-\frac{(3ai-be)\_R^2}{a}-\frac{2(ah-bd)\_R}{a}-\frac{ag-3bc}{a}\right)\ln(x-\_}{\_R^3}}{16b^2}$
default	$\frac{\frac{(ai+be)x^3}{4ab}+\frac{(ah+bd)x^2}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{aj+bf}{4b^2}}{-b\,x^4+a}+\frac{\frac{(-ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{(-2ah+2bd)\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{(-3}{}}{4ba}$

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETU
RNVERBOSE)
```

```
[Out] (1/4*(a*i+b*e)/a/b*x^3+1/4*(a*h+b*d)/a/b*x^2+1/4*(a*g+b*c)/a/b*x+1/4*(a*j+b
*f)/b^2)/(-b*x^4+a)-1/16/b^2*sum((-4*j*_R^3-1/a*(3*a*i-b*e)*_R^2-2/a*(a*h-b
*d)*_R-1/a*(a*g-3*b*c))/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,
x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx$$

$$= -\frac{(b^2e+abi)x^3+abf+a^2j+(b^2d+abh)x^2+(b^2c+abg)x}{4\,(ab^3x^4-a^2b^2)} + \frac{\frac{2\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be-a\sqrt{b}g+3\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\left(b^{\frac{3}{2}}d-a\sqrt{b}h+2\,a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}b}-\frac{2\left(b^{\frac{3}{2}}d-a\sqrt{b}h-2\,a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}b}-}{16\,ab}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algor
ithm="maxima")
```

```
[Out] -1/4*((b^2*e + a*b*i)*x^3 + a*b*f + a^2*j + (b^2*d + a*b*h)*x^2 + (b^2*c +
a*b*g)*x)/(a*b^3*x^4 - a^2*b^2) + 1/16*(2*(3*b^(3/2)*c - sqrt(a)*b*e - a*sq
rt(b)*g + 3*a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqr
t(sqrt(a)*sqrt(b))*sqrt(b)) + 2*(b^(3/2)*d - a*sqrt(b)*h + 2*a^(3/2)*j)*log
(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - 2*(b^(3/2)*d - a*sqrt(b)*h - 2*a^(3/2
)*j)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (3*b^(3/2)*c + sqrt(a)*b*e -
a*sqrt(b)*g - 3*a^(3/2)*i)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)
*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(184) = 368$.

Time = 0.28 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.16

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx = \frac{j\log\left(|bx^4-a|\right)}{4\,b^2} - \frac{\sqrt{2}\left(3\,b^3c-ab^2g-2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d+2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(\right.}{2\,(-\frac{a}{b})}\right.}{16\,(-ab^3)^{\frac{3}{4}}\,ab} - \frac{\sqrt{2}\left(3\,b^3c-ab^2g+2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d-2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-\sqrt{-ab}b^2e-3\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(\right.}{2\,(-\frac{a}{b})}\right.}{16\,(-ab^3)^{\frac{3}{4}}\,ab} - \frac{\sqrt{2}(3\,b^3c-ab^2g-\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi)\log\left(x^2+\sqrt{2}x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,ab} + \frac{\sqrt{2}(3\,b^3c-ab^2g-\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi)\log\left(x^2-\sqrt{2}x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,ab} - \frac{(be+ai)x^3+(bd+ah)x^2+(bc+ag)x+\frac{abf+a^2j}{b}}{4\,(bx^4-a)ab}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algor
ithm="giac")
```

```
[Out] 1/4*j*log(abs(b*x^4 - a))/b^2 - 1/16*sqrt(2)*(3*b^3*c - a*b^2*g - 2*sqrt(2)
*(-a*b^3)^(1/4)*b^2*d + 2*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - sqrt(-a*b)*b^2*e +
 3*sqrt(-a*b)*a*b*i)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)
^(1/4))/((-a*b^3)^(3/4)*a*b) - 1/16*sqrt(2)*(3*b^3*c - a*b^2*g + 2*sqrt(2)*
(-a*b^3)^(1/4)*b^2*d - 2*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - sqrt(-a*b)*b^2*e -
3*sqrt(-a*b)*a*b*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^
(1/4))/((-a*b^3)^(3/4)*a*b) - 1/32*sqrt(2)*(3*b^3*c - a*b^2*g - sqrt(-a*b)*
b^2*e + 3*sqrt(-a*b)*a*b*i)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/
((-a*b^3)^(3/4)*a*b) + 1/32*sqrt(2)*(3*b^3*c - a*b^2*g - sqrt(-a*b)*b^2*e +
 3*sqrt(-a*b)*a*b*i)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^
3)^(3/4)*a*b) - 1/4*((b*e + a*i)*x^3 + (b*d + a*h)*x^2 + (b*c + a*g)*x + (a
*b*f + a^2*j)/b)/((b*x^4 - a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 10.22 (sec) , antiderivative size = 3943, normalized size of antiderivative = 17.52

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{(a - bx^4)^2} \, dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^2
,x)
```

```
[Out] ((b*f + a*j)/(4*b^2) + (x*(b*c + a*g))/(4*a*b) + (x^2*(b*d + a*h))/(4*a*b)
+ (x^3*(b*e + a*i))/(4*a*b))/(a - b*x^4) + symsum(log((27*a^4*i^3 - a*b^3*e
^3 - 12*b^4*c*d^2 + 9*b^4*c^2*e + 16*a^4*g*j^2 - 12*a^2*b^2*c*h^2 + a^2*b^2
*e*g^2 + 9*a^2*b^2*e^2*i - 48*a^4*h*i*j + 4*a*b^3*d^2*g - 27*a*b^3*c^2*i -
48*a^3*b*c*j^2 - 27*a^3*b*e*i^2 + 4*a^3*b*g*h^2 - 3*a^3*b*g^2*i + 18*a^2*b^
2*c*g*i - 16*a^2*b^2*d*e*j - 8*a^2*b^2*d*g*h + 24*a*b^3*c*d*h - 6*a*b^3*c*e
*g + 48*a^3*b*d*i*j + 16*a^3*b*e*h*j)/(64*a^3*b^2) - root(65536*a^7*b^8*z^4
 - 65536*a^7*b^6*j*z^3 - 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6*c*i*z^2 + 4096
*a^5*b^6*d*h*z^2 + 1024*a^5*b^6*e*g*z^2 - 3072*a^4*b^7*c*e*z^2 + 24576*a^7*
b^4*j^2*z^2 - 2048*a^6*b^5*h^2*z^2 - 2048*a^4*b^7*d^2*z^2 + 1536*a^6*b^3*g*
i*j*z - 4608*a^5*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^5*b^4*e*h*i*z -
 512*a^5*b^4*e*g*j*z + 1536*a^4*b^5*c*e*j*z - 768*a^4*b^5*d*e*i*z + 768*a^4
*b^5*c*g*h*z - 768*a^3*b^6*c*d*g*z + 1024*a^6*b^3*h^2*j*z - 1152*a^6*b^3*h*
i^2*z - 128*a^5*b^4*g^2*h*z + 1024*a^4*b^5*d^2*j*z + 1152*a^5*b^4*d*i^2*z -
 128*a^4*b^5*e^2*h*z - 1152*a^3*b^6*c^2*h*z + 128*a^4*b^5*d*g^2*z + 128*a^3
*b^6*d*e^2*z + 1152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z - 192*a^5*b^2*e*h*
i*j + 192*a^4*b^3*d*e*i*j - 192*a^4*b^3*c*g*h*j + 96*a^4*b^3*d*g*h*i - 288*
a^3*b^4*c*d*h*i + 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*i - 32*a^3*b^4*d*e
*g*h + 96*a^2*b^5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b^2*g*h^2*i - 288*a
^5*b^2*d*i^2*j + 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2 + 256*a^5*b^2*d*h
*j^2 + 64*a^5*b^2*e*g*j^2 + 288*a^3*b^4*c^2*h*j - 32*a^4*b^3*d*g^2*j - 12*a
^4*b^3*e*g^2*i + 144*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i + 16*a^4*b^3*e*g*
h^2 - 108*a^4*b^3*c*g*i^2 - 32*a^3*b^4*d*e^2*j - 192*a^4*b^3*c*e*j^2 - 288*
a^2*b^5*c^2*d*j - 108*a^2*b^5*c^2*e*i + 144*a^2*b^5*c*d^2*i - 48*a^3*b^4*c*
e*h^2 + 16*a^2*b^5*d^2*e*g - 12*a^2*b^5*c*e^2*g + 288*a^6*b*h*i^2*j - 192*a
^6*b*g*i*j^2 - 48*a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^5*b^2*g^2*i^2 - 12
8*a^4*b^3*d^2*j^2 - 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*i^2 + 96*a^3*b^4*d
^2*h^2 + 2*a^3*b^4*e^2*g^2 - 54*a^2*b^5*c^2*g^2 - 128*a^6*b*h^2*j^2 + 108*a
^5*b^2*e*i^3 + 12*a^3*b^4*e^3*i - 64*a^4*b^3*d*h^3 - 64*a^2*b^5*d^3*h + 12*
a^3*b^4*c*g^3 + 18*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 - 81*a^6*b*i^4 + 16*a*b^6
*d^4 + 256*a^7*j^4 - 81*b^7*c^4 - a^4*b^3*g^4 - a^2*b^5*e^4, z, m)*(root(65
536*a^7*b^8*z^4 - 65536*a^7*b^6*j*z^3 - 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6
*c*i*z^2 + 4096*a^5*b^6*d*h*z^2 + 1024*a^5*b^6*e*g*z^2 - 3072*a^4*b^7*c*e*z
^2 + 24576*a^7*b^4*j^2*z^2 - 2048*a^6*b^5*h^2*z^2 - 2048*a^4*b^7*d^2*z^2 +
1536*a^6*b^3*g*i*j*z - 4608*a^5*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^
```

5*b^4*e*h*i*z - 512*a^5*b^4*e*g*j*z + 1536*a^4*b^5*c*e*j*z - 768*a^4*b^5*d*
e*i*z + 768*a^4*b^5*c*g*h*z - 768*a^3*b^6*c*d*g*z + 1024*a^6*b^3*h^2*j*z -
1152*a^6*b^3*h*i^2*z - 128*a^5*b^4*g^2*h*z + 1024*a^4*b^5*d^2*j*z + 1152*a^
5*b^4*d*i^2*z - 128*a^4*b^5*e^2*h*z - 1152*a^3*b^6*c^2*h*z + 128*a^4*b^5*d*
g^2*z + 128*a^3*b^6*d*e^2*z + 1152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z - 1
92*a^5*b^2*e*h*i*j + 192*a^4*b^3*d*e*i*j - 192*a^4*b^3*c*g*h*j + 96*a^4*b^3
*d*g*h*i - 288*a^3*b^4*c*d*h*i + 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*i -
32*a^3*b^4*d*e*g*h + 96*a^2*b^5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b^2*
g*h^2*i - 288*a^5*b^2*d*i^2*j + 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2 +
256*a^5*b^2*d*h*j^2 + 64*a^5*b^2*e*g*j^2 + 288*a^3*b^4*c^2*h*j - 32*a^4*b^3
*d*g^2*j - 12*a^4*b^3*e*g^2*i + 144*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i +
16*a^4*b^3*e*g*h^2 - 108*a^4*b^3*c*g*i^2 - 32*a^3*b^4*d*e^2*j - 192*a^4*b^3
*c*e*j^2 - 288*a^2*b^5*c^2*d*j - 108*a^2*b^5*c^2*e*i + 144*a^2*b^5*c*d^2*i
- 48*a^3*b^4*c*e*h^2 + 16*a^2*b^5*d^2*e*g - 12*a^2*b^5*c*e^2*g + 288*a^6*b*
h*i^2*j - 192*a^6*b*g*i*j^2 - 48*a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^5*b
^2*g^2*i^2 - 128*a^4*b^3*d^2*j^2 - 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*i^2
+ 96*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 - 54*a^2*b^5*c^2*g^2 - 128*a^6*b*
h^2*j^2 + 108*a^5*b^2*e*i^3 + 12*a^3*b^4*e^3*i - 64*a^4*b^3*d*h^3 - 64*a^2*
b^5*d^3*h + 12*a^3*b^4*c*g^3 + 18*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 - 81*a^6*b
*i^4 + 16*a*b^6*d^4 + 256*a^7*j^4 - 81*b^7*c^4 - a^4*b^3*g^4 - a^2*b^5*e^4,
z, m)*((768*a^3*b^5*c - 256*a^4*b^4*g)/(64*a^3*b^2) - (x*(128*a^3*b^5*d -
128*a^4*b^4*h))/(16*a^3*b^2)) - (64*a^2*b^4*d*e + 384*a^3*b^3*c*j - 192*a^3
*b^3*d*i - 64*a^3*b^3*e*h - 128*a^4*b^2*g*j + 192*a^4*b^2*h*i)/(64*a^3*b^2)
+ (x*(36*a*b^5*c^2 + 4*a^2*b^4*e^2 + 4*a^3*b^3*g^2 + 36*a^4*b^2*i^2 - 24*a
^2*b^4*c*g + 64*a^3*b^3*d*j - 24*a^3*b^3*e*i - 64*a^4*b^2*h*j))/(16*a^3*b^2
)) + (x*(2*a^3*b*h^3 - 2*b^4*d^3 - 8*a^4*h*j^2 + 9*a^4*i^2*j - 6*a^2*b^2*d*
h^2 + a^2*b^2*e^2*j + 3*b^4*c*d*e + 6*a*b^3*d^2*h + 9*a*b^3*c^2*j + 8*a^3*b
*d*j^2 + a^3*b*g^2*j - 6*a^2*b^2*c*g*j + 9*a^2*b^2*c*h*i + 3*a^2*b^2*d*g*i
+ a^2*b^2*e*g*h - 9*a*b^3*c*d*i - 3*a*b^3*c*e*h - a*b^3*d*e*g - 6*a^3*b*e*i
*j - 3*a^3*b*g*h*i))/(16*a^3*b^2))*root(65536*a^7*b^8*z^4 - 65536*a^7*b^6*j
*z^3 - 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6*c*i*z^2 + 4096*a^5*b^6*d*h*z^2 +
1024*a^5*b^6*e*g*z^2 - 3072*a^4*b^7*c*e*z^2 + 24576*a^7*b^4*j^2*z^2 - 2048
*a^6*b^5*h^2*z^2 - 2048*a^4*b^7*d^2*z^2 + 1536*a^6*b^3*g*i*j*z - 4608*a^5*b
^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^5*b^4*e*h*i*z - 512*a^5*b^4*e*g*j
*z + 1536*a^4*b^5*c*e*j*z - 768*a^4*b^5*d*e*i*z + 768*a^4*b^5*c*g*h*z - 768
*a^3*b^6*c*d*g*z + 1024*a^6*b^3*h^2*j*z - 1152*a^6*b^3*h*i^2*z - 128*a^5*b^
4*g^2*h*z + 1024*a^4*b^5*d^2*j*z + 1152*a^5*b^4*d*i^2*z - 128*a^4*b^5*e^2*h
*z - 1152*a^3*b^6*c^2*h*z + 128*a^4*b^5*d*g^2*z + 128*a^3*b^6*d*e^2*z + 115
2*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z - 192*a^5*b^2*e*h*i*j + 192*a^4*b^3*
d*e*i*j - 192*a^4*b^3*c*g*h*j + 96*a^4*b^3*d*g*h*i - 288*a^3*b^4*c*d*h*i +
192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*i - 32*a^3*b^4*d*e*g*h + 96*a^2*b^5*
c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b^2*g*h^2*i - 288*a^5*b^2*d*i^2*j + 3
2*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2 + 256*a^5*b^2*d*h*j^2 + 64*a^5*b^2*
e*g*j^2 + 288*a^3*b^4*c^2*h*j - 32*a^4*b^3*d*g^2*j - 12*a^4*b^3*e*g^2*i + 1
44*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i + 16*a^4*b^3*e*g*h^2 - 108*a^4*b^3*

```
c*g*i^2 - 32*a^3*b^4*d*e^2*j - 192*a^4*b^3*c*e*j^2 - 288*a^2*b^5*c^2*d*j -
108*a^2*b^5*c^2*e*i + 144*a^2*b^5*c*d^2*i - 48*a^3*b^4*c*e*h^2 + 16*a^2*b^5
*d^2*e*g - 12*a^2*b^5*c*e^2*g + 288*a^6*b*h*i^2*j - 192*a^6*b*g*i*j^2 - 48*
a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^5*b^2*g^2*i^2 - 128*a^4*b^3*d^2*j^2
- 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*i^2 + 96*a^3*b^4*d^2*h^2 + 2*a^3*b^4
*e^2*g^2 - 54*a^2*b^5*c^2*g^2 - 128*a^6*b*h^2*j^2 + 108*a^5*b^2*e*i^3 + 12*
a^3*b^4*e^3*i - 64*a^4*b^3*d*h^3 - 64*a^2*b^5*d^3*h + 12*a^3*b^4*c*g^3 + 18
*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 - 81*a^6*b*i^4 + 16*a*b^6*d^4 + 256*a^7*j^4
 - 81*b^7*c^4 - a^4*b^3*g^4 - a^2*b^5*e^4, z, m), m, 1, 4)
```

3.195 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2} dx$

Optimal result

Integrand size = 35, antiderivative size = 353

$$\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2} dx &= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab(a+bx^4)} + \frac{(bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} \\
&\quad - \frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} \\
&\quad + \frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} \\
&\quad - \frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}} \\
&\quad + \frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}
\end{aligned}$$

```
[Out] 1/4*x*(b*c-a*g+(-a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(b*x^4+a)+1/4*(a*h+b*d)*ar
ctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2
)+a^(1/2)+x^2*b^(1/2))*(3*b*c+a*g-e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2
)+1/32*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(3*b*c+a*g-e*a^(1/
2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/16*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4
))*(3*b*c+a*g+e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/16*arctan(1+b^(1
/4)*x*2^(1/2)/a^(1/4))*(3*b*c+a*g+e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2
)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1872, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}\,dx$$
$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{8\sqrt{2}a^{7/4}b^{5/4}}$$
$$+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+bd)}{4a^{3/2}b^{3/2}}$$
$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}$$
$$+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}$$
$$+\frac{x(x(bd-ah)-ag+bc+bex^2+bfx^3)}{4ab\,(a+bx^4)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + (
(b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) - ((3*b*c +
Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[
b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqr
t[2]*a^(7/4)*b^(5/4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
```

```
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\int \frac{-b(3bc+ag)-2b(bd+ah)x-b^2ex^2}{a+bx^4}\,dx}{4ab^2} \\
&= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\int \left(-\frac{2b(bd+ah)x}{a+bx^4} + \frac{-b(3bc+ag)-b^2ex^2}{a+bx^4}\right)dx}{4ab^2} \\
&= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\int \frac{-b(3bc+ag)-b^2ex^2}{a+bx^4}\,dx}{4ab^2} + \frac{(bd+ah)\int \frac{x}{a+bx^4}\,dx}{2ab} \\
&= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab\,(a+bx^4)} + \frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{8a^{3/2}b^{3/2}} \\
&\quad + \frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{8a^{3/2}b^{3/2}} + \frac{(bd+ah)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{4ab}
\end{aligned}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab\left(a+bx^4\right)}+\frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$
$$-\frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}b^{5/4}}$$
$$-\frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}b^{5/4}}$$
$$+\frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{3/2}}$$
$$+\frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{3/2}}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab\left(a+bx^4\right)}+\frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$
$$-\frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}$$
$$+\frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}$$
$$+\frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}$$
$$-\frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab\,(a+bx^4)} + \frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} - \frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} - \frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.02

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}\,dx$$

$$= \frac{-\frac{8a^{3/4}\sqrt{b}(-bx(c+x(d+ex))+a(f+x(g+hx)))}{a+bx^4} - 2\left(3\sqrt{2}b^{5/4}c+4\sqrt[4]{a}bd+\sqrt{2}\sqrt{a}b^{3/4}e+\sqrt{2}a\sqrt[4]{b}g+4a^{5/4}h\right)\arctan\left(1\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(a +
 b*x^4) - 2*(3*Sqrt[2]*b^(5/4)*c + 4*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*
e + Sqrt[2]*a*b^(1/4)*g + 4*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/
4)] + 2*(3*Sqrt[2]*b^(5/4)*c - 4*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e +
Sqrt[2]*a*b^(1/4)*g - 4*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
+ Sqrt[2]*b^(1/4)*(-3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(3*b*c - Sqrt[a]*Sqrt[b]
*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^(7/
4)*b^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\frac{e\,x^3}{4a}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(\_R^2 e+\frac{2(ah+bd)\_R}{b}+\frac{ag+3bc}{b}\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$\frac{\frac{e\,x^3}{4a}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b\,x^4+a}+\frac{\frac{(ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{(2ah+}{}}{4}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4/a*e*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/(b*x^4+a)+1
/16/b/a*sum((_R^2*e+2/b*(a*h+b*d)*_R+1/b*(a*g+3*b*c))/_R^3*ln(x-_R),_R=Root
Of(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.06

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^4\right)^2}\,dx=\frac{bex^3+(bd-ah)x^2-af+(bc-ag)x}{4\,(ab^2x^4+a^2b)}$$

$$+\frac{\frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima"
)
```

```
[Out] 1/4*(b*e*x^3 + (b*d - a*h)*x^2 - a*f + (b*c - a*g)*x)/(a*b^2*x^4 + a^2*b) +
 1/32*(sqrt(2)*(3*b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g)*log(sqrt(b)*x^2 +
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(3*b^(3/2)
*c - sqrt(a)*b*e + a*sqrt(b)*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x
 + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a^
(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g - 4*sqrt(a)*b^(3/2)*d - 4*a^(3/
2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sq
rt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)
*a^(1/4)*b^(7/4)*c + sqrt(2)*a^(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g
+ 4*sqrt(a)*b^(3/2)*d + 4*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*
x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*s
qrt(b))*b^(3/4)))/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.11

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}\,dx = \frac{bex^3+bdx^2-ahx^2+bcx-agx-af}{4\,(bx^4+a)ab}$$
$$+\frac{\sqrt{2}\Big(2\sqrt{2}\sqrt{ab}b^2d+2\sqrt{2}\sqrt{ab}abh+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg+(ab^3)^{\frac{3}{4}}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+\frac{\sqrt{2}\Big(2\sqrt{2}\sqrt{ab}b^2d+2\sqrt{2}\sqrt{ab}abh+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg+(ab^3)^{\frac{3}{4}}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+\frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$
$$-\frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*e*x^3 + b*d*x^2 - a*h*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 + a)*a*b) +
 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a
*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(
2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqr
t(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c +
(a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a
/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (
a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b
*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3
)
```

Mupad [B] (verification not implemented)

Time = 9.94 (sec) , antiderivative size = 1623, normalized size of antiderivative = 4.60

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x)
```

```
[Out] symsum(log((12*b^3*c*d^2 - a*b^2*e^3 - 9*b^3*c^2*e + 4*a^3*g*h^2 + 4*a*b^2*
d^2*g + 12*a^2*b*c*h^2 - a^2*b*e*g^2 + 24*a*b^2*c*d*h - 6*a*b^2*c*e*g + 8*a
```

^2*b*d*g*h)/(64*a^3*b) - root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z^2 + 1024*a^5*b^4*e*g*z^2 + 3072*a^4*b^5*c*e*z^2 + 2048*a^6*b^3*h^2*z^2 + 2048*a^4*b^5*d^2*z^2 - 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b^2*g^2*h*z + 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z - 128*a^4*b^3*d*g^2*z + 128*a^3*b^4*d*e^2*z - 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h - 96*a^2*b^3*c*d*e*h - 48*a^3*b^2*c*e*h^2 - 16*a^2*b^3*d^2*e*g + 12*a^2*b^3*c*e^2*g - 16*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e + 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g + 96*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 + 54*a^2*b^3*c^2*g^2 + 64*a^2*b^3*d^3*h + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 + 81*b^5*c^4 + a^2*b^3*e^4 + a^4*b*g^4, z, k)*(root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z^2 + 1024*a^5*b^4*e*g*z^2 + 3072*a^4*b^5*c*e*z^2 + 2048*a^6*b^3*h^2*z^2 + 2048*a^4*b^5*d^2*z^2 - 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b^2*g^2*h*z + 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z - 128*a^4*b^3*d*g^2*z + 128*a^3*b^4*d*e^2*z - 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h - 96*a^2*b^3*c*d*e*h - 48*a^3*b^2*c*e*h^2 - 16*a^2*b^3*d^2*e*g + 12*a^2*b^3*c*e^2*g - 16*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e + 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g + 96*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 + 54*a^2*b^3*c^2*g^2 + 64*a^2*b^3*d^3*h + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 + 81*b^5*c^4 + a^2*b^3*e^4 + a^4*b*g^4, z, k)*((768*a^3*b^4*c + 256*a^4*b^3*g)/(64*a^3*b) - (x*(128*a^3*b^4*d + 128*a^4*b^3*h))/(16*a^3*b)) + (64*a^2*b^3*d*e + 64*a^3*b^2*e*h)/(64*a^3*b) + (x*(36*a*b^4*c^2 - 4*a^2*b^3*e^2 + 4*a^3*b^2*g^2 + 24*a^2*b^3*c*g))/(16*a^3*b)) + (x*(2*b^3*d^3 + 2*a^3*h^3 - 3*b^3*c*d*e + 6*a*b^2*d^2*h + 6*a^2*b*d*h^2 - 3*a*b^2*c*e*h - a*b^2*d*e*g - a^2*b*e*g*h))/(16*a^3*b))*root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z^2 + 1024*a^5*b^4*e*g*z^2 + 3072*a^4*b^5*c*e*z^2 + 2048*a^6*b^3*h^2*z^2 + 2048*a^4*b^5*d^2*z^2 - 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b^2*g^2*h*z + 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z - 128*a^4*b^3*d*g^2*z + 128*a^3*b^4*d*e^2*z - 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h - 96*a^2*b^3*c*d*e*h - 48*a^3*b^2*c*e*h^2 - 16*a^2*b^3*d^2*e*g + 12*a^2*b^3*c*e^2*g - 16*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e + 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g + 96*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 + 54*a^2*b^3*c^2*g^2 + 64*a^2*b^3*d^3*h + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 + 81*b^5*c^4 + a^2*b^3*e^4 + a^4*b*g^4, z, k), k, 1, 4) + ((e*x^3)/(4*a) - f/(4*b) + (x*(b*c - a*g))/(4*a*b) + (x^2*(b*d - a*h))/(4*a*b))/(a + b*x^4)

3.196 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2} dx$

Optimal result

Integrand size = 40, antiderivative size = 395

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2} dx = \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{4ab(a+bx^4)} + \frac{(bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} - \frac{\left(\sqrt{b}(3bc+ag)+\sqrt{a}(be+3ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{7/4}} + \frac{\left(\sqrt{b}(3bc+ag)+\sqrt{a}(be+3ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{7/4}} - \frac{\left(\sqrt{b}(3bc+ag)-\sqrt{a}(be+3ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{7/4}} + \frac{\left(\sqrt{b}(3bc+ag)-\sqrt{a}(be+3ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{7/4}}$$

```
[Out] 1/4*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+b*f*x^3)/a/b/(b*x^4+a)+1/4*(a*h+
b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-1/32*ln(-a^(1/4)*b^(1/4)*x
*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a^
(7/4)/b^(7/4)*2^(1/2)+1/32*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2)
)*(-(3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+1/16*a
rctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2
))/a^(7/4)/b^(7/4)*2^(1/2)+1/16*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*a*i
+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1872, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx$$
$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}$$
$$+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}+\frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+bd)}{4a^{3/2}b^{3/2}}$$
$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}$$
$$+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}$$
$$+\frac{x(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3)}{4ab\,(a+bx^4)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(4*a*b*(a + b*x
^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) - ((
Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a
]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4
)*b^(7/4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + (
(Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
```

```
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

integral

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\int \frac{-b(3bc+ag)-2b(bd+ah)x-b(be+3ai)x^2}{a+bx^4}\,dx}{4ab^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\int \left(-\frac{2b(bd+ah)x}{a+bx^4} + \frac{-b(3bc+ag)-b(be+3ai)x^2}{a+bx^4}\right)dx}{4ab^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{4ab\,(a+bx^4)} - \frac{\int \frac{-b(3bc+ag)-b(be+3ai)x^2}{a+bx^4}\,dx}{4ab^2} + \frac{(bd+ah)\int \frac{x}{a+bx^4}\,dx}{2ab}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{4ab\,(a+bx^4)} + \frac{(bd+ah)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{4ab} - \frac{\left(be-\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{8ab^2} + \frac{\left(be+\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{8ab^2}$$

$$=\frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{4ab\left(a+bx^4\right)}+\frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$
$$+\frac{\left(be-\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{5/4}b^{7/4}}$$
$$+\frac{\left(be-\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{5/4}b^{7/4}}$$
$$+\frac{\left(be+\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16ab^2}$$
$$+\frac{\left(be+\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16ab^2}$$

$$=\frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{4ab\left(a+bx^4\right)}+\frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$
$$+\frac{\left(be-\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{5/4}b^{7/4}}$$
$$-\frac{\left(be-\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{5/4}b^{7/4}}$$
$$+\frac{\left(be+\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}b^{7/4}}$$
$$-\frac{\left(be+\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}b^{7/4}}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{4ab\,(a+bx^4)} + \frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$
$$- \frac{\left(be+\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}b^{7/4}}$$
$$+ \frac{\left(be+\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}b^{7/4}}$$
$$+ \frac{\left(be-\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{5/4}b^{7/4}}$$
$$- \frac{\left(be-\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{5/4}b^{7/4}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.05

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx$$
$$= \frac{-\frac{8a^{3/4}b^{3/4}(-bx(c+x(d+ex))+a(f+x(g+x(h+ix))))}{a+bx^4} - 2\left(3\sqrt{2}b^{3/2}c+4\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+4a^{5/4}\sqrt[4]{b}h+3\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x
]
```

```
[Out] ((-8*a^(3/4)*b^(3/4)*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + x*(h + i*x))
)))/(a + b*x^4) - 2*(3*Sqrt[2]*b^(3/2)*c + 4*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sq
rt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b^(3/2)*c - 4*a^(1
/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g - 4*a^(5/4)*b^(1/
4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[
2]*(-3*b^(3/2)*c + Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b^(3/2)*c - Sqrt[a]*b*
e + a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/(32*a^(7/4)*b^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.34

method	result
risch	$\frac{-\frac{(ai-be)x^3}{4ab}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 b+a)}\frac{\left((3ai+be)\_R^2+2(ah+bd)\_R+ag+3bc\right)\ln(x-\_R)}{\_R^3}}{16a\,b^2}$
default	$\frac{-\frac{(ai-be)x^3}{4ab}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b\,x^4+a}+\frac{\frac{(ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}}{}$

[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)

[Out] (-1/4*(a*i-b*e)/a/b*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/(b*x^4+a)+1/16/a/b^2*sum(((3*a*i+b*e)*_R^2+2*(a*h+b*d)*_R+a*g+3*b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx=\text{Timed out}$$

[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx=\text{Timed out}$$

[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.05

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx = \frac{(be-ai)x^3+(bd-ah)x^2-af+(bc-ag)x}{4\,(ab^2x^4+a^2b)}$$

$$+\frac{\frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g-3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g-3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="m
axima")
```

```
[Out] 1/4*((b*e - a*i)*x^3 + (b*d - a*h)*x^2 - a*f + (b*c - a*g)*x)/(a*b^2*x^4 +
a^2*b) + 1/32*(sqrt(2)*(3*b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g - 3*a^(3/2)
*i)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)
) - sqrt(2)*(3*b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g - 3*a^(3/2)*i)*log(sqr
t(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sq
rt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a^(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/
4)*g + 3*sqrt(2)*a^(7/4)*b^(1/4)*i - 4*sqrt(a)*b^(3/2)*d - 4*a^(3/2)*sqrt(b
)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a
)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*
b^(7/4)*c + sqrt(2)*a^(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g + 3*sqrt(
2)*a^(7/4)*b^(1/4)*i + 4*sqrt(a)*b^(3/2)*d + 4*a^(3/2)*sqrt(b)*h)*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.16

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx = \frac{bex^3-aix^3+bdx^2-ahx^2+bcx-agx-af}{4\,(bx^4+a)ab}$$
$$+\frac{\sqrt{2}\Big(2\sqrt{2}\sqrt{ab}b^3d+2\sqrt{2}\sqrt{ab}ab^2h+3\,(ab^3)^{\frac{1}{4}}\,b^3c+(ab^3)^{\frac{1}{4}}\,ab^2g+(ab^3)^{\frac{3}{4}}\,be+3\,(ab^3)^{\frac{3}{4}}\,ai\Big)\arctan\left(\frac{\sqrt{2}\big(2x \ldots}{2 \ldots}\right.}{16\,a^2b^4}$$
$$+\frac{\sqrt{2}\Big(2\sqrt{2}\sqrt{ab}b^3d+2\sqrt{2}\sqrt{ab}ab^2h+3\,(ab^3)^{\frac{1}{4}}\,b^3c+(ab^3)^{\frac{1}{4}}\,ab^2g+(ab^3)^{\frac{3}{4}}\,be+3\,(ab^3)^{\frac{3}{4}}\,ai\Big)\arctan\left(\frac{\sqrt{2}\big(2x \ldots}{2 \ldots}\right.}{16\,a^2b^4}$$
$$+\frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{1}{4}}\,b^3c+(ab^3)^{\frac{1}{4}}\,ab^2g-(ab^3)^{\frac{3}{4}}\,be-3\,(ab^3)^{\frac{3}{4}}\,ai\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^4}$$
$$-\frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{1}{4}}\,b^3c+(ab^3)^{\frac{1}{4}}\,ab^2g-(ab^3)^{\frac{3}{4}}\,be-3\,(ab^3)^{\frac{3}{4}}\,ai\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^4}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="g
iac")
```

```
[Out] 1/4*(b*e*x^3 - a*i*x^3 + b*d*x^2 - a*h*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 +
 a)*a*b) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^3*d + 2*sqrt(2)*sqrt(a*b)*a*
b^2*h + 3*(a*b^3)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b^2*g + (a*b^3)^(3/4)*b*e +
 3*(a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^
(1/4))/(a^2*b^4) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^3*d + 2*sqrt(2)*sqrt
(a*b)*a*b^2*h + 3*(a*b^3)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b^2*g + (a*b^3)^(3/
4)*b*e + 3*(a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4)
)/(a/b)^(1/4))/(a^2*b^4) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^3*c + (a*b^3)^(1
/4)*a*b^2*g - (a*b^3)^(3/4)*b*e - 3*(a*b^3)^(3/4)*a*i)*log(x^2 + sqrt(2)*x*
(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^3*c +
(a*b^3)^(1/4)*a*b^2*g - (a*b^3)^(3/4)*b*e - 3*(a*b^3)^(3/4)*a*i)*log(x^2 -
sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 10.17 (sec) , antiderivative size = 2605, normalized size of antiderivative = 6.59

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x)
```

```
[Out] symsum(log(- root(65536*a^7*b^7*z^4 + 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b^5*c
*i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^5*e*g*z^2 + 3072*a^4*b^6*c*e*z^2
 + 2048*a^6*b^4*h^2*z^2 + 2048*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z + 768*
a^4*b^4*d*e*i*z - 768*a^4*b^4*c*g*h*z - 768*a^3*b^5*c*d*g*z + 1152*a^6*b^2*
h*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^3*d*i^2*z + 128*a^4*b^4*e^2*h*z
- 1152*a^3*b^5*c^2*h*z - 128*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z - 1152*a
^2*b^6*c^2*d*z - 96*a^4*b^2*d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*c*e*
g*i - 32*a^3*b^3*d*e*g*h - 96*a^2*b^4*c*d*e*h + 12*a^4*b^2*e*g^2*i - 144*a^
4*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i - 16*a^4*b^2*e*g*h^2 + 108*a^4*b^2*c*g*i
^2 + 108*a^2*b^4*c^2*e*i - 144*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 - 16*a^
2*b^4*d^2*e*g + 12*a^2*b^4*c*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*e +
108*a^5*b*e*i^3 + 108*a*b^5*c^3*g + 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^2*i^
2 + 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 54*a^2*b^4*c^2*g^2 + 18*a^5*b*
g^2*i^2 + 12*a^3*b^3*e^3*i + 64*a^4*b^2*d*h^3 + 64*a^2*b^4*d^3*h + 12*a^3*b
^3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4 + 16*a*b^5*d^4 + 81*a^6*i^4 + 81
*b^6*c^4 + a^4*b^2*g^4 + a^2*b^4*e^4, z, l)*(root(65536*a^7*b^7*z^4 + 3072*
a^6*b^4*g*i*z^2 + 9216*a^5*b^5*c*i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^
5*e*g*z^2 + 3072*a^4*b^6*c*e*z^2 + 2048*a^6*b^4*h^2*z^2 + 2048*a^4*b^6*d^2*
z^2 + 768*a^5*b^3*e*h*i*z + 768*a^4*b^4*d*e*i*z - 768*a^4*b^4*c*g*h*z - 768
*a^3*b^5*c*d*g*z + 1152*a^6*b^2*h*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^
3*d*i^2*z + 128*a^4*b^4*e^2*h*z - 1152*a^3*b^5*c^2*h*z - 128*a^4*b^4*d*g^2*
z + 128*a^3*b^5*d*e^2*z - 1152*a^2*b^6*c^2*d*z - 96*a^4*b^2*d*g*h*i - 288*a
^3*b^3*c*d*h*i + 72*a^3*b^3*c*e*g*i - 32*a^3*b^3*d*e*g*h - 96*a^2*b^4*c*d*e
*h + 12*a^4*b^2*e*g^2*i - 144*a^4*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i - 16*a^4
*b^2*e*g*h^2 + 108*a^4*b^2*c*g*i^2 + 108*a^2*b^4*c^2*e*i - 144*a^2*b^4*c*d^
2*i - 48*a^3*b^3*c*e*h^2 - 16*a^2*b^4*d^2*e*g + 12*a^2*b^4*c*e^2*g - 48*a^5
*b*g*h^2*i - 48*a*b^5*c*d^2*e + 108*a^5*b*e*i^3 + 108*a*b^5*c^3*g + 54*a^4*
b^2*e^2*i^2 + 162*a^3*b^3*c^2*i^2 + 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2
+ 54*a^2*b^4*c^2*g^2 + 18*a^5*b*g^2*i^2 + 12*a^3*b^3*e^3*i + 64*a^4*b^2*d*h
^3 + 64*a^2*b^4*d^3*h + 12*a^3*b^3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4
+ 16*a*b^5*d^4 + 81*a^6*i^4 + 81*b^6*c^4 + a^4*b^2*g^4 + a^2*b^4*e^4, z, l)
*((768*a^3*b^5*c + 256*a^4*b^4*g)/(64*a^3*b^2) - (x*(128*a^3*b^4*d + 128*a^
4*b^3*h))/(16*a^3*b)) + (64*a^2*b^4*d*e + 192*a^3*b^3*d*i + 64*a^3*b^3*e*h
+ 192*a^4*b^2*h*i)/(64*a^3*b^2) + (x*(36*a*b^4*c^2 - 36*a^4*b*i^2 - 4*a^2*b
^3*e^2 + 4*a^3*b^2*g^2 + 24*a^2*b^3*c*g - 24*a^3*b^2*e*i))/(16*a^3*b)) - (2
7*a^4*i^3 + a*b^3*e^3 - 12*b^4*c*d^2 + 9*b^4*c^2*e - 12*a^2*b^2*c*h^2 + a^2
```

*b^2*e*g^2 + 9*a^2*b^2*e^2*i - 4*a*b^3*d^2*g + 27*a*b^3*c^2*i + 27*a^3*b*e*
i^2 - 4*a^3*b*g*h^2 + 3*a^3*b*g^2*i + 18*a^2*b^2*c*g*i - 8*a^2*b^2*d*g*h -
24*a*b^3*c*d*h + 6*a*b^3*c*e*g)/(64*a^3*b^2) - (x*(3*b^3*c*d*e - 2*a^3*h^3
- 2*b^3*d^3 + 3*a^3*g*h*i - 6*a*b^2*d^2*h - 6*a^2*b*d*h^2 + 9*a*b^2*c*d*i +
3*a*b^2*c*e*h + a*b^2*d*e*g + 9*a^2*b*c*h*i + 3*a^2*b*d*g*i + a^2*b*e*g*h)
)/(16*a^3*b))*root(65536*a^7*b^7*z^4 + 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b^5*
c*i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^5*e*g*z^2 + 3072*a^4*b^6*c*e*z^
2 + 2048*a^6*b^4*h^2*z^2 + 2048*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z + 768
*a^4*b^4*d*e*i*z - 768*a^4*b^4*c*g*h*z - 768*a^3*b^5*c*d*g*z + 1152*a^6*b^2
*h*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^3*d*i^2*z + 128*a^4*b^4*e^2*h*z
- 1152*a^3*b^5*c^2*h*z - 128*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z - 1152*
a^2*b^6*c^2*d*z - 96*a^4*b^2*d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*c*e
*g*i - 32*a^3*b^3*d*e*g*h - 96*a^2*b^4*c*d*e*h + 12*a^4*b^2*e*g^2*i - 144*a
^4*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i - 16*a^4*b^2*e*g*h^2 + 108*a^4*b^2*c*g*
i^2 + 108*a^2*b^4*c^2*e*i - 144*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 - 16*a
^2*b^4*d^2*e*g + 12*a^2*b^4*c*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*e +
108*a^5*b*e*i^3 + 108*a*b^5*c^3*g + 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^2*i
^2 + 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 54*a^2*b^4*c^2*g^2 + 18*a^5*b
*g^2*i^2 + 12*a^3*b^3*e^3*i + 64*a^4*b^2*d*h^3 + 64*a^2*b^4*d^3*h + 12*a^3*
b^3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4 + 16*a*b^5*d^4 + 81*a^6*i^4 + 8
1*b^6*c^4 + a^4*b^2*g^4 + a^2*b^4*e^4, z, l), l, 1, 4) + ((x*(b*c - a*g))/(
4*a*b) - f/(4*b) + (x^2*(b*d - a*h))/(4*a*b) + (x^3*(b*e - a*i))/(4*a*b))/(
a + b*x^4)

3.197 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2} dx$

Optimal result

Integrand size = 45, antiderivative size = 417

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx \\
&= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{4ab\,(a+bx^4)} + \frac{(bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} \\
&\quad - \frac{\left(\sqrt{b}(3bc+ag)+\sqrt{a}(be+3ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{7/4}} \\
&\quad + \frac{\left(\sqrt{b}(3bc+ag)+\sqrt{a}(be+3ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{7/4}} \\
&\quad - \frac{\left(\sqrt{b}(3bc+ag)-\sqrt{a}(be+3ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{7/4}} \\
&\quad + \frac{\left(\sqrt{b}(3bc+ag)-\sqrt{a}(be+3ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{7/4}} + \frac{j\log(a+bx^4)}{4b^2}
\end{aligned}$$

```
[Out] 1/4*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+(-a*j+b*f)*x^3)/a/b/(b*x^4+a)+1/
4*(a*h+b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)+1/4*j*ln(b*x^4+a)/b
^2-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+b*e)*a^
(1/2)+(a*g+3*b*c)*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+1/32*ln(a^(1/4)*b^(1/4)*
x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a
^(7/4)/b^(7/4)*2^(1/2)+1/16*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*a*i+b*
e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+1/16*arctan(1+b^(1/
4)*x*2^(1/2)/a^(1/4))*((3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a^(7/4)/b^(
7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1872, 1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 211, 266}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx$$
$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}$$
$$+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}+\frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+bd)}{4a^{3/2}b^{3/2}}$$
$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}$$
$$+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}$$
$$+\frac{j\log(a+bx^4)}{4b^2}+\frac{x(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc)}{4ab(a+bx^4)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^2
,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(4*a*b*
(a + b*x^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/
2)) - ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g)
+ Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*b^(7/4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7
/4)) + ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + (j*Log[
a + b*x^4])/(4*b^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
```

```
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{4ab\,(a+bx^4)} - \frac{\int \frac{-b(3bc+ag)-2b(bd+ah)x-b(be+3ai)x^2-4abjx^3}{a+bx^4}\,dx}{4ab^2} \\
&= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{4ab\,(a+bx^4)} - \frac{\int \left(\frac{-b(3bc+ag)-b(be+3ai)x^2}{a+bx^4}+\frac{x(-2b(bd+ah)-4abjx^2)}{a+bx^4}\right)dx}{4ab^2} \\
&= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{4ab\,(a+bx^4)} - \frac{\int \frac{-b(3bc+ag)-b(be+3ai)x^2}{a+bx^4}\,dx}{4ab^2} - \frac{\int \frac{x(-2b(bd+ah)-4abjx^2)}{a+bx^4}\,dx}{4ab^2}
\end{aligned}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{4ab\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{-2b(bd+ah)-4abjx}{a+bx^2}\,dx, x, x^2\right)}{8ab^2} - \frac{\left(be-\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{8ab^2} + \frac{\left(be+\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{8ab^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{4ab} + \frac{\left(be-\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{5/4}b^{7/4}} + \frac{\left(be-\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{5/4}b^{7/4}} + \frac{\left(be+\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16ab^2} + \frac{\left(be+\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16ab^2} + \frac{j\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, x^2\right)}{2b}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} + \frac{\left(be-\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{5/4}b^{7/4}} - \frac{\left(be-\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{5/4}b^{7/4}} + \frac{j\log\left(a+bx^4\right)}{4b^2} + \frac{\left(be+\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}b^{7/4}} - \frac{\left(be+\frac{\sqrt{b}(3bc+ag)}{\sqrt{a}}+3ai\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}b^{7/4}}$$

$$= \frac{x(bc - ag + (bd - ah)x + (be - ai)x^2 + (bf - aj)x^3)}{4ab\,(a + bx^4)} + \frac{(bd + ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} - \frac{\left(be + \frac{\sqrt{b}(3bc+ag)}{\sqrt{a}} + 3ai\right)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}b^{7/4}} + \frac{\left(be + \frac{\sqrt{b}(3bc+ag)}{\sqrt{a}} + 3ai\right)\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}b^{7/4}} + \frac{\left(be - \frac{\sqrt{b}(3bc+ag)}{\sqrt{a}} + 3ai\right)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{16\sqrt{2}a^{5/4}b^{7/4}} - \frac{\left(be - \frac{\sqrt{b}(3bc+ag)}{\sqrt{a}} + 3ai\right)\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{16\sqrt{2}a^{5/4}b^{7/4}} + \frac{j\log(a + bx^4)}{4b^2}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.10

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{(a + bx^4)^2}\,dx$$

$$= \frac{\frac{8(a^2j+b^2x(c+x(d+ex))-ab(f+x(g+x(h+ix))))}{a(a+bx^4)} - \frac{2\sqrt[4]{b}\left(3\sqrt{2}b^{3/2}c+4\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+4a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i\right)\arctan\left(1-\right.}{a^{7/4}}}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4)^2,x]
```

```
[Out] ((8*(a^2*j + b^2*x*(c + x*(d + e*x)) - a*b*(f + x*(g + x*(h + i*x)))))/(a*(
a + b*x^4)) - (2*b^(1/4)*(3*Sqrt[2]*b^(3/2)*c + 4*a^(1/4)*b^(5/4)*d + Sqrt[
2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3
/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*b^(1/4)*(3*Sqr
t[2]*b^(3/2)*c - 4*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt
[b]*g - 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*x)/a^(1/4)])/a^(7/4) + (Sqrt[2]*b^(1/4)*(-3*b^(3/2)*c + Sqrt[a]*b*e - a
*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/a^(7/4) + (Sqrt[2]*b^(1/4)*(3*b^(3/2)*c - Sqrt[a]*b*e + a*Sqrt[b]*g
- 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7
/4) + 8*j*Log[a + b*x^4])/(32*b^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.37

method	result
risch	$\frac{-\frac{(ai-be)x^3}{4ab}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}+\frac{aj-bf}{4b^2}}{b\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{\left(4j\_R^3+\frac{(3ai+be)\_R^2}{a}+\frac{2(ah+bd)\_R}{a}+\frac{ag+3bc}{a}\right)\ln(x-\_R)}{\_R^3}}{16b^2}$
default	$\frac{-\frac{(ai-be)x^3}{4ab}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}+\frac{aj-bf}{4b^2}}{b\,x^4+a}+\frac{\frac{(ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}}{}$

[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)

[Out] (-1/4*(a*i-b*e)/a/b*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x+1/4*(a*j-b*f)/b^2)/(b*x^4+a)+1/16/b^2*sum((4*j*_R^3+1/a*(3*a*i+b*e)*_R^2+2/a*(a*h+b*d)*_R+(a*g+3*b*c)/a)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx = \text{Timed out}$$

[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx = \text{Timed out}$$

[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.10

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx = \frac{(b^2e-abi)x^3-abf+a^2j+(b^2d-abh)x^2+(b^2c-abg)x}{4\,(ab^3x^4+a^2b^2)} + \frac{\frac{\sqrt{2}\left(4\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}j+3\,b^2c-\sqrt{a}b^{\frac{3}{2}}e+abg-3\,a^{\frac{3}{2}}\sqrt{b}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(4\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}j-3\,b^2c+\sqrt{a}b^{\frac{3}{2}}e-abg+3\,a^{\frac{3}{2}}\sqrt{b}i\right)\log\left(\sqrt{b}x^2\right.}{a^{\frac{3}{4}}b^{\frac{5}{4}}}}{}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algori
thm="maxima")
```

```
[Out] 1/4*((b^2*e - a*b*i)*x^3 - a*b*f + a^2*j + (b^2*d - a*b*h)*x^2 + (b^2*c - a
*b*g)*x)/(a*b^3*x^4 + a^2*b^2) + 1/32*(sqrt(2)*(4*sqrt(2)*a^(7/4)*b^(1/4)*j
 + 3*b^2*c - sqrt(a)*b^(3/2)*e + a*b*g - 3*a^(3/2)*sqrt(b)*i)*log(sqrt(b)*x
^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(4*sq
rt(2)*a^(7/4)*b^(1/4)*j - 3*b^2*c + sqrt(a)*b^(3/2)*e - a*b*g + 3*a^(3/2)*s
qrt(b)*i)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b
^(5/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e + sqrt
(2)*a^(5/4)*b^(5/4)*g + 3*sqrt(2)*a^(7/4)*b^(3/4)*i - 4*sqrt(a)*b^2*d - 4*a
^(3/2)*b*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt
(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(3*sqrt(2)*a
^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e + sqrt(2)*a^(5/4)*b^(5/4)*g +
3*sqrt(2)*a^(7/4)*b^(3/4)*i + 4*sqrt(a)*b^2*d + 4*a^(3/2)*b*h)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3
/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.17

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx$$

$$= \frac{j\log\left(|bx^4+a|\right)}{4\,b^2} + \frac{(be-ai)x^3+(bd-ah)x^2+(bc-ag)x-\frac{abf-a^2j}{b}}{4\,(bx^4+a)ab}$$

$$+ \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^3d+2\sqrt{2}\sqrt{ab}ab^2h+3\,(ab^3)^{\frac{1}{4}}\,b^3c+(ab^3)^{\frac{1}{4}}\,ab^2g+(ab^3)^{\frac{3}{4}}\,be+3\,(ab^3)^{\frac{3}{4}}\,ai\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\cdots\right.}{2\,(\cdots}\right.}{16\,a^2b^4}$$

$$+ \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^3d+2\sqrt{2}\sqrt{ab}ab^2h+3\,(ab^3)^{\frac{1}{4}}\,b^3c+(ab^3)^{\frac{1}{4}}\,ab^2g+(ab^3)^{\frac{3}{4}}\,be+3\,(ab^3)^{\frac{3}{4}}\,ai\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\cdots\right.}{2\,(\cdots}\right.}{16\,a^2b^4}$$

$$+ \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^3c+(ab^3)^{\frac{1}{4}}\,ab^2g-(ab^3)^{\frac{3}{4}}\,be-3\,(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^4}$$

$$- \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^3c+(ab^3)^{\frac{1}{4}}\,ab^2g-(ab^3)^{\frac{3}{4}}\,be-3\,(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^4}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algori
thm="giac")
```

```
[Out] 1/4*j*log(abs(b*x^4 + a))/b^2 + 1/4*((b*e - a*i)*x^3 + (b*d - a*h)*x^2 + (b
*c - a*g)*x - (a*b*f - a^2*j)/b)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(2*sqrt(2
)*sqrt(a*b)*b^3*d + 2*sqrt(2)*sqrt(a*b)*a*b^2*h + 3*(a*b^3)^(1/4)*b^3*c + (
a*b^3)^(1/4)*a*b^2*g + (a*b^3)^(3/4)*b*e + 3*(a*b^3)^(3/4)*a*i)*arctan(1/2*
sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 1/16*sqrt(2)*(
2*sqrt(2)*sqrt(a*b)*b^3*d + 2*sqrt(2)*sqrt(a*b)*a*b^2*h + 3*(a*b^3)^(1/4)*b
^3*c + (a*b^3)^(1/4)*a*b^2*g + (a*b^3)^(3/4)*b*e + 3*(a*b^3)^(3/4)*a*i)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 1/32*s
qrt(2)*(3*(a*b^3)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b^2*g - (a*b^3)^(3/4)*b*e -
 3*(a*b^3)^(3/4)*a*i)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4
) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b^2*g - (a*b^3)^(
3/4)*b*e - 3*(a*b^3)^(3/4)*a*i)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b)
)/(a^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 10.39 (sec) , antiderivative size = 3939, normalized size of antiderivative = 9.45

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^2
,x)
```

```
[Out] ((x*(b*c - a*g))/(4*a*b) - (b*f - a*j)/(4*b^2) + (x^2*(b*d - a*h))/(4*a*b)
+ (x^3*(b*e - a*i))/(4*a*b))/(a + b*x^4) + symsum(log(- root(65536*a^7*b^8*
z^4 - 65536*a^7*b^6*j*z^3 + 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6*c*i*z^2 + 4
096*a^5*b^6*d*h*z^2 + 1024*a^5*b^6*e*g*z^2 + 3072*a^4*b^7*c*e*z^2 + 24576*a
^7*b^4*j^2*z^2 + 2048*a^6*b^5*h^2*z^2 + 2048*a^4*b^7*d^2*z^2 - 1536*a^6*b^3
*g*i*j*z - 4608*a^5*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^5*b^4*e*h*i*
z - 512*a^5*b^4*e*g*j*z - 1536*a^4*b^5*c*e*j*z + 768*a^4*b^5*d*e*i*z - 768*
a^4*b^5*c*g*h*z - 768*a^3*b^6*c*d*g*z - 1024*a^6*b^3*h^2*j*z + 1152*a^6*b^3
*h*i^2*z - 128*a^5*b^4*g^2*h*z - 1024*a^4*b^5*d^2*j*z + 1152*a^5*b^4*d*i^2*
z + 128*a^4*b^5*e^2*h*z - 1152*a^3*b^6*c^2*h*z - 128*a^4*b^5*d*g^2*z + 128*
a^3*b^6*d*e^2*z - 1152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z - 192*a^5*b^2*e
*h*i*j - 192*a^4*b^3*d*e*i*j + 192*a^4*b^3*c*g*h*j - 96*a^4*b^3*d*g*h*i - 2
88*a^3*b^4*c*d*h*i + 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*i - 32*a^3*b^4*
d*e*g*h - 96*a^2*b^5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b^2*g*h^2*i - 28
8*a^5*b^2*d*i^2*j - 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2 + 256*a^5*b^2*
d*h*j^2 + 64*a^5*b^2*e*g*j^2 + 288*a^3*b^4*c^2*h*j + 32*a^4*b^3*d*g^2*j + 1
2*a^4*b^3*e*g^2*i - 144*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i - 16*a^4*b^3*e
*g*h^2 + 108*a^4*b^3*c*g*i^2 - 32*a^3*b^4*d*e^2*j + 192*a^4*b^3*c*e*j^2 + 2
88*a^2*b^5*c^2*d*j + 108*a^2*b^5*c^2*e*i - 144*a^2*b^5*c*d^2*i - 48*a^3*b^4
*c*e*h^2 - 16*a^2*b^5*d^2*e*g + 12*a^2*b^5*c*e^2*g - 288*a^6*b*h*i^2*j + 19
2*a^6*b*g*i*j^2 - 48*a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^5*b^2*g^2*i^2 +
 128*a^4*b^3*d^2*j^2 + 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*i^2 + 96*a^3*b^
4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 54*a^2*b^5*c^2*g^2 + 128*a^6*b*h^2*j^2 + 10
8*a^5*b^2*e*i^3 + 12*a^3*b^4*e^3*i + 64*a^4*b^3*d*h^3 + 64*a^2*b^5*d^3*h +
12*a^3*b^4*c*g^3 + 18*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 + 81*a^6*b*i^4 + 16*a*
b^6*d^4 + 256*a^7*j^4 + 81*b^7*c^4 + a^4*b^3*g^4 + a^2*b^5*e^4, z, m)*(root
(65536*a^7*b^8*z^4 - 65536*a^7*b^6*j*z^3 + 3072*a^6*b^5*g*i*z^2 + 9216*a^5*
b^6*c*i*z^2 + 4096*a^5*b^6*d*h*z^2 + 1024*a^5*b^6*e*g*z^2 + 3072*a^4*b^7*c*
e*z^2 + 24576*a^7*b^4*j^2*z^2 + 2048*a^6*b^5*h^2*z^2 + 2048*a^4*b^7*d^2*z^2
 - 1536*a^6*b^3*g*i*j*z - 4608*a^5*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768
*a^5*b^4*e*h*i*z - 512*a^5*b^4*e*g*j*z - 1536*a^4*b^5*c*e*j*z + 768*a^4*b^5
*d*e*i*z - 768*a^4*b^5*c*g*h*z - 768*a^3*b^6*c*d*g*z - 1024*a^6*b^3*h^2*j*z
 + 1152*a^6*b^3*h*i^2*z - 128*a^5*b^4*g^2*h*z - 1024*a^4*b^5*d^2*j*z + 1152
*a^5*b^4*d*i^2*z + 128*a^4*b^5*e^2*h*z - 1152*a^3*b^6*c^2*h*z - 128*a^4*b^5
*d*g^2*z + 128*a^3*b^6*d*e^2*z - 1152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z
```

- 192*a^5*b^2*e*h*i*j - 192*a^4*b^3*d*e*i*j + 192*a^4*b^3*c*g*h*j - 96*a^4*
b^3*d*g*h*i - 288*a^3*b^4*c*d*h*i + 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*
i - 32*a^3*b^4*d*e*g*h - 96*a^2*b^5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b
^2*g*h^2*i - 288*a^5*b^2*d*i^2*j - 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2
 + 256*a^5*b^2*d*h*j^2 + 64*a^5*b^2*e*g*j^2 + 288*a^3*b^4*c^2*h*j + 32*a^4*
b^3*d*g^2*j + 12*a^4*b^3*e*g^2*i - 144*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i
 - 16*a^4*b^3*e*g*h^2 + 108*a^4*b^3*c*g*i^2 - 32*a^3*b^4*d*e^2*j + 192*a^4*
b^3*c*e*j^2 + 288*a^2*b^5*c^2*d*j + 108*a^2*b^5*c^2*e*i - 144*a^2*b^5*c*d^2
*i - 48*a^3*b^4*c*e*h^2 - 16*a^2*b^5*d^2*e*g + 12*a^2*b^5*c*e^2*g - 288*a^6
*b*h*i^2*j + 192*a^6*b*g*i*j^2 - 48*a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^
5*b^2*g^2*i^2 + 128*a^4*b^3*d^2*j^2 + 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*
i^2 + 96*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 54*a^2*b^5*c^2*g^2 + 128*a^6
*b*h^2*j^2 + 108*a^5*b^2*e*i^3 + 12*a^3*b^4*e^3*i + 64*a^4*b^3*d*h^3 + 64*a
^2*b^5*d^3*h + 12*a^3*b^4*c*g^3 + 18*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 + 81*a^
6*b*i^4 + 16*a*b^6*d^4 + 256*a^7*j^4 + 81*b^7*c^4 + a^4*b^3*g^4 + a^2*b^5*e
^4, z, m)*((768*a^3*b^5*c + 256*a^4*b^4*g)/(64*a^3*b^2) - (x*(128*a^3*b^5*d
 + 128*a^4*b^4*h))/(16*a^3*b^2)) + (64*a^2*b^4*d*e - 384*a^3*b^3*c*j + 192*
a^3*b^3*d*i + 64*a^3*b^3*e*h - 128*a^4*b^2*g*j + 192*a^4*b^2*h*i)/(64*a^3*b
^2) + (x*(36*a*b^5*c^2 - 4*a^2*b^4*e^2 + 4*a^3*b^3*g^2 - 36*a^4*b^2*i^2 + 2
4*a^2*b^4*c*g + 64*a^3*b^3*d*j - 24*a^3*b^3*e*i + 64*a^4*b^2*h*j))/(16*a^3*
b^2)) - (27*a^4*i^3 + a*b^3*e^3 - 12*b^4*c*d^2 + 9*b^4*c^2*e + 16*a^4*g*j^2
 - 12*a^2*b^2*c*h^2 + a^2*b^2*e*g^2 + 9*a^2*b^2*e^2*i - 48*a^4*h*i*j - 4*a*
b^3*d^2*g + 27*a*b^3*c^2*i + 48*a^3*b*c*j^2 + 27*a^3*b*e*i^2 - 4*a^3*b*g*h^
2 + 3*a^3*b*g^2*i + 18*a^2*b^2*c*g*i - 16*a^2*b^2*d*e*j - 8*a^2*b^2*d*g*h -
 24*a*b^3*c*d*h + 6*a*b^3*c*e*g - 48*a^3*b*d*i*j - 16*a^3*b*e*h*j)/(64*a^3*
b^2) - (x*(9*a^4*i^2*j - 2*a^3*b*h^3 - 8*a^4*h*j^2 - 2*b^4*d^3 - 6*a^2*b^2*
d*h^2 + a^2*b^2*e^2*j + 3*b^4*c*d*e - 6*a*b^3*d^2*h - 9*a*b^3*c^2*j - 8*a^3
*b*d*j^2 - a^3*b*g^2*j - 6*a^2*b^2*c*g*j + 9*a^2*b^2*c*h*i + 3*a^2*b^2*d*g*
i + a^2*b^2*e*g*h + 9*a*b^3*c*d*i + 3*a*b^3*c*e*h + a*b^3*d*e*g + 6*a^3*b*e
*i*j + 3*a^3*b*g*h*i))/(16*a^3*b^2))*root(65536*a^7*b^8*z^4 - 65536*a^7*b^6
*j*z^3 + 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6*c*i*z^2 + 4096*a^5*b^6*d*h*z^2
 + 1024*a^5*b^6*e*g*z^2 + 3072*a^4*b^7*c*e*z^2 + 24576*a^7*b^4*j^2*z^2 + 20
48*a^6*b^5*h^2*z^2 + 2048*a^4*b^7*d^2*z^2 - 1536*a^6*b^3*g*i*j*z - 4608*a^5
*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^5*b^4*e*h*i*z - 512*a^5*b^4*e*g
*j*z - 1536*a^4*b^5*c*e*j*z + 768*a^4*b^5*d*e*i*z - 768*a^4*b^5*c*g*h*z - 7
68*a^3*b^6*c*d*g*z - 1024*a^6*b^3*h^2*j*z + 1152*a^6*b^3*h*i^2*z - 128*a^5*
b^4*g^2*h*z - 1024*a^4*b^5*d^2*j*z + 1152*a^5*b^4*d*i^2*z + 128*a^4*b^5*e^2
*h*z - 1152*a^3*b^6*c^2*h*z - 128*a^4*b^5*d*g^2*z + 128*a^3*b^6*d*e^2*z - 1
152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z - 192*a^5*b^2*e*h*i*j - 192*a^4*b^
3*d*e*i*j + 192*a^4*b^3*c*g*h*j - 96*a^4*b^3*d*g*h*i - 288*a^3*b^4*c*d*h*i
+ 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*i - 32*a^3*b^4*d*e*g*h - 96*a^2*b^
5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b^2*g*h^2*i - 288*a^5*b^2*d*i^2*j -
 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2 + 256*a^5*b^2*d*h*j^2 + 64*a^5*b^
2*e*g*j^2 + 288*a^3*b^4*c^2*h*j + 32*a^4*b^3*d*g^2*j + 12*a^4*b^3*e*g^2*i -
 144*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i - 16*a^4*b^3*e*g*h^2 + 108*a^4*b^

```
3*c*g*i^2 - 32*a^3*b^4*d*e^2*j + 192*a^4*b^3*c*e*j^2 + 288*a^2*b^5*c^2*d*j
+ 108*a^2*b^5*c^2*e*i - 144*a^2*b^5*c*d^2*i - 48*a^3*b^4*c*e*h^2 - 16*a^2*b
^5*d^2*e*g + 12*a^2*b^5*c*e^2*g - 288*a^6*b*h*i^2*j + 192*a^6*b*g*i*j^2 - 4
8*a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^5*b^2*g^2*i^2 + 128*a^4*b^3*d^2*j^
2 + 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*i^2 + 96*a^3*b^4*d^2*h^2 + 2*a^3*b
^4*e^2*g^2 + 54*a^2*b^5*c^2*g^2 + 128*a^6*b*h^2*j^2 + 108*a^5*b^2*e*i^3 + 1
2*a^3*b^4*e^3*i + 64*a^4*b^3*d*h^3 + 64*a^2*b^5*d^3*h + 12*a^3*b^4*c*g^3 +
18*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 + 81*a^6*b*i^4 + 16*a*b^6*d^4 + 256*a^7*j
^4 + 81*b^7*c^4 + a^4*b^3*g^4 + a^2*b^5*e^4, z, m), m, 1, 4)
```

3.198 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3} dx$

Optimal result

Integrand size = 36, antiderivative size = 241

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3} dx = \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{8ab(a-bx^4)^2} + \frac{4af+x(7bc-ag+2(3bd-ah)x+5bex^2)}{32a^2b(a-bx^4)} + \frac{\left(21bc-5\sqrt{a}\sqrt{b}e-3ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{5/4}} + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e-3ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{5/4}} + \frac{(3bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}$$

```
[Out] 1/8*x*(b*c+a*g+(a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(-b*x^4+a)^2+1/32*(4*a*f+x*
(7*b*c-a*g+2*(-a*h+3*b*d)*x+5*b*e*x^2))/a^2/b/(-b*x^4+a)+1/16*(-a*h+3*b*d)*
arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)+1/64*arctan(b^(1/4)*x/a^(1/4))
*(21*b*c-3*a*g-5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)+1/64*arctanh(b^(1/4)*x
/a^(1/4))*(21*b*c-3*a*g+5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1872, 1868, 1890, 281, 214, 1181, 211}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(3bd-ah)}{16a^{5/2}b^{3/2}} + \frac{x(2x(3bd-ah)-ag+7bc+5bex^2)+4af}{32a^2b\,(a-bx^4)} + \frac{x(x(ah+bd)+ag+bc+bex^2+bfx^3)}{8ab\,(a-bx^4)^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) +
 (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + 5*b*e*x^2))/(32*a^2*b*(a - b
*x^4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)
])/(64*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(5/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{8ab\left(a-bx^4\right)^2} - \frac{\int \frac{-b(7bc-ag)-2b(3bd-ah)x-5b^2ex^2-4b^2fx^3}{(a-bx^4)^2}\,dx}{8ab^2} \\ &= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{8ab\left(a-bx^4\right)^2} \\ &\quad + \frac{4af+x(7bc-ag+2(3bd-ah)x+5bex^2)}{32a^2b\left(a-bx^4\right)} + \frac{\int \frac{3b(7bc-ag)+4b(3bd-ah)x+5b^2ex^2}{a-bx^4}\,dx}{32a^2b^2}\end{aligned}$$

$$
\begin{aligned}
&= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{8ab\left(a-bx^4\right)^2} \\
&\quad + \frac{4af+x(7bc-ag+2(3bd-ah)x+5bex^2)}{32a^2b\left(a-bx^4\right)} \\
&\quad + \frac{\int\left(\frac{4b(3bd-ah)x}{a-bx^4}+\frac{3b(7bc-ag)+5b^2ex^2}{a-bx^4}\right)dx}{32a^2b^2} \\
&= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{8ab\left(a-bx^4\right)^2} \\
&\quad + \frac{4af+x(7bc-ag+2(3bd-ah)x+5bex^2)}{32a^2b\left(a-bx^4\right)} \\
&\quad + \frac{\int\frac{3b(7bc-ag)+5b^2ex^2}{a-bx^4}dx}{32a^2b^2} + \frac{(3bd-ah)\int\frac{x}{a-bx^4}dx}{8a^2b} \\
&= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{8ab\left(a-bx^4\right)^2} \\
&\quad + \frac{4af+x(7bc-ag+2(3bd-ah)x+5bex^2)}{32a^2b\left(a-bx^4\right)} \\
&\quad - \frac{\left(21bc-5\sqrt{a}\sqrt{b}e-3ag\right)\int\frac{1}{-\sqrt{a}\sqrt{b}-bx^2}dx}{64a^{5/2}\sqrt{b}} \\
&\quad + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e-3ag\right)\int\frac{1}{\sqrt{a}\sqrt{b}-bx^2}dx}{64a^{5/2}\sqrt{b}} + \frac{(3bd-ah)\mathrm{Subst}\left(\int\frac{1}{a-bx^2}dx,x,x^2\right)}{16a^2b} \\
&= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{8ab\left(a-bx^4\right)^2} \\
&\quad + \frac{4af+x(7bc-ag+2(3bd-ah)x+5bex^2)}{32a^2b\left(a-bx^4\right)} \\
&\quad + \frac{\left(21bc-5\sqrt{a}\sqrt{b}e-3ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{5/4}} \\
&\quad + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e-3ag\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{5/4}} + \frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.28

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx$$

$$= \frac{\frac{4a^{3/4}\sqrt{b}x(7bc+bx(6d+5ex)-a(g+2hx))}{a-bx^4}+\frac{16a^{7/4}\sqrt{b}(bx(c+x(d+ex))+a(f+x(g+hx)))}{(a-bx^4)^2}+2\sqrt[4]{b}\left(21bc-5\sqrt{a}\sqrt{b}e-3ag\right)\arctan\left(\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x]
```

```
[Out] ((4*a^(3/4)*Sqrt[b]*x*(7*b*c + b*x*(6*d + 5*e*x) - a*(g + 2*h*x)))/(a - b*x
^4) + (16*a^(7/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + h*x))))/(a
 - b*x^4)^2 + 2*b^(1/4)*(21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1
/4)*x)/a^(1/4)] + (-21*b^(5/4)*c - 12*a^(1/4)*b*d - 5*Sqrt[a]*b^(3/4)*e + 3
*a*b^(1/4)*g + 4*a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (21*b^(5/4)*c - 12*a
^(1/4)*b*d + 5*Sqrt[a]*b^(3/4)*e - 3*a*b^(1/4)*g + 4*a^(5/4)*h)*Log[a^(1/4)
 + b^(1/4)*x] - 4*a^(1/4)*(-3*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a
^(11/4)*b^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.72

method	result
risch	$\frac{-\frac{5be\,x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-b\,x^4+a)^2}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)}\frac{\left(5\_R^2e-\frac{4(ah-3bd)\_R}{b}-\frac{3(ag-}{b}\right.}{\_R^3}}{128a^2b}$
default	$\frac{-\frac{5be\,x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-b\,x^4+a)^2}+\frac{\frac{(-3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+}{}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-5/32*b*e/a^2*x^7+1/16*(a*h-3*b*d)/a^2*x^6+1/32*(a*g-7*b*c)/a^2*x^5+9/32/a
*e*x^3+1/16*(a*h+5*b*d)/a/b*x^2+1/32*(3*a*g+11*b*c)/a/b*x+1/8*f/b)/(-b*x^4+
a)^2-1/128/a^2/b*sum((5*_R^2*e-4/b*(a*h-3*b*d)*_R-3/b*(a*g-7*b*c))/_R^3*ln(
x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas
")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.31

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx = -\frac{5\,b^2ex^7+2\,(3\,b^2d-abh)x^6-9\,abex^3+(7\,b^2c-abg)x^5-4\,a^2f-2\,(5\,abd+a^2h)x^2-(11\,abc+3\,a^2g)x}{32\,(a^2b^3x^8-2\,a^3b^2x^4+a^4b)} + \frac{\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be-3\,a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\,b^{\frac{3}{2}}c+5\,\sqrt{a}be-3\,a\sqrt{\cdots}\right.}{\sqrt{a}\sqrt{\cdots}}}{128\,a^2b}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima
")
[Out] -1/32*(5*b^2*e*x^7 + 2*(3*b^2*d - a*b*h)*x^6 - 9*a*b*e*x^3 + (7*b^2*c - a*b
*g)*x^5 - 4*a^2*f - 2*(5*a*b*d + a^2*h)*x^2 - (11*a*b*c + 3*a^2*g)*x)/(a^2*
b^3*x^8 - 2*a^3*b^2*x^4 + a^4*b) + 1/128*(4*(3*b*d - a*h)*log(sqrt(b)*x^2 +
 sqrt(a))/(sqrt(a)*sqrt(b)) - 4*(3*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(s
qrt(a)*sqrt(b)) + 2*(21*b^(3/2)*c - 5*sqrt(a)*b*e - 3*a*sqrt(b)*g)*arctan(s
qrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (
21*b^(3/2)*c + 5*sqrt(a)*b*e - 3*a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)
*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
b))*sqrt(b)))/(a^2*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(201) = 402.

Time = 0.28 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.80

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx = -\frac{\sqrt{2}\left(21\,b^2c-3\,abg-12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+4\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,ah+5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2} - \frac{\sqrt{2}\left(21\,b^2c-3\,abg+12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-4\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,ah-5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2} - \frac{\sqrt{2}\left(21\,b^2c-3\,abg-5\,\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2} + \frac{\sqrt{2}\left(21\,b^2c-3\,abg-5\,\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2} - \frac{5\,b^2ex^7+6\,b^2dx^6-2\,abhx^6+7\,b^2cx^5-abgx^5-9\,abex^3-10\,abdx^2-2\,a^2hx^2-11\,abcx-3\,a^2gx-4\,a^2}{32\,(bx^4-a)^2a^2b}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*(21*b^2*c - 3*a*b*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 4*sqrt
(2)*(-a*b^3)^(1/4)*a*h + 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2
)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*
c - 3*a*b*g + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 4*sqrt(2)*(-a*b^3)^(1/4)*a*h
- 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^
(1/4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a
*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2
) + 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)
*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b^2*e*x^7 + 6*
b^2*d*x^6 - 2*a*b*h*x^6 + 7*b^2*c*x^5 - a*b*g*x^5 - 9*a*b*e*x^3 - 10*a*b*d*
x^2 - 2*a^2*h*x^2 - 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 9.89 (sec) , antiderivative size = 1687, normalized size of antiderivative = 7.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a-bx^4\right)^3}\,dx = \text{Too large to display}$$

[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x)

[Out] (f/(8*b) + (9*e*x^3)/(32*a) - (x^5*(7*b*c - a*g))/(32*a^2) - (x^6*(3*b*d - a*h))/(16*a^2) + (x*(11*b*c + 3*a*g))/(32*a*b) + (x^2*(5*b*d + a*h))/(16*a*b) - (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4) + symsum(log(- root(268435456*a^11*b^6*z^4 + 3145728*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 - 6881280*a^6*b^5*c*e*z^2 - 524288*a^8*b^3*h^2*z^2 - 4718592*a^6*b^5*d^2*z^2 + 258048*a^5*b^3*c*g*h*z - 774144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z - 51200*a^5*b^3*e^2*h*z - 903168*a^4*b^4*c^2*h*z + 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z + 2709504*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h + 40320*a^2*b^3*c*d*e*h + 8640*a^2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 - 6300*a^2*b^3*c*e^2*g + 960*a^4*b*e*g*h^2 - 60480*a*b^4*c*d^2*e - 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 13824*a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 - 23814*a^2*b^3*c^2*g^2 - 27648*a^2*b^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 - 625*a^2*b^3*e^4 - 81*a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 - 194481*b^5*c^4, z, k)*(root(268435456*a^11*b^6*z^4 + 3145728*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 - 6881280*a^6*b^5*c*e*z^2 - 524288*a^8*b^3*h^2*z^2 - 4718592*a^6*b^5*d^2*z^2 + 258048*a^5*b^3*c*g*h*z - 774144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z - 51200*a^5*b^3*e^2*h*z - 903168*a^4*b^4*c^2*h*z + 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z + 2709504*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h + 40320*a^2*b^3*c*d*e*h + 8640*a^2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 - 6300*a^2*b^3*c*e^2*g + 960*a^4*b*e*g*h^2 - 60480*a*b^4*c*d^2*e - 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 13824*a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 - 23814*a^2*b^3*c^2*g^2 - 27648*a^2*b^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 - 625*a^2*b^3*e^4 - 81*a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 - 194481*b^5*c^4, z, k)*((344064*a^5*b^4*c - 49152*a^6*b^3*g)/(32768*a^6*b) - (x*(24576*a^5*b^4*d - 8192*a^6*b^3*h))/(4096*a^6*b)) - (15360*a^3*b^3*d*e - 5120*a^4*b^2*e*h)/(32768*a^6*b) + (x*(7056*a^2*b^4*c^2 + 400*a^3*b^3*e^2 + 144*a^4*b^2*g^2 - 2016*a^3*b^3*c*g))/(4096*a^6*b)) - (125*a*b^2*e^3 + 3024*b^3*c*d^2 - 2205*b^3*c^2*e - 48*a^3*g*h^2 - 432*a*b^2*d^2*g + 336*a^2*b*c*h^2 - 45*a^2*b*e*g^2 - 2016*a*b^2*c*d*h + 630*a*b^2*c*e*g + 288*a^2*b*d*g*h)/(32768*a^6*b) - (x*(216*b^3*d^3 - 8*a^3*h^3 - 315*b^3*c*d*e - 216*a*b^2*d^2*h + 72*a^2*b*d*h^2 + 105*a*b^2*c*e*h + 45*a*b^2*d*e*g - 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^6*z^4 + 3145728*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 - 6881280*a^6*b^5*c*e*z^2 - 524288*a^8*b^3*h^2*z^2 - 4718592*a^6*b^5*d^2*z^2 + 258048*a^5*b^3*c*g*h*z - 774144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z - 51200*a^5*b^3*e^2*h*z - 903168*a^4*b^4*c^2*h*z + 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z + 270

```
9504*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h + 40320*a^2*b^3*c*d*e*h + 8640*
a^2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 - 6300*a^2*b^3*c*e^2*g + 960*a^4*b*e
*g*h^2 - 60480*a*b^4*c*d^2*e - 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 1382
4*a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 - 23814*a^2*b^3*c^2*g^2 - 27648*a^2
*b^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 - 625*a^2*b^3*e^4 - 8
1*a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 - 194481*b^5*c^4, z, k), k, 1,
4)
```

3.199 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 268

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx \\
&= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{8ab\,(a-bx^4)^2} \\
&\quad + \frac{4af+x(7bc-ag+2(3bd-ah)x+(5be-3ai)x^2)}{32a^2b\,(a-bx^4)} \\
&\quad - \frac{\left(5be-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{7/4}} \\
&\quad + \frac{\left(5be+\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{7/4}} + \frac{(3bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}
\end{aligned}$$

```
[Out] 1/8*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+b*f*x^3)/a/b/(-b*x^4+a)^2+1/32*(4*
a*f+x*(7*b*c-a*g+2*(-a*h+3*b*d)*x+(-3*a*i+5*b*e)*x^2))/a^2/b/(-b*x^4+a)+1/1
6*(-a*h+3*b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)-1/64*arctan(b^(
1/4)*x/a^(1/4))*(5*b*e-3*a*i-3*(-a*g+7*b*c)*b^(1/2)/a^(1/2))/a^(9/4)/b^(7/4
)+1/64*arctanh(b^(1/4)*x/a^(1/4))*(5*b*e-3*a*i+3*(-a*g+7*b*c)*b^(1/2)/a^(1/
2))/a^(9/4)/b^(7/4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1872, 1868, 1890, 281, 214, 1181, 211}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx$$
$$=-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}$$
$$+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(3bd-ah)}{16a^{5/2}b^{3/2}}$$
$$+\frac{x(2x(3bd-ah)+x^2(5be-3ai)-ag+7bc)+4af}{32a^2b\,(a-bx^4)}$$
$$+\frac{x(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3)}{8ab\,(a-bx^4)^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(8*a*b*(a - b*x
^4)^2) + (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + (5*b*e - 3*a*i)*x^2)
)/(32*a^2*b*(a - b*x^4)) - ((5*b*e - (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*
a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((5*b*e + (3*Sqrt[
b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4
)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(
3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{8ab\left(a-bx^4\right)^2} - \frac{\int \frac{-b(7bc-ag)-2b(3bd-ah)x-b(5be-3ai)x^2-4b^2fx^3}{\left(a-bx^4\right)^2}\,dx}{8ab^2}\\ &= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x(7bc-ag+2(3bd-ah)x+(5be-3ai)x^2)}{32a^2b\left(a-bx^4\right)} + \frac{\int \frac{3b(7bc-ag)+4b(3bd-ah)x+b(5be-3ai)x^2}{a-bx^4}\,dx}{32a^2b^2}\end{aligned}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x(7bc-ag+2(3bd-ah)x+(5be-3ai)x^2)}{32a^2b\left(a-bx^4\right)} + \frac{\int \left(\frac{4b(3bd-ah)x}{a-bx^4}+\frac{3b(7bc-ag)+b(5be-3ai)x^2}{a-bx^4}\right)dx}{32a^2b^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x(7bc-ag+2(3bd-ah)x+(5be-3ai)x^2)}{32a^2b\left(a-bx^4\right)} + \frac{\int \frac{3b(7bc-ag)+b(5be-3ai)x^2}{a-bx^4}dx}{32a^2b^2} + \frac{(3bd-ah)\int \frac{x}{a-bx^4}dx}{8a^2b}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x(7bc-ag+2(3bd-ah)x+(5be-3ai)x^2)}{32a^2b\left(a-bx^4\right)} + \frac{(3bd-ah)\text{Subst}\left(\int \frac{1}{a-bx^2}dx,x,x^2\right)}{16a^2b} + \frac{\left(5be-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}dx}{64a^2b} + \frac{\left(5be+\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}dx}{64a^2b}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x(7bc-ag+2(3bd-ah)x+(5be-3ai)x^2)}{32a^2b\left(a-bx^4\right)} - \frac{\left(5be-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{7/4}} + \frac{\left(5be+\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{7/4}} + \frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.34

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx$$
$$= \frac{-\frac{4a^{3/4}b^{3/4}x(-b(7c+x(6d+5ex))+a(g+x(2h+3ix)))}{a-bx^4}+\frac{16a^{7/4}b^{3/4}(bx(c+x(d+ex))+a(f+x(g+x(h+ix))))}{(a-bx^4)^2}+2\left(21b^{3/2}c-5\sqrt{a}be-\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x
]
```

```
[Out] ((-4*a^(3/4)*b^(3/4)*x*(-(b*(7*c + x*(6*d + 5*e*x))) + a*(g + x*(2*h + 3*i*
x))))/(a - b*x^4) + (16*a^(7/4)*b^(3/4)*(b*x*(c + x*(d + e*x)) + a*(f + x*(
g + x*(h + i*x)))))/(a - b*x^4)^2 + 2*(21*b^(3/2)*c - 5*Sqrt[a]*b*e - 3*a*S
qrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] + (-21*b^(3/2)*c - 12*a
^(1/4)*b^(5/4)*d - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*
a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] + (21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d
+ 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^
(1/4) + b^(1/4)*x] - 4*a^(1/4)*b^(1/4)*(-3*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]
*x^2])/(128*a^(11/4)*b^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\frac{(3ai-5be)x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{(ai+9be)x^3}{32ab}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-bx^4+a)^2}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)}\frac{\left(-(3ai-5be)\_R^2-4(\cdots\right.}{\cdots}}{128a^2b^2\cdots}$
default	$\frac{\frac{(3ai-5be)x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{(ai+9be)x^3}{32ab}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-bx^4+a)^2}+\frac{\frac{(-3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\cdots\right.\right.}{4a}}{\cdots}$

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERB
OSE)
```

```
[Out] (1/32*(3*a*i-5*b*e)/a^2*x^7+1/16*(a*h-3*b*d)/a^2*x^6+1/32*(a*g-7*b*c)/a^2*x
^5+1/32*(a*i+9*b*e)/a/b*x^3+1/16*(a*h+5*b*d)/a/b*x^2+1/32*(3*a*g+11*b*c)/a/
b*x+1/8*f/b)/(-b*x^4+a)^2-1/128/a^2/b^2*sum((-(3*a*i-5*b*e)*_R^2-4*(a*h-3*b
*d)*_R-3*a*g+21*b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="
fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.28

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx = -\frac{(5\,b^2e-3\,abi)x^7+2\,(3\,b^2d-abh)x^6+(7\,b^2c-abg)x^5-(9\,abe+a^2i)x^3-4\,a^2f-2\,(5\,abd+a^2h)x^2-(}{32\,(a^2b^3x^8-2\,a^3b^2x^4+a^4b)} + \frac{\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be-3\,a\sqrt{b}g+3\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\,b^{\frac{3}{2}}c+5\,\sqrt{a}be-}{}}{128\,a^2b}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="
maxima")
[Out] -1/32*((5*b^2*e - 3*a*b*i)*x^7 + 2*(3*b^2*d - a*b*h)*x^6 + (7*b^2*c - a*b*g
)*x^5 - (9*a*b*e + a^2*i)*x^3 - 4*a^2*f - 2*(5*a*b*d + a^2*h)*x^2 - (11*a*b
*c + 3*a^2*g)*x)/(a^2*b^3*x^8 - 2*a^3*b^2*x^4 + a^4*b) + 1/128*(4*(3*b*d -
a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 4*(3*b*d - a*h)*log(sqr
t(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(21*b^(3/2)*c - 5*sqrt(a)*b*e - 3
*a*sqrt(b)*g + 3*a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (21*b^(3/2)*c + 5*sqrt(a)*b*e - 3*a*sqrt
(b)*g - 3*a^(3/2)*i)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + s
qrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^2*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(228) = 456$.

Time = 0.28 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.97

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx = -\frac{\sqrt{2}\left(21\,b^3c-3\,ab^2g-12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d+4\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-5\,\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\right.}{}\right.}{128\,(-ab^3)^{\frac{3}{4}}\,a^2b} - \frac{\sqrt{2}\left(21\,b^3c-3\,ab^2g+12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d-4\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-5\,\sqrt{-ab}b^2e-3\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\right.}{}\right.}{128\,(-ab^3)^{\frac{3}{4}}\,a^2b} - \frac{\sqrt{2}(21\,b^3c-3\,ab^2g-5\,\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2b} + \frac{\sqrt{2}(21\,b^3c-3\,ab^2g-5\,\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2b} - \frac{5\,b^2ex^7-3\,abix^7+6\,b^2dx^6-2\,abhx^6+7\,b^2cx^5-abgx^5-9\,abex^3-a^2ix^3-10\,abdx^2-2\,a^2hx^2-11\,a}{32\,(bx^4-a)^2a^2b}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="
giac")
```

```
[Out] -1/128*sqrt(2)*(21*b^3*c - 3*a*b^2*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b^2*d + 4*
sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 5*sqrt(-a*b)*b^2*e + 3*sqrt(-a*b)*a*b*i)*arc
tan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*
a^2*b) - 1/128*sqrt(2)*(21*b^3*c - 3*a*b^2*g + 12*sqrt(2)*(-a*b^3)^(1/4)*b^
2*d - 4*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 5*sqrt(-a*b)*b^2*e - 3*sqrt(-a*b)*a*
b*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3
)^(3/4)*a^2*b) - 1/256*sqrt(2)*(21*b^3*c - 3*a*b^2*g - 5*sqrt(-a*b)*b^2*e +
 3*sqrt(-a*b)*a*b*i)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^
3)^(3/4)*a^2*b) + 1/256*sqrt(2)*(21*b^3*c - 3*a*b^2*g - 5*sqrt(-a*b)*b^2*e
+ 3*sqrt(-a*b)*a*b*i)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b
^3)^(3/4)*a^2*b) - 1/32*(5*b^2*e*x^7 - 3*a*b*i*x^7 + 6*b^2*d*x^6 - 2*a*b*h*
x^6 + 7*b^2*c*x^5 - a*b*g*x^5 - 9*a*b*e*x^3 - a^2*i*x^3 - 10*a*b*d*x^2 - 2*
a^2*h*x^2 - 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 2680, normalized size of antiderivative = 10.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x)
[Out] symsum(log((27*a^4*i^3 - 125*a*b^3*e^3 - 3024*b^4*c*d^2 + 2205*b^4*c^2*e -
336*a^2*b^2*c*h^2 + 45*a^2*b^2*e*g^2 + 225*a^2*b^2*e^2*i + 432*a*b^3*d^2*g
- 1323*a*b^3*c^2*i - 135*a^3*b*e*i^2 + 48*a^3*b*g*h^2 - 27*a^3*b*g^2*i + 37
8*a^2*b^2*c*g*i - 288*a^2*b^2*d*g*h + 2016*a*b^3*c*d*h - 630*a*b^3*c*e*g)/(
32768*a^6*b^2) - root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2 + 412
8768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 - 6
881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^2*z^2 +
 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*e*i*z -
774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55
296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z + 5529
6*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2*d*z + 3456
*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*
b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^
2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4*b^2*c*g*i^2 -
 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^4*d^2*e*g - 672
0*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*
c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*b^2*e^2*i^2 + 138
24*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 - 23814*a^2
*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i - 27648*a^2*b^4*d^3*h
 - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 - 81*a^4*b
^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 - 81*a^6*i^4 - 1
94481*b^6*c^4, z, l)*(root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2
+ 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^
2 - 6881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^2*
z^2 + 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*e*i
*z - 774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z
 + 55296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z +
 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2*d*z +
 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760
*a^3*b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 5184*a^3*b
^3*d^2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4*b^2*c*g*
i^2 - 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^4*d^2*e*g
- 6720*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a
*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*b^2*e^2*i^2
+ 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 - 2381
4*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i - 27648*a^2*b^4*
```

d^3*h - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 - 81*
a^4*b^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 - 81*a^6*i^
4 - 194481*b^6*c^4, z, l)*((344064*a^5*b^5*c - 49152*a^6*b^4*g)/(32768*a^6*
b^2) - (x*(24576*a^5*b^4*d - 8192*a^6*b^3*h))/(4096*a^6*b)) - (15360*a^3*b^
4*d*e - 9216*a^4*b^3*d*i - 5120*a^4*b^3*e*h + 3072*a^5*b^2*h*i)/(32768*a^6*
b^2) + (x*(144*a^5*b*i^2 + 7056*a^2*b^4*c^2 + 400*a^3*b^3*e^2 + 144*a^4*b^2
*g^2 - 2016*a^3*b^3*c*g - 480*a^4*b^2*e*i))/(4096*a^6*b)) - (x*(216*b^3*d^3
 - 8*a^3*h^3 - 315*b^3*c*d*e + 9*a^3*g*h*i - 216*a*b^2*d^2*h + 72*a^2*b*d*h
^2 + 189*a*b^2*c*d*i + 105*a*b^2*c*e*h + 45*a*b^2*d*e*g - 63*a^2*b*c*h*i -
27*a^2*b*d*g*i - 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^7*z^4
 - 589824*a^8*b^4*g*i*z^2 + 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z
^2 + 983040*a^7*b^5*e*g*z^2 - 6881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*
z^2 - 4718592*a^6*b^6*d^2*z^2 + 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*
h*z - 184320*a^5*b^4*d*e*i*z - 774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2
*z - 18432*a^6*b^3*g^2*h*z + 55296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z
- 903168*a^4*b^5*c^2*h*z + 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z +
 2709504*a^3*b^6*c^2*d*z + 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7
560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^
4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e
*g*h^2 - 2268*a^4*b^2*c*g*i^2 - 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2
*i + 8640*a^2*b^4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 5
76*a^5*b*g*h^2*i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3
*g - 1350*a^4*b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 +
450*a^3*b^3*e^2*g^2 - 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*
b^3*e^3*i - 27648*a^2*b^4*d^3*h - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 +
 22050*a*b^5*c^2*e^2 - 81*a^4*b^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 2
0736*a*b^5*d^4 - 81*a^6*i^4 - 194481*b^6*c^4, z, l), l, 1, 4) + (f/(8*b) -
(x^5*(7*b*c - a*g))/(32*a^2) - (x^6*(3*b*d - a*h))/(16*a^2) - (x^7*(5*b*e -
 3*a*i))/(32*a^2) + (x*(11*b*c + 3*a*g))/(32*a*b) + (x^2*(5*b*d + a*h))/(16
*a*b) + (x^3*(9*b*e + a*i))/(32*a*b))/(a^2 + b^2*x^8 - 2*a*b*x^4)

3.200 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 285

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx = \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{8ab\,(a-bx^4)^2} + \frac{4a(bf-aj)+x(b(7bc-ag)+2b(3bd-ah)x+b(5be-3ai)x^2)}{32a^2b^2\,(a-bx^4)} - \frac{\left(5be-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{7/4}} + \frac{\left(5be+\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{7/4}} + \frac{(3bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}$$

```
[Out] 1/8*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+(a*j+b*f)*x^3)/a/b/(-b*x^4+a)^2+1/
32*(4*a*(-a*j+b*f)+x*(b*(-a*g+7*b*c)+2*b*(-a*h+3*b*d)*x+b*(-3*a*i+5*b*e)*x^
2))/a^2/b^2/(-b*x^4+a)+1/16*(-a*h+3*b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/
2)/b^(3/2)-1/64*arctan(b^(1/4)*x/a^(1/4))*(5*b*e-3*a*i-3*(-a*g+7*b*c)*b^(1/
2)/a^(1/2))/a^(9/4)/b^(7/4)+1/64*arctanh(b^(1/4)*x/a^(1/4))*(5*b*e-3*a*i+3*
(-a*g+7*b*c)*b^(1/2)/a^(1/2))/a^(9/4)/b^(7/4)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1872, 1868, 1890, 281, 214, 1181, 211}

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{(a - bx^4)^3} dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}} - 3ai + 5be\right)}{64a^{9/4}b^{7/4}}$$
$$+ \frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}} - 3ai + 5be\right)}{64a^{9/4}b^{7/4}} + \frac{\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(3bd - ah)}{16a^{5/2}b^{3/2}}$$
$$+ \frac{x(b(7bc - ag) + 2bx(3bd - ah) + bx^2(5be - 3ai)) + 4a(bf - aj)}{32a^2b^2(a - bx^4)}$$
$$+ \frac{x(x(ah + bd) + x^2(ai + be) + x^3(aj + bf) + ag + bc)}{8ab(a - bx^4)^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^3
,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(8*a*b*
(a - b*x^4)^2) + (4*a*(b*f - a*j) + x*(b*(7*b*c - a*g) + 2*b*(3*b*d - a*h)*
x + b*(5*b*e - 3*a*i)*x^2))/(32*a^2*b^2*(a - b*x^4)) - ((5*b*e - (3*Sqrt[b]
*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b
^(7/4)) + ((5*b*e + (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(
1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^
2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{8ab\left(a-bx^4\right)^2} \\
&\quad -\frac{\int \frac{-b(7bc-ag)-2b(3bd-ah)x-b(5be-3ai)x^2-4b(bf-aj)x^3}{\left(a-bx^4\right)^2}\,dx}{8ab^2} \\
&= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{8ab\left(a-bx^4\right)^2} \\
&\quad +\frac{4a(bf-aj)+x(b(7bc-ag)+2b(3bd-ah)x+b(5be-3ai)x^2)}{32a^2b^2\left(a-bx^4\right)} \\
&\quad +\frac{\int \frac{3b(7bc-ag)+4b(3bd-ah)x+b(5be-3ai)x^2}{a-bx^4}\,dx}{32a^2b^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{8ab\left(a-bx^4\right)^2} \\
&\quad + \frac{4a(bf-aj)+x(b(7bc-ag)+2b(3bd-ah)x+b(5be-3ai)x^2)}{32a^2b^2\left(a-bx^4\right)} \\
&\quad + \frac{\int \left(\frac{4b(3bd-ah)x}{a-bx^4}+\frac{3b(7bc-ag)+b(5be-3ai)x^2}{a-bx^4}\right)dx}{32a^2b^2} \\
&= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{8ab\left(a-bx^4\right)^2} \\
&\quad + \frac{4a(bf-aj)+x(b(7bc-ag)+2b(3bd-ah)x+b(5be-3ai)x^2)}{32a^2b^2\left(a-bx^4\right)} \\
&\quad + \frac{\int \frac{3b(7bc-ag)+b(5be-3ai)x^2}{a-bx^4}dx}{32a^2b^2} + \frac{(3bd-ah)\int \frac{x}{a-bx^4}dx}{8a^2b} \\
&= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{8ab\left(a-bx^4\right)^2} \\
&\quad + \frac{4a(bf-aj)+x(b(7bc-ag)+2b(3bd-ah)x+b(5be-3ai)x^2)}{32a^2b^2\left(a-bx^4\right)} \\
&\quad + \frac{(3bd-ah)\text{Subst}\left(\int \frac{1}{a-bx^2}dx,x,x^2\right)}{16a^2b} + \frac{\left(5be-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}dx}{64a^2b} \\
&\quad + \frac{\left(5be+\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}dx}{64a^2b} \\
&= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{8ab\left(a-bx^4\right)^2} \\
&\quad + \frac{4a(bf-aj)+x(b(7bc-ag)+2b(3bd-ah)x+b(5be-3ai)x^2)}{32a^2b^2\left(a-bx^4\right)} \\
&\quad - \frac{\left(5be-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{7/4}} \\
&\quad + \frac{\left(5be+\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{7/4}} + \frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx$$
$$= \frac{-\frac{4a^{3/4}(8a^2j-b^2x(7c+x(6d+5ex))+abx(g+x(2h+3ix)))}{a-bx^4}+\frac{16a^{7/4}(a^2j+b^2x(c+x(d+ex))+ab(f+x(g+x(h+ix))))}{(a-bx^4)^2}+2\sqrt[4]{b}\Big(21b^{3/2}c-5\sqrt{\cdots}}{\cdots}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4)^3,x]
```

```
[Out] ((-4*a^(3/4)*(8*a^2*j - b^2*x*(7*c + x*(6*d + 5*e*x)) + a*b*x*(g + x*(2*h +
 3*i*x))))/(a - b*x^4) + (16*a^(7/4)*(a^2*j + b^2*x*(c + x*(d + e*x)) + a*b
*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^2 + 2*b^(1/4)*(21*b^(3/2)*c - 5*Sq
rt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] + b^(1
/4)*(-21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g +
 4*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] + b^(1/4)*(21*
b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 4*a^(5/4
)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 4*a^(1/4)*Sqrt[b]*(-3
*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a^(11/4)*b^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.72

method	result
risch	$\frac{\frac{(3ai-5be)x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{jx^4}{4b}+\frac{(ai+9be)x^3}{32ab}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}-\frac{aj-bf}{8b^2}}{(-bx^4+a)^2}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)}\Big(-(3ai-5be)\_\cdots}{\cdots}$
default	$\frac{\frac{(3ai-5be)x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{jx^4}{4b}+\frac{(ai+9be)x^3}{32ab}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}-\frac{aj-bf}{8b^2}}{(-bx^4+a)^2}+\frac{\frac{(-3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\cdots\right)}{4a}}{\cdots}$

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETU
RNVERBOSE)
```

```
[Out] (1/32*(3*a*i-5*b*e)/a^2*x^7+1/16*(a*h-3*b*d)/a^2*x^6+1/32*(a*g-7*b*c)/a^2*x
^5+1/4*j*x^4/b+1/32*(a*i+9*b*e)/a/b*x^3+1/16*(a*h+5*b*d)/a/b*x^2+1/32*(3*a*
g+11*b*c)/a/b*x-1/8*(a*j-b*f)/b^2)/(-b*x^4+a)^2-1/128/a^2/b^2*sum((-(3*a*i-
```

```
5*b*e)*_R^2-4*(a*h-3*b*d)*_R-3*a*g+21*b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,
x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.32

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx$$
$$= \frac{8\,a^2bjx^4 - (5\,b^3e - 3\,ab^2i)x^7 - 2\,(3\,b^3d - ab^2h)x^6 - (7\,b^3c - ab^2g)x^5 + 4\,a^2bf - 4\,a^3j + (9\,ab^2e + a^2bi)x^3}{32\,(a^2b^4x^8 - 2\,a^3b^3x^4 + a^4b^2)}$$
$$+ \frac{\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be-3\,a\sqrt{b}g+3\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \left(21\,b^{\frac{3}{2}}c+5\,\sqrt{a}be}{128\,a^2b}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algor
ithm="maxima")
```

```
[Out] 1/32*(8*a^2*b*j*x^4 - (5*b^3*e - 3*a*b^2*i)*x^7 - 2*(3*b^3*d - a*b^2*h)*x^6
 - (7*b^3*c - a*b^2*g)*x^5 + 4*a^2*b*f - 4*a^3*j + (9*a*b^2*e + a^2*b*i)*x^
3 + 2*(5*a*b^2*d + a^2*b*h)*x^2 + (11*a*b^2*c + 3*a^2*b*g)*x)/(a^2*b^4*x^8
- 2*a^3*b^3*x^4 + a^4*b^2) + 1/128*(4*(3*b*d - a*h)*log(sqrt(b)*x^2 + sqrt(
a))/(sqrt(a)*sqrt(b)) - 4*(3*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)
*sqrt(b)) + 2*(21*b^(3/2)*c - 5*sqrt(a)*b*e - 3*a*sqrt(b)*g + 3*a^(3/2)*i)*
arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt
(b)) - (21*b^(3/2)*c + 5*sqrt(a)*b*e - 3*a*sqrt(b)*g - 3*a^(3/2)*i)*log((sq
rt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^2*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(245) = 490$.

Time = 0.29 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.97

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx =$$

$$-\frac{\sqrt{2}\left(21\,b^3c-3\,ab^2g-12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d+4\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-5\,\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\,x\right.}{2}\right.}{128\,(-ab^3)^{\frac{3}{4}}\,a^2b}$$

$$-\frac{\sqrt{2}\left(21\,b^3c-3\,ab^2g+12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d-4\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-5\,\sqrt{-ab}b^2e-3\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\,x\right.}{2}\right.}{128\,(-ab^3)^{\frac{3}{4}}\,a^2b}$$

$$-\frac{\sqrt{2}\left(21\,b^3c-3\,ab^2g-5\,\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2b}$$

$$+\frac{\sqrt{2}\left(21\,b^3c-3\,ab^2g-5\,\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2b}$$

$$-\frac{5\,b^3ex^7-3\,ab^2ix^7+6\,b^3dx^6-2\,ab^2hx^6+7\,b^3cx^5-ab^2gx^5-8\,a^2bjx^4-9\,ab^2ex^3-a^2bix^3-10\,ab^2dx^2-}{32\,(bx^4-a)^2a^2b^2}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algor
ithm="giac")
```

```
[Out] -1/128*sqrt(2)*(21*b^3*c - 3*a*b^2*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b^2*d + 4*
sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 5*sqrt(-a*b)*b^2*e + 3*sqrt(-a*b)*a*b*i)*arc
tan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*
a^2*b) - 1/128*sqrt(2)*(21*b^3*c - 3*a*b^2*g + 12*sqrt(2)*(-a*b^3)^(1/4)*b^
2*d - 4*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 5*sqrt(-a*b)*b^2*e - 3*sqrt(-a*b)*a*
b*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3
```

```
)^(3/4)*a^2*b) - 1/256*sqrt(2)*(21*b^3*c - 3*a*b^2*g - 5*sqrt(-a*b)*b^2*e +
 3*sqrt(-a*b)*a*b*i)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^
3)^(3/4)*a^2*b) + 1/256*sqrt(2)*(21*b^3*c - 3*a*b^2*g - 5*sqrt(-a*b)*b^2*e
+ 3*sqrt(-a*b)*a*b*i)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b
^3)^(3/4)*a^2*b) - 1/32*(5*b^3*e*x^7 - 3*a*b^2*i*x^7 + 6*b^3*d*x^6 - 2*a*b^
2*h*x^6 + 7*b^3*c*x^5 - a*b^2*g*x^5 - 8*a^2*b*j*x^4 - 9*a*b^2*e*x^3 - a^2*b
*i*x^3 - 10*a*b^2*d*x^2 - 2*a^2*b*h*x^2 - 11*a*b^2*c*x - 3*a^2*b*g*x - 4*a^
2*b*f + 4*a^3*j)/((b*x^4 - a)^2*a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 10.02 (sec) , antiderivative size = 2696, normalized size of antiderivative = 9.46

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^3
,x)
```

```
[Out] symsum(log((27*a^4*i^3 - 125*a*b^3*e^3 - 3024*b^4*c*d^2 + 2205*b^4*c^2*e -
336*a^2*b^2*c*h^2 + 45*a^2*b^2*e*g^2 + 225*a^2*b^2*e^2*i + 432*a*b^3*d^2*g
- 1323*a*b^3*c^2*i - 135*a^3*b*e*i^2 + 48*a^3*b*g*h^2 - 27*a^3*b*g^2*i + 37
8*a^2*b^2*c*g*i - 288*a^2*b^2*d*g*h + 2016*a*b^3*c*d*h - 630*a*b^3*c*e*g)/(
32768*a^6*b^2) - root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2 + 412
8768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 - 6
881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^2*z^2 +
 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*e*i*z -
774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55
296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z + 5529
6*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2*d*z + 3456
*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*
b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^
2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4*b^2*c*g*i^2 -
 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^4*d^2*e*g - 672
0*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*
c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*b^2*e^2*i^2 + 138
24*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 - 23814*a^2
*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i - 27648*a^2*b^4*d^3*h
 - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 - 81*a^4*b
^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 - 81*a^6*i^4 - 1
94481*b^6*c^4, z, m)*(root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2
+ 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^
2 - 6881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^2*
z^2 + 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*e*i
*z - 774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z
```

+ 55296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z + 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2*d*z + 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4*b^2*c*g*i^2 - 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 - 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i - 27648*a^2*b^4*d^3*h - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 - 81*a^4*b^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 - 81*a^6*i^4 - 194481*b^6*c^4, z, m)*((344064*a^5*b^5*c - 49152*a^6*b^4*g)/(32768*a^6*b^2) - (x*(24576*a^5*b^4*d - 8192*a^6*b^3*h))/(4096*a^6*b)) - (15360*a^3*b^4*d*e - 9216*a^4*b^3*d*i - 5120*a^4*b^3*e*h + 3072*a^5*b^2*h*i)/(32768*a^6*b^2) + (x*(144*a^5*b*i^2 + 7056*a^2*b^4*c^2 + 400*a^3*b^3*e^2 + 144*a^4*b^2*g^2 - 2016*a^3*b^3*c*g - 480*a^4*b^2*e*i))/(4096*a^6*b)) - (x*(216*b^3*d^3 - 8*a^3*h^3 - 315*b^3*c*d*e + 9*a^3*g*h*i - 216*a*b^2*d^2*h + 72*a^2*b*d*h^2 + 189*a*b^2*c*d*i + 105*a*b^2*c*e*h + 45*a*b^2*d*e*g - 63*a^2*b*c*h*i - 27*a^2*b*d*g*i - 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2 + 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 - 6881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^2*z^2 + 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*e*i*z - 774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z + 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2*d*z + 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4*b^2*c*g*i^2 - 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 - 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i - 27648*a^2*b^4*d^3*h - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 - 81*a^4*b^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 - 81*a^6*i^4 - 194481*b^6*c^4, z, m), m, 1, 4) + ((b*f - a*j)/(8*b^2) + (j*x^4)/(4*b) - (x^5*(7*b*c - a*g))/(32*a^2) - (x^6*(3*b*d - a*h))/(16*a^2) - (x^7*(5*b*e - 3*a*i))/(32*a^2) + (x*(11*b*c + 3*a*g))/(32*a*b) + (x^2*(5*b*d + a*h))/(16*a*b) + (x^3*(9*b*e + a*i))/(32*a*b))/(a^2 + b^2*x^8 - 2*a*b*x^4)

3.201 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 413

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx = \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab(a+bx^4)^2} - \frac{4af-x(7bc+ag+2(3bd+ah)x+5bex^2)}{32a^2b(a+bx^4)} + \frac{(3bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} - \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} - \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}} + \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}}$$

```
[Out] 1/8*x*(b*c-a*g+(-a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(b*x^4+a)^2+1/32*(-4*a*f+x
*(7*b*c+a*g+2*(a*h+3*b*d)*x+5*b*e*x^2))/a^2/b/(b*x^4+a)+1/16*(a*h+3*b*d)*ar
ctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)-1/256*ln(-a^(1/4)*b^(1/4)*x*2^(1/
2)+a^(1/2)+x^2*b^(1/2))*(21*b*c+3*a*g-5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)
*2^(1/2)+1/256*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(21*b*c+3*
a*g-5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+1/128*arctan(-1+b^(1/4)*x
*2^(1/2)/a^(1/4))*(21*b*c+3*a*g+5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)*2^(1/
2)+1/128*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(21*b*c+3*a*g+5*e*a^(1/2)*b^(1
/2))/a^(11/4)/b^(5/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {1872, 1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx$$
$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}} + \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+3bd)}{16a^{5/2}b^{3/2}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}} - \frac{4af-x(2x(ah+3bd)+ag+7bc+5bex^2)}{32a^2b\,(a+bx^4)} + \frac{x(x(bd-ah)-ag+bc+bex^2+bfx^3)}{8ab\,(a+bx^4)^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) -
 (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + 5*b*e*x^2))/(32*a^2*b*(a + b
*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
 - ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3
*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)
) - ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt
[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/(128*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
```

```
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\left(a+bx^4\right)^2}-\frac{\int \frac{-b(7bc+ag)-2b(3bd+ah)x-5b^2ex^2-4b^2fx^3}{\left(a+bx^4\right)^2}\,dx}{8ab^2}\\
&= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\left(a+bx^4\right)^2}\\
&\quad-\frac{4af-x(7bc+ag+2(3bd+ah)x+5bex^2)}{32a^2b\left(a+bx^4\right)}+\frac{\int \frac{3b(7bc+ag)+4b(3bd+ah)x+5b^2ex^2}{a+bx^4}\,dx}{32a^2b^2}\\
&= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\left(a+bx^4\right)^2}\\
&\quad-\frac{4af-x(7bc+ag+2(3bd+ah)x+5bex^2)}{32a^2b\left(a+bx^4\right)}\\
&\quad+\frac{\int \left(\frac{4b(3bd+ah)x}{a+bx^4}+\frac{3b(7bc+ag)+5b^2ex^2}{a+bx^4}\right)dx}{32a^2b^2}\\
&= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\left(a+bx^4\right)^2}\\
&\quad-\frac{4af-x(7bc+ag+2(3bd+ah)x+5bex^2)}{32a^2b\left(a+bx^4\right)}\\
&\quad+\frac{\int \frac{3b(7bc+ag)+5b^2ex^2}{a+bx^4}\,dx}{32a^2b^2}+\frac{(3bd+ah)\int \frac{x}{a+bx^4}\,dx}{8a^2b}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\left(a+bx^4\right)^2} \\
&\quad - \frac{4af-x(7bc+ag+2(3bd+ah)x+5bex^2)}{32a^2b\left(a+bx^4\right)} \\
&\quad + \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{64a^{5/2}b^{3/2}} \\
&\quad + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{64a^{5/2}b^{3/2}} + \frac{(3bd+ah)\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{16a^2b} \\
&= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\left(a+bx^4\right)^2} \\
&\quad - \frac{4af-x(7bc+ag+2(3bd+ah)x+5bex^2)}{32a^2b\left(a+bx^4\right)} + \frac{(3bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} \\
&\quad - \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{128\sqrt{2}a^{11/4}b^{5/4}} \\
&\quad - \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{128\sqrt{2}a^{11/4}b^{5/4}} \\
&\quad + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{128a^{5/2}b^{3/2}} \\
&\quad + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{128a^{5/2}b^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\left(a+bx^4\right)^2} \\
&\quad - \frac{4af-x(7bc+ag+2(3bd+ah)x+5bex^2)}{32a^2b\left(a+bx^4\right)} + \frac{(3bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} \\
&\quad - \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}} \\
&\quad + \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}} \\
&\quad + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} \\
&\quad - \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab\left(a+bx^4\right)^2} \\
&\quad - \frac{4af-x(7bc+ag+2(3bd+ah)x+5bex^2)}{32a^2b\left(a+bx^4\right)} + \frac{(3bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} \\
&\quad - \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} \\
&\quad + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} \\
&\quad - \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}} \\
&\quad + \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx$$
$$= \frac{\frac{8a^{3/4}\sqrt{b}x(7bc+bx(6d+5ex)+a(g+2hx))}{a+bx^4}-\frac{32a^{7/4}\sqrt{b}(-bx(c+x(d+ex))+a(f+x(g+hx)))}{(a+bx^4)^2}-2\Big(21\sqrt{2}b^{5/4}c+24\sqrt[4]{a}bd+5\sqrt{2}\sqrt{a}b}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*Sqrt[b]*x*(7*b*c + b*x*(6*d + 5*e*x) + a*(g + 2*h*x)))/(a + b*x
^4) - (32*a^(7/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))
/(a + b*x^4)^2 - 2*(21*Sqrt[2]*b^(5/4)*c + 24*a^(1/4)*b*d + 5*Sqrt[2]*Sqrt[
a]*b^(3/4)*e + 3*Sqrt[2]*a*b^(1/4)*g + 8*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)] + 2*(21*Sqrt[2]*b^(5/4)*c - 24*a^(1/4)*b*d + 5*Sqrt[2]*Sqr
t[a]*b^(3/4)*e + 3*Sqrt[2]*a*b^(1/4)*g - 8*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*(-21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*
(21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.41

method	result
risch	$\frac{\frac{5be\,x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{\left(5\_R^2e+\frac{4(ah+3bd)\_R}{b}+\frac{3ag+}{b}\right.}{\_R^3}}{128a^2b}$
default	$\frac{\frac{5be\,x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\frac{(3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\right.\right.}{8a}}{}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (5/32*b*e/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x^5+9/32/a*
e*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8*f/b)/(b*x^4+a)
^2+1/128/a^2/b*sum((5*_R^2*e+4/b*(a*h+3*b*d)*_R+3/b*(a*g+7*b*c))/_R^3*ln(x-
_R),_R=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.08

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx = \frac{5\,b^2ex^7 + 2\,(3\,b^2d+abh)x^6 + 9\,abex^3 + (7\,b^2c+abg)x^5 - 4\,a^2f + 2\,(5\,abd - a^2h)x^2 + (11\,abc - 3\,a^2g)x}{32\,(a^2b^3x^8 + 2\,a^3b^2x^4 + a^4b)} + \frac{\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c - 5\,\sqrt{a}be + 3\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c - 5\,\sqrt{a}be + 3\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(21\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c\right.}{}}{}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima"
)
[Out] 1/32*(5*b^2*e*x^7 + 2*(3*b^2*d + a*b*h)*x^6 + 9*a*b*e*x^3 + (7*b^2*c + a*b*
g)*x^5 - 4*a^2*f + 2*(5*a*b*d - a^2*h)*x^2 + (11*a*b*c - 3*a^2*g)*x)/(a^2*b
^3*x^8 + 2*a^3*b^2*x^4 + a^4*b) + 1/256*(sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*
b*e + 3*a*sqrt(b)*g)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))
/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g)
*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) +
```

```
 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*
a^(5/4)*b^(3/4)*g - 24*sqrt(a)*b^(3/2)*d - 8*a^(3/2)*sqrt(b)*h)*arctan(1/2*
sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(
3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 5*s
qrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 24*sqrt(a)*b^(3/2)
*d + 8*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)
*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(
a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.10

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx$$
$$= \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+4\sqrt{2}\sqrt{ab}abh+21\,(ab^3)^{\frac{1}{4}}\,b^2c+3\,(ab^3)^{\frac{1}{4}}\,abg+5\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$
$$+\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+4\sqrt{2}\sqrt{ab}abh+21\,(ab^3)^{\frac{1}{4}}\,b^2c+3\,(ab^3)^{\frac{1}{4}}\,abg+5\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$
$$+\frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^2c+3\,(ab^3)^{\frac{1}{4}}\,abg-5\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$
$$-\frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^2c+3\,(ab^3)^{\frac{1}{4}}\,abg-5\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$
$$+\frac{5\,b^2ex^7+6\,b^2dx^6+2\,abhx^6+7\,b^2cx^5+abgx^5+9\,abex^3+10\,abdx^2-2\,a^2hx^2+11\,abcx-3\,a^2gx-4\,a}{32\,(bx^4+a)^2a^2b}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*
(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)
*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)
*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^
(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*
c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1
/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 2*a*b*h*x^6
+ 7*b^2*c*x^5 + a*b*g*x^5 + 9*a*b*e*x^3 + 10*a*b*d*x^2 - 2*a^2*h*x^2 + 11*a
*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 10.02 (sec) , antiderivative size = 1686, normalized size of antiderivative = 4.08

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x)

[Out] ((9*e*x^3)/(32*a) - f/(8*b) + (x^5*(7*b*c + a*g))/(32*a^2) + (x^6*(3*b*d + a*h))/(16*a^2) + (x*(11*b*c - 3*a*g))/(32*a*b) + (x^2*(5*b*d - a*h))/(16*a*b) + (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4) + symsum(log((3024*b^3*c*d^2 - 125*a*b^2*e^3 - 2205*b^3*c^2*e + 48*a^3*g*h^2 + 432*a*b^2*d^2*g + 336*a^2*b*c*h^2 - 45*a^2*b*e*g^2 + 2016*a*b^2*c*d*h - 630*a*b^2*c*e*g + 288*a^2*b*d*g*h)/(32768*a^6*b) - root(268435456*a^11*b^6*z^4 + 3145728*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 + 6881280*a^6*b^5*c*e*z^2 + 524288*a^8*b^3*h^2*z^2 + 4718592*a^6*b^5*d^2*z^2 - 258048*a^5*b^3*c*g*h*z - 774144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z + 51200*a^5*b^3*e^2*h*z - 903168*a^4*b^4*c^2*h*z - 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z - 2709504*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h - 40320*a^2*b^3*c*d*e*h - 8640*a^2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 + 6300*a^2*b^3*c*e^2*g - 960*a^4*b*e*g*h^2 - 60480*a*b^4*c*d^2*e + 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 13824*a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 + 23814*a^2*b^3*c^2*g^2 + 27648*a^2*b^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 + 625*a^2*b^3*e^4 + 81*a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 + 194481*b^5*c^4, z, k)*(root(268435456*a^11*b^6*z^4 + 3145728*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 + 6881280*a^6*b^5*c*e*z^2 + 524288*a^8*b^3*h^2*z^2 + 4718592*a^6*b^5*d^2*z^2 - 258048*a^5*b^3*c*g*h*z - 774144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z + 51200*a^5*b^3*e^2*h*z - 903168*a^4*b^4*c^2*h*z - 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z - 2709504*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h - 40320*a^2*b^3*c*d*e*h - 8640*a^2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 + 6300*a^2*b^3*c*e^2*g - 960*a^4*b*e*g*h^2 - 60480*a*b^4*c*d^2*e + 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 13824*a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 + 23814*a^2*b^3*c^2*g^2 + 27648*a^2*b^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 + 625*a^2*b^3*e^4 + 81*a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 + 194481*b^5*c^4, z, k)*((344064*a^5*b^4*c + 49152*a^6*b^3*g)/(32768*a^6*b) - (x*(24576*a^5*b^4*d + 8192*a^6*b^3*h))/(4096*a^6*b)) + (15360*a^3*b^3*d*e + 5120*a^4*b^2*e*h)/(32768*a^6*b) + (x*(7056*a^2*b^4*c^2 - 400*a^3*b^3*e^2 + 144*a^4*b^2*g^2 + 2016*a^3*b^3*c*g))/(4096*a^6*b)) + (x*(216*b^3*d^3 + 8*a^3*h^3 - 315*b^3*c*d*e + 216*a*b^2*d^2*h + 72*a^2*b*d*h^2 - 105*a*b^2*c*e*h - 45*a*b^2*d*e*g - 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^6*z^4 + 3145728*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 + 6881280*a^6*b^5*c*e*z^2 + 524288*a^8*b^3*h^2*z^2 + 4718592*a^6*b^5*d^2*z^2 - 258048*a^5*b^3*c*g*h*z - 774144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z + 51200*a^5*b^3*e^2*h*z - 903168*a^4*b^4*c^2*h*z - 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z - 27095

```
04*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h - 40320*a^2*b^3*c*d*e*h - 8640*a^
2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 + 6300*a^2*b^3*c*e^2*g - 960*a^4*b*e*g
*h^2 - 60480*a*b^4*c*d^2*e + 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 13824*
a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 + 23814*a^2*b^3*c^2*g^2 + 27648*a^2*b
^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 + 625*a^2*b^3*e^4 + 81*
a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 + 194481*b^5*c^4, z, k), k, 1, 4)
```

3.202 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3} dx$

Optimal result

Integrand size = 40, antiderivative size = 463

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3} dx \\
&= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{8ab(a+bx^4)^2} \\
&- \frac{4af-x(7bc+ag+2(3bd+ah)x+(5be+3ai)x^2)}{32a^2b(a+bx^4)} + \frac{(3bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} \\
&- \frac{\left(3\sqrt{b}(7bc+ag)+\sqrt{a}(5be+3ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{7/4}} \\
&+ \frac{\left(3\sqrt{b}(7bc+ag)+\sqrt{a}(5be+3ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{7/4}} \\
&- \frac{\left(3\sqrt{b}(7bc+ag)-\sqrt{a}(5be+3ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{7/4}} \\
&+ \frac{\left(3\sqrt{b}(7bc+ag)-\sqrt{a}(5be+3ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{7/4}}
\end{aligned}$$

```
[Out] 1/8*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+b*f*x^3)/a/b/(b*x^4+a)^2+1/32*(-
4*a*f+x*(7*b*c+a*g+2*(a*h+3*b*d)*x+(3*a*i+5*b*e)*x^2))/a^2/b/(b*x^4+a)+1/16
*(a*h+3*b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)-1/256*ln(-a^(1/4)*
b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+5*b*e)*a^(1/2)+3*(a*g+7*b*c
)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/256*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1
/2)+x^2*b^(1/2))*(-(3*a*i+5*b*e)*a^(1/2)+3*(a*g+7*b*c)*b^(1/2))/a^(11/4)/b^
```

```
(7/4)*2^(1/2)+1/128*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*a*i+5*b*e)*a^(
1/2)+3*(a*g+7*b*c)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/128*arctan(1+b^(1/4)
*x*2^(1/2)/a^(1/4))*((3*a*i+5*b*e)*a^(1/2)+3*(a*g+7*b*c)*b^(1/2))/a^(11/4)/
b^(7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {1872, 1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx$$
$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}} + \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+3bd)}{16a^{5/2}b^{3/2}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}} - \frac{4af-x(2x(ah+3bd)+x^2(3ai+5be)+ag+7bc)}{32a^2b\,(a+bx^4)} + \frac{x(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3)}{8ab\,(a+bx^4)^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(8*a*b*(a + b*x
^4)^2) - (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + (5*b*e + 3*a*i)*x^2)
)/(32*a^2*b*(a + b*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(1
6*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3
*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) - ((3*Sqrt[b]*(7*b*c + a*g) -
 Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]
*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(
128*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{8ab\left(a+bx^4\right)^2} - \frac{\int \frac{-b(7bc+ag)-2b(3bd+ah)x-b(5be+3ai)x^2-4b^2fx^3}{\left(a+bx^4\right)^2}\,dx}{8ab^2}\\ &= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x(7bc+ag+2(3bd+ah)x+(5be+3ai)x^2)}{32a^2b\left(a+bx^4\right)} + \frac{\int \frac{3b(7bc+ag)+4b(3bd+ah)x+b(5be+3ai)x^2}{a+bx^4}\,dx}{32a^2b^2}\end{aligned}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x(7bc+ag+2(3bd+ah)x+(5be+3ai)x^2)}{32a^2b\left(a+bx^4\right)} + \frac{\int \left(\frac{4b(3bd+ah)x}{a+bx^4}+\frac{3b(7bc+ag)+b(5be+3ai)x^2}{a+bx^4}\right)dx}{32a^2b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x(7bc+ag+2(3bd+ah)x+(5be+3ai)x^2)}{32a^2b\left(a+bx^4\right)} + \frac{\int \frac{3b(7bc+ag)+b(5be+3ai)x^2}{a+bx^4}dx}{32a^2b^2} + \frac{(3bd+ah)\int \frac{x}{a+bx^4}dx}{8a^2b}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x(7bc+ag+2(3bd+ah)x+(5be+3ai)x^2)}{32a^2b\left(a+bx^4\right)} + \frac{(3bd+ah)\text{Subst}\left(\int \frac{1}{a+bx^2}dx,x,x^2\right)}{16a^2b} - \frac{\left(5be-\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}dx}{64a^2b^2} + \frac{\left(5be+\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}dx}{64a^2b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x(7bc+ag+2(3bd+ah)x+(5be+3ai)x^2)}{32a^2b\left(a+bx^4\right)} + \frac{(3bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} + \frac{\left(5be-\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{128\sqrt{2}a^{9/4}b^{7/4}} + \frac{\left(5be-\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{128\sqrt{2}a^{9/4}b^{7/4}} + \frac{\left(5be+\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{128a^2b^2} + \frac{\left(5be+\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{128a^2b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x(7bc+ag+2(3bd+ah)x+(5be+3ai)x^2)}{32a^2b\left(a+bx^4\right)} + \frac{(3bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} + \frac{\left(5be-\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{7/4}} - \frac{\left(5be-\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{7/4}} + \frac{\left(5be+\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{9/4}b^{7/4}} - \frac{\left(5be+\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{9/4}b^{7/4}}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{8ab\,(a+bx^4)^2} - \frac{4af-x(7bc+ag+2(3bd+ah)x+(5be+3ai)x^2)}{32a^2b\,(a+bx^4)} + \frac{(3bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} - \frac{\left(5be+\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{9/4}b^{7/4}} + \frac{\left(5be+\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{9/4}b^{7/4}} + \frac{\left(5be-\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{7/4}} - \frac{\left(5be-\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{7/4}}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.02

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx$$

$$= \frac{\frac{8a^{3/4}b^{3/4}x(7bc+ag+bx(6d+5ex)+ax(2h+3ix))}{a+bx^4} - \frac{32a^{7/4}b^{3/4}(-bx(c+x(d+ex))+a(f+x(g+x(h+ix))))}{(a+bx^4)^2} - 2\left(21\sqrt{2}b^{3/2}c+24\sqrt[4]{a}b^{5/4}d\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x
]
```

```
[Out] ((8*a^(3/4)*b^(3/4)*x*(7*b*c + a*g + b*x*(6*d + 5*e*x) + a*x*(2*h + 3*i*x))
)/(a + b*x^4) - (32*a^(7/4)*b^(3/4)*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g
 + x*(h + i*x)))))/(a + b*x^4)^2 - 2*(21*Sqrt[2]*b^(3/2)*c + 24*a^(1/4)*b^(
5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*
h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(21*Sq
rt[2]*b^(3/2)*c - 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*
a*Sqrt[b]*g - 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-21*b^(3/2)*c + 5*Sqrt[a]*b*e - 3*a*Sqrt[b
]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] +
 Sqrt[2]*(21*b^(3/2)*c - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{(3ai+5be)x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}-\frac{(ai-9be)x^3}{32ab}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{\left((3ai+5be)\_R^2+4(ah\right.}{}}{128a^2b^2}$
default	$\frac{\frac{(3ai+5be)x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}-\frac{(ai-9be)x^3}{32ab}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\frac{(3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right.\right.}{8}}{}$

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBO
SE)
```

```
[Out] (1/32*(3*a*i+5*b*e)/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x
^5-1/32*(a*i-9*b*e)/a/b*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*g-11*b*c)/a/
b*x-1/8*f/b)/(b*x^4+a)^2+1/128/a^2/b^2*sum(((3*a*i+5*b*e)*_R^2+4*(a*h+3*b*d
)*_R+3*a*g+21*b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx=\text{Timed out}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx=\text{Timed out}$$

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.07

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\left(a+bx^4\right)^3}\,dx$$
$$= \frac{(5\,b^2e+3\,abi)x^7+2\,(3\,b^2d+abh)x^6+(7\,b^2c+abg)x^5+(9\,abe-a^2i)x^3-4\,a^2f+2\,(5\,abd-a^2h)x^2+(11}{32\,(a^2b^3x^8+2\,a^3b^2x^4+a^4b)}$$
$$+\frac{\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be+3\,a\sqrt{b}g-3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be+3\,a\sqrt{b}g-3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+}{}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="m
axima")
```

```
[Out] 1/32*((5*b^2*e + 3*a*b*i)*x^7 + 2*(3*b^2*d + a*b*h)*x^6 + (7*b^2*c + a*b*g)
*x^5 + (9*a*b*e - a^2*i)*x^3 - 4*a^2*f + 2*(5*a*b*d - a^2*h)*x^2 + (11*a*b*
c - 3*a^2*g)*x)/(a^2*b^3*x^8 + 2*a^3*b^2*x^4 + a^4*b) + 1/256*(sqrt(2)*(21*
b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g - 3*a^(3/2)*i)*log(sqrt(b)*x^2 +
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*b^(3/2
)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g - 3*a^(3/2)*i)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^
(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 3*sqr
t(2)*a^(7/4)*b^(1/4)*i - 24*sqrt(a)*b^(3/2)*d - 8*a^(3/2)*sqrt(b)*h)*arctan
(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))
/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c
+ 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 3*sqrt(2)*a^(
7/4)*b^(1/4)*i + 24*sqrt(a)*b^(3/2)*d + 8*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqr
t(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4
)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.14

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx$$
$$= \frac{5\,b^2ex^7+3\,abix^7+6\,b^2dx^6+2\,abhx^6+7\,b^2cx^5+abgx^5+9\,abex^3-a^2ix^3+10\,abdx^2-2\,a^2hx^2+11\,abc}{32\,(bx^4+a)^2a^2b}$$
$$+\frac{\sqrt{2}\Big(12\,\sqrt{2}\sqrt{ab}b^3d+4\,\sqrt{2}\sqrt{ab}ab^2h+21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g+5\,(ab^3)^{\frac{3}{4}}\,be+3\,(ab^3)^{\frac{3}{4}}\,ai\Big)\arctan\Bigg(}{128\,a^3b^4}$$
$$+\frac{\sqrt{2}\Big(12\,\sqrt{2}\sqrt{ab}b^3d+4\,\sqrt{2}\sqrt{ab}ab^2h+21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g+5\,(ab^3)^{\frac{3}{4}}\,be+3\,(ab^3)^{\frac{3}{4}}\,ai\Big)\arctan\Bigg(}{128\,a^3b^4}$$
$$+\frac{\sqrt{2}\Big(21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g-5\,(ab^3)^{\frac{3}{4}}\,be-3\,(ab^3)^{\frac{3}{4}}\,ai\Big)\log\Big(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\Big)}{256\,a^3b^4}$$
$$-\frac{\sqrt{2}\Big(21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g-5\,(ab^3)^{\frac{3}{4}}\,be-3\,(ab^3)^{\frac{3}{4}}\,ai\Big)\log\Big(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\Big)}{256\,a^3b^4}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="g
iac")
```

```
[Out] 1/32*(5*b^2*e*x^7 + 3*a*b*i*x^7 + 6*b^2*d*x^6 + 2*a*b*h*x^6 + 7*b^2*c*x^5 +
 a*b*g*x^5 + 9*a*b*e*x^3 - a^2*i*x^3 + 10*a*b*d*x^2 - 2*a^2*h*x^2 + 11*a*b*
c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b) + 1/128*sqrt(2)*(12*sqrt(2
)*sqrt(a*b)*b^3*d + 4*sqrt(2)*sqrt(a*b)*a*b^2*h + 21*(a*b^3)^(1/4)*b^3*c +
3*(a*b^3)^(1/4)*a*b^2*g + 5*(a*b^3)^(3/4)*b*e + 3*(a*b^3)^(3/4)*a*i)*arctan
(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^4) + 1/128*sqr
t(2)*(12*sqrt(2)*sqrt(a*b)*b^3*d + 4*sqrt(2)*sqrt(a*b)*a*b^2*h + 21*(a*b^3)
^(1/4)*b^3*c + 3*(a*b^3)^(1/4)*a*b^2*g + 5*(a*b^3)^(3/4)*b*e + 3*(a*b^3)^(3
/4)*a*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b
^4) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^3*c + 3*(a*b^3)^(1/4)*a*b^2*g - 5*(
a*b^3)^(3/4)*b*e - 3*(a*b^3)^(3/4)*a*i)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + s
qrt(a/b))/(a^3*b^4) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^3*c + 3*(a*b^3)^(1/
4)*a*b^2*g - 5*(a*b^3)^(3/4)*b*e - 3*(a*b^3)^(3/4)*a*i)*log(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4)
```

Mupad [B] (verification not implemented)

Time = 10.48 (sec) , antiderivative size = 2680, normalized size of antiderivative = 5.79

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x)
```

```
[Out] symsum(log(- root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 4128768
*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 + 68812
80*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2 + 614
40*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z - 7741
44*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55296*
a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 55296*a^
5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 3456*a^4
*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*
d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*
i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g*i^2 + 264
60*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*g - 6720*a^
3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^
2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*i^2 + 13824*a
^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 + 23814*a^2*b^4
*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^2*b^4*d^3*h + 3
072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 + 81*a^4*b^2*g
^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 81*a^6*i^4 + 19448
1*b^6*c^4, z, l)*(root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 41
28768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 +
6881280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2
+ 61440*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z -
 774144*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 5
5296*a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 552
96*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 345
6*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3
*b^3*d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d
^2*g*i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g*i^2
+ 26460*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*g - 67
20*a^3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5
*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*i^2 + 13
824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 + 23814*a^
2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^2*b^4*d^3*
h + 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 + 81*a^4*
b^2*g^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 81*a^6*i^4 +
194481*b^6*c^4, z, l)*((344064*a^5*b^5*c + 49152*a^6*b^4*g)/(32768*a^6*b^2)
 - (x*(24576*a^5*b^4*d + 8192*a^6*b^3*h))/(4096*a^6*b)) + (15360*a^3*b^4*d*
```

e + 9216*a^4*b^3*d*i + 5120*a^4*b^3*e*h + 3072*a^5*b^2*h*i)/(32768*a^6*b^2) - (x*(144*a^5*b*i^2 - 7056*a^2*b^4*c^2 + 400*a^3*b^3*e^2 - 144*a^4*b^2*g^2 - 2016*a^3*b^3*c*g + 480*a^4*b^2*e*i))/(4096*a^6*b)) - (27*a^4*i^3 + 125*a*b^3*e^3 - 3024*b^4*c*d^2 + 2205*b^4*c^2*e - 336*a^2*b^2*c*h^2 + 45*a^2*b^2*e*g^2 + 225*a^2*b^2*e^2*i - 432*a*b^3*d^2*g + 1323*a*b^3*c^2*i + 135*a^3*b*e*i^2 - 48*a^3*b*g*h^2 + 27*a^3*b*g^2*i + 378*a^2*b^2*c*g*i - 288*a^2*b^2*d*g*h - 2016*a*b^3*c*d*h + 630*a*b^3*c*e*g)/(32768*a^6*b^2) - (x*(315*b^3*c*d*e - 8*a^3*h^3 - 216*b^3*d^3 + 9*a^3*g*h*i - 216*a*b^2*d^2*h - 72*a^2*b*d*h^2 + 189*a*b^2*c*d*i + 105*a*b^2*c*e*h + 45*a*b^2*d*e*g + 63*a^2*b*c*h*i + 27*a^2*b*d*g*i + 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 + 6881280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2 + 61440*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z - 774144*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55296*a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g*i^2 + 26460*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 + 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^2*b^4*d^3*h + 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 + 81*a^4*b^2*g^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 81*a^6*i^4 + 194481*b^6*c^4, z, l), l, 1, 4) + ((x^5*(7*b*c + a*g))/(32*a^2) - f/(8*b) + (x^6*(3*b*d + a*h))/(16*a^2) + (x^7*(5*b*e + 3*a*i))/(32*a^2) + (x*(11*b*c - 3*a*g))/(32*a*b) + (x^2*(5*b*d - a*h))/(16*a*b) + (x^3*(9*b*e - a*i))/(32*a*b))/(a^2 + b^2*x^8 + 2*a*b*x^4)

3.203 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 480

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}\,dx \\
&= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{8ab\,(a+bx^4)^2} \\
&\quad - \frac{4a(bf+aj)-x(b(7bc+ag)+2b(3bd+ah)x+b(5be+3ai)x^2)}{32a^2b^2\,(a+bx^4)} \\
&\quad + \frac{(3bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} - \frac{\left(3\sqrt{b}(7bc+ag)+\sqrt{a}(5be+3ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{7/4}} \\
&\quad + \frac{\left(3\sqrt{b}(7bc+ag)+\sqrt{a}(5be+3ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{7/4}} \\
&\quad - \frac{\left(3\sqrt{b}(7bc+ag)-\sqrt{a}(5be+3ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{7/4}} \\
&\quad + \frac{\left(3\sqrt{b}(7bc+ag)-\sqrt{a}(5be+3ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{7/4}}
\end{aligned}$$

```
[Out] 1/8*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+(-a*j+b*f)*x^3)/a/b/(b*x^4+a)^2+
1/32*(-4*a*(a*j+b*f)+x*(b*(a*g+7*b*c)+2*b*(a*h+3*b*d)*x+b*(3*a*i+5*b*e)*x^2
))/a^2/b^2/(b*x^4+a)+1/16*(a*h+3*b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b
^(3/2)-1/256*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+5*
b*e)*a^(1/2)+3*(a*g+7*b*c)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/256*ln(a^(1/
4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+5*b*e)*a^(1/2)+3*(a*g+7*
b*c)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/128*arctan(-1+b^(1/4)*x*2^(1/2)/a^
```

```
(1/4))*((3*a*i+5*b*e)*a^(1/2)+3*(a*g+7*b*c)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/
2)+1/128*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*a*i+5*b*e)*a^(1/2)+3*(a*g+
7*b*c)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.244, Rules used = {1872, 1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}\,dx$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}} + \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+3bd)}{16a^{5/2}b^{3/2}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}} - \frac{4a(aj+bf)-x(b(ag+7bc)+2bx(ah+3bd)+bx^2(3ai+5be))}{32a^2b^2(a+bx^4)} + \frac{x(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc)}{8ab(a+bx^4)^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^3
,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(8*a*b*
(a + b*x^4)^2) - (4*a*(b*f + a*j) - x*(b*(7*b*c + a*g) + 2*b*(3*b*d + a*h)*
x + b*(5*b*e + 3*a*i)*x^2))/(32*a^2*b^2*(a + b*x^4)) + ((3*b*d + a*h)*ArcTa
n[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g)
+ Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqr
t[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i
))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) -
 ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqr
t[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{8ab\,(a+bx^4)^2} \\
&\quad - \frac{\int \frac{-b(7bc+ag)-2b(3bd+ah)x-b(5be+3ai)x^2-4b(bf+aj)x^3}{(a+bx^4)^2}\,dx}{8ab^2} \\
&= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{8ab\,(a+bx^4)^2} \\
&\quad - \frac{4a(bf+aj)-x(b(7bc+ag)+2b(3bd+ah)x+b(5be+3ai)x^2)}{32a^2b^2\,(a+bx^4)} \\
&\quad + \frac{\int \frac{3b(7bc+ag)+4b(3bd+ah)x+b(5be+3ai)x^2}{a+bx^4}\,dx}{32a^2b^2}
\end{aligned}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{8ab\left(a+bx^4\right)^2} - \frac{4a(bf+aj)-x(b(7bc+ag)+2b(3bd+ah)x+b(5be+3ai)x^2)}{32a^2b^2\left(a+bx^4\right)} + \frac{\int\left(\frac{4b(3bd+ah)x}{a+bx^4}+\frac{3b(7bc+ag)+b(5be+3ai)x^2}{a+bx^4}\right)dx}{32a^2b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{8ab\left(a+bx^4\right)^2} - \frac{4a(bf+aj)-x(b(7bc+ag)+2b(3bd+ah)x+b(5be+3ai)x^2)}{32a^2b^2\left(a+bx^4\right)} + \frac{\int\frac{3b(7bc+ag)+b(5be+3ai)x^2}{a+bx^4}dx}{32a^2b^2} + \frac{(3bd+ah)\int\frac{x}{a+bx^4}dx}{8a^2b}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{8ab\left(a+bx^4\right)^2} - \frac{4a(bf+aj)-x(b(7bc+ag)+2b(3bd+ah)x+b(5be+3ai)x^2)}{32a^2b^2\left(a+bx^4\right)} + \frac{(3bd+ah)\text{Subst}\left(\int\frac{1}{a+bx^2}dx,x,x^2\right)}{16a^2b} - \frac{\left(5be-\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\int\frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}dx}{64a^2b^2} + \frac{\left(5be+\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\int\frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}dx}{64a^2b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{8ab\left(a+bx^4\right)^2} - \frac{4a(bf+aj)-x(b(7bc+ag)+2b(3bd+ah)x+b(5be+3ai)x^2)}{32a^2b^2\left(a+bx^4\right)} + \frac{(3bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} + \frac{\left(5be-\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{128\sqrt{2}a^{9/4}b^{7/4}} + \frac{\left(5be-\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{128\sqrt{2}a^{9/4}b^{7/4}} + \frac{\left(5be+\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{128a^2b^2} + \frac{\left(5be+\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{128a^2b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{8ab\left(a+bx^4\right)^2} - \frac{4a(bf+aj)-x(b(7bc+ag)+2b(3bd+ah)x+b(5be+3ai)x^2)}{32a^2b^2\left(a+bx^4\right)} + \frac{(3bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} + \frac{\left(5be-\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{7/4}} - \frac{\left(5be-\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{7/4}} + \frac{\left(5be+\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{9/4}b^{7/4}} - \frac{\left(5be+\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{9/4}b^{7/4}}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{8ab\left(a+bx^4\right)^2} - \frac{4a(bf+aj)-x(b(7bc+ag)+2b(3bd+ah)x+b(5be+3ai)x^2)}{32a^2b^2\left(a+bx^4\right)} + \frac{(3bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} - \frac{\left(5be+\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{9/4}b^{7/4}} + \frac{\left(5be+\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{9/4}b^{7/4}} + \frac{\left(5be-\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{7/4}} - \frac{\left(5be-\frac{3\sqrt{b}(7bc+ag)}{\sqrt{a}}+3ai\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{7/4}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a+bx^4\right)^3}\,dx$$

$$= \frac{\frac{8a^{3/4}(-8a^2j+b^2x(7c+x(6d+5ex))+abx(g+x(2h+3ix)))}{a+bx^4}+\frac{32a^{7/4}(a^2j+b^2x(c+x(d+ex))-ab(f+x(g+x(h+ix))))}{(a+bx^4)^2}-2\sqrt[4]{b}\Big(21\sqrt{2}b^{3/2}c+}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*(-8*a^2*j + b^2*x*(7*c + x*(6*d + 5*e*x)) + a*b*x*(g + x*(2*h +
 3*i*x))))/(a + b*x^4) + (32*a^(7/4)*(a^2*j + b^2*x*(c + x*(d + e*x)) - a*b
*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^2 - 2*b^(1/4)*(21*Sqrt[2]*b^(3/2)*
c + 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g +
8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a
^(1/4)] + 2*b^(1/4)*(21*Sqrt[2]*b^(3/2)*c - 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2
]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g - 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(
3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*(-21*b^(3
/2)*c + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(21*b^(3/2)*c - 5*Sqrt[a
]*b*e + 3*a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/(256*a^(11/4)*b^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.42

method	result
risch	$\frac{\frac{(3ai+5be)x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}-\frac{j\,x^4}{4b}-\frac{(ai-9be)x^3}{32ab}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{aj+bf}{8b^2}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{\left((3ai+5be)\_\right.}{}}{}$
default	$\frac{\frac{(3ai+5be)x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}-\frac{j\,x^4}{4b}-\frac{(ai-9be)x^3}{32ab}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{aj+bf}{8b^2}}{(b\,x^4+a)^2}+\frac{\frac{(3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{}}{}$

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETUR
NVERBOSE)
```

```
[Out] (1/32*(3*a*i+5*b*e)/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x
^5-1/4*j*x^4/b-1/32*(a*i-9*b*e)/a/b*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*
g-11*b*c)/a/b*x-1/8*(a*j+b*f)/b^2)/(b*x^4+a)^2+1/128/a^2/b^2*sum(((3*a*i+5*
b*e)*_R^2+4*(a*h+3*b*d)*_R+3*a*g+21*b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x
)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.11

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{(a + bx^4)^3}\,dx =$$

$$-\frac{8\,a^2bjx^4 - (5\,b^3e + 3\,ab^2i)x^7 - 2\,(3\,b^3d + ab^2h)x^6 - (7\,b^3c + ab^2g)x^5 + 4\,a^2bf + 4\,a^3j - (9\,ab^2e - a^2bi)x}{32\,(a^2b^4x^8 + 2\,a^3b^3x^4 + a^4b^2)}$$

$$+\frac{\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c - 5\,\sqrt{a}be + 3\,a\sqrt{b}g - 3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c - 5\,\sqrt{a}be + 3\,a\sqrt{b}g - 3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \cdots}{\cdots}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algori
thm="maxima")
```

```
[Out] -1/32*(8*a^2*b*j*x^4 - (5*b^3*e + 3*a*b^2*i)*x^7 - 2*(3*b^3*d + a*b^2*h)*x^
6 - (7*b^3*c + a*b^2*g)*x^5 + 4*a^2*b*f + 4*a^3*j - (9*a*b^2*e - a^2*b*i)*x
^3 - 2*(5*a*b^2*d - a^2*b*h)*x^2 - (11*a*b^2*c - 3*a^2*b*g)*x)/(a^2*b^4*x^8
 + 2*a^3*b^3*x^4 + a^4*b^2) + 1/256*(sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e
+ 3*a*sqrt(b)*g - 3*a^(3/2)*i)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x
+ sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*
sqrt(b)*g - 3*a^(3/2)*i)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt
(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 5*sqrt(2)*a^(3/4
)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 3*sqrt(2)*a^(7/4)*b^(1/4)*i - 2
4*sqrt(a)*b^(3/2)*d - 8*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x
+ sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqr
t(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4
)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 3*sqrt(2)*a^(7/4)*b^(1/4)*i + 24*sqrt(a
)*b^(3/2)*d + 8*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2
)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^
(3/4)))/(a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.17

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}\,dx$$

$$= \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^3d+4\sqrt{2}\sqrt{ab}ab^2h+21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g+5\,(ab^3)^{\frac{3}{4}}\,be+3\,(ab^3)^{\frac{3}{4}}\,ai\right)\arctan\left(\frac{\sqrt{2}}{}\right.}{128\,a^3b^4}$$

$$+ \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^3d+4\sqrt{2}\sqrt{ab}ab^2h+21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g+5\,(ab^3)^{\frac{3}{4}}\,be+3\,(ab^3)^{\frac{3}{4}}\,ai\right)\arctan\left(\right.}{128\,a^3b^4}$$

$$+ \frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g-5\,(ab^3)^{\frac{3}{4}}\,be-3\,(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^4}$$

$$- \frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g-5\,(ab^3)^{\frac{3}{4}}\,be-3\,(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^4}$$

$$+ \frac{5\,b^3ex^7+3\,ab^2ix^7+6\,b^3dx^6+2\,ab^2hx^6+7\,b^3cx^5+ab^2gx^5-8\,a^2bjx^4+9\,ab^2ex^3-a^2bix^3+10\,ab^2dx^2}{32\,(bx^4+a)^2a^2b^2}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algori
thm="giac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^3*d + 4*sqrt(2)*sqrt(a*b)*a*b^2*h + 2
1*(a*b^3)^(1/4)*b^3*c + 3*(a*b^3)^(1/4)*a*b^2*g + 5*(a*b^3)^(3/4)*b*e + 3*(
a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(a^3*b^4) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^3*d + 4*sqrt(2)*sqrt(a
*b)*a*b^2*h + 21*(a*b^3)^(1/4)*b^3*c + 3*(a*b^3)^(1/4)*a*b^2*g + 5*(a*b^3)^
(3/4)*b*e + 3*(a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(a^3*b^4) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^3*c + 3*(a*
b^3)^(1/4)*a*b^2*g - 5*(a*b^3)^(3/4)*b*e - 3*(a*b^3)^(3/4)*a*i)*log(x^2 + s
qrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) - 1/256*sqrt(2)*(21*(a*b^3)^(1/
4)*b^3*c + 3*(a*b^3)^(1/4)*a*b^2*g - 5*(a*b^3)^(3/4)*b*e - 3*(a*b^3)^(3/4)*
a*i)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) + 1/32*(5*b^3*e
*x^7 + 3*a*b^2*i*x^7 + 6*b^3*d*x^6 + 2*a*b^2*h*x^6 + 7*b^3*c*x^5 + a*b^2*g*
x^5 - 8*a^2*b*j*x^4 + 9*a*b^2*e*x^3 - a^2*b*i*x^3 + 10*a*b^2*d*x^2 - 2*a^2*
b*h*x^2 + 11*a*b^2*c*x - 3*a^2*b*g*x - 4*a^2*b*f - 4*a^3*j)/((b*x^4 + a)^2*
a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 10.19 (sec) , antiderivative size = 2695, normalized size of antiderivative = 5.61

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{(a + bx^4)^3} dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^3
,x)
```

```
[Out] symsum(log(- root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 4128768
*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 + 68812
80*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2 + 614
40*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z - 7741
44*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55296*
a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 55296*a^
5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 3456*a^4
*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*
d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*
i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g*i^2 + 264
60*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*g - 6720*a^
3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^
2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*i^2 + 13824*a
^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 + 23814*a^2*b^4
*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^2*b^4*d^3*h + 3
072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 + 81*a^4*b^2*g
^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 81*a^6*i^4 + 19448
1*b^6*c^4, z, m)*(root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 41
28768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 +
6881280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2
+ 61440*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z -
 774144*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 5
5296*a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 552
96*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 345
6*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3
*b^3*d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d
^2*g*i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g*i^2
+ 26460*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*g - 67
20*a^3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5
*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*i^2 + 13
824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 + 23814*a^
2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^2*b^4*d^3*
h + 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 + 81*a^4*
b^2*g^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 81*a^6*i^4 +
194481*b^6*c^4, z, m)*((344064*a^5*b^5*c + 49152*a^6*b^4*g)/(32768*a^6*b^2)
```

- (x*(24576*a^5*b^4*d + 8192*a^6*b^3*h))/(4096*a^6*b)) + (15360*a^3*b^4*d*e + 9216*a^4*b^3*d*i + 5120*a^4*b^3*e*h + 3072*a^5*b^2*h*i)/(32768*a^6*b^2) - (x*(144*a^5*b*i^2 - 7056*a^2*b^4*c^2 + 400*a^3*b^3*e^2 - 144*a^4*b^2*g^2 - 2016*a^3*b^3*c*g + 480*a^4*b^2*e*i))/(4096*a^6*b)) - (27*a^4*i^3 + 125*a*b^3*e^3 - 3024*b^4*c*d^2 + 2205*b^4*c^2*e - 336*a^2*b^2*c*h^2 + 45*a^2*b^2*e*g^2 + 225*a^2*b^2*e^2*i - 432*a*b^3*d^2*g + 1323*a*b^3*c^2*i + 135*a^3*b*e*i^2 - 48*a^3*b*g*h^2 + 27*a^3*b*g^2*i + 378*a^2*b^2*c*g*i - 288*a^2*b^2*d*g*h - 2016*a*b^3*c*d*h + 630*a*b^3*c*e*g)/(32768*a^6*b^2) - (x*(315*b^3*c*d*e - 8*a^3*h^3 - 216*b^3*d^3 + 9*a^3*g*h*i - 216*a*b^2*d^2*h - 72*a^2*b*d*h^2 + 189*a*b^2*c*d*i + 105*a*b^2*c*e*h + 45*a*b^2*d*e*g + 63*a^2*b*c*h*i + 27*a^2*b*d*g*i + 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 + 6881280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2 + 61440*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z - 774144*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55296*a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g*i^2 + 26460*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 + 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^2*b^4*d^3*h + 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 + 81*a^4*b^2*g^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 81*a^6*i^4 + 194481*b^6*c^4, z, m), m, 1, 4) + ((x^5*(7*b*c + a*g))/(32*a^2) - (j*x^4)/(4*b) - (b*f + a*j)/(8*b^2) + (x^6*(3*b*d + a*h))/(16*a^2) + (x^7*(5*b*e + 3*a*i))/(32*a^2) + (x*(11*b*c - 3*a*g))/(32*a*b) + (x^2*(5*b*d - a*h))/(16*a*b) + (x^3*(9*b*e - a*i))/(32*a*b))/(a^2 + b^2*x^8 + 2*a*b*x^4)

3.204 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 293

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx &= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} \\ &+ \frac{x(7(11bc-ag)+12(5bd-ah)x+45bex^2)}{384a^3b\,(a-bx^4)} \\ &+ \frac{8af+x(11bc-ag+2(5bd-ah)x+9bex^2)}{96a^2b\,(a-bx^4)^2} \\ &+ \frac{\left(77bc-15\sqrt{a}\sqrt{b}e-7ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{5/4}} \\ &+ \frac{\left(77bc+15\sqrt{a}\sqrt{b}e-7ag\right)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{5/4}} \\ &+ \frac{(5bd-ah)\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}\end{aligned}$$

```
[Out] 1/12*x*(b*c+a*g+(a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(-b*x^4+a)^3+1/384*x*(-7*a
*g+77*b*c+12*(-a*h+5*b*d)*x+45*b*e*x^2)/a^3/b/(-b*x^4+a)+1/96*(8*a*f+x*(11*
b*c-a*g+2*(-a*h+5*b*d)*x+9*b*e*x^2))/a^2/b/(-b*x^4+a)^2+1/32*(-a*h+5*b*d)*a
rctanh(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)+1/256*arctan(b^(1/4)*x/a^(1/4))
*(77*b*c-7*a*g-15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)+1/256*arctanh(b^(1/4)
*x/a^(1/4))*(77*b*c-7*a*g+15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1872, 1868, 1869, 1890, 281, 214, 1181, 211}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(5bd-ah)}{32a^{7/2}b^{3/2}} + \frac{x(7(11bc-ag)+12x(5bd-ah)+45bex^2)}{384a^3b(a-bx^4)} + \frac{x(2x(5bd-ah)-ag+11bc+9bex^2)+8af}{96a^2b(a-bx^4)^2} + \frac{x(x(ah+bd)+ag+bc+bex^2+bfx^3)}{12ab(a-bx^4)^3}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3)
+ (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 45*b*e*x^2))/(384*a^3*b*(a -
b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d - a*h)*x + 9*b*e*x^2))/(96*a^
2*b*(a - b*x^4)^2) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/
4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e -
7*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((5*b*d - a*h
)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab\left(a-bx^4\right)^3} - \frac{\int \frac{-b(11bc-ag)-2b(5bd-ah)x-9b^2ex^2-8b^2fx^3}{\left(a-bx^4\right)^3}\,dx}{12ab^2}$$

$$=\frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab\left(a-bx^4\right)^3}+\frac{8af+x(11bc-ag+2(5bd-ah)x+9bex^2)}{96a^2b\left(a-bx^4\right)^2}+\frac{\int\frac{7b(11bc-ag)+12b(5bd-ah)x+45b^2ex^2}{(a-bx^4)^2}dx}{96a^2b^2}$$

$$=\frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab\left(a-bx^4\right)^3}+\frac{x(7(11bc-ag)+12(5bd-ah)x+45bex^2)}{384a^3b\left(a-bx^4\right)}+\frac{8af+x(11bc-ag+2(5bd-ah)x+9bex^2)}{96a^2b\left(a-bx^4\right)^2}-\frac{\int\frac{-21b(11bc-ag)-24b(5bd-ah)x-45b^2ex^2}{a-bx^4}dx}{384a^3b^2}$$

$$=\frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab\left(a-bx^4\right)^3}+\frac{x(7(11bc-ag)+12(5bd-ah)x+45bex^2)}{384a^3b\left(a-bx^4\right)}+\frac{8af+x(11bc-ag+2(5bd-ah)x+9bex^2)}{96a^2b\left(a-bx^4\right)^2}-\frac{\int\left(-\frac{24b(5bd-ah)x}{a-bx^4}+\frac{-21b(11bc-ag)-45b^2ex^2}{a-bx^4}\right)dx}{384a^3b^2}$$

$$=\frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab\left(a-bx^4\right)^3}+\frac{x(7(11bc-ag)+12(5bd-ah)x+45bex^2)}{384a^3b\left(a-bx^4\right)}+\frac{8af+x(11bc-ag+2(5bd-ah)x+9bex^2)}{96a^2b\left(a-bx^4\right)^2}-\frac{\int\frac{-21b(11bc-ag)-45b^2ex^2}{a-bx^4}dx}{384a^3b^2}+\frac{(5bd-ah)\int\frac{x}{a-bx^4}dx}{16a^3b}$$

$$=\frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab\left(a-bx^4\right)^3}+\frac{x(7(11bc-ag)+12(5bd-ah)x+45bex^2)}{384a^3b\left(a-bx^4\right)}+\frac{8af+x(11bc-ag+2(5bd-ah)x+9bex^2)}{96a^2b\left(a-bx^4\right)^2}-\frac{\left(77bc-15\sqrt{a}\sqrt{b}e-7ag\right)\int\frac{1}{-\sqrt{a}\sqrt{b}-bx^2}dx}{256a^{7/2}\sqrt{b}}+\frac{\left(77bc+15\sqrt{a}\sqrt{b}e-7ag\right)\int\frac{1}{\sqrt{a}\sqrt{b}-bx^2}dx}{256a^{7/2}\sqrt{b}}+\frac{(5bd-ah)\text{Subst}\left(\int\frac{1}{a-bx^2}dx,x,x^2\right)}{32a^3b}$$

$$
\begin{aligned}
&= \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab\,(a-bx^4)^3} \\
&\quad + \frac{x(7(11bc-ag)+12(5bd-ah)x+45bex^2)}{384a^3b\,(a-bx^4)} \\
&\quad + \frac{8af+x(11bc-ag+2(5bd-ah)x+9bex^2)}{96a^2b\,(a-bx^4)^2} \\
&\quad + \frac{\left(77bc-15\sqrt{a}\sqrt{b}e-7ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{5/4}} \\
&\quad + \frac{\left(77bc+15\sqrt{a}\sqrt{b}e-7ag\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{5/4}} + \frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.23

$$
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx
$$

$$
= \frac{\frac{4a^{3/4}\sqrt{b}x(77bc-7ag+60bdx-12ahx+45bex^2)}{a-bx^4} + \frac{16a^{7/4}\sqrt{b}x(11bc+bx(10d+9ex)-a(g+2hx))}{(a-bx^4)^2} + \frac{128a^{11/4}\sqrt{b}(bx(c+x(d+ex))+a(f+x(g+hx)))}{(a-bx^4)^3}}{}
$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x]
```

```
[Out] ((4*a^(3/4)*Sqrt[b]*x*(77*b*c - 7*a*g + 60*b*d*x - 12*a*h*x + 45*b*e*x^2))/
(a - b*x^4) + (16*a^(7/4)*Sqrt[b]*x*(11*b*c + b*x*(10*d + 9*e*x) - a*(g + 2
*h*x)))/(a - b*x^4)^2 + (128*a^(11/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f
 + x*(g + h*x))))/(a - b*x^4)^3 + 6*b^(1/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e
- 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - 3*(77*b^(5/4)*c + 40*a^(1/4)*b*d + 1
5*Sqrt[a]*b^(3/4)*e - 7*a*b^(1/4)*g - 8*a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x]
 + 3*(77*b^(5/4)*c - 40*a^(1/4)*b*d + 15*Sqrt[a]*b^(3/4)*e - 7*a*b^(1/4)*g
+ 8*a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] - 24*a^(1/4)*(-5*b*d + a*h)*Log[Sqr
t[a] + Sqrt[b]*x^2])/(1536*a^(15/4)*b^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.75

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}-\frac{21be\,x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{(ah+11bd)x^2}{32ab}+\frac{(7ag+51bc)x}{128ab}+\frac{f}{12b}}{(-b\,x^4+a)^3}-\_R=$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}-\frac{21be\,x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{(ah+11bd)x^2}{32ab}+\frac{(7ag+51bc)x}{128ab}+\frac{f}{12b}}{(-b\,x^4+a)^3}+\frac{(-7ag}{}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (15/128*e/a^3*b^2*x^11-1/32*(a*h-5*b*d)/a^3*b*x^10-7/384*(a*g-11*b*c)/a^3*b
*x^9-21/64*b*e/a^2*x^7+1/12/a^2*(a*h-5*b*d)*x^6+3/64/a^2*(a*g-11*b*c)*x^5+1
13/384/a*e*x^3+1/32*(a*h+11*b*d)/a/b*x^2+1/128*(7*a*g+51*b*c)/a/b*x+1/12*f/
b)/(-b*x^4+a)^3-1/512/a^3/b*sum((15*_R^2*e-8/b*(a*h-5*b*d)*_R-7*(a*g-11*b*c
)/b)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx = \text{Timed out}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx = \text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.33

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a - bx^4)^4}\,dx =$$

$$-\frac{45\,b^3ex^{11} - 126\,ab^2ex^7 + 12\,(5\,b^3d - ab^2h)x^{10} + 7\,(11\,b^3c - ab^2g)x^9 + 113\,a^2bex^3 - 32\,(5\,ab^2d - a^2bh)x^6}{384\,(a^3b^4x^{12} - 3\,a^4b^3x^8 + 3\,a^5b^2x}$$

$$+\frac{\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{8\,(5\,bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be-7\,a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(77\,b^{\frac{3}{2}}c+15\,\sqrt{a}be-7\,a\sqrt{}\right.}{\sqrt{a}\sqrt{}}}{512\,a^3b}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima
")
```

```
[Out] -1/384*(45*b^3*e*x^11 - 126*a*b^2*e*x^7 + 12*(5*b^3*d - a*b^2*h)*x^10 + 7*(
11*b^3*c - a*b^2*g)*x^9 + 113*a^2*b*e*x^3 - 32*(5*a*b^2*d - a^2*b*h)*x^6 -
18*(11*a*b^2*c - a^2*b*g)*x^5 + 32*a^3*f + 12*(11*a^2*b*d + a^3*h)*x^2 + 3*
(51*a^2*b*c + 7*a^3*g)*x)/(a^3*b^4*x^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a
^6*b) + 1/512*(8*(5*b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b))
 - 8*(5*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(77*b^(
3/2)*c - 15*sqrt(a)*b*e - 7*a*sqrt(b)*g)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (77*b^(3/2)*c + 15*sqrt(a)*
b*e - 7*a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + s
qrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^3*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.69

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx =$$

$$-\frac{\sqrt{2}\Big(77\,b^2c-7\,abg-40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+8\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,ah+15\,\sqrt{-ab}be\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$-\frac{\sqrt{2}\Big(77\,b^2c-7\,abg+40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-8\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,ah-15\,\sqrt{-ab}be\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$-\frac{\sqrt{2}(77\,b^2c-7\,abg-15\,\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$+\frac{\sqrt{2}(77\,b^2c-7\,abg-15\,\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$-\frac{45\,b^3ex^{11}+60\,b^3dx^{10}-12\,ab^2hx^{10}+77\,b^3cx^9-7\,ab^2gx^9-126\,ab^2ex^7-160\,ab^2dx^6+32\,a^2bhx^6-198}{384\,(bx^4-a)^3a}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^2*c - 7*a*b*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 8*sqrt
(2)*(-a*b^3)^(1/4)*a*h + 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2
*c - 7*a*b*g + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 8*sqrt(2)*(-a*b^3)^(1/4)*a*h
 - 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b
)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqr
t(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)
*a^3) + 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqrt(-a*b)*b*e)*log(x^2 - s
qrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/384*(45*b^3*e*
x^11 + 60*b^3*d*x^10 - 12*a*b^2*h*x^10 + 77*b^3*c*x^9 - 7*a*b^2*g*x^9 - 126
*a*b^2*e*x^7 - 160*a*b^2*d*x^6 + 32*a^2*b*h*x^6 - 198*a*b^2*c*x^5 + 18*a^2*
b*g*x^5 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 12*a^3*h*x^2 + 153*a^2*b*c*x
+ 21*a^3*g*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 1747, normalized size of antiderivative = 5.96

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a - bx^4)^4} dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x)
```

```
[Out] symsum(log(- root(68719476736*a^15*b^6*z^4 - 1211105280*a^8*b^5*c*e*z^2 + 3
35544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 - 838860800*a^8*b^5*d^
2*z^2 - 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z + 17661952*a^6
*b^3*c*g*h*z + 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 80281
6*a^7*b^2*g^2*h*z - 3686400*a^6*b^3*e^2*h*z + 4014080*a^6*b^3*d*g^2*z + 184
32000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h + 2956800*a^2*b^3*c*d*e*h +
672000*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 - 485100*a^2*b^3*c*e^2*g +
26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e - 81920*a^4*b*d*h^3 + 12782924*
a*b^4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 - 1743126*a^2*
b^3*c^2*g^2 - 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*b^4*
c^2*e^2 - 50625*a^2*b^3*e^4 - 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096*a^5
*h^4 - 35153041*b^5*c^4, z, k)*(root(68719476736*a^15*b^6*z^4 - 1211105280*
a^8*b^5*c*e*z^2 + 335544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 - 8
38860800*a^8*b^5*d^2*z^2 - 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d
*g*z + 17661952*a^6*b^3*c*g*h*z + 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*
b^4*c^2*h*z - 802816*a^7*b^2*g^2*h*z - 3686400*a^6*b^3*e^2*h*z + 4014080*a^
6*b^3*d*g^2*z + 18432000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h + 2956800
*a^2*b^3*c*d*e*h + 672000*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 - 485100
*a^2*b^3*c*e^2*g + 26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e - 81920*a^4*
b*d*h^3 + 12782924*a*b^4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2
*g^2 - 1743126*a^2*b^3*c^2*g^2 - 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g
^3 + 2668050*a*b^4*c^2*e^2 - 50625*a^2*b^3*e^4 - 2401*a^4*b*g^4 + 2560000*a
*b^4*d^4 + 4096*a^5*h^4 - 35153041*b^5*c^4, z, k)*((20185088*a^7*b^4*c - 18
35008*a^8*b^3*g)/(2097152*a^9*b) - (x*(655360*a^7*b^4*d - 131072*a^8*b^3*h)
)/(131072*a^9*b)) - (614400*a^4*b^3*d*e - 122880*a^5*b^2*e*h)/(2097152*a^9*
b) + (x*(189728*a^3*b^4*c^2 + 7200*a^4*b^3*e^2 + 1568*a^5*b^2*g^2 - 34496*a
^4*b^3*c*g))/(131072*a^9*b)) - (3375*a*b^2*e^3 + 123200*b^3*c*d^2 - 88935*b
^3*c^2*e - 448*a^3*g*h^2 - 11200*a*b^2*d^2*g + 4928*a^2*b*c*h^2 - 735*a^2*b
*e*g^2 - 49280*a*b^2*c*d*h + 16170*a*b^2*c*e*g + 4480*a^2*b*d*g*h)/(2097152
*a^9*b) - (x*(4000*b^3*d^3 - 32*a^3*h^3 - 5775*b^3*c*d*e - 2400*a*b^2*d^2*h
 + 480*a^2*b*d*h^2 + 1155*a*b^2*c*e*h + 525*a*b^2*d*e*g - 105*a^2*b*e*g*h))
/(131072*a^9*b))*root(68719476736*a^15*b^6*z^4 - 1211105280*a^8*b^5*c*e*z^2
 + 335544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 - 838860800*a^8*b^
5*d^2*z^2 - 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z + 17661952
*a^6*b^3*c*g*h*z + 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 8
02816*a^7*b^2*g^2*h*z - 3686400*a^6*b^3*e^2*h*z + 4014080*a^6*b^3*d*g^2*z +
```

```
 18432000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h + 2956800*a^2*b^3*c*d*e*
h + 672000*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 - 485100*a^2*b^3*c*e^2*
g + 26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e - 81920*a^4*b*d*h^3 + 12782
924*a*b^4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 - 1743126*
a^2*b^3*c^2*g^2 - 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*
b^4*c^2*e^2 - 50625*a^2*b^3*e^4 - 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096
*a^5*h^4 - 35153041*b^5*c^4, z, k), k, 1, 4) + (f/(12*b) + (113*e*x^3)/(384
*a) - (3*x^5*(11*b*c - a*g))/(64*a^2) - (x^6*(5*b*d - a*h))/(12*a^2) + (7*b
*x^9*(11*b*c - a*g))/(384*a^3) + (x*(51*b*c + 7*a*g))/(128*a*b) + (b*x^10*(
5*b*d - a*h))/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) + (x^2*(11*b*d + a*h))/(
32*a*b) - (21*b*e*x^7)/(64*a^2))/(a^3 - b^3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^
8)
```

3.205 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4} dx$

Optimal result

Integrand size = 41, antiderivative size = 331

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4} dx
&= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{12ab\left(a-bx^4\right)^3} \\
&+ \frac{x(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2)}{384a^3b\left(a-bx^4\right)} \\
&+ \frac{8af+x(11bc-ag+2(5bd-ah)x+3(3be-ai)x^2)}{96a^2b\left(a-bx^4\right)^2} \\
&+ \frac{\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{13/4}b^{7/4}} \\
&+ \frac{\left(15be+\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{13/4}b^{7/4}} + \frac{(5bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}
\end{aligned}
$$

```
[Out] 1/12*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+b*f*x^3)/a/b/(-b*x^4+a)^3+1/384*x
*(-7*a*g+77*b*c+12*(-a*h+5*b*d)*x+15*(-a*i+3*b*e)*x^2)/a^3/b/(-b*x^4+a)+1/9
6*(8*a*f+x*(11*b*c-a*g+2*(-a*h+5*b*d)*x+3*(-a*i+3*b*e)*x^2))/a^2/b/(-b*x^4+
a)^2+1/32*(-a*h+5*b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)+1/256*a
rctanh(b^(1/4)*x/a^(1/4))*(15*b*e-5*a*i+7*(-a*g+11*b*c)*b^(1/2)/a^(1/2))/a^
(13/4)/b^(7/4)+1/256*arctan(b^(1/4)*x/a^(1/4))*(5*a*i-15*b*e+7*(-a*g+11*b*c
)*b^(1/2)/a^(1/2))/a^(13/4)/b^(7/4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {1872, 1868, 1869, 1890, 281, 214, 1181, 211}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx$$
$$= \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{256a^{13/4}b^{7/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(5bd-ah)}{32a^{7/2}b^{3/2}} + \frac{x(7(11bc-ag)+12x(5bd-ah)+15x^2(3be-ai))}{384a^3b\,(a-bx^4)} + \frac{x(2x(5bd-ah)+3x^2(3be-ai)-ag+11bc)+8af}{96a^2b\,(a-bx^4)^2} + \frac{x(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3)}{12ab\,(a-bx^4)^3}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(12*a*b*(a - b*
x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 15*(3*b*e - a*i)*x^2)
)/(384*a^3*b*(a - b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d - a*h)*x +
3*(3*b*e - a*i)*x^2))/(96*a^2*b*(a - b*x^4)^2) + (((7*Sqrt[b]*(11*b*c - a*g
))/Sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^
(7/4)) + ((15*b*e + (7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*a*i)*ArcTanh[(b^
(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((5*b*d - a*h)*ArcTanh[(Sqrt[b]
*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{12ab\left(a-bx^4\right)^3} - \frac{\int \frac{-b(11bc-ag)-2b(5bd-ah)x-3b(3be-ai)x^2-8b^2fx^3}{\left(a-bx^4\right)^3}\,dx}{12ab^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{12ab\left(a-bx^4\right)^3} + \frac{8af+x(11bc-ag+2(5bd-ah)x+3(3be-ai)x^2)}{96a^2b\left(a-bx^4\right)^2} + \frac{\int \frac{7b(11bc-ag)+12b(5bd-ah)x+15b(3be-ai)x^2}{\left(a-bx^4\right)^2}\,dx}{96a^2b^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{12ab\left(a-bx^4\right)^3} + \frac{x(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2)}{384a^3b\left(a-bx^4\right)} + \frac{8af+x(11bc-ag+2(5bd-ah)x+3(3be-ai)x^2)}{96a^2b\left(a-bx^4\right)^2} - \frac{\int \frac{-21b(11bc-ag)-24b(5bd-ah)x-15b(3be-ai)x^2}{a-bx^4}\,dx}{384a^3b^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{12ab\left(a-bx^4\right)^3} + \frac{x(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2)}{384a^3b\left(a-bx^4\right)} + \frac{8af+x(11bc-ag+2(5bd-ah)x+3(3be-ai)x^2)}{96a^2b\left(a-bx^4\right)^2} - \frac{\int \left(-\frac{24b(5bd-ah)x}{a-bx^4}+\frac{-21b(11bc-ag)-15b(3be-ai)x^2}{a-bx^4}\right)dx}{384a^3b^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{12ab\left(a-bx^4\right)^3} + \frac{x(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2)}{384a^3b\left(a-bx^4\right)} + \frac{8af+x(11bc-ag+2(5bd-ah)x+3(3be-ai)x^2)}{96a^2b\left(a-bx^4\right)^2} - \frac{\int \frac{-21b(11bc-ag)-15b(3be-ai)x^2}{a-bx^4}\,dx}{384a^3b^2} + \frac{(5bd-ah)\int \frac{x}{a-bx^4}\,dx}{16a^3b}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{12ab(a-bx^4)^3} + \frac{x(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2)}{384a^3b(a-bx^4)} + \frac{8af+x(11bc-ag+2(5bd-ah)x+3(3be-ai)x^2)}{96a^2b(a-bx^4)^2} + \frac{(5bd-ah)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{32a^3b} + \frac{\left(15be+\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx}{256a^3b} - \frac{\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{256a^3b}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{12ab(a-bx^4)^3} + \frac{x(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2)}{384a^3b(a-bx^4)} + \frac{8af+x(11bc-ag+2(5bd-ah)x+3(3be-ai)x^2)}{96a^2b(a-bx^4)^2} + \frac{\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{13/4}b^{7/4}} + \frac{\left(15be+\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{13/4}b^{7/4}} + \frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.27

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx = \frac{-\frac{4ab^{3/4}x(-77bc+7ag-15bx(4d+3ex)+3ax(4h+5ix))}{a-bx^4} - \frac{16a^2b^{3/4}x(-b(11c+x(10d+9ex))+a(g+x(2h+3ix)))}{(a-bx^4)^2} + \frac{128a^3b^{3/4}(bx(c+x(d+ex))+\cdots}{(a-bx^4)\cdots}}{\cdots}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x
]
```

```
[Out] ((-4*a*b^(3/4)*x*(-77*b*c + 7*a*g - 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i
*x)))/(a - b*x^4) - (16*a^2*b^(3/4)*x*(-(b*(11*c + x*(10*d + 9*e*x))) + a*(
g + x*(2*h + 3*i*x))))/(a - b*x^4)^2 + (128*a^3*b^(3/4)*(b*x*(c + x*(d + e*
x)) + a*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^3 + 6*a^(1/4)*(77*b^(3/2)*c
 - 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)
] + 3*a^(1/4)*(-77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d - 15*Sqrt[a]*b*e + 7*a*
```

```
Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] - 3
*a^(1/4)*(-77*b^(3/2)*c + 40*a^(1/4)*b^(5/4)*d - 15*Sqrt[a]*b*e + 7*a*Sqrt[
b]*g - 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 24*Sqr
t[a]*b^(1/4)*(-5*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(1536*a^4*b^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.57 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.73

method	result
risch	$\frac{-\frac{5(ai-3be)b\,x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}+\frac{7(ai-3be)x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{(5ai+113be)x^3}{384ab}+\frac{(ah+11bd)x^2}{32ab}+\frac{(7ag+51bc}{128ab}}{(-b\,x^4+a)^3}$
default	$\frac{-\frac{5(ai-3be)b\,x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}+\frac{7(ai-3be)x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{(5ai+113be)x^3}{384ab}+\frac{(ah+11bd)x^2}{32ab}+\frac{(7ag+51bc}{128ab}}{(-b\,x^4+a)^3}$

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERB
OSE)
```

```
[Out] (-5/128*(a*i-3*b*e)/a^3*b*x^11-1/32*(a*h-5*b*d)/a^3*b*x^10-7/384*(a*g-11*b*
c)/a^3*b*x^9+7/64*(a*i-3*b*e)/a^2*x^7+1/12/a^2*(a*h-5*b*d)*x^6+3/64/a^2*(a*
g-11*b*c)*x^5+1/384*(5*a*i+113*b*e)/a/b*x^3+1/32*(a*h+11*b*d)/a/b*x^2+1/128
*(7*a*g+51*b*c)/a/b*x+1/12*f/b)/(-b*x^4+a)^3-1/512/a^3/b^2*sum((-5*(a*i-3*b
*e)*_R^2-8*(a*h-5*b*d)*_R-7*a*g+77*b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx = \text{Timed out}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx = \text{Timed out}$$

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.30

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx = -\frac{15\,(3\,b^3e-ab^2i)x^{11}+12\,(5\,b^3d-ab^2h)x^{10}+7\,(11\,b^3c-ab^2g)x^9-42\,(3\,ab^2e-a^2bi)x^7-32\,(5\,ab^2d-a^2}{384\,(a^3b^4x^{12}-3\,a^4}$$
$$+\frac{\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be-7\,a\sqrt{b}g+5\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\,b^{\frac{3}{2}}c+15\,\sqrt{a}b}{}}{512\,a^3b}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="
maxima")
[Out] -1/384*(15*(3*b^3*e - a*b^2*i)*x^11 + 12*(5*b^3*d - a*b^2*h)*x^10 + 7*(11*b
^3*c - a*b^2*g)*x^9 - 42*(3*a*b^2*e - a^2*b*i)*x^7 - 32*(5*a*b^2*d - a^2*b*
h)*x^6 - 18*(11*a*b^2*c - a^2*b*g)*x^5 + 32*a^3*f + (113*a^2*b*e + 5*a^3*i)
*x^3 + 12*(11*a^2*b*d + a^3*h)*x^2 + 3*(51*a^2*b*c + 7*a^3*g)*x)/(a^3*b^4*x
^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6*b) + 1/512*(8*(5*b*d - a*h)*log(s
qrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 8*(5*b*d - a*h)*log(sqrt(b)*x^2 -
 sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(77*b^(3/2)*c - 15*sqrt(a)*b*e - 7*a*sqrt(b
)*g + 5*a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sq
rt(a)*sqrt(b))*sqrt(b)) - (77*b^(3/2)*c + 15*sqrt(a)*b*e - 7*a*sqrt(b)*g -
5*a^(3/2)*i)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt
(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^3*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(284) = 568$.

Time = 0.29 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.82

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx =$$

$$-\frac{\sqrt{2}\left(77\,b^3c-7\,ab^2g-40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d+8\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-15\,\sqrt{-ab}b^2e+5\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}(\ldots}{\ldots}\right.}{512\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$-\frac{\sqrt{2}\left(77\,b^3c-7\,ab^2g+40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d-8\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-15\,\sqrt{-ab}b^2e-5\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}(\ldots}{\ldots}\right.}{512\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$-\frac{\sqrt{2}(77\,b^3c-7\,ab^2g-15\,\sqrt{-ab}b^2e+5\,\sqrt{-ab}abi)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$+\frac{\sqrt{2}(77\,b^3c-7\,ab^2g-15\,\sqrt{-ab}b^2e+5\,\sqrt{-ab}abi)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$-\frac{45\,b^3ex^{11}-15\,ab^2ix^{11}+60\,b^3dx^{10}-12\,ab^2hx^{10}+77\,b^3cx^9-7\,ab^2gx^9-126\,ab^2ex^7+42\,a^2bix^7-160\,a\ldots}{\ldots}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="
giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^3*c - 7*a*b^2*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b^2*d + 8*
sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 15*sqrt(-a*b)*b^2*e + 5*sqrt(-a*b)*a*b*i)*ar
ctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)
*a^3*b) - 1/512*sqrt(2)*(77*b^3*c - 7*a*b^2*g + 40*sqrt(2)*(-a*b^3)^(1/4)*b
^2*d - 8*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 15*sqrt(-a*b)*b^2*e - 5*sqrt(-a*b)*
a*b*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b
^3)^(3/4)*a^3*b) - 1/1024*sqrt(2)*(77*b^3*c - 7*a*b^2*g - 15*sqrt(-a*b)*b^2
*e + 5*sqrt(-a*b)*a*b*i)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-
a*b^3)^(3/4)*a^3*b) + 1/1024*sqrt(2)*(77*b^3*c - 7*a*b^2*g - 15*sqrt(-a*b)*
b^2*e + 5*sqrt(-a*b)*a*b*i)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/
((-a*b^3)^(3/4)*a^3*b) - 1/384*(45*b^3*e*x^11 - 15*a*b^2*i*x^11 + 60*b^3*d*
x^10 - 12*a*b^2*h*x^10 + 77*b^3*c*x^9 - 7*a*b^2*g*x^9 - 126*a*b^2*e*x^7 + 4
2*a^2*b*i*x^7 - 160*a*b^2*d*x^6 + 32*a^2*b*h*x^6 - 198*a*b^2*c*x^5 + 18*a^2
*b*g*x^5 + 113*a^2*b*e*x^3 + 5*a^3*i*x^3 + 132*a^2*b*d*x^2 + 12*a^3*h*x^2 +
 153*a^2*b*c*x + 21*a^3*g*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 10.32 (sec) , antiderivative size = 2747, normalized size of antiderivative = 8.30

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x)
```

```
[Out] (f/(12*b) - (3*x^5*(11*b*c - a*g))/(64*a^2) - (x^6*(5*b*d - a*h))/(12*a^2)
- (7*x^7*(3*b*e - a*i))/(64*a^2) + (7*b*x^9*(11*b*c - a*g))/(384*a^3) + (x*
(51*b*c + 7*a*g))/(128*a*b) + (b*x^10*(5*b*d - a*h))/(32*a^3) + (5*b*x^11*(
3*b*e - a*i))/(128*a^3) + (x^2*(11*b*d + a*h))/(32*a*b) + (x^3*(113*b*e + 5
*a*i))/(384*a*b))/(a^3 - b^3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log
((125*a^4*i^3 - 3375*a*b^3*e^3 - 123200*b^4*c*d^2 + 88935*b^4*c^2*e - 4928*
a^2*b^2*c*h^2 + 735*a^2*b^2*e*g^2 + 3375*a^2*b^2*e^2*i + 11200*a*b^3*d^2*g
- 29645*a*b^3*c^2*i - 1125*a^3*b*e*i^2 + 448*a^3*b*g*h^2 - 245*a^3*b*g^2*i
+ 5390*a^2*b^2*c*g*i - 4480*a^2*b^2*d*g*h + 49280*a*b^3*c*d*h - 16170*a*b^3
*c*e*g)/(2097152*a^9*b^2) - root(68719476736*a^15*b^7*z^4 - 1211105280*a^8*
b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 11010
0480*a^9*b^5*e*g*z^2 - 36700160*a^10*b^4*g*i*z^2 - 838860800*a^8*b^6*d^2*z^
2 - 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*
c*d*g*z + 17661952*a^6*b^4*c*g*h*z - 12288000*a^6*b^4*d*e*i*z + 485703680*a
^4*b^6*c^2*d*z - 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816
*a^7*b^3*g^2*h*z - 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z + 4014
080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z + 89600*a^4*b^2*d*g*h*i - 98
5600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h + 29
56800*a^2*b^4*c*d*e*h - 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i + 98
560*a^4*b^2*c*h^2*i + 26880*a^4*b^2*e*g*h^2 - 53900*a^4*b^2*c*g*i^2 - 17787
00*a^2*b^4*c^2*e*i + 2464000*a^2*b^4*c*d^2*i + 672000*a^2*b^4*d^2*e*g - 295
680*a^3*b^3*c*e*h^2 - 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000
*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g - 33750*a^4*b^2*e^
2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2
*g^2 - 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i -
 2048000*a^2*b^4*d^3*h - 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 26680
50*a*b^5*c^2*e^2 - 2401*a^4*b^2*g^4 - 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 +
2560000*a*b^5*d^4 - 625*a^6*i^4 - 35153041*b^6*c^4, z, l)*(root(68719476736
*a^15*b^7*z^4 - 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 33
5544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 - 36700160*a^10*b^4*g*i
*z^2 - 838860800*a^8*b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*
b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z + 17661952*a^6*b^4*c*g*h*z - 1228800
0*a^6*b^4*d*e*i*z + 485703680*a^4*b^6*c^2*d*z - 409600*a^8*b^2*h*i^2*z - 97
140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z - 3686400*a^6*b^4*e^2*h*z +
 2048000*a^7*b^3*d*i^2*z + 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2
*z + 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*
```

```
i - 268800*a^3*b^3*d*e*g*h + 2956800*a^2*b^4*c*d*e*h - 14700*a^4*b^2*e*g^2*
i - 224000*a^3*b^3*d^2*g*i + 98560*a^4*b^2*c*h^2*i + 26880*a^4*b^2*e*g*h^2
- 53900*a^4*b^2*c*g*i^2 - 1778700*a^2*b^4*c^2*e*i + 2464000*a^2*b^4*c*d^2*i
 + 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 - 485100*a^2*b^4*c*e^2*g
 - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924
*a*b^5*c^3*g - 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*
b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 - 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*
g^2*i^2 + 67500*a^3*b^3*e^3*i - 2048000*a^2*b^4*d^3*h - 81920*a^4*b^2*d*h^3
 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 - 2401*a^4*b^2*g^4 - 50625*
a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 - 625*a^6*i^4 - 35153041*b
^6*c^4, z, l)*((20185088*a^7*b^5*c - 1835008*a^8*b^4*g)/(2097152*a^9*b^2) -
 (x*(655360*a^7*b^4*d - 131072*a^8*b^3*h))/(131072*a^9*b)) - (614400*a^4*b^
4*d*e - 204800*a^5*b^3*d*i - 122880*a^5*b^3*e*h + 40960*a^6*b^2*h*i)/(20971
52*a^9*b^2) + (x*(800*a^6*b*i^2 + 189728*a^3*b^4*c^2 + 7200*a^4*b^3*e^2 + 1
568*a^5*b^2*g^2 - 34496*a^4*b^3*c*g - 4800*a^5*b^2*e*i))/(131072*a^9*b)) -
(x*(4000*b^3*d^3 - 32*a^3*h^3 - 5775*b^3*c*d*e + 35*a^3*g*h*i - 2400*a*b^2*
d^2*h + 480*a^2*b*d*h^2 + 1925*a*b^2*c*d*i + 1155*a*b^2*c*e*h + 525*a*b^2*d
*e*g - 385*a^2*b*c*h*i - 175*a^2*b*d*g*i - 105*a^2*b*e*g*h))/(131072*a^9*b)
)*root(68719476736*a^15*b^7*z^4 - 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^
9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 - 367
00160*a^10*b^4*g*i*z^2 - 838860800*a^8*b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*
z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z + 17661952*a^6*b^4
*c*g*h*z - 12288000*a^6*b^4*d*e*i*z + 485703680*a^4*b^6*c^2*d*z - 409600*a^
8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z - 3686400
*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z + 4014080*a^6*b^4*d*g^2*z + 1843
2000*a^5*b^5*d*e^2*z + 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323
400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h + 2956800*a^2*b^4*c*d*e*h - 14
700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i + 98560*a^4*b^2*c*h^2*i + 2688
0*a^4*b^2*e*g*h^2 - 53900*a^4*b^2*c*g*i^2 - 1778700*a^2*b^4*c^2*e*i + 24640
00*a^2*b^4*c*d^2*i + 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 - 4851
00*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*
b*e*i^3 + 12782924*a*b^5*c^3*g - 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2
*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 - 1743126*a^2*b^4*c^2
*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i - 2048000*a^2*b^4*d^3*h - 8
1920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 - 2401*a^
4*b^2*g^4 - 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 - 625*a^
6*i^4 - 35153041*b^6*c^4, z, l), l, 1, 4)
```

3.206 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4} dx$

Optimal result

Integrand size = 46, antiderivative size = 349

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4} dx \\
&= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{12ab(a-bx^4)^3} \\
&\quad+\frac{x(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2)}{384a^3b(a-bx^4)} \\
&\quad+\frac{4a(2bf-aj)+x(b(11bc-ag)+2b(5bd-ah)x+3b(3be-ai)x^2)}{96a^2b^2(a-bx^4)^2} \\
&\quad+\frac{\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{13/4}b^{7/4}} \\
&\quad+\frac{\left(15be+\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{13/4}b^{7/4}}+\frac{(5bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}
\end{aligned}$$

```
[Out] 1/12*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+(a*j+b*f)*x^3)/a/b/(-b*x^4+a)^3+1
/384*x*(-7*a*g+77*b*c+12*(-a*h+5*b*d)*x+15*(-a*i+3*b*e)*x^2)/a^3/b/(-b*x^4+
a)+1/96*(4*a*(-a*j+2*b*f)+x*(b*(-a*g+11*b*c)+2*b*(-a*h+5*b*d)*x+3*b*(-a*i+3
*b*e)*x^2))/a^2/b^2/(-b*x^4+a)^2+1/32*(-a*h+5*b*d)*arctanh(x^2*b^(1/2)/a^(1
/2))/a^(7/2)/b^(3/2)+1/256*arctanh(b^(1/4)*x/a^(1/4))*(15*b*e-5*a*i+7*(-a*g
+11*b*c)*b^(1/2)/a^(1/2))/a^(13/4)/b^(7/4)+1/256*arctan(b^(1/4)*x/a^(1/4))*
(5*a*i-15*b*e+7*(-a*g+11*b*c)*b^(1/2)/a^(1/2))/a^(13/4)/b^(7/4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1872, 1868, 1869, 1890, 281, 214, 1181, 211}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx$$
$$=\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}}$$
$$+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{256a^{13/4}b^{7/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(5bd-ah)}{32a^{7/2}b^{3/2}}$$
$$+\frac{x(7(11bc-ag)+12x(5bd-ah)+15x^2(3be-ai))}{384a^3b\,(a-bx^4)}$$
$$+\frac{x(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai))+4a(2bf-aj)}{96a^2b^2\,(a-bx^4)^2}$$
$$+\frac{x(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc)}{12ab\,(a-bx^4)^3}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^4
,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(12*a*b
*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 15*(3*b*e - a
*i)*x^2))/(384*a^3*b*(a - b*x^4)) + (4*a*(2*b*f - a*j) + x*(b*(11*b*c - a*g
) + 2*b*(5*b*d - a*h)*x + 3*b*(3*b*e - a*i)*x^2))/(96*a^2*b^2*(a - b*x^4)^2
) + (((7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)
*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((15*b*e + (7*Sqrt[b]*(11*b*c - a*g)
)/Sqrt[a] - 5*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + (
(5*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{12ab\left(a-bx^4\right)^3} - \frac{\int \frac{-b(11bc-ag)-2b(5bd-ah)x-3b(3be-ai)x^2-4b(2bf-aj)x^3}{\left(a-bx^4\right)^3}\,dx}{12ab^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{12ab\left(a-bx^4\right)^3} + \frac{4a(2bf-aj)+x(b(11bc-ag)+2b(5bd-ah)x+3b(3be-ai)x^2)}{96a^2b^2\left(a-bx^4\right)^2} + \frac{\int \frac{7b(11bc-ag)+12b(5bd-ah)x+15b(3be-ai)x^2}{\left(a-bx^4\right)^2}\,dx}{96a^2b^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{12ab\left(a-bx^4\right)^3} + \frac{x(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2)}{384a^3b\left(a-bx^4\right)} + \frac{4a(2bf-aj)+x(b(11bc-ag)+2b(5bd-ah)x+3b(3be-ai)x^2)}{96a^2b^2\left(a-bx^4\right)^2} - \frac{\int \frac{-21b(11bc-ag)-24b(5bd-ah)x-15b(3be-ai)x^2}{a-bx^4}\,dx}{384a^3b^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{12ab\left(a-bx^4\right)^3} + \frac{x(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2)}{384a^3b\left(a-bx^4\right)} + \frac{4a(2bf-aj)+x(b(11bc-ag)+2b(5bd-ah)x+3b(3be-ai)x^2)}{96a^2b^2\left(a-bx^4\right)^2} - \frac{\int \left(-\frac{24b(5bd-ah)x}{a-bx^4}+\frac{-21b(11bc-ag)-15b(3be-ai)x^2}{a-bx^4}\right)dx}{384a^3b^2}$$

$$= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{12ab\left(a-bx^4\right)^3} + \frac{x(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2)}{384a^3b\left(a-bx^4\right)} + \frac{4a(2bf-aj)+x(b(11bc-ag)+2b(5bd-ah)x+3b(3be-ai)x^2)}{96a^2b^2\left(a-bx^4\right)^2} - \frac{\int \frac{-21b(11bc-ag)-15b(3be-ai)x^2}{a-bx^4}\,dx}{384a^3b^2} + \frac{(5bd-ah)\int \frac{x}{a-bx^4}\,dx}{16a^3b}$$

$$
\begin{aligned}
&= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{12ab\left(a-bx^4\right)^3} \\
&\quad+ \frac{x(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2)}{384a^3b\left(a-bx^4\right)} \\
&\quad+ \frac{4a(2bf-aj)+x(b(11bc-ag)+2b(5bd-ah)x+3b(3be-ai)x^2)}{96a^2b^2\left(a-bx^4\right)^2} \\
&\quad+ \frac{(5bd-ah)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{32a^3b} + \frac{\left(15be+\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx}{256a^3b} \\
&\quad- \frac{\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{256a^3b} \\
&= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{12ab\left(a-bx^4\right)^3} \\
&\quad+ \frac{x(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2)}{384a^3b\left(a-bx^4\right)} \\
&\quad+ \frac{4a(2bf-aj)+x(b(11bc-ag)+2b(5bd-ah)x+3b(3be-ai)x^2)}{96a^2b^2\left(a-bx^4\right)^2} \\
&\quad+ \frac{\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{13/4}b^{7/4}} \\
&\quad+ \frac{\left(15be+\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{13/4}b^{7/4}} + \frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.26

$$
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a-bx^4\right)^4}\,dx
$$

$$
= \frac{-\frac{4abx(-77bc+7ag-15bx(4d+3ex)+3ax(4h+5ix))}{a-bx^4} - \frac{16a^2\left(12a^2j-b^2x(11c+x(10d+9ex))+abx(g+x(2h+3ix))\right)}{\left(a-bx^4\right)^2} + \frac{128a^3\left(a^2j+b^2x(c+x(d+e\ldots\right.}{(a-\ldots}}{}
$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4)^4,x]
```

```
[Out] ((-4*a*b*x*(-77*b*c + 7*a*g - 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i*x)))/
(a - b*x^4) - (16*a^2*(12*a^2*j - b^2*x*(11*c + x*(10*d + 9*e*x)) + a*b*x*(
g + x*(2*h + 3*i*x))))/(a - b*x^4)^2 + (128*a^3*(a^2*j + b^2*x*(c + x*(d +
e*x)) + a*b*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^3 + 6*a^(1/4)*b^(1/4)*(
77*b^(3/2)*c - 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*ArcTan[(b^(1/4
)*x)/a^(1/4)] + 3*a^(1/4)*b^(1/4)*(-77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d - 1
```

```
5*Sqrt[a]*b*e + 7*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1
/4) - b^(1/4)*x] + 3*a^(1/4)*b^(1/4)*(77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d +
 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h - 5*a^(3/2)*i)*Log[a^
(1/4) + b^(1/4)*x] - 24*Sqrt[a]*Sqrt[b]*(-5*b*d + a*h)*Log[Sqrt[a] + Sqrt[b
]*x^2])/(1536*a^4*b^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.60 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.73

method	result
risch	$\frac{-\frac{5(ai-3be)b\,x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}+\frac{7(ai-3be)x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{j\,x^4}{8b}+\frac{(5ai+113be)x^3}{384ab}+\frac{(ah+11bd)x^2}{32ab}+\frac{(7a}{(-b\,x^4+a)^3}$
default	$\frac{-\frac{5(ai-3be)b\,x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}+\frac{7(ai-3be)x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{j\,x^4}{8b}+\frac{(5ai+113be)x^3}{384ab}+\frac{(ah+11bd)x^2}{32ab}+\frac{(7a}{(-b\,x^4+a)^3}$

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETU
RNVERBOSE)

[Out] (-5/128*(a*i-3*b*e)/a^3*b*x^11-1/32*(a*h-5*b*d)/a^3*b*x^10-7/384*(a*g-11*b*
c)/a^3*b*x^9+7/64*(a*i-3*b*e)/a^2*x^7+1/12/a^2*(a*h-5*b*d)*x^6+3/64/a^2*(a*
g-11*b*c)*x^5+1/8*j*x^4/b+1/384*(5*a*i+113*b*e)/a/b*x^3+1/32*(a*h+11*b*d)/a
/b*x^2+1/128*(7*a*g+51*b*c)/a/b*x-1/24*(a*j-2*b*f)/b^2)/(-b*x^4+a)^3-1/512/
a^3/b^2*sum((-5*(a*i-3*b*e)*_R^2-8*(a*h-5*b*d)*_R-7*a*g+77*b*c)/_R^3*ln(x-_
R),_R=RootOf(_Z^4*b-a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx = \text{Timed out}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algor
ithm="fricas")

[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx = \text{Timed out}$$

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,
x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx = -\frac{15\,(3\,b^4e-ab^3i)x^{11}+12\,(5\,b^4d-ab^3h)x^{10}+7\,(11\,b^4c-ab^3g)x^9+48\,a^3bjx^4-42\,(3\,ab^3e-a^2b^2i)x^7-3}{384} + \frac{\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be-7\,a\sqrt{b}g+5\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\,b^{\frac{3}{2}}c+15\,\sqrt{a}b}{}}{512\,a^3b}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algor
ithm="maxima")
```

```
[Out] -1/384*(15*(3*b^4*e - a*b^3*i)*x^11 + 12*(5*b^4*d - a*b^3*h)*x^10 + 7*(11*b
^4*c - a*b^3*g)*x^9 + 48*a^3*b*j*x^4 - 42*(3*a*b^3*e - a^2*b^2*i)*x^7 - 32*
(5*a*b^3*d - a^2*b^2*h)*x^6 - 18*(11*a*b^3*c - a^2*b^2*g)*x^5 + 32*a^3*b*f
- 16*a^4*j + (113*a^2*b^2*e + 5*a^3*b*i)*x^3 + 12*(11*a^2*b^2*d + a^3*b*h)*
x^2 + 3*(51*a^2*b^2*c + 7*a^3*b*g)*x)/(a^3*b^5*x^12 - 3*a^4*b^4*x^8 + 3*a^5
*b^3*x^4 - a^6*b^2) + 1/512*(8*(5*b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sq
rt(a)*sqrt(b)) - 8*(5*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b
)) + 2*(77*b^(3/2)*c - 15*sqrt(a)*b*e - 7*a*sqrt(b)*g + 5*a^(3/2)*i)*arctan
(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) -
 (77*b^(3/2)*c + 15*sqrt(a)*b*e - 7*a*sqrt(b)*g - 5*a^(3/2)*i)*log((sqrt(b)
*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^3*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(302) = 604$.

Time = 0.28 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.81

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx =$$

$$-\frac{\sqrt{2}\left(77\,b^3c-7\,ab^2g-40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d+8\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-15\,\sqrt{-ab}b^2e+5\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}(}{}\right.}{512\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$-\frac{\sqrt{2}\left(77\,b^3c-7\,ab^2g+40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d-8\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-15\,\sqrt{-ab}b^2e-5\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}(}{}\right.}{512\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$-\frac{\sqrt{2}(77\,b^3c-7\,ab^2g-15\,\sqrt{-ab}b^2e+5\,\sqrt{-ab}abi)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$+\frac{\sqrt{2}(77\,b^3c-7\,ab^2g-15\,\sqrt{-ab}b^2e+5\,\sqrt{-ab}abi)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$-\frac{45\,b^4ex^{11}-15\,ab^3ix^{11}+60\,b^4dx^{10}-12\,ab^3hx^{10}+77\,b^4cx^9-7\,ab^3gx^9-126\,ab^3ex^7+42\,a^2b^2ix^7-160}{}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algor
ithm="giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^3*c - 7*a*b^2*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b^2*d + 8*
sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 15*sqrt(-a*b)*b^2*e + 5*sqrt(-a*b)*a*b*i)*ar
ctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)
*a^3*b) - 1/512*sqrt(2)*(77*b^3*c - 7*a*b^2*g + 40*sqrt(2)*(-a*b^3)^(1/4)*b
^2*d - 8*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 15*sqrt(-a*b)*b^2*e - 5*sqrt(-a*b)*
a*b*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b
^3)^(3/4)*a^3*b) - 1/1024*sqrt(2)*(77*b^3*c - 7*a*b^2*g - 15*sqrt(-a*b)*b^2
*e + 5*sqrt(-a*b)*a*b*i)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-
a*b^3)^(3/4)*a^3*b) + 1/1024*sqrt(2)*(77*b^3*c - 7*a*b^2*g - 15*sqrt(-a*b)*
b^2*e + 5*sqrt(-a*b)*a*b*i)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/
((-a*b^3)^(3/4)*a^3*b) - 1/384*(45*b^4*e*x^11 - 15*a*b^3*i*x^11 + 60*b^4*d*
x^10 - 12*a*b^3*h*x^10 + 77*b^4*c*x^9 - 7*a*b^3*g*x^9 - 126*a*b^3*e*x^7 + 4
2*a^2*b^2*i*x^7 - 160*a*b^3*d*x^6 + 32*a^2*b^2*h*x^6 - 198*a*b^3*c*x^5 + 18
*a^2*b^2*g*x^5 + 48*a^3*b*j*x^4 + 113*a^2*b^2*e*x^3 + 5*a^3*b*i*x^3 + 132*a
^2*b^2*d*x^2 + 12*a^3*b*h*x^2 + 153*a^2*b^2*c*x + 21*a^3*b*g*x + 32*a^3*b*f
 - 16*a^4*j)/((b*x^4 - a)^3*a^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 10.57 (sec) , antiderivative size = 2764, normalized size of antiderivative = 7.92

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^4
,x)
```

```
[Out] symsum(log((125*a^4*i^3 - 3375*a*b^3*e^3 - 123200*b^4*c*d^2 + 88935*b^4*c^2
*e - 4928*a^2*b^2*c*h^2 + 735*a^2*b^2*e*g^2 + 3375*a^2*b^2*e^2*i + 11200*a*
b^3*d^2*g - 29645*a*b^3*c^2*i - 1125*a^3*b*e*i^2 + 448*a^3*b*g*h^2 - 245*a^
3*b*g^2*i + 5390*a^2*b^2*c*g*i - 4480*a^2*b^2*d*g*h + 49280*a*b^3*c*d*h - 1
6170*a*b^3*c*e*g)/(2097152*a^9*b^2) - root(68719476736*a^15*b^7*z^4 - 12111
05280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z
^2 + 110100480*a^9*b^5*e*g*z^2 - 36700160*a^10*b^4*g*i*z^2 - 838860800*a^8*
b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 8830976
0*a^5*b^5*c*d*g*z + 17661952*a^6*b^4*c*g*h*z - 12288000*a^6*b^4*d*e*i*z + 4
85703680*a^4*b^6*c^2*d*z - 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*
z - 802816*a^7*b^3*g^2*h*z - 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^
2*z + 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z + 89600*a^4*b^2*d*
g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*
e*g*h + 2956800*a^2*b^4*c*d*e*h - 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^
2*g*i + 98560*a^4*b^2*c*h^2*i + 26880*a^4*b^2*e*g*h^2 - 53900*a^4*b^2*c*g*i
^2 - 1778700*a^2*b^4*c^2*e*i + 2464000*a^2*b^4*c*d^2*i + 672000*a^2*b^4*d^2
*e*g - 295680*a^3*b^3*c*e*h^2 - 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i
 - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g - 33750*
a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a
^3*b^3*e^2*g^2 - 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b
^3*e^3*i - 2048000*a^2*b^4*d^3*h - 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g
^3 + 2668050*a*b^5*c^2*e^2 - 2401*a^4*b^2*g^4 - 50625*a^2*b^4*e^4 + 4096*a^
5*b*h^4 + 2560000*a*b^5*d^4 - 625*a^6*i^4 - 35153041*b^6*c^4, z, m)*(root(6
8719476736*a^15*b^7*z^4 - 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*
i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 - 36700160*a^
10*b^4*g*i*z^2 - 838860800*a^8*b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*z^2 + 24
57600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z + 17661952*a^6*b^4*c*g*h*z
 - 12288000*a^6*b^4*d*e*i*z + 485703680*a^4*b^6*c^2*d*z - 409600*a^8*b^2*h*
i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z - 3686400*a^6*b^4
*e^2*h*z + 2048000*a^7*b^3*d*i^2*z + 4014080*a^6*b^4*d*g^2*z + 18432000*a^5
*b^5*d*e^2*z + 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*
b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h + 2956800*a^2*b^4*c*d*e*h - 14700*a^4*
b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i + 98560*a^4*b^2*c*h^2*i + 26880*a^4*b^
2*e*g*h^2 - 53900*a^4*b^2*c*g*i^2 - 1778700*a^2*b^4*c^2*e*i + 2464000*a^2*b
^4*c*d^2*i + 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 - 485100*a^2*b
```

^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g - 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 - 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i - 2048000*a^2*b^4*d^3*h - 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 - 2401*a^4*b^2*g^4 - 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 - 625*a^6*i^4 - 35153041*b^6*c^4, z, m)*((20185088*a^7*b^5*c - 1835008*a^8*b^4*g)/(2097152*a^9*b^2) - (x*(655360*a^7*b^4*d - 131072*a^8*b^3*h))/(131072*a^9*b)) - (614400*a^4*b^4*d*e - 204800*a^5*b^3*d*i - 122880*a^5*b^3*e*h + 40960*a^6*b^2*h*i)/(2097152*a^9*b^2) + (x*(800*a^6*b*i^2 + 189728*a^3*b^4*c^2 + 7200*a^4*b^3*e^2 + 1568*a^5*b^2*g^2 - 34496*a^4*b^3*c*g - 4800*a^5*b^2*e*i))/(131072*a^9*b)) - (x*(4000*b^3*d^3 - 32*a^3*h^3 - 5775*b^3*c*d*e + 35*a^3*g*h*i - 2400*a*b^2*d^2*h + 480*a^2*b*d*h^2 + 1925*a*b^2*c*d*i + 1155*a*b^2*c*e*h + 525*a*b^2*d*e*g - 385*a^2*b*c*h*i - 175*a^2*b*d*g*i - 105*a^2*b*e*g*h))/(131072*a^9*b))*root(68719476736*a^15*b^7*z^4 - 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 - 36700160*a^10*b^4*g*i*z^2 - 838860800*a^8*b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z + 17661952*a^6*b^4*c*g*h*z - 12288000*a^6*b^4*d*e*i*z + 485703680*a^4*b^6*c^2*d*z - 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z - 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z + 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z + 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h + 2956800*a^2*b^4*c*d*e*h - 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i + 98560*a^4*b^2*c*h^2*i + 26880*a^4*b^2*e*g*h^2 - 53900*a^4*b^2*c*g*i^2 - 1778700*a^2*b^4*c^2*e*i + 2464000*a^2*b^4*c*d^2*i + 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 - 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g - 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 - 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i - 2048000*a^2*b^4*d^3*h - 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 - 2401*a^4*b^2*g^4 - 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 - 625*a^6*i^4 - 35153041*b^6*c^4, z, m), m, 1, 4) + ((2*b*f - a*j)/(24*b^2) + (j*x^4)/(8*b) - (3*x^5*(11*b*c - a*g))/(64*a^2) - (x^6*(5*b*d - a*h))/(12*a^2) - (7*x^7*(3*b*e - a*i))/(64*a^2) + (7*b*x^9*(11*b*c - a*g))/(384*a^3) + (x*(51*b*c + 7*a*g))/(128*a*b) + (b*x^10*(5*b*d - a*h))/(32*a^3) + (5*b*x^11*(3*b*e - a*i))/(128*a^3) + (x^2*(11*b*d + a*h))/(32*a*b) + (x^3*(113*b*e + 5*a*i))/(384*a*b))/(a^3 - b^3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8)

3.207 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4} dx$

Optimal result

Integrand size = 35, antiderivative size = 462

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4} dx = \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+45bex^2)}{384a^3b(a+bx^4)} - \frac{8af-x(11bc+ag+2(5bd+ah)x+9bex^2)}{96a^2b(a+bx^4)^2} + \frac{(5bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} - \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}} + \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}} - \frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}} + \frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}}$$

```
[Out] 1/12*x*(b*c-a*g+(-a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(b*x^4+a)^3+1/384*x*(7*a*
g+77*b*c+12*(a*h+5*b*d)*x+45*b*e*x^2)/a^3/b/(b*x^4+a)+1/96*(-8*a*f+x*(11*b*
c+a*g+2*(a*h+5*b*d)*x+9*b*e*x^2))/a^2/b/(b*x^4+a)^2+1/32*(a*h+5*b*d)*arctan
(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+
a^(1/2)+x^2*b^(1/2))*(77*b*c+7*a*g-15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2
^(1/2)+1/1024*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(77*b*c+7*a
```

```
*g-15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x
*2^(1/2)/a^(1/4))*(77*b*c+7*a*g+15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2^(1
/2)+1/512*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(77*b*c+7*a*g+15*e*a^(1/2)*b^
(1/2))/a^(15/4)/b^(5/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {1872, 1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx$$
$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}} + \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+5bd)}{32a^{7/2}b^{3/2}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}} + \frac{x(7(ag+11bc)+12x(ah+5bd)+45bex^2)}{384a^3b\,(a+bx^4)} - \frac{8af-x(2x(ah+5bd)+ag+11bc+9bex^2)}{96a^2b\,(a+bx^4)^2} + \frac{x(x(bd-ah)-ag+bc+bex^2+bfx^3)}{12ab\,(a+bx^4)^3}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3)
+ (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 45*b*e*x^2))/(384*a^3*b*(a +
b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d + a*h)*x + 9*b*e*x^2))/(96*a^
2*b*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7
/2)*b^(3/2)) - ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a
]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*
a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) +
```

```
((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3} - \frac{\int \frac{-b(11bc+ag)-2b(5bd+ah)x-9b^2ex^2-8b^2fx^3}{\left(a+bx^4\right)^3}\,dx}{12ab^2}$$

$$=\frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3}-\frac{8af-x(11bc+ag+2(5bd+ah)x+9bex^2)}{96a^2b\left(a+bx^4\right)^2}+\frac{\int\frac{7b(11bc+ag)+12b(5bd+ah)x+45b^2ex^2}{(a+bx^4)^2}\,dx}{96a^2b^2}$$

$$=\frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3}+\frac{x(7(11bc+ag)+12(5bd+ah)x+45bex^2)}{384a^3b\left(a+bx^4\right)}-\frac{8af-x(11bc+ag+2(5bd+ah)x+9bex^2)}{96a^2b\left(a+bx^4\right)^2}-\frac{\int\frac{-21b(11bc+ag)-24b(5bd+ah)x-45b^2ex^2}{a+bx^4}\,dx}{384a^3b^2}$$

$$=\frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3}+\frac{x(7(11bc+ag)+12(5bd+ah)x+45bex^2)}{384a^3b\left(a+bx^4\right)}-\frac{8af-x(11bc+ag+2(5bd+ah)x+9bex^2)}{96a^2b\left(a+bx^4\right)^2}-\frac{\int\left(-\frac{24b(5bd+ah)x}{a+bx^4}+\frac{-21b(11bc+ag)-45b^2ex^2}{a+bx^4}\right)\,dx}{384a^3b^2}$$

$$=\frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3}+\frac{x(7(11bc+ag)+12(5bd+ah)x+45bex^2)}{384a^3b\left(a+bx^4\right)}-\frac{8af-x(11bc+ag+2(5bd+ah)x+9bex^2)}{96a^2b\left(a+bx^4\right)^2}-\frac{\int\frac{-21b(11bc+ag)-45b^2ex^2}{a+bx^4}\,dx}{384a^3b^2}+\frac{(5bd+ah)\int\frac{x}{a+bx^4}\,dx}{16a^3b}$$

$$=\frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3}+\frac{x(7(11bc+ag)+12(5bd+ah)x+45bex^2)}{384a^3b\left(a+bx^4\right)}-\frac{8af-x(11bc+ag+2(5bd+ah)x+9bex^2)}{96a^2b\left(a+bx^4\right)^2}+\frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\int\frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{256a^{7/2}b^{3/2}}-\frac{\left(15\sqrt{b}e-\frac{7(11bc+ag)}{\sqrt{a}}\right)\int\frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{256a^3b^{3/2}}+\frac{(5bd+ah)\text{Subst}\left(\int\frac{1}{a+bx^2}\,dx,x,x^2\right)}{32a^3b}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+45bex^2)}{384a^3b\left(a+bx^4\right)} - \frac{8af-x(11bc+ag+2(5bd+ah)x+9bex^2)}{96a^2b\left(a+bx^4\right)^2} + \frac{(5bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} - \frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{512\sqrt{2}a^{15/4}b^{5/4}} - \frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{512\sqrt{2}a^{15/4}b^{5/4}} + \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{512a^{7/2}b^{3/2}} + \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{512a^{7/2}b^{3/2}}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\left(a+bx^4\right)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+45bex^2)}{384a^3b\left(a+bx^4\right)} - \frac{8af-x(11bc+ag+2(5bd+ah)x+9bex^2)}{96a^2b\left(a+bx^4\right)^2} + \frac{(5bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} - \frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}} + \frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}} + \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}} - \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}}$$

$$= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+45bex^2)}{384a^3b\,(a+bx^4)} - \frac{8af-x(11bc+ag+2(5bd+ah)x+9bex^2)}{96a^2b\,(a+bx^4)^2} + \frac{(5bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} - \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}} + \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}} - \frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}} + \frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx$$

$$= \frac{\frac{8a^{3/4}\sqrt{b}x(77bc+7ag+60bdx+12ahx+45bex^2)}{a+bx^4} + \frac{32a^{7/4}\sqrt{b}x(11bc+bx(10d+9ex)+a(g+2hx))}{(a+bx^4)^2} - \frac{256a^{11/4}\sqrt{b}(-bx(c+x(d+ex))+a(f+x(g+hx)}{(a+bx^4)^3}}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x]
```

```
[Out] ((8*a^(3/4)*Sqrt[b]*x*(77*b*c + 7*a*g + 60*b*d*x + 12*a*h*x + 45*b*e*x^2))/
(a + b*x^4) + (32*a^(7/4)*Sqrt[b]*x*(11*b*c + b*x*(10*d + 9*e*x) + a*(g + 2
*h*x)))/(a + b*x^4)^2 - (256*a^(11/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a
*(f + x*(g + h*x))))/(a + b*x^4)^3 - 6*(77*Sqrt[2]*b^(5/4)*c + 80*a^(1/4)*b
*d + 15*Sqrt[2]*Sqrt[a]*b^(3/4)*e + 7*Sqrt[2]*a*b^(1/4)*g + 16*a^(5/4)*h)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b^(5/4)*c - 80*a^(1/
4)*b*d + 15*Sqrt[2]*Sqrt[a]*b^(3/4)*e + 7*Sqrt[2]*a*b^(1/4)*g - 16*a^(5/4)*
h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*b^(1/4)*(77*b*c - 15
*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2] + 3*Sqrt[2]*b^(1/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.47

method	result
risch	$\frac{\frac{15 e b^2 x^{11}}{128 a^3}+\frac{(a h+5 b d) b x^{10}}{32 a^3}+\frac{7(a g+11 b c) b x^9}{384 a^3}+\frac{21 b e x^7}{64 a^2}+\frac{(a h+5 b d) x^6}{12 a^2}+\frac{3(a g+11 b c) x^5}{64 a^2}+\frac{113 e x^3}{384 a}-\frac{(a h-11 b d) x^2}{32 a b}-\frac{(7 a g-51 b c) x}{128 a b}-\frac{f}{12 b}}{(b x^4+a)^3}+$
default	$\frac{\frac{15 e b^2 x^{11}}{128 a^3}+\frac{(a h+5 b d) b x^{10}}{32 a^3}+\frac{7(a g+11 b c) b x^9}{384 a^3}+\frac{21 b e x^7}{64 a^2}+\frac{(a h+5 b d) x^6}{12 a^2}+\frac{3(a g+11 b c) x^5}{64 a^2}+\frac{113 e x^3}{384 a}-\frac{(a h-11 b d) x^2}{32 a b}-\frac{(7 a g-51 b c) x}{128 a b}-\frac{f}{12 b}}{(b x^4+a)^3}+$

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)

[Out] (15/128*e/a^3*b^2*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c)/a^3*b*x^9+21/64*b*e/a^2*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a*g+11*b*c)*x^5+113/384/a*e*x^3-1/32*(a*h-11*b*d)/a/b*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12*f/b)/(b*x^4+a)^3+1/512/a^3/b*sum((15*_R^2*e+8/b*(a*h+5*b*d)*_R+7*(a*g+11*b*c)/b)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx=\text{Timed out}$$

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx=\text{Timed out}$$

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.12

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a + bx^4)^4}\,dx$$
$$= \frac{45\,b^3ex^{11} + 126\,ab^2ex^7 + 12\,(5\,b^3d + ab^2h)x^{10} + 7\,(11\,b^3c + ab^2g)x^9 + 113\,a^2bex^3 + 32\,(5\,ab^2d + a^2bh)x^6 + \cdots}{384\,(a^3b^4x^{12} + 3\,a^4b^3x^8 + 3\,a^5b^2x^4 \cdots}$$
$$+ \frac{\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c - 15\,\sqrt{a}be + 7\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c - 15\,\sqrt{a}be + 7\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b^{\cdots}\right.}{\cdots}}{\cdots}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima"
)
```

```
[Out] 1/384*(45*b^3*e*x^11 + 126*a*b^2*e*x^7 + 12*(5*b^3*d + a*b^2*h)*x^10 + 7*(1
1*b^3*c + a*b^2*g)*x^9 + 113*a^2*b*e*x^3 + 32*(5*a*b^2*d + a^2*b*h)*x^6 + 1
8*(11*a*b^2*c + a^2*b*g)*x^5 - 32*a^3*f + 12*(11*a^2*b*d - a^3*h)*x^2 + 3*(
51*a^2*b*c - 7*a^3*g)*x)/(a^3*b^4*x^12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^
6*b) + 1/1024*(sqrt(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*e + 7*a*sqrt(b)*g)*log(
sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt
(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*e + 7*a*sqrt(b)*g)*log(sqrt(b)*x^2 - sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b
^(7/4)*c + 15*sqrt(2)*a^(3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g - 80*
sqrt(a)*b^(3/2)*d - 16*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x +
 sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt
(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*sqrt(2)*a^(3/4)*b^(5/4
)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 80*sqrt(a)*b^(3/2)*d + 16*a^(3/2)*sqrt(
b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(
a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^3*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.11

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx$$

$$= \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+8\sqrt{2}\sqrt{ab}abh+77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$

$$+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+8\sqrt{2}\sqrt{ab}abh+77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$

$$+\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$

$$-\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$

$$+\frac{45\,b^3ex^{11}+60\,b^3dx^{10}+12\,ab^2hx^{10}+77\,b^3cx^9+7\,ab^2gx^9+126\,ab^2ex^7+160\,ab^2dx^6+32\,a^2bhx^6+198}{384\,(bx^4+a)^3a}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*
(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2
)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4
)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^
3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 + sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)
*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a
/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 1
2*a*b^2*h*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 126*a*b^2*e*x^7 + 160*a*b^2
*d*x^6 + 32*a^2*b*h*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*e*x^
3 + 132*a^2*b*d*x^2 - 12*a^3*h*x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)
/((b*x^4 + a)^3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 10.10 (sec) , antiderivative size = 1743, normalized size of antiderivative = 3.77

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x)

[Out] symsum(log((123200*b^3*c*d^2 - 3375*a*b^2*e^3 - 88935*b^3*c^2*e + 448*a^3*g*h^2 + 11200*a*b^2*d^2*g + 4928*a^2*b*c*h^2 - 735*a^2*b*e*g^2 + 49280*a*b^2*c*d*h - 16170*a*b^2*c*e*g + 4480*a^2*b*d*g*h)/(2097152*a^9*b) - root(68719476736*a^15*b^6*z^4 + 1211105280*a^8*b^5*c*e*z^2 + 335544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 + 838860800*a^8*b^5*d^2*z^2 + 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z - 17661952*a^6*b^3*c*g*h*z - 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 802816*a^7*b^2*g^2*h*z + 3686400*a^6*b^3*e^2*h*z - 4014080*a^6*b^3*d*g^2*z + 18432000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h - 2956800*a^2*b^3*c*d*e*h - 672000*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 + 485100*a^2*b^3*c*e^2*g - 26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e + 81920*a^4*b*d*h^3 + 12782924*a*b^4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 + 1743126*a^2*b^3*c^2*g^2 + 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*b^4*c^2*e^2 + 50625*a^2*b^3*e^4 + 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096*a^5*h^4 + 35153041*b^5*c^4, z, k)*(root(68719476736*a^15*b^6*z^4 + 1211105280*a^8*b^5*c*e*z^2 + 335544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 + 838860800*a^8*b^5*d^2*z^2 + 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z - 17661952*a^6*b^3*c*g*h*z - 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 802816*a^7*b^2*g^2*h*z + 3686400*a^6*b^3*e^2*h*z - 4014080*a^6*b^3*d*g^2*z + 18432000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h - 2956800*a^2*b^3*c*d*e*h - 672000*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 + 485100*a^2*b^3*c*e^2*g - 26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e + 81920*a^4*b*d*h^3 + 12782924*a*b^4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 + 1743126*a^2*b^3*c^2*g^2 + 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*b^4*c^2*e^2 + 50625*a^2*b^3*e^4 + 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096*a^5*h^4 + 35153041*b^5*c^4, z, k)*((20185088*a^7*b^4*c + 1835008*a^8*b^3*g)/(2097152*a^9*b) - (x*(655360*a^7*b^4*d + 131072*a^8*b^3*h))/(131072*a^9*b)) + (614400*a^4*b^3*d*e + 122880*a^5*b^2*e*h)/(2097152*a^9*b) + (x*(189728*a^3*b^4*c^2 - 7200*a^4*b^3*e^2 + 1568*a^5*b^2*g^2 + 34496*a^4*b^3*c*g))/(131072*a^9*b)) + (x*(4000*b^3*d^3 + 32*a^3*h^3 - 5775*b^3*c*d*e + 2400*a*b^2*d^2*h + 480*a^2*b*d*h^2 - 1155*a*b^2*c*e*h - 525*a*b^2*d*e*g - 105*a^2*b*e*g*h))/(131072*a^9*b))*root(68719476736*a^15*b^6*z^4 + 1211105280*a^8*b^5*c*e*z^2 + 335544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 + 838860800*a^8*b^5*d^2*z^2 + 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z - 17661952*a^6*b^3*c*g*h*z - 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 802816*a^7*b^2*g^2*h*z + 3686400*a^6*b^3*e^2*h*z - 4014080*a^6*b^3*d*g^2*z + 1

```
8432000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h - 2956800*a^2*b^3*c*d*e*h
- 672000*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 + 485100*a^2*b^3*c*e^2*g
- 26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e + 81920*a^4*b*d*h^3 + 1278292
4*a*b^4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 + 1743126*a^
2*b^3*c^2*g^2 + 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*b^
4*c^2*e^2 + 50625*a^2*b^3*e^4 + 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096*a
^5*h^4 + 35153041*b^5*c^4, z, k), k, 1, 4) + ((113*e*x^3)/(384*a) - f/(12*b
) + (3*x^5*(11*b*c + a*g))/(64*a^2) + (x^6*(5*b*d + a*h))/(12*a^2) + (7*b*x
^9*(11*b*c + a*g))/(384*a^3) + (x*(51*b*c - 7*a*g))/(128*a*b) + (b*x^10*(5*
b*d + a*h))/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) + (x^2*(11*b*d - a*h))/(32
*a*b) + (21*b*e*x^7)/(64*a^2))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8)
```

3.208 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 516

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx \\
&= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{12ab\,(a+bx^4)^3} \\
&\quad+\frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\,(a+bx^4)} \\
&\quad-\frac{8af-x(11bc+ag+2(5bd+ah)x+3(3be+ai)x^2)}{96a^2b\,(a+bx^4)^2}+\frac{(5bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} \\
&\quad-\frac{\left(7\sqrt{b}(11bc+ag)+5\sqrt{a}(3be+ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{7/4}} \\
&\quad+\frac{\left(7\sqrt{b}(11bc+ag)+5\sqrt{a}(3be+ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{7/4}} \\
&\quad-\frac{\left(7\sqrt{b}(11bc+ag)-5\sqrt{a}(3be+ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{7/4}} \\
&\quad+\frac{\left(7\sqrt{b}(11bc+ag)-5\sqrt{a}(3be+ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{7/4}}
\end{aligned}$$

```
[Out] 1/12*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+b*f*x^3)/a/b/(b*x^4+a)^3+1/384*
x*(7*a*g+77*b*c+12*(a*h+5*b*d)*x+15*(a*i+3*b*e)*x^2)/a^3/b/(b*x^4+a)+1/96*(
-8*a*f+x*(11*b*c+a*g+2*(a*h+5*b*d)*x+3*(a*i+3*b*e)*x^2))/a^2/b/(b*x^4+a)^2+
1/32*(a*h+5*b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)-1/1024*ln(-a^(
```

```
1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*(a*i+3*b*e)*a^(1/2)+7*(a*g+
11*b*c)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)+1/1024*ln(a^(1/4)*b^(1/4)*x*2^(1/
2)+a^(1/2)+x^2*b^(1/2))*(-5*(a*i+3*b*e)*a^(1/2)+7*(a*g+11*b*c)*b^(1/2))/a^(
15/4)/b^(7/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*(a*i+3*
b*e)*a^(1/2)+7*(a*g+11*b*c)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)+1/512*arctan(
1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*(a*i+3*b*e)*a^(1/2)+7*(a*g+11*b*c)*b^(1/2))
/a^(15/4)/b^(7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1872, 1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx$$
$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}} + \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+5bd)}{32a^{7/2}b^{3/2}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}} + \frac{x(7(ag+11bc)+12x(ah+5bd)+15x^2(ai+3be))}{384a^3b\,(a+bx^4)} - \frac{8af-x(2x(ah+5bd)+3x^2(ai+3be)+ag+11bc)}{96a^2b\,(a+bx^4)^2} + \frac{x(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3)}{12ab\,(a+bx^4)^3}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(12*a*b*(a + b*
x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 15*(3*b*e + a*i)*x^2)
)/(384*a^3*b*(a + b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d + a*h)*x +
3*(3*b*e + a*i)*x^2))/(96*a^2*b*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqr
t[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((7*Sqrt[b]*(11*b*c + a*g) + 5*S
qrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]
*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*
```

```
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) - (
(7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt
[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{12ab\,(a+bx^4)^3} - \frac{\int \frac{-b(11bc+ag)-2b(5bd+ah)x-3b(3be+ai)x^2-8b^2fx^3}{(a+bx^4)^3}\,dx}{12ab^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{12ab\,(a+bx^4)^3} - \frac{8af-x(11bc+ag+2(5bd+ah)x+3(3be+ai)x^2)}{96a^2b\,(a+bx^4)^2} + \frac{\int \frac{7b(11bc+ag)+12b(5bd+ah)x+15b(3be+ai)x^2}{(a+bx^4)^2}\,dx}{96a^2b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\,(a+bx^4)} - \frac{8af-x(11bc+ag+2(5bd+ah)x+3(3be+ai)x^2)}{96a^2b\,(a+bx^4)^2} - \frac{\int \frac{-21b(11bc+ag)-24b(5bd+ah)x-15b(3be+ai)x^2}{a+bx^4}\,dx}{384a^3b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\,(a+bx^4)} - \frac{8af-x(11bc+ag+2(5bd+ah)x+3(3be+ai)x^2)}{96a^2b\,(a+bx^4)^2} - \frac{\int \left(-\frac{24b(5bd+ah)x}{a+bx^4} + \frac{-21b(11bc+ag)-15b(3be+ai)x^2}{a+bx^4}\right)dx}{384a^3b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\,(a+bx^4)} - \frac{8af-x(11bc+ag+2(5bd+ah)x+3(3be+ai)x^2)}{96a^2b\,(a+bx^4)^2} - \frac{\int \frac{-21b(11bc+ag)-15b(3be+ai)x^2}{a+bx^4}\,dx}{384a^3b^2} + \frac{(5bd+ah)\int \frac{x}{a+bx^4}\,dx}{16a^3b}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{12ab\left(a+bx^4\right)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\left(a+bx^4\right)} - \frac{8af-x(11bc+ag+2(5bd+ah)x+3(3be+ai)x^2)}{96a^2b\left(a+bx^4\right)^2} + \frac{(5bd+ah)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{32a^3b} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}-5(3be+ai)\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{256a^3b^2} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}+5(3be+ai)\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{256a^3b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{12ab\left(a+bx^4\right)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\left(a+bx^4\right)} - \frac{8af-x(11bc+ag+2(5bd+ah)x+3(3be+ai)x^2)}{96a^2b\left(a+bx^4\right)^2} + \frac{(5bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} - \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}-5(3be+ai)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{512\sqrt{2}a^{13/4}b^{7/4}} - \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}-5(3be+ai)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{512\sqrt{2}a^{13/4}b^{7/4}} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}+5(3be+ai)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{512a^3b^2} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}+5(3be+ai)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{512a^3b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{12ab\left(a+bx^4\right)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\left(a+bx^4\right)} - \frac{8af-x(11bc+ag+2(5bd+ah)x+3(3be+ai)x^2)}{96a^2b\left(a+bx^4\right)^2} + \frac{(5bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} - \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}-5(3be+ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{13/4}b^{7/4}} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}-5(3be+ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{13/4}b^{7/4}} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}+5(3be+ai)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{13/4}b^{7/4}} - \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}+5(3be+ai)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{13/4}b^{7/4}}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{12ab\left(a+bx^4\right)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\left(a+bx^4\right)} - \frac{8af-x(11bc+ag+2(5bd+ah)x+3(3be+ai)x^2)}{96a^2b\left(a+bx^4\right)^2} + \frac{(5bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} - \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}+5(3be+ai)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{13/4}b^{7/4}} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}+5(3be+ai)\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{13/4}b^{7/4}} - \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}-5(3be+ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{13/4}b^{7/4}} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}-5(3be+ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{13/4}b^{7/4}}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.03

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx$$
$$=\frac{\frac{32a^{7/4}b^{3/4}x(11bc+ag+bx(10d+9ex)+ax(2h+3ix))}{(a+bx^4)^2}+\frac{8a^{3/4}b^{3/4}x(77bc+7ag+15bx(4d+3ex)+3ax(4h+5ix))}{a+bx^4}-\frac{256a^{11/4}b^{3/4}(-bx(c+x(d+e\ldots}{(a+bx\ldots}}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,x
]
```

```
[Out] ((32*a^(7/4)*b^(3/4)*x*(11*b*c + a*g + b*x*(10*d + 9*e*x) + a*x*(2*h + 3*i*
x)))/(a + b*x^4)^2 + (8*a^(3/4)*b^(3/4)*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3
*e*x) + 3*a*x*(4*h + 5*i*x)))/(a + b*x^4) - (256*a^(11/4)*b^(3/4)*(-(b*x*(c
 + x*(d + e*x))) + a*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^3 - 6*(77*Sqrt
[2]*b^(3/2)*c + 80*a^(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a
*Sqrt[b]*g + 16*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b^(3/2)*c - 80*a^(1/4)*b^(5/4)*d + 15
*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sqrt[b]*g - 16*a^(5/4)*b^(1/4)*h + 5*Sqr
t[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*(-77*b^
(3/2)*c + 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqrt[2]*(77*b^(3/2)*c - 15*Sqrt[a]*
b*e + 7*a*Sqrt[b]*g - 5*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2])/(3072*a^(15/4)*b^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.46

method	result
risch	$\frac{\frac{5(ai+3be)b\,x^{11}}{128a^3}+\frac{(ah+5bd)b\,x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{7(ai+3be)x^7}{64a^2}+\frac{(ah+5bd)x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}-\frac{(5ai-113be)x^3}{384ab}-\frac{(ah-11bd)x^2}{32ab}-\frac{(7ag-51bc)x}{128ab}\ldots}{(b\,x^4+a)^3}$
default	$\frac{\frac{5(ai+3be)b\,x^{11}}{128a^3}+\frac{(ah+5bd)b\,x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{7(ai+3be)x^7}{64a^2}+\frac{(ah+5bd)x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}-\frac{(5ai-113be)x^3}{384ab}-\frac{(ah-11bd)x^2}{32ab}-\frac{(7ag-51bc)x}{128ab}\ldots}{(b\,x^4+a)^3}$

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBO
SE)
```

```
[Out] (5/128*(a*i+3*b*e)/a^3*b*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c
)/a^3*b*x^9+7/64*(a*i+3*b*e)/a^2*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a*g
+11*b*c)*x^5-1/384*(5*a*i-113*b*e)/a/b*x^3-1/32*(a*h-11*b*d)/a/b*x^2-1/128*
(7*a*g-51*b*c)/a/b*x-1/12*f/b)/(b*x^4+a)^3+1/512/a^3/b^2*sum((5*(a*i+3*b*e)
*_R^2+8*(a*h+5*b*d)*_R+7*a*g+77*b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx = \text{Timed out}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx = \text{Timed out}$$

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.12

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx$$
$$= \frac{15\,(3\,b^3e+ab^2i)x^{11}+12\,(5\,b^3d+ab^2h)x^{10}+7\,(11\,b^3c+ab^2g)x^9+42\,(3\,ab^2e+a^2bi)x^7+32\,(5\,ab^2d+a^2bh}{384\,(a^3b^4x^{12}+3\,a^4b^3}$$
$$+\frac{\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be+7\,a\sqrt{b}g-5\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be+7\,a\sqrt{b}g-5\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+}{}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="m
axima")
```

```
[Out] 1/384*(15*(3*b^3*e + a*b^2*i)*x^11 + 12*(5*b^3*d + a*b^2*h)*x^10 + 7*(11*b^
3*c + a*b^2*g)*x^9 + 42*(3*a*b^2*e + a^2*b*i)*x^7 + 32*(5*a*b^2*d + a^2*b*h
)*x^6 + 18*(11*a*b^2*c + a^2*b*g)*x^5 - 32*a^3*f + (113*a^2*b*e - 5*a^3*i)*
x^3 + 12*(11*a^2*b*d - a^3*h)*x^2 + 3*(51*a^2*b*c - 7*a^3*g)*x)/(a^3*b^4*x^
12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*b) + 1/1024*(sqrt(2)*(77*b^(3/2)*c
 - 15*sqrt(a)*b*e + 7*a*sqrt(b)*g - 5*a^(3/2)*i)*log(sqrt(b)*x^2 + sqrt(2)*
a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*b^(3/2)*c - 15
*sqrt(a)*b*e + 7*a*sqrt(b)*g - 5*a^(3/2)*i)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c
 + 15*sqrt(2)*a^(3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*a
^(7/4)*b^(1/4)*i - 80*sqrt(a)*b^(3/2)*d - 16*a^(3/2)*sqrt(b)*h)*arctan(1/2*
sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(
3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*
sqrt(2)*a^(3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*a^(7/4)
*b^(1/4)*i + 80*sqrt(a)*b^(3/2)*d + 16*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2
)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*s
qrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^3*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.17

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx$$

$$= \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^3d+8\sqrt{2}\sqrt{ab}ab^2h+77\,(ab^3)^{\frac{1}{4}}\,b^3c+7\,(ab^3)^{\frac{1}{4}}\,ab^2g+15\,(ab^3)^{\frac{3}{4}}\,be+5\,(ab^3)^{\frac{3}{4}}\,ai\right)\arctan\left(\right.}{512\,a^4b^4}$$
$$+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^3d+8\sqrt{2}\sqrt{ab}ab^2h+77\,(ab^3)^{\frac{1}{4}}\,b^3c+7\,(ab^3)^{\frac{1}{4}}\,ab^2g+15\,(ab^3)^{\frac{3}{4}}\,be+5\,(ab^3)^{\frac{3}{4}}\,ai\right)\arctan}{512\,a^4b^4}$$
$$+\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^3c+7\,(ab^3)^{\frac{1}{4}}\,ab^2g-15\,(ab^3)^{\frac{3}{4}}\,be-5\,(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^4}$$
$$-\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^3c+7\,(ab^3)^{\frac{1}{4}}\,ab^2g-15\,(ab^3)^{\frac{3}{4}}\,be-5\,(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^4}$$
$$+\frac{45\,b^3ex^{11}+15\,ab^2ix^{11}+60\,b^3dx^{10}+12\,ab^2hx^{10}+77\,b^3cx^9+7\,ab^2gx^9+126\,ab^2ex^7+42\,a^2bix^7+160\,a}{}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="g
iac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^3*d + 8*sqrt(2)*sqrt(a*b)*a*b^2*h + 7
7*(a*b^3)^(1/4)*b^3*c + 7*(a*b^3)^(1/4)*a*b^2*g + 15*(a*b^3)^(3/4)*b*e + 5*
(a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(a^4*b^4) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^3*d + 8*sqrt(2)*sqrt(
a*b)*a*b^2*h + 77*(a*b^3)^(1/4)*b^3*c + 7*(a*b^3)^(1/4)*a*b^2*g + 15*(a*b^3
)^(3/4)*b*e + 5*(a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^
(1/4))/(a/b)^(1/4))/(a^4*b^4) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^3*c + 7*
(a*b^3)^(1/4)*a*b^2*g - 15*(a*b^3)^(3/4)*b*e - 5*(a*b^3)^(3/4)*a*i)*log(x^2
 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^4) - 1/1024*sqrt(2)*(77*(a*b^3
)^(1/4)*b^3*c + 7*(a*b^3)^(1/4)*a*b^2*g - 15*(a*b^3)^(3/4)*b*e - 5*(a*b^3)^
(3/4)*a*i)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^4) + 1/384*(
45*b^3*e*x^11 + 15*a*b^2*i*x^11 + 60*b^3*d*x^10 + 12*a*b^2*h*x^10 + 77*b^3*
c*x^9 + 7*a*b^2*g*x^9 + 126*a*b^2*e*x^7 + 42*a^2*b*i*x^7 + 160*a*b^2*d*x^6
+ 32*a^2*b*h*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*e*x^3 - 5*a
^3*i*x^3 + 132*a^2*b*d*x^2 - 12*a^3*h*x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32
*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 10.16 (sec) , antiderivative size = 2741, normalized size of antiderivative = 5.31

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,x)
```

```
[Out] ((3*x^5*(11*b*c + a*g))/(64*a^2) - f/(12*b) + (x^6*(5*b*d + a*h))/(12*a^2)
+ (7*x^7*(3*b*e + a*i))/(64*a^2) + (7*b*x^9*(11*b*c + a*g))/(384*a^3) + (x*
(51*b*c - 7*a*g))/(128*a*b) + (b*x^10*(5*b*d + a*h))/(32*a^3) + (5*b*x^11*(
3*b*e + a*i))/(128*a^3) + (x^2*(11*b*d - a*h))/(32*a*b) + (x^3*(113*b*e - 5
*a*i))/(384*a*b))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log
(- root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 + 403701760*a
^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 + 36
700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432*a^10*b^4*h^2
*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 17661952*a^6*b^
4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2*d*z + 409600*a
^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z + 368640
0*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*b^4*d*g^2*z + 184
32000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 32
3400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*a^2*b^4*c*d*e*h + 1
4700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a^4*b^2*c*h^2*i - 268
80*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a^2*b^4*c^2*e*i - 2464
000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 + 485
100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5
*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^
```

2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 + 1743126*a^2*b^4*c^
2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i + 2048000*a^2*b^4*d^3*h +
81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 + 2401*a
^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 + 625*a
^6*i^4 + 35153041*b^6*c^4, z, l)*(root(68719476736*a^15*b^7*z^4 + 121110528
0*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 +
110100480*a^9*b^5*e*g*z^2 + 36700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*
d^2*z^2 + 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^
5*b^5*c*d*g*z - 17661952*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 48570
3680*a^4*b^6*c^2*d*z + 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z -
802816*a^7*b^3*g^2*h*z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z
- 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*
i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*
h - 2956800*a^2*b^4*c*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*
i - 98560*a^4*b^2*c*h^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 +
1778700*a^2*b^4*c^2*e*i - 2464000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g
- 295680*a^3*b^3*c*e*h^2 + 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7
392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*
b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b
^3*e^2*g^2 + 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e
^3*i + 2048000*a^2*b^4*d^3*h + 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 +
2668050*a*b^5*c^2*e^2 + 2401*a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4096*a^5*b*
h^4 + 2560000*a*b^5*d^4 + 625*a^6*i^4 + 35153041*b^6*c^4, z, l)*((20185088*
a^7*b^5*c + 1835008*a^8*b^4*g)/(2097152*a^9*b^2) - (x*(655360*a^7*b^4*d + 1
31072*a^8*b^3*h))/(131072*a^9*b)) + (614400*a^4*b^4*d*e + 204800*a^5*b^3*d*
i + 122880*a^5*b^3*e*h + 40960*a^6*b^2*h*i)/(2097152*a^9*b^2) - (x*(800*a^6
*b*i^2 - 189728*a^3*b^4*c^2 + 7200*a^4*b^3*e^2 - 1568*a^5*b^2*g^2 - 34496*a
^4*b^3*c*g + 4800*a^5*b^2*e*i))/(131072*a^9*b)) - (125*a^4*i^3 + 3375*a*b^3
*e^3 - 123200*b^4*c*d^2 + 88935*b^4*c^2*e - 4928*a^2*b^2*c*h^2 + 735*a^2*b^
2*e*g^2 + 3375*a^2*b^2*e^2*i - 11200*a*b^3*d^2*g + 29645*a*b^3*c^2*i + 1125
*a^3*b*e*i^2 - 448*a^3*b*g*h^2 + 245*a^3*b*g^2*i + 5390*a^2*b^2*c*g*i - 448
0*a^2*b^2*d*g*h - 49280*a*b^3*c*d*h + 16170*a*b^3*c*e*g)/(2097152*a^9*b^2)
- (x*(5775*b^3*c*d*e - 32*a^3*h^3 - 4000*b^3*d^3 + 35*a^3*g*h*i - 2400*a*b^
2*d^2*h - 480*a^2*b*d*h^2 + 1925*a*b^2*c*d*i + 1155*a*b^2*c*e*h + 525*a*b^2
*d*e*g + 385*a^2*b*c*h*i + 175*a^2*b*d*g*i + 105*a^2*b*e*g*h))/(131072*a^9*
b))*root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 + 403701760*
a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 + 3
6700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432*a^10*b^4*h^
2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 17661952*a^6*b
^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2*d*z + 409600*
a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z + 36864
00*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*b^4*d*g^2*z + 18
432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 3
23400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*a^2*b^4*c*d*e*h +
14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a^4*b^2*c*h^2*i - 26

```
880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a^2*b^4*c^2*e*i - 246
4000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 + 48
5100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^
5*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d
^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 + 1743126*a^2*b^4*c
^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i + 2048000*a^2*b^4*d^3*h +
 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 + 2401*
a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 + 625*
a^6*i^4 + 35153041*b^6*c^4, z, l), l, 1, 4)
```

3.209 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4} dx$

Optimal result

Integrand size = 45, antiderivative size = 534

$$\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4} dx
&= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{12ab\,(a+bx^4)^3} \\
&+ \frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\,(a+bx^4)} \\
&- \frac{4a(2bf+aj)-x(b(11bc+ag)+2b(5bd+ah)x+3b(3be+ai)x^2)}{96a^2b^2\,(a+bx^4)^2} \\
&+ \frac{(5bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} - \frac{\left(7\sqrt{b}(11bc+ag)+5\sqrt{a}(3be+ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{7/4}} \\
&+ \frac{\left(7\sqrt{b}(11bc+ag)+5\sqrt{a}(3be+ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{7/4}} \\
&- \frac{\left(7\sqrt{b}(11bc+ag)-5\sqrt{a}(3be+ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{7/4}} \\
&+ \frac{\left(7\sqrt{b}(11bc+ag)-5\sqrt{a}(3be+ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{7/4}}
\end{aligned}$$

```
[Out] 1/12*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+(-a*j+b*f)*x^3)/a/b/(b*x^4+a)^3
+1/384*x*(7*a*g+77*b*c+12*(a*h+5*b*d)*x+15*(a*i+3*b*e)*x^2)/a^3/b/(b*x^4+a)
+1/96*(-4*a*(a*j+2*b*f)+x*(b*(a*g+11*b*c)+2*b*(a*h+5*b*d)*x+3*b*(a*i+3*b*e)
*x^2))/a^2/b^2/(b*x^4+a)^2+1/32*(a*h+5*b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(
```

```
7/2)/b^(3/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*
(a*i+3*b*e)*a^(1/2)+7*(a*g+11*b*c)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)+1/1024
*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*(a*i+3*b*e)*a^(1/2)+
7*(a*g+11*b*c)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*
2^(1/2)/a^(1/4))*(5*(a*i+3*b*e)*a^(1/2)+7*(a*g+11*b*c)*b^(1/2))/a^(15/4)/b^
(7/4)*2^(1/2)+1/512*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*(a*i+3*b*e)*a^(1
/2)+7*(a*g+11*b*c)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1872, 1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx$$
$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}$$
$$+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+5bd)}{32a^{7/2}b^{3/2}}$$
$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}$$
$$+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}$$
$$+\frac{x(7(ag+11bc)+12x(ah+5bd)+15x^2(ai+3be))}{384a^3b\,(a+bx^4)}$$
$$-\frac{4a(aj+2bf)-x(b(ag+11bc)+2bx(ah+5bd)+3bx^2(ai+3be))}{96a^2b^2\,(a+bx^4)^2}$$
$$+\frac{x(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc)}{12ab\,(a+bx^4)^3}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^4
,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(12*a*b
*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 15*(3*b*e + a
*i)*x^2))/(384*a^3*b*(a + b*x^4)) - (4*a*(2*b*f + a*j) - x*(b*(11*b*c + a*g
) + 2*b*(5*b*d + a*h)*x + 3*b*(3*b*e + a*i)*x^2))/(96*a^2*b^2*(a + b*x^4)^2
) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((
7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(
```

```
1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*
g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256
*Sqrt[2]*a^(15/4)*b^(7/4)) - ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e
+ a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2
]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15
/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{12ab\,(a+bx^4)^3} - \frac{\int \frac{-b(11bc+ag)-2b(5bd+ah)x-3b(3be+ai)x^2-4b(2bf+aj)x^3}{(a+bx^4)^3}\,dx}{12ab^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{12ab\,(a+bx^4)^3} - \frac{4a(2bf+aj)-x(b(11bc+ag)+2b(5bd+ah)x+3b(3be+ai)x^2)}{96a^2b^2\,(a+bx^4)^2} + \frac{\int \frac{7b(11bc+ag)+12b(5bd+ah)x+15b(3be+ai)x^2}{(a+bx^4)^2}\,dx}{96a^2b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\,(a+bx^4)} - \frac{4a(2bf+aj)-x(b(11bc+ag)+2b(5bd+ah)x+3b(3be+ai)x^2)}{96a^2b^2\,(a+bx^4)^2} - \frac{\int \frac{-21b(11bc+ag)-24b(5bd+ah)x-15b(3be+ai)x^2}{a+bx^4}\,dx}{384a^3b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\,(a+bx^4)} - \frac{4a(2bf+aj)-x(b(11bc+ag)+2b(5bd+ah)x+3b(3be+ai)x^2)}{96a^2b^2\,(a+bx^4)^2} - \frac{\int \left(-\frac{24b(5bd+ah)x}{a+bx^4}+\frac{-21b(11bc+ag)-15b(3be+ai)x^2}{a+bx^4}\right)dx}{384a^3b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\,(a+bx^4)} - \frac{4a(2bf+aj)-x(b(11bc+ag)+2b(5bd+ah)x+3b(3be+ai)x^2)}{96a^2b^2\,(a+bx^4)^2} - \frac{\int \frac{-21b(11bc+ag)-15b(3be+ai)x^2}{a+bx^4}\,dx}{384a^3b^2} + \frac{(5bd+ah)\int \frac{x}{a+bx^4}\,dx}{16a^3b}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{12ab\left(a+bx^4\right)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\left(a+bx^4\right)} - \frac{4a(2bf+aj)-x(b(11bc+ag)+2b(5bd+ah)x+3b(3be+ai)x^2)}{96a^2b^2\left(a+bx^4\right)^2} + \frac{(5bd+ah)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{32a^3b} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}-5(3be+ai)\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{256a^3b^2} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}+5(3be+ai)\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{256a^3b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{12ab\left(a+bx^4\right)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\left(a+bx^4\right)} - \frac{4a(2bf+aj)-x(b(11bc+ag)+2b(5bd+ah)x+3b(3be+ai)x^2)}{96a^2b^2\left(a+bx^4\right)^2} + \frac{(5bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} - \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}-5(3be+ai)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{512\sqrt{2}a^{13/4}b^{7/4}} - \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}-5(3be+ai)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{512\sqrt{2}a^{13/4}b^{7/4}} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}+5(3be+ai)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{512a^3b^2} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}+5(3be+ai)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{512a^3b^2}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\,(a+bx^4)} - \frac{4a(2bf+aj)-x(b(11bc+ag)+2b(5bd+ah)x+3b(3be+ai)x^2)}{96a^2b^2\,(a+bx^4)^2} + \frac{(5bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} - \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}-5(3be+ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{13/4}b^{7/4}} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}-5(3be+ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{13/4}b^{7/4}} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}+5(3be+ai)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{13/4}b^{7/4}} - \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}+5(3be+ai)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{13/4}b^{7/4}}$$

$$= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{12ab\,(a+bx^4)^3} + \frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\,(a+bx^4)} - \frac{4a(2bf+aj)-x(b(11bc+ag)+2b(5bd+ah)x+3b(3be+ai)x^2)}{96a^2b^2\,(a+bx^4)^2} + \frac{(5bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} - \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}+5(3be+ai)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{13/4}b^{7/4}} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}+5(3be+ai)\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{13/4}b^{7/4}} - \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}-5(3be+ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{13/4}b^{7/4}} + \frac{\left(\frac{7\sqrt{b}(11bc+ag)}{\sqrt{a}}-5(3be+ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{13/4}b^{7/4}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx$$
$$=\frac{\frac{8a^{3/4}bx(77bc+7ag+15bx(4d+3ex)+3ax(4h+5ix))}{a+bx^4}-\frac{32a^{7/4}(12a^2j-b^2x(11c+x(10d+9ex))-abx(g+x(2h+3ix)))}{(a+bx^4)^2}+\frac{256a^{11/4}(a^2j+b^2x(c+x(\cdots}{(a\cdots}}{\cdots}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4)^4,x]
```

```
[Out] ((8*a^(3/4)*b*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i*x
)))/(a + b*x^4) - (32*a^(7/4)*(12*a^2*j - b^2*x*(11*c + x*(10*d + 9*e*x)) -
 a*b*x*(g + x*(2*h + 3*i*x))))/(a + b*x^4)^2 + (256*a^(11/4)*(a^2*j + b^2*x
*(c + x*(d + e*x)) - a*b*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^3 - 6*b^(1
/4)*(77*Sqrt[2]*b^(3/2)*c + 80*a^(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e +
 7*Sqrt[2]*a*Sqrt[b]*g + 16*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*b^(1/4)*(77*Sqrt[2]*b^(3/2)*c - 80*a^
(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sqrt[b]*g - 16*a^(5/
4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
 + 3*Sqrt[2]*b^(1/4)*(-77*b^(3/2)*c + 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^
(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqrt[2]
*b^(1/4)*(77*b^(3/2)*c - 15*Sqrt[a]*b*e + 7*a*Sqrt[b]*g - 5*a^(3/2)*i)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.58 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.47

method	result
risch	$\frac{\frac{5(ai+3be)b\,x^{11}}{128a^3}+\frac{(ah+5bd)b\,x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{7(ai+3be)x^7}{64a^2}+\frac{(ah+5bd)x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}-\frac{j\,x^4}{8b}-\frac{(5ai-113be)x^3}{384ab}-\frac{(ah-11bd)x^2}{32ab}-\frac{(7ag-51\cdots}{128a\cdots}}{(b\,x^4+a)^3}$
default	$\frac{\frac{5(ai+3be)b\,x^{11}}{128a^3}+\frac{(ah+5bd)b\,x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{7(ai+3be)x^7}{64a^2}+\frac{(ah+5bd)x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}-\frac{j\,x^4}{8b}-\frac{(5ai-113be)x^3}{384ab}-\frac{(ah-11bd)x^2}{32ab}-\frac{(7ag-51\cdots}{128a\cdots}}{(b\,x^4+a)^3}$

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETUR
NVERBOSE)
```

```
[Out] (5/128*(a*i+3*b*e)/a^3*b*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c
)/a^3*b*x^9+7/64*(a*i+3*b*e)/a^2*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a*g
+11*b*c)*x^5-1/8*j*x^4/b-1/384*(5*a*i-113*b*e)/a/b*x^3-1/32*(a*h-11*b*d)/a/
b*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/24*(a*j+2*b*f)/b^2)/(b*x^4+a)^3+1/512/a^
3/b^2*sum((5*(a*i+3*b*e)*_R^2+8*(a*h+5*b*d)*_R+7*a*g+77*b*c)/_R^3*ln(x-_R),
_R=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx = \text{Timed out}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx = \text{Timed out}$$

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x
)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 613, normalized size of antiderivative = 1.15

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx$$
$$= \frac{15\,(3\,b^4e+ab^3i)x^{11}+12\,(5\,b^4d+ab^3h)x^{10}+7\,(11\,b^4c+ab^3g)x^9-48\,a^3bjx^4+42\,(3\,ab^3e+a^2b^2i)x^7+32}{384\,(}$$

$$+\frac{\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be+7\,a\sqrt{b}g-5\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be+7\,a\sqrt{b}g-5\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}}{}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algori
thm="maxima")
```

```
[Out] 1/384*(15*(3*b^4*e + a*b^3*i)*x^11 + 12*(5*b^4*d + a*b^3*h)*x^10 + 7*(11*b^
4*c + a*b^3*g)*x^9 - 48*a^3*b*j*x^4 + 42*(3*a*b^3*e + a^2*b^2*i)*x^7 + 32*(
5*a*b^3*d + a^2*b^2*h)*x^6 + 18*(11*a*b^3*c + a^2*b^2*g)*x^5 - 32*a^3*b*f -
 16*a^4*j + (113*a^2*b^2*e - 5*a^3*b*i)*x^3 + 12*(11*a^2*b^2*d - a^3*b*h)*x
^2 + 3*(51*a^2*b^2*c - 7*a^3*b*g)*x)/(a^3*b^5*x^12 + 3*a^4*b^4*x^8 + 3*a^5*
b^3*x^4 + a^6*b^2) + 1/1024*(sqrt(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*e + 7*a*s
qrt(b)*g - 5*a^(3/2)*i)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(
a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*e + 7*a*sqrt(b
)*g - 5*a^(3/2)*i)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(
a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*sqrt(2)*a^(3/4)*b^(
5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*a^(7/4)*b^(1/4)*i - 80*sqr
t(a)*b^(3/2)*d - 16*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sq
rt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b)
)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*sqrt(2)*a^(3/4)*b^(5/4)*e
 + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*a^(7/4)*b^(1/4)*i + 80*sqrt(a)*b
^(3/2)*d + 16*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*
a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3
/4)))/(a^3*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.19

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx$$

$$= \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^3d+8\sqrt{2}\sqrt{ab}ab^2h+77\,(ab^3)^{\frac{1}{4}}\,b^3c+7\,(ab^3)^{\frac{1}{4}}\,ab^2g+15\,(ab^3)^{\frac{3}{4}}\,be+5\,(ab^3)^{\frac{3}{4}}\,ai\right)\arctan\left(\frac{\sqrt{2}}{}\right.}{512\,a^4b^4}$$

$$+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^3d+8\sqrt{2}\sqrt{ab}ab^2h+77\,(ab^3)^{\frac{1}{4}}\,b^3c+7\,(ab^3)^{\frac{1}{4}}\,ab^2g+15\,(ab^3)^{\frac{3}{4}}\,be+5\,(ab^3)^{\frac{3}{4}}\,ai\right)\arctan\left(\right.}{512\,a^4b^4}$$

$$+\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^3c+7\,(ab^3)^{\frac{1}{4}}\,ab^2g-15\,(ab^3)^{\frac{3}{4}}\,be-5\,(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^4}$$

$$-\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^3c+7\,(ab^3)^{\frac{1}{4}}\,ab^2g-15\,(ab^3)^{\frac{3}{4}}\,be-5\,(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^4}$$

$$+\frac{45\,b^4ex^{11}+15\,ab^3ix^{11}+60\,b^4dx^{10}+12\,ab^3hx^{10}+77\,b^4cx^9+7\,ab^3gx^9+126\,ab^3ex^7+42\,a^2b^2ix^7+160\,ab}{}$$

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algori
thm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^3*d + 8*sqrt(2)*sqrt(a*b)*a*b^2*h + 7
7*(a*b^3)^(1/4)*b^3*c + 7*(a*b^3)^(1/4)*a*b^2*g + 15*(a*b^3)^(3/4)*b*e + 5*
(a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(a^4*b^4) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^3*d + 8*sqrt(2)*sqrt(
a*b)*a*b^2*h + 77*(a*b^3)^(1/4)*b^3*c + 7*(a*b^3)^(1/4)*a*b^2*g + 15*(a*b^3
)^(3/4)*b*e + 5*(a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^
(1/4))/(a/b)^(1/4))/(a^4*b^4) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^3*c + 7*
(a*b^3)^(1/4)*a*b^2*g - 15*(a*b^3)^(3/4)*b*e - 5*(a*b^3)^(3/4)*a*i)*log(x^2
 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^4) - 1/1024*sqrt(2)*(77*(a*b^3
)^(1/4)*b^3*c + 7*(a*b^3)^(1/4)*a*b^2*g - 15*(a*b^3)^(3/4)*b*e - 5*(a*b^3)^
(3/4)*a*i)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^4) + 1/384*(
45*b^4*e*x^11 + 15*a*b^3*i*x^11 + 60*b^4*d*x^10 + 12*a*b^3*h*x^10 + 77*b^4*
c*x^9 + 7*a*b^3*g*x^9 + 126*a*b^3*e*x^7 + 42*a^2*b^2*i*x^7 + 160*a*b^3*d*x^
6 + 32*a^2*b^2*h*x^6 + 198*a*b^3*c*x^5 + 18*a^2*b^2*g*x^5 - 48*a^3*b*j*x^4
+ 113*a^2*b^2*e*x^3 - 5*a^3*b*i*x^3 + 132*a^2*b^2*d*x^2 - 12*a^3*b*h*x^2 +
153*a^2*b^2*c*x - 21*a^3*b*g*x - 32*a^3*b*f - 16*a^4*j)/((b*x^4 + a)^3*a^3*
b^2)
```

Mupad [B] (verification not implemented)

Time = 10.45 (sec) , antiderivative size = 2757, normalized size of antiderivative = 5.16

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^4
,x)
```

```
[Out] symsum(log(- root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 + 4
03701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*
g*z^2 + 36700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432*a^
10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 17661
952*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2*d*z
+ 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*
z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*b^4*d*g
^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*
d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*a^2*b^4*c
*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a^4*b^2*c*h
^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a^2*b^4*c^2*
e*i - 2464000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e
*h^2 + 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e
+ 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*b^2*e^2*i^2 + 614400*
```

a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 + 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i + 2048000*a^2*b^4*d^3*h + 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 + 2401*a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 + 625*a^6*i^4 + 35153041*b^6*c^4, z, m)*(root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 + 36700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 17661952*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2*d*z + 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*a^2*b^4*c*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a^4*b^2*c*h^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a^2*b^4*c^2*e*i - 2464000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 + 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 + 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i + 2048000*a^2*b^4*d^3*h + 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 + 2401*a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 + 625*a^6*i^4 + 35153041*b^6*c^4, z, m)*((20185088*a^7*b^5*c + 1835008*a^8*b^4*g)/(2097152*a^9*b^2) - (x*(655360*a^7*b^4*d + 131072*a^8*b^3*h))/(131072*a^9*b)) + (614400*a^4*b^4*d*e + 204800*a^5*b^3*d*i + 122880*a^5*b^3*e*h + 40960*a^6*b^2*h*i)/(2097152*a^9*b^2) - (x*(800*a^6*b*i^2 - 189728*a^3*b^4*c^2 + 7200*a^4*b^3*e^2 - 1568*a^5*b^2*g^2 - 34496*a^4*b^3*c*g + 4800*a^5*b^2*e*i))/(131072*a^9*b)) - (125*a^4*i^3 + 3375*a*b^3*e^3 - 123200*b^4*c*d^2 + 88935*b^4*c^2*e - 4928*a^2*b^2*c*h^2 + 735*a^2*b^2*e*g^2 + 3375*a^2*b^2*e^2*i - 11200*a*b^3*d^2*g + 29645*a*b^3*c^2*i + 1125*a^3*b*e*i^2 - 448*a^3*b*g*h^2 + 245*a^3*b*g^2*i + 5390*a^2*b^2*c*g*i - 4480*a^2*b^2*d*g*h - 49280*a*b^3*c*d*h + 16170*a*b^3*c*e*g)/(2097152*a^9*b^2) - (x*(5775*b^3*c*d*e - 32*a^3*h^3 - 4000*b^3*d^3 + 35*a^3*g*h*i - 2400*a*b^2*d^2*h - 480*a^2*b*d*h^2 + 1925*a*b^2*c*d*i + 1155*a*b^2*c*e*h + 525*a*b^2*d*e*g + 385*a^2*b*c*h*i + 175*a^2*b*d*g*i + 105*a^2*b*e*g*h))/(131072*a^9*b))*root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 + 36700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 17661952*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2*d*z + 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*a^2*b^4*c*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a^4*b^2*c* h^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a^2*b^4*c^2 *e*i - 2464000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c* e*h^2 + 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*b^2*e^2*i^2 + 614400 *a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 + 1743126 *a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i + 2048000*a^2*b ^4*d^3*h + 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e ^2 + 2401*a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5* d^4 + 625*a^6*i^4 + 35153041*b^6*c^4, z, m), m, 1, 4) + ((3*x^5*(11*b*c + a *g))/(64*a^2) - (j*x^4)/(8*b) - (2*b*f + a*j)/(24*b^2) + (x^6*(5*b*d + a*h))/(12*a^2) + (7*x^7*(3*b*e + a*i))/(64*a^2) + (7*b*x^9*(11*b*c + a*g))/(384 *a^3) + (x*(51*b*c - 7*a*g))/(128*a*b) + (b*x^10*(5*b*d + a*h))/(32*a^3) + (5*b*x^11*(3*b*e + a*i))/(128*a^3) + (x^2*(11*b*d - a*h))/(32*a*b) + (x^3*(113*b*e - 5*a*i))/(384*a*b))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8)

3.210 $\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx=\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}$$

```
[Out] 1/2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+1/2*c*(cos(2*arctan(b^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*a
rctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^
(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1899, 226, 281, 223, 212}

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx=\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

```
[In] Int[(c + d*x)/Sqrt[a + b*x^4],x]
```

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{\sqrt{a+bx^4}}+\frac{dx}{\sqrt{a+bx^4}}\right)\,dx\\
&= c\int \frac{1}{\sqrt{a+bx^4}}\,dx+d\int \frac{x}{\sqrt{a+bx^4}}\,dx\\
&= \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)
\end{aligned}$$

$$= \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}} + \frac{1}{2}d\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)$$

$$= \frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

$$\int\frac{c+dx}{\sqrt{a+bx^4}}\,dx = \frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{cx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}$$

```
[In] Integrate[(c + d*x)/Sqrt[a + b*x^4],x]
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}$	96
elliptic	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}$	99

```
[In] int((d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1
/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.60

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx = \frac{4\,b^{\frac{3}{2}}c\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+a\sqrt{b}d\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)}{4\,ab}$$

```
[In] integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^(3/2)*c*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + a*sq
rt(b)*d*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a))/(a*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx = \frac{d\,\text{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2)
, (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx = \int \frac{dx+c}{\sqrt{bx^4+a}}\,dx$$

```
[In] integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx = \int \frac{dx+c}{\sqrt{bx^4+a}}\,dx$$

[In] integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((d*x + c)/sqrt(b*x^4 + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx = \int \frac{c+d\,x}{\sqrt{b\,x^4+a}}\,dx$$

[In] int((c + d*x)/(a + b*x^4)^(1/2),x)

[Out] int((c + d*x)/(a + b*x^4)^(1/2), x)

3.211 $\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 87

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx=\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}+\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}$$

```
[Out] 1/2*d*arctan(x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(1/2)+a^(1/4)*c*EllipticF(b^(1
/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/(-b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1899, 230, 227, 281, 223, 209}

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx=\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

```
[In] Int[(c + d*x)/Sqrt[a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[1 -
 (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b
*x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{\sqrt{a-bx^4}}+\frac{dx}{\sqrt{a-bx^4}}\right)dx\\
&= c\int \frac{1}{\sqrt{a-bx^4}}\,dx+d\int \frac{x}{\sqrt{a-bx^4}}\,dx\\
&= \frac{1}{2}d\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,x^2\right)+\frac{\left(c\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{\sqrt{a-bx^4}}\\
&= \frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{x^2}{\sqrt{a-bx^4}}\right)
\end{aligned}$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}} + \frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx = \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}} + \frac{cx\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)}{\sqrt{a-bx^4}}$$

```
[In] Integrate[(c + d*x)/Sqrt[a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 - (b*x^
4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/Sqrt[a - b*x^4]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{c\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}+\frac{d\arctan\left(\frac{x^2\sqrt{b}}{\sqrt{-b\,x^4+a}}\right)}{2\sqrt{b}}$	90
elliptic	$\frac{c\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}+\frac{d\ln\left(-\frac{2b\,x^2}{\sqrt{-b}}+2\sqrt{-b\,x^4+a}\right)}{2\sqrt{-b}}$	99

```
[In] int((d*x+c)/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] c/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^
(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)+1/2*
d*arctan(x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx = \frac{4\sqrt{-b}bc\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1) - a\sqrt{-b}d\log\left(2\,bx^4-2\,\sqrt{-bx^4+a}\sqrt{-b}x^2-a\right)}{4\,ab}$$

```
[In] integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="fricas")
[Out] 1/4*(4*sqrt(-b)*b*c*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) - a*s
qrt(-b)*d*log(2*b*x^4 - 2*sqrt(-b*x^4 + a)*sqrt(-b)*x^2 - a))/(a*b)
```

Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx = d\left(\begin{cases} -\frac{i\,\mathrm{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{for}\ \left|\frac{bx^4}{a}\right|>1 \\ \frac{\mathrm{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((d*x+c)/(-b*x**4+a)**(1/2),x)
[Out] d*Piecewise((-I*acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x**4/a) > 1)
, (asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True)) + c*x*gamma(1/4)*hyper((1
/4, 1/2), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx = \int \frac{dx+c}{\sqrt{-bx^4+a}}\,dx$$

```
[In] integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x + c)/sqrt(-b*x^4 + a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx=\int \frac{dx+c}{\sqrt{-bx^4+a}}\,dx$$

```
[In] integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)/sqrt(-b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx=\int \frac{c+d\,x}{\sqrt{a-b\,x^4}}\,dx$$

```
[In] int((c + d*x)/(a - b*x^4)^(1/2),x)
[Out] int((c + d*x)/(a - b*x^4)^(1/2), x)
```

3.212 $\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx=\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{-a+bx^4}}\right)}{2\sqrt{b}}+\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{-a+bx^4}}$$

```
[Out] 1/2*d*arctanh(x^2*b^(1/2)/(b*x^4-a)^(1/2))/b^(1/2)+a^(1/4)*c*EllipticF(b^(1
/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/(b*x^4-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1899, 230, 227, 281, 223, 212}

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx=\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{bx^4-a}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{bx^4-a}}\right)}{2\sqrt{b}}$$

```
[In] Int[(c + d*x)/Sqrt[-a + b*x^4],x]
```

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[1
 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[-a
+ b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{c}{\sqrt{-a+bx^4}}+\frac{dx}{\sqrt{-a+bx^4}}\right)dx\\
&= c\int \frac{1}{\sqrt{-a+bx^4}}\,dx+d\int \frac{x}{\sqrt{-a+bx^4}}\,dx\\
&= \frac{1}{2}d\text{Subst}\left(\int \frac{1}{\sqrt{-a+bx^2}}\,dx,x,x^2\right)+\frac{\left(c\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{\sqrt{-a+bx^4}}\\
&= \frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{-a+bx^4}}+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{-a+bx^4}}\right)
\end{aligned}
$$

$$=\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{-a+bx^4}}\right)}{2\sqrt{b}}+\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{-a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int\frac{c+dx}{\sqrt{-a+bx^4}}\,dx=\frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{-a+bx^4}}\right)}{2\sqrt{b}}+\frac{cx\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)}{\sqrt{-a+bx^4}}$$

```
[In] Integrate[(c + d*x)/Sqrt[-a + b*x^4],x]
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 - (b*
x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/Sqrt[-a + b*x^4]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{c\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4-a}}+\frac{d\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4-a}\right)}{2\sqrt{b}}$	95
elliptic	$\frac{c\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4-a}}+\frac{d\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4-a}\right)}{2\sqrt{b}}$	98

```
[In] int((d*x+c)/(b*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
[Out] c/(-1/a^(1/2)*b^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)*(1-x^2*b^(1/2)/a
^(1/2))^(1/2)/(b*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*b^(1/2))^(1/2),I)+1/2
*d*ln(x^2*b^(1/2)+(b*x^4-a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx = -\frac{4\,b^{\frac{3}{2}}c\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1) - a\sqrt{b}d\log\left(2\,bx^4+2\,\sqrt{bx^4-a}\sqrt{b}x^2-a\right)}{4\,ab}$$

```
[In] integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^(3/2)*c*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) - a*sqr
t(b)*d*log(2*b*x^4 + 2*sqrt(b*x^4 - a)*sqrt(b)*x^2 - a))/(a*b)
```

Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx = d\left(\begin{cases}\frac{\operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{for}\ \left|\frac{bx^4}{a}\right|>1\\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{otherwise}\end{cases}\right) - \frac{icx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((d*x+c)/(b*x**4-a)**(1/2),x)
```

```
[Out] d*Piecewise((acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x**4/a) > 1), (
-I*asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True)) - I*c*x*gamma(1/4)*hyper(
(1/4, 1/2), (5/4,), b*x**4/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx = \int \frac{dx+c}{\sqrt{bx^4-a}}\,dx$$

```
[In] integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^4 - a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{-a+bx^4}} \, dx = \int \frac{dx+c}{\sqrt{bx^4-a}} \, dx$$

```
[In] integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)/sqrt(b*x^4 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{-a+bx^4}} \, dx = \int \frac{c+d\,x}{\sqrt{b\,x^4-a}} \, dx$$

```
[In] int((c + d*x)/(b*x^4 - a)^(1/2),x)
[Out] int((c + d*x)/(b*x^4 - a)^(1/2), x)
```

3.213 $\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 127

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}} + \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}}$$

```
[Out] 1/2*d*arctan(x^2*b^(1/2)/(-b*x^4-a)^(1/2))/b^(1/2)+1/2*c*(cos(2*arctan(b^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*a
rctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^
(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/(-b*x^4-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1899, 226, 281, 223, 209}

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}} + \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}$$

```
[In] Int[(c + d*x)/Sqrt[-a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[-a - b*x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{\sqrt{-a-bx^4}}+\frac{dx}{\sqrt{-a-bx^4}}\right)\,dx\\
&= c\int \frac{1}{\sqrt{-a-bx^4}}\,dx+d\int \frac{x}{\sqrt{-a-bx^4}}\,dx\\
&= \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}}+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{\sqrt{-a-bx^2}}\,dx,x,x^2\right)
\end{aligned}$$

$$= \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}} + \frac{1}{2}d\text{Subst}\left(\int\frac{1}{1+bx^2}\,dx,x,\frac{x^2}{\sqrt{-a-bx^4}}\right)$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}} + \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.67

$$\int\frac{c+dx}{\sqrt{-a-bx^4}}\,dx = \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}} + \frac{cx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{-a-bx^4}}$$

```
[In] Integrate[(c + d*x)/Sqrt[-a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[-a - b*x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{c\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4-a}}+\frac{d\arctan\left(\frac{x^2\sqrt{b}}{\sqrt{-b\,x^4-a}}\right)}{2\sqrt{b}}$	101
elliptic	$\frac{c\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4-a}}+\frac{d\ln\left(-\frac{2b\,x^2}{\sqrt{-b}}+2\sqrt{-b\,x^4-a}\right)}{2\sqrt{-b}}$	110

```
[In] int((d*x+c)/(-b*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] c/(-I/a^(1/2)*b^(1/2))^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(-b*x^4-a)^(1/2)*EllipticF(x*(-I/a^(1/2)*b^(1/2))^(1/2),I
)+1/2*d*arctan(x^2*b^(1/2)/(-b*x^4-a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = -\frac{4\sqrt{-b}bc\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1\right)+a\sqrt{-b}d\log\left(-2\,bx^4+2\,\sqrt{-bx^4-a}\sqrt{-b}x^2-a\right)}{4\,ab}$$

```
[In] integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(-b)*b*c*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) +
a*sqrt(-b)*d*log(-2*b*x^4 + 2*sqrt(-b*x^4 - a)*sqrt(-b)*x^2 - a))/(a*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.52

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = -\frac{id\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} - \frac{icx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((d*x+c)/(-b*x**4-a)**(1/2),x)
```

```
[Out] -I*d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) - I*c*x*gamma(1/4)*hyper((1/4,
 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = \int \frac{dx+c}{\sqrt{-bx^4-a}}\,dx$$

```
[In] integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^4 - a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = \int \frac{dx+c}{\sqrt{-bx^4-a}}\,dx$$

```
[In] integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)/sqrt(-b*x^4 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = \int \frac{c+d\,x}{\sqrt{-b\,x^4-a}}\,dx$$

```
[In] int((c + d*x)/(- a - b*x^4)^(1/2),x)
[Out] int((c + d*x)/(- a - b*x^4)^(1/2), x)
```

3.214 $\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 257

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \frac{\sqrt[4]{a}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] 1/2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+e*x*(b*x^4+a)^(1/2)/b^(1
/2)/(a^(1/2)+x^2*b^(1/2))-a^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)
^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*
x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(e+c*b^(1/2)/a^(1/2))*((b*x^
4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1899, 281, 223, 212, 1212, 226, 1210}

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

```
[In] Int[(c + d*x + e*x^2)/Sqrt[a + b*x^4],x]
```

```
[Out] (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (d*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*c)/Sqrt[a] +
 e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{dx}{\sqrt{a+bx^4}}+\frac{c+ex^2}{\sqrt{a+bx^4}}\right)dx\\
&= d\int \frac{x}{\sqrt{a+bx^4}}\,dx+\int \frac{c+ex^2}{\sqrt{a+bx^4}}\,dx\\
&= \frac{1}{2}d\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\left(\sqrt{a}e\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{b}}+\left(c+\frac{\sqrt{a}e}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx\\
&= \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}\\
&\quad+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}\\
&\quad+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)
\end{aligned}$$

$$= \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.51

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{cx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}} + \frac{ex^3\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}$$

```
[In] Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x^4],x]
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4] + (e
*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(3
*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.75

method	result
default	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}} + \frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}} + \frac{d\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4}\right.}{2\sqrt{b}}$
elliptic	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}} + \frac{d\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}} + \frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{}}{}}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$

```
[In] int((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+I
*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b
^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*d*ln(x^2*b^(
1/2)+(b*x^4+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.50

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \frac{4\,a\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+a\sqrt{b}dx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+4\,(bc-ae)\sqrt{b}x(\cdots}{4\,abx}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) +
a*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 4*(b*c -
a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 4*sqrt
(b*x^4 + a)*a*e)/(a*b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.40

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \frac{d\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2)
, (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma(3
/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/
4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \int \frac{e\,x^2+d\,x+c}{\sqrt{b\,x^4+a}}\,dx$$

```
[In] int((c + d*x + e*x^2)/(a + b*x^4)^(1/2),x)
[Out] int((c + d*x + e*x^2)/(a + b*x^4)^(1/2), x)
```

3.215 $\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 14

$$\int \frac{ag - bgx^4}{(a + bx^4)^{3/2}}\,dx = \frac{gx}{\sqrt{a + bx^4}}$$

```
[Out] g*x/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {391}

$$\int \frac{ag - bgx^4}{(a + bx^4)^{3/2}}\,dx = \frac{gx}{\sqrt{a + bx^4}}$$

```
[In] Int[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^4]
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\text{integral} = \frac{gx}{\sqrt{a + bx^4}}$$

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{ag - bgx^4}{(a + bx^4)^{3/2}}\,dx = \frac{gx}{\sqrt{a + bx^4}}$$

```
[In] Integrate[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]
[Out] (g*x)/Sqrt[a + b*x^4]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{gx}{\sqrt{b\,x^{4}+a}}$	13
default	$\frac{gx}{\sqrt{b\,x^{4}+a}}$	13
trager	$\frac{gx}{\sqrt{b\,x^{4}+a}}$	13
elliptic	$\frac{gx}{\sqrt{b\,x^{4}+a}}$	13
pseudoelliptic	$\frac{gx}{\sqrt{b\,x^{4}+a}}$	13

```
[In] int((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] g*x/(b*x^4+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{ag - bgx^4}{(a + bx^4)^{3/2}}\,dx = \frac{gx}{\sqrt{bx^4 + a}}$$

```
[In] integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="fricas")
[Out] g*x/sqrt(b*x^4 + a)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.84 (sec) , antiderivative size = 80, normalized size of antiderivative = 5.71

$$\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate((-b*g*x**4+a*g)/(b*x**4+a)**(3/2),x)
```

```
[Out] g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{gx}{\sqrt{bx^4+a}}$$

```
[In] integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] g*x/sqrt(b*x^4 + a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{gx}{\sqrt{bx^4+a}}$$

```
[In] integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] g*x/sqrt(b*x^4 + a)
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{g\,x}{\sqrt{b\,x^4+a}}$$

```
[In] int((a*g - b*g*x^4)/(a + b*x^4)^(3/2),x)
[Out] (g*x)/(a + b*x^4)^(1/2)
```

3.216 $\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

```
[Out] 1/2*(2*a*g*x+e*x^2)/a/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1870}

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

```
[In] Int[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (2*a*g*x + e*x^2)/(2*a*Sqrt[a + b*x^4])
```

Rule 1870

```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, Simp[-(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

Rubi steps

$$\text{integral}=\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{x(2ag+ex)}{2a\sqrt{a+bx^4}}$$

```
[In] Integrate[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]
[Out] (x*(2*a*g + e*x))/(2*a*Sqrt[a + b*x^4])
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result
gosper	$\frac{x(2ag+ex)}{2\sqrt{b\,x^4+a}\,a}$
trager	$\frac{x(2ag+ex)}{2\sqrt{b\,x^4+a}\,a}$
elliptic	$\frac{e\,x^2}{2a\sqrt{b\,x^4+a}}+\frac{gx}{\sqrt{b\,x^4+a}}$
default	$ag\left(\frac{x}{2a\sqrt{(x^4+\frac{a}{b})b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+\frac{e\,x^2}{2a\sqrt{b\,x^4+a}}-gb\left(-\frac{x}{2b\sqrt{(x^4+\frac{a}{b})b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+}}{2b\sqrt{}}\right.$

```
[In] int((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(2*a*g+e*x)/(b*x^4+a)^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{\sqrt{bx^4+a}(2\,agx+ex^2)}{2\,(abx^4+a^2)}$$

```
[In] integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="fricas")
[Out] 1/2*sqrt(b*x^4 + a)*(2*a*g*x + e*x^2)/(a*b*x^4 + a^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.84 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.59

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{gx\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate((-b*g*x**4+a*g+e*x)/(b*x**4+a)**(3/2),x)
[Out] g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/
a))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{2\,agx+ex^2}{2\,\sqrt{bx^4+a}a}$$

```
[In] integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] 1/2*(2*a*g*x + e*x^2)/(sqrt(b*x^4 + a)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{\left(2\,g+\frac{ex}{a}\right)x}{2\,\sqrt{bx^4+a}}$$

```
[In] integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] 1/2*(2*g + e*x/a)*x/sqrt(b*x^4 + a)
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{g\,x+\frac{e\,x^2}{2\,a}}{\sqrt{b\,x^4+a}}$$

```
[In] int((a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x)
[Out] (g*x + (e*x^2)/(2*a))/(a + b*x^4)^(1/2)
```

3.217 $\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 25

$$\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx = -\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

```
[Out] 1/2*(2*b*g*x-f)/b/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1870}

$$\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx = -\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

```
[In] Int[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] -1/2*(f - 2*b*g*x)/(b*Sqrt[a + b*x^4])
```

Rule 1870

```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, Simp[-(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

Rubi steps

$$\text{integral} = -\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{-f+2bgx}{2b\sqrt{a+bx^4}}$$

```
[In] Integrate[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
[Out] (-f + 2*b*g*x)/(2*b*Sqrt[a + b*x^4])
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result
gosper	$\frac{2bgx-f}{2b\sqrt{b\,x^{4}+a}}$
trager	$\frac{2bgx-f}{2b\sqrt{b\,x^{4}+a}}$
elliptic	$-\frac{f}{2b\sqrt{b\,x^{4}+a}}+\frac{gx}{\sqrt{b\,x^{4}+a}}$
default	$-\frac{f}{2b\sqrt{b\,x^{4}+a}}+ag\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)-gb\left(-\frac{x}{2b\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{}}{2b\sqrt{}}\right.$

```
[In] int((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(2*b*g*x-f)/b/(b*x^4+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{\sqrt{bx^4+a}(2\,bgx-f)}{2\,(b^2x^4+ab)}$$

```
[In] integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="fricas")
[Out] 1/2*sqrt(b*x^4 + a)*(2*b*g*x - f)/(b^2*x^4 + a*b)
```

Sympy [A] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.36

$$\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx = f\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate((-b*g*x**4+f*x**3+a*g)/(b*x**4+a)**(3/2),x)
```

```
[Out] f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{2\,bgx-f}{2\,\sqrt{bx^4+a}b}$$

```
[In] integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*g*x - f)/(sqrt(b*x^4 + a)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{2\,gx-\frac{f}{b}}{2\,\sqrt{bx^4+a}}$$

```
[In] integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*g*x - f/b)/sqrt(b*x^4 + a)
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{ag+fx^3-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx=\frac{g\,x-\frac{f}{2\,b}}{\sqrt{b\,x^4+a}}$$

```
[In] int((a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x)
[Out] (g*x - f/(2*b))/(a + b*x^4)^(1/2)
```

3.218 $\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}} dx$

Optimal result

Integrand size = 31, antiderivative size = 38

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}} dx = -\frac{af-2abgx-bex^2}{2ab\sqrt{a+bx^4}}$$

```
[Out] 1/2*(2*a*b*g*x+b*e*x^2-a*f)/a/b/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1870}

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}} dx = -\frac{-2abgx+af-bex^2}{2ab\sqrt{a+bx^4}}$$

```
[In] Int[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] -1/2*(a*f - 2*a*b*g*x - b*e*x^2)/(a*b*Sqrt[a + b*x^4])
```

Rule 1870

```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, Simp[-(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

Rubi steps

$$\text{integral} = -\frac{af-2abgx-bex^2}{2ab\sqrt{a+bx^4}}$$

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{ag+ex+fx^3-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx=\frac{-af+2abgx+bex^2}{2ab\sqrt{a+bx^4}}$$

[In] Integrate[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (-(a*f) + 2*a*b*g*x + b*e*x^2)/(2*a*b*Sqrt[a + b*x^4])

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result
gosper	$\frac{2abgx+be\,x^2-af}{2ab\sqrt{b\,x^4+a}}$
trager	$\frac{2abgx+be\,x^2-af}{2ab\sqrt{b\,x^4+a}}$
elliptic	$-\frac{-be\,x^2+af}{2\sqrt{b\,x^4+a}\,ab}+\frac{gx}{\sqrt{b\,x^4+a}}$
default	$-\frac{f}{2b\sqrt{b\,x^4+a}}+ag\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+\frac{e\,x^2}{2a\sqrt{b\,x^4+a}}-gb\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\right.$

[In] int((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2*(2*a*b*g*x+b*e*x^2-a*f)/a/b/(b*x^4+a)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{ag+ex+fx^3-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx=\frac{\sqrt{bx^4+a}\left(2\,abgx+bex^2-af\right)}{2\left(ab^2x^4+a^2b\right)}$$

[In] integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] 1/2*sqrt(b*x^4 + a)*(2*a*b*g*x + b*e*x^2 - a*f)/(a*b^2*x^4 + a^2*b)

Sympy [A] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.50

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx = f\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate((-b*g*x**4+f*x**3+a*g+e*x)/(b*x**4+a)**(3/2),x)
```

```
[Out] f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*
x**4/a))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{\sqrt{bx^4+a}(2\,abgx+bex^2-af)}{2\,(ab^2x^4+a^2b)}$$

```
[In] integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*(2*a*b*g*x + b*e*x^2 - a*f)/(a*b^2*x^4 + a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{(2\,g+\frac{ex}{a})x-\frac{f}{b}}{2\,\sqrt{bx^4+a}}$$

```
[In] integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((2*g + e*x/a)*x - f/b)/sqrt(b*x^4 + a)
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{ag + ex + fx^3 - bgx^4}{(a + bx^4)^{3/2}} \, dx = \frac{g\,x - \frac{f}{2\,b} + \frac{e\,x^2}{2\,a}}{\sqrt{b\,x^4 + a}}$$

```
[In] int((a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x)
[Out] (g*x - f/(2*b) + (e*x^2)/(2*a))/(a + b*x^4)^(1/2)
```

3.219 $\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = -\frac{x}{\sqrt{1+x^4}}$$

```
[Out] -x/(x^4+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {391}

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = -\frac{x}{\sqrt{x^4+1}}$$

```
[In] Int[(-1 + x^4)/(1 + x^4)^(3/2),x]
```

```
[Out] -(x/Sqrt[1 + x^4])
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\text{integral} = -\frac{x}{\sqrt{1+x^4}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = -\frac{x}{\sqrt{1+x^4}}$$

```
[In] Integrate[(-1 + x^4)/(1 + x^4)^(3/2),x]
[Out] -(x/Sqrt[1 + x^4])
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{x}{\sqrt{x^4+1}}$	11
default	$-\frac{x}{\sqrt{x^4+1}}$	11
trager	$-\frac{x}{\sqrt{x^4+1}}$	11
risch	$-\frac{x}{\sqrt{x^4+1}}$	11
elliptic	$-\frac{x}{\sqrt{x^4+1}}$	11
pseudoelliptic	$-\frac{x}{\sqrt{x^4+1}}$	11
meijerg	$-x\,{}_2F_1\left(\frac{1}{4},\frac{3}{2};\frac{5}{4};-x^4\right)+\frac{x^5\,{}_2F_1\left(\frac{5}{4},\frac{3}{2};\frac{9}{4};-x^4\right)}{5}$	32

```
[In] int((x^4-1)/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -x/(x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = -\frac{x}{\sqrt{x^4+1}}$$

```
[In] integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="fricas")
[Out] -x/sqrt(x^4 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.83

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((x**4-1)/(x**4+1)**(3/2),x)
```

```
[Out] x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(I*pi))/(4*gamma(9/
4)) - x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi))/(4*gamma
(5/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = -\frac{x}{\sqrt{x^4+1}}$$

```
[In] integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -x/sqrt(x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = -\frac{x}{\sqrt{x^4+1}}$$

```
[In] integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] -x/sqrt(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}} \, dx = -\frac{x}{\sqrt{x^4+1}}$$

```
[In] int((x^4 - 1)/(x^4 + 1)^(3/2),x)
[Out] -x/(x^4 + 1)^(1/2)
```

3.220 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 385

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx &= \frac{f\sqrt{a+bx^4}}{2b}+\frac{gx\sqrt{a+bx^4}}{3b}+\frac{hx^2\sqrt{a+bx^4}}{4b} \\ &+\frac{ix^3\sqrt{a+bx^4}}{5b}+\frac{(5be-3ai)x\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{(2bd-ah)\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}} \\ &-\frac{\sqrt[4]{a}(5be-3ai)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}} \\ &+\frac{\sqrt[4]{a}\left(15be+\frac{5\sqrt{b}(3bc-ag)}{\sqrt{a}}-9ai\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

```
[Out] 1/4*(-a*h+2*b*d)*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/2*f*(b*x^4+
a)^(1/2)/b+1/3*g*x*(b*x^4+a)^(1/2)/b+1/4*h*x^2*(b*x^4+a)^(1/2)/b+1/5*i*x^3*
(b*x^4+a)^(1/2)/b+1/5*(-3*a*i+5*b*e)*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2
*b^(1/2))-1/5*a^(1/4)*(-3*a*i+5*b*e)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(
1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1
/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2
)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)+1/30*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4
)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4
)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(15*b*e-9*a*i+5*(-a*g+3*b*
c)*b^(1/2)/a^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^
4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1899, 1833, 1829, 655, 223, 212, 1902, 1212, 226, 1210}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\frac{5\sqrt{b}(3bc-ag)}{\sqrt{a}}-9ai+15be\right)}{30b^{7/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(5be-3ai)E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}} + \frac{\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)(2bd-ah)}{4b^{3/2}} + \frac{x\sqrt{a+bx^4}(5be-3ai)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{f\sqrt{a+bx^4}}{2b} + \frac{gx\sqrt{a+bx^4}}{3b} + \frac{hx^2\sqrt{a+bx^4}}{4b} + \frac{ix^3\sqrt{a+bx^4}}{5b}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/Sqrt[a + b*x^4],x]
```

```
[Out] (f*Sqrt[a + b*x^4])/(2*b) + (g*x*Sqrt[a + b*x^4])/(3*b) + (h*x^2*Sqrt[a + b
*x^4])/(4*b) + (i*x^3*Sqrt[a + b*x^4])/(5*b) + ((5*b*e - 3*a*i)*x*Sqrt[a +
b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*b*d - a*h)*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*(5*b*e - 3*a*i)*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(15*b*
e + (5*Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 9*a*i)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x(d+fx^2+hx^4)}{\sqrt{a+bx^4}}+\frac{c+ex^2+gx^4+ix^6}{\sqrt{a+bx^4}}\right)dx \\
&= \int \frac{x(d+fx^2+hx^4)}{\sqrt{a+bx^4}}\,dx+\int \frac{c+ex^2+gx^4+ix^6}{\sqrt{a+bx^4}}\,dx \\
&= \frac{ix^3\sqrt{a+bx^4}}{5b}+\frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{\sqrt{a+bx^2}}\,dx,x,x^2\right)+\frac{\int \frac{5bc+(5be-3ai)x^2+5bgx^4}{\sqrt{a+bx^4}}\,dx}{5b} \\
&= \frac{gx\sqrt{a+bx^4}}{3b}+\frac{hx^2\sqrt{a+bx^4}}{4b}+\frac{ix^3\sqrt{a+bx^4}}{5b} \\
&\quad +\frac{\int \frac{5b(3bc-ag)+3b(5be-3ai)x^2}{\sqrt{a+bx^4}}\,dx}{15b^2}+\frac{\text{Subst}\left(\int \frac{2bd-ah+2bfx}{\sqrt{a+bx^2}}\,dx,x,x^2\right)}{4b} \\
&= \frac{f\sqrt{a+bx^4}}{2b}+\frac{gx\sqrt{a+bx^4}}{3b}+\frac{hx^2\sqrt{a+bx^4}}{4b}+\frac{ix^3\sqrt{a+bx^4}}{5b} \\
&\quad +\frac{(2bd-ah)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)}{4b}-\frac{\left(\sqrt{a}(5be-3ai)\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{5b^{3/2}} \\
&\quad +\frac{\left(5\sqrt{b}(3bc-ag)+3\sqrt{a}(5be-3ai)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{15b^{3/2}} \\
&= \frac{f\sqrt{a+bx^4}}{2b}+\frac{gx\sqrt{a+bx^4}}{3b}+\frac{hx^2\sqrt{a+bx^4}}{4b}+\frac{ix^3\sqrt{a+bx^4}}{5b}+\frac{(5be-3ai)x\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} \\
&\quad -\frac{\sqrt[4]{a}(5be-3ai)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}} \\
&\quad +\frac{\left(5\sqrt{b}(3bc-ag)+3\sqrt{a}(5be-3ai)\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}} \\
&\quad +\frac{(2bd-ah)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)}{4b}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{f\sqrt{a+bx^4}}{2b} + \frac{gx\sqrt{a+bx^4}}{3b} + \frac{hx^2\sqrt{a+bx^4}}{4b} + \frac{ix^3\sqrt{a+bx^4}}{5b} \\
&+ \frac{(5be-3ai)x\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{(2bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}} \\
&- \frac{\sqrt[4]{a}(5be-3ai)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}} \\
&+ \frac{\left(5\sqrt{b}(3bc-ag)+3\sqrt{a}(5be-3ai)\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.23 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.73

$$
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx
$$

$$
= \frac{30a\sqrt{b}f+20a\sqrt{b}gx+15a\sqrt{b}hx^2+12a\sqrt{b}ix^3+30b^{3/2}fx^4+20b^{3/2}gx^5+15b^{3/2}hx^6+12b^{3/2}ix^7+30bd\sqrt{a}}{}
$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/Sqrt[a + b*x^4]
,x]

[Out] (30*a*Sqrt[b]*f + 20*a*Sqrt[b]*g*x + 15*a*Sqrt[b]*h*x^2 + 12*a*Sqrt[b]*i*x^
3 + 30*b^(3/2)*f*x^4 + 20*b^(3/2)*g*x^5 + 15*b^(3/2)*h*x^6 + 12*b^(3/2)*i*x
^7 + 30*b*d*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 15*a*h
*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*Sqrt[b]*(-3*b*
c + a*g)*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a
)] + 4*Sqrt[b]*(5*b*e - 3*a*i)*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/
2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^(3/2)*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.75

method	result
elliptic	$\frac{i\,x^{3}\sqrt{b\,x^{4}+a}}{5b}+\frac{h\,x^{2}\sqrt{b\,x^{4}+a}}{4b}+\frac{gx\sqrt{b\,x^{4}+a}}{3b}+\frac{f\sqrt{b\,x^{4}+a}}{2b}+\frac{\left(c-\frac{ag}{3b}\right)\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{\left(d-\frac{ah}{2b}\right)\ln\left(2\right.}{}$
risch	$\frac{\left(12i\,x^{3}+15h\,x^{2}+20gx+30f\right)\sqrt{b\,x^{4}+a}}{60b}-\frac{\frac{10ag\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{30bc\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{i(18a}{}}{}$
default	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+i\left(\frac{x^{3}\sqrt{b\,x^{4}+a}}{5b}-\frac{3ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)$

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/5*i*x^3*(b*x^4+a)^(1/2)/b+1/4*h*x^2*(b*x^4+a)^(1/2)/b+1/3*g*x*(b*x^4+a)^(
1/2)/b+1/2*f*(b*x^4+a)^(1/2)/b+(c-1/3*a/b*g)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I
/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)
*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*(d-1/2*a/b*h)*ln(2*x^2*b^(1/2
)+2*(b*x^4+a)^(1/2))/b^(1/2)+I*(e-3/5*a/b*i)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1
/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+
a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a
^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.53

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$$
$$=\frac{24\,(5\,abe-3\,a^2i)\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+8\,(15\,b^2c-15\,abe-5\,abg+9\,a^2i)\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(a}{}$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorith
m="fricas")
```

```
[Out] 1/120*(24*(5*a*b*e - 3*a^2*i)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/
b)^(1/4)/x), -1) + 8*(15*b^2*c - 15*a*b*e - 5*a*b*g + 9*a^2*i)*sqrt(b)*x*(-
a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) - 15*(2*a*b*d - a^2*h)*sq
rt(b)*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(12*a*b*i*x^4
 + 15*a*b*h*x^3 + 20*a*b*g*x^2 + 30*a*b*f*x + 60*a*b*e - 36*a^2*i)*sqrt(b*x
^4 + a))/(a*b^2*x)
```

Sympy [A] (verification not implemented)

Time = 3.29 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.68

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{\sqrt{a}hx^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{ah\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + f\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right)$$
$$+ \frac{d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$
$$+ \frac{gx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{ix^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*h*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*h*asinh(sqrt(b)*x**2/sqrt(a))/(
4*b**(3/2)) + f*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(
2*b), True)) + d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*h
yper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) +
 e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(7/4)) + g*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp
_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + i*x**7*gamma(7/4)*hyper((1/2, 7/4)
, (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx = \int \frac{ix^6+hx^5+gx^4+fx^3+ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a)
, x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx=\int \frac{ix^6+hx^5+gx^4+fx^3+ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$$
$$=\int \frac{i\,x^6+h\,x^5+g\,x^4+f\,x^3+e\,x^2+d\,x+c}{\sqrt{b\,x^4+a}}\,dx$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^(1/2),x)
```

```
[Out] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^(1/2), x)
```

3.221 $\int \frac{1+x}{1+x^5} dx$

Optimal result

Integrand size = 11, antiderivative size = 109

$$\int \frac{1+x}{1+x^5} dx = -\frac{1}{5}\sqrt[5]{-1}(1+\sqrt[5]{-1}) \log \left(\sqrt[5]{-1}-x\right) + \frac{1}{5}(-1)^{4/5} \left(1 -(-1)^{4/5}\right) \log \left(-(-1)^{4/5}-x\right)+\frac{1}{5}(-1)^{2/5} \left(1-(-1)^{2/5}\right) \log \left((-1)^{2/5}+x\right)-\frac{1}{5}(-1)^{3/5} \left(1+(-1)^{3/5}\right) \log \left(-(-\right.$$

```
[Out] -1/5*(-1)^(1/5)*(1+(-1)^(1/5))*ln((-1)^(1/5)-x)+1/5*(-1)^(4/5)*(1-(-1)^(4/5
))*ln(-(-1)^(4/5)-x)+1/5*(-1)^(2/5)*(1-(-1)^(2/5))*ln((-1)^(2/5)+x)-1/5*(-1
)^(3/5)*(1+(-1)^(3/5))*ln(-(-1)^(3/5)+x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1600, 2093}

$$\int \frac{1+x}{1+x^5} dx = -\frac{1}{5}\sqrt[5]{-1}(1+\sqrt[5]{-1}) \log \left(\sqrt[5]{-1}-x\right) + \frac{1}{5}(-1)^{4/5} \left(1 -(-1)^{4/5}\right) \log \left(-x-(-1)^{4/5}\right)+\frac{1}{5}(-1)^{2/5} \left(1-(-1)^{2/5}\right) \log \left(x+(-1)^{2/5}\right)-\frac{1}{5}(-1)^{3/5} \left(1+(-1)^{3/5}\right) \log \left(x-(\right.$$

```
[In] Int[(1 + x)/(1 + x^5),x]
```

```
[Out] -1/5*((-1)^(1/5)*(1 + (-1)^(1/5))*Log[(-1)^(1/5) - x]) + ((-1)^(4/5)*(1 - (
-1)^(4/5))*Log[-(-1)^(4/5) - x])/5 + ((-1)^(2/5)*(1 - (-1)^(2/5))*Log[(-1)^
(2/5) + x])/5 - ((-1)^(3/5)*(1 + (-1)^(3/5))*Log[-(-1)^(3/5) + x])/5
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2093

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[1/a^
(3*p), Int[ExpandIntegrand[1/((a - b*x)^p/(a^5 - b^5*x^5)^p), x], x], x] /;
 NeQ[a, 0] && EqQ[c, b^2/a] && EqQ[d, b^3/a^2] && EqQ[e, b^4/a^3]] /; FreeQ
[p, x] && PolyQ[P4, x, 4] && ILtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \frac{1}{1-x+x^2-x^3+x^4}\,dx$$

$$= \int \left(\frac{-1+(-1)^{4/5}}{5\left(-1+\sqrt[5]{-1}x\right)}+\frac{-1-(-1)^{3/5}}{5\left(-1-(-1)^{2/5}x\right)}+\frac{-1+(-1)^{2/5}}{5\left(-1+(-1)^{3/5}x\right)}+\frac{-1-\sqrt[5]{-1}}{5\left(-1-(-1)^{4/5}x\right)}\right)dx$$

$$= -\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(\sqrt[5]{-1}-x\right)+\frac{1}{5}(-1)^{4/5}\left(1 -(-1)^{4/5}\right)\log\left(-(-1)^{4/5}-x\right)+\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left((-1)^{2/5}+x\right)-\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$\int \frac{1+x}{1+x^5}\,dx = \text{RootSum}\left[1-\#1+\#1^2-\#1^3+\#1^4\&,\frac{\log(x-\#1)}{-1+2\#1-3\#1^2+4\#1^3}\&\right]$$

```
[In] Integrate[(1 + x)/(1 + x^5),x]
[Out] RootSum[1 - #1 + #1^2 - #1^3 + #1^4 & , Log[x - #1]/(-1 + 2*#1 - 3*#1^2 + 4
*#1^3) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.49 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.41

method	result
risch	$\sum_{\_R=\mathrm{RootOf}(\_Z^4-\_Z^3+\_Z^2-\_Z+1)} \frac{\ln(x-\_R)}{4\_R^3-3\_R^2+2\_R-1}$
default	$\frac{\sqrt{5}\ln\left(x\sqrt{5}+2x^2-x+2\right)}{10}+\frac{2\left(-\frac{\sqrt{5}\left(\sqrt{5}-1\right)}{2}+5-\sqrt{5}\right)\arctan\left(\frac{\sqrt{5}+4x-1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-\frac{\sqrt{5}\ln\left(-x\sqrt{5}+2x^2-x+2\right)}{10}-\frac{2\left(-\frac{\sqrt{5}\left(-\sqrt{5}-1\right)}{2}-\sqrt{5}\right.}{5\sqrt{}}$
meijerg	$\frac{x\ln\left(1+\left(x^5\right)^{\frac{1}{5}}\right)}{5\left(x^5\right)^{\frac{1}{5}}}-\frac{x\cos\left(\frac{\pi}{5}\right)\ln\left(1-2\cos\left(\frac{\pi}{5}\right)\left(x^5\right)^{\frac{1}{5}}+\left(x^5\right)^{\frac{2}{5}}\right)}{5\left(x^5\right)^{\frac{1}{5}}}+\frac{2x\sin\left(\frac{\pi}{5}\right)\arctan\left(\frac{\sin\left(\frac{\pi}{5}\right)\left(x^5\right)^{\frac{1}{5}}}{1-\cos\left(\frac{\pi}{5}\right)\left(x^5\right)^{\frac{1}{5}}}\right)}{5\left(x^5\right)^{\frac{1}{5}}}+\frac{x\cos\left(\frac{2\pi}{5}\right)\ln\left(1+2\cos\left(\frac{2}{}\right.\right.}{5\left(x^5\right)^{\frac{1}{}}}$

```
[In] int((1+x)/(x^5+1),x,method=_RETURNVERBOSE)
```

```
[Out] sum(1/(4*_R^3-3*_R^2+2*_R-1)*ln(x-_R),_R=RootOf(_Z^4-_Z^3+_Z^2-_Z+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(73) = 146$.

Time = 0.99 (sec) , antiderivative size = 835, normalized size of antiderivative = 7.66

$$\int \frac{1+x}{1+x^5}\,dx = \text{Too large to display}$$

```
[In] integrate((1+x)/(x^5+1),x, algorithm="fricas")
```

```
[Out] -1/10*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))*log(3/8*(sqrt(5) + 5*sqrt(-2/
25*sqrt(5) - 1/5))^3 + 1/8*(3*sqrt(5) + 15*sqrt(-2/25*sqrt(5) - 1/5) + 8)*(
sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2 + 3/8*((sqrt(5) + 5*sqrt(-2/25*sqr
t(5) - 1/5))^2 - 12)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 11*x + 1) -
1/10*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))*log(-3/8*(sqrt(5) + 5*sqrt(-2/
25*sqrt(5) - 1/5))^3 + (sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 + 11*x - 9
/2*sqrt(5) - 45/2*sqrt(-2/25*sqrt(5) - 1/5) - 14) + 1/10*(sqrt(5) + 5*sqrt(
-3/100*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*(sqrt(5) + 5*sqrt(-
2/25*sqrt(5) - 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 3/100*(sqrt(
5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2))*log(-1/8*(3*sqrt(5) + 15*sqrt(-2/25*s
qrt(5) - 1/5) + 8)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - (sqrt(5) + 5
*sqrt(-2/25*sqrt(5) - 1/5))^2 - 3/8*((sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5)
)^2 - 12)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 5/4*sqrt(-3/100*(sqrt(5
) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) -
 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 3/100*(sqrt(5) - 5*sqrt(-2
/25*sqrt(5) - 1/5))^2)*((3*sqrt(5) + 15*sqrt(-2/25*sqrt(5) - 1/5) + 8)*(sqr
t(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 8*sqrt(5) + 40*sqrt(-2/25*sqrt(5) - 1
/5) + 36) + 22*x + 9/2*sqrt(5) + 45/2*sqrt(-2/25*sqrt(5) - 1/5) + 2) + 1/10
*(sqrt(5) - 5*sqrt(-3/100*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*
(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1
```

```
/5)) - 3/100*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2))*log(-1/8*(3*sqrt(5
) + 15*sqrt(-2/25*sqrt(5) - 1/5) + 8)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5
))^2 - (sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 3/8*((sqrt(5) + 5*sqrt(-
2/25*sqrt(5) - 1/5))^2 - 12)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 5/4*
sqrt(-3/100*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*(sqrt(5) + 5*s
qrt(-2/25*sqrt(5) - 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 3/100*(
sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2)*((3*sqrt(5) + 15*sqrt(-2/25*sqrt(
5) - 1/5) + 8)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 8*sqrt(5) + 40*sqr
t(-2/25*sqrt(5) - 1/5) + 36) + 22*x + 9/2*sqrt(5) + 45/2*sqrt(-2/25*sqrt(5)
 - 1/5) + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1287 vs. 2(109) = 218.

Time = 0.58 (sec) , antiderivative size = 1287, normalized size of antiderivative = 11.81

$$\int \frac{1+x}{1+x^5}\,dx = \text{Too large to display}$$

```
[In] integrate((1+x)/(x**5+1),x)
```

```
[Out] sqrt(5)*log(x**2 + x*(-48/11 - 21*sqrt(5)/11 + 4*sqrt(10)*sqrt(sqrt(5) + 3)
/11 + 45*sqrt(2)*sqrt(sqrt(5) + 3)/22) - 1381*sqrt(10)*sqrt(sqrt(5) + 3)/48
4 - 3045*sqrt(2)*sqrt(sqrt(5) + 3)/484 + 2213*sqrt(5)/242 + 5217/242)/10 -
sqrt(5)*log(x**2 + x*(-48/11 - 45*sqrt(2)*sqrt(3 - sqrt(5))/22 + 4*sqrt(10)
*sqrt(3 - sqrt(5))/11 + 21*sqrt(5)/11) - 2213*sqrt(5)/242 - 1381*sqrt(10)*s
qrt(3 - sqrt(5))/484 + 3045*sqrt(2)*sqrt(3 - sqrt(5))/484 + 5217/242)/10 +
2*sqrt(-sqrt(10)*sqrt(3 - sqrt(5))/50 + 3/20)*atan(44*x/(-8*sqrt(5)*sqrt(-2
*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sq
rt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15
)) - 96/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*s
qrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt
(10)*sqrt(3 - sqrt(5)) + 15)) - 45*sqrt(2)*sqrt(3 - sqrt(5))/(-8*sqrt(5)*sq
rt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(
-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5))
 + 15)) + 8*sqrt(10)*sqrt(3 - sqrt(5))/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3
- sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - s
qrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15)) + 42*sqrt(5)/(
-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - s
qrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt
(3 - sqrt(5)) + 15))) + 2*sqrt(-sqrt(10)*sqrt(sqrt(5) + 3)/50 + 3/20)*atan(
44*x/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(
10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)
*sqrt(sqrt(5) + 3) + 15)) - 96/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3
) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqr
```

```
t(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15)) - 42*sqrt(5)/(8*sqrt(5)
*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqrt(5
) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) +
 3) + 15)) + 8*sqrt(10)*sqrt(sqrt(5) + 3)/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(
sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(1
0)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15)) + 45*sqrt(2)
*sqrt(sqrt(5) + 3)/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18
*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sq
rt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15)))
```

Maxima [F]

$$\int \frac{1+x}{1+x^5}\,dx = \int \frac{x+1}{x^5+1}\,dx$$

```
[In] integrate((1+x)/(x^5+1),x, algorithm="maxima")
[Out] integrate((x + 1)/(x^5 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{1+x}{1+x^5}\,dx &= \frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)\\ &\quad+\frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)\\ &\quad-\frac{1}{10}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)+\frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)\end{aligned}$$

```
[In] integrate((1+x)/(x^5+1),x, algorithm="giac")
[Out] 1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) +
 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10))
- 1/10*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/10*sqrt(5)*log(x^2 +
1/2*x*(sqrt(5) - 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \frac{1+x}{1+x^5} \, dx = \sum_{k=1}^{4} \ln \left(\operatorname{root}\left(z^4 - \frac{z}{25} + \frac{1}{125}, z, k\right) \left(-4x + \operatorname{root}\left(z^4 - \frac{z}{25} + \frac{1}{125}, z, k\right) \left(25 \operatorname{root}\left(z^4 - \frac{z}{25} + \frac{1}{125}, z, k\right) + 15x - 15\right) + 1\right)\right) \operatorname{root}\left(z^4 - \frac{z}{25} + \frac{1}{125}, z, k\right)$$

```
[In] int((x + 1)/(x^5 + 1),x)
```

```
[Out] symsum(log(root(z^4 - z/25 + 1/125, z, k)*(root(z^4 - z/25 + 1/125, z, k)*(
25*root(z^4 - z/25 + 1/125, z, k) + 15*x - 15) - 4*x + 1))*root(z^4 - z/25
+ 1/125, z, k), k, 1, 4)
```

3.222 $\int \frac{1-x}{1-x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \frac{1-x}{1-x^5} \, dx = -\frac{1}{5}(-1)^{2/5} \left(1 -(-1)^{2/5}\right) \log \left((-1)^{2/5}-x\right)+\frac{1}{5}(-1)^{3/5} \left(1+(-1)^{3/5}\right) \log \left(-(-1)^{3/5}-x\right)+\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right) \log \left(\sqrt[5]{-1}+x\right)$$

```
[Out] -1/5*(-1)^(2/5)*(1-(-1)^(2/5))*ln((-1)^(2/5)-x)+1/5*(-1)^(3/5)*(1+(-1)^(3/5
))*ln(-(-1)^(3/5)-x)+1/5*(-1)^(1/5)*(1+(-1)^(1/5))*ln((-1)^(1/5)+x)-1/5*(-1
)^(4/5)*(1-(-1)^(4/5))*ln(-(-1)^(4/5)+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1600, 2093}

$$\int \frac{1-x}{1-x^5} \, dx = -\frac{1}{5}(-1)^{2/5} \left(1 -(-1)^{2/5}\right) \log \left((-1)^{2/5}-x\right)+\frac{1}{5}(-1)^{3/5} \left(1+(-1)^{3/5}\right) \log \left(-x-(-1)^{3/5}\right)+\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right) \log \left(x+\sqrt[5]{-1}\right)$$

```
[In] Int[(1 - x)/(1 - x^5),x]
```

```
[Out] -1/5*((-1)^(2/5)*(1 - (-1)^(2/5))*Log[(-1)^(2/5) - x]) + ((-1)^(3/5)*(1 + (
-1)^(3/5))*Log[-(-1)^(3/5) - x])/5 + ((-1)^(1/5)*(1 + (-1)^(1/5))*Log[(-1)^
(1/5) + x])/5 - ((-1)^(4/5)*(1 - (-1)^(4/5))*Log[-(-1)^(4/5) + x])/5
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2093

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[1/a^
(3*p), Int[ExpandIntegrand[1/((a - b*x)^p/(a^5 - b^5*x^5)^p), x], x], x] /;
 NeQ[a, 0] && EqQ[c, b^2/a] && EqQ[d, b^3/a^2] && EqQ[e, b^4/a^3]] /; FreeQ
[p, x] && PolyQ[P4, x, 4] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{1+x+x^2+x^3+x^4}\,dx\\ &= \int \left(\frac{1-(-1)^{4/5}}{5\left(1+\sqrt[5]{-1}x\right)}+\frac{1+(-1)^{3/5}}{5\left(1-(-1)^{2/5}x\right)}+\frac{1-(-1)^{2/5}}{5\left(1+(-1)^{3/5}x\right)}+\frac{1+\sqrt[5]{-1}}{5\left(1-(-1)^{4/5}x\right)}\right)dx\\ &= -\frac{1}{5}(-1)^{2/5}\left(1\right.\\ &\quad\left.-(-1)^{2/5}\right)\log\left((-1)^{2/5}-x\right)+\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\log\left(-(-1)^{3/5}-x\right)+\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(\sqrt{}\right.\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.43

$$\int \frac{1-x}{1-x^5}\,dx = \text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\&,\frac{\log(x-\#1)}{1+2\#1+3\#1^2+4\#1^3}\&\right]$$

```
[In] Integrate[(1 - x)/(1 - x^5),x]
[Out] RootSum[1 + #1 + #1^2 + #1^3 + #1^4 & , Log[x - #1]/(1 + 2*#1 + 3*#1^2 + 4*
#1^3) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.38

method	result
risch	$\sum_{\_R=\text{RootOf}(\_Z^4+\_Z^3+\_Z^2+\_Z+1)} \frac{\ln(x-\_R)}{4\_R^3+3\_R^2+2\_R+1}$
default	$-\frac{\sqrt{5}\,\ln\left(-x\sqrt{5}+2x^2+x+2\right)}{10}-\frac{2\left(-\frac{\sqrt{5}\,(-\sqrt{5}+1)}{2}+\sqrt{5}-5\right)\arctan\left(\frac{-\sqrt{5}+4x+1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}+\frac{\sqrt{5}\,\ln\left(x\sqrt{5}+2x^2+x+2\right)}{10}+\frac{2\left(-\frac{\sqrt{5}\,(\sqrt{5}+1)}{2}\right.}{}$
meijerg	$-\frac{x\left(\ln\left(1-(x^5)^{\frac{1}{5}}\right)+\cos\left(\frac{2\pi}{5}\right)\ln\left(1-2\cos\left(\frac{2\pi}{5}\right)(x^5)^{\frac{1}{5}}+(x^5)^{\frac{2}{5}}\right)-2\sin\left(\frac{2\pi}{5}\right)\arctan\left(\frac{\sin\left(\frac{2\pi}{5}\right)(x^5)^{\frac{1}{5}}}{1-\cos\left(\frac{2\pi}{5}\right)(x^5)^{\frac{1}{5}}}\right)-\cos\left(\frac{\pi}{5}\right)\ln\left(1+2\cos\left(\frac{\pi}{5}\right)(x^5)\right.\right.}{5(x^5)^{\frac{1}{5}}}$

```
[In] int((1-x)/(-x^5+1),x,method=_RETURNVERBOSE)
```

```
[Out] sum(1/(4*_R^3+3*_R^2+2*_R+1)*ln(x-_R),_R=RootOf(_Z^4+_Z^3+_Z^2+_Z+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(73) = 146$.

Time = 1.02 (sec) , antiderivative size = 799, normalized size of antiderivative = 7.33

$$\int \frac{1-x}{1-x^5}\,dx = \text{Too large to display}$$

```
[In] integrate((1-x)/(-x^5+1),x, algorithm="fricas")
```

```
[Out] -1/10*(sqrt(5) - sqrt(2*sqrt(5) - 5))*log(3/8*(sqrt(5) + sqrt(2*sqrt(5) - 5
))^3 + 1/8*(3*sqrt(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(5
) - 5))^2 + 3/8*((sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 12)*(sqrt(5) - sqrt(2*
sqrt(5) - 5)) + 11*x - 1) - 1/10*(sqrt(5) + sqrt(2*sqrt(5) - 5))*log(-3/8*(
sqrt(5) + sqrt(2*sqrt(5) - 5))^3 - (sqrt(5) + sqrt(2*sqrt(5) - 5))^2 + 11*x
 - 9/2*sqrt(5) - 9/2*sqrt(2*sqrt(5) - 5) + 14) + 1/10*(sqrt(5) + 5*sqrt(-3/
100*(sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 1/50*(sqrt(5) + sqrt(2*sqrt(5) - 5)
)*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 3/100*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2
))*log(-1/8*(3*sqrt(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(
5) - 5))^2 + (sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 3/8*((sqrt(5) + sqrt(2*sqr
t(5) - 5))^2 - 12)*(sqrt(5) - sqrt(2*sqrt(5) - 5)) + 5/4*sqrt(-3/100*(sqrt(
5) + sqrt(2*sqrt(5) - 5))^2 - 1/50*(sqrt(5) + sqrt(2*sqrt(5) - 5))*(sqrt(5)
 - sqrt(2*sqrt(5) - 5)) - 3/100*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2)*((3*sqrt
(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 8*sqrt(5
) - 8*sqrt(2*sqrt(5) - 5) + 36) + 22*x + 9/2*sqrt(5) + 9/2*sqrt(2*sqrt(5) -
 5) - 2) + 1/10*(sqrt(5) - 5*sqrt(-3/100*(sqrt(5) + sqrt(2*sqrt(5) - 5))^2
- 1/50*(sqrt(5) + sqrt(2*sqrt(5) - 5))*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 3/
100*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2))*log(-1/8*(3*sqrt(5) + 3*sqrt(2*sqrt
(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2 + (sqrt(5) + sqrt(2*sqrt(5)
 - 5))^2 - 3/8*((sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 12)*(sqrt(5) - sqrt(2*s
```

```
qrt(5) - 5)) - 5/4*sqrt(-3/100*(sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 1/50*(sq
rt(5) + sqrt(2*sqrt(5) - 5))*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 3/100*(sqrt(
5) - sqrt(2*sqrt(5) - 5))^2)*((3*sqrt(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt
(5) - sqrt(2*sqrt(5) - 5)) - 8*sqrt(5) - 8*sqrt(2*sqrt(5) - 5) + 36) + 22*x
 + 9/2*sqrt(5) + 9/2*sqrt(2*sqrt(5) - 5) - 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1287 vs. $2(109) = 218$.

Time = 0.55 (sec) , antiderivative size = 1287, normalized size of antiderivative = 11.81

$$\int \frac{1-x}{1-x^5}\,dx = \text{Too large to display}$$

```
[In] integrate((1-x)/(-x**5+1),x)
```

```
[Out] sqrt(5)*log(x**2 + x*(-21*sqrt(5)/11 - 4*sqrt(10)*sqrt(3 - sqrt(5))/11 + 45
*sqrt(2)*sqrt(3 - sqrt(5))/22 + 48/11) - 2213*sqrt(5)/242 - 1381*sqrt(10)*s
qrt(3 - sqrt(5))/484 + 3045*sqrt(2)*sqrt(3 - sqrt(5))/484 + 5217/242)/10 -
sqrt(5)*log(x**2 + x*(-45*sqrt(2)*sqrt(sqrt(5) + 3)/22 - 4*sqrt(10)*sqrt(sq
rt(5) + 3)/11 + 21*sqrt(5)/11 + 48/11) - 1381*sqrt(10)*sqrt(sqrt(5) + 3)/48
4 - 3045*sqrt(2)*sqrt(sqrt(5) + 3)/484 + 2213*sqrt(5)/242 + 5217/242)/10 +
2*sqrt(-sqrt(10)*sqrt(3 - sqrt(5))/50 + 3/20)*atan(44*x/(-8*sqrt(5)*sqrt(-2
*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sq
rt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15
)) - 42*sqrt(5)/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sq
rt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt
(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15)) - 8*sqrt(10)*sqrt(3 - sqrt(5))/(-8*sq
rt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5
))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 -
sqrt(5)) + 15)) + 45*sqrt(2)*sqrt(3 - sqrt(5))/(-8*sqrt(5)*sqrt(-2*sqrt(10)
*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sq
rt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15)) + 96/(
-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - s
qrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt
(3 - sqrt(5)) + 15))) + 2*sqrt(-sqrt(10)*sqrt(sqrt(5) + 3)/50 + 3/20)*atan(
44*x/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(
10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)
*sqrt(sqrt(5) + 3) + 15)) - 45*sqrt(2)*sqrt(sqrt(5) + 3)/(8*sqrt(5)*sqrt(-2
*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) +
 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15
)) - 8*sqrt(10)*sqrt(sqrt(5) + 3)/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5)
+ 3) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(
sqrt(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15)) + 42*sqrt(5)/(8*sqrt
(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqr
```

```
t(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5
) + 3) + 15)) + 96/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18
*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sq
rt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15)))
```

Maxima [F]

$$\int \frac{1-x}{1-x^5}\,dx = \int \frac{x-1}{x^5-1}\,dx$$

```
[In] integrate((1-x)/(-x^5+1),x, algorithm="maxima")
[Out] integrate((x - 1)/(x^5 - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int \frac{1-x}{1-x^5}\,dx = \frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\sqrt{5}+10}}\right) + \frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{-2\sqrt{5}+10}}\right) + \frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right) - \frac{1}{10}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)$$

```
[In] integrate((1-x)/(-x^5+1),x, algorithm="giac")
[Out] 1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x - sqrt(5) + 1)/sqrt(2*sqrt(5) + 10)) +
 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x + sqrt(5) + 1)/sqrt(-2*sqrt(5) + 10))
+ 1/10*sqrt(5)*log(x^2 + 1/2*x*(sqrt(5) + 1) + 1) - 1/10*sqrt(5)*log(x^2 -
1/2*x*(sqrt(5) - 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

$$\int \frac{1-x}{1-x^5}\,dx = \sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)\left(4x + \mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)\left(25\,\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)+15x+15\right)+1\right)\right)\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)$$

```
[In] int((x - 1)/(x^5 - 1),x)
```

```
[Out] symsum(log(-root(z^4 + z/25 + 1/125, z, k)*(4*x + root(z^4 + z/25 + 1/125,
z, k)*(25*root(z^4 + z/25 + 1/125, z, k) + 15*x + 15) + 1))*root(z^4 + z/25
 + 1/125, z, k), k, 1, 4)
```

3.223 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 208

$$\begin{aligned}\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx =\ & \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^6} \\ & - \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^6}{6b^5} \\ & + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^9}{9b^4} \\ & + \frac{(b^2d-abe+a^2f)\,x^{12}}{12b^3} + \frac{(be-af)x^{15}}{15b^2} + \frac{fx^{18}}{18b} \\ & - \frac{a^3(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3b^7}\end{aligned}$$

```
[Out] 1/3*a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^3/b^6-1/6*a*(-a^3*f+a^2*b*e-a*b^2*
d+b^3*c)*x^6/b^5+1/9*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^9/b^4+1/12*(a^2*f-a*b
*e+b^2*d)*x^12/b^3+1/15*(-a*f+b*e)*x^15/b^2+1/18*f*x^18/b-1/3*a^3*(-a^3*f+a
^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/b^7
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used

$= \{1835, 1634\}$

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{x^{12}(a^2f-abe+b^2d)}{12b^3} - \frac{a^3\log(a+bx^3)(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^7} + \frac{a^2x^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^6} - \frac{ax^6(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5} + \frac{x^9(a^3(-f)+a^2be-ab^2d+b^3c)}{9b^4} + \frac{x^{15}(be-af)}{15b^2} + \frac{fx^{18}}{18b}$$

```
[In] Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^6) - (a*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^6)/(6*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9
)/(9*b^4) + ((b^2*d - a*b*e + a^2*f)*x^12)/(12*b^3) + ((b*e - a*f)*x^15)/(1
5*b^2) + (f*x^18)/(18*b) - (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a +
 b*x^3])/(3*b^7)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)}{b^6} + \frac{a(-b^3c+ab^2d-a^2be+a^3f)\,x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{b^4} + \frac{(b^2d-abe+a^2f)\,x^3}{b^3} + \frac{(be-af)x^4}{b^2} + \frac{fx^5}{b} + \frac{a^3(-b^3c+ab^2d-a^2be+a^3f)}{b^6(a+bx)}\right) dx, x, x^3\right)$$

$$
\begin{aligned}
&= \frac{a^2(b^3c - ab^2d + a^2be - a^3f)\,x^3}{3b^6} - \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^6}{6b^5} \\
&\quad + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^9}{9b^4} + \frac{(b^2d - abe + a^2f)\,x^{12}}{12b^3} \\
&\quad + \frac{(be - af)x^{15}}{15b^2} + \frac{fx^{18}}{18b} - \frac{a^3(b^3c - ab^2d + a^2be - a^3f)\log(a + bx^3)}{3b^7}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.90

$$
\int \frac{x^{11}(c + dx^3 + ex^6 + fx^9)}{a + bx^3}\,dx
= \frac{bx^3(-60a^5f + 30a^4b(2e + fx^3) - 10a^3b^2(6d + 3ex^3 + 2fx^6) + 5a^2b^3(12c + 6dx^3 + 4ex^6 + 3fx^9) + b^5x^6(20
$$

```
[In] Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(-60*a^5*f + 30*a^4*b*(2*e + f*x^3) - 10*a^3*b^2*(6*d + 3*e*x^3 + 2*
f*x^6) + 5*a^2*b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9) + b^5*x^6*(20*c + 1
5*d*x^3 + 12*e*x^6 + 10*f*x^9) - a*b^4*x^3*(30*c + 20*d*x^3 + 15*e*x^6 + 12
*f*x^9)) + 60*a^3*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a + b*x^3])/(1
80*b^7)
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.94

method	result
norman	$-\frac{(f a^3-a^2be+a b^2d-b^3c)x^9}{9b^4}-\frac{(af-be)x^{15}}{15b^2}+\frac{f x^{18}}{18b}+\frac{(a^2f-aeb+b^2d)x^{12}}{12b^3}+\frac{a(f a^3-a^2be+a b^2d-b^3c)x^6}{6b^5}-\frac{a^2(f a^3-$
default	$-\frac{-\frac{1}{6}f x^{18}b^5+\frac{1}{5}a b^4 f x^{15}-\frac{1}{5}b^5 e x^{15}-\frac{1}{4}a^2b^3 f x^{12}+\frac{1}{4}a b^4 e x^{12}-\frac{1}{4}b^5 d x^{12}+\frac{1}{3}a^3b^2 f x^9-\frac{1}{3}a^2b^3 e x^9+\frac{1}{3}a b^4 d x^9-\frac{1}{3}b^5 c x^9-\frac{1}{2}f x^6 a}{3b^6}$
parallelrisch	$10f x^{18}b^6-12x^{15}a b^5f+12x^{15}b^6e+15x^{12}a^2b^4f-15x^{12}a b^5e+15x^{12}b^6d-20x^9a^3b^3f+20x^9a^2b^4e-20x^9a b^5d+20x^9b^6c+30x^6a^4b$
risch	$\frac{f x^{18}}{18b}-\frac{af x^{15}}{15b^2}+\frac{e x^{15}}{15b}+\frac{a^2f x^{12}}{12b^3}-\frac{ae x^{12}}{12b^2}+\frac{d x^{12}}{12b}-\frac{a^3f x^9}{9b^4}+\frac{a^2e x^9}{9b^3}-\frac{ad x^9}{9b^2}+\frac{c x^9}{9b}+\frac{f x^6a^4}{6b^5}-\frac{a^3e x^6}{6b^4}+$

```
[In] int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/b^4*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)*x^9-1/15/b^2*(a*f-b*e)*x^15+1/18*f*x
^18/b+1/12*(a^2*f-a*b*e+b^2*d)*x^12/b^3+1/6*a/b^5*(a^3*f-a^2*b*e+a*b^2*d-b^
3*c)*x^6-1/3*a^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^6*x^3+1/3*a^3*(a^3*f-a^2*b
*e+a*b^2*d-b^3*c)/b^7*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.01

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{10\,b^6fx^{18}+12\,(b^6e-ab^5f)x^{15}+15\,(b^6d-ab^5e+a^2b^4f)x^{12}+20\,(b^6c-ab^5d+a^2b^4e-a^3b^3f)x^9-30\,(a}{}$$

[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")

[Out] 1/180*(10*b^6*f*x^18 + 12*(b^6*e - a*b^5*f)*x^15 + 15*(b^6*d - a*b^5*e + a^2*b^4*f)*x^12 + 20*(b^6*c - a*b^5*d + a^2*b^4*e - a^3*b^3*f)*x^9 - 30*(a*b^5*c - a^2*b^4*d + a^3*b^3*e - a^4*b^2*f)*x^6 + 60*(a^2*b^4*c - a^3*b^3*d + a^4*b^2*e - a^5*b*f)*x^3 - 60*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*log(b*x^3 + a))/b^7

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx &= \frac{a^3(a^3f-a^2be+ab^2d-b^3c)\log{(a+bx^3)}}{3b^7} \\ &\quad + x^{15}\left(-\frac{af}{15b^2}+\frac{e}{15b}\right)+x^{12}\left(\frac{a^2f}{12b^3}-\frac{ae}{12b^2}+\frac{d}{12b}\right) \\ &\quad + x^9\left(-\frac{a^3f}{9b^4}+\frac{a^2e}{9b^3}-\frac{ad}{9b^2}+\frac{c}{9b}\right) \\ &\quad + x^6\left(\frac{a^4f}{6b^5}-\frac{a^3e}{6b^4}+\frac{a^2d}{6b^3}-\frac{ac}{6b^2}\right) \\ &\quad + x^3\left(-\frac{a^5f}{3b^6}+\frac{a^4e}{3b^5}-\frac{a^3d}{3b^4}+\frac{a^2c}{3b^3}\right)+\frac{fx^{18}}{18b}\end{aligned}$$

[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)

[Out] a**3*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**7) + x**15*(-a*f/(15*b**2) + e/(15*b)) + x**12*(a**2*f/(12*b**3) - a*e/(12*b**2) + d/(12*b)) + x**9*(-a**3*f/(9*b**4) + a**2*e/(9*b**3) - a*d/(9*b**2) + c/(9*b)) + x**6*(a**4*f/(6*b**5) - a**3*e/(6*b**4) + a**2*d/(6*b**3) - a*c/(6*b**2)) + x**3*(-a**5*f/(3*b**6) + a**4*e/(3*b**5) - a**3*d/(3*b**4) + a**2*c/(3*b**3)) + f*x**18/(18*b)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{10\,b^5fx^{18}+12\,(b^5e-ab^4f)x^{15}+15\,(b^5d-ab^4e+a^2b^3f)x^{12}+20\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^9-30\,(ab^4}{180\,b^6}$$
$$-\frac{(a^3b^3c-a^4b^2d+a^5be-a^6f)\log\left(bx^3+a\right)}{3\,b^7}$$

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/180*(10*b^5*f*x^18 + 12*(b^5*e - a*b^4*f)*x^15 + 15*(b^5*d - a*b^4*e + a^
2*b^3*f)*x^12 + 20*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^9 - 30*(a*b^
4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^6 + 60*(a^2*b^3*c - a^3*b^2*d + a^
4*b*e - a^5*f)*x^3)/b^6 - 1/3*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*log
(b*x^3 + a)/b^7
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.15

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{10\,b^5fx^{18}+12\,b^5ex^{15}-12\,ab^4fx^{15}+15\,b^5dx^{12}-15\,ab^4ex^{12}+15\,a^2b^3fx^{12}+20\,b^5cx^9-20\,ab^4dx^9+20\,a^2b}{}$$
$$-\frac{(a^3b^3c-a^4b^2d+a^5be-a^6f)\log\left(|bx^3+a|\right)}{3\,b^7}$$

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/180*(10*b^5*f*x^18 + 12*b^5*e*x^15 - 12*a*b^4*f*x^15 + 15*b^5*d*x^12 - 15
*a*b^4*e*x^12 + 15*a^2*b^3*f*x^12 + 20*b^5*c*x^9 - 20*a*b^4*d*x^9 + 20*a^2*
b^3*e*x^9 - 20*a^3*b^2*f*x^9 - 30*a*b^4*c*x^6 + 30*a^2*b^3*d*x^6 - 30*a^3*b
^2*e*x^6 + 30*a^4*b*f*x^6 + 60*a^2*b^3*c*x^3 - 60*a^3*b^2*d*x^3 + 60*a^4*b*
e*x^3 - 60*a^5*f*x^3)/b^6 - 1/3*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*l
og(abs(b*x^3 + a))/b^7
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.14

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^{15}\left(\frac{e}{15\,b}-\frac{a\,f}{15\,b^2}\right)+x^{12}\left(\frac{d}{12\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{12\,b}\right) + x^9\left(\frac{c}{9\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{9\,b}\right) + \frac{\ln\left(b\,x^3+a\right)\,\left(f\,a^6-e\,a^5\,b+d\,a^4\,b^2-c\,a^3\,b^3\right)}{3\,b^7} + \frac{f\,x^{18}}{18\,b}+\frac{a^2\,x^3\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{3\,b^2} - \frac{a\,x^6\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{6\,b}$$

```
[In] int((x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^15*(e/(15*b) - (a*f)/(15*b^2)) + x^12*(d/(12*b) - (a*(e/b - (a*f)/b^2))/(
12*b)) + x^9*(c/(9*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(9*b)) + (log(a
 + b*x^3)*(a^6*f - a^3*b^3*c + a^4*b^2*d - a^5*b*e))/(3*b^7) + (f*x^18)/(18
*b) + (a^2*x^3*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(3*b^2) - (a*
x^6*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(6*b)
```

3.224 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 170

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^6}{6b^4} + \frac{(b^2d-abe+a^2f)\,x^9}{9b^3} + \frac{(be-af)x^{12}}{12b^2} + \frac{fx^{15}}{15b} + \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3b^6}$$

```
[Out] -1/3*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^3/b^5+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b
^3*c)*x^6/b^4+1/9*(a^2*f-a*b*e+b^2*d)*x^9/b^3+1/12*(-a*f+b*e)*x^12/b^2+1/15
*f*x^15/b+1/3*a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/b^6
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1835, 1634}

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{x^9(a^2f-abe+b^2d)}{9b^3} + \frac{a^2\log(a+bx^3)\,(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^6} - \frac{ax^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5} + \frac{x^6(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^4} + \frac{x^{12}(be-af)}{12b^2} + \frac{fx^{15}}{15b}$$

```
[In] Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -1/3*(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/b^5 + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x^6)/(6*b^4) + ((b^2*d - a*b*e + a^2*f)*x^9)/(9*b^3) + ((b
*e - a*f)*x^12)/(12*b^2) + (f*x^15)/(15*b) + (a^2*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx+ex^2+fx^3)}{a+bx}\,dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-b^3c+ab^2d-a^2be+a^3f)}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4} + \frac{(b^2d-abe+a^2f)\,x^2}{b^3} + \frac{(be-af)x^3}{b^2} + \frac{fx^4}{b} - \frac{a^2(-b^3c+ab^2d-a^2be+a^3f)}{b^5(a+bx)}\right) dx, x, x^3\right) \\
&= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^6}{6b^4} + \frac{(b^2d-abe+a^2f)\,x^9}{9b^3} + \frac{(be-af)x^{12}}{12b^2} + \frac{fx^{15}}{15b} + \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3b^6}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.91

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{bx^3(60a^4f-30a^3b(2e+fx^3)+10a^2b^2(6d+3ex^3+2fx^6)-5ab^3(12c+6dx^3+4ex^6+3fx^9)+b^4x^3(30c+}{180b^6}$$

```
[In] Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(60*a^4*f - 30*a^3*b*(2*e + f*x^3) + 10*a^2*b^2*(6*d + 3*e*x^3 + 2*f
*x^6) - 5*a*b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9) + b^4*x^3*(30*c + 20*d
*x^3 + 15*e*x^6 + 12*f*x^9)) - 60*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f
)*Log[a + b*x^3])/(180*b^6)
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.94

method	result
norman	$-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)x^6}{6b^4}-\frac{(af-be)x^{12}}{12b^2}+\frac{f\,x^{15}}{15b}+\frac{(a^2f-aeb+b^2d)x^9}{9b^3}+\frac{a(f\,a^3-a^2be+a\,b^2d-b^3c)x^3}{3b^5}-\frac{a^2(f\,a^3-c}{}$
default	$\frac{\frac{1}{5}f\,x^{15}b^4-\frac{1}{4}a\,b^3f\,x^{12}+\frac{1}{4}b^4e\,x^{12}+\frac{1}{3}a^2b^2f\,x^9-\frac{1}{3}a\,b^3e\,x^9+\frac{1}{3}b^4d\,x^9-\frac{1}{2}f\,x^6a^3b+\frac{1}{2}a^2b^2e\,x^6-\frac{1}{2}a\,b^3d\,x^6+\frac{1}{2}b^4c\,x^6+a^4f\,x^3-a^3be\,x^3}{3b^5}$
parallelrisch	$-\frac{-12f\,x^{15}b^5+15x^{12}a\,b^4f-15x^{12}b^5e-20x^9a^2b^3f+20x^9a\,b^4e-20x^9b^5d+30x^6a^3b^2f-30x^6a^2b^3e+30x^6a\,b^4d-30x^6b^5c-60a^4bf}{180b}$
risch	$\frac{f\,x^{15}}{15b}-\frac{af\,x^{12}}{12b^2}+\frac{e\,x^{12}}{12b}+\frac{a^2f\,x^9}{9b^3}-\frac{ae\,x^9}{9b^2}+\frac{d\,x^9}{9b}-\frac{f\,x^6a^3}{6b^4}+\frac{a^2e\,x^6}{6b^3}-\frac{ad\,x^6}{6b^2}+\frac{c\,x^6}{6b}+\frac{a^4f\,x^3}{3b^5}-\frac{a^3e\,x^3}{3b^4}+\frac{a^2c}{3b}$

```
[In] int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b^4*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)*x^6-1/12/b^2*(a*f-b*e)*x^12+1/15*f*x
^15/b+1/9*(a^2*f-a*b*e+b^2*d)*x^9/b^3+1/3*a/b^5*(a^3*f-a^2*b*e+a*b^2*d-b^3*
c)*x^3-1/3*a^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^6*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{12\,b^5fx^{15}+15\,(b^5e-ab^4f)x^{12}+20\,(b^5d-ab^4e+a^2b^3f)x^9+30\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^6-60\,(ab^4c}{180\,b^6}$$

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/180*(12*b^5*f*x^15 + 15*(b^5*e - a*b^4*f)*x^12 + 20*(b^5*d - a*b^4*e + a^
2*b^3*f)*x^9 + 30*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^6 - 60*(a*b^4
*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 + 60*(a^2*b^3*c - a^3*b^2*d + a^4
*b*e - a^5*f)*log(b*x^3 + a))/b^6
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = -\frac{a^2(a^3f-a^2be+ab^2d-b^3c)\log{(a+bx^3)}}{3b^6} + x^{12}\left(-\frac{af}{12b^2}+\frac{e}{12b}\right)+x^9\left(\frac{a^2f}{9b^3}-\frac{ae}{9b^2}+\frac{d}{9b}\right) + x^6\left(-\frac{a^3f}{6b^4}+\frac{a^2e}{6b^3}-\frac{ad}{6b^2}+\frac{c}{6b}\right) + x^3\left(\frac{a^4f}{3b^5}-\frac{a^3e}{3b^4}+\frac{a^2d}{3b^3}-\frac{ac}{3b^2}\right)+\frac{fx^{15}}{15b}$$

```
[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] -a**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**6) + x*
*12*(-a*f/(12*b**2) + e/(12*b)) + x**9*(a**2*f/(9*b**3) - a*e/(9*b**2) + d/
(9*b)) + x**6*(-a**3*f/(6*b**4) + a**2*e/(6*b**3) - a*d/(6*b**2) + c/(6*b))
 + x**3*(a**4*f/(3*b**5) - a**3*e/(3*b**4) + a**2*d/(3*b**3) - a*c/(3*b**2)
) + f*x**15/(15*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{12\,b^4fx^{15}+15\,(b^4e-ab^3f)x^{12}+20\,(b^4d-ab^3e+a^2b^2f)x^9+30\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^6-60\,(ab^3}{180\,b^5} + \frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\log{(bx^3+a)}}{3\,b^6}$$

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/180*(12*b^4*f*x^15 + 15*(b^4*e - a*b^3*f)*x^12 + 20*(b^4*d - a*b^3*e + a^
2*b^2*f)*x^9 + 30*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^6 - 60*(a*b^3*c
 - a^2*b^2*d + a^3*b*e - a^4*f)*x^3)/b^5 + 1/3*(a^2*b^3*c - a^3*b^2*d + a^4
*b*e - a^5*f)*log(b*x^3 + a)/b^6
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.13

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$=\frac{12\,b^4fx^{15}+15\,b^4ex^{12}-15\,ab^3fx^{12}+20\,b^4dx^9-20\,ab^3ex^9+20\,a^2b^2fx^9+30\,b^4cx^6-30\,ab^3dx^6+30\,a^2b^2e}{180\,b^5}$$
$$+\frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\log(|bx^3+a|)}{3\,b^6}$$

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/180*(12*b^4*f*x^15 + 15*b^4*e*x^12 - 15*a*b^3*f*x^12 + 20*b^4*d*x^9 - 20*
a*b^3*e*x^9 + 20*a^2*b^2*f*x^9 + 30*b^4*c*x^6 - 30*a*b^3*d*x^6 + 30*a^2*b^2
*e*x^6 - 30*a^3*b*f*x^6 - 60*a*b^3*c*x^3 + 60*a^2*b^2*d*x^3 - 60*a^3*b*e*x^
3 + 60*a^4*f*x^3)/b^5 + 1/3*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*log(a
bs(b*x^3 + a))/b^6
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.11

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^{12}\left(\frac{e}{12\,b}-\frac{a\,f}{12\,b^2}\right)+x^9\left(\frac{d}{9\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{9\,b}\right)$$
$$+x^6\left(\frac{c}{6\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{6\,b}\right)$$
$$-\frac{\ln\left(b\,x^3+a\right)\,\left(f\,a^5-e\,a^4\,b+d\,a^3\,b^2-c\,a^2\,b^3\right)}{3\,b^6}$$
$$+\frac{f\,x^{15}}{15\,b}-\frac{a\,x^3\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{3\,b}$$

```
[In] int((x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^12*(e/(12*b) - (a*f)/(12*b^2)) + x^9*(d/(9*b) - (a*(e/b - (a*f)/b^2))/(9*
b)) + x^6*(c/(6*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(6*b)) - (log(a +
b*x^3)*(a^5*f - a^2*b^3*c + a^3*b^2*d - a^4*b*e))/(3*b^6) + (f*x^15)/(15*b)
 - (a*x^3*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(3*b)
```

3.225 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 132

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^4} + \frac{(b^2d-abe+a^2f)\,x^6}{6b^3} + \frac{(be-af)x^9}{9b^2} + \frac{fx^{12}}{12b} - \frac{a(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3b^5}$$

```
[Out] 1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^3/b^4+1/6*(a^2*f-a*b*e+b^2*d)*x^6/b^3+
1/9*(-a*f+b*e)*x^9/b^2+1/12*f*x^12/b-1/3*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*l
n(b*x^3+a)/b^5
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{x^6(a^2f-abe+b^2d)}{6b^3} - \frac{a\log(a+bx^3)\,(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5} + \frac{x^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^4} + \frac{x^9(be-af)}{9b^2} + \frac{fx^{12}}{12b}$$

```
[In] Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f
)*x^6)/(6*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (f*x^12)/(12*b) - (a*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^5)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx+ex^2+fx^3)}{a+bx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^4}+\frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^2}{b^2}\right.\right.\\
&\qquad\left.\left.+\frac{fx^3}{b}+\frac{a(-b^3c+ab^2d-a^2be+a^3f)}{b^4(a+bx)}\right)\,dx, x, x^3\right)\\
&= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^4}+\frac{(b^2d-abe+a^2f)\,x^6}{6b^3}\\
&\quad+\frac{(be-af)x^9}{9b^2}+\frac{fx^{12}}{12b}-\frac{a(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3b^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.90

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{bx^3(-12a^3f+6a^2b(2e+fx^3)-2ab^2(6d+3ex^3+2fx^6)+b^3(12c+6dx^3+4ex^6+3fx^9))+12a(-b^3c+a}{36b^5}$$

```
[In] Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(-12*a^3*f + 6*a^2*b*(2*e + f*x^3) - 2*a*b^2*(6*d + 3*e*x^3 + 2*f*x^
6) + b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9)) + 12*a*(-(b^3*c) + a*b^2*d -
 a^2*b*e + a^3*f)*Log[a + b*x^3])/(36*b^5)
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.93

method	result
norman	$-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)x^3}{3b^4}-\frac{(af-be)x^9}{9b^2}+\frac{f\,x^{12}}{12b}+\frac{(a^2f-aeb+b^2d)x^6}{6b^3}+\frac{a(f\,a^3-a^2be+a\,b^2d-b^3c)\ln(b\,x^3+a)}{3b^5}$
default	$-\frac{-\frac{1}{4}b^3f\,x^{12}+\frac{1}{3}a\,b^2f\,x^9-\frac{1}{3}b^3e\,x^9-\frac{1}{2}x^6f\,a^2b+\frac{1}{2}a\,b^2e\,x^6-\frac{1}{2}b^3d\,x^6+f\,a^3x^3-a^2be\,x^3+a\,b^2d\,x^3-b^3c\,x^3}{3b^4}+\frac{a(f\,a^3-a^2be+a\,b^2}{3b}$
parallelrisch	$\frac{3f\,x^{12}b^4-4x^9a\,b^3f+4x^9b^4e+6x^6a^2b^2f-6x^6a\,b^3e+6b^4d\,x^6-12a^3bf\,x^3+12a^2b^2e\,x^3-12a\,b^3d\,x^3+12b^4c\,x^3+12\ln(b\,x^3+a)a^4f}{36b^5}$
risch	$\frac{f\,x^{12}}{12b}-\frac{af\,x^9}{9b^2}+\frac{e\,x^9}{9b}+\frac{x^6f\,a^2}{6b^3}-\frac{ae\,x^6}{6b^2}+\frac{d\,x^6}{6b}-\frac{f\,a^3x^3}{3b^4}+\frac{a^2e\,x^3}{3b^3}-\frac{ad\,x^3}{3b^2}+\frac{c\,x^3}{3b}+\frac{a^4\ln(b\,x^3+a)f}{3b^5}-\frac{a^3\ln(}{}$

```
[In] int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b^4*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)*x^3-1/9/b^2*(a*f-b*e)*x^9+1/12*f*x^1
2/b+1/6*(a^2*f-a*b*e+b^2*d)*x^6/b^3+1/3*a/b^5*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)
*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{3\,b^4fx^{12}+4\,(b^4e-ab^3f)x^9+6\,(b^4d-ab^3e+a^2b^2f)x^6+12\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^3-12\,(ab^3c-a}{36\,b^5}$$

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^4*f*x^12 + 4*(b^4*e - a*b^3*f)*x^9 + 6*(b^4*d - a*b^3*e + a^2*b^2
*f)*x^6 + 12*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 - 12*(a*b^3*c - a^
2*b^2*d + a^3*b*e - a^4*f)*log(b*x^3 + a))/b^5
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{a(a^3f-a^2be+ab^2d-b^3c)\log\left(a+bx^3\right)}{3b^5}+x^9\left(-\frac{af}{9b^2}+\frac{e}{9b}\right)+x^6\left(\frac{a^2f}{6b^3}-\frac{ae}{6b^2}+\frac{d}{6b}\right)+x^3\left(-\frac{a^3f}{3b^4}+\frac{a^2e}{3b^3}-\frac{ad}{3b^2}+\frac{c}{3b}\right)+\frac{fx^{12}}{12b}$$

```
[In] integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] a*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**5) + x**9*(
-a*f/(9*b**2) + e/(9*b)) + x**6*(a**2*f/(6*b**3) - a*e/(6*b**2) + d/(6*b))
+ x**3*(-a**3*f/(3*b**4) + a**2*e/(3*b**3) - a*d/(3*b**2) + c/(3*b)) + f*x*
*12/(12*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{3\,b^3fx^{12}+4\,(b^3e-ab^2f)x^9+6\,(b^3d-ab^2e+a^2bf)x^6+12\,(b^3c-ab^2d+a^2be-a^3f)x^3}{36\,b^4} - \frac{(ab^3c-a^2b^2d+a^3be-a^4f)\log(bx^3+a)}{3\,b^5}$$

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/36*(3*b^3*f*x^12 + 4*(b^3*e - a*b^2*f)*x^9 + 6*(b^3*d - a*b^2*e + a^2*b*f
)*x^6 + 12*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/b^4 - 1/3*(a*b^3*c - a^
2*b^2*d + a^3*b*e - a^4*f)*log(b*x^3 + a)/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.09

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{3\,b^3fx^{12}+4\,b^3ex^9-4\,ab^2fx^9+6\,b^3dx^6-6\,ab^2ex^6+6\,a^2bfx^6+12\,b^3cx^3-12\,ab^2dx^3+12\,a^2bex^3-12\,a^3f}{36\,b^4} - \frac{(ab^3c-a^2b^2d+a^3be-a^4f)\log(|bx^3+a|)}{3\,b^5}$$

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/36*(3*b^3*f*x^12 + 4*b^3*e*x^9 - 4*a*b^2*f*x^9 + 6*b^3*d*x^6 - 6*a*b^2*e*
x^6 + 6*a^2*b*f*x^6 + 12*b^3*c*x^3 - 12*a*b^2*d*x^3 + 12*a^2*b*e*x^3 - 12*a
^3*f*x^3)/b^4 - 1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*log(abs(b*x^3 +
 a))/b^5
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.07

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^9\left(\frac{e}{9\,b}-\frac{a\,f}{9\,b^2}\right)+x^6\left(\frac{d}{6\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{6\,b}\right) + x^3\left(\frac{c}{3\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{3\,b}\right)+\frac{f\,x^{12}}{12\,b} + \frac{\ln\left(b\,x^3+a\right)\,\left(f\,a^4-e\,a^3\,b+d\,a^2\,b^2-c\,a\,b^3\right)}{3\,b^5}$$

```
[In] int((x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^9*(e/(9*b) - (a*f)/(9*b^2)) + x^6*(d/(6*b) - (a*(e/b - (a*f)/b^2))/(6*b))
 + x^3*(c/(3*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(3*b)) + (f*x^12)/(12
*b) + (log(a + b*x^3)*(a^4*f + a^2*b^2*d - a*b^3*c - a^3*b*e))/(3*b^5)
```

3.226 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3} dx = \frac{(b^2d-abe+a^2f)\,x^3}{3b^3} + \frac{(be-af)x^6}{6b^2} + \frac{fx^9}{9b} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3b^4}$$

```
[Out] 1/3*(a^2*f-a*b*e+b^2*d)*x^3/b^3+1/6*(-a*f+b*e)*x^6/b^2+1/9*f*x^9/b+1/3*(-a^
3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/b^4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1833, 1864}

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3} dx = \frac{x^3(a^2f-abe+b^2d)}{3b^3} + \frac{\log(a+bx^3)(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^4} + \frac{x^6(be-af)}{6b^2} + \frac{fx^9}{9b}$$

```
[In] Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x^3)/(3*b^3) + ((b*e - a*f)*x^6)/(6*b^2) + (f*x^9)
/(9*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^4)
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{a+bx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^2d-abe+a^2f}{b^3}+\frac{(be-af)x}{b^2}+\frac{fx^2}{b}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3(a+bx)}\right)\,dx, x, x^3\right)\\
&= \frac{(b^2d-abe+a^2f)\,x^3}{3b^3}+\frac{(be-af)x^6}{6b^2}+\frac{fx^9}{9b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3b^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{bx^3(6a^2f-3ab(2e+fx^3)+b^2(6d+3ex^3+2fx^6))+6(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{18b^4}$$

```
[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
[Out] (b*x^3*(6*a^2*f - 3*a*b*(2*e + f*x^3) + b^2*(6*d + 3*e*x^3 + 2*f*x^6)) + 6*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(18*b^4)
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93

method	result
norman	$-\frac{(af-be)x^6}{6b^2}+\frac{f\,x^9}{9b}+\frac{(a^2f-aeb+b^2d)x^3}{3b^3}-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)\ln(b\,x^3+a)}{3b^4}$
default	$\frac{\frac{1}{3}b^2f\,x^9-\frac{1}{2}abf\,x^6+\frac{1}{2}b^2e\,x^6+a^2f\,x^3-abe\,x^3+d\,x^3b^2}{3b^3}+\frac{(-f\,a^3+a^2be-a\,b^2d+b^3c)\ln(b\,x^3+a)}{3b^4}$
parallelrisch	$-\frac{-2b^3f\,x^9+3x^6a\,b^2f-3x^6b^3e-6a^2bf\,x^3+6a\,b^2e\,x^3-6b^3d\,x^3+6\ln(b\,x^3+a)a^3f-6\ln(b\,x^3+a)a^2be+6\ln(b\,x^3+a)a\,b^2d-6\ln(b\,x}{18b^4}$
risch	$\frac{f\,x^9}{9b}-\frac{af\,x^6}{6b^2}+\frac{e\,x^6}{6b}+\frac{a^2f\,x^3}{3b^3}-\frac{ae\,x^3}{3b^2}+\frac{d\,x^3}{3b}-\frac{\ln(b\,x^3+a)f\,a^3}{3b^4}+\frac{\ln(b\,x^3+a)a^2e}{3b^3}-\frac{\ln(b\,x^3+a)ad}{3b^2}+\frac{c\ln(b\,x^3+a)}{3b}$

[In] int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/6/b^2*(a*f-b*e)*x^6+1/9*f*x^9/b+1/3*(a^2*f-a*b*e+b^2*d)*x^3/b^3-1/3/b^4*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)*ln(b*x^3+a)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{2\,b^3fx^9+3\,(b^3e-ab^2f)x^6+6\,(b^3d-ab^2e+a^2bf)x^3+6\,(b^3c-ab^2d+a^2be-a^3f)\log\left(bx^3+a\right)}{18\,b^4}$$

[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")

[Out] 1/18*(2*b^3*f*x^9 + 3*(b^3*e - a*b^2*f)*x^6 + 6*(b^3*d - a*b^2*e + a^2*b*f)*x^3 + 6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a))/b^4

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^6\left(-\frac{af}{6b^2}+\frac{e}{6b}\right)+x^3\left(\frac{a^2f}{3b^3}-\frac{ae}{3b^2}+\frac{d}{3b}\right) + \frac{fx^9}{9b}-\frac{(a^3f-a^2be+ab^2d-b^3c)\log\left(a+bx^3\right)}{3b^4}$$

[In] integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)

[Out] x**6*(-a*f/(6*b**2) + e/(6*b)) + x**3*(a**2*f/(3*b**3) - a*e/(3*b**2) + d/(3*b)) + f*x**9/(9*b) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**4)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{2\,b^2fx^9+3\,(b^2e-abf)x^6+6\,(b^2d-abe+a^2f)x^3}{18\,b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(bx^3+a\right)}{3\,b^4}$$

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/18*(2*b^2*f*x^9 + 3*(b^2*e - a*b*f)*x^6 + 6*(b^2*d - a*b*e + a^2*f)*x^3)/
b^3 + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{2\,b^2fx^9+3\,b^2ex^6-3\,abfx^6+6\,b^2dx^3-6\,abex^3+6\,a^2fx^3}{18\,b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(|bx^3+a|\right)}{3\,b^4}$$

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/18*(2*b^2*f*x^9 + 3*b^2*e*x^6 - 3*a*b*f*x^6 + 6*b^2*d*x^3 - 6*a*b*e*x^3 +
 6*a^2*f*x^3)/b^3 + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(abs(b*x^3 +
 a))/b^4
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^6\left(\frac{e}{6\,b}-\frac{a\,f}{6\,b^2}\right)+x^3\left(\frac{d}{3\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{3\,b}\right) + \frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^4}+\frac{f\,x^9}{9\,b}$$

```
[In] int((x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^6*(e/(6*b) - (a*f)/(6*b^2)) + x^3*(d/(3*b) - (a*(e/b - (a*f)/b^2))/(3*b))
 + (log(a + b*x^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^4) + (f*x^9)/(
9*b)
```

3.227 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)}\,dx = \frac{(be-af)x^3}{3b^2}+\frac{fx^6}{6b}+\frac{c\log(x)}{a} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3ab^3}$$

```
[Out] 1/3*(-a*f+b*e)*x^3/b^2+1/6*f*x^6/b+c*ln(x)/a-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^
3*c)*ln(b*x^3+a)/a/b^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)}\,dx = -\frac{\log(a+bx^3)(a^3(-f)+a^2be-ab^2d+b^3c)}{3ab^3} + \frac{x^3(be-af)}{3b^2}+\frac{c\log(x)}{a}+\frac{fx^6}{6b}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x]
```

```
[Out] ((b*e - a*f)*x^3)/(3*b^2) + (f*x^6)/(6*b) + (c*Log[x])/a - ((b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a*b^3)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
```

```
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x(a+bx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{be-af}{b^2}+\frac{c}{ax}+\frac{fx}{b}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2(a+bx)}\right)dx, x, x^3\right)\\ &= \frac{(be-af)x^3}{3b^2}+\frac{fx^6}{6b}+\frac{c\log(x)}{a}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3ab^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)}\,dx = \frac{abx^3(2be-2af+bfx^3)+6b^3c\log(x)-2(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{6ab^3}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x]
[Out] (a*b*x^3*(2*b*e - 2*a*f + b*f*x^3) + 6*b^3*c*Log[x] - 2*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*Log[a + b*x^3])/(6*a*b^3)
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

method	result
default	$\frac{(-f\,x^3 b+af-be)^2}{6b^3 f}+\frac{c\ln(x)}{a}+\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)\ln(b\,x^3+a)}{3a\,b^3}$
norman	$-\frac{(af-be)x^3}{3b^2}+\frac{f\,x^6}{6b}+\frac{c\ln(x)}{a}+\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)\ln(b\,x^3+a)}{3a\,b^3}$
parallelrisch	$\frac{x^6a\,b^2f-2a^2bf\,x^3+2a\,b^2e\,x^3+6c\ln(x)b^3+2\ln(b\,x^3+a)a^3f-2\ln(b\,x^3+a)a^2be+2\ln(b\,x^3+a)a\,b^2d-2\ln(b\,x^3+a)b^3c}{6a\,b^3}$
risch	$\frac{f\,x^6}{6b}-\frac{fa\,x^3}{3b^2}+\frac{e\,x^3}{3b}+\frac{f\,a^2}{6b^3}-\frac{ae}{3b^2}+\frac{e^2}{6bf}+\frac{c\ln(x)}{a}+\frac{a^2\ln(-b\,x^3-a)f}{3b^3}-\frac{a\ln(-b\,x^3-a)e}{3b^2}+\frac{\ln(-b\,x^3-a)d}{3b}-\ln($

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/6*(-b*f*x^3+a*f-b*e)^2/b^3/f+c*ln(x)/a+1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/
a/b^3*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x\,(a+bx^3)}\,dx = \frac{ab^2fx^6+6\,b^3c\log(x)+2\,(ab^2e-a^2bf)x^3-2\,(b^3c-ab^2d+a^2be-a^3f)\log(bx^3+a)}{6\,ab^3}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="fricas")
[Out] 1/6*(a*b^2*f*x^6 + 6*b^3*c*log(x) + 2*(a*b^2*e - a^2*b*f)*x^3 - 2*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a))/(a*b^3)
```

Sympy [A] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^3+ex^6+fx^9}{x\,(a+bx^3)}\,dx = x^3\left(-\frac{af}{3b^2}+\frac{e}{3b}\right)+\frac{fx^6}{6b}+\frac{c\log(x)}{a}+\frac{(a^3f-a^2be+ab^2d-b^3c)\log\left(\frac{a}{b}+x^3\right)}{3ab^3}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a),x)
[Out] x**3*(-a*f/(3*b**2) + e/(3*b)) + f*x**6/(6*b) + c*log(x)/a + (a**3*f - a**2
*b*e + a*b**2*d - b**3*c)*log(a/b + x**3)/(3*a*b**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)}\,dx = \frac{c\log(x^3)}{3a} + \frac{bfx^6+2(be-af)x^3}{6b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log(bx^3+a)}{3ab^3}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*c*log(x^3)/a + 1/6*(b*f*x^6 + 2*(b*e - a*f)*x^3)/b^2 - 1/3*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a)/(a*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)}\,dx = \frac{c\log(|x|)}{a} + \frac{bfx^6+2bex^3-2afx^3}{6b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log(|bx^3+a|)}{3ab^3}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] c*log(abs(x))/a + 1/6*(b*f*x^6 + 2*b*e*x^3 - 2*a*f*x^3)/b^2 - 1/3*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*log(abs(b*x^3 + a))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)}\,dx = x^3\left(\frac{e}{3b}-\frac{af}{3b^2}\right) + \frac{fx^6}{6b} + \frac{c\ln(x)}{a} - \frac{\ln(bx^3+a)\left(-fa^3+ea^2b-dab^2+cb^3\right)}{3ab^3}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x)
```

```
[Out] x^3*(e/(3*b) - (a*f)/(3*b^2)) + (f*x^6)/(6*b) + (c*log(x))/a - (log(a + b*x
^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a*b^3)
```

3.228 $\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)} dx$

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)} dx = -\frac{c}{3ax^3}+\frac{fx^3}{3b}-\frac{(bc-ad)\log(x)}{a^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^2b^2}$$

```
[Out] -1/3*c/a/x^3+1/3*f*x^3/b-(-a*d+b*c)*ln(x)/a^2+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b
^3*c)*ln(b*x^3+a)/a^2/b^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)} dx = -\frac{\log(x)(bc-ad)}{a^2} + \frac{\log(a+bx^3)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^2b^2}-\frac{c}{3ax^3}+\frac{fx^3}{3b}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*c/(a*x^3) + (f*x^3)/(3*b) - ((b*c - a*d)*Log[x])/a^2 + ((b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^2*b^2)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
```

```
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{f}{b}+\frac{c}{ax^2}+\frac{-bc+ad}{a^2x}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b(a+bx)}\right)dx, x, x^3\right)\\ &= -\frac{c}{3ax^3}+\frac{fx^3}{3b}-\frac{(bc-ad)\log(x)}{a^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^2b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)}\,dx = \frac{1}{3}\left(-\frac{c}{ax^3}+\frac{fx^3}{b}+\frac{3(-bc+ad)\log(x)}{a^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{a^2b^2}\right)$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x]
[Out] (-(c/(a*x^3)) + (f*x^3)/b + (3*(-(b*c) + a*d)*Log[x])/a^2 + ((b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(a^2*b^2))/3
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

method	result	s
default	$\frac{f\,x^3}{3b}-\frac{c}{3a\,x^3}+\frac{(ad-bc)\ln(x)}{a^2}-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)\ln(b\,x^3+a)}{3a^2b^2}$	7
norman	$\frac{-\frac{c}{3a}+\frac{f\,x^6}{3b}}{x^3}+\frac{(ad-bc)\ln(x)}{a^2}-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)\ln(b\,x^3+a)}{3a^2b^2}$	7
risch	$\frac{f\,x^3}{3b}-\frac{c}{3a\,x^3}+\frac{d\ln(x)}{a}-\frac{bc\ln(x)}{a^2}-\frac{a\ln(b\,x^3+a)f}{3b^2}+\frac{e\ln(b\,x^3+a)}{3b}-\frac{d\ln(b\,x^3+a)}{3a}+\frac{b\ln(b\,x^3+a)c}{3a^2}$	9
parallelrisch	$\frac{x^6f\,a^2b+3\ln(x)x^3a\,b^2d-3\ln(x)x^3b^3c-\ln(b\,x^3+a)x^3a^3f+\ln(b\,x^3+a)x^3a^2be-\ln(b\,x^3+a)x^3a\,b^2d+\ln(b\,x^3+a)x^3b^3c-a\,b^2c}{3a^2b^2x^3}$	1

[In] int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 1/3*f*x^3/b-1/3*c/a/x^3+(a*d-b*c)/a^2*ln(x)-1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^2/b^2*ln(b*x^3+a)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)}\,dx = \frac{a^2bfx^6+(b^3c-ab^2d+a^2be-a^3f)x^3\log\left(bx^3+a\right)-3\left(b^3c-ab^2d\right)x^3\log\left(x\right)-ab^2c}{3\,a^2b^2x^3}$$

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="fricas")

[Out] 1/3*(a^2*b*f*x^6 + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3*log(b*x^3 + a) - 3*(b^3*c - a*b^2*d)*x^3*log(x) - a*b^2*c)/(a^2*b^2*x^3)

Sympy [A] (verification not implemented)

Time = 49.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)}\,dx = \frac{fx^3}{3b}-\frac{c}{3ax^3}+\frac{(ad-bc)\log(x)}{a^2} - \frac{(a^3f-a^2be+ab^2d-b^3c)\log\left(\frac{a}{b}+x^3\right)}{3a^2b^2}$$

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a),x)

[Out] f*x**3/(3*b) - c/(3*a*x**3) + (a*d - b*c)*log(x)/a**2 - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a/b + x**3)/(3*a**2*b**2)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)}\,dx = \frac{fx^3}{3\,b} - \frac{(bc-ad)\log(x^3)}{3\,a^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log(bx^3+a)}{3\,a^2b^2} - \frac{c}{3\,ax^3}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*f*x^3/b - 1/3*(b*c - a*d)*log(x^3)/a^2 + 1/3*(b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*log(b*x^3 + a)/(a^2*b^2) - 1/3*c/(a*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)}\,dx = \frac{fx^3}{3\,b} - \frac{(bc-ad)\log(|x|)}{a^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log(|bx^3+a|)}{3\,a^2b^2} + \frac{bcx^3-adx^3-ac}{3\,a^2x^3}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*f*x^3/b - (b*c - a*d)*log(abs(x))/a^2 + 1/3*(b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*log(abs(b*x^3 + a))/(a^2*b^2) + 1/3*(b*c*x^3 - a*d*x^3 - a*c)/(a^2
*x^3)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)}\,dx = \frac{f\,x^3}{3\,b} - \frac{c}{3\,a\,x^3} + \frac{\ln(x)\,(a\,d-b\,c)}{a^2} + \frac{\ln(b\,x^3+a)\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{3\,a^2\,b^2}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x)
```

```
[Out] (f*x^3)/(3*b) - c/(3*a*x^3) + (log(x)*(a*d - b*c))/a^2 + (log(a + b*x^3)*(b
^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^2*b^2)
```

3.229 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)}\,dx = -\frac{c}{6ax^6}+\frac{bc-ad}{3a^2x^3}+\frac{(b^2c-abd+a^2e)\log(x)}{a^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^3b}$$

```
[Out] -1/6*c/a/x^6+1/3*(-a*d+b*c)/a^2/x^3+(a^2*e-a*b*d+b^2*c)*ln(x)/a^3-1/3*(-a^3
*f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/a^3/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)}\,dx = \frac{bc-ad}{3a^2x^3}+\frac{\log(x)(a^2e-abd+b^2c)}{a^3} - \frac{\log(a+bx^3)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^3b}-\frac{c}{6ax^6}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x]
```

```
[Out] -1/6*c/(a*x^6) + (b*c - a*d)/(3*a^2*x^3) + ((b^2*c - a*b*d + a^2*e)*Log[x])
/a^3 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^3*b)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
```

```
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{ax^3}+\frac{-bc+ad}{a^2x^2}+\frac{b^2c-abd+a^2e}{a^3x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3(a+bx)}\right)dx, x, x^3\right)\\ &= -\frac{c}{6ax^6}+\frac{bc-ad}{3a^2x^3}+\frac{(b^2c-abd+a^2e)\log(x)}{a^3}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)}\,dx = \frac{-\frac{a(ac-2bcx^3+2adx^3)}{x^6}+6(b^2c-abd+a^2e)\log(x)+\left(-2b^2c+2abd-2a^2e+\frac{2a^3f}{b}\right)\log(a+bx^3)}{6a^3}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x]
[Out] (-((a*(a*c - 2*b*c*x^3 + 2*a*d*x^3))/x^6) + 6*(b^2*c - a*b*d + a^2*e)*Log[x
] + (-2*b^2*c + 2*a*b*d - 2*a^2*e + (2*a^3*f)/b)*Log[a + b*x^3])/(6*a^3)
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

method	result
default	$-\frac{c}{6a\,x^{6}}-\frac{ad-bc}{3a^{2}x^{3}}+\frac{\left(a^{2}e-abd+b^{2}c\right)\ln\left(x\right)}{a^{3}}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)\ln\left(b\,x^{3}+a\right)}{3a^{3}b}$
norman	$\frac{-\frac{c}{6a}-\frac{(ad-bc)x^{3}}{3a^{2}}}{x^{6}}+\frac{\left(a^{2}e-abd+b^{2}c\right)\ln\left(x\right)}{a^{3}}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)\ln\left(b\,x^{3}+a\right)}{3a^{3}b}$
risch	$\frac{-\frac{c}{6a}-\frac{(ad-bc)x^{3}}{3a^{2}}}{x^{6}}+\frac{e\ln\left(x\right)}{a}-\frac{\ln\left(x\right)bd}{a^{2}}+\frac{\ln\left(x\right)b^{2}c}{a^{3}}+\frac{\ln\left(-b\,x^{3}-a\right)f}{3b}-\frac{\ln\left(-b\,x^{3}-a\right)e}{3a}+\frac{b\ln\left(-b\,x^{3}-a\right)d}{3a^{2}}-\frac{b^{2}\ln\left(-b\,x^{3}\right.}{3a^{3}}$
parallelrisch	$\frac{6\ln\left(x\right)x^{6}a^{2}be-6\ln\left(x\right)x^{6}a\,b^{2}d+6\ln\left(x\right)x^{6}b^{3}c+2\ln\left(b\,x^{3}+a\right)x^{6}a^{3}f-2\ln\left(b\,x^{3}+a\right)x^{6}a^{2}be+2\ln\left(b\,x^{3}+a\right)x^{6}a\,b^{2}d-2\ln\left(b\,x^{3}+a\right)x^{6}b^{3}}{6a^{3}x^{6}b}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*c/a/x^6-1/3*(a*d-b*c)/a^2/x^3+(a^2*e-a*b*d+b^2*c)*ln(x)/a^3+1/3*(a^3*f
-a^2*b*e+a*b^2*d-b^3*c)/a^3/b*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)}\,dx = -\frac{2\,(b^3c-ab^2d+a^2be-a^3f)x^6\log\left(bx^3+a\right)-6\,(b^3c-ab^2d+a^2be)x^6\log\left(x\right)+a^2bc-2\,(ab^2c-a^2bd)x^3}{6\,a^3bx^6}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6*log(b*x^3 + a) - 6*(b^3*c -
 a*b^2*d + a^2*b*e)*x^6*log(x) + a^2*b*c - 2*(a*b^2*c - a^2*b*d)*x^3)/(a^3*
b*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)}\,dx = \frac{(b^2c-abd+a^2e)\log(x^3)}{3\,a^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log(bx^3+a)}{3\,a^3b} + \frac{2\,(bc-ad)x^3-ac}{6\,a^2x^6}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*(b^2*c - a*b*d + a^2*e)*log(x^3)/a^3 - 1/3*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*log(b*x^3 + a)/(a^3*b) + 1/6*(2*(b*c - a*d)*x^3 - a*c)/(a^2*x^6)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.29

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)}\,dx = \frac{(b^2c-abd+a^2e)\log(|x|)}{a^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log(|bx^3+a|)}{3\,a^3b} - \frac{3\,b^2cx^6-3\,abdx^6+3\,a^2ex^6-2\,abcx^3+2\,a^2dx^3+a^2c}{6\,a^3x^6}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="giac")
[Out] (b^2*c - a*b*d + a^2*e)*log(abs(x))/a^3 - 1/3*(b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*log(abs(b*x^3 + a))/(a^3*b) - 1/6*(3*b^2*c*x^6 - 3*a*b*d*x^6 + 3*a^2
*e*x^6 - 2*a*b*c*x^3 + 2*a^2*d*x^3 + a^2*c)/(a^3*x^6)
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)}\,dx = \frac{\ln(x)\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{a^3} - \frac{\frac{c}{6\,a}+\frac{x^3\,(a\,d-b\,c)}{3\,a^2}}{x^6} - \frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^3\,b}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x)
[Out] (log(x)*(b^2*c + a^2*e - a*b*d))/a^3 - (c/(6*a) + (x^3*(a*d - b*c))/(3*a^2)
)/x^6 - (log(a + b*x^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^3*b)
```

3.230 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)} dx$

Optimal result

Integrand size = 30, antiderivative size = 128

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)} dx = -\frac{c}{9ax^9}+\frac{bc-ad}{6a^2x^6}-\frac{b^2c-abd+a^2e}{3a^3x^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log(x)}{a^4} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(a+bx^3\right)}{3a^4}$$

```
[Out] -1/9*c/a/x^9+1/6*(-a*d+b*c)/a^2/x^6+1/3*(-a^2*e+a*b*d-b^2*c)/a^3/x^3-(-a^3*
f+a^2*b*e-a*b^2*d+b^3*c)*ln(x)/a^4+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*
x^3+a)/a^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)} dx = \frac{bc-ad}{6a^2x^6}-\frac{a^2e-abd+b^2c}{3a^3x^3} + \frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4} - \frac{\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^4}-\frac{c}{9ax^9}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x]
```

```
[Out] -1/9*c/(a*x^9) + (b*c - a*d)/(6*a^2*x^6) - (b^2*c - a*b*d + a^2*e)/(3*a^3*x
^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[x])/a^4 + ((b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^4)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{ax^4}+\frac{-bc+ad}{a^2x^3}+\frac{b^2c-abd+a^2e}{a^3x^2}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x}\right.\right.\\ &\qquad \left.\left.-\frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^4(a+bx)}\right)\,dx, x, x^3\right)\\ &= -\frac{c}{9ax^9}+\frac{bc-ad}{6a^2x^6}-\frac{b^2c-abd+a^2e}{3a^3x^3}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log(x)}{a^4}\\ &\quad +\frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)}\,dx &= -\frac{c}{9ax^9}+\frac{bc-ad}{6a^2x^6}+\frac{-b^2c+abd-a^2e}{3a^3x^3}\\ &\quad +\frac{(-b^3c+ab^2d-a^2be+a^3f)\log(x)}{a^4}\\ &\quad +\frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^4}\end{aligned}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x]
```

```
[Out] -1/9*c/(a*x^9) + (b*c - a*d)/(6*a^2*x^6) + (-(b^2*c) + a*b*d - a^2*e)/(3*a^
3*x^3) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[x])/a^4 + ((b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^4)
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

method	result
default	$-\frac{c}{9a\,x^{9}}-\frac{ad-bc}{6a^{2}x^{6}}-\frac{a^{2}e-abd+b^{2}c}{3a^{3}x^{3}}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)\ln\left(x\right)}{a^{4}}-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)\ln\left(b\,x^{3}+a\right)}{3a^{4}}$
norman	$\frac{-\frac{c}{9a}-\frac{(ad-bc)x^{3}}{6a^{2}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{3a^{3}}}{x^{9}}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)\ln\left(x\right)}{a^{4}}-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)\ln\left(b\,x^{3}+a\right)}{3a^{4}}$
risch	$\frac{-\frac{c}{9a}-\frac{(ad-bc)x^{3}}{6a^{2}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{3a^{3}}}{x^{9}}+\frac{\ln(x)f}{a}-\frac{\ln(x)be}{a^{2}}+\frac{\ln(x)b^{2}d}{a^{3}}-\frac{\ln(x)b^{3}c}{a^{4}}-\frac{\ln\left(b\,x^{3}+a\right)f}{3a}+\frac{\ln\left(b\,x^{3}+a\right)be}{3a^{2}}-\ln$
parallelrisch	$\frac{18\ln(x)x^{9}a^{3}f-18\ln(x)x^{9}a^{2}be+18\ln(x)x^{9}a\,b^{2}d-18\ln(x)x^{9}b^{3}c-6\ln\left(b\,x^{3}+a\right)x^{9}a^{3}f+6\ln\left(b\,x^{3}+a\right)x^{9}a^{2}be-6\ln\left(b\,x^{3}+a\right)x^{9}a\,b^{2}d}{18a^{4}x^{9}}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*c/a/x^9-1/6*(a*d-b*c)/a^2/x^6-1/3*(a^2*e-a*b*d+b^2*c)/a^3/x^3+(a^3*f-a
^2*b*e+a*b^2*d-b^3*c)/a^4*ln(x)-1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4*ln(b*
x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)}\,dx = \frac{6\left(b^3c-ab^2d+a^2be-a^3f\right)x^9\log\left(bx^3+a\right)-18\left(b^3c-ab^2d+a^2be-a^3f\right)x^9\log\left(x\right)-6\left(ab^2c-a^2bd+a^3e\right)}{18\,a^4x^9}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9*log(b*x^3 + a) - 18*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x^9*log(x) - 6*(a*b^2*c - a^2*b*d + a^3*e)*x^6
 - 2*a^3*c + 3*(a^2*b*c - a^3*d)*x^3)/(a^4*x^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\log(bx^3+a)}{3a^4} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log(x^3)}{3a^4} - \frac{6(b^2c-abd+a^2e)x^6-3(abc-a^2d)x^3+2a^2c}{18a^3x^9}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a)/a^4 - 1/3*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*log(x^3)/a^4 - 1/18*(6*(b^2*c - a*b*d + a^2*e)*x^
6 - 3*(a*b*c - a^2*d)*x^3 + 2*a^2*c)/(a^3*x^9)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.41

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)}\,dx = -\frac{(b^3c-ab^2d+a^2be-a^3f)\log(|x|)}{a^4} + \frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\log(|bx^3+a|)}{3a^4b} + \frac{11b^3cx^9-11ab^2dx^9+11a^2bex^9-11a^3fx^9-6ab^2cx^6+6a^2bdx^6-6a^3ex^6+3a^2bcx^3-3a^3dx^3-2}{18a^4x^9}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(abs(x))/a^4 + 1/3*(b^4*c - a*b^3*d
 + a^2*b^2*e - a^3*b*f)*log(abs(b*x^3 + a))/(a^4*b) + 1/18*(11*b^3*c*x^9 -
11*a*b^2*d*x^9 + 11*a^2*b*e*x^9 - 11*a^3*f*x^9 - 6*a*b^2*c*x^6 + 6*a^2*b*d*
x^6 - 6*a^3*e*x^6 + 3*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^4*x^9)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)}\,dx = \frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^4} - \frac{\frac{c}{9\,a}+\frac{x^3\,\left(a\,d-b\,c\right)}{6\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{3\,a^3}}{x^9} - \frac{\ln\left(x\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^4}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x)
```

```
[Out] (log(a + b*x^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^4) - (c/(9*a) + (
x^3*(a*d - b*c))/(6*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(3*a^3))/x^9 - (lo
g(x)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^4
```

3.231 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)} dx$

Optimal result

Integrand size = 30, antiderivative size = 164

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)} dx = -\frac{c}{12ax^{12}}+\frac{bc-ad}{9a^2x^9}-\frac{b^2c-abd+a^2e}{6a^3x^6}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\log(x)}{a^5}-\frac{b(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^5}$$

```
[Out] -1/12*c/a/x^12+1/9*(-a*d+b*c)/a^2/x^9+1/6*(-a^2*e+a*b*d-b^2*c)/a^3/x^6+1/3*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^3+b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(
x)/a^5-1/3*b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/a^5
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)} dx = \frac{bc-ad}{9a^2x^9}-\frac{a^2e-abd+b^2c}{6a^3x^6}-\frac{b\log(a+bx^3)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^5}+\frac{b\log(x)(a^3(-f)+a^2be-ab^2d+b^3c)}{a^5}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^4x^3}-\frac{c}{12ax^{12}}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x]
```

```
[Out] -1/12*c/(a*x^12) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(6*a^3
*x^6) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) + (b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*Log[x])/a^5 - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
Log[a + b*x^3])/(3*a^5)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5(a+bx)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{ax^5}+\frac{-bc+ad}{a^2x^4}+\frac{b^2c-abd+a^2e}{a^3x^3}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^2}\right.\right.\\
&\qquad \left.\left.-\frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^5x}+\frac{b^2(-b^3c+ab^2d-a^2be+a^3f)}{a^5(a+bx)}\right)\,dx,x,x^3\right)\\
&= -\frac{c}{12ax^{12}}+\frac{bc-ad}{9a^2x^9}-\frac{b^2c-abd+a^2e}{6a^3x^6}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}\\
&\quad +\frac{b(b^3c-ab^2d+a^2be-a^3f)\log(x)}{a^5}-\frac{b(b^3c-ab^2d+a^2be-a^3f)\log\left(a+bx^3\right)}{3a^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)}\,dx$$
$$=\frac{12ab^3cx^9-6a^2b^2x^6(c+2dx^3)+2a^3bx^3(2c+3dx^3+6ex^6)-a^4(3c+4dx^3+6ex^6+12fx^9)+36b(b^3c-a}{36a^5x^{12}}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x]
```

```
[Out] (12*a*b^3*c*x^9 - 6*a^2*b^2*x^6*(c + 2*d*x^3) + 2*a^3*b*x^3*(2*c + 3*d*x^3
+ 6*e*x^6) - a^4*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) + 36*b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*x^12*Log[x] - 12*b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f
)*x^12*Log[a + b*x^3])/(36*a^5*x^12)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.95

method	result
default	$-\frac{c}{12a\,x^{12}}-\frac{ad-bc}{9a^2x^9}-\frac{a^2e-abd+b^2c}{6a^3x^6}-\frac{f\,a^3-a^2be+a\,b^2d-b^3c}{3a^4x^3}-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)b\ln(x)}{a^5}+\frac{b(f\,a^3-a^2be+a\,b^2}{3a}$
norman	$\frac{-\frac{c}{12a}-\frac{(ad-bc)x^3}{9a^2}-\frac{(a^2e-abd+b^2c)x^6}{6a^3}-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)x^9}{3a^4}}{x^{12}}-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)b\ln(x)}{a^5}+\frac{b(f\,a^3-a^2be+a\,b^2d-}{3a^5}$
risch	$\frac{-\frac{c}{12a}-\frac{(ad-bc)x^3}{9a^2}-\frac{(a^2e-abd+b^2c)x^6}{6a^3}-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)x^9}{3a^4}}{x^{12}}-\frac{b\ln(x)f}{a^2}+\frac{b^2\ln(x)e}{a^3}-\frac{b^3\ln(x)d}{a^4}+\frac{b^4\ln(x)c}{a^5}+\frac{b\ln}{}$
parallelrisch	$-\frac{36\ln(x)x^{12}a^3bf-36\ln(x)x^{12}a^2b^2e+36\ln(x)x^{12}a\,b^3d-36\ln(x)x^{12}b^4c-12\ln(b\,x^3+a)x^{12}a^3bf+12\ln(b\,x^3+a)x^{12}a^2b^2e-12}{}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*c/a/x^12-1/9*(a*d-b*c)/a^2/x^9-1/6*(a^2*e-a*b*d+b^2*c)/a^3/x^6-1/3*(a
^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^3-(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b*ln(
x)+1/3*b*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)}\,dx = -\frac{12\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^{12}\log(bx^3+a)-36\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^{12}\log(x)-12\,(ab^3c}{36\,a^5x^{12}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/36*(12*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12*log(b*x^3 + a) - 36*
(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12*log(x) - 12*(a*b^3*c - a^2*b^2
*d + a^3*b*e - a^4*f)*x^9 + 6*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 3*a^4*c -
 4*(a^3*b*c - a^4*d)*x^3)/(a^5*x^12)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)}\,dx &= -\frac{\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)\log\left(bx^3+a\right)}{3\,a^5}+\frac{\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)\log\left(x^3\right)}{3\,a^5}\\ &\quad+\frac{12\left(b^3c-ab^2d+a^2be-a^3f\right)x^9-6\left(ab^2c-a^2bd+a^3e\right)x^6-3\,a^3c+4\left(a^2bc-a^3d\right)x^3}{36\,a^4x^{12}}\end{aligned}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="maxima")
[Out] -1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(b*x^3 + a)/a^5 + 1/3*(b^4*
c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(x^3)/a^5 + 1/36*(12*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^9 - 6*(a*b^2*c - a^2*b*d + a^3*e)*x^6 - 3*a^3*c + 4*(
a^2*b*c - a^3*d)*x^3)/(a^4*x^12)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.40

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)}\,dx &= \frac{\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)\log\left(|x|\right)}{a^5}-\frac{\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)\log\left(|bx^3+a|\right)}{3\,a^5b}\\ &\quad-\frac{25\,b^4cx^{12}-25\,ab^3dx^{12}+25\,a^2b^2ex^{12}-25\,a^3bfx^{12}-12\,ab^3cx^9+12\,a^2b^2dx^9-12\,a^3bex^9+12\,a^4fx^9+6\,a}{36\,a^5x^{12}}\end{aligned}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="giac")
[Out] (b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(abs(x))/a^5 - 1/3*(b^5*c - a*b^
4*d + a^2*b^3*e - a^3*b^2*f)*log(abs(b*x^3 + a))/(a^5*b) - 1/36*(25*b^4*c*x
```

```
^12 - 25*a*b^3*d*x^12 + 25*a^2*b^2*e*x^12 - 25*a^3*b*f*x^12 - 12*a*b^3*c*x^
9 + 12*a^2*b^2*d*x^9 - 12*a^3*b*e*x^9 + 12*a^4*f*x^9 + 6*a^2*b^2*c*x^6 - 6*
a^3*b*d*x^6 + 6*a^4*e*x^6 - 4*a^3*b*c*x^3 + 4*a^4*d*x^3 + 3*a^4*c)/(a^5*x^1
2)
```

Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)}\,dx = \frac{\ln(x)\,(-f\,a^3\,b+e\,a^2\,b^2-d\,a\,b^3+c\,b^4)}{a^5} - \frac{\ln(b\,x^3+a)\,(-f\,a^3\,b+e\,a^2\,b^2-d\,a\,b^3+c\,b^4)}{3\,a^5} - \frac{\frac{c}{12\,a}-\frac{x^9\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{3\,a^4}+\frac{x^3\,(a\,d-b\,c)}{9\,a^2}+\frac{x^6\,(e\,a^2-d\,a\,b+c\,b^2)}{6\,a^3}}{x^{12}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x)
```

```
[Out] (log(x)*(b^4*c + a^2*b^2*e - a*b^3*d - a^3*b*f))/a^5 - (log(a + b*x^3)*(b^4
*c + a^2*b^2*e - a*b^3*d - a^3*b*f))/(3*a^5) - (c/(12*a) - (x^9*(b^3*c - a^
3*f - a*b^2*d + a^2*b*e))/(3*a^4) + (x^3*(a*d - b*c))/(9*a^2) + (x^6*(b^2*c
 + a^2*e - a*b*d))/(6*a^3))/x^12
```

3.232 $\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 205

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx = -\frac{c}{15ax^{15}}+\frac{bc-ad}{12a^2x^{12}}-\frac{b^2c-abd+a^2e}{9a^3x^9}+\frac{b^3c-ab^2d+a^2be-a^3f}{6a^4x^6}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{3a^5x^3}-\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\log(x)}{a^6}+\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^6}$$

```
[Out] -1/15*c/a/x^15+1/12*(-a*d+b*c)/a^2/x^12+1/9*(-a^2*e+a*b*d-b^2*c)/a^3/x^9+1/
6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^6-1/3*b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)/a^5/x^3-b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(x)/a^6+1/3*b^2*(-a^3*f+a^2
*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/a^6
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used

$= \{1835, 1634\}$

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{16}\left(a+bx^3\right)}\,dx = \frac{bc-ad}{12a^2x^{12}} - \frac{a^2e-abd+b^2c}{9a^3x^9} + \frac{b^2\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^6} - \frac{b^2\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6} - \frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^5x^3} + \frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^4x^6} - \frac{c}{15ax^{15}}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x]
```

```
[Out] -1/15*c/(a*x^15) + (b*c - a*d)/(12*a^2*x^12) - (b^2*c - a*b*d + a^2*e)/(9*a
^3*x^9) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*x^6) - (b*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f))/(3*a^5*x^3) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*Log[x])/a^6 + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])
/(3*a^6)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^6(a+bx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{ax^6} + \frac{-bc+ad}{a^2x^5} + \frac{b^2c-abd+a^2e}{a^3x^4} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^3} - \frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^5x^2} + \frac{b^2(-b^3c+ab^2d-a^2be+a^3f)}{a^6x} - \frac{b^3(-b^3c+ab^2d-a^2be+a^3f)}{a^6(a+bx)}\right) dx, x, x^3\right)$$

$$
= -\frac{c}{15ax^{15}} + \frac{bc-ad}{12a^2x^{12}} - \frac{b^2c-abd+a^2e}{9a^3x^9} + \frac{b^3c-ab^2d+a^2be-a^3f}{6a^4x^6} - \frac{b(b^3c-ab^2d+a^2be-a^3f)}{3a^5x^3} - \frac{b^2(b^3c-ab^2d+a^2be-a^3f)\log(x)}{a^6} + \frac{b^2(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^6}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95

$$
\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx = -\frac{\frac{a(60b^4cx^{12}-30ab^3x^9(c+2dx^3)+10a^2b^2x^6(2c+3dx^3+6ex^6)-5a^3bx^3(3c+4dx^3+6ex^6+12fx^9)+a^4(12c+15dx^3+20ex^6+30fx^9))}{x^{15}} + 180b^2(b}{180a^6}
$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x]
```

```
[Out] -1/180*((a*(60*b^4*c*x^12 - 30*a*b^3*x^9*(c + 2*d*x^3) + 10*a^2*b^2*x^6*(2*
c + 3*d*x^3 + 6*e*x^6) - 5*a^3*b*x^3*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) +
 a^4*(12*c + 15*d*x^3 + 20*e*x^6 + 30*f*x^9)))/x^15 + 180*b^2*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*Log[x] - 60*b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
Log[a + b*x^3])/a^6
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.94

method	result
default	$-\frac{c}{15a\,x^{15}}-\frac{ad-bc}{12a^2x^{12}}-\frac{a^2e-abd+b^2c}{9a^3x^9}-\frac{f\,a^3-a^2be+a\,b^2d-b^3c}{6a^4x^6}+\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)b^2\ln(x)}{a^6}+\frac{(f\,a^3-a^2be+a\,b^2}{3a^5x^3}$
norman	$\frac{-\frac{c}{15a}-\frac{(ad-bc)x^3}{12a^2}-\frac{(a^2e-abd+b^2c)x^6}{9a^3}-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)x^9}{6a^4}+\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)b\,x^{12}}{3a^5}}{x^{15}}+\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)b^2\ln}{a^6}$
risch	$\frac{-\frac{c}{15a}-\frac{(ad-bc)x^3}{12a^2}-\frac{(a^2e-abd+b^2c)x^6}{9a^3}-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)x^9}{6a^4}+\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)b\,x^{12}}{3a^5}}{x^{15}}+\frac{b^2\ln(x)f}{a^3}-\frac{b^3\ln(x)e}{a^4}+b$
parallelrisch	$\frac{180\ln(x)x^{15}a^3b^2f-180\ln(x)x^{15}a^2b^3e+180\ln(x)x^{15}a\,b^4d-180\ln(x)x^{15}b^5c-60\ln(b\,x^3+a)x^{15}a^3b^2f+60\ln(b\,x^3+a)x^{15}a^2b^3e-}{}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*c/a/x^15-1/12*(a*d-b*c)/a^2/x^12-1/9*(a^2*e-a*b*d+b^2*c)/a^3/x^9-1/6*
(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^6+(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^6*b^2
*ln(x)+1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b/x^3-1/3*b^2*(a^3*f-a^2*b*e+a
*b^2*d-b^3*c)/a^6*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx = \frac{60\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{15}\log(bx^3+a)-180\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{15}\log(x)-60\,(ab^4c}{}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/180*(60*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^15*log(b*x^3 + a) - 1
80*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^15*log(x) - 60*(a*b^4*c - a^
2*b^3*d + a^3*b^2*e - a^4*b*f)*x^12 + 30*(a^2*b^3*c - a^3*b^2*d + a^4*b*e -
 a^5*f)*x^9 - 20*(a^3*b^2*c - a^4*b*d + a^5*e)*x^6 - 12*a^5*c + 15*(a^4*b*c
 - a^5*d)*x^3)/(a^6*x^15)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**16/(b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx = \frac{(b^5c-ab^4d+a^2b^3e-a^3b^2f)\log(bx^3+a)}{3\,a^6} - \frac{(b^5c-ab^4d+a^2b^3e-a^3b^2f)\log(x^3)}{3\,a^6} - \frac{60\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^{12}-30\,(ab^3c-a^2b^2d+a^3be-a^4f)x^9+20\,(a^2b^2c-a^3bd+a^4e)x^6+1}{180\,a^5x^{15}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*log(b*x^3 + a)/a^6 - 1/3*(b^5
*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*log(x^3)/a^6 - 1/180*(60*(b^4*c - a*b
^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 30*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*
f)*x^9 + 20*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 12*a^4*c - 15*(a^3*b*c - a^
4*d)*x^3)/(a^5*x^15)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.37

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx$$
$$= -\frac{(b^5c-ab^4d+a^2b^3e-a^3b^2f)\log(|x|)}{a^6}+\frac{(b^6c-ab^5d+a^2b^4e-a^3b^3f)\log(|bx^3+a|)}{3\,a^6b}$$
$$+\frac{137\,b^5cx^{15}-137\,ab^4dx^{15}+137\,a^2b^3ex^{15}-137\,a^3b^2fx^{15}-60\,ab^4cx^{12}+60\,a^2b^3dx^{12}-60\,a^3b^2ex^{12}+60\,a^4}{}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*log(abs(x))/a^6 + 1/3*(b^6*c - a
*b^5*d + a^2*b^4*e - a^3*b^3*f)*log(abs(b*x^3 + a))/(a^6*b) + 1/180*(137*b^
5*c*x^15 - 137*a*b^4*d*x^15 + 137*a^2*b^3*e*x^15 - 137*a^3*b^2*f*x^15 - 60*
a*b^4*c*x^12 + 60*a^2*b^3*d*x^12 - 60*a^3*b^2*e*x^12 + 60*a^4*b*f*x^12 + 30
*a^2*b^3*c*x^9 - 30*a^3*b^2*d*x^9 + 30*a^4*b*e*x^9 - 30*a^5*f*x^9 - 20*a^3*
b^2*c*x^6 + 20*a^4*b*d*x^6 - 20*a^5*e*x^6 + 15*a^4*b*c*x^3 - 15*a^5*d*x^3 -
 12*a^5*c)/(a^6*x^15)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx$$
$$=\frac{\ln(b\,x^3+a)\,\left(-f\,a^3\,b^2+e\,a^2\,b^3-d\,a\,b^4+c\,b^5\right)}{3\,a^6}$$
$$-\frac{\frac{c}{15\,a}-\frac{x^9\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{6\,a^4}+\frac{x^3\,(a\,d-b\,c)}{12\,a^2}+\frac{x^6\,(e\,a^2-d\,a\,b+c\,b^2)}{9\,a^3}+\frac{b\,x^{12}\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{3\,a^5}}{x^{15}}$$
$$-\frac{\ln(x)\,\left(-f\,a^3\,b^2+e\,a^2\,b^3-d\,a\,b^4+c\,b^5\right)}{a^6}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x)
```

```
[Out] (log(a + b*x^3)*(b^5*c + a^2*b^3*e - a^3*b^2*f - a*b^4*d))/(3*a^6) - (c/(15
*a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(6*a^4) + (x^3*(a*d - b*c))
/(12*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(9*a^3) + (b*x^12*(b^3*c - a^3*f
- a*b^2*d + a^2*b*e))/(3*a^5))/x^15 - (log(x)*(b^5*c + a^2*b^3*e - a^3*b^2*
f - a*b^4*d))/a^6
```

3.233 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 348

$$\begin{aligned}\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx &= \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{b^6}-\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^5}\\ &+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^7}{7b^4}+\frac{(b^2d-abe+a^2f)\,x^{10}}{10b^3}+\frac{(be-af)x^{13}}{13b^2}\\ &+\frac{fx^{16}}{16b}+\frac{a^{7/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{19/3}}\\ &-\frac{a^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{19/3}}\\ &+\frac{a^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{19/3}}\end{aligned}$$

```
[Out] a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^6-1/4*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)*x^4/b^5+1/7*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^7/b^4+1/10*(a^2*f-a*b*e+b^2
*d)*x^10/b^3+1/13*(-a*f+b*e)*x^13/b^2+1/16*f*x^16/b-1/3*a^(7/3)*(-a^3*f+a^2
*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(19/3)+1/6*a^(7/3)*(-a^3*f+a^2*
b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(19/3)+1/3*a
^(7/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1
/3)*3^(1/2))/b^(19/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1850, 1502, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{x^{10}(a^2f-abe+b^2d)}{10b^3}+\frac{a^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{b^6} - \frac{ax^4(a^3(-f)+a^2be-ab^2d+b^3c)}{4b^5}+\frac{x^7(a^3(-f)+a^2be-ab^2d+b^3c)}{7b^4} + \frac{a^{7/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}b^{19/3}} + \frac{a^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^{19/3}} - \frac{a^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^{19/3}}+\frac{x^{13}(be-af)}{13b^2}+\frac{fx^{16}}{16b}$$

```
[In] Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7)/(7*b
^4) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^3) + ((b*e - a*f)*x^13)/(13*b^2)
 + (f*x^16)/(16*b) + (a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(19/3)) - (a^(7/3)*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(19/3)) + (
a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(6*b^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{fx^{16}}{16b} + \frac{\int \frac{x^9(16bc+16bdx^3+16(be-af)x^6)}{a+bx^3}\,dx}{16b}$$

$$= \frac{fx^{16}}{16b} + \frac{\int \left(\frac{16a^2(b^3c-ab^2d+a^2be-a^3f)}{b^5} - \frac{16a(b^3c-ab^2d+a^2be-a^3f)x^3}{b^4} + \frac{16(b^3c-ab^2d+a^2be-a^3f)x^6}{b^3} + \frac{16(b^2d-abe+a^2f)x^9}{b^2} + \frac{1}{}\right.}{16b}$$

$$= \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{b^6} - \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^7}{7b^4} + \frac{(b^2d-abe+a^2f)\,x^{10}}{10b^3} + \frac{(be-af)x^{13}}{13b^2} + \frac{fx^{16}}{16b} - \frac{(a^3(b^3c-ab^2d+a^2be-a^3f))\int \frac{1}{a+bx^3}\,dx}{b^6}$$

$$= \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{b^6} - \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^7}{7b^4} + \frac{(b^2d-abe+a^2f)\,x^{10}}{10b^3} + \frac{(be-af)x^{13}}{13b^2} + \frac{fx^{16}}{16b} - \frac{\left(a^{7/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^6} - \frac{\left(a^{7/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b^6}$$

$$= \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{b^6} - \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^7}{7b^4} + \frac{(b^2d-abe+a^2f)\,x^{10}}{10b^3} + \frac{(be-af)x^{13}}{13b^2} + \frac{fx^{16}}{16b} - \frac{a^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{19/3}} + \frac{\left(a^{7/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{19/3}} - \frac{\left(a^{8/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^6}$$

$$= \frac{a^2(b^3c - ab^2d + a^2be - a^3f)\,x}{b^6} - \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^4}{4b^5} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^7}{7b^4} + \frac{(b^2d - abe + a^2f)\,x^{10}}{10b^3} + \frac{(be - af)x^{13}}{13b^2} + \frac{fx^{16}}{16b} - \frac{a^{7/3}(b^3c - ab^2d + a^2be - a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3b^{19/3}} + \frac{a^{7/3}(b^3c - ab^2d + a^2be - a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6b^{19/3}} - \frac{\left(a^{7/3}(b^3c - ab^2d + a^2be - a^3f)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{19/3}}$$

$$= \frac{a^2(b^3c - ab^2d + a^2be - a^3f)\,x}{b^6} - \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^4}{4b^5} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^7}{7b^4} + \frac{(b^2d - abe + a^2f)\,x^{10}}{10b^3} + \frac{(be - af)x^{13}}{13b^2} + \frac{fx^{16}}{16b} + \frac{a^{7/3}(b^3c - ab^2d + a^2be - a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{19/3}} - \frac{a^{7/3}(b^3c - ab^2d + a^2be - a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3b^{19/3}} + \frac{a^{7/3}(b^3c - ab^2d + a^2be - a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6b^{19/3}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.01

$$\int \frac{x^9(c + dx^3 + ex^6 + fx^9)}{a + bx^3}\,dx = -\frac{a^2(-b^3c + ab^2d - a^2be + a^3f)\,x}{b^6} + \frac{a(-b^3c + ab^2d - a^2be + a^3f)\,x^4}{4b^5} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^7}{7b^4} + \frac{(b^2d - abe + a^2f)\,x^{10}}{10b^3} + \frac{(be - af)x^{13}}{13b^2} + \frac{fx^{16}}{16b} + \frac{a^{7/3}(-b^3c + ab^2d - a^2be + a^3f)\arctan\left(\frac{-\sqrt[3]{a} + 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{19/3}} + \frac{a^{7/3}(-b^3c + ab^2d - a^2be + a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3b^{19/3}} - \frac{a^{7/3}(-b^3c + ab^2d - a^2be + a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6b^{19/3}}$$

```
[In] Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -((a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^6) + (a*(-(b^3*c) + a*b^
2*d - a^2*b*e + a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
x^7)/(7*b^4) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^3) + ((b*e - a*f)*x^13)
/(13*b^2) + (f*x^16)/(16*b) + (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(19/3)) +
 (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
(3*b^(19/3)) - (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(19/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.70

method	result
risch	$\frac{f\,x^{16}}{16b}-\frac{x^{13}af}{13b^{2}}+\frac{x^{13}e}{13b}+\frac{x^{10}a^{2}f}{10b^{3}}-\frac{x^{10}ae}{10b^{2}}+\frac{x^{10}d}{10b}-\frac{x^{7}a^{3}f}{7b^{4}}+\frac{x^{7}a^{2}e}{7b^{3}}-\frac{x^{7}ad}{7b^{2}}+\frac{x^{7}c}{7b}+\frac{a^{4}f\,x^{4}}{4b^{5}}-\frac{a^{3}e\,x^{4}}{4b^{4}}+\frac{a^{2}d\,x^{4}}{4b^{3}}-$
default	$-\frac{-\frac{1}{16}f\,x^{16}b^{5}+\frac{1}{13}x^{13}a\,b^{4}f-\frac{1}{13}x^{13}b^{5}e-\frac{1}{10}x^{10}a^{2}b^{3}f+\frac{1}{10}x^{10}a\,b^{4}e-\frac{1}{10}x^{10}b^{5}d+\frac{1}{7}x^{7}a^{3}b^{2}f-\frac{1}{7}x^{7}a^{2}b^{3}e+\frac{1}{7}x^{7}a\,b^{4}d-\frac{1}{7}x^{7}b^{5}c-\frac{1}{4}a^{4}bf\,x^{4}+}{b^{6}}$

```
[In] int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*f*x^16/b-1/13/b^2*x^13*a*f+1/13/b*x^13*e+1/10/b^3*x^10*a^2*f-1/10/b^2*
x^10*a*e+1/10/b*x^10*d-1/7/b^4*x^7*a^3*f+1/7/b^3*x^7*a^2*e-1/7/b^2*x^7*a*d+
1/7/b*x^7*c+1/4/b^5*a^4*f*x^4-1/4/b^4*a^3*e*x^4+1/4/b^3*a^2*d*x^4-1/4/b^2*a
*c*x^4-1/b^6*a^5*f*x+1/b^5*a^4*e*x-1/b^4*a^3*d*x+1/b^3*a^2*c*x+1/3/b^7*a^3*
sum((a^3*f-a^2*b*e+a*b^2*d-b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.98

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{1365\,b^5fx^{16}+1680\,(b^5e-ab^4f)x^{13}+2184\,(b^5d-ab^4e+a^2b^3f)x^{10}+3120\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x}{}$$

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/21840*(1365*b^5*f*x^16 + 1680*(b^5*e - a*b^4*f)*x^13 + 2184*(b^5*d - a*b^
4*e + a^2*b^3*f)*x^10 + 3120*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^7
- 5460*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4 - 7280*sqrt(3)*(a^2*
b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*
(a/b)^(2/3) - sqrt(3)*a)/a) + 3640*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f
)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 7280*(a^2*b^3*c - a^
3*b^2*d + a^4*b*e - a^5*f)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 21840*(a^2*b^
3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.35

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= x^{13}\left(-\frac{af}{13b^2}+\frac{e}{13b}\right)+x^{10}\left(\frac{a^2f}{10b^3}-\frac{ae}{10b^2}+\frac{d}{10b}\right)+x^7\left(-\frac{a^3f}{7b^4}+\frac{a^2e}{7b^3}-\frac{ad}{7b^2}+\frac{c}{7b}\right)$$
$$+x^4\left(\frac{a^4f}{4b^5}-\frac{a^3e}{4b^4}+\frac{a^2d}{4b^3}-\frac{ac}{4b^2}\right)+x\left(-\frac{a^5f}{b^6}+\frac{a^4e}{b^5}-\frac{a^3d}{b^4}+\frac{a^2c}{b^3}\right)$$
$$+\mathrm{RootSum}\left(27t^3b^{19}-a^{16}f^3+3a^{15}bef^2-3a^{14}b^2df^2-3a^{14}b^2e^2f+3a^{13}b^3cf^2+6a^{13}b^3def+a^{13}b^3e^3-6\right.$$
$$+\frac{fx^{16}}{16b}$$

```
[In] integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**13*(-a*f/(13*b**2) + e/(13*b)) + x**10*(a**2*f/(10*b**3) - a*e/(10*b**2)
 + d/(10*b)) + x**7*(-a**3*f/(7*b**4) + a**2*e/(7*b**3) - a*d/(7*b**2) + c/
(7*b)) + x**4*(a**4*f/(4*b**5) - a**3*e/(4*b**4) + a**2*d/(4*b**3) - a*c/(4
*b**2)) + x*(-a**5*f/b**6 + a**4*e/b**5 - a**3*d/b**4 + a**2*c/b**3) + Root
Sum(27*_t**3*b**19 - a**16*f**3 + 3*a**15*b*e*f**2 - 3*a**14*b**2*d*f**2 -
```

```
3*a**14*b**2*e**2*f + 3*a**13*b**3*c*f**2 + 6*a**13*b**3*d*e*f + a**13*b**3
*e**3 - 6*a**12*b**4*c*e*f - 3*a**12*b**4*d**2*f - 3*a**12*b**4*d*e**2 + 6*
a**11*b**5*c*d*f + 3*a**11*b**5*c*e**2 + 3*a**11*b**5*d**2*e - 3*a**10*b**6
*c**2*f - 6*a**10*b**6*c*d*e - a**10*b**6*d**3 + 3*a**9*b**7*c**2*e + 3*a**
9*b**7*c*d**2 - 3*a**8*b**8*c**2*d + a**7*b**9*c**3, Lambda(_t, _t*log(3*_t
*b**6/(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c) + x))) + f*x**16/(16*
b)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.01

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{455\,b^5fx^{16}+560\,(b^5e-ab^4f)x^{13}+728\,(b^5d-ab^4e+a^2b^3f)x^{10}+1040\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^7-1}{7280\,b^6}$$
$$-\frac{\sqrt{3}(a^3b^3c-a^4b^2d+a^5be-a^6f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^7\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(a^3b^3c-a^4b^2d+a^5be-a^6f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^7\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(a^3b^3c-a^4b^2d+a^5be-a^6f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^7\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/7280*(455*b^5*f*x^16 + 560*(b^5*e - a*b^4*f)*x^13 + 728*(b^5*d - a*b^4*e
+ a^2*b^3*f)*x^10 + 1040*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^7 - 18
20*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4 + 7280*(a^2*b^3*c - a^3*
b^2*d + a^4*b*e - a^5*f)*x)/b^6 - 1/3*sqrt(3)*(a^3*b^3*c - a^4*b^2*d + a^5*
b*e - a^6*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*(a/b)
^(2/3)) + 1/6*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(b^7*(a/b)^(2/3)) - 1/3*(a^3*b^3*c - a^4*b^2*d + a^5*b*
e - a^6*f)*log(x + (a/b)^(1/3))/(b^7*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.28

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = -\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}a^2b^3c-(-ab^2)^{\frac{1}{3}}a^3b^2d+(-ab^2)^{\frac{1}{3}}a^4be-(-ab^2)^{\frac{1}{3}}a^5f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^7} - \frac{\left((-ab^2)^{\frac{1}{3}}a^2b^3c-(-ab^2)^{\frac{1}{3}}a^3b^2d+(-ab^2)^{\frac{1}{3}}a^4be-(-ab^2)^{\frac{1}{3}}a^5f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^7} + \frac{(a^3b^{13}c-a^4b^{12}d+a^5b^{11}e-a^6b^{10}f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{16}} + \frac{455b^{15}fx^{16}+560b^{15}ex^{13}-560ab^{14}fx^{13}+728b^{15}dx^{10}-728ab^{14}ex^{10}+728a^2b^{13}fx^{10}+1040b^{15}cx^7-}{}$$

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*a^2*b^3*c - (-a*b^2)^(1/3)*a^3*b^2*d + (-a*b^2
)^(1/3)*a^4*b*e - (-a*b^2)^(1/3)*a^5*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/b^7 - 1/6*((-a*b^2)^(1/3)*a^2*b^3*c - (-a*b^2)^(1/3)*a^3
*b^2*d + (-a*b^2)^(1/3)*a^4*b*e - (-a*b^2)^(1/3)*a^5*f)*log(x^2 + x*(-a/b)^
(1/3) + (-a/b)^(2/3))/b^7 + 1/3*(a^3*b^13*c - a^4*b^12*d + a^5*b^11*e - a^6
*b^10*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^16) + 1/7280*(455*b^1
5*f*x^16 + 560*b^15*e*x^13 - 560*a*b^14*f*x^13 + 728*b^15*d*x^10 - 728*a*b^
14*e*x^10 + 728*a^2*b^13*f*x^10 + 1040*b^15*c*x^7 - 1040*a*b^14*d*x^7 + 104
0*a^2*b^13*e*x^7 - 1040*a^3*b^12*f*x^7 - 1820*a*b^14*c*x^4 + 1820*a^2*b^13*
d*x^4 - 1820*a^3*b^12*e*x^4 + 1820*a^4*b^11*f*x^4 + 7280*a^2*b^13*c*x - 728
0*a^3*b^12*d*x + 7280*a^4*b^11*e*x - 7280*a^5*b^10*f*x)/b^16
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.03

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= x^{13}\left(\frac{e}{13\,b}-\frac{a\,f}{13\,b^2}\right)+x^{10}\left(\frac{d}{10\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{10\,b}\right)+x^7\left(\frac{c}{7\,b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{7\,b}\right)$$
$$+\frac{f\,x^{16}}{16\,b}-\frac{a^{7/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{19/3}}$$
$$+\frac{a^2\,x\left(\frac{c}{b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b^2}-\frac{a\,x^4\left(\frac{c}{b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{4\,b}$$
$$-\frac{a^{7/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{19/3}}$$
$$+\frac{a^{7/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{19/3}}$$

```
[In] int((x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^13*(e/(13*b) - (a*f)/(13*b^2)) + x^10*(d/(10*b) - (a*(e/b - (a*f)/b^2))/(
10*b)) + x^7*(c/(7*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(7*b)) + (f*x^1
6)/(16*b) - (a^(7/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^
2*b*e))/(3*b^(19/3)) + (a^2*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b)
)/b^2 - (a*x^4*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(4*b) - (a^(7
/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*
(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(19/3)) + (a^(7/3)*log(3^(1/2)*a^
(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a
*b^2*d + a^2*b*e))/(3*b^(19/3))
```

3.234 $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 316

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3} dx = -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4}+\frac{(b^2d-abe+a^2f)\,x^8}{8b^3} + \frac{(be-af)x^{11}}{11b^2}+\frac{fx^{14}}{14b}-\frac{a^{5/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{17/3}} - \frac{a^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{17/3}} + \frac{a^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{17/3}}$$

```
[Out] -1/2*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/b^5+1/5*(-a^3*f+a^2*b*e-a*b^2*d+b
^3*c)*x^5/b^4+1/8*(a^2*f-a*b*e+b^2*d)*x^8/b^3+1/11*(-a*f+b*e)*x^11/b^2+1/14
*f*x^14/b-1/3*a^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/
b^(17/3)+1/6*a^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/b^(17/3)-1/3*a^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arct
an(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(17/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1850, 1502, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{x^8(a^2f-abe+b^2d)}{8b^3} - \frac{ax^2(a^3(-f)+a^2be-ab^2d+b^3c)}{2b^5} + \frac{x^5(a^3(-f)+a^2be-ab^2d+b^3c)}{5b^4} - \frac{a^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}b^{17/3}} + \frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^{17/3}} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^{17/3}} + \frac{x^{11}(be-af)}{11b^2} + \frac{fx^{14}}{14b}$$

```
[In] Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -1/2*(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/b^5 + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^8)/(8*b^3) + ((b
*e - a*f)*x^11)/(11*b^2) + (f*x^14)/(14*b) - (a^(5/3)*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
b^(17/3)) - (a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1
/3)*x])/(3*b^(17/3)) + (a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(17/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{fx^{14}}{14b} + \frac{\int \frac{x^7(14bc+14bdx^3+14(be-af)x^6)}{a+bx^3}\,dx}{14b}$$

$$= \frac{fx^{14}}{14b} + \frac{\int \left(-\frac{14a(b^3c-ab^2d+a^2be-a^3f)x}{b^4} + \frac{14(b^3c-ab^2d+a^2be-a^3f)x^4}{b^3} + \frac{14(b^2d-abe+a^2f)x^7}{b^2} + \frac{14(be-af)x^{10}}{b} - \frac{14(-a^2b^3c+a}{b^4(}\right.}{14b}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)x^2}{2b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)x^5}{5b^4} + \frac{(b^2d-abe+a^2f)x^8}{8b^3} + \frac{(be-af)x^{11}}{11b^2} + \frac{fx^{14}}{14b} + \frac{(a^2(b^3c-ab^2d+a^2be-a^3f))\int \frac{x}{a+bx^3}\,dx}{b^5}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)x^2}{2b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)x^5}{5b^4} + \frac{(b^2d-abe+a^2f)x^8}{8b^3} + \frac{(be-af)x^{11}}{11b^2} + \frac{fx^{14}}{14b} - \frac{\left(a^{5/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{16/3}} + \frac{\left(a^{5/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b^{16/3}}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)x^2}{2b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)x^5}{5b^4} + \frac{(b^2d-abe+a^2f)x^8}{8b^3} + \frac{(be-af)x^{11}}{11b^2} + \frac{fx^{14}}{14b} - \frac{a^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{17/3}} + \frac{\left(a^{5/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{17/3}} + \frac{(a^2(b^3c-ab^2d+a^2be-a^3f))\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^{16/3}}$$

$$= -\frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^2}{2b^5} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^5}{5b^4} + \frac{(b^2d - abe + a^2f)\,x^8}{8b^3} + \frac{(be - af)x^{11}}{11b^2} + \frac{fx^{14}}{14b} - \frac{a^{5/3}(b^3c - ab^2d + a^2be - a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3b^{17/3}} + \frac{a^{5/3}(b^3c - ab^2d + a^2be - a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6b^{17/3}} + \frac{\left(a^{5/3}(b^3c - ab^2d + a^2be - a^3f)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{17/3}}$$

$$= -\frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^2}{2b^5} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^5}{5b^4} + \frac{(b^2d - abe + a^2f)\,x^8}{8b^3} + \frac{(be - af)x^{11}}{11b^2} + \frac{fx^{14}}{14b} - \frac{a^{5/3}(b^3c - ab^2d + a^2be - a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{17/3}} - \frac{a^{5/3}(b^3c - ab^2d + a^2be - a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3b^{17/3}} + \frac{a^{5/3}(b^3c - ab^2d + a^2be - a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6b^{17/3}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.98

$$\int \frac{x^7(c + dx^3 + ex^6 + fx^9)}{a + bx^3}\,dx = \frac{a(-b^3c + ab^2d - a^2be + a^3f)\,x^2}{2b^5} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^5}{5b^4} + \frac{(b^2d - abe + a^2f)\,x^8}{8b^3} + \frac{(be - af)x^{11}}{11b^2} + \frac{fx^{14}}{14b} + \frac{a^{5/3}(-b^3c + ab^2d - a^2be + a^3f)\arctan\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^{17/3}} + \frac{a^{5/3}(-b^3c + ab^2d - a^2be + a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3b^{17/3}} - \frac{a^{5/3}(-b^3c + ab^2d - a^2be + a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6b^{17/3}}$$

[In] Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]

[Out] (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(2*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^8)/(8*b^3) + ((b*e - a*f)*x^11)/(11*b^2) + (f*x^14)/(14*b) + (a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(17/3)) + (a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(17/3)) - (a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(17/3))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.65

method	result
risch	$\frac{f x^{14}}{14b}-\frac{x^{11}af}{11b^2}+\frac{e x^{11}}{11b}+\frac{x^8a^2f}{8b^3}-\frac{ae x^8}{8b^2}+\frac{x^8d}{8b}-\frac{x^5a^3f}{5b^4}+\frac{a^2e x^5}{5b^3}-\frac{x^5ad}{5b^2}+\frac{x^5c}{5b}+\frac{x^2a^4f}{2b^5}-\frac{a^3e x^2}{2b^4}+\frac{x^2a^2d}{2b^3}-\frac{a}{2}$
default	$\frac{\frac{f x^{14}b^4}{14}+\frac{\left(-a b^3f+b^4e\right)x^{11}}{11}+\frac{\left(a^2b^2f-a b^3e+b^4d\right)x^8}{8}+\frac{\left(-a^3bf+a^2e b^2-a b^3d+b^4c\right)x^5}{5}+\frac{\left(a^4f-a^3be+a^2b^2d-a b^3c\right)x^2}{2}}{b^5}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{}$

[In] int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 1/14*f*x^14/b-1/11/b^2*x^11*a*f+1/11/b*e*x^11+1/8/b^3*x^8*a^2*f-1/8/b^2*a*e*x^8+1/8/b*x^8*d-1/5/b^4*x^5*a^3*f+1/5/b^3*a^2*e*x^5-1/5/b^2*x^5*a*d+1/5/b*x^5*c+1/2/b^5*x^2*a^4*f-1/2/b^4*a^3*e*x^2+1/2/b^3*x^2*a^2*d-1/2/b^2*a*c*x^2+1/3/b^6*a^2*sum((-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.02

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$

$$= \frac{660\,b^4fx^{14}+840\,(b^4e-ab^3f)x^{11}+1155\,(b^4d-ab^3e+a^2b^2f)x^8+1848\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^5-}{}$$

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/9240*(660*b^4*f*x^14 + 840*(b^4*e - a*b^3*f)*x^11 + 1155*(b^4*d - a*b^3*e
 + a^2*b^2*f)*x^8 + 1848*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^5 - 4620
*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^2 + 3080*sqrt(3)*(a*b^3*c - a^2*
b^2*d + a^3*b*e - a^4*f)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2
)^(1/3) - sqrt(3)*a)/a) + 1540*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(a^2
/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) - 3080*(a*
b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^
(2/3)))/b^5
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.62

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^{11}\left(-\frac{af}{11b^2}+\frac{e}{11b}\right)+x^8\left(\frac{a^2f}{8b^3}-\frac{ae}{8b^2}+\frac{d}{8b}\right)$$
$$+x^5\left(-\frac{a^3f}{5b^4}+\frac{a^2e}{5b^3}-\frac{ad}{5b^2}+\frac{c}{5b}\right)+x^2\left(\frac{a^4f}{2b^5}-\frac{a^3e}{2b^4}+\frac{a^2d}{2b^3}-\frac{ac}{2b^2}\right)$$
$$+\mathrm{RootSum}\left(27t^3b^{17}-a^{14}f^3+3a^{13}bef^2-3a^{12}b^2df^2-3a^{12}b^2e^2f+3a^{11}b^3cf^2+6a^{11}b^3def+a^{11}b^3e^3-6\right.$$
$$+\frac{fx^{14}}{14b}$$

```
[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**11*(-a*f/(11*b**2) + e/(11*b)) + x**8*(a**2*f/(8*b**3) - a*e/(8*b**2) +
d/(8*b)) + x**5*(-a**3*f/(5*b**4) + a**2*e/(5*b**3) - a*d/(5*b**2) + c/(5*b
)) + x**2*(a**4*f/(2*b**5) - a**3*e/(2*b**4) + a**2*d/(2*b**3) - a*c/(2*b**
2)) + RootSum(27*_t**3*b**17 - a**14*f**3 + 3*a**13*b*e*f**2 - 3*a**12*b**2
*d*f**2 - 3*a**12*b**2*e**2*f + 3*a**11*b**3*c*f**2 + 6*a**11*b**3*d*e*f +
a**11*b**3*e**3 - 6*a**10*b**4*c*e*f - 3*a**10*b**4*d**2*f - 3*a**10*b**4*d
*e**2 + 6*a**9*b**5*c*d*f + 3*a**9*b**5*c*e**2 + 3*a**9*b**5*d**2*e - 3*a**
```

```
8*b**6*c**2*f - 6*a**8*b**6*c*d*e - a**8*b**6*d**3 + 3*a**7*b**7*c**2*e + 3
*a**7*b**7*c*d**2 - 3*a**6*b**8*c**2*d + a**5*b**9*c**3, Lambda(_t, _t*log(
9*_t**2*b**11/(a**9*f**2 - 2*a**8*b*e*f + 2*a**7*b**2*d*f + a**7*b**2*e**2
- 2*a**6*b**3*c*f - 2*a**6*b**3*d*e + 2*a**5*b**4*c*e + a**5*b**4*d**2 - 2*
a**4*b**5*c*d + a**3*b**6*c**2) + x))) + f*x**14/(14*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.99

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{\sqrt{3}(a^2b^3c-a^3b^2d+a^4be-a^5f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{220\,b^4fx^{14}+280\,(b^4e-ab^3f)x^{11}+385\,(b^4d-ab^3e+a^2b^2f)x^8+616\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^5-1}{3080\,b^5} + \frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(1/3)) + 1/3080*(220*b^4*f*x^14 +
 280*(b^4*e - a*b^3*f)*x^11 + 385*(b^4*d - a*b^3*e + a^2*b^2*f)*x^8 + 616*(
b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^5 - 1540*(a*b^3*c - a^2*b^2*d + a^
3*b*e - a^4*f)*x^2)/b^5 + 1/6*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*log
(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^6*(a/b)^(1/3)) - 1/3*(a^2*b^3*c - a^
3*b^2*d + a^4*b*e - a^5*f)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.37

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx =$$
$$-\frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}ab^3c-(-ab^2)^{\frac{2}{3}}a^2b^2d+(-ab^2)^{\frac{2}{3}}a^3be-(-ab^2)^{\frac{2}{3}}a^4f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^7}$$
$$+\frac{\left((-ab^2)^{\frac{2}{3}}ab^3c-(-ab^2)^{\frac{2}{3}}a^2b^2d+(-ab^2)^{\frac{2}{3}}a^3be-(-ab^2)^{\frac{2}{3}}a^4f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^7}$$
$$-\frac{\left(a^2b^{12}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^{11}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^4b^{10}e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^5b^9f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{14}}$$
$$+\frac{220b^{13}fx^{14}+280b^{13}ex^{11}-280ab^{12}fx^{11}+385b^{13}dx^8-385ab^{12}ex^8+385a^2b^{11}fx^8+616b^{13}cx^5-616}{308}$$

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*a*b^3*c - (-a*b^2)^(2/3)*a^2*b^2*d + (-a*b^2)^
(2/3)*a^3*b*e - (-a*b^2)^(2/3)*a^4*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/b^7 + 1/6*((-a*b^2)^(2/3)*a*b^3*c - (-a*b^2)^(2/3)*a^2*b^2
*d + (-a*b^2)^(2/3)*a^3*b*e - (-a*b^2)^(2/3)*a^4*f)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/b^7 - 1/3*(a^2*b^12*c*(-a/b)^(1/3) - a^3*b^11*d*(-a/b)^(1
/3) + a^4*b^10*e*(-a/b)^(1/3) - a^5*b^9*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(ab
s(x - (-a/b)^(1/3)))/(a*b^14) + 1/3080*(220*b^13*f*x^14 + 280*b^13*e*x^11 -
 280*a*b^12*f*x^11 + 385*b^13*d*x^8 - 385*a*b^12*e*x^8 + 385*a^2*b^11*f*x^8
 + 616*b^13*c*x^5 - 616*a*b^12*d*x^5 + 616*a^2*b^11*e*x^5 - 616*a^3*b^10*f*
x^5 - 1540*a*b^12*c*x^2 + 1540*a^2*b^11*d*x^2 - 1540*a^3*b^10*e*x^2 + 1540*
a^4*b^9*f*x^2)/b^14
```

Mupad [B] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.99

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$

$$= x^{11}\left(\frac{e}{11\,b}-\frac{a\,f}{11\,b^2}\right)+x^8\left(\frac{d}{8\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{8\,b}\right)+x^5\left(\frac{c}{5\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{5\,b}\right)+\frac{f\,x^{14}}{14\,b}$$
$$-\frac{a^{5/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{17/3}}-\frac{a\,x^2\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{2\,b}$$
$$+\frac{a^{5/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{17/3}}$$
$$-\frac{a^{5/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{17/3}}$$

```
[In] int((x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^11*(e/(11*b) - (a*f)/(11*b^2)) + x^8*(d/(8*b) - (a*(e/b - (a*f)/b^2))/(8*
b)) + x^5*(c/(5*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(5*b)) + (f*x^14)/
(14*b) - (a^(5/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b
*e))/(3*b^(17/3)) - (a*x^2*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(
2*b) + (a^(5/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1
i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(17/3)) - (a^(5/3)*lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c
 - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(17/3))
```

3.235 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 312

$$\begin{aligned}\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx &= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4}+\frac{(b^2d-abe+a^2f)\,x^7}{7b^3}\\&+\frac{(be-af)x^{10}}{10b^2}+\frac{fx^{13}}{13b}-\frac{a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{16/3}}\\&+\frac{a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{16/3}}\\&-\frac{a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{16/3}}\end{aligned}$$

```
[Out] -a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^5+1/4*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
x^4/b^4+1/7*(a^2*f-a*b*e+b^2*d)*x^7/b^3+1/10*(-a*f+b*e)*x^10/b^2+1/13*f*x^1
3/b+1/3*a^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(16/
3)-1/6*a^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/b^(16/3)-1/3*a^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3
*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(16/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1850, 1502, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$=\frac{x^7(a^2f-abe+b^2d)}{7b^3}-\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{b^5}+\frac{x^4(a^3(-f)+a^2be-ab^2d+b^3c)}{4b^4}-\frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}b^{16/3}}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^{16/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^{16/3}}+\frac{x^{10}(be-af)}{10b^2}+\frac{fx^{13}}{13b}$$

```
[In] Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e -
 a*f)*x^10)/(10*b^2) + (f*x^13)/(13*b) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1
6/3)) + (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(16/3)) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(16/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{fx^{13}}{13b} + \frac{\int \frac{x^6(13bc+13bdx^3+13(be-af)x^6)}{a+bx^3}\,dx}{13b}$$

$$= \frac{fx^{13}}{13b} + \frac{\int \left(-\frac{13a(b^3c-ab^2d+a^2be-a^3f)}{b^4} + \frac{13(b^3c-ab^2d+a^2be-a^3f)x^3}{b^3} + \frac{13(b^2d-abe+a^2f)x^6}{b^2} + \frac{13(be-af)x^9}{b} - \frac{13(-a^2b^3c+a^3b}{b^4(a+}\right.}{13b}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4} + \frac{(b^2d-abe+a^2f)\,x^7}{7b^3} + \frac{(be-af)x^{10}}{10b^2} + \frac{fx^{13}}{13b} + \frac{(a^2(b^3c-ab^2d+a^2be-a^3f))\int \frac{1}{a+bx^3}\,dx}{b^5}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4} + \frac{(b^2d-abe+a^2f)\,x^7}{7b^3} + \frac{(be-af)x^{10}}{10b^2} + \frac{fx^{13}}{13b} + \frac{\left(a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^5} + \frac{\left(a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b^5}$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4} + \frac{(b^2d-abe+a^2f)\,x^7}{7b^3} + \frac{(be-af)x^{10}}{10b^2} + \frac{fx^{13}}{13b} + \frac{a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{16/3}} - \frac{\left(a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{16/3}} + \frac{\left(a^{5/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^5}$$

$$
\begin{aligned}
&= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4}\\
&+\frac{(b^2d-abe+a^2f)\,x^7}{7b^3}+\frac{(be-af)x^{10}}{10b^2}+\frac{fx^{13}}{13b}\\
&+\frac{a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{16/3}}\\
&-\frac{a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{16/3}}\\
&+\frac{\left(a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{16/3}}\\
&= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4}\\
&+\frac{(b^2d-abe+a^2f)\,x^7}{7b^3}+\frac{(be-af)x^{10}}{10b^2}+\frac{fx^{13}}{13b}\\
&-\frac{a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{16/3}}\\
&+\frac{a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{16/3}}\\
&-\frac{a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{16/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.98

$$
\begin{aligned}
&\int\frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx\\
&= \frac{a(-b^3c+ab^2d-a^2be+a^3f)\,x}{b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4}+\frac{(b^2d-abe+a^2f)\,x^7}{7b^3}\\
&+\frac{(be-af)x^{10}}{10b^2}+\frac{fx^{13}}{13b}+\frac{a^{4/3}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^{16/3}}\\
&-\frac{a^{4/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{16/3}}\\
&+\frac{a^{4/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{16/3}}
\end{aligned}
$$

```
[In] Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^5 + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e -
 a*f)*x^10)/(10*b^2) + (f*x^13)/(13*b) + (a^(4/3)*(-(b^3*c) + a*b^2*d - a^2
*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(16/3
)) - (a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(16/3)) + (a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(16/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.62

method	result
risch	$\frac{f\,x^{13}}{13b}-\frac{x^{10}af}{10b^{2}}+\frac{e\,x^{10}}{10b}+\frac{x^{7}a^{2}f}{7b^{3}}-\frac{ae\,x^{7}}{7b^{2}}+\frac{x^{7}d}{7b}-\frac{x^{4}a^{3}f}{4b^{4}}+\frac{a^{2}e\,x^{4}}{4b^{3}}-\frac{x^{4}ad}{4b^{2}}+\frac{x^{4}c}{4b}+\frac{x\,a^{4}f}{b^{5}}-\frac{a^{3}ex}{b^{4}}+\frac{x\,a^{2}d}{b^{3}}-\frac{acx}{b^{2}}$ …
default	$\frac{\frac{1}{13}f\,x^{13}b^{4}-\frac{1}{10}x^{10}a\,b^{3}f+\frac{1}{10}x^{10}b^{4}e+\frac{1}{7}x^{7}a^{2}b^{2}f-\frac{1}{7}x^{7}a\,b^{3}e+\frac{1}{7}b^{4}d\,x^{7}-\frac{1}{4}a^{3}bf\,x^{4}+\frac{1}{4}a^{2}b^{2}e\,x^{4}-\frac{1}{4}a\,b^{3}d\,x^{4}+\frac{1}{4}b^{4}c\,x^{4}+a^{4}fx-a^{3}bex+a^{2}b^{2}d \dots}{b^{5}}$ …

```
[In] int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*f*x^13/b-1/10/b^2*x^10*a*f+1/10/b*e*x^10+1/7/b^3*x^7*a^2*f-1/7/b^2*a*e
*x^7+1/7/b*x^7*d-1/4/b^4*x^4*a^3*f+1/4/b^3*a^2*e*x^4-1/4/b^2*x^4*a*d+1/4/b*
x^4*c+1/b^5*x*a^4*f-1/b^4*a^3*e*x+1/b^3*x*a^2*d-1/b^2*a*c*x+1/3/b^6*a^2*sum
((-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.97

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$=\frac{420\,b^4fx^{13}+546\,(b^4e-ab^3f)x^{10}+780\,(b^4d-ab^3e+a^2b^2f)x^7+1365\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^4-1}{}$$

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/5460*(420*b^4*f*x^13 + 546*(b^4*e - a*b^3*f)*x^10 + 780*(b^4*d - a*b^3*e
+ a^2*b^2*f)*x^7 + 1365*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^4 - 1820*
sqrt(3)*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(-a/b)^(1/3)*arctan(1/3*(2*
sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*(a*b^3*c - a^2*b^2*d + a^3*b
*e - a^4*f)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 1820*(a
*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) -
5460*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.36

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^{10}\left(-\frac{af}{10b^2}+\frac{e}{10b}\right)+x^7\left(\frac{a^2f}{7b^3}-\frac{ae}{7b^2}+\frac{d}{7b}\right)$$
$$+x^4\left(-\frac{a^3f}{4b^4}+\frac{a^2e}{4b^3}-\frac{ad}{4b^2}+\frac{c}{4b}\right)+x\left(\frac{a^4f}{b^5}-\frac{a^3e}{b^4}+\frac{a^2d}{b^3}-\frac{ac}{b^2}\right)$$
$$+\mathrm{RootSum}\left(27t^3b^{16}+a^{13}f^3-3a^{12}bef^2+3a^{11}b^2df^2+3a^{11}b^2e^2f-3a^{10}b^3cf^2-6a^{10}b^3def-a^{10}b^3e^3+6\right.$$
$$+\frac{fx^{13}}{13b}$$

```
[In] integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**10*(-a*f/(10*b**2) + e/(10*b)) + x**7*(a**2*f/(7*b**3) - a*e/(7*b**2) +
d/(7*b)) + x**4*(-a**3*f/(4*b**4) + a**2*e/(4*b**3) - a*d/(4*b**2) + c/(4*b
)) + x*(a**4*f/b**5 - a**3*e/b**4 + a**2*d/b**3 - a*c/b**2) + RootSum(27*_t
**3*b**16 + a**13*f**3 - 3*a**12*b*e*f**2 + 3*a**11*b**2*d*f**2 + 3*a**11*b
**2*e**2*f - 3*a**10*b**3*c*f**2 - 6*a**10*b**3*d*e*f - a**10*b**3*e**3 + 6
*a**9*b**4*c*e*f + 3*a**9*b**4*d**2*f + 3*a**9*b**4*d*e**2 - 6*a**8*b**5*c*
d*f - 3*a**8*b**5*c*e**2 - 3*a**8*b**5*d**2*e + 3*a**7*b**6*c**2*f + 6*a**7
*b**6*c*d*e + a**7*b**6*d**3 - 3*a**6*b**7*c**2*e - 3*a**6*b**7*c*d**2 + 3*
a**5*b**8*c**2*d - a**4*b**9*c**3, Lambda(_t, _t*log(-3*_t*b**5/(a**4*f - a
**3*b*e + a**2*b**2*d - a*b**3*c) + x))) + f*x**13/(13*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{140\,b^4fx^{13}+182\,(b^4e-ab^3f)x^{10}+260\,(b^4d-ab^3e+a^2b^2f)x^7+455\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^4-182}{1820\,b^5}$$
$$+\frac{\sqrt{3}(a^2b^3c-a^3b^2d+a^4be-a^5f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/1820*(140*b^4*f*x^13 + 182*(b^4*e - a*b^3*f)*x^10 + 260*(b^4*d - a*b^3*e
+ a^2*b^2*f)*x^7 + 455*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^4 - 1820*(
a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5 + 1/3*sqrt(3)*(a^2*b^3*c - a^
3*b^2*d + a^4*b*e - a^5*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(b^6*(a/b)^(2/3)) - 1/6*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^6*(a/b)^(2/3)) + 1/3*(a^2*b^3*c - a^3*
b^2*d + a^4*b*e - a^5*f)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.26

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$=\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ab^3c-\left(-ab^2\right)^{\frac{1}{3}}a^2b^2d+\left(-ab^2\right)^{\frac{1}{3}}a^3be-\left(-ab^2\right)^{\frac{1}{3}}a^4f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^6}$$
$$+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^3c-\left(-ab^2\right)^{\frac{1}{3}}a^2b^2d+\left(-ab^2\right)^{\frac{1}{3}}a^3be-\left(-ab^2\right)^{\frac{1}{3}}a^4f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^6}$$
$$-\frac{\left(a^2b^{11}c-a^3b^{10}d+a^4b^9e-a^5b^8f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{13}}$$
$$+\frac{140\,b^{12}fx^{13}+182\,b^{12}ex^{10}-182\,ab^{11}fx^{10}+260\,b^{12}dx^7-260\,ab^{11}ex^7+260\,a^2b^{10}fx^7+455\,b^{12}cx^4-455}{1820\,b}$$

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*a*b^3*c - (-a*b^2)^(1/3)*a^2*b^2*d + (-a*b^2)^(
1/3)*a^3*b*e - (-a*b^2)^(1/3)*a^4*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/b^6 + 1/6*((-a*b^2)^(1/3)*a*b^3*c - (-a*b^2)^(1/3)*a^2*b^2*
d + (-a*b^2)^(1/3)*a^3*b*e - (-a*b^2)^(1/3)*a^4*f)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/b^6 - 1/3*(a^2*b^11*c - a^3*b^10*d + a^4*b^9*e - a^5*b^8*f
)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^13) + 1/1820*(140*b^12*f*x^1
3 + 182*b^12*e*x^10 - 182*a*b^11*f*x^10 + 260*b^12*d*x^7 - 260*a*b^11*e*x^7
 + 260*a^2*b^10*f*x^7 + 455*b^12*c*x^4 - 455*a*b^11*d*x^4 + 455*a^2*b^10*e*
x^4 - 455*a^3*b^9*f*x^4 - 1820*a*b^11*c*x + 1820*a^2*b^10*d*x - 1820*a^3*b^
9*e*x + 1820*a^4*b^8*f*x)/b^13
```

Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$

$$= x^{10}\left(\frac{e}{10\,b}-\frac{a\,f}{10\,b^2}\right)+x^7\left(\frac{d}{7\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{7\,b}\right)+x^4\left(\frac{c}{4\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{4\,b}\right)+\frac{f\,x^{13}}{13\,b}$$
$$+\frac{a^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{16/3}}-\frac{a\,x\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b}$$
$$+\frac{a^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{16/3}}$$
$$-\frac{a^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{16/3}}$$

```
[In] int((x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^10*(e/(10*b) - (a*f)/(10*b^2)) + x^7*(d/(7*b) - (a*(e/b - (a*f)/b^2))/(7*
b)) + x^4*(c/(4*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(4*b)) + (f*x^13)/
(13*b) + (a^(4/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b
*e))/(3*b^(16/3)) - (a*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/b +
 (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 -
 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(16/3)) - (a^(4/3)*log(3^(1
/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3
*f - a*b^2*d + a^2*b*e))/(3*b^(16/3))
```

3.236 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 279

$$\begin{aligned}\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx &= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^4}+\frac{(b^2d-abe+a^2f)\,x^5}{5b^3}+\frac{(be-af)x^8}{8b^2}\\&\quad+\frac{fx^{11}}{11b}+\frac{a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{14/3}}\\&\quad+\frac{a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{14/3}}\\&\quad-\frac{a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{14/3}}\end{aligned}$$

```
[Out] 1/2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/b^4+1/5*(a^2*f-a*b*e+b^2*d)*x^5/b^3+
1/8*(-a*f+b*e)*x^8/b^2+1/11*f*x^11/b+1/3*a^(2/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^
3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(14/3)-1/6*a^(2/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3
*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(14/3)+1/3*a^(2/3)*(-a^3*f+
a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^
(14/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1850, 1502, 298, 31, 648, 631, 210, 642}

$$\begin{aligned}
&\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx \\
&= \frac{x^5(a^2f-abe+b^2d)}{5b^3}+\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{2b^4} \\
&\quad+\frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}b^{14/3}} \\
&\quad-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^{14/3}} \\
&\quad+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^{14/3}}+\frac{x^8(be-af)}{8b^2}+\frac{fx^{11}}{11b}
\end{aligned}$$

```
[In] Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^4) + ((b^2*d - a*b*e + a^2*f
)*x^5)/(5*b^3) + ((b*e - a*f)*x^8)/(8*b^2) + (f*x^11)/(11*b) + (a^(2/3)*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*b^(14/3)) + (a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
Log[a^(1/3) + b^(1/3)*x])/(3*b^(14/3)) - (a^(2/3)*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{fx^{11}}{11b} + \frac{\int \frac{x^4(11bc+11bdx^3+11(be-af)x^6)}{a+bx^3}\,dx}{11b} \\ &= \frac{fx^{11}}{11b} \\ &\quad + \frac{\int \left(\frac{11(b^3c-ab^2d+a^2be-a^3f)x}{b^3} + \frac{11(b^2d-abe+a^2f)x^4}{b^2} + \frac{11(be-af)x^7}{b} + \frac{11(-ab^3c+a^2b^2d-a^3be+a^4f)x}{b^3(a+bx^3)}\right)dx}{11b}\end{aligned}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^4}+\frac{(b^2d-abe+a^2f)\,x^5}{5b^3} + \frac{(be-af)x^8}{8b^2}+\frac{fx^{11}}{11b}-\frac{(a(b^3c-ab^2d+a^2be-a^3f))\int\frac{x}{a+bx^3}\,dx}{b^4}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^4}+\frac{(b^2d-abe+a^2f)\,x^5}{5b^3}+\frac{(be-af)x^8}{8b^2} + \frac{fx^{11}}{11b}+\frac{\left(a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{13/3}} - \frac{\left(a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b^{13/3}}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^4}+\frac{(b^2d-abe+a^2f)\,x^5}{5b^3}+\frac{(be-af)x^8}{8b^2} + \frac{fx^{11}}{11b}+\frac{a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{14/3}} - \frac{\left(a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{14/3}} - \frac{(a(b^3c-ab^2d+a^2be-a^3f))\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^{13/3}}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^4}+\frac{(b^2d-abe+a^2f)\,x^5}{5b^3}+\frac{(be-af)x^8}{8b^2} + \frac{fx^{11}}{11b}+\frac{a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{14/3}} - \frac{a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{14/3}} - \frac{\left(a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{14/3}}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^4}+\frac{(b^2d-abe+a^2f)\,x^5}{5b^3}+\frac{(be-af)x^8}{8b^2} + \frac{fx^{11}}{11b}+\frac{a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{14/3}} + \frac{a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{14/3}} - \frac{a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{14/3}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.95

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$

$$= \frac{660b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\,x^2+264b^{5/3}(b^2d-abe+a^2f)\,x^5+165b^{8/3}(be-af)x^8+120b^{11/3}fx^{11}\cdots}{\cdots}$$

[In] Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]

[Out] (660*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2 + 264*b^(5/3)*(b^2*d - a*b*e + a^2*f)*x^5 + 165*b^(8/3)*(b*e - a*f)*x^8 + 120*b^(11/3)*f*x^11 - 440*Sqrt[3]*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 440*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1320*b^(14/3))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.56

method	result
risch	$\frac{f x^{11}}{11b}-\frac{x^8 f a}{8b^2}+\frac{x^8 e}{8b}+\frac{x^5 f a^2}{5b^3}-\frac{x^5 a e}{5b^2}+\frac{d x^5}{5b}-\frac{x^2 f a^3}{2b^4}+\frac{x^2 a^2 e}{2b^3}-\frac{x^2 a d}{2b^2}+\frac{c x^2}{2b}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(f a^3-a\cdots\right.}{\cdots}\right)}{3b^5}\cdots$
default	$-\frac{-\frac{b^3 f x^{11}}{11}+\frac{\left(f a b^2-b^3 e\right)x^8}{8}+\frac{\left(-f a^2 b+a b^2 e-b^3 d\right)x^5}{5}+\frac{x^2\left(f a^3-a^2 b e+a b^2 d-b^3 c\right)}{2}}{b^4}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\cdots\right)}{\cdots}$

[In] int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 1/11*f*x^11/b-1/8/b^2*x^8*f*a+1/8/b*x^8*e+1/5/b^3*x^5*f*a^2-1/5/b^2*x^5*a*e+1/5*d*x^5/b-1/2/b^4*x^2*f*a^3+1/2/b^3*x^2*a^2*e-1/2/b^2*x^2*a*d+1/2*c*x^2/

```
b+1/3/b^5*a*sum((a^3*f-a^2*b*e+a*b^2*d-b^3*c)/_R*ln(x-_R),_R=RootOf(_Z^3*b+
a))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.01

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{120\,b^3fx^{11}+165\,(b^3e-ab^2f)x^8+264\,(b^3d-ab^2e+a^2bf)x^5+660\,(b^3c-ab^2d+a^2be-a^3f)x^2-440\,\sqrt{3}}{}$$

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/1320*(120*b^3*f*x^11 + 165*(b^3*e - a*b^2*f)*x^8 + 264*(b^3*d - a*b^2*e +
 a^2*b*f)*x^5 + 660*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2 - 440*sqrt(3)*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b
*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1/3))
- 440*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^
2/b^2)^(2/3)))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.68

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= x^8\left(-\frac{af}{8b^2}+\frac{e}{8b}\right)+x^5\left(\frac{a^2f}{5b^3}-\frac{ae}{5b^2}+\frac{d}{5b}\right)+x^2\left(-\frac{a^3f}{2b^4}+\frac{a^2e}{2b^3}-\frac{ad}{2b^2}+\frac{c}{2b}\right)$$
$$+\mathrm{RootSum}\left(27t^3b^{14}+a^{11}f^3-3a^{10}bef^2+3a^9b^2df^2+3a^9b^2e^2f-3a^8b^3cf^2-6a^8b^3def-a^8b^3e^3+6a^7b^4c\right.$$
$$+\frac{fx^{11}}{11b}$$

```
[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**8*(-a*f/(8*b**2) + e/(8*b)) + x**5*(a**2*f/(5*b**3) - a*e/(5*b**2) + d/(
5*b)) + x**2*(-a**3*f/(2*b**4) + a**2*e/(2*b**3) - a*d/(2*b**2) + c/(2*b))
+ RootSum(27*_t**3*b**14 + a**11*f**3 - 3*a**10*b*e*f**2 + 3*a**9*b**2*d*f*
*2 + 3*a**9*b**2*e**2*f - 3*a**8*b**3*c*f**2 - 6*a**8*b**3*d*e*f - a**8*b**
```

```
3*e**3 + 6*a**7*b**4*c*e*f + 3*a**7*b**4*d**2*f + 3*a**7*b**4*d*e**2 - 6*a*
*6*b**5*c*d*f - 3*a**6*b**5*c*e**2 - 3*a**6*b**5*d**2*e + 3*a**5*b**6*c**2*
f + 6*a**5*b**6*c*d*e + a**5*b**6*d**3 - 3*a**4*b**7*c**2*e - 3*a**4*b**7*c
*d**2 + 3*a**3*b**8*c**2*d - a**2*b**9*c**3, Lambda(_t, _t*log(9*_t**2*b**9
/(a**7*f**2 - 2*a**6*b*e*f + 2*a**5*b**2*d*f + a**5*b**2*e**2 - 2*a**4*b**3
*c*f - 2*a**4*b**3*d*e + 2*a**3*b**4*c*e + a**3*b**4*d**2 - 2*a**2*b**5*c*d
 + a*b**6*c**2) + x))) + f*x**11/(11*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.96

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = -\frac{\sqrt{3}(ab^3c-a^2b^2d+a^3be-a^4f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{40\,b^3fx^{11}+55\,(b^3e-ab^2f)x^8+88\,(b^3d-ab^2e+a^2bf)x^5+220\,(b^3c-ab^2d+a^2be-a^3f)x^2}{440\,b^4} - \frac{(ab^3c-a^2b^2d+a^3be-a^4f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(ab^3c-a^2b^2d+a^3be-a^4f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(1/3)) + 1/440*(40*b^3*f*x^11 + 55
*(b^3*e - a*b^2*f)*x^8 + 88*(b^3*d - a*b^2*e + a^2*b*f)*x^5 + 220*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*x^2)/b^4 - 1/6*(a*b^3*c - a^2*b^2*d + a^3*b*e -
a^4*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(1/3)) + 1/3*(a*b^
3*c - a^2*b^2*d + a^3*b*e - a^4*f)*log(x + (a/b)^(1/3))/(b^5*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.36

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}\,b^3c-(-ab^2)^{\frac{2}{3}}\,ab^2d+(-ab^2)^{\frac{2}{3}}\,a^2be-(-ab^2)^{\frac{2}{3}}\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^6} - \frac{\left((-ab^2)^{\frac{2}{3}}\,b^3c-(-ab^2)^{\frac{2}{3}}\,ab^2d+(-ab^2)^{\frac{2}{3}}\,a^2be-(-ab^2)^{\frac{2}{3}}\,a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^6} + \frac{\left(ab^{10}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^9d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^3b^8e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^4b^7f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{11}} + \frac{40\,b^{10}fx^{11}+55\,b^{10}ex^8-55\,ab^9fx^8+88\,b^{10}dx^5-88\,ab^9ex^5+88\,a^2b^8fx^5+220\,b^{10}cx^2-220\,ab^9dx^2+22}{440\,b^{11}}$$

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + (-a*b^2)^(2/3)
*a^2*b*e - (-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/b^6 - 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + (-a*
b^2)^(2/3)*a^2*b*e - (-a*b^2)^(2/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b
)^(2/3))/b^6 + 1/3*(a*b^10*c*(-a/b)^(1/3) - a^2*b^9*d*(-a/b)^(1/3) + a^3*b^
8*e*(-a/b)^(1/3) - a^4*b^7*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^
(1/3)))/(a*b^11) + 1/440*(40*b^10*f*x^11 + 55*b^10*e*x^8 - 55*a*b^9*f*x^8 +
 88*b^10*d*x^5 - 88*a*b^9*e*x^5 + 88*a^2*b^8*f*x^5 + 220*b^10*c*x^2 - 220*a
*b^9*d*x^2 + 220*a^2*b^8*e*x^2 - 220*a^3*b^7*f*x^2)/b^11
```

Mupad [B] (verification not implemented)

Time = 10.58 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.96

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= x^8\left(\frac{e}{8\,b}-\frac{a\,f}{8\,b^2}\right)+x^5\left(\frac{d}{5\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{5\,b}\right)+x^2\left(\frac{c}{2\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{2\,b}\right)$$
$$+\frac{f\,x^{11}}{11\,b}+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{14/3}}$$
$$-\frac{a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{14/3}}$$
$$+\frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{14/3}}$$

```
[In] int((x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^8*(e/(8*b) - (a*f)/(8*b^2)) + x^5*(d/(5*b) - (a*(e/b - (a*f)/b^2))/(5*b))
 + x^2*(c/(2*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(2*b)) + (f*x^11)/(11
*b) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e)
)/(3*b^(14/3)) - (a^(2/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*(
(3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(14/3)) + (
a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1
/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(14/3))
```

3.237 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 274

$$\begin{aligned}
&\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx \\
&= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4}+\frac{(b^2d-abe+a^2f)\,x^4}{4b^3}+\frac{(be-af)x^7}{7b^2} \\
&\quad+\frac{fx^{10}}{10b}+\frac{\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{13/3}} \\
&\quad-\frac{\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{13/3}} \\
&\quad+\frac{\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{13/3}}
\end{aligned}$$

```
[Out] (-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^4+1/4*(a^2*f-a*b*e+b^2*d)*x^4/b^3+1/7*(-
a*f+b*e)*x^7/b^2+1/10*f*x^10/b-1/3*a^(1/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*l
n(a^(1/3)+b^(1/3)*x)/b^(13/3)+1/6*a^(1/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(13/3)+1/3*a^(1/3)*(-a^3*f+a^2*b*
e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(13/3)
*3^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1850, 1502, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$=\frac{x^4(a^2f-abe+b^2d)}{4b^3}+\frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}b^{13/3}}$$
$$-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^{13/3}}+\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4}$$
$$+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^{13/3}}+\frac{x^7(be-af)}{7b^2}+\frac{fx^{10}}{10b}$$

```
[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a^2*f)*x^4)
/(4*b^3) + ((b*e - a*f)*x^7)/(7*b^2) + (f*x^10)/(10*b) + (a^(1/3)*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*b^(13/3)) - (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^
(1/3) + b^(1/3)*x])/(3*b^(13/3)) + (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{fx^{10}}{10b}+\frac{\int \frac{x^3(10bc+10bdx^3+10(be-af)x^6)}{a+bx^3}\,dx}{10b}\\
&= \frac{fx^{10}}{10b}+\frac{\int \left(\frac{10(b^3c-ab^2d+a^2be-a^3f)}{b^3}+\frac{10(b^2d-abe+a^2f)x^3}{b^2}+\frac{10(be-af)x^6}{b}+\frac{10(-ab^3c+a^2b^2d-a^3be+a^4f)}{b^3(a+bx^3)}\right)dx}{10b}\\
&= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4}+\frac{(b^2d-abe+a^2f)\,x^4}{4b^3}\\
&\quad+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}-\frac{(a(b^3c-ab^2d+a^2be-a^3f))\int \frac{1}{a+bx^3}\,dx}{b^4}
\end{aligned}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4}+\frac{(b^2d-abe+a^2f)\,x^4}{4b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}-\frac{\left(\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^4}-\frac{\left(\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b^4}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4}+\frac{(b^2d-abe+a^2f)\,x^4}{4b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}-\frac{\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{13/3}}+\frac{\left(\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{13/3}}-\frac{\left(a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^4}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4}+\frac{(b^2d-abe+a^2f)\,x^4}{4b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}-\frac{\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{13/3}}+\frac{\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{13/3}}-\frac{\left(\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{13/3}}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4}+\frac{(b^2d-abe+a^2f)\,x^4}{4b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}+\frac{\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{13/3}}-\frac{\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{13/3}}+\frac{\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{13/3}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.96

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$

$$= \frac{420\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\,x+105b^{4/3}(b^2d-abe+a^2f)\,x^4+60b^{7/3}(be-af)x^7+42b^{10/3}fx^{10}-140}{}$$

[In] Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]

[Out] (420*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x + 105*b^(4/3)*(b^2*d - a
*b*e + a^2*f)*x^4 + 60*b^(7/3)*(b*e - a*f)*x^7 + 42*b^(10/3)*f*x^10 - 140*S
qrt[3]*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)
*Log[a^(1/3) + b^(1/3)*x] - 70*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(420*b^(13/3))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.53

method	result
risch	$\frac{f x^{10}}{10b}-\frac{x^7 f a}{7b^2}+\frac{x^7 e}{7b}+\frac{x^4 f a^2}{4b^3}-\frac{x^4 a e}{4b^2}+\frac{d x^4}{4b}-\frac{x f a^3}{b^4}+\frac{x a^2 e}{b^3}-\frac{x a d}{b^2}+\frac{c x}{b}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(f a^3-a^2be+a b}{}\right.}{3b^5}$
default	$-\frac{-\frac{1}{10}b^3 f x^{10}+\frac{1}{7}x^7 a b^2 f-\frac{1}{7}x^7 b^3 e-\frac{1}{4}a^2 b f x^4+\frac{1}{4}a b^2 e x^4-\frac{1}{4}d x^4 b^3+f a^3 x-a^2 b e x+a b^2 d x-b^3 c x}{b^4}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x\right.}{6b\left(\frac{a}{b}\right)}\right.}{}$

[In] int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 1/10*f*x^10/b-1/7/b^2*x^7*f*a+1/7/b*x^7*e+1/4/b^3*x^4*f*a^2-1/4/b^2*x^4*a*e
+1/4*d*x^4/b-1/b^4*x*f*a^3+1/b^3*x*a^2*e-1/b^2*x*a*d+c*x/b+1/3/b^5*a*sum((a
^3*f-a^2*b*e+a*b^2*d-b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.91

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{42\,b^3fx^{10}+60\,(b^3e-ab^2f)x^7+105\,(b^3d-ab^2e+a^2bf)x^4-140\,\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\left(\frac{a}{b}\right)^{\frac{1}{3}}\operatorname{arcta}}{}$$

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/420*(42*b^3*f*x^10 + 60*(b^3*e - a*b^2*f)*x^7 + 105*(b^3*d - a*b^2*e + a^
2*b*f)*x^4 - 140*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a/b)^(1/3)*ar
ctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.37

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= x^7\left(-\frac{af}{7b^2}+\frac{e}{7b}\right)+x^4\left(\frac{a^2f}{4b^3}-\frac{ae}{4b^2}+\frac{d}{4b}\right)+x\left(-\frac{a^3f}{b^4}+\frac{a^2e}{b^3}-\frac{ad}{b^2}+\frac{c}{b}\right)$$
$$+\operatorname{RootSum}\Big(27t^3b^{13}-a^{10}f^3+3a^9bef^2-3a^8b^2df^2-3a^8b^2e^2f+3a^7b^3cf^2+6a^7b^3def+a^7b^3e^3-6a^6b^4c$$
$$+\frac{fx^{10}}{10b}$$

```
[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**7*(-a*f/(7*b**2) + e/(7*b)) + x**4*(a**2*f/(4*b**3) - a*e/(4*b**2) + d/(
4*b)) + x*(-a**3*f/b**4 + a**2*e/b**3 - a*d/b**2 + c/b) + RootSum(27*_t**3*
b**13 - a**10*f**3 + 3*a**9*b*e*f**2 - 3*a**8*b**2*d*f**2 - 3*a**8*b**2*e**
2*f + 3*a**7*b**3*c*f**2 + 6*a**7*b**3*d*e*f + a**7*b**3*e**3 - 6*a**6*b**4
*c*e*f - 3*a**6*b**4*d**2*f - 3*a**6*b**4*d*e**2 + 6*a**5*b**5*c*d*f + 3*a*
*5*b**5*c*e**2 + 3*a**5*b**5*d**2*e - 3*a**4*b**6*c**2*f - 6*a**4*b**6*c*d*
e - a**4*b**6*d**3 + 3*a**3*b**7*c**2*e + 3*a**3*b**7*c*d**2 - 3*a**2*b**8*
c**2*d + a*b**9*c**3, Lambda(_t, _t*log(3*_t*b**4/(a**3*f - a**2*b*e + a*b*
*2*d - b**3*c) + x))) + f*x**10/(10*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.97

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{14\,b^3fx^{10}+20\,(b^3e-ab^2f)x^7+35\,(b^3d-ab^2e+a^2bf)x^4+140\,(b^3c-ab^2d+a^2be-a^3f)x}{140\,b^4} - \frac{\sqrt{3}(ab^3c-a^2b^2d+a^3be-a^4f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(ab^3c-a^2b^2d+a^3be-a^4f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(ab^3c-a^2b^2d+a^3be-a^4f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/140*(14*b^3*f*x^10 + 20*(b^3*e - a*b^2*f)*x^7 + 35*(b^3*d - a*b^2*e + a^2
*b*f)*x^4 + 140*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 - 1/3*sqrt(3)*(a
*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/(b^5*(a/b)^(2/3)) + 1/6*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4
*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(2/3)) - 1/3*(a*b^3*c
 - a^2*b^2*d + a^3*b*e - a^4*f)*log(x + (a/b)^(1/3))/(b^5*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.24

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = -\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}b^3c-(-ab^2)^{\frac{1}{3}}ab^2d+(-ab^2)^{\frac{1}{3}}a^2be-(-ab^2)^{\frac{1}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^5} - \frac{\left((-ab^2)^{\frac{1}{3}}b^3c-(-ab^2)^{\frac{1}{3}}ab^2d+(-ab^2)^{\frac{1}{3}}a^2be-(-ab^2)^{\frac{1}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^5} + \frac{(ab^9c-a^2b^8d+a^3b^7e-a^4b^6f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{10}} + \frac{14b^9fx^{10}+20b^9ex^7-20ab^8fx^7+35b^9dx^4-35ab^8ex^4+35a^2b^7fx^4+140b^9cx-140ab^8dx+140a^2\cdots}{140b^{10}}$$

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d + (-a*b^2)^(1/3
)*a^2*b*e - (-a*b^2)^(1/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/b^5 - 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d + (-a
*b^2)^(1/3)*a^2*b*e - (-a*b^2)^(1/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3))/b^5 + 1/3*(a*b^9*c - a^2*b^8*d + a^3*b^7*e - a^4*b^6*f)*(-a/b)^(1
/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^10) + 1/140*(14*b^9*f*x^10 + 20*b^9*e*x
^7 - 20*a*b^8*f*x^7 + 35*b^9*d*x^4 - 35*a*b^8*e*x^4 + 35*a^2*b^7*f*x^4 + 14
0*b^9*c*x - 140*a*b^8*d*x + 140*a^2*b^7*e*x - 140*a^3*b^6*f*x)/b^10
```

Mupad [B] (verification not implemented)

Time = 10.12 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.96

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$=x^7\left(\frac{e}{7\,b}-\frac{a\,f}{7\,b^2}\right)+x^4\left(\frac{d}{4\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{4\,b}\right)+x\left(\frac{c}{b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)$$
$$+\frac{f\,x^{10}}{10\,b}-\frac{a^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{13/3}}$$
$$-\frac{a^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{13/3}}$$
$$+\frac{a^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{13/3}}$$

```
[In] int((x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^7*(e/(7*b) - (a*f)/(7*b^2)) + x^4*(d/(4*b) - (a*(e/b - (a*f)/b^2))/(4*b))
 + x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b) + (f*x^10)/(10*b) - (a^(
1/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(13
/3)) - (a^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1
i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(13/3)) + (a^(1/3)*lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c
 - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(13/3))
```

3.238 $\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 245

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{(b^2d-abe+a^2f)\,x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{11/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{11/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{11/3}}$$

```
[Out] 1/2*(a^2*f-a*b*e+b^2*d)*x^2/b^3+1/5*(-a*f+b*e)*x^5/b^2+1/8*f*x^8/b-1/3*(-a^
3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(11/3)+1/6*(-a^3
*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)
/b^(11/3)-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(11/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1850, 1502, 298, 31, 648, 631, 210, 642}

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$

$$=\frac{x^2(a^2f-abe+b^2d)}{2b^3}-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}\sqrt[3]{a}b^{11/3}}$$
$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3\sqrt[3]{a}b^{11/3}}$$
$$+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6\sqrt[3]{a}b^{11/3}}+\frac{x^5(be-af)}{5b^2}+\frac{fx^8}{8b}$$

```
[In] Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x^2)/(2*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^8)
/(8*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(11/3)) - ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(11/3)) + ((b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*
a^(1/3)*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{fx^8}{8b} + \frac{\int \frac{x\left(8bc+8bdx^3+8(be-af)x^6\right)}{a+bx^3}\,dx}{8b} \\
&= \frac{fx^8}{8b} + \frac{\int \left(\frac{8\left(b^2d-abe+a^2f\right)x}{b^2} + \frac{8(be-af)x^4}{b} + \frac{8\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^2(a+bx^3)}\right)dx}{8b} \\
&= \frac{\left(b^2d-abe+a^2f\right)x^2}{2b^3} + \frac{(be-af)x^5}{5b^2} + \frac{fx^8}{8b} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{x}{a+bx^3}\,dx}{b^3}
\end{aligned}
$$

$$= \frac{(b^2d-abe+a^2f)\,x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}b^{10/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}b^{10/3}}$$

$$= \frac{(b^2d-abe+a^2f)\,x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{11/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}b^{11/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^{10/3}}$$

$$= \frac{(b^2d-abe+a^2f)\,x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{11/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{11/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{11/3}}$$

$$= \frac{(b^2d-abe+a^2f)\,x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{11/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{11/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{11/3}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.94

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$

$$= \frac{60b^{2/3}(b^2d-abe+a^2f)\,x^2+24b^{5/3}(be-af)x^5+15b^{8/3}fx^8+\frac{40\sqrt{3}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+}{120b^{11/3}}$$

```
[In] Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (60*b^(2/3)*(b^2*d - a*b*e + a^2*f)*x^2 + 24*b^(5/3)*(b*e - a*f)*x^5 + 15*b
^(8/3)*f*x^8 + (40*Sqrt[3]*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-(b^3*c) + a*b^2*d - a^2
*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (20*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(12
0*b^(11/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.46

method	result
risch	$\frac{f x^8}{8b}-\frac{x^5 a f}{5b^2}+\frac{x^5 e}{5b}+\frac{a^2 f x^2}{2b^3}-\frac{a e x^2}{2b^2}+\frac{d x^2}{2b}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(-f a^3+a^2be-a b^2 d+b^3 c)\ln(x-\_R)}{\_R}}{3b^4}$
default	$\frac{\frac{b^2 f x^8}{8}+\frac{(-afb+b^2e)x^5}{5}+\frac{(a^2f-aeb+b^2d)x^2}{2}}{b^3}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)(f a^3-}{b^3}$

```
[In] int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*f*x^8/b-1/5/b^2*x^5*a*f+1/5/b*x^5*e+1/2/b^3*a^2*f*x^2-1/2/b^2*a*e*x^2+1
/2*d*x^2/b+1/3/b^4*sum((-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/_R*ln(x-_R),_R=RootOf
```

```
(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.32

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$

$$= \left[\frac{15\,ab^4fx^8+24\,(ab^4e-a^2b^3f)x^5+60\,(ab^4d-a^2b^3e+a^3b^2f)x^2-60\,\sqrt{\tfrac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)\sqrt{}}{}\right.$$

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/120*(15*a*b^4*f*x^8 + 24*(a*b^4*e - a^2*b^3*f)*x^5 + 60*(a*b^4*d - a^2*b
^3*e + a^3*b^2*f)*x^2 - 60*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4
*b*f)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*
(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/
3)*x)/(b*x^3 + a)) + 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*l
og(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^5), 1/120*(15*a*
b^4*f*x^8 + 24*(a*b^4*e - a^2*b^3*f)*x^5 + 60*(a*b^4*d - a^2*b^3*e + a^3*b^
2*f)*x^2 - 120*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt((
a*b^2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3
)/a)/b) + 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*log(b^2*x^2
- (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^5)]
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.74

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^5\left(-\frac{af}{5b^2}+\frac{e}{5b}\right)+x^2\left(\frac{a^2f}{2b^3}-\frac{ae}{2b^2}+\frac{d}{2b}\right)$$

$$+\mathrm{RootSum}\left(27t^3ab^{11}-a^9f^3+3a^8bef^2-3a^7b^2df^2-3a^7b^2e^2f+3a^6b^3cf^2+6a^6b^3def+a^6b^3e^3-6a^5b^4ce\right.$$

$$+\frac{fx^8}{8b}$$

```
[In] integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**5*(-a*f/(5*b**2) + e/(5*b)) + x**2*(a**2*f/(2*b**3) - a*e/(2*b**2) + d/(
2*b)) + RootSum(27*_t**3*a*b**11 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**
2*d*f**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a*
*6*b**3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2
+ 6*a**4*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6
*c**2*f - 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*
b**7*c*d**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(9*_t**2*a*b**7
/(a**6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3
*c*f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d +
b**6*c**2) + x))) + f*x**8/(8*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.92

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{5\,b^2fx^8+8\,(b^2e-abf)x^5+20\,(b^2d-abe+a^2f)x^2}{40\,b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) + 1/40*(5*b^2*f*x^8 + 8*(b^2*e -
 a*b*f)*x^5 + 20*(b^2*d - a*b*e + a^2*f)*x^2)/b^3 + 1/6*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3)) -
 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(1
/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.17

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{1}{3}}\,b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{1}{3}}\,b^3} - \frac{\left(b^8c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^7d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^6e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^5f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^8} + \frac{5\,b^7fx^8+8\,b^7ex^5-8\,ab^6fx^5+20\,b^7dx^2-20\,ab^6ex^2+20\,a^2b^5fx^2}{40\,b^8}$$

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^3) - 1/6*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^3
) - 1/3*(b^8*c*(-a/b)^(1/3) - a*b^7*d*(-a/b)^(1/3) + a^2*b^6*e*(-a/b)^(1/3)
 - a^3*b^5*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^8)
+ 1/40*(5*b^7*f*x^8 + 8*b^7*e*x^5 - 8*a*b^6*f*x^5 + 20*b^7*d*x^2 - 20*a*b^6
*e*x^2 + 20*a^2*b^5*f*x^2)/b^8
```

Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.92

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= x^5\left(\frac{e}{5\,b}-\frac{a\,f}{5\,b^2}\right)+x^2\left(\frac{d}{2\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{2\,b}\right)+\frac{f\,x^8}{8\,b} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{1/3}\,b^{11/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{1/3}\,b^{11/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{1/3}\,b^{11/3}}$$

```
[In] int((x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^5*(e/(5*b) - (a*f)/(5*b^2)) + x^2*(d/(2*b) - (a*(e/b - (a*f)/b^2))/(2*b))
 + (f*x^8)/(8*b) - (log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2
*b*e))/(3*a^(1/3)*b^(11/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/
3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(1/3)*
b^(11/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/
2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(1/3)*b^(11/3))
```

3.239 $\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 240

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx =& \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b} \\ &-\frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}} \\ &+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}} \\ &-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}\end{aligned}$$

```
[Out] (a^2*f-a*b*e+b^2*d)*x/b^3+1/4*(-a*f+b*e)*x^4/b^2+1/7*f*x^7/b+1/3*(-a^3*f+a^
2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(10/3)-1/6*(-a^3*f+a^2
*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(10
/3)-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/a^(2/3)/b^(10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used

$= \{1901, 206, 31, 648, 631, 210, 642\}$

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx = \frac{x(a^2f-abe+b^2d)}{b^3} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{2/3}b^{10/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{2/3}b^{10/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{2/3}b^{10/3}} + \frac{x^4(be-af)}{4b^2} + \frac{fx^7}{7b}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^4)/(4*b^2) + (f*x^7)/(7*b)
 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{b^2d-abe+a^2f}{b^3}+\frac{(be-af)x^3}{b^2}+\frac{fx^6}{b}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\,(a+bx^3)}\right)dx\\
&= \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{a+bx^3}\,dx}{b^3}\\
&= \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}\\
&\quad+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b^3}\\
&\quad+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^3}\\
&= \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}\\
&\quad+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}\\
&\quad-\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{10/3}}\\
&\quad+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}b^3}
\end{aligned}$$

$$= \frac{(b^2d - abe + a^2f)\,x}{b^3} + \frac{(be - af)x^4}{4b^2} + \frac{fx^7}{7b} + \frac{(b^3c - ab^2d + a^2be - a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}} - \frac{(b^3c - ab^2d + a^2be - a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}b^{10/3}}$$

$$= \frac{(b^2d - abe + a^2f)\,x}{b^3} + \frac{(be - af)x^4}{4b^2} + \frac{fx^7}{7b} - \frac{(b^3c - ab^2d + a^2be - a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}} + \frac{(b^3c - ab^2d + a^2be - a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}} - \frac{(b^3c - ab^2d + a^2be - a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

$$\int \frac{c + dx^3 + ex^6 + fx^9}{a + bx^3}\,dx$$

$$= \frac{84\sqrt[3]{b}(b^2d - abe + a^2f)\,x + 21b^{4/3}(be - af)x^4 + 12b^{7/3}fx^7 + \frac{28\sqrt{3}(-b^3c + ab^2d - a^2be + a^3f)\arctan\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{28}{}}{84b^{10/3}}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x]
```

```
[Out] (84*b^(1/3)*(b^2*d - a*b*e + a^2*f)*x + 21*b^(4/3)*(b*e - a*f)*x^4 + 12*b^(
7/3)*f*x^7 + (28*Sqrt[3]*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(-(b^3*c) + a*b^2*d - a^2*
b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(84*b
^(10/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.43

method	result
risch	$\frac{f\,x^{7}}{7b}-\frac{x^{4} a f}{4b^{2}}+\frac{x^{4} e}{4b}+\frac{a^{2} f x}{b^{3}}-\frac{a e x}{b^{2}}+\frac{d x}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-f\,a^{3}+a^{2} b e-a\,b^{2} d+b^{3} c\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3b^{4}}$
default	$\frac{\frac{1}{7}b^{2} f\,x^{7}-\frac{1}{4}a b f\,x^{4}+\frac{1}{4}b^{2} e\,x^{4}+a^{2} f x-a b e x+b^{2} d x}{b^{3}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\left(-f\,a^{3}\right.}{b^{3}}$

[In] int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 1/7*f*x^7/b-1/4/b^2*x^4*a*f+1/4/b*x^4*e+1/b^3*a^2*f*x-1/b^2*a*e*x+d*x/b+1/3
/b^4*sum((-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.50

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx$$

$$=\left[\frac{12\,a^2b^3fx^7+21\,(a^2b^3e-a^3b^2f)x^4-42\,\sqrt{\frac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3+3\,(-a^2b)^{\frac{1}{3}}ax}{}\right.}{}\right.$$

[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")

[Out] [1/84*(12*a^2*b^3*f*x^7 + 21*(a^2*b^3*e - a^3*b^2*f)*x^4 - 42*sqrt(1/3)*(a*
b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*

```
x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*
x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-
a^2*b)^(1/3)*a) + 28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log
(a*b*x + (-a^2*b)^(2/3)) + 84*(a^2*b^3*d - a^3*b^2*e + a^4*b*f)*x)/(a^2*b^4
), 1/84*(12*a^2*b^3*f*x^7 + 21*(a^2*b^3*e - a^3*b^2*f)*x^4 + 84*sqrt(1/3)*(
a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(s
qrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^
2) - 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-
a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 84*(a^2*b^3*d - a^3*b^2*e + a
^4*b*f)*x)/(a^2*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.42

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx = x^4\left(-\frac{af}{4b^2}+\frac{e}{4b}\right)+x\left(\frac{a^2f}{b^3}-\frac{ae}{b^2}+\frac{d}{b}\right) + \text{RootSum}\left(27t^3a^2b^{10}+a^9f^3-3a^8bef^2+3a^7b^2df^2+3a^7b^2e^2f-3a^6b^3cf^2-6a^6b^3def-a^6b^3e^3+6a^5b^4\right. + \frac{fx^7}{7b}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**4*(-a*f/(4*b**2) + e/(4*b)) + x*(a**2*f/b**3 - a*e/b**2 + d/b) + RootSum
(27*_t**3*a**2*b**10 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f**2 + 3
*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b**3*e**3
 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a**4*b**
5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*f + 6*
a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c*d**2
+ 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(-3*_t*a*b**3/(a**3*f - a**
2*b*e + a*b**2*d - b**3*c) + x))) + f*x**7/(7*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx = \frac{4\,b^2fx^7+7\,(b^2e-abf)x^4+28\,(b^2d-abe+a^2f)x}{28\,b^3} + \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/28*(4*b^2*f*x^7 + 7*(b^2*e - a*b*f)*x^4 + 28*(b^2*d - a*b*e + a^2*f)*x)/b
^3 + 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) - 1/6*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1
/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3
))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx = -\frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{2}{3}}\,b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{2}{3}}\,b^2} - \frac{(b^7c-ab^6d+a^2b^5e-a^3b^4f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^7} + \frac{4\,b^6fx^7+7\,b^6ex^4-7\,ab^5fx^4+28\,b^6dx-28\,ab^5ex+28\,a^2b^4fx}{28\,b^7}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^2) - 1/6*(b^3*c - a*b^2*d + a
^2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^
2) - 1/3*(b^7*c - a*b^6*d + a^2*b^5*e - a^3*b^4*f)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a*b^7) + 1/28*(4*b^6*f*x^7 + 7*b^6*e*x^4 - 7*a*b^5*f*x^4 +
 28*b^6*d*x - 28*a*b^5*e*x + 28*a^2*b^4*f*x)/b^7
```

Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.92

$$
\begin{aligned}
&\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx \\
&= x^4\left(\frac{e}{4\,b}-\frac{a\,f}{4\,b^2}\right)+x\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)+\frac{f\,x^7}{7\,b} \\
&\quad+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{2/3}\,b^{10/3}} \\
&\quad+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{2/3}\,b^{10/3}} \\
&\quad-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{2/3}\,b^{10/3}}
\end{aligned}
$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x)
```

```
[Out] x^4*(e/(4*b) - (a*f)/(4*b^2)) + x*(d/b - (a*(e/b - (a*f)/b^2))/b) + (f*x^7)
/(7*b) + (log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*
a^(2/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1
/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(2/3)*b^(10/3))
- (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(
b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(2/3)*b^(10/3))
```

3.240 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 227

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx =& -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}\\ &+\frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{8/3}}\\ &+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{8/3}}\\ &-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{8/3}}\end{aligned}$$

```
[Out] -c/a/x+1/2*(-a*f+b*e)*x^2/b^2+1/5*f*x^5/b+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(8/3)-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(8/3)+1/3*(-a^3*f+a^2*b
*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)
/b^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used

= {1848, 298, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{4/3}b^{8/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{4/3}b^{8/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{4/3}b^{8/3}} + \frac{x^2(be-af)}{2b^2} - \frac{c}{ax} + \frac{fx^5}{5b}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x^2)/(2*b^2) + (f*x^5)/(5*b) + ((b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*a^(4/3)*b^(8/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b
^(1/3)*x])/(3*a^(4/3)*b^(8/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{b}+\frac{(-b^3c+ab^2d-a^2be+a^3f)\,x}{ab^2\,(a+bx^3)}\right)dx\\
&= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{(-b^3c+ab^2d-a^2be+a^3f)\int\frac{x}{a+bx^3}\,dx}{ab^2}\\
&= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}b^{7/3}}\\
&\quad -\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{4/3}b^{7/3}}\\
&= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{8/3}}\\
&\quad -\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}b^{8/3}}\\
&\quad -\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2ab^{7/3}}
\end{aligned}$$

$$= -\frac{c}{ax} + \frac{(be-af)x^2}{2b^2} + \frac{fx^5}{5b} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{8/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{8/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{4/3}b^{8/3}}$$

$$= -\frac{c}{ax} + \frac{(be-af)x^2}{2b^2} + \frac{fx^5}{5b} + \frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{8/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{8/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{8/3}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx$$

$$= \frac{-30\sqrt[3]{a}b^{8/3}c + 15a^{4/3}b^{2/3}(be-af)x^3 + 6a^{4/3}b^{5/3}fx^6 + 10\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\,x\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) \ldots}{3\ldots}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x]
```

```
[Out] (-30*a^(1/3)*b^(8/3)*c + 15*a^(4/3)*b^(2/3)*(b*e - a*f)*x^3 + 6*a^(4/3)*b^(
5/3)*f*x^6 + 10*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*L
og[a^(1/3) + b^(1/3)*x] - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(30*a^(4/3)*b^(8/3)*x)
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.70

method	result
default	$-\frac{-\frac{bf\,x^5}{5}+\frac{(af-be)x^2}{2}}{b^2}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)}{a\,b^2}-\frac{c}{a}$
risch	$\frac{f\,x^5}{5b}-\frac{x^2af}{2b^2}+\frac{e\,x^2}{2b}-\frac{c}{ax}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a^9f^3-3a^8be\,f^2+3a^7b^2d\,f^2+3a^7b^2e^2f-3a^6b^3c\,f^2-6a^6b^3def-a^6b^3e^3+6a^5b^4cef+3a^5b^4d^2f+3\right.}}{}$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/b^2*(-1/5*b*f*x^5+1/2*(a*f-b*e)*x^2)+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a/b^2-c/a/x

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.47

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)}\,dx$$
$$=\left[\frac{6\,a^2b^3fx^6-30\,ab^4c+15\,(a^2b^3e-a^3b^2f)x^3-15\,\sqrt{\tfrac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-}{}\right.}{}\right.$$

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="fricas")

[Out] [1/30*(6*a^2*b^3*f*x^6 - 30*a*b^4*c + 15*(a^2*b^3*e - a^3*b^2*f)*x^3 - 15*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x*sqrt((-a*b^2)^(1/3)/

```
a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b
^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 5*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b^2*x^2 + (-a*b^2)
^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b
^2)^(2/3)*x*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^4*x), 1/30*(6*a^2*b^3*f*x^6 -
 30*a*b^4*c + 15*(a^2*b^3*e - a^3*b^2*f)*x^3 - 30*sqrt(1/3)*(a*b^4*c - a^2*
b^3*d + a^3*b^2*e - a^4*b*f)*x*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*
b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 5*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^
(2/3)) + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b*x -
(-a*b^2)^(1/3)))/(a^2*b^4*x)]
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.80

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx = x^2\left(-\frac{af}{2b^2}+\frac{e}{2b}\right) + \mathrm{RootSum}\left(27t^3a^4b^8+a^9f^3-3a^8bef^2+3a^7b^2df^2+3a^7b^2e^2f-3a^6b^3cf^2-6a^6b^3def-a^6b^3e^3+6a^5b^4\right. + \frac{fx^5}{5b}-\frac{c}{ax}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a),x)
```

```
[Out] x**2*(-a*f/(2*b**2) + e/(2*b)) + RootSum(27*_t**3*a**4*b**8 + a**9*f**3 - 3
*a**8*b*e*f**2 + 3*a**7*b**2*d*f**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f*
*2 - 6*a**6*b**3*d*e*f - a**6*b**3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d
**2*f + 3*a**5*b**4*d*e**2 - 6*a**4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**
4*b**5*d**2*e + 3*a**3*b**6*c**2*f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3
*a**2*b**7*c**2*e - 3*a**2*b**7*c*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambd
a(_t, _t*log(9*_t**2*a**3*b**5/(a**6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f
+ a**4*b**2*e**2 - 2*a**3*b**3*c*f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a*
*2*b**4*d**2 - 2*a*b**5*c*d + b**6*c**2) + x))) + f*x**5/(5*b) - c/(a*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx = \frac{2\,bfx^5+5\,(be-af)x^2}{10\,b^2}-\frac{c}{ax} - \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab^3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/10*(2*b*f*x^5 + 5*(b*e - a*f)*x^2)/b^2 - c/(a*x) - 1/3*sqrt(3)*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(a*b^3*(a/b)^(1/3)) - 1/6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(1/3)) + 1/3*(b^3*c - a*b^2*d + a
^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.17

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx = -\frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{1}{3}}ab^2}-\frac{c}{ax} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{1}{3}}ab^2} + \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2b^2} + \frac{2\,b^4fx^5+5\,b^4ex^2-5\,ab^3fx^2}{10\,b^5}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a*b^2) - c/(a*x) + 1/6*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b
^2)^(1/3)*a*b^2) + 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) + a^2*b*e
*(-a/b)^(1/3) - a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))
/(a^2*b^2) + 1/10*(2*b^4*f*x^5 + 5*b^4*e*x^2 - 5*a*b^3*f*x^2)/b^5
```

Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx = x^2\left(\frac{e}{2\,b}-\frac{a\,f}{2\,b^2}\right)-\frac{c}{a\,x}+\frac{f\,x^5}{5\,b}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{4/3}\,b^{8/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{4/3}\,b^{8/3}} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{4/3}\,b^{8/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x)
```

```
[Out] x^2*(e/(2*b) - (a*f)/(2*b^2)) - c/(a*x) + (f*x^5)/(5*b) + (log(b^(1/3)*x +
a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(4/3)*b^(8/3)) - (log(3^
(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a
^3*f - a*b^2*d + a^2*b*e))/(3*a^(4/3)*b^(8/3)) + (log(3^(1/2)*a^(1/3)*1i -
2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^
2*b*e))/(3*a^(4/3)*b^(8/3))
```

3.241 $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 224

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)}\,dx = -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b} + \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}b^{7/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}b^{7/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}b^{7/3}}$$

```
[Out] -1/2*c/a/x^2+(-a*f+b*e)*x/b^2+1/4*f*x^4/b-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(7/3)+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(7/3)+1/3*(-a^3*f+a^2*b
*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)
/b^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used

$= \{1848, 206, 31, 648, 631, 210, 642\}$

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{5/3}b^{7/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{5/3}b^{7/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{5/3}b^{7/3}} + \frac{x(be-af)}{b^2} - \frac{c}{2ax^2} + \frac{fx^4}{4b}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*c/(a*x^2) + ((b*e - a*f)*x)/b^2 + (f*x^4)/(4*b) + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*a^(5/3)*b^(7/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(5/3)*b^(7/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{be-af}{b^2}+\frac{c}{ax^3}+\frac{fx^3}{b}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2\left(a+bx^3\right)}\right)dx\\
&= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int\frac{1}{a+bx^3}\,dx}{ab^2}\\
&= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{5/3}b^2}\\
&\quad -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}b^2}\\
&= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}b^{7/3}}\\
&\quad +\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{5/3}b^{7/3}}\\
&\quad -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{4/3}b^2}
\end{aligned}$$

$$= -\frac{c}{2ax^2} + \frac{(be-af)x}{b^2} + \frac{fx^4}{4b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}b^{7/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}b^{7/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{5/3}b^{7/3}}$$

$$= -\frac{c}{2ax^2} + \frac{(be-af)x}{b^2} + \frac{fx^4}{4b} + \frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}b^{7/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}b^{7/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}b^{7/3}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)}\,dx = \frac{1}{12}\left(-\frac{6c}{ax^2} + \frac{12(be-af)x}{b^2} + \frac{3fx^4}{b} + \frac{4\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}b^{7/3}} + \frac{4(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}b^{7/3}} + \frac{2(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}b^{7/3}}\right)$$

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x]

[Out] ((-6*c)/(a*x^2) + (12*(b*e - a*f)*x)/b^2 + (3*f*x^4)/b + (4*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(5/3)*b^(7/3)) + (4*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(5/3)*b^(7/3)) + (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(5/3)*b^(7/3)))/12

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.69

method	result
default	$-\frac{-\frac{1}{4} b f x^4+a f x-b e x}{b^2}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\left(f a^3-a^2 b e+a b^2 d-b^3 c\right)}{a b^2}-\frac{c}{2a}$
risch	$\frac{f x^4}{4b}-\frac{a f x}{b^2}+\frac{e x}{b}-\frac{c}{2a x^2}+\frac{\_R=\mathrm{RootOf}\left(-a^9 f^3+3a^8 b e f^2-3a^7 b^2 d f^2-3a^7 b^2 e^2 f+3a^6 b^3 c f^2+6a^6 b^3 d e f+a^6 b^3 e^3-6a^5 b^4 c e f-3a^5 b^4 d^2 f\right.}{}$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/b^2*(-1/4*b*f*x^4+a*f*x-b*e*x)+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/ b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*a rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/a/b^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c) -1/2*c/a/x^2

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.52

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)}\,dx = \left[\frac{3\,a^3b^2fx^6 - 6\,a^2b^3c - 6\,\sqrt{\frac{1}{3}}(ab^4c - a^2b^3d + a^3b^2e - a^4bf)x^2\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3 - 3\,\left(a^2b\right)^{\frac{1}{3}}ax - a^2 + 3\,\sqrt{\frac{1}{3}}\left(2\,abx\right.}{bx^3}\right.}{} \right.$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a^3*b^2*f*x^6 - 6*a^2*b^3*c - 6*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a
^3*b^2*e - a^4*b*f)*x^2*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(
1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*(a^2*b)^(2/3)*x^2*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x + (a^2*b)^(
2/3)) + 12*(a^3*b^2*e - a^4*b*f)*x^3)/(a^3*b^3*x^2), 1/12*(3*a^3*b^2*f*x^6
- 6*a^2*b^3*c - 12*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^
2*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)
*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2
*b)^(2/3)*x^2*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x + (a^2*b)^(2/3)) +
12*(a^3*b^2*e - a^4*b*f)*x^3)/(a^3*b^3*x^2)]
```

Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.46

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)}\,dx = x\left(-\frac{af}{b^2}+\frac{e}{b}\right) + \operatorname{RootSum}\left(27t^3a^5b^7 - a^9f^3 + 3a^8bef^2 - 3a^7b^2df^2 - 3a^7b^2e^2f + 3a^6b^3cf^2 + 6a^6b^3def + a^6b^3e^3 - 6a^5b^4\right. + \frac{fx^4}{4b} - \frac{c}{2ax^2}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a),x)
```

```
[Out] x*(-a*f/b**2 + e/b) + RootSum(27*_t**3*a**5*b**7 - a**9*f**3 + 3*a**8*b*e*f
**2 - 3*a**7*b**2*d*f**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6
*b**3*d*e*f + a**6*b**3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a
**5*b**4*d*e**2 + 6*a**4*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2
*e - 3*a**3*b**6*c**2*f - 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*
c**2*e + 3*a**2*b**7*c*d**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*lo
g(3*_t*a**2*b**2/(a**3*f - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x**4/(4
*b) - c/(2*a*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)}\,dx = \frac{bfx^4+4(be-af)x}{4b^2} - \frac{c}{2ax^2} - \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*f*x^4 + 4*(b*e - a*f)*x)/b^2 - 1/2*c/(a*x^2) - 1/3*sqrt(3)*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(a*b^3*(a/b)^(2/3)) + 1/6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x^2 -
 x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) - 1/3*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\,(a+bx^3)}\,dx = \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{2}{3}}\,ab} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{2}{3}}\,ab} + \frac{(b^3c-ab^2d+a^2be-a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2b^2} - \frac{c}{2\,ax^2} + \frac{b^3fx^4+4\,b^3ex-4\,ab^2fx}{4\,b^4}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) + 1/6*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b
) + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b^2) - 1/2*c/(a*x^2) + 1/4*(b^3*f*x^4 + 4*b^3*e*x - 4*a*b^2*f
*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\,(a+bx^3)}\,dx = x\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)-\frac{c}{2\,a\,x^2}+\frac{f\,x^4}{4\,b}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{5/3}\,b^{7/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{5/3}\,b^{7/3}} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{5/3}\,b^{7/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x)
```

```
[Out] x*(e/b - (a*f)/b^2) - c/(2*a*x^2) + (f*x^4)/(4*b) - (log(b^(1/3)*x + a^(1/3
))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(5/3)*b^(7/3)) - (log(3^(1/2)*
```

```
a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f -
 a*b^2*d + a^2*b*e))/(3*a^(5/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1
/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e)
)/(3*a^(5/3)*b^(7/3))
```

3.242 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 227

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)}\,dx=-\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}b^{5/3}}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}b^{5/3}}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}b^{5/3}}$$

```
[Out] -1/4*c/a/x^4+(-a*d+b*c)/a^2/x+1/2*f*x^2/b-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(5/3)+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(5/3)-1/3*(-a^3*f+a^2*b
*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)
/b^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used

$= \{1848, 298, 31, 648, 631, 210, 642\}$

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)}\,dx = \frac{bc-ad}{a^2x} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{7/3}b^{5/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{7/3}b^{5/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{7/3}b^{5/3}} - \frac{c}{4ax^4} + \frac{fx^2}{2b}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x]
```

```
[Out] -1/4*c/(a*x^4) + (b*c - a*d)/(a^2*x) + (f*x^2)/(2*b) - ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*a^(7/3)*b^(5/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(7/3)*b^(5/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{c}{ax^5}+\frac{-bc+ad}{a^2x^2}+\frac{fx}{b}-\frac{(-b^3c+ab^2d-a^2be+a^3f)\,x}{a^2b\,(a+bx^3)}\right)dx\\
&= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{(-b^3c+ab^2d-a^2be+a^3f)\int\frac{x}{a+bx^3}\,dx}{a^2b}\\
&= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{7/3}b^{4/3}}\\
&\quad+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{7/3}b^{4/3}}\\
&= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}b^{5/3}}\\
&\quad+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{7/3}b^{5/3}}\\
&\quad+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^2b^{4/3}}\\
&= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}b^{5/3}}\\
&\quad+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}b^{5/3}}\\
&\quad+\frac{(b^3c-ab^2d+a^2be-a^3f)\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{7/3}b^{5/3}}
\end{aligned}
$$

$$= -\frac{c}{4ax^4} + \frac{bc-ad}{a^2x} + \frac{fx^2}{2b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}b^{5/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}b^{5/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}b^{5/3}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)}\,dx = \frac{1}{12}\left(-\frac{3c}{ax^4} + \frac{12(bc-ad)}{a^2x} + \frac{6fx^2}{b} + \frac{4\sqrt{3}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}b^{5/3}} + \frac{4(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{7/3}b^{5/3}} + \frac{2(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{7/3}b^{5/3}}\right)$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x]
```

```
[Out] ((-3*c)/(a*x^4) + (12*(b*c - a*d))/(a^2*x) + (6*f*x^2)/b + (4*Sqrt[3]*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]])/(a^(7/3)*b^(5/3)) + (4*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/
3) + b^(1/3)*x])/(a^(7/3)*b^(5/3)) + (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(7/3)*b^(5/3)))/12
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.70

method	result
default	$\frac{f\,x^{2}}{2b}-\frac{c}{4a\,x^{4}}-\frac{ad-bc}{a^{2}x}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)}{a^{2}b}$
risch	$\frac{f\,x^{2}}{2b}+\frac{-\frac{(ad-bc)b\,x^{3}}{a^{2}}-\frac{cb}{4a}}{b\,x^{4}}+\frac{\_R=\operatorname{RootOf}\left(a^{7}b^{2}\_Z^{3}-a^{9}f^{3}+3a^{8}be\,f^{2}-3a^{7}b^{2}d\,f^{2}-3a^{7}b^{2}e^{2}f+3a^{6}b^{3}c\,f^{2}+6a^{6}b^{3}def+a^{6}b^{3}e^{3}-6a^{5}b^{4}cef-3a\right.}{}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*x^2/b-1/4*c/a/x^4-(a*d-b*c)/a^2/x-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3
))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^
(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c
)/a^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 556, normalized size of antiderivative = 2.45

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)}\,dx$$

$$=\left[\frac{6\,a^{3}b^{2}fx^{6}-6\sqrt{\frac{1}{3}}(ab^{4}c-a^{2}b^{3}d+a^{3}b^{2}e-a^{4}bf)x^{4}\sqrt{-\frac{(ab^{2})^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^{2}x^{3}-ab-3\sqrt{\frac{1}{3}}\left(abx+2\,(ab^{2})^{\frac{2}{3}}x^{2}-(ab^{2})^{\frac{1}{3}}a\right)\sqrt{-\cdots}}{bx^{3}+a}\right.}{}\right.$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*a^3*b^2*f*x^6 - 6*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4
*b*f)*x^4*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x
```

```
+ 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)
^(2/3)*x)/(b*x^3 + a)) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3
)*x^4*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*x^4*log(b*x + (a*b^2)^(1/3)) - 3*a^2*b^3*
c + 12*(a*b^4*c - a^2*b^3*d)*x^3)/(a^3*b^3*x^4), 1/12*(6*a^3*b^2*f*x^6 - 12
*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4*sqrt((a*b^2)^(1/
3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) +
2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*x^4*log(b^2*x^2 - (a*b^
2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^
2)^(2/3)*x^4*log(b*x + (a*b^2)^(1/3)) - 3*a^2*b^3*c + 12*(a*b^4*c - a^2*b^3
*d)*x^3)/(a^3*b^3*x^4)]
```

Sympy [A] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.81

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)}\,dx$$
$$=\operatorname{RootSum}\left(27t^3a^7b^5-a^9f^3+3a^8bef^2-3a^7b^2df^2-3a^7b^2e^2f+3a^6b^3cf^2+6a^6b^3def+a^6b^3e^3-6a^5b^4cef\right.$$
$$+\frac{fx^2}{2b}+\frac{-ac+x^3(-4ad+4bc)}{4a^2x^4}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**7*b**5 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**2*d*f*
*2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a**6*b**
3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2 + 6*a*
*4*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*
f - 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c
*d**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**5*b**3/(a
**6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*
f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**
6*c**2) + x))) + f*x**2/(2*b) + (-a*c + x**3*(-4*a*d + 4*b*c))/(4*a**2*x**4
)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)}\,dx=\frac{fx^2}{2\,b}+\frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{4\,(bc-ad)x^3-ac}{4\,a^2x^4}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*f*x^2/b + 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sq
rt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3)) + 1/6*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2
*(a/b)^(1/3)) - 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3)
)/(a^2*b^2*(a/b)^(1/3)) + 1/4*(4*(b*c - a*d)*x^3 - a*c)/(a^2*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.14

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)}\,dx=\frac{fx^2}{2\,b}+\frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{1}{3}}\,a^2b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{1}{3}}\,a^2b}-\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b}+\frac{4\,bcx^3-4\,adx^3-ac}{4\,a^2x^4}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b + 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sq
rt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^2*b) - 1/6*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-
a*b^2)^(1/3)*a^2*b) - 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) + a^2*
b*e*(-a/b)^(1/3) - a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3
)))/(a^3*b) + 1/4*(4*b*c*x^3 - 4*a*d*x^3 - a*c)/(a^2*x^4)
```

Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)}\,dx$$
$$=\frac{f\,x^2}{2\,b}-\frac{\frac{b\,c}{4\,a}+\frac{b\,x^3\,(a\,d-b\,c)}{a^2}}{b\,x^4}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{7/3}\,b^{5/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{7/3}\,b^{5/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{7/3}\,b^{5/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x)
```

```
[Out] (f*x^2)/(2*b) - ((b*c)/(4*a) + (b*x^3*(a*d - b*c))/a^2)/(b*x^4) - (log(b^(1
/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(7/3)*b^(5/3)) +
 (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b
^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(7/3)*b^(5/3)) - (log(3^(1/2)*a^(1/
3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^
2*d + a^2*b*e))/(3*a^(7/3)*b^(5/3))
```

3.243 $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 225

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)}\,dx = -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}b^{4/3}} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}b^{4/3}} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}b^{4/3}}$$

```
[Out] -1/5*c/a/x^5+1/2*(-a*d+b*c)/a^2/x^2+f*x/b+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(4/3)-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(4/3)-1/3*(-a^3*f+a^2*b
*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)
/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used

$= \{1848, 206, 31, 648, 631, 210, 642\}$

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)}\,dx = \frac{bc-ad}{2a^2x^2} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{8/3}b^{4/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{8/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{8/3}b^{4/3}} - \frac{c}{5ax^5} + \frac{fx}{b}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x]
```

```
[Out] -1/5*c/(a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b - ((b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(8/3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/
3)*x])/(3*a^(8/3)*b^(4/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{f}{b}+\frac{c}{ax^6}+\frac{-bc+ad}{a^2x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b\,(a+bx^3)}\right)dx\\
&= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{a+bx^3}\,dx}{a^2b}\\
&= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{8/3}b}\\
&\quad+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{8/3}b}\\
&= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}b^{4/3}}\\
&\quad-\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{8/3}b^{4/3}}\\
&\quad+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{7/3}b}\\
&= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}b^{4/3}}\\
&\quad-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}b^{4/3}}\\
&\quad+\frac{(b^3c-ab^2d+a^2be-a^3f)\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{8/3}b^{4/3}}
\end{aligned}
$$

$$= -\frac{c}{5ax^5} + \frac{bc-ad}{2a^2x^2} + \frac{fx}{b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}b^{4/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}b^{4/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}b^{4/3}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)}\,dx = -\frac{c}{5ax^5} + \frac{bc-ad}{2a^2x^2} + \frac{fx}{b} + \frac{(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{8/3}b^{4/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}b^{4/3}} + \frac{(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}b^{4/3}}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x]
```

```
[Out] -1/5*c/(a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b + ((-(b^3*c) + a*b^2*d -
 a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(
8/3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(8/3)*b^(4/3)) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*b^(4/3))
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.69

method	result
default	$\frac{fx}{b}-\frac{c}{5a\,x^{5}}-\frac{ad-bc}{2x^{2}a^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\left(-f\,a^{3}+a^{2}be-a\,b^{2}d+b^{3}c\right)}{a^{2}b}$
risch	$\frac{fx}{b}+\frac{-\frac{(ad-bc)b\,x^{3}}{2a^{2}}-\frac{cb}{5a}}{b\,x^{5}}+\frac{\_R=\mathrm{RootOf}\left(a^{8}b\_Z^{3}+a^{9}f^{3}-3a^{8}be\,f^{2}+3a^{7}b^{2}d\,f^{2}+3a^{7}b^{2}e^{2}f-3a^{6}b^{3}c\,f^{2}-6a^{6}b^{3}def-a^{6}b^{3}e^{3}+6a^{5}b^{4}cef+3a^{5}b\right.}{}$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] f*x/b-1/5*c/a/x^5-1/2*(a*d-b*c)/x^2/a^2+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/a^2/b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.60

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\,(a+bx^3)}\,dx$$

$$= \left[\frac{30\,a^4bfx^6-15\,\sqrt{\frac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^5\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3+3\,(-a^2b)^{\frac{1}{3}}ax-a^2-3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(-a^2b)\right.}{bx^3+a}\right.}{}\right.$$

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="fricas")

[Out] [1/30*(30*a^4*b*f*x^6 - 15*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^5*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*(b^3*c

```
- a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x + (-a^2*b)^(2/3))
 - 6*a^3*b^2*c + 15*(a^2*b^3*c - a^3*b^2*d)*x^3)/(a^4*b^2*x^5), 1/30*(30*a^
4*b*f*x^6 + 30*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^5*sq
rt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)
*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-
a^2*b)^(2/3)*x^5*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x + (-a^2*b)^(
2/3)) - 6*a^3*b^2*c + 15*(a^2*b^3*c - a^3*b^2*d)*x^3)/(a^4*b^2*x^5)]
```

Sympy [A] (verification not implemented)

Time = 7.49 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.46

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)}\,dx$$
$$=\operatorname{RootSum}\Big(27t^3a^8b^4+a^9f^3-3a^8bef^2+3a^7b^2df^2+3a^7b^2e^2f-3a^6b^3cf^2-6a^6b^3def-a^6b^3e^3+6a^5b^4cef$$
$$+\frac{fx}{b}+\frac{-2ac+x^3(-5ad+5bc)}{10a^2x^5}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**8*b**4 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f*
*2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b**
3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a*
*4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*
f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c
*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(-3*_t*a**3*b/(a**3*f
 - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x/b + (-2*a*c + x**3*(-5*a*d +
5*b*c))/(10*a**2*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\,(a+bx^3)}\,dx = \frac{fx}{b} + \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\,(bc-ad)x^3-2\,ac}{10\,a^2x^5}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] f*x/b + 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) - 1/6*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)
^(2/3)) + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a^2
*b^2*(a/b)^(2/3)) + 1/10*(5*(b*c - a*d)*x^3 - 2*a*c)/(a^2*x^5)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\,(a+bx^3)}\,dx = \frac{fx}{b} - \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}a^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}a^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b} + \frac{5\,bcx^3-5\,adx^3-2\,ac}{10\,a^2x^5}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="giac")
```

```
[Out] f*x/b - 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/6*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(
2/3)*a^2) - 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a^3*b) + 1/10*(5*b*c*x^3 - 5*a*d*x^3 - 2*a*c)/(a^2*x^5)
```

Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6 (a+bx^3)} \, dx = \frac{f\,x}{b}-\frac{\frac{b\,c}{5\,a}+\frac{b\,x^3\,\left(a\,d-b\,c\right)}{2\,a^2}}{b\,x^5}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{8/3}\,b^{4/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{8/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{8/3}\,b^{4/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x)
```

```
[Out] (f*x)/b - ((b*c)/(5*a) + (b*x^3*(a*d - b*c))/(2*a^2))/(b*x^5) + (log(b^(1/3
)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(8/3)*b^(4/3)) + (
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3
*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(8/3)*b^(4/3)) - (log(3^(1/2)*a^(1/3)
*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*
d + a^2*b*e))/(3*a^(8/3)*b^(4/3))
```

3.244 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 242

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx = -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x} + \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}b^{2/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}b^{2/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}b^{2/3}}$$

```
[Out] -1/7*c/a/x^7+1/4*(-a*d+b*c)/a^2/x^4+(-a^2*e+a*b*d-b^2*c)/a^3/x+1/3*(-a^3*f+
a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(2/3)-1/6*(-a^3*f+a
^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^
(2/3)+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a
^(1/3)*3^(1/2))/a^(10/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used

$= \{1848, 298, 31, 648, 631, 210, 642\}$

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx = \frac{bc-ad}{4a^2x^4} - \frac{a^2e-abd+b^2c}{a^3x} + \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{10/3}b^{2/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{10/3}b^{2/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{10/3}b^{2/3}} - \frac{c}{7ax^7}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x]
```

```
[Out] -1/7*c/(a*x^7) + (b*c - a*d)/(4*a^2*x^4) - (b^2*c - a*b*d + a^2*e)/(a^3*x)
+ ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(Sqrt[3]*a^(10/3)*b^(2/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a
^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(10/3)*b^(2/3)) - ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(10/3
)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{c}{ax^8} + \frac{-bc+ad}{a^2x^5} + \frac{b^2c-abd+a^2e}{a^3x^2} + \frac{(-b^3c+ab^2d-a^2be+a^3f)\,x}{a^3(a+bx^3)}\right) dx$$

$$= -\frac{c}{7ax^7} + \frac{bc-ad}{4a^2x^4} - \frac{b^2c-abd+a^2e}{a^3x} + \frac{(-b^3c+ab^2d-a^2be+a^3f)\int \frac{x}{a+bx^3}\,dx}{a^3}$$

$$= -\frac{c}{7ax^7} + \frac{bc-ad}{4a^2x^4} - \frac{b^2c-abd+a^2e}{a^3x} + \frac{(b^3c-ab^2d+a^2be-a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{10/3}\sqrt[3]{b}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{10/3}\sqrt[3]{b}}$$

$$= -\frac{c}{7ax^7} + \frac{bc-ad}{4a^2x^4} - \frac{b^2c-abd+a^2e}{a^3x} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}b^{2/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{10/3}b^{2/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^3\sqrt[3]{b}}$$

$$= -\frac{c}{7ax^7} + \frac{bc-ad}{4a^2x^4} - \frac{b^2c-abd+a^2e}{a^3x} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}b^{2/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}b^{2/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{10/3}b^{2/3}}$$

$$= -\frac{c}{7ax^7} + \frac{bc-ad}{4a^2x^4} - \frac{b^2c-abd+a^2e}{a^3x} + \frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}b^{2/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}b^{2/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx$$

$$= \frac{-\frac{12a^{7/3}c}{x^7} + \frac{21a^{4/3}(bc-ad)}{x^4} - \frac{84\sqrt[3]{a}(b^2c-abd+a^2e)}{x} + \frac{28\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{28(b^3c-ab^2d+a^2be-a^3f)\,\text{lo}}{b^{2/3}}}{84a^{10/3}}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x]
```

```
[Out] ((-12*a^(7/3)*c)/x^7 + (21*a^(4/3)*(b*c - a*d))/x^4 - (84*a^(1/3)*(b^2*c -
a*b*d + a^2*e))/x + (28*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (28*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (14*(-(b^3*c) + a*b^2*d -
a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(
84*a^(10/3))
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.70

method	result
default	$-\frac{c}{7a\,x^{7}}-\frac{ad-bc}{4a^{2}x^{4}}-\frac{a^{2}e-abd+b^{2}c}{a^{3}x}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\left(f\,a^{3}-a^{2}be+\right.}{a^{3}}$
risch	$\frac{-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{a^{3}}-\frac{(ad-bc)x^{3}}{4a^{2}}-\frac{c}{7a}}{x^{7}}+\frac{\left(\_R=\operatorname{RootOf}\left(a^{10}b^{2}\_Z^{3}+a^{9}f^{3}-3a^{8}be\,f^{2}+3a^{7}b^{2}d\,f^{2}+3a^{7}b^{2}e^{2}f-3a^{6}b^{3}c\,f^{2}-6a^{6}b^{3}def-a^{6}b^{3}e^{3}+\right.\right.}{}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*c/a/x^7-1/4*(a*d-b*c)/a^2/x^4-(a^2*e-a*b*d+b^2*c)/a^3/x+(-1/3/b/(a/b)^
(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/a^3*(a^
3*f-a^2*b*e+a*b^2*d-b^3*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.52

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx$$

$$= \left[-\frac{42\sqrt{\frac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^7\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2(-ab^2)^{\frac{2}{3}}x^2+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3}{bx^3+a}\right.}{} \right.$$

$$-\frac{84\sqrt{\frac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^7\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx+(-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{b}\right)+14(b^3c-ab^2}{}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/84*(42*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^7*sqrt((
-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/
3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*
x^3 + a)) + 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b
^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b*x - (-a*b^2)^(1/3)) + 84*(a*b^4*c - a^
2*b^3*d + a^3*b^2*e)*x^6 + 12*a^3*b^2*c - 21*(a^2*b^3*c - a^3*b^2*d)*x^3)/(
a^4*b^2*x^7), -1/84*(84*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*
f)*x^7*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sq
rt(-(-a*b^2)^(1/3)/a)/b) + 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^
(2/3)*x^7*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b*x - (-a*b^2)^(1/3)) + 8
4*(a*b^4*c - a^2*b^3*d + a^3*b^2*e)*x^6 + 12*a^3*b^2*c - 21*(a^2*b^3*c - a^
3*b^2*d)*x^3)/(a^4*b^2*x^7)]
```

Sympy [A] (verification not implemented)

Time = 43.35 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.79

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)}\,dx$$
$$= \operatorname{RootSum}\Big(27t^3a^{10}b^2+a^9f^3-3a^8bef^2+3a^7b^2df^2+3a^7b^2e^2f-3a^6b^3cf^2-6a^6b^3def-a^6b^3e^3+6a^5b^4ce$$
$$+\frac{-4a^2c+x^6(-28a^2e+28abd-28b^2c)+x^3(-7a^2d+7abc)}{28a^3x^7}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**10*b**2 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f
**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b*
*3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a
**4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2
*f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*
c*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**7*b/(a**
6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*f
- 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**6*
c**2) + x))) + (-4*a**2*c + x**6*(-28*a**2*e + 28*a*b*d - 28*b**2*c) + x**3
*(-7*a**2*d + 7*a*b*c))/(28*a**3*x**7)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)}\,dx = -\frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{28\,(b^2c-abd+a^2e)x^6-7\,(abc-a^2d)x^3+4\,a^2c}{28\,a^3x^7}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(1/3)) - 1/6*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(1/3)) + 1
/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(1
/3)) - 1/28*(28*(b^2*c - a*b*d + a^2*e)*x^6 - 7*(a*b*c - a^2*d)*x^3 + 4*a^2
*c)/(a^3*x^7)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.12

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx$$
$$= -\frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{1}{3}}a^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{1}{3}}a^3} + \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^4} - \frac{28b^2cx^6-28abdx^6+28a^2ex^6-7abcx^3+7a^2dx^3+4a^2c}{28a^3x^7}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^3) + 1/6*(b^3*c - a*b^2*d + a
^2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^
3) + 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) + a^2*b*e*(-a/b)^(1/3)
- a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - 1/28*(2
8*b^2*c*x^6 - 28*a*b*d*x^6 + 28*a^2*e*x^6 - 7*a*b*c*x^3 + 7*a^2*d*x^3 + 4*a
^2*c)/(a^3*x^7)
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\,(a+bx^3)}\,dx$$
$$= \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{10/3}\,b^{2/3}} - \frac{\frac{c}{7\,a}+\frac{x^3\,(a\,d-b\,c)}{4\,a^2}+\frac{x^6\,(e\,a^2-d\,a\,b+c\,b^2)}{a^3}}{x^7}$$
$$- \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{10/3}\,b^{2/3}}$$
$$+ \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{10/3}\,b^{2/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(10/3)*
b^(2/3)) - (c/(7*a) + (x^3*(a*d - b*c))/(4*a^2) + (x^6*(b^2*c + a^2*e - a*b
*d))/a^3)/x^7 - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*
1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(10/3)*b^(2/3)) + (l
og(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*
c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(10/3)*b^(2/3))
```

3.245 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 244

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx = -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{11/3}\sqrt[3]{b}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{11/3}\sqrt[3]{b}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{11/3}\sqrt[3]{b}}$$

```
[Out] -1/8*c/a/x^8+1/5*(-a*d+b*c)/a^2/x^5+1/2*(-a^2*e+a*b*d-b^2*c)/a^3/x^2-1/3*(-
a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(1/3)+1/6*(-a
^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11
/3)/b^(1/3)+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3
)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1848, 206, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx=\frac{bc-ad}{5a^2x^5}-\frac{a^2e-abd+b^2c}{2a^3x^2}+\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{11/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{11/3}\sqrt[3]{b}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{11/3}\sqrt[3]{b}}-\frac{c}{8ax^8}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x]
```

```
[Out] -1/8*c/(a*x^8) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/(2*a^3*x
^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(11/3)*b^(1/3)) - ((b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(11/3)*b^(1/3)) + ((b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(
11/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{ax^9}+\frac{-bc+ad}{a^2x^6}+\frac{b^2c-abd+a^2e}{a^3x^3}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3\left(a+bx^3\right)}\right)dx \\
&= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{1}{a+bx^3}\,dx}{a^3} \\
&= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{11/3}} \\
&\quad -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{11/3}}
\end{aligned}$$

$$= -\frac{c}{8ax^8} + \frac{bc-ad}{5a^2x^5} - \frac{b^2c-abd+a^2e}{2a^3x^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{11/3}\sqrt[3]{b}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{10/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{11/3}\sqrt[3]{b}}$$

$$= -\frac{c}{8ax^8} + \frac{bc-ad}{5a^2x^5} - \frac{b^2c-abd+a^2e}{2a^3x^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{11/3}\sqrt[3]{b}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{11/3}\sqrt[3]{b}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{11/3}\sqrt[3]{b}}$$

$$= -\frac{c}{8ax^8} + \frac{bc-ad}{5a^2x^5} - \frac{b^2c-abd+a^2e}{2a^3x^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{11/3}\sqrt[3]{b}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{11/3}\sqrt[3]{b}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{11/3}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx$$

$$= \frac{-\frac{15a^{8/3}c}{x^8} + \frac{24a^{5/3}(bc-ad)}{x^5} - \frac{60a^{2/3}(b^2c-abd+a^2e)}{x^2} + \frac{40\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{40(-b^3c+ab^2d-a^2be+a^3f}{\sqrt[3]{b}}}{120a^{11/3}}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x]
```

[Out] ((-15*a^(8/3)*c)/x^8 + (24*a^(5/3)*(b*c - a*d))/x^5 - (60*a^(2/3)*(b^2*c - a*b*d + a^2*e))/x^2 + (40*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (40*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(120*a^(11/3))

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.70

method	result
default	$-\frac{c}{8 a x^{8}}-\frac{a d-b c}{5 a^{2} x^{5}}-\frac{a^{2} e-a b d+b^{2} c}{2 a^{3} x^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\left(f a^{3}-a^{2} b e+a b^{2}\right.}{a^{3}}$
risch	$\frac{-\frac{\left(a^{2} e-a b d+b^{2} c\right) x^{6}}{2 a^{3}}-\frac{(a d-b c) x^{3}}{5 a^{2}}-\frac{c}{8 a}}{x^{8}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{11} b \_Z^{3}-a^{9} f^{3}+3 a^{8} b e f^{2}-3 a^{7} b^{2} d f^{2}-3 a^{7} b^{2} e^{2} f+3 a^{6} b^{3} c f^{2}+6 a^{6} b^{3} d e f+a^{6} b^{3} e^{3}-6 a\right.}\right.}{}$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/8*c/a/x^8-1/5*(a*d-b*c)/a^2/x^5-1/2*(a^2*e-a*b*d+b^2*c)/a^3/x^2+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/a^3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 595, normalized size of antiderivative = 2.44

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx$$

$$= \left[-\frac{60\sqrt{\frac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^8\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{\cdots}}{bx^3+a}\right.\cdots}{\cdots}\right.$$

$$-\frac{120\sqrt{\frac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^8\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)-20\,(b^3c-a\cdots}{\cdots}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/120*(60*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8*sqrt(
-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*
(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*
x^3 + a)) - 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^8*log(a*
b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*(a^2*b)^(2/3)*x^8*log(a*b*x + (a^2*b)^(2/3)) + 60*(a^2*b^3*c - a^3
*b^2*d + a^4*b*e)*x^6 + 15*a^4*b*c - 24*(a^3*b^2*c - a^4*b*d)*x^3)/(a^5*b*x
^8), -1/120*(120*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8*
sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*
b)^(2/3)*x^8*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^8*log(a*b*x + (a^2*b)^(2/3)) +
60*(a^2*b^3*c - a^3*b^2*d + a^4*b*e)*x^6 + 15*a^4*b*c - 24*(a^3*b^2*c - a^4
*b*d)*x^3)/(a^5*b*x^8)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)}\,dx=&-\frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\&+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\&-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\&-\frac{20\,(b^2c-abd+a^2e)x^6-8\,(abc-a^2d)x^3+5\,a^2c}{40\,a^3x^8}\end{aligned}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) + 1/6*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) - 1
/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2
/3)) - 1/40*(20*(b^2*c - a*b*d + a^2*e)*x^6 - 8*(a*b*c - a^2*d)*x^3 + 5*a^2
*c)/(a^3*x^8)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.20

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)}\,dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^4} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-\left(-ab^2\right)^{\frac{1}{3}}ab^2d+\left(-ab^2\right)^{\frac{1}{3}}a^2be-\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^4b} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-\left(-ab^2\right)^{\frac{1}{3}}ab^2d+\left(-ab^2\right)^{\frac{1}{3}}a^2be-\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^4b} - \frac{20\,b^2cx^6-20\,abdx^6+20\,a^2ex^6-8\,abcx^3+8\,a^2dx^3+5\,a^2c}{40\,a^3x^8}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/
3)))/a^4 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d + (-a
*b^2)^(1/3)*a^2*b*e - (-a*b^2)^(1/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/(a^4*b) - 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)
*a*b^2*d + (-a*b^2)^(1/3)*a^2*b*e - (-a*b^2)^(1/3)*a^3*f)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/40*(20*b^2*c*x^6 - 20*a*b*d*x^6 + 20*a^
2*e*x^6 - 8*a*b*c*x^3 + 8*a^2*d*x^3 + 5*a^2*c)/(a^3*x^8)
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)}\,dx = -\frac{\frac{c}{8\,a}+\frac{x^3\,\left(a\,d-b\,c\right)}{5\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{2\,a^3}}{x^8} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{11/3}\,b^{1/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{11/3}\,b^{1/3}} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{11/3}\,b^{1/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^
3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(11/3)*b^(1/3)) - (log(b^(1/3)*x + a
```

```
^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(11/3)*b^(1/3)) - (log(3^
(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a
^3*f - a*b^2*d + a^2*b*e))/(3*a^(11/3)*b^(1/3)) - (c/(8*a) + (x^3*(a*d - b*
c))/(5*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(2*a^3))/x^8
```

3.246 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 277

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx = &-\frac{c}{10ax^{10}}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{4a^3x^4}\\ &+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}\\ &-\frac{\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{13/3}}\\ &-\frac{\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{13/3}}\\ &+\frac{\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{13/3}}\end{aligned}$$

```
[Out] -1/10*c/a/x^10+1/7*(-a*d+b*c)/a^2/x^7+1/4*(-a^2*e+a*b*d-b^2*c)/a^3/x^4+(-a^
3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x-1/3*b^(1/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)+1/6*b^(1/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(13/3)-1/3*b^(1/3)*(-a^3*f+a^2*
b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/
3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1848, 298, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx$$
$$=\frac{bc-ad}{7a^2x^7}-\frac{a^2e-abd+b^2c}{4a^3x^4}-\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{13/3}}$$
$$-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{13/3}}$$
$$+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{13/3}}$$
$$+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^4x}-\frac{c}{10ax^{10}}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x]
```

```
[Out] -1/10*c/(a*x^10) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/(4*a^3
*x^4) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (b^(1/3)*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(Sqrt[3]*a^(13/3)) - (b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/
3) + b^(1/3)*x])/(3*a^(13/3)) + (b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{ax^{11}}+\frac{-bc+ad}{a^2x^8}+\frac{b^2c-abd+a^2e}{a^3x^5}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^2}\right.\\
&\qquad\left.-\frac{b(-b^3c+ab^2d-a^2be+a^3f)\,x}{a^4\,(a+bx^3)}\right)dx\\
&= -\frac{c}{10ax^{10}}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{4a^3x^4}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}\\
&\quad+\frac{(b(b^3c-ab^2d+a^2be-a^3f))\int \frac{x}{a+bx^3}\,dx}{a^4}\\
&= -\frac{c}{10ax^{10}}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{4a^3x^4}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}\\
&\quad-\frac{\left(b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{13/3}}\\
&\quad+\frac{\left(b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{13/3}}
\end{aligned}$$

$$= -\frac{c}{10ax^{10}} + \frac{bc-ad}{7a^2x^7} - \frac{b^2c-abd+a^2e}{4a^3x^4} + \frac{b^3c-ab^2d+a^2be-a^3f}{a^4x} - \frac{\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{13/3}} + \frac{\left(\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{13/3}} + \frac{\left(b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^4}$$

$$= -\frac{c}{10ax^{10}} + \frac{bc-ad}{7a^2x^7} - \frac{b^2c-abd+a^2e}{4a^3x^4} + \frac{b^3c-ab^2d+a^2be-a^3f}{a^4x} - \frac{\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{13/3}} + \frac{\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{13/3}} + \frac{\left(\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{13/3}}$$

$$= -\frac{c}{10ax^{10}} + \frac{bc-ad}{7a^2x^7} - \frac{b^2c-abd+a^2e}{4a^3x^4} + \frac{b^3c-ab^2d+a^2be-a^3f}{a^4x} - \frac{\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{13/3}} - \frac{\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{13/3}} + \frac{\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{13/3}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx$$

$$= \frac{-\frac{42a^{10/3}c}{x^{10}} + \frac{60a^{7/3}(bc-ad)}{x^7} - \frac{105a^{4/3}(b^2c-abd+a^2e)}{x^4} + \frac{420\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)}{x} - 140\sqrt{3}\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3}{}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x]
```

[Out] ((-42*a^(10/3)*c)/x^10 + (60*a^(7/3)*(b*c - a*d))/x^7 - (105*a^(4/3)*(b^2*c - a*b*d + a^2*e))/x^4 + (420*a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/x - 140*Sqrt[3]*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 70*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(420*a^(13/3))

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.74

method	result
default	$-\frac{c}{10 a x^{10}}-\frac{a d-b c}{7 a^{2} x^{7}}-\frac{a^{2} e-a b d+b^{2} c}{4 a^{3} x^{4}}-\frac{f a^{3}-a^{2} b e+a b^{2} d-b^{3} c}{a^{4} x}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\, \arctan\left(\frac{\sqrt{3}}{\cdots}\right.}{3 b \left(\frac{a}{b}\cdots\right.}\right.}{a^{4}}$
risch	$\frac{-\frac{\left(f a^{3}-a^{2} b e+a b^{2} d-b^{3} c\right) x^{9}}{a^{4}}-\frac{\left(a^{2} e-a b d+b^{2} c\right) x^{6}}{4 a^{3}}-\frac{(a d-b c) x^{3}}{7 a^{2}}-\frac{c}{10 a}}{x^{10}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{13} \_Z^{3}-a^{9} b f^{3}+3 a^{8} b^{2} e f^{2}-3 a^{7} b^{3} d f^{2}-3 a^{7} b^{3} e^{2} f+\cdots\right.}\right.}{\cdots}$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/10*c/a/x^10-1/7*(a*d-b*c)/a^2/x^7-1/4*(a^2*e-a*b*d+b^2*c)/a^3/x^4-(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4*b

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx = \frac{140\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)x^{10}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+70\left(b^3c-ab^2d+a^2be-a^3f\right)x^{10}\cdots}{\cdots}$$

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="fricas")

```
[Out] 1/420*(140*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^10*(b/a)^(1/3)*arc
tan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 70*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*x^10*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) -
140*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^10*(b/a)^(1/3)*log(b*x + a*(b/a)^
(2/3)) + 420*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9 - 105*(a*b^2*c - a^2*b
*d + a^3*e)*x^6 - 42*a^3*c + 60*(a^2*b*c - a^3*d)*x^3)/(a^4*x^10)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx &= \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &+ \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &- \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &+ \frac{140(b^3c-ab^2d+a^2be-a^3f)x^9-35(ab^2c-a^2bd+a^3e)x^6-14a^3c+20(a^2bc-a^3d)x^3}{140a^4x^{10}}\end{aligned}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a^4*(a/b)^(1/3)) + 1/6*(b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*(a/b)^(1/3)) - 1/3*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(1/3)) +
1/140*(140*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9 - 35*(a*b^2*c - a^2*b*d
+ a^3*e)*x^6 - 14*a^3*c + 20*(a^2*b*c - a^3*d)*x^3)/(a^4*x^10)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.34

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx$$
$$= -\frac{\left(b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3bf\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^5}$$
$$- \frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}\,b^3c - (-ab^2)^{\frac{2}{3}}\,ab^2d + (-ab^2)^{\frac{2}{3}}\,a^2be - (-ab^2)^{\frac{2}{3}}\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^5b}$$
$$+ \frac{\left((-ab^2)^{\frac{2}{3}}\,b^3c - (-ab^2)^{\frac{2}{3}}\,ab^2d + (-ab^2)^{\frac{2}{3}}\,a^2be - (-ab^2)^{\frac{2}{3}}\,a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^5b}$$
$$+ \frac{140\,b^3cx^9 - 140\,ab^2dx^9 + 140\,a^2bex^9 - 140\,a^3fx^9 - 35\,ab^2cx^6 + 35\,a^2bdx^6 - 35\,a^3ex^6 + 20\,a^2bcx^3 - 20}{140\,a^4x^{10}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(b^4*c*(-a/b)^(1/3) - a*b^3*d*(-a/b)^(1/3) + a^2*b^2*e*(-a/b)^(1/3) -
a^3*b*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^5 - 1/3*sqr
t(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + (-a*b^2)^(2/3)*a^2*b*
e - (-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/(a^5*b) + 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + (-a*b^2
)^(2/3)*a^2*b*e - (-a*b^2)^(2/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/(a^5*b) + 1/140*(140*b^3*c*x^9 - 140*a*b^2*d*x^9 + 140*a^2*b*e*x^9 -
140*a^3*f*x^9 - 35*a*b^2*c*x^6 + 35*a^2*b*d*x^6 - 35*a^3*e*x^6 + 20*a^2*b*c
*x^3 - 20*a^3*d*x^3 - 14*a^3*c)/(a^4*x^10)
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx$$
$$= -\frac{\frac{c}{10\,a} - \frac{x^9\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{a^4} + \frac{x^3\,(a\,d-b\,c)}{7\,a^2} + \frac{x^6\,(e\,a^2-d\,a\,b+c\,b^2)}{4\,a^3}}{x^{10}}$$
$$- \frac{b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{3\,a^{13/3}}$$
$$+ \frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{3\,a^{13/3}}$$
$$- \frac{b^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{3\,a^{13/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x)
```

```
[Out] (b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(13/3)) - (b^(1/3)*log(b^(1/
3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(13/3)) - (c/(10*
a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^4 + (x^3*(a*d - b*c))/(7*a
^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(4*a^3))/x^10 - (b^(1/3)*log(3^(1/2)*a^
(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a
*b^2*d + a^2*b*e))/(3*a^(13/3))
```

3.247 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 280

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx = -\frac{c}{11ax^{11}}+\frac{bc-ad}{8a^2x^8}-\frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} - \frac{b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{14/3}} + \frac{b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{14/3}} - \frac{b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{14/3}}$$

```
[Out] -1/11*c/a/x^11+1/8*(-a*d+b*c)/a^2/x^8+1/5*(-a^2*e+a*b*d-b^2*c)/a^3/x^5+1/2*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^2+1/3*b^(2/3)*(-a^3*f+a^2*b*e-a*b^2*d+
b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)-1/6*b^(2/3)*(-a^3*f+a^2*b*e-a*b^2*d+b
^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)-1/3*b^(2/3)*(-a^3*
f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/
a^(14/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1848, 206, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx$$
$$= \frac{bc-ad}{8a^2x^8} - \frac{a^2e-abd+b^2c}{5a^3x^5} - \frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{14/3}}$$
$$- \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{14/3}}$$
$$+ \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{14/3}}$$
$$+ \frac{a^3(-f)+a^2be-ab^2d+b^3c}{2a^4x^2} - \frac{c}{11ax^{11}}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x]
```

```
[Out] -1/11*c/(a*x^11) + (b*c - a*d)/(8*a^2*x^8) - (b^2*c - a*b*d + a^2*e)/(5*a^3
*x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(2*a^4*x^2) - (b^(2/3)*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(14/3)) + (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a
^(1/3) + b^(1/3)*x])/(3*a^(14/3)) - (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a
^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{c}{ax^{12}} + \frac{-bc+ad}{a^2x^9} + \frac{b^2c-abd+a^2e}{a^3x^6} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^3} - \frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^4(a+bx^3)}\right) dx$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{(b(b^3c-ab^2d+a^2be-a^3f))\int \frac{1}{a+bx^3}\,dx}{a^4}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{(b(b^3c-ab^2d+a^2be-a^3f))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{14/3}} + \frac{(b(b^3c-ab^2d+a^2be-a^3f))\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{14/3}}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{14/3}} - \frac{\left(b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{14/3}} + \frac{(b(b^3c-ab^2d+a^2be-a^3f))\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{13/3}}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{14/3}} - \frac{b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{14/3}} + \frac{\left(b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{14/3}}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} - \frac{b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{14/3}} + \frac{b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{14/3}} - \frac{b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{14/3}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx$$

$$= \frac{-\frac{120a^{11/3}c}{x^{11}} + \frac{165a^{8/3}(bc-ad)}{x^8} - \frac{264a^{5/3}(b^2c-abd+a^2e)}{x^5} + \frac{660a^{2/3}(b^3c-ab^2d+a^2be-a^3f)}{x^2} - 440\sqrt{3}b^{2/3}(b^3c-ab^2d+a^2be-}{}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x]
```

```
[Out] ((-120*a^(11/3)*c)/x^11 + (165*a^(8/3)*(b*c - a*d))/x^8 - (264*a^(5/3)*(b^2
*c - a*b*d + a^2*e))/x^5 + (660*a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
)/x^2 - 440*Sqrt[3]*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 440*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*b^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*
e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1320*a^(14/3))
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.73

method	result
default	$-\frac{c}{11a\,x^{11}}-\frac{ad-bc}{8a^2x^8}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{f\,a^3-a^2be+a\,b^2d-b^3c}{2a^4x^2}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\cdots\right.}{\cdots}\right.}{3b\left(\frac{a}{b}\right)\cdots}\right.}{a^4}$
risch	$\frac{-\frac{\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x^9}{2a^4}-\frac{\left(a^2e-abd+b^2c\right)x^6}{5a^3}-\frac{(ad-bc)x^3}{8a^2}-\frac{c}{11a}}{x^{11}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{14}\_Z^3+a^9b^2f^3-3a^8b^3e\,f^2+3a^7b^4d\,f^2+3a^7b^4e^2f\cdots\right.}\right.}{\cdots}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/11*c/a/x^11-1/8*(a*d-b*c)/a^2/x^8-1/5*(a^2*e-a*b*d+b^2*c)/a^3/x^5-1/2*(a
^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^2-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/
6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4
*b
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.05

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\,(a+bx^3)}\,dx =$$

$$-\frac{440\,\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)x^{11}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-220\,(b^3c-ab^2d+a^2be-a^3\cdots}{\cdots}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/1320*(440*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^11*(-b^2/a^2)^(1
/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 220*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x^11*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b
^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 440*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*x^11*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) - 660*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^9 + 264*(a*b^2*c - a^2*b*d + a^3*e)*x^6 + 120*a^3
*c - 165*(a^2*b*c - a^3*d)*x^3)/(a^4*x^11)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)}\,dx=\frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{220\,(b^3c-ab^2d+a^2be-a^3f)x^9-88\,(ab^2c-a^2bd+a^3e)x^6-40\,a^3c+55\,(a^2bc-a^3d)x^3}{440\,a^4x^{11}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a^4*(a/b)^(2/3)) - 1/6*(b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*(a/b)^(2/3)) + 1/3*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(2/3)) +
1/440*(220*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9 - 88*(a*b^2*c - a^2*b*d
+ a^3*e)*x^6 - 40*a^3*c + 55*(a^2*b*c - a^3*d)*x^3)/(a^4*x^11)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.19

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx$$
$$=\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}b^3c-(-ab^2)^{\frac{1}{3}}ab^2d+(-ab^2)^{\frac{1}{3}}a^2be-(-ab^2)^{\frac{1}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^5}$$
$$-\frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^5}$$
$$+\frac{\left((-ab^2)^{\frac{1}{3}}b^3c-(-ab^2)^{\frac{1}{3}}ab^2d+(-ab^2)^{\frac{1}{3}}a^2be-(-ab^2)^{\frac{1}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^5}$$
$$+\frac{220b^3cx^9-220ab^2dx^9+220a^2bex^9-220a^3fx^9-88ab^2cx^6+88a^2bdx^6-88a^3ex^6+55a^2bcx^3-55}{440a^4x^{11}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d + (-a*b^2)^(1/3)
*a^2*b*e - (-a*b^2)^(1/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/a^5 - 1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*(-a/b)^(1/3)*
log(abs(x - (-a/b)^(1/3)))/a^5 + 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)
*a*b^2*d + (-a*b^2)^(1/3)*a^2*b*e - (-a*b^2)^(1/3)*a^3*f)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/a^5 + 1/440*(220*b^3*c*x^9 - 220*a*b^2*d*x^9 + 220*
a^2*b*e*x^9 - 220*a^3*f*x^9 - 88*a*b^2*c*x^6 + 88*a^2*b*d*x^6 - 88*a^3*e*x^
6 + 55*a^2*b*c*x^3 - 55*a^3*d*x^3 - 40*a^3*c)/(a^4*x^11)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx$$
$$=\frac{b^{2/3}\ln\left(b^{1/3}x+a^{1/3}\right)\left(-fa^3+ea^2b-dab^2+cb^3\right)}{3a^{14/3}}$$
$$-\frac{\frac{c}{11a}-\frac{x^9\left(-fa^3+ea^2b-dab^2+cb^3\right)}{2a^4}+\frac{x^3(ad-bc)}{8a^2}+\frac{x^6\left(ea^2-dab+cb^2\right)}{5a^3}}{x^{11}}$$
$$+\frac{b^{2/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)\left(-fa^3+ea^2b-dab^2+cb^3\right)}{3a^{14/3}}$$
$$-\frac{b^{2/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)\left(-fa^3+ea^2b-dab^2+cb^3\right)}{3a^{14/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x)

[Out] (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a
^(14/3)) - (c/(11*a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(2*a^4) +
(x^3*(a*d - b*c))/(8*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(5*a^3))/x^11 + (
b^(2/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1
/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(14/3)) - (b^(2/3)*log(3^(1/2
)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f
 - a*b^2*d + a^2*b*e))/(3*a^(14/3))
```

3.248 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 313

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)}\,dx &= -\frac{c}{13ax^{13}}+\frac{bc-ad}{10a^2x^{10}}-\frac{b^2c-abd+a^2e}{7a^3x^7}\\&\quad+\frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{a^5x}\\&\quad+\frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{16/3}}\\&\quad+\frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{16/3}}\\&\quad-\frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{16/3}}\end{aligned}$$

```
[Out] -1/13*c/a/x^13+1/10*(-a*d+b*c)/a^2/x^10+1/7*(-a^2*e+a*b*d-b^2*c)/a^3/x^7+1/
4*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^4-b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a
^5/x+1/3*b^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(16
/3)-1/6*b^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/a^(16/3)+1/3*b^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/
3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(16/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1848, 298, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)}\,dx$$
$$= \frac{bc-ad}{10a^2x^{10}} - \frac{a^2e-abd+b^2c}{7a^3x^7} + \frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{16/3}}$$
$$- \frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{16/3}}$$
$$+ \frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{16/3}}$$
$$- \frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{a^5x} + \frac{a^3(-f)+a^2be-ab^2d+b^3c}{4a^4x^4} - \frac{c}{13ax^{13}}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x]
```

```
[Out] -1/13*c/(a*x^13) + (b*c - a*d)/(10*a^2*x^10) - (b^2*c - a*b*d + a^2*e)/(7*a
^3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4) - (b*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f))/(a^5*x) + (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(16/3))
+ (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3
*a^(16/3)) - (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(16/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{ax^{14}}+\frac{-bc+ad}{a^2x^{11}}+\frac{b^2c-abd+a^2e}{a^3x^8}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^5}\right.\\
&\qquad\left.-\frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^5x^2}+\frac{b^2(-b^3c+ab^2d-a^2be+a^3f)\,x}{a^5\,(a+bx^3)}\right)dx\\
&= -\frac{c}{13ax^{13}}+\frac{bc-ad}{10a^2x^{10}}-\frac{b^2c-abd+a^2e}{7a^3x^7}+\frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4}\\
&\quad-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{a^5x}-\frac{\left(b^2(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{x}{a+bx^3}\,dx}{a^5}\\
&= -\frac{c}{13ax^{13}}+\frac{bc-ad}{10a^2x^{10}}-\frac{b^2c-abd+a^2e}{7a^3x^7}+\frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4}\\
&\quad-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{a^5x}+\frac{\left(b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{16/3}}\\
&\quad-\frac{\left(b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{16/3}}
\end{aligned}$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b(b^3c-ab^2d+a^2be-a^3f)}{a^5x} + \frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{16/3}} - \frac{\left(b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{16/3}} - \frac{\left(b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^5}$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b(b^3c-ab^2d+a^2be-a^3f)}{a^5x} + \frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{16/3}} - \frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{16/3}} - \frac{\left(b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{16/3}}$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b(b^3c-ab^2d+a^2be-a^3f)}{a^5x} + \frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{16/3}} + \frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{16/3}} - \frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{16/3}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)}\,dx = -\frac{c}{13ax^{13}}+\frac{bc-ad}{10a^2x^{10}}-\frac{b^2c-abd+a^2e}{7a^3x^7}+\frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4}+\frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^5x}+\frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{16/3}}+\frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{16/3}}+\frac{b^{4/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{16/3}}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x]
```

```
[Out] -1/13*c/(a*x^13) + (b*c - a*d)/(10*a^2*x^10) - (b^2*c - a*b*d + a^2*e)/(7*a
^3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4) + (b*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f))/(a^5*x) + (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(16/3)) + (
b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^
(16/3)) + (b^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(16/3))
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.76

method	result
default	$-\frac{c}{13a\,x^{13}}-\frac{ad-bc}{10a^2x^{10}}-\frac{a^2e-abd+b^2c}{7a^3x^7}-\frac{f\,a^3-a^2be+a\,b^2d-b^3c}{4a^4x^4}+\frac{b(f\,a^3-a^2be+a\,b^2d-b^3c)}{a^5x}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right.\right.}{6b\left(\right.}\right.}{}$
risch	$\frac{\frac{\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)b\,x^{12}}{a^5}-\frac{\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x^9}{4a^4}-\frac{\left(a^2e-abd+b^2c\right)x^6}{7a^3}-\frac{(ad-bc)x^3}{10a^2}-\frac{c}{13a}}{x^{13}}+\frac{\left(\_R=\text{RootOf}\left(a^{16}\_Z^3+a^9b^4f^3-3a^8b\right.\right.}{}$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/13*c/a/x^13-1/10*(a*d-b*c)/a^2/x^10-1/7*(a^2*e-a*b*d+b^2*c)/a^3/x^7-1/4*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^4+b*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5/x+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b^2

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)}\,dx = -\frac{1820\,\sqrt{3}(b^4c-ab^3d+a^2b^2e-a^3bf)x^{13}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)-910\,(b^4c-ab^3d+a^2b^2e}{}$$

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x, algorithm="fricas")

[Out] -1/5460*(1820*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^13*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^13*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) + 1820*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^13*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) + 5460*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 1365*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 780*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 420*a^4*c - 546*(a^3*b*c - a^4*d)*x^3)/(a^5*x^13)

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)}\,dx=-\frac{\sqrt{3}(b^4c-ab^3d+a^2b^2e-a^3bf)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^5\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^5\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^5\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{1820\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^{12}-455\,(ab^3c-a^2b^2d+a^3be-a^4f)x^9+260\,(a^2b^2c-a^3bd+a^4e)x^6}{1820\,a^5x^{13}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(1/3)) - 1/6*(b^4*c - a*b^3*d + a^
2*b^2*e - a^3*b*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(a/b)^(1/3))
 + 1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(x + (a/b)^(1/3))/(a^5*(a
/b)^(1/3)) - 1/1820*(1820*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 45
5*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 260*(a^2*b^2*c - a^3*b*d +
a^4*e)*x^6 + 140*a^4*c - 182*(a^3*b*c - a^4*d)*x^3)/(a^5*x^13)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.32

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)}\,dx$$
$$= \frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}b^3c-(-ab^2)^{\frac{2}{3}}ab^2d+(-ab^2)^{\frac{2}{3}}a^2be-(-ab^2)^{\frac{2}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^6}$$
$$+\frac{\left(b^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^3e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^2f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^6}$$
$$-\frac{\left((-ab^2)^{\frac{2}{3}}b^3c-(-ab^2)^{\frac{2}{3}}ab^2d+(-ab^2)^{\frac{2}{3}}a^2be-(-ab^2)^{\frac{2}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^6}$$
$$-\frac{1820\,b^4cx^{12}-1820\,ab^3dx^{12}+1820\,a^2b^2ex^{12}-1820\,a^3bfx^{12}-455\,ab^3cx^9+455\,a^2b^2dx^9-455\,a^3bex^9+4}{1820\,a^5x^{13}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + (-a*b^2)^(2/3)
*a^2*b*e - (-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/a^6 + 1/3*(b^5*c*(-a/b)^(1/3) - a*b^4*d*(-a/b)^(1/3) + a^2*b^3*
e*(-a/b)^(1/3) - a^3*b^2*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1
/3)))/a^6 - 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + (-a*b^2)^(
2/3)*a^2*b*e - (-a*b^2)^(2/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/a^6 - 1/1820*(1820*b^4*c*x^12 - 1820*a*b^3*d*x^12 + 1820*a^2*b^2*e*x^12
- 1820*a^3*b*f*x^12 - 455*a*b^3*c*x^9 + 455*a^2*b^2*d*x^9 - 455*a^3*b*e*x^9
 + 455*a^4*f*x^9 + 260*a^2*b^2*c*x^6 - 260*a^3*b*d*x^6 + 260*a^4*e*x^6 - 18
2*a^3*b*c*x^3 + 182*a^4*d*x^3 + 140*a^4*c)/(a^5*x^13)
```

Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)}\,dx$$
$$= \frac{b^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{16/3}}$$
$$-\frac{\frac{c}{13\,a}-\frac{x^9\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{4\,a^4}+\frac{x^3\,\left(a\,d-b\,c\right)}{10\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{7\,a^3}+\frac{b\,x^{12}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^5}}{x^{13}}$$
$$-\frac{b^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{16/3}}$$
$$+\frac{b^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{16/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x)
```

```
[Out] (b^(4/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a
^(16/3)) - (c/(13*a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(4*a^4) +
(x^3*(a*d - b*c))/(10*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(7*a^3) + (b*x^1
2*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^5)/x^13 - (b^(4/3)*log(3^(1/2)*a^(
1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*
b^2*d + a^2*b*e))/(3*a^(16/3)) + (b^(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3
)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/
(3*a^(16/3))
```

3.249 $\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 315

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx &= -\frac{c}{14ax^{14}}+\frac{bc-ad}{11a^2x^{11}}-\frac{b^2c-abd+a^2e}{8a^3x^8}\\ &+\frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{2a^5x^2}\\ &+\frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{17/3}}\\ &-\frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{17/3}}\\ &+\frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{17/3}}\end{aligned}$$

```
[Out] -1/14*c/a/x^14+1/11*(-a*d+b*c)/a^2/x^11+1/8*(-a^2*e+a*b*d-b^2*c)/a^3/x^8+1/
5*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^5-1/2*b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)/a^5/x^2-1/3*b^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)
/a^(17/3)+1/6*b^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(17/3)+1/3*b^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(17/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1848, 206, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx$$

$$=\frac{bc-ad}{11a^2x^{11}}-\frac{a^2e-abd+b^2c}{8a^3x^8}+\frac{b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{17/3}}$$
$$+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{17/3}}$$
$$-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{17/3}}$$
$$-\frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{2a^5x^2}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{5a^4x^5}-\frac{c}{14ax^{14}}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x]
```

```
[Out] -1/14*c/(a*x^14) + (b*c - a*d)/(11*a^2*x^11) - (b^2*c - a*b*d + a^2*e)/(8*a
^3*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) - (b*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f))/(2*a^5*x^2) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(17/
3)) - (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x]
)/(3*a^(17/3)) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(17/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{ax^{15}} + \frac{-bc+ad}{a^2x^{12}} + \frac{b^2c-abd+a^2e}{a^3x^9} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^6} \right. \\
&\qquad \left. - \frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^5x^3} + \frac{b^2(-b^3c+ab^2d-a^2be+a^3f)}{a^5\,(a+bx^3)}\right) dx \\
&= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} \\
&\quad - \frac{b(b^3c-ab^2d+a^2be-a^3f)}{2a^5x^2} - \frac{(b^2(b^3c-ab^2d+a^2be-a^3f))\int \frac{1}{a+bx^3}\,dx}{a^5} \\
&= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} \\
&\quad - \frac{b(b^3c-ab^2d+a^2be-a^3f)}{2a^5x^2} - \frac{(b^2(b^3c-ab^2d+a^2be-a^3f))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{17/3}} \\
&\quad - \frac{(b^2(b^3c-ab^2d+a^2be-a^3f))\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{17/3}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} \\
&\quad - \frac{b(b^3c-ab^2d+a^2be-a^3f)}{2a^5x^2} - \frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{17/3}} \\
&\quad + \frac{\left(b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{17/3}} \\
&\quad - \frac{\left(b^2(b^3c-ab^2d+a^2be-a^3f)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{16/3}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} \\
&\quad - \frac{b(b^3c-ab^2d+a^2be-a^3f)}{2a^5x^2} - \frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{17/3}} \\
&\quad + \frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{17/3}} \\
&\quad - \frac{\left(b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{17/3}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} \\
&\quad - \frac{b(b^3c-ab^2d+a^2be-a^3f)}{2a^5x^2} + \frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{17/3}} \\
&\quad - \frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{17/3}} \\
&\quad + \frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{17/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx = -\frac{c}{14ax^{14}}+\frac{bc-ad}{11a^2x^{11}}-\frac{b^2c-abd+a^2e}{8a^3x^8}+\frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5}+\frac{b(-b^3c+ab^2d-a^2be+a^3f)}{2a^5x^2}+\frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{17/3}}+\frac{b^{5/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{17/3}}+\frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{17/3}}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x]
```

```
[Out] -1/14*c/(a*x^14) + (b*c - a*d)/(11*a^2*x^11) - (b^2*c - a*b*d + a^2*e)/(8*a
^3*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) + (b*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f))/(2*a^5*x^2) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(17/3))
 + (b^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x]
)/(3*a^(17/3)) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(17/3))
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.76

method	result
default	$-\frac{c}{14a\,x^{14}}-\frac{ad-bc}{11a^2x^{11}}-\frac{a^2e-abd+b^2c}{8a^3x^8}-\frac{f\,a^3-a^2be+a\,b^2d-b^3c}{5a^4x^5}+\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)b}{2a^5x^2}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right.\right.}{6b(}\right.}{}$
risch	$\frac{\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)b\,x^{12}}{2a^5}-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)x^9}{5a^4}-\frac{(a^2e-abd+b^2c)x^6}{8a^3}-\frac{(ad-bc)x^3}{11a^2}-\frac{c}{14a}}{x^{14}}+\frac{\left(\_R=\mathrm{RootOf}\left(a^{17}\_Z^3-a^9b^5f^3+3a\right.\right.}{}$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/14*c/a/x^14-1/11*(a*d-b*c)/a^2/x^11-1/8*(a^2*e-a*b*d+b^2*c)/a^3/x^8-1/5*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^5+1/2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b/x^2+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b^2

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}\left(a+bx^3\right)}\,dx =$$

$$-\frac{3080\,\sqrt{3}(b^4c-ab^3d+a^2b^2e-a^3bf)x^{14}\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-1540\,(b^4c-ab^3d+a^2b^2e-}{}$$

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="fricas")

[Out] -1/9240*(3080*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^14*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 1540*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^14*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 3080*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^14*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 4620*(b^

```
4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 1848*(a*b^3*c - a^2*b^2*d + a^3
*b*e - a^4*f)*x^9 + 1155*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 660*a^4*c - 84
0*(a^3*b*c - a^4*d)*x^3)/(a^5*x^14)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**15/(b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx = -\frac{\sqrt{3}(b^4c-ab^3d+a^2b^2e-a^3bf)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{1540(b^4c-ab^3d+a^2b^2e-a^3bf)x^{12}-616(ab^3c-a^2b^2d+a^3be-a^4f)x^9+385(a^2b^2c-a^3bd+a^4e)x^6}{3080a^5x^{14}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(2/3)) + 1/6*(b^4*c - a*b^3*d + a^
2*b^2*e - a^3*b*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(a/b)^(2/3))
 - 1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(x + (a/b)^(1/3))/(a^5*(a
/b)^(2/3)) - 1/3080*(1540*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 61
6*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 385*(a^2*b^2*c - a^3*b*d +
a^4*e)*x^6 + 220*a^4*c - 280*(a^3*b*c - a^4*d)*x^3)/(a^5*x^14)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.23

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx = -\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}b^4c-(-ab^2)^{\frac{1}{3}}ab^3d+(-ab^2)^{\frac{1}{3}}a^2b^2e-(-ab^2)^{\frac{1}{3}}a^3bf\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^6} + \frac{(b^5c-ab^4d+a^2b^3e-a^3b^2f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^6} - \frac{\left((-ab^2)^{\frac{1}{3}}b^4c-(-ab^2)^{\frac{1}{3}}ab^3d+(-ab^2)^{\frac{1}{3}}a^2b^2e-(-ab^2)^{\frac{1}{3}}a^3bf\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^6} - \frac{1540\,b^4cx^{12}-1540\,ab^3dx^{12}+1540\,a^2b^2ex^{12}-1540\,a^3bfx^{12}-616\,ab^3cx^9+616\,a^2b^2dx^9-616\,a^3bex^9+}{3080\,a^5x^{14}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*b^4*c - (-a*b^2)^(1/3)*a*b^3*d + (-a*b^2)^(1/3
)*a^2*b^2*e - (-a*b^2)^(1/3)*a^3*b*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/a^6 + 1/3*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6 - 1/6*((-a*b^2)^(1/3)*b^4*c - (-a*b^2
)^(1/3)*a*b^3*d + (-a*b^2)^(1/3)*a^2*b^2*e - (-a*b^2)^(1/3)*a^3*b*f)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^6 - 1/3080*(1540*b^4*c*x^12 - 1540*a*b
^3*d*x^12 + 1540*a^2*b^2*e*x^12 - 1540*a^3*b*f*x^12 - 616*a*b^3*c*x^9 + 616
*a^2*b^2*d*x^9 - 616*a^3*b*e*x^9 + 616*a^4*f*x^9 + 385*a^2*b^2*c*x^6 - 385*
a^3*b*d*x^6 + 385*a^4*e*x^6 - 280*a^3*b*c*x^3 + 280*a^4*d*x^3 + 220*a^4*c)/
(a^5*x^14)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx$$
$$= -\frac{\frac{c}{14\,a}-\frac{x^9\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{5\,a^4}+\frac{x^3\,\left(a\,d-b\,c\right)}{11\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{8\,a^3}+\frac{b\,x^{12}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{2\,a^5}}{x^{14}}$$
$$-\frac{b^{5/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{17/3}}$$
$$-\frac{b^{5/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{17/3}}$$
$$+\frac{b^{5/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{17/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x)
```

```
[Out] (b^(5/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 +
1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(17/3)) - (b^(5/3)*log(b^(1/
3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(17/3)) - (b^(5/3
)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b
^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(17/3)) - (c/(14*a) - (x^9*(b^3*c -
 a^3*f - a*b^2*d + a^2*b*e))/(5*a^4) + (x^3*(a*d - b*c))/(11*a^2) + (x^6*(b
^2*c + a^2*e - a*b*d))/(8*a^3) + (b*x^12*(b^3*c - a^3*f - a*b^2*d + a^2*b*e
))/(2*a^5))/x^14
```

3.250 $\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 351

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{17}\left(a+bx^3\right)}\,dx =& -\frac{c}{16ax^{16}}+\frac{bc-ad}{13a^2x^{13}}-\frac{b^2c-abd+a^2e}{10a^3x^{10}} \\ &+\frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{4a^5x^4} \\ &+\frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x} \\ &-\frac{b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{19/3}} \\ &-\frac{b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{19/3}} \\ &+\frac{b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{19/3}}\end{aligned}$$

```
[Out] -1/16*c/a/x^16+1/13*(-a*d+b*c)/a^2/x^13+1/10*(-a^2*e+a*b*d-b^2*c)/a^3/x^10+
1/7*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^7-1/4*b*(-a^3*f+a^2*b*e-a*b^2*d+b^
3*c)/a^5/x^4+b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^6/x-1/3*b^(7/3)*(-a^3*f+a
^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(19/3)+1/6*b^(7/3)*(-a^3*f+a^
2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(19/3)-1/3
*b^(7/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/a^(19/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1848, 298, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx$$
$$= \frac{bc-ad}{13a^2x^{13}} - \frac{a^2e-abd+b^2c}{10a^3x^{10}} - \frac{b^{7/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{19/3}}$$
$$+ \frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{19/3}}$$
$$- \frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{19/3}} + \frac{b^2(a^3(-f)+a^2be-ab^2d+b^3c)}{a^6x}$$
$$- \frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^5x^4} + \frac{a^3(-f)+a^2be-ab^2d+b^3c}{7a^4x^7} - \frac{c}{16ax^{16}}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x]
```

```
[Out] -1/16*c/(a*x^16) + (b*c - a*d)/(13*a^2*x^13) - (b^2*c - a*b*d + a^2*e)/(10*
a^3*x^10) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^7) - (b*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f))/(4*a^5*x^4) + (b^2*(b^3*c - a*b^2*d + a^2*b*e -
a^3*f))/(a^6*x) - (b^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(19/3)) - (b^(7/3)*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(19/3)) + (b^(
7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(6*a^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{c}{ax^{17}} + \frac{-bc+ad}{a^2x^{14}} + \frac{b^2c-abd+a^2e}{a^3x^{11}} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^8} \right. \\ &\quad - \frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^5x^5} + \frac{b^2(-b^3c+ab^2d-a^2be+a^3f)}{a^6x^2} \\ &\quad \left. - \frac{b^3(-b^3c+ab^2d-a^2be+a^3f)\,x}{a^6\,(a+bx^3)}\right) dx \\ &= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} \\ &\quad - \frac{b(b^3c-ab^2d+a^2be-a^3f)}{4a^5x^4} + \frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x} \\ &\quad + \frac{(b^3(b^3c-ab^2d+a^2be-a^3f))\int \frac{x}{a+bx^3}\,dx}{a^6}\end{aligned}$$

$$=-\frac{c}{16ax^{16}}+\frac{bc-ad}{13a^2x^{13}}-\frac{b^2c-abd+a^2e}{10a^3x^{10}}+\frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{4a^5x^4}+\frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x}-\frac{\left(b^{8/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{19/3}}+\frac{\left(b^{8/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{19/3}}$$

$$=-\frac{c}{16ax^{16}}+\frac{bc-ad}{13a^2x^{13}}-\frac{b^2c-abd+a^2e}{10a^3x^{10}}+\frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{4a^5x^4}+\frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x}-\frac{b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{19/3}}+\frac{\left(b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{19/3}}+\frac{\left(b^{8/3}(b^3c-ab^2d+a^2be-a^3f)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^6}$$

$$=-\frac{c}{16ax^{16}}+\frac{bc-ad}{13a^2x^{13}}-\frac{b^2c-abd+a^2e}{10a^3x^{10}}+\frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{4a^5x^4}+\frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x}-\frac{b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{19/3}}+\frac{b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{19/3}}+\frac{\left(b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{19/3}}$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b(b^3c-ab^2d+a^2be-a^3f)}{4a^5x^4} + \frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x} - \frac{b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{19/3}} - \frac{b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{19/3}} + \frac{b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{19/3}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx = -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} + \frac{b(-b^3c+ab^2d-a^2be+a^3f)}{4a^5x^4} + \frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x} + \frac{b^{7/3}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{19/3}} + \frac{b^{7/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{19/3}} + \frac{b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{19/3}}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x]
```

```
[Out] -1/16*c/(a*x^16) + (b*c - a*d)/(13*a^2*x^13) - (b^2*c - a*b*d + a^2*e)/(10*
a^3*x^10) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^7) + (b*(-(b^3*c)
+ a*b^2*d - a^2*b*e + a^3*f))/(4*a^5*x^4) + (b^2*(b^3*c - a*b^2*d + a^2*b*e
 - a^3*f))/(a^6*x) + (b^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan
[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(19/3)) + (b^(7/3)*(-(b^3
*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(19/3)) + (
b^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(6*a^(19/3))
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.79

method	result
default	$-\frac{c}{16a\,x^{16}}-\frac{ad-bc}{13a^{2}x^{13}}-\frac{a^{2}e-abd+b^{2}c}{10a^{3}x^{10}}-\frac{f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c}{7a^{4}x^{7}}-\frac{(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c)b^{2}}{a^{6}x}+\frac{(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c)b}{4a^{5}x^{4}}-$
risch	$\frac{-\frac{c}{16a}-\frac{(ad-bc)x^{3}}{13a^{2}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{10a^{3}}-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{9}}{7a^{4}}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)b\,x^{12}}{4a^{5}}-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)b^{2}x^{15}}{a^{6}}}{x^{16}}+$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/16*c/a/x^16-1/13*(a*d-b*c)/a^2/x^13-1/10*(a^2*e-a*b*d+b^2*c)/a^3/x^10-1/7*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^7-(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^6*b^2/x+1/4*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b/x^4-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b^3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^6

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^{3}+ex^{6}+fx^{9}}{x^{17}\left(a+bx^{3}\right)}\,dx$$
$$=\frac{7280\sqrt{3}(b^{5}c-ab^{4}d+a^{2}b^{3}e-a^{3}b^{2}f)x^{16}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+3640\,(b^{5}c-ab^{4}d+a^{2}b^{3}e-a^{3}}{}$$

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="fricas")

[Out] 1/21840*(7280*sqrt(3)*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^16*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 3640*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^16*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 7280*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^16*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) + 21840*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b

```
^2*f)*x^15 - 5460*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^12 + 3120*(
a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x^9 - 2184*(a^3*b^2*c - a^4*b*d +
a^5*e)*x^6 - 1365*a^5*c + 1680*(a^4*b*c - a^5*d)*x^3)/(a^6*x^16)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**17/(b*x**3+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx = \frac{\sqrt{3}(b^5c-ab^4d+a^2b^3e-a^3b^2f)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(b^5c-ab^4d+a^2b^3e-a^3b^2f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(b^5c-ab^4d+a^2b^3e-a^3b^2f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{7280\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{15}-1820\,(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^{12}+1040\,(a^2b^3c-a^3b^2d+}{7280\,a^6x^{16}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^6*(a/b)^(1/3)) + 1/6*(b^5*c - a*b^4*d + a
^2*b^3*e - a^3*b^2*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^(1/
3)) - 1/3*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*log(x + (a/b)^(1/3))/(a
^6*(a/b)^(1/3)) + 1/7280*(7280*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^
15 - 1820*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^12 + 1040*(a^2*b^3*
c - a^3*b^2*d + a^4*b*e - a^5*f)*x^9 - 728*(a^3*b^2*c - a^4*b*d + a^5*e)*x^
6 - 455*a^5*c + 560*(a^4*b*c - a^5*d)*x^3)/(a^6*x^16)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.33

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}\left(a+bx^3\right)}\,dx = -\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^4c-\left(-ab^2\right)^{\frac{2}{3}}ab^3d+\left(-ab^2\right)^{\frac{2}{3}}a^2b^2e-\left(-ab^2\right)^{\frac{2}{3}}a^3bf\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^7} - \frac{\left(b^6c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^5d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^4e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^7} + \frac{\left(\left(-ab^2\right)^{\frac{2}{3}}b^4c-\left(-ab^2\right)^{\frac{2}{3}}ab^3d+\left(-ab^2\right)^{\frac{2}{3}}a^2b^2e-\left(-ab^2\right)^{\frac{2}{3}}a^3bf\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^7} + \frac{7280\,b^5cx^{15}-7280\,ab^4dx^{15}+7280\,a^2b^3ex^{15}-7280\,a^3b^2fx^{15}-1820\,ab^4cx^{12}+1820\,a^2b^3dx^{12}-1820\,a^3b^2}{}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*b^4*c - (-a*b^2)^(2/3)*a*b^3*d + (-a*b^2)^(2/3
)*a^2*b^2*e - (-a*b^2)^(2/3)*a^3*b*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/a^7 - 1/3*(b^6*c*(-a/b)^(1/3) - a*b^5*d*(-a/b)^(1/3) + a^2
*b^4*e*(-a/b)^(1/3) - a^3*b^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/a^7 + 1/6*((-a*b^2)^(2/3)*b^4*c - (-a*b^2)^(2/3)*a*b^3*d + (-a*b
^2)^(2/3)*a^2*b^2*e - (-a*b^2)^(2/3)*a^3*b*f)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/a^7 + 1/7280*(7280*b^5*c*x^15 - 7280*a*b^4*d*x^15 + 7280*a^2*b^
3*e*x^15 - 7280*a^3*b^2*f*x^15 - 1820*a*b^4*c*x^12 + 1820*a^2*b^3*d*x^12 -
1820*a^3*b^2*e*x^12 + 1820*a^4*b*f*x^12 + 1040*a^2*b^3*c*x^9 - 1040*a^3*b^2
*d*x^9 + 1040*a^4*b*e*x^9 - 1040*a^5*f*x^9 - 728*a^3*b^2*c*x^6 + 728*a^4*b*
d*x^6 - 728*a^5*e*x^6 + 560*a^4*b*c*x^3 - 560*a^5*d*x^3 - 455*a^5*c)/(a^6*x
^16)
```

Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}\left(a+bx^3\right)}\,dx =$$

$$-\frac{\frac{c}{16\,a}-\frac{x^9\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{7\,a^4}+\frac{x^3\,\left(a\,d-b\,c\right)}{13\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{10\,a^3}+\frac{b\,x^{12}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{4\,a^5}-\frac{b^2\,x^{15}\,\left(-f\,a^3+e\right.}{a}}{x^{16}}$$

$$-\frac{b^{7/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{19/3}}$$

$$+\frac{b^{7/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{19/3}}$$

$$-\frac{b^{7/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{19/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x)
```

```
[Out] (b^(7/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(19/3)) - (b^(7/3)*log(b^(1/
3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(19/3)) - (c/(16*
a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(7*a^4) + (x^3*(a*d - b*c))/
(13*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(10*a^3) + (b*x^12*(b^3*c - a^3*f
- a*b^2*d + a^2*b*e))/(4*a^5) - (b^2*x^15*(b^3*c - a^3*f - a*b^2*d + a^2*b*
e))/a^6)/x^16 - (b^(7/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((
3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(19/3))
```

3.251 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 220

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x^3}{3b^6} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^6}{6b^5} + \frac{(b^2d-2abe+3a^2f)\,x^9}{9b^4} + \frac{(be-2af)x^{12}}{12b^3} + \frac{fx^{15}}{15b^2} + \frac{a^3(b^3c-ab^2d+a^2be-a^3f)}{3b^7\,(a+bx^3)} + \frac{a^2(3b^3c-4ab^2d+5a^2be-6a^3f)\log(a+bx^3)}{3b^7}$$

```
[Out] -1/3*a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*x^3/b^6+1/6*(-4*a^3*f+3*a^2*b
*e-2*a*b^2*d+b^3*c)*x^6/b^5+1/9*(3*a^2*f-2*a*b*e+b^2*d)*x^9/b^4+1/12*(-2*a*
f+b*e)*x^12/b^3+1/15*f*x^15/b^2+1/3*a^3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b^7/
(b*x^3+a)+1/3*a^2*(-6*a^3*f+5*a^2*b*e-4*a*b^2*d+3*b^3*c)*ln(b*x^3+a)/b^7
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used

= {1835, 1634}

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{x^9(3a^2f-2abe+b^2d)}{9b^4}+\frac{a^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^7\,(a+bx^3)} + \frac{a^2\log\left(a+bx^3\right)\left(-6a^3f+5a^2be-4ab^2d+3b^3c\right)}{3b^7} - \frac{ax^3(-5a^3f+4a^2be-3ab^2d+2b^3c)}{3b^6} + \frac{x^6(-4a^3f+3a^2be-2ab^2d+b^3c)}{6b^5} + \frac{x^{12}(be-2af)}{12b^3}+\frac{fx^{15}}{15b^2}$$

```
[In] Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x^3)/b^6 + ((b^3*c - 2*
a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6)/(6*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*
x^9)/(9*b^4) + ((b*e - 2*a*f)*x^12)/(12*b^3) + (f*x^15)/(15*b^2) + (a^3*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^7*(a + b*x^3)) + (a^2*(3*b^3*c - 4*a
*b^2*d + 5*a^2*b*e - 6*a^3*f)*Log[a + b*x^3])/(3*b^7)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-2b^3c+3ab^2d-4a^2be+5a^3f)}{b^6}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x}{b^5} + \frac{(b^2d-2abe+3a^2f)\,x^2}{b^4}+\frac{(be-2af)x^3}{b^3}+\frac{fx^4}{b^2}+\frac{a^3(-b^3c+ab^2d-a^2be+a^3f)}{b^6(a+bx)^2} - \frac{a^2(-3b^3c+4ab^2d-5a^2be+6a^3f)}{b^6(a+bx)}\right) dx, x, x^3\right)$$

$$= -\frac{a(2b^3c - 3ab^2d + 4a^2be - 5a^3f)\,x^3}{3b^6} + \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)\,x^6}{6b^5} + \frac{(b^2d - 2abe + 3a^2f)\,x^9}{9b^4} + \frac{(be - 2af)x^{12}}{12b^3} + \frac{fx^{15}}{15b^2} + \frac{a^3(b^3c - ab^2d + a^2be - a^3f)}{3b^7\,(a + bx^3)} + \frac{a^2(3b^3c - 4ab^2d + 5a^2be - 6a^3f)\log(a + bx^3)}{3b^7}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.93

$$\int \frac{x^{11}(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2}\,dx$$

$$= \frac{60ab(-2b^3c + 3ab^2d - 4a^2be + 5a^3f)\,x^3 + 30b^2(b^3c - 2ab^2d + 3a^2be - 4a^3f)\,x^6 + 20b^3(b^2d - 2abe + 3a^2f)}{}$$

```
[In] Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (60*a*b*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x^3 + 30*b^2*(b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6 + 20*b^3*(b^2*d - 2*a*b*e + 3*a^2*f)*x
^9 + 15*b^4*(b*e - 2*a*f)*x^12 + 12*b^5*f*x^15 - (60*a^3*(-(b^3*c) + a*b^2*
d - a^2*b*e + a^3*f))/(a + b*x^3) + 60*a^2*(3*b^3*c - 4*a*b^2*d + 5*a^2*b*e
 - 6*a^3*f)*Log[a + b*x^3])/(180*b^7)
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.99

method	result
default	$\frac{\frac{f\,x^{15} b^4}{15}+\frac{\left(-2a\,b^3 f+b^4 e\right)x^{12}}{12}+\frac{\left(3a^2 b^2 f-2a\,b^3 e+b^4 d\right)x^9}{9}+\frac{\left(-4a^3 bf+3a^2 e\,b^2-2a\,b^3 d+b^4 c\right)x^6}{6}+\frac{\left(5a^4 f-4a^3 be+3a^2 b^2 d-2a\,b^3 c\right)x^3}{3}}{b^6} - a^2$
norman	$\frac{-\frac{a\left(6f\,a^5-5a^4 eb+4a^3 d\,b^2-3a^2 c\,b^3\right)}{3b^7}+\frac{f\,x^{18}}{15b}-\frac{(6af-5be)x^{15}}{60b^2}+\frac{\left(6a^2 f-5aeb+4b^2 d\right)x^{12}}{36b^3}-\frac{\left(6f\,a^3-5a^2 be+4a\,b^2 d-3b^3 c\right)x^9}{18b^4}+a\left(6f\,a^3-5a^2 b\right.}{b\,x^3+a}$
risch	$\frac{f\,x^{15}}{15b^2}-\frac{af\,x^{12}}{6b^3}+\frac{e\,x^{12}}{12b^2}+\frac{a^2 f\,x^9}{3b^4}-\frac{2ae\,x^9}{9b^3}+\frac{d\,x^9}{9b^2}-\frac{2f\,x^6 a^3}{3b^5}+\frac{a^2 e\,x^6}{2b^4}-\frac{ad\,x^6}{3b^3}+\frac{c\,x^6}{6b^2}+\frac{5a^4 f\,x^3}{3b^6}-\frac{4a^3 e\,x^3}{3b^5}+$
parallelrisch	$-\frac{-12f\,x^{18}b^6+360a^6 f-15x^{15}b^6 e-20x^{12}b^6 d-30x^9 b^6 c+360\ln\left(b\,x^3+a\right)a^6 f-180a^3 b^3 c-50x^9 a^2 b^4 e+40x^9 a\,b^5 d-180x^6 a^4 b^2 f+1}{}$

```
[In] int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^6*(1/15*f*x^15*b^4+1/12*(-2*a*b^3*f+b^4*e)*x^12+1/9*(3*a^2*b^2*f-2*a*b^
3*e+b^4*d)*x^9+1/6*(-4*a^3*b*f+3*a^2*b^2*e-2*a*b^3*d+b^4*c)*x^6+1/3*(5*a^4*
f-4*a^3*b*e+3*a^2*b^2*d-2*a*b^3*c)*x^3)-1/3*a^2/b^6*((6*a^3*f-5*a^2*b*e+4*a
*b^2*d-3*b^3*c)/b*ln(b*x^3+a)+a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.38

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{12\,b^6fx^{18}+3\,(5\,b^6e-6\,ab^5f)x^{15}+5\,(4\,b^6d-5\,ab^5e+6\,a^2b^4f)x^{12}+10\,(3\,b^6c-4\,ab^5d+5\,a^2b^4e-6\,a^3b^3f}{}$$

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/180*(12*b^6*f*x^18 + 3*(5*b^6*e - 6*a*b^5*f)*x^15 + 5*(4*b^6*d - 5*a*b^5*
e + 6*a^2*b^4*f)*x^12 + 10*(3*b^6*c - 4*a*b^5*d + 5*a^2*b^4*e - 6*a^3*b^3*f
)*x^9 + 60*a^3*b^3*c - 60*a^4*b^2*d + 60*a^5*b*e - 60*a^6*f - 30*(3*a*b^5*c
 - 4*a^2*b^4*d + 5*a^3*b^3*e - 6*a^4*b^2*f)*x^6 - 60*(2*a^2*b^4*c - 3*a^3*b
^3*d + 4*a^4*b^2*e - 5*a^5*b*f)*x^3 + 60*(3*a^3*b^3*c - 4*a^4*b^2*d + 5*a^5
*b*e - 6*a^6*f + (3*a^2*b^4*c - 4*a^3*b^3*d + 5*a^4*b^2*e - 6*a^5*b*f)*x^3)
*log(b*x^3 + a))/(b^8*x^3 + a*b^7)
```

Sympy [A] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = &-\frac{a^2\cdot(6a^3f-5a^2be+4ab^2d-3b^3c)\log(a+bx^3)}{3b^7} \\ &+x^{12}\left(-\frac{af}{6b^3}+\frac{e}{12b^2}\right)+x^9\left(\frac{a^2f}{3b^4}-\frac{2ae}{9b^3}+\frac{d}{9b^2}\right) \\ &+x^6\left(-\frac{2a^3f}{3b^5}+\frac{a^2e}{2b^4}-\frac{ad}{3b^3}+\frac{c}{6b^2}\right) \\ &+x^3\cdot\left(\frac{5a^4f}{3b^6}-\frac{4a^3e}{3b^5}+\frac{a^2d}{b^4}-\frac{2ac}{3b^3}\right) \\ &+\frac{-a^6f+a^5be-a^4b^2d+a^3b^3c}{3ab^7+3b^8x^3}+\frac{fx^{15}}{15b^2}\end{aligned}$$

```
[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -a**2*(6*a**3*f - 5*a**2*b*e + 4*a*b**2*d - 3*b**3*c)*log(a + b*x**3)/(3*b*
*7) + x**12*(-a*f/(6*b**3) + e/(12*b**2)) + x**9*(a**2*f/(3*b**4) - 2*a*e/(
9*b**3) + d/(9*b**2)) + x**6*(-2*a**3*f/(3*b**5) + a**2*e/(2*b**4) - a*d/(3
*b**3) + c/(6*b**2)) + x**3*(5*a**4*f/(3*b**6) - 4*a**3*e/(3*b**5) + a**2*d
/b**4 - 2*a*c/(3*b**3)) + (-a**6*f + a**5*b*e - a**4*b**2*d + a**3*b**3*c)/
(3*a*b**7 + 3*b**8*x**3) + f*x**15/(15*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.01

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{a^3b^3c-a^4b^2d+a^5be-a^6f}{3\,(b^8x^3+ab^7)}$$
$$+\frac{12\,b^4fx^{15}+15\,(b^4e-2\,ab^3f)x^{12}+20\,(b^4d-2\,ab^3e+3\,a^2b^2f)x^9+30\,(b^4c-2\,ab^3d+3\,a^2b^2e-4\,a^3bf)x^6}{180\,b^6}$$
$$+\frac{(3\,a^2b^3c-4\,a^3b^2d+5\,a^4be-6\,a^5f)\log(bx^3+a)}{3\,b^7}$$

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)/(b^8*x^3 + a*b^7) + 1/180*(12
*b^4*f*x^15 + 15*(b^4*e - 2*a*b^3*f)*x^12 + 20*(b^4*d - 2*a*b^3*e + 3*a^2*b
^2*f)*x^9 + 30*(b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^6 - 60*(2*a*
b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*x^3)/b^6 + 1/3*(3*a^2*b^3*c - 4*
a^3*b^2*d + 5*a^4*b*e - 6*a^5*f)*log(b*x^3 + a)/b^7
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.33

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{(3\,a^2b^3c-4\,a^3b^2d+5\,a^4be-6\,a^5f)\log(|bx^3+a|)}{3\,b^7}$$
$$-\frac{3\,a^2b^4cx^3-4\,a^3b^3dx^3+5\,a^4b^2ex^3-6\,a^5bfx^3+2\,a^3b^3c-3\,a^4b^2d+4\,a^5be-5\,a^6f}{3\,(bx^3+a)b^7}$$
$$+\frac{12\,b^8fx^{15}+15\,b^8ex^{12}-30\,ab^7fx^{12}+20\,b^8dx^9-40\,ab^7ex^9+60\,a^2b^6fx^9+30\,b^8cx^6-60\,ab^7dx^6+90\,a^2b^6}{180\,b^{10}}$$

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*b^3*c - 4*a^3*b^2*d + 5*a^4*b*e - 6*a^5*f)*log(abs(b*x^3 + a))/b
^7 - 1/3*(3*a^2*b^4*c*x^3 - 4*a^3*b^3*d*x^3 + 5*a^4*b^2*e*x^3 - 6*a^5*b*f*x
^3 + 2*a^3*b^3*c - 3*a^4*b^2*d + 4*a^5*b*e - 5*a^6*f)/((b*x^3 + a)*b^7) + 1
/180*(12*b^8*f*x^15 + 15*b^8*e*x^12 - 30*a*b^7*f*x^12 + 20*b^8*d*x^9 - 40*a
*b^7*e*x^9 + 60*a^2*b^6*f*x^9 + 30*b^8*c*x^6 - 60*a*b^7*d*x^6 + 90*a^2*b^6*
e*x^6 - 120*a^3*b^5*f*x^6 - 120*a*b^7*c*x^3 + 180*a^2*b^6*d*x^3 - 240*a^3*b
^5*e*x^3 + 300*a^4*b^4*f*x^3)/b^10
```

Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.62

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^{12}\left(\frac{e}{12\,b^2}-\frac{a\,f}{6\,b^3}\right) - x^3\left(\frac{2\,a\left(\frac{c}{b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)}{3\,b} - \frac{a^2\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{3\,b^2}\right) - x^9\left(\frac{a^2\,f}{9\,b^4}-\frac{d}{9\,b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{9\,b}\right) + x^6\left(\frac{c}{6\,b^2} - \frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{6\,b^2} + \frac{a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{3\,b}\right) - \frac{\ln\left(b\,x^3+a\right)\,\left(6\,f\,a^5-5\,e\,a^4\,b+4\,d\,a^3\,b^2-3\,c\,a^2\,b^3\right)}{3\,b^7} + \frac{f\,x^{15}}{15\,b^2} - \frac{f\,a^6-e\,a^5\,b+d\,a^4\,b^2-c\,a^3\,b^3}{3\,b\,\left(b^7\,x^3+a\,b^6\right)}$$

```
[In] int((x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^12*(e/(12*b^2) - (a*f)/(6*b^3)) - x^3*((2*a*(c/b^2 - (a^2*(e/b^2 - (2*a*f
)/b^3))/b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b
))/(3*b) - (a^2*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(3*b
^2)) - x^9*((a^2*f)/(9*b^4) - d/(9*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(9*b)
) + x^6*(c/(6*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))/(6*b^2) + (a*((a^2*f)/b^4
- d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(3*b)) - (log(a + b*x^3)*(6*a^5*f
```

```
- 3*a^2*b^3*c + 4*a^3*b^2*d - 5*a^4*b*e))/(3*b^7) + (f*x^15)/(15*b^2) - (a
^6*f - a^3*b^3*c + a^4*b^2*d - a^5*b*e)/(3*b*(a*b^6 + b^7*x^3))
```

3.252 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 180

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^3}{3b^5}+\frac{(b^2d-2abe+3a^2f)\,x^6}{6b^4}+\frac{(be-2af)x^9}{9b^3}+\frac{fx^{12}}{12b^2}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)}{3b^6\,(a+bx^3)}-\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\log(a+bx^3)}{3b^6}$$

```
[Out] 1/3*(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*x^3/b^5+1/6*(3*a^2*f-2*a*b*e+b^2*d
)*x^6/b^4+1/9*(-2*a*f+b*e)*x^9/b^3+1/12*f*x^12/b^2-1/3*a^2*(-a^3*f+a^2*b*e-
a*b^2*d+b^3*c)/b^6/(b*x^3+a)-1/3*a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*l
n(b*x^3+a)/b^6
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{x^6(3a^2f-2abe+b^2d)}{6b^4}-\frac{a^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^6\,(a+bx^3)}-\frac{a\log(a+bx^3)\,(-5a^3f+4a^2be-3ab^2d+2b^3c)}{3b^6}+\frac{x^3(-4a^3f+3a^2be-2ab^2d+b^3c)}{3b^5}+\frac{x^9(be-2af)}{9b^3}+\frac{fx^{12}}{12b^2}$$

```
[In] Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3)/(3*b^5) + ((b^2*d - 2*a*b*e
 + 3*a^2*f)*x^6)/(6*b^4) + ((b*e - 2*a*f)*x^9)/(9*b^3) + (f*x^12)/(12*b^2)
- (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^6*(a + b*x^3)) - (a*(2*b^3
*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx)^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^3c-2ab^2d+3a^2be-4a^3f}{b^5}+\frac{(b^2d-2abe+3a^2f)\,x}{b^4}+\frac{(be-2af)x^2}{b^3}\right.\right.\\ &\qquad +\frac{fx^3}{b^2}-\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)}{b^5(a+bx)^2}\\ &\qquad \left.\left.+\frac{a(-2b^3c+3ab^2d-4a^2be+5a^3f)}{b^5(a+bx)}\right)dx, x, x^3\right)\\ &= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^3}{3b^5}+\frac{(b^2d-2abe+3a^2f)\,x^6}{6b^4}+\frac{(be-2af)x^9}{9b^3}+\frac{fx^{12}}{12b^2}\\ &\quad -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)}{3b^6\,(a+bx^3)}-\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\log\left(a+bx^3\right)}{3b^6}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.93

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$=\frac{12b(b^3c-2ab^2d+3a^2be-4a^3f)\,x^3+6b^2(b^2d-2abe+3a^2f)\,x^6+4b^3(be-2af)x^9+3b^4fx^{12}+\frac{12a^2(-b^3c+}{a}}{36b^6}$$

```
[In] Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (12*b*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3 + 6*b^2*(b^2*d - 2*a*b*
e + 3*a^2*f)*x^6 + 4*b^3*(b*e - 2*a*f)*x^9 + 3*b^4*f*x^12 + (12*a^2*(-(b^3*
c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) + 12*a*(-2*b^3*c + 3*a*b^2*d -
 4*a^2*b*e + 5*a^3*f)*Log[a + b*x^3])/(36*b^6)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.99

method	result
default	$-\frac{-\frac{b^3 f x^{12}}{12}+\frac{(2fab^2-b^3e)x^9}{9}+\frac{(-3fa^2b+2ab^2e-b^3d)x^6}{6}+\frac{x^3(4fa^3-3a^2be+2ab^2d-b^3c)}{3}}{b^5}+\frac{a\left(\frac{(5fa^3-4a^2be+3ab^2d-2b^3c)\ln(b}{b}\right.}{3b}$
norman	$\frac{-\frac{(5fa^3-4a^2be+3ab^2d-2b^3c)x^6}{6b^4}+\frac{fx^{15}}{12b}-\frac{(5af-4be)x^{12}}{36b^2}+\frac{(5a^2f-4aeb+3b^2d)x^9}{18b^3}-\frac{(5fa^5-4a^4eb+3a^3db^2-2a^2cb^3)x^3}{3ab^5}}{bx^3+a}+\frac{a(5fa^3-}{}$
risch	$\frac{fx^{12}}{12b^2}-\frac{2afx^9}{9b^3}+\frac{ex^9}{9b^2}+\frac{x^6fa^2}{2b^4}-\frac{aex^6}{3b^3}+\frac{dx^6}{6b^2}-\frac{4fa^3x^3}{3b^5}+\frac{a^2ex^3}{b^4}-\frac{2adx^3}{3b^3}+\frac{cx^3}{3b^2}+\frac{a^5f}{3b^6(bx^3+a)}-\frac{a}{3b^5(b}$
parallelrisch	$\frac{3fx^{15}b^5-5x^{12}ab^4f+4x^{12}b^5e+10x^9a^2b^3f-8x^9ab^4e+6x^9b^5d-30x^6a^3b^2f+24x^6a^2b^3e-18x^6ab^4d+12x^6b^5c+60\ln(bx^3+a)x^{}}{}$

```
[In] int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*(-1/12*b^3*f*x^12+1/9*(2*a*b^2*f-b^3*e)*x^9+1/6*(-3*a^2*b*f+2*a*b^2*
e-b^3*d)*x^6+1/3*x^3*(4*a^3*f-3*a^2*b*e+2*a*b^2*d-b^3*c))+1/3*a/b^5*((5*a^3
*f-4*a^2*b*e+3*a*b^2*d-2*b^3*c)/b*ln(b*x^3+a)+a*(a^3*f-a^2*b*e+a*b^2*d-b^3*
c)/b/(b*x^3+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.43

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{3\,b^5fx^{15}+(4\,b^5e-5\,ab^4f)x^{12}+2\,(3\,b^5d-4\,ab^4e+5\,a^2b^3f)x^9+6\,(2\,b^5c-3\,ab^4d+4\,a^2b^3e-5\,a^3b^2f)x^6}{}$$

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^5*f*x^15 + (4*b^5*e - 5*a*b^4*f)*x^12 + 2*(3*b^5*d - 4*a*b^4*e +
5*a^2*b^3*f)*x^9 + 6*(2*b^5*c - 3*a*b^4*d + 4*a^2*b^3*e - 5*a^3*b^2*f)*x^6
- 12*a^2*b^3*c + 12*a^3*b^2*d - 12*a^4*b*e + 12*a^5*f + 12*(a*b^4*c - 2*a^2
*b^3*d + 3*a^3*b^2*e - 4*a^4*b*f)*x^3 - 12*(2*a^2*b^3*c - 3*a^3*b^2*d + 4*a
^4*b*e - 5*a^5*f + (2*a*b^4*c - 3*a^2*b^3*d + 4*a^3*b^2*e - 5*a^4*b*f)*x^3)
*log(b*x^3 + a))/(b^7*x^3 + a*b^6)
```

Sympy [A] (verification not implemented)

Time = 12.66 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.05

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{a(5a^3f-4a^2be+3ab^2d-2b^3c)\log(a+bx^3)}{3b^6} + x^9\left(-\frac{2af}{9b^3}+\frac{e}{9b^2}\right) + x^6\left(\frac{a^2f}{2b^4}-\frac{ae}{3b^3}+\frac{d}{6b^2}\right) + x^3\left(-\frac{4a^3f}{3b^5}+\frac{a^2e}{b^4}-\frac{2ad}{3b^3}+\frac{c}{3b^2}\right) + \frac{a^5f-a^4be+a^3b^2d-a^2b^3c}{3ab^6+3b^7x^3} + \frac{fx^{12}}{12b^2}$$

```
[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] a*(5*a**3*f - 4*a**2*b*e + 3*a*b**2*d - 2*b**3*c)*log(a + b*x**3)/(3*b**6)
+ x**9*(-2*a*f/(9*b**3) + e/(9*b**2)) + x**6*(a**2*f/(2*b**4) - a*e/(3*b**3
) + d/(6*b**2)) + x**3*(-4*a**3*f/(3*b**5) + a**2*e/b**4 - 2*a*d/(3*b**3) +
 c/(3*b**2)) + (a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(3*a*b**6 +
3*b**7*x**3) + f*x**12/(12*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{a^2b^3c-a^3b^2d+a^4be-a^5f}{3\,(b^7x^3+ab^6)} + \frac{3\,b^3fx^{12}+4\,(b^3e-2\,ab^2f)x^9+6\,(b^3d-2\,ab^2e+3\,a^2bf)x^6+12\,(b^3c-2\,ab^2d+3\,a^2be-4\,a^3f)x^3}{36\,b^5} - \frac{(2\,ab^3c-3\,a^2b^2d+4\,a^3be-5\,a^4f)\log(bx^3+a)}{3\,b^6}$$

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)/(b^7*x^3 + a*b^6) + 1/36*(3*
b^3*f*x^12 + 4*(b^3*e - 2*a*b^2*f)*x^9 + 6*(b^3*d - 2*a*b^2*e + 3*a^2*b*f)*
x^6 + 12*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3)/b^5 - 1/3*(2*a*b^3*
c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*log(b*x^3 + a)/b^6
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.34

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{(2\,ab^3c-3\,a^2b^2d+4\,a^3be-5\,a^4f)\log(|bx^3+a|)}{3\,b^6} + \frac{2\,ab^4cx^3-3\,a^2b^3dx^3+4\,a^3b^2ex^3-5\,a^4bfx^3+a^2b^3c-2\,a^3b^2d+3\,a^4be-4\,a^5f}{3\,(bx^3+a)b^6} + \frac{3\,b^6fx^{12}+4\,b^6ex^9-8\,ab^5fx^9+6\,b^6dx^6-12\,ab^5ex^6+18\,a^2b^4fx^6+12\,b^6cx^3-24\,ab^5dx^3+36\,a^2b^4ex^3 -}{36\,b^8}$$

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(2*a*b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*log(abs(b*x^3 + a))/b^
6 + 1/3*(2*a*b^4*c*x^3 - 3*a^2*b^3*d*x^3 + 4*a^3*b^2*e*x^3 - 5*a^4*b*f*x^3
+ a^2*b^3*c - 2*a^3*b^2*d + 3*a^4*b*e - 4*a^5*f)/((b*x^3 + a)*b^6) + 1/36*(
3*b^6*f*x^12 + 4*b^6*e*x^9 - 8*a*b^5*f*x^9 + 6*b^6*d*x^6 - 12*a*b^5*e*x^6 +
 18*a^2*b^4*f*x^6 + 12*b^6*c*x^3 - 24*a*b^5*d*x^3 + 36*a^2*b^4*e*x^3 - 48*a
^3*b^3*f*x^3)/b^8
```

Mupad [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.29

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^9\left(\frac{e}{9\,b^2}-\frac{2\,a\,f}{9\,b^3}\right)-x^6\left(\frac{a^2\,f}{6\,b^4}-\frac{d}{6\,b^2}+\frac{a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b}\right) + x^3\left(\frac{c}{3\,b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b^2} + \frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{3\,b}\right) + \frac{f\,x^{12}}{12\,b^2}+\frac{f\,a^5-e\,a^4\,b+d\,a^3\,b^2-c\,a^2\,b^3}{3\,b\,\left(b^6\,x^3+a\,b^5\right)} + \frac{\ln\left(b\,x^3+a\right)\,\left(5\,f\,a^4-4\,e\,a^3\,b+3\,d\,a^2\,b^2-2\,c\,a\,b^3\right)}{3\,b^6}$$

```
[In] int((x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^9*(e/(9*b^2) - (2*a*f)/(9*b^3)) - x^6*((a^2*f)/(6*b^4) - d/(6*b^2) + (a*(
e/b^2 - (2*a*f)/b^3))/(3*b)) + x^3*(c/(3*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))
```

```
/(3*b^2) + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(3*b
)) + (f*x^12)/(12*b^2) + (a^5*f - a^2*b^3*c + a^3*b^2*d - a^4*b*e)/(3*b*(a*
b^5 + b^6*x^3)) + (log(a + b*x^3)*(5*a^4*f + 3*a^2*b^2*d - 2*a*b^3*c - 4*a^
3*b*e))/(3*b^6)
```

3.253 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 140

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx = \frac{(b^2d-2abe+3a^2f)\,x^3}{3b^4}+\frac{(be-2af)x^6}{6b^3}+\frac{fx^9}{9b^2}+\frac{a(b^3c-ab^2d+a^2be-a^3f)}{3b^5\,(a+bx^3)}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\log{(a+bx^3)}}{3b^5}$$

```
[Out] 1/3*(3*a^2*f-2*a*b*e+b^2*d)*x^3/b^4+1/6*(-2*a*f+b*e)*x^6/b^3+1/9*f*x^9/b^2+
1/3*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b^5/(b*x^3+a)+1/3*(-4*a^3*f+3*a^2*b*e-
2*a*b^2*d+b^3*c)*ln(b*x^3+a)/b^5
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx = \frac{x^3(3a^2f-2abe+b^2d)}{3b^4}+\frac{a(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5\,(a+bx^3)}+\frac{\log{(a+bx^3)}\,(-4a^3f+3a^2be-2ab^2d+b^3c)}{3b^5}+\frac{x^6(be-2af)}{6b^3}+\frac{fx^9}{9b^2}$$

```
[In] Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^6)/(6*b^3) + (
f*x^9)/(9*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^5*(a + b*x^3)
) + ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*Log[a + b*x^3])/(3*b^5)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx)^2}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^2d-2abe+3a^2f}{b^4}+\frac{(be-2af)x}{b^3}+\frac{fx^2}{b^2}\right.\right.\\
&\qquad \left.\left.+\frac{a(-b^3c+ab^2d-a^2be+a^3f)}{b^4(a+bx)^2}+\frac{b^3c-2ab^2d+3a^2be-4a^3f}{b^4(a+bx)}\right)dx, x, x^3\right)\\
&= \frac{(b^2d-2abe+3a^2f)\,x^3}{3b^4}+\frac{(be-2af)x^6}{6b^3}+\frac{fx^9}{9b^2}\\
&\quad +\frac{a(b^3c-ab^2d+a^2be-a^3f)}{3b^5\,(a+bx^3)}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\log\left(a+bx^3\right)}{3b^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{\left(a+bx^3\right)^2}\,dx$$
$$= \frac{6b(b^2d-2abe+3a^2f)\,x^3+3b^2(be-2af)x^6+2b^3fx^9+\frac{6a\left(b^3c-ab^2d+a^2be-a^3f\right)}{a+bx^3}+6(b^3c-2ab^2d+3a^2be-4a}{18b^5}$$

```
[In] Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (6*b*(b^2*d - 2*a*b*e + 3*a^2*f)*x^3 + 3*b^2*(b*e - 2*a*f)*x^6 + 2*b^3*f*x^
9 + (6*a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a + b*x^3) + 6*(b^3*c - 2*a*
b^2*d + 3*a^2*b*e - 4*a^3*f)*Log[a + b*x^3])/(18*b^5)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

method	result
default	$\frac{\frac{b^2 f x^9}{9}+\frac{(-2afb+b^2e)x^6}{6}+\frac{x^3(3a^2f-2aeb+b^2d)}{3}}{b^4}-\frac{\frac{(4fa^3-3a^2be+2ab^2d-b^3c)\ln(bx^3+a)}{b}+\frac{a(fa^3-a^2be+ab^2d-b^3c)}{b(bx^3+a)}}{3b^4}$
norman	$\frac{\frac{fx^{12}}{9b}-\frac{(4af-3be)x^9}{18b^2}+\frac{(4a^2f-3aeb+2b^2d)x^6}{6b^3}+\frac{(4a^4f-3a^3be+2a^2b^2d-ab^3c)x^3}{3ab^4}}{bx^3+a}-\frac{(4fa^3-3a^2be+2ab^2d-b^3c)\ln(bx^3+a)}{3b^5}$
risch	$\frac{fx^9}{9b^2}-\frac{afx^6}{3b^3}+\frac{ex^6}{6b^2}+\frac{a^2fx^3}{b^4}-\frac{2aex^3}{3b^3}+\frac{dx^3}{3b^2}-\frac{a^4f}{3b^5(bx^3+a)}+\frac{a^3e}{3b^4(bx^3+a)}-\frac{a^2d}{3b^3(bx^3+a)}+\frac{ac}{3b^2(bx^3+a)}-$
parallelrisch	$-\frac{-2fx^{12}b^4+4x^9ab^3f-3x^9b^4e-12x^6a^2b^2f+9x^6ab^3e-6b^4dx^6+24\ln(bx^3+a)x^3a^3bf-18\ln(bx^3+a)x^3a^2b^2e+12\ln(bx^3+a}{}$

```
[In] int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/9*b^2*f*x^9+1/6*(-2*a*b*f+b^2*e)*x^6+1/3*x^3*(3*a^2*f-2*a*b*e+b^2*
d))-1/3/b^4*((4*a^3*f-3*a^2*b*e+2*a*b^2*d-b^3*c)/b*ln(b*x^3+a)+a*(a^3*f-a^2
*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.44

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$=\frac{2b^4fx^{12}+(3b^4e-4ab^3f)x^9+3(2b^4d-3ab^3e+4a^2b^2f)x^6+6ab^3c-6a^2b^2d+6a^3be-6a^4f+6(ab^3}{18(}$$

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^4*f*x^12 + (3*b^4*e - 4*a*b^3*f)*x^9 + 3*(2*b^4*d - 3*a*b^3*e + 4
*a^2*b^2*f)*x^6 + 6*a*b^3*c - 6*a^2*b^2*d + 6*a^3*b*e - 6*a^4*f + 6*(a*b^3*
d - 2*a^2*b^2*e + 3*a^3*b*f)*x^3 + 6*(a*b^3*c - 2*a^2*b^2*d + 3*a^3*b*e - 4
*a^4*f + (b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^3)*log(b*x^3 + a))
/(b^6*x^3 + a*b^5)
```

Sympy [A] (verification not implemented)

Time = 11.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^6\left(-\frac{af}{3b^3}+\frac{e}{6b^2}\right)+x^3\left(\frac{a^2f}{b^4}-\frac{2ae}{3b^3}+\frac{d}{3b^2}\right) + \frac{-a^4f+a^3be-a^2b^2d+ab^3c}{3ab^5+3b^6x^3}+\frac{fx^9}{9b^2} - \frac{(4a^3f-3a^2be+2ab^2d-b^3c)\log(a+bx^3)}{3b^5}$$

```
[In] integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**6*(-a*f/(3*b**3) + e/(6*b**2)) + x**3*(a**2*f/b**4 - 2*a*e/(3*b**3) + d/
(3*b**2)) + (-a**4*f + a**3*b*e - a**2*b**2*d + a*b**3*c)/(3*a*b**5 + 3*b**
6*x**3) + f*x**9/(9*b**2) - (4*a**3*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c)*l
og(a + b*x**3)/(3*b**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{ab^3c-a^2b^2d+a^3be-a^4f}{3(b^6x^3+ab^5)} + \frac{2b^2fx^9+3(b^2e-2abf)x^6+6(b^2d-2abe+3a^2f)x^3}{18b^4} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\log(bx^3+a)}{3b^5}$$

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)/(b^6*x^3 + a*b^5) + 1/18*(2*b^2
*f*x^9 + 3*(b^2*e - 2*a*b*f)*x^6 + 6*(b^2*d - 2*a*b*e + 3*a^2*f)*x^3)/b^4 +
 1/3*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*log(b*x^3 + a)/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.51

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{\frac{(bx^3+a)^3\left(2\,f+\frac{3\,\left(b^2e-4\,abf\right)}{\left(bx^3+a\right)b}+\frac{6\,\left(b^4d-3\,ab^3e+6\,a^2b^2f\right)}{\left(bx^3+a\right)^2b^2}\right)}{b^4}-\frac{6\,\left(b^3c-2\,ab^2d+3\,a^2be-4\,a^3f\right)\log\left(\frac{\left|bx^3+a\right|}{\left(bx^3+a\right)^2|b|}\right)}{b^4}+\frac{6\,\left(\frac{ab^6c}{bx^3+a}-\frac{a^2b^5d}{bx^3+a}+\frac{a^3b^4e}{bx^3+a}-\right.}{b^7}}{18\,b}$$

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/18*((b*x^3 + a)^3*(2*f + 3*(b^2*e - 4*a*b*f)/((b*x^3 + a)*b) + 6*(b^4*d -
 3*a*b^3*e + 6*a^2*b^2*f)/((b*x^3 + a)^2*b^2))/b^4 - 6*(b^3*c - 2*a*b^2*d +
 3*a^2*b*e - 4*a^3*f)*log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b^4 + 6*(a
*b^6*c/(b*x^3 + a) - a^2*b^5*d/(b*x^3 + a) + a^3*b^4*e/(b*x^3 + a) - a^4*b^
3*f/(b*x^3 + a))/b^7)/b
```

Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.11

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^6\left(\frac{e}{6\,b^2}-\frac{a\,f}{3\,b^3}\right)-x^3\left(\frac{a^2\,f}{3\,b^4}-\frac{d}{3\,b^2}+\frac{2\,a\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(-4\,f\,a^3+3\,e\,a^2\,b-2\,d\,a\,b^2+c\,b^3\right)}{3\,b^5}-\frac{f\,a^4-e\,a^3\,b+d\,a^2\,b^2-c\,a\,b^3}{3\,b\,\left(b^5\,x^3+a\,b^4\right)}+\frac{f\,x^9}{9\,b^2}$$

```
[In] int((x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^6*(e/(6*b^2) - (a*f)/(3*b^3)) - x^3*((a^2*f)/(3*b^4) - d/(3*b^2) + (2*a*(
e/b^2 - (2*a*f)/b^3))/(3*b)) + (log(a + b*x^3)*(b^3*c - 4*a^3*f - 2*a*b^2*d
 + 3*a^2*b*e))/(3*b^5) - (a^4*f + a^2*b^2*d - a*b^3*c - a^3*b*e)/(3*b*(a*b^
4 + b^5*x^3)) + (f*x^9)/(9*b^2)
```

3.254 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 103

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx = \frac{(be-2af)x^3}{3b^3} + \frac{fx^6}{6b^2} - \frac{b^3c-ab^2d+a^2be-a^3f}{3b^4(a+bx^3)} + \frac{(b^2d-2abe+3a^2f)\log(a+bx^3)}{3b^4}$$

```
[Out] 1/3*(-2*a*f+b*e)*x^3/b^3+1/6*f*x^6/b^2+1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^
4/(b*x^3+a)+1/3*(3*a^2*f-2*a*b*e+b^2*d)*ln(b*x^3+a)/b^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1833, 1864}

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx = \frac{\log(a+bx^3)(3a^2f-2abe+b^2d)}{3b^4} - \frac{a^3(-f)+a^2be-ab^2d+b^3c}{3b^4(a+bx^3)} + \frac{x^3(be-2af)}{3b^3} + \frac{fx^6}{6b^2}$$

```
[In] Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x^3)/(3*b^3) + (f*x^6)/(6*b^2) - (b^3*c - a*b^2*d + a^2*b*e
- a^3*f)/(3*b^4*(a + b*x^3)) + ((b^2*d - 2*a*b*e + 3*a^2*f)*Log[a + b*x^3])
/(3*b^4)
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{(a+bx)^2}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{be-2af}{b^3}+\frac{fx}{b^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3(a+bx)^2}+\frac{b^2d-2abe+3a^2f}{b^3(a+bx)}\right)dx, x, x^3\right)\\
&= \frac{(be-2af)x^3}{3b^3}+\frac{fx^6}{6b^2}-\frac{b^3c-ab^2d+a^2be-a^3f}{3b^4\left(a+bx^3\right)}+\frac{(b^2d-2abe+3a^2f)\log\left(a+bx^3\right)}{3b^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{2b(be-2af)x^3+b^2fx^6+\frac{2(-b^3c+ab^2d-a^2be+a^3f)}{a+bx^3}+2(b^2d-2abe+3a^2f)\log\left(a+bx^3\right)}{6b^4}$$

```
[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
[Out] (2*b*(b*e - 2*a*f)*x^3 + b^2*f*x^6 + (2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3
*f))/(a + b*x^3) + 2*(b^2*d - 2*a*b*e + 3*a^2*f)*Log[a + b*x^3])/(6*b^4)
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

method	result
norman	$\frac{\frac{3fa^3-2a^2be+ab^2d-b^3c}{3b^4}+\frac{fx^9}{6b}-\frac{(3af-2be)x^6}{6b^2}}{bx^3+a}+\frac{(3a^2f-2aeb+b^2d)\ln(bx^3+a)}{3b^4}$
default	$\frac{(-fx^3b+2af-be)^2}{6b^4f}+\frac{\frac{(3a^2f-2aeb+b^2d)\ln(bx^3+a)}{b}-\frac{-fa^3+a^2be-ab^2d+b^3c}{b(bx^3+a)}}{3b^3}$
parallelrisch	$\frac{b^3fx^9-3x^6ab^2f+2x^6b^3e+6\ln(bx^3+a)x^3a^2bf-4\ln(bx^3+a)x^3ab^2e+2\ln(bx^3+a)x^3b^3d+6\ln(bx^3+a)a^3f-4\ln(bx^3+a)a^2be-}{6b^4(bx^3+a)}$
risch	$\frac{fx^6}{6b^2}-\frac{2fax^3}{3b^3}+\frac{ex^3}{3b^2}+\frac{2fa^2}{3b^4}-\frac{2ae}{3b^3}+\frac{e^2}{6b^2f}+\frac{fa^3}{3b^4(bx^3+a)}-\frac{a^2e}{3b^3(bx^3+a)}+\frac{ad}{3b^2(bx^3+a)}-\frac{c}{3b(bx^3+a)}+\frac{\ln(b}{}$

```
[In] int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
[Out] (1/3*(3*a^3*f-2*a^2*b*e+a*b^2*d-b^3*c)/b^4+1/6*f*x^9/b-1/6*(3*a*f-2*b*e)/b^
2*x^6)/(b*x^3+a)+1/3*(3*a^2*f-2*a*b*e+b^2*d)*ln(b*x^3+a)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.39

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{b^3fx^9+(2\,b^3e-3\,ab^2f)x^6-2\,b^3c+2\,ab^2d-2\,a^2be+2\,a^3f+2\,(ab^2e-2\,a^2bf)x^3+2\,(ab^2d-2\,a^2be+3\,a}{6\,(b^5x^3+ab^4)}$$

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
[Out] 1/6*(b^3*f*x^9 + (2*b^3*e - 3*a*b^2*f)*x^6 - 2*b^3*c + 2*a*b^2*d - 2*a^2*b*
e + 2*a^3*f + 2*(a*b^2*e - 2*a^2*b*f)*x^3 + 2*(a*b^2*d - 2*a^2*b*e + 3*a^3*
f + (b^3*d - 2*a*b^2*e + 3*a^2*b*f)*x^3)*log(b*x^3 + a))/(b^5*x^3 + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^3\left(-\frac{2af}{3b^3}+\frac{e}{3b^2}\right)+\frac{a^3f-a^2be+ab^2d-b^3c}{3ab^4+3b^5x^3} + \frac{fx^6}{6b^2}+\frac{(3a^2f-2abe+b^2d)\log(a+bx^3)}{3b^4}$$

```
[In] integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
[Out] x**3*(-2*a*f/(3*b**3) + e/(3*b**2)) + (a**3*f - a**2*b*e + a*b**2*d - b**3*
c)/(3*a*b**4 + 3*b**5*x**3) + f*x**6/(6*b**2) + (3*a**2*f - 2*a*b*e + b**2*
d)*log(a + b*x**3)/(3*b**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{b^3c-ab^2d+a^2be-a^3f}{3\,(b^5x^3+ab^4)} + \frac{bfx^6+2\,(be-2\,af)x^3}{6\,b^3} + \frac{(b^2d-2\,abe+3\,a^2f)\log\left(bx^3+a\right)}{3\,b^4}$$

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(b^5*x^3 + a*b^4) + 1/6*(b*f*x^6 +
 2*(b*e - 2*a*f)*x^3)/b^3 + 1/3*(b^2*d - 2*a*b*e + 3*a^2*f)*log(b*x^3 + a)/
b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(95) = 190.

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.99

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{1}{6}\,f\left(\frac{(bx^3+a)^2\left(\frac{6\,a}{bx^3+a}-1\right)}{b^4} + \frac{6\,a^2\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^4} - \frac{2\,a^3}{(bx^3+a)b^4}\right) + \frac{1}{3}\,e\left(\frac{2\,a\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^3} + \frac{bx^3+a}{b^3} - \frac{a^2}{(bx^3+a)b^3}\right) - \frac{d\left(\frac{\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b} - \frac{a}{(bx^3+a)b}\right)}{3\,b} - \frac{c}{3\,(bx^3+a)b}$$

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/6*f*((b*x^3 + a)^2*(6*a/(b*x^3 + a) - 1)/b^4 + 6*a^2*log(abs(b*x^3 + a)/
((b*x^3 + a)^2*abs(b)))/b^4 - 2*a^3/((b*x^3 + a)*b^4)) + 1/3*e*(2*a*log(abs
(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b^3 + (b*x^3 + a)/b^3 - a^2/((b*x^3 + a
)*b^3)) - 1/3*d*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 +
 a)*b))/b - 1/3*c/((b*x^3 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^3\left(\frac{e}{3\,b^2}-\frac{2\,a\,f}{3\,b^3}\right)+\frac{f\,x^6}{6\,b^2}-\frac{-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3}{3\,b\,\left(b^4\,x^3+a\,b^3\right)}+\frac{\ln\left(b\,x^3+a\right)\,\left(3\,f\,a^2-2\,e\,a\,b+d\,b^2\right)}{3\,b^4}$$

```
[In] int((x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^3*(e/(3*b^2) - (2*a*f)/(3*b^3)) + (f*x^6)/(6*b^2) - (b^3*c - a^3*f - a*b^
2*d + a^2*b*e)/(3*b*(a*b^3 + b^4*x^3)) + (log(a + b*x^3)*(b^2*d + 3*a^2*f -
 2*a*b*e))/(3*b^4)
```

3.255 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 100

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^2}\,dx = \frac{fx^3}{3b^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{3ab^3(a+bx^3)}+\frac{c\log(x)}{a^2}-\frac{(b^3c-a^2be+2a^3f)\log(a+bx^3)}{3a^2b^3}$$

```
[Out] 1/3*f*x^3/b^2+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a/b^3/(b*x^3+a)+c*ln(x)/a^
2-1/3*(2*a^3*f-a^2*b*e+b^3*c)*ln(b*x^3+a)/a^2/b^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^2}\,dx = \frac{c\log(x)}{a^2}-\frac{\log(a+bx^3)(2a^3f-a^2be+b^3c)}{3a^2b^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3ab^3(a+bx^3)}+\frac{fx^3}{3b^2}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x]
```

```
[Out] (f*x^3)/(3*b^2) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a*b^3*(a + b*x^3))
 + (c*Log[x])/a^2 - ((b^3*c - a^2*b*e + 2*a^3*f)*Log[a + b*x^3])/(3*a^2*b^3
)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x(a+bx)^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{f}{b^2}+\frac{c}{a^2x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2(a+bx)^2}+\frac{-b^3c+a^2be-2a^3f}{a^2b^2(a+bx)}\right)dx, x, x^3\right)\\ &= \frac{fx^3}{3b^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{3ab^3(a+bx^3)}+\frac{c\log(x)}{a^2}-\frac{(b^3c-a^2be+2a^3f)\log(a+bx^3)}{3a^2b^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^2}\,dx = \frac{3c\log(x)+\frac{\frac{a\left(b^3c-a^3f+a^2b\left(e+fx^3\right)+ab^2\left(-d+fx^6\right)\right)}{a+bx^3}+\left(-b^3c+a^2be-2a^3f\right)\log\left(a+bx^3\right)}{b^3}}{3a^2}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x]
[Out] (3*c*Log[x] + ((a*(b^3*c - a^3*f + a^2*b*(e + f*x^3) + a*b^2*(-d + f*x^6)))
/(a + b*x^3) + (-(b^3*c) + a^2*b*e - 2*a^3*f)*Log[a + b*x^3])/b^3)/(3*a^2)
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

method	result
default	$\frac{f\,x^{3}}{3b^{2}}+\frac{c\ln\left(x\right)}{a^{2}}-\frac{\frac{\left(2f\,a^{3}-a^{2}be+b^{3}c\right)\ln\left(b\,x^{3}+a\right)}{b}+\frac{a\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)}{b\left(b\,x^{3}+a\right)}}{3a^{2}b^{2}}$
norman	$\frac{\frac{f\,x^{6}}{3b}-\frac{2f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c}{3a\,b^{3}}}{b\,x^{3}+a}+\frac{c\ln\left(x\right)}{a^{2}}-\frac{\left(2f\,a^{3}-a^{2}be+b^{3}c\right)\ln\left(b\,x^{3}+a\right)}{3a^{2}b^{3}}$
risch	$\frac{f\,x^{3}}{3b^{2}}-\frac{a^{2}f}{3b^{3}\left(b\,x^{3}+a\right)}+\frac{ae}{3b^{2}\left(b\,x^{3}+a\right)}-\frac{d}{3b\left(b\,x^{3}+a\right)}+\frac{c}{3a\left(b\,x^{3}+a\right)}+\frac{c\ln\left(x\right)}{a^{2}}-\frac{2\ln\left(b\,x^{3}+a\right)af}{3b^{3}}+\frac{\ln\left(b\,x^{3}+a\right)e}{3b^{2}}-\frac{c\ln}{}$
parallelrisch	$\frac{x^{6}a^{2}b^{2}f+3\ln\left(x\right)x^{3}b^{4}c-2\ln\left(b\,x^{3}+a\right)x^{3}a^{3}bf+\ln\left(b\,x^{3}+a\right)x^{3}a^{2}b^{2}e-\ln\left(b\,x^{3}+a\right)x^{3}b^{4}c+3\ln\left(x\right)a\,b^{3}c-2\ln\left(b\,x^{3}+a\right)a^{4}f+\ln\left(b\,x^{3}+\right.}{3a^{2}b^{3}\left(b\,x^{3}+a\right)}$

[In] int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/3*f*x^3/b^2+c*ln(x)/a^2-1/3/a^2/b^2*((2*a^3*f-a^2*b*e+b^3*c)/b*ln(b*x^3+a
)+a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.45

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^2}\,dx$$
$$=\frac{a^2b^2fx^6+a^3bfx^3+ab^3c-a^2b^2d+a^3be-a^4f-\left(ab^3c-a^3be+2\,a^4f+\left(b^4c-a^2b^2e+2\,a^3bf\right)x^3\right)\log\left(bx\right.}{3\left(a^2b^4x^3+a^3b^3\right)}$$

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/3*(a^2*b^2*f*x^6 + a^3*b*f*x^3 + a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f -
(a*b^3*c - a^3*b*e + 2*a^4*f + (b^4*c - a^2*b^2*e + 2*a^3*b*f)*x^3)*log(b*x
^3 + a) + 3*(b^4*c*x^3 + a*b^3*c)*log(x))/(a^2*b^4*x^3 + a^3*b^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**2,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^2}\,dx = \frac{fx^3}{3\,b^2} + \frac{b^3c-ab^2d+a^2be-a^3f}{3\,(ab^4x^3+a^2b^3)} + \frac{c\log\left(x^3\right)}{3\,a^2} - \frac{\left(b^3c-a^2be+2\,a^3f\right)\log\left(bx^3+a\right)}{3\,a^2b^3}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*f*x^3/b^2 + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a*b^4*x^3 + a^2*b^
3) + 1/3*c*log(x^3)/a^2 - 1/3*(b^3*c - a^2*b*e + 2*a^3*f)*log(b*x^3 + a)/(a
^2*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.23

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^2}\,dx = \frac{fx^3}{3\,b^2} + \frac{c\log\left(|x|\right)}{a^2} - \frac{\left(b^3c-a^2be+2\,a^3f\right)\log\left(|bx^3+a|\right)}{3\,a^2b^3} + \frac{b^4cx^3-a^2b^2ex^3+2\,a^3bfx^3+2\,ab^3c-a^2b^2d+a^4f}{3\,(bx^3+a)a^2b^3}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*f*x^3/b^2 + c*log(abs(x))/a^2 - 1/3*(b^3*c - a^2*b*e + 2*a^3*f)*log(abs
(b*x^3 + a))/(a^2*b^3) + 1/3*(b^4*c*x^3 - a^2*b^2*e*x^3 + 2*a^3*b*f*x^3 + 2
*a*b^3*c - a^2*b^2*d + a^4*f)/((b*x^3 + a)*a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^2}\,dx = \frac{f\,x^3}{3\,b^2} + \frac{c\,\ln\left(x\right)}{a^2} + \frac{-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3}{3\,a\,b\,\left(b^3\,x^3+a\,b^2\right)} - \frac{\ln\left(b\,x^3+a\right)\,\left(2\,f\,a^3-e\,a^2\,b+c\,b^3\right)}{3\,a^2\,b^3}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x)
```

```
[Out] (f*x^3)/(3*b^2) + (c*log(x))/a^2 + (b^3*c - a^3*f - a*b^2*d + a^2*b*e)/(3*a
*b*(a*b^2 + b^3*x^3)) - (log(a + b*x^3)*(b^3*c + 2*a^3*f - a^2*b*e))/(3*a^2
*b^3)
```

3.256 $\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 109

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx=-\frac{c}{3a^2x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{3a^2b^2\left(a+bx^3\right)}-\frac{(2bc-ad)\log(x)}{a^3}+\frac{\left(2b^3c-ab^2d+a^3f\right)\log\left(a+bx^3\right)}{3a^3b^2}$$

```
[Out] -1/3*c/a^2/x^3+1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^2/b^2/(b*x^3+a)-(-a*d+2*
b*c)*ln(x)/a^3+1/3*(a^3*f-a*b^2*d+2*b^3*c)*ln(b*x^3+a)/a^3/b^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx=\frac{\log\left(a+bx^3\right)\left(a^3f-ab^2d+2b^3c\right)}{3a^3b^2}-\frac{\log(x)(2bc-ad)}{a^3}-\frac{c}{3a^2x^3}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^2b^2\left(a+bx^3\right)}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -1/3*c/(a^2*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^2*b^2*(a + b*x^
3)) - ((2*b*c - a*d)*Log[x])/a^3 + ((2*b^3*c - a*b^2*d + a^3*f)*Log[a + b*x
^3])/(3*a^3*b^2)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^2x^2}+\frac{-2bc+ad}{a^3x}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b(a+bx)^2}+\frac{2b^3c-ab^2d+a^3f}{a^3b(a+bx)}\right)dx, x, x^3\right)$$

$$= -\frac{c}{3a^2x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{3a^2b^2\left(a+bx^3\right)}-\frac{(2bc-ad)\log(x)}{a^3}+\frac{\left(2b^3c-ab^2d+a^3f\right)\log\left(a+bx^3\right)}{3a^3b^2}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx = \frac{-\frac{ac}{x^3}+\frac{a\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^2(a+bx^3)}+3(-2bc+ad)\log(x)+\frac{(2b^3c-ab^2d+a^3f)\log(a+bx^3)}{b^2}}{3a^3}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x]
[Out] (-((a*c)/x^3) + (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(b^2*(a + b*x^3)
) + 3*(-2*b*c + a*d)*Log[x] + ((2*b^3*c - a*b^2*d + a^3*f)*Log[a + b*x^3])/
b^2)/(3*a^3)
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

method	result
default	$-\frac{c}{3a^{2}x^{3}}+\frac{(ad-2bc)\ln(x)}{a^{3}}+\frac{\frac{\left(f\,a^{3}-a\,b^{2}d+2b^{3}c\right)\ln\left(b\,x^{3}+a\right)}{b^{2}}+\frac{a\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)}{b^{2}\left(b\,x^{3}+a\right)}}{3a^{3}}$
norman	$\frac{-\frac{c}{3a}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-2b^{3}c\right)x^{3}}{3a^{2}b^{2}}}{x^{3}(b\,x^{3}+a)}+\frac{(ad-2bc)\ln(x)}{a^{3}}+\frac{(f\,a^{3}-a\,b^{2}d+2b^{3}c)\ln(b\,x^{3}+a)}{3a^{3}b^{2}}$
risch	$\frac{-\frac{c}{3a}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-2b^{3}c\right)x^{3}}{3a^{2}b^{2}}}{x^{3}(b\,x^{3}+a)}+\frac{d\ln(x)}{a^{2}}-\frac{2bc\ln(x)}{a^{3}}+\frac{\ln(-b\,x^{3}-a)f}{3b^{2}}-\frac{\ln(-b\,x^{3}-a)d}{3a^{2}}+\frac{2b\ln(-b\,x^{3}-a)c}{3a^{3}}$
parallelrisch	$\frac{3\ln(x)x^{6}a\,b^{3}d-6\ln(x)x^{6}b^{4}c+\ln(b\,x^{3}+a)x^{6}a^{3}bf-\ln(b\,x^{3}+a)x^{6}a\,b^{3}d+2\ln(b\,x^{3}+a)x^{6}b^{4}c+3\ln(x)x^{3}a^{2}b^{2}d-6\ln(x)x^{3}a\,b^{3}c+\ln}{3a^{3}b^{2}x^{3}(b\,x^{3}+a)}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*c/a^2/x^3+(a*d-2*b*c)/a^3*ln(x)+1/3/a^3*((a^3*f-a*b^2*d+2*b^3*c)/b^2*l
n(b*x^3+a)+a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^2/(b*x^3+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.58

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx = -\frac{a^2b^2c+(2\,ab^3c-a^2b^2d+a^3be-a^4f)x^3-((2\,b^4c-ab^3d+a^3bf)x^6+(2\,ab^3c-a^2b^2d+a^4f)x^3)\log{(bx}}{3\,(a^3b^3x^6+a^4b^2x^3)}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(a^2*b^2*c + (2*a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^3 - ((2*b^4*c
 - a*b^3*d + a^3*b*f)*x^6 + (2*a*b^3*c - a^2*b^2*d + a^4*f)*x^3)*log(b*x^3
+ a) + 3*((2*b^4*c - a*b^3*d)*x^6 + (2*a*b^3*c - a^2*b^2*d)*x^3)*log(x))/(a
^3*b^3*x^6 + a^4*b^2*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx=-\frac{ab^2c+\left(2\,b^3c-ab^2d+a^2be-a^3f\right)x^3}{3\left(a^2b^3x^6+a^3b^2x^3\right)}-\frac{\left(2\,bc-ad\right)\log\left(x^3\right)}{3\,a^3}+\frac{\left(2\,b^3c-ab^2d+a^3f\right)\log\left(bx^3+a\right)}{3\,a^3b^2}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
[Out] -1/3*(a*b^2*c + (2*b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(a^2*b^3*x^6 + a
^3*b^2*x^3) - 1/3*(2*b*c - a*d)*log(x^3)/a^3 + 1/3*(2*b^3*c - a*b^2*d + a^3
*f)*log(b*x^3 + a)/(a^3*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.19

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx=-\frac{\left(2\,bc-ad\right)\log\left(|x|\right)}{a^3}+\frac{\left(2\,b^3c-ab^2d+a^3f\right)\log\left(|bx^3+a|\right)}{3\,a^3b^2}-\frac{a^2bfx^6+4\,b^3cx^3-2\,ab^2dx^3+2\,a^2bex^3-a^3fx^3+2\,ab^2c}{6\left(bx^6+ax^3\right)a^2b^2}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="giac")
[Out] -(2*b*c - a*d)*log(abs(x))/a^3 + 1/3*(2*b^3*c - a*b^2*d + a^3*f)*log(abs(b*
x^3 + a))/(a^3*b^2) - 1/6*(a^2*b*f*x^6 + 4*b^3*c*x^3 - 2*a*b^2*d*x^3 + 2*a^
2*b*e*x^3 - a^3*f*x^3 + 2*a*b^2*c)/((b*x^6 + a*x^3)*a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx=\frac{\ln\left(x\right)\,\left(a\,d-2\,b\,c\right)}{a^3}-\frac{\frac{c}{3\,a}+\frac{x^3\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+2\,c\,b^3\right)}{3\,a^2\,b^2}}{b\,x^6+a\,x^3}+\frac{\ln\left(b\,x^3+a\right)\,\left(f\,a^3-d\,a\,b^2+2\,c\,b^3\right)}{3\,a^3\,b^2}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] (log(x)*(a*d - 2*b*c))/a^3 - (c/(3*a) + (x^3*(2*b^3*c - a^3*f - a*b^2*d + a
^2*b*e))/(3*a^2*b^2))/(a*x^3 + b*x^6) + (log(a + b*x^3)*(2*b^3*c + a^3*f -
a*b^2*d))/(3*a^3*b^2)
```

3.257 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 130

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^2}\,dx = -\frac{c}{6a^2x^6}+\frac{2bc-ad}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^3b(a+bx^3)} + \frac{(3b^2c-2abd+a^2e)\log(x)}{a^4} - \frac{(3b^2c-2abd+a^2e)\log(a+bx^3)}{3a^4}$$

```
[Out] -1/6*c/a^2/x^6+1/3*(-a*d+2*b*c)/a^3/x^3+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/
a^3/b/(b*x^3+a)+(a^2*e-2*a*b*d+3*b^2*c)*ln(x)/a^4-1/3*(a^2*e-2*a*b*d+3*b^2*
c)*ln(b*x^3+a)/a^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^2}\,dx = \frac{2bc-ad}{3a^3x^3}-\frac{c}{6a^2x^6}-\frac{\log(a+bx^3)(a^2e-2abd+3b^2c)}{3a^4} + \frac{\log(x)(a^2e-2abd+3b^2c)}{a^4}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^3b(a+bx^3)}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -1/6*c/(a^2*x^6) + (2*b*c - a*d)/(3*a^3*x^3) + (b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)/(3*a^3*b*(a + b*x^3)) + ((3*b^2*c - 2*a*b*d + a^2*e)*Log[x])/a^4 -
((3*b^2*c - 2*a*b*d + a^2*e)*Log[a + b*x^3])/(3*a^4)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx)^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^2x^3}+\frac{-2bc+ad}{a^3x^2}+\frac{3b^2c-2abd+a^2e}{a^4x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3(a+bx)^2}\right.\right.\\ &\qquad\left.\left.-\frac{b(3b^2c-2abd+a^2e)}{a^4(a+bx)}\right)dx,x,x^3\right)\\ &= -\frac{c}{6a^2x^6}+\frac{2bc-ad}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^3b\,(a+bx^3)}\\ &\quad+\frac{(3b^2c-2abd+a^2e)\log(x)}{a^4}-\frac{(3b^2c-2abd+a^2e)\log\left(a+bx^3\right)}{3a^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx = -\frac{\frac{a^2c}{x^6}+\frac{2a(-2bc+ad)}{x^3}+\frac{2a(-b^3c+ab^2d-a^2be+a^3f)}{b(a+bx^3)}-6(3b^2c-2abd+a^2e)\log(x)+2(3b^2c-2abd+a^2e)\log\left(a+\right.}{6a^4}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -1/6*((a^2*c)/x^6 + (2*a*(-2*b*c + a*d))/x^3 + (2*a*(-(b^3*c) + a*b^2*d - a
^2*b*e + a^3*f))/(b*(a + b*x^3)) - 6*(3*b^2*c - 2*a*b*d + a^2*e)*Log[x] + 2
*(3*b^2*c - 2*a*b*d + a^2*e)*Log[a + b*x^3])/a^4
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

method	result
default	$-\frac{c}{6a^2x^6}-\frac{ad-2bc}{3a^3x^3}+\frac{(a^2e-2abd+3b^2c)\ln(x)}{a^4}+\frac{(-a^2e+2abd-3b^2c)\ln(b\,x^3+a)-\frac{a\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)}{b\left(b\,x^3+a\right)}}{3a^4}$
norman	$\frac{-\frac{c}{6a}-\frac{(2ad-3bc)x^3}{6a^2}+\frac{\left(f\,a^3-a^2be+2a\,b^2d-3b^3c\right)x^9}{3a^4}}{x^6(b\,x^3+a)}+\frac{(a^2e-2abd+3b^2c)\ln(x)}{a^4}-\frac{(a^2e-2abd+3b^2c)\ln(b\,x^3+a)}{3a^4}$
risch	$\frac{-\frac{\left(f\,a^3-a^2be+2a\,b^2d-3b^3c\right)x^6}{3a^3b}-\frac{(2ad-3bc)x^3}{6a^2}-\frac{c}{6a}}{x^6(b\,x^3+a)}+\frac{e\ln(x)}{a^2}-\frac{2\ln(x)bd}{a^3}+\frac{3\ln(x)b^2c}{a^4}-\frac{e\ln(b\,x^3+a)}{3a^2}+\frac{2\ln(b\,x^3+a)bd}{3a^3}-$
parallelrisch	$6\ln(x)x^9a^2be-12\ln(x)x^9a\,b^2d+18\ln(x)x^9b^3c-2\ln(b\,x^3+a)x^9a^2be+4\ln(b\,x^3+a)x^9a\,b^2d-6\ln(b\,x^3+a)x^9b^3c+2x^9a^3f-2x^9a^2b$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*c/a^2/x^6-1/3*(a*d-2*b*c)/a^3/x^3+(a^2*e-2*a*b*d+3*b^2*c)*ln(x)/a^4+1/
3/a^4*((-a^2*e+2*a*b*d-3*b^2*c)*ln(b*x^3+a)-a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)
/b/(b*x^3+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.60

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx$$
$$=\frac{2\left(3\,ab^3c-2\,a^2b^2d+a^3be-a^4f\right)x^6-a^3bc+\left(3\,a^2b^2c-2\,a^3bd\right)x^3-2\left(\left(3\,b^4c-2\,ab^3d+a^2b^2e\right)x^9+\left(3\,ab^3c\right.\right.}{6\left(a^4b^2x^9+\right.}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(3*a*b^3*c - 2*a^2*b^2*d + a^3*b*e - a^4*f)*x^6 - a^3*b*c + (3*a^2*b
^2*c - 2*a^3*b*d)*x^3 - 2*((3*b^4*c - 2*a*b^3*d + a^2*b^2*e)*x^9 + (3*a*b^3
*c - 2*a^2*b^2*d + a^3*b*e)*x^6)*log(b*x^3 + a) + 6*((3*b^4*c - 2*a*b^3*d +
 a^2*b^2*e)*x^9 + (3*a*b^3*c - 2*a^2*b^2*d + a^3*b*e)*x^6)*log(x))/(a^4*b^2
*x^9 + a^5*b*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx=\frac{2\left(3\,b^3c-2\,ab^2d+a^2be-a^3f\right)x^6-a^2bc+\left(3\,ab^2c-2\,a^2bd\right)x^3}{6\left(a^3b^2x^9+a^4bx^6\right)}-\frac{\left(3\,b^2c-2\,abd+a^2e\right)\log\left(bx^3+a\right)}{3\,a^4}+\frac{\left(3\,b^2c-2\,abd+a^2e\right)\log\left(x^3\right)}{3\,a^4}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(3*b^3*c - 2*a*b^2*d + a^2*b*e - a^3*f)*x^6 - a^2*b*c + (3*a*b^2*c -
 2*a^2*b*d)*x^3)/(a^3*b^2*x^9 + a^4*b*x^6) - 1/3*(3*b^2*c - 2*a*b*d + a^2*e
)*log(b*x^3 + a)/a^4 + 1/3*(3*b^2*c - 2*a*b*d + a^2*e)*log(x^3)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.51

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx=\frac{\left(3\,b^2c-2\,abd+a^2e\right)\log\left(|x|\right)}{a^4}-\frac{\left(3\,b^3c-2\,ab^2d+a^2be\right)\log\left(|bx^3+a|\right)}{3\,a^4b}+\frac{3\,b^4cx^3-2\,ab^3dx^3+a^2b^2ex^3+4\,ab^3c-3\,a^2b^2d+2\,a^3be-a^4f}{3\left(bx^3+a\right)a^4b}-\frac{9\,b^2cx^6-6\,abdx^6+3\,a^2ex^6-4\,abcx^3+2\,a^2dx^3+a^2c}{6\,a^4x^6}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] (3*b^2*c - 2*a*b*d + a^2*e)*log(abs(x))/a^4 - 1/3*(3*b^3*c - 2*a*b^2*d + a^
2*b*e)*log(abs(b*x^3 + a))/(a^4*b) + 1/3*(3*b^4*c*x^3 - 2*a*b^3*d*x^3 + a^2
*b^2*e*x^3 + 4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)/((b*x^3 + a)*a^4*
b) - 1/6*(9*b^2*c*x^6 - 6*a*b*d*x^6 + 3*a^2*e*x^6 - 4*a*b*c*x^3 + 2*a^2*d*x
^3 + a^2*c)/(a^4*x^6)
```

Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx=\frac{\ln\left(x\right)\,\left(e\,a^2-2\,d\,a\,b+3\,c\,b^2\right)}{a^4}-\frac{\ln\left(b\,x^3+a\right)\,\left(e\,a^2-2\,d\,a\,b+3\,c\,b^2\right)}{3\,a^4}-\frac{\frac{c}{6\,a}+\frac{x^3\,\left(2\,a\,d-3\,b\,c\right)}{6\,a^2}-\frac{x^6\,\left(-f\,a^3+e\,a^2\,b-2\,d\,a\,b^2+3\,c\,b^3\right)}{3\,a^3\,b}}{b\,x^9+a\,x^6}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x)
```

```
[Out] (log(x)*(3*b^2*c + a^2*e - 2*a*b*d))/a^4 - (log(a + b*x^3)*(3*b^2*c + a^2*e
 - 2*a*b*d))/(3*a^4) - (c/(6*a) + (x^3*(2*a*d - 3*b*c))/(6*a^2) - (x^6*(3*b
^3*c - a^3*f - 2*a*b^2*d + a^2*b*e))/(3*a^3*b))/(a*x^6 + b*x^9)
```

3.258 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 175

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^2}\,dx = -\frac{c}{9a^2x^9}+\frac{2bc-ad}{6a^3x^6}-\frac{3b^2c-2abd+a^2e}{3a^4x^3} -\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4\left(a+bx^3\right)} -\frac{\left(4b^3c-3ab^2d+2a^2be-a^3f\right)\log(x)}{a^5} +\frac{\left(4b^3c-3ab^2d+2a^2be-a^3f\right)\log\left(a+bx^3\right)}{3a^5}$$

```
[Out] -1/9*c/a^2/x^9+1/6*(-a*d+2*b*c)/a^3/x^6+1/3*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x^
3+1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/(b*x^3+a)-(-a^3*f+2*a^2*b*e-3*a*b^2
*d+4*b^3*c)*ln(x)/a^5+1/3*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)*ln(b*x^3+a)/
a^5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used

$= \{1835, 1634\}$

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^2}\,dx = \frac{2bc-ad}{6a^3x^6} - \frac{c}{9a^2x^9} - \frac{a^2e-2abd+3b^2c}{3a^4x^3} + \frac{\log\left(a+bx^3\right)\left(a^3(-f)+2a^2be-3ab^2d+4b^3c\right)}{3a^5} - \frac{\log(x)\left(a^3(-f)+2a^2be-3ab^2d+4b^3c\right)}{a^5} - \frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^4\left(a+bx^3\right)}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x]
```

```
[Out] -1/9*c/(a^2*x^9) + (2*b*c - a*d)/(6*a^3*x^6) - (3*b^2*c - 2*a*b*d + a^2*e)/
(3*a^4*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*(a + b*x^3)) - ((4
*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*Log[x])/a^5 + ((4*b^3*c - 3*a*b^2*d
 + 2*a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^5)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^2x^4} + \frac{-2bc+ad}{a^3x^3} + \frac{3b^2c-2abd+a^2e}{a^4x^2} + \frac{-4b^3c+3ab^2d-2a^2be+a^3f}{a^5x} - \frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^4(a+bx)^2} - \frac{b(-4b^3c+3ab^2d-2a^2be+a^3f)}{a^5(a+bx)}\right) dx, x, x^3\right)$$

$$= -\frac{c}{9a^2x^9} + \frac{2bc-ad}{6a^3x^6} - \frac{3b^2c-2abd+a^2e}{3a^4x^3} - \frac{b^3c-ab^2d+a^2be-a^3f}{3a^4\left(a+bx^3\right)} - \frac{\left(4b^3c-3ab^2d+2a^2be-a^3f\right)\log(x)}{a^5} + \frac{\left(4b^3c-3ab^2d+2a^2be-a^3f\right)\log\left(a+bx^3\right)}{3a^5}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^2}\,dx$$
$$= \frac{-\frac{2a^3c}{x^9}-\frac{3a^2(-2bc+ad)}{x^6}-\frac{6a(3b^2c-2abd+a^2e)}{x^3}+\frac{6a(-b^3c+ab^2d-a^2be+a^3f)}{a+bx^3}+18(-4b^3c+3ab^2d-2a^2be+a^3f)\log(x)}{18a^5}$$

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x]

[Out] ((-2*a^3*c)/x^9 - (3*a^2*(-2*b*c + a*d))/x^6 - (6*a*(3*b^2*c - 2*a*b*d + a^2*e))/x^3 + (6*a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) + 18*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f)*Log[x] + 6*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*Log[a + b*x^3])/(18*a^5)

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.97

method	result
default	$-\frac{c}{9a^2x^9}-\frac{ad-2bc}{6a^3x^6}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}+\frac{(fa^3-2a^2be+3ab^2d-4b^3c)\ln(x)}{a^5}-\frac{b\left(\frac{(fa^3-2a^2be+3ab^2d-4b^3c)\ln(bx^3+a)}{b}\right.}{3a^5}$
norman	$\frac{-\frac{c}{9a}-\frac{(3ad-4bc)x^3}{18a^2}-\frac{(2a^2e-3abd+4b^2c)x^6}{6a^3}+\frac{b(-fa^3+2a^2be-3ab^2d+4b^3c)x^{12}}{3a^5}}{x^9(bx^3+a)}+\frac{(fa^3-2a^2be+3ab^2d-4b^3c)\ln(x)}{a^5}-\frac{(fa^3-}{}$
risch	$\frac{\frac{(fa^3-2a^2be+3ab^2d-4b^3c)x^9}{3a^4}-\frac{(2a^2e-3abd+4b^2c)x^6}{6a^3}-\frac{(3ad-4bc)x^3}{18a^2}-\frac{c}{9a}}{x^9(bx^3+a)}+\frac{\ln(x)f}{a^2}-\frac{2\ln(x)be}{a^3}+\frac{3\ln(x)b^2d}{a^4}-\frac{4\ln(x)b^3c}{a^5}$
parallelrisch	$\frac{-6x^6a^4be+9x^6a^3b^2d-12x^6a^2b^3c-3x^3a^4bd-72\ln(x)x^{12}b^5c+24\ln(bx^3+a)x^{12}b^5c-2a^4bc+4a^3b^2cx^3-36\ln(x)x^{12}a^2b^3e+54\ln}{}$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/9*c/a^2/x^9-1/6*(a*d-2*b*c)/a^3/x^6-1/3*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^3+(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5*ln(x)-1/3/a^5*b*((a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/b*ln(b*x^3+a)-a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a)
)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.49

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^2}\,dx = -\frac{6\,(4\,ab^3c-3\,a^2b^2d+2\,a^3be-a^4f)x^9+3\,(4\,a^2b^2c-3\,a^3bd+2\,a^4e)x^6+2\,a^4c-(4\,a^3bc-3\,a^4d)x^3-6\,((\ldots$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(6*(4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*x^9 + 3*(4*a^2*b^2*c
 - 3*a^3*b*d + 2*a^4*e)*x^6 + 2*a^4*c - (4*a^3*b*c - 3*a^4*d)*x^3 - 6*((4*b
^4*c - 3*a*b^3*d + 2*a^2*b^2*e - a^3*b*f)*x^12 + (4*a*b^3*c - 3*a^2*b^2*d +
 2*a^3*b*e - a^4*f)*x^9)*log(b*x^3 + a) + 18*((4*b^4*c - 3*a*b^3*d + 2*a^2*
b^2*e - a^3*b*f)*x^12 + (4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*x^9)*
log(x))/(a^5*b*x^12 + a^6*x^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^2}\,dx = -\frac{6\,(4\,b^3c-3\,ab^2d+2\,a^2be-a^3f)x^9+3\,(4\,ab^2c-3\,a^2bd+2\,a^3e)x^6+2\,a^3c-(4\,a^2bc-3\,a^3d)x^3}{18\,(a^4bx^{12}+a^5x^9)} + \frac{(4\,b^3c-3\,ab^2d+2\,a^2be-a^3f)\log(bx^3+a)}{3\,a^5} - \frac{(4\,b^3c-3\,ab^2d+2\,a^2be-a^3f)\log(x^3)}{3\,a^5}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/18*(6*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*x^9 + 3*(4*a*b^2*c - 3*a
^2*b*d + 2*a^3*e)*x^6 + 2*a^3*c - (4*a^2*b*c - 3*a^3*d)*x^3)/(a^4*b*x^12 +
a^5*x^9) + 1/3*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*log(b*x^3 + a)/a^5
 - 1/3*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*log(x^3)/a^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.54

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^2}\,dx = -\frac{(4\,b^3c-3\,ab^2d+2\,a^2be-a^3f)\log(|x|)}{a^5} + \frac{(4\,b^4c-3\,ab^3d+2\,a^2b^2e-a^3bf)\log(|bx^3+a|)}{3\,a^5b} - \frac{4\,b^4cx^3-3\,ab^3dx^3+2\,a^2b^2ex^3-a^3bfx^3+5\,ab^3c-4\,a^2b^2d+3\,a^3be-2\,a^4f}{3\,(bx^3+a)a^5} + \frac{44\,b^3cx^9-33\,ab^2dx^9+22\,a^2bex^9-11\,a^3fx^9-18\,ab^2cx^6+12\,a^2bdx^6-6\,a^3ex^6+6\,a^2bcx^3-3\,a^3dx^3-}{18\,a^5x^9}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*log(abs(x))/a^5 + 1/3*(4*b^4*c -
 3*a*b^3*d + 2*a^2*b^2*e - a^3*b*f)*log(abs(b*x^3 + a))/(a^5*b) - 1/3*(4*b^
4*c*x^3 - 3*a*b^3*d*x^3 + 2*a^2*b^2*e*x^3 - a^3*b*f*x^3 + 5*a*b^3*c - 4*a^2
*b^2*d + 3*a^3*b*e - 2*a^4*f)/((b*x^3 + a)*a^5) + 1/18*(44*b^3*c*x^9 - 33*a
*b^2*d*x^9 + 22*a^2*b*e*x^9 - 11*a^3*f*x^9 - 18*a*b^2*c*x^6 + 12*a^2*b*d*x^
6 - 6*a^3*e*x^6 + 6*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^5*x^9)
```

Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^2}\,dx = \frac{\ln(b\,x^3+a)\,\left(-f\,a^3+2\,e\,a^2\,b-3\,d\,a\,b^2+4\,c\,b^3\right)}{3\,a^5} - \frac{\frac{c}{9\,a}+\frac{x^9\,\left(-f\,a^3+2\,e\,a^2\,b-3\,d\,a\,b^2+4\,c\,b^3\right)}{3\,a^4}+\frac{x^3\,\left(3\,a\,d-4\,b\,c\right)}{18\,a^2}+\frac{x^6\,\left(2\,e\,a^2-3\,d\,a\,b+4\,c\,b^2\right)}{6\,a^3}}{b\,x^{12}+a\,x^9} - \frac{\ln(x)\,\left(-f\,a^3+2\,e\,a^2\,b-3\,d\,a\,b^2+4\,c\,b^3\right)}{a^5}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x)
```

```
[Out] (log(a + b*x^3)*(4*b^3*c - a^3*f - 3*a*b^2*d + 2*a^2*b*e))/(3*a^5) - (c/(9*
a) + (x^9*(4*b^3*c - a^3*f - 3*a*b^2*d + 2*a^2*b*e))/(3*a^4) + (x^3*(3*a*d
- 4*b*c))/(18*a^2) + (x^6*(4*b^2*c + 2*a^2*e - 3*a*b*d))/(6*a^3))/(a*x^9 +
b*x^12) - (log(x)*(4*b^3*c - a^3*f - 3*a*b^2*d + 2*a^2*b*e))/a^5
```

3.259 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 214

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2}\,dx = &-\frac{c}{12a^2x^{12}}+\frac{2bc-ad}{9a^3x^9}-\frac{3b^2c-2abd+a^2e}{6a^4x^6}\\ &+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}+\frac{b(b^3c-ab^2d+a^2be-a^3f)}{3a^5\left(a+bx^3\right)}\\ &+\frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)\log(x)}{a^6}\\ &-\frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)\log\left(a+bx^3\right)}{3a^6}\end{aligned}$$

```
[Out] -1/12*c/a^2/x^12+1/9*(-a*d+2*b*c)/a^3/x^9+1/6*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/
x^6+1/3*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x^3+1/3*b*(-a^3*f+a^2*b*e-
a*b^2*d+b^3*c)/a^5/(b*x^3+a)+b*(-2*a^3*f+3*a^2*b*e-4*a*b^2*d+5*b^3*c)*ln(x)
/a^6-1/3*b*(-2*a^3*f+3*a^2*b*e-4*a*b^2*d+5*b^3*c)*ln(b*x^3+a)/a^6
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used

$= \{1835, 1634\}$

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2}\,dx = \frac{2bc-ad}{9a^3x^9}-\frac{c}{12a^2x^{12}}-\frac{a^2e-2abd+3b^2c}{6a^4x^6} - \frac{b\log\left(a+bx^3\right)\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{3a^6} + \frac{b\log(x)\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{a^6} + \frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^5\left(a+bx^3\right)} + \frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{3a^5x^3}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x]
```

```
[Out] -1/12*c/(a^2*x^12) + (2*b*c - a*d)/(9*a^3*x^9) - (3*b^2*c - 2*a*b*d + a^2*e
)/(6*a^4*x^6) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) + (b*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*a^5*(a + b*x^3)) + (b*(5*b^3*c - 4*
a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x])/a^6 - (b*(5*b^3*c - 4*a*b^2*d + 3*a^
2*b*e - 2*a^3*f)*Log[a + b*x^3])/(3*a^6)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^2x^5}+\frac{-2bc+ad}{a^3x^4}+\frac{3b^2c-2abd+a^2e}{a^4x^3} + \frac{-4b^3c+3ab^2d-2a^2be+a^3f}{a^5x^2}-\frac{b(-5b^3c+4ab^2d-3a^2be+2a^3f)}{a^6x} + \frac{b^2(-b^3c+ab^2d-a^2be+a^3f)}{a^5(a+bx)^2}+\frac{b^2(-5b^3c+4ab^2d-3a^2be+2a^3f)}{a^6(a+bx)}\right)\,dx, x, x^3\right)$$

$$
\begin{aligned}
&= -\frac{c}{12a^2x^{12}} + \frac{2bc-ad}{9a^3x^9} - \frac{3b^2c-2abd+a^2e}{6a^4x^6} + \frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3} \\
&\quad + \frac{b(b^3c-ab^2d+a^2be-a^3f)}{3a^5(a+bx^3)} + \frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)\log(x)}{a^6} \\
&\quad - \frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)\log(a+bx^3)}{3a^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

$$
\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^2}\,dx = -\frac{\frac{3a^4c}{x^{12}} + \frac{4a^3(-2bc+ad)}{x^9} + \frac{6a^2(3b^2c-2abd+a^2e)}{x^6} + \frac{12a(-4b^3c+3ab^2d-2a^2be+a^3f)}{x^3} + \frac{12ab(-b^3c+ab^2d-a^2be+a^3f)}{a+bx^3} - 36b(5b^3c-}{36a^6}
$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x]
```

```
[Out] -1/36*((3*a^4*c)/x^12 + (4*a^3*(-2*b*c + a*d))/x^9 + (6*a^2*(3*b^2*c - 2*a*
b*d + a^2*e))/x^6 + (12*a*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f))/x^3 +
 (12*a*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) - 36*b*(5*b^3*
c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x] + 12*b*(5*b^3*c - 4*a*b^2*d + 3
*a^2*b*e - 2*a^3*f)*Log[a + b*x^3])/a^6
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.98

method	result
default	$-\frac{c}{12a^2x^{12}} - \frac{ad-2bc}{9a^3x^9} - \frac{a^2e-2abd+3b^2c}{6a^4x^6} - \frac{f\,a^3-2a^2be+3a\,b^2d-4b^3c}{3a^5x^3} - \frac{b(2f\,a^3-3a^2be+4a\,b^2d-5b^3c)\ln(x)}{a^6} + \frac{b^2\left(\frac{(2f}{}\right.}{}$
norman	$\frac{-\frac{c}{12a}-\frac{\left(2f\,a^3-3a^2be+4a\,b^2d-5b^3c\right)x^9}{6a^4}-\frac{(4ad-5bc)x^3}{36a^2}-\frac{\left(3a^2e-4abd+5b^2c\right)x^6}{18a^3}+\frac{b\left(2a^3bf-3a^2e\,b^2+4a\,b^3d-5b^4c\right)x^{15}}{3a^6}}{x^{12}(b\,x^3+a)} - \frac{b(2f\,a^3-3a^2b}{}$
risch	$\frac{-\frac{c}{12a}-\frac{(4ad-5bc)x^3}{36a^2}-\frac{\left(3a^2e-4abd+5b^2c\right)x^6}{18a^3}-\frac{\left(2f\,a^3-3a^2be+4a\,b^2d-5b^3c\right)x^9}{6a^4}-\frac{b\left(2f\,a^3-3a^2be+4a\,b^2d-5b^3c\right)x^{12}}{3a^5}}{x^{12}(b\,x^3+a)} - \frac{2b\ln(x)f}{a^3} + \frac{3b}{}$
parallelrisch	$-\frac{3a^5bc-180\ln(x)x^{12}a\,b^5c-24\ln(b\,x^3+a)x^{12}a^4b^2f+36\ln(b\,x^3+a)x^{12}a^3b^3e-48\ln(b\,x^3+a)x^{12}a^2b^4d+60\ln(b\,x^3+a)x^{12}a\,b^5c+}{}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*c/a^2/x^12-1/9*(a*d-2*b*c)/a^3/x^9-1/6*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^
6-1/3*(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5/x^3-b*(2*a^3*f-3*a^2*b*e+4*a*
b^2*d-5*b^3*c)/a^6*ln(x)+1/3*b^2/a^6*((2*a^3*f-3*a^2*b*e+4*a*b^2*d-5*b^3*c)
/b*ln(b*x^3+a)-a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.45

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^2}\,dx = \frac{12\,(5\,ab^4c-4\,a^2b^3d+3\,a^3b^2e-2\,a^4bf)x^{12}+6\,(5\,a^2b^3c-4\,a^3b^2d+3\,a^4be-2\,a^5f)x^9-2\,(5\,a^3b^2c-4\,a^4bc}{}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*(5*a*b^4*c - 4*a^2*b^3*d + 3*a^3*b^2*e - 2*a^4*b*f)*x^12 + 6*(5*a^
2*b^3*c - 4*a^3*b^2*d + 3*a^4*b*e - 2*a^5*f)*x^9 - 2*(5*a^3*b^2*c - 4*a^4*b
*d + 3*a^5*e)*x^6 - 3*a^5*c + (5*a^4*b*c - 4*a^5*d)*x^3 - 12*((5*b^5*c - 4*
a*b^4*d + 3*a^2*b^3*e - 2*a^3*b^2*f)*x^15 + (5*a*b^4*c - 4*a^2*b^3*d + 3*a^
3*b^2*e - 2*a^4*b*f)*x^12)*log(b*x^3 + a) + 36*((5*b^5*c - 4*a*b^4*d + 3*a^
2*b^3*e - 2*a^3*b^2*f)*x^15 + (5*a*b^4*c - 4*a^2*b^3*d + 3*a^3*b^2*e - 2*a^
4*b*f)*x^12)*log(x))/(a^6*b*x^15 + a^7*x^12)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^2}\,dx = \frac{12\,(5\,b^4c-4\,ab^3d+3\,a^2b^2e-2\,a^3bf)x^{12}+6\,(5\,ab^3c-4\,a^2b^2d+3\,a^3be-2\,a^4f)x^9-2\,(5\,a^2b^2c-4\,a^3bd+}{36\,(a^5bx^{15}+a^6x^{12})} - \frac{(5\,b^4c-4\,ab^3d+3\,a^2b^2e-2\,a^3bf)\log(bx^3+a)}{3\,a^6} + \frac{(5\,b^4c-4\,ab^3d+3\,a^2b^2e-2\,a^3bf)\log(x^3)}{3\,a^6}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/36*(12*(5*b^4*c - 4*a*b^3*d + 3*a^2*b^2*e - 2*a^3*b*f)*x^12 + 6*(5*a*b^3*
c - 4*a^2*b^2*d + 3*a^3*b*e - 2*a^4*f)*x^9 - 2*(5*a^2*b^2*c - 4*a^3*b*d + 3
*a^4*e)*x^6 - 3*a^4*c + (5*a^3*b*c - 4*a^4*d)*x^3)/(a^5*b*x^15 + a^6*x^12)
- 1/3*(5*b^4*c - 4*a*b^3*d + 3*a^2*b^2*e - 2*a^3*b*f)*log(b*x^3 + a)/a^6 +
1/3*(5*b^4*c - 4*a*b^3*d + 3*a^2*b^2*e - 2*a^3*b*f)*log(x^3)/a^6
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.51

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2}\,dx = \frac{\left(5\,b^4c-4\,ab^3d+3\,a^2b^2e-2\,a^3bf\right)\log\left(|x|\right)}{a^6} - \frac{\left(5\,b^5c-4\,ab^4d+3\,a^2b^3e-2\,a^3b^2f\right)\log\left(|bx^3+a|\right)}{3\,a^6b} + \frac{5\,b^5cx^3-4\,ab^4dx^3+3\,a^2b^3ex^3-2\,a^3b^2fx^3+6\,ab^4c-5\,a^2b^3d+4\,a^3b^2e-3\,a^4bf}{3\,(bx^3+a)a^6} - \frac{125\,b^4cx^{12}-100\,ab^3dx^{12}+75\,a^2b^2ex^{12}-50\,a^3bfx^{12}-48\,ab^3cx^9+36\,a^2b^2dx^9-24\,a^3bex^9+12\,a^4fx^9+}{36\,a^6x^{12}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] (5*b^4*c - 4*a*b^3*d + 3*a^2*b^2*e - 2*a^3*b*f)*log(abs(x))/a^6 - 1/3*(5*b^
5*c - 4*a*b^4*d + 3*a^2*b^3*e - 2*a^3*b^2*f)*log(abs(b*x^3 + a))/(a^6*b) +
1/3*(5*b^5*c*x^3 - 4*a*b^4*d*x^3 + 3*a^2*b^3*e*x^3 - 2*a^3*b^2*f*x^3 + 6*a*
b^4*c - 5*a^2*b^3*d + 4*a^3*b^2*e - 3*a^4*b*f)/((b*x^3 + a)*a^6) - 1/36*(12
5*b^4*c*x^12 - 100*a*b^3*d*x^12 + 75*a^2*b^2*e*x^12 - 50*a^3*b*f*x^12 - 48*
a*b^3*c*x^9 + 36*a^2*b^2*d*x^9 - 24*a^3*b*e*x^9 + 12*a^4*f*x^9 + 18*a^2*b^2
*c*x^6 - 12*a^3*b*d*x^6 + 6*a^4*e*x^6 - 8*a^3*b*c*x^3 + 4*a^4*d*x^3 + 3*a^4
*c)/(a^6*x^12)
```

Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2}\,dx = \frac{\ln\left(x\right)\,\left(-2\,f\,a^3\,b+3\,e\,a^2\,b^2-4\,d\,a\,b^3+5\,c\,b^4\right)}{a^6} - \frac{\ln\left(b\,x^3+a\right)\,\left(-2\,f\,a^3\,b+3\,e\,a^2\,b^2-4\,d\,a\,b^3+5\,c\,b^4\right)}{3\,a^6} - \frac{\frac{c}{12\,a}-\frac{x^9\,\left(-2\,f\,a^3+3\,e\,a^2\,b-4\,d\,a\,b^2+5\,c\,b^3\right)}{6\,a^4}+\frac{x^3\,\left(4\,a\,d-5\,b\,c\right)}{36\,a^2}+\frac{x^6\,\left(3\,e\,a^2-4\,d\,a\,b+5\,c\,b^2\right)}{18\,a^3}-\frac{b\,x^{12}\,\left(-2\,f\,a^3+3\,e\,a^2\,b-4\,d\,a\,b^2+5\,c\,b^3\right)}{3\,a^5}}{b\,x^{15}+a\,x^{12}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x)
```

```
[Out] (log(x)*(5*b^4*c + 3*a^2*b^2*e - 4*a*b^3*d - 2*a^3*b*f))/a^6 - (log(a + b*x
^3)*(5*b^4*c + 3*a^2*b^2*e - 4*a*b^3*d - 2*a^3*b*f))/(3*a^6) - (c/(12*a) -
(x^9*(5*b^3*c - 2*a^3*f - 4*a*b^2*d + 3*a^2*b*e))/(6*a^4) + (x^3*(4*a*d - 5
*b*c))/(36*a^2) + (x^6*(5*b^2*c + 3*a^2*e - 4*a*b*d))/(18*a^3) - (b*x^12*(5
*b^3*c - 2*a^3*f - 4*a*b^2*d + 3*a^2*b*e))/(3*a^5))/(a*x^12 + b*x^15)
```

3.260 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 369

$$\begin{aligned}\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx &= -\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x}{b^6}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^4}{4b^5}\\&\quad+\frac{(b^2d-2abe+3a^2f)\,x^7}{7b^4}+\frac{(be-2af)x^{10}}{10b^3}+\frac{fx^{13}}{13b^2}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^6\,(a+bx^3)}\\&\quad-\frac{a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{19/3}}\\&\quad+\frac{a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{19/3}}\\&\quad-\frac{a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{19/3}}\end{aligned}$$

```
[Out] -a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*x/b^6+1/4*(-4*a^3*f+3*a^2*b*e-2*a
*b^2*d+b^3*c)*x^4/b^5+1/7*(3*a^2*f-2*a*b*e+b^2*d)*x^7/b^4+1/10*(-2*a*f+b*e)
*x^10/b^3+1/13*f*x^13/b^2-1/3*a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^6/(b*x
^3+a)+1/9*a^(4/3)*(-16*a^3*f+13*a^2*b*e-10*a*b^2*d+7*b^3*c)*ln(a^(1/3)+b^(1
/3)*x)/b^(19/3)-1/18*a^(4/3)*(-16*a^3*f+13*a^2*b*e-10*a*b^2*d+7*b^3*c)*ln(a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(19/3)-1/9*a^(4/3)*(-16*a^3*f+13*a^
2*b*e-10*a*b^2*d+7*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))
/b^(19/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1842, 1901, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= \frac{x^7(3a^2f-2abe+b^2d)}{7b^4}-\frac{a^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^6(a+bx^3)}$$
$$-\frac{ax(-5a^3f+4a^2be-3ab^2d+2b^3c)}{b^6}+\frac{x^4(-4a^3f+3a^2be-2ab^2d+b^3c)}{4b^5}$$
$$-\frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-16a^3f+13a^2be-10ab^2d+7b^3c)}{3\sqrt{3}b^{19/3}}$$
$$-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-16a^3f+13a^2be-10ab^2d+7b^3c)}{18b^{19/3}}$$
$$+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-16a^3f+13a^2be-10ab^2d+7b^3c)}{9b^{19/3}}+\frac{x^{10}(be-2af)}{10b^3}+\frac{fx^{13}}{13b^2}$$

```
[In] Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] -((a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x)/b^6) + ((b^3*c - 2*a*b^
2*d + 3*a^2*b*e - 4*a^3*f)*x^4)/(4*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^7)
/(7*b^4) + ((b*e - 2*a*f)*x^10)/(10*b^3) + (f*x^13)/(13*b^2) - (a^2*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^6*(a + b*x^3)) - (a^(4/3)*(7*b^3*c - 1
0*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(3*Sqrt[3]*b^(19/3)) + (a^(4/3)*(7*b^3*c - 10*a*b^2*d + 13*a^2*b
*e - 16*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(19/3)) - (a^(4/3)*(7*b^3*c -
 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(18*b^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^6\,(a+bx^3)}$$
$$-\frac{\int \frac{-a^3(b^3c-ab^2d+a^2be-a^3f)+3a^2b(b^3c-ab^2d+a^2be-a^3f)x^3-3ab^2(b^3c-ab^2d+a^2be-a^3f)x^6-3ab^3(b^2d-abe+a^2f)x^9-3ab^4(be-af)x^{12}-3a}{a+bx^3}}{3ab^6}$$

$$= -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^6\,(a+bx^3)} - \frac{\int \Big(3a^2(2b^3c-3ab^2d+4a^2be-5a^3f) - 3ab(b^3c-2ab^2d+3a^2be-4a^3f)\,x^3 - 3ab^2(b^2d-2abe}{3ab^6}$$

$$= -\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x}{b^6} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^4}{4b^5} + \frac{(b^2d-2abe+3a^2f)\,x^7}{7b^4} + \frac{(be-2af)x^{10}}{10b^3} + \frac{fx^{13}}{13b^2} - \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^6\,(a+bx^3)} + \frac{\left(a^2(7b^3c-10ab^2d+13a^2be-16a^3f)\right)\int \frac{1}{a+bx^3}\,dx}{3b^6}$$

$$= -\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x}{b^6} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^4}{4b^5} + \frac{(b^2d-2abe+3a^2f)\,x^7}{7b^4} + \frac{(be-2af)x^{10}}{10b^3} + \frac{fx^{13}}{13b^2} - \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^6\,(a+bx^3)} + \frac{\left(a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9b^6} + \frac{\left(a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9b^6}$$

$$= -\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x}{b^6} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^4}{4b^5} + \frac{(b^2d-2abe+3a^2f)\,x^7}{7b^4} + \frac{(be-2af)x^{10}}{10b^3} + \frac{fx^{13}}{13b^2} - \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^6\,(a+bx^3)} + \frac{a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{19/3}} - \frac{\left(a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18b^{19/3}} + \frac{\left(a^{5/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^6}$$

$$
= -\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x}{b^6}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^4}{4b^5}
+\frac{(b^2d-2abe+3a^2f)\,x^7}{7b^4}+\frac{(be-2af)x^{10}}{10b^3}+\frac{fx^{13}}{13b^2}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^6\,(a+bx^3)}
+\frac{a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{19/3}}
-\frac{a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{19/3}}
+\frac{\left(a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3b^{19/3}}
$$

$$
= -\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x}{b^6}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^4}{4b^5}
+\frac{(b^2d-2abe+3a^2f)\,x^7}{7b^4}+\frac{(be-2af)x^{10}}{10b^3}+\frac{fx^{13}}{13b^2}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^6\,(a+bx^3)}
-\frac{a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{19/3}}
+\frac{a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{19/3}}
-\frac{a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{19/3}}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.99

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{a(-2b^3c+3ab^2d-4a^2be+5a^3f)\,x}{b^6}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^4}{4b^5} + \frac{(b^2d-2abe+3a^2f)\,x^7}{7b^4}+\frac{(be-2af)x^{10}}{10b^3}+\frac{fx^{13}}{13b^2}+\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)\,x}{3b^6\,(a+bx^3)} + \frac{a^{4/3}(-7b^3c+10ab^2d-13a^2be+16a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{19/3}} - \frac{a^{4/3}(-7b^3c+10ab^2d-13a^2be+16a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{19/3}} + \frac{a^{4/3}(-7b^3c+10ab^2d-13a^2be+16a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{19/3}}$$

```
[In] Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x)/b^6 + ((b^3*c - 2*a*b^2*
d + 3*a^2*b*e - 4*a^3*f)*x^4)/(4*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^7)/(
7*b^4) + ((b*e - 2*a*f)*x^10)/(10*b^3) + (f*x^13)/(13*b^2) + (a^2*(-(b^3*c)
 + a*b^2*d - a^2*b*e + a^3*f)*x)/(3*b^6*(a + b*x^3)) + (a^(4/3)*(-7*b^3*c +
 10*a*b^2*d - 13*a^2*b*e + 16*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]])/(3*Sqrt[3]*b^(19/3)) - (a^(4/3)*(-7*b^3*c + 10*a*b^2*d - 13*a^2*b*e
+ 16*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(19/3)) + (a^(4/3)*(-7*b^3*c + 1
0*a*b^2*d - 13*a^2*b*e + 16*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*b^(19/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.66

method	result
risch	$\frac{f\,x^{13}}{13b^2}-\frac{x^{10}af}{5b^3}+\frac{x^{10}e}{10b^2}+\frac{3x^7a^2f}{7b^4}-\frac{2x^7ae}{7b^3}+\frac{d\,x^7}{7b^2}-\frac{a^3f\,x^4}{b^5}+\frac{3a^2e\,x^4}{4b^4}-\frac{ad\,x^4}{2b^3}+\frac{c\,x^4}{4b^2}+\frac{5a^4fx}{b^6}-\frac{4a^3ex}{b^5}+\frac{3a^2dx}{b^4}$
default	$\frac{\frac{1}{13}f\,x^{13}b^4-\frac{1}{5}x^{10}a\,b^3f+\frac{1}{10}x^{10}b^4e+\frac{3}{7}x^7a^2b^2f-\frac{2}{7}x^7a\,b^3e+\frac{1}{7}b^4d\,x^7-a^3bf\,x^4+\frac{3}{4}a^2b^2e\,x^4-\frac{1}{2}a\,b^3d\,x^4+\frac{1}{4}b^4c\,x^4+5a^4fx-4a^3bex+3a^2b^2}{b^6}$

```
[In] int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*f*x^13/b^2-1/5/b^3*x^10*a*f+1/10/b^2*x^10*e+3/7/b^4*x^7*a^2*f-2/7/b^3*
x^7*a*e+1/7/b^2*d*x^7-1/b^5*a^3*f*x^4+3/4/b^4*a^2*e*x^4-1/2/b^3*a*d*x^4+1/4
/b^2*c*x^4+5/b^6*a^4*f*x-4/b^5*a^3*e*x+3/b^4*a^2*d*x-2/b^3*a*c*x+(1/3*f*a^5
-1/3*a^4*e*b+1/3*a^3*d*b^2-1/3*a^2*c*b^3)*x/b^6/(b*x^3+a)+1/9/b^7*a^2*sum((
-16*a^3*f+13*a^2*b*e-10*a*b^2*d+7*b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.32

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \frac{1260\,b^5fx^{16}+126\,(13\,b^5e-16\,ab^4f)x^{13}+234\,(10\,b^5d-13\,ab^4e+16\,a^2b^3f)x^{10}+585\,(7\,b^5c-10\,ab^4d+13}{}$$

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16380*(1260*b^5*f*x^16 + 126*(13*b^5*e - 16*a*b^4*f)*x^13 + 234*(10*b^5*d
 - 13*a*b^4*e + 16*a^2*b^3*f)*x^10 + 585*(7*b^5*c - 10*a*b^4*d + 13*a^2*b^3
*e - 16*a^3*b^2*f)*x^7 - 4095*(7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16
*a^4*b*f)*x^4 - 1820*sqrt(3)*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*
a^5*f + (7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16*a^4*b*f)*x^3)*(-a/b)^
(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*(7*a^2*b
^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f + (7*a*b^4*c - 10*a^2*b^3*d + 1
3*a^3*b^2*e - 16*a^4*b*f)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3)) - 1820*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f + (7*a
*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16*a^4*b*f)*x^3)*(-a/b)^(1/3)*log(x
- (-a/b)^(1/3)) - 5460*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f)
*x)/(b^7*x^3 + a*b^6)
```

Sympy [A] (verification not implemented)

Time = 85.66 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.36

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= x^{10}\left(-\frac{af}{5b^3}+\frac{e}{10b^2}\right)+x^7\cdot\left(\frac{3a^2f}{7b^4}-\frac{2ae}{7b^3}+\frac{d}{7b^2}\right)+x^4\left(-\frac{a^3f}{b^5}+\frac{3a^2e}{4b^4}-\frac{ad}{2b^3}+\frac{c}{4b^2}\right)$$
$$+x\left(\frac{5a^4f}{b^6}-\frac{4a^3e}{b^5}+\frac{3a^2d}{b^4}-\frac{2ac}{b^3}\right)+\frac{x(a^5f-a^4be+a^3b^2d-a^2b^3c)}{3ab^6+3b^7x^3}$$
$$+\mathrm{RootSum}\left(729t^3b^{19}+4096a^{13}f^3-9984a^{12}bef^2+7680a^{11}b^2df^2+8112a^{11}b^2e^2f-5376a^{10}b^3cf^2-124\right.$$
$$+\frac{fx^{13}}{13b^2}$$

```
[In] integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**10*(-a*f/(5*b**3) + e/(10*b**2)) + x**7*(3*a**2*f/(7*b**4) - 2*a*e/(7*b*
*3) + d/(7*b**2)) + x**4*(-a**3*f/b**5 + 3*a**2*e/(4*b**4) - a*d/(2*b**3) +
 c/(4*b**2)) + x*(5*a**4*f/b**6 - 4*a**3*e/b**5 + 3*a**2*d/b**4 - 2*a*c/b**
3) + x*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(3*a*b**6 + 3*b**7*x
**3) + RootSum(729*_t**3*b**19 + 4096*a**13*f**3 - 9984*a**12*b*e*f**2 + 76
80*a**11*b**2*d*f**2 + 8112*a**11*b**2*e**2*f - 5376*a**10*b**3*c*f**2 - 12
480*a**10*b**3*d*e*f - 2197*a**10*b**3*e**3 + 8736*a**9*b**4*c*e*f + 4800*a
**9*b**4*d**2*f + 5070*a**9*b**4*d*e**2 - 6720*a**8*b**5*c*d*f - 3549*a**8*
b**5*c*e**2 - 3900*a**8*b**5*d**2*e + 2352*a**7*b**6*c**2*f + 5460*a**7*b**
6*c*d*e + 1000*a**7*b**6*d**3 - 1911*a**6*b**7*c**2*e - 2100*a**6*b**7*c*d*
*2 + 1470*a**5*b**8*c**2*d - 343*a**4*b**9*c**3, Lambda(_t, _t*log(-9*_t*b*
*6/(16*a**4*f - 13*a**3*b*e + 10*a**2*b**2*d - 7*a*b**3*c) + x))) + f*x**13
/(13*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)x}{3\,(b^7x^3+ab^6)}$$

$$+\frac{140\,b^4fx^{13}+182\,(b^4e-2\,ab^3f)x^{10}+260\,(b^4d-2\,ab^3e+3\,a^2b^2f)x^7+455\,(b^4c-2\,ab^3d+3\,a^2b^2e-4\,a^3b}{1820\,b^6}$$

$$+\frac{\sqrt{3}(7\,a^2b^3c-10\,a^3b^2d+13\,a^4be-16\,a^5f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$$-\frac{(7\,a^2b^3c-10\,a^3b^2d+13\,a^4be-16\,a^5f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$$+\frac{(7\,a^2b^3c-10\,a^3b^2d+13\,a^4be-16\,a^5f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x/(b^7*x^3 + a*b^6) + 1/1820
*(140*b^4*f*x^13 + 182*(b^4*e - 2*a*b^3*f)*x^10 + 260*(b^4*d - 2*a*b^3*e +
3*a^2*b^2*f)*x^7 + 455*(b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^4 -
1820*(2*a*b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*x)/b^6 + 1/9*sqrt(3)*(
7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f)*arctan(1/3*sqrt(3)*(2*x
 - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*(a/b)^(2/3)) - 1/18*(7*a^2*b^3*c - 10*a^3
*b^2*d + 13*a^4*b*e - 16*a^5*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^7
*(a/b)^(2/3)) + 1/9*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f)*lo
g(x + (a/b)^(1/3))/(b^7*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.20

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= \frac{\sqrt{3}\left(7\left(-ab^2\right)^{\frac{1}{3}}ab^3c-10\left(-ab^2\right)^{\frac{1}{3}}a^2b^2d+13\left(-ab^2\right)^{\frac{1}{3}}a^3be-16\left(-ab^2\right)^{\frac{1}{3}}a^4f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^7}$$
$$-\frac{\left(7a^2b^3c-10a^3b^2d+13a^4be-16a^5f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9ab^6}$$
$$+\frac{\left(7\left(-ab^2\right)^{\frac{1}{3}}ab^3c-10\left(-ab^2\right)^{\frac{1}{3}}a^2b^2d+13\left(-ab^2\right)^{\frac{1}{3}}a^3be-16\left(-ab^2\right)^{\frac{1}{3}}a^4f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right.}{18b^7}$$
$$-\frac{a^2b^3cx-a^3b^2dx+a^4bex-a^5fx}{3\left(bx^3+a\right)b^6}$$
$$+\frac{140b^{24}fx^{13}+182b^{24}ex^{10}-364ab^{23}fx^{10}+260b^{24}dx^7-520ab^{23}ex^7+780a^2b^{22}fx^7+455b^{24}cx^4-910}{1820}$$

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*a*b^3*c - 10*(-a*b^2)^(1/3)*a^2*b^2*d + 13*(-
a*b^2)^(1/3)*a^3*b*e - 16*(-a*b^2)^(1/3)*a^4*f)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/b^7 - 1/9*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b
*e - 16*a^5*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^6) + 1/18*(7*(-
a*b^2)^(1/3)*a*b^3*c - 10*(-a*b^2)^(1/3)*a^2*b^2*d + 13*(-a*b^2)^(1/3)*a^3*
b*e - 16*(-a*b^2)^(1/3)*a^4*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7
 - 1/3*(a^2*b^3*c*x - a^3*b^2*d*x + a^4*b*e*x - a^5*f*x)/((b*x^3 + a)*b^6)
+ 1/1820*(140*b^24*f*x^13 + 182*b^24*e*x^10 - 364*a*b^23*f*x^10 + 260*b^24*
d*x^7 - 520*a*b^23*e*x^7 + 780*a^2*b^22*f*x^7 + 455*b^24*c*x^4 - 910*a*b^23
*d*x^4 + 1365*a^2*b^22*e*x^4 - 1820*a^3*b^21*f*x^4 - 3640*a*b^23*c*x + 5460
*a^2*b^22*d*x - 7280*a^3*b^21*e*x + 9100*a^4*b^20*f*x)/b^26
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.30

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= x^{10}\left(\frac{e}{10\,b^2}-\frac{a\,f}{5\,b^3}\right)-x\left(\frac{2\,a\left(\frac{c}{b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)}{b}-\frac{a^2\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b^2}\right)-x^7\left(\frac{a^2\,f}{7\,b^4}-\frac{d}{7\,b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{7\,b}\right)$$
$$+x^4\left(\frac{c}{4\,b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{4\,b^2}+\frac{a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{2\,b}\right)$$
$$+\frac{f\,x^{13}}{13\,b^2}+\frac{x\left(\frac{f\,a^5}{3}-\frac{e\,a^4\,b}{3}+\frac{d\,a^3\,b^2}{3}-\frac{c\,a^2\,b^3}{3}\right)}{b^7\,x^3+a\,b^6}$$
$$+\frac{a^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-16\,f\,a^3+13\,e\,a^2\,b-10\,d\,a\,b^2+7\,c\,b^3\right)}{9\,b^{19/3}}$$
$$+\frac{a^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-16\,f\,a^3+13\,e\,a^2\,b-10\,d\,a\,b^2+7\,c\,b^3\right)}{9\,b^{19/3}}$$
$$-\frac{a^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-16\,f\,a^3+13\,e\,a^2\,b-10\,d\,a\,b^2+7\,c\,b^3\right)}{9\,b^{19/3}}$$

```
[In] int((x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^10*(e/(10*b^2) - (a*f)/(5*b^3)) - x*((2*a*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/
b^3))/b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b))
/b - (a^2*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b^2) - x^7
*((a^2*f)/(7*b^4) - d/(7*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(7*b)) + x^4*(c
/(4*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))/(4*b^2) + (a*((a^2*f)/b^4 - d/b^2 +
```

```
(2*a*(e/b^2 - (2*a*f)/b^3))/b))/(2*b)) + (f*x^13)/(13*b^2) + (x*((a^5*f)/3
- (a^2*b^3*c)/3 + (a^3*b^2*d)/3 - (a^4*b*e)/3))/(a*b^6 + b^7*x^3) + (a^(4/3
)*log(b^(1/3)*x + a^(1/3))*(7*b^3*c - 16*a^3*f - 10*a*b^2*d + 13*a^2*b*e))/
(9*b^(19/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3
^(1/2)*1i)/2 - 1/2)*(7*b^3*c - 16*a^3*f - 10*a*b^2*d + 13*a^2*b*e))/(9*b^(1
9/3)) - (a^(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*
1i)/2 + 1/2)*(7*b^3*c - 16*a^3*f - 10*a*b^2*d + 13*a^2*b*e))/(9*b^(19/3))
```

3.261 $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 335

$$\begin{aligned}\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx &= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^2}{2b^5}+\frac{(b^2d-2abe+3a^2f)\,x^5}{5b^4}\\&+\frac{(be-2af)x^8}{8b^3}+\frac{fx^{11}}{11b^2}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^5\,(a+bx^3)}\\&+\frac{a^{2/3}(5b^3c-8ab^2d+11a^2be-14a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{17/3}}\\&+\frac{a^{2/3}(5b^3c-8ab^2d+11a^2be-14a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{17/3}}\\&-\frac{a^{2/3}(5b^3c-8ab^2d+11a^2be-14a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{17/3}}\end{aligned}$$

```
[Out] 1/2*(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*x^2/b^5+1/5*(3*a^2*f-2*a*b*e+b^2*d
)*x^5/b^4+1/8*(-2*a*f+b*e)*x^8/b^3+1/11*f*x^11/b^2+1/3*a*(-a^3*f+a^2*b*e-a*
b^2*d+b^3*c)*x^2/b^5/(b*x^3+a)+1/9*a^(2/3)*(-14*a^3*f+11*a^2*b*e-8*a*b^2*d+
5*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(17/3)-1/18*a^(2/3)*(-14*a^3*f+11*a^2*b*e-
8*a*b^2*d+5*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(17/3)+1/9*a
^(2/3)*(-14*a^3*f+11*a^2*b*e-8*a*b^2*d+5*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))/b^(17/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1842, 1865, 1850, 1502, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$=\frac{x^5(3a^2f-2abe+b^2d)}{5b^4}+\frac{x^2(-4a^3f+3a^2be-2ab^2d+b^3c)}{2b^5}$$
$$+\frac{ax^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5(a+bx^3)}$$
$$+\frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-14a^3f+11a^2be-8ab^2d+5b^3c)}{3\sqrt{3}b^{17/3}}$$
$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-14a^3f+11a^2be-8ab^2d+5b^3c)}{18b^{17/3}}$$
$$+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-14a^3f+11a^2be-8ab^2d+5b^3c)}{9b^{17/3}}+\frac{x^8(be-2af)}{8b^3}+\frac{fx^{11}}{11b^2}$$

```
[In] Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2)/(2*b^5) + ((b^2*d - 2*a*b*e
 + 3*a^2*f)*x^5)/(5*b^4) + ((b*e - 2*a*f)*x^8)/(8*b^3) + (f*x^11)/(11*b^2)
+ (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*b^5*(a + b*x^3)) + (a^(2/3
)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(17/3)) + (a^(2/3)*(5*b^3*c - 8*a*b^2*
d + 11*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x]])/(9*b^(17/3)) - (a^(2/3
)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(18*b^(17/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
```

```
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1865

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rubi steps

$$\text{integral} = \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3b^5\,(a+bx^3)} - \frac{\int \frac{2a^2b(b^3c-ab^2d+a^2be-a^3f)x-3ab^2(b^3c-ab^2d+a^2be-a^3f)x^4-3ab^3(b^2d-abe+a^2f)x^7-3ab^4(be-af)x^{10}-3ab^5fx^{13}}{a+bx^3}\,dx}{3ab^6}$$

$$= \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3b^5\,(a+bx^3)} - \frac{\int \frac{x(2a^2b(b^3c-ab^2d+a^2be-a^3f)-3ab^2(b^3c-ab^2d+a^2be-a^3f)x^3-3ab^3(b^2d-abe+a^2f)x^6-3ab^4(be-af)x^9-3ab^5fx^{12})}{a+bx^3}\,dx}{3ab^6}$$

$$= \frac{fx^{11}}{11b^2} + \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3b^5\,(a+bx^3)} - \frac{\int \frac{x(22a^2b^2(b^3c-ab^2d+a^2be-a^3f)-33ab^3(b^3c-ab^2d+a^2be-a^3f)x^3-33ab^4(b^2d-abe+a^2f)x^6-33ab^5(be-2af)x^9)}{a+bx^3}\,dx}{33ab^7}$$

$$= \frac{(be-2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3b^5\,(a+bx^3)} - \frac{\int \frac{x(176a^2b^3(b^3c-ab^2d+a^2be-a^3f)-264ab^4(b^3c-ab^2d+a^2be-a^3f)x^3-264ab^5(b^2d-2abe+3a^2f)x^6)}{a+bx^3}\,dx}{264ab^8}$$

$$= \frac{(be-2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3b^5\,(a+bx^3)} - \frac{\int \left(-264ab^3(b^3c-2ab^2d+3a^2be-4a^3f)\,x - 264ab^4(b^2d-2abe+3a^2f)\,x^4 - \frac{88(-5a^2b^6c+8a^3b^5d-\cdots}{a+\cdots}\right.}{264ab^8}$$

$$= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^2}{2b^5} + \frac{(b^2d-2abe+3a^2f)\,x^5}{5b^4} + \frac{(be-2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3b^5\,(a+bx^3)} - \frac{(a(5b^3c-8ab^2d+11a^2be-14a^3f))\int \frac{x}{a+bx^3}\,dx}{3b^5}$$

$$= \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)\,x^2}{2b^5} + \frac{(b^2d - 2abe + 3a^2f)\,x^5}{5b^4} + \frac{(be - 2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3b^5\,(a + bx^3)} + \frac{\left(a^{2/3}(5b^3c - 8ab^2d + 11a^2be - 14a^3f)\right)\int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x}\,dx}{9b^{16/3}} - \frac{\left(a^{2/3}(5b^3c - 8ab^2d + 11a^2be - 14a^3f)\right)\int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{9b^{16/3}}$$

$$= \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)\,x^2}{2b^5} + \frac{(b^2d - 2abe + 3a^2f)\,x^5}{5b^4} + \frac{(be - 2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3b^5\,(a + bx^3)} + \frac{a^{2/3}(5b^3c - 8ab^2d + 11a^2be - 14a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{9b^{17/3}} - \frac{\left(a^{2/3}(5b^3c - 8ab^2d + 11a^2be - 14a^3f)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{18b^{17/3}} - \frac{\left(a(5b^3c - 8ab^2d + 11a^2be - 14a^3f)\right)\int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{6b^{16/3}}$$

$$= \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)\,x^2}{2b^5} + \frac{(b^2d - 2abe + 3a^2f)\,x^5}{5b^4} + \frac{(be - 2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3b^5\,(a + bx^3)} + \frac{a^{2/3}(5b^3c - 8ab^2d + 11a^2be - 14a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{9b^{17/3}} - \frac{a^{2/3}(5b^3c - 8ab^2d + 11a^2be - 14a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{18b^{17/3}} - \frac{\left(a^{2/3}(5b^3c - 8ab^2d + 11a^2be - 14a^3f)\right)\text{Subst}\left(\int \frac{1}{-3 - x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3b^{17/3}}$$

$$= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^2}{2b^5}+\frac{(b^2d-2abe+3a^2f)\,x^5}{5b^4}+\frac{(be-2af)x^8}{8b^3}+\frac{fx^{11}}{11b^2}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^5\,(a+bx^3)}+\frac{a^{2/3}(5b^3c-8ab^2d+11a^2be-14a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{17/3}}+\frac{a^{2/3}(5b^3c-8ab^2d+11a^2be-14a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{17/3}}-\frac{a^{2/3}(5b^3c-8ab^2d+11a^2be-14a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{17/3}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.95

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \frac{1980b^{2/3}(b^3c-2ab^2d+3a^2be-4a^3f)\,x^2+792b^{5/3}(b^2d-2abe+3a^2f)\,x^5+495b^{8/3}(be-2af)x^8+360b^{1}}{}$$

```
[In] Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (1980*b^(2/3)*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2 + 792*b^(5/3)*(
b^2*d - 2*a*b*e + 3*a^2*f)*x^5 + 495*b^(8/3)*(b*e - 2*a*f)*x^8 + 360*b^(11/
3)*f*x^11 + (1320*a*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a + b
*x^3) - 440*Sqrt[3]*a^(2/3)*(-5*b^3*c + 8*a*b^2*d - 11*a^2*b*e + 14*a^3*f)*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 440*a^(2/3)*(-5*b^3*c + 8*a*b
^2*d - 11*a^2*b*e + 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*a^(2/3)*(-5*b^
3*c + 8*a*b^2*d - 11*a^2*b*e + 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(3960*b^(17/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.61

method	result
risch	$\frac{f x^{11}}{11b^2} - \frac{x^8 f a}{4b^3} + \frac{x^8 e}{8b^2} + \frac{3x^5 f a^2}{5b^4} - \frac{2x^5 a e}{5b^3} + \frac{d x^5}{5b^2} - \frac{2x^2 f a^3}{b^5} + \frac{3x^2 a^2 e}{2b^4} - \frac{x^2 a d}{b^3} + \frac{x^2 c}{2b^2} + \frac{\left(-\frac{1}{3}a^4 f+\frac{1}{3}a^3 b e-\frac{1}{3}a^2 b^2 d+\frac{1}{3}a b}{b^5(b x^3+a)}$
default	$-\frac{-\frac{b^3 f x^{11}}{11}+\frac{\left(2 f a b^2-b^3 e\right)x^8}{8}+\frac{\left(-3 f a^2 b+2 a b^2 e-b^3 d\right)x^5}{5}+\frac{x^2\left(4 f a^3-3 a^2 b e+2 a b^2 d-b^3 c\right)}{2}}{b^5} + \frac{a\left(\frac{\left(-\frac{1}{3} f a^3+\frac{1}{3}a^2 b e-\frac{1}{3}a b^2 d+\frac{1}{3}b^3 c\right)x^2}{b x^3+a}+\left(\frac{14}{3}\right.\right.}{}$

[In] int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/11*f*x^11/b^2-1/4/b^3*x^8*f*a+1/8/b^2*x^8*e+3/5/b^4*x^5*f*a^2-2/5/b^3*x^5
*a*e+1/5/b^2*d*x^5-2/b^5*x^2*f*a^3+3/2/b^4*x^2*a^2*e-1/b^3*x^2*a*d+1/2/b^2*
x^2*c+(-1/3*a^4*f+1/3*a^3*b*e-1/3*a^2*b^2*d+1/3*a*b^3*c)*x^2/b^5/(b*x^3+a)+
1/9/b^6*a*sum((14*a^3*f-11*a^2*b*e+8*a*b^2*d-5*b^3*c)/_R*ln(x-_R),_R=RootOf
(_Z^3*b+a))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.36

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \frac{360\,b^4 f x^{14} + 45\,(11\,b^4 e - 14\,ab^3 f)x^{11} + 99\,(8\,b^4 d - 11\,ab^3 e + 14\,a^2 b^2 f)x^8 + 396\,(5\,b^4 c - 8\,ab^3 d + 11\,a^2 b^2 e -}{}$$

[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/3960*(360*b^4*f*x^14 + 45*(11*b^4*e - 14*a*b^3*f)*x^11 + 99*(8*b^4*d - 11
*a*b^3*e + 14*a^2*b^2*f)*x^8 + 396*(5*b^4*c - 8*a*b^3*d + 11*a^2*b^2*e - 14
*a^3*b*f)*x^5 + 660*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f)*x^2 -

```
 440*sqrt(3)*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f + (5*b^4*c -
8*a*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f)*x^3)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*
sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*(5*a*b^3*c - 8*a^2*b^2*d
 + 11*a^3*b*e - 14*a^4*f + (5*b^4*c - 8*a*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f
)*x^3)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1/
3)) - 440*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f + (5*b^4*c - 8*a
*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2
/b^2)^(2/3)))/(b^6*x^3 + a*b^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.97

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{(ab^3c-a^2b^2d+a^3be-a^4f)x^2}{3\,(b^6x^3+ab^5)} - \frac{\sqrt{3}(5\,ab^3c-8\,a^2b^2d+11\,a^3be-14\,a^4f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{40\,b^3fx^{11}+55\,(b^3e-2\,ab^2f)x^8+88\,(b^3d-2\,ab^2e+3\,a^2bf)x^5+220\,(b^3c-2\,ab^2d+3\,a^2be-4\,a^3f)x^2}{440\,b^5} - \frac{(5\,ab^3c-8\,a^2b^2d+11\,a^3be-14\,a^4f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(5\,ab^3c-8\,a^2b^2d+11\,a^3be-14\,a^4f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^2/(b^6*x^3 + a*b^5) - 1/9*sqr
t(3)*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f)*arctan(1/3*sqrt(3)*(
```

```
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(1/3)) + 1/440*(40*b^3*f*x^11 +
55*(b^3*e - 2*a*b^2*f)*x^8 + 88*(b^3*d - 2*a*b^2*e + 3*a^2*b*f)*x^5 + 220*(
b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2)/b^5 - 1/18*(5*a*b^3*c - 8*a^2
*b^2*d + 11*a^3*b*e - 14*a^4*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^6
*(a/b)^(1/3)) + 1/9*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f)*log(x
 + (a/b)^(1/3))/(b^6*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.30

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= \frac{\left(5\,ab^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 8\,a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 11\,a^3be\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 14\,a^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^5}$$
$$+ \frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{2}{3}}b^3c - 8\left(-ab^2\right)^{\frac{2}{3}}ab^2d + 11\left(-ab^2\right)^{\frac{2}{3}}a^2be - 14\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^7}$$
$$+ \frac{ab^3cx^2 - a^2b^2dx^2 + a^3bex^2 - a^4fx^2}{3\left(bx^3+a\right)b^5}$$
$$- \frac{\left(5\left(-ab^2\right)^{\frac{2}{3}}b^3c - 8\left(-ab^2\right)^{\frac{2}{3}}ab^2d + 11\left(-ab^2\right)^{\frac{2}{3}}a^2be - 14\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^7}$$
$$+ \frac{40\,b^{20}fx^{11} + 55\,b^{20}ex^8 - 110\,ab^{19}fx^8 + 88\,b^{20}dx^5 - 176\,ab^{19}ex^5 + 264\,a^2b^{18}fx^5 + 220\,b^{20}cx^2 - 440\,ab^{19}dx}{440\,b^{22}}$$

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(5*a*b^3*c*(-a/b)^(1/3) - 8*a^2*b^2*d*(-a/b)^(1/3) + 11*a^3*b*e*(-a/b)^
(1/3) - 14*a^4*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b
^5) + 1/9*sqrt(3)*(5*(-a*b^2)^(2/3)*b^3*c - 8*(-a*b^2)^(2/3)*a*b^2*d + 11*(
-a*b^2)^(2/3)*a^2*b*e - 14*(-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/b^7 + 1/3*(a*b^3*c*x^2 - a^2*b^2*d*x^2 + a^3*b*
e*x^2 - a^4*f*x^2)/((b*x^3 + a)*b^5) - 1/18*(5*(-a*b^2)^(2/3)*b^3*c - 8*(-a
*b^2)^(2/3)*a*b^2*d + 11*(-a*b^2)^(2/3)*a^2*b*e - 14*(-a*b^2)^(2/3)*a^3*f)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7 + 1/440*(40*b^20*f*x^11 + 55*b
^20*e*x^8 - 110*a*b^19*f*x^8 + 88*b^20*d*x^5 - 176*a*b^19*e*x^5 + 264*a^2*b
^18*f*x^5 + 220*b^20*c*x^2 - 440*a*b^19*d*x^2 + 660*a^2*b^18*e*x^2 - 880*a^
3*b^17*f*x^2)/b^22
```

Mupad [B] (verification not implemented)

Time = 10.37 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.08

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^8\left(\frac{e}{8\,b^2}-\frac{a\,f}{4\,b^3}\right)-x^5\left(\frac{a^2\,f}{5\,b^4}-\frac{d}{5\,b^2}+\frac{2\,a\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{5\,b}\right)$$
$$+x^2\left(\frac{c}{2\,b^2}-\frac{a^2\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{2\,b^2}+\frac{a\,\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)$$
$$+\frac{f\,x^{11}}{11\,b^2}-\frac{x^2\,\left(\frac{f\,a^4}{3}-\frac{e\,a^3\,b}{3}+\frac{d\,a^2\,b^2}{3}-\frac{c\,a\,b^3}{3}\right)}{b^6\,x^3+a\,b^5}$$
$$+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-14\,f\,a^3+11\,e\,a^2\,b-8\,d\,a\,b^2+5\,c\,b^3\right)}{9\,b^{17/3}}$$
$$-\frac{a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+11\,e\,a^2\,b-8\,d\,a\,b^2+5\,c\,b^3\right)}{9\,b^{17/3}}$$
$$+\frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+11\,e\,a^2\,b-8\,d\,a\,b^2+5\,c\,b^3\right)}{9\,b^{17/3}}$$

```
[In] int((x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^8*(e/(8*b^2) - (a*f)/(4*b^3)) - x^5*((a^2*f)/(5*b^4) - d/(5*b^2) + (2*a*(
e/b^2 - (2*a*f)/b^3))/(5*b)) + x^2*(c/(2*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))
/(2*b^2) + (a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b) + (
f*x^11)/(11*b^2) - (x^2*((a^4*f)/3 + (a^2*b^2*d)/3 - (a*b^3*c)/3 - (a^3*b*e
)/3))/(a*b^5 + b^6*x^3) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*b^3*c - 14*a
^3*f - 8*a*b^2*d + 11*a^2*b*e))/(9*b^(17/3)) - (a^(2/3)*log(3^(1/2)*a^(1/3)
*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*b^3*c - 14*a^3*f - 8
*a*b^2*d + 11*a^2*b*e))/(9*b^(17/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*
b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c - 14*a^3*f - 8*a*b^2*d
 + 11*a^2*b*e))/(9*b^(17/3))
```

3.262 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 328

$$\begin{aligned}
\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx
&= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x}{b^5}+\frac{(b^2d-2abe+3a^2f)\,x^4}{4b^4} \\
&+\frac{(be-2af)x^7}{7b^3}+\frac{fx^{10}}{10b^2}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^5\,(a+bx^3)} \\
&+\frac{\sqrt[3]{a}(4b^3c-7ab^2d+10a^2be-13a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{16/3}} \\
&-\frac{\sqrt[3]{a}(4b^3c-7ab^2d+10a^2be-13a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{16/3}} \\
&+\frac{\sqrt[3]{a}(4b^3c-7ab^2d+10a^2be-13a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{16/3}}
\end{aligned}$$

```
[Out] (-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*x/b^5+1/4*(3*a^2*f-2*a*b*e+b^2*d)*x^4/
b^4+1/7*(-2*a*f+b*e)*x^7/b^3+1/10*f*x^10/b^2+1/3*a*(-a^3*f+a^2*b*e-a*b^2*d+
b^3*c)*x/b^5/(b*x^3+a)-1/9*a^(1/3)*(-13*a^3*f+10*a^2*b*e-7*a*b^2*d+4*b^3*c)
*ln(a^(1/3)+b^(1/3)*x)/b^(16/3)+1/18*a^(1/3)*(-13*a^3*f+10*a^2*b*e-7*a*b^2*
d+4*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(16/3)+1/9*a^(1/3)*(
-13*a^3*f+10*a^2*b*e-7*a*b^2*d+4*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/b^(16/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1842, 1901, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$=\frac{x^4(3a^2f-2abe+b^2d)}{4b^4}+\frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-13a^3f+10a^2be-7ab^2d+4b^3c)}{3\sqrt{3}b^{16/3}}$$
$$-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-13a^3f+10a^2be-7ab^2d+4b^3c)}{9b^{16/3}}$$
$$+\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5(a+bx^3)}+\frac{x(-4a^3f+3a^2be-2ab^2d+b^3c)}{b^5}$$
$$+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-13a^3f+10a^2be-7ab^2d+4b^3c)}{18b^{16/3}}$$
$$+\frac{x^7(be-2af)}{7b^3}+\frac{fx^{10}}{10b^2}$$

```
[In] Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a*b*e + 3*a
^2*f)*x^4)/(4*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^10)/(10*b^2) + (a*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^5*(a + b*x^3)) + (a^(1/3)*(4*b^3
*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(3*Sqrt[3]*b^(16/3)) - (a^(1/3)*(4*b^3*c - 7*a*b^2*d + 10*a
^2*b*e - 13*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(16/3)) + (a^(1/3)*(4*b^3
*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(18*b^(16/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

integral

$$= \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x}{3b^5\,(a + bx^3)} - \frac{\int \frac{a^2(b^3c-ab^2d+a^2be-a^3f)-3ab(b^3c-ab^2d+a^2be-a^3f)x^3-3ab^2(b^2d-abe+a^2f)x^6-3ab^3(be-af)x^9-3ab^4fx^{12}}{a+bx^3}\,dx}{3ab^5}$$

$$=\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^5\,(a+bx^3)}$$

$$-\frac{\int\Big(-3a(b^3c-2ab^2d+3a^2be-4a^3f)-3ab(b^2d-2abe+3a^2f)\,x^3-3ab^2(be-2af)x^6-3ab^3fx^9+\frac{4a}{}}{3ab^5}$$

$$=\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x}{b^5}+\frac{(b^2d-2abe+3a^2f)\,x^4}{4b^4}+\frac{(be-2af)x^7}{7b^3}+\frac{fx^{10}}{10b^2}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^5\,(a+bx^3)}-\frac{\left(a(4b^3c-7ab^2d+10a^2be-13a^3f)\right)\int\frac{1}{a+bx^3}\,dx}{3b^5}$$

$$=\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x}{b^5}+\frac{(b^2d-2abe+3a^2f)\,x^4}{4b^4}+\frac{(be-2af)x^7}{7b^3}+\frac{fx^{10}}{10b^2}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^5\,(a+bx^3)}-\frac{\left(\sqrt[3]{a}(4b^3c-7ab^2d+10a^2be-13a^3f)\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9b^5}-\frac{\left(\sqrt[3]{a}(4b^3c-7ab^2d+10a^2be-13a^3f)\right)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9b^5}$$

$$=\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x}{b^5}+\frac{(b^2d-2abe+3a^2f)\,x^4}{4b^4}+\frac{(be-2af)x^7}{7b^3}+\frac{fx^{10}}{10b^2}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^5\,(a+bx^3)}-\frac{\sqrt[3]{a}(4b^3c-7ab^2d+10a^2be-13a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{16/3}}+\frac{\left(\sqrt[3]{a}(4b^3c-7ab^2d+10a^2be-13a^3f)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18b^{16/3}}-\frac{\left(a^{2/3}(4b^3c-7ab^2d+10a^2be-13a^3f)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^5}$$

$$=\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x}{b^5}+\frac{(b^2d-2abe+3a^2f)\,x^4}{4b^4}+\frac{(be-2af)x^7}{7b^3}+\frac{fx^{10}}{10b^2}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^5\,(a+bx^3)}-\frac{\sqrt[3]{a}(4b^3c-7ab^2d+10a^2be-13a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{16/3}}+\frac{\sqrt[3]{a}(4b^3c-7ab^2d+10a^2be-13a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{16/3}}-\frac{\left(\sqrt[3]{a}(4b^3c-7ab^2d+10a^2be-13a^3f)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3b^{16/3}}$$

$$= \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)\,x}{b^5} + \frac{(b^2d - 2abe + 3a^2f)\,x^4}{4b^4} + \frac{(be - 2af)x^7}{7b^3} + \frac{fx^{10}}{10b^2} + \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x}{3b^5\,(a + bx^3)} + \frac{\sqrt[3]{a}(4b^3c - 7ab^2d + 10a^2be - 13a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{16/3}} - \frac{\sqrt[3]{a}(4b^3c - 7ab^2d + 10a^2be - 13a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{9b^{16/3}} + \frac{\sqrt[3]{a}(4b^3c - 7ab^2d + 10a^2be - 13a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{18b^{16/3}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.96

$$\int \frac{x^6(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2}\,dx$$

$$= \frac{1260\sqrt[3]{b}(b^3c - 2ab^2d + 3a^2be - 4a^3f)\,x + 315b^{4/3}(b^2d - 2abe + 3a^2f)\,x^4 + 180b^{7/3}(be - 2af)x^7 + 126b^{10/3}\ldots}{\ldots}$$

```
[In] Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (1260*b^(1/3)*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x + 315*b^(4/3)*(b^
2*d - 2*a*b*e + 3*a^2*f)*x^4 + 180*b^(7/3)*(b*e - 2*a*f)*x^7 + 126*b^(10/3)
*f*x^10 + (420*a*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a + b*x^3)
 - 140*Sqrt[3]*a^(1/3)*(-4*b^3*c + 7*a*b^2*d - 10*a^2*b*e + 13*a^3*f)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-4*b^3*c + 7*a*b^2*d
- 10*a^2*b*e + 13*a^3*f)*Log[a^(1/3) + b^(1/3)*x] - 70*a^(1/3)*(-4*b^3*c +
7*a*b^2*d - 10*a^2*b*e + 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(1260*b^(16/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.59

method	result
risch	$\frac{f\,x^{10}}{10b^2}-\frac{2x^7fa}{7b^3}+\frac{x^7e}{7b^2}+\frac{3x^4fa^2}{4b^4}-\frac{x^4ae}{2b^3}+\frac{d\,x^4}{4b^2}-\frac{4xfa^3}{b^5}+\frac{3xa^2e}{b^4}-\frac{2xad}{b^3}+\frac{xc}{b^2}+\frac{\left(-\frac{1}{3}a^4f+\frac{1}{3}a^3be-\frac{1}{3}a^2b^2d+\frac{1}{3}ab^3\right.}{b^5(b\,x^3+a)}$
default	$-\frac{-\frac{1}{10}b^3fx^{10}+\frac{2}{7}x^7ab^2f-\frac{1}{7}x^7b^3e-\frac{3}{4}a^2bfx^4+\frac{1}{2}ab^2ex^4-\frac{1}{4}dx^4b^3+4fa^3x-3a^2bex+2ab^2dx-b^3cx}{b^5}+\frac{a\left(\frac{\left(-\frac{1}{3}fa^3+\frac{1}{3}a^2be-\frac{1}{3}ab^2d\right.}{b\,x^3+a}\right.}{}$

[In] int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/10*f*x^10/b^2-2/7/b^3*x^7*f*a+1/7/b^2*x^7*e+3/4/b^4*x^4*f*a^2-1/2/b^3*x^4 *a*e+1/4/b^2*d*x^4-4/b^5*x*f*a^3+3/b^4*x*a^2*e-2/b^3*x*a*d+1/b^2*x*c+(-1/3* a^4*f+1/3*a^3*b*e-1/3*a^2*b^2*d+1/3*a*b^3*c)*x/b^5/(b*x^3+a)+1/9/b^6*a*sum((13*a^3*f-10*a^2*b*e+7*a*b^2*d-4*b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.29

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$=\frac{126\,b^4fx^{13}+18\,(10\,b^4e-13\,ab^3f)x^{10}+45\,(7\,b^4d-10\,ab^3e+13\,a^2b^2f)x^7+315\,(4\,b^4c-7\,ab^3d+10\,a^2b^2e}{}$$

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(126*b^4*f*x^13 + 18*(10*b^4*e - 13*a*b^3*f)*x^10 + 45*(7*b^4*d - 10
*a*b^3*e + 13*a^2*b^2*f)*x^7 + 315*(4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13
*a^3*b*f)*x^4 - 140*sqrt(3)*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*
f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3*b*f)*x^3)*(a/b)^(1/3)*arct
an(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*(4*a*b^3*c - 7*a^2*b
^2*d + 10*a^3*b*e - 13*a^4*f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3
*b*f)*x^3)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*(4*a*b^
3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b
^2*e - 13*a^3*b*f)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*(4*a*b^3*c -
 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f)*x)/(b^6*x^3 + a*b^5)
```

Sympy [A] (verification not implemented)

Time = 76.44 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.37

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^7\left(-\frac{2af}{7b^3}+\frac{e}{7b^2}\right)+x^4\cdot\left(\frac{3a^2f}{4b^4}-\frac{ae}{2b^3}+\frac{d}{4b^2}\right)$$
$$+x\left(-\frac{4a^3f}{b^5}+\frac{3a^2e}{b^4}-\frac{2ad}{b^3}+\frac{c}{b^2}\right)+\frac{x(-a^4f+a^3be-a^2b^2d+ab^3c)}{3ab^5+3b^6x^3}$$
$$+\text{RootSum}\left(729t^3b^{16}-2197a^{10}f^3+5070a^9bef^2-3549a^8b^2df^2-3900a^8b^2e^2f+2028a^7b^3cf^2+5460a^7b^3\right.$$
$$+\frac{fx^{10}}{10b^2}$$

```
[In] integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**7*(-2*a*f/(7*b**3) + e/(7*b**2)) + x**4*(3*a**2*f/(4*b**4) - a*e/(2*b**3
) + d/(4*b**2)) + x*(-4*a**3*f/b**5 + 3*a**2*e/b**4 - 2*a*d/b**3 + c/b**2)
+ x*(-a**4*f + a**3*b*e - a**2*b**2*d + a*b**3*c)/(3*a*b**5 + 3*b**6*x**3)
+ RootSum(729*_t**3*b**16 - 2197*a**10*f**3 + 5070*a**9*b*e*f**2 - 3549*a**
8*b**2*d*f**2 - 3900*a**8*b**2*e**2*f + 2028*a**7*b**3*c*f**2 + 5460*a**7*b
**3*d*e*f + 1000*a**7*b**3*e**3 - 3120*a**6*b**4*c*e*f - 1911*a**6*b**4*d**
2*f - 2100*a**6*b**4*d*e**2 + 2184*a**5*b**5*c*d*f + 1200*a**5*b**5*c*e**2
+ 1470*a**5*b**5*d**2*e - 624*a**4*b**6*c**2*f - 1680*a**4*b**6*c*d*e - 343
*a**4*b**6*d**3 + 480*a**3*b**7*c**2*e + 588*a**3*b**7*c*d**2 - 336*a**2*b*
*8*c**2*d + 64*a*b**9*c**3, Lambda(_t, _t*log(9*_t*b**5/(13*a**3*f - 10*a**
2*b*e + 7*a*b**2*d - 4*b**3*c) + x))) + f*x**10/(10*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.98

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{(ab^3c-a^2b^2d+a^3be-a^4f)x}{3\,(b^6x^3+ab^5)} + \frac{14\,b^3fx^{10}+20\,(b^3e-2\,ab^2f)x^7+35\,(b^3d-2\,ab^2e+3\,a^2bf)x^4+140\,(b^3c-2\,ab^2d+3\,a^2be-4\,a^3f)x}{140\,b^5} - \frac{\sqrt{3}(4\,ab^3c-7\,a^2b^2d+10\,a^3be-13\,a^4f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(4\,ab^3c-7\,a^2b^2d+10\,a^3be-13\,a^4f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(4\,ab^3c-7\,a^2b^2d+10\,a^3be-13\,a^4f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x/(b^6*x^3 + a*b^5) + 1/140*(14
*b^3*f*x^10 + 20*(b^3*e - 2*a*b^2*f)*x^7 + 35*(b^3*d - 2*a*b^2*e + 3*a^2*b*
f)*x^4 + 140*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 - 1/9*sqrt(3)
*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f)*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(2/3)) + 1/18*(4*a*b^3*c - 7*a^2*b^2
*d + 10*a^3*b*e - 13*a^4*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^6*(a/
b)^(2/3)) - 1/9*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f)*log(x + (
a/b)^(1/3))/(b^6*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.18

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(4\,(-ab^2)^{\frac{1}{3}}\,b^3c-7\,(-ab^2)^{\frac{1}{3}}\,ab^2d+10\,(-ab^2)^{\frac{1}{3}}\,a^2be-13\,(-ab^2)^{\frac{1}{3}}\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^6} + \frac{(4\,ab^3c-7\,a^2b^2d+10\,a^3be-13\,a^4f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^5} - \frac{\left(4\,(-ab^2)^{\frac{1}{3}}\,b^3c-7\,(-ab^2)^{\frac{1}{3}}\,ab^2d+10\,(-ab^2)^{\frac{1}{3}}\,a^2be-13\,(-ab^2)^{\frac{1}{3}}\,a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^6} + \frac{ab^3cx-a^2b^2dx+a^3bex-a^4fx}{3\,(bx^3+a)b^5} + \frac{14\,b^{18}fx^{10}+20\,b^{18}ex^7-40\,ab^{17}fx^7+35\,b^{18}dx^4-70\,ab^{17}ex^4+105\,a^2b^{16}fx^4+140\,b^{18}cx-280\,ab^{17}dx+}{140\,b^{20}}$$

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(4*(-a*b^2)^(1/3)*b^3*c - 7*(-a*b^2)^(1/3)*a*b^2*d + 10*(-a*b^
2)^(1/3)*a^2*b*e - 13*(-a*b^2)^(1/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/b^6 + 1/9*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13
*a^4*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) - 1/18*(4*(-a*b^2)^
(1/3)*b^3*c - 7*(-a*b^2)^(1/3)*a*b^2*d + 10*(-a*b^2)^(1/3)*a^2*b*e - 13*(-a
*b^2)^(1/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^6 + 1/3*(a*b^
3*c*x - a^2*b^2*d*x + a^3*b*e*x - a^4*f*x)/((b*x^3 + a)*b^5) + 1/140*(14*b^
18*f*x^10 + 20*b^18*e*x^7 - 40*a*b^17*f*x^7 + 35*b^18*d*x^4 - 70*a*b^17*e*x
^4 + 105*a^2*b^16*f*x^4 + 140*b^18*c*x - 280*a*b^17*d*x + 420*a^2*b^16*e*x
- 560*a^3*b^15*f*x)/b^20
```

Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.09

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= x^7\left(\frac{e}{7\,b^2}-\frac{2\,a\,f}{7\,b^3}\right)+x\left(\frac{c}{b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)$$
$$-x^4\left(\frac{a^2\,f}{4\,b^4}-\frac{d}{4\,b^2}+\frac{a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{2\,b}\right)-\frac{x\left(\frac{f\,a^4}{3}-\frac{e\,a^3\,b}{3}+\frac{d\,a^2\,b^2}{3}-\frac{c\,a\,b^3}{3}\right)}{b^6\,x^3+a\,b^5}$$
$$+\frac{f\,x^{10}}{10\,b^2}-\frac{a^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-13\,f\,a^3+10\,e\,a^2\,b-7\,d\,a\,b^2+4\,c\,b^3\right)}{9\,b^{16/3}}$$
$$-\frac{a^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-13\,f\,a^3+10\,e\,a^2\,b-7\,d\,a\,b^2+4\,c\,b^3\right)}{9\,b^{16/3}}$$
$$+\frac{a^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-13\,f\,a^3+10\,e\,a^2\,b-7\,d\,a\,b^2+4\,c\,b^3\right)}{9\,b^{16/3}}$$

```
[In] int((x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^7*(e/(7*b^2) - (2*a*f)/(7*b^3)) + x*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/b^3))/
b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b) - x^4*
((a^2*f)/(4*b^4) - d/(4*b^2) + (a*(e/b^2 - (2*a*f)/b^3))/(2*b)) - (x*((a^4*
f)/3 + (a^2*b^2*d)/3 - (a*b^3*c)/3 - (a^3*b*e)/3))/(a*b^5 + b^6*x^3) + (f*x
^10)/(10*b^2) - (a^(1/3)*log(b^(1/3)*x + a^(1/3))*(4*b^3*c - 13*a^3*f - 7*a
*b^2*d + 10*a^2*b*e))/(9*b^(16/3)) - (a^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^
(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*b^3*c - 13*a^3*f - 7*a*b^2*d +
 10*a^2*b*e))/(9*b^(16/3)) + (a^(1/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x
+ a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(4*b^3*c - 13*a^3*f - 7*a*b^2*d + 10*a^2*
b*e))/(9*b^(16/3))
```

3.263 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 298

$$\begin{aligned}
&\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx \\
&= \frac{(b^2d-2abe+3a^2f)\,x^2}{2b^4}+\frac{(be-2af)x^5}{5b^3}+\frac{fx^8}{8b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)} \\
&-\frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{14/3}} \\
&-\frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{14/3}} \\
&+\frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{14/3}}
\end{aligned}$$

```
[Out] 1/2*(3*a^2*f-2*a*b*e+b^2*d)*x^2/b^4+1/5*(-2*a*f+b*e)*x^5/b^3+1/8*f*x^8/b^2-
1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/b^4/(b*x^3+a)-1/9*(-11*a^3*f+8*a^2*b
*e-5*a*b^2*d+2*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(14/3)+1/18*(-11*a^3*
f+8*a^2*b*e-5*a*b^2*d+2*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^
(1/3)/b^(14/3)-1/9*(-11*a^3*f+8*a^2*b*e-5*a*b^2*d+2*b^3*c)*arctan(1/3*(a^(1
/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(14/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1842, 1865, 1850, 1502, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$=\frac{x^2(3a^2f-2abe+b^2d)}{2b^4}-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-11a^3f+8a^2be-5ab^2d+2b^3c)}{3\sqrt{3}\sqrt[3]{a}b^{14/3}}$$
$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-11a^3f+8a^2be-5ab^2d+2b^3c)}{9\sqrt[3]{a}b^{14/3}}-\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^4(a+bx^3)}$$
$$+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-11a^3f+8a^2be-5ab^2d+2b^3c)}{18\sqrt[3]{a}b^{14/3}}+\frac{x^5(be-2af)}{5b^3}+\frac{fx^8}{8b^2}$$

```
[In] Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^2)/(2*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) + (
f*x^8)/(8*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*b^4*(a + b*x^
3)) - ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(14/3)) - ((2*b^3*c - 5*a*
b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3)*b^(14/3)
) + ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1865

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rubi steps

$$\text{integral} = -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)} - \frac{\int \frac{-2ab(b^3c-ab^2d+a^2be-a^3f)x-3ab^2(b^2d-abe+a^2f)x^4-3ab^3(be-af)x^7-3ab^4fx^{10}}{a+bx^3}\,dx}{3ab^5}$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)} - \frac{\int \frac{x(-2ab(b^3c-ab^2d+a^2be-a^3f)-3ab^2(b^2d-abe+a^2f)x^3-3ab^3(be-af)x^6-3ab^4fx^9)}{a+bx^3}\,dx}{3ab^5}$$

$$= \frac{fx^8}{8b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)} - \frac{\int \frac{x(-16ab^2(b^3c-ab^2d+a^2be-a^3f)-24ab^3(b^2d-abe+a^2f)x^3-24ab^4(be-2af)x^6)}{a+bx^3}\,dx}{24ab^6}$$

$$= \frac{fx^8}{8b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)} - \frac{\int \left(-24ab^2(b^2d-2abe+3a^2f)\,x-24ab^3(be-2af)x^4+\frac{8(-2ab^5c+5a^2b^4d-8a^3b^3e+11a^4b^2f)x}{a+bx^3}\right)dx}{24ab^6}$$

$$= \frac{(b^2d-2abe+3a^2f)\,x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)} + \frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\int \frac{x}{a+bx^3}\,dx}{3b^4}$$

$$= \frac{(b^2d-2abe+3a^2f)\,x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)} - \frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{13/3}} + \frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9\sqrt[3]{a}b^{13/3}}$$

$$= \frac{(b^2d-2abe+3a^2f)\,x^2}{2b^4}+\frac{(be-2af)x^5}{5b^3}+\frac{fx^8}{8b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)} - \frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{14/3}} + \frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18\sqrt[3]{a}b^{14/3}} + \frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{13/3}}$$

$$= \frac{(b^2d-2abe+3a^2f)\,x^2}{2b^4}+\frac{(be-2af)x^5}{5b^3}+\frac{fx^8}{8b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)} - \frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{14/3}} + \frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{14/3}} + \frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\,\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}b^{14/3}}$$

$$= \frac{(b^2d-2abe+3a^2f)\,x^2}{2b^4}+\frac{(be-2af)x^5}{5b^3}+\frac{fx^8}{8b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)} - \frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{14/3}} - \frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{14/3}} + \frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{14/3}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.95

$$\int\frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \frac{180b^{2/3}(b^2d-2abe+3a^2f)\,x^2+72b^{5/3}(be-2af)x^5+45b^{8/3}fx^8-\frac{120b^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{a+bx^3}+\frac{40\sqrt{3}(-2b^3c+}{}}{}$$

```
[In] Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (180*b^(2/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x^2 + 72*b^(5/3)*(b*e - 2*a*f)*x^5
 + 45*b^(8/3)*f*x^8 - (120*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)
/(a + b*x^3) + (40*Sqrt[3]*(-2*b^3*c + 5*a*b^2*d - 8*a^2*b*e + 11*a^3*f)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-2*b^3*c + 5*a*b^
2*d - 8*a^2*b*e + 11*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (20*(2*b^3*
c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/a^(1/3))/(360*b^(14/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.53

method	result
risch	$\frac{f x^8}{8b^2} - \frac{2x^5 a f}{5b^3} + \frac{x^5 e}{5b^2} + \frac{3a^2 f x^2}{2b^4} - \frac{a e x^2}{b^3} + \frac{d x^2}{2b^2} + \frac{\left(\frac{1}{3} f a^3 - \frac{1}{3} a^2 b e + \frac{1}{3} a b^2 d - \frac{1}{3} b^3 c\right) x^2}{b^4 (b x^3 + a)} + \frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{\left(-11 f a^3 + 8 \cdots\right.}{\cdots}}{9b^5 \cdots}$
default	$\frac{\frac{b^2 f x^8}{8} + \frac{\left(-2afb+b^2e\right)x^5}{5} + \frac{x^2\left(3a^2f-2aeb+b^2d\right)}{2}}{b^4} - \frac{\frac{\left(-\frac{1}{3} f a^3 + \frac{1}{3} a^2 b e - \frac{1}{3} a b^2 d + \frac{1}{3} b^3 c\right) x^2}{b x^3 + a} + \left(\frac{11}{3} f a^3 - \frac{8}{3} a^2 b e + \frac{5}{3} a b^2 d - \frac{2}{3} b^3 c\right) \left(-\frac{\ln\left(x + \left(\frac{a}{b}\right.\right.}{3b\left(\frac{a}{b}\right) \cdots} \cdots\right.}{b^4}$

```
[In] int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*f*x^8/b^2-2/5/b^3*x^5*a*f+1/5/b^2*x^5*e+3/2/b^4*a^2*f*x^2-1/b^3*a*e*x^2
+1/2*d*x^2/b^2+(1/3*f*a^3-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b^3*c)*x^2/b^4/(b*x^3
+a)+1/9/b^5*sum((-11*a^3*f+8*a^2*b*e-5*a*b^2*d+2*b^3*c)/_R*ln(x-_R),_R=Root
Of(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 920, normalized size of antiderivative = 3.09

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \left[\frac{45\,ab^5fx^{11}+9\,(8\,ab^5e-11\,a^2b^4f)x^8+36\,(5\,ab^5d-8\,a^2b^4e+11\,a^3b^3f)x^5-60\,(2\,ab^5c-5\,a^2b^4d+8\,a^3b^3e}{} \right.$$

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/360*(45*a*b^5*f*x^11 + 9*(8*a*b^5*e - 11*a^2*b^4*f)*x^8 + 36*(5*a*b^5*d
- 8*a^2*b^4*e + 11*a^3*b^3*f)*x^5 - 60*(2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3
*e - 11*a^4*b^2*f)*x^2 - 60*sqrt(1/3)*(2*a^2*b^4*c - 5*a^3*b^3*d + 8*a^4*b^
2*e - 11*a^5*b*f + (2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2*f)*x
^3)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a
*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)
*x)/(b*x^3 + a)) + 20*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f + (2*
b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^
2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^
3*b*e - 11*a^4*f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a
*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^7*x^3 + a^2*b^6), 1/360*(45*a*b^
5*f*x^11 + 9*(8*a*b^5*e - 11*a^2*b^4*f)*x^8 + 36*(5*a*b^5*d - 8*a^2*b^4*e +
 11*a^3*b^3*f)*x^5 - 60*(2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2
*f)*x^2 - 120*sqrt(1/3)*(2*a^2*b^4*c - 5*a^3*b^3*d + 8*a^4*b^2*e - 11*a^5*b
*f + (2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2*f)*x^3)*sqrt((a*b^
2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)
/b) + 20*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f + (2*b^4*c - 5*a*b
^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(
1/3)*b*x + (a*b^2)^(2/3)) - 40*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^
4*f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*l
og(b*x + (a*b^2)^(1/3)))/(a*b^7*x^3 + a^2*b^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.93

$$\begin{aligned}
&\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx \\
&= -\frac{(b^3c-ab^2d+a^2be-a^3f)x^2}{3\,(b^5x^3+ab^4)} \\
&\quad+\frac{\sqrt{3}(2\,b^3c-5\,ab^2d+8\,a^2be-11\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\
&\quad+\frac{5\,b^2fx^8+8\,(b^2e-2\,abf)x^5+20\,(b^2d-2\,abe+3\,a^2f)x^2}{40\,b^4} \\
&\quad+\frac{(2\,b^3c-5\,ab^2d+8\,a^2be-11\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\
&\quad-\frac{(2\,b^3c-5\,ab^2d+8\,a^2be-11\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}
\end{aligned}$$

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2/(b^5*x^3 + a*b^4) + 1/9*sqrt(3
)*(2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a
/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(1/3)) + 1/40*(5*b^2*f*x^8 + 8*(b^2*e -
2*a*b*f)*x^5 + 20*(b^2*d - 2*a*b*e + 3*a^2*f)*x^2)/b^4 + 1/18*(2*b^3*c - 5*
a*b^2*d + 8*a^2*b*e - 11*a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5
*(a/b)^(1/3)) - 1/9*(2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*log(x + (a
/b)^(1/3))/(b^5*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.13

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= \frac{\sqrt{3}(2\,b^3c - 5\,ab^2d + 8\,a^2be - 11\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{1}{3}}\,b^4} - \frac{(2\,b^3c - 5\,ab^2d + 8\,a^2be - 11\,a^3f)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{1}{3}}\,b^4} - \frac{\left(2\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 8\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 11\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4} - \frac{b^3cx^2 - ab^2dx^2 + a^2bex^2 - a^3fx^2}{3\,(bx^3+a)b^4} + \frac{5\,b^{14}fx^8 + 8\,b^{14}ex^5 - 16\,ab^{13}fx^5 + 20\,b^{14}dx^2 - 40\,ab^{13}ex^2 + 60\,a^2b^{12}fx^2}{40\,b^{16}}$$

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^4) - 1/18*(2*b^3*c -
5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
((-a*b^2)^(1/3)*b^4) - 1/9*(2*b^3*c*(-a/b)^(1/3) - 5*a*b^2*d*(-a/b)^(1/3) +
 8*a^2*b*e*(-a/b)^(1/3) - 11*a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/(a*b^4) - 1/3*(b^3*c*x^2 - a*b^2*d*x^2 + a^2*b*e*x^2 - a^3*f*
x^2)/((b*x^3 + a)*b^4) + 1/40*(5*b^14*f*x^8 + 8*b^14*e*x^5 - 16*a*b^13*f*x^
5 + 20*b^14*d*x^2 - 40*a*b^13*e*x^2 + 60*a^2*b^12*f*x^2)/b^16
```

Mupad [B] (verification not implemented)

Time = 10.32 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.96

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= x^5\left(\frac{e}{5\,b^2}-\frac{2\,a\,f}{5\,b^3}\right)-x^2\left(\frac{a^2\,f}{2\,b^4}-\frac{d}{2\,b^2}+\frac{a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)$$
$$+\frac{f\,x^8}{8\,b^2}-\frac{x^2\left(-\frac{f\,a^3}{3}+\frac{e\,a^2\,b}{3}-\frac{d\,a\,b^2}{3}+\frac{c\,b^3}{3}\right)}{b^5\,x^3+a\,b^4}$$
$$-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-11\,f\,a^3+8\,e\,a^2\,b-5\,d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{1/3}\,b^{14/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-11\,f\,a^3+8\,e\,a^2\,b-5\,d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{1/3}\,b^{14/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-11\,f\,a^3+8\,e\,a^2\,b-5\,d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{1/3}\,b^{14/3}}$$

```
[In] int((x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^5*(e/(5*b^2) - (2*a*f)/(5*b^3)) - x^2*((a^2*f)/(2*b^4) - d/(2*b^2) + (a*(
e/b^2 - (2*a*f)/b^3))/b) + (f*x^8)/(8*b^2) - (x^2*((b^3*c)/3 - (a^3*f)/3 -
(a*b^2*d)/3 + (a^2*b*e)/3))/(a*b^4 + b^5*x^3) - (log(b^(1/3)*x + a^(1/3))*(
2*b^3*c - 11*a^3*f - 5*a*b^2*d + 8*a^2*b*e))/(9*a^(1/3)*b^(14/3)) + (log(3^
(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*b^3*c -
 11*a^3*f - 5*a*b^2*d + 8*a^2*b*e))/(9*a^(1/3)*b^(14/3)) - (log(3^(1/2)*a^(
1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*b^3*c - 11*a^3*f
 - 5*a*b^2*d + 8*a^2*b*e))/(9*a^(1/3)*b^(14/3))
```

3.264 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 288

$$\begin{aligned}
&\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx \\
&= \frac{(b^2d-2abe+3a^2f)\,x}{b^4}+\frac{(be-2af)x^4}{4b^3}+\frac{fx^7}{7b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^4\,(a+bx^3)} \\
&\quad-\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{13/3}} \\
&\quad+\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{13/3}} \\
&\quad-\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{13/3}}
\end{aligned}$$

```
[Out] (3*a^2*f-2*a*b*e+b^2*d)*x/b^4+1/4*(-2*a*f+b*e)*x^4/b^3+1/7*f*x^7/b^2-1/3*(-
a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^4/(b*x^3+a)+1/9*(-10*a^3*f+7*a^2*b*e-4*a*b
^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(13/3)-1/18*(-10*a^3*f+7*a^2*b*
e-4*a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(13/
3)-1/9*(-10*a^3*f+7*a^2*b*e-4*a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(13/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1842, 1901, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= \frac{x(3a^2f-2abe+b^2d)}{b^4}-\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^4(a+bx^3)}$$
$$-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-10a^3f+7a^2be-4ab^2d+b^3c)}{3\sqrt{3}a^{2/3}b^{13/3}}$$
$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-10a^3f+7a^2be-4ab^2d+b^3c)}{18a^{2/3}b^{13/3}}$$
$$+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-10a^3f+7a^2be-4ab^2d+b^3c)}{9a^{2/3}b^{13/3}}+\frac{x^4(be-2af)}{4b^3}+\frac{fx^7}{7b^2}$$

```
[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^4)/(4*b^3) + (f*x^7)
/(7*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^4*(a + b*x^3)) - ((
b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(13/3)) + ((b^3*c - 4*a*b^2*d + 7*a^
2*b*e - 10*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(13/3)) - ((b^3*c
- 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(18*a^(2/3)*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^4\,(a+bx^3)} - \frac{\int \frac{-a(b^3c-ab^2d+a^2be-a^3f)-3ab(b^2d-abe+a^2f)x^3-3ab^2(be-af)x^6-3ab^3fx^9}{a+bx^3}\,dx}{3ab^4}\\
&= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^4\,(a+bx^3)} - \frac{\int \left(-3a(b^2d-2abe+3a^2f)-3ab(be-2af)x^3-3ab^2fx^6+\frac{-ab^3c+4a^2b^2d-7a^3be+10a^4f}{a+bx^3}\right)dx}{3ab^4}
\end{aligned}$$

$$=\frac{(b^2d-2abe+3a^2f)\,x}{b^4}+\frac{(be-2af)x^4}{4b^3}+\frac{fx^7}{7b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^4\,(a+bx^3)}+\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\int\frac{1}{a+bx^3}\,dx}{3b^4}$$

$$=\frac{(b^2d-2abe+3a^2f)\,x}{b^4}+\frac{(be-2af)x^4}{4b^3}+\frac{fx^7}{7b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^4\,(a+bx^3)}+\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{2/3}b^4}+\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{2/3}b^4}$$

$$=\frac{(b^2d-2abe+3a^2f)\,x}{b^4}+\frac{(be-2af)x^4}{4b^3}+\frac{fx^7}{7b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^4\,(a+bx^3)}+\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{13/3}}-\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{2/3}b^{13/3}}+\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}b^4}$$

$$=\frac{(b^2d-2abe+3a^2f)\,x}{b^4}+\frac{(be-2af)x^4}{4b^3}+\frac{fx^7}{7b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^4\,(a+bx^3)}+\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{13/3}}-\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{13/3}}+\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{2/3}b^{13/3}}$$

$$=\frac{(b^2d-2abe+3a^2f)\,x}{b^4}+\frac{(be-2af)x^4}{4b^3}+\frac{fx^7}{7b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^4\,(a+bx^3)}-\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{13/3}}+\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{13/3}}-\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{13/3}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.96

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \frac{252\sqrt[3]{b}(b^2d-2abe+3a^2f)\,x+63b^{4/3}(be-2af)x^4+36b^{7/3}fx^7-\frac{84\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)x}{a+bx^3}+\frac{28\sqrt{3}(-b^3c+4ab^2c}{}}{}$$

```
[In] Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (252*b^(1/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x + 63*b^(4/3)*(b*e - 2*a*f)*x^4 +
 36*b^(7/3)*f*x^7 - (84*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a +
 b*x^3) + (28*Sqrt[3]*(-(b^3*c) + 4*a*b^2*d - 7*a^2*b*e + 10*a^3*f)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(b^3*c - 4*a*b^2*d + 7*
a^2*b*e - 10*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(-(b^3*c) + 4*a
*b^2*d - 7*a^2*b*e + 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/a^(2/3))/(252*b^(13/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.51

method	result
risch	$\frac{f\,x^7}{7b^2}-\frac{x^4af}{2b^3}+\frac{x^4e}{4b^2}+\frac{3a^2fx}{b^4}-\frac{2aex}{b^3}+\frac{dx}{b^2}+\frac{\left(\frac{1}{3}f\,a^3-\frac{1}{3}a^2be+\frac{1}{3}a\,b^2d-\frac{1}{3}b^3c\right)x}{b^4(b\,x^3+a)}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-10f\,a^3+7a^2be-4a\right.}{-}}{9b^5}$
default	$\frac{\frac{1}{7}b^2f\,x^7-\frac{1}{2}abf\,x^4+\frac{1}{4}b^2e\,x^4+3a^2fx-2abex+b^2dx}{b^4}-\frac{\frac{\left(-\frac{1}{3}f\,a^3+\frac{1}{3}a^2be-\frac{1}{3}a\,b^2d+\frac{1}{3}b^3c\right)x}{b\,x^3+a}+\left(10f\,a^3-7a^2be+4a\,b^2d-b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\right.}{b^4}$

```
[In] int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*f*x^7/b^2-1/2/b^3*x^4*a*f+1/4/b^2*x^4*e+3/b^4*a^2*f*x-2/b^3*a*e*x+d*x/b
^2+(1/3*f*a^3-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b^3*c)*x/b^4/(b*x^3+a)+1/9/b^5*su
m((-10*a^3*f+7*a^2*b*e-4*a*b^2*d+b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 946, normalized size of antiderivative = 3.28

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \left[\frac{36\,a^2b^4fx^{10}+9\,(7\,a^2b^4e-10\,a^3b^3f)x^7+63\,(4\,a^2b^4d-7\,a^3b^3e+10\,a^4b^2f)x^4-42\,\sqrt{\tfrac{1}{3}}(a^2b^4c-4\,a^3b^3d+\cdots}{\cdots}\right.$$

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/252*(36*a^2*b^4*f*x^10 + 9*(7*a^2*b^4*e - 10*a^3*b^3*f)*x^7 + 63*(4*a^2*
b^4*d - 7*a^3*b^3*e + 10*a^4*b^2*f)*x^4 - 42*sqrt(1/3)*(a^2*b^4*c - 4*a^3*b
^3*d + 7*a^4*b^2*e - 10*a^5*b*f + (a*b^5*c - 4*a^2*b^4*d + 7*a^3*b^3*e - 10
*a^4*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a
*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sq
rt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(a*b^3*c - 4*a^2*b^2*d + 7*a^3*b*e
- 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3)*(-a^2*b)^(
2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(a*b^3*c - 4*a
^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3
*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(a^2*b^4*c - 4*a
^3*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f)*x)/(a^2*b^6*x^3 + a^3*b^5), 1/252*(36*
a^2*b^4*f*x^10 + 9*(7*a^2*b^4*e - 10*a^3*b^3*f)*x^7 + 63*(4*a^2*b^4*d - 7*a
^3*b^3*e + 10*a^4*b^2*f)*x^4 + 84*sqrt(1/3)*(a^2*b^4*c - 4*a^3*b^3*d + 7*a^
4*b^2*e - 10*a^5*b*f + (a*b^5*c - 4*a^2*b^4*d + 7*a^3*b^3*e - 10*a^4*b^2*f)
*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*
b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 14*(a*b^3*c - 4*a^2*b^2*d + 7*a^
3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3)*(-a^
2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(a*b^3*c
 - 4*a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e -
10*a^3*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(a^2*b^4*c
 - 4*a^3*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f)*x)/(a^2*b^6*x^3 + a^3*b^5)]
```

Sympy [A] (verification not implemented)

Time = 67.31 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.39

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= x^4\left(-\frac{af}{2b^3}+\frac{e}{4b^2}\right)+x\left(\frac{3a^2f}{b^4}-\frac{2ae}{b^3}+\frac{d}{b^2}\right)+\frac{x(a^3f-a^2be+ab^2d-b^3c)}{3ab^4+3b^5x^3}$$
$$+\mathrm{RootSum}\left(729t^3a^2b^{13}+1000a^9f^3-2100a^8bef^2+1200a^7b^2df^2+1470a^7b^2e^2f-300a^6b^3cf^2-1680a^6b^3\right.$$
$$+\frac{fx^7}{7b^2}$$

```
[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**4*(-a*f/(2*b**3) + e/(4*b**2)) + x*(3*a**2*f/b**4 - 2*a*e/b**3 + d/b**2)
 + x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a*b**4 + 3*b**5*x**3) + Roo
tSum(729*_t**3*a**2*b**13 + 1000*a**9*f**3 - 2100*a**8*b*e*f**2 + 1200*a**7
*b**2*d*f**2 + 1470*a**7*b**2*e**2*f - 300*a**6*b**3*c*f**2 - 1680*a**6*b**
3*d*e*f - 343*a**6*b**3*e**3 + 420*a**5*b**4*c*e*f + 480*a**5*b**4*d**2*f +
 588*a**5*b**4*d*e**2 - 240*a**4*b**5*c*d*f - 147*a**4*b**5*c*e**2 - 336*a*
*4*b**5*d**2*e + 30*a**3*b**6*c**2*f + 168*a**3*b**6*c*d*e + 64*a**3*b**6*d
**3 - 21*a**2*b**7*c**2*e - 48*a**2*b**7*c*d**2 + 12*a*b**8*c**2*d - b**9*c
**3, Lambda(_t, _t*log(-9*_t*a*b**4/(10*a**3*f - 7*a**2*b*e + 4*a*b**2*d -
b**3*c) + x))) + f*x**7/(7*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.94

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)x}{3\,(b^5x^3+ab^4)}+\frac{4\,b^2fx^7+7\,(b^2e-2\,abf)x^4+28\,(b^2d-2\,abe+3\,a^2f)x}{28\,b^4}$$
$$+\frac{\sqrt{3}(b^3c-4\,ab^2d+7\,a^2be-10\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(b^3c-4\,ab^2d+7\,a^2be-10\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(b^3c-4\,ab^2d+7\,a^2be-10\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x/(b^5*x^3 + a*b^4) + 1/28*(4*b^2*
f*x^7 + 7*(b^2*e - 2*a*b*f)*x^4 + 28*(b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + 1
/9*sqrt(3)*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2/3)) - 1/18*(b^3*c - 4*a*b^2*d
+ 7*a^2*b*e - 10*a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(
2/3)) + 1/9*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*log(x + (a/b)^(1/3))
/(b^5*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= -\frac{\sqrt{3}(b^3c-4\,ab^2d+7\,a^2be-10\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}\,b^3} - \frac{(b^3c-4\,ab^2d+7\,a^2be-10\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}\,b^3} - \frac{(b^3c-4\,ab^2d+7\,a^2be-10\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4} - \frac{b^3cx-ab^2dx+a^2bex-a^3fx}{3\,(bx^3+a)b^4} + \frac{4\,b^{12}fx^7+7\,b^{12}ex^4-14\,ab^{11}fx^4+28\,b^{12}dx-56\,ab^{11}ex+84\,a^2b^{10}fx}{28\,b^{14}}$$

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^3) - 1/18*(b^3*c - 4*a
*b^2*d + 7*a^2*b*e - 10*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-
a*b^2)^(2/3)*b^3) - 1/9*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*(-a/b)^(
1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 1/3*(b^3*c*x - a*b^2*d*x + a^2*b*
e*x - a^3*f*x)/((b*x^3 + a)*b^4) + 1/28*(4*b^12*f*x^7 + 7*b^12*e*x^4 - 14*a
*b^11*f*x^4 + 28*b^12*d*x - 56*a*b^11*e*x + 84*a^2*b^10*f*x)/b^14
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.97

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= x^4\left(\frac{e}{4\,b^2}-\frac{a\,f}{2\,b^3}\right)-x\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)-\frac{x\,\left(-\frac{f\,a^3}{3}+\frac{e\,a^2\,b}{3}-\frac{d\,a\,b^2}{3}+\frac{c\,b^3}{3}\right)}{b^5\,x^3+a\,b^4}$$
$$+\frac{f\,x^7}{7\,b^2}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-10\,f\,a^3+7\,e\,a^2\,b-4\,d\,a\,b^2+c\,b^3\right)}{9\,a^{2/3}\,b^{13/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-10\,f\,a^3+7\,e\,a^2\,b-4\,d\,a\,b^2+c\,b^3\right)}{9\,a^{2/3}\,b^{13/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-10\,f\,a^3+7\,e\,a^2\,b-4\,d\,a\,b^2+c\,b^3\right)}{9\,a^{2/3}\,b^{13/3}}$$

```
[In] int((x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^4*(e/(4*b^2) - (a*f)/(2*b^3)) - x*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2
*a*f)/b^3))/b) - (x*((b^3*c)/3 - (a^3*f)/3 - (a*b^2*d)/3 + (a^2*b*e)/3))/(a
*b^4 + b^5*x^3) + (f*x^7)/(7*b^2) + (log(b^(1/3)*x + a^(1/3))*(b^3*c - 10*a
^3*f - 4*a*b^2*d + 7*a^2*b*e))/(9*a^(2/3)*b^(13/3)) + (log(3^(1/2)*a^(1/3)*
1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - 10*a^3*f - 4*a*
b^2*d + 7*a^2*b*e))/(9*a^(2/3)*b^(13/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1
/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - 10*a^3*f - 4*a*b^2*d + 7*a
^2*b*e))/(9*a^(2/3)*b^(13/3))
```

3.265 $\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 271

$$\begin{aligned}\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx &= \frac{(be-2af)x^2}{2b^3}+\frac{fx^5}{5b^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3ab^3\,(a+bx^3)}\\&\quad-\frac{(b^3c+2ab^2d-5a^2be+8a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{11/3}}\\&\quad-\frac{(b^3c+2ab^2d-5a^2be+8a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{11/3}}\\&\quad+\frac{(b^3c+2ab^2d-5a^2be+8a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{11/3}}\end{aligned}$$

```
[Out] 1/2*(-2*a*f+b*e)*x^2/b^3+1/5*f*x^5/b^2+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x
^2/a/b^3/(b*x^3+a)-1/9*(8*a^3*f-5*a^2*b*e+2*a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/
3)*x)/a^(4/3)/b^(11/3)+1/18*(8*a^3*f-5*a^2*b*e+2*a*b^2*d+b^3*c)*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(11/3)-1/9*(8*a^3*f-5*a^2*b*e+2*a*
b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(1
1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1842, 1608, 1502, 298, 31, 648, 631, 210, 642}

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$=\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3ab^3(a+bx^3)}-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(8a^3f-5a^2be+2ab^2d+b^3c)}{3\sqrt{3}a^{4/3}b^{11/3}}$$
$$+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(8a^3f-5a^2be+2ab^2d+b^3c)}{18a^{4/3}b^{11/3}}$$
$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(8a^3f-5a^2be+2ab^2d+b^3c)}{9a^{4/3}b^{11/3}}+\frac{x^2(be-2af)}{2b^3}+\frac{fx^5}{5b^2}$$

```
[In] Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x^2)/(2*b^3) + (f*x^5)/(5*b^2) + ((b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*x^2)/(3*a*b^3*(a + b*x^3)) - ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3
)*b^(11/3)) - ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(1/3) + b^(1
/3)*x])/(9*a^(4/3)*b^(11/3)) + ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3ab^3\,(a + bx^3)} - \frac{\int \frac{-b(b^3c+2ab^2d-2a^2be+2a^3f)x-3ab^2(be-af)x^4-3ab^3fx^7}{a+bx^3}\,dx}{3ab^4}$$

$$= \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3ab^3\,(a+bx^3)} - \frac{\int \frac{x\left(-b\left(b^3c+2ab^2d-2a^2be+2a^3f\right)-3ab^2(be-af)x^3-3ab^3fx^6\right)}{a+bx^3}\,dx}{3ab^4}$$

$$= \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3ab^3\,(a+bx^3)} - \frac{\int \left(-3ab(be-2af)x - 3ab^2fx^4 + \frac{(-b^4c-2ab^3d+5a^2b^2e-8a^3bf)x}{a+bx^3}\right)\,dx}{3ab^4}$$

$$= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3ab^3\,(a+bx^3)} + \frac{(b^3c+2ab^2d-5a^2be+8a^3f)\int \frac{x}{a+bx^3}\,dx}{3ab^3}$$

$$= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3ab^3\,(a+bx^3)} - \frac{(b^3c+2ab^2d-5a^2be+8a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}b^{10/3}} + \frac{(b^3c+2ab^2d-5a^2be+8a^3f)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{4/3}b^{10/3}}$$

$$= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3ab^3\,(a+bx^3)} - \frac{(b^3c+2ab^2d-5a^2be+8a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{11/3}} + \frac{(b^3c+2ab^2d-5a^2be+8a^3f)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{4/3}b^{11/3}} + \frac{(b^3c+2ab^2d-5a^2be+8a^3f)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6ab^{10/3}}$$

$$= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3ab^3\,(a+bx^3)} - \frac{(b^3c+2ab^2d-5a^2be+8a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{11/3}} + \frac{(b^3c+2ab^2d-5a^2be+8a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{11/3}} + \frac{(b^3c+2ab^2d-5a^2be+8a^3f)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{4/3}b^{11/3}}$$

$$= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3ab^3\,(a+bx^3)} - \frac{(b^3c+2ab^2d-5a^2be+8a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{11/3}} - \frac{(b^3c+2ab^2d-5a^2be+8a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{11/3}} + \frac{(b^3c+2ab^2d-5a^2be+8a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{11/3}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.94

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \frac{45b^{2/3}(be-2af)x^2+18b^{5/3}fx^5+\frac{30b^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{a(a+bx^3)}-\frac{10\sqrt{3}(b^3c+2ab^2d-5a^2be+8a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}-}{90b^{11/3}}$$

```
[In] Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (45*b^(2/3)*(b*e - 2*a*f)*x^2 + 18*b^(5/3)*f*x^5 + (30*b^(2/3)*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^2)/(a*(a + b*x^3)) - (10*Sqrt[3]*(b^3*c + 2*a*b^2
*d - 5*a^2*b*e + 8*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(4
/3) - (10*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(1/3) + b^(1/3)*x
])/a^(4/3) + (5*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(90*b^(11/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.48

method	result
risch	$\frac{f x^5}{5b^2}-\frac{x^2 a f}{b^3}+\frac{e x^2}{2b^2}-\frac{(f a^3-a^2be+a b^2d-b^3c)x^2}{3a b^3(b x^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(8f a^3-5a^2be+2a b^2d+b^3c)\ln(x-\_R)}{\_R}}{9b^4a}$
default	$-\frac{-\frac{bf x^5}{5}+\frac{(2af-be)x^2}{2}}{b^3}+\frac{-\frac{(f a^3-a^2be+a b^2d-b^3c)x^2}{3a(b x^3+a)}+\frac{(8f a^3-5a^2be+2a b^2d+b^3c)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\mathrm{arc}}{}\right)}{3a}}{b^3}$

```
[In] int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*f*x^5/b^2-1/b^3*x^2*a*f+1/2/b^2*e*x^2-1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)
/a*x^2/b^3/(b*x^3+a)+1/9/b^4/a*sum((8*a^3*f-5*a^2*b*e+2*a*b^2*d+b^3*c)/_R*l
n(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 874, normalized size of antiderivative = 3.23

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$=\left[\frac{18\,a^2b^4fx^8+9\,(5\,a^2b^4e-8\,a^3b^3f)x^5+15\,(2\,ab^5c-2\,a^2b^4d+5\,a^3b^3e-8\,a^4b^2f)x^2+15\,\sqrt{\frac{1}{3}}(a^2b^4c+2\,a^3b}{}\right.$$

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/90*(18*a^2*b^4*f*x^8 + 9*(5*a^2*b^4*e - 8*a^3*b^3*f)*x^5 + 15*(2*a*b^5*c
 - 2*a^2*b^4*d + 5*a^3*b^3*e - 8*a^4*b^2*f)*x^2 + 15*sqrt(1/3)*(a^2*b^4*c +
 2*a^3*b^3*d - 5*a^4*b^2*e + 8*a^5*b*f + (a*b^5*c + 2*a^2*b^4*d - 5*a^3*b^3
*e + 8*a^4*b^2*f)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt
(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)
```

```
/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 5*(a*b^3*c + 2*a^2*b^2*d - 5*a^3*b
*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*b*f)*x^3)*(-a*b^2)^
(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*(a*b^3*c + 2*
a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*
b*f)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^3 + a^3*b^5)
, 1/90*(18*a^2*b^4*f*x^8 + 9*(5*a^2*b^4*e - 8*a^3*b^3*f)*x^5 + 15*(2*a*b^5*
c - 2*a^2*b^4*d + 5*a^3*b^3*e - 8*a^4*b^2*f)*x^2 + 30*sqrt(1/3)*(a^2*b^4*c
+ 2*a^3*b^3*d - 5*a^4*b^2*e + 8*a^5*b*f + (a*b^5*c + 2*a^2*b^4*d - 5*a^3*b^
3*e + 8*a^4*b^2*f)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (
-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 5*(a*b^3*c + 2*a^2*b^2*d - 5*a^
3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*b*f)*x^3)*(-a*b^
2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*(a*b^3*c +
 2*a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a
^3*b*f)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^3 + a^3*b
^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.96

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)x^2}{3\,(ab^4x^3+a^2b^3)} + \frac{2\,bfx^5+5\,(be-2\,af)x^2}{10\,b^3} + \frac{\sqrt{3}(b^3c+2\,ab^2d-5\,a^2be+8\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(b^3c+2\,ab^2d-5\,a^2be+8\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(b^3c+2\,ab^2d-5\,a^2be+8\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2/(a*b^4*x^3 + a^2*b^3) + 1/10*(2
*b*f*x^5 + 5*(b*e - 2*a*f)*x^2)/b^3 + 1/9*sqrt(3)*(b^3*c + 2*a*b^2*d - 5*a^
2*b*e + 8*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^4
*(a/b)^(1/3)) + 1/18*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*log(x^2 - x*
(a/b)^(1/3) + (a/b)^(2/3))/(a*b^4*(a/b)^(1/3)) - 1/9*(b^3*c + 2*a*b^2*d - 5
*a^2*b*e + 8*a^3*f)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.15

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{\sqrt{3}(b^3c+2\,ab^2d-5\,a^2be+8\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}ab^3} - \frac{(b^3c+2\,ab^2d-5\,a^2be+8\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}ab^3} - \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+8\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^3} + \frac{b^3cx^2-ab^2dx^2+a^2bex^2-a^3fx^2}{3\,(bx^3+a)ab^3} + \frac{2\,b^8fx^5+5\,b^8ex^2-10\,ab^7fx^2}{10\,b^{10}}$$

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a*b^3) - 1/18*(b^3*c + 2*a
*b^2*d - 5*a^2*b*e + 8*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a
*b^2)^(1/3)*a*b^3) - 1/9*(b^3*c*(-a/b)^(1/3) + 2*a*b^2*d*(-a/b)^(1/3) - 5*a
^2*b*e*(-a/b)^(1/3) + 8*a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b^3) + 1/3*(b^3*c*x^2 - a*b^2*d*x^2 + a^2*b*e*x^2 - a^3*f*x^2
)/((b*x^3 + a)*a*b^3) + 1/10*(2*b^8*f*x^5 + 5*b^8*e*x^2 - 10*a*b^7*f*x^2)/b
^10
```

Mupad [B] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.91

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= x^2\left(\frac{e}{2\,b^2}-\frac{a\,f}{b^3}\right)+\frac{f\,x^5}{5\,b^2}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(8\,f\,a^3-5\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{9\,a^{4/3}\,b^{11/3}}$$
$$+\frac{x^2\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a\,\left(b^4\,x^3+a\,b^3\right)}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(8\,f\,a^3-5\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{9\,a^{4/3}\,b^{11/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(8\,f\,a^3-5\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{9\,a^{4/3}\,b^{11/3}}$$

```
[In] int((x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^2*(e/(2*b^2) - (a*f)/b^3) + (f*x^5)/(5*b^2) - (log(b^(1/3)*x + a^(1/3))*(
b^3*c + 8*a^3*f + 2*a*b^2*d - 5*a^2*b*e))/(9*a^(4/3)*b^(11/3)) + (x^2*(b^3*
c - a^3*f - a*b^2*d + a^2*b*e))/(3*a*(a*b^3 + b^4*x^3)) + (log(3^(1/2)*a^(1
/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c + 8*a^3*f + 2
*a*b^2*d - 5*a^2*b*e))/(9*a^(4/3)*b^(11/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b
^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c + 8*a^3*f + 2*a*b^2*d - 5
*a^2*b*e))/(9*a^(4/3)*b^(11/3))
```

3.266 $\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 264

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx = \frac{(be-2af)x}{b^3}+\frac{fx^4}{4b^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3ab^3\,(a+bx^3)} - \frac{(2b^3c+ab^2d-4a^2be+7a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}} + \frac{(2b^3c+ab^2d-4a^2be+7a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}} - \frac{(2b^3c+ab^2d-4a^2be+7a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}$$

```
[Out] (-2*a*f+b*e)*x/b^3+1/4*f*x^4/b^2+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a/b^3
/(b*x^3+a)+1/9*(7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^
(5/3)/b^(10/3)-1/18*(7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^3*c)*ln(a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(10/3)-1/9*(7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^
3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(10/3)*3^(
1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1872, 1425, 396, 206, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx = \frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{3ab^3(a+bx^3)} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(7a^3f-4a^2be+ab^2d+2b^3c)}{3\sqrt{3}a^{5/3}b^{10/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(7a^3f-4a^2be+ab^2d+2b^3c)}{18a^{5/3}b^{10/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(7a^3f-4a^2be+ab^2d+2b^3c)}{9a^{5/3}b^{10/3}} + \frac{x(be-2af)}{b^3} + \frac{fx^4}{4b^2}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^4)/(4*b^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*x)/(3*a*b^3*(a + b*x^3)) - ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(10/
3)) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
(9*a^(5/3)*b^(10/3)) - ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1425

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{3ab^3\,(a+bx^3)} - \frac{\int \frac{-2b^3c - ab^2d + a^2be - a^3f - 3ab(be-af)x^3 - 3ab^2fx^6}{a+bx^3}\,dx}{3ab^3}$$

$$= \frac{fx^4}{4b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{3ab^3\,(a+bx^3)} - \frac{\int \frac{4b(-2b^3c - ab^2d + a^2be - a^3f) - (-12a^2b^2f + 12ab^2(be-af))x^3}{a+bx^3}\,dx}{12ab^4}$$

$$= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{3ab^3\,(a+bx^3)} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f)\int \frac{1}{a+bx^3}\,dx}{3ab^3}$$

$$= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{3ab^3\,(a+bx^3)} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b^3} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b^3}$$

$$= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{3ab^3\,(a+bx^3)} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}} - \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{10/3}} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}b^3}$$

$$= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{3ab^3\,(a+bx^3)} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}} - \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}b^{10/3}}$$

$$= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3ab^3\,(a+bx^3)} - \frac{(2b^3c+ab^2d-4a^2be+7a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}} + \frac{(2b^3c+ab^2d-4a^2be+7a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}} - \frac{(2b^3c+ab^2d-4a^2be+7a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx$$

$$= \frac{36\sqrt[3]{b}(be-2af)x + 9b^{4/3}fx^4 + \frac{12\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)x}{a(a+bx^3)} - \frac{4\sqrt{3}(2b^3c+ab^2d-4a^2be+7a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{4(2b^3c+\ldots}{\ldots}}{36b^{10/3}}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(1/3)*(b*e - 2*a*f)*x + 9*b^(4/3)*f*x^4 + (12*b^(1/3)*(b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*x)/(a*(a + b*x^3)) - (4*Sqrt[3]*(2*b^3*c + a*b^2*d - 4
*a^2*b*e + 7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) +
(4*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(5
/3) - (2*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(36*b^(10/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.47

method	result
risch	$\frac{f\,x^{4}}{4b^{2}}-\frac{2xaf}{b^{3}}+\frac{ex}{b^{2}}-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)x}{3a\,b^{3}\left(b\,x^{3}+a\right)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(7f\,a^{3}-4a^{2}be+a\,b^{2}d+2b^{3}c\right)\ln\left(x-\_R\right)}{\_R^{2}}}{9b^{4}a}$
default	$-\frac{-\frac{1}{4}bf\,x^{4}+2afx-bex}{b^{3}}+\frac{-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)x}{3a\left(b\,x^{3}+a\right)}+\frac{\left(7f\,a^{3}-4a^{2}be+a\,b^{2}d+2b^{3}c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\mathrm{arct}}{}\right.}{3a}}{b^{3}}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*f*x^4/b^2-2/b^3*x*a*f+1/b^2*e*x-1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a*x/b
^3/(b*x^3+a)+1/9/b^4/a*sum((7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^3*c)/_R^2*ln(x-_R
),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 861, normalized size of antiderivative = 3.26

$$\int \frac{c+dx^3+ex^6+fx^9}{\left(a+bx^3\right)^2}\,dx$$

$$= \left[\frac{9\,a^3b^3fx^7+9\,(4\,a^3b^3e-7\,a^4b^2f)x^4+6\,\sqrt{\frac{1}{3}}(2\,a^2b^4c+a^3b^3d-4\,a^4b^2e+7\,a^5bf+(2\,ab^5c+a^2b^4d-4\,a^3b}{}\right.$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/36*(9*a^3*b^3*f*x^7 + 9*(4*a^3*b^3*e - 7*a^4*b^2*f)*x^4 + 6*sqrt(1/3)*(2
*a^2*b^4*c + a^3*b^3*d - 4*a^4*b^2*e + 7*a^5*b*f + (2*a*b^5*c + a^2*b^4*d -
 4*a^3*b^3*e + 7*a^4*b^2*f)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*
(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b
)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 2*(2*a*b^3*c + a^2*b^2*d
```

```
- 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7*a^3*b*f)*x^3)*
(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3
*c + a^2*b^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7
*a^3*b*f)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^2*b^4*c - a
^3*b^3*d + 4*a^4*b^2*e - 7*a^5*b*f)*x)/(a^3*b^5*x^3 + a^4*b^4), 1/36*(9*a^3
*b^3*f*x^7 + 9*(4*a^3*b^3*e - 7*a^4*b^2*f)*x^4 + 12*sqrt(1/3)*(2*a^2*b^4*c
+ a^3*b^3*d - 4*a^4*b^2*e + 7*a^5*b*f + (2*a*b^5*c + a^2*b^4*d - 4*a^3*b^3*
e + 7*a^4*b^2*f)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/
3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(2*a*b^3*c + a^2*b^2
*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7*a^3*b*f)*x^
3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*
b^3*c + a^2*b^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e
+ 7*a^3*b*f)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^2*b^4*c
- a^3*b^3*d + 4*a^4*b^2*e - 7*a^5*b*f)*x)/(a^3*b^5*x^3 + a^4*b^4)]
```

Sympy [A] (verification not implemented)

Time = 3.10 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.43

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx = x\left(-\frac{2af}{b^3}+\frac{e}{b^2}\right)+\frac{x(-a^3f+a^2be-ab^2d+b^3c)}{3a^2b^3+3ab^4x^3}$$
$$+\mathrm{RootSum}\Big(729t^3a^5b^{10}-343a^9f^3+588a^8bef^2-147a^7b^2df^2-336a^7b^2e^2f-294a^6b^3cf^2+168a^6b^3def+$$
$$+\frac{fx^4}{4b^2}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x*(-2*a*f/b**3 + e/b**2) + x*(-a**3*f + a**2*b*e - a*b**2*d + b**3*c)/(3*a*
*2*b**3 + 3*a*b**4*x**3) + RootSum(729*_t**3*a**5*b**10 - 343*a**9*f**3 + 5
88*a**8*b*e*f**2 - 147*a**7*b**2*d*f**2 - 336*a**7*b**2*e**2*f - 294*a**6*b
**3*c*f**2 + 168*a**6*b**3*d*e*f + 64*a**6*b**3*e**3 + 336*a**5*b**4*c*e*f
- 21*a**5*b**4*d**2*f - 48*a**5*b**4*d*e**2 - 84*a**4*b**5*c*d*f - 96*a**4*
b**5*c*e**2 + 12*a**4*b**5*d**2*e - 84*a**3*b**6*c**2*f + 48*a**3*b**6*c*d*
e - a**3*b**6*d**3 + 48*a**2*b**7*c**2*e - 6*a**2*b**7*c*d**2 - 12*a*b**8*c
**2*d - 8*b**9*c**3, Lambda(_t, _t*log(9*_t*a**2*b**3/(7*a**3*f - 4*a**2*b*
e + a*b**2*d + 2*b**3*c) + x))) + f*x**4/(4*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)x}{3\,(ab^4x^3+a^2b^3)}+\frac{bfx^4+4\,(be-2\,af)x}{4\,b^3}+\frac{\sqrt{3}(2\,b^3c+ab^2d-4\,a^2be+7\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(2\,b^3c+ab^2d-4\,a^2be+7\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(2\,b^3c+ab^2d-4\,a^2be+7\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x/(a*b^4*x^3 + a^2*b^3) + 1/4*(b*f*
x^4 + 4*(b*e - 2*a*f)*x)/b^3 + 1/9*sqrt(3)*(2*b^3*c + a*b^2*d - 4*a^2*b*e +
 7*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^4*(a/b)^
(2/3)) - 1/18*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(a*b^4*(a/b)^(2/3)) + 1/9*(2*b^3*c + a*b^2*d - 4*a^2*b*
e + 7*a^3*f)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}(2\,b^3c+ab^2d-4\,a^2be+7\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}ab^2}-\frac{(2\,b^3c+ab^2d-4\,a^2be+7\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}ab^2}-\frac{(2\,b^3c+ab^2d-4\,a^2be+7\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^3}+\frac{b^3cx-ab^2dx+a^2bex-a^3fx}{3\,(bx^3+a)ab^3}+\frac{b^6fx^4+4\,b^6ex-8\,ab^5fx}{4\,b^8}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) - 1/18*(2*b^3*c +
a*b^2*d - 4*a^2*b*e + 7*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-
a*b^2)^(2/3)*a*b^2) - 1/9*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*(-a/b)^
(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^3) + 1/3*(b^3*c*x - a*b^2*d*x + a^2
*b*e*x - a^3*f*x)/((b*x^3 + a)*a*b^3) + 1/4*(b^6*f*x^4 + 4*b^6*e*x - 8*a*b^
5*f*x)/b^8
```

Mupad [B] (verification not implemented)

Time = 9.77 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx$$
$$= x\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)+\frac{f\,x^4}{4\,b^2}+\frac{x\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{3\,a\,\left(b^4\,x^3+a\,b^3\right)}$$
$$+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(7\,f\,a^3-4\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{5/3}\,b^{10/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(7\,f\,a^3-4\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{5/3}\,b^{10/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(7\,f\,a^3-4\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{5/3}\,b^{10/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x)
```

```
[Out] x*(e/b^2 - (2*a*f)/b^3) + (f*x^4)/(4*b^2) + (x*(b^3*c - a^3*f - a*b^2*d + a
^2*b*e))/(3*a*(a*b^3 + b^4*x^3)) + (log(b^(1/3)*x + a^(1/3))*(2*b^3*c + 7*a
^3*f + a*b^2*d - 4*a^2*b*e))/(9*a^(5/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i
 + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*b^3*c + 7*a^3*f + a*b^2
*d - 4*a^2*b*e))/(9*a^(5/3)*b^(10/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)
*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*b^3*c + 7*a^3*f + a*b^2*d - 4*a^2*b
*e))/(9*a^(5/3)*b^(10/3))
```

3.267 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 265

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^2}\,dx = -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)} + \frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{8/3}} + \frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{8/3}} - \frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{8/3}}$$

```
[Out] -c/a^2/x+1/2*f*x^2/b^2-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^2/b^2/(b*x^
3+a)+1/9*(5*a^3*f-2*a^2*b*e-a*b^2*d+4*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/
b^(8/3)-1/18*(5*a^3*f-2*a^2*b*e-a*b^2*d+4*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/a^(7/3)/b^(8/3)+1/9*(5*a^3*f-2*a^2*b*e-a*b^2*d+4*b^3*c)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1843, 1502, 298, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^2}\,dx = -\frac{c}{a^2x}-\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^2b^2(a+bx^3)} + \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(5a^3f-2a^2be-ab^2d+4b^3c)}{3\sqrt{3}a^{7/3}b^{8/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(5a^3f-2a^2be-ab^2d+4b^3c)}{18a^{7/3}b^{8/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(5a^3f-2a^2be-ab^2d+4b^3c)}{9a^{7/3}b^{8/3}} + \frac{fx^2}{2b^2}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -(c/(a^2*x)) + (f*x^2)/(2*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/
(3*a^2*b^2*(a + b*x^3)) + ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(8/3)) +
((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(
7/3)*b^(8/3)) - ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(7/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^2b^2\,(a+bx^3)}-\frac{\int\frac{-3b^3c+b\left(\frac{b^3c}{a}-b^2d-2abe+2a^2f\right)x^3-3ab^2fx^6}{x^2(a+bx^3)}\,dx}{3ab^3}\\ &= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^2b^2\,(a+bx^3)}-\frac{\int\left(-\frac{3b^3c}{ax^2}-3abfx+\frac{b(4b^3c-ab^2d-2a^2be+5a^3f)x}{a(a+bx^3)}\right)dx}{3ab^3}\end{aligned}$$

$$=-\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^2b^2\,(a+bx^3)}-\frac{(4b^3c-ab^2d-2a^2be+5a^3f)\int\frac{x}{a+bx^3}\,dx}{3a^2b^2}$$

$$=-\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^2b^2\,(a+bx^3)}+\frac{(4b^3c-ab^2d-2a^2be+5a^3f)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{7/3}b^{7/3}}-\frac{(4b^3c-ab^2d-2a^2be+5a^3f)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{7/3}b^{7/3}}$$

$$=-\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^2b^2\,(a+bx^3)}+\frac{(4b^3c-ab^2d-2a^2be+5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{8/3}}-\frac{(4b^3c-ab^2d-2a^2be+5a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{7/3}b^{8/3}}-\frac{(4b^3c-ab^2d-2a^2be+5a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^2b^{7/3}}$$

$$=-\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^2b^2\,(a+bx^3)}+\frac{(4b^3c-ab^2d-2a^2be+5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{8/3}}-\frac{(4b^3c-ab^2d-2a^2be+5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{8/3}}-\frac{(4b^3c-ab^2d-2a^2be+5a^3f)\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{7/3}b^{8/3}}$$

$$=-\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^2b^2\,(a+bx^3)}+\frac{(4b^3c-ab^2d-2a^2be+5a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{8/3}}+\frac{(4b^3c-ab^2d-2a^2be+5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{8/3}}-\frac{(4b^3c-ab^2d-2a^2be+5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{8/3}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^2}\,dx = \frac{1}{18}\left(-\frac{18c}{a^2x}+\frac{9fx^2}{b^2}+\frac{6(-b^3c+ab^2d-a^2be+a^3f)\,x^2}{a^2b^2\,(a+bx^3)} + \frac{2\sqrt{3}(4b^3c-ab^2d-2a^2be+5a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}b^{8/3}} + \frac{2(4b^3c-ab^2d-2a^2be+5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{7/3}b^{8/3}} - \frac{(4b^3c-ab^2d-2a^2be+5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{7/3}b^{8/3}}\right)$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] ((-18*c)/(a^2*x) + (9*f*x^2)/b^2 + (6*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f
)*x^2)/(a^2*b^2*(a + b*x^3)) + (2*Sqrt[3]*(4*b^3*c - a*b^2*d - 2*a^2*b*e +
5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(7/3)*b^(8/3)) + (
2*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(7
/3)*b^(8/3)) - ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(7/3)*b^(8/3)))/18
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.71

method	result
default	$\frac{f\,x^{2}}{2b^{2}}-\frac{c}{a^{2}x}-\frac{\frac{\left(-\frac{1}{3}f\,a^{3}+\frac{1}{3}a^{2}be-\frac{1}{3}a\,b^{2}d+\frac{1}{3}b^{3}c\right)x^{2}}{b\,x^{3}+a}+\left(\frac{5}{3}f\,a^{3}-\frac{1}{3}a\,b^{2}d+\frac{4}{3}b^{3}c-\frac{2}{3}a^{2}be\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\ldots}{\ldots}\right.}{a^{2}b^{2}}$
risch	$\frac{f\,x^{2}}{2b^{2}}+\frac{\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-4b^{3}c\right)x^{3}}{3a^{2}}-\frac{b^{2}c}{a}}{b^{2}x\left(b\,x^{3}+a\right)}+\frac{\_R=\mathrm{RootOf}\left(a^{7}b^{2}\_Z^{3}-125a^{9}f^{3}+150a^{8}be\,f^{2}+75a^{7}b^{2}d\,f^{2}-60a^{7}b^{2}e^{2}f-300a^{6}b^{3}c\,f^{2}-60a^{6}b^{3}de\ldots}{\ldots}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*x^2/b^2-c/a^2/x-1/a^2/b^2*((-1/3*f*a^3+1/3*a^2*b*e-1/3*a*b^2*d+1/3*b^
3*c)*x^2/(b*x^3+a)+(5/3*f*a^3-1/3*a*b^2*d+4/3*b^3*c-2/3*a^2*b*e)*(-1/3/b/(a
/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 860, normalized size of antiderivative = 3.25

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^2}\,dx$$

$$=\left[\frac{9\,a^3b^3fx^6-18\,a^2b^4c-3\left(8\,ab^5c-2\,a^2b^4d+2\,a^3b^3e-5\,a^4b^2f\right)x^3+3\,\sqrt{\frac{1}{3}}\left(\left(4\,ab^5c-a^2b^4d-2\,a^3b^3e+5\,a^4\ldots\right.\right.}{\ldots}\right.$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(9*a^3*b^3*f*x^6 - 18*a^2*b^4*c - 3*(8*a*b^5*c - 2*a^2*b^4*d + 2*a^3*
b^3*e - 5*a^4*b^2*f)*x^3 + 3*sqrt(1/3)*((4*a*b^5*c - a^2*b^4*d - 2*a^3*b^3*
```

```
e + 5*a^4*b^2*f)*x^4 + (4*a^2*b^4*c - a^3*b^3*d - 2*a^4*b^2*e + 5*a^5*b*f)*
x)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*
b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*
x)/(b*x^3 + a)) - ((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a
*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a
*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*
a^3*b*f)*x^4 + (4*a*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/
3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^5*x^4 + a^4*b^4*x), 1/18*(9*a^3*b^3*f*x
^6 - 18*a^2*b^4*c - 3*(8*a*b^5*c - 2*a^2*b^4*d + 2*a^3*b^3*e - 5*a^4*b^2*f)
*x^3 + 6*sqrt(1/3)*((4*a*b^5*c - a^2*b^4*d - 2*a^3*b^3*e + 5*a^4*b^2*f)*x^4
 + (4*a^2*b^4*c - a^3*b^3*d - 2*a^4*b^2*e + 5*a^5*b*f)*x)*sqrt((a*b^2)^(1/3
)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - (
(4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a*b^3*c - a^2*b^2*d
- 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (
a*b^2)^(2/3)) + 2*((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a
*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b*x + (a*b^2
)^(1/3)))/(a^3*b^5*x^4 + a^4*b^4*x)]
```

Sympy [A] (verification not implemented)

Time = 63.97 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.72

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^2}\,dx = \frac{-3ab^2c+x^3(a^3f-a^2be+ab^2d-4b^3c)}{3a^3b^2x+3a^2b^3x^4} + \text{RootSum}\left(729t^3a^7b^8-125a^9f^3+150a^8bef^2+75a^7b^2df^2-60a^7b^2e^2f-300a^6b^3cf^2-60a^6b^3def+8a\right. + \frac{fx^2}{2b^2}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] (-3*a*b**2*c + x**3*(a**3*f - a**2*b*e + a*b**2*d - 4*b**3*c))/(3*a**3*b**2
*x + 3*a**2*b**3*x**4) + RootSum(729*_t**3*a**7*b**8 - 125*a**9*f**3 + 150*
a**8*b*e*f**2 + 75*a**7*b**2*d*f**2 - 60*a**7*b**2*e**2*f - 300*a**6*b**3*c
*f**2 - 60*a**6*b**3*d*e*f + 8*a**6*b**3*e**3 + 240*a**5*b**4*c*e*f - 15*a*
*5*b**4*d**2*f + 12*a**5*b**4*d*e**2 + 120*a**4*b**5*c*d*f - 48*a**4*b**5*c
*e**2 + 6*a**4*b**5*d**2*e - 240*a**3*b**6*c**2*f - 48*a**3*b**6*c*d*e + a*
*3*b**6*d**3 + 96*a**2*b**7*c**2*e - 12*a**2*b**7*c*d**2 + 48*a*b**8*c**2*d
 - 64*b**9*c**3, Lambda(_t, _t*log(81*_t**2*a**5*b**5/(25*a**6*f**2 - 20*a*
*5*b*e*f - 10*a**4*b**2*d*f + 4*a**4*b**2*e**2 + 40*a**3*b**3*c*f + 4*a**3*
b**3*d*e - 16*a**2*b**4*c*e + a**2*b**4*d**2 - 8*a*b**5*c*d + 16*b**6*c**2)
 + x))) + f*x**2/(2*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^2}\,dx = \frac{fx^2}{2\,b^2} - \frac{3\,ab^2c+(4\,b^3c-ab^2d+a^2be-a^3f)x^3}{3\,(a^2b^3x^4+a^3b^2x)} - \frac{\sqrt{3}(4\,b^3c-ab^2d-2\,a^2be+5\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(4\,b^3c-ab^2d-2\,a^2be+5\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(4\,b^3c-ab^2d-2\,a^2be+5\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*f*x^2/b^2 - 1/3*(3*a*b^2*c + (4*b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)
/(a^2*b^3*x^4 + a^3*b^2*x) - 1/9*sqrt(3)*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5
*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/b)^
(1/3)) - 1/18*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(1/3)) + 1/9*(4*b^3*c - a*b^2*d - 2*a^2*
b*e + 5*a^3*f)*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.14

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^2}\,dx$$
$$= \frac{fx^2}{2b^2} - \frac{\sqrt{3}(4b^3c-ab^2d-2a^2be+5a^3f)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}a^2b^2}$$
$$+ \frac{(4b^3c-ab^2d-2a^2be+5a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}a^2b^2}$$
$$+ \frac{\left(4b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^3b^2}$$
$$- \frac{4b^3cx^3-ab^2dx^3+a^2bex^3-a^3fx^3+3ab^2c}{3(bx^4+ax)a^2b^2}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b^2 - 1/9*sqrt(3)*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^2*b^2) +
 1/18*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*b^2) + 1/9*(4*b^3*c*(-a/b)^(1/3) - a*b^2*
d*(-a/b)^(1/3) - 2*a^2*b*e*(-a/b)^(1/3) + 5*a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3
)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^2) - 1/3*(4*b^3*c*x^3 - a*b^2*d*x^3 + a
^2*b*e*x^3 - a^3*f*x^3 + 3*a*b^2*c)/((b*x^4 + a*x)*a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^2}\,dx$$
$$= \frac{f\,x^2}{2\,b^2} - \frac{\frac{x^3\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{3\,a^2}+\frac{b^2\,c}{a}}{b^3\,x^4+a\,b^2\,x}$$
$$+ \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{9\,a^{7/3}\,b^{8/3}}$$
$$- \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{9\,a^{7/3}\,b^{8/3}}$$
$$+ \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{9\,a^{7/3}\,b^{8/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] (f*x^2)/(2*b^2) - ((x^3*(4*b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^2) + (b
^2*c)/a)/(b^3*x^4 + a*b^2*x) + (log(b^(1/3)*x + a^(1/3))*(4*b^3*c + 5*a^3*f
 - a*b^2*d - 2*a^2*b*e))/(9*a^(7/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*
b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(4*b^3*c + 5*a^3*f - a*b^2*d -
2*a^2*b*e))/(9*a^(7/3)*b^(8/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a
^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*b^3*c + 5*a^3*f - a*b^2*d - 2*a^2*b*e))/(
9*a^(7/3)*b^(8/3))
```

3.268 $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 260

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^2}\,dx = -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^2b^2(a+bx^3)} + \frac{(5b^3c-2ab^2d-a^2be+4a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}b^{7/3}} - \frac{(5b^3c-2ab^2d-a^2be+4a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}b^{7/3}} + \frac{(5b^3c-2ab^2d-a^2be+4a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}b^{7/3}}$$

```
[Out] -1/2*c/a^2/x^2+f*x/b^2-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^2/b^2/(b*x^3+
a)-1/9*(4*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^
(7/3)+1/18*(4*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/a^(8/3)/b^(7/3)+1/9*(4*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1502, 206, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^2}\,dx = -\frac{c}{2a^2x^2}-\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^2b^2(a+bx^3)} + \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(4a^3f-a^2be-2ab^2d+5b^3c)}{3\sqrt{3}a^{8/3}b^{7/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(4a^3f-a^2be-2ab^2d+5b^3c)}{18a^{8/3}b^{7/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(4a^3f-a^2be-2ab^2d+5b^3c)}{9a^{8/3}b^{7/3}}+\frac{fx}{b^2}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/2*c/(a^2*x^2) + (f*x)/b^2 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a
^2*b^2*(a + b*x^3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(7/3)) - ((5*
b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)
*b^(7/3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{3a^2b^2\,(a+bx^3)} - \frac{\int \frac{-3b^3c + b\left(\frac{2b^3c}{a} - 2b^2d - abe + a^2f\right)x^3 - 3ab^2fx^6}{x^3(a+bx^3)}\,dx}{3ab^3}$$

$$= -\frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{3a^2b^2\,(a+bx^3)} - \frac{\int \left(-3abf - \frac{3b^3c}{ax^3} + \frac{b(5b^3c - 2ab^2d - a^2be + 4a^3f)}{a(a+bx^3)}\right)dx}{3ab^3}$$

$$= -\frac{c}{2a^2x^2} + \frac{fx}{b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{3a^2b^2\,(a + bx^3)} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)\int \frac{1}{a+bx^3}\,dx}{3a^2b^2}$$

$$= -\frac{c}{2a^2x^2} + \frac{fx}{b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{3a^2b^2\,(a + bx^3)} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{8/3}b^2} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{8/3}b^2}$$

$$= -\frac{c}{2a^2x^2} + \frac{fx}{b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{3a^2b^2\,(a + bx^3)} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{9a^{8/3}b^{7/3}} + \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{8/3}b^{7/3}} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{7/3}b^2}$$

$$= -\frac{c}{2a^2x^2} + \frac{fx}{b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{3a^2b^2\,(a + bx^3)} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{9a^{8/3}b^{7/3}} + \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{18a^{8/3}b^{7/3}} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{8/3}b^{7/3}}$$

$$= -\frac{c}{2a^2x^2} + \frac{fx}{b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{3a^2b^2\,(a + bx^3)} + \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}b^{7/3}} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{9a^{8/3}b^{7/3}} + \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{18a^{8/3}b^{7/3}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^2}\,dx = \frac{1}{18}\left(-\frac{9c}{a^2x^2}+\frac{18fx}{b^2}+\frac{6(-b^3c+ab^2d-a^2be+a^3f)\,x}{a^2b^2\left(a+bx^3\right)} + \frac{2\sqrt{3}(5b^3c-2ab^2d-a^2be+4a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}b^{7/3}} - \frac{2(5b^3c-2ab^2d-a^2be+4a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{8/3}b^{7/3}} + \frac{(5b^3c-2ab^2d-a^2be+4a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{8/3}b^{7/3}}\right)$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*c)/(a^2*x^2) + (18*f*x)/b^2 + (6*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f
)*x)/(a^2*b^2*(a + b*x^3)) + (2*Sqrt[3]*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*
a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(8/3)*b^(7/3)) - (2*
(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(8/3
)*b^(7/3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(8/3)*b^(7/3)))/18
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.70

method	result
default	$\frac{fx}{b^2}-\frac{c}{2a^2x^2}-\frac{\frac{\left(-\frac{1}{3}f\,a^3+\frac{1}{3}a^2be-\frac{1}{3}a\,b^2d+\frac{1}{3}b^3c\right)x}{b\,x^3+a}+\frac{\left(4f\,a^3-a^2be-2a\,b^2d+5b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\ldots\right)}{3b\ldots}\right)}{3}}{a^2b^2}$
risch	$\frac{fx}{b^2}+\frac{\frac{\left(2f\,a^3-2a^2be+2a\,b^2d-5b^3c\right)x^3}{6a^2}-\frac{b^2c}{2a}}{b^2x^2\left(b\,x^3+a\right)}+\frac{\_R=\text{RootOf}\left(a^8b\_Z^3+64a^9f^3-48a^8be\,f^2-96a^7b^2d\,f^2+12a^7b^2e^2f+240a^6b^3c\,f^2+48a^6b^3def\ldots\right.}{\ldots}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f*x/b^2-1/2*c/a^2/x^2-1/a^2/b^2*((-1/3*f*a^3+1/3*a^2*b*e-1/3*a*b^2*d+1/3*b^
3*c)*x/(b*x^3+a)+1/3*(4*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)*(1/3/b/(a/b)^(2/3)
*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/
b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 902, normalized size of antiderivative = 3.47

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^2}\,dx$$

$$=\left[\frac{18\,a^4b^2fx^6-9\,a^3b^3c-3\,(5\,a^2b^4c-2\,a^3b^3d+2\,a^4b^2e-8\,a^5bf)x^3+3\,\sqrt{\frac{1}{3}}((5\,ab^5c-2\,a^2b^4d-a^3b^3e+4\,a^4\ldots}{\ldots}\right.$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(18*a^4*b^2*f*x^6 - 9*a^3*b^3*c - 3*(5*a^2*b^4*c - 2*a^3*b^3*d + 2*a^
4*b^2*e - 8*a^5*b*f)*x^3 + 3*sqrt(1/3)*((5*a*b^5*c - 2*a^2*b^4*d - a^3*b^3*
```

```
e + 4*a^4*b^2*f)*x^5 + (5*a^2*b^4*c - 2*a^3*b^3*d - a^4*b^2*e + 4*a^5*b*f)*
x^2)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3
*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^
(1/3)/b))/(b*x^3 + a)) + ((5*b^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^5
 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*(-a^2*b)^(2/3)*log(a*
b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((5*b^4*c - 2*a*b^3*d - a^
2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^
2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^4*x^5 + a^5*b^3*x^2),
 1/18*(18*a^4*b^2*f*x^6 - 9*a^3*b^3*c - 3*(5*a^2*b^4*c - 2*a^3*b^3*d + 2*a^
4*b^2*e - 8*a^5*b*f)*x^3 - 6*sqrt(1/3)*((5*a*b^5*c - 2*a^2*b^4*d - a^3*b^3*
e + 4*a^4*b^2*f)*x^5 + (5*a^2*b^4*c - 2*a^3*b^3*d - a^4*b^2*e + 4*a^5*b*f)*
x^2)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b
)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) + ((5*b^4*c - 2*a*b^3*d - a^2*b^2*e
 + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*(-a^
2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((5*b^4*c
 - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*
b*e + 4*a^4*f)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^4*x^
5 + a^5*b^3*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^3 (a + bx^3)^2} \, dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{3\,ab^2c+\left(5\,b^3c-2\,ab^2d+2\,a^2be-2\,a^3f\right)x^3}{6\left(a^2b^3x^5+a^3b^2x^2\right)}+\frac{fx}{b^2} - \frac{\sqrt{3}\left(5\,b^3c-2\,ab^2d-a^2be+4\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(5\,b^3c-2\,ab^2d-a^2be+4\,a^3f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(5\,b^3c-2\,ab^2d-a^2be+4\,a^3f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a*b^2*c + (5*b^3*c - 2*a*b^2*d + 2*a^2*b*e - 2*a^3*f)*x^3)/(a^2*b^3
*x^5 + a^3*b^2*x^2) + f*x/b^2 - 1/9*sqrt(3)*(5*b^3*c - 2*a*b^2*d - a^2*b*e
+ 4*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/
b)^(2/3)) + 1/18*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*log(x^2 - x*(a/b
)^(1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(2/3)) - 1/9*(5*b^3*c - 2*a*b^2*d - a
^2*b*e + 4*a^3*f)*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^2}\,dx = \frac{fx}{b^2} + \frac{\sqrt{3}(5\,b^3c-2\,ab^2d-a^2be+4\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}a^2b} + \frac{(5\,b^3c-2\,ab^2d-a^2be+4\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}a^2b} + \frac{(5\,b^3c-2\,ab^2d-a^2be+4\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^2} - \frac{c}{2\,a^2x^2} - \frac{b^3cx-ab^2dx+a^2bex-a^3fx}{3\,(bx^3+a)a^2b^2}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] f*x/b^2 + 1/9*sqrt(3)*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*arctan(1/3*
sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b) + 1/18*(5
*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/((-a*b^2)^(2/3)*a^2*b) + 1/9*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f
)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^2) - 1/2*c/(a^2*x^2) - 1/3
*(b^3*c*x - a*b^2*d*x + a^2*b*e*x - a^3*f*x)/((b*x^3 + a)*a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^2}\,dx = \frac{f\,x}{b^2} - \frac{\frac{x^3\,\left(-2\,f\,a^3+2\,e\,a^2\,b-2\,d\,a\,b^2+5\,c\,b^3\right)}{6\,a^2}+\frac{b^2\,c}{2\,a}}{b^3\,x^5+a\,b^2\,x^2} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(4\,f\,a^3-e\,a^2\,b-2\,d\,a\,b^2+5\,c\,b^3\right)}{9\,a^{8/3}\,b^{7/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(4\,f\,a^3-e\,a^2\,b-2\,d\,a\,b^2+5\,c\,b^3\right)}{9\,a^{8/3}\,b^{7/3}} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(4\,f\,a^3-e\,a^2\,b-2\,d\,a\,b^2+5\,c\,b^3\right)}{9\,a^{8/3}\,b^{7/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] (f*x)/b^2 - ((x^3*(5*b^3*c - 2*a^3*f - 2*a*b^2*d + 2*a^2*b*e))/(6*a^2) + (b
^2*c)/(2*a))/(b^3*x^5 + a*b^2*x^2) - (log(b^(1/3)*x + a^(1/3))*(5*b^3*c + 4
*a^3*f - 2*a*b^2*d - a^2*b*e))/(9*a^(8/3)*b^(7/3)) - (log(3^(1/2)*a^(1/3)*1
i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c + 4*a^3*f - 2*a*
b^2*d - a^2*b*e))/(9*a^(8/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)
*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*b^3*c + 4*a^3*f - 2*a*b^2*d - a^2*b
*e))/(9*a^(8/3)*b^(7/3))
```

3.269 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 269

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^2}\,dx = -\frac{c}{4a^2x^4}+\frac{2bc-ad}{a^3x}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^3b\,(a+bx^3)} - \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}b^{5/3}} - \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}b^{5/3}} + \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{10/3}b^{5/3}}$$

```
[Out] -1/4*c/a^2/x^4+(-a*d+2*b*c)/a^3/x+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^
3/b/(b*x^3+a)-1/9*(2*a^3*f+a^2*b*e-4*a*b^2*d+7*b^3*c)*ln(a^(1/3)+b^(1/3)*x)
/a^(10/3)/b^(5/3)+1/18*(2*a^3*f+a^2*b*e-4*a*b^2*d+7*b^3*c)*ln(a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(5/3)-1/9*(2*a^3*f+a^2*b*e-4*a*b^2*d+7
*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/b^(5/3)*
3^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1843, 1502, 298, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^2}\,dx=\frac{2bc-ad}{a^3x}-\frac{c}{4a^2x^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^3f+a^2be-4ab^2d+7b^3c\right)}{3\sqrt{3}a^{10/3}b^{5/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^3f+a^2be-4ab^2d+7b^3c\right)}{18a^{10/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^3f+a^2be-4ab^2d+7b^3c\right)}{9a^{10/3}b^{5/3}}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] -1/4*c/(a^2*x^4) + (2*b*c - a*d)/(a^3*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*x^2)/(3*a^3*b*(a + b*x^3)) - ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f
)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)*b^
(5/3)) - ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x
])/(9*a^(10/3)*b^(5/3)) + ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(10/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^3b\,(a+bx^3)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-b^2\left(\frac{b^3c}{a^2}-\frac{b^2d}{a}+be+2af\right)x^6}{x^5(a+bx^3)}\,dx}{3ab^3}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^3b\,(a+bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^5}-\frac{3b^3(-2bc+ad)}{a^2x^2}-\frac{b^2(7b^3c-4ab^2d+a^2be+2a^3f)x}{a^2(a+bx^3)}\right)\,dx}{3ab^3}$$

$$= -\frac{c}{4a^2x^4} + \frac{2bc-ad}{a^3x} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^3b\,(a+bx^3)} + \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\int\frac{x}{a+bx^3}\,dx}{3a^3b}$$

$$= -\frac{c}{4a^2x^4} + \frac{2bc-ad}{a^3x} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^3b\,(a+bx^3)} - \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{10/3}b^{4/3}} + \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{10/3}b^{4/3}}$$

$$= -\frac{c}{4a^2x^4} + \frac{2bc-ad}{a^3x} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^3b\,(a+bx^3)} - \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}b^{5/3}} + \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{10/3}b^{5/3}} + \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^3b^{4/3}}$$

$$= -\frac{c}{4a^2x^4} + \frac{2bc-ad}{a^3x} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^3b\,(a+bx^3)} - \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}b^{5/3}} + \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{10/3}b^{5/3}} + \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\,\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{10/3}b^{5/3}}$$

$$= -\frac{c}{4a^2x^4} + \frac{2bc-ad}{a^3x} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^3b\,(a+bx^3)} - \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}b^{5/3}} - \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}b^{5/3}} + \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{10/3}b^{5/3}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{9a^{4/3}c}{x^4} - \frac{36\sqrt[3]{a}(-2bc+ad)}{x} - \frac{12\sqrt[3]{a}(-b^3c+ab^2d-a^2be+a^3f)x^2}{b(a+bx^3)} - \frac{4\sqrt{3}(7b^3c-4ab^2d+a^2be+2a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}} - \frac{4(7b^3c-4}{}}{36a^{10/3}}$$

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x]

[Out] ((-9*a^(4/3)*c)/x^4 - (36*a^(1/3)*(-2*b*c + a*d))/x - (12*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)) - (4*Sqrt[3]*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(5/3) - (4*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(5/3) + (2*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3))/(36*a^(10/3))

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.72

method	result
default	$-\frac{c}{4a^2x^4}-\frac{ad-2bc}{a^3x}+\frac{-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)x^2}{3b(b\,x^3+a)}+\frac{(2f\,a^3+a^2be-4a\,b^2d+7b^3c)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\mathrm{arct}}{}\right.}{3b}}{a^3}$
risch	$\frac{-\frac{(f\,a^3-a^2be+4a\,b^2d-7b^3c)x^6}{3a^3b}-\frac{(4ad-7bc)x^3}{4a^2}-\frac{c}{4a}}{x^4(b\,x^3+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}b^5\_Z^3+8a^9f^3+12a^8bef^2-48a^7b^2df^2+6a^7b^2e^2f+84a^6b^3cf^2-48\right.}\right.}{}$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/4*c/a^2/x^4-(a*d-2*b*c)/a^3/x+1/a^3*(-1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b*x^2/(b*x^3+a)+1/3*(2*a^3*f+a^2*b*e-4*a*b^2*d+7*b^3*c)/b*(-1/3/b/(a/b)^(1/

```
3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/
3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 902, normalized size of antiderivative = 3.35

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^2}\,dx$$

$$= \left[-\frac{9\,a^3b^3c-12\,(7\,ab^5c-4\,a^2b^4d+a^3b^3e-a^4b^2f)x^6-9\,(7\,a^2b^4c-4\,a^3b^3d)x^3-6\,\sqrt{\tfrac{1}{3}}((7\,ab^5c-4\,a^2b^4d+}{}\right.$$

$$-\frac{9\,a^3b^3c-12\,(7\,ab^5c-4\,a^2b^4d+a^3b^3e-a^4b^2f)x^6-9\,(7\,a^2b^4c-4\,a^3b^3d)x^3-12\,\sqrt{\tfrac{1}{3}}((7\,ab^5c-4\,a^2b^4d+}{}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/36*(9*a^3*b^3*c - 12*(7*a*b^5*c - 4*a^2*b^4*d + a^3*b^3*e - a^4*b^2*f)*
x^6 - 9*(7*a^2*b^4*c - 4*a^3*b^3*d)*x^3 - 6*sqrt(1/3)*((7*a*b^5*c - 4*a^2*b
^4*d + a^3*b^3*e + 2*a^4*b^2*f)*x^7 + (7*a^2*b^4*c - 4*a^3*b^3*d + a^4*b^2*
e + 2*a^5*b*f)*x^4)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/
3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a)
 - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 2*((7*b^4*c - 4*a*b^3*d + a^2*b^2*e +
 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*(-a*b^
2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 4*((7*b^4*c -
 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*
e + 2*a^4*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^4*x^7 +
a^5*b^3*x^4), -1/36*(9*a^3*b^3*c - 12*(7*a*b^5*c - 4*a^2*b^4*d + a^3*b^3*e
- a^4*b^2*f)*x^6 - 9*(7*a^2*b^4*c - 4*a^3*b^3*d)*x^3 - 12*sqrt(1/3)*((7*a*b
^5*c - 4*a^2*b^4*d + a^3*b^3*e + 2*a^4*b^2*f)*x^7 + (7*a^2*b^4*c - 4*a^3*b^
3*d + a^4*b^2*e + 2*a^5*b*f)*x^4)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*
(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 2*((7*b^4*c - 4*a*b^3
```

```
*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^
4*f)*x^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3))
 + 4*((7*b^4*c - 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^
2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))
/(a^4*b^4*x^7 + a^5*b^3*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^2}\,dx &= \frac{4\,(7\,b^3c-4\,ab^2d+a^2be-a^3f)x^6-3\,a^2bc+3\,(7\,ab^2c-4\,a^2bd)x^3}{12\,(a^3b^2x^7+a^4bx^4)}\\ &\quad+\frac{\sqrt{3}(7\,b^3c-4\,ab^2d+a^2be+2\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}\\ &\quad+\frac{(7\,b^3c-4\,ab^2d+a^2be+2\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}\\ &\quad-\frac{(7\,b^3c-4\,ab^2d+a^2be+2\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}\end{aligned}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="maxima")
[Out] 1/12*(4*(7*b^3*c - 4*a*b^2*d + a^2*b*e - a^3*f)*x^6 - 3*a^2*b*c + 3*(7*a*b^
2*c - 4*a^2*b*d)*x^3)/(a^3*b^2*x^7 + a^4*b*x^4) + 1/9*sqrt(3)*(7*b^3*c - 4*
a*b^2*d + a^2*b*e + 2*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/(a^3*b^2*(a/b)^(1/3)) + 1/18*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(1/3)) - 1/9*(7*b^3
*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(1/
3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.14

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^2}\,dx$$
$$= \frac{\sqrt{3}(7b^3c-4ab^2d+a^2be+2a^3f)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}a^3b} - \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}a^3b} - \frac{\left(7b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^4b} + \frac{b^3cx^2-ab^2dx^2+a^2bex^2-a^3fx^2}{3(bx^3+a)a^3b} + \frac{8bcx^3-4adx^3-ac}{4a^3x^4}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^3*b) - 1/18*(7*b^3*c - 4
*a*b^2*d + a^2*b*e + 2*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a
*b^2)^(1/3)*a^3*b) - 1/9*(7*b^3*c*(-a/b)^(1/3) - 4*a*b^2*d*(-a/b)^(1/3) + a
^2*b*e*(-a/b)^(1/3) + 2*a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^4*b) + 1/3*(b^3*c*x^2 - a*b^2*d*x^2 + a^2*b*e*x^2 - a^3*f*x^2)/
((b*x^3 + a)*a^3*b) + 1/4*(8*b*c*x^3 - 4*a*d*x^3 - a*c)/(a^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^2}\,dx$$
$$= -\frac{\frac{c}{4a}+\frac{x^3(4ad-7bc)}{4a^2}-\frac{x^6(-fa^3+ea^2b-4dab^2+7cb^3)}{3a^3b}}{bx^7+ax^4} - \frac{\ln\left(b^{1/3}x+a^{1/3}\right)(2fa^3+ea^2b-4dab^2+7cb^3)}{9a^{10/3}b^{5/3}} + \frac{\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(2fa^3+ea^2b-4dab^2+7cb^3)}{9a^{10/3}b^{5/3}} - \frac{\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(2fa^3+ea^2b-4dab^2+7cb^3)}{9a^{10/3}b^{5/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(7*
b^3*c + 2*a^3*f - 4*a*b^2*d + a^2*b*e))/(9*a^(10/3)*b^(5/3)) - (log(b^(1/3)
*x + a^(1/3))*(7*b^3*c + 2*a^3*f - 4*a*b^2*d + a^2*b*e))/(9*a^(10/3)*b^(5/3
)) - (c/(4*a) + (x^3*(4*a*d - 7*b*c))/(4*a^2) - (x^6*(7*b^3*c - a^3*f - 4*a
*b^2*d + a^2*b*e))/(3*a^3*b))/(a*x^4 + b*x^7) - (log(3^(1/2)*a^(1/3)*1i - 2
*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(7*b^3*c + 2*a^3*f - 4*a*b^2*d
 + a^2*b*e))/(9*a^(10/3)*b^(5/3))
```

3.270 $\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 270

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^2}\,dx = -\frac{c}{5a^2x^5}+\frac{2bc-ad}{2a^3x^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)} - \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}b^{4/3}} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}b^{4/3}} - \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{11/3}b^{4/3}}$$

```
[Out] -1/5*c/a^2/x^5+1/2*(-a*d+2*b*c)/a^3/x^2+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
x/a^3/b/(b*x^3+a)+1/9*(a^3*f+2*a^2*b*e-5*a*b^2*d+8*b^3*c)*ln(a^(1/3)+b^(1/3
)*x)/a^(11/3)/b^(4/3)-1/18*(a^3*f+2*a^2*b*e-5*a*b^2*d+8*b^3*c)*ln(a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(4/3)-1/9*(a^3*f+2*a^2*b*e-5*a*b^2
*d+8*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(4
/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1502, 206, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^2}\,dx = \frac{2bc-ad}{2a^3x^2}-\frac{c}{5a^2x^5}+\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^3b\left(a+bx^3\right)} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{3\sqrt{3}a^{11/3}b^{4/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{18a^{11/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{9a^{11/3}b^{4/3}}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] -1/5*c/(a^2*x^5) + (2*b*c - a*d)/(2*a^3*x^2) + ((b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*x)/(3*a^3*b*(a + b*x^3)) - ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3
*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(11/3)*
b^(4/3)) + ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)
*x])/(9*a^(11/3)*b^(4/3)) - ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(11/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{3a^3b\,(a + bx^3)} - \frac{\int \frac{-3b^3c + 3b^3\left(\frac{bc}{a} - d\right)x^3 - b^2\left(\frac{2b^3c}{a^2} - \frac{2b^2d}{a} + 2be + af\right)x^6}{x^6(a + bx^3)}\,dx}{3ab^3}$$

$$= \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{3a^3b\,(a + bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^6} - \frac{3b^3(-2bc + ad)}{a^2x^3} - \frac{b^2(8b^3c - 5ab^2d + 2a^2be + a^3f)}{a^2(a + bx^3)}\right)dx}{3ab^3}$$

$$= -\frac{c}{5a^2x^5} + \frac{2bc-ad}{2a^3x^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^3b\,(a+bx^3)} + \frac{(8b^3c-5ab^2d+2a^2be+a^3f)\int \frac{1}{a+bx^3}\,dx}{3a^3b}$$

$$= -\frac{c}{5a^2x^5} + \frac{2bc-ad}{2a^3x^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^3b\,(a+bx^3)} + \frac{(8b^3c-5ab^2d+2a^2be+a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{11/3}b} + \frac{(8b^3c-5ab^2d+2a^2be+a^3f)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{11/3}b}$$

$$= -\frac{c}{5a^2x^5} + \frac{2bc-ad}{2a^3x^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^3b\,(a+bx^3)} + \frac{(8b^3c-5ab^2d+2a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}b^{4/3}} - \frac{(8b^3c-5ab^2d+2a^2be+a^3f)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{11/3}b^{4/3}} + \frac{(8b^3c-5ab^2d+2a^2be+a^3f)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{10/3}b}$$

$$= -\frac{c}{5a^2x^5} + \frac{2bc-ad}{2a^3x^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^3b\,(a+bx^3)} + \frac{(8b^3c-5ab^2d+2a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}b^{4/3}} - \frac{(8b^3c-5ab^2d+2a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{11/3}b^{4/3}} + \frac{(8b^3c-5ab^2d+2a^2be+a^3f)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{11/3}b^{4/3}}$$

$$= -\frac{c}{5a^2x^5} + \frac{2bc-ad}{2a^3x^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^3b\,(a+bx^3)} - \frac{(8b^3c-5ab^2d+2a^2be+a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}b^{4/3}} + \frac{(8b^3c-5ab^2d+2a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}b^{4/3}} - \frac{(8b^3c-5ab^2d+2a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{11/3}b^{4/3}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{18a^{5/3}c}{x^5}-\frac{45a^{2/3}(-2bc+ad)}{x^2}-\frac{30a^{2/3}\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{b(a+bx^3)}-\frac{10\sqrt{3}\left(8b^3c-5ab^2d+2a^2be+a^3f\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}}+\frac{10(8b^3c-5}{}}{90a^{11/3}}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] ((-18*a^(5/3)*c)/x^5 - (45*a^(2/3)*(-2*b*c + a*d))/x^2 - (30*a^(2/3)*(-(b^3
*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)) - (10*Sqrt[3]*(8*b^3*c
- 5*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
])/b^(4/3) + (10*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^
(1/3)*x])/b^(4/3) - (5*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3))/(90*a^(11/3))
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.71

method	result
default	$-\frac{c}{5a^2x^5}-\frac{ad-2bc}{2a^3x^2}+\frac{-\frac{\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x}{3b\left(b\,x^3+a\right)}+\frac{\left(f\,a^3+2a^2be-5a\,b^2d+8b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\right.}{3}\right.}{3b}}{a^3}$
risch	$\frac{-\frac{\left(2f\,a^3-2a^2be+5a\,b^2d-8b^3c\right)x^6}{6a^3b}-\frac{(5ad-8bc)x^3}{10a^2}-\frac{c}{5a}}{x^5\left(b\,x^3+a\right)}+\left(\sum_{\_R=\text{RootOf}\left(a^{11}b^4\_Z^3-a^9f^3-6a^8be\,f^2+15a^7b^2d\,f^2-12a^7b^2e^2f-24a^6b^3c\,f^2+60a\right.}\right.$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*c/a^2/x^5-1/2*(a*d-2*b*c)/a^3/x^2+1/a^3*(-1/3*(a^3*f-a^2*b*e+a*b^2*d-b
^3*c)/b*x/(b*x^3+a)+1/3*(a^3*f+2*a^2*b*e-5*a*b^2*d+8*b^3*c)/b*(1/3/b/(a/b)^
```

```
(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 897, normalized size of antiderivative = 3.32

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^2}\,dx$$

$$= \left[-\frac{18\,a^4b^2c-15\,(8\,a^2b^4c-5\,a^3b^3d+2\,a^4b^2e-2\,a^5bf)x^6-9\,(8\,a^3b^3c-5\,a^4b^2d)x^3-15\,\sqrt{\frac{1}{3}}((8\,ab^5c-5\,a}{}\right.$$

$$-\frac{18\,a^4b^2c-15\,(8\,a^2b^4c-5\,a^3b^3d+2\,a^4b^2e-2\,a^5bf)x^6-9\,(8\,a^3b^3c-5\,a^4b^2d)x^3-30\,\sqrt{\frac{1}{3}}((8\,ab^5c-5\,a^2}{}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/90*(18*a^4*b^2*c - 15*(8*a^2*b^4*c - 5*a^3*b^3*d + 2*a^4*b^2*e - 2*a^5*
b*f)*x^6 - 9*(8*a^3*b^3*c - 5*a^4*b^2*d)*x^3 - 15*sqrt(1/3)*((8*a*b^5*c - 5
*a^2*b^4*d + 2*a^3*b^3*e + a^4*b^2*f)*x^8 + (8*a^2*b^4*c - 5*a^3*b^3*d + 2*
a^4*b^2*e + a^5*b*f)*x^5)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)
^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)
*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + 5*((8*b^4*c - 5*a*b^3*d + 2*a^2*
b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*(
a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 10*((8*b^4*
c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2*d + 2*a
^3*b*e + a^4*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^3*x^8
 + a^6*b^2*x^5), -1/90*(18*a^4*b^2*c - 15*(8*a^2*b^4*c - 5*a^3*b^3*d + 2*a^
4*b^2*e - 2*a^5*b*f)*x^6 - 9*(8*a^3*b^3*c - 5*a^4*b^2*d)*x^3 - 30*sqrt(1/3)
*((8*a*b^5*c - 5*a^2*b^4*d + 2*a^3*b^3*e + a^4*b^2*f)*x^8 + (8*a^2*b^4*c -
5*a^3*b^3*d + 2*a^4*b^2*e + a^5*b*f)*x^5)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt
(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 5*
```

```
((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2
*d + 2*a^3*b*e + a^4*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x +
(a^2*b)^(1/3)*a) - 10*((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 +
(8*a*b^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x
+ (a^2*b)^(2/3)))/(a^5*b^3*x^8 + a^6*b^2*x^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^2}\,dx &= \frac{5\,(8\,b^3c-5\,ab^2d+2\,a^2be-2\,a^3f)x^6-6\,a^2bc+3\,(8\,ab^2c-5\,a^2bd)x^3}{30\,(a^3b^2x^8+a^4bx^5)} \\ &+ \frac{\sqrt{3}(8\,b^3c-5\,ab^2d+2\,a^2be+a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &- \frac{(8\,b^3c-5\,ab^2d+2\,a^2be+a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &+ \frac{(8\,b^3c-5\,ab^2d+2\,a^2be+a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="maxima")
[Out] 1/30*(5*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e - 2*a^3*f)*x^6 - 6*a^2*b*c + 3*(8*
a*b^2*c - 5*a^2*b*d)*x^3)/(a^3*b^2*x^8 + a^4*b*x^5) + 1/9*sqrt(3)*(8*b^3*c
- 5*a*b^2*d + 2*a^2*b*e + a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/
b)^(1/3))/(a^3*b^2*(a/b)^(2/3)) - 1/18*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a
^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(2/3)) + 1/9*(8
*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)
^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^2}\,dx = -\frac{\sqrt{3}(8\,b^3c-5\,ab^2d+2\,a^2be+a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,a^3} - \frac{(8\,b^3c-5\,ab^2d+2\,a^2be+a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,a^3} - \frac{(8\,b^3c-5\,ab^2d+2\,a^2be+a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4b} + \frac{b^3cx-ab^2dx+a^2bex-a^3fx}{3\,(bx^3+a)a^3b} + \frac{10\,bcx^3-5\,adx^3-2\,ac}{10\,a^3x^5}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/18*(8*b^3*c - 5*
a*b^2*d + 2*a^2*b*e + a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*
b^2)^(2/3)*a^3) - 1/9*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*(-a/b)^(1/3
)*log(abs(x - (-a/b)^(1/3)))/(a^4*b) + 1/3*(b^3*c*x - a*b^2*d*x + a^2*b*e*x
 - a^3*f*x)/((b*x^3 + a)*a^3*b) + 1/10*(10*b*c*x^3 - 5*a*d*x^3 - 2*a*c)/(a^
3*x^5)
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^2}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{9\,a^{11/3}\,b^{4/3}} - \frac{\frac{c}{5\,a}+\frac{x^3\,(5\,a\,d-8\,b\,c)}{10\,a^2}-\frac{x^6\,\left(-2\,f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{6\,a^3\,b}}{b\,x^8+a\,x^5} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{9\,a^{11/3}\,b^{4/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{9\,a^{11/3}\,b^{4/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(8*b^3*c + a^3*f - 5*a*b^2*d + 2*a^2*b*e))/(9*a^(
11/3)*b^(4/3)) - (c/(5*a) + (x^3*(5*a*d - 8*b*c))/(10*a^2) - (x^6*(8*b^3*c
- 2*a^3*f - 5*a*b^2*d + 2*a^2*b*e))/(6*a^3*b))/(a*x^5 + b*x^8) + (log(3^(1/
2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(8*b^3*c + a^
3*f - 5*a*b^2*d + 2*a^2*b*e))/(9*a^(11/3)*b^(4/3)) - (log(3^(1/2)*a^(1/3)*1
i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(8*b^3*c + a^3*f - 5*a*b^
2*d + 2*a^2*b*e))/(9*a^(11/3)*b^(4/3))
```

3.271 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 297

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2}\,dx = -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4(a+bx^3)} + \frac{(10b^3c-7ab^2d+4a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{13/3}b^{2/3}} + \frac{(10b^3c-7ab^2d+4a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{13/3}b^{2/3}} - \frac{(10b^3c-7ab^2d+4a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{13/3}b^{2/3}}$$

```
[Out] -1/7*c/a^2/x^7+1/4*(-a*d+2*b*c)/a^3/x^4+(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x-1/3*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^4/(b*x^3+a)+1/9*(-a^3*f+4*a^2*b*e-7*a*
b^2*d+10*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)/b^(2/3)-1/18*(-a^3*f+4*a^2*b
*e-7*a*b^2*d+10*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(13/3)/b
^(2/3)+1/9*(-a^3*f+4*a^2*b*e-7*a*b^2*d+10*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1843, 1848, 298, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2}\,dx$$
$$=\frac{2bc-ad}{4a^3x^4}-\frac{c}{7a^2x^7}-\frac{a^2e-2abd+3b^2c}{a^4x}$$
$$+\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+4a^2be-7ab^2d+10b^3c)}{3\sqrt{3}a^{13/3}b^{2/3}}$$
$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+4a^2be-7ab^2d+10b^3c)}{18a^{13/3}b^{2/3}}$$
$$+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+4a^2be-7ab^2d+10b^3c)}{9a^{13/3}b^{2/3}}-\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^4(a+bx^3)}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x]
```

```
[Out] -1/7*c/(a^2*x^7) + (2*b*c - a*d)/(4*a^3*x^4) - (3*b^2*c - 2*a*b*d + a^2*e)/
(a^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^4*(a + b*x^3)) + (
(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(13/3)*b^(2/3)) + ((10*b^3*c - 7*a*b^2*d +
4*a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(13/3)*b^(2/3)) - ((10*b^
3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(18*a^(13/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4\,(a+bx^3)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}}{x^8(a+bx^3)}\,dx}{3ab^3}$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4\,(a+bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^8}-\frac{3b^3(-2bc+ad)}{a^2x^5}-\frac{3b^3(3b^2c-2abd+a^2e)}{a^3x^2}-\frac{b^3(-10b^3c+7ab^2d-4a^2be+a^3f)x}{a^3(a+bx^3)}\right)dx}{3ab^3}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4\,(a+bx^3)} - \frac{(10b^3c-7ab^2d+4a^2be-a^3f)\int\frac{x}{a+bx^3}\,dx}{3a^4}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4\,(a+bx^3)} + \frac{(10b^3c-7ab^2d+4a^2be-a^3f)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{13/3}\sqrt[3]{b}} - \frac{(10b^3c-7ab^2d+4a^2be-a^3f)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{13/3}\sqrt[3]{b}}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4\,(a+bx^3)} + \frac{(10b^3c-7ab^2d+4a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{13/3}b^{2/3}} - \frac{(10b^3c-7ab^2d+4a^2be-a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{13/3}b^{2/3}} - \frac{(10b^3c-7ab^2d+4a^2be-a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^4\sqrt[3]{b}}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4\,(a+bx^3)} + \frac{(10b^3c-7ab^2d+4a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{13/3}b^{2/3}} - \frac{(10b^3c-7ab^2d+4a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{13/3}b^{2/3}} - \frac{(10b^3c-7ab^2d+4a^2be-a^3f)\,\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{13/3}b^{2/3}}$$

$$= -\frac{c}{7a^2x^7} + \frac{2bc - ad}{4a^3x^4} - \frac{3b^2c - 2abd + a^2e}{a^4x} - \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{3a^4\,(a + bx^3)} + \frac{(10b^3c - 7ab^2d + 4a^2be - a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{13/3}b^{2/3}} + \frac{(10b^3c - 7ab^2d + 4a^2be - a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{9a^{13/3}b^{2/3}} - \frac{(10b^3c - 7ab^2d + 4a^2be - a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{18a^{13/3}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.95

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^8\,(a + bx^3)^2}\,dx$$

$$= \frac{-\frac{36a^{7/3}c}{x^7} - \frac{63a^{4/3}(-2bc+ad)}{x^4} - \frac{252\sqrt[3]{a}(3b^2c-2abd+a^2e)}{x} + \frac{84\sqrt[3]{a}(-b^3c+ab^2d-a^2be+a^3f)x^2}{a+bx^3} + \frac{28\sqrt{3}(10b^3c-7ab^2d+4a^2be-a^3f)\,\mathrm{a}}{b^{2/3}}}{252a}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x]
```

```
[Out] ((-36*a^(7/3)*c)/x^7 - (63*a^(4/3)*(-2*b*c + a*d))/x^4 - (252*a^(1/3)*(3*b^
2*c - 2*a*b*d + a^2*e))/x + (84*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3
*f)*x^2)/(a + b*x^3) + (28*Sqrt[3]*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*
f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (28*(10*b^3*c - 7
*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (14*(-10*
b^3*c + 7*a*b^2*d - 4*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/b^(2/3))/(252*a^(13/3))
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.72

method	result
default	$-\frac{c}{7a^2x^7}-\frac{ad-2bc}{4a^3x^4}-\frac{a^2e-2abd+3b^2c}{a^4x}+\frac{\frac{\left(\frac{1}{3}fa^3-\frac{1}{3}a^2be+\frac{1}{3}ab^2d-\frac{1}{3}b^3c\right)x^2}{bx^3+a}+\left(-\frac{4}{3}a^2be+\frac{7}{3}ab^2d-\frac{10}{3}b^3c+\frac{1}{3}fa^3\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.}{a^4}$
risch	$\frac{\frac{\left(fa^3-4a^2be+7ab^2d-10b^3c\right)x^9}{3a^4}-\frac{\left(4a^2e-7abd+10b^2c\right)x^6}{4a^3}-\frac{(7ad-10bc)x^3}{28a^2}-\frac{c}{7a}}{x^7(bx^3+a)}+\left(\sum_{\_R=\mathrm{RootOf}\left(a^{13}b^2\_Z^3+a^9f^3-12a^8bef^2+21a^7b^2df^2+48\right.}\right.$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/7*c/a^2/x^7-1/4*(a*d-2*b*c)/a^3/x^4-(a^2*e-2*a*b*d+3*b^2*c)/a^4/x+1/a^4*((1/3*f*a^3-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b^3*c)*x^2/(b*x^3+a)+(-4/3*a^2*b*e+7/3*a*b^2*d-10/3*b^3*c+1/3*f*a^3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 982, normalized size of antiderivative = 3.31

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [-1/252*(84*(10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^3*e - a^4*b^2*f)*x^9 + 36*a^4*b^2*c + 63*(10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e)*x^6 - 9*(10*a^3*b^3*c - 7*a^4*b^2*d)*x^3 + 42*sqrt(1/3)*((10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^3*e - a^4*b^2*f)*x^10 + (10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e - a^5*b*f)*x^7)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 14*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^7)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^7)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^3*x^10 + a^6*b^2*x^7), -1/252*(84*(10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^3*e - a^4*b^2*f)*x

```
^9 + 36*a^4*b^2*c + 63*(10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e)*x^6 - 9*(
10*a^3*b^3*c - 7*a^4*b^2*d)*x^3 + 84*sqrt(1/3)*((10*a*b^5*c - 7*a^2*b^4*d +
 4*a^3*b^3*e - a^4*b^2*f)*x^10 + (10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e
- a^5*b*f)*x^7)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^
(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 14*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e
 - a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^7)*(-a*
b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*((10*b^4
*c - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d +
4*a^3*b*e - a^4*f)*x^7)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^3*
x^10 + a^6*b^2*x^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^2}\,dx=$$
$$-\frac{28\,(10\,b^3c-7\,ab^2d+4\,a^2be-a^3f)x^9+21\,(10\,ab^2c-7\,a^2bd+4\,a^3e)x^6+12\,a^3c-3\,(10\,a^2bc-7\,a^3d)x^3}{84\,(a^4bx^{10}+a^5x^7)}$$
$$-\frac{\sqrt{3}(10\,b^3c-7\,ab^2d+4\,a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$-\frac{(10\,b^3c-7\,ab^2d+4\,a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^4b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+\frac{(10\,b^3c-7\,ab^2d+4\,a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^4b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/84*(28*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*x^9 + 21*(10*a*b^2*c -
 7*a^2*b*d + 4*a^3*e)*x^6 + 12*a^3*c - 3*(10*a^2*b*c - 7*a^3*d)*x^3)/(a^4*b
*x^10 + a^5*x^7) - 1/9*sqrt(3)*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(1/3)) - 1/
18*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/
b)^(2/3))/(a^4*b*(a/b)^(1/3)) + 1/9*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3
*f)*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.10

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8 (a+bx^3)^2} \, dx = -\frac{\sqrt{3}(10\,b^3c-7\,ab^2d+4\,a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}a^4} + \frac{(10\,b^3c-7\,ab^2d+4\,a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}a^4} + \frac{\left(10\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5} - \frac{b^3cx^2-ab^2dx^2+a^2bex^2-a^3fx^2}{3\,(bx^3+a)a^4} - \frac{84\,b^2cx^6-56\,abdx^6+28\,a^2ex^6-14\,abcx^3+7\,a^2dx^3+4\,a^2c}{28\,a^4x^7}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^4) + 1/18*(10*b^3*c -
7*a*b^2*d + 4*a^2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-
a*b^2)^(1/3)*a^4) + 1/9*(10*b^3*c*(-a/b)^(1/3) - 7*a*b^2*d*(-a/b)^(1/3) + 4
*a^2*b*e*(-a/b)^(1/3) - a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/a^5 - 1/3*(b^3*c*x^2 - a*b^2*d*x^2 + a^2*b*e*x^2 - a^3*f*x^2)/((b*
x^3 + a)*a^4) - 1/28*(84*b^2*c*x^6 - 56*a*b*d*x^6 + 28*a^2*e*x^6 - 14*a*b*c
*x^3 + 7*a^2*d*x^3 + 4*a^2*c)/(a^4*x^7)
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2}\,dx$$
$$=\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+4\,e\,a^2\,b-7\,d\,a\,b^2+10\,c\,b^3\right)}{9\,a^{13/3}\,b^{2/3}}-\frac{\frac{c}{7\,a}+\frac{x^9\,\left(-f\,a^3+4\,e\,a^2\,b-7\,d\,a\,b^2+10\,c\,b^3\right)}{3\,a^4}+\frac{x^3\,\left(7\,a\,d-10\,b\,c\right)}{28\,a^2}+\frac{x^6\,\left(4\,e\,a^2-7\,d\,a\,b+10\,c\,b^2\right)}{4\,a^3}}{b\,x^{10}+a\,x^7}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+4\,e\,a^2\,b-7\,d\,a\,b^2+10\,c\,b^3\right)}{9\,a^{13/3}\,b^{2/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+4\,e\,a^2\,b-7\,d\,a\,b^2+10\,c\,b^3\right)}{9\,a^{13/3}\,b^{2/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^2*b*e))/(9*a^
(13/3)*b^(2/3)) - (c/(7*a) + (x^9*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^2*b*e
))/(3*a^4) + (x^3*(7*a*d - 10*b*c))/(28*a^2) + (x^6*(10*b^2*c + 4*a^2*e - 7
*a*b*d))/(4*a^3))/(a*x^7 + b*x^10) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
- a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^2*b*e
))/(9*a^(13/3)*b^(2/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*
((3^(1/2)*1i)/2 - 1/2)*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^2*b*e))/(9*a^(13
/3)*b^(2/3))
```

3.272 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 297

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^2}\,dx = -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^4(a+bx^3)} + \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{14/3}\sqrt[3]{b}} - \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{14/3}\sqrt[3]{b}} + \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{14/3}\sqrt[3]{b}}$$

```
[Out] -1/8*c/a^2/x^8+1/5*(-a*d+2*b*c)/a^3/x^5+1/2*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x^
2-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^4/(b*x^3+a)-1/9*(-2*a^3*f+5*a^2*b*
e-8*a*b^2*d+11*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)/b^(1/3)+1/18*(-2*a^3*f
+5*a^2*b*e-8*a*b^2*d+11*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^
(14/3)/b^(1/3)+1/9*(-2*a^3*f+5*a^2*b*e-8*a*b^2*d+11*b^3*c)*arctan(1/3*(a^(1
/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1843, 1848, 206, 31, 648, 631, 210, 642}

$$\begin{aligned}
&\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx \\
&= \frac{2bc-ad}{5a^3x^5}-\frac{c}{8a^2x^8}-\frac{a^2e-2abd+3b^2c}{2a^4x^2} \\
&+\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{3\sqrt{3}a^{14/3}\sqrt[3]{b}} \\
&-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{9a^{14/3}\sqrt[3]{b}} \\
&+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{18a^{14/3}\sqrt[3]{b}} \\
&-\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^4\left(a+bx^3\right)}
\end{aligned}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x]
```

```
[Out] -1/8*c/(a^2*x^8) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d + a^2*e)/
(2*a^4*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^4*(a + b*x^3)) +
 ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(14/3)*b^(1/3)) - ((11*b^3*c - 8*a*b^2*
d + 5*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(14/3)*b^(1/3)) + (
(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(18*a^(14/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^4(a+bx^3)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{2b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}}{x^9(a+bx^3)}\,dx}{3ab^3}$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^4(a+bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^9}-\frac{3b^3(-2bc+ad)}{a^2x^6}-\frac{3b^3(3b^2c-2abd+a^2e)}{a^3x^3}-\frac{b^3(-11b^3c+8ab^2d-5a^2be+2a^3f)}{a^3(a+bx^3)}\right)dx}{3ab^3}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^4(a+bx^3)} - \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\int\frac{1}{a+bx^3}\,dx}{3a^4}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^4(a+bx^3)} - \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{14/3}} - \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{14/3}}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^4(a+bx^3)} - \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{14/3}\sqrt[3]{b}} - \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{13/3}} + \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{14/3}\sqrt[3]{b}}$$

$$= -\frac{c}{8a^2x^8} + \frac{2bc-ad}{5a^3x^5} - \frac{3b^2c-2abd+a^2e}{2a^4x^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^4(a+bx^3)} - \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{14/3}\sqrt[3]{b}} + \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{14/3}\sqrt[3]{b}} - \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\,\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{14/3}\sqrt[3]{b}}$$

$$= -\frac{c}{8a^2x^8} + \frac{2bc-ad}{5a^3x^5} - \frac{3b^2c-2abd+a^2e}{2a^4x^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^4(a+bx^3)} + \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{14/3}\sqrt[3]{b}} - \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{14/3}\sqrt[3]{b}} + \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{14/3}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{45a^{8/3}c}{x^8} - \frac{72a^{5/3}(-2bc+ad)}{x^5} - \frac{180a^{2/3}(3b^2c-2abd+a^2e)}{x^2} + \frac{120a^{2/3}(-b^3c+ab^2d-a^2be+a^3f)x}{a+bx^3} + \frac{40\sqrt{3}(11b^3c-8ab^2d+5a^2be-2a^3f)\,\mathrm{arc}}{\sqrt[3]{b}}}{360a}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x]
```

```
[Out] ((-45*a^(8/3)*c)/x^8 - (72*a^(5/3)*(-2*b*c + a*d))/x^5 - (180*a^(2/3)*(3*b^
2*c - 2*a*b*d + a^2*e))/x^2 + (120*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*x)/(a + b*x^3) + (40*Sqrt[3]*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a
^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (40*(-11*b^3*c
 + 8*a*b^2*d - 5*a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (20
*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/b^(1/3))/(360*a^(14/3))
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.72

method	result
default	$-\frac{c}{8a^2x^8}-\frac{ad-2bc}{5a^3x^5}-\frac{a^2e-2abd+3b^2c}{2a^4x^2}+\frac{\frac{\left(\frac{1}{3}fa^3-\frac{1}{3}a^2be+\frac{1}{3}ab^2d-\frac{1}{3}b^3c\right)x}{bx^3+a}+\frac{\left(2fa^3-5a^2be+8ab^2d-11b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\right.}{}\right.}{3}}{a^4}$
risch	$\frac{\frac{\left(2fa^3-5a^2be+8ab^2d-11b^3c\right)x^9}{6a^4}-\frac{\left(5a^2e-8abd+11b^2c\right)x^6}{10a^3}-\frac{(8ad-11bc)x^3}{40a^2}-\frac{c}{8a}}{x^8(bx^3+a)}+\left(\sum_{\_R=\operatorname{RootOf}\left(a^{14}b\_Z^3-8a^9f^3+60a^8bef^2-96a^7b^2df^2\right.}\right.$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/8*c/a^2/x^8-1/5*(a*d-2*b*c)/a^3/x^5-1/2*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^2+1/a^4*((1/3*f*a^3-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b^3*c)*x/(b*x^3+a)+1/3*(2*a^3*f-5*a^2*b*e+8*a*b^2*d-11*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 959, normalized size of antiderivative = 3.23

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [-1/360*(60*(11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*f)*x^9 + 45*a^5*b*c + 36*(11*a^3*b^3*c - 8*a^4*b^2*d + 5*a^5*b*e)*x^6 - 9*(11*a^4*b^2*c - 8*a^5*b*d)*x^3 + 60*sqrt(1/3)*((11*a*b^5*c - 8*a^2*b^4*d + 5*a^3*b^3*e - 2*a^4*b^2*f)*x^11 + (11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*f)*x^8)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 20*((11*b^4*c - 8*a*b^3*d + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e - 2*a^4*f)*x^8)*(a^2*b)^(2/3

```
)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*((11*b^4*c - 8*a*b^
3*d + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e
 - 2*a^4*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^2*x^11 +
a^7*b*x^8), -1/360*(60*(11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*
f)*x^9 + 45*a^5*b*c + 36*(11*a^3*b^3*c - 8*a^4*b^2*d + 5*a^5*b*e)*x^6 - 9*(
11*a^4*b^2*c - 8*a^5*b*d)*x^3 + 120*sqrt(1/3)*((11*a*b^5*c - 8*a^2*b^4*d +
5*a^3*b^3*e - 2*a^4*b^2*f)*x^11 + (11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e
 - 2*a^5*b*f)*x^8)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*
x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*((11*b^4*c - 8*a*b^3*d
 + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e -
2*a^4*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a
) + 40*((11*b^4*c - 8*a*b^3*d + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11*a*b^3*c
 - 8*a^2*b^2*d + 5*a^3*b*e - 2*a^4*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b
)^(2/3)))/(a^6*b^2*x^11 + a^7*b*x^8)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx = -\frac{20\,(11\,b^3c-8\,ab^2d+5\,a^2be-2\,a^3f)x^9+12\,(11\,ab^2c-8\,a^2bd+5\,a^3e)x^6+15\,a^3c-3\,(11\,a^2bc-8\,a^3d)x^3}{120\,(a^4bx^{11}+a^5x^8)} - \frac{\sqrt{3}(11\,b^3c-8\,ab^2d+5\,a^2be-2\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(11\,b^3c-8\,ab^2d+5\,a^2be-2\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(11\,b^3c-8\,ab^2d+5\,a^2be-2\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/120*(20*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*x^9 + 12*(11*a*b^2*
c - 8*a^2*b*d + 5*a^3*e)*x^6 + 15*a^3*c - 3*(11*a^2*b*c - 8*a^3*d)*x^3)/(a^
4*b*x^11 + a^5*x^8) - 1/9*sqrt(3)*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3
*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
 + 1/18*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)) - 1/9*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*
e - 2*a^3*f)*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.15

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^2}\,dx = \frac{(11\,b^3c-8\,ab^2d+5\,a^2be-2\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5}$$
$$-\frac{\sqrt{3}\left(11\,(-ab^2)^{\frac{1}{3}}\,b^3c-8\,(-ab^2)^{\frac{1}{3}}\,ab^2d+5\,(-ab^2)^{\frac{1}{3}}\,a^2be-2\,(-ab^2)^{\frac{1}{3}}\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^5b}$$
$$-\frac{b^3cx-ab^2dx+a^2bex-a^3fx}{3\,(bx^3+a)a^4}$$
$$-\frac{\left(11\,(-ab^2)^{\frac{1}{3}}\,b^3c-8\,(-ab^2)^{\frac{1}{3}}\,ab^2d+5\,(-ab^2)^{\frac{1}{3}}\,a^2be-2\,(-ab^2)^{\frac{1}{3}}\,a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^5b}$$
$$-\frac{60\,b^2cx^6-40\,abdx^6+20\,a^2ex^6-16\,abcx^3+8\,a^2dx^3+5\,a^2c}{40\,a^4x^8}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/a^5 - 1/9*sqrt(3)*(11*(-a*b^2)^(1/3)*b^3*c - 8*(-a*b^2)^(1/3)
*a*b^2*d + 5*(-a*b^2)^(1/3)*a^2*b*e - 2*(-a*b^2)^(1/3)*a^3*f)*arctan(1/3*sq
rt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b) - 1/3*(b^3*c*x - a*b^2*d*x
 + a^2*b*e*x - a^3*f*x)/((b*x^3 + a)*a^4) - 1/18*(11*(-a*b^2)^(1/3)*b^3*c -
 8*(-a*b^2)^(1/3)*a*b^2*d + 5*(-a*b^2)^(1/3)*a^2*b*e - 2*(-a*b^2)^(1/3)*a^3
*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) - 1/40*(60*b^2*c*x^6 -
 40*a*b*d*x^6 + 20*a^2*e*x^6 - 16*a*b*c*x^3 + 8*a^2*d*x^3 + 5*a^2*c)/(a^4*x
^8)
```

Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^2}\,dx$$
$$= -\frac{\frac{c}{8a}+\frac{x^9\,(-2\,f\,a^3+5\,e\,a^2\,b-8\,d\,a\,b^2+11\,c\,b^3)}{6\,a^4}+\frac{x^3\,(8\,a\,d-11\,b\,c)}{40\,a^2}+\frac{x^6\,(5\,e\,a^2-8\,d\,a\,b+11\,c\,b^2)}{10\,a^3}}{b\,x^{11}+a\,x^8}$$
$$-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(-2\,f\,a^3+5\,e\,a^2\,b-8\,d\,a\,b^2+11\,c\,b^3)}{9\,a^{14/3}\,b^{1/3}}$$
$$-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(-2\,f\,a^3+5\,e\,a^2\,b-8\,d\,a\,b^2+11\,c\,b^3)}{9\,a^{14/3}\,b^{1/3}}$$
$$+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(-2\,f\,a^3+5\,e\,a^2\,b-8\,d\,a\,b^2+11\,c\,b^3)}{9\,a^{14/3}\,b^{1/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(11
*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(9*a^(14/3)*b^(1/3)) - (log(b^(1
/3)*x + a^(1/3))*(11*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(9*a^(14/3)*
b^(1/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2
 - 1/2)*(11*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(9*a^(14/3)*b^(1/3))
- (c/(8*a) + (x^9*(11*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(6*a^4) + (
x^3*(8*a*d - 11*b*c))/(40*a^2) + (x^6*(11*b^2*c + 5*a^2*e - 8*a*b*d))/(10*a
^3))/(a*x^8 + b*x^11)
```

3.273 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 334

$$\begin{aligned}
&\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^2}\,dx\\
&= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}\\
&\quad+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^5\,(a+bx^3)}\\
&\quad-\frac{\sqrt[3]{b}(13b^3c-10ab^2d+7a^2be-4a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{16/3}}\\
&\quad-\frac{\sqrt[3]{b}(13b^3c-10ab^2d+7a^2be-4a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{16/3}}\\
&\quad+\frac{\sqrt[3]{b}(13b^3c-10ab^2d+7a^2be-4a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{16/3}}
\end{aligned}$$

```
[Out] -1/10*c/a^2/x^10+1/7*(-a*d+2*b*c)/a^3/x^7+1/4*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/
x^4+(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x+1/3*b*(-a^3*f+a^2*b*e-a*b^2*
d+b^3*c)*x^2/a^5/(b*x^3+a)-1/9*b^(1/3)*(-4*a^3*f+7*a^2*b*e-10*a*b^2*d+13*b^
3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(16/3)+1/18*b^(1/3)*(-4*a^3*f+7*a^2*b*e-10*a*b
^2*d+13*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(16/3)-1/9*b^(1/
3)*(-4*a^3*f+7*a^2*b*e-10*a*b^2*d+13*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x
)/a^(1/3)*3^(1/2))/a^(16/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1843, 1848, 298, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2}\,dx$$
$$= \frac{2bc-ad}{7a^3x^7}-\frac{c}{10a^2x^{10}}-\frac{a^2e-2abd+3b^2c}{4a^4x^4}$$
$$-\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-4a^3f+7a^2be-10ab^2d+13b^3c\right)}{3\sqrt{3}a^{16/3}}$$
$$-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-4a^3f+7a^2be-10ab^2d+13b^3c\right)}{9a^{16/3}}$$
$$+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-4a^3f+7a^2be-10ab^2d+13b^3c\right)}{18a^{16/3}}$$
$$+\frac{bx^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{a^5x}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x]
```

```
[Out] -1/10*c/(a^2*x^10) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d + a^2*e
)/(4*a^4*x^4) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) + (b*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^5*(a + b*x^3)) - (b^(1/3)*(13*b^3
*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(3*Sqrt[3]*a^(16/3)) - (b^(1/3)*(13*b^3*c - 10*a*b^2*d + 7*a
^2*b*e - 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(16/3)) + (b^(1/3)*(13*b^3
*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(18*a^(16/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^5\,(a+bx^3)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{3b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}-\frac{b^4\left(b^3c-ab^2d+a^2be-a^3f\right)x^{12}}{a^4}}{x^{11}(a+bx^3)}\,dx}{3ab^3}$$

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^5\,(a+bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^{11}}-\frac{3b^3(-2bc+ad)}{a^2x^8}-\frac{3b^3(3b^2c-2abd+a^2e)}{a^3x^5}-\frac{3b^3(-4b^3c+3ab^2d-2a^2be+a^3f)}{a^4x^2}+\frac{b^4(-13b^3c+10ab^2d-7a^2be+4a^3f)x}{a^4(a+bx^3)}\right)dx}{3ab^3}$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^5\,(a+bx^3)}+\frac{(b(13b^3c-10ab^2d+7a^2be-4a^3f))\int\frac{x}{a+bx^3}\,dx}{3a^5}$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^5\,(a+bx^3)}-\frac{\left(b^{2/3}(13b^3c-10ab^2d+7a^2be-4a^3f)\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{16/3}} + \frac{\left(b^{2/3}(13b^3c-10ab^2d+7a^2be-4a^3f)\right)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{16/3}}$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^5\,(a+bx^3)}-\frac{\sqrt[3]{b}(13b^3c-10ab^2d+7a^2be-4a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{16/3}} + \frac{\left(\sqrt[3]{b}(13b^3c-10ab^2d+7a^2be-4a^3f)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{16/3}} + \frac{\left(b^{2/3}(13b^3c-10ab^2d+7a^2be-4a^3f)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^5}$$

$$= -\frac{c}{10a^2x^{10}} + \frac{2bc-ad}{7a^3x^7} - \frac{3b^2c-2abd+a^2e}{4a^4x^4} + \frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}$$
$$+ \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^5\,(a+bx^3)} - \frac{\sqrt[3]{b}(13b^3c-10ab^2d+7a^2be-4a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{16/3}}$$
$$+ \frac{\sqrt[3]{b}(13b^3c-10ab^2d+7a^2be-4a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{16/3}}$$
$$+ \frac{\left(\sqrt[3]{b}(13b^3c-10ab^2d+7a^2be-4a^3f)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{16/3}}$$

$$= -\frac{c}{10a^2x^{10}} + \frac{2bc-ad}{7a^3x^7} - \frac{3b^2c-2abd+a^2e}{4a^4x^4}$$
$$+ \frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^5\,(a+bx^3)}$$
$$- \frac{\sqrt[3]{b}(13b^3c-10ab^2d+7a^2be-4a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{16/3}}$$
$$- \frac{\sqrt[3]{b}(13b^3c-10ab^2d+7a^2be-4a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{16/3}}$$
$$+ \frac{\sqrt[3]{b}(13b^3c-10ab^2d+7a^2be-4a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{16/3}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{126a^{10/3}c}{x^{10}} - \frac{180a^{7/3}(-2bc+ad)}{x^7} - \frac{315a^{4/3}(3b^2c-2abd+a^2e)}{x^4} - \frac{1260\sqrt[3]{a}(-4b^3c+3ab^2d-2a^2be+a^3f)}{x} - \frac{420\sqrt[3]{a}b(-b^3c+ab^2d-a^2be\cdots}{a+bx^3}}{}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x]
```

```
[Out] ((-126*a^(10/3)*c)/x^10 - (180*a^(7/3)*(-2*b*c + a*d))/x^7 - (315*a^(4/3)*(
3*b^2*c - 2*a*b*d + a^2*e))/x^4 - (1260*a^(1/3)*(-4*b^3*c + 3*a*b^2*d - 2*a
^2*b*e + a^3*f))/x - (420*a^(1/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*
x^2)/(a + b*x^3) - 140*Sqrt[3]*b^(1/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e -
 4*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(1/3)*(-13*b^
3*c + 10*a*b^2*d - 7*a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 70*b^(1/
3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(1260*a^(16/3))
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.75

method	result
default	$-\frac{c}{10a^2x^{10}}-\frac{ad-2bc}{7a^3x^7}-\frac{a^2e-2abd+3b^2c}{4a^4x^4}-\frac{f\,a^3-2a^2be+3a\,b^2d-4b^3c}{a^5x}-\frac{b\left(\frac{\left(\frac{1}{3}f\,a^3-\frac{1}{3}a^2be+\frac{1}{3}a\,b^2d-\frac{1}{3}b^3c\right)x^2}{b\,x^3+a}+\left(\frac{4}{3}f\,a^3-\frac{7}{3}a^2be+\right.\right.}{}$
risch	$\frac{-\frac{b\left(4f\,a^3-7a^2be+10a\,b^2d-13b^3c\right)x^{12}}{3a^5}-\frac{\left(4f\,a^3-7a^2be+10a\,b^2d-13b^3c\right)x^9}{4a^4}-\frac{\left(7a^2e-10abd+13b^2c\right)x^6}{28a^3}-\frac{(10ad-13bc)x^3}{70a^2}-\frac{c}{10a}}{x^{10}\left(b\,x^3+a\right)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}}\right.}{}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*c/a^2/x^10-1/7*(a*d-2*b*c)/a^3/x^7-1/4*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^
4-(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5/x-1/a^5*b*((1/3*f*a^3-1/3*a^2*b*e
+1/3*a*b^2*d-1/3*b^3*c)*x^2/(b*x^3+a)+(4/3*f*a^3-7/3*a^2*b*e+10/3*a*b^2*d-1
3/3*b^3*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(
a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.67 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.32

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2}\,dx = \frac{420\,(13\,b^4c-10\,ab^3d+7\,a^2b^2e-4\,a^3bf)x^{12}+315\,(13\,ab^3c-10\,a^2b^2d+7\,a^3be-4\,a^4f)x^9-45\,(13\,a^2b^2c-}{}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(420*(13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^12 + 315*(1
3*a*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^9 - 45*(13*a^2*b^2*c - 10
*a^3*b*d + 7*a^4*e)*x^6 - 126*a^4*c + 18*(13*a^3*b*c - 10*a^4*d)*x^3 + 140*
sqrt(3)*((13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^13 + (13*a*b^3
*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^10)*(b/a)^(1/3)*arctan(2/3*sqrt(
```

```
3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 70*((13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e
- 4*a^3*b*f)*x^13 + (13*a*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^10)
*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 140*((13*b^4*c
- 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^13 + (13*a*b^3*c - 10*a^2*b^2*d +
 7*a^3*b*e - 4*a^4*f)*x^10)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)))/(a^5*b*x^
13 + a^6*x^10)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2}\,dx = \frac{140\,(13\,b^4c-10\,ab^3d+7\,a^2b^2e-4\,a^3bf)x^{12}+105\,(13\,ab^3c-10\,a^2b^2d+7\,a^3be-4\,a^4f)x^9-15\,(13\,a^2b^2c}{420\,(a^5bx^{13}+a^6x^{10})} + \frac{\sqrt{3}(13\,b^3c-10\,ab^2d+7\,a^2be-4\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^5\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(13\,b^3c-10\,ab^2d+7\,a^2be-4\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^5\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(13\,b^3c-10\,ab^2d+7\,a^2be-4\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^5\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/420*(140*(13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^12 + 105*(13
*a*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^9 - 15*(13*a^2*b^2*c - 10*
a^3*b*d + 7*a^4*e)*x^6 - 42*a^4*c + 6*(13*a^3*b*c - 10*a^4*d)*x^3)/(a^5*b*x
^13 + a^6*x^10) + 1/9*sqrt(3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)
```

```
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(1/3)) + 1/
18*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(a^5*(a/b)^(1/3)) - 1/9*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4
*a^3*f)*log(x + (a/b)^(1/3))/(a^5*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.29

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2}\,dx = -\frac{\left(13\,b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-10\,ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+7\,a^2b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^3bf\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^6} - \frac{\sqrt{3}\left(13\left(-ab^2\right)^{\frac{2}{3}}b^3c-10\left(-ab^2\right)^{\frac{2}{3}}ab^2d+7\left(-ab^2\right)^{\frac{2}{3}}a^2be-4\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^6b} + \frac{b^4cx^2-ab^3dx^2+a^2b^2ex^2-a^3bfx^2}{3\left(bx^3+a\right)a^5} + \frac{\left(13\left(-ab^2\right)^{\frac{2}{3}}b^3c-10\left(-ab^2\right)^{\frac{2}{3}}ab^2d+7\left(-ab^2\right)^{\frac{2}{3}}a^2be-4\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^6b} + \frac{560\,b^3cx^9-420\,ab^2dx^9+280\,a^2bex^9-140\,a^3fx^9-105\,ab^2cx^6+70\,a^2bdx^6-35\,a^3ex^6+40\,a^2bcx^3-20\,}{140\,a^5x^{10}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(13*b^4*c*(-a/b)^(1/3) - 10*a*b^3*d*(-a/b)^(1/3) + 7*a^2*b^2*e*(-a/b)^
(1/3) - 4*a^3*b*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6
 - 1/9*sqrt(3)*(13*(-a*b^2)^(2/3)*b^3*c - 10*(-a*b^2)^(2/3)*a*b^2*d + 7*(-a
*b^2)^(2/3)*a^2*b*e - 4*(-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a
/b)^(1/3))/(-a/b)^(1/3))/(a^6*b) + 1/3*(b^4*c*x^2 - a*b^3*d*x^2 + a^2*b^2*e
*x^2 - a^3*b*f*x^2)/((b*x^3 + a)*a^5) + 1/18*(13*(-a*b^2)^(2/3)*b^3*c - 10*
(-a*b^2)^(2/3)*a*b^2*d + 7*(-a*b^2)^(2/3)*a^2*b*e - 4*(-a*b^2)^(2/3)*a^3*f)
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^6*b) + 1/140*(560*b^3*c*x^9 -
420*a*b^2*d*x^9 + 280*a^2*b*e*x^9 - 140*a^3*f*x^9 - 105*a*b^2*c*x^6 + 70*a^
2*b*d*x^6 - 35*a^3*e*x^6 + 40*a^2*b*c*x^3 - 20*a^3*d*x^3 - 14*a^3*c)/(a^5*x
^10)
```

Mupad [B] (verification not implemented)

Time = 9.80 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2}\,dx = -\frac{\frac{c}{10\,a}-\frac{x^9\,\left(-4\,f\,a^3+7\,e\,a^2\,b-10\,d\,a\,b^2+13\,c\,b^3\right)}{4\,a^4}+\frac{x^3\,\left(10\,a\,d-13\,b\,c\right)}{70\,a^2}+\frac{x^6\,\left(7\,e\,a^2-10\,d\,a\,b+13\,c\,b^2\right)}{28\,a^3}-\frac{b\,x^{12}\,\left(-4\,f\,a^3+7\,e\,a^2\,b-10\,d\,a\,b^2+\right.}{3\,a^5}}{b\,x^{13}+a\,x^{10}}$$
$$-\frac{b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-4\,f\,a^3+7\,e\,a^2\,b-10\,d\,a\,b^2+13\,c\,b^3\right)}{9\,a^{16/3}}$$
$$+\frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-4\,f\,a^3+7\,e\,a^2\,b-10\,d\,a\,b^2+13\,c\,b^3\right)}{9\,a^{16/3}}$$
$$-\frac{b^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-4\,f\,a^3+7\,e\,a^2\,b-10\,d\,a\,b^2+13\,c\,b^3\right)}{9\,a^{16/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x)
```

```
[Out] (b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
1/2)*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2*b*e))/(9*a^(16/3)) - (b^(1/3)
*log(b^(1/3)*x + a^(1/3))*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2*b*e))/(9
*a^(16/3)) - (c/(10*a) - (x^9*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2*b*e)
)/(4*a^4) + (x^3*(10*a*d - 13*b*c))/(70*a^2) + (x^6*(13*b^2*c + 7*a^2*e - 1
0*a*b*d))/(28*a^3) - (b*x^12*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2*b*e))
/(3*a^5))/(a*x^10 + b*x^13) - (b^(1/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x
 + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2
*b*e))/(9*a^(16/3))
```

3.274 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 335

$$
\begin{aligned}
\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^2}\,dx
&= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5} \\
&\quad+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^5(a+bx^3)} \\
&\quad-\frac{b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{17/3}} \\
&\quad+\frac{b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{17/3}} \\
&\quad-\frac{b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{17/3}}
\end{aligned}
$$

```
[Out] -1/11*c/a^2/x^11+1/8*(-a*d+2*b*c)/a^3/x^8+1/5*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/
x^5+1/2*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x^2+1/3*b*(-a^3*f+a^2*b*e-
a*b^2*d+b^3*c)*x/a^5/(b*x^3+a)+1/9*b^(2/3)*(-5*a^3*f+8*a^2*b*e-11*a*b^2*d+1
4*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(17/3)-1/18*b^(2/3)*(-5*a^3*f+8*a^2*b*e-11
*a*b^2*d+14*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(17/3)-1/9*b
^(2/3)*(-5*a^3*f+8*a^2*b*e-11*a*b^2*d+14*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))/a^(17/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1843, 1848, 206, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^2}\,dx$$
$$=\frac{2bc-ad}{8a^3x^8}-\frac{c}{11a^2x^{11}}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}$$
$$-\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-5a^3f+8a^2be-11ab^2d+14b^3c)}{3\sqrt{3}a^{17/3}}$$
$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-5a^3f+8a^2be-11ab^2d+14b^3c)}{18a^{17/3}}$$
$$+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-5a^3f+8a^2be-11ab^2d+14b^3c)}{9a^{17/3}}$$
$$+\frac{bx(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^5(a+bx^3)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{2a^5x^2}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x]
```

```
[Out] -1/11*c/(a^2*x^11) + (2*b*c - a*d)/(8*a^3*x^8) - (3*b^2*c - 2*a*b*d + a^2*e
)/(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(2*a^5*x^2) + (b*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^5*(a + b*x^3)) - (b^(2/3)*(14*b
^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(3*Sqrt[3]*a^(17/3)) + (b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8
*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(17/3)) - (b^(2/3)*(14*b
^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(18*a^(17/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^5\,(a+bx^3)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3(b^2c-abd+a^2e)x^6}{a^2}+\frac{3b^3(b^3c-ab^2d+a^2be-a^3f)x^9}{a^3}-\frac{2b^4(b^3c-ab^2d+a^2be-a^3f)x^{12}}{a^4}}{x^{12}(a+bx^3)}\,dx}{3ab^3}$$

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^5\,(a+bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^{12}}-\frac{3b^3(-2bc+ad)}{a^2x^9}-\frac{3b^3(3b^2c-2abd+a^2e)}{a^3x^6}-\frac{3b^3(-4b^3c+3ab^2d-2a^2be+a^3f)}{a^4x^3}+\frac{b^4(-14b^3c+11ab^2d-8a^2be+5a^3f)}{a^4(a+bx^3)}\right)dx}{3ab^3}$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^5\,(a+bx^3)}+\frac{(b(14b^3c-11ab^2d+8a^2be-5a^3f))\int \frac{1}{a+bx^3}\,dx}{3a^5}$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^5\,(a+bx^3)}+\frac{(b(14b^3c-11ab^2d+8a^2be-5a^3f))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{17/3}} + \frac{(b(14b^3c-11ab^2d+8a^2be-5a^3f))\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{17/3}}$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^5\,(a+bx^3)}+\frac{b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{17/3}} - \frac{\left(b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{17/3}} + \frac{(b(14b^3c-11ab^2d+8a^2be-5a^3f))\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{16/3}}$$

$$= -\frac{c}{11a^2x^{11}} + \frac{2bc-ad}{8a^3x^8} - \frac{3b^2c-2abd+a^2e}{5a^4x^5} + \frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^5\,(a+bx^3)} + \frac{b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{17/3}} - \frac{b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{17/3}} + \frac{\left(b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{17/3}}$$

$$= -\frac{c}{11a^2x^{11}} + \frac{2bc-ad}{8a^3x^8} - \frac{3b^2c-2abd+a^2e}{5a^4x^5} + \frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^5\,(a+bx^3)} - \frac{b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{17/3}} + \frac{b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{17/3}} - \frac{b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{17/3}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{360a^{11/3}c}{x^{11}} - \frac{495a^{8/3}(-2bc+ad)}{x^8} - \frac{792a^{5/3}(3b^2c-2abd+a^2e)}{x^5} - \frac{1980a^{2/3}(-4b^3c+3ab^2d-2a^2be+a^3f)}{x^2} - \frac{1320a^{2/3}b(-b^3c+ab^2d-a^2be+}{a+bx^3}}{}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x]
```

```
[Out] ((-360*a^(11/3)*c)/x^11 - (495*a^(8/3)*(-2*b*c + a*d))/x^8 - (792*a^(5/3)*(
3*b^2*c - 2*a*b*d + a^2*e))/x^5 - (1980*a^(2/3)*(-4*b^3*c + 3*a*b^2*d - 2*a
^2*b*e + a^3*f))/x^2 - (1320*a^(2/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*x)/(a + b*x^3) - 440*Sqrt[3]*b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e
- 5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 440*b^(2/3)*(14*b^
3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*b^(2
/3)*(-14*b^3*c + 11*a*b^2*d - 8*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(3960*a^(17/3))
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.75

method	result
default	$-\frac{c}{11a^2x^{11}}-\frac{ad-2bc}{8a^3x^8}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}-\frac{f\,a^3-2a^2be+3a\,b^2d-4b^3c}{2a^5x^2}-\frac{b\left(\frac{\left(\frac{1}{3}f\,a^3-\frac{1}{3}a^2be+\frac{1}{3}a\,b^2d-\frac{1}{3}b^3c\right)x}{b\,x^3+a}+\frac{\left(5f\,a^3-8a^2be+11\right.}{}\right.}{}$
risch	$\frac{-\frac{b\left(5f\,a^3-8a^2be+11a\,b^2d-14b^3c\right)x^{12}}{6a^5}-\frac{\left(5f\,a^3-8a^2be+11a\,b^2d-14b^3c\right)x^9}{10a^4}-\frac{\left(8a^2e-11abd+14b^2c\right)x^6}{40a^3}-\frac{(11ad-14bc)x^3}{88a^2}-\frac{c}{11a}}{x^{11}(b\,x^3+a)}+\frac{\left(\_R=\text{Root}\right.}{}$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/11*c/a^2/x^11-1/8*(a*d-2*b*c)/a^3/x^8-1/5*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^5-1/2*(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5/x^2-1/a^5*b*((1/3*f*a^3-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b^3*c)*x/(b*x^3+a)+1/3*(5*a^3*f-8*a^2*b*e+11*a*b^2*d-14*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.42

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx$$

$$=\frac{660\,(14\,b^4c-11\,ab^3d+8\,a^2b^2e-5\,a^3bf)x^{12}+396\,(14\,ab^3c-11\,a^2b^2d+8\,a^3be-5\,a^4f)x^9-99\,(14\,a^2b^2c}{}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3960*(660*(14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^12 + 396*(1
4*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^9 - 99*(14*a^2*b^2*c - 11
*a^3*b*d + 8*a^4*e)*x^6 - 360*a^4*c + 45*(14*a^3*b*c - 11*a^4*d)*x^3 - 440*
sqrt(3)*((14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^14 + (14*a*b^3
*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^11)*(-b^2/a^2)^(1/3)*arctan(1/3*
(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 220*((14*b^4*c - 11*a*b^3
*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e
 - 5*a^4*f)*x^11)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a
^2*(-b^2/a^2)^(2/3)) - 440*((14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*
f)*x^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^11)*(-b^2/a^2
)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^5*b*x^14 + a^6*x^11)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx$$
$$=\frac{220\,(14\,b^4c-11\,ab^3d+8\,a^2b^2e-5\,a^3bf)x^{12}+132\,(14\,ab^3c-11\,a^2b^2d+8\,a^3be-5\,a^4f)x^9-33\,(14\,a^2b^2c-}{1320\,(a^5bx^{14}+a^6x^{11})}$$
$$+\frac{\sqrt{3}(14\,b^3c-11\,ab^2d+8\,a^2be-5\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(14\,b^3c-11\,ab^2d+8\,a^2be-5\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(14\,b^3c-11\,ab^2d+8\,a^2be-5\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1320*(220*(14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^12 + 132*(1
4*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^9 - 33*(14*a^2*b^2*c - 11
*a^3*b*d + 8*a^4*e)*x^6 - 120*a^4*c + 15*(14*a^3*b*c - 11*a^4*d)*x^3)/(a^5*
b*x^14 + a^6*x^11) + 1/9*sqrt(3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3
*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(2/3)) -
 1/18*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*log(x^2 - x*(a/b)^(1/3)
 + (a/b)^(2/3))/(a^5*(a/b)^(2/3)) + 1/9*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e
- 5*a^3*f)*log(x + (a/b)^(1/3))/(a^5*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.15

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx$$
$$= \frac{\sqrt{3}\left(14\left(-ab^2\right)^{\frac{1}{3}}b^3c-11\left(-ab^2\right)^{\frac{1}{3}}ab^2d+8\left(-ab^2\right)^{\frac{1}{3}}a^2be-5\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^6}$$
$$- \frac{\left(14b^4c-11ab^3d+8a^2b^2e-5a^3bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^6}$$
$$+ \frac{\left(14\left(-ab^2\right)^{\frac{1}{3}}b^3c-11\left(-ab^2\right)^{\frac{1}{3}}ab^2d+8\left(-ab^2\right)^{\frac{1}{3}}a^2be-5\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^6}$$
$$+ \frac{b^4cx-ab^3dx+a^2b^2ex-a^3bfx}{3\left(bx^3+a\right)a^5}$$
$$+ \frac{880b^3cx^9-660ab^2dx^9+440a^2bex^9-220a^3fx^9-264ab^2cx^6+176a^2bdx^6-88a^3ex^6+110a^2bcx^3-}{440a^5x^{11}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(14*(-a*b^2)^(1/3)*b^3*c - 11*(-a*b^2)^(1/3)*a*b^2*d + 8*(-a*b^
2)^(1/3)*a^2*b*e - 5*(-a*b^2)^(1/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/a^6 - 1/9*(14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^
3*b*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6 + 1/18*(14*(-a*b^2)^(1/3
)*b^3*c - 11*(-a*b^2)^(1/3)*a*b^2*d + 8*(-a*b^2)^(1/3)*a^2*b*e - 5*(-a*b^2)
^(1/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^6 + 1/3*(b^4*c*x -
 a*b^3*d*x + a^2*b^2*e*x - a^3*b*f*x)/((b*x^3 + a)*a^5) + 1/440*(880*b^3*c*
x^9 - 660*a*b^2*d*x^9 + 440*a^2*b*e*x^9 - 220*a^3*f*x^9 - 264*a*b^2*c*x^6 +
 176*a^2*b*d*x^6 - 88*a^3*e*x^6 + 110*a^2*b*c*x^3 - 55*a^3*d*x^3 - 40*a^3*c
)/(a^5*x^11)
```

Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx = \frac{b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-5\,f\,a^3+8\,e\,a^2\,b-11\,d\,a\,b^2+14\,c\,b^3\right)}{9\,a^{17/3}} - \frac{\frac{c}{11\,a}-\frac{x^9\,\left(-5\,f\,a^3+8\,e\,a^2\,b-11\,d\,a\,b^2+14\,c\,b^3\right)}{10\,a^4}+\frac{x^3\,\left(11\,a\,d-14\,b\,c\right)}{88\,a^2}+\frac{x^6\,\left(8\,e\,a^2-11\,d\,a\,b+14\,c\,b^2\right)}{40\,a^3}-\frac{b\,x^{12}\,\left(-5\,f\,a^3+8\,e\,a^2\,b-11\,d\,a\,b^2+14\right.}{6\,a^5}}{b\,x^{14}+a\,x^{11}} + \frac{b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-5\,f\,a^3+8\,e\,a^2\,b-11\,d\,a\,b^2+14\,c\,b^3\right)}{9\,a^{17/3}} - \frac{b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-5\,f\,a^3+8\,e\,a^2\,b-11\,d\,a\,b^2+14\,c\,b^3\right)}{9\,a^{17/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x)
```

```
[Out] (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(14*b^3*c - 5*a^3*f - 11*a*b^2*d + 8*a^2*
b*e))/(9*a^(17/3)) - (c/(11*a) - (x^9*(14*b^3*c - 5*a^3*f - 11*a*b^2*d + 8*
a^2*b*e))/(10*a^4) + (x^3*(11*a*d - 14*b*c))/(88*a^2) + (x^6*(14*b^2*c + 8*
a^2*e - 11*a*b*d))/(40*a^3) - (b*x^12*(14*b^3*c - 5*a^3*f - 11*a*b^2*d + 8*
a^2*b*e))/(6*a^5))/(a*x^11 + b*x^14) + (b^(2/3)*log(3^(1/2)*a^(1/3)*1i + 2*
b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(14*b^3*c - 5*a^3*f - 11*a*b^2*
d + 8*a^2*b*e))/(9*a^(17/3)) - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*
x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(14*b^3*c - 5*a^3*f - 11*a*b^2*d + 8*a^
2*b*e))/(9*a^(17/3))
```

3.275 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 375

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx = -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4} - \frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{a^6x}-\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^6\left(a+bx^3\right)} + \frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{19/3}} + \frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{19/3}} - \frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{19/3}}$$

```
[Out] -1/13*c/a^2/x^13+1/10*(-a*d+2*b*c)/a^3/x^10+1/7*(-a^2*e+2*a*b*d-3*b^2*c)/a^
4/x^7+1/4*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x^4-b*(-2*a^3*f+3*a^2*b*
e-4*a*b^2*d+5*b^3*c)/a^6/x-1/3*b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^6/(
b*x^3+a)+1/9*b^(4/3)*(-7*a^3*f+10*a^2*b*e-13*a*b^2*d+16*b^3*c)*ln(a^(1/3)+b
^(1/3)*x)/a^(19/3)-1/18*b^(4/3)*(-7*a^3*f+10*a^2*b*e-13*a*b^2*d+16*b^3*c)*l
n(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(19/3)+1/9*b^(4/3)*(-7*a^3*f+10*
a^2*b*e-13*a*b^2*d+16*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/
2))/a^(19/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1843, 1848, 298, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx$$
$$= \frac{2bc-ad}{10a^3x^{10}} - \frac{c}{13a^2x^{13}} - \frac{a^2e-2abd+3b^2c}{7a^4x^7} + \frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-7a^3f+10a^2be-13ab^2d+16b^3c\right)}{3\sqrt{3}a^{19/3}} - \frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-7a^3f+10a^2be-13ab^2d+16b^3c\right)}{18a^{19/3}} + \frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-7a^3f+10a^2be-13ab^2d+16b^3c\right)}{9a^{19/3}} - \frac{b^2x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^6\left(a+bx^3\right)} - \frac{b(-2a^3f+3a^2be-4ab^2d+5b^3c)}{a^6x} + \frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{4a^5x^4}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x]
```

```
[Out] -1/13*c/(a^2*x^13) + (2*b*c - a*d)/(10*a^3*x^10) - (3*b^2*c - 2*a*b*d + a^2
*e)/(7*a^4*x^7) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(4*a^5*x^4) - (
b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(a^6*x) - (b^2*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*x^2)/(3*a^6*(a + b*x^3)) + (b^(4/3)*(16*b^3*c - 13*a
*b^2*d + 10*a^2*b*e - 7*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(3*Sqrt[3]*a^(19/3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e
- 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(19/3)) - (b^(4/3)*(16*b^3*c - 13
*a*b^2*d + 10*a^2*b*e - 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(18*a^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^6\,(a+bx^3)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{3b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}-\frac{3b^4\left(b^3c-ab^2d+a^2be-a^3f\right)x^{12}}{a^4}+\frac{b^5\left(b^3c-ab^2d+a^2be-a^3f\right)x^{15}}{a^5}}{x^{14}(a+bx^3)}\,dx}{3ab^3}$$

$$= -\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^6\,(a+bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^{14}}-\frac{3b^3(-2bc+ad)}{a^2x^{11}}-\frac{3b^3(3b^2c-2abd+a^2e)}{a^3x^8}-\frac{3b^3(-4b^3c+3ab^2d-2a^2be+a^3f)}{a^4x^5}+\frac{3b^4(-5b^3c+4ab^2d-3a^2be+2a^3f)}{a^5x^2}\right.}{3ab^3}$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4} - \frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{a^6x}-\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^6\,(a+bx^3)} - \frac{\left(b^2(16b^3c-13ab^2d+10a^2be-7a^3f)\right)\int\frac{x}{a+bx^3}\,dx}{3a^6}$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4} - \frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{a^6x}-\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^6\,(a+bx^3)} + \frac{\left(b^{5/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{19/3}} - \frac{\left(b^{5/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\right)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{19/3}}$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4} - \frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{a^6x}-\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^6\,(a+bx^3)} + \frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{19/3}} - \frac{\left(b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{19/3}} - \frac{\left(b^{5/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^6}$$

$$= -\frac{c}{13a^2x^{13}} + \frac{2bc-ad}{10a^3x^{10}} - \frac{3b^2c-2abd+a^2e}{7a^4x^7} + \frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4} - \frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{a^6x} - \frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^6\,(a+bx^3)} + \frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{19/3}} - \frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{19/3}} - \frac{\left(b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{19/3}}$$

$$= -\frac{c}{13a^2x^{13}} + \frac{2bc-ad}{10a^3x^{10}} - \frac{3b^2c-2abd+a^2e}{7a^4x^7} + \frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4} - \frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{a^6x} - \frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^6\,(a+bx^3)} + \frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{19/3}} + \frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{19/3}} - \frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{19/3}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.99

$$
\begin{aligned}
&\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx \\
&= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4} \\
&\quad+\frac{b(-5b^3c+4ab^2d-3a^2be+2a^3f)}{a^6x}+\frac{b^2(-b^3c+ab^2d-a^2be+a^3f)\,x^2}{3a^6\left(a+bx^3\right)} \\
&\quad+\frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{19/3}} \\
&\quad+\frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{19/3}} \\
&\quad+\frac{b^{4/3}(-16b^3c+13ab^2d-10a^2be+7a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{19/3}}
\end{aligned}
$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x]
```

```
[Out] -1/13*c/(a^2*x^13) + (2*b*c - a*d)/(10*a^3*x^10) - (3*b^2*c - 2*a*b*d + a^2
*e)/(7*a^4*x^7) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(4*a^5*x^4) + (
b*(-5*b^3*c + 4*a*b^2*d - 3*a^2*b*e + 2*a^3*f))/(a^6*x) + (b^2*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f)*x^2)/(3*a^6*(a + b*x^3)) + (b^(4/3)*(16*b^3*c -
13*a*b^2*d + 10*a^2*b*e - 7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]])/(3*Sqrt[3]*a^(19/3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e -
7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(19/3)) + (b^(4/3)*(-16*b^3*c + 13*
a*b^2*d - 10*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(18*a^(19/3))
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.77

method	result
default	$-\frac{c}{13a^2x^{13}}-\frac{ad-2bc}{10a^3x^{10}}-\frac{a^2e-2abd+3b^2c}{7a^4x^7}-\frac{f a^3-2a^2be+3a b^2d-4b^3c}{4a^5x^4}+\frac{b(2f a^3-3a^2be+4a b^2d-5b^3c)}{a^6x}+\frac{b^2\left(\frac{\left(\frac{1}{3}f a^3-\frac{1}{3}a^2\right.}{}\right.}{}$
risch	$\frac{-\frac{c}{13a}-\frac{(13ad-16bc)x^3}{130a^2}-\frac{\left(10a^2e-13abd+16b^2c\right)x^6}{70a^3}-\frac{\left(7f a^3-10a^2be+13a b^2d-16b^3c\right)x^9}{28a^4}+\frac{b\left(7f a^3-10a^2be+13a b^2d-16b^3c\right)x^{12}}{4a^5}+\frac{b^2\left(7f a^3-10\right.}{}}{x^{13}(b x^3+a)}$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

```
[Out] -1/13*c/a^2/x^13-1/10*(a*d-2*b*c)/a^3/x^10-1/7*(a^2*e-2*a*b*d+3*b^2*c)/a^4/
x^7-1/4*(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5/x^4+b*(2*a^3*f-3*a^2*b*e+4*
a*b^2*d-5*b^3*c)/a^6/x+b^2/a^6*((1/3*f*a^3-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b^3*
c)*x^2/(b*x^3+a)+(7/3*f*a^3-10/3*a^2*b*e+13/3*a*b^2*d-16/3*b^3*c)*(-1/3/b/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.35

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx = -\frac{5460\,(16\,b^5c-13\,ab^4d+10\,a^2b^3e-7\,a^3b^2f)x^{15}+4095\,(16\,ab^4c-13\,a^2b^3d+10\,a^3b^2e-7\,a^4bf)x^{12}-58}{}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16380*(5460*(16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^15 +
4095*(16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^12 - 585*(16*
a^2*b^3*c - 13*a^3*b^2*d + 10*a^4*b*e - 7*a^5*f)*x^9 + 234*(16*a^3*b^2*c -
13*a^4*b*d + 10*a^5*e)*x^6 + 1260*a^5*c - 126*(16*a^4*b*c - 13*a^5*d)*x^3 +
 1820*sqrt(3)*((16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^16 +
```

```
(16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^13)*(-b/a)^(1/3)*a
rctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*((16*b^5*c - 13*a*b^4
*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^16 + (16*a*b^4*c - 13*a^2*b^3*d + 10*a^3
*b^2*e - 7*a^4*b*f)*x^13)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b
/a)^(1/3)) + 1820*((16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^1
6 + (16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^13)*(-b/a)^(1/
3)*log(b*x + a*(-b/a)^(2/3)))/(a^6*b*x^16 + a^7*x^13)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx = -\frac{1820\,(16\,b^5c-13\,ab^4d+10\,a^2b^3e-7\,a^3b^2f)x^{15}+1365\,(16\,ab^4c-13\,a^2b^3d+10\,a^3b^2e-7\,a^4bf)x^{12}-195}{5460} - \frac{\sqrt{3}(16\,b^4c-13\,ab^3d+10\,a^2b^2e-7\,a^3bf)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(16\,b^4c-13\,ab^3d+10\,a^2b^2e-7\,a^3bf)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(16\,b^4c-13\,ab^3d+10\,a^2b^2e-7\,a^3bf)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/5460*(1820*(16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^15 + 1
365*(16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^12 - 195*(16*a
^2*b^3*c - 13*a^3*b^2*d + 10*a^4*b*e - 7*a^5*f)*x^9 + 78*(16*a^3*b^2*c - 13
```

*a^4*b*d + 10*a^5*e)*x^6 + 420*a^5*c - 42*(16*a^4*b*c - 13*a^5*d)*x^3)/(a^6*b*x^16 + a^7*x^13) - 1/9*sqrt(3)*(16*b^4*c - 13*a*b^3*d + 10*a^2*b^2*e - 7*a^3*b*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^6*(a/b)^(1/3)) - 1/18*(16*b^4*c - 13*a*b^3*d + 10*a^2*b^2*e - 7*a^3*b*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^(1/3)) + 1/9*(16*b^4*c - 13*a*b^3*d + 10*a^2*b^2*e - 7*a^3*b*f)*log(x + (a/b)^(1/3))/(a^6*(a/b)^(1/3))

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.27

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^2}\,dx$$
$$= \frac{\sqrt{3}\left(16\,(-ab^2)^{\frac{2}{3}}\,b^3c-13\,(-ab^2)^{\frac{2}{3}}\,ab^2d+10\,(-ab^2)^{\frac{2}{3}}\,a^2be-7\,(-ab^2)^{\frac{2}{3}}\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^7}$$
$$+\frac{\left(16\,b^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-13\,ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+10\,a^2b^3e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7\,a^3b^2f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^7}$$
$$-\frac{\left(16\,(-ab^2)^{\frac{2}{3}}\,b^3c-13\,(-ab^2)^{\frac{2}{3}}\,ab^2d+10\,(-ab^2)^{\frac{2}{3}}\,a^2be-7\,(-ab^2)^{\frac{2}{3}}\,a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^7}$$
$$-\frac{b^5cx^2-ab^4dx^2+a^2b^3ex^2-a^3b^2fx^2}{3\,(bx^3+a)a^6}$$
$$-\frac{9100\,b^4cx^{12}-7280\,ab^3dx^{12}+5460\,a^2b^2ex^{12}-3640\,a^3bfx^{12}-1820\,ab^3cx^9+1365\,a^2b^2dx^9-910\,a^3bex^9}{1820\,a^6x^{13}}$$

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/9*sqrt(3)*(16*(-a*b^2)^(2/3)*b^3*c - 13*(-a*b^2)^(2/3)*a*b^2*d + 10*(-a*b^2)^(2/3)*a^2*b*e - 7*(-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^7 + 1/9*(16*b^5*c*(-a/b)^(1/3) - 13*a*b^4*d*(-a/b)^(1/3) + 10*a^2*b^3*e*(-a/b)^(1/3) - 7*a^3*b^2*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^7 - 1/18*(16*(-a*b^2)^(2/3)*b^3*c - 13*(-a*b^2)^(2/3)*a*b^2*d + 10*(-a*b^2)^(2/3)*a^2*b*e - 7*(-a*b^2)^(2/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^7 - 1/3*(b^5*c*x^2 - a*b^4*d*x^2 + a^2*b^3*e*x^2 - a^3*b^2*f*x^2)/((b*x^3 + a)*a^6) - 1/1820*(9100*b^4*c*x^12 - 7280*a*b^3*d*x^12 + 5460*a^2*b^2*e*x^12 - 3640*a^3*b*f*x^12 - 1820*a*b^3*c*x^9 + 1365*a^2*b^2*d*x^9 - 910*a^3*b*e*x^9 + 455*a^4*f*x^9 + 780*a^2*b^2*c*x^6 - 520*a^3*b*d*x^6 + 260*a^4*e*x^6 - 364*a^3*b*c*x^3 + 182*a^4*d*x^3 + 140*a^4*c)/(a^6*x^13)

Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx = \frac{b^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^{19/3}} - \frac{\frac{c}{13\,a}-\frac{x^9\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{28\,a^4}+\frac{x^3\,\left(13\,a\,d-16\,b\,c\right)}{130\,a^2}+\frac{x^6\,\left(10\,e\,a^2-13\,d\,a\,b+16\,c\,b^2\right)}{70\,a^3}+\frac{b\,x^{12}\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2\right.}{4\,a^5}}{b\,x^{16}+a\,x^{13}} - \frac{b^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^{19/3}} + \frac{b^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^{19/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x)
```

```
[Out] (b^(4/3)*log(b^(1/3)*x + a^(1/3))*(16*b^3*c - 7*a^3*f - 13*a*b^2*d + 10*a^2
*b*e))/(9*a^(19/3)) - (c/(13*a) - (x^9*(16*b^3*c - 7*a^3*f - 13*a*b^2*d + 1
0*a^2*b*e))/(28*a^4) + (x^3*(13*a*d - 16*b*c))/(130*a^2) + (x^6*(16*b^2*c +
 10*a^2*e - 13*a*b*d))/(70*a^3) + (b*x^12*(16*b^3*c - 7*a^3*f - 13*a*b^2*d
+ 10*a^2*b*e))/(4*a^5) + (b^2*x^15*(16*b^3*c - 7*a^3*f - 13*a*b^2*d + 10*a^
2*b*e))/(3*a^6))/(a*x^13 + b*x^16) - (b^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^
(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(16*b^3*c - 7*a^3*f - 13*a*b^2*d
+ 10*a^2*b*e))/(9*a^(19/3)) + (b^(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x
 + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(16*b^3*c - 7*a^3*f - 13*a*b^2*d + 10*a^
2*b*e))/(9*a^(19/3))
```

3.276 $\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 266

$$\begin{aligned}\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x^3}{3b^7}\\ &+\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^6}{6b^6}\\ &+\frac{(b^2d-3abe+6a^2f)\,x^9}{9b^5}+\frac{(be-3af)x^{12}}{12b^4}\\ &+\frac{fx^{15}}{15b^3}-\frac{a^4(b^3c-ab^2d+a^2be-a^3f)}{6b^8\,(a+bx^3)^2}\\ &+\frac{a^3(4b^3c-5ab^2d+6a^2be-7a^3f)}{3b^8\,(a+bx^3)}\\ &+\frac{a^2(6b^3c-10ab^2d+15a^2be-21a^3f)\log(a+bx^3)}{3b^8}\end{aligned}$$

```
[Out] -1/3*a*(-15*a^3*f+10*a^2*b*e-6*a*b^2*d+3*b^3*c)*x^3/b^7+1/6*(-10*a^3*f+6*a^
2*b*e-3*a*b^2*d+b^3*c)*x^6/b^6+1/9*(6*a^2*f-3*a*b*e+b^2*d)*x^9/b^5+1/12*(-3
*a*f+b*e)*x^12/b^4+1/15*f*x^15/b^3-1/6*a^4*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b
^8/(b*x^3+a)^2+1/3*a^3*(-7*a^3*f+6*a^2*b*e-5*a*b^2*d+4*b^3*c)/b^8/(b*x^3+a)
+1/3*a^2*(-21*a^3*f+15*a^2*b*e-10*a*b^2*d+6*b^3*c)*ln(b*x^3+a)/b^8
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{x^9(6a^2f-3abe+b^2d)}{9b^5} + \frac{a^3(-7a^3f+6a^2be-5ab^2d+4b^3c)}{3b^8\,(a+bx^3)} + \frac{a^2\log(a+bx^3)\,(-21a^3f+15a^2be-10ab^2d+6b^3c)}{3b^8} - \frac{ax^3(-15a^3f+10a^2be-6ab^2d+3b^3c)}{3b^7} + \frac{x^6(-10a^3f+6a^2be-3ab^2d+b^3c)}{6b^6} - \frac{a^4(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^8\,(a+bx^3)^2} + \frac{x^{12}(be-3af)}{12b^4} + \frac{fx^{15}}{15b^3}$$

```
[In] Int[(x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] -1/3*(a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x^3)/b^7 + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^6)/(6*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*
f)*x^9)/(9*b^5) + ((b*e - 3*a*f)*x^12)/(12*b^4) + (f*x^15)/(15*b^3) - (a^4*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^8*(a + b*x^3)^2) + (a^3*(4*b^3*c
- 5*a*b^2*d + 6*a^2*b*e - 7*a^3*f))/(3*b^8*(a + b*x^3)) + (a^2*(6*b^3*c - 1
0*a*b^2*d + 15*a^2*b*e - 21*a^3*f)*Log[a + b*x^3])/(3*b^8)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx)^3}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-3b^3c+6ab^2d-10a^2be+15a^3f)}{b^7} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6} + \frac{(b^2d-3abe+6a^2f)\,x^2}{b^5} + \frac{(be-3af)x^3}{b^4} + \frac{fx^4}{b^3} - \frac{a^4(-b^3c+ab^2d-a^2be+a^3f)}{b^7(a+bx)^3} + \frac{a^3(-4b^3c+5ab^2d-6a^2be+7a^3f)}{b^7(a+bx)^2} - \frac{a^2(-6b^3c+10ab^2d-15a^2be+21a^3f)}{b^7(a+bx)}\right) dx, x, x^3\right)$$
$$= -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x^3}{3b^7} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^6}{6b^6} + \frac{(b^2d-3abe+6a^2f)\,x^9}{9b^5} + \frac{(be-3af)x^{12}}{12b^4} + \frac{fx^{15}}{15b^3} - \frac{a^4(b^3c-ab^2d+a^2be-a^3f)}{6b^8\,(a+bx^3)^2} + \frac{a^3(4b^3c-5ab^2d+6a^2be-7a^3f)}{3b^8\,(a+bx^3)} + \frac{a^2(6b^3c-10ab^2d+15a^2be-21a^3f)\log\,(a+bx^3)}{3b^8}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.92

$$\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{60ab(-3b^3c+6ab^2d-10a^2be+15a^3f)\,x^3+30b^2(b^3c-3ab^2d+6a^2be-10a^3f)\,x^6+20b^3(b^2d-3abe+6\cdots}{}$$

```
[In] Integrate[(x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (60*a*b*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x^3 + 30*b^2*(b^3*c
- 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^6 + 20*b^3*(b^2*d - 3*a*b*e + 6*a^2*f
)*x^9 + 15*b^4*(b*e - 3*a*f)*x^12 + 12*b^5*f*x^15 + (30*a^4*(-(b^3*c) + a*b
^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 - (60*a^3*(-4*b^3*c + 5*a*b^2*d - 6*
a^2*b*e + 7*a^3*f))/(a + b*x^3) + 60*a^2*(6*b^3*c - 10*a*b^2*d + 15*a^2*b*e
 - 21*a^3*f)*Log[a + b*x^3])/(180*b^8)
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.98

method	result
norman	$\frac{-\frac{a^{2}\left(21 f\,a^{5}-15 a^{4} e b+10 a^{3} d\,b^{2}-6 a^{2} c\,b^{3}\right)}{2 b^{8}}+\frac{f\,x^{21}}{15 b}-\frac{\left(7 a f-5 b e\right) x^{18}}{60 b^{2}}+\frac{\left(21 a^{2} f-15 a e b+10 b^{2} d\right) x^{15}}{90 b^{3}}-\frac{\left(21 f\,a^{3}-15 a^{2} b e+10 a\,b^{2} d-6 b^{3} c\right) x^{12}}{36 b^{4}}+\frac{a(}{}}{\left(b\,x^{3}+a\right)^{2}}$
default	$\frac{\frac{f\,x^{15} b^{4}}{15}+\frac{\left(-3 a\,b^{3} f+b^{4} e\right) x^{12}}{12}+\frac{\left(6 a^{2} b^{2} f-3 a\,b^{3} e+b^{4} d\right) x^{9}}{9}+\frac{\left(-10 a^{3} b f+6 a^{2} e\,b^{2}-3 a\,b^{3} d+b^{4} c\right) x^{6}}{6}+\frac{\left(15 a^{4} f-10 a^{3} b e+6 a^{2} b^{2} d-3 a\,b^{3} c\right) x^{3}}{3}}{b^{7}}-$
risch	$\frac{f\,x^{15}}{15 b^{3}}-\frac{x^{12} a f}{4 b^{4}}+\frac{e\,x^{12}}{12 b^{3}}+\frac{2 x^{9} a^{2} f}{3 b^{5}}-\frac{a e\,x^{9}}{3 b^{4}}+\frac{x^{9} d}{9 b^{3}}-\frac{5 x^{6} a^{3} f}{3 b^{6}}+\frac{a^{2} e\,x^{6}}{b^{5}}-\frac{x^{6} a d}{2 b^{4}}+\frac{x^{6} c}{6 b^{3}}+\frac{5 a^{4} f\,x^{3}}{b^{7}}-\frac{10 a^{3} e\,x^{3}}{3 b^{6}}+$
parallelrisch	$-\frac{1200 x^{3} a^{4} b^{3} d-720 x^{3} a^{3} b^{4} c+21 x^{18} a\,b^{6} f-42 x^{15} a^{2} b^{5} f+30 x^{15} a\,b^{6} e+1260 \ln \left(b\,x^{3}+a\right) x^{6} a^{5} b^{2} f-900 \ln \left(b\,x^{3}+a\right) x^{6} a^{4} b^{3} e+600 \ln}{}$

```
[In] int(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*a^2*(21*a^5*f-15*a^4*b*e+10*a^3*b^2*d-6*a^2*b^3*c)/b^8+1/15*f/b*x^21-
1/60*(7*a*f-5*b*e)/b^2*x^18+1/90*(21*a^2*f-15*a*b*e+10*b^2*d)/b^3*x^15-1/36
*(21*a^3*f-15*a^2*b*e+10*a*b^2*d-6*b^3*c)/b^4*x^12+1/9*a/b^5*(21*a^3*f-15*a
^2*b*e+10*a*b^2*d-6*b^3*c)*x^9-2/3*a*(21*a^5*f-15*a^4*b*e+10*a^3*b^2*d-6*a^
2*b^3*c)/b^7*x^3)/(b*x^3+a)^2-1/3*a^2*(21*a^3*f-15*a^2*b*e+10*a*b^2*d-6*b^3
*c)/b^8*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.49

$$\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{12\,b^7 f x^{21} + 3\,(5\,b^7 e - 7\,ab^6 f)x^{18} + 2\,(10\,b^7 d - 15\,ab^6 e + 21\,a^2 b^5 f)x^{15} + 5\,(6\,b^7 c - 10\,ab^6 d + 15\,a^2 b^5 e - 21\,a}{}$$

```
[In] integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/180*(12*b^7*f*x^21 + 3*(5*b^7*e - 7*a*b^6*f)*x^18 + 2*(10*b^7*d - 15*a*b^
6*e + 21*a^2*b^5*f)*x^15 + 5*(6*b^7*c - 10*a*b^6*d + 15*a^2*b^5*e - 21*a^3*
b^4*f)*x^12 - 20*(6*a*b^6*c - 10*a^2*b^5*d + 15*a^3*b^4*e - 21*a^4*b^3*f)*x
^9 + 210*a^4*b^3*c - 270*a^5*b^2*d + 330*a^6*b*e - 390*a^7*f - 30*(11*a^2*b
^5*c - 21*a^3*b^4*d + 34*a^4*b^3*e - 50*a^5*b^2*f)*x^6 + 60*(a^3*b^4*c + a^
4*b^3*d - 4*a^5*b^2*e + 8*a^6*b*f)*x^3 + 60*(6*a^4*b^3*c - 10*a^5*b^2*d + 1
5*a^6*b*e - 21*a^7*f + (6*a^2*b^5*c - 10*a^3*b^4*d + 15*a^4*b^3*e - 21*a^5*
b^2*f)*x^6 + 2*(6*a^3*b^4*c - 10*a^4*b^3*d + 15*a^5*b^2*e - 21*a^6*b*f)*x^3
)*log(b*x^3 + a))/(b^10*x^6 + 2*a*b^9*x^3 + a^2*b^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**14*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.03

$$\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{7\,a^4b^3c-9\,a^5b^2d+11\,a^6be-13\,a^7f+2\,(4\,a^3b^4c-5\,a^4b^3d+6\,a^5b^2e-7\,a^6bf)x^3}{6\,(b^{10}x^6+2\,ab^9x^3+a^2b^8)} + \frac{12\,b^4fx^{15}+15\,(b^4e-3\,ab^3f)x^{12}+20\,(b^4d-3\,ab^3e+6\,a^2b^2f)x^9+30\,(b^4c-3\,ab^3d+6\,a^2b^2e-10\,a^3bf}{180\,b^7} + \frac{(6\,a^2b^3c-10\,a^3b^2d+15\,a^4be-21\,a^5f)\log(bx^3+a)}{3\,b^8}$$

```
[In] integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/6*(7*a^4*b^3*c - 9*a^5*b^2*d + 11*a^6*b*e - 13*a^7*f + 2*(4*a^3*b^4*c - 5
*a^4*b^3*d + 6*a^5*b^2*e - 7*a^6*b*f)*x^3)/(b^10*x^6 + 2*a*b^9*x^3 + a^2*b^
8) + 1/180*(12*b^4*f*x^15 + 15*(b^4*e - 3*a*b^3*f)*x^12 + 20*(b^4*d - 3*a*b
^3*e + 6*a^2*b^2*f)*x^9 + 30*(b^4*c - 3*a*b^3*d + 6*a^2*b^2*e - 10*a^3*b*f)
*x^6 - 60*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3*b*e - 15*a^4*f)*x^3)/b^7 + 1/3*
(6*a^2*b^3*c - 10*a^3*b^2*d + 15*a^4*b*e - 21*a^5*f)*log(b*x^3 + a)/b^8
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.28

$$\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{(6\,a^2b^3c-10\,a^3b^2d+15\,a^4be-21\,a^5f)\log(|bx^3+a|)}{3\,b^8} - \frac{18\,a^2b^5cx^6-30\,a^3b^4dx^6+45\,a^4b^3ex^6-63\,a^5b^2fx^6+28\,a^3b^4cx^3-50\,a^4b^3dx^3+78\,a^5b^2ex^3-112\,a^6bfx}{6\,(bx^3+a)^2b^8} + \frac{12\,b^{12}fx^{15}+15\,b^{12}ex^{12}-45\,ab^{11}fx^{12}+20\,b^{12}dx^9-60\,ab^{11}ex^9+120\,a^2b^{10}fx^9+30\,b^{12}cx^6-90\,ab^{11}dx^6}{180\,b^{15}}$$

```
[In] integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(6*a^2*b^3*c - 10*a^3*b^2*d + 15*a^4*b*e - 21*a^5*f)*log(abs(b*x^3 + a)
)/b^8 - 1/6*(18*a^2*b^5*c*x^6 - 30*a^3*b^4*d*x^6 + 45*a^4*b^3*e*x^6 - 63*a^
5*b^2*f*x^6 + 28*a^3*b^4*c*x^3 - 50*a^4*b^3*d*x^3 + 78*a^5*b^2*e*x^3 - 112*
a^6*b*f*x^3 + 11*a^4*b^3*c - 21*a^5*b^2*d + 34*a^6*b*e - 50*a^7*f)/((b*x^3
+ a)^2*b^8) + 1/180*(12*b^12*f*x^15 + 15*b^12*e*x^12 - 45*a*b^11*f*x^12 + 2
0*b^12*d*x^9 - 60*a*b^11*e*x^9 + 120*a^2*b^10*f*x^9 + 30*b^12*c*x^6 - 90*a*
b^11*d*x^6 + 180*a^2*b^10*e*x^6 - 300*a^3*b^9*f*x^6 - 180*a*b^11*c*x^3 + 36
0*a^2*b^10*d*x^3 - 600*a^3*b^9*e*x^3 + 900*a^4*b^8*f*x^3)/b^15
```

Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.69

$$\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= x^{12}\left(\frac{e}{12\,b^3}-\frac{a\,f}{4\,b^4}\right)+x^6\left(\frac{c}{6\,b^3}-\frac{a^3\,f}{6\,b^6}-\frac{a^2\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{2\,b^2}+\frac{a\,\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{2\,b}\right)$$
$$-x^9\left(\frac{a^2\,f}{3\,b^5}-\frac{d}{9\,b^3}+\frac{a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{3\,b}\right)$$
$$-\frac{\frac{13\,f\,a^7-11\,e\,a^6\,b+9\,d\,a^5\,b^2-7\,c\,a^4\,b^3}{6\,b}+x^3\,\left(\frac{7\,f\,a^6}{3}-2\,e\,a^5\,b+\frac{5\,d\,a^4\,b^2}{3}-\frac{4\,c\,a^3\,b^3}{3}\right)}{a^2\,b^7+2\,a\,b^8\,x^3+b^9\,x^6}$$
$$-x^3\left(\frac{a\,\left(\frac{c}{b^3}-\frac{a^3\,f}{b^6}-\frac{3\,a^2\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{3\,a\,\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)}{b}-\frac{a^2\,\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b^2}+\frac{a^3\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{3\,b^3}\right)$$
$$-\frac{\ln\left(b\,x^3+a\right)\,\left(21\,f\,a^5-15\,e\,a^4\,b+10\,d\,a^3\,b^2-6\,c\,a^2\,b^3\right)}{3\,b^8}+\frac{f\,x^{15}}{15\,b^3}$$

```
[In] int((x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^12*(e/(12*b^3) - (a*f)/(4*b^4)) + x^6*(c/(6*b^3) - (a^3*f)/(6*b^6) - (a^2
*(e/b^3 - (3*a*f)/b^4))/(2*b^2) + (a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 -
 (3*a*f)/b^4))/b))/(2*b)) - x^9*((a^2*f)/(3*b^5) - d/(9*b^3) + (a*(e/b^3 -
(3*a*f)/b^4))/(3*b)) - ((13*a^7*f - 7*a^4*b^3*c + 9*a^5*b^2*d - 11*a^6*b*e)
/(6*b) + x^3*((7*a^6*f)/3 - (4*a^3*b^3*c)/3 + (5*a^4*b^2*d)/3 - 2*a^5*b*e))
/(a^2*b^7 + b^9*x^6 + 2*a*b^8*x^3) - x^3*((a*(c/b^3 - (a^3*f)/b^6 - (3*a^2*
(e/b^3 - (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3
*a*f)/b^4))/b))/b))/b - (a^2*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)
/b^4))/b))/b^2 + (a^3*(e/b^3 - (3*a*f)/b^4))/(3*b^3)) - (log(a + b*x^3)*(21
*a^5*f - 6*a^2*b^3*c + 10*a^3*b^2*d - 15*a^4*b*e))/(3*b^8) + (f*x^15)/(15*b
^3)
```

3.277 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 226

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^3}{3b^6} + \frac{(b^2d-3abe+6a^2f)\,x^6}{6b^5} + \frac{(be-3af)x^9}{9b^4} + \frac{fx^{12}}{12b^3} + \frac{a^3(b^3c-ab^2d+a^2be-a^3f)}{6b^7\,(a+bx^3)^2} - \frac{a^2(3b^3c-4ab^2d+5a^2be-6a^3f)}{3b^7\,(a+bx^3)} - \frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\log(a+bx^3)}{3b^7}$$

```
[Out] 1/3*(-10*a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*x^3/b^6+1/6*(6*a^2*f-3*a*b*e+b^2*
d)*x^6/b^5+1/9*(-3*a*f+b*e)*x^9/b^4+1/12*f*x^12/b^3+1/6*a^3*(-a^3*f+a^2*b*e
-a*b^2*d+b^3*c)/b^7/(b*x^3+a)^2-1/3*a^2*(-6*a^3*f+5*a^2*b*e-4*a*b^2*d+3*b^3
*c)/b^7/(b*x^3+a)-1/3*a*(-15*a^3*f+10*a^2*b*e-6*a*b^2*d+3*b^3*c)*ln(b*x^3+a
)/b^7
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{x^6(6a^2f-3abe+b^2d)}{6b^5} - \frac{a^2(-6a^3f+5a^2be-4ab^2d+3b^3c)}{3b^7(a+bx^3)} + \frac{a^3(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^7(a+bx^3)^2} - \frac{a\log(a+bx^3)(-15a^3f+10a^2be-6ab^2d+3b^3c)}{3b^7} + \frac{x^3(-10a^3f+6a^2be-3ab^2d+b^3c)}{3b^6} + \frac{x^9(be-3af)}{9b^4} + \frac{fx^{12}}{12b^3}$$

```
[In] Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*d - 3*a*b*
e + 6*a^2*f)*x^6)/(6*b^5) + ((b*e - 3*a*f)*x^9)/(9*b^4) + (f*x^12)/(12*b^3)
 + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^7*(a + b*x^3)^2) - (a^2*(
3*b^3*c - 4*a*b^2*d + 5*a^2*b*e - 6*a^3*f))/(3*b^7*(a + b*x^3)) - (a*(3*b^3
*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*Log[a + b*x^3])/(3*b^7)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx)^3}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^3c - 3ab^2d + 6a^2be - 10a^3f}{b^6} + \frac{(b^2d - 3abe + 6a^2f)\,x}{b^5} + \frac{(be - 3af)x^2}{b^4} + \frac{fx^3}{b^3} + \frac{a^3(-b^3c + ab^2d - a^2be + a^3f)}{b^6(a+bx)^3} - \frac{a^2(-3b^3c + 4ab^2d - 5a^2be + 6a^3f)}{b^6(a+bx)^2} + \frac{a(-3b^3c + 6ab^2d - 10a^2be + 15a^3f)}{b^6(a+bx)}\right) dx, x, x^3\right)$$

$$= \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)\,x^3}{3b^6} + \frac{(b^2d - 3abe + 6a^2f)\,x^6}{6b^5} + \frac{(be - 3af)x^9}{9b^4} + \frac{fx^{12}}{12b^3} + \frac{a^3(b^3c - ab^2d + a^2be - a^3f)}{6b^7\,(a+bx^3)^2} - \frac{a^2(3b^3c - 4ab^2d + 5a^2be - 6a^3f)}{3b^7\,(a+bx^3)} - \frac{a(3b^3c - 6ab^2d + 10a^2be - 15a^3f)\log\left(a+bx^3\right)}{3b^7}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.92

$$\int \frac{x^{11}(c + dx^3 + ex^6 + fx^9)}{\left(a+bx^3\right)^3}\,dx$$

$$= \frac{12b(b^3c - 3ab^2d + 6a^2be - 10a^3f)\,x^3 + 6b^2(b^2d - 3abe + 6a^2f)\,x^6 + 4b^3(be - 3af)x^9 + 3b^4fx^{12} + \frac{6a^3(b^3c-a\cdots}{(a\cdots}}{36b^7}$$

```
[In] Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (12*b*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3 + 6*b^2*(b^2*d - 3*a*b
*e + 6*a^2*f)*x^6 + 4*b^3*(b*e - 3*a*f)*x^9 + 3*b^4*f*x^12 + (6*a^3*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f))/(a + b*x^3)^2 + (12*a^2*(-3*b^3*c + 4*a*b^2*d
 - 5*a^2*b*e + 6*a^3*f))/(a + b*x^3) + 12*a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*
b*e + 15*a^3*f)*Log[a + b*x^3])/(36*b^7)
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.96

method	result
norman	$\frac{\frac{a^2\left(15a^4f-10a^3be+6a^2b^2d-3a\,b^3c\right)}{2b^7}-\frac{\left(15f\,a^3-10a^2be+6a\,b^2d-3b^3c\right)x^9}{9b^4}+\frac{f\,x^{18}}{12b}-\frac{(3af-2be)x^{15}}{18b^2}+\frac{\left(15a^2f-10aeb+6b^2d\right)x^{12}}{36b^3}+\frac{2a\left(15a^4\right.}{}}{\left(b\,x^3+a\right)^2}$
default	$-\frac{-\frac{b^3f\,x^{12}}{12}+\frac{\left(3fa\,b^2-b^3e\right)x^9}{9}+\frac{\left(-6f\,a^2b+3a\,b^2e-b^3d\right)x^6}{6}+\frac{\left(10f\,a^3-6a^2be+3a\,b^2d-b^3c\right)x^3}{3}}{b^6}+\frac{a\left(\frac{\left(15f\,a^3-10a^2be+6a\,b^2d-3b^3c\right)\ln}{b}\right.}{}$
risch	$\frac{f\,x^{12}}{12b^3}-\frac{af\,x^9}{3b^4}+\frac{e\,x^9}{9b^3}+\frac{x^6f\,a^2}{b^5}-\frac{ae\,x^6}{2b^4}+\frac{d\,x^6}{6b^3}-\frac{10f\,a^3x^3}{3b^6}+\frac{2a^2e\,x^3}{b^5}-\frac{ad\,x^3}{b^4}+\frac{c\,x^3}{3b^3}+\frac{\left(2f\,a^5-\frac{5}{3}a^4eb+\frac{4}{3}a^3d\right.}{}$
parallelrisch	$\frac{3f\,x^{18}b^6+270a^6f+4x^{15}b^6e+6x^{12}b^6d+12x^9b^6c+180\ln\left(b\,x^3+a\right)a^6f-36\ln\left(b\,x^3+a\right)x^6a\,b^5c-54a^3b^3c-120\ln\left(b\,x^3+a\right)x^6a^3b^3e}{}$

```
[In] int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*a^2*(15*a^4*f-10*a^3*b*e+6*a^2*b^2*d-3*a*b^3*c)/b^7-1/9/b^4*(15*a^3*f-
10*a^2*b*e+6*a*b^2*d-3*b^3*c)*x^9+1/12*f*x^18/b-1/18*(3*a*f-2*b*e)/b^2*x^15
+1/36*(15*a^2*f-10*a*b*e+6*b^2*d)/b^3*x^12+2/3*a*(15*a^4*f-10*a^3*b*e+6*a^2
*b^2*d-3*a*b^3*c)/b^6*x^3)/(b*x^3+a)^2+1/3*a*(15*a^3*f-10*a^2*b*e+6*a*b^2*d
-3*b^3*c)/b^7*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.56

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$=\frac{3\,b^6fx^{18}+2\,(2\,b^6e-3\,ab^5f)x^{15}+(6\,b^6d-10\,ab^5e+15\,a^2b^4f)x^{12}+4\,(3\,b^6c-6\,ab^5d+10\,a^2b^4e-15\,a^3b^3}{}$$

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^6*f*x^18 + 2*(2*b^6*e - 3*a*b^5*f)*x^15 + (6*b^6*d - 10*a*b^5*e +
 15*a^2*b^4*f)*x^12 + 4*(3*b^6*c - 6*a*b^5*d + 10*a^2*b^4*e - 15*a^3*b^3*f)
*x^9 - 30*a^3*b^3*c + 42*a^4*b^2*d - 54*a^5*b*e + 66*a^6*f + 6*(4*a*b^5*c -
 11*a^2*b^4*d + 21*a^3*b^3*e - 34*a^4*b^2*f)*x^6 - 12*(2*a^2*b^4*c - a^3*b^
3*d - a^4*b^2*e + 4*a^5*b*f)*x^3 - 12*(3*a^3*b^3*c - 6*a^4*b^2*d + 10*a^5*b
*e - 15*a^6*f + (3*a*b^5*c - 6*a^2*b^4*d + 10*a^3*b^3*e - 15*a^4*b^2*f)*x^6
 + 2*(3*a^2*b^4*c - 6*a^3*b^3*d + 10*a^4*b^2*e - 15*a^5*b*f)*x^3)*log(b*x^3
 + a))/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.03

$$\begin{aligned}&\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx\\&=-\frac{5\,a^3b^3c-7\,a^4b^2d+9\,a^5be-11\,a^6f+2\,(3\,a^2b^4c-4\,a^3b^3d+5\,a^4b^2e-6\,a^5bf)x^3}{6\,(b^9x^6+2\,ab^8x^3+a^2b^7)}\\&\quad+\frac{3\,b^3fx^{12}+4\,(b^3e-3\,ab^2f)x^9+6\,(b^3d-3\,ab^2e+6\,a^2bf)x^6+12\,(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f)x^3}{36\,b^6}\\&\quad-\frac{(3\,ab^3c-6\,a^2b^2d+10\,a^3be-15\,a^4f)\log(bx^3+a)}{3\,b^7}\end{aligned}$$

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
[Out] -1/6*(5*a^3*b^3*c - 7*a^4*b^2*d + 9*a^5*b*e - 11*a^6*f + 2*(3*a^2*b^4*c - 4
*a^3*b^3*d + 5*a^4*b^2*e - 6*a^5*b*f)*x^3)/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7
) + 1/36*(3*b^3*f*x^12 + 4*(b^3*e - 3*a*b^2*f)*x^9 + 6*(b^3*d - 3*a*b^2*e +
 6*a^2*b*f)*x^6 + 12*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/b^6 -
1/3*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3*b*e - 15*a^4*f)*log(b*x^3 + a)/b^7
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.29

$$\begin{aligned}\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= -\frac{(3\,ab^3c-6\,a^2b^2d+10\,a^3be-15\,a^4f)\log(|bx^3+a|)}{3\,b^7}\\&\quad+\frac{9\,ab^5cx^6-18\,a^2b^4dx^6+30\,a^3b^3ex^6-45\,a^4b^2fx^6+12\,a^2b^4cx^3-28\,a^3b^3dx^3+50\,a^4b^2ex^3-78\,a^5bfx^3+}{6\,(bx^3+a)^2b^7}\\&\quad+\frac{3\,b^9fx^{12}+4\,b^9ex^9-12\,ab^8fx^9+6\,b^9dx^6-18\,ab^8ex^6+36\,a^2b^7fx^6+12\,b^9cx^3-36\,ab^8dx^3+72\,a^2b^7ex^3}{36\,b^{12}}\end{aligned}$$

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3*b*e - 15*a^4*f)*log(abs(b*x^3 + a))/
b^7 + 1/6*(9*a*b^5*c*x^6 - 18*a^2*b^4*d*x^6 + 30*a^3*b^3*e*x^6 - 45*a^4*b^2
*f*x^6 + 12*a^2*b^4*c*x^3 - 28*a^3*b^3*d*x^3 + 50*a^4*b^2*e*x^3 - 78*a^5*b*
f*x^3 + 4*a^3*b^3*c - 11*a^4*b^2*d + 21*a^5*b*e - 34*a^6*f)/((b*x^3 + a)^2*
b^7) + 1/36*(3*b^9*f*x^12 + 4*b^9*e*x^9 - 12*a*b^8*f*x^9 + 6*b^9*d*x^6 - 18
*a*b^8*e*x^6 + 36*a^2*b^7*f*x^6 + 12*b^9*c*x^3 - 36*a*b^8*d*x^3 + 72*a^2*b^
7*e*x^3 - 120*a^3*b^6*f*x^3)/b^12
```

Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.30

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= x^9\left(\frac{e}{9\,b^3}-\frac{a\,f}{3\,b^4}\right)+x^3\left(\frac{c}{3\,b^3}-\frac{a^3\,f}{3\,b^6}-\frac{a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)$$
$$-x^6\left(\frac{a^2\,f}{2\,b^5}-\frac{d}{6\,b^3}+\frac{a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{2\,b}\right)$$
$$+\frac{\frac{11\,f\,a^6-9\,e\,a^5\,b+7\,d\,a^4\,b^2-5\,c\,a^3\,b^3}{6\,b}+x^3\left(2\,f\,a^5-\frac{5\,e\,a^4\,b}{3}+\frac{4\,d\,a^3\,b^2}{3}-c\,a^2\,b^3\right)}{a^2\,b^6+2\,a\,b^7\,x^3+b^8\,x^6}$$
$$+\frac{f\,x^{12}}{12\,b^3}+\frac{\ln\left(b\,x^3+a\right)\,\left(15\,f\,a^4-10\,e\,a^3\,b+6\,d\,a^2\,b^2-3\,c\,a\,b^3\right)}{3\,b^7}$$

```
[In] int((x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^9*(e/(9*b^3) - (a*f)/(3*b^4)) + x^3*(c/(3*b^3) - (a^3*f)/(3*b^6) - (a^2*(
e/b^3 - (3*a*f)/b^4))/b^2 + (a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*
f)/b^4))/b))/b) - x^6*((a^2*f)/(2*b^5) - d/(6*b^3) + (a*(e/b^3 - (3*a*f)/b^
4))/(2*b)) + ((11*a^6*f - 5*a^3*b^3*c + 7*a^4*b^2*d - 9*a^5*b*e)/(6*b) + x^
3*(2*a^5*f - a^2*b^3*c + (4*a^3*b^2*d)/3 - (5*a^4*b*e)/3))/(a^2*b^6 + b^8*x
^6 + 2*a*b^7*x^3) + (f*x^12)/(12*b^3) + (log(a + b*x^3)*(15*a^4*f + 6*a^2*b
^2*d - 3*a*b^3*c - 10*a^3*b*e))/(3*b^7)
```

3.278 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 186

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx = \frac{(b^2d-3abe+6a^2f)\,x^3}{3b^5} + \frac{(be-3af)x^6}{6b^4} + \frac{fx^9}{9b^3} - \frac{a^2(b^3c-ab^2d+a^2be-a^3f)}{6b^6\,(a+bx^3)^2} + \frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)}{3b^6\,(a+bx^3)} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\log(a+bx^3)}{3b^6}$$

```
[Out] 1/3*(6*a^2*f-3*a*b*e+b^2*d)*x^3/b^5+1/6*(-3*a*f+b*e)*x^6/b^4+1/9*f*x^9/b^3-
1/6*a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b^6/(b*x^3+a)^2+1/3*a*(-5*a^3*f+4*a^
2*b*e-3*a*b^2*d+2*b^3*c)/b^6/(b*x^3+a)+1/3*(-10*a^3*f+6*a^2*b*e-3*a*b^2*d+b
^3*c)*ln(b*x^3+a)/b^6
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used

$= \{1835, 1634\}$

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{x^3(6a^2f-3abe+b^2d)}{3b^5}+\frac{a(-5a^3f+4a^2be-3ab^2d+2b^3c)}{3b^6\,(a+bx^3)} - \frac{a^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^6\,(a+bx^3)^2} + \frac{\log\,(a+bx^3)\,(-10a^3f+6a^2be-3ab^2d+b^3c)}{3b^6} + \frac{x^6(be-3af)}{6b^4}+\frac{fx^9}{9b^3}$$

```
[In] Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^6)/(6*b^4) + (
f*x^9)/(9*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^6*(a + b*x^
3)^2) + (a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f))/(3*b^6*(a + b*x^3))
 + ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx)^3}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^2d-3abe+6a^2f}{b^5}+\frac{(be-3af)x}{b^4}+\frac{fx^2}{b^3}-\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)}{b^5(a+bx)^3}\right.\right.\\
&\quad\left.\left.+\frac{a(-2b^3c+3ab^2d-4a^2be+5a^3f)}{b^5(a+bx)^2}+\frac{b^3c-3ab^2d+6a^2be-10a^3f}{b^5(a+bx)}\right)\,dx, x, x^3\right)\\
&= \frac{(b^2d-3abe+6a^2f)\,x^3}{3b^5}+\frac{(be-3af)x^6}{6b^4}+\frac{fx^9}{9b^3}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)}{6b^6\,(a+bx^3)^2}\\
&\quad+\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)}{3b^6\,(a+bx^3)}+\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\log\,(a+bx^3)}{3b^6}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.91

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{6b(b^2d-3abe+6a^2f)\,x^3+3b^2(be-3af)x^6+2b^3fx^9+\frac{3a^2(-b^3c+ab^2d-a^2be+a^3f)}{(a+bx^3)^2}-\frac{6a(-2b^3c+3ab^2d-4a^2be+5a^3f)}{a+bx^3}}{18b^6}$$

```
[In] Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (6*b*(b^2*d - 3*a*b*e + 6*a^2*f)*x^3 + 3*b^2*(b*e - 3*a*f)*x^6 + 2*b^3*f*x^
9 + (3*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 - (6*a*(-2
*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f))/(a + b*x^3) + 6*(b^3*c - 3*a*b^2
*d + 6*a^2*b*e - 10*a^3*f)*Log[a + b*x^3])/(18*b^6)
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.95

method	result
norman	$\frac{-\frac{a^2(10fa^3-6a^2be+3ab^2d-b^3c)}{2b^6}+\frac{fx^{15}}{9b}-\frac{(5af-3be)x^{12}}{18b^2}+\frac{(10a^2f-6aeb+3b^2d)x^9}{9b^3}-\frac{2a(10fa^3-6a^2be+3ab^2d-b^3c)x^3}{3b^5}}{(bx^3+a)^2}-\frac{(10fa^3-6a\cdots}{}$
default	$\frac{\frac{b^2fx^9}{9}+\frac{(-3afb+b^2e)x^6}{6}+\frac{(6a^2f-3aeb+b^2d)x^3}{3}}{b^5}-\frac{\frac{(10fa^3-6a^2be+3ab^2d-b^3c)\ln(bx^3+a)}{b}-\frac{a^2(fa^3-a^2be+ab^2d-b^3c)}{2b(bx^3+a)^2}+\frac{a(5fa^3-\cdots}{}}{3b^5}$
risch	$\frac{fx^9}{9b^3}-\frac{afx^6}{2b^4}+\frac{ex^6}{6b^3}+\frac{2a^2fx^3}{b^5}-\frac{aex^3}{b^4}+\frac{dx^3}{3b^3}+\frac{(-\frac{5}{3}a^4f+\frac{4}{3}a^3be-a^2b^2d+\frac{2}{3}ab^3c)x^3-\frac{a^2(9fa^3-7a^2be+5ab^2d-3b^3c)}{6b}}{b^5(bx^3+a)^2}$
parallelrisch	$-\frac{-2fx^{15}b^5+90fa^5-72a^3b^2ex^3+36a^2b^3dx^3-12ab^4cx^3-9a^2cb^3+27a^3db^2-54a^4eb+120a^4bfx^3+120\ln(bx^3+a)x^3a^4bf-72\cdots}{}$

```
[In] int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*a^2*(10*a^3*f-6*a^2*b*e+3*a*b^2*d-b^3*c)/b^6+1/9*f*x^15/b-1/18*(5*a*f
-3*b*e)/b^2*x^12+1/9*(10*a^2*f-6*a*b*e+3*b^2*d)/b^3*x^9-2/3*a*(10*a^3*f-6*a
^2*b*e+3*a*b^2*d-b^3*c)/b^5*x^3)/(b*x^3+a)^2-1/3*(10*a^3*f-6*a^2*b*e+3*a*b^
2*d-b^3*c)/b^6*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.59

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{2\,b^5fx^{15}+(3\,b^5e-5\,ab^4f)x^{12}+2\,(3\,b^5d-6\,ab^4e+10\,a^2b^3f)x^9+3\,(4\,ab^4d-11\,a^2b^3e+21\,a^3b^2f)x^6+9\,a}{}$$

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^5*f*x^15 + (3*b^5*e - 5*a*b^4*f)*x^12 + 2*(3*b^5*d - 6*a*b^4*e +
10*a^2*b^3*f)*x^9 + 3*(4*a*b^4*d - 11*a^2*b^3*e + 21*a^3*b^2*f)*x^6 + 9*a^2
*b^3*c - 15*a^3*b^2*d + 21*a^4*b*e - 27*a^5*f + 6*(2*a*b^4*c - 2*a^2*b^3*d
+ a^3*b^2*e + a^4*b*f)*x^3 + 6*((b^5*c - 3*a*b^4*d + 6*a^2*b^3*e - 10*a^3*b
^2*f)*x^6 + a^2*b^3*c - 3*a^3*b^2*d + 6*a^4*b*e - 10*a^5*f + 2*(a*b^4*c - 3
*a^2*b^3*d + 6*a^3*b^2*e - 10*a^4*b*f)*x^3)*log(b*x^3 + a))/(b^8*x^6 + 2*a*
b^7*x^3 + a^2*b^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.03

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{3\,a^2b^3c-5\,a^3b^2d+7\,a^4be-9\,a^5f+2\,(2\,ab^4c-3\,a^2b^3d+4\,a^3b^2e-5\,a^4bf)x^3}{6\,(b^8x^6+2\,ab^7x^3+a^2b^6)}$$
$$+\frac{2\,b^2fx^9+3\,(b^2e-3\,abf)x^6+6\,(b^2d-3\,abe+6\,a^2f)x^3}{18\,b^5}$$
$$+\frac{(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f)\log\left(bx^3+a\right)}{3\,b^6}$$

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*b^3*c - 5*a^3*b^2*d + 7*a^4*b*e - 9*a^5*f + 2*(2*a*b^4*c - 3*a^2
*b^3*d + 4*a^3*b^2*e - 5*a^4*b*f)*x^3)/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6) +
1/18*(2*b^2*f*x^9 + 3*(b^2*e - 3*a*b*f)*x^6 + 6*(b^2*d - 3*a*b*e + 6*a^2*f)
*x^3)/b^5 + 1/3*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*log(b*x^3 + a)/b
^6
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.24

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f)\log\left(|bx^3+a|\right)}{3\,b^6} - \frac{3\,b^5cx^6-9\,ab^4dx^6+18\,a^2b^3ex^6-30\,a^3b^2fx^6+2\,ab^4cx^3-12\,a^2b^3dx^3+28\,a^3b^2ex^3-50\,a^4bfx^3-4\,a^3b^2}{6\,(bx^3+a)^2b^6} + \frac{2\,b^6fx^9+3\,b^6ex^6-9\,ab^5fx^6+6\,b^6dx^3-18\,ab^5ex^3+36\,a^2b^4fx^3}{18\,b^9}$$

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*log(abs(b*x^3 + a))/b^6 - 1/
6*(3*b^5*c*x^6 - 9*a*b^4*d*x^6 + 18*a^2*b^3*e*x^6 - 30*a^3*b^2*f*x^6 + 2*a*
b^4*c*x^3 - 12*a^2*b^3*d*x^3 + 28*a^3*b^2*e*x^3 - 50*a^4*b*f*x^3 - 4*a^3*b^
2*d + 11*a^4*b*e - 21*a^5*f)/((b*x^3 + a)^2*b^6) + 1/18*(2*b^6*f*x^9 + 3*b^
6*e*x^6 - 9*a*b^5*f*x^6 + 6*b^6*d*x^3 - 18*a*b^5*e*x^3 + 36*a^2*b^4*f*x^3)/
b^9
```

Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = x^6\left(\frac{e}{6\,b^3}-\frac{a\,f}{2\,b^4}\right) - \frac{x^3\left(\frac{5\,f\,a^4}{3}-\frac{4\,e\,a^3\,b}{3}+d\,a^2\,b^2-\frac{2\,c\,a\,b^3}{3}\right)+\frac{9\,f\,a^5-7\,e\,a^4\,b+5\,d\,a^3\,b^2-3\,c\,a^2\,b^3}{6\,b}}{a^2\,b^5+2\,a\,b^6\,x^3+b^7\,x^6} - x^3\left(\frac{a^2\,f}{b^5}-\frac{d}{3\,b^3}+\frac{a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right) + \frac{\ln\left(b\,x^3+a\right)\,\left(-10\,f\,a^3+6\,e\,a^2\,b-3\,d\,a\,b^2+c\,b^3\right)}{3\,b^6} + \frac{f\,x^9}{9\,b^3}$$

```
[In] int((x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^6*(e/(6*b^3) - (a*f)/(2*b^4)) - (x^3*((5*a^4*f)/3 + a^2*b^2*d - (2*a*b^3*
c)/3 - (4*a^3*b*e)/3) + (9*a^5*f - 3*a^2*b^3*c + 5*a^3*b^2*d - 7*a^4*b*e)/(
6*b))/(a^2*b^5 + b^7*x^6 + 2*a*b^6*x^3) - x^3*((a^2*f)/b^5 - d/(3*b^3) + (a
*(e/b^3 - (3*a*f)/b^4))/b) + (log(a + b*x^3)*(b^3*c - 10*a^3*f - 3*a*b^2*d
+ 6*a^2*b*e))/(3*b^6) + (f*x^9)/(9*b^3)
```

3.279 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 146

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx=\frac{(be-3af)x^3}{3b^4}+\frac{fx^6}{6b^3}+\frac{a(b^3c-ab^2d+a^2be-a^3f)}{6b^5(a+bx^3)^2}-\frac{b^3c-2ab^2d+3a^2be-4a^3f}{3b^5(a+bx^3)}+\frac{(b^2d-3abe+6a^2f)\log(a+bx^3)}{3b^5}$$

```
[Out] 1/3*(-3*a*f+b*e)*x^3/b^4+1/6*f*x^6/b^3+1/6*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
/b^5/(b*x^3+a)^2+1/3*(4*a^3*f-3*a^2*b*e+2*a*b^2*d-b^3*c)/b^5/(b*x^3+a)+1/3*
(6*a^2*f-3*a*b*e+b^2*d)*ln(b*x^3+a)/b^5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx=\frac{\log(a+bx^3)(6a^2f-3abe+b^2d)}{3b^5}-\frac{-4a^3f+3a^2be-2ab^2d+b^3c}{3b^5(a+bx^3)}+\frac{a(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5(a+bx^3)^2}+\frac{x^3(be-3af)}{3b^4}+\frac{fx^6}{6b^3}$$

```
[In] Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x^3)/(3*b^4) + (f*x^6)/(6*b^3) + (a*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f))/(6*b^5*(a + b*x^3)^2) - (b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3
*f)/(3*b^5*(a + b*x^3)) + ((b^2*d - 3*a*b*e + 6*a^2*f)*Log[a + b*x^3])/(3*b
^5)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx)^3}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{be-3af}{b^4}+\frac{fx}{b^3}+\frac{a(-b^3c+ab^2d-a^2be+a^3f)}{b^4(a+bx)^3}\right.\right.\\
&\qquad\left.\left.+\frac{b^3c-2ab^2d+3a^2be-4a^3f}{b^4(a+bx)^2}+\frac{b^2d-3abe+6a^2f}{b^4(a+bx)}\right)\,dx,x,x^3\right)\\
&= \frac{(be-3af)x^3}{3b^4}+\frac{fx^6}{6b^3}+\frac{a(b^3c-ab^2d+a^2be-a^3f)}{6b^5\left(a+bx^3\right)^2}\\
&\quad-\frac{b^3c-2ab^2d+3a^2be-4a^3f}{3b^5\left(a+bx^3\right)}+\frac{\left(b^2d-3abe+6a^2f\right)\log\left(a+bx^3\right)}{3b^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{\left(a+bx^3\right)^3}\,dx$$
$$=\frac{7a^4f+a^3b(-5e+2fx^3)+a^2b^2(3d-4ex^3-11fx^6)+b^4x^3(-2c+2ex^6+fx^9)-ab^3(c-4x^3(d+ex^3-}{6b^5\left(a+bx^3\right)^2}$$

```
[In] Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (7*a^4*f + a^3*b*(-5*e + 2*f*x^3) + a^2*b^2*(3*d - 4*e*x^3 - 11*f*x^6) + b^
4*x^3*(-2*c + 2*e*x^6 + f*x^9) - a*b^3*(c - 4*x^3*(d + e*x^3 - f*x^6)) + 2*
(b^2*d - 3*a*b*e + 6*a^2*f)*(a + b*x^3)^2*Log[a + b*x^3])/(6*b^5*(a + b*x^3
)^2)
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94

method	result
norman	$\frac{\frac{a\left(18 f a^3-9 a^2 b e+3 a b^2 d-b^3 c\right)}{6 b^5}+\frac{f x^{12}}{6 b}-\frac{(2 a f-b e) x^9}{3 b^2}+\frac{\left(12 f a^3-6 a^2 b e+2 a b^2 d-b^3 c\right) x^3}{3 b^4}}{\left(b x^3+a\right)^2}+\frac{\left(6 a^2 f-3 a e b+b^2 d\right) \ln \left(b x^3+a\right)}{3 b^5}$
default	$\frac{\left(-f x^3 b+3 a f-b e\right)^2}{6 b^5 f}+\frac{\frac{\left(6 a^2 f-3 a e b+b^2 d\right) \ln \left(b x^3+a\right)}{b}-\frac{a\left(f a^3-a^2 b e+a b^2 d-b^3 c\right)}{2 b\left(b x^3+a\right)^2}-\frac{-4 f a^3+3 a^2 b e-2 a b^2 d+b^3 c}{b\left(b x^3+a\right)}}{3 b^4}$
risch	$\frac{f x^6}{6 b^3}-\frac{f a x^3}{b^4}+\frac{e x^3}{3 b^3}+\frac{3 f a^2}{2 b^5}-\frac{a e}{b^4}+\frac{e^2}{6 b^3 f}+\frac{\left(\frac{4}{3} f a^3-a^2 b e+\frac{2}{3} a b^2 d-\frac{1}{3} b^3 c\right) x^3+\frac{a\left(7 f a^3-5 a^2 b e+3 a b^2 d-b^3 c\right)}{6 b}}{b^4\left(b x^3+a\right)^2}+\frac{2 \ln (b x$
parallelrisch	$f x^{12} b^4-4 x^9 a b^3 f+2 x^9 b^4 e+12 \ln \left(b x^3+a\right) x^6 a^2 b^2 f-6 \ln \left(b x^3+a\right) x^6 a b^3 e+2 \ln \left(b x^3+a\right) x^6 b^4 d+24 \ln \left(b x^3+a\right) x^3 a^3 b f-12 \ln (b x^3$

```
[In] int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/6*a*(18*a^3*f-9*a^2*b*e+3*a*b^2*d-b^3*c)/b^5+1/6*f*x^12/b-1/3*(2*a*f-b*e
)/b^2*x^9+1/3*(12*a^3*f-6*a^2*b*e+2*a*b^2*d-b^3*c)/b^4*x^3)/(b*x^3+a)^2+1/3
*(6*a^2*f-3*a*b*e+b^2*d)*ln(b*x^3+a)/b^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.54

$$\int \frac{x^5\left(c+d x^3+e x^6+f x^9\right)}{\left(a+b x^3\right)^3} d x$$
$$=\frac{b^4 f x^{12}+2\left(b^4 e-2 a b^3 f\right) x^9+\left(4 a b^3 e-11 a^2 b^2 f\right) x^6-a b^3 c+3 a^2 b^2 d-5 a^3 b e+7 a^4 f-2\left(b^4 c-2 a b^3 d+2\right.}{6\left(b^7 x\right.}$$

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*f*x^12 + 2*(b^4*e - 2*a*b^3*f)*x^9 + (4*a*b^3*e - 11*a^2*b^2*f)*x^
6 - a*b^3*c + 3*a^2*b^2*d - 5*a^3*b*e + 7*a^4*f - 2*(b^4*c - 2*a*b^3*d + 2*
a^2*b^2*e - a^3*b*f)*x^3 + 2*((b^4*d - 3*a*b^3*e + 6*a^2*b^2*f)*x^6 + a^2*b
^2*d - 3*a^3*b*e + 6*a^4*f + 2*(a*b^3*d - 3*a^2*b^2*e + 6*a^3*b*f)*x^3)*log
(b*x^3 + a))/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= -\frac{ab^3c-3\,a^2b^2d+5\,a^3be-7\,a^4f+2\,(b^4c-2\,ab^3d+3\,a^2b^2e-4\,a^3bf)x^3}{6\,(b^7x^6+2\,ab^6x^3+a^2b^5)} \\ &\quad+\frac{bfx^6+2\,(be-3\,af)x^3}{6\,b^4}+\frac{(b^2d-3\,abe+6\,a^2f)\log(bx^3+a)}{3\,b^5}\end{aligned}$$

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
[Out] -1/6*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a^4*f + 2*(b^4*c - 2*a*b^3*d +
3*a^2*b^2*e - 4*a^3*b*f)*x^3)/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5) + 1/6*(b*f*
x^6 + 2*(b*e - 3*a*f)*x^3)/b^4 + 1/3*(b^2*d - 3*a*b*e + 6*a^2*f)*log(b*x^3
+ a)/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= \frac{(b^2d-3\,abe+6\,a^2f)\log(|bx^3+a|)}{3\,b^5}+\frac{b^3fx^6+2\,b^3ex^3-6\,ab^2fx^3}{6\,b^6} \\ &\quad-\frac{ab^3c-3\,a^2b^2d+5\,a^3be-7\,a^4f+2\,(b^4c-2\,ab^3d+3\,a^2b^2e-4\,a^3bf)x^3}{6\,(bx^3+a)^2b^5}\end{aligned}$$

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
[Out] 1/3*(b^2*d - 3*a*b*e + 6*a^2*f)*log(abs(b*x^3 + a))/b^5 + 1/6*(b^3*f*x^6 +
2*b^3*e*x^3 - 6*a*b^2*f*x^3)/b^6 - 1/6*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e -
 7*a^4*f + 2*(b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^3)/((b*x^3 + a
)^2*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.04

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= x^3\left(\frac{e}{3\,b^3}-\frac{a\,f}{b^4}\right)+\frac{\frac{7\,f\,a^4-5\,e\,a^3\,b+3\,d\,a^2\,b^2-c\,a\,b^3}{6\,b}-x^3\left(-\frac{4\,f\,a^3}{3}+e\,a^2\,b-\frac{2\,d\,a\,b^2}{3}+\frac{c\,b^3}{3}\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}$$
$$+\frac{f\,x^6}{6\,b^3}+\frac{\ln\left(b\,x^3+a\right)\,\left(6\,f\,a^2-3\,e\,a\,b+d\,b^2\right)}{3\,b^5}$$

```
[In] int((x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^3*(e/(3*b^3) - (a*f)/b^4) + ((7*a^4*f + 3*a^2*b^2*d - a*b^3*c - 5*a^3*b*e
)/(6*b) - x^3*((b^3*c)/3 - (4*a^3*f)/3 - (2*a*b^2*d)/3 + a^2*b*e))/(a^2*b^4
 + b^6*x^6 + 2*a*b^5*x^3) + (f*x^6)/(6*b^3) + (log(a + b*x^3)*(b^2*d + 6*a^
2*f - 3*a*b*e))/(3*b^5)
```

3.280 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 109

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{fx^3}{3b^3} - \frac{b^3c-ab^2d+a^2be-a^3f}{6b^4(a+bx^3)^2} - \frac{b^2d-2abe+3a^2f}{3b^4(a+bx^3)} + \frac{(be-3af)\log(a+bx^3)}{3b^4}$$

```
[Out] 1/3*f*x^3/b^3+1/6*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^4/(b*x^3+a)^2+1/3*(-3*a^2
*f+2*a*b*e-b^2*d)/b^4/(b*x^3+a)+1/3*(-3*a*f+b*e)*ln(b*x^3+a)/b^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1833, 1864}

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = -\frac{3a^2f-2abe+b^2d}{3b^4(a+bx^3)} - \frac{a^3(-f)+a^2be-ab^2d+b^3c}{6b^4(a+bx^3)^2} + \frac{(be-3af)\log(a+bx^3)}{3b^4} + \frac{fx^3}{3b^3}$$

```
[In] Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (f*x^3)/(3*b^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*b^4*(a + b*x^3)^2)
 - (b^2*d - 2*a*b*e + 3*a^2*f)/(3*b^4*(a + b*x^3)) + ((b*e - 3*a*f)*Log[a +
b*x^3])/(3*b^4)
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{(a+bx)^3}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{f}{b^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3(a+bx)^3}+\frac{b^2d-2abe+3a^2f}{b^3(a+bx)^2}+\frac{be-3af}{b^3(a+bx)}\right)\,dx, x, x^3\right)\\
&= \frac{fx^3}{3b^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{6b^4\left(a+bx^3\right)^2}-\frac{b^2d-2abe+3a^2f}{3b^4\left(a+bx^3\right)}+\frac{(be-3af)\log\left(a+bx^3\right)}{3b^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$=\frac{-5a^3f+a^2b(3e-4fx^3)+ab^2(-d+4ex^3+4fx^6)-b^3(c+2dx^3-2fx^9)+2(be-3af)\left(a+bx^3\right)^2\log\left(a\right.}{6b^4\left(a+bx^3\right)^2}$$

```
[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (-5*a^3*f + a^2*b*(3*e - 4*f*x^3) + a*b^2*(-d + 4*e*x^3 + 4*f*x^6) - b^3*(c
 + 2*d*x^3 - 2*f*x^9) + 2*(b*e - 3*a*f)*(a + b*x^3)^2*Log[a + b*x^3])/(6*b^
4*(a + b*x^3)^2)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

method	result
norman	$\frac{-\frac{9fa^3-3a^2be+ab^2d+b^3c}{6b^4}+\frac{fx^9}{3b}-\frac{\left(6a^2f-2aeb+b^2d\right)x^3}{3b^3}}{(bx^3+a)^2}-\frac{(3af-be)\ln\left(bx^3+a\right)}{3b^4}$
risch	$\frac{fx^3}{3b^3}+\frac{(-a^2f+\frac{2}{3}aeb-\frac{1}{3}b^2d)x^3-\frac{5fa^3-3a^2be+ab^2d+b^3c}{6b}}{b^3(bx^3+a)^2}-\frac{\ln(bx^3+a)af}{b^4}+\frac{\ln(bx^3+a)e}{3b^3}$
default	$\frac{fx^3}{3b^3}-\frac{\frac{(3af-be)\ln\left(bx^3+a\right)}{b}-\frac{fa^3-a^2be+ab^2d-b^3c}{2b\left(bx^3+a\right)^2}-\frac{-3a^2f+2aeb-b^2d}{b\left(bx^3+a\right)}}{3b^3}$
parallelrisch	$-\frac{-2b^3fx^9+6\ln(bx^3+a)x^6ab^2f-2\ln(bx^3+a)x^6b^3e+12\ln(bx^3+a)x^3a^2bf-4\ln(bx^3+a)x^3ab^2e+12a^2bfx^3-4ab^2ex^3+2b}{6b^4(bx^3+a)^2}$

```
[In] int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/6*(9*a^3*f-3*a^2*b*e+a*b^2*d+b^3*c)/b^4+1/3*f*x^9/b-1/3*(6*a^2*f-2*a*b*
e+b^2*d)/b^3*x^3)/(b*x^3+a)^2-1/3*(3*a*f-b*e)/b^4*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.45

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$=\frac{2b^3fx^9+4ab^2fx^6-b^3c-ab^2d+3a^2be-5a^3f-2(b^3d-2ab^2e+2a^2bf)x^3+2((b^3e-3ab^2f)x^6+a^2}{6(b^6x^6+2ab^5x^3+a^2b^4)}$$

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*f*x^9 + 4*a*b^2*f*x^6 - b^3*c - a*b^2*d + 3*a^2*b*e - 5*a^3*f -
2*(b^3*d - 2*a*b^2*e + 2*a^2*b*f)*x^3 + 2*((b^3*e - 3*a*b^2*f)*x^6 + a^2*b*
e - 3*a^3*f + 2*(a*b^2*e - 3*a^2*b*f)*x^3)*log(b*x^3 + a))/(b^6*x^6 + 2*a*b
^5*x^3 + a^2*b^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{fx^3}{3\,b^3} - \frac{b^3c+ab^2d-3\,a^2be+5\,a^3f+2\,(b^3d-2\,ab^2e+3\,a^2bf)x^3}{6\,(b^6x^6+2\,ab^5x^3+a^2b^4)} + \frac{(be-3\,af)\log\left(bx^3+a\right)}{3\,b^4}$$

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/3*f*x^3/b^3 - 1/6*(b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f + 2*(b^3*d - 2*a
*b^2*e + 3*a^2*b*f)*x^3)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) + 1/3*(b*e - 3*a
*f)*log(b*x^3 + a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{fx^3}{3\,b^3} + \frac{(be-3\,af)\log\left(|bx^3+a|\right)}{3\,b^4} - \frac{b^3c+ab^2d-3\,a^2be+5\,a^3f+2\,(b^3d-2\,ab^2e+3\,a^2bf)x^3}{6\,(bx^3+a)^2b^4}$$

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
[Out] 1/3*f*x^3/b^3 + 1/3*(b*e - 3*a*f)*log(abs(b*x^3 + a))/b^4 - 1/6*(b^3*c + a*
b^2*d - 3*a^2*b*e + 5*a^3*f + 2*(b^3*d - 2*a*b^2*e + 3*a^2*b*f)*x^3)/((b*x^
3 + a)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.03

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{f\,x^3}{3\,b^3} - \frac{x^3\,\left(f\,a^2-\frac{2\,e\,a\,b}{3}+\frac{d\,b^2}{3}\right)+\frac{5\,f\,a^3-3\,e\,a^2\,b+d\,a\,b^2+c\,b^3}{6\,b}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6} - \frac{\ln\left(b\,x^3+a\right)\,\left(3\,a\,f-b\,e\right)}{3\,b^4}$$

```
[In] int((x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
[Out] (f*x^3)/(3*b^3) - (x^3*((b^2*d)/3 + a^2*f - (2*a*b*e)/3) + (b^3*c + 5*a^3*f
 + a*b^2*d - 3*a^2*b*e)/(6*b))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) - (log(a +
 b*x^3)*(3*a*f - b*e))/(3*b^4)
```

3.281 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 114

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^3} dx = \frac{b^3c-ab^2d+a^2be-a^3f}{6ab^3(a+bx^3)^2}+\frac{b^3c-a^2be+2a^3f}{3a^2b^3(a+bx^3)}+\frac{c\log(x)}{a^3}-\frac{1}{3}\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a+bx^3\right)$$

```
[Out] 1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a/b^3/(b*x^3+a)^2+1/3*(2*a^3*f-a^2*b*e+b
^3*c)/a^2/b^3/(b*x^3+a)+c*ln(x)/a^3-1/3*(c/a^3-f/b^3)*ln(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^3} dx = -\frac{1}{3}\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a+bx^3\right)+\frac{c\log(x)}{a^3}+\frac{2a^3f-a^2be+b^3c}{3a^2b^3(a+bx^3)}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6ab^3(a+bx^3)^2}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x]
```

```
[Out] (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a*b^3*(a + b*x^3)^2) + (b^3*c - a^2*
b*e + 2*a^3*f)/(3*a^2*b^3*(a + b*x^3)) + (c*Log[x])/a^3 - ((c/a^3 - f/b^3)*
Log[a + b*x^3])/3
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2(a+bx)^3}+\frac{-b^3c+a^2be-2a^3f}{a^2b^2(a+bx)^2}\right.\right.\\ &\qquad\left.\left.+\frac{-b^3c+a^3f}{a^3b^2(a+bx)}\right)\,dx,x,x^3\right)\\ &= \frac{b^3c-ab^2d+a^2be-a^3f}{6ab^3\left(a+bx^3\right)^2}+\frac{b^3c-a^2be+2a^3f}{3a^2b^3\left(a+bx^3\right)}+\frac{c\log(x)}{a^3}-\frac{1}{3}\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a+bx^3\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^3}\,dx = \frac{6c\log(x)+\frac{\frac{a\left(3ab^3c+3a^4f+2b^4cx^3-a^2b^2\left(d+2ex^3\right)-a^3b\left(e-4fx^3\right)\right)}{\left(a+bx^3\right)^2}+2\left(-b^3c+a^3f\right)\log\left(a+bx^3\right)}{b^3}}{6a^3}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x]
```

```
[Out] (6*c*Log[x] + ((a*(3*a*b^3*c + 3*a^4*f + 2*b^4*c*x^3 - a^2*b^2*(d + 2*e*x^3
) - a^3*b*(e - 4*f*x^3)))/(a + b*x^3)^2 + 2*(-(b^3*c) + a^3*f)*Log[a + b*x^
3])/b^3)/(6*a^3)
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

method	result
norman	$\frac{\frac{3 f a^{3}-a^{2} b e-a b^{2} d+3 b^{3} c}{6 a b^{3}}+\frac{\left(2 f a^{3}-a^{2} b e+b^{3} c\right) x^{3}}{3 a^{2} b^{2}}}{\left(b x^{3}+a\right)^{2}}+\frac{c \ln \left(x\right)}{a^{3}}+\frac{\left(f a^{3}-b^{3} c\right) \ln \left(b x^{3}+a\right)}{3 a^{3} b^{3}}$
default	$\frac{c \ln \left(x\right)}{a^{3}}+\frac{\frac{\left(f a^{3}-b^{3} c\right) \ln \left(b x^{3}+a\right)}{b^{3}}-\frac{a^{2} \left(f a^{3}-a^{2} b e+a b^{2} d-b^{3} c\right)}{2 b^{3} \left(b x^{3}+a\right)^{2}}+\frac{a \left(2 f a^{3}-a^{2} b e+b^{3} c\right)}{b^{3} \left(b x^{3}+a\right)}}{3 a^{3}}$
risch	$\frac{\frac{3 f a^{3}-a^{2} b e-a b^{2} d+3 b^{3} c}{6 a b^{3}}+\frac{\left(2 f a^{3}-a^{2} b e+b^{3} c\right) x^{3}}{3 a^{2} b^{2}}}{\left(b x^{3}+a\right)^{2}}+\frac{c \ln \left(x\right)}{a^{3}}+\frac{\ln \left(-b x^{3}-a\right) f}{3 b^{3}}-\frac{\ln \left(-b x^{3}-a\right) c}{3 a^{3}}$
parallelrisch	$\frac{6 \ln \left(x\right) x^{6} b^{5} c+2 \ln \left(b x^{3}+a\right) x^{6} a^{3} b^{2} f-2 \ln \left(b x^{3}+a\right) x^{6} b^{5} c+12 \ln \left(x\right) x^{3} a b^{4} c+4 \ln \left(b x^{3}+a\right) x^{3} a^{4} b f-4 \ln \left(b x^{3}+a\right) x^{3} a b^{4} c+4 a^{4} b f x^{3}-}{6 a^{3} b^{3} \left(b x^{3}+a\right)^{2}}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/6*(3*a^3*f-a^2*b*e-a*b^2*d+3*b^3*c)/a/b^3+1/3*(2*a^3*f-a^2*b*e+b^3*c)/a^
2/b^2*x^3)/(b*x^3+a)^2+c*ln(x)/a^3+1/3*(a^3*f-b^3*c)/a^3/b^3*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.64

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^3}\,dx$$
$$=\frac{3\,a^2b^3c-a^3b^2d-a^4be+3\,a^5f+2\,(ab^4c-a^3b^2e+2\,a^4bf)x^3-2\,((b^5c-a^3b^2f)x^6+a^2b^3c-a^5f+2\,(ab^4c}{6\,(a^3b^5x^6+2\,a^4b^4x^3+a^5b^3)}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*b^3*c - a^3*b^2*d - a^4*b*e + 3*a^5*f + 2*(a*b^4*c - a^3*b^2*e +
 2*a^4*b*f)*x^3 - 2*((b^5*c - a^3*b^2*f)*x^6 + a^2*b^3*c - a^5*f + 2*(a*b^4
*c - a^4*b*f)*x^3)*log(b*x^3 + a) + 6*(b^5*c*x^6 + 2*a*b^4*c*x^3 + a^2*b^3*
c)*log(x))/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^3}\,dx=\frac{3\,ab^3c-a^2b^2d-a^3be+3\,a^4f+2\,(b^4c-a^2b^2e+2\,a^3bf)x^3}{6\,(a^2b^5x^6+2\,a^3b^4x^3+a^4b^3)}+\frac{c\log\left(x^3\right)}{3\,a^3}-\frac{(b^3c-a^3f)\log\left(bx^3+a\right)}{3\,a^3b^3}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*b^3*c - a^2*b^2*d - a^3*b*e + 3*a^4*f + 2*(b^4*c - a^2*b^2*e + 2*a
^3*b*f)*x^3)/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3) + 1/3*c*log(x^3)/a^3 -
 1/3*(b^3*c - a^3*f)*log(b*x^3 + a)/(a^3*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.11

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^3}\,dx=\frac{c\log\left(|x|\right)}{a^3}-\frac{(b^3c-a^3f)\log\left(|bx^3+a|\right)}{3\,a^3b^3}+\frac{3\,b^4cx^6-3\,a^3bfx^6+8\,ab^3cx^3-2\,a^3bex^3-2\,a^4fx^3+6\,a^2b^2c-a^3bd-a^4e}{6\,(bx^3+a)^2a^3b^2}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] c*log(abs(x))/a^3 - 1/3*(b^3*c - a^3*f)*log(abs(b*x^3 + a))/(a^3*b^3) + 1/6
*(3*b^4*c*x^6 - 3*a^3*b*f*x^6 + 8*a*b^3*c*x^3 - 2*a^3*b*e*x^3 - 2*a^4*f*x^3
 + 6*a^2*b^2*c - a^3*b*d - a^4*e)/((b*x^3 + a)^2*a^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^3}\,dx=\frac{\frac{3\,f\,a^3-e\,a^2\,b-d\,a\,b^2+3\,c\,b^3}{6\,a\,b^3}+\frac{x^3\,\left(2\,f\,a^3-e\,a^2\,b+c\,b^3\right)}{3\,a^2\,b^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{c\,\ln\left(x\right)}{a^3}-\frac{\ln\left(b\,x^3+a\right)\,\left(b^3\,c-a^3\,f\right)}{3\,a^3\,b^3}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x)
```

```
[Out] ((3*b^3*c + 3*a^3*f - a*b^2*d - a^2*b*e)/(6*a*b^3) + (x^3*(b^3*c + 2*a^3*f
- a^2*b*e))/(3*a^2*b^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + (c*log(x))/a^3 - (lo
g(a + b*x^3)*(b^3*c - a^3*f))/(3*a^3*b^3)
```

3.282 $\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 134

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^3} dx = -\frac{c}{3a^3x^3} - \frac{b^3c-ab^2d+a^2be-a^3f}{6a^2b^2(a+bx^3)^2} - \frac{2b^3c-ab^2d+a^3f}{3a^3b^2(a+bx^3)} - \frac{(3bc-ad)\log(x)}{a^4} + \frac{(3bc-ad)\log(a+bx^3)}{3a^4}$$

```
[Out] -1/3*c/a^3/x^3+1/6*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^2/b^2/(b*x^3+a)^2+1/3*(-
a^3*f+a*b^2*d-2*b^3*c)/a^3/b^2/(b*x^3+a)-(-a*d+3*b*c)*ln(x)/a^4+1/3*(-a*d+3
*b*c)*ln(b*x^3+a)/a^4
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^3} dx = \frac{(3bc-ad)\log(a+bx^3)}{3a^4} - \frac{\log(x)(3bc-ad)}{a^4} - \frac{a^3f-ab^2d+2b^3c}{3a^3b^2(a+bx^3)} - \frac{c}{3a^3x^3} - \frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^2b^2(a+bx^3)^2}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -1/3*c/(a^3*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^2*b^2*(a + b*x^
3)^2) - (2*b^3*c - a*b^2*d + a^3*f)/(3*a^3*b^2*(a + b*x^3)) - ((3*b*c - a*d
)*Log[x])/a^4 + ((3*b*c - a*d)*Log[a + b*x^3])/(3*a^4)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx)^3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x^2}+\frac{-3bc+ad}{a^4x}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b(a+bx)^3}+\frac{2b^3c-ab^2d+a^3f}{a^3b(a+bx)^2}\right.\right.\\ &\qquad\left.\left.-\frac{b(-3bc+ad)}{a^4(a+bx)}\right)dx, x, x^3\right)\\ &= -\frac{c}{3a^3x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{6a^2b^2(a+bx^3)^2}-\frac{2b^3c-ab^2d+a^3f}{3a^3b^2(a+bx^3)}\\ &\quad-\frac{(3bc-ad)\log(x)}{a^4}+\frac{(3bc-ad)\log(a+bx^3)}{3a^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^3}\,dx = \frac{-\frac{2ac}{x^3}+\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)}{b^2(a+bx^3)^2}-\frac{2a(2b^3c-ab^2d+a^3f)}{b^2(a+bx^3)}+6(-3bc+ad)\log(x)+2(3bc-ad)\log(a+bx^3)}{6a^4}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] ((-2*a*c)/x^3 + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(b^2*(a + b*x^
3)^2) - (2*a*(2*b^3*c - a*b^2*d + a^3*f))/(b^2*(a + b*x^3)) + 6*(-3*b*c + a
*d)*Log[x] + 2*(3*b*c - a*d)*Log[a + b*x^3])/(6*a^4)
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

method	result
norman	$\frac{-\frac{c}{3a}+\frac{\left(a^{2}e-2abd+6b^{2}c\right)x^{6}}{3a^{3}}+\frac{\left(f\,a^{3}+a^{2}be-3a\,b^{2}d+9b^{3}c\right)x^{9}}{6a^{4}}}{x^{3}\left(b\,x^{3}+a\right)^{2}}+\frac{(ad-3bc)\ln(x)}{a^{4}}-\frac{(ad-3bc)\ln\left(b\,x^{3}+a\right)}{3a^{4}}$
default	$-\frac{c}{3a^{3}x^{3}}+\frac{(ad-3bc)\ln(x)}{a^{4}}+\frac{(-ad+3bc)\ln\left(b\,x^{3}+a\right)+\frac{a^{2}\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)}{2b^{2}\left(b\,x^{3}+a\right)^{2}}-\frac{a\left(f\,a^{3}-a\,b^{2}d+2b^{3}c\right)}{b^{2}\left(b\,x^{3}+a\right)}}{3a^{4}}$
risch	$\frac{-\frac{\left(f\,a^{3}-a\,b^{2}d+3b^{3}c\right)x^{6}}{3a^{3}b}-\frac{\left(f\,a^{3}+a^{2}be-3a\,b^{2}d+9b^{3}c\right)x^{3}}{6a^{2}b^{2}}-\frac{c}{3a}}{x^{3}\left(b\,x^{3}+a\right)^{2}}+\frac{d\ln(x)}{a^{3}}-\frac{3bc\ln(x)}{a^{4}}-\frac{d\ln\left(b\,x^{3}+a\right)}{3a^{3}}+\frac{bc\ln\left(b\,x^{3}+a\right)}{a^{4}}$
parallelrisch	$6\ln(x)x^{9}a\,b^{2}d-18\ln(x)x^{9}b^{3}c-2\ln\left(b\,x^{3}+a\right)x^{9}a\,b^{2}d+6\ln\left(b\,x^{3}+a\right)x^{9}b^{3}c+x^{9}a^{3}f+x^{9}a^{2}be-3x^{9}a\,b^{2}d+9b^{3}c\,x^{9}+12\ln(x)x^{6}a^{2}bd$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*c/a+1/3*(a^2*e-2*a*b*d+6*b^2*c)/a^3*x^6+1/6*(a^3*f+a^2*b*e-3*a*b^2*d+
9*b^3*c)/a^4*x^9)/x^3/(b*x^3+a)^2+(a*d-3*b*c)/a^4*ln(x)-1/3*(a*d-3*b*c)/a^4
*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.87

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^3}\,dx = -\frac{2\,(3\,ab^4c-a^2b^3d+a^4bf)x^6+2\,a^3b^2c+(9\,a^2b^3c-3\,a^3b^2d+a^4be+a^5f)x^3-2\,((3\,b^5c-ab^4d)x^9+2\,(3}{6\,(a}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(3*a*b^4*c - a^2*b^3*d + a^4*b*f)*x^6 + 2*a^3*b^2*c + (9*a^2*b^3*c
- 3*a^3*b^2*d + a^4*b*e + a^5*f)*x^3 - 2*((3*b^5*c - a*b^4*d)*x^9 + 2*(3*a*
b^4*c - a^2*b^3*d)*x^6 + (3*a^2*b^3*c - a^3*b^2*d)*x^3)*log(b*x^3 + a) + 6*
((3*b^5*c - a*b^4*d)*x^9 + 2*(3*a*b^4*c - a^2*b^3*d)*x^6 + (3*a^2*b^3*c - a
^3*b^2*d)*x^3)*log(x))/(a^4*b^4*x^9 + 2*a^5*b^3*x^6 + a^6*b^2*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^3}\,dx &= -\frac{2\,(3\,b^4c-ab^3d+a^3bf)x^6+2\,a^2b^2c+(9\,ab^3c-3\,a^2b^2d+a^3be+a^4f)x^3}{6\,(a^3b^4x^9+2\,a^4b^3x^6+a^5b^2x^3)} \\ &\quad+\frac{(3\,bc-ad)\log\left(bx^3+a\right)}{3\,a^4}-\frac{(3\,bc-ad)\log\left(x^3\right)}{3\,a^4}\end{aligned}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
[Out] -1/6*(2*(3*b^4*c - a*b^3*d + a^3*b*f)*x^6 + 2*a^2*b^2*c + (9*a*b^3*c - 3*a^
2*b^2*d + a^3*b*e + a^4*f)*x^3)/(a^3*b^4*x^9 + 2*a^4*b^3*x^6 + a^5*b^2*x^3)
 + 1/3*(3*b*c - a*d)*log(b*x^3 + a)/a^4 - 1/3*(3*b*c - a*d)*log(x^3)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^3}\,dx &= -\frac{(3\,bc-ad)\log\left(|x|\right)}{a^4}+\frac{(3\,b^2c-abd)\log\left(|bx^3+a|\right)}{3\,a^4b}+\frac{3\,bcx^3-adx^3-ac}{3\,a^4x^3} \\ &\quad-\frac{9\,b^5cx^6-3\,ab^4dx^6+22\,ab^4cx^3-8\,a^2b^3dx^3+2\,a^4bfx^3+14\,a^2b^3c-6\,a^3b^2d+a^4be+a^5f}{6\,(bx^3+a)^2a^4b^2}\end{aligned}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="giac")
[Out] -(3*b*c - a*d)*log(abs(x))/a^4 + 1/3*(3*b^2*c - a*b*d)*log(abs(b*x^3 + a))/
(a^4*b) + 1/3*(3*b*c*x^3 - a*d*x^3 - a*c)/(a^4*x^3) - 1/6*(9*b^5*c*x^6 - 3*
a*b^4*d*x^6 + 22*a*b^4*c*x^3 - 8*a^2*b^3*d*x^3 + 2*a^4*b*f*x^3 + 14*a^2*b^3
*c - 6*a^3*b^2*d + a^4*b*e + a^5*f)/((b*x^3 + a)^2*a^4*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\,(a+bx^3)^3}\,dx=\frac{\ln(x)\,(a\,d-3\,b\,c)}{a^4}-\frac{\ln(b\,x^3+a)\,(a\,d-3\,b\,c)}{3\,a^4}-\frac{\frac{c}{3\,a}+\frac{x^6\,(f\,a^3-d\,a\,b^2+3\,c\,b^3)}{3\,a^3\,b}+\frac{x^3\,(f\,a^3+e\,a^2\,b-3\,d\,a\,b^2+9\,c\,b^3)}{6\,a^2\,b^2}}{a^2\,x^3+2\,a\,b\,x^6+b^2\,x^9}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x)
```

```
[Out] (log(x)*(a*d - 3*b*c))/a^4 - (log(a + b*x^3)*(a*d - 3*b*c))/(3*a^4) - (c/(3
*a) + (x^6*(3*b^3*c + a^3*f - a*b^2*d))/(3*a^3*b) + (x^3*(9*b^3*c + a^3*f -
 3*a*b^2*d + a^2*b*e))/(6*a^2*b^2))/(a^2*x^3 + b^2*x^9 + 2*a*b*x^6)
```

3.283 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 163

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^3}\,dx = -\frac{c}{6a^3x^6}+\frac{3bc-ad}{3a^4x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{6a^3b(a+bx^3)^2}+\frac{3b^2c-2abd+a^2e}{3a^4(a+bx^3)}+\frac{(6b^2c-3abd+a^2e)\log(x)}{a^5}-\frac{(6b^2c-3abd+a^2e)\log(a+bx^3)}{3a^5}$$

```
[Out] -1/6*c/a^3/x^6+1/3*(-a*d+3*b*c)/a^4/x^3+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/
a^3/b/(b*x^3+a)^2+1/3*(a^2*e-2*a*b*d+3*b^2*c)/a^4/(b*x^3+a)+(a^2*e-3*a*b*d+
6*b^2*c)*ln(x)/a^5-1/3*(a^2*e-3*a*b*d+6*b^2*c)*ln(b*x^3+a)/a^5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^3}\,dx = \frac{3bc-ad}{3a^4x^3}-\frac{c}{6a^3x^6}-\frac{\log(a+bx^3)(a^2e-3abd+6b^2c)}{3a^5}+\frac{\log(x)(a^2e-3abd+6b^2c)}{a^5}+\frac{a^2e-2abd+3b^2c}{3a^4(a+bx^3)}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^3b(a+bx^3)^2}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] -1/6*c/(a^3*x^6) + (3*b*c - a*d)/(3*a^4*x^3) + (b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)/(6*a^3*b*(a + b*x^3)^2) + (3*b^2*c - 2*a*b*d + a^2*e)/(3*a^4*(a + b
*x^3)) + ((6*b^2*c - 3*a*b*d + a^2*e)*Log[x])/a^5 - ((6*b^2*c - 3*a*b*d + a
^2*e)*Log[a + b*x^3])/(3*a^5)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx)^3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x^3}+\frac{-3bc+ad}{a^4x^2}+\frac{6b^2c-3abd+a^2e}{a^5x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3(a+bx)^3}\right.\right.\\ &\qquad \left.\left.-\frac{b(3b^2c-2abd+a^2e)}{a^4(a+bx)^2}-\frac{b(6b^2c-3abd+a^2e)}{a^5(a+bx)}\right)dx, x, x^3\right)\\ &= -\frac{c}{6a^3x^6}+\frac{3bc-ad}{3a^4x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{6a^3b\left(a+bx^3\right)^2}+\frac{3b^2c-2abd+a^2e}{3a^4\left(a+bx^3\right)}\\ &\quad +\frac{(6b^2c-3abd+a^2e)\log(x)}{a^5}-\frac{(6b^2c-3abd+a^2e)\log\left(a+bx^3\right)}{3a^5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^3}\,dx = \frac{-\frac{a^2c}{x^6}-\frac{2a(-3bc+ad)}{x^3}+\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)}{b(a+bx^3)^2}+\frac{2a\left(3b^2c-2abd+a^2e\right)}{a+bx^3}+6(6b^2c-3abd+a^2e)\log(x)-2(6b^2c-3ab}{6a^5}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] (-((a^2*c)/x^6) - (2*a*(-3*b*c + a*d))/x^3 + (a^2*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f))/(b*(a + b*x^3)^2) + (2*a*(3*b^2*c - 2*a*b*d + a^2*e))/(a + b*x^
3) + 6*(6*b^2*c - 3*a*b*d + a^2*e)*Log[x] - 2*(6*b^2*c - 3*a*b*d + a^2*e)*L
og[a + b*x^3])/(6*a^5)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.94

method	result
default	$-\frac{c}{6a^3x^6}-\frac{ad-3bc}{3a^4x^3}+\frac{(a^2e-3abd+6b^2c)\ln(x)}{a^5}+\frac{(-a^2e+3abd-6b^2c)\ln(bx^3+a)-\frac{a^2\left(fa^3-a^2be+ab^2d-b^3c\right)}{2b\left(bx^3+a\right)^2}+\frac{a\left(a^2e-2abd+\right)}{bx^3+a}}{3a^5}$
norman	$\frac{-\frac{c}{6a}-\frac{(ad-2bc)x^3}{3a^2}+\frac{\left(fa^3-2a^2be+6ab^2d-12b^3c\right)x^9}{3a^4}+\frac{b\left(fa^3-3a^2be+9ab^2d-18b^3c\right)x^{12}}{6a^5}}{x^6(bx^3+a)^2}+\frac{(a^2e-3abd+6b^2c)\ln(x)}{a^5}-(a^2e-3abd-$
risch	$\frac{\frac{b\left(a^2e-3abd+6b^2c\right)x^9}{3a^4}-\frac{\left(fa^3-3a^2be+9ab^2d-18b^3c\right)x^6}{6a^3b}-\frac{(ad-2bc)x^3}{3a^2}-\frac{c}{6a}}{x^6(bx^3+a)^2}+\frac{e\ln(x)}{a^3}-\frac{3\ln(x)bd}{a^4}+\frac{6\ln(x)b^2c}{a^5}-\frac{e\ln(bx^3+a)}{3a^3}$
parallelrisch	$\frac{-4a^3bex^9+2a^4fx^9+12a^2b^2dx^9-18b^4cx^{12}+36\ln(x)x^6a^2b^2c+6\ln(bx^3+a)x^6a^3bd-12\ln(bx^3+a)x^6a^2b^2c-a^4c-2a^4dx^3+4a^3}{}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*c/a^3/x^6-1/3*(a*d-3*b*c)/a^4/x^3+(a^2*e-3*a*b*d+6*b^2*c)*ln(x)/a^5+1/
3/a^5*((-a^2*e+3*a*b*d-6*b^2*c)*ln(b*x^3+a)-1/2*a^2*(a^3*f-a^2*b*e+a*b^2*d-
b^3*c)/b/(b*x^3+a)^2+a*(a^2*e-2*a*b*d+3*b^2*c)/(b*x^3+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(153) = 306.

Time = 0.28 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^3}\,dx$$
$$=\frac{2\left(6\,ab^4c-3\,a^2b^3d+a^3b^2e\right)x^9+\left(18\,a^2b^3c-9\,a^3b^2d+3\,a^4be-a^5f\right)x^6-a^4bc+2\left(2\,a^3b^2c-a^4bd\right)x^3-2\left(\left(6\right.\right.}{}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e)*x^9 + (18*a^2*b^3*c - 9*a^3*b^
2*d + 3*a^4*b*e - a^5*f)*x^6 - a^4*b*c + 2*(2*a^3*b^2*c - a^4*b*d)*x^3 - 2*
((6*b^5*c - 3*a*b^4*d + a^2*b^3*e)*x^12 + 2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*
b^2*e)*x^9 + (6*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e)*x^6)*log(b*x^3 + a) + 6*
((6*b^5*c - 3*a*b^4*d + a^2*b^3*e)*x^12 + 2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*
b^2*e)*x^9 + (6*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e)*x^6)*log(x))/(a^5*b^3*x^
12 + 2*a^6*b^2*x^9 + a^7*b*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^3}\,dx = \frac{2\,(6\,b^4c-3\,ab^3d+a^2b^2e)x^9+(18\,ab^3c-9\,a^2b^2d+3\,a^3be-a^4f)x^6-a^3bc+2\,(2\,a^2b^2c-a^3bd)x^3}{6\,(a^4b^3x^{12}+2\,a^5b^2x^9+a^6bx^6)} - \frac{(6\,b^2c-3\,abd+a^2e)\log(bx^3+a)}{3\,a^5} + \frac{(6\,b^2c-3\,abd+a^2e)\log(x^3)}{3\,a^5}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/6*(2*(6*b^4*c - 3*a*b^3*d + a^2*b^2*e)*x^9 + (18*a*b^3*c - 9*a^2*b^2*d +
3*a^3*b*e - a^4*f)*x^6 - a^3*b*c + 2*(2*a^2*b^2*c - a^3*b*d)*x^3)/(a^4*b^3*
x^12 + 2*a^5*b^2*x^9 + a^6*b*x^6) - 1/3*(6*b^2*c - 3*a*b*d + a^2*e)*log(b*x
^3 + a)/a^5 + 1/3*(6*b^2*c - 3*a*b*d + a^2*e)*log(x^3)/a^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.13

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^3}\,dx = \frac{(6\,b^2c-3\,abd+a^2e)\log(|x|)}{a^5} - \frac{(6\,b^3c-3\,ab^2d+a^2be)\log(|bx^3+a|)}{3\,a^5b} + \frac{12\,b^4cx^9-6\,ab^3dx^9+2\,a^2b^2ex^9+18\,ab^3cx^6-9\,a^2b^2dx^6+3\,a^3bex^6-a^4fx^6+4\,a^2b^2cx^3-2\,a^3bdx^3-a}{6\,(bx^6+ax^3)^2a^4b}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="giac")
[Out] (6*b^2*c - 3*a*b*d + a^2*e)*log(abs(x))/a^5 - 1/3*(6*b^3*c - 3*a*b^2*d + a^
2*b*e)*log(abs(b*x^3 + a))/(a^5*b) + 1/6*(12*b^4*c*x^9 - 6*a*b^3*d*x^9 + 2*
a^2*b^2*e*x^9 + 18*a*b^3*c*x^6 - 9*a^2*b^2*d*x^6 + 3*a^3*b*e*x^6 - a^4*f*x^
6 + 4*a^2*b^2*c*x^3 - 2*a^3*b*d*x^3 - a^3*b*c)/((b*x^6 + a*x^3)^2*a^4*b)
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^3}\,dx$$
$$=\frac{\ln(x)\left(e\,a^2-3\,d\,a\,b+6\,c\,b^2\right)}{a^5}-\frac{\ln\left(b\,x^3+a\right)\left(e\,a^2-3\,d\,a\,b+6\,c\,b^2\right)}{3\,a^5}$$
$$-\frac{\frac{c}{6\,a}+\frac{x^3\,(a\,d-2\,b\,c)}{3\,a^2}-\frac{b\,x^9\,(e\,a^2-3\,d\,a\,b+6\,c\,b^2)}{3\,a^4}-\frac{x^6\,(-f\,a^3+3\,e\,a^2\,b-9\,d\,a\,b^2+18\,c\,b^3)}{6\,a^3\,b}}{a^2\,x^6+2\,a\,b\,x^9+b^2\,x^{12}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x)
```

```
[Out] (log(x)*(6*b^2*c + a^2*e - 3*a*b*d))/a^5 - (log(a + b*x^3)*(6*b^2*c + a^2*e
 - 3*a*b*d))/(3*a^5) - (c/(6*a) + (x^3*(a*d - 2*b*c))/(3*a^2) - (b*x^9*(6*b
^2*c + a^2*e - 3*a*b*d))/(3*a^4) - (x^6*(18*b^3*c - a^3*f - 9*a*b^2*d + 3*a
^2*b*e))/(6*a^3*b))/(a^2*x^6 + b^2*x^12 + 2*a*b*x^9)
```

3.284 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 218

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx = &-\frac{c}{9a^3x^9}+\frac{3bc-ad}{6a^4x^6}-\frac{6b^2c-3abd+a^2e}{3a^5x^3}\\ &-\frac{b^3c-ab^2d+a^2be-a^3f}{6a^4\left(a+bx^3\right)^2}-\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5\left(a+bx^3\right)}\\ &-\frac{\left(10b^3c-6ab^2d+3a^2be-a^3f\right)\log(x)}{a^6}\\ &+\frac{\left(10b^3c-6ab^2d+3a^2be-a^3f\right)\log\left(a+bx^3\right)}{3a^6}\end{aligned}$$

```
[Out] -1/9*c/a^3/x^9+1/6*(-a*d+3*b*c)/a^4/x^6+1/3*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x^
3+1/6*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/(b*x^3+a)^2+1/3*(a^3*f-2*a^2*b*e+3*
a*b^2*d-4*b^3*c)/a^5/(b*x^3+a)-(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)*ln(x)/
a^6+1/3*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)*ln(b*x^3+a)/a^6
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used

$= \{1835, 1634\}$

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx = \frac{3bc-ad}{6a^4x^6}-\frac{c}{9a^3x^9}-\frac{a^2e-3abd+6b^2c}{3a^5x^3} + \frac{\log\left(a+bx^3\right)\left(a^3(-f)+3a^2be-6ab^2d+10b^3c\right)}{3a^6} - \frac{\log(x)\left(a^3(-f)+3a^2be-6ab^2d+10b^3c\right)}{a^6} - \frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{3a^5\left(a+bx^3\right)} - \frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^4\left(a+bx^3\right)^2}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x]
```

```
[Out] -1/9*c/(a^3*x^9) + (3*b*c - a*d)/(6*a^4*x^6) - (6*b^2*c - 3*a*b*d + a^2*e)/
(3*a^5*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*(a + b*x^3)^2) - (
4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*(a + b*x^3)) - ((10*b^3*c -
 6*a*b^2*d + 3*a^2*b*e - a^3*f)*Log[x])/a^6 + ((10*b^3*c - 6*a*b^2*d + 3*a^
2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^6)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx)^3}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x^4}+\frac{-3bc+ad}{a^4x^3}+\frac{6b^2c-3abd+a^2e}{a^5x^2} + \frac{-10b^3c+6ab^2d-3a^2be+a^3f}{a^6x}-\frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^4(a+bx)^3} - \frac{b(-4b^3c+3ab^2d-2a^2be+a^3f)}{a^5(a+bx)^2} - \frac{b(-10b^3c+6ab^2d-3a^2be+a^3f)}{a^6(a+bx)}\right) dx, x, x^3\right)$$

$$= -\frac{c}{9a^3x^9} + \frac{3bc-ad}{6a^4x^6} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} - \frac{b^3c-ab^2d+a^2be-a^3f}{6a^4(a+bx^3)^2} - \frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5(a+bx^3)} - \frac{(10b^3c-6ab^2d+3a^2be-a^3f)\log(x)}{a^6} + \frac{(10b^3c-6ab^2d+3a^2be-a^3f)\log(a+bx^3)}{3a^6}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{2a^3c}{x^9} - \frac{3a^2(-3bc+ad)}{x^6} - \frac{6a(6b^2c-3abd+a^2e)}{x^3} + \frac{3a^2(-b^3c+ab^2d-a^2be+a^3f)}{(a+bx^3)^2} + \frac{6a(-4b^3c+3ab^2d-2a^2be+a^3f)}{a+bx^3} + 18(-10b^3c+}{18a^6}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x]
```

```
[Out] ((-2*a^3*c)/x^9 - (3*a^2*(-3*b*c + a*d))/x^6 - (6*a*(6*b^2*c - 3*a*b*d + a^
2*e))/x^3 + (3*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 +
(6*a*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f))/(a + b*x^3) + 18*(-10*b^3*
c + 6*a*b^2*d - 3*a^2*b*e + a^3*f)*Log[x] + 6*(10*b^3*c - 6*a*b^2*d + 3*a^2
*b*e - a^3*f)*Log[a + b*x^3])/(18*a^6)
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.97

method	result
default	$-\frac{c}{9a^3x^9} - \frac{ad-3bc}{6a^4x^6} - \frac{a^2e-3abd+6b^2c}{3a^5x^3} + \frac{(f\,a^3-3a^2be+6a\,b^2d-10b^3c)\ln(x)}{a^6} - \frac{b\left(\frac{(f\,a^3-3a^2be+6a\,b^2d-10b^3c)\ln(b\,x^3+a)}{b}\right.}{}$
norman	$\frac{-\frac{c}{9a}-\frac{(3ad-5bc)x^3}{18a^2}-\frac{(3a^2e-6abd+10b^2c)x^6}{9a^3}+\frac{(a^3b^2f-3a^2b^3e+6a\,b^4d-10b^5c)x^9}{2a^4b^2}+\frac{(a^3b^2f-3a^2b^3e+6a\,b^4d-10b^5c)x^{12}}{3a^5b}}{x^9(b\,x^3+a)^2} + (f\,a^3-3a$
risch	$\frac{-\frac{c}{9a}-\frac{(3ad-5bc)x^3}{18a^2}-\frac{(3a^2e-6abd+10b^2c)x^6}{9a^3}+\frac{(f\,a^3-3a^2be+6a\,b^2d-10b^3c)x^9}{2a^4}+\frac{b(f\,a^3-3a^2be+6a\,b^2d-10b^3c)x^{12}}{3a^5}}{x^9(b\,x^3+a)^2} + \frac{\ln(x)f}{a^3} - 3$
parallelrisch	$-6x^6a^5b^2e+12x^6a^4b^3d-20x^6a^3b^4c-90x^9a^2b^5c+9x^9a^5b^2f-27x^9a^4b^3e-3x^3a^5b^2d+5x^3a^4b^3c+54x^9a^3b^4d-12\ln(b\,x^3+a)x^{12}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*c/a^3/x^9-1/6*(a*d-3*b*c)/a^4/x^6-1/3*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^3+
(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6*ln(x)-1/3*b/a^6*((a^3*f-3*a^2*b*e+
```

```
6*a*b^2*d-10*b^3*c)/b*ln(b*x^3+a)-1/2*a^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(
b*x^3+a)^2-a*(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/b/(b*x^3+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.82

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx = -\frac{6\,(10\,ab^4c-6\,a^2b^3d+3\,a^3b^2e-a^4bf)x^{12}+9\,(10\,a^2b^3c-6\,a^3b^2d+3\,a^4be-a^5f)x^9+2\,(10\,a^3b^2c-6\,a^4be}{}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/18*(6*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^12 + 9*(10*a^
2*b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*x^9 + 2*(10*a^3*b^2*c - 6*a^4*b*
d + 3*a^5*e)*x^6 + 2*a^5*c - (5*a^4*b*c - 3*a^5*d)*x^3 - 6*((10*b^5*c - 6*a
*b^4*d + 3*a^2*b^3*e - a^3*b^2*f)*x^15 + 2*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^
3*b^2*e - a^4*b*f)*x^12 + (10*a^2*b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*
x^9)*log(b*x^3 + a) + 18*((10*b^5*c - 6*a*b^4*d + 3*a^2*b^3*e - a^3*b^2*f)*
x^15 + 2*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^12 + (10*a^2*
b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*x^9)*log(x))/(a^6*b^2*x^15 + 2*a^7
*b*x^12 + a^8*x^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx = -\frac{6\left(10\,b^4c-6\,ab^3d+3\,a^2b^2e-a^3bf\right)x^{12}+9\left(10\,ab^3c-6\,a^2b^2d+3\,a^3be-a^4f\right)x^9+2\left(10\,a^2b^2c-6\,a^3bd\right.}{18\left(a^5b^2x^{15}+2\,a^6bx^{12}+a^7x^9\right)} + \frac{\left(10\,b^3c-6\,ab^2d+3\,a^2be-a^3f\right)\log\left(bx^3+a\right)}{3\,a^6} - \frac{\left(10\,b^3c-6\,ab^2d+3\,a^2be-a^3f\right)\log\left(x^3\right)}{3\,a^6}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(6*(10*b^4*c - 6*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^12 + 9*(10*a*b^3*
c - 6*a^2*b^2*d + 3*a^3*b*e - a^4*f)*x^9 + 2*(10*a^2*b^2*c - 6*a^3*b*d + 3*
a^4*e)*x^6 + 2*a^4*c - (5*a^3*b*c - 3*a^4*d)*x^3)/(a^5*b^2*x^15 + 2*a^6*b*x
^12 + a^7*x^9) + 1/3*(10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*log(b*x^3 +
 a)/a^6 - 1/3*(10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*log(x^3)/a^6
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.45

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx = -\frac{\left(10\,b^3c-6\,ab^2d+3\,a^2be-a^3f\right)\log\left(|x|\right)}{a^6} + \frac{\left(10\,b^4c-6\,ab^3d+3\,a^2b^2e-a^3bf\right)\log\left(|bx^3+a|\right)}{3\,a^6b} - \frac{30\,b^5cx^6-18\,ab^4dx^6+9\,a^2b^3ex^6-3\,a^3b^2fx^6+68\,ab^4cx^3-42\,a^2b^3dx^3+22\,a^3b^2ex^3-8\,a^4bfx^3+39\,a^2\cdots}{6\left(bx^3+a\right)^2a^6} + \frac{110\,b^3cx^9-66\,ab^2dx^9+33\,a^2bex^9-11\,a^3fx^9-36\,ab^2cx^6+18\,a^2bdx^6-6\,a^3ex^6+9\,a^2bcx^3-3\,a^3dx^3\cdots}{18\,a^6x^9}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -(10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*log(abs(x))/a^6 + 1/3*(10*b^4*c
 - 6*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*log(abs(b*x^3 + a))/(a^6*b) - 1/6*(30
*b^5*c*x^6 - 18*a*b^4*d*x^6 + 9*a^2*b^3*e*x^6 - 3*a^3*b^2*f*x^6 + 68*a*b^4*
c*x^3 - 42*a^2*b^3*d*x^3 + 22*a^3*b^2*e*x^3 - 8*a^4*b*f*x^3 + 39*a^2*b^3*c
- 25*a^3*b^2*d + 14*a^4*b*e - 6*a^5*f)/((b*x^3 + a)^2*a^6) + 1/18*(110*b^3*
c*x^9 - 66*a*b^2*d*x^9 + 33*a^2*b*e*x^9 - 11*a^3*f*x^9 - 36*a*b^2*c*x^6 + 1
8*a^2*b*d*x^6 - 6*a^3*e*x^6 + 9*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^6*x
^9)
```

Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx = \frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{3\,a^6} - \frac{\frac{c}{9\,a}+\frac{x^9\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{2\,a^4}+\frac{x^3\,\left(3\,a\,d-5\,b\,c\right)}{18\,a^2}+\frac{x^6\,\left(3\,e\,a^2-6\,d\,a\,b+10\,c\,b^2\right)}{9\,a^3}+\frac{b\,x^{12}\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{3\,a^5}}{a^2\,x^9+2\,a\,b\,x^{12}+b^2\,x^{15}} - \frac{\ln\left(x\right)\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{a^6}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x)
```

```
[Out] (log(a + b*x^3)*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/(3*a^6) - (c/(9
*a) + (x^9*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/(2*a^4) + (x^3*(3*a*
d - 5*b*c))/(18*a^2) + (x^6*(10*b^2*c + 3*a^2*e - 6*a*b*d))/(9*a^3) + (b*x^
12*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/(3*a^5))/(a^2*x^9 + b^2*x^15
 + 2*a*b*x^12) - (log(x)*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/a^6
```

3.285 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 258

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^3}\,dx &= -\frac{c}{12a^3x^{12}}+\frac{3bc-ad}{9a^4x^9}-\frac{6b^2c-3abd+a^2e}{6a^5x^6} \\ &\quad+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3}+\frac{b(b^3c-ab^2d+a^2be-a^3f)}{6a^5(a+bx^3)^2} \\ &\quad+\frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{3a^6(a+bx^3)} \\ &\quad+\frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)\log(x)}{a^7} \\ &\quad-\frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)\log(a+bx^3)}{3a^7}\end{aligned}$$

```
[Out] -1/12*c/a^3/x^12+1/9*(-a*d+3*b*c)/a^4/x^9+1/6*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/
x^6+1/3*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x^3+1/6*b*(-a^3*f+a^2*b*e
-a*b^2*d+b^3*c)/a^5/(b*x^3+a)^2+1/3*b*(-2*a^3*f+3*a^2*b*e-4*a*b^2*d+5*b^3*c
)/a^6/(b*x^3+a)+b*(-3*a^3*f+6*a^2*b*e-10*a*b^2*d+15*b^3*c)*ln(x)/a^7-1/3*b*
(-3*a^3*f+6*a^2*b*e-10*a*b^2*d+15*b^3*c)*ln(b*x^3+a)/a^7
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1835, 1634}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx = \frac{3bc-ad}{9a^4x^9}-\frac{c}{12a^3x^{12}}-\frac{a^2e-3abd+6b^2c}{6a^5x^6} - \frac{b\log\left(a+bx^3\right)\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{3a^7} + \frac{b\log(x)\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{a^7} + \frac{b(-2a^3f+3a^2be-4ab^2d+5b^3c)}{3a^6\left(a+bx^3\right)} + \frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{3a^6x^3} + \frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^5\left(a+bx^3\right)^2}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x]
```

```
[Out] -1/12*c/(a^3*x^12) + (3*b*c - a*d)/(9*a^4*x^9) - (6*b^2*c - 3*a*b*d + a^2*e
)/(6*a^5*x^6) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) + (b
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*a^5*(a + b*x^3)^2) + (b*(5*b^3*c -
 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(3*a^6*(a + b*x^3)) + (b*(15*b^3*c - 10*
a*b^2*d + 6*a^2*b*e - 3*a^3*f)*Log[x])/a^7 - (b*(15*b^3*c - 10*a*b^2*d + 6*
a^2*b*e - 3*a^3*f)*Log[a + b*x^3])/(3*a^7)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5(a+bx)^3}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x^5}+\frac{-3bc+ad}{a^4x^4}+\frac{6b^2c-3abd+a^2e}{a^5x^3}\right.\right.\\
&\quad+\frac{-10b^3c+6ab^2d-3a^2be+a^3f}{a^6x^2}-\frac{b(-15b^3c+10ab^2d-6a^2be+3a^3f)}{a^7x}\\
&\quad+\frac{b^2(-b^3c+ab^2d-a^2be+a^3f)}{a^5(a+bx)^3}+\frac{b^2(-5b^3c+4ab^2d-3a^2be+2a^3f)}{a^6(a+bx)^2}\\
&\quad\left.\left.+\frac{b^2(-15b^3c+10ab^2d-6a^2be+3a^3f)}{a^7(a+bx)}\right)\,dx, x, x^3\right)\\
&= -\frac{c}{12a^3x^{12}}+\frac{3bc-ad}{9a^4x^9}-\frac{6b^2c-3abd+a^2e}{6a^5x^6}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3}\\
&\quad+\frac{b(b^3c-ab^2d+a^2be-a^3f)}{6a^5\left(a+bx^3\right)^2}+\frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{3a^6\left(a+bx^3\right)}\\
&\quad+\frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)\log(x)}{a^7}\\
&\quad-\frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)\log\left(a+bx^3\right)}{3a^7}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx$$
$$= \frac{-\frac{a\left(-180b^5cx^{15}+30ab^4x^{12}\left(-9c+4dx^3\right)-12a^2b^3x^9\left(5c-15dx^3+6ex^6\right)-2a^4bx^3\left(3c+5dx^3+12ex^6-27fx^9\right)+a^5\left(3c+4dx^3+6ex^6+12fx^9\right)+a^3b^2x\right.}{x^{12}\left(a+bx^3\right)^2}}{}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x]
```

```
[Out] (-((a*(-180*b^5*c*x^15 + 30*a*b^4*x^12*(-9*c + 4*d*x^3) - 12*a^2*b^3*x^9*(5
*c - 15*d*x^3 + 6*e*x^6) - 2*a^4*b*x^3*(3*c + 5*d*x^3 + 12*e*x^6 - 27*f*x^9
) + a^5*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) + a^3*b^2*x^6*(15*c + 40*d*x^3
 - 108*e*x^6 + 36*f*x^9)))/(x^12*(a + b*x^3)^2)) + 36*b*(15*b^3*c - 10*a*b^
2*d + 6*a^2*b*e - 3*a^3*f)*Log[x] + 12*b*(-15*b^3*c + 10*a*b^2*d - 6*a^2*b*
e + 3*a^3*f)*Log[a + b*x^3])/(36*a^7)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.98

method	result
default	$-\frac{c}{12a^3x^{12}}-\frac{ad-3bc}{9a^4x^9}-\frac{a^2e-3abd+6b^2c}{6a^5x^6}-\frac{f\,a^3-3a^2be+6a\,b^2d-10b^3c}{3a^6x^3}-\frac{b\left(3f\,a^3-6a^2be+10a\,b^2d-15b^3c\right)\ln(x)}{a^7}+\frac{b^2(}{}$
norman	$\frac{-\frac{c}{12a}-\frac{(2ad-3bc)x^3}{18a^2}-\frac{\left(6a^2e-10abd+15b^2c\right)x^6}{36a^3}-\frac{\left(3f\,a^3-6a^2be+10a\,b^2d-15b^3c\right)x^9}{9a^4}+\frac{\left(-3a^3b^3f+6a^2b^4e-10a\,b^5d+15b^6c\right)x^{12}}{2a^5b^2}+\frac{(-3a^3b^3}{}}{x^{12}(b\,x^3+a)^2}$
risch	$\frac{-\frac{c}{12a}-\frac{(2ad-3bc)x^3}{18a^2}-\frac{\left(6a^2e-10abd+15b^2c\right)x^6}{36a^3}-\frac{\left(3f\,a^3-6a^2be+10a\,b^2d-15b^3c\right)x^9}{9a^4}-\frac{b\left(3f\,a^3-6a^2be+10a\,b^2d-15b^3c\right)x^{12}}{2a^5}-\frac{b^2(3f\,a^3-6a}{}}{x^{12}(b\,x^3+a)^2}$
parallelrisch	$-\frac{-60x^9a^3b^5c+6x^6a^6b^2e-10x^6a^5b^3d+15x^6a^4b^4c+4x^3a^6b^2d-6x^3a^5b^3c-540\ln(x)x^{18}b^8c+180\ln(b\,x^3+a)x^{18}b^8c+3a^6b^2c+108}{}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*c/a^3/x^12-1/9*(a*d-3*b*c)/a^4/x^9-1/6*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^
6-1/3*(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6/x^3-b*(3*a^3*f-6*a^2*b*e+10*
a*b^2*d-15*b^3*c)/a^7*ln(x)+1/3*b^2/a^7*((3*a^3*f-6*a^2*b*e+10*a*b^2*d-15*b
^3*c)/b*ln(b*x^3+a)-1/2*a^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a)^2-a*(
2*a^3*f-3*a^2*b*e+4*a*b^2*d-5*b^3*c)/b/(b*x^3+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.74

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx$$
$$=\frac{12\,(15\,ab^5c-10\,a^2b^4d+6\,a^3b^3e-3\,a^4b^2f)x^{15}+18\,(15\,a^2b^4c-10\,a^3b^3d+6\,a^4b^2e-3\,a^5bf)x^{12}+4\,(15\,a^3b^3}{}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/36*(12*(15*a*b^5*c - 10*a^2*b^4*d + 6*a^3*b^3*e - 3*a^4*b^2*f)*x^15 + 18*
(15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2*e - 3*a^5*b*f)*x^12 + 4*(15*a^3*b^
3*c - 10*a^4*b^2*d + 6*a^5*b*e - 3*a^6*f)*x^9 - 3*a^6*c - (15*a^4*b^2*c - 1
0*a^5*b*d + 6*a^6*e)*x^6 + 2*(3*a^5*b*c - 2*a^6*d)*x^3 - 12*((15*b^6*c - 10
*a*b^5*d + 6*a^2*b^4*e - 3*a^3*b^3*f)*x^18 + 2*(15*a*b^5*c - 10*a^2*b^4*d +
 6*a^3*b^3*e - 3*a^4*b^2*f)*x^15 + (15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2
*e - 3*a^5*b*f)*x^12)*log(b*x^3 + a) + 36*((15*b^6*c - 10*a*b^5*d + 6*a^2*b
^4*e - 3*a^3*b^3*f)*x^18 + 2*(15*a*b^5*c - 10*a^2*b^4*d + 6*a^3*b^3*e - 3*a
^4*b^2*f)*x^15 + (15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2*e - 3*a^5*b*f)*x^
12)*log(x))/(a^7*b^2*x^18 + 2*a^8*b*x^15 + a^9*x^12)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^{13}\,(a + bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{c + dx^3 + ex^6 + fx^9}{x^{13}\,(a + bx^3)^3}\,dx &= \frac{12\,(15\,b^5c - 10\,ab^4d + 6\,a^2b^3e - 3\,a^3b^2f)x^{15} + 18\,(15\,ab^4c - 10\,a^2b^3d + 6\,a^3b^2e - 3\,a^4bf)x^{12} + 4\,(15\,a^2b^3c}{36\,(a^6b^2x^{18} + 2\,a^7bx^{15}} \\ &\quad - \frac{(15\,b^4c - 10\,ab^3d + 6\,a^2b^2e - 3\,a^3bf)\log\left(bx^3 + a\right)}{3\,a^7} \\ &\quad + \frac{(15\,b^4c - 10\,ab^3d + 6\,a^2b^2e - 3\,a^3bf)\log\left(x^3\right)}{3\,a^7}\end{aligned}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(12*(15*b^5*c - 10*a*b^4*d + 6*a^2*b^3*e - 3*a^3*b^2*f)*x^15 + 18*(15*
a*b^4*c - 10*a^2*b^3*d + 6*a^3*b^2*e - 3*a^4*b*f)*x^12 + 4*(15*a^2*b^3*c -
10*a^3*b^2*d + 6*a^4*b*e - 3*a^5*f)*x^9 - (15*a^3*b^2*c - 10*a^4*b*d + 6*a^
5*e)*x^6 - 3*a^5*c + 2*(3*a^4*b*c - 2*a^5*d)*x^3)/(a^6*b^2*x^18 + 2*a^7*b*x
^15 + a^8*x^12) - 1/3*(15*b^4*c - 10*a*b^3*d + 6*a^2*b^2*e - 3*a^3*b*f)*log
(b*x^3 + a)/a^7 + 1/3*(15*b^4*c - 10*a*b^3*d + 6*a^2*b^2*e - 3*a^3*b*f)*log
(x^3)/a^7
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.44

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx = \frac{\left(15\,b^4c-10\,ab^3d+6\,a^2b^2e-3\,a^3bf\right)\log\left(|x|\right)}{a^7} - \frac{\left(15\,b^5c-10\,ab^4d+6\,a^2b^3e-3\,a^3b^2f\right)\log\left(|bx^3+a|\right)}{3\,a^7b} + \frac{45\,b^6cx^6-30\,ab^5dx^6+18\,a^2b^4ex^6-9\,a^3b^3fx^6+100\,ab^5cx^3-68\,a^2b^4dx^3+42\,a^3b^3ex^3-22\,a^4b^2fx^3+56}{6\left(bx^3+a\right)^2a^7} - \frac{375\,b^4cx^{12}-250\,ab^3dx^{12}+150\,a^2b^2ex^{12}-75\,a^3bfx^{12}-120\,ab^3cx^9+72\,a^2b^2dx^9-36\,a^3bex^9+12\,a^4fx^9}{36\,a^7x^{12}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (15*b^4*c - 10*a*b^3*d + 6*a^2*b^2*e - 3*a^3*b*f)*log(abs(x))/a^7 - 1/3*(15
*b^5*c - 10*a*b^4*d + 6*a^2*b^3*e - 3*a^3*b^2*f)*log(abs(b*x^3 + a))/(a^7*b
) + 1/6*(45*b^6*c*x^6 - 30*a*b^5*d*x^6 + 18*a^2*b^4*e*x^6 - 9*a^3*b^3*f*x^6
 + 100*a*b^5*c*x^3 - 68*a^2*b^4*d*x^3 + 42*a^3*b^3*e*x^3 - 22*a^4*b^2*f*x^3
 + 56*a^2*b^4*c - 39*a^3*b^3*d + 25*a^4*b^2*e - 14*a^5*b*f)/((b*x^3 + a)^2*
a^7) - 1/36*(375*b^4*c*x^12 - 250*a*b^3*d*x^12 + 150*a^2*b^2*e*x^12 - 75*a^
3*b*f*x^12 - 120*a*b^3*c*x^9 + 72*a^2*b^2*d*x^9 - 36*a^3*b*e*x^9 + 12*a^4*f
*x^9 + 36*a^2*b^2*c*x^6 - 18*a^3*b*d*x^6 + 6*a^4*e*x^6 - 12*a^3*b*c*x^3 + 4
*a^4*d*x^3 + 3*a^4*c)/(a^7*x^12)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx = \frac{\ln\left(x\right)\,\left(-3\,f\,a^3\,b+6\,e\,a^2\,b^2-10\,d\,a\,b^3+15\,c\,b^4\right)}{a^7} - \frac{\ln\left(b\,x^3+a\right)\,\left(-3\,f\,a^3\,b+6\,e\,a^2\,b^2-10\,d\,a\,b^3+15\,c\,b^4\right)}{3\,a^7} - \frac{\frac{c}{12\,a}-\frac{x^9\,\left(-3\,f\,a^3+6\,e\,a^2\,b-10\,d\,a\,b^2+15\,c\,b^3\right)}{9\,a^4}+\frac{x^3\,\left(2\,a\,d-3\,b\,c\right)}{18\,a^2}+\frac{x^6\,\left(6\,e\,a^2-10\,d\,a\,b+15\,c\,b^2\right)}{36\,a^3}-\frac{b\,x^{12}\,\left(-3\,f\,a^3+6\,e\,a^2\,b-10\,d\,a\,b^2+15\,c}{2\,a^5}}{a^2\,x^{12}+2\,a\,b\,x^{15}+b^2\,x^{18}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x)
```

```
[Out] (log(x)*(15*b^4*c + 6*a^2*b^2*e - 10*a*b^3*d - 3*a^3*b*f))/a^7 - (log(a + b
*x^3)*(15*b^4*c + 6*a^2*b^2*e - 10*a*b^3*d - 3*a^3*b*f))/(3*a^7) - (c/(12*a
) - (x^9*(15*b^3*c - 3*a^3*f - 10*a*b^2*d + 6*a^2*b*e))/(9*a^4) + (x^3*(2*a
*d - 3*b*c))/(18*a^2) + (x^6*(15*b^2*c + 6*a^2*e - 10*a*b*d))/(36*a^3) - (b
*x^12*(15*b^3*c - 3*a^3*f - 10*a*b^2*d + 6*a^2*b*e))/(2*a^5) - (b^2*x^15*(1
5*b^3*c - 3*a^3*f - 10*a*b^2*d + 6*a^2*b*e))/(3*a^6))/(a^2*x^12 + b^2*x^18
+ 2*a*b*x^15)
```

3.286 $\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 416

$$\begin{aligned}
\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx
&= -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x}{b^7}+\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^4}{4b^6}\\
&\quad+\frac{(b^2d-3abe+6a^2f)\,x^7}{7b^5}+\frac{(be-3af)x^{10}}{10b^4}+\frac{fx^{13}}{13b^3}\\
&\quad+\frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2}-\frac{a^2(19b^3c-25ab^2d+31a^2be-37a^3f)\,x}{18b^7\,(a+bx^3)}\\
&\quad-\frac{a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{22/3}}\\
&\quad+\frac{a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{22/3}}\\
&\quad-\frac{a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{22/3}}
\end{aligned}$$

```
[Out] -a*(-15*a^3*f+10*a^2*b*e-6*a*b^2*d+3*b^3*c)*x/b^7+1/4*(-10*a^3*f+6*a^2*b*e-
3*a*b^2*d+b^3*c)*x^4/b^6+1/7*(6*a^2*f-3*a*b*e+b^2*d)*x^7/b^5+1/10*(-3*a*f+b
*e)*x^10/b^4+1/13*f*x^13/b^3+1/6*a^3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^7/(
b*x^3+a)^2-1/18*a^2*(-37*a^3*f+31*a^2*b*e-25*a*b^2*d+19*b^3*c)*x/b^7/(b*x^3
+a)+1/27*a^(4/3)*(-152*a^3*f+104*a^2*b*e-65*a*b^2*d+35*b^3*c)*ln(a^(1/3)+b^
(1/3)*x)/b^(22/3)-1/54*a^(4/3)*(-152*a^3*f+104*a^2*b*e-65*a*b^2*d+35*b^3*c)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(22/3)-1/27*a^(4/3)*(-152*a^3*
f+104*a^2*b*e-65*a*b^2*d+35*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)
*3^(1/2))/b^(22/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1842, 1872, 1901, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{x^7(6a^2f-3abe+b^2d)}{7b^5} - \frac{a^2x(-37a^3f+31a^2be-25ab^2d+19b^3c)}{18b^7(a+bx^3)} + \frac{a^3x(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^7(a+bx^3)^2} - \frac{ax(-15a^3f+10a^2be-6ab^2d+3b^3c)}{b^7} + \frac{x^4(-10a^3f+6a^2be-3ab^2d+b^3c)}{4b^6} - \frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-152a^3f+104a^2be-65ab^2d+35b^3c)}{9\sqrt{3}b^{22/3}} - \frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-152a^3f+104a^2be-65ab^2d+35b^3c)}{54b^{22/3}} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-152a^3f+104a^2be-65ab^2d+35b^3c)}{27b^{22/3}} + \frac{x^{10}(be-3af)}{10b^4} + \frac{fx^{13}}{13b^3}$$

```
[In] Int[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] -((a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x)/b^7) + ((b^3*c - 3*a*
b^2*d + 6*a^2*b*e - 10*a^3*f)*x^4)/(4*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x
^7)/(7*b^5) + ((b*e - 3*a*f)*x^10)/(10*b^4) + (f*x^13)/(13*b^3) + (a^3*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^7*(a + b*x^3)^2) - (a^2*(19*b^3*c -
 25*a*b^2*d + 31*a^2*b*e - 37*a^3*f)*x)/(18*b^7*(a + b*x^3)) - (a^(4/3)*(35
*b^3*c - 65*a*b^2*d + 104*a^2*b*e - 152*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(22/3)) + (a^(4/3)*(35*b^3*c - 65*a*b^2
*d + 104*a^2*b*e - 152*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(22/3)) - (a^
(4/3)*(35*b^3*c - 65*a*b^2*d + 104*a^2*b*e - 152*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(22/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
```

```
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2} - \frac{\int \frac{a^4(b^3c-ab^2d+a^2be-a^3f)-6a^3b(b^3c-ab^2d+a^2be-a^3f)x^3+6a^2b^2(b^3c-ab^2d+a^2be-a^3f)x^6-6ab^3(b^3c-ab^2d+a^2be-a^3f)x^9-6ab^4(b^2d-ab\cdots}{(a+bx^3)^2}}{6ab^7}$$

$$= \frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2} - \frac{a^2(19b^3c-25ab^2d+31a^2be-37a^3f)\,x}{18b^7\,(a+bx^3)} + \frac{\int \frac{2a^4b^6(8b^3c-11ab^2d+14a^2be-17a^3f)-18a^3b^7(2b^3c-3ab^2d+4a^2be-5a^3f)x^3+18a^2b^8(b^3c-2ab^2d+3a^2be-4a^3f)x^6+18a^2b^9(b^2d-\cdots}{a+bx^3}}{18a^2b^{13}}$$

$$= \frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2} - \frac{a^2(19b^3c-25ab^2d+31a^2be-37a^3f)\,x}{18b^7\,(a+bx^3)} + \frac{\int \Big(-18a^3b^6(3b^3c-6ab^2d+10a^2be-15a^3f)+18a^2b^7(b^3c-3ab^2d+6a^2be-10a^3f)\,x^3+18a^2b^8\cdots}{1\cdots}$$

$$= -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x}{b^7} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^4}{4b^6} + \frac{(b^2d-3abe+6a^2f)\,x^7}{7b^5} + \frac{(be-3af)x^{10}}{10b^4} + \frac{fx^{13}}{13b^3} + \frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2} - \frac{a^2(19b^3c-25ab^2d+31a^2be-37a^3f)\,x}{18b^7\,(a+bx^3)} + \frac{(a^2(35b^3c-65ab^2d+104a^2be-152a^3f))\int \frac{1}{a+bx^3}\,dx}{9b^7}$$

$$= -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x}{b^7}+\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^4}{4b^6}$$
$$+\frac{(b^2d-3abe+6a^2f)\,x^7}{7b^5}+\frac{(be-3af)x^{10}}{10b^4}+\frac{fx^{13}}{13b^3}$$
$$+\frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2}-\frac{a^2(19b^3c-25ab^2d+31a^2be-37a^3f)\,x}{18b^7\,(a+bx^3)}$$
$$+\frac{\left(a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27b^7}$$
$$+\frac{\left(a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\right)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27b^7}$$

$$= -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x}{b^7}+\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^4}{4b^6}$$
$$+\frac{(b^2d-3abe+6a^2f)\,x^7}{7b^5}+\frac{(be-3af)x^{10}}{10b^4}+\frac{fx^{13}}{13b^3}$$
$$+\frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2}-\frac{a^2(19b^3c-25ab^2d+31a^2be-37a^3f)\,x}{18b^7\,(a+bx^3)}$$
$$+\frac{a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{22/3}}$$
$$-\frac{\left(a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54b^{22/3}}$$
$$+\frac{\left(a^{5/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18b^7}$$

$$= -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x}{b^7}+\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^4}{4b^6}$$
$$+\frac{(b^2d-3abe+6a^2f)\,x^7}{7b^5}+\frac{(be-3af)x^{10}}{10b^4}+\frac{fx^{13}}{13b^3}$$
$$+\frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2}-\frac{a^2(19b^3c-25ab^2d+31a^2be-37a^3f)\,x}{18b^7\,(a+bx^3)}$$
$$+\frac{a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{22/3}}$$
$$-\frac{a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{22/3}}$$
$$+\frac{\left(a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9b^{22/3}}$$

$$= -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x}{b^7}+\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^4}{4b^6} + \frac{(b^2d-3abe+6a^2f)\,x^7}{7b^5}+\frac{(be-3af)x^{10}}{10b^4}+\frac{fx^{13}}{13b^3} + \frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2}-\frac{a^2(19b^3c-25ab^2d+31a^2be-37a^3f)\,x}{18b^7\,(a+bx^3)} - \frac{a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{22/3}} + \frac{a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{22/3}} - \frac{a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{22/3}}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.99

$$\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{a(-3b^3c+6ab^2d-10a^2be+15a^3f)\,x}{b^7}+\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^4}{4b^6} + \frac{(b^2d-3abe+6a^2f)\,x^7}{7b^5}+\frac{(be-3af)x^{10}}{10b^4}+\frac{fx^{13}}{13b^3} + \frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2}+\frac{a^2(-19b^3c+25ab^2d-31a^2be+37a^3f)\,x}{18b^7\,(a+bx^3)} + \frac{a^{4/3}(-35b^3c+65ab^2d-104a^2be+152a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{22/3}} - \frac{a^{4/3}(-35b^3c+65ab^2d-104a^2be+152a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{22/3}} + \frac{a^{4/3}(-35b^3c+65ab^2d-104a^2be+152a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{22/3}}$$

```
[In] Integrate[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x)/b^7 + ((b^3*c - 3*a*b^
2*d + 6*a^2*b*e - 10*a^3*f)*x^4)/(4*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^7
)/(7*b^5) + ((b*e - 3*a*f)*x^10)/(10*b^4) + (f*x^13)/(13*b^3) + (a^3*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^7*(a + b*x^3)^2) + (a^2*(-19*b^3*c +
```

25*a*b^2*d - 31*a^2*b*e + 37*a^3*f)*x)/(18*b^7*(a + b*x^3)) + (a^(4/3)*(-35*b^3*c + 65*a*b^2*d - 104*a^2*b*e + 152*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(22/3)) - (a^(4/3)*(-35*b^3*c + 65*a*b^2*d - 104*a^2*b*e + 152*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(22/3)) + (a^(4/3)*(-35*b^3*c + 65*a*b^2*d - 104*a^2*b*e + 152*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(22/3))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.68

method	result
risch	$\frac{f x^{13}}{13b^3} - \frac{3x^{10}af}{10b^4} + \frac{x^{10}e}{10b^3} + \frac{6x^7a^2f}{7b^5} - \frac{3x^7ae}{7b^4} + \frac{d x^7}{7b^3} - \frac{5a^3 f x^4}{2b^6} + \frac{3a^2 e x^4}{2b^5} - \frac{3ad x^4}{4b^4} + \frac{c x^4}{4b^3} + \frac{15a^4 fx}{b^7} - \frac{10a^3 ex}{b^6} +$
default	$\frac{\frac{1}{13} f x^{13} b^4 - \frac{3}{10} x^{10} a b^3 f + \frac{1}{10} x^{10} b^4 e + \frac{6}{7} x^7 a^2 b^2 f - \frac{3}{7} x^7 a b^3 e + \frac{1}{7} b^4 d x^7 - \frac{5}{2} a^3 b f x^4 + \frac{3}{2} a^2 b^2 e x^4 - \frac{3}{4} a b^3 d x^4 + \frac{1}{4} b^4 c x^4 + 15a^4 fx - 10a^3 bex +}{b^7}$

[In] int(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/13*f*x^13/b^3-3/10/b^4*x^10*a*f+1/10/b^3*x^10*e+6/7/b^5*x^7*a^2*f-3/7/b^4*x^7*a*e+1/7/b^3*d*x^7-5/2/b^6*a^3*f*x^4+3/2/b^5*a^2*e*x^4-3/4/b^4*a*d*x^4+1/4/b^3*c*x^4+15/b^7*a^4*f*x-10/b^6*a^3*e*x+6/b^5*a^2*d*x-3/b^4*a*c*x+((37/18*a^5*b*f-31/18*a^4*e*b^2+25/18*a^3*d*b^3-19/18*a^2*c*b^4)*x^4+1/9*a^3*(17*a^3*f-14*a^2*b*e+11*a*b^2*d-8*b^3*c)*x)/b^7/(b*x^3+a)^2-1/27/b^8*a^2*sum((152*a^3*f-104*a^2*b*e+65*a*b^2*d-35*b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.60

$$\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{3780\,b^6fx^{19}+378\,(13\,b^6e-19\,ab^5f)x^{16}+108\,(65\,b^6d-104\,ab^5e+152\,a^2b^4f)x^{13}+351\,(35\,b^6c-65\,ab^5d+\cdots}{\cdots}$$

```
[In] integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/49140*(3780*b^6*f*x^19 + 378*(13*b^6*e - 19*a*b^5*f)*x^16 + 108*(65*b^6*d
 - 104*a*b^5*e + 152*a^2*b^4*f)*x^13 + 351*(35*b^6*c - 65*a*b^5*d + 104*a^2
*b^4*e - 152*a^3*b^3*f)*x^10 - 3510*(35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^
3*e - 152*a^4*b^2*f)*x^7 - 9555*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*
e - 152*a^5*b*f)*x^4 - 1820*sqrt(3)*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*
b*e - 152*a^6*f + (35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*
f)*x^6 + 2*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)
*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*
(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e - 152*a^6*f + (35*a*b^5*c - 65*a
^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*f)*x^6 + 2*(35*a^2*b^4*c - 65*a^3*b^
3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3)) - 1820*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e - 152*
a^6*f + (35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*f)*x^6 + 2
*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)*(-a/b)^(1
/3)*log(x - (-a/b)^(1/3)) - 5460*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e
 - 152*a^6*f)*x)/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**12*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.02

$$\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = -\frac{(19\,a^2b^4c-25\,a^3b^3d+31\,a^4b^2e-37\,a^5bf)x^4+2\,(8\,a^3b^3c-11\,a^4b^2d+14\,a^5be-17\,a^6f)x}{18\,(b^9x^6+2\,ab^8x^3+a^2b^7)} + \frac{140\,b^4fx^{13}+182\,(b^4e-3\,ab^3f)x^{10}+260\,(b^4d-3\,ab^3e+6\,a^2b^2f)x^7+455\,(b^4c-3\,ab^3d+6\,a^2b^2e-10}{1820\,b^7} + \frac{\sqrt{3}(35\,a^2b^3c-65\,a^3b^2d+104\,a^4be-152\,a^5f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^8\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(35\,a^2b^3c-65\,a^3b^2d+104\,a^4be-152\,a^5f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^8\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(35\,a^2b^3c-65\,a^3b^2d+104\,a^4be-152\,a^5f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^8\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((19*a^2*b^4*c - 25*a^3*b^3*d + 31*a^4*b^2*e - 37*a^5*b*f)*x^4 + 2*(8
*a^3*b^3*c - 11*a^4*b^2*d + 14*a^5*b*e - 17*a^6*f)*x)/(b^9*x^6 + 2*a*b^8*x^
3 + a^2*b^7) + 1/1820*(140*b^4*f*x^13 + 182*(b^4*e - 3*a*b^3*f)*x^10 + 260*
(b^4*d - 3*a*b^3*e + 6*a^2*b^2*f)*x^7 + 455*(b^4*c - 3*a*b^3*d + 6*a^2*b^2*
e - 10*a^3*b*f)*x^4 - 1820*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3*b*e - 15*a^4*f
)*x)/b^7 + 1/27*sqrt(3)*(35*a^2*b^3*c - 65*a^3*b^2*d + 104*a^4*b*e - 152*a^
5*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^8*(a/b)^(2/3))
- 1/54*(35*a^2*b^3*c - 65*a^3*b^2*d + 104*a^4*b*e - 152*a^5*f)*log(x^2 - x*
(a/b)^(1/3) + (a/b)^(2/3))/(b^8*(a/b)^(2/3)) + 1/27*(35*a^2*b^3*c - 65*a^3*
b^2*d + 104*a^4*b*e - 152*a^5*f)*log(x + (a/b)^(1/3))/(b^8*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.18

$$\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= \frac{\sqrt{3}\left(35\left(-ab^2\right)^{\frac{1}{3}}ab^3c-65\left(-ab^2\right)^{\frac{1}{3}}a^2b^2d+104\left(-ab^2\right)^{\frac{1}{3}}a^3be-152\left(-ab^2\right)^{\frac{1}{3}}a^4f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^8}$$
$$- \frac{\left(35\,a^2b^3c-65\,a^3b^2d+104\,a^4be-152\,a^5f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^7}$$
$$+ \frac{\left(35\left(-ab^2\right)^{\frac{1}{3}}ab^3c-65\left(-ab^2\right)^{\frac{1}{3}}a^2b^2d+104\left(-ab^2\right)^{\frac{1}{3}}a^3be-152\left(-ab^2\right)^{\frac{1}{3}}a^4f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)\right.}{54\,b^8}$$
$$- \frac{19\,a^2b^4cx^4-25\,a^3b^3dx^4+31\,a^4b^2ex^4-37\,a^5bfx^4+16\,a^3b^3cx-22\,a^4b^2dx+28\,a^5bex-34\,a^6fx}{18\left(bx^3+a\right)^2b^7}$$
$$+ \frac{140\,b^{36}fx^{13}+182\,b^{36}ex^{10}-546\,ab^{35}fx^{10}+260\,b^{36}dx^7-780\,ab^{35}ex^7+1560\,a^2b^{34}fx^7+455\,b^{36}cx^4-136}{18}$$

```
[In] integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(35*(-a*b^2)^(1/3)*a*b^3*c - 65*(-a*b^2)^(1/3)*a^2*b^2*d + 104
*(-a*b^2)^(1/3)*a^3*b*e - 152*(-a*b^2)^(1/3)*a^4*f)*arctan(1/3*sqrt(3)*(2*x
 + (-a/b)^(1/3))/(-a/b)^(1/3))/b^8 - 1/27*(35*a^2*b^3*c - 65*a^3*b^2*d + 10
4*a^4*b*e - 152*a^5*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/
54*(35*(-a*b^2)^(1/3)*a*b^3*c - 65*(-a*b^2)^(1/3)*a^2*b^2*d + 104*(-a*b^2)^
(1/3)*a^3*b*e - 152*(-a*b^2)^(1/3)*a^4*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)
^(2/3))/b^8 - 1/18*(19*a^2*b^4*c*x^4 - 25*a^3*b^3*d*x^4 + 31*a^4*b^2*e*x^4
- 37*a^5*b*f*x^4 + 16*a^3*b^3*c*x - 22*a^4*b^2*d*x + 28*a^5*b*e*x - 34*a^6*
f*x)/((b*x^3 + a)^2*b^7) + 1/1820*(140*b^36*f*x^13 + 182*b^36*e*x^10 - 546*
a*b^35*f*x^10 + 260*b^36*d*x^7 - 780*a*b^35*e*x^7 + 1560*a^2*b^34*f*x^7 + 4
55*b^36*c*x^4 - 1365*a*b^35*d*x^4 + 2730*a^2*b^34*e*x^4 - 4550*a^3*b^33*f*x
^4 - 5460*a*b^35*c*x + 10920*a^2*b^34*d*x - 18200*a^3*b^33*e*x + 27300*a^4*
b^32*f*x)/b^39
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.38

$$\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = x^{10}\left(\frac{e}{10\,b^3}-\frac{3\,a\,f}{10\,b^4}\right) + x^4\left(\frac{c}{4\,b^3}-\frac{a^3\,f}{4\,b^6}-\frac{3\,a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{4\,b^2}+\frac{3\,a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{4\,b}\right) + \frac{x\left(\frac{17\,f\,a^6}{9}-\frac{14\,e\,a^5\,b}{9}+\frac{11\,d\,a^4\,b^2}{9}-\frac{8\,c\,a^3\,b^3}{9}\right)-x^4\left(-\frac{37\,f\,a^5\,b}{18}+\frac{31\,e\,a^4\,b^2}{18}-\frac{25\,d\,a^3\,b^3}{18}+\frac{19\,c\,a^2\,b^4}{18}\right)}{a^2\,b^7+2\,a\,b^8\,x^3+b^9\,x^6} - x\left(\frac{3\,a\left(\frac{c}{b^3}-\frac{a^3\,f}{b^6}-\frac{3\,a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{3\,a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)}{b} - \frac{3\,a^2\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b^2}+\frac{a^3\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^3}\right) - x^7\left(\frac{3\,a^2\,f}{7\,b^5}-\frac{d}{7\,b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{7\,b}\right)+\frac{f\,x^{13}}{13\,b^3} + \frac{a^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-152\,f\,a^3+104\,e\,a^2\,b-65\,d\,a\,b^2+35\,c\,b^3\right)}{27\,b^{22/3}} + \frac{a^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-152\,f\,a^3+104\,e\,a^2\,b-65\,d\,a\,b^2+35\,c\,b^3\right)}{27\,b^{22/3}} - \frac{a^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-152\,f\,a^3+104\,e\,a^2\,b-65\,d\,a\,b^2+35\,c\,b^3\right)}{27\,b^{22/3}}$$

```
[In] int((x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^10*(e/(10*b^3) - (3*a*f)/(10*b^4)) + x^4*(c/(4*b^3) - (a^3*f)/(4*b^6) - (
3*a^2*(e/b^3 - (3*a*f)/b^4))/(4*b^2) + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(
```

```
e/b^3 - (3*a*f)/b^4))/b))/(4*b)) + (x*((17*a^6*f)/9 - (8*a^3*b^3*c)/9 + (11
*a^4*b^2*d)/9 - (14*a^5*b*e)/9) - x^4*((19*a^2*b^4*c)/18 - (25*a^3*b^3*d)/1
8 + (31*a^4*b^2*e)/18 - (37*a^5*b*f)/18))/(a^2*b^7 + b^9*x^6 + 2*a*b^8*x^3)
 - x*((3*a*(c/b^3 - (a^3*f)/b^6 - (3*a^2*(e/b^3 - (3*a*f)/b^4))/b^2 + (3*a*
((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b))/b - (3*a^2*((3
*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b^2 + (a^3*(e/b^3 - (
3*a*f)/b^4))/b^3) - x^7*((3*a^2*f)/(7*b^5) - d/(7*b^3) + (3*a*(e/b^3 - (3*a
*f)/b^4))/(7*b)) + (f*x^13)/(13*b^3) + (a^(4/3)*log(b^(1/3)*x + a^(1/3))*(3
5*b^3*c - 152*a^3*f - 65*a*b^2*d + 104*a^2*b*e))/(27*b^(22/3)) + (a^(4/3)*l
og(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(35*b
^3*c - 152*a^3*f - 65*a*b^2*d + 104*a^2*b*e))/(27*b^(22/3)) - (a^(4/3)*log(
3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(35*b^3*
c - 152*a^3*f - 65*a*b^2*d + 104*a^2*b*e))/(27*b^(22/3))
```

3.287 $\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 384

$$\begin{aligned}\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6}+\frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5}+\frac{(be-3af)x^8}{8b^4}\\&+\frac{fx^{11}}{11b^3}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2}+\frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,x^2}{9b^6\,(a+bx^3)}\\&+\frac{a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{20/3}}\\&+\frac{a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{20/3}}\\&-\frac{a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{20/3}}\end{aligned}$$

```
[Out] 1/2*(-10*a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*x^2/b^6+1/5*(6*a^2*f-3*a*b*e+b^2*
d)*x^5/b^5+1/8*(-3*a*f+b*e)*x^8/b^4+1/11*f*x^11/b^3-1/6*a^2*(-a^3*f+a^2*b*e
-a*b^2*d+b^3*c)*x^2/b^6/(b*x^3+a)^2+1/9*a*(-16*a^3*f+13*a^2*b*e-10*a*b^2*d+
7*b^3*c)*x^2/b^6/(b*x^3+a)+1/27*a^(2/3)*(-119*a^3*f+77*a^2*b*e-44*a*b^2*d+2
0*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(20/3)-1/54*a^(2/3)*(-119*a^3*f+77*a^2*b*e
-44*a*b^2*d+20*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(20/3)+1/
27*a^(2/3)*(-119*a^3*f+77*a^2*b*e-44*a*b^2*d+20*b^3*c)*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(20/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1842, 1865, 1850, 1502, 298, 31, 648, 631, 210, 642}

$$\begin{aligned}
&\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} \, dx \\
&= \frac{x^5(6a^2f-3abe+b^2d)}{5b^5}+\frac{x^2(-10a^3f+6a^2be-3ab^2d+b^3c)}{2b^6} \\
&+ \frac{ax^2(-16a^3f+13a^2be-10ab^2d+7b^3c)}{9b^6\,(a+bx^3)}-\frac{a^2x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^6\,(a+bx^3)^2} \\
&+ \frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-119a^3f+77a^2be-44ab^2d+20b^3c)}{9\sqrt{3}b^{20/3}} \\
&- \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-119a^3f+77a^2be-44ab^2d+20b^3c)}{54b^{20/3}} \\
&+ \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-119a^3f+77a^2be-44ab^2d+20b^3c)}{27b^{20/3}}+\frac{x^8(be-3af)}{8b^4}+\frac{fx^{11}}{11b^3}
\end{aligned}$$

```
[In] Int[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*d - 3*a*b*
e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4) + (f*x^11)/(11*b^3)
 - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^6*(a + b*x^3)^2) + (a
*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*x^2)/(9*b^6*(a + b*x^3)) +
(a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(20/3)) + (a^(2/3)*(20*b^3*c
 - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(20
/3)) - (a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(20/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
```

```
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1865

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rubi steps

$$\text{integral} = -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} - \frac{\int \frac{-2a^3b(b^3c-ab^2d+a^2be-a^3f)x+6a^2b^2(b^3c-ab^2d+a^2be-a^3f)x^4-6ab^3(b^3c-ab^2d+a^2be-a^3f)x^7-6ab^4(b^2d-abe+a^2f)x^{10}-6ab^5(be-af)x}{(a+bx^3)^2}}{6ab^7}$$

$$= -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} - \frac{\int \frac{x(-2a^3b(b^3c-ab^2d+a^2be-a^3f)+6a^2b^2(b^3c-ab^2d+a^2be-a^3f)x^3-6ab^3(b^3c-ab^2d+a^2be-a^3f)x^6-6ab^4(b^2d-abe+a^2f)x^9-6ab}{(a+bx^3)^2}}{6ab^7}$$

$$= -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} + \frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,x^2}{9b^6\,(a+bx^3)} + \frac{\int \frac{-2a^3b^7(11b^3c-17ab^2d+23a^2be-29a^3f)x+18a^2b^8(b^3c-2ab^2d+3a^2be-4a^3f)x^4+18a^2b^9(b^2d-2abe+3a^2f)x^7+18a^2b^{10}(be-2af)}{a+bx^3}}{18a^2b^{13}}$$

$$= -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} + \frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,x^2}{9b^6\,(a+bx^3)} + \frac{\int \frac{x(-2a^3b^7(11b^3c-17ab^2d+23a^2be-29a^3f)+18a^2b^8(b^3c-2ab^2d+3a^2be-4a^3f)x^3+18a^2b^9(b^2d-2abe+3a^2f)x^6+18a^2b^{10}(be-2af}{a+bx^3}}{18a^2b^{13}}$$

$$= \frac{fx^{11}}{11b^3} - \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} + \frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,x^2}{9b^6\,(a+bx^3)} + \frac{\int \frac{x(-22a^3b^8(11b^3c-17ab^2d+23a^2be-29a^3f)+198a^2b^9(b^3c-2ab^2d+3a^2be-4a^3f)x^3+198a^2b^{10}(b^2d-2abe+3a^2f)x^6+198a^2b^{11}(be}{a+bx^3}}{198a^2b^{14}}$$

$$= \frac{(be-3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} - \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} + \frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,x^2}{9b^6\,(a+bx^3)} + \frac{\int \frac{x(-176a^3b^9(11b^3c-17ab^2d+23a^2be-29a^3f)+1584a^2b^{10}(b^3c-2ab^2d+3a^2be-4a^3f)x^3+1584a^2b^{11}(b^2d-3abe+6a^2f)x^6)}{a+bx^3}\,dx}{1584a^2b^{15}}$$

$$= \frac{(be-3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} - \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2}$$
$$+ \frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,x^2}{9b^6\,(a+bx^3)}$$
$$+ \frac{\int \Big(1584a^2b^9(b^3c-3ab^2d+6a^2be-10a^3f)\,x + 1584a^2b^{10}(b^2d-3abe+6a^2f)\,x^4 + \frac{176(-20a^3b^{12}c\cdots}{}}{1584a^2b^{15}}$$

$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6} + \frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5} + \frac{(be-3af)x^8}{8b^4}$$
$$+ \frac{fx^{11}}{11b^3} - \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} + \frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,x^2}{9b^6\,(a+bx^3)}$$
$$- \frac{(a(20b^3c-44ab^2d+77a^2be-119a^3f))\int \frac{x}{a+bx^3}\,dx}{9b^6}$$

$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6} + \frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5} + \frac{(be-3af)x^8}{8b^4}$$
$$+ \frac{fx^{11}}{11b^3} - \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} + \frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,x^2}{9b^6\,(a+bx^3)}$$
$$+ \frac{\left(a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27b^{19/3}}$$
$$- \frac{\left(a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27b^{19/3}}$$

$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6} + \frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5} + \frac{(be-3af)x^8}{8b^4}$$
$$+ \frac{fx^{11}}{11b^3} - \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2} + \frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,x^2}{9b^6\,(a+bx^3)}$$
$$+ \frac{a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{20/3}}$$
$$- \frac{\left(a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54b^{20/3}}$$
$$- \frac{(a(20b^3c-44ab^2d+77a^2be-119a^3f))\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18b^{19/3}}$$

$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6}+\frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5}+\frac{(be-3af)x^8}{8b^4}$$
$$+\frac{fx^{11}}{11b^3}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2}+\frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,x^2}{9b^6\,(a+bx^3)}$$
$$+\frac{a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{20/3}}$$
$$-\frac{a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{20/3}}$$
$$-\frac{\left(a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9b^{20/3}}$$

$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6}+\frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5}+\frac{(be-3af)x^8}{8b^4}$$
$$+\frac{fx^{11}}{11b^3}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2}+\frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,x^2}{9b^6\,(a+bx^3)}$$
$$+\frac{a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{20/3}}$$
$$+\frac{a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{20/3}}$$
$$-\frac{a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{20/3}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.99

$$\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6}+\frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5}+\frac{(be-3af)x^8}{8b^4}+\frac{fx^{11}}{11b^3}+\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)\,x^2}{6b^6\,(a+bx^3)^2}+\frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,x^2}{9b^6\,(a+bx^3)}-\frac{a^{2/3}(-20b^3c+44ab^2d-77a^2be+119a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{20/3}}-\frac{a^{2/3}(-20b^3c+44ab^2d-77a^2be+119a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{20/3}}+\frac{a^{2/3}(-20b^3c+44ab^2d-77a^2be+119a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{20/3}}$$

```
[In] Integrate[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*d - 3*a*b*
e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4) + (f*x^11)/(11*b^3)
 + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(6*b^6*(a + b*x^3)^2) +
 (a*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*x^2)/(9*b^6*(a + b*x^3))
 - (a^(2/3)*(-20*b^3*c + 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(20/3)) - (a^(2/3)*(-20*b^3*c
+ 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(20/
3)) + (a^(2/3)*(-20*b^3*c + 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(20/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.64

method	result
risch	$\frac{f x^{11}}{11b^3} - \frac{3x^8 fa}{8b^4} + \frac{x^8 e}{8b^3} + \frac{6x^5 f a^2}{5b^5} - \frac{3x^5 ae}{5b^4} + \frac{d x^5}{5b^3} - \frac{5x^2 f a^3}{b^6} + \frac{3x^2 a^2 e}{b^5} - \frac{3x^2 ad}{2b^4} + \frac{x^2 c}{2b^3} + \frac{\left(-\frac{16}{9}a^4bf+\frac{13}{9}a^3b^2e-\frac{10}{9}a^2b^3\right.}{}$
default	$-\frac{-\frac{b^3 f x^{11}}{11}+\frac{(3fab^2-b^3e)x^8}{8}+\frac{(-6fa^2b+3ab^2e-b^3d)x^5}{5}+\frac{(10fa^3-6a^2be+3ab^2d-b^3c)x^2}{2}}{b^6} + \frac{a\left(\frac{\left(-\frac{16}{9}a^3bf+\frac{13}{9}a^2eb^2-\frac{10}{9}ab^3d+\frac{7}{9}b^4c\right)x^5}{(bx^3+}\right.}{}$

```
[In] int(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*f*x^11/b^3-3/8/b^4*x^8*f*a+1/8/b^3*x^8*e+6/5/b^5*x^5*f*a^2-3/5/b^4*x^5
*a*e+1/5/b^3*d*x^5-5/b^6*x^2*f*a^3+3/b^5*x^2*a^2*e-3/2/b^4*x^2*a*d+1/2/b^3*
x^2*c+((-16/9*a^4*b*f+13/9*a^3*b^2*e-10/9*a^2*b^3*d+7/9*a*b^4*c)*x^5-1/18*a
^2*(29*a^3*f-23*a^2*b*e+17*a*b^2*d-11*b^3*c)*x^2)/b^6/(b*x^3+a)^2+1/27/b^7*
a*sum((119*a^3*f-77*a^2*b*e+44*a*b^2*d-20*b^3*c)/_R*ln(x-_R),_R=RootOf(_Z^3
*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.65

$$\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= \frac{1080\,b^5 f x^{17} + 135\,(11\,b^5 e - 17\,ab^4 f)x^{14} + 54\,(44\,b^5 d - 77\,ab^4 e + 119\,a^2 b^3 f)x^{11} + 297\,(20\,b^5 c - 44\,ab^4 d + 7}{}$$

```
[In] integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/11880*(1080*b^5*f*x^17 + 135*(11*b^5*e - 17*a*b^4*f)*x^14 + 54*(44*b^5*d
- 77*a*b^4*e + 119*a^2*b^3*f)*x^11 + 297*(20*b^5*c - 44*a*b^4*d + 77*a^2*b^
3*e - 119*a^3*b^2*f)*x^8 + 1056*(20*a*b^4*c - 44*a^2*b^3*d + 77*a^3*b^2*e -
 119*a^4*b*f)*x^5 + 660*(20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*a^5
*f)*x^2 - 440*sqrt(3)*((20*b^5*c - 44*a*b^4*d + 77*a^2*b^3*e - 119*a^3*b^2*
f)*x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*a^5*f + 2*(20*a*b^4
*c - 44*a^2*b^3*d + 77*a^3*b^2*e - 119*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3)*arcta
```

```
n(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*((20*b^5*c - 44
*a*b^4*d + 77*a^2*b^3*e - 119*a^3*b^2*f)*x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d
+ 77*a^4*b*e - 119*a^5*f + 2*(20*a*b^4*c - 44*a^2*b^3*d + 77*a^3*b^2*e - 11
9*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2
/b^2)^(1/3)) - 440*((20*b^5*c - 44*a*b^4*d + 77*a^2*b^3*e - 119*a^3*b^2*f)*
x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*a^5*f + 2*(20*a*b^4*c
- 44*a^2*b^3*d + 77*a^3*b^2*e - 119*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x
+ b*(-a^2/b^2)^(2/3)))/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**10*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= \frac{2\,(7\,ab^4c-10\,a^2b^3d+13\,a^3b^2e-16\,a^4bf)x^5+(11\,a^2b^3c-17\,a^3b^2d+23\,a^4be-29\,a^5f)x^2}{18\,(b^8x^6+2\,ab^7x^3+a^2b^6)} \\ &\quad - \frac{\sqrt{3}(20\,ab^3c-44\,a^2b^2d+77\,a^3be-119\,a^4f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^7\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &\quad + \frac{40\,b^3fx^{11}+55\,(b^3e-3\,ab^2f)x^8+88\,(b^3d-3\,ab^2e+6\,a^2bf)x^5+220\,(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f)x^2}{440\,b^6} \\ &\quad - \frac{(20\,ab^3c-44\,a^2b^2d+77\,a^3be-119\,a^4f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^7\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &\quad + \frac{(20\,ab^3c-44\,a^2b^2d+77\,a^3be-119\,a^4f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}\end{aligned}$$

```
[In] integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16*a^4*b*f)*x^5 + (11*a^
2*b^3*c - 17*a^3*b^2*d + 23*a^4*b*e - 29*a^5*f)*x^2)/(b^8*x^6 + 2*a*b^7*x^3
 + a^2*b^6) - 1/27*sqrt(3)*(20*a*b^3*c - 44*a^2*b^2*d + 77*a^3*b*e - 119*a^
4*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*(a/b)^(1/3))
+ 1/440*(40*b^3*f*x^11 + 55*(b^3*e - 3*a*b^2*f)*x^8 + 88*(b^3*d - 3*a*b^2*e
 + 6*a^2*b*f)*x^5 + 220*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/b^6
 - 1/54*(20*a*b^3*c - 44*a^2*b^2*d + 77*a^3*b*e - 119*a^4*f)*log(x^2 - x*(a
/b)^(1/3) + (a/b)^(2/3))/(b^7*(a/b)^(1/3)) + 1/27*(20*a*b^3*c - 44*a^2*b^2*
d + 77*a^3*b*e - 119*a^4*f)*log(x + (a/b)^(1/3))/(b^7*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.26

$$\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{\left(20\,ab^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 44\,a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 77\,a^3be\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 119\,a^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^6}$$
$$+ \frac{\sqrt{3}\left(20\,(-ab^2)^{\frac{2}{3}}\,b^3c - 44\,(-ab^2)^{\frac{2}{3}}\,ab^2d + 77\,(-ab^2)^{\frac{2}{3}}\,a^2be - 119\,(-ab^2)^{\frac{2}{3}}\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^8}$$
$$- \frac{\left(20\,(-ab^2)^{\frac{2}{3}}\,b^3c - 44\,(-ab^2)^{\frac{2}{3}}\,ab^2d + 77\,(-ab^2)^{\frac{2}{3}}\,a^2be - 119\,(-ab^2)^{\frac{2}{3}}\,a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^8}$$
$$+ \frac{14\,ab^4cx^5 - 20\,a^2b^3dx^5 + 26\,a^3b^2ex^5 - 32\,a^4bfx^5 + 11\,a^2b^3cx^2 - 17\,a^3b^2dx^2 + 23\,a^4bex^2 - 29\,a^5fx^2}{18\,(bx^3+a)^2b^6}$$
$$+ \frac{40\,b^{30}fx^{11} + 55\,b^{30}ex^8 - 165\,ab^{29}fx^8 + 88\,b^{30}dx^5 - 264\,ab^{29}ex^5 + 528\,a^2b^{28}fx^5 + 220\,b^{30}cx^2 - 660\,ab^{29}dx^2 + \ldots}{440\,b^{33}}$$

```
[In] integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*(20*a*b^3*c*(-a/b)^(1/3) - 44*a^2*b^2*d*(-a/b)^(1/3) + 77*a^3*b*e*(-a/
b)^(1/3) - 119*a^4*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^6) + 1/27*sqrt(3)*(20*(-a*b^2)^(2/3)*b^3*c - 44*(-a*b^2)^(2/3)*a*b^2*d
 + 77*(-a*b^2)^(2/3)*a^2*b*e - 119*(-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^8 - 1/54*(20*(-a*b^2)^(2/3)*b^3*c - 4
4*(-a*b^2)^(2/3)*a*b^2*d + 77*(-a*b^2)^(2/3)*a^2*b*e - 119*(-a*b^2)^(2/3)*a
^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^8 + 1/18*(14*a*b^4*c*x^5 -
 20*a^2*b^3*d*x^5 + 26*a^3*b^2*e*x^5 - 32*a^4*b*f*x^5 + 11*a^2*b^3*c*x^2 -
17*a^3*b^2*d*x^2 + 23*a^4*b*e*x^2 - 29*a^5*f*x^2)/((b*x^3 + a)^2*b^6) + 1/4
40*(40*b^30*f*x^11 + 55*b^30*e*x^8 - 165*a*b^29*f*x^8 + 88*b^30*d*x^5 - 264
*a*b^29*e*x^5 + 528*a^2*b^28*f*x^5 + 220*b^30*c*x^2 - 660*a*b^29*d*x^2 + 13
20*a^2*b^28*e*x^2 - 2200*a^3*b^27*f*x^2)/b^33
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.11

$$\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= x^8\left(\frac{e}{8\,b^3}-\frac{3\,a\,f}{8\,b^4}\right)+x^2\left(\frac{c}{2\,b^3}-\frac{a^3\,f}{2\,b^6}-\frac{3\,a^2\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{2\,b^2}+\frac{3\,a\,\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{2\,b}\right)$$
$$-\frac{\left(\frac{16\,f\,a^4\,b}{9}-\frac{13\,e\,a^3\,b^2}{9}+\frac{10\,d\,a^2\,b^3}{9}-\frac{7\,c\,a\,b^4}{9}\right)\,x^5+\left(\frac{29\,f\,a^5}{18}-\frac{23\,e\,a^4\,b}{18}+\frac{17\,d\,a^3\,b^2}{18}-\frac{11\,c\,a^2\,b^3}{18}\right)\,x^2}{a^2\,b^6+2\,a\,b^7\,x^3+b^8\,x^6}$$
$$-x^5\left(\frac{3\,a^2\,f}{5\,b^5}-\frac{d}{5\,b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{5\,b}\right)+\frac{f\,x^{11}}{11\,b^3}$$
$$+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-119\,f\,a^3+77\,e\,a^2\,b-44\,d\,a\,b^2+20\,c\,b^3\right)}{27\,b^{20/3}}$$
$$-\frac{a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-119\,f\,a^3+77\,e\,a^2\,b-44\,d\,a\,b^2+20\,c\,b^3\right)}{27\,b^{20/3}}$$
$$+\frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-119\,f\,a^3+77\,e\,a^2\,b-44\,d\,a\,b^2+20\,c\,b^3\right)}{27\,b^{20/3}}$$

```
[In] int((x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^8*(e/(8*b^3) - (3*a*f)/(8*b^4)) + x^2*(c/(2*b^3) - (a^3*f)/(2*b^6) - (3*a
^2*(e/b^3 - (3*a*f)/b^4))/(2*b^2) + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b
^3 - (3*a*f)/b^4))/b))/(2*b)) - (x^2*((29*a^5*f)/18 - (11*a^2*b^3*c)/18 + (
17*a^3*b^2*d)/18 - (23*a^4*b*e)/18) + x^5*((10*a^2*b^3*d)/9 - (13*a^3*b^2*e
)/9 - (7*a*b^4*c)/9 + (16*a^4*b*f)/9))/(a^2*b^6 + b^8*x^6 + 2*a*b^7*x^3) -
x^5*((3*a^2*f)/(5*b^5) - d/(5*b^3) + (3*a*(e/b^3 - (3*a*f)/b^4))/(5*b)) + (
f*x^11)/(11*b^3) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(20*b^3*c - 119*a^3*f
- 44*a*b^2*d + 77*a^2*b*e))/(27*b^(20/3)) - (a^(2/3)*log(3^(1/2)*a^(1/3)*1i
 + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(20*b^3*c - 119*a^3*f - 44
*a*b^2*d + 77*a^2*b*e))/(27*b^(20/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2
*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(20*b^3*c - 119*a^3*f - 44*a*b
^2*d + 77*a^2*b*e))/(27*b^(20/3))
```

3.288 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 375

$$\begin{aligned}
&\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx \\
&= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6}+\frac{(b^2d-3abe+6a^2f)\,x^4}{4b^5}+\frac{(be-3af)x^7}{7b^4}+\frac{fx^{10}}{10b^3} \\
&\quad -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^6\,(a+bx^3)^2}+\frac{a(13b^3c-19ab^2d+25a^2be-31a^3f)\,x}{18b^6\,(a+bx^3)} \\
&\quad +\frac{\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{19/3}} \\
&\quad -\frac{\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{19/3}} \\
&\quad +\frac{\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{19/3}}
\end{aligned}$$

```
[Out] (-10*a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*x/b^6+1/4*(6*a^2*f-3*a*b*e+b^2*d)*x^4
/b^5+1/7*(-3*a*f+b*e)*x^7/b^4+1/10*f*x^10/b^3-1/6*a^2*(-a^3*f+a^2*b*e-a*b^2
*d+b^3*c)*x/b^6/(b*x^3+a)^2+1/18*a*(-31*a^3*f+25*a^2*b*e-19*a*b^2*d+13*b^3*
c)*x/b^6/(b*x^3+a)-1/27*a^(1/3)*(-104*a^3*f+65*a^2*b*e-35*a*b^2*d+14*b^3*c)
*ln(a^(1/3)+b^(1/3)*x)/b^(19/3)+1/54*a^(1/3)*(-104*a^3*f+65*a^2*b*e-35*a*b^
2*d+14*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(19/3)+1/27*a^(1/
3)*(-104*a^3*f+65*a^2*b*e-35*a*b^2*d+14*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3
)*x)/a^(1/3)*3^(1/2))/b^(19/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1842, 1872, 1901, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$=\frac{x^4(6a^2f-3abe+b^2d)}{4b^5}+\frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-104a^3f+65a^2be-35ab^2d+14b^3c)}{9\sqrt{3}b^{19/3}}$$
$$-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-104a^3f+65a^2be-35ab^2d+14b^3c)}{27b^{19/3}}$$
$$+\frac{ax(-31a^3f+25a^2be-19ab^2d+13b^3c)}{18b^6(a+bx^3)}$$
$$-\frac{a^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^6(a+bx^3)^2}+\frac{x(-10a^3f+6a^2be-3ab^2d+b^3c)}{b^6}$$
$$+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-104a^3f+65a^2be-35ab^2d+14b^3c)}{54b^{19/3}}$$
$$+\frac{x^7(be-3af)}{7b^4}+\frac{fx^{10}}{10b^3}$$

```
[In] Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*a*b*e + 6*
a^2*f)*x^4)/(4*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*x^10)/(10*b^3) - (a^
2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^6*(a + b*x^3)^2) + (a*(13*b^3
*c - 19*a*b^2*d + 25*a^2*b*e - 31*a^3*f)*x)/(18*b^6*(a + b*x^3)) + (a^(1/3)
*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(19/3)) - (a^(1/3)*(14*b^3*c - 35*a*
b^2*d + 65*a^2*b*e - 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(19/3)) + (
a^(1/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
```

```
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^6\,(a+bx^3)^2} - \frac{\int \frac{-a^3(b^3c-ab^2d+a^2be-a^3f)+6a^2b(b^3c-ab^2d+a^2be-a^3f)x^3-6ab^2(b^3c-ab^2d+a^2be-a^3f)x^6-6ab^3(b^2d-abe+a^2f)x^9-6ab^4(be-af)x^{12}-}{(a+bx^3)^2}}{6ab^6}$$

$$= -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^6\,(a+bx^3)^2} + \frac{a(13b^3c-19ab^2d+25a^2be-31a^3f)\,x}{18b^6\,(a+bx^3)} + \frac{\int \frac{-2a^3b^5(5b^3c-8ab^2d+11a^2be-14a^3f)+18a^2b^6(b^3c-2ab^2d+3a^2be-4a^3f)x^3+18a^2b^7(b^2d-2abe+3a^2f)x^6+18a^2b^8(be-2af)x^9}{a+bx^3}}{18a^2b^{11}}$$

$$= -\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^6\,(a+bx^3)^2} + \frac{a(13b^3c-19ab^2d+25a^2be-31a^3f)\,x}{18b^6\,(a+bx^3)} + \frac{\int \Big(18a^2b^5(b^3c-3ab^2d+6a^2be-10a^3f)+18a^2b^6(b^2d-3abe+6a^2f)\,x^3+18a^2b^7(be-3af)x^6}{18a^2b^{11}}$$

$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6} + \frac{(b^2d-3abe+6a^2f)\,x^4}{4b^5} + \frac{(be-3af)x^7}{7b^4} + \frac{fx^{10}}{10b^3} - \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^6\,(a+bx^3)^2} + \frac{a(13b^3c-19ab^2d+25a^2be-31a^3f)\,x}{18b^6\,(a+bx^3)} - \frac{(a(14b^3c-35ab^2d+65a^2be-104a^3f))\int \frac{1}{a+bx^3}\,dx}{9b^6}$$

$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6} + \frac{(b^2d-3abe+6a^2f)\,x^4}{4b^5} + \frac{(be-3af)x^7}{7b^4} + \frac{fx^{10}}{10b^3} - \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^6\,(a+bx^3)^2} + \frac{a(13b^3c-19ab^2d+25a^2be-31a^3f)\,x}{18b^6\,(a+bx^3)} - \frac{\left(\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27b^6} - \frac{\left(\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27b^6}$$

$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6}+\frac{(b^2d-3abe+6a^2f)\,x^4}{4b^5}+\frac{(be-3af)x^7}{7b^4}+\frac{fx^{10}}{10b^3}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^6\,(a+bx^3)^2}+\frac{a(13b^3c-19ab^2d+25a^2be-31a^3f)\,x}{18b^6\,(a+bx^3)}-\frac{\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{19/3}}+\frac{\left(\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54b^{19/3}}-\frac{\left(a^{2/3}(14b^3c-35ab^2d+65a^2be-104a^3f)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18b^6}$$

$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6}+\frac{(b^2d-3abe+6a^2f)\,x^4}{4b^5}+\frac{(be-3af)x^7}{7b^4}+\frac{fx^{10}}{10b^3}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^6\,(a+bx^3)^2}+\frac{a(13b^3c-19ab^2d+25a^2be-31a^3f)\,x}{18b^6\,(a+bx^3)}-\frac{\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{19/3}}+\frac{\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{19/3}}-\frac{\left(\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9b^{19/3}}$$

$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6}+\frac{(b^2d-3abe+6a^2f)\,x^4}{4b^5}+\frac{(be-3af)x^7}{7b^4}+\frac{fx^{10}}{10b^3}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^6\,(a+bx^3)^2}+\frac{a(13b^3c-19ab^2d+25a^2be-31a^3f)\,x}{18b^6\,(a+bx^3)}+\frac{\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{19/3}}-\frac{\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{19/3}}+\frac{\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{19/3}}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.97

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= \frac{3780\sqrt[3]{b}(b^3c-3ab^2d+6a^2be-10a^3f)\,x+945b^{4/3}(b^2d-3abe+6a^2f)\,x^4+540b^{7/3}(be-3af)x^7+378b^{10}}{}$$

```
[In] Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (3780*b^(1/3)*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x + 945*b^(4/3)*(b
^2*d - 3*a*b*e + 6*a^2*f)*x^4 + 540*b^(7/3)*(b*e - 3*a*f)*x^7 + 378*b^(10/3
)*f*x^10 + (630*a^2*b^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(a +
b*x^3)^2 + (210*a*b^(1/3)*(13*b^3*c - 19*a*b^2*d + 25*a^2*b*e - 31*a^3*f)*x
)/(a + b*x^3) - 140*Sqrt[3]*a^(1/3)*(-14*b^3*c + 35*a*b^2*d - 65*a^2*b*e +
104*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-14*b
^3*c + 35*a*b^2*d - 65*a^2*b*e + 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x] - 70*a
^(1/3)*(-14*b^3*c + 35*a*b^2*d - 65*a^2*b*e + 104*a^3*f)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(3780*b^(19/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.62

method	result
risch	$\frac{f\,x^{10}}{10b^3}-\frac{3x^7 fa}{7b^4}+\frac{x^7 e}{7b^3}+\frac{3x^4 f\,a^2}{2b^5}-\frac{3x^4 ae}{4b^4}+\frac{d\,x^4}{4b^3}-\frac{10xf\,a^3}{b^6}+\frac{6x\,a^2 e}{b^5}-\frac{3xad}{b^4}+\frac{xc}{b^3}+\frac{\left(-\frac{31}{18}a^4 bf+\frac{25}{18}a^3b^2e-\frac{19}{18}a^2b^3d+\right.}{}$
	$a\left(\frac{\left(-\frac{31}{18}a^3bf+\frac{25}{18}a^2eb^2-\frac{19}{18}\right.}{}\right.$
default	$-\frac{-\frac{1}{10}b^3 f\,x^{10}+\frac{3}{7}x^7 a\,b^2 f-\frac{1}{7}x^7b^3e-\frac{3}{2}a^2bf\,x^4+\frac{3}{4}a\,b^2 e\,x^4-\frac{1}{4}d\,x^4b^3+10f\,a^3x-6a^2bex+3a\,b^2dx-b^3cx}{b^6}+$

```
[In] int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*f*x^10/b^3-3/7/b^4*x^7*f*a+1/7/b^3*x^7*e+3/2/b^5*x^4*f*a^2-3/4/b^4*x^4
*a*e+1/4/b^3*d*x^4-10/b^6*x*f*a^3+6/b^5*x*a^2*e-3/b^4*x*a*d+1/b^3*x*c+((-31
/18*a^4*b*f+25/18*a^3*b^2*e-19/18*a^2*b^3*d+13/18*a*b^4*c)*x^4-1/9*a^2*(14*
a^3*f-11*a^2*b*e+8*a*b^2*d-5*b^3*c)*x)/b^6/(b*x^3+a)^2+1/27/b^7*a*sum((104*
a^3*f-65*a^2*b*e+35*a*b^2*d-14*b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.61

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$=\frac{378\,b^5fx^{16}+108\,(5\,b^5e-8\,ab^4f)x^{13}+27\,(35\,b^5d-65\,ab^4e+104\,a^2b^3f)x^{10}+270\,(14\,b^5c-35\,ab^4d+65\,a^2}{}$$

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3780*(378*b^5*f*x^16 + 108*(5*b^5*e - 8*a*b^4*f)*x^13 + 27*(35*b^5*d - 65
*a*b^4*e + 104*a^2*b^3*f)*x^10 + 270*(14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e
- 104*a^3*b^2*f)*x^7 + 735*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*
a^4*b*f)*x^4 - 140*sqrt(3)*((14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e - 104*a^3
*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*d + 65*a^4*b*e - 104*a^5*f + 2*(14*
a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*a^4*b*f)*x^3)*(a/b)^(1/3)*arcta
n(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*((14*b^5*c - 35*a*b^4
*d + 65*a^2*b^3*e - 104*a^3*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*d + 65*a
^4*b*e - 104*a^5*f + 2*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*a^4*
b*f)*x^3)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*((14*b^5
*c - 35*a*b^4*d + 65*a^2*b^3*e - 104*a^3*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3
*b^2*d + 65*a^4*b*e - 104*a^5*f + 2*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2
*e - 104*a^4*b*f)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*(14*a^2*b^3*c
 - 35*a^3*b^2*d + 65*a^4*b*e - 104*a^5*f)*x)/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b
^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{(13\,ab^4c-19\,a^2b^3d+25\,a^3b^2e-31\,a^4bf)x^4+2\,(5\,a^2b^3c-8\,a^3b^2d+11\,a^4be-14\,a^5f)x}{18\,(b^8x^6+2\,ab^7x^3+a^2b^6)}$$
$$+\frac{14\,b^3fx^{10}+20\,(b^3e-3\,ab^2f)x^7+35\,(b^3d-3\,ab^2e+6\,a^2bf)x^4+140\,(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f)x}{140\,b^6}$$
$$-\frac{\sqrt{3}(14\,ab^3c-35\,a^2b^2d+65\,a^3be-104\,a^4f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^7\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(14\,ab^3c-35\,a^2b^2d+65\,a^3be-104\,a^4f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^7\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(14\,ab^3c-35\,a^2b^2d+65\,a^3be-104\,a^4f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^7\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((13*a*b^4*c - 19*a^2*b^3*d + 25*a^3*b^2*e - 31*a^4*b*f)*x^4 + 2*(5*a^
2*b^3*c - 8*a^3*b^2*d + 11*a^4*b*e - 14*a^5*f)*x)/(b^8*x^6 + 2*a*b^7*x^3 +
a^2*b^6) + 1/140*(14*b^3*f*x^10 + 20*(b^3*e - 3*a*b^2*f)*x^7 + 35*(b^3*d -
3*a*b^2*e + 6*a^2*b*f)*x^4 + 140*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)
*x)/b^6 - 1/27*sqrt(3)*(14*a*b^3*c - 35*a^2*b^2*d + 65*a^3*b*e - 104*a^4*f)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*(a/b)^(2/3)) + 1/
54*(14*a*b^3*c - 35*a^2*b^2*d + 65*a^3*b*e - 104*a^4*f)*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(b^7*(a/b)^(2/3)) - 1/27*(14*a*b^3*c - 35*a^2*b^2*d + 6
5*a^3*b*e - 104*a^4*f)*log(x + (a/b)^(1/3))/(b^7*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.16

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}\left(14\left(-ab^2\right)^{\frac{1}{3}}b^3c-35\left(-ab^2\right)^{\frac{1}{3}}ab^2d+65\left(-ab^2\right)^{\frac{1}{3}}a^2be-104\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^7} + \frac{\left(14\,ab^3c-35\,a^2b^2d+65\,a^3be-104\,a^4f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^6} - \frac{\left(14\left(-ab^2\right)^{\frac{1}{3}}b^3c-35\left(-ab^2\right)^{\frac{1}{3}}ab^2d+65\left(-ab^2\right)^{\frac{1}{3}}a^2be-104\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^7} + \frac{13\,ab^4cx^4-19\,a^2b^3dx^4+25\,a^3b^2ex^4-31\,a^4bfx^4+10\,a^2b^3cx-16\,a^3b^2dx+22\,a^4bex-28\,a^5fx}{18\left(bx^3+a\right)^2b^6} + \frac{14\,b^{27}fx^{10}+20\,b^{27}ex^7-60\,ab^{26}fx^7+35\,b^{27}dx^4-105\,ab^{26}ex^4+210\,a^2b^{25}fx^4+140\,b^{27}cx-420\,ab^{26}dx\cdots}{140\,b^{30}}$$

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(14*(-a*b^2)^(1/3)*b^3*c - 35*(-a*b^2)^(1/3)*a*b^2*d + 65*(-a
*b^2)^(1/3)*a^2*b*e - 104*(-a*b^2)^(1/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/b^7 + 1/27*(14*a*b^3*c - 35*a^2*b^2*d + 65*a^3*b
*e - 104*a^4*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^6) - 1/54*(14*
(-a*b^2)^(1/3)*b^3*c - 35*(-a*b^2)^(1/3)*a*b^2*d + 65*(-a*b^2)^(1/3)*a^2*b*
e - 104*(-a*b^2)^(1/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7
+ 1/18*(13*a*b^4*c*x^4 - 19*a^2*b^3*d*x^4 + 25*a^3*b^2*e*x^4 - 31*a^4*b*f*x
^4 + 10*a^2*b^3*c*x - 16*a^3*b^2*d*x + 22*a^4*b*e*x - 28*a^5*f*x)/((b*x^3 +
 a)^2*b^6) + 1/140*(14*b^27*f*x^10 + 20*b^27*e*x^7 - 60*a*b^26*f*x^7 + 35*b
^27*d*x^4 - 105*a*b^26*e*x^4 + 210*a^2*b^25*f*x^4 + 140*b^27*c*x - 420*a*b^
26*d*x + 840*a^2*b^25*e*x - 1400*a^3*b^24*f*x)/b^30
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.12

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= x^7\left(\frac{e}{7b^3}-\frac{3af}{7b^4}\right)+x\left(\frac{c}{b^3}-\frac{a^3f}{b^6}-\frac{3a^2\left(\frac{e}{b^3}-\frac{3af}{b^4}\right)}{b^2}+\frac{3a\left(\frac{3a^2f}{b^5}-\frac{d}{b^3}+\frac{3a\left(\frac{e}{b^3}-\frac{3af}{b^4}\right)}{b}\right)}{b}\right)$$
$$-x^4\left(\frac{3a^2f}{4b^5}-\frac{d}{4b^3}+\frac{3a\left(\frac{e}{b^3}-\frac{3af}{b^4}\right)}{4b}\right)$$
$$-\frac{\left(\frac{31fa^4b}{18}-\frac{25ea^3b^2}{18}+\frac{19da^2b^3}{18}-\frac{13cab^4}{18}\right)x^4+\left(\frac{14fa^5}{9}-\frac{11ea^4b}{9}+\frac{8da^3b^2}{9}-\frac{5ca^2b^3}{9}\right)x}{a^2b^6+2ab^7x^3+b^8x^6}$$
$$+\frac{fx^{10}}{10b^3}-\frac{a^{1/3}\ln\left(b^{1/3}x+a^{1/3}\right)\left(-104fa^3+65ea^2b-35dab^2+14cb^3\right)}{27b^{19/3}}$$
$$-\frac{a^{1/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-104fa^3+65ea^2b-35dab^2+14cb^3\right)}{27b^{19/3}}$$
$$+\frac{a^{1/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-104fa^3+65ea^2b-35dab^2+14cb^3\right)}{27b^{19/3}}$$

```
[In] int((x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^7*(e/(7*b^3) - (3*a*f)/(7*b^4)) + x*(c/b^3 - (a^3*f)/b^6 - (3*a^2*(e/b^3
- (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b
^4))/b))/b) - x^4*((3*a^2*f)/(4*b^5) - d/(4*b^3) + (3*a*(e/b^3 - (3*a*f)/b^
4))/(4*b)) - (x*((14*a^5*f)/9 - (5*a^2*b^3*c)/9 + (8*a^3*b^2*d)/9 - (11*a^4
*b*e)/9) + x^4*((19*a^2*b^3*d)/18 - (25*a^3*b^2*e)/18 - (13*a*b^4*c)/18 + (
31*a^4*b*f)/18))/(a^2*b^6 + b^8*x^6 + 2*a*b^7*x^3) + (f*x^10)/(10*b^3) - (a
^(1/3)*log(b^(1/3)*x + a^(1/3))*(14*b^3*c - 104*a^3*f - 35*a*b^2*d + 65*a^2
*b*e))/(27*b^(19/3)) - (a^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1
/3))*((3^(1/2)*1i)/2 - 1/2)*(14*b^3*c - 104*a^3*f - 35*a*b^2*d + 65*a^2*b*e
))/(27*b^(19/3)) + (a^(1/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))
*((3^(1/2)*1i)/2 + 1/2)*(14*b^3*c - 104*a^3*f - 35*a*b^2*d + 65*a^2*b*e))/(
27*b^(19/3))
```

3.289 $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 345

$$\begin{aligned}
\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx
&= \frac{(b^2d-3abe+6a^2f)\,x^2}{2b^5}+\frac{(be-3af)x^5}{5b^4}+\frac{fx^8}{8b^3} \\
&\quad+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2}-\frac{(4b^3c-7ab^2d+10a^2be-13a^3f)\,x^2}{9b^5\,(a+bx^3)} \\
&\quad-\frac{(5b^3c-20ab^2d+44a^2be-77a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a}b^{17/3}} \\
&\quad-\frac{(5b^3c-20ab^2d+44a^2be-77a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{17/3}} \\
&\quad+\frac{(5b^3c-20ab^2d+44a^2be-77a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54\sqrt[3]{a}b^{17/3}}
\end{aligned}$$

```
[Out] 1/2*(6*a^2*f-3*a*b*e+b^2*d)*x^2/b^5+1/5*(-3*a*f+b*e)*x^5/b^4+1/8*f*x^8/b^3+
1/6*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/b^5/(b*x^3+a)^2-1/9*(-13*a^3*f+10*
a^2*b*e-7*a*b^2*d+4*b^3*c)*x^2/b^5/(b*x^3+a)-1/27*(-77*a^3*f+44*a^2*b*e-20*
a*b^2*d+5*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(17/3)+1/54*(-77*a^3*f+44*
a^2*b*e-20*a*b^2*d+5*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/
3)/b^(17/3)-1/27*(-77*a^3*f+44*a^2*b*e-20*a*b^2*d+5*b^3*c)*arctan(1/3*(a^(1
/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(17/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1842, 1865, 1850, 1502, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{x^2(6a^2f-3abe+b^2d)}{2b^5} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-77a^3f+44a^2be-20ab^2d+5b^3c)}{9\sqrt{3}\sqrt[3]{a}b^{17/3}}$$
$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-77a^3f+44a^2be-20ab^2d+5b^3c)}{27\sqrt[3]{a}b^{17/3}}$$
$$-\frac{x^2(-13a^3f+10a^2be-7ab^2d+4b^3c)}{9b^5(a+bx^3)} + \frac{ax^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5(a+bx^3)^2}$$
$$+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-77a^3f+44a^2be-20ab^2d+5b^3c)}{54\sqrt[3]{a}b^{17/3}}$$
$$+\frac{x^5(be-3af)}{5b^4} + \frac{fx^8}{8b^3}$$

```
[In] Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x^2)/(2*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^4) + (
f*x^8)/(8*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^5*(a + b*
x^3)^2) - ((4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*x^2)/(9*b^5*(a + b
*x^3)) - ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(1/3)*b^(17/3)) - ((5*b^3*c -
 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1/3)*
b^(17/3)) + ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(1/3)*b^(17/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
```

```
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1865

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2} \\
&\quad -\frac{\int \frac{2a^2b(b^3c-ab^2d+a^2be-a^3f)x-6ab^2(b^3c-ab^2d+a^2be-a^3f)x^4-6ab^3(b^2d-abe+a^2f)x^7-6ab^4(be-af)x^{10}-6ab^5fx^{13}}{(a+bx^3)^2}\,dx}{6ab^6} \\
&= \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2} \\
&\quad -\frac{\int \frac{x(2a^2b(b^3c-ab^2d+a^2be-a^3f)-6ab^2(b^3c-ab^2d+a^2be-a^3f)x^3-6ab^3(b^2d-abe+a^2f)x^6-6ab^4(be-af)x^9-6ab^5fx^{12})}{(a+bx^3)^2}\,dx}{6ab^6} \\
&= \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2}-\frac{(4b^3c-7ab^2d+10a^2be-13a^3f)\,x^2}{9b^5\,(a+bx^3)} \\
&\quad +\frac{\int \frac{2a^2b^6(5b^3c-11ab^2d+17a^2be-23a^3f)x+18a^2b^7(b^2d-2abe+3a^2f)x^4+18a^2b^8(be-2af)x^7+18a^2b^9fx^{10}}{a+bx^3}\,dx}{18a^2b^{11}} \\
&= \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2}-\frac{(4b^3c-7ab^2d+10a^2be-13a^3f)\,x^2}{9b^5\,(a+bx^3)} \\
&\quad +\frac{\int \frac{x(2a^2b^6(5b^3c-11ab^2d+17a^2be-23a^3f)+18a^2b^7(b^2d-2abe+3a^2f)x^3+18a^2b^8(be-2af)x^6+18a^2b^9fx^9)}{a+bx^3}\,dx}{18a^2b^{11}} \\
&= \frac{fx^8}{8b^3}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2}-\frac{(4b^3c-7ab^2d+10a^2be-13a^3f)\,x^2}{9b^5\,(a+bx^3)} \\
&\quad +\frac{\int \frac{x(16a^2b^7(5b^3c-11ab^2d+17a^2be-23a^3f)+144a^2b^8(b^2d-2abe+3a^2f)x^3+144a^2b^9(be-3af)x^6)}{a+bx^3}\,dx}{144a^2b^{12}} \\
&= \frac{fx^8}{8b^3}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2}-\frac{(4b^3c-7ab^2d+10a^2be-13a^3f)\,x^2}{9b^5\,(a+bx^3)} \\
&\quad +\frac{\int \left(144a^2b^7(b^2d-3abe+6a^2f)\,x+144a^2b^8(be-3af)x^4-\frac{16(-5a^2b^{10}c+20a^3b^9d-44a^4b^8e+77a^5b^7f)x}{a+bx^3}\right)\,d}{144a^2b^{12}}
\end{aligned}$$

$$= \frac{(b^2d - 3abe + 6a^2f)\,x^2}{2b^5} + \frac{(be - 3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6b^5\,(a + bx^3)^2} - \frac{(4b^3c - 7ab^2d + 10a^2be - 13a^3f)\,x^2}{9b^5\,(a + bx^3)} + \frac{(5b^3c - 20ab^2d + 44a^2be - 77a^3f)\int \frac{x}{a+bx^3}\,dx}{9b^5}$$

$$= \frac{(b^2d - 3abe + 6a^2f)\,x^2}{2b^5} + \frac{(be - 3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6b^5\,(a + bx^3)^2} - \frac{(4b^3c - 7ab^2d + 10a^2be - 13a^3f)\,x^2}{9b^5\,(a + bx^3)} - \frac{(5b^3c - 20ab^2d + 44a^2be - 77a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27\sqrt[3]{a}b^{16/3}} + \frac{(5b^3c - 20ab^2d + 44a^2be - 77a^3f)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27\sqrt[3]{a}b^{16/3}}$$

$$= \frac{(b^2d - 3abe + 6a^2f)\,x^2}{2b^5} + \frac{(be - 3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6b^5\,(a + bx^3)^2} - \frac{(4b^3c - 7ab^2d + 10a^2be - 13a^3f)\,x^2}{9b^5\,(a + bx^3)} - \frac{(5b^3c - 20ab^2d + 44a^2be - 77a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{17/3}} + \frac{(5b^3c - 20ab^2d + 44a^2be - 77a^3f)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54\sqrt[3]{a}b^{17/3}} + \frac{(5b^3c - 20ab^2d + 44a^2be - 77a^3f)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18b^{16/3}}$$

$$= \frac{(b^2d - 3abe + 6a^2f)\,x^2}{2b^5} + \frac{(be - 3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6b^5\,(a + bx^3)^2} - \frac{(4b^3c - 7ab^2d + 10a^2be - 13a^3f)\,x^2}{9b^5\,(a + bx^3)} - \frac{(5b^3c - 20ab^2d + 44a^2be - 77a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{17/3}} + \frac{(5b^3c - 20ab^2d + 44a^2be - 77a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54\sqrt[3]{a}b^{17/3}} + \frac{(5b^3c - 20ab^2d + 44a^2be - 77a^3f)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9\sqrt[3]{a}b^{17/3}}$$

$$
\begin{aligned}
&= \frac{(b^2d-3abe+6a^2f)\,x^2}{2b^5}+\frac{(be-3af)x^5}{5b^4}+\frac{fx^8}{8b^3}\\
&\quad+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2}-\frac{(4b^3c-7ab^2d+10a^2be-13a^3f)\,x^2}{9b^5\,(a+bx^3)}\\
&\quad-\frac{(5b^3c-20ab^2d+44a^2be-77a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a}b^{17/3}}\\
&\quad-\frac{(5b^3c-20ab^2d+44a^2be-77a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{17/3}}\\
&\quad+\frac{(5b^3c-20ab^2d+44a^2be-77a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54\sqrt[3]{a}b^{17/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.95

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= \frac{540b^{2/3}(b^2d-3abe+6a^2f)\,x^2+216b^{5/3}(be-3af)x^5+135b^{8/3}fx^8+\frac{180ab^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{(a+bx^3)^2}-\frac{120b^{2/3}(4}{}}{}$$

```
[In] Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (540*b^(2/3)*(b^2*d - 3*a*b*e + 6*a^2*f)*x^2 + 216*b^(5/3)*(b*e - 3*a*f)*x^
5 + 135*b^(8/3)*f*x^8 + (180*a*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
x^2)/(a + b*x^3)^2 - (120*b^(2/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^
3*f)*x^2)/(a + b*x^3) + (40*Sqrt[3]*(-5*b^3*c + 20*a*b^2*d - 44*a^2*b*e + 7
7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-5*b^3
*c + 20*a*b^2*d - 44*a^2*b*e + 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3)
+ (20*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(1080*b^(17/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.57

method	result
risch	$\frac{f\,x^{8}}{8b^{3}}-\frac{3x^{5}af}{5b^{4}}+\frac{x^{5}e}{5b^{3}}+\frac{3a^{2}f\,x^{2}}{b^{5}}-\frac{3ae\,x^{2}}{2b^{4}}+\frac{x^{2}d}{2b^{3}}+\frac{\left(\frac{13}{9}a^{3}bf-\frac{10}{9}a^{2}e\,b^{2}+\frac{7}{9}a\,b^{3}d-\frac{4}{9}b^{4}c\right)x^{5}+\frac{a\left(23f\,a^{3}-17a^{2}be+11a\,b^{2}d-5b^{3}c\right)x}{18}}{b^{5}\left(b\,x^{3}+a\right)^{2}}$ $\frac{\left(-\frac{13}{9}a^{3}bf+\frac{10}{9}a^{2}e\,b^{2}-\frac{7}{9}a\,b^{3}d+\frac{4}{9}b^{4}c\right)x^{5}-\frac{a\left(23f\,a^{3}-17a^{2}be+11a\,b^{2}d-5b^{3}c\right)x^{2}}{18}}{\left(b\,x^{3}+a\right)^{2}}+\left(\frac{77}{9}f\right.$
default	$\frac{\frac{b^{2}f\,x^{8}}{8}+\frac{\left(-3afb+b^{2}e\right)x^{5}}{5}+\frac{\left(6a^{2}f-3aeb+b^{2}d\right)x^{2}}{2}}{b^{5}}-$

[In] int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/8*f*x^8/b^3-3/5/b^4*x^5*a*f+1/5/b^3*x^5*e+3/b^5*a^2*f*x^2-3/2/b^4*a*e*x^2
+1/2/b^3*x^2*d+((13/9*a^3*b*f-10/9*a^2*e*b^2+7/9*a*b^3*d-4/9*b^4*c)*x^5+1/1
8*a*(23*a^3*f-17*a^2*b*e+11*a*b^2*d-5*b^3*c)*x^2)/b^5/(b*x^3+a)^2+1/27/b^6*
sum((-77*a^3*f+44*a^2*b*e-20*a*b^2*d+5*b^3*c)/_R*ln(x-_R),_R=RootOf(_Z^3*b+
a))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. 2(298) = 596.

Time = 0.29 (sec) , antiderivative size = 1278, normalized size of antiderivative = 3.70

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] [1/1080*(135*a*b^6*f*x^14 + 54*(4*a*b^6*e - 7*a^2*b^5*f)*x^11 + 27*(20*a*b^
6*d - 44*a^2*b^5*e + 77*a^3*b^4*f)*x^8 - 96*(5*a*b^6*c - 20*a^2*b^5*d + 44*
a^3*b^4*e - 77*a^4*b^3*f)*x^5 - 60*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3
*e - 77*a^5*b^2*f)*x^2 - 60*sqrt(1/3)*(5*a^3*b^4*c - 20*a^4*b^3*d + 44*a^5*
b^2*e - 77*a^6*b*f + (5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e - 77*a^4*b^3*
f)*x^6 + 2*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5*b^2*f)*x^3)*
sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2

```
)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/
(b*x^3 + a)) + 20*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b^2*f)*x^6
 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4*c - 20*a
^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b
^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e -
 77*a^3*b^2*f)*x^6 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2
*(5*a*b^4*c - 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*
log(b*x + (a*b^2)^(1/3)))/(a*b^9*x^6 + 2*a^2*b^8*x^3 + a^3*b^7), 1/1080*(13
5*a*b^6*f*x^14 + 54*(4*a*b^6*e - 7*a^2*b^5*f)*x^11 + 27*(20*a*b^6*d - 44*a^
2*b^5*e + 77*a^3*b^4*f)*x^8 - 96*(5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e -
 77*a^4*b^3*f)*x^5 - 60*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5
*b^2*f)*x^2 - 120*sqrt(1/3)*(5*a^3*b^4*c - 20*a^4*b^3*d + 44*a^5*b^2*e - 77
*a^6*b*f + (5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e - 77*a^4*b^3*f)*x^6 + 2
*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5*b^2*f)*x^3)*sqrt((a*b^
2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)
/b) + 20*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b^2*f)*x^6 + 5*a^2*
b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4*c - 20*a^2*b^3*d
+ 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)
*b*x + (a*b^2)^(2/3)) - 40*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b
^2*f)*x^6 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4
*c - 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b*x +
 (a*b^2)^(1/3)))/(a*b^9*x^6 + 2*a^2*b^8*x^3 + a^3*b^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.96

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= -\frac{2\,(4\,b^4c-7\,ab^3d+10\,a^2b^2e-13\,a^3bf)x^5+(5\,ab^3c-11\,a^2b^2d+17\,a^3be-23\,a^4f)x^2}{18\,(b^7x^6+2\,ab^6x^3+a^2b^5)}$$
$$+\frac{\sqrt{3}(5\,b^3c-20\,ab^2d+44\,a^2be-77\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+\frac{5\,b^2fx^8+8\,(b^2e-3\,abf)x^5+20\,(b^2d-3\,abe+6\,a^2f)x^2}{40\,b^5}$$
$$+\frac{(5\,b^3c-20\,ab^2d+44\,a^2be-77\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$-\frac{(5\,b^3c-20\,ab^2d+44\,a^2be-77\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(2*(4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3*b*f)*x^5 + (5*a*b^3*c
 - 11*a^2*b^2*d + 17*a^3*b*e - 23*a^4*f)*x^2)/(b^7*x^6 + 2*a*b^6*x^3 + a^2*
b^5) + 1/27*sqrt(3)*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*arctan(1
/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(1/3)) + 1/40*(5*b^2
*f*x^8 + 8*(b^2*e - 3*a*b*f)*x^5 + 20*(b^2*d - 3*a*b*e + 6*a^2*f)*x^2)/b^5
+ 1/54*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*log(x^2 - x*(a/b)^(1/
3) + (a/b)^(2/3))/(b^6*(a/b)^(1/3)) - 1/27*(5*b^3*c - 20*a*b^2*d + 44*a^2*b
*e - 77*a^3*f)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.11

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{\sqrt{3}(5\,b^3c-20\,ab^2d+44\,a^2be-77\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}b^5}$$
$$-\frac{(5\,b^3c-20\,ab^2d+44\,a^2be-77\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}b^5}$$
$$-\frac{\left(5\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-20\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+44\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-77\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^5}$$
$$-\frac{8\,b^4cx^5-14\,ab^3dx^5+20\,a^2b^2ex^5-26\,a^3bfx^5+5\,ab^3cx^2-11\,a^2b^2dx^2+17\,a^3bex^2-23\,a^4fx^2}{18\,(bx^3+a)^2b^5}$$
$$+\frac{5\,b^{21}fx^8+8\,b^{21}ex^5-24\,ab^{20}fx^5+20\,b^{21}dx^2-60\,ab^{20}ex^2+120\,a^2b^{19}fx^2}{40\,b^{24}}$$

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*arctan(1/3*sqrt
(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^5) - 1/54*(5*b^3*c
 - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3))/((-a*b^2)^(1/3)*b^5) - 1/27*(5*b^3*c*(-a/b)^(1/3) - 20*a*b^2*d*(-a/b)^
(1/3) + 44*a^2*b*e*(-a/b)^(1/3) - 77*a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(a
bs(x - (-a/b)^(1/3)))/(a*b^5) - 1/18*(8*b^4*c*x^5 - 14*a*b^3*d*x^5 + 20*a^2
*b^2*e*x^5 - 26*a^3*b*f*x^5 + 5*a*b^3*c*x^2 - 11*a^2*b^2*d*x^2 + 17*a^3*b*e
*x^2 - 23*a^4*f*x^2)/((b*x^3 + a)^2*b^5) + 1/40*(5*b^21*f*x^8 + 8*b^21*e*x^
5 - 24*a*b^20*f*x^5 + 20*b^21*d*x^2 - 60*a*b^20*e*x^2 + 120*a^2*b^19*f*x^2)
/b^24
```

Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.98

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= x^5\left(\frac{e}{5\,b^3}-\frac{3\,a\,f}{5\,b^4}\right)$$
$$+\frac{x^2\left(\frac{23\,f\,a^4}{18}-\frac{17\,e\,a^3\,b}{18}+\frac{11\,d\,a^2\,b^2}{18}-\frac{5\,c\,a\,b^3}{18}\right)-x^5\left(-\frac{13\,f\,a^3\,b}{9}+\frac{10\,e\,a^2\,b^2}{9}-\frac{7\,d\,a\,b^3}{9}+\frac{4\,c\,b^4}{9}\right)}{a^2\,b^5+2\,a\,b^6\,x^3+b^7\,x^6}$$
$$-x^2\left(\frac{3\,a^2\,f}{2\,b^5}-\frac{d}{2\,b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{2\,b}\right)+\frac{f\,x^8}{8\,b^3}$$
$$-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-77\,f\,a^3+44\,e\,a^2\,b-20\,d\,a\,b^2+5\,c\,b^3\right)}{27\,a^{1/3}\,b^{17/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-77\,f\,a^3+44\,e\,a^2\,b-20\,d\,a\,b^2+5\,c\,b^3\right)}{27\,a^{1/3}\,b^{17/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-77\,f\,a^3+44\,e\,a^2\,b-20\,d\,a\,b^2+5\,c\,b^3\right)}{27\,a^{1/3}\,b^{17/3}}$$

```
[In] int((x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^5*(e/(5*b^3) - (3*a*f)/(5*b^4)) + (x^2*((23*a^4*f)/18 + (11*a^2*b^2*d)/18
 - (5*a*b^3*c)/18 - (17*a^3*b*e)/18) - x^5*((4*b^4*c)/9 + (10*a^2*b^2*e)/9
- (7*a*b^3*d)/9 - (13*a^3*b*f)/9))/(a^2*b^5 + b^7*x^6 + 2*a*b^6*x^3) - x^2*
((3*a^2*f)/(2*b^5) - d/(2*b^3) + (3*a*(e/b^3 - (3*a*f)/b^4))/(2*b)) + (f*x^
8)/(8*b^3) - (log(b^(1/3)*x + a^(1/3))*(5*b^3*c - 77*a^3*f - 20*a*b^2*d + 4
4*a^2*b*e))/(27*a^(1/3)*b^(17/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x -
 a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*b^3*c - 77*a^3*f - 20*a*b^2*d + 44*a^2*
b*e))/(27*a^(1/3)*b^(17/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/
3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c - 77*a^3*f - 20*a*b^2*d + 44*a^2*b*e))/
(27*a^(1/3)*b^(17/3))
```

3.290 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 336

$$\begin{aligned}\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= \frac{(b^2d-3abe+6a^2f)\,x}{b^5}+\frac{(be-3af)x^4}{4b^4}+\frac{fx^7}{7b^3} \\ &\quad+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^5\,(a+bx^3)^2}-\frac{(7b^3c-13ab^2d+19a^2be-25a^3f)\,x}{18b^5\,(a+bx^3)} \\ &\quad-\frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{16/3}} \\ &\quad+\frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{16/3}} \\ &\quad-\frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{2/3}b^{16/3}}\end{aligned}$$

```
[Out] (6*a^2*f-3*a*b*e+b^2*d)*x/b^5+1/4*(-3*a*f+b*e)*x^4/b^4+1/7*f*x^7/b^3+1/6*a*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^5/(b*x^3+a)^2-1/18*(-25*a^3*f+19*a^2*b*e
-13*a*b^2*d+7*b^3*c)*x/b^5/(b*x^3+a)+1/27*(-65*a^3*f+35*a^2*b*e-14*a*b^2*d+
2*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(16/3)-1/54*(-65*a^3*f+35*a^2*b*e-
14*a*b^2*d+2*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(16
/3)-1/27*(-65*a^3*f+35*a^2*b*e-14*a*b^2*d+2*b^3*c)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(16/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1842, 1872, 1901, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{x(6a^2f-3abe+b^2d)}{b^5} - \frac{x(-25a^3f+19a^2be-13ab^2d+7b^3c)}{18b^5(a+bx^3)} + \frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5(a+bx^3)^2} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-65a^3f+35a^2be-14ab^2d+2b^3c)}{9\sqrt{3}a^{2/3}b^{16/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-65a^3f+35a^2be-14ab^2d+2b^3c)}{54a^{2/3}b^{16/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-65a^3f+35a^2be-14ab^2d+2b^3c)}{27a^{2/3}b^{16/3}} + \frac{x^4(be-3af)}{4b^4} + \frac{fx^7}{7b^3}$$

```
[In] Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x)/b^5 + ((b*e - 3*a*f)*x^4)/(4*b^4) + (f*x^7)
/(7*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^5*(a + b*x^3)^2)
- ((7*b^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^3*f)*x)/(18*b^5*(a + b*x^3)) -
 ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3
)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*b^(16/3)) + ((2*b^3*c - 14*a*b^
2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(16/3)
) - ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(54*a^(2/3)*b^(16/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

integral

$$= \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^5\,(a+bx^3)^2} - \frac{\int \frac{a^2(b^3c-ab^2d+a^2be-a^3f)-6ab(b^3c-ab^2d+a^2be-a^3f)x^3-6ab^2(b^2d-abe+a^2f)x^6-6ab^3(be-af)x^9-6ab^4fx^{12}}{(a+bx^3)^2}\,dx}{6ab^5}$$

$$= \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^5\,(a+bx^3)^2} - \frac{(7b^3c-13ab^2d+19a^2be-25a^3f)\,x}{18b^5\,(a+bx^3)} + \frac{\int \frac{2a^2b^4(2b^3c-5ab^2d+8a^2be-11a^3f)+18a^2b^5(b^2d-2abe+3a^2f)x^3+18a^2b^6(be-2af)x^6+18a^2b^7fx^9}{a+bx^3}\,dx}{18a^2b^9}$$

$$= \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^5\,(a+bx^3)^2} - \frac{(7b^3c-13ab^2d+19a^2be-25a^3f)\,x}{18b^5\,(a+bx^3)} + \frac{\int \left(18a^2b^4(b^2d-3abe+6a^2f)+18a^2b^5(be-3af)x^3+18a^2b^6fx^6-\frac{2(-2a^2b^7c+14a^3b^6d-35a^4b^5e+65a^5b^4f)}{a+bx^3}\right) d}{18a^2b^9}$$

$$= \frac{(b^2d-3abe+6a^2f)\,x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^5\,(a+bx^3)^2} - \frac{(7b^3c-13ab^2d+19a^2be-25a^3f)\,x}{18b^5\,(a+bx^3)} + \frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\int \frac{1}{a+bx^3}\,dx}{9b^5}$$

$$= \frac{(b^2d-3abe+6a^2f)\,x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^5\,(a+bx^3)^2} - \frac{(7b^3c-13ab^2d+19a^2be-25a^3f)\,x}{18b^5\,(a+bx^3)} + \frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{2/3}b^5} + \frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{2/3}b^5}$$

$$=\frac{(b^2d-3abe+6a^2f)\,x}{b^5}+\frac{(be-3af)x^4}{4b^4}+\frac{fx^7}{7b^3}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^5\,(a+bx^3)^2}-\frac{(7b^3c-13ab^2d+19a^2be-25a^3f)\,x}{18b^5\,(a+bx^3)}+\frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{16/3}}-\frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{2/3}b^{16/3}}+\frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18\sqrt[3]{a}b^5}$$

$$=\frac{(b^2d-3abe+6a^2f)\,x}{b^5}+\frac{(be-3af)x^4}{4b^4}+\frac{fx^7}{7b^3}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^5\,(a+bx^3)^2}-\frac{(7b^3c-13ab^2d+19a^2be-25a^3f)\,x}{18b^5\,(a+bx^3)}+\frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{16/3}}-\frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{2/3}b^{16/3}}+\frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{2/3}b^{16/3}}$$

$$=\frac{(b^2d-3abe+6a^2f)\,x}{b^5}+\frac{(be-3af)x^4}{4b^4}+\frac{fx^7}{7b^3}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^5\,(a+bx^3)^2}-\frac{(7b^3c-13ab^2d+19a^2be-25a^3f)\,x}{18b^5\,(a+bx^3)}-\frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{16/3}}+\frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{16/3}}-\frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{2/3}b^{16/3}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.96

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= \frac{756\sqrt[3]{b}(b^2d-3abe+6a^2f)\,x+189b^{4/3}(be-3af)x^4+108b^{7/3}fx^7+\frac{126a\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)x}{(a+bx^3)^2}-\frac{42\sqrt[3]{b}(7b^3}{}}{}$$

```
[In] Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (756*b^(1/3)*(b^2*d - 3*a*b*e + 6*a^2*f)*x + 189*b^(4/3)*(b*e - 3*a*f)*x^4
+ 108*b^(7/3)*f*x^7 + (126*a*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)
/(a + b*x^3)^2 - (42*b^(1/3)*(7*b^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^3*f)
*x)/(a + b*x^3) + (28*Sqrt[3]*(-2*b^3*c + 14*a*b^2*d - 35*a^2*b*e + 65*a^3*
f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(2*b^3*c - 14
*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(
-2*b^3*c + 14*a*b^2*d - 35*a^2*b*e + 65*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/a^(2/3))/(756*b^(16/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.55

method	result
risch	$\frac{fx^7}{7b^3}-\frac{3x^4af}{4b^4}+\frac{x^4e}{4b^3}+\frac{6a^2fx}{b^5}-\frac{3aex}{b^4}+\frac{xd}{b^3}+\frac{\left(\frac{25}{18}a^3bf-\frac{19}{18}a^2eb^2+\frac{13}{18}ab^3d-\frac{7}{18}b^4c\right)x^4+\frac{a\left(11fa^3-8a^2be+5ab^2d-2b^3c\right)x}{9}}{b^5(bx^3+a)^2}+$ $(65$
default	$\frac{\frac{1}{7}b^2fx^7-\frac{3}{4}abfx^4+\frac{1}{4}b^2ex^4+6a^2fx-3abex+b^2dx}{b^5}-\frac{\frac{\left(-\frac{25}{18}a^3bf+\frac{19}{18}a^2eb^2-\frac{13}{18}ab^3d+\frac{7}{18}b^4c\right)x^4-\frac{a\left(11fa^3-8a^2be+5ab^2d-2b^3c\right)x}{9}}{(bx^3+a)^2}+}{}$

```
[In] int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*f*x^7/b^3-3/4/b^4*x^4*a*f+1/4/b^3*x^4*e+6/b^5*a^2*f*x-3/b^4*a*e*x+1/b^3
*x*d+((25/18*a^3*b*f-19/18*a^2*e*b^2+13/18*a*b^3*d-7/18*b^4*c)*x^4+1/9*a*(1
1*a^3*f-8*a^2*b*e+5*a*b^2*d-2*b^3*c)*x)/b^5/(b*x^3+a)^2+1/27/b^6*sum((-65*a
^3*f+35*a^2*b*e-14*a*b^2*d+2*b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(291) = 582$.

Time = 0.29 (sec) , antiderivative size = 1318, normalized size of antiderivative = 3.92

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/756*(108*a^2*b^5*f*x^13 + 27*(7*a^2*b^5*e - 13*a^3*b^4*f)*x^10 + 54*(14*
a^2*b^5*d - 35*a^3*b^4*e + 65*a^4*b^3*f)*x^7 - 147*(2*a^2*b^5*c - 14*a^3*b^
4*d + 35*a^4*b^3*e - 65*a^5*b^2*f)*x^4 - 42*sqrt(1/3)*(2*a^3*b^4*c - 14*a^4
*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f + (2*a*b^6*c - 14*a^2*b^5*d + 35*a^3*b^4
*e - 65*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c - 14*a^3*b^4*d + 35*a^4*b^3*e - 65*
a^5*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*
x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqr
t((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*((2*b^5*c - 14*a*b^4*d + 35*a^2*b^3*
e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e - 65*a^5*f
+ 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*b*f)*x^3)*(-a^2*b)^(2
/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*((2*b^5*c - 14*
a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d + 3
5*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*
b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(2*a^3*b^4*c - 14
*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f)*x)/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a
^4*b^6), 1/756*(108*a^2*b^5*f*x^13 + 27*(7*a^2*b^5*e - 13*a^3*b^4*f)*x^10 +
 54*(14*a^2*b^5*d - 35*a^3*b^4*e + 65*a^4*b^3*f)*x^7 - 147*(2*a^2*b^5*c - 1
4*a^3*b^4*d + 35*a^4*b^3*e - 65*a^5*b^2*f)*x^4 + 84*sqrt(1/3)*(2*a^3*b^4*c
- 14*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f + (2*a*b^6*c - 14*a^2*b^5*d + 35
*a^3*b^4*e - 65*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c - 14*a^3*b^4*d + 35*a^4*b^3
*e - 65*a^5*b^2*f)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b
)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 14*((2*b^5*c -
 14*a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d
 + 35*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*
a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3
)*a) + 28*((2*b^5*c - 14*a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2
*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d
 + 35*a^3*b^2*e - 65*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3
```

```
)) - 84*(2*a^3*b^4*c - 14*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f)*x)/(a^2*b^
8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= -\frac{(7\,b^4c-13\,ab^3d+19\,a^2b^2e-25\,a^3bf)x^4+2\,(2\,ab^3c-5\,a^2b^2d+8\,a^3be-11\,a^4f)x}{18\,(b^7x^6+2\,ab^6x^3+a^2b^5)} \\ &\quad+\frac{4\,b^2fx^7+7\,(b^2e-3\,abf)x^4+28\,(b^2d-3\,abe+6\,a^2f)x}{28\,b^5} \\ &\quad+\frac{\sqrt{3}(2\,b^3c-14\,ab^2d+35\,a^2be-65\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &\quad-\frac{(2\,b^3c-14\,ab^2d+35\,a^2be-65\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &\quad+\frac{(2\,b^3c-14\,ab^2d+35\,a^2be-65\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
[Out] -1/18*((7*b^4*c - 13*a*b^3*d + 19*a^2*b^2*e - 25*a^3*b*f)*x^4 + 2*(2*a*b^3*
c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f)*x)/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5
) + 1/28*(4*b^2*f*x^7 + 7*(b^2*e - 3*a*b*f)*x^4 + 28*(b^2*d - 3*a*b*e + 6*a
^2*f)*x)/b^5 + 1/27*sqrt(3)*(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(2/3)) - 1/5
4*(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(b^6*(a/b)^(2/3)) + 1/27*(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e -
65*a^3*f)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.01

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= -\frac{\sqrt{3}(2\,b^3c-14\,ab^2d+35\,a^2be-65\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,b^4} - \frac{(2\,b^3c-14\,ab^2d+35\,a^2be-65\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,b^4} - \frac{(2\,b^3c-14\,ab^2d+35\,a^2be-65\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^5} - \frac{7\,b^4cx^4-13\,ab^3dx^4+19\,a^2b^2ex^4-25\,a^3bfx^4+4\,ab^3cx-10\,a^2b^2dx+16\,a^3bex-22\,a^4fx}{18\,(bx^3+a)^2b^5} + \frac{4\,b^{18}fx^7+7\,b^{18}ex^4-21\,ab^{17}fx^4+28\,b^{18}dx-84\,ab^{17}ex+168\,a^2b^{16}fx}{28\,b^{21}}$$

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^4) - 1/54*(2*b^3*
c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/((-a*b^2)^(2/3)*b^4) - 1/27*(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a
^3*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) - 1/18*(7*b^4*c*x^4 -
 13*a*b^3*d*x^4 + 19*a^2*b^2*e*x^4 - 25*a^3*b*f*x^4 + 4*a*b^3*c*x - 10*a^2*
b^2*d*x + 16*a^3*b*e*x - 22*a^4*f*x)/((b*x^3 + a)^2*b^5) + 1/28*(4*b^18*f*x
^7 + 7*b^18*e*x^4 - 21*a*b^17*f*x^4 + 28*b^18*d*x - 84*a*b^17*e*x + 168*a^2
*b^16*f*x)/b^21
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= x^4\left(\frac{e}{4\,b^3}-\frac{3\,a\,f}{4\,b^4}\right)-x\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)$$
$$-\frac{x^4\left(-\frac{25\,f\,a^3\,b}{18}+\frac{19\,e\,a^2\,b^2}{18}-\frac{13\,d\,a\,b^3}{18}+\frac{7\,c\,b^4}{18}\right)-x\left(\frac{11\,f\,a^4}{9}-\frac{8\,e\,a^3\,b}{9}+\frac{5\,d\,a^2\,b^2}{9}-\frac{2\,c\,a\,b^3}{9}\right)}{a^2\,b^5+2\,a\,b^6\,x^3+b^7\,x^6}$$
$$+\frac{f\,x^7}{7\,b^3}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-65\,f\,a^3+35\,e\,a^2\,b-14\,d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{2/3}\,b^{16/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-65\,f\,a^3+35\,e\,a^2\,b-14\,d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{2/3}\,b^{16/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-65\,f\,a^3+35\,e\,a^2\,b-14\,d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{2/3}\,b^{16/3}}$$

```
[In] int((x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^4*(e/(4*b^3) - (3*a*f)/(4*b^4)) - x*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3
- (3*a*f)/b^4))/b) - (x^4*((7*b^4*c)/18 + (19*a^2*b^2*e)/18 - (13*a*b^3*d)/
18 - (25*a^3*b*f)/18) - x*((11*a^4*f)/9 + (5*a^2*b^2*d)/9 - (2*a*b^3*c)/9 -
 (8*a^3*b*e)/9))/(a^2*b^5 + b^7*x^6 + 2*a*b^6*x^3) + (f*x^7)/(7*b^3) + (log
(b^(1/3)*x + a^(1/3))*(2*b^3*c - 65*a^3*f - 14*a*b^2*d + 35*a^2*b*e))/(27*a
^(2/3)*b^(16/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/
2)*1i)/2 - 1/2)*(2*b^3*c - 65*a^3*f - 14*a*b^2*d + 35*a^2*b*e))/(27*a^(2/3)
*b^(16/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)
/2 + 1/2)*(2*b^3*c - 65*a^3*f - 14*a*b^2*d + 35*a^2*b*e))/(27*a^(2/3)*b^(16
/3))
```

3.291 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 316

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{(be-3af)x^2}{2b^4}+\frac{fx^5}{5b^3}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2}+\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\,x^2}{9ab^4\,(a+bx^3)} - \frac{(b^3c+5ab^2d-20a^2be+44a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{14/3}} - \frac{(b^3c+5ab^2d-20a^2be+44a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{14/3}} + \frac{(b^3c+5ab^2d-20a^2be+44a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{14/3}}$$

```
[Out] 1/2*(-3*a*f+b*e)*x^2/b^4+1/5*f*x^5/b^3-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x
^2/b^4/(b*x^3+a)^2+1/9*(-10*a^3*f+7*a^2*b*e-4*a*b^2*d+b^3*c)*x^2/a/b^4/(b*x
^3+a)-1/27*(44*a^3*f-20*a^2*b*e+5*a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(4
/3)/b^(14/3)+1/54*(44*a^3*f-20*a^2*b*e+5*a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(14/3)-1/27*(44*a^3*f-20*a^2*b*e+5*a*b^2*d
+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(14/3)*
3^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1842, 1865, 1608, 1502, 298, 31, 648, 631, 210, 642}

$$\begin{aligned}
&\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx \\
&= \frac{x^2(-10a^3f+7a^2be-4ab^2d+b^3c)}{9ab^4(a+bx^3)} - \frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^4(a+bx^3)^2} \\
&\quad - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(44a^3f-20a^2be+5ab^2d+b^3c)}{9\sqrt{3}a^{4/3}b^{14/3}} \\
&\quad + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(44a^3f-20a^2be+5ab^2d+b^3c)}{54a^{4/3}b^{14/3}} \\
&\quad - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(44a^3f-20a^2be+5ab^2d+b^3c)}{27a^{4/3}b^{14/3}} + \frac{x^2(be-3af)}{2b^4} + \frac{fx^5}{5b^3}
\end{aligned}$$

```
[In] Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x^2)/(2*b^4) + (f*x^5)/(5*b^3) - ((b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*x^2)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10
*a^3*f)*x^2)/(9*a*b^4*(a + b*x^3)) - ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3
)*b^(14/3)) - ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3) + b^
(1/3)*x])/(27*a^(4/3)*b^(14/3)) + ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3
*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1865

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} \\
&\quad -\frac{\int \frac{-2ab\left(b^3c-ab^2d+a^2be-a^3f\right)x-6ab^2\left(b^2d-abe+a^2f\right)x^4-6ab^3(be-af)x^7-6ab^4fx^{10}}{\left(a+bx^3\right)^2}\,dx}{6ab^5} \\
&= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} \\
&\quad -\frac{\int \frac{x\left(-2ab\left(b^3c-ab^2d+a^2be-a^3f\right)-6ab^2\left(b^2d-abe+a^2f\right)x^3-6ab^3(be-af)x^6-6ab^4fx^9\right)}{\left(a+bx^3\right)^2}\,dx}{6ab^5} \\
&= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2}+\frac{\left(b^3c-4ab^2d+7a^2be-10a^3f\right)x^2}{9ab^4\left(a+bx^3\right)} \\
&\quad +\frac{\int \frac{2ab^5\left(b^3c+5ab^2d-11a^2be+17a^3f\right)x+18a^2b^6(be-2af)x^4+18a^2b^7fx^7}{a+bx^3}\,dx}{18a^2b^9} \\
&= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2}+\frac{\left(b^3c-4ab^2d+7a^2be-10a^3f\right)x^2}{9ab^4\left(a+bx^3\right)} \\
&\quad +\frac{\int \frac{x\left(2ab^5\left(b^3c+5ab^2d-11a^2be+17a^3f\right)+18a^2b^6(be-2af)x^3+18a^2b^7fx^6\right)}{a+bx^3}\,dx}{18a^2b^9} \\
&= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2}+\frac{\left(b^3c-4ab^2d+7a^2be-10a^3f\right)x^2}{9ab^4\left(a+bx^3\right)} \\
&\quad +\frac{\int \left(18a^2b^5(be-3af)x+18a^2b^6fx^4+\frac{2\left(ab^8c+5a^2b^7d-20a^3b^6e+44a^4b^5f\right)x}{a+bx^3}\right)dx}{18a^2b^9} \\
&= \frac{(be-3af)x^2}{2b^4}+\frac{fx^5}{5b^3}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} \\
&\quad +\frac{\left(b^3c-4ab^2d+7a^2be-10a^3f\right)x^2}{9ab^4\left(a+bx^3\right)}+\frac{\left(b^3c+5ab^2d-20a^2be+44a^3f\right)\int \frac{x}{a+bx^3}\,dx}{9ab^4}
\end{aligned}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-4ab^2d+7a^2be-10a^3f)\,x^2}{9ab^4\,(a+bx^3)} - \frac{(b^3c+5ab^2d-20a^2be+44a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{4/3}b^{13/3}} + \frac{(b^3c+5ab^2d-20a^2be+44a^3f)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{4/3}b^{13/3}}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-4ab^2d+7a^2be-10a^3f)\,x^2}{9ab^4\,(a+bx^3)} - \frac{(b^3c+5ab^2d-20a^2be+44a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{14/3}} + \frac{(b^3c+5ab^2d-20a^2be+44a^3f)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{4/3}b^{14/3}} + \frac{(b^3c+5ab^2d-20a^2be+44a^3f)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18ab^{13/3}}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-4ab^2d+7a^2be-10a^3f)\,x^2}{9ab^4\,(a+bx^3)} - \frac{(b^3c+5ab^2d-20a^2be+44a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{14/3}} + \frac{(b^3c+5ab^2d-20a^2be+44a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{14/3}} + \frac{(b^3c+5ab^2d-20a^2be+44a^3f)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{4/3}b^{14/3}}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-4ab^2d+7a^2be-10a^3f)\,x^2}{9ab^4\,(a+bx^3)} - \frac{(b^3c+5ab^2d-20a^2be+44a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{14/3}} - \frac{(b^3c+5ab^2d-20a^2be+44a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{14/3}} + \frac{(b^3c+5ab^2d-20a^2be+44a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{14/3}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.95

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= \frac{135b^{2/3}(be-3af)x^2 + 54b^{5/3}fx^5 - \frac{45b^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{(a+bx^3)^2} + \frac{30b^{2/3}(b^3c-4ab^2d+7a^2be-10a^3f)x^2}{a(a+bx^3)} - \frac{10\sqrt{3}(b^3c+5ab^2 \cdots}{\cdots}}{\cdots}$$

```
[In] Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (135*b^(2/3)*(b*e - 3*a*f)*x^2 + 54*b^(5/3)*f*x^5 - (45*b^(2/3)*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*x^2)/(a + b*x^3)^2 + (30*b^(2/3)*(b^3*c - 4*a*b^2*
d + 7*a^2*b*e - 10*a^3*f)*x^2)/(a*(a + b*x^3)) - (10*Sqrt[3]*(b^3*c + 5*a*b
^2*d - 20*a^2*b*e + 44*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/
a^(4/3) - (10*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3) + b^(
1/3)*x])/a^(4/3) + (5*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(270*b^(14/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.53

method	result
risch	$\frac{f x^5}{5b^3} - \frac{3x^2 a f}{2b^4} + \frac{e x^2}{2b^3} + \frac{-\frac{b\left(10 f a^3-7a^2 b e+4 a b^2 d-b^3 c\right)x^5}{9a}+\left(-\frac{17}{18} f a^3+\frac{11}{18} a^2 b e-\frac{5}{18} a b^2 d-\frac{1}{18} b^3 c\right)x^2}{b^4\left(b x^3+a\right)^2} + \sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{(44 \ldots}{\ldots}$
default	$-\frac{-\frac{b f x^5}{5}+\frac{(3af-be)x^2}{2}}{b^4} + \frac{\frac{-\frac{b\left(10 f a^3-7a^2 b e+4 a b^2 d-b^3 c\right)x^5}{9a}+\left(-\frac{17}{18} f a^3+\frac{11}{18} a^2 b e-\frac{5}{18} a b^2 d-\frac{1}{18} b^3 c\right)x^2}{\left(b x^3+a\right)^2} + \frac{\left(44 f a^3-20a^2 b e+5 a b^2 d+b^3 c\right)\left(\ln \ldots\right)}{\ldots}}{b^4}$

```
[In] int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*f*x^5/b^3-3/2/b^4*x^2*a*f+1/2/b^3*e*x^2+(-1/9*b*(10*a^3*f-7*a^2*b*e+4*a
*b^2*d-b^3*c)/a*x^5+(-17/18*f*a^3+11/18*a^2*b*e-5/18*a*b^2*d-1/18*b^3*c)*x^
2)/b^4/(b*x^3+a)^2+1/27/b^5/a*sum((44*a^3*f-20*a^2*b*e+5*a*b^2*d+b^3*c)/_R*
ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. 2(271) = 542.

Time = 0.29 (sec) , antiderivative size = 1224, normalized size of antiderivative = 3.87

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} \, dx = \text{Too large to display}$$

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/270*(54*a^2*b^5*f*x^11 + 27*(5*a^2*b^5*e - 11*a^3*b^4*f)*x^8 + 6*(5*a*b^
6*c - 20*a^2*b^5*d + 80*a^3*b^4*e - 176*a^4*b^3*f)*x^5 - 15*(a^2*b^5*c + 5*
a^3*b^4*d - 20*a^4*b^3*e + 44*a^5*b^2*f)*x^2 + 15*sqrt(1/3)*(a^3*b^4*c + 5*
a^4*b^3*d - 20*a^5*b^2*e + 44*a^6*b*f + (a*b^6*c + 5*a^2*b^5*d - 20*a^3*b^4
*e + 44*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 5*a^3*b^4*d - 20*a^4*b^3*e + 44*a^5
*b^2*f)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b
*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-
```

```
a*b^2)^(2/3)*x)/(b*x^3 + a)) + 5*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e + 44*a^
3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(a*b^4*c
 + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2
 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*((b^5*c + 5*a*b^4*d - 20*a^2*b
^3*e + 44*a^3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f
+ 2*(a*b^4*c + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)
*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6), 1/270*
(54*a^2*b^5*f*x^11 + 27*(5*a^2*b^5*e - 11*a^3*b^4*f)*x^8 + 6*(5*a*b^6*c - 2
0*a^2*b^5*d + 80*a^3*b^4*e - 176*a^4*b^3*f)*x^5 - 15*(a^2*b^5*c + 5*a^3*b^4
*d - 20*a^4*b^3*e + 44*a^5*b^2*f)*x^2 + 30*sqrt(1/3)*(a^3*b^4*c + 5*a^4*b^3
*d - 20*a^5*b^2*e + 44*a^6*b*f + (a*b^6*c + 5*a^2*b^5*d - 20*a^3*b^4*e + 44
*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 5*a^3*b^4*d - 20*a^4*b^3*e + 44*a^5*b^2*f)
*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqr
t(-(-a*b^2)^(1/3)/a)/b) + 5*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e + 44*a^3*b^2
*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(a*b^4*c + 5*
a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-
a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e
+ 44*a^3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(
a*b^4*c + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)*log(
b*x - (-a*b^2)^(1/3)))/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx=\text{Timed out}$$

```
[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.98

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{2\,(b^4c-4\,ab^3d+7\,a^2b^2e-10\,a^3bf)x^5-(ab^3c+5\,a^2b^2d-11\,a^3be+17\,a^4f)x^2}{18\,(ab^6x^6+2\,a^2b^5x^3+a^3b^4)} + \frac{2\,bfx^5+5\,(be-3\,af)x^2}{10\,b^4} + \frac{\sqrt{3}(b^3c+5\,ab^2d-20\,a^2be+44\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(b^3c+5\,ab^2d-20\,a^2be+44\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(b^3c+5\,ab^2d-20\,a^2be+44\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^5 - (a*b^3*c + 5*a
^2*b^2*d - 11*a^3*b*e + 17*a^4*f)*x^2)/(a*b^6*x^6 + 2*a^2*b^5*x^3 + a^3*b^4
) + 1/10*(2*b*f*x^5 + 5*(b*e - 3*a*f)*x^2)/b^4 + 1/27*sqrt(3)*(b^3*c + 5*a*
b^2*d - 20*a^2*b*e + 44*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)
^(1/3))/(a*b^5*(a/b)^(1/3)) + 1/54*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3
*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^5*(a/b)^(1/3)) - 1/27*(b^3*
c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*log(x + (a/b)^(1/3))/(a*b^5*(a/b)^(1
/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.14

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{\sqrt{3}(b^3c+5\,ab^2d-20\,a^2be+44\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{1}{3}}\,ab^4} - \frac{(b^3c+5\,ab^2d-20\,a^2be+44\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{1}{3}}\,ab^4} - \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-20\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+44\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^4} + \frac{2\,b^4cx^5-8\,ab^3dx^5+14\,a^2b^2ex^5-20\,a^3bfx^5-ab^3cx^2-5\,a^2b^2dx^2+11\,a^3bex^2-17\,a^4fx^2}{18\,(bx^3+a)^2ab^4} + \frac{2\,b^{12}fx^5+5\,b^{12}ex^2-15\,ab^{11}fx^2}{10\,b^{15}}$$

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a*b^4) - 1/54*(b^3*c +
5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/((-a*b^2)^(1/3)*a*b^4) - 1/27*(b^3*c*(-a/b)^(1/3) + 5*a*b^2*d*(-a/b)^(1/3)
 - 20*a^2*b*e*(-a/b)^(1/3) + 44*a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a^2*b^4) + 1/18*(2*b^4*c*x^5 - 8*a*b^3*d*x^5 + 14*a^2*b^2
*e*x^5 - 20*a^3*b*f*x^5 - a*b^3*c*x^2 - 5*a^2*b^2*d*x^2 + 11*a^3*b*e*x^2 -
17*a^4*f*x^2)/((b*x^3 + a)^2*a*b^4) + 1/10*(2*b^12*f*x^5 + 5*b^12*e*x^2 - 1
5*a*b^11*f*x^2)/b^15
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.93

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= x^2\left(\frac{e}{2\,b^3}-\frac{3\,a\,f}{2\,b^4}\right)-\frac{x^2\left(\frac{17\,f\,a^3}{18}-\frac{11\,e\,a^2\,b}{18}+\frac{5\,d\,a\,b^2}{18}+\frac{c\,b^3}{18}\right)-\frac{x^5\,\left(-10\,f\,a^3\,b+7\,e\,a^2\,b^2-4\,d\,a\,b^3+c\,b^4\right)}{9\,a}}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}$$
$$+\frac{f\,x^5}{5\,b^3}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(44\,f\,a^3-20\,e\,a^2\,b+5\,d\,a\,b^2+c\,b^3\right)}{27\,a^{4/3}\,b^{14/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(44\,f\,a^3-20\,e\,a^2\,b+5\,d\,a\,b^2+c\,b^3\right)}{27\,a^{4/3}\,b^{14/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(44\,f\,a^3-20\,e\,a^2\,b+5\,d\,a\,b^2+c\,b^3\right)}{27\,a^{4/3}\,b^{14/3}}$$

```
[In] int((x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^2*(e/(2*b^3) - (3*a*f)/(2*b^4)) - (x^2*((b^3*c)/18 + (17*a^3*f)/18 + (5*a
*b^2*d)/18 - (11*a^2*b*e)/18) - (x^5*(b^4*c + 7*a^2*b^2*e - 4*a*b^3*d - 10*
a^3*b*f))/(9*a))/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + (f*x^5)/(5*b^3) - (log
(b^(1/3)*x + a^(1/3))*(b^3*c + 44*a^3*f + 5*a*b^2*d - 20*a^2*b*e))/(27*a^(4
/3)*b^(14/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*
1i)/2 + 1/2)*(b^3*c + 44*a^3*f + 5*a*b^2*d - 20*a^2*b*e))/(27*a^(4/3)*b^(14
/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1
/2)*(b^3*c + 44*a^3*f + 5*a*b^2*d - 20*a^2*b*e))/(27*a^(4/3)*b^(14/3))
```

3.292 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 307

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx = \frac{(be-3af)x}{b^4}+\frac{fx^4}{4b^3}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2}+\frac{(b^3c-7ab^2d+13a^2be-19a^3f)\,x}{18ab^4\,(a+bx^3)} - \frac{(b^3c+2ab^2d-14a^2be+35a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{13/3}} + \frac{(b^3c+2ab^2d-14a^2be+35a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{13/3}} - \frac{(b^3c+2ab^2d-14a^2be+35a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{13/3}}$$

```
[Out] (-3*a*f+b*e)*x/b^4+1/4*f*x^4/b^3-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^4/(
b*x^3+a)^2+1/18*(-19*a^3*f+13*a^2*b*e-7*a*b^2*d+b^3*c)*x/a/b^4/(b*x^3+a)+1/
27*(35*a^3*f-14*a^2*b*e+2*a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(1
3/3)-1/54*(35*a^3*f-14*a^2*b*e+2*a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/a^(5/3)/b^(13/3)-1/27*(35*a^3*f-14*a^2*b*e+2*a*b^2*d+b^3*c)*
arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(13/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1842, 1872, 1425, 396, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$=\frac{x(-19a^3f+13a^2be-7ab^2d+b^3c)}{18ab^4(a+bx^3)}-\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^4(a+bx^3)^2}$$
$$-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(35a^3f-14a^2be+2ab^2d+b^3c)}{9\sqrt{3}a^{5/3}b^{13/3}}$$
$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(35a^3f-14a^2be+2ab^2d+b^3c)}{54a^{5/3}b^{13/3}}$$
$$+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(35a^3f-14a^2be+2ab^2d+b^3c)}{27a^{5/3}b^{13/3}}+\frac{x(be-3af)}{b^4}+\frac{fx^4}{4b^3}$$

```
[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x)/b^4 + (f*x^4)/(4*b^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*x)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 7*a*b^2*d + 13*a^2*b*e - 19*a^3*f)
*x)/(18*a*b^4*(a + b*x^3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(13/
3)) + ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x]
)/(27*a^(5/3)*b^(13/3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1425

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
```

```
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2} \\
&\quad -\frac{\int \frac{-a(b^3c-ab^2d+a^2be-a^3f)-6ab(b^2d-abe+a^2f)x^3-6ab^2(be-af)x^6-6ab^3fx^9}{(a+bx^3)^2}\,dx}{6ab^4} \\
&= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2}+\frac{(b^3c-7ab^2d+13a^2be-19a^3f)\,x}{18ab^4\,(a+bx^3)} \\
&\quad +\frac{\int \frac{2ab^3(b^3c+2ab^2d-5a^2be+8a^3f)+18a^2b^4(be-2af)x^3+18a^2b^5fx^6}{a+bx^3}\,dx}{18a^2b^7} \\
&= \frac{fx^4}{4b^3}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2}+\frac{(b^3c-7ab^2d+13a^2be-19a^3f)\,x}{18ab^4\,(a+bx^3)} \\
&\quad +\frac{\int \frac{8ab^4(b^3c+2ab^2d-5a^2be+8a^3f)-(72a^3b^5f-72a^2b^5(be-2af))x^3}{a+bx^3}\,dx}{72a^2b^8} \\
&= \frac{(be-3af)x}{b^4}+\frac{fx^4}{4b^3}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2} \\
&\quad +\frac{(b^3c-7ab^2d+13a^2be-19a^3f)\,x}{18ab^4\,(a+bx^3)}+\frac{(b^3c+2ab^2d-14a^2be+35a^3f)\int \frac{1}{a+bx^3}\,dx}{9ab^4} \\
&= \frac{(be-3af)x}{b^4}+\frac{fx^4}{4b^3}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2} \\
&\quad +\frac{(b^3c-7ab^2d+13a^2be-19a^3f)\,x}{18ab^4\,(a+bx^3)} \\
&\quad +\frac{(b^3c+2ab^2d-14a^2be+35a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{5/3}b^4} \\
&\quad +\frac{(b^3c+2ab^2d-14a^2be+35a^3f)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{5/3}b^4}
\end{aligned}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-7ab^2d+13a^2be-19a^3f)\,x}{18ab^4\,(a+bx^3)} + \frac{(b^3c+2ab^2d-14a^2be+35a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{13/3}} - \frac{(b^3c+2ab^2d-14a^2be+35a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{5/3}b^{13/3}} + \frac{(b^3c+2ab^2d-14a^2be+35a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{4/3}b^4}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-7ab^2d+13a^2be-19a^3f)\,x}{18ab^4\,(a+bx^3)} + \frac{(b^3c+2ab^2d-14a^2be+35a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{13/3}} - \frac{(b^3c+2ab^2d-14a^2be+35a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{13/3}} + \frac{(b^3c+2ab^2d-14a^2be+35a^3f)\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{5/3}b^{13/3}}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4\,(a+bx^3)^2} + \frac{(b^3c-7ab^2d+13a^2be-19a^3f)\,x}{18ab^4\,(a+bx^3)} - \frac{(b^3c+2ab^2d-14a^2be+35a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{13/3}} + \frac{(b^3c+2ab^2d-14a^2be+35a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{13/3}} - \frac{(b^3c+2ab^2d-14a^2be+35a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{13/3}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.96

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= \frac{108\sqrt[3]{b}(be-3af)x+27b^{4/3}fx^4-\frac{18\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)x}{(a+bx^3)^2}+\frac{6\sqrt[3]{b}(b^3c-7ab^2d+13a^2be-19a^3f)x}{a(a+bx^3)}-\frac{4\sqrt{3}(b^3c+2ab^2d-14a}{}}{}$$

```
[In] Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (108*b^(1/3)*(b*e - 3*a*f)*x + 27*b^(4/3)*f*x^4 - (18*b^(1/3)*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*x)/(a + b*x^3)^2 + (6*b^(1/3)*(b^3*c - 7*a*b^2*d + 1
3*a^2*b*e - 19*a^3*f)*x)/(a*(a + b*x^3)) - (4*Sqrt[3]*(b^3*c + 2*a*b^2*d -
14*a^2*b*e + 35*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3)
 + (4*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])
/a^(5/3) - (2*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(108*b^(13/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.52

method	result
risch	$\frac{f x^4}{4b^3}-\frac{3xaf}{b^4}+\frac{ex}{b^3}+\frac{-\frac{b\left(19f a^3-13a^2be+7a b^2d-b^3c\right)x^4}{18a}+\left(-\frac{8}{9}f a^3+\frac{5}{9}a^2be-\frac{2}{9}a b^2d-\frac{1}{9}b^3c\right)x}{b^4(b x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(35f a^3-14\right.}{}}{27b^5}$
default	$-\frac{-\frac{1}{4}bf x^4+3afx-bex}{b^4}+\frac{-\frac{b\left(19f a^3-13a^2be+7a b^2d-b^3c\right)x^4}{18a}+\left(-\frac{8}{9}f a^3+\frac{5}{9}a^2be-\frac{2}{9}a b^2d-\frac{1}{9}b^3c\right)x}{\left(b x^3+a\right)^2}+\frac{\left(35f a^3-14a^2be+2a b^2d+b^3c\right)\left(\frac{\ln\left(x+\right.}{3b\left(\right.}\right.}{}}{b^4}$

```
[In] int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*f*x^4/b^3-3/b^4*x*a*f+1/b^3*e*x+(-1/18*b*(19*a^3*f-13*a^2*b*e+7*a*b^2*d
-b^3*c)/a*x^4+(-8/9*f*a^3+5/9*a^2*b*e-2/9*a*b^2*d-1/9*b^3*c)*x)/b^4/(b*x^3+
a)^2+1/27/b^5/a*sum((35*a^3*f-14*a^2*b*e+2*a*b^2*d+b^3*c)/_R^2*ln(x-_R),_R=
RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(264) = 528$.

Time = 0.31 (sec) , antiderivative size = 1213, normalized size of antiderivative = 3.95

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/108*(27*a^3*b^4*f*x^10 + 54*(2*a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 3*(2*a^2*b
^5*c - 14*a^3*b^4*d + 98*a^4*b^3*e - 245*a^5*b^2*f)*x^4 + 6*sqrt(1/3)*(a^3*
b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*e + 35*a^6*b*f + (a*b^6*c + 2*a^2*b^5*d -
14*a^3*b^4*e + 35*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 2*a^3*b^4*d - 14*a^4*b^3*
e + 35*a^5*b^2*f)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1
/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)
*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 2*((b^5*c + 2*a*b^4*d - 14*a^2*b^3*
e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^5*f + 2
*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(2/3)*log
(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*((b^5*c + 2*a*b^4*d - 14*
a^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a
^5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(
2/3)*log(a*b*x + (a^2*b)^(2/3)) - 12*(a^3*b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*
e + 35*a^6*b*f)*x)/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5), 1/108*(27*a^3*b
^4*f*x^10 + 54*(2*a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 3*(2*a^2*b^5*c - 14*a^3*b^
4*d + 98*a^4*b^3*e - 245*a^5*b^2*f)*x^4 + 12*sqrt(1/3)*(a^3*b^4*c + 2*a^4*b
^3*d - 14*a^5*b^2*e + 35*a^6*b*f + (a*b^6*c + 2*a^2*b^5*d - 14*a^3*b^4*e +
35*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 2*a^3*b^4*d - 14*a^4*b^3*e + 35*a^5*b^2*
f)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)
^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*((b^5*c + 2*a*b^4*d - 14*a^2*b^3*e
 + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^5*f + 2*
(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*((b^5*c + 2*a*b^4*d - 14*a
^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^
5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(2
/3)*log(a*b*x + (a^2*b)^(2/3)) - 12*(a^3*b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*e
 + 35*a^6*b*f)*x)/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.99

$$\begin{aligned}
&\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx \\
&= \frac{(b^4c-7\,ab^3d+13\,a^2b^2e-19\,a^3bf)x^4-2\,(ab^3c+2\,a^2b^2d-5\,a^3be+8\,a^4f)x}{18\,(ab^6x^6+2\,a^2b^5x^3+a^3b^4)} \\
&\quad+\frac{bfx^4+4\,(be-3\,af)x}{4\,b^4}+\frac{\sqrt{3}(b^3c+2\,ab^2d-14\,a^2be+35\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\
&\quad-\frac{(b^3c+2\,ab^2d-14\,a^2be+35\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\
&\quad+\frac{(b^3c+2\,ab^2d-14\,a^2be+35\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}
\end{aligned}$$

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/18*((b^4*c - 7*a*b^3*d + 13*a^2*b^2*e - 19*a^3*b*f)*x^4 - 2*(a*b^3*c + 2*
a^2*b^2*d - 5*a^3*b*e + 8*a^4*f)*x)/(a*b^6*x^6 + 2*a^2*b^5*x^3 + a^3*b^4) +
 1/4*(b*f*x^4 + 4*(b*e - 3*a*f)*x)/b^4 + 1/27*sqrt(3)*(b^3*c + 2*a*b^2*d -
14*a^2*b*e + 35*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/
(a*b^5*(a/b)^(2/3)) - 1/54*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*log(
x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^5*(a/b)^(2/3)) + 1/27*(b^3*c + 2*a*
b^2*d - 14*a^2*b*e + 35*a^3*f)*log(x + (a/b)^(1/3))/(a*b^5*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.02

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= -\frac{\sqrt{3}(b^3c+2\,ab^2d-14\,a^2be+35\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}ab^3} - \frac{(b^3c+2\,ab^2d-14\,a^2be+35\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}ab^3} - \frac{(b^3c+2\,ab^2d-14\,a^2be+35\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^4} + \frac{b^4cx^4-7\,ab^3dx^4+13\,a^2b^2ex^4-19\,a^3bfx^4-2\,ab^3cx-4\,a^2b^2dx+10\,a^3bex-16\,a^4fx}{18\left(bx^3+a\right)^2ab^4} + \frac{b^9fx^4+4\,b^9ex-12\,ab^8fx}{4\,b^{12}}$$

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^3) - 1/54*(b^3*c +
 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)
)/((-a*b^2)^(2/3)*a*b^3) - 1/27*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)
*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^4) + 1/18*(b^4*c*x^4 - 7*a*
b^3*d*x^4 + 13*a^2*b^2*e*x^4 - 19*a^3*b*f*x^4 - 2*a*b^3*c*x - 4*a^2*b^2*d*x
 + 10*a^3*b*e*x - 16*a^4*f*x)/((b*x^3 + a)^2*a*b^4) + 1/4*(b^9*f*x^4 + 4*b^
9*e*x - 12*a*b^8*f*x)/b^12
```

Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.94

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= x\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)-\frac{x\left(\frac{8\,f\,a^3}{9}-\frac{5\,e\,a^2\,b}{9}+\frac{2\,d\,a\,b^2}{9}+\frac{c\,b^3}{9}\right)-\frac{x^4\,\left(-19\,f\,a^3\,b+13\,e\,a^2\,b^2-7\,d\,a\,b^3+c\,b^4\right)}{18\,a}}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}$$
$$+\frac{f\,x^4}{4\,b^3}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(35\,f\,a^3-14\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{27\,a^{5/3}\,b^{13/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(35\,f\,a^3-14\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{27\,a^{5/3}\,b^{13/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(35\,f\,a^3-14\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{27\,a^{5/3}\,b^{13/3}}$$

```
[In] int((x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x*(e/b^3 - (3*a*f)/b^4) - (x*((b^3*c)/9 + (8*a^3*f)/9 + (2*a*b^2*d)/9 - (5*
a^2*b*e)/9) - (x^4*(b^4*c + 13*a^2*b^2*e - 7*a*b^3*d - 19*a^3*b*f))/(18*a))
/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + (f*x^4)/(4*b^3) + (log(b^(1/3)*x + a^(
1/3))*(b^3*c + 35*a^3*f + 2*a*b^2*d - 14*a^2*b*e))/(27*a^(5/3)*b^(13/3)) +
(log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^
3*c + 35*a^3*f + 2*a*b^2*d - 14*a^2*b*e))/(27*a^(5/3)*b^(13/3)) - (log(3^(1
/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c + 35*
a^3*f + 2*a*b^2*d - 14*a^2*b*e))/(27*a^(5/3)*b^(13/3))
```

3.293 $\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 301

$$\begin{aligned}&\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx\\&=\frac{fx^2}{2b^3}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2}+\frac{(2b^3c+ab^2d-4a^2be+7a^3f)\,x^2}{9a^2b^3\,(a+bx^3)}\\&-\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{11/3}}\\&-\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{11/3}}\\&+\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{11/3}}\end{aligned}$$

```
[Out] 1/2*f*x^2/b^3+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a/b^3/(b*x^3+a)^2+1/9*
(7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^3*c)*x^2/a^2/b^3/(b*x^3+a)-1/27*(-20*a^3*f+5
*a^2*b*e+a*b^2*d+2*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(11/3)+1/54*(-20*
a^3*f+5*a^2*b*e+a*b^2*d+2*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
a^(7/3)/b^(11/3)-1/27*(-20*a^3*f+5*a^2*b*e+a*b^2*d+2*b^3*c)*arctan(1/3*(a^(
1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(11/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1842, 1608, 1496, 470, 298, 31, 648, 631, 210, 642}

$$\begin{aligned}\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= \frac{x^2(7a^3f-4a^2be+ab^2d+2b^3c)}{9a^2b^3(a+bx^3)}+\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6ab^3(a+bx^3)^2}\\ &\quad-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-20a^3f+5a^2be+ab^2d+2b^3c)}{9\sqrt{3}a^{7/3}b^{11/3}}\\ &\quad+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-20a^3f+5a^2be+ab^2d+2b^3c)}{54a^{7/3}b^{11/3}}\\ &\quad-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-20a^3f+5a^2be+ab^2d+2b^3c)}{27a^{7/3}b^{11/3}}+\frac{fx^2}{2b^3}\end{aligned}$$

```
[In] Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (f*x^2)/(2*b^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a*b^3*(a + b
*x^3)^2) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x^2)/(9*a^2*b^3*(a +
b*x^3)) - ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(11/3)) - ((2*b^3*c + a*
b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(11/3
)) + ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1496

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-d)^((m - Mod[m, n])/n - 1)*(c*d^2
 - b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*((d + e*x^n)^(q + 1)/(n*e^(2*p + (m -
 Mod[m, n])/n)*(q + 1))), x] + Dist[1/(n*e^(2*p + (m - Mod[m, n])/n)*(q + 1
)), Int[x^Mod[m, n]*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^n)
)*(n*e^(2*p + (m - Mod[m, n])/n)*(q + 1)*x^(m - Mod[m, n])*(a + b*x^n + c*x
^(2*n))^p - (-d)^((m - Mod[m, n])/n - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d*(Mod[
m, n] + 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))], x], x], x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGt
Q[p, 0] && ILtQ[q, -1] && IGtQ[m, 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
```

```
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2}-\frac{\int \frac{-2b(2b^3c+ab^2d-a^2be+a^3f)x-6ab^2(be-af)x^4-6ab^3fx^7}{(a+bx^3)^2}\,dx}{6ab^4}\\
&= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2}-\frac{\int \frac{x\left(-2b(2b^3c+ab^2d-a^2be+a^3f)-6ab^2(be-af)x^3-6ab^3fx^6\right)}{(a+bx^3)^2}\,dx}{6ab^4}\\
&= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2}+\frac{(2b^3c+ab^2d-4a^2be+7a^3f)\,x^2}{9a^2b^3\,(a+bx^3)}\\
&\quad+\frac{\int \frac{x\left(2b^3\left(\frac{2b^3c}{a}+b^2d+5abe-11a^2f\right)+18ab^4fx^3\right)}{a+bx^3}\,dx}{18ab^6}\\
&= \frac{fx^2}{2b^3}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2}+\frac{(2b^3c+ab^2d-4a^2be+7a^3f)\,x^2}{9a^2b^3\,(a+bx^3)}\\
&\quad+\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\int \frac{x}{a+bx^3}\,dx}{9a^2b^3}\\
&= \frac{fx^2}{2b^3}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2}+\frac{(2b^3c+ab^2d-4a^2be+7a^3f)\,x^2}{9a^2b^3\,(a+bx^3)}\\
&\quad-\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{7/3}b^{10/3}}\\
&\quad+\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{7/3}b^{10/3}}
\end{aligned}$$

$$
=\frac{fx^2}{2b^3}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2}+\frac{(2b^3c+ab^2d-4a^2be+7a^3f)\,x^2}{9a^2b^3\,(a+bx^3)}
-\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{11/3}}
+\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{7/3}b^{11/3}}
+\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^2b^{10/3}}
$$

$$
=\frac{fx^2}{2b^3}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2}+\frac{(2b^3c+ab^2d-4a^2be+7a^3f)\,x^2}{9a^2b^3\,(a+bx^3)}
-\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{11/3}}
+\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{11/3}}
+\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{7/3}b^{11/3}}
$$

$$
=\frac{fx^2}{2b^3}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2}+\frac{(2b^3c+ab^2d-4a^2be+7a^3f)\,x^2}{9a^2b^3\,(a+bx^3)}
-\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{11/3}}
-\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{11/3}}
+\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{11/3}}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.94

$$
\int\frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx
$$

$$
=\frac{27b^{2/3}fx^2+\frac{9b^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{a(a+bx^3)^2}+\frac{6b^{2/3}(2b^3c+ab^2d-4a^2be+7a^3f)x^2}{a^2(a+bx^3)}-\frac{2\sqrt{3}(2b^3c+ab^2d+5a^2be-20a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}}}{54b^{11/3}}
$$

```
[In] Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (27*b^(2/3)*f*x^2 + (9*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a*
(a + b*x^3)^2) + (6*b^(2/3)*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x^2)/
(a^2*(a + b*x^3)) - (2*Sqrt[3]*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(7/3) - (2*(2*b^3*c + a*b^2*d
 + 5*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(7/3) + ((2*b^3*c + a*
b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/a^(7/3))/(54*b^(11/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.50

method	result
risch	$\frac{f x^2}{2b^3}+\frac{\frac{b\left(7f a^3-4a^2be+a b^2d+2b^3c\right)x^5}{9a^2}+\frac{\left(11f a^3-5a^2be-a b^2d+7b^3c\right)x^2}{18a}}{b^3\left(b x^3+a\right)^2}-\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^3+a\right)}\frac{\left(20f a^3-5a^2be-a b^2d-2b^3c\right)\ln\left(x-\_R\right)}{\_R}}{27b^4a^2}$
default	$\frac{f x^2}{2b^3}-\frac{\frac{-\frac{b\left(7f a^3-4a^2be+a b^2d+2b^3c\right)x^5}{9a^2}-\frac{\left(11f a^3-5a^2be-a b^2d+7b^3c\right)x^2}{18a}}{\left(b x^3+a\right)^2}+\frac{\left(20f a^3-5a^2be-a b^2d-2b^3c\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x\cdots\right.}{6b\left(\frac{a}{b}\right)\cdots}\cdots\right)}{9a^2}}{b^3}$

```
[In] int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*x^2/b^3+(1/9*b*(7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^3*c)/a^2*x^5+1/18*(11*a
^3*f-5*a^2*b*e-a*b^2*d+7*b^3*c)/a*x^2)/b^3/(b*x^3+a)^2-1/27/b^4/a^2*sum((20
*a^3*f-5*a^2*b*e-a*b^2*d-2*b^3*c)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(258) = 516$.

Time = 0.29 (sec) , antiderivative size = 1158, normalized size of antiderivative = 3.85

$$\int \frac{x(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} \, dx = \text{Too large to display}$$

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(27*a^3*b^4*f*x^8 + 6*(2*a*b^6*c + a^2*b^5*d - 4*a^3*b^4*e + 16*a^4*b
^3*f)*x^5 + 3*(7*a^2*b^5*c - a^3*b^4*d - 5*a^4*b^3*e + 20*a^5*b^2*f)*x^2 -
3*sqrt(1/3)*(2*a^3*b^4*c + a^4*b^3*d + 5*a^5*b^2*e - 20*a^6*b*f + (2*a*b^6*
c + a^2*b^5*d + 5*a^3*b^4*e - 20*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c + a^3*b^4*
d + 5*a^4*b^3*e - 20*a^5*b^2*f)*x^3)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3
- a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(
a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((2*b^5*c + a*b^4*d + 5
*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e - 20*a
^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^2)^(2
/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 2*((2*b^5*c + a*b^4*
d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e -
 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^
2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5),
 1/54*(27*a^3*b^4*f*x^8 + 6*(2*a*b^6*c + a^2*b^5*d - 4*a^3*b^4*e + 16*a^4*b
^3*f)*x^5 + 3*(7*a^2*b^5*c - a^3*b^4*d - 5*a^4*b^3*e + 20*a^5*b^2*f)*x^2 -
6*sqrt(1/3)*(2*a^3*b^4*c + a^4*b^3*d + 5*a^5*b^2*e - 20*a^6*b*f + (2*a*b^6*
c + a^2*b^5*d + 5*a^3*b^4*e - 20*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c + a^3*b^4*
d + 5*a^4*b^3*e - 20*a^5*b^2*f)*x^3)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(1/3
)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + ((2*b^5*c + a*b^4*d +
5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e - 20*
a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^2)^(
2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 2*((2*b^5*c + a*b^4
*d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e
- 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b
^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5)
]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= \frac{2\,(2\,b^4c+ab^3d-4\,a^2b^2e+7\,a^3bf)x^5+(7\,ab^3c-a^2b^2d-5\,a^3be+11\,a^4f)x^2}{18\,(a^2b^5x^6+2\,a^3b^4x^3+a^4b^3)} \\ &\quad+\frac{fx^2}{2\,b^3}+\frac{\sqrt{3}(2\,b^3c+ab^2d+5\,a^2be-20\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &\quad+\frac{(2\,b^3c+ab^2d+5\,a^2be-20\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &\quad-\frac{(2\,b^3c+ab^2d+5\,a^2be-20\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}\end{aligned}$$

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/18*(2*(2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7*a^3*b*f)*x^5 + (7*a*b^3*c - a^
2*b^2*d - 5*a^3*b*e + 11*a^4*f)*x^2)/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3
) + 1/2*f*x^2/b^3 + 1/27*sqrt(3)*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^4*(a/b)^(1/3))
+ 1/54*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*log(x^2 - x*(a/b)^(1/3) +
 (a/b)^(2/3))/(a^2*b^4*(a/b)^(1/3)) - 1/27*(2*b^3*c + a*b^2*d + 5*a^2*b*e -
 20*a^3*f)*log(x + (a/b)^(1/3))/(a^2*b^4*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.11

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$=\frac{fx^2}{2\,b^3}+\frac{\sqrt{3}(2\,b^3c+ab^2d+5\,a^2be-20\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}a^2b^3}$$
$$-\frac{(2\,b^3c+ab^2d+5\,a^2be-20\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}a^2b^3}$$
$$-\frac{\left(2\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-20\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^3}$$
$$+\frac{4\,b^4cx^5+2\,ab^3dx^5-8\,a^2b^2ex^5+14\,a^3bfx^5+7\,ab^3cx^2-a^2b^2dx^2-5\,a^3bex^2+11\,a^4fx^2}{18\,(bx^3+a)^2a^2b^3}$$

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b^3 + 1/27*sqrt(3)*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*arc
tan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^2*b^3)
 - 1/54*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*b^3) - 1/27*(2*b^3*c*(-a/b)^(1/3) + a*
b^2*d*(-a/b)^(1/3) + 5*a^2*b*e*(-a/b)^(1/3) - 20*a^3*f*(-a/b)^(1/3))*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3) + 1/18*(4*b^4*c*x^5 + 2*a*b^3*d
*x^5 - 8*a^2*b^2*e*x^5 + 14*a^3*b*f*x^5 + 7*a*b^3*c*x^2 - a^2*b^2*d*x^2 - 5
*a^3*b*e*x^2 + 11*a^4*f*x^2)/((b*x^3 + a)^2*a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.93

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$=\frac{\frac{x^2\,(11\,f\,a^3-5\,e\,a^2\,b-d\,a\,b^2+7\,c\,b^3)}{18\,a}+\frac{x^5\,(7\,f\,a^3\,b-4\,e\,a^2\,b^2+d\,a\,b^3+2\,c\,b^4)}{9\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{f\,x^2}{2\,b^3}$$
$$-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-20\,f\,a^3+5\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{7/3}\,b^{11/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-20\,f\,a^3+5\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{7/3}\,b^{11/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-20\,f\,a^3+5\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{7/3}\,b^{11/3}}$$

```
[In] int((x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] ((x^2*(7*b^3*c + 11*a^3*f - a*b^2*d - 5*a^2*b*e))/(18*a) + (x^5*(2*b^4*c -
4*a^2*b^2*e + a*b^3*d + 7*a^3*b*f))/(9*a^2))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x
^3) + (f*x^2)/(2*b^3) - (log(b^(1/3)*x + a^(1/3))*(2*b^3*c - 20*a^3*f + a*b
^2*d + 5*a^2*b*e))/(27*a^(7/3)*b^(11/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1
/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*b^3*c - 20*a^3*f + a*b^2*d + 5*a
^2*b*e))/(27*a^(7/3)*b^(11/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^
(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*b^3*c - 20*a^3*f + a*b^2*d + 5*a^2*b*e))/(
27*a^(7/3)*b^(11/3))
```

3.294 $\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 27, antiderivative size = 292

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3} dx &= \frac{fx}{b^3}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6ab^3(a+bx^3)^2}\\&\quad+\frac{(5b^3c+ab^2d-7a^2be+13a^3f)\,x}{18a^2b^3(a+bx^3)}\\&\quad-\frac{(5b^3c+ab^2d+2a^2be-14a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{10/3}}\\&\quad+\frac{(5b^3c+ab^2d+2a^2be-14a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{10/3}}\\&\quad-\frac{(5b^3c+ab^2d+2a^2be-14a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{10/3}}\end{aligned}$$

```
[Out] f*x/b^3+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a/b^3/(b*x^3+a)^2+1/18*(13*a^3
*f-7*a^2*b*e+a*b^2*d+5*b^3*c)*x/a^2/b^3/(b*x^3+a)+1/27*(-14*a^3*f+2*a^2*b*e
+a*b^2*d+5*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(10/3)-1/54*(-14*a^3*f+2*
a^2*b*e+a*b^2*d+5*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/
b^(10/3)-1/27*(-14*a^3*f+2*a^2*b*e+a*b^2*d+5*b^3*c)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1872, 1423, 396, 206, 31, 648, 631, 210, 642}

$$\begin{aligned} &\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3}\,dx \\ &= \frac{x(13a^3f-7a^2be+ab^2d+5b^3c)}{18a^2b^3(a+bx^3)}+\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6ab^3(a+bx^3)^2} \\ &\quad -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-14a^3f+2a^2be+ab^2d+5b^3c)}{9\sqrt{3}a^{8/3}b^{10/3}} \\ &\quad -\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-14a^3f+2a^2be+ab^2d+5b^3c)}{54a^{8/3}b^{10/3}} \\ &\quad +\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-14a^3f+2a^2be+ab^2d+5b^3c)}{27a^{8/3}b^{10/3}}+\frac{fx}{b^3} \end{aligned}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x]
```

```
[Out] (f*x)/b^3 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a*b^3*(a + b*x^3)^2)
 + ((5*b^3*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(18*a^2*b^3*(a + b*x^3))
- ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(10/3)) + ((5*b^3*c + a*b^2*d +
2*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(10/3)) - ((5
*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(54*a^(8/3)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Sim
p[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
```

```
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2}-\frac{\int\frac{-5b^3c-ab^2d+a^2be-a^3f-6ab(be-af)x^3-6ab^2fx^6}{\left(a+bx^3\right)^2}\,dx}{6ab^3}\\
&= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2}+\frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)}\\
&\quad+\frac{\int\frac{2b^2\left(5b^3c+ab^2d+2a^2be-5a^3f\right)+18a^2b^3fx^3}{a+bx^3}\,dx}{18a^2b^5}\\
&= \frac{fx}{b^3}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2}+\frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)}\\
&\quad+\frac{\left(5b^3c+ab^2d+2a^2be-14a^3f\right)\int\frac{1}{a+bx^3}\,dx}{9a^2b^3}\\
&= \frac{fx}{b^3}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2}+\frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)}\\
&\quad+\frac{\left(5b^3c+ab^2d+2a^2be-14a^3f\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{8/3}b^3}\\
&\quad+\frac{\left(5b^3c+ab^2d+2a^2be-14a^3f\right)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{8/3}b^3}\\
&= \frac{fx}{b^3}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2}+\frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)}\\
&\quad+\frac{\left(5b^3c+ab^2d+2a^2be-14a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{10/3}}\\
&\quad-\frac{\left(5b^3c+ab^2d+2a^2be-14a^3f\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}b^{10/3}}\\
&\quad+\frac{\left(5b^3c+ab^2d+2a^2be-14a^3f\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{7/3}b^3}
\end{aligned}
$$

$$= \frac{fx}{b^3} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{6ab^3\,(a+bx^3)^2} + \frac{(5b^3c + ab^2d - 7a^2be + 13a^3f)\,x}{18a^2b^3\,(a+bx^3)} + \frac{(5b^3c + ab^2d + 2a^2be - 14a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{10/3}} - \frac{(5b^3c + ab^2d + 2a^2be - 14a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{10/3}} + \frac{(5b^3c + ab^2d + 2a^2be - 14a^3f)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{8/3}b^{10/3}}$$

$$= \frac{fx}{b^3} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{6ab^3\,(a+bx^3)^2} + \frac{(5b^3c + ab^2d - 7a^2be + 13a^3f)\,x}{18a^2b^3\,(a+bx^3)} - \frac{(5b^3c + ab^2d + 2a^2be - 14a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{10/3}} + \frac{(5b^3c + ab^2d + 2a^2be - 14a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{10/3}} - \frac{(5b^3c + ab^2d + 2a^2be - 14a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{10/3}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3}\,dx$$

$$= \frac{54\sqrt[3]{b}fx + \frac{9\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)x}{a(a+bx^3)^2} + \frac{3\sqrt[3]{b}(5b^3c+ab^2d-7a^2be+13a^3f)x}{a^2(a+bx^3)} - \frac{2\sqrt{3}(5b^3c+ab^2d+2a^2be-14a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}}}{54b^{10/3}}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x]
```

```
[Out] (54*b^(1/3)*f*x + (9*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a*(a +
 b*x^3)^2) + (3*b^(1/3)*(5*b^3*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(a^2*
(a + b*x^3)) - (2*Sqrt[3]*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*ArcTan
[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(8/3) + (2*(5*b^3*c + a*b^2*d + 2*
a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) - ((5*b^3*c + a*b^2*d
 + 2*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^
(8/3))/(54*b^(10/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.50

method	result
risch	$\frac{fx}{b^3}+\frac{\frac{b\left(13f\,a^3-7a^2be+a\,b^2d+5b^3c\right)x^4}{18a^2}+\frac{\left(5f\,a^3-2a^2be-a\,b^2d+4b^3c\right)x}{9a}}{b^3\left(b\,x^3+a\right)^2}-\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^3+a\right)}\frac{\left(14f\,a^3-2a^2be-a\,b^2d-5b^3c\right)\ln\left(x-\_R\right)}{\_R^2}}{27b^4a^2}$
default	$\frac{fx}{b^3}-\frac{\frac{-\frac{b\left(13f\,a^3-7a^2be+a\,b^2d+5b^3c\right)x^4}{18a^2}-\frac{\left(5f\,a^3-2a^2be-a\,b^2d+4b^3c\right)x}{9a}}{\left(b\,x^3+a\right)^2}+\frac{\left(14f\,a^3-2a^2be-a\,b^2d-5b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\ldots\right.\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{9a^2}}{b^3}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f*x/b^3+(1/18*b*(13*a^3*f-7*a^2*b*e+a*b^2*d+5*b^3*c)/a^2*x^4+1/9*(5*a^3*f-2
*a^2*b*e-a*b^2*d+4*b^3*c)/a*x)/b^3/(b*x^3+a)^2-1/27/b^4/a^2*sum((14*a^3*f-2
*a^2*b*e-a*b^2*d-5*b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(251) = 502.

Time = 0.28 (sec) , antiderivative size = 1184, normalized size of antiderivative = 4.05

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(54*a^4*b^3*f*x^7 + 3*(5*a^2*b^5*c + a^3*b^4*d - 7*a^4*b^3*e + 49*a^5
*b^2*f)*x^4 - 3*sqrt(1/3)*(5*a^3*b^4*c + a^4*b^3*d + 2*a^5*b^2*e - 14*a^6*b
*f + (5*a*b^6*c + a^2*b^5*d + 2*a^3*b^4*e - 14*a^4*b^3*f)*x^6 + 2*(5*a^2*b^
5*c + a^3*b^4*d + 2*a^4*b^3*e - 14*a^5*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/b)*l
og((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2
*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - ((5*
b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*d
 + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2*e - 14*a^4*b
```

```
*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a)
+ 2*((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a
^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2*e -
14*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 6*(4*a^3*b^4*
c - a^4*b^3*d - 2*a^5*b^2*e + 14*a^6*b*f)*x)/(a^4*b^6*x^6 + 2*a^5*b^5*x^3 +
 a^6*b^4), 1/54*(54*a^4*b^3*f*x^7 + 3*(5*a^2*b^5*c + a^3*b^4*d - 7*a^4*b^3*
e + 49*a^5*b^2*f)*x^4 + 6*sqrt(1/3)*(5*a^3*b^4*c + a^4*b^3*d + 2*a^5*b^2*e
- 14*a^6*b*f + (5*a*b^6*c + a^2*b^5*d + 2*a^3*b^4*e - 14*a^4*b^3*f)*x^6 + 2
*(5*a^2*b^5*c + a^3*b^4*d + 2*a^4*b^3*e - 14*a^5*b^2*f)*x^3)*sqrt(-(-a^2*b)
^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-
a^2*b)^(1/3)/b)/a^2) - ((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^
6 + 5*a^2*b^3*c + a^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3
*d + 2*a^3*b^2*e - 14*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(
2/3)*x - (-a^2*b)^(1/3)*a) + 2*((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b
^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c +
 a^2*b^3*d + 2*a^3*b^2*e - 14*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^
2*b)^(2/3)) + 6*(4*a^3*b^4*c - a^4*b^3*d - 2*a^5*b^2*e + 14*a^6*b*f)*x)/(a^
4*b^6*x^6 + 2*a^5*b^5*x^3 + a^6*b^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^3 + ex^6 + fx^9}{(a + bx^3)^3} \, dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3}\,dx = \frac{(5\,b^4c+ab^3d-7\,a^2b^2e+13\,a^3bf)x^4+2\,(4\,ab^3c-a^2b^2d-2\,a^3be+5\,a^4f)x}{18\,(a^2b^5x^6+2\,a^3b^4x^3+a^4b^3)} + \frac{fx}{b^3} + \frac{\sqrt{3}(5\,b^3c+ab^2d+2\,a^2be-14\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(5\,b^3c+ab^2d+2\,a^2be-14\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(5\,b^3c+ab^2d+2\,a^2be-14\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((5*b^4*c + a*b^3*d - 7*a^2*b^2*e + 13*a^3*b*f)*x^4 + 2*(4*a*b^3*c - a
^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)
+ f*x/b^3 + 1/27*sqrt(3)*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*arctan(
1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^4*(a/b)^(2/3)) - 1/54*(
5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
2/3))/(a^2*b^4*(a/b)^(2/3)) + 1/27*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*
f)*log(x + (a/b)^(1/3))/(a^2*b^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3}\,dx$$
$$= \frac{fx}{b^3} - \frac{\sqrt{3}(5\,b^3c+ab^2d+2\,a^2be-14\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}a^2b^2} - \frac{(5\,b^3c+ab^2d+2\,a^2be-14\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{2}{3}}a^2b^2} - \frac{(5\,b^3c+ab^2d+2\,a^2be-14\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^3} + \frac{5\,b^4cx^4+ab^3dx^4-7\,a^2b^2ex^4+13\,a^3bfx^4+8\,ab^3cx-2\,a^2b^2dx-4\,a^3bex+10\,a^4fx}{18\,\left(bx^3+a\right)^2a^2b^3}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
[Out] f*x/b^3 - 1/27*sqrt(3)*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*arctan(1/
3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b^2) - 1/5
4*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a
/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b^2) - 1/27*(5*b^3*c + a*b^2*d + 2*a^2*b*e -
 14*a^3*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3) + 1/18*(5*b^4*
c*x^4 + a*b^3*d*x^4 - 7*a^2*b^2*e*x^4 + 13*a^3*b*f*x^4 + 8*a*b^3*c*x - 2*a^
2*b^2*d*x - 4*a^3*b*e*x + 10*a^4*f*x)/((b*x^3 + a)^2*a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3}\,dx$$
$$= \frac{\frac{x\,\left(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{9\,a}+\frac{x^4\,\left(13\,f\,a^3\,b-7\,e\,a^2\,b^2+d\,a\,b^3+5\,c\,b^4\right)}{18\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{f\,x}{b^3} + \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-14\,f\,a^3+2\,e\,a^2\,b+d\,a\,b^2+5\,c\,b^3\right)}{27\,a^{8/3}\,b^{10/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+2\,e\,a^2\,b+d\,a\,b^2+5\,c\,b^3\right)}{27\,a^{8/3}\,b^{10/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+2\,e\,a^2\,b+d\,a\,b^2+5\,c\,b^3\right)}{27\,a^{8/3}\,b^{10/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x)
```

```
[Out] ((x*(4*b^3*c + 5*a^3*f - a*b^2*d - 2*a^2*b*e))/(9*a) + (x^4*(5*b^4*c - 7*a^
2*b^2*e + a*b^3*d + 13*a^3*b*f))/(18*a^2))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3
) + (f*x)/b^3 + (log(b^(1/3)*x + a^(1/3))*(5*b^3*c - 14*a^3*f + a*b^2*d + 2
*a^2*b*e))/(27*a^(8/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x -
a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c - 14*a^3*f + a*b^2*d + 2*a^2*b*e))
/(27*a^(8/3)*b^(10/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*(
(3^(1/2)*1i)/2 + 1/2)*(5*b^3*c - 14*a^3*f + a*b^2*d + 2*a^2*b*e))/(27*a^(8/
3)*b^(10/3))
```

3.295 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 303

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^3}\,dx =& -\frac{c}{a^3x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2}\\&-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)}\\&+\frac{\left(14b^3c-2ab^2d-a^2be-5a^3f\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{8/3}}\\&+\frac{\left(14b^3c-2ab^2d-a^2be-5a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{8/3}}\\&-\frac{\left(14b^3c-2ab^2d-a^2be-5a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}b^{8/3}}\end{aligned}$$

```
[Out] -c/a^3/x-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^2/b^2/(b*x^3+a)^2-1/9*(4*
a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)*x^2/a^3/b^2/(b*x^3+a)+1/27*(-5*a^3*f-a^2*b
*e-2*a*b^2*d+14*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(8/3)-1/54*(-5*a^3*
f-a^2*b*e-2*a*b^2*d+14*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(
10/3)/b^(8/3)+1/27*(-5*a^3*f-a^2*b*e-2*a*b^2*d+14*b^3*c)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/b^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1843, 1498, 464, 298, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^3}\,dx$$
$$= -\frac{c}{a^3x} - \frac{x^2(4a^3f-a^2be-2ab^2d+5b^3c)}{9a^3b^2(a+bx^3)} - \frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^2b^2(a+bx^3)^2}$$
$$+ \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-5a^3f-a^2be-2ab^2d+14b^3c)}{9\sqrt{3}a^{10/3}b^{8/3}}$$
$$- \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-5a^3f-a^2be-2ab^2d+14b^3c)}{54a^{10/3}b^{8/3}}$$
$$+ \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-5a^3f-a^2be-2ab^2d+14b^3c)}{27a^{10/3}b^{8/3}}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -(c/(a^3*x)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^2*b^2*(a + b*
x^3)^2) - ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*x^2)/(9*a^3*b^2*(a + b
*x^3)) + ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(8/3)) + ((14*b^3*c - 2*
a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(8/3)
) - ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1498

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-d)^((m - Mod[m, n])/n - 1)*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*((d + e*x^n)^(q + 1)/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1))), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^n))*
(n*(-d)^(-(m - Mod[m, n])/n + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p
- ((c*d^2 - b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*
(Mod[m, n] + 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))], x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &
& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\int\frac{-6b^3c+2b\left(\frac{2b^3c}{a}-2b^2d-abe+a^2f\right)x^3-6ab^2fx^6}{x^2\left(a+bx^3\right)^2}\,dx}{6ab^3}\\
&= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)}\\
&\quad+\frac{\int\frac{18ab^5c-2b^3\left(5b^3c-2ab^2d-a^2be-5a^3f\right)x^3}{x^2\left(a+bx^3\right)}\,dx}{18a^3b^5}\\
&= -\frac{c}{a^3x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)}\\
&\quad-\frac{\left(14b^3c-2ab^2d-a^2be-5a^3f\right)\int\frac{x}{a+bx^3}\,dx}{9a^3b^2}\\
&= -\frac{c}{a^3x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)}\\
&\quad+\frac{\left(14b^3c-2ab^2d-a^2be-5a^3f\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{10/3}b^{7/3}}\\
&\quad-\frac{\left(14b^3c-2ab^2d-a^2be-5a^3f\right)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{10/3}b^{7/3}}\\
&= -\frac{c}{a^3x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)}\\
&\quad+\frac{\left(14b^3c-2ab^2d-a^2be-5a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{8/3}}\\
&\quad-\frac{\left(14b^3c-2ab^2d-a^2be-5a^3f\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{10/3}b^{8/3}}\\
&\quad-\frac{\left(14b^3c-2ab^2d-a^2be-5a^3f\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^3b^{7/3}}
\end{aligned}$$

$$= -\frac{c}{a^3x} - \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^2b^2\,(a+bx^3)^2} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)\,x^2}{9a^3b^2\,(a+bx^3)}$$
$$+ \frac{(14b^3c - 2ab^2d - a^2be - 5a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{10/3}b^{8/3}}$$
$$- \frac{(14b^3c - 2ab^2d - a^2be - 5a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54a^{10/3}b^{8/3}}$$
$$- \frac{(14b^3c - 2ab^2d - a^2be - 5a^3f)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{10/3}b^{8/3}}$$

$$= -\frac{c}{a^3x} - \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^2b^2\,(a+bx^3)^2} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)\,x^2}{9a^3b^2\,(a+bx^3)}$$
$$+ \frac{(14b^3c - 2ab^2d - a^2be - 5a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{8/3}}$$
$$+ \frac{(14b^3c - 2ab^2d - a^2be - 5a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{10/3}b^{8/3}}$$
$$- \frac{(14b^3c - 2ab^2d - a^2be - 5a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54a^{10/3}b^{8/3}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.94

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^2\,(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{54\sqrt[3]{a}c}{x} + \frac{9a^{4/3}(-b^3c+ab^2d-a^2be+a^3f)x^2}{b^2(a+bx^3)^2} - \frac{6\sqrt[3]{a}(5b^3c-2ab^2d-a^2be+4a^3f)x^2}{b^2(a+bx^3)} + \frac{2\sqrt{3}(14b^3c-2ab^2d-a^2be-5a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{8/3}}}{54a^{10/3}}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(1/3)*c)/x + (9*a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)
/(b^2*(a + b*x^3)^2) - (6*a^(1/3)*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)
*x^2)/(b^2*(a + b*x^3)) + (2*Sqrt[3]*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^
3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(8/3) - (2*(-14*b^3*c +
 2*a*b^2*d + a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(8/3) + ((-14*b
^3*c + 2*a*b^2*d + a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/b^(8/3))/(54*a^(10/3))
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.72

method	result
default	$-\frac{c}{a^3 x}+\frac{\frac{-\frac{\left(4f\,a^3-a^2be-2a\,b^2d+5b^3c\right)x^5}{9b}-\frac{a\left(5f\,a^3+a^2be-7a\,b^2d+13b^3c\right)x^2}{18b^2}}{\left(b\,x^3+a\right)^2}+\frac{\left(5f\,a^3+a^2be+2a\,b^2d-14b^3c\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)\right.}{6b\left(\right.}\right.}{9b^2}}{a^3}$
risch	$\frac{-\frac{\left(4f\,a^3-a^2be-2a\,b^2d+14b^3c\right)x^6}{9a^3b}-\frac{\left(5f\,a^3+a^2be-7a\,b^2d+49b^3c\right)x^3}{18a^2b^2}-\frac{c}{a}}{x\left(b\,x^3+a\right)^2}+\frac{\left(\_R=\text{RootOf}\left(a^{10}b^8\_Z^3+125a^9f^3+75a^8be\,f^2+150a^7b^2d\,f^2+15a^7\right.\right.}{}$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] -c/a^3/x+1/a^3*((-1/9*(4*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)/b*x^5-1/18*a*(5*a^3*f+a^2*b*e-7*a*b^2*d+13*b^3*c)/b^2*x^2)/(b*x^3+a)^2+1/9*(5*a^3*f+a^2*b*e+2*a*b^2*d-14*b^3*c)/b^2*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(262) = 524.

Time = 0.31 (sec) , antiderivative size = 1206, normalized size of antiderivative = 3.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="fricas")

[Out] [-1/54*(54*a^3*b^4*c + 6*(14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e + 4*a^4*b^3*f)*x^6 + 3*(49*a^2*b^5*c - 7*a^3*b^4*d + a^4*b^3*e + 5*a^5*b^2*f)*x^3 + 3*sqrt(1/3)*((14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 2*(14*a^2*b^5*c - 2*a^3*b^4*d - a^4*b^3*e - 5*a^5*b^2*f)*x^4 + (14*a^3*b^4*c - 2*a^4*b^3*d - a^5*b^2*e - 5*a^6*b*f)*x)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((14*b^5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2*a^2*b^3*d - a^3*b^2*

```
e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4*b*e - 5*a^5*f)*x)*(-
a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*((14*b^
5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2*a^2*b^3*
d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4*b*e - 5*
a^5*f)*x)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^6*x^7 + 2*a^5*b^
5*x^4 + a^6*b^4*x), -1/54*(54*a^3*b^4*c + 6*(14*a*b^6*c - 2*a^2*b^5*d - a^3
*b^4*e + 4*a^4*b^3*f)*x^6 + 3*(49*a^2*b^5*c - 7*a^3*b^4*d + a^4*b^3*e + 5*a
^5*b^2*f)*x^3 + 6*sqrt(1/3)*((14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e - 5*a^4*
b^3*f)*x^7 + 2*(14*a^2*b^5*c - 2*a^3*b^4*d - a^4*b^3*e - 5*a^5*b^2*f)*x^4 +
 (14*a^3*b^4*c - 2*a^4*b^3*d - a^5*b^2*e - 5*a^6*b*f)*x)*sqrt(-(-a*b^2)^(1/
3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b)
+ ((14*b^5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2
*a^2*b^3*d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4
*b*e - 5*a^5*f)*x)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^
2)^(2/3)) - 2*((14*b^5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14
*a*b^4*c - 2*a^2*b^3*d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3
*b^2*d - a^4*b*e - 5*a^5*f)*x)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a
^4*b^6*x^7 + 2*a^5*b^5*x^4 + a^6*b^4*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^3}\,dx$$
$$= -\frac{2\,(14\,b^4c-2\,ab^3d-a^2b^2e+4\,a^3bf)x^6+18\,a^2b^2c+(49\,ab^3c-7\,a^2b^2d+a^3be+5\,a^4f)x^3}{18\,(a^3b^4x^7+2\,a^4b^3x^4+a^5b^2x)}$$
$$-\frac{\sqrt{3}(14\,b^3c-2\,ab^2d-a^2be-5\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$-\frac{(14\,b^3c-2\,ab^2d-a^2be-5\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+\frac{(14\,b^3c-2\,ab^2d-a^2be-5\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(2*(14*b^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^6 + 18*a^2*b^2*c
+ (49*a*b^3*c - 7*a^2*b^2*d + a^3*b*e + 5*a^4*f)*x^3)/(a^3*b^4*x^7 + 2*a^4*
b^3*x^4 + a^5*b^2*x) - 1/27*sqrt(3)*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3
*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^3*(a/b)^(1/3
)) - 1/54*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(a^3*b^3*(a/b)^(1/3)) + 1/27*(14*b^3*c - 2*a*b^2*d - a^2*b
*e - 5*a^3*f)*log(x + (a/b)^(1/3))/(a^3*b^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.11

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^3}\,dx$$
$$= -\frac{\sqrt{3}(14\,b^3c-2\,ab^2d-a^2be-5\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}a^3b^2} - \frac{c}{a^3x}$$
$$+ \frac{(14\,b^3c-2\,ab^2d-a^2be-5\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}a^3b^2}$$
$$+ \frac{\left(14\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4b^2}$$
$$- \frac{10\,b^4cx^5-4\,ab^3dx^5-2\,a^2b^2ex^5+8\,a^3bfx^5+13\,ab^3cx^2-7\,a^2b^2dx^2+a^3bex^2+5\,a^4fx^2}{18\,\left(bx^3+a\right)^2a^3b^2}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^3*b^2) - c/(a^3*x) +
1/54*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(1/3)*a^3*b^2) + 1/27*(14*b^3*c*(-a/b)^(1/3) - 2*a*
b^2*d*(-a/b)^(1/3) - a^2*b*e*(-a/b)^(1/3) - 5*a^3*f*(-a/b)^(1/3))*(-a/b)^(1
/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b^2) - 1/18*(10*b^4*c*x^5 - 4*a*b^3*d*x
^5 - 2*a^2*b^2*e*x^5 + 8*a^3*b*f*x^5 + 13*a*b^3*c*x^2 - 7*a^2*b^2*d*x^2 + a
^3*b*e*x^2 + 5*a^4*f*x^2)/((b*x^3 + a)^2*a^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 11.93 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^3}\,dx$$
$$= -\frac{\frac{c}{a}+\frac{x^6\,\left(4\,f\,a^3-e\,a^2\,b-2\,d\,a\,b^2+14\,c\,b^3\right)}{9\,a^3\,b}+\frac{x^3\,\left(5\,f\,a^3+e\,a^2\,b-7\,d\,a\,b^2+49\,c\,b^3\right)}{18\,a^2\,b^2}}{a^2\,x+2\,a\,b\,x^4+b^2\,x^7}$$
$$- \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(5\,f\,a^3+e\,a^2\,b+2\,d\,a\,b^2-14\,c\,b^3\right)}{27\,a^{10/3}\,b^{8/3}}$$
$$+ \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,f\,a^3+e\,a^2\,b+2\,d\,a\,b^2-14\,c\,b^3\right)}{27\,a^{10/3}\,b^{8/3}}$$
$$- \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,f\,a^3+e\,a^2\,b+2\,d\,a\,b^2-14\,c\,b^3\right)}{27\,a^{10/3}\,b^{8/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*
a^3*f - 14*b^3*c + 2*a*b^2*d + a^2*b*e))/(27*a^(10/3)*b^(8/3)) - (log(b^(1/
3)*x + a^(1/3))*(5*a^3*f - 14*b^3*c + 2*a*b^2*d + a^2*b*e))/(27*a^(10/3)*b^
(8/3)) - (c/a + (x^6*(14*b^3*c + 4*a^3*f - 2*a*b^2*d - a^2*b*e))/(9*a^3*b)
+ (x^3*(49*b^3*c + 5*a^3*f - 7*a*b^2*d + a^2*b*e))/(18*a^2*b^2))/(a^2*x + b
^2*x^7 + 2*a*b*x^4) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(5*a^3*f - 14*b^3*c + 2*a*b^2*d + a^2*b*e))/(27*a^(10/3)
*b^(8/3))
```

3.296 $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 301

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^3} dx = -\frac{c}{2a^3x^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)x}{6a^2b^2(a+bx^3)^2} - \frac{(11b^3c-5ab^2d-a^2be+7a^3f)x}{18a^3b^2(a+bx^3)} + \frac{(20b^3c-5ab^2d-a^2be-2a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}b^{7/3}} - \frac{(20b^3c-5ab^2d-a^2be-2a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}b^{7/3}} + \frac{(20b^3c-5ab^2d-a^2be-2a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}b^{7/3}}$$

```
[Out] -1/2*c/a^3/x^2-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^2/b^2/(b*x^3+a)^2-1/1
8*(7*a^3*f-a^2*b*e-5*a*b^2*d+11*b^3*c)*x/a^3/b^2/(b*x^3+a)-1/27*(-2*a^3*f-a
^2*b*e-5*a*b^2*d+20*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(7/3)+1/54*(-2*
a^3*f-a^2*b*e-5*a*b^2*d+20*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/a^(11/3)/b^(7/3)+1/27*(-2*a^3*f-a^2*b*e-5*a*b^2*d+20*b^3*c)*arctan(1/3*(a^
(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1843, 1498, 464, 206, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^3}\,dx$$
$$= -\frac{c}{2a^3x^2} - \frac{x(7a^3f-a^2be-5ab^2d+11b^3c)}{18a^3b^2(a+bx^3)} - \frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^2b^2(a+bx^3)^2}$$
$$+ \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-2a^3f-a^2be-5ab^2d+20b^3c)}{9\sqrt{3}a^{11/3}b^{7/3}}$$
$$+ \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-2a^3f-a^2be-5ab^2d+20b^3c)}{54a^{11/3}b^{7/3}}$$
$$- \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-2a^3f-a^2be-5ab^2d+20b^3c)}{27a^{11/3}b^{7/3}}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -1/2*c/(a^3*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^2*b^2*(a +
b*x^3)^2) - ((11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3*f)*x)/(18*a^3*b^2*(a +
 b*x^3)) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(7/3)) - ((20*b^3*c -
5*a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(7/
3)) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1498

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-d)^((m - Mod[m, n])/n - 1)*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*((d + e*x^n)^(q + 1)/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1))), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^n))*
(n*(-d)^(-(m - Mod[m, n])/n + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p
- ((c*d^2 - b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*
(Mod[m, n] + 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))], x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &
& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^2b^2(a+bx^3)^2}-\frac{\int\frac{-6b^3c+b\left(\frac{5b^3c}{a}-5b^2d-abe+a^2f\right)x^3-6ab^2fx^6}{x^3(a+bx^3)^2}\,dx}{6ab^3}\\
&= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^2b^2(a+bx^3)^2}-\frac{(11b^3c-5ab^2d-a^2be+7a^3f)\,x}{18a^3b^2(a+bx^3)}\\
&\quad+\frac{\int\frac{18ab^5c-2b^3(11b^3c-5ab^2d-a^2be-2a^3f)x^3}{x^3(a+bx^3)}\,dx}{18a^3b^5}\\
&= -\frac{c}{2a^3x^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^2b^2(a+bx^3)^2}-\frac{(11b^3c-5ab^2d-a^2be+7a^3f)\,x}{18a^3b^2(a+bx^3)}\\
&\quad-\frac{(20b^3c-5ab^2d-a^2be-2a^3f)\int\frac{1}{a+bx^3}\,dx}{9a^3b^2}\\
&= -\frac{c}{2a^3x^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^2b^2(a+bx^3)^2}-\frac{(11b^3c-5ab^2d-a^2be+7a^3f)\,x}{18a^3b^2(a+bx^3)}\\
&\quad-\frac{(20b^3c-5ab^2d-a^2be-2a^3f)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{11/3}b^2}\\
&\quad-\frac{(20b^3c-5ab^2d-a^2be-2a^3f)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{11/3}b^2}\\
&= -\frac{c}{2a^3x^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^2b^2(a+bx^3)^2}-\frac{(11b^3c-5ab^2d-a^2be+7a^3f)\,x}{18a^3b^2(a+bx^3)}\\
&\quad-\frac{(20b^3c-5ab^2d-a^2be-2a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}b^{7/3}}\\
&\quad+\frac{(20b^3c-5ab^2d-a^2be-2a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{11/3}b^{7/3}}\\
&\quad-\frac{(20b^3c-5ab^2d-a^2be-2a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{10/3}b^2}
\end{aligned}$$

$$= -\frac{c}{2a^3x^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{6a^2b^2\,(a+bx^3)^2} - \frac{(11b^3c - 5ab^2d - a^2be + 7a^3f)\,x}{18a^3b^2\,(a+bx^3)}$$
$$- \frac{(20b^3c - 5ab^2d - a^2be - 2a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{11/3}b^{7/3}}$$
$$+ \frac{(20b^3c - 5ab^2d - a^2be - 2a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54a^{11/3}b^{7/3}}$$
$$- \frac{(20b^3c - 5ab^2d - a^2be - 2a^3f)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{11/3}b^{7/3}}$$

$$= -\frac{c}{2a^3x^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{6a^2b^2\,(a+bx^3)^2} - \frac{(11b^3c - 5ab^2d - a^2be + 7a^3f)\,x}{18a^3b^2\,(a+bx^3)}$$
$$+ \frac{(20b^3c - 5ab^2d - a^2be - 2a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}b^{7/3}}$$
$$- \frac{(20b^3c - 5ab^2d - a^2be - 2a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{11/3}b^{7/3}}$$
$$+ \frac{(20b^3c - 5ab^2d - a^2be - 2a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54a^{11/3}b^{7/3}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.94

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^3\,(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{27a^{2/3}c}{x^2} + \frac{9a^{5/3}(-b^3c + ab^2d - a^2be + a^3f)x}{b^2(a+bx^3)^2} - \frac{3a^{2/3}(11b^3c - 5ab^2d - a^2be + 7a^3f)x}{b^2(a+bx^3)} + \frac{2\sqrt{3}(20b^3c - 5ab^2d - a^2be - 2a^3f)\arctan\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{b^{7/3}}}{54a^{11/3}}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(2/3)*c)/x^2 + (9*a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)
/(b^2*(a + b*x^3)^2) - (3*a^(2/3)*(11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3*f
)*x)/(b^2*(a + b*x^3)) + (2*Sqrt[3]*(20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3
*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(7/3) + (2*(-20*b^3*c +
5*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(7/3) - ((-20*b^
3*c + 5*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/b^(7/3))/(54*a^(11/3))
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.72

method	result
default	$-\frac{c}{2a^{3}x^{2}}+\frac{\frac{-\frac{\left(7f\,a^{3}-a^{2}be-5a\,b^{2}d+11b^{3}c\right)x^{4}}{18b}-\frac{a\left(2f\,a^{3}+a^{2}be-4a\,b^{2}d+7b^{3}c\right)x}{9b^{2}}}{\left(b\,x^{3}+a\right)^{2}}+\frac{\left(2f\,a^{3}+a^{2}be+5a\,b^{2}d-20b^{3}c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.}{6b\left(\frac{a}{b}\right.}\right.}{9b^{2}}}{a^{3}}$
risch	$\frac{-\frac{\left(7f\,a^{3}-a^{2}be-5a\,b^{2}d+20b^{3}c\right)x^{6}}{18a^{3}b}-\frac{\left(2f\,a^{3}+a^{2}be-4a\,b^{2}d+16b^{3}c\right)x^{3}}{9a^{2}b^{2}}-\frac{c}{2a}}{x^{2}\left(b\,x^{3}+a\right)^{2}}+\left(\sum_{\_R=\operatorname{RootOf}\left(a^{11}b^{7}\_Z^{3}-8a^{9}f^{3}-12a^{8}be\,f^{2}-60a^{7}b^{2}d\,f^{2}-6a^{7}b^{2}e\right.}\right.$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c/a^3/x^2+1/a^3*((-1/18*(7*a^3*f-a^2*b*e-5*a*b^2*d+11*b^3*c)/b*x^4-1/9
*a*(2*a^3*f+a^2*b*e-4*a*b^2*d+7*b^3*c)/b^2*x)/(b*x^3+a)^2+1/9*(2*a^3*f+a^2*
b*e+5*a*b^2*d-20*b^3*c)/b^2*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b
)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(258) = 516.

Time = 0.29 (sec) , antiderivative size = 1217, normalized size of antiderivative = 4.04

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(27*a^4*b^3*c + 3*(20*a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e + 7*a^5*b^
2*f)*x^6 + 6*(16*a^3*b^4*c - 4*a^4*b^3*d + a^5*b^2*e + 2*a^6*b*f)*x^3 + 3*s
qrt(1/3)*((20*a*b^6*c - 5*a^2*b^5*d - a^3*b^4*e - 2*a^4*b^3*f)*x^8 + 2*(20*
a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e - 2*a^5*b^2*f)*x^5 + (20*a^3*b^4*c - 5*
a^4*b^3*d - a^5*b^2*e - 2*a^6*b*f)*x^2)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x
^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x -
 (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - ((20*b^5*c - 5*a*b
^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2
```

```
*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((20*b^
5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*
d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*
a^5*f)*x^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^5*x^8 + 2*a^6*
b^4*x^5 + a^7*b^3*x^2), -1/54*(27*a^4*b^3*c + 3*(20*a^2*b^5*c - 5*a^3*b^4*d
 - a^4*b^3*e + 7*a^5*b^2*f)*x^6 + 6*(16*a^3*b^4*c - 4*a^4*b^3*d + a^5*b^2*e
 + 2*a^6*b*f)*x^3 + 6*sqrt(1/3)*((20*a*b^6*c - 5*a^2*b^5*d - a^3*b^4*e - 2*
a^4*b^3*f)*x^8 + 2*(20*a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e - 2*a^5*b^2*f)*x
^5 + (20*a^3*b^4*c - 5*a^4*b^3*d - a^5*b^2*e - 2*a^6*b*f)*x^2)*sqrt((a^2*b)
^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*
b)^(1/3)/b)/a^2) - ((20*b^5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 +
2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c -
5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(
2/3)*x + (a^2*b)^(1/3)*a) + 2*((20*b^5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^
2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a
^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)*(a^2*b)^(2/3)*log(a*b*x +
(a^2*b)^(2/3)))/(a^5*b^5*x^8 + 2*a^6*b^4*x^5 + a^7*b^3*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^3 (a + bx^3)^3} \, dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx = -\frac{(20\,b^4c-5\,ab^3d-a^2b^2e+7\,a^3bf)x^6+9\,a^2b^2c+2\,(16\,ab^3c-4\,a^2b^2d+a^3be+2\,a^4f)x^3}{18\,(a^3b^4x^8+2\,a^4b^3x^5+a^5b^2x^2)} - \frac{\sqrt{3}(20\,b^3c-5\,ab^2d-a^2be-2\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(20\,b^3c-5\,ab^2d-a^2be-2\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(20\,b^3c-5\,ab^2d-a^2be-2\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((20*b^4*c - 5*a*b^3*d - a^2*b^2*e + 7*a^3*b*f)*x^6 + 9*a^2*b^2*c + 2
*(16*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^3)/(a^3*b^4*x^8 + 2*a^4*b
^3*x^5 + a^5*b^2*x^2) - 1/27*sqrt(3)*(20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^
3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^3*(a/b)^(2/
3)) + 1/54*(20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*log(x^2 - x*(a/b)^(1/
3) + (a/b)^(2/3))/(a^3*b^3*(a/b)^(2/3)) - 1/27*(20*b^3*c - 5*a*b^2*d - a^2*
b*e - 2*a^3*f)*log(x + (a/b)^(1/3))/(a^3*b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^3}\,dx = \frac{\sqrt{3}(20\,b^3c-5\,ab^2d-a^2be-2\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^3b} + \frac{(20\,b^3c-5\,ab^2d-a^2be-2\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^3b} + \frac{(20\,b^3c-5\,ab^2d-a^2be-2\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4b^2} - \frac{20\,b^4cx^6-5\,ab^3dx^6-a^2b^2ex^6+7\,a^3bfx^6+32\,ab^3cx^3-8\,a^2b^2dx^3+2\,a^3bex^3+4\,a^4fx^3+9\,a^2b^2c}{18\,(bx^4+ax)^2a^3b^2}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3*b) + 1/54*(20*b^3*c
- 5*a*b^2*d - a^2*b*e - 2*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(
(-a*b^2)^(2/3)*a^3*b) + 1/27*(20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b^2) - 1/18*(20*b^4*c*x^6 - 5*a*b
^3*d*x^6 - a^2*b^2*e*x^6 + 7*a^3*b*f*x^6 + 32*a*b^3*c*x^3 - 8*a^2*b^2*d*x^3
 + 2*a^3*b*e*x^3 + 4*a^4*f*x^3 + 9*a^2*b^2*c)/((b*x^4 + a*x)^2*a^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 10.05 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^3}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(2\,f\,a^3+e\,a^2\,b+5\,d\,a\,b^2-20\,c\,b^3\right)}{27\,a^{11/3}\,b^{7/3}} - \frac{\frac{c}{2\,a}+\frac{x^3\,\left(2\,f\,a^3+e\,a^2\,b-4\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^2\,b^2}+\frac{x^6\,\left(7\,f\,a^3-e\,a^2\,b-5\,d\,a\,b^2+20\,c\,b^3\right)}{18\,a^3\,b}}{a^2\,x^2+2\,a\,b\,x^5+b^2\,x^8} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,f\,a^3+e\,a^2\,b+5\,d\,a\,b^2-20\,c\,b^3\right)}{27\,a^{11/3}\,b^{7/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,f\,a^3+e\,a^2\,b+5\,d\,a\,b^2-20\,c\,b^3\right)}{27\,a^{11/3}\,b^{7/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(2*a^3*f - 20*b^3*c + 5*a*b^2*d + a^2*b*e))/(27*a
^(11/3)*b^(7/3)) - (c/(2*a) + (x^3*(16*b^3*c + 2*a^3*f - 4*a*b^2*d + a^2*b*
e))/(9*a^2*b^2) + (x^6*(20*b^3*c + 7*a^3*f - 5*a*b^2*d - a^2*b*e))/(18*a^3*
b))/(a^2*x^2 + b^2*x^8 + 2*a*b*x^5) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
 - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*a^3*f - 20*b^3*c + 5*a*b^2*d + a^2*b*
e))/(27*a^(11/3)*b^(7/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3)
)*((3^(1/2)*1i)/2 + 1/2)*(2*a^3*f - 20*b^3*c + 5*a*b^2*d + a^2*b*e))/(27*a^
(11/3)*b^(7/3))
```

3.297 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 317

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^3}\,dx = -\frac{c}{4a^3x^4}+\frac{3bc-ad}{a^4x}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^3b(a+bx^3)^2}+\frac{(8b^3c-5ab^2d+2a^2be+a^3f)\,x^2}{9a^4b(a+bx^3)} - \frac{(35b^3c-14ab^2d+2a^2be+a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{13/3}b^{5/3}} - \frac{(35b^3c-14ab^2d+2a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{13/3}b^{5/3}} + \frac{(35b^3c-14ab^2d+2a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{13/3}b^{5/3}}$$

```
[Out] -1/4*c/a^3/x^4+(-a*d+3*b*c)/a^4/x+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^
3/b/(b*x^3+a)^2+1/9*(a^3*f+2*a^2*b*e-5*a*b^2*d+8*b^3*c)*x^2/a^4/b/(b*x^3+a)
-1/27*(a^3*f+2*a^2*b*e-14*a*b^2*d+35*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)/
b^(5/3)+1/54*(a^3*f+2*a^2*b*e-14*a*b^2*d+35*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/a^(13/3)/b^(5/3)-1/27*(a^3*f+2*a^2*b*e-14*a*b^2*d+35*b^3*
c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)/b^(5/3)*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1843, 1498, 1502, 298, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}dx$$
$$=\frac{3bc-ad}{a^4x}-\frac{c}{4a^3x^4}+\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^3b\left(a+bx^3\right)^2}-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3f+2a^2be-14ab^2d+35b^3c)}{9\sqrt{3}a^{13/3}b^{5/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3f+2a^2be-14ab^2d+35b^3c)}{54a^{13/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3f+2a^2be-14ab^2d+35b^3c)}{27a^{13/3}b^{5/3}}+\frac{x^2(a^3f+2a^2be-5ab^2d+8b^3c)}{9a^4b\left(a+bx^3\right)}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] -1/4*c/(a^3*x^4) + (3*b*c - a*d)/(a^4*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*x^2)/(6*a^3*b*(a + b*x^3)^2) + ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3
*f)*x^2)/(9*a^4*b*(a + b*x^3)) - ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*
f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(13/3)*b
^(5/3)) - ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3
)*x]])/(27*a^(13/3)*b^(5/3)) + ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(13/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1498

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-d)^((m - Mod[m, n])/n - 1)*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*((d + e*x^n)^(q + 1)/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1))), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^n))*
(n*(-d)^(-(m - Mod[m, n])/n + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p
- ((c*d^2 - b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*
(Mod[m, n] + 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))], x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &
& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
```

```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^3b\,(a+bx^3)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-2b^2\left(\frac{2b^3c}{a^2}-\frac{2b^2d}{a}+2be+af\right)x^6}{x^5(a+bx^3)^2}\,dx}{6ab^3}\\
&= \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^3b\,(a+bx^3)^2} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f)\,x^2}{9a^4b\,(a+bx^3)} \\
&\quad - \frac{\int \frac{-18a^2b^5c+18ab^5(2bc-ad)x^3-2b^4(8b^3c-5ab^2d+2a^2be+a^3f)x^6}{x^5(a+bx^3)}\,dx}{18a^4b^5}\\
&= \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^3b\,(a+bx^3)^2} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f)\,x^2}{9a^4b\,(a+bx^3)} \\
&\quad - \frac{\int \left(-\frac{18ab^5c}{x^5}+\frac{18b^5(3bc-ad)}{x^2}-\frac{2b^4(35b^3c-14ab^2d+2a^2be+a^3f)x}{a+bx^3}\right)dx}{18a^4b^5}\\
&= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^3b\,(a+bx^3)^2} \\
&\quad + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f)\,x^2}{9a^4b\,(a+bx^3)} + \frac{(35b^3c - 14ab^2d + 2a^2be + a^3f)\int \frac{x}{a+bx^3}\,dx}{9a^4b}\\
&= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^3b\,(a+bx^3)^2} \\
&\quad + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f)\,x^2}{9a^4b\,(a+bx^3)} \\
&\quad - \frac{(35b^3c - 14ab^2d + 2a^2be + a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{13/3}b^{4/3}} \\
&\quad + \frac{(35b^3c - 14ab^2d + 2a^2be + a^3f)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{13/3}b^{4/3}}
\end{aligned}
$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc - ad}{a^4x} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^3b\,(a + bx^3)^2} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f)\,x^2}{9a^4b\,(a + bx^3)} - \frac{(35b^3c - 14ab^2d + 2a^2be + a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{13/3}b^{5/3}} + \frac{(35b^3c - 14ab^2d + 2a^2be + a^3f)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{13/3}b^{5/3}} + \frac{(35b^3c - 14ab^2d + 2a^2be + a^3f)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^4b^{4/3}}$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc - ad}{a^4x} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^3b\,(a + bx^3)^2} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f)\,x^2}{9a^4b\,(a + bx^3)} - \frac{(35b^3c - 14ab^2d + 2a^2be + a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{13/3}b^{5/3}} + \frac{(35b^3c - 14ab^2d + 2a^2be + a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54a^{13/3}b^{5/3}} + \frac{(35b^3c - 14ab^2d + 2a^2be + a^3f)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{13/3}b^{5/3}}$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc - ad}{a^4x} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^3b\,(a + bx^3)^2} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f)\,x^2}{9a^4b\,(a + bx^3)} - \frac{(35b^3c - 14ab^2d + 2a^2be + a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{13/3}b^{5/3}} - \frac{(35b^3c - 14ab^2d + 2a^2be + a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{13/3}b^{5/3}} + \frac{(35b^3c - 14ab^2d + 2a^2be + a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54a^{13/3}b^{5/3}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}\,dx$$

$$= \frac{-\frac{27a^{4/3}c}{x^4} - \frac{108\sqrt[3]{a}(-3bc+ad)}{x} - \frac{18a^{4/3}(-b^3c+ab^2d-a^2be+a^3f)x^2}{b(a+bx^3)^2} + \frac{12\sqrt[3]{a}(8b^3c-5ab^2d+2a^2be+a^3f)x^2}{b(a+bx^3)} - \frac{4\sqrt{3}(35b^3c-14ab^2d+2a^2b}{}}{10}$$

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x]

[Out] ((-27*a^(4/3)*c)/x^4 - (108*a^(1/3)*(-3*b*c + a*d))/x - (18*a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)^2) + (12*a^(1/3)*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)) - (4*Sqrt[3]*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(5/3) - (4*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(5/3) + (2*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3))/(108*a^(13/3))

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.73

method	result
default	$-\frac{c}{4a^3x^4} - \frac{ad-3bc}{a^4x} + \frac{\frac{\left(\frac{1}{9}fa^3+\frac{2}{9}a^2be-\frac{5}{9}ab^2d+\frac{8}{9}b^3c\right)x^5-\frac{a\left(fa^3-7a^2be+13ab^2d-19b^3c\right)x^2}{18b}}{\left(bx^3+a\right)^2} + \frac{\left(fa^3+2a^2be-14ab^2d+35b^3c\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right.\right.}{3b\left(\frac{a}{b}\right.}\right.}{}}{a^4}$
risch	$\frac{\frac{\left(fa^3+2a^2be-14ab^2d+35b^3c\right)x^9}{9a^4}-\frac{\left(2fa^3-14a^2be+98ab^2d-245b^3c\right)x^6}{36a^3b}-\frac{(2ad-5bc)x^3}{2a^2}-\frac{c}{4a}}{x^4\left(bx^3+a\right)^2} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{13}b^5\_Z^3+a^9f^3+6a^8bef^2-42a\right.}\right.}{}$

[In] int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] -1/4*c/a^3/x^4-(a*d-3*b*c)/a^4/x+1/a^4*(((1/9*f*a^3+2/9*a^2*b*e-5/9*a*b^2*d

```
+8/9*b^3*c)*x^5-1/18*a*(a^3*f-7*a^2*b*e+13*a*b^2*d-19*b^3*c)/b*x^2)/(b*x^3+
a)^2+1/9*(a^3*f+2*a^2*b*e-14*a*b^2*d+35*b^3*c)/b*(-1/3/b/(a/b)^(1/3)*ln(x+(
a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)
/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(274) = 548$.

Time = 0.29 (sec) , antiderivative size = 1254, normalized size of antiderivative = 3.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/108*(12*(35*a*b^6*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^9 - 27*a
^4*b^3*c + 3*(245*a^2*b^5*c - 98*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^
6 + 54*(5*a^3*b^4*c - 2*a^4*b^3*d)*x^3 + 6*sqrt(1/3)*((35*a*b^6*c - 14*a^2*
b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^10 + 2*(35*a^2*b^5*c - 14*a^3*b^4*d + 2*
a^4*b^3*e + a^5*b^2*f)*x^7 + (35*a^3*b^4*c - 14*a^4*b^3*d + 2*a^5*b^2*e + a
^6*b*f)*x^4)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b
*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-
a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*((35*b^5*c - 14*a*b^4*d + 2*a^2*b^3*e + a^
3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e + a^4*b*f)*x^7 +
 (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(-a*b^2)^(2/3)*log(
b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*((35*b^5*c - 14*a*b^4*d
+ 2*a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*
e + a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(
-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^5*x^10 + 2*a^6*b^4*x^7 + a^
7*b^3*x^4), 1/108*(12*(35*a*b^6*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)
*x^9 - 27*a^4*b^3*c + 3*(245*a^2*b^5*c - 98*a^3*b^4*d + 14*a^4*b^3*e - 2*a^
5*b^2*f)*x^6 + 54*(5*a^3*b^4*c - 2*a^4*b^3*d)*x^3 + 12*sqrt(1/3)*((35*a*b^6
*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^10 + 2*(35*a^2*b^5*c - 14*a^
3*b^4*d + 2*a^4*b^3*e + a^5*b^2*f)*x^7 + (35*a^3*b^4*c - 14*a^4*b^3*d + 2*a
^5*b^2*e + a^6*b*f)*x^4)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x +
(-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*((35*b^5*c - 14*a*b^4*d + 2*
a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e +
a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(-a*b
^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*((35*b^5*c
 - 14*a*b^4*d + 2*a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*
d + 2*a^3*b^2*e + a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e +
 a^5*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^5*x^10 + 2*a^
6*b^4*x^7 + a^7*b^3*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}\,dx$$

$$= \frac{4\,(35\,b^4c-14\,ab^3d+2\,a^2b^2e+a^3bf)x^9+(245\,ab^3c-98\,a^2b^2d+14\,a^3be-2\,a^4f)x^6-9\,a^3bc+18\,(5\,a^2b^2c-}{36\,(a^4b^3x^{10}+2\,a^5b^2x^7+a^6bx^4)}$$
$$+\frac{\sqrt{3}(35\,b^3c-14\,ab^2d+2\,a^2be+a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+\frac{(35\,b^3c-14\,ab^2d+2\,a^2be+a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$-\frac{(35\,b^3c-14\,ab^2d+2\,a^2be+a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(4*(35*b^4*c - 14*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^9 + (245*a*b^3*c
- 98*a^2*b^2*d + 14*a^3*b*e - 2*a^4*f)*x^6 - 9*a^3*b*c + 18*(5*a^2*b^2*c -
2*a^3*b*d)*x^3)/(a^4*b^3*x^10 + 2*a^5*b^2*x^7 + a^6*b*x^4) + 1/27*sqrt(3)*(
35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/(a^4*b^2*(a/b)^(1/3)) + 1/54*(35*b^3*c - 14*a*b^2*d + 2
*a^2*b*e + a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b^2*(a/b)^(1/
3)) - 1/27*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*log(x + (a/b)^(1/3))
/(a^4*b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.11

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}\,dx = \frac{\sqrt{3}(35\,b^3c-14\,ab^2d+2\,a^2be+a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}a^4b} - \frac{(35\,b^3c-14\,ab^2d+2\,a^2be+a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}a^4b} - \frac{\left(35\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-14\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5b} + \frac{16\,b^4cx^5-10\,ab^3dx^5+4\,a^2b^2ex^5+2\,a^3bfx^5+19\,ab^3cx^2-13\,a^2b^2dx^2+7\,a^3bex^2-a^4fx^2}{18\,\left(bx^3+a\right)^2a^4b} + \frac{12\,bcx^3-4\,adx^3-ac}{4\,a^4x^4}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^4*b) - 1/54*(35*b^3*c
 - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/((-a*b^2)^(1/3)*a^4*b) - 1/27*(35*b^3*c*(-a/b)^(1/3) - 14*a*b^2*d*(-a/b)^(
1/3) + 2*a^2*b*e*(-a/b)^(1/3) + a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a^5*b) + 1/18*(16*b^4*c*x^5 - 10*a*b^3*d*x^5 + 4*a^2*b^2*
e*x^5 + 2*a^3*b*f*x^5 + 19*a*b^3*c*x^2 - 13*a^2*b^2*d*x^2 + 7*a^3*b*e*x^2 -
 a^4*f*x^2)/((b*x^3 + a)^2*a^4*b) + 1/4*(12*b*c*x^3 - 4*a*d*x^3 - a*c)/(a^4
*x^4)
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}\,dx$$
$$= -\frac{\frac{c}{4\,a}-\frac{x^9\,\left(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3\right)}{9\,a^4}+\frac{x^3\,\left(2\,a\,d-5\,b\,c\right)}{2\,a^2}-\frac{x^6\,\left(-2\,f\,a^3+14\,e\,a^2\,b-98\,d\,a\,b^2+245\,c\,b^3\right)}{36\,a^3\,b}}{a^2\,x^4+2\,a\,b\,x^7+b^2\,x^{10}}$$
$$-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3\right)}{27\,a^{13/3}\,b^{5/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3\right)}{27\,a^{13/3}\,b^{5/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3\right)}{27\,a^{13/3}\,b^{5/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(35
*b^3*c + a^3*f - 14*a*b^2*d + 2*a^2*b*e))/(27*a^(13/3)*b^(5/3)) - (log(b^(1
/3)*x + a^(1/3))*(35*b^3*c + a^3*f - 14*a*b^2*d + 2*a^2*b*e))/(27*a^(13/3)*
b^(5/3)) - (c/(4*a) - (x^9*(35*b^3*c + a^3*f - 14*a*b^2*d + 2*a^2*b*e))/(9*
a^4) + (x^3*(2*a*d - 5*b*c))/(2*a^2) - (x^6*(245*b^3*c - 2*a^3*f - 98*a*b^2
*d + 14*a^2*b*e))/(36*a^3*b))/(a^2*x^4 + b^2*x^10 + 2*a*b*x^7) - (log(3^(1/
2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(35*b^3*c + a
^3*f - 14*a*b^2*d + 2*a^2*b*e))/(27*a^(13/3)*b^(5/3))
```

3.298 $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 316

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^3}\,dx = -\frac{c}{5a^3x^5}+\frac{3bc-ad}{2a^4x^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^3b\,(a+bx^3)^2}+\frac{(17b^3c-11ab^2d+5a^2be+a^3f)\,x}{18a^4b\,(a+bx^3)}-\frac{(44b^3c-20ab^2d+5a^2be+a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{14/3}b^{4/3}}+\frac{(44b^3c-20ab^2d+5a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}b^{4/3}}-\frac{(44b^3c-20ab^2d+5a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{14/3}b^{4/3}}$$

```
[Out] -1/5*c/a^3/x^5+1/2*(-a*d+3*b*c)/a^4/x^2+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
x/a^3/b/(b*x^3+a)^2+1/18*(a^3*f+5*a^2*b*e-11*a*b^2*d+17*b^3*c)*x/a^4/b/(b*x
^3+a)+1/27*(a^3*f+5*a^2*b*e-20*a*b^2*d+44*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(1
4/3)/b^(4/3)-1/54*(a^3*f+5*a^2*b*e-20*a*b^2*d+44*b^3*c)*ln(a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)/b^(4/3)-1/27*(a^3*f+5*a^2*b*e-20*a*b^2*d+44
*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)/b^(4/3)*
3^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1843, 1498, 1502, 206, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6 (a+bx^3)^3} dx$$
$$= \frac{3bc-ad}{2a^4x^2} - \frac{c}{5a^3x^5} + \frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^3b(a+bx^3)^2}$$
$$- \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3f+5a^2be-20ab^2d+44b^3c)}{9\sqrt{3}a^{14/3}b^{4/3}}$$
$$- \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3f+5a^2be-20ab^2d+44b^3c)}{54a^{14/3}b^{4/3}}$$
$$+ \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3f+5a^2be-20ab^2d+44b^3c)}{27a^{14/3}b^{4/3}} + \frac{x(a^3f+5a^2be-11ab^2d+17b^3c)}{18a^4b(a+bx^3)}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] -1/5*c/(a^3*x^5) + (3*b*c - a*d)/(2*a^4*x^2) + ((b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*x)/(6*a^3*b*(a + b*x^3)^2) + ((17*b^3*c - 11*a*b^2*d + 5*a^2*b*e +
 a^3*f)*x)/(18*a^4*b*(a + b*x^3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a
^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(14/3
)*b^(4/3)) + ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(
1/3)*x])/(27*a^(14/3)*b^(4/3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*
f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(14/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1498

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-d)^((m - Mod[m, n])/n - 1)*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*((d + e*x^n)^(q + 1)/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1))), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^n))*
(n*(-d)^(-(m - Mod[m, n])/n + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p
- ((c*d^2 - b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*
(Mod[m, n] + 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))], x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &
& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
```

```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^3b\,(a+bx^3)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-b^2\left(\frac{5b^3c}{a^2}-\frac{5b^2d}{a}+5be+af\right)x^6}{x^6(a+bx^3)^2}\,dx}{6ab^3}\\
&= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^3b\,(a+bx^3)^2} + \frac{(17b^3c-11ab^2d+5a^2be+a^3f)\,x}{18a^4b\,(a+bx^3)}\\
&\quad - \frac{\int \frac{-18a^2b^5c+18ab^5(2bc-ad)x^3-2b^4(17b^3c-11ab^2d+5a^2be+a^3f)x^6}{x^6(a+bx^3)}\,dx}{18a^4b^5}\\
&= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^3b\,(a+bx^3)^2} + \frac{(17b^3c-11ab^2d+5a^2be+a^3f)\,x}{18a^4b\,(a+bx^3)}\\
&\quad - \frac{\int \left(-\frac{18ab^5c}{x^6}+\frac{18b^5(3bc-ad)}{x^3}-\frac{2b^4(44b^3c-20ab^2d+5a^2be+a^3f)}{a+bx^3}\right)dx}{18a^4b^5}\\
&= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^3b\,(a+bx^3)^2}\\
&\quad + \frac{(17b^3c-11ab^2d+5a^2be+a^3f)\,x}{18a^4b\,(a+bx^3)} + \frac{(44b^3c-20ab^2d+5a^2be+a^3f)\int \frac{1}{a+bx^3}\,dx}{9a^4b}\\
&= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^3b\,(a+bx^3)^2}\\
&\quad + \frac{(17b^3c-11ab^2d+5a^2be+a^3f)\,x}{18a^4b\,(a+bx^3)}\\
&\quad + \frac{(44b^3c-20ab^2d+5a^2be+a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{14/3}b}\\
&\quad + \frac{(44b^3c-20ab^2d+5a^2be+a^3f)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{14/3}b}
\end{aligned}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^3b\,(a+bx^3)^2} + \frac{(17b^3c-11ab^2d+5a^2be+a^3f)\,x}{18a^4b\,(a+bx^3)} + \frac{(44b^3c-20ab^2d+5a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}b^{4/3}} - \frac{(44b^3c-20ab^2d+5a^2be+a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{14/3}b^{4/3}} + \frac{(44b^3c-20ab^2d+5a^2be+a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{13/3}b}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^3b\,(a+bx^3)^2} + \frac{(17b^3c-11ab^2d+5a^2be+a^3f)\,x}{18a^4b\,(a+bx^3)} + \frac{(44b^3c-20ab^2d+5a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}b^{4/3}} - \frac{(44b^3c-20ab^2d+5a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{14/3}b^{4/3}} + \frac{(44b^3c-20ab^2d+5a^2be+a^3f)\,\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{14/3}b^{4/3}}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^3b\,(a+bx^3)^2} + \frac{(17b^3c-11ab^2d+5a^2be+a^3f)\,x}{18a^4b\,(a+bx^3)} - \frac{(44b^3c-20ab^2d+5a^2be+a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{14/3}b^{4/3}} + \frac{(44b^3c-20ab^2d+5a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}b^{4/3}} - \frac{(44b^3c-20ab^2d+5a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{14/3}b^{4/3}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{54a^{5/3}c}{x^5} - \frac{135a^{2/3}(-3bc+ad)}{x^2} - \frac{45a^{5/3}(-b^3c+ab^2d-a^2be+a^3f)x}{b(a+bx^3)^2} + \frac{15a^{2/3}(17b^3c-11ab^2d+5a^2be+a^3f)x}{b(a+bx^3)} - \frac{10\sqrt{3}(44b^3c-20ab^2d+5a^2}{}}{2}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(5/3)*c)/x^5 - (135*a^(2/3)*(-3*b*c + a*d))/x^2 - (45*a^(5/3)*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)^2) + (15*a^(2/3)*(17*b^
3*c - 11*a*b^2*d + 5*a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)) - (10*Sqrt[3]*(44*
b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(4/3) + (10*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(
1/3) + b^(1/3)*x])/b^(4/3) - (5*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3))/(270*a^(14/3))
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.72

method	result
default	$-\frac{c}{5a^3x^5} - \frac{ad-3bc}{2a^4x^2} + \frac{\frac{\left(\frac{1}{18}fa^3+\frac{5}{18}a^2be-\frac{11}{18}ab^2d+\frac{17}{18}b^3c\right)x^4 - \frac{a\left(fa^3-4a^2be+7ab^2d-10b^3c\right)x}{9b}}{\left(bx^3+a\right)^2} + \left(fa^3+5a^2be-20ab^2d+44b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right.\right.}{3b\left(\frac{a}{b}\right)}\right.}{a^4}$
risch	$\frac{\frac{\left(fa^3+5a^2be-20ab^2d+44b^3c\right)x^9}{18a^4} - \frac{\left(5fa^3-20a^2be+80ab^2d-176b^3c\right)x^6}{45a^3b} - \frac{(5ad-11bc)x^3}{10a^2} - \frac{c}{5a}}{x^5\left(bx^3+a\right)^2} + \left(\_R=\text{RootOf}\left(a^{14}b^4\_Z^3-a^9f^3-15a^8bef^2+\right.\right.$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*c/a^3/x^5-1/2*(a*d-3*b*c)/a^4/x^2+1/a^4*(((1/18*f*a^3+5/18*a^2*b*e-11/
```

```
18*a*b^2*d+17/18*b^3*c)*x^4-1/9*a*(a^3*f-4*a^2*b*e+7*a*b^2*d-10*b^3*c)/b*x)
/(b*x^3+a)^2+1/9*(a^3*f+5*a^2*b*e-20*a*b^2*d+44*b^3*c)/b*(1/3/b/(a/b)^(2/3)
*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/
b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(271) = 542$.

Time = 0.30 (sec) , antiderivative size = 1247, normalized size of antiderivative = 3.95

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^6 (a + bx^3)^3} \, dx = \text{Too large to display}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="fricas")
[Out] [1/270*(15*(44*a^2*b^5*c - 20*a^3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f)*x^9 - 54
*a^5*b^2*c + 6*(176*a^3*b^4*c - 80*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*x^
6 + 27*(11*a^4*b^3*c - 5*a^5*b^2*d)*x^3 + 15*sqrt(1/3)*((44*a*b^6*c - 20*a^
2*b^5*d + 5*a^3*b^4*e + a^4*b^3*f)*x^11 + 2*(44*a^2*b^5*c - 20*a^3*b^4*d +
5*a^4*b^3*e + a^5*b^2*f)*x^8 + (44*a^3*b^4*c - 20*a^4*b^3*d + 5*a^5*b^2*e +
 a^6*b*f)*x^5)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x
- a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(
a^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*((44*b^5*c - 20*a*b^4*d + 5*a^2*b^3*e + a
^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a^3*b^2*e + a^4*b*f)*x^8
+ (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x^5)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*((44*b^5*c - 20*a*b^4*d +
 5*a^2*b^3*e + a^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a^3*b^2*e
 + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x^5)*(a
^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^4*x^11 + 2*a^7*b^3*x^8 + a^8
*b^2*x^5), 1/270*(15*(44*a^2*b^5*c - 20*a^3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f
)*x^9 - 54*a^5*b^2*c + 6*(176*a^3*b^4*c - 80*a^4*b^3*d + 20*a^5*b^2*e - 5*a
^6*b*f)*x^6 + 27*(11*a^4*b^3*c - 5*a^5*b^2*d)*x^3 + 30*sqrt(1/3)*((44*a*b^6
*c - 20*a^2*b^5*d + 5*a^3*b^4*e + a^4*b^3*f)*x^11 + 2*(44*a^2*b^5*c - 20*a^
3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f)*x^8 + (44*a^3*b^4*c - 20*a^4*b^3*d + 5*a
^5*b^2*e + a^6*b*f)*x^5)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^
(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*((44*b^5*c - 20*a
*b^4*d + 5*a^2*b^3*e + a^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a
^3*b^2*e + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)
*x^5)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(
(44*b^5*c - 20*a*b^4*d + 5*a^2*b^3*e + a^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20
*a^2*b^3*d + 5*a^3*b^2*e + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*
a^4*b*e + a^5*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^4*x^
11 + 2*a^7*b^3*x^8 + a^8*b^2*x^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3}\,dx$$

$$=\frac{5\,(44\,b^4c-20\,ab^3d+5\,a^2b^2e+a^3bf)x^9+2\,(176\,ab^3c-80\,a^2b^2d+20\,a^3be-5\,a^4f)x^6-18\,a^3bc+9\,(11\,a^2b^2}{90\,(a^4b^3x^{11}+2\,a^5b^2x^8+a^6bx^5)}$$

$$+\frac{\sqrt{3}(44\,b^3c-20\,ab^2d+5\,a^2be+a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$$-\frac{(44\,b^3c-20\,ab^2d+5\,a^2be+a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$$+\frac{(44\,b^3c-20\,ab^2d+5\,a^2be+a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/90*(5*(44*b^4*c - 20*a*b^3*d + 5*a^2*b^2*e + a^3*b*f)*x^9 + 2*(176*a*b^3*
c - 80*a^2*b^2*d + 20*a^3*b*e - 5*a^4*f)*x^6 - 18*a^3*b*c + 9*(11*a^2*b^2*c
 - 5*a^3*b*d)*x^3)/(a^4*b^3*x^11 + 2*a^5*b^2*x^8 + a^6*b*x^5) + 1/27*sqrt(3
)*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/
b)^(1/3))/(a/b)^(1/3))/(a^4*b^2*(a/b)^(2/3)) - 1/54*(44*b^3*c - 20*a*b^2*d
+ 5*a^2*b*e + a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b^2*(a/b)^
(2/3)) + 1/27*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*log(x + (a/b)^(1/
3))/(a^4*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3}\,dx$$
$$= -\frac{\sqrt{3}(44\,b^3c-20\,ab^2d+5\,a^2be+a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^4} - \frac{(44\,b^3c-20\,ab^2d+5\,a^2be+a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^4} - \frac{(44\,b^3c-20\,ab^2d+5\,a^2be+a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5b} + \frac{17\,b^4cx^4-11\,ab^3dx^4+5\,a^2b^2ex^4+a^3bfx^4+20\,ab^3cx-14\,a^2b^2dx+8\,a^3bex-2\,a^4fx}{18\left(bx^3+a\right)^2a^4b} + \frac{15\,bcx^3-5\,adx^3-2\,ac}{10\,a^4x^5}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^4) - 1/54*(44*b^3*c
- 20*a*b^2*d + 5*a^2*b*e + a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
((-a*b^2)^(2/3)*a^4) - 1/27*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b) + 1/18*(17*b^4*c*x^4 - 11*a*b^
3*d*x^4 + 5*a^2*b^2*e*x^4 + a^3*b*f*x^4 + 20*a*b^3*c*x - 14*a^2*b^2*d*x + 8
*a^3*b*e*x - 2*a^4*f*x)/((b*x^3 + a)^2*a^4*b) + 1/10*(15*b*c*x^3 - 5*a*d*x^
3 - 2*a*c)/(a^4*x^5)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3}\,dx$$
$$=\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{27\,a^{14/3}\,b^{4/3}}$$
$$-\frac{\frac{c}{5\,a}-\frac{x^9\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{18\,a^4}+\frac{x^3\,\left(5\,a\,d-11\,b\,c\right)}{10\,a^2}-\frac{x^6\,\left(-5\,f\,a^3+20\,e\,a^2\,b-80\,d\,a\,b^2+176\,c\,b^3\right)}{45\,a^3\,b}}{a^2\,x^5+2\,a\,b\,x^8+b^2\,x^{11}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{27\,a^{14/3}\,b^{4/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{27\,a^{14/3}\,b^{4/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(44*b^3*c + a^3*f - 20*a*b^2*d + 5*a^2*b*e))/(27*
a^(14/3)*b^(4/3)) - (c/(5*a) - (x^9*(44*b^3*c + a^3*f - 20*a*b^2*d + 5*a^2*
b*e))/(18*a^4) + (x^3*(5*a*d - 11*b*c))/(10*a^2) - (x^6*(176*b^3*c - 5*a^3*
f - 80*a*b^2*d + 20*a^2*b*e))/(45*a^3*b))/(a^2*x^5 + b^2*x^11 + 2*a*b*x^8)
+ (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(
44*b^3*c + a^3*f - 20*a*b^2*d + 5*a^2*b*e))/(27*a^(14/3)*b^(4/3)) - (log(3^
(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(44*b^3*c
+ a^3*f - 20*a*b^2*d + 5*a^2*b*e))/(27*a^(14/3)*b^(4/3))
```

3.299 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 343

$$\begin{aligned}
&\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^3}\,dx \\
&= -\frac{c}{7a^3x^7}+\frac{3bc-ad}{4a^4x^4}-\frac{6b^2c-3abd+a^2e}{a^5x} \\
&\quad -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^4(a+bx^3)^2}-\frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\,x^2}{9a^5(a+bx^3)} \\
&\quad +\frac{(65b^3c-35ab^2d+14a^2be-2a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{16/3}b^{2/3}} \\
&\quad +\frac{(65b^3c-35ab^2d+14a^2be-2a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{16/3}b^{2/3}} \\
&\quad -\frac{(65b^3c-35ab^2d+14a^2be-2a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{16/3}b^{2/3}}
\end{aligned}$$

```
[Out] -1/7*c/a^3/x^7+1/4*(-a*d+3*b*c)/a^4/x^4+(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x-1/6*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^4/(b*x^3+a)^2-1/9*(-2*a^3*f+5*a^2*b*e-
8*a*b^2*d+11*b^3*c)*x^2/a^5/(b*x^3+a)+1/27*(-2*a^3*f+14*a^2*b*e-35*a*b^2*d+
65*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(16/3)/b^(2/3)-1/54*(-2*a^3*f+14*a^2*b*e-
35*a*b^2*d+65*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(16/3)/b^(
2/3)+1/27*(-2*a^3*f+14*a^2*b*e-35*a*b^2*d+65*b^3*c)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)/a^(1/3)*3^(1/2))/a^(16/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1843, 1848, 298, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^3}\,dx$$
$$= \frac{3bc-ad}{4a^4x^4}-\frac{c}{7a^3x^7}-\frac{a^2e-3abd+6b^2c}{a^5x}+\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-2a^3f+14a^2be-35ab^2d+65b^3c)}{9\sqrt{3}a^{16/3}b^{2/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-2a^3f+14a^2be-35ab^2d+65b^3c)}{54a^{16/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-2a^3f+14a^2be-35ab^2d+65b^3c)}{27a^{16/3}b^{2/3}}-\frac{x^2(-2a^3f+5a^2be-8ab^2d+11b^3c)}{9a^5(a+bx^3)}-\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^4(a+bx^3)^2}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x]
```

```
[Out] -1/7*c/(a^3*x^7) + (3*b*c - a*d)/(4*a^4*x^4) - (6*b^2*c - 3*a*b*d + a^2*e)/
(a^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^4*(a + b*x^3)^2) -
 ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*x^2)/(9*a^5*(a + b*x^3)) + (
(65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(16/3)*b^(2/3)) + ((65*b^3*c - 35*a*b^2
*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(16/3)*b^(2/3))
- ((65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(54*a^(16/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^4\,(a+bx^3)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{4b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}}{x^8(a+bx^3)^2}\,dx}{6ab^3}$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^4\,(a+bx^3)^2} - \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\,x^2}{9a^5\,(a+bx^3)} + \frac{\int \frac{18b^6c-18b^6\left(\frac{2bc}{a}-d\right)x^3+18b^6\left(\frac{3b^2c}{a^2}-\frac{2bd}{a}+e\right)x^6-\frac{2b^6\left(11b^3c-8ab^2d+5a^2be-2a^3f\right)x^9}{a^3}}{x^8(a+bx^3)}\,dx}{18a^2b^6}$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^4\,(a+bx^3)^2} - \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\,x^2}{9a^5\,(a+bx^3)} + \frac{\int \left(\frac{18b^6c}{ax^8}+\frac{18b^6(-3bc+ad)}{a^2x^5}+\frac{18b^6(6b^2c-3abd+a^2e)}{a^3x^2}+\frac{2b^6(-65b^3c+35ab^2d-14a^2be+2a^3f)x}{a^3(a+bx^3)}\right)dx}{18a^2b^6}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{4a^4x^4} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^4\,(a+bx^3)^2} - \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\,x^2}{9a^5\,(a+bx^3)} - \frac{(65b^3c-35ab^2d+14a^2be-2a^3f)\int \frac{x}{a+bx^3}\,dx}{9a^5}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{4a^4x^4} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^4\,(a+bx^3)^2} - \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\,x^2}{9a^5\,(a+bx^3)} + \frac{(65b^3c-35ab^2d+14a^2be-2a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{16/3}\sqrt[3]{b}} - \frac{(65b^3c-35ab^2d+14a^2be-2a^3f)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{16/3}\sqrt[3]{b}}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{4a^4x^4} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^4\,(a+bx^3)^2} - \frac{(11b^3c-8ab^2d+5a^2be-2a^3f)\,x^2}{9a^5\,(a+bx^3)} + \frac{(65b^3c-35ab^2d+14a^2be-2a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{16/3}b^{2/3}} - \frac{(65b^3c-35ab^2d+14a^2be-2a^3f)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{16/3}b^{2/3}} - \frac{(65b^3c-35ab^2d+14a^2be-2a^3f)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^5\sqrt[3]{b}}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc - ad}{4a^4x^4} - \frac{6b^2c - 3abd + a^2e}{a^5x} - \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^4\,(a + bx^3)^2} - \frac{(11b^3c - 8ab^2d + 5a^2be - 2a^3f)\,x^2}{9a^5\,(a + bx^3)} + \frac{(65b^3c - 35ab^2d + 14a^2be - 2a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{16/3}b^{2/3}} - \frac{(65b^3c - 35ab^2d + 14a^2be - 2a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54a^{16/3}b^{2/3}} - \frac{(65b^3c - 35ab^2d + 14a^2be - 2a^3f)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{16/3}b^{2/3}}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc - ad}{4a^4x^4} - \frac{6b^2c - 3abd + a^2e}{a^5x} - \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^4\,(a + bx^3)^2} - \frac{(11b^3c - 8ab^2d + 5a^2be - 2a^3f)\,x^2}{9a^5\,(a + bx^3)} + \frac{(65b^3c - 35ab^2d + 14a^2be - 2a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{16/3}b^{2/3}} + \frac{(65b^3c - 35ab^2d + 14a^2be - 2a^3f)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{16/3}b^{2/3}} - \frac{(65b^3c - 35ab^2d + 14a^2be - 2a^3f)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54a^{16/3}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.96

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^8\,(a + bx^3)^3}\,dx$$

$$= \frac{-\frac{108a^{7/3}c}{x^7} - \frac{189a^{4/3}(-3bc+ad)}{x^4} - \frac{756\sqrt[3]{a}(6b^2c-3abd+a^2e)}{x} + \frac{126a^{4/3}(-b^3c+ab^2d-a^2be+a^3f)x^2}{(a+bx^3)^2} + \frac{84\sqrt[3]{a}(-11b^3c+8ab^2d-5a^2be+}{a+bx^3}}{}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x]
[Out] ((-108*a^(7/3)*c)/x^7 - (189*a^(4/3)*(-3*b*c + a*d))/x^4 - (756*a^(1/3)*(6*
b^2*c - 3*a*b*d + a^2*e))/x + (126*a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*x^2)/(a + b*x^3)^2 + (84*a^(1/3)*(-11*b^3*c + 8*a*b^2*d - 5*a^2*b*e
+ 2*a^3*f)*x^2)/(a + b*x^3) + (28*Sqrt[3]*(65*b^3*c - 35*a*b^2*d + 14*a^2*b
```

```
*e - 2*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (28*(6
5*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2
/3) + (14*(-65*b^3*c + 35*a*b^2*d - 14*a^2*b*e + 2*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(756*a^(16/3))
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.74

method	result
default	$-\frac{c}{7a^{3}x^{7}}-\frac{ad-3bc}{4a^{4}x^{4}}-\frac{a^{2}e-3abd+6b^{2}c}{a^{5}x}+\frac{\frac{\left(\frac{2}{9}a^{3}bf-\frac{5}{9}a^{2}e\,b^{2}+\frac{8}{9}a\,b^{3}d-\frac{11}{9}b^{4}c\right)x^{5}+\frac{a\left(7f\,a^{3}-13a^{2}be+19a\,b^{2}d-25b^{3}c\right)x^{2}}{18}}{\left(b\,x^{3}+a\right)^{2}}+\left(\frac{2}{9}f\,a^{3}-\frac{14}{9}a^{2}b\right.}{}$
risch	$\frac{\frac{b\left(2f\,a^{3}-14a^{2}be+35a\,b^{2}d-65b^{3}c\right)x^{12}}{9a^{5}}+\frac{7\left(2f\,a^{3}-14a^{2}be+35a\,b^{2}d-65b^{3}c\right)x^{9}}{36a^{4}}-\frac{\left(14a^{2}e-35abd+65b^{2}c\right)x^{6}}{14a^{3}}-\frac{(7ad-13bc)x^{3}}{28a^{2}}-\frac{c}{7a}}{x^{7}\left(b\,x^{3}+a\right)^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}}\right.}{}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*c/a^3/x^7-1/4*(a*d-3*b*c)/a^4/x^4-(a^2*e-3*a*b*d+6*b^2*c)/a^5/x+1/a^5*
(((2/9*a^3*b*f-5/9*a^2*e*b^2+8/9*a*b^3*d-11/9*b^4*c)*x^5+1/18*a*(7*a^3*f-13
*a^2*b*e+19*a*b^2*d-25*b^3*c)*x^2)/(b*x^3+a)^2+(2/9*f*a^3-14/9*a^2*b*e+35/9
*a*b^2*d-65/9*b^3*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(298) = 596.

Time = 0.30 (sec) , antiderivative size = 1340, normalized size of antiderivative = 3.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/756*(84*(65*a*b^6*c - 35*a^2*b^5*d + 14*a^3*b^4*e - 2*a^4*b^3*f)*x^12 +
 147*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^9 + 108*a
```

```
^5*b^2*c + 54*(65*a^3*b^4*c - 35*a^4*b^3*d + 14*a^5*b^2*e)*x^6 - 27*(13*a^4
*b^3*c - 7*a^5*b^2*d)*x^3 + 42*sqrt(1/3)*((65*a*b^6*c - 35*a^2*b^5*d + 14*a
^3*b^4*e - 2*a^4*b^3*f)*x^13 + 2*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*b^3*
e - 2*a^5*b^2*f)*x^10 + (65*a^3*b^4*c - 35*a^4*b^3*d + 14*a^5*b^2*e - 2*a^6
*b*f)*x^7)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x
 + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*
b^2)^(2/3)*x)/(b*x^3 + a)) + 14*((65*b^5*c - 35*a*b^4*d + 14*a^2*b^3*e - 2*
a^3*b^2*f)*x^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d + 14*a^3*b^2*e - 2*a^4*b*f)*
x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*a^4*b*e - 2*a^5*f)*x^7)*(-a*b^2)^(
2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*((65*b^5*c - 3
5*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*f)*x^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d
 + 14*a^3*b^2*e - 2*a^4*b*f)*x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*a^4*b
*e - 2*a^5*f)*x^7)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^6*b^4*x^13
+ 2*a^7*b^3*x^10 + a^8*b^2*x^7), -1/756*(84*(65*a*b^6*c - 35*a^2*b^5*d + 14
*a^3*b^4*e - 2*a^4*b^3*f)*x^12 + 147*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*
b^3*e - 2*a^5*b^2*f)*x^9 + 108*a^5*b^2*c + 54*(65*a^3*b^4*c - 35*a^4*b^3*d
+ 14*a^5*b^2*e)*x^6 - 27*(13*a^4*b^3*c - 7*a^5*b^2*d)*x^3 + 84*sqrt(1/3)*((
65*a*b^6*c - 35*a^2*b^5*d + 14*a^3*b^4*e - 2*a^4*b^3*f)*x^13 + 2*(65*a^2*b^
5*c - 35*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^10 + (65*a^3*b^4*c - 35*
a^4*b^3*d + 14*a^5*b^2*e - 2*a^6*b*f)*x^7)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(s
qrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 14*((65*b^5*
c - 35*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*f)*x^13 + 2*(65*a*b^4*c - 35*a^2*
b^3*d + 14*a^3*b^2*e - 2*a^4*b*f)*x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*
a^4*b*e - 2*a^5*f)*x^7)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (
-a*b^2)^(2/3)) - 28*((65*b^5*c - 35*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*f)*x
^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d + 14*a^3*b^2*e - 2*a^4*b*f)*x^10 + (65*a
^2*b^3*c - 35*a^3*b^2*d + 14*a^4*b*e - 2*a^5*f)*x^7)*(-a*b^2)^(2/3)*log(b*x
 - (-a*b^2)^(1/3)))/(a^6*b^4*x^13 + 2*a^7*b^3*x^10 + a^8*b^2*x^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^8 \left(a + bx^3\right)^3} \, dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^3}\,dx = -\frac{28\,(65\,b^4c-35\,ab^3d+14\,a^2b^2e-2\,a^3bf)x^{12}+49\,(65\,ab^3c-35\,a^2b^2d+14\,a^3be-2\,a^4f)x^9+18\,(65\,a^2b^2c}{252\,(a^5b^2x^{13}+2\,a^6bx^{10}+a^7x^7)}$$
$$-\frac{\sqrt{3}(65\,b^3c-35\,ab^2d+14\,a^2be-2\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$-\frac{(65\,b^3c-35\,ab^2d+14\,a^2be-2\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^5b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+\frac{(65\,b^3c-35\,ab^2d+14\,a^2be-2\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^5b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/252*(28*(65*b^4*c - 35*a*b^3*d + 14*a^2*b^2*e - 2*a^3*b*f)*x^12 + 49*(65
*a*b^3*c - 35*a^2*b^2*d + 14*a^3*b*e - 2*a^4*f)*x^9 + 18*(65*a^2*b^2*c - 35
*a^3*b*d + 14*a^4*e)*x^6 + 36*a^4*c - 9*(13*a^3*b*c - 7*a^4*d)*x^3)/(a^5*b^
2*x^13 + 2*a^6*b*x^10 + a^7*x^7) - 1/27*sqrt(3)*(65*b^3*c - 35*a*b^2*d + 14
*a^2*b*e - 2*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^
5*b*(a/b)^(1/3)) - 1/54*(65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*log(
x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*b*(a/b)^(1/3)) + 1/27*(65*b^3*c - 3
5*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*log(x + (a/b)^(1/3))/(a^5*b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.09

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^3}\,dx$$
$$= -\frac{\sqrt{3}(65\,b^3c-35\,ab^2d+14\,a^2be-2\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^5} + \frac{(65\,b^3c-35\,ab^2d+14\,a^2be-2\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{1}{3}}a^5} + \frac{\left(65\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-35\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+14\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^6} - \frac{22\,b^4cx^5-16\,ab^3dx^5+10\,a^2b^2ex^5-4\,a^3bfx^5+25\,ab^3cx^2-19\,a^2b^2dx^2+13\,a^3bex^2-7\,a^4fx^2}{18\left(bx^3+a\right)^2a^5} - \frac{168\,b^2cx^6-84\,abdx^6+28\,a^2ex^6-21\,abcx^3+7\,a^2dx^3+4\,a^2c}{28\,a^5x^7}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^5) + 1/54*(65*b^3
*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/((-a*b^2)^(1/3)*a^5) + 1/27*(65*b^3*c*(-a/b)^(1/3) - 35*a*b^2*d*(-a/b
)^(1/3) + 14*a^2*b*e*(-a/b)^(1/3) - 2*a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(
abs(x - (-a/b)^(1/3)))/a^6 - 1/18*(22*b^4*c*x^5 - 16*a*b^3*d*x^5 + 10*a^2*b
^2*e*x^5 - 4*a^3*b*f*x^5 + 25*a*b^3*c*x^2 - 19*a^2*b^2*d*x^2 + 13*a^3*b*e*x
^2 - 7*a^4*f*x^2)/((b*x^3 + a)^2*a^5) - 1/28*(168*b^2*c*x^6 - 84*a*b*d*x^6
+ 28*a^2*e*x^6 - 21*a*b*c*x^3 + 7*a^2*d*x^3 + 4*a^2*c)/(a^5*x^7)
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^3}\,dx=\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-2\,f\,a^3+14\,e\,a^2\,b-35\,d\,a\,b^2+65\,c\,b^3\right)}{27\,a^{16/3}\,b^{2/3}}$$
$$-\frac{\frac{c}{7\,a}+\frac{7\,x^9\,\left(-2\,f\,a^3+14\,e\,a^2\,b-35\,d\,a\,b^2+65\,c\,b^3\right)}{36\,a^4}+\frac{x^3\,\left(7\,a\,d-13\,b\,c\right)}{28\,a^2}+\frac{x^6\,\left(14\,e\,a^2-35\,d\,a\,b+65\,c\,b^2\right)}{14\,a^3}+\frac{b\,x^{12}\,\left(-2\,f\,a^3+14\,e\,a^2\,b-35\,d\,a\,b^2+\right.}{9\,a^5}}{a^2\,x^7+2\,a\,b\,x^{10}+b^2\,x^{13}}$$
$$-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-2\,f\,a^3+14\,e\,a^2\,b-35\,d\,a\,b^2+65\,c\,b^3\right)}{27\,a^{16/3}\,b^{2/3}}$$
$$+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-2\,f\,a^3+14\,e\,a^2\,b-35\,d\,a\,b^2+65\,c\,b^3\right)}{27\,a^{16/3}\,b^{2/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(65*b^3*c - 2*a^3*f - 35*a*b^2*d + 14*a^2*b*e))/(
27*a^(16/3)*b^(2/3)) - (c/(7*a) + (7*x^9*(65*b^3*c - 2*a^3*f - 35*a*b^2*d +
 14*a^2*b*e))/(36*a^4) + (x^3*(7*a*d - 13*b*c))/(28*a^2) + (x^6*(65*b^2*c +
 14*a^2*e - 35*a*b*d))/(14*a^3) + (b*x^12*(65*b^3*c - 2*a^3*f - 35*a*b^2*d
+ 14*a^2*b*e))/(9*a^5))/(a^2*x^7 + b^2*x^13 + 2*a*b*x^10) - (log(3^(1/2)*a^
(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(65*b^3*c - 2*a^3*
f - 35*a*b^2*d + 14*a^2*b*e))/(27*a^(16/3)*b^(2/3)) + (log(3^(1/2)*a^(1/3)*
1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(65*b^3*c - 2*a^3*f - 35
*a*b^2*d + 14*a^2*b*e))/(27*a^(16/3)*b^(2/3))
```

3.300 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 341

$$\begin{aligned}
\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^3}\,dx
&= -\frac{c}{8a^3x^8}+\frac{3bc-ad}{5a^4x^5}-\frac{6b^2c-3abd+a^2e}{2a^5x^2} \\
&\quad -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^4(a+bx^3)^2}-\frac{(23b^3c-17ab^2d+11a^2be-5a^3f)\,x}{18a^5(a+bx^3)} \\
&\quad +\frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{17/3}\sqrt[3]{b}} \\
&\quad -\frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{17/3}\sqrt[3]{b}} \\
&\quad +\frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{17/3}\sqrt[3]{b}}
\end{aligned}$$

```
[Out] -1/8*c/a^3/x^8+1/5*(-a*d+3*b*c)/a^4/x^5+1/2*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x^
2-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^4/(b*x^3+a)^2-1/18*(-5*a^3*f+11*a^
2*b*e-17*a*b^2*d+23*b^3*c)*x/a^5/(b*x^3+a)-1/27*(-5*a^3*f+20*a^2*b*e-44*a*b
^2*d+77*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(17/3)/b^(1/3)+1/54*(-5*a^3*f+20*a^2
*b*e-44*a*b^2*d+77*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(17/3
)/b^(1/3)+1/27*(-5*a^3*f+20*a^2*b*e-44*a*b^2*d+77*b^3*c)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(17/3)/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1843, 1848, 206, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^3}\,dx$$
$$=\frac{3bc-ad}{5a^4x^5}-\frac{c}{8a^3x^8}-\frac{a^2e-3abd+6b^2c}{2a^5x^2}$$
$$+\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-5a^3f+20a^2be-44ab^2d+77b^3c)}{9\sqrt{3}a^{17/3}\sqrt[3]{b}}$$
$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-5a^3f+20a^2be-44ab^2d+77b^3c)}{27a^{17/3}\sqrt[3]{b}}$$
$$+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-5a^3f+20a^2be-44ab^2d+77b^3c)}{54a^{17/3}\sqrt[3]{b}}$$
$$-\frac{x(-5a^3f+11a^2be-17ab^2d+23b^3c)}{18a^5(a+bx^3)}-\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^4(a+bx^3)^2}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x]
```

```
[Out] -1/8*c/(a^3*x^8) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/
(2*a^5*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^4*(a + b*x^3)^2)
 - ((23*b^3*c - 17*a*b^2*d + 11*a^2*b*e - 5*a^3*f)*x)/(18*a^5*(a + b*x^3))
+ ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(17/3)*b^(1/3)) - ((77*b^3*c - 44*a*
b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(17/3)*b^(1/3
)) + ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(54*a^(17/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^4\,(a+bx^3)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{5b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}}{x^9(a+bx^3)^2}\,dx}{6ab^3}$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^4\,(a+bx^3)^2} - \frac{(23b^3c-17ab^2d+11a^2be-5a^3f)\,x}{18a^5\,(a+bx^3)} + \frac{\int \frac{18b^6c-18b^6\left(\frac{2bc}{a}-d\right)x^3+18b^6\left(\frac{3b^2c}{a^2}-\frac{2bd}{a}+e\right)x^6-\frac{2b^6\left(23b^3c-17ab^2d+11a^2be-5a^3f\right)x^9}{a^3}}{x^9(a+bx^3)}\,dx}{18a^2b^6}$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^4\,(a+bx^3)^2} - \frac{(23b^3c-17ab^2d+11a^2be-5a^3f)\,x}{18a^5\,(a+bx^3)} + \frac{\int \left(\frac{18b^6c}{ax^9}+\frac{18b^6(-3bc+ad)}{a^2x^6}+\frac{18b^6\left(6b^2c-3abd+a^2e\right)}{a^3x^3}+\frac{2b^6\left(-77b^3c+44ab^2d-20a^2be+5a^3f\right)}{a^3(a+bx^3)}\right)dx}{18a^2b^6}$$

$$= -\frac{c}{8a^3x^8}+\frac{3bc-ad}{5a^4x^5}-\frac{6b^2c-3abd+a^2e}{2a^5x^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^4\,(a+bx^3)^2} - \frac{(23b^3c-17ab^2d+11a^2be-5a^3f)\,x}{18a^5\,(a+bx^3)} - \frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\int \frac{1}{a+bx^3}\,dx}{9a^5}$$

$$= -\frac{c}{8a^3x^8}+\frac{3bc-ad}{5a^4x^5}-\frac{6b^2c-3abd+a^2e}{2a^5x^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^4\,(a+bx^3)^2} - \frac{(23b^3c-17ab^2d+11a^2be-5a^3f)\,x}{18a^5\,(a+bx^3)} - \frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{17/3}} - \frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{17/3}}$$

$$= -\frac{c}{8a^3x^8} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{2a^5x^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^4\,(a+bx^3)^2} - \frac{(23b^3c-17ab^2d+11a^2be-5a^3f)\,x}{18a^5\,(a+bx^3)} - \frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{17/3}\sqrt[3]{b}} - \frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{16/3}} + \frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{17/3}\sqrt[3]{b}}$$

$$= -\frac{c}{8a^3x^8} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{2a^5x^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^4\,(a+bx^3)^2} - \frac{(23b^3c-17ab^2d+11a^2be-5a^3f)\,x}{18a^5\,(a+bx^3)} - \frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{17/3}\sqrt[3]{b}} + \frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{17/3}\sqrt[3]{b}} - \frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{17/3}\sqrt[3]{b}}$$

$$= -\frac{c}{8a^3x^8} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{2a^5x^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^4\,(a+bx^3)^2} - \frac{(23b^3c-17ab^2d+11a^2be-5a^3f)\,x}{18a^5\,(a+bx^3)} + \frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{17/3}\sqrt[3]{b}} - \frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{17/3}\sqrt[3]{b}} + \frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{17/3}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^3}\,dx$$

$$= \frac{-\frac{135a^{8/3}c}{x^8}-\frac{216a^{5/3}(-3bc+ad)}{x^5}-\frac{540a^{2/3}(6b^2c-3abd+a^2e)}{x^2}+\frac{180a^{5/3}(-b^3c+ab^2d-a^2be+a^3f)x}{(a+bx^3)^2}+\frac{60a^{2/3}(-23b^3c+17ab^2d-11a^2be+5\cdots}{a+bx^3}}{\cdots}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x]
```

```
[Out] ((-135*a^(8/3)*c)/x^8 - (216*a^(5/3)*(-3*b*c + a*d))/x^5 - (540*a^(2/3)*(6*
b^2*c - 3*a*b*d + a^2*e))/x^2 + (180*a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e
+ a^3*f)*x)/(a + b*x^3)^2 + (60*a^(2/3)*(-23*b^3*c + 17*a*b^2*d - 11*a^2*b*
e + 5*a^3*f)*x)/(a + b*x^3) + (40*Sqrt[3]*(77*b^3*c - 44*a*b^2*d + 20*a^2*b
*e - 5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (40*(-
77*b^3*c + 44*a*b^2*d - 20*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(
1/3) + (20*(77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(1080*a^(17/3))
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.74

method	result
default	$-\frac{c}{8a^3x^8}-\frac{ad-3bc}{5a^4x^5}-\frac{a^2e-3abd+6b^2c}{2a^5x^2}+\frac{\frac{\left(\frac{5}{18}a^3bf-\frac{11}{18}a^2eb^2+\frac{17}{18}ab^3d-\frac{23}{18}b^4c\right)x^4+\frac{a\left(4fa^3-7a^2be+10ab^2d-13b^3c\right)x}{9}}{\left(bx^3+a\right)^2}+\frac{\left(5fa^3-20a^2be\cdots\right.}{\cdots}}{\cdots}$
risch	$\frac{\frac{b\left(5fa^3-20a^2be+44ab^2d-77b^3c\right)x^{12}}{18a^5}+\frac{4\left(5fa^3-20a^2be+44ab^2d-77b^3c\right)x^9}{45a^4}-\frac{\left(20a^2e-44abd+77b^2c\right)x^6}{40a^3}-\frac{(4ad-7bc)x^3}{20a^2}-\frac{c}{8a}}{x^8\left(bx^3+a\right)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\cdots}\right.}{\cdots}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*c/a^3/x^8-1/5*(a*d-3*b*c)/a^4/x^5-1/2*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^2+
1/a^5*(((5/18*a^3*b*f-11/18*a^2*e*b^2+17/18*a*b^3*d-23/18*b^4*c)*x^4+1/9*a*
(4*a^3*f-7*a^2*b*e+10*a*b^2*d-13*b^3*c)*x)/(b*x^3+a)^2+1/9*(5*a^3*f-20*a^2*
b*e+44*a*b^2*d-77*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(
2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(294) = 588$.

Time = 0.28 (sec) , antiderivative size = 1317, normalized size of antiderivative = 3.86

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^9 (a + bx^3)^3} \, dx = \text{Too large to display}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/1080*(60*(77*a^2*b^5*c - 44*a^3*b^4*d + 20*a^4*b^3*e - 5*a^5*b^2*f)*x^1
2 + 96*(77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*x^9 + 135*a
^6*b*c + 27*(77*a^4*b^3*c - 44*a^5*b^2*d + 20*a^6*b*e)*x^6 - 54*(7*a^5*b^2*
c - 4*a^6*b*d)*x^3 + 60*sqrt(1/3)*((77*a*b^6*c - 44*a^2*b^5*d + 20*a^3*b^4*
e - 5*a^4*b^3*f)*x^14 + 2*(77*a^2*b^5*c - 44*a^3*b^4*d + 20*a^4*b^3*e - 5*a
^5*b^2*f)*x^11 + (77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*x
^8)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*s
qrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3
)/b))/(b*x^3 + a)) - 20*((77*b^5*c - 44*a*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*
f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b^2*e - 5*a^4*b*f)*x^11 + (
77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e - 5*a^5*f)*x^8)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*((77*b^5*c - 44*a*b^4*d +
 20*a^2*b^3*e - 5*a^3*b^2*f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b
^2*e - 5*a^4*b*f)*x^11 + (77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e - 5*a^5*
f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^7*b^3*x^14 + 2*a^8*b^2
*x^11 + a^9*b*x^8), -1/1080*(60*(77*a^2*b^5*c - 44*a^3*b^4*d + 20*a^4*b^3*e
 - 5*a^5*b^2*f)*x^12 + 96*(77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*b^2*e - 5*a
^6*b*f)*x^9 + 135*a^6*b*c + 27*(77*a^4*b^3*c - 44*a^5*b^2*d + 20*a^6*b*e)*x
^6 - 54*(7*a^5*b^2*c - 4*a^6*b*d)*x^3 + 120*sqrt(1/3)*((77*a*b^6*c - 44*a^2
*b^5*d + 20*a^3*b^4*e - 5*a^4*b^3*f)*x^14 + 2*(77*a^2*b^5*c - 44*a^3*b^4*d
+ 20*a^4*b^3*e - 5*a^5*b^2*f)*x^11 + (77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*
b^2*e - 5*a^6*b*f)*x^8)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(
2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*((77*b^5*c - 44*a
*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d +
20*a^3*b^2*e - 5*a^4*b*f)*x^11 + (77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e
- 5*a^5*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)
*a) + 40*((77*b^5*c - 44*a*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*f)*x^14 + 2*(77
```

```
*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b^2*e - 5*a^4*b*f)*x^11 + (77*a^2*b^3*c -
44*a^3*b^2*d + 20*a^4*b*e - 5*a^5*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)
^(2/3)))/(a^7*b^3*x^14 + 2*a^8*b^2*x^11 + a^9*b*x^8)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^3}\,dx = -\frac{20\,(77\,b^4c-44\,ab^3d+20\,a^2b^2e-5\,a^3bf)x^{12}+32\,(77\,ab^3c-44\,a^2b^2d+20\,a^3be-5\,a^4f)x^9+9\,(77\,a^2b^2c}{360\,(a^5b^2x^{14}+2\,a^6bx^{11}+a^7x^8)} - \frac{\sqrt{3}(77\,b^3c-44\,ab^2d+20\,a^2be-5\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(77\,b^3c-44\,ab^2d+20\,a^2be-5\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^5b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(77\,b^3c-44\,ab^2d+20\,a^2be-5\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^5b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/360*(20*(77*b^4*c - 44*a*b^3*d + 20*a^2*b^2*e - 5*a^3*b*f)*x^12 + 32*(77
*a*b^3*c - 44*a^2*b^2*d + 20*a^3*b*e - 5*a^4*f)*x^9 + 9*(77*a^2*b^2*c - 44*
a^3*b*d + 20*a^4*e)*x^6 + 45*a^4*c - 18*(7*a^3*b*c - 4*a^4*d)*x^3)/(a^5*b^2
*x^14 + 2*a^6*b*x^11 + a^7*x^8) - 1/27*sqrt(3)*(77*b^3*c - 44*a*b^2*d + 20*
a^2*b*e - 5*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5
*b*(a/b)^(2/3)) + 1/54*(77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*b*(a/b)^(2/3)) - 1/27*(77*b^3*c - 44
*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*log(x + (a/b)^(1/3))/(a^5*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.14

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^3}\,dx = \frac{(77\,b^3c-44\,ab^2d+20\,a^2be-5\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^6} - \frac{\sqrt{3}\left(77\,(-ab^2)^{\frac{1}{3}}\,b^3c-44\,(-ab^2)^{\frac{1}{3}}\,ab^2d+20\,(-ab^2)^{\frac{1}{3}}\,a^2be-5\,(-ab^2)^{\frac{1}{3}}\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^6b} - \frac{\left(77\,(-ab^2)^{\frac{1}{3}}\,b^3c-44\,(-ab^2)^{\frac{1}{3}}\,ab^2d+20\,(-ab^2)^{\frac{1}{3}}\,a^2be-5\,(-ab^2)^{\frac{1}{3}}\,a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^6b} - \frac{23\,b^4cx^4-17\,ab^3dx^4+11\,a^2b^2ex^4-5\,a^3bfx^4+26\,ab^3cx-20\,a^2b^2dx+14\,a^3bex-8\,a^4fx}{18\,(bx^3+a)^2a^5} - \frac{120\,b^2cx^6-60\,abdx^6+20\,a^2ex^6-24\,abcx^3+8\,a^2dx^3+5\,a^2c}{40\,a^5x^8}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*(77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/a^6 - 1/27*sqrt(3)*(77*(-a*b^2)^(1/3)*b^3*c - 44*(-a*b^2)^
(1/3)*a*b^2*d + 20*(-a*b^2)^(1/3)*a^2*b*e - 5*(-a*b^2)^(1/3)*a^3*f)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^6*b) - 1/54*(77*(-a*b^2)^
(1/3)*b^3*c - 44*(-a*b^2)^(1/3)*a*b^2*d + 20*(-a*b^2)^(1/3)*a^2*b*e - 5*(-a
*b^2)^(1/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^6*b) - 1/18*
(23*b^4*c*x^4 - 17*a*b^3*d*x^4 + 11*a^2*b^2*e*x^4 - 5*a^3*b*f*x^4 + 26*a*b^
3*c*x - 20*a^2*b^2*d*x + 14*a^3*b*e*x - 8*a^4*f*x)/((b*x^3 + a)^2*a^5) - 1/
40*(120*b^2*c*x^6 - 60*a*b*d*x^6 + 20*a^2*e*x^6 - 24*a*b*c*x^3 + 8*a^2*d*x^
3 + 5*a^2*c)/(a^5*x^8)
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\,(a+bx^3)^3}\,dx =$$
$$-\frac{\frac{c}{8\,a}+\frac{4\,x^9\,\left(-5\,f\,a^3+20\,e\,a^2\,b-44\,d\,a\,b^2+77\,c\,b^3\right)}{45\,a^4}+\frac{x^3\,\left(4\,a\,d-7\,b\,c\right)}{20\,a^2}+\frac{x^6\,\left(20\,e\,a^2-44\,d\,a\,b+77\,c\,b^2\right)}{40\,a^3}+\frac{b\,x^{12}\,\left(-5\,f\,a^3+20\,e\,a^2\,b-44\,d\,a\,b^2+\right.}{18\,a^5}}{a^2\,x^8+2\,a\,b\,x^{11}+b^2\,x^{14}}$$
$$-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-5\,f\,a^3+20\,e\,a^2\,b-44\,d\,a\,b^2+77\,c\,b^3\right)}{27\,a^{17/3}\,b^{1/3}}$$
$$-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-5\,f\,a^3+20\,e\,a^2\,b-44\,d\,a\,b^2+77\,c\,b^3\right)}{27\,a^{17/3}\,b^{1/3}}$$
$$+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-5\,f\,a^3+20\,e\,a^2\,b-44\,d\,a\,b^2+77\,c\,b^3\right)}{27\,a^{17/3}\,b^{1/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(77
*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e))/(27*a^(17/3)*b^(1/3)) - (log(b
^(1/3)*x + a^(1/3))*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e))/(27*a^(
17/3)*b^(1/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)
*1i)/2 - 1/2)*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e))/(27*a^(17/3)*
b^(1/3)) - (c/(8*a) + (4*x^9*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e)
)/(45*a^4) + (x^3*(4*a*d - 7*b*c))/(20*a^2) + (x^6*(77*b^2*c + 20*a^2*e - 4
4*a*b*d))/(40*a^3) + (b*x^12*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e)
)/(18*a^5))/(a^2*x^8 + b^2*x^14 + 2*a*b*x^11)
```

3.301 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 381

$$\begin{aligned}
&\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^3}\,dx \\
&= -\frac{c}{10a^3x^{10}}+\frac{3bc-ad}{7a^4x^7}-\frac{6b^2c-3abd+a^2e}{4a^5x^4}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} \\
&\quad+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^5(a+bx^3)^2}+\frac{b(14b^3c-11ab^2d+8a^2be-5a^3f)\,x^2}{9a^6(a+bx^3)} \\
&\quad-\frac{\sqrt[3]{b}(104b^3c-65ab^2d+35a^2be-14a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{19/3}} \\
&\quad-\frac{\sqrt[3]{b}(104b^3c-65ab^2d+35a^2be-14a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{19/3}} \\
&\quad+\frac{\sqrt[3]{b}(104b^3c-65ab^2d+35a^2be-14a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{19/3}}
\end{aligned}$$

```
[Out] -1/10*c/a^3/x^10+1/7*(-a*d+3*b*c)/a^4/x^7+1/4*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/
x^4+(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x+1/6*b*(-a^3*f+a^2*b*e-a*b^2
*d+b^3*c)*x^2/a^5/(b*x^3+a)^2+1/9*b*(-5*a^3*f+8*a^2*b*e-11*a*b^2*d+14*b^3*c
)*x^2/a^6/(b*x^3+a)-1/27*b^(1/3)*(-14*a^3*f+35*a^2*b*e-65*a*b^2*d+104*b^3*c
)*ln(a^(1/3)+b^(1/3)*x)/a^(19/3)+1/54*b^(1/3)*(-14*a^3*f+35*a^2*b*e-65*a*b^
2*d+104*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(19/3)-1/27*b^(1
/3)*(-14*a^3*f+35*a^2*b*e-65*a*b^2*d+104*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))/a^(19/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1843, 1848, 298, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^3}\,dx$$
$$= \frac{3bc-ad}{7a^4x^7}-\frac{c}{10a^3x^{10}}-\frac{a^2e-3abd+6b^2c}{4a^5x^4}$$
$$-\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-14a^3f+35a^2be-65ab^2d+104b^3c)}{9\sqrt{3}a^{19/3}}$$
$$-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-14a^3f+35a^2be-65ab^2d+104b^3c)}{27a^{19/3}}$$
$$+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-14a^3f+35a^2be-65ab^2d+104b^3c)}{54a^{19/3}}$$
$$+\frac{bx^2(-5a^3f+8a^2be-11ab^2d+14b^3c)}{9a^6(a+bx^3)}$$
$$+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{a^6x}+\frac{bx^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^5(a+bx^3)^2}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x]
```

```
[Out] -1/10*c/(a^3*x^10) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e
)/(4*a^5*x^4) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^5*(a + b*x^3)^2) + (b*(14*b^3*c
- 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*x^2)/(9*a^6*(a + b*x^3)) - (b^(1/3)*(10
4*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(19/3)) - (b^(1/3)*(104*b^3*c - 65*a*b^2
*d + 35*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(19/3)) + (b^(1
/3)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(54*a^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^5\,(a+bx^3)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{6b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}-\frac{4b^4\left(b^3c-ab^2d+a^2be-a^3f\right)x^{12}}{a^4}}{x^{11}(a+bx^3)^2}\,dx}{6ab^3}$$

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^5\,(a+bx^3)^2} + \frac{b(14b^3c-11ab^2d+8a^2be-5a^3f)\,x^2}{9a^6\,(a+bx^3)} + \frac{\int \frac{18b^7c-18b^7\left(\frac{2bc}{a}-d\right)x^3+18b^7\left(\frac{3b^2c}{a^2}-\frac{2bd}{a}+e\right)x^6-18b^7\left(\frac{4b^3c}{a^3}-\frac{3b^2d}{a^2}+\frac{2be}{a}-f\right)x^9+\frac{2b^8\left(14b^3c-11ab^2d+8a^2be-5a^3f\right)x^{12}}{a^4}}{x^{11}(a+bx^3)}\,dx}{18a^2b^7}$$

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^5\,(a+bx^3)^2} + \frac{b(14b^3c-11ab^2d+8a^2be-5a^3f)\,x^2}{9a^6\,(a+bx^3)} + \frac{\int \left(\frac{18b^7c}{ax^{11}}+\frac{18b^7(-3bc+ad)}{a^2x^8}+\frac{18b^7(6b^2c-3abd+a^2e)}{a^3x^5}+\frac{18b^7(-10b^3c+6ab^2d-3a^2be+a^3f)}{a^4x^2}-\frac{2b^8(-104b^3c+65ab^2d-35a^2be+14a^3f)x}{a^4(a+bx^3)}\right.}{18a^2b^7}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^5\,(a+bx^3)^2} + \frac{b(14b^3c-11ab^2d+8a^2be-5a^3f)\,x^2}{9a^6\,(a+bx^3)} + \frac{(b(104b^3c-65ab^2d+35a^2be-14a^3f))\int \frac{x}{a+bx^3}\,dx}{9a^6}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^5\,(a+bx^3)^2} + \frac{b(14b^3c-11ab^2d+8a^2be-5a^3f)\,x^2}{9a^6\,(a+bx^3)} - \frac{\left(b^{2/3}(104b^3c-65ab^2d+35a^2be-14a^3f)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{19/3}} + \frac{\left(b^{2/3}(104b^3c-65ab^2d+35a^2be-14a^3f)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{19/3}}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^5\,(a+bx^3)^2} + \frac{b(14b^3c-11ab^2d+8a^2be-5a^3f)\,x^2}{9a^6\,(a+bx^3)} - \frac{\sqrt[3]{b}(104b^3c-65ab^2d+35a^2be-14a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{19/3}} + \frac{\left(\sqrt[3]{b}(104b^3c-65ab^2d+35a^2be-14a^3f)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{19/3}} + \frac{\left(b^{2/3}(104b^3c-65ab^2d+35a^2be-14a^3f)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^6}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^5\,(a+bx^3)^2} + \frac{b(14b^3c-11ab^2d+8a^2be-5a^3f)\,x^2}{9a^6\,(a+bx^3)} - \frac{\sqrt[3]{b}(104b^3c-65ab^2d+35a^2be-14a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{19/3}} + \frac{\sqrt[3]{b}(104b^3c-65ab^2d+35a^2be-14a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{19/3}} + \frac{\left(\sqrt[3]{b}(104b^3c-65ab^2d+35a^2be-14a^3f)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{19/3}}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^5\,(a+bx^3)^2} + \frac{b(14b^3c-11ab^2d+8a^2be-5a^3f)\,x^2}{9a^6\,(a+bx^3)} - \frac{\sqrt[3]{b}(104b^3c-65ab^2d+35a^2be-14a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{19/3}} - \frac{\sqrt[3]{b}(104b^3c-65ab^2d+35a^2be-14a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{19/3}} + \frac{\sqrt[3]{b}(104b^3c-65ab^2d+35a^2be-14a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{19/3}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{378a^{10/3}c}{x^{10}} - \frac{540a^{7/3}(-3bc+ad)}{x^7} - \frac{945a^{4/3}(6b^2c-3abd+a^2e)}{x^4} - \frac{3780\sqrt[3]{a}(-10b^3c+6ab^2d-3a^2be+a^3f)}{x} - \frac{630a^{4/3}b(-b^3c+ab^2d-a^2be+}{(a+bx^3)^2}}{}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x]
```

```
[Out] ((-378*a^(10/3)*c)/x^10 - (540*a^(7/3)*(-3*b*c + a*d))/x^7 - (945*a^(4/3)*(
6*b^2*c - 3*a*b*d + a^2*e))/x^4 - (3780*a^(1/3)*(-10*b^3*c + 6*a*b^2*d - 3*
a^2*b*e + a^3*f))/x - (630*a^(4/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)
*x^2)/(a + b*x^3)^2 - (420*a^(1/3)*b*(-14*b^3*c + 11*a*b^2*d - 8*a^2*b*e +
5*a^3*f)*x^2)/(a + b*x^3) - 140*Sqrt[3]*b^(1/3)*(104*b^3*c - 65*a*b^2*d + 3
5*a^2*b*e - 14*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(
1/3)*(-104*b^3*c + 65*a*b^2*d - 35*a^2*b*e + 14*a^3*f)*Log[a^(1/3) + b^(1/3
)*x] + 70*b^(1/3)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(3780*a^(19/3))
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.76

method	result
default	$-\frac{c}{10a^3x^{10}} - \frac{ad-3bc}{7a^4x^7} - \frac{a^2e-3abd+6b^2c}{4a^5x^4} - \frac{f a^3-3a^2be+6a b^2d-10b^3c}{a^6x} - \frac{b\left(\frac{\frac{b\left(5f a^3-8a^2be+11a b^2d-14b^3c\right)x^5}{9}+\left(\frac{13}{18}a^4f-\frac{19}{18}a^3be\right.}{\left(b x^3+a\right)^2}\right.}{}$
risch	$\frac{-\frac{c}{10a}-\frac{(5ad-8bc)x^3}{35a^2}-\frac{\left(35a^2e-65abd+104b^2c\right)x^6}{140a^3}-\frac{\left(14f a^3-35a^2be+65a b^2d-104b^3c\right)x^9}{14a^4}-\frac{7b\left(14f a^3-35a^2be+65a b^2d-104b^3c\right)x^{12}}{36a^5}-\frac{b^2\left(14f a^3\right.}{}}{x^{10}(b x^3+a)^2}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*c/a^3/x^10-1/7*(a*d-3*b*c)/a^4/x^7-1/4*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^
4-(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6/x-b/a^6*((1/9*b*(5*a^3*f-8*a^2*b
*e+11*a*b^2*d-14*b^3*c)*x^5+(13/18*a^4*f-19/18*a^3*b*e+25/18*a^2*b^2*d-31/1
8*a*b^3*c)*x^2)/(b*x^3+a)^2+(14/9*f*a^3-35/9*a^2*b*e+65/9*a*b^2*d-104/9*b^3
*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.63

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^3}\,dx$$

$$= \frac{420\,(104\,b^5c-65\,ab^4d+35\,a^2b^3e-14\,a^3b^2f)x^{15}+735\,(104\,ab^4c-65\,a^2b^3d+35\,a^3b^2e-14\,a^4bf)x^{12}+27}{}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3780*(420*(104*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^15 + 7
35*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^12 + 270*(104
*a^2*b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)*x^9 - 27*(104*a^3*b^2*c
- 65*a^4*b*d + 35*a^5*e)*x^6 - 378*a^5*c + 108*(8*a^4*b*c - 5*a^5*d)*x^3 +
140*sqrt(3)*((104*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^16 +
2*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^13 + (104*a^2*
b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)*x^10)*(b/a)^(1/3)*arctan(2/3*
sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 70*((104*b^5*c - 65*a*b^4*d + 35*a^2
*b^3*e - 14*a^3*b^2*f)*x^16 + 2*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e
- 14*a^4*b*f)*x^13 + (104*a^2*b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)
*x^10)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 140*((104
*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^16 + 2*(104*a*b^4*c -
65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^13 + (104*a^2*b^3*c - 65*a^3*b^
2*d + 35*a^4*b*e - 14*a^5*f)*x^10)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)))/(a
^6*b^2*x^16 + 2*a^7*b*x^13 + a^8*x^10)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx = \frac{140\,(104\,b^5c-65\,ab^4d+35\,a^2b^3e-14\,a^3b^2f)x^{15}+245\,(104\,ab^4c-65\,a^2b^3d+35\,a^3b^2e-14\,a^4bf)x^{12}+90\,(}{1260\,(a^6b^2x^{16}} + \frac{\sqrt{3}(104\,b^3c-65\,ab^2d+35\,a^2be-14\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(104\,b^3c-65\,ab^2d+35\,a^2be-14\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(104\,b^3c-65\,ab^2d+35\,a^2be-14\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/1260*(140*(104*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^15 + 2
45*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^12 + 90*(104*
a^2*b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)*x^9 - 9*(104*a^3*b^2*c -
65*a^4*b*d + 35*a^5*e)*x^6 - 126*a^5*c + 36*(8*a^4*b*c - 5*a^5*d)*x^3)/(a^6
*b^2*x^16 + 2*a^7*b*x^13 + a^8*x^10) + 1/27*sqrt(3)*(104*b^3*c - 65*a*b^2*d
 + 35*a^2*b*e - 14*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(a^6*(a/b)^(1/3)) + 1/54*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^(1/3)) - 1/27*(104*b^3*
c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*log(x + (a/b)^(1/3))/(a^6*(a/b)^(1/
3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.26

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx = -\frac{\left(104\,b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-65\,ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+35\,a^2b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-14\,a^3bf\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7} - \frac{\sqrt{3}\left(104\left(-ab^2\right)^{\frac{2}{3}}b^3c-65\left(-ab^2\right)^{\frac{2}{3}}ab^2d+35\left(-ab^2\right)^{\frac{2}{3}}a^2be-14\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^7b} + \frac{\left(104\left(-ab^2\right)^{\frac{2}{3}}b^3c-65\left(-ab^2\right)^{\frac{2}{3}}ab^2d+35\left(-ab^2\right)^{\frac{2}{3}}a^2be-14\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right.}{54\,a^7b} + \frac{28\,b^5cx^5-22\,ab^4dx^5+16\,a^2b^3ex^5-10\,a^3b^2fx^5+31\,ab^4cx^2-25\,a^2b^3dx^2+19\,a^3b^2ex^2-13\,a^4bfx^2}{18\left(bx^3+a\right)^2a^6} + \frac{1400\,b^3cx^9-840\,ab^2dx^9+420\,a^2bex^9-140\,a^3fx^9-210\,ab^2cx^6+105\,a^2bdx^6-35\,a^3ex^6+60\,a^2bcx^3-}{140\,a^6x^{10}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(104*b^4*c*(-a/b)^(1/3) - 65*a*b^3*d*(-a/b)^(1/3) + 35*a^2*b^2*e*(-a/
b)^(1/3) - 14*a^3*b*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))
/a^7 - 1/27*sqrt(3)*(104*(-a*b^2)^(2/3)*b^3*c - 65*(-a*b^2)^(2/3)*a*b^2*d +
 35*(-a*b^2)^(2/3)*a^2*b*e - 14*(-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^7*b) + 1/54*(104*(-a*b^2)^(2/3)*b^3*c -
 65*(-a*b^2)^(2/3)*a*b^2*d + 35*(-a*b^2)^(2/3)*a^2*b*e - 14*(-a*b^2)^(2/3)*
a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^7*b) + 1/18*(28*b^5*c*x^
5 - 22*a*b^4*d*x^5 + 16*a^2*b^3*e*x^5 - 10*a^3*b^2*f*x^5 + 31*a*b^4*c*x^2 -
 25*a^2*b^3*d*x^2 + 19*a^3*b^2*e*x^2 - 13*a^4*b*f*x^2)/((b*x^3 + a)^2*a^6)
+ 1/140*(1400*b^3*c*x^9 - 840*a*b^2*d*x^9 + 420*a^2*b*e*x^9 - 140*a^3*f*x^9
 - 210*a*b^2*c*x^6 + 105*a^2*b*d*x^6 - 35*a^3*e*x^6 + 60*a^2*b*c*x^3 - 20*a
^3*d*x^3 - 14*a^3*c)/(a^6*x^10)
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx =$$

$$-\frac{\frac{c}{10\,a}-\frac{x^9\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{14\,a^4}+\frac{x^3\,\left(5\,a\,d-8\,b\,c\right)}{35\,a^2}+\frac{x^6\,\left(35\,e\,a^2-65\,d\,a\,b+104\,c\,b^2\right)}{140\,a^3}-\frac{7\,b\,x^{12}\,(-14\,f\,a^3+35\,e\,a^2\,b-65\,d}{36\,a^5}}{a^2\,x^{10}+2\,a\,b\,x^{13}+b^2\,x^{16}}$$

$$-\frac{b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{27\,a^{19/3}}$$

$$+\frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{27\,a^{19/3}}$$

$$-\frac{b^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{27\,a^{19/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x)
```

```
[Out] (b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
1/2)*(104*b^3*c - 14*a^3*f - 65*a*b^2*d + 35*a^2*b*e))/(27*a^(19/3)) - (b^(
1/3)*log(b^(1/3)*x + a^(1/3))*(104*b^3*c - 14*a^3*f - 65*a*b^2*d + 35*a^2*b
*e))/(27*a^(19/3)) - (c/(10*a) - (x^9*(104*b^3*c - 14*a^3*f - 65*a*b^2*d +
35*a^2*b*e))/(14*a^4) + (x^3*(5*a*d - 8*b*c))/(35*a^2) + (x^6*(104*b^2*c +
35*a^2*e - 65*a*b*d))/(140*a^3) - (7*b*x^12*(104*b^3*c - 14*a^3*f - 65*a*b^
2*d + 35*a^2*b*e))/(36*a^5) - (b^2*x^15*(104*b^3*c - 14*a^3*f - 65*a*b^2*d
+ 35*a^2*b*e))/(9*a^6))/(a^2*x^10 + b^2*x^16 + 2*a*b*x^13) - (b^(1/3)*log(3
^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(104*b^3*
c - 14*a^3*f - 65*a*b^2*d + 35*a^2*b*e))/(27*a^(19/3))
```

3.302 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 380

$$\begin{aligned}
&\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^3} dx \\
&= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} \\
&\quad + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5(a+bx^3)^2} + \frac{b(29b^3c-23ab^2d+17a^2be-11a^3f)\,x}{18a^6(a+bx^3)} \\
&\quad - \frac{b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{20/3}} \\
&\quad + \frac{b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{20/3}} \\
&\quad - \frac{b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{20/3}}
\end{aligned}$$

```
[Out] -1/11*c/a^3/x^11+1/8*(-a*d+3*b*c)/a^4/x^8+1/5*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/
x^5+1/2*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x^2+1/6*b*(-a^3*f+a^2*b*e
-a*b^2*d+b^3*c)*x/a^5/(b*x^3+a)^2+1/18*b*(-11*a^3*f+17*a^2*b*e-23*a*b^2*d+2
9*b^3*c)*x/a^6/(b*x^3+a)+1/27*b^(2/3)*(-20*a^3*f+44*a^2*b*e-77*a*b^2*d+119*
b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(20/3)-1/54*b^(2/3)*(-20*a^3*f+44*a^2*b*e-77
*a*b^2*d+119*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(20/3)-1/27
*b^(2/3)*(-20*a^3*f+44*a^2*b*e-77*a*b^2*d+119*b^3*c)*arctan(1/3*(a^(1/3)-2*
b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(20/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1843, 1848, 206, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^3}\,dx$$
$$= \frac{3bc-ad}{8a^4x^8} - \frac{c}{11a^3x^{11}} - \frac{a^2e-3abd+6b^2c}{5a^5x^5} - \frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-20a^3f+44a^2be-77ab^2d+119b^3c)}{9\sqrt{3}a^{20/3}} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-20a^3f+44a^2be-77ab^2d+119b^3c)}{54a^{20/3}} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-20a^3f+44a^2be-77ab^2d+119b^3c)}{27a^{20/3}} + \frac{bx(-11a^3f+17a^2be-23ab^2d+29b^3c)}{18a^6(a+bx^3)} + \frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{2a^6x^2} + \frac{bx(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^5(a+bx^3)^2}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x]
```

```
[Out] -1/11*c/(a^3*x^11) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d + a^2*e
)/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(2*a^6*x^2) + (b
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*(a + b*x^3)^2) + (b*(29*b^3*
c - 23*a*b^2*d + 17*a^2*b*e - 11*a^3*f)*x)/(18*a^6*(a + b*x^3)) - (b^(2/3)*
(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3
)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(20/3)) + (b^(2/3)*(119*b^3*c - 77*a*
b^2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(20/3)) - (b
^(2/3)*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(20/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

integral

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{6b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}-\frac{5b^4\left(b^3c-ab^2d+a^2be-a^3f\right)x^{12}}{a^4}}{x^{12}\left(a+bx^3\right)^2}\,dx}{6ab^3}$$

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5\left(a+bx^3\right)^2} + \frac{b(29b^3c-23ab^2d+17a^2be-11a^3f)\,x}{18a^6\left(a+bx^3\right)} + \frac{\int \frac{18b^7c-18b^7\left(\frac{2bc}{a}-d\right)x^3+18b^7\left(\frac{3b^2c}{a^2}-\frac{2bd}{a}+e\right)x^6-18b^7\left(\frac{4b^3c}{a^3}-\frac{3b^2d}{a^2}+\frac{2be}{a}-f\right)x^9+\frac{2b^8\left(29b^3c-23ab^2d+17a^2be-11a^3f\right)x^{12}}{a^4}}{x^{12}\left(a+bx^3\right)}\,dx}{18a^2b^7}$$

$$= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5\left(a+bx^3\right)^2} + \frac{b(29b^3c-23ab^2d+17a^2be-11a^3f)\,x}{18a^6\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^7c}{ax^{12}}+\frac{18b^7(-3bc+ad)}{a^2x^9}+\frac{18b^7\left(6b^2c-3abd+a^2e\right)}{a^3x^6}+\frac{18b^7\left(-10b^3c+6ab^2d-3a^2be+a^3f\right)}{a^4x^3}-\frac{2b^8\left(-119b^3c+77ab^2d-44a^2be+20a^3f\right)}{a^4\left(a+bx^3\right)}\right)}{18a^2b^7}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5\left(a+bx^3\right)^2} + \frac{b(29b^3c-23ab^2d+17a^2be-11a^3f)\,x}{18a^6\left(a+bx^3\right)} + \frac{\left(b(119b^3c-77ab^2d+44a^2be-20a^3f)\right)\int \frac{1}{a+bx^3}\,dx}{9a^6}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5\left(a+bx^3\right)^2} + \frac{b(29b^3c-23ab^2d+17a^2be-11a^3f)\,x}{18a^6\left(a+bx^3\right)} + \frac{\left(b(119b^3c-77ab^2d+44a^2be-20a^3f)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{20/3}} + \frac{\left(b(119b^3c-77ab^2d+44a^2be-20a^3f)\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{20/3}}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5\left(a+bx^3\right)^2} + \frac{b(29b^3c-23ab^2d+17a^2be-11a^3f)\,x}{18a^6\left(a+bx^3\right)} + \frac{b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{20/3}} - \frac{\left(b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{20/3}} + \frac{\left(b(119b^3c-77ab^2d+44a^2be-20a^3f)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{19/3}}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5\left(a+bx^3\right)^2} + \frac{b(29b^3c-23ab^2d+17a^2be-11a^3f)\,x}{18a^6\left(a+bx^3\right)} + \frac{b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{20/3}} - \frac{b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{20/3}} + \frac{\left(b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{20/3}}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5\left(a+bx^3\right)^2} + \frac{b(29b^3c-23ab^2d+17a^2be-11a^3f)\,x}{18a^6\left(a+bx^3\right)} - \frac{b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{20/3}} + \frac{b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{20/3}} - \frac{b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{20/3}}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^3}\,dx$$
$$= -\frac{c}{11a^3x^{11}}+\frac{3bc-ad}{8a^4x^8}-\frac{6b^2c-3abd+a^2e}{5a^5x^5}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5\left(a+bx^3\right)^2}+\frac{b(29b^3c-23ab^2d+17a^2be-11a^3f)\,x}{18a^6\left(a+bx^3\right)}+\frac{b^{2/3}(-119b^3c+77ab^2d-44a^2be+20a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{20/3}}+\frac{b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{20/3}}+\frac{b^{2/3}(-119b^3c+77ab^2d-44a^2be+20a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{20/3}}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x]
```

```
[Out] -1/11*c/(a^3*x^11) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d + a^2*e
)/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(2*a^6*x^2) + (b
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*(a + b*x^3)^2) + (b*(29*b^3*
c - 23*a*b^2*d + 17*a^2*b*e - 11*a^3*f)*x)/(18*a^6*(a + b*x^3)) + (b^(2/3)*
(-119*b^3*c + 77*a*b^2*d - 44*a^2*b*e + 20*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)
/a^(1/3))/Sqrt[3]])/(9*Sqrt[3]*a^(20/3)) + (b^(2/3)*(119*b^3*c - 77*a*b^2*d
 + 44*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(20/3)) + (b^(2/3
)*(-119*b^3*c + 77*a*b^2*d - 44*a^2*b*e + 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(54*a^(20/3))
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.76

method	result
default	$-\frac{c}{11a^3x^{11}}-\frac{ad-3bc}{8a^4x^8}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}-\frac{f\,a^3-3a^2be+6a\,b^2d-10b^3c}{2a^6x^2}-\frac{b\left(\frac{\left(\frac{11}{18}a^3bf-\frac{17}{18}a^2e\,b^2+\frac{23}{18}a\,b^3d-\frac{29}{18}b^4c\right)x^4+\frac{a\left(7f\,a^3-\right.}{}}{\left(b\,x^3+a\right)^2}\right.}{}$
risch	$\frac{-\frac{c}{11a}-\frac{(11ad-17bc)x^3}{88a^2}-\frac{\left(44a^2e-77abd+119b^2c\right)x^6}{220a^3}-\frac{\left(20f\,a^3-44a^2be+77a\,b^2d-119b^3c\right)x^9}{40a^4}-\frac{4b\left(20f\,a^3-44a^2be+77a\,b^2d-119b^3c\right)x^{12}}{45a^5}-\frac{b^2\left(20\right.}{}}{x^{11}\left(b\,x^3+a\right)^2}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/11*c/a^3/x^11-1/8*(a*d-3*b*c)/a^4/x^8-1/5*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^
5-1/2*(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6/x^2-b/a^6*(((11/18*a^3*b*f-1
7/18*a^2*e*b^2+23/18*a*b^3*d-29/18*b^4*c)*x^4+1/9*a*(7*a^3*f-10*a^2*b*e+13*
a*b^2*d-16*b^3*c)*x)/(b*x^3+a)^2+1/9*(20*a^3*f-44*a^2*b*e+77*a*b^2*d-119*b^
3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.72

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^3}\,dx$$

$$= \frac{660\,(119\,b^5c-77\,ab^4d+44\,a^2b^3e-20\,a^3b^2f)x^{15}+1056\,(119\,ab^4c-77\,a^2b^3d+44\,a^3b^2e-20\,a^4bf)x^{12}+29\cdots}{\cdots}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/11880*(660*(119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^15 +
1056*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^12 + 297*(1
19*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^9 - 54*(119*a^3*b^2*
c - 77*a^4*b*d + 44*a^5*e)*x^6 - 1080*a^5*c + 135*(17*a^4*b*c - 11*a^5*d)*x
^3 - 440*sqrt(3)*((119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^
17 + 2*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^14 + (119
*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^11)*(-b^2/a^2)^(1/3)*a
rctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 220*((119*b^5*c
 - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^17 + 2*(119*a*b^4*c - 77*a^2
*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^14 + (119*a^2*b^3*c - 77*a^3*b^2*d +
44*a^4*b*e - 20*a^5*f)*x^11)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2
)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 440*((119*b^5*c - 77*a*b^4*d + 44*a^2*b^3
*e - 20*a^3*b^2*f)*x^17 + 2*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20
*a^4*b*f)*x^14 + (119*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^1
1)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^6*b^2*x^17 + 2*a^7*b*
x^14 + a^8*x^11)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^3}\,dx$$

$$= \frac{220\,(119\,b^5c-77\,ab^4d+44\,a^2b^3e-20\,a^3b^2f)x^{15}+352\,(119\,ab^4c-77\,a^2b^3d+44\,a^3b^2e-20\,a^4bf)x^{12}+99}{3960\,(a^6b^2}$$

$$+\frac{\sqrt{3}(119\,b^3c-77\,ab^2d+44\,a^2be-20\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^6\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$$-\frac{(119\,b^3c-77\,ab^2d+44\,a^2be-20\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^6\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$$+\frac{(119\,b^3c-77\,ab^2d+44\,a^2be-20\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^6\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3960*(220*(119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^15 + 3
52*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^12 + 99*(119*
a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^9 - 18*(119*a^3*b^2*c -
 77*a^4*b*d + 44*a^5*e)*x^6 - 360*a^5*c + 45*(17*a^4*b*c - 11*a^5*d)*x^3)/(
a^6*b^2*x^17 + 2*a^7*b*x^14 + a^8*x^11) + 1/27*sqrt(3)*(119*b^3*c - 77*a*b^
2*d + 44*a^2*b*e - 20*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/(a^6*(a/b)^(2/3)) - 1/54*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^
3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^(2/3)) + 1/27*(119*b
^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*log(x + (a/b)^(1/3))/(a^6*(a/b)^
(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.14

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^3}\,dx$$
$$= \frac{\sqrt{3}\left(119\left(-ab^2\right)^{\frac{1}{3}}b^3c-77\left(-ab^2\right)^{\frac{1}{3}}ab^2d+44\left(-ab^2\right)^{\frac{1}{3}}a^2be-20\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^7}$$
$$- \frac{\left(119b^4c-77ab^3d+44a^2b^2e-20a^3bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^7}$$
$$+ \frac{\left(119\left(-ab^2\right)^{\frac{1}{3}}b^3c-77\left(-ab^2\right)^{\frac{1}{3}}ab^2d+44\left(-ab^2\right)^{\frac{1}{3}}a^2be-20\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^7}$$
$$+ \frac{29b^5cx^4-23ab^4dx^4+17a^2b^3ex^4-11a^3b^2fx^4+32ab^4cx-26a^2b^3dx+20a^3b^2ex-14a^4bfx}{18\left(bx^3+a\right)^2a^6}$$
$$+ \frac{2200b^3cx^9-1320ab^2dx^9+660a^2bex^9-220a^3fx^9-528ab^2cx^6+264a^2bdx^6-88a^3ex^6+165a^2bcx^3 -}{440a^6x^{11}}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(119*(-a*b^2)^(1/3)*b^3*c - 77*(-a*b^2)^(1/3)*a*b^2*d + 44*(-a
*b^2)^(1/3)*a^2*b*e - 20*(-a*b^2)^(1/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-
a/b)^(1/3))/(-a/b)^(1/3))/a^7 - 1/27*(119*b^4*c - 77*a*b^3*d + 44*a^2*b^2*e
 - 20*a^3*b*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^7 + 1/54*(119*(-a*
b^2)^(1/3)*b^3*c - 77*(-a*b^2)^(1/3)*a*b^2*d + 44*(-a*b^2)^(1/3)*a^2*b*e -
20*(-a*b^2)^(1/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^7 + 1/1
8*(29*b^5*c*x^4 - 23*a*b^4*d*x^4 + 17*a^2*b^3*e*x^4 - 11*a^3*b^2*f*x^4 + 32
*a*b^4*c*x - 26*a^2*b^3*d*x + 20*a^3*b^2*e*x - 14*a^4*b*f*x)/((b*x^3 + a)^2
*a^6) + 1/440*(2200*b^3*c*x^9 - 1320*a*b^2*d*x^9 + 660*a^2*b*e*x^9 - 220*a^
3*f*x^9 - 528*a*b^2*c*x^6 + 264*a^2*b*d*x^6 - 88*a^3*e*x^6 + 165*a^2*b*c*x^
3 - 55*a^3*d*x^3 - 40*a^3*c)/(a^6*x^11)
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^3}\,dx$$

$$= \frac{b^{2/3}\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-20\,f\,a^3+44\,e\,a^2\,b-77\,d\,a\,b^2+119\,c\,b^3\right)}{27\,a^{20/3}}$$

$$-\frac{\frac{c}{11\,a}-\frac{x^9\,\left(-20\,f\,a^3+44\,e\,a^2\,b-77\,d\,a\,b^2+119\,c\,b^3\right)}{40\,a^4}+\frac{x^3\,\left(11\,a\,d-17\,b\,c\right)}{88\,a^2}+\frac{x^6\,\left(44\,e\,a^2-77\,d\,a\,b+119\,c\,b^2\right)}{220\,a^3}-\frac{4\,b\,x^{12}\,\left(-20\,f\,a^3+44\,e\,a^2\,b-\cdots\right.}{45\,a^5}}{a^2\,x^{11}+2\,a\,b\,x^{14}+b^2\,x^{17}}$$

$$+\frac{b^{2/3}\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-20\,f\,a^3+44\,e\,a^2\,b-77\,d\,a\,b^2+119\,c\,b^3\right)}{27\,a^{20/3}}$$

$$-\frac{b^{2/3}\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-20\,f\,a^3+44\,e\,a^2\,b-77\,d\,a\,b^2+119\,c\,b^3\right)}{27\,a^{20/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x)
```

```
[Out] (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(119*b^3*c - 20*a^3*f - 77*a*b^2*d + 44*a
^2*b*e))/(27*a^(20/3)) - (c/(11*a) - (x^9*(119*b^3*c - 20*a^3*f - 77*a*b^2*
d + 44*a^2*b*e))/(40*a^4) + (x^3*(11*a*d - 17*b*c))/(88*a^2) + (x^6*(119*b^
2*c + 44*a^2*e - 77*a*b*d))/(220*a^3) - (4*b*x^12*(119*b^3*c - 20*a^3*f - 7
7*a*b^2*d + 44*a^2*b*e))/(45*a^5) - (b^2*x^15*(119*b^3*c - 20*a^3*f - 77*a*
b^2*d + 44*a^2*b*e))/(18*a^6))/(a^2*x^11 + b^2*x^17 + 2*a*b*x^14) + (b^(2/3
)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(1
19*b^3*c - 20*a^3*f - 77*a*b^2*d + 44*a^2*b*e))/(27*a^(20/3)) - (b^(2/3)*lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(119*b
^3*c - 20*a^3*f - 77*a*b^2*d + 44*a^2*b*e))/(27*a^(20/3))
```

3.303 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 424

$$\begin{aligned}
&\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx\\
&=-\frac{c}{13a^3x^{13}}+\frac{3bc-ad}{10a^4x^{10}}-\frac{6b^2c-3abd+a^2e}{7a^5x^7}\\
&\quad+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4}-\frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)}{a^7x}\\
&\quad-\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^6\left(a+bx^3\right)^2}-\frac{b^2(17b^3c-14ab^2d+11a^2be-8a^3f)\,x^2}{9a^7\left(a+bx^3\right)}\\
&\quad+\frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{22/3}}\\
&\quad+\frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{22/3}}\\
&\quad-\frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{22/3}}
\end{aligned}$$

```
[Out] -1/13*c/a^3/x^13+1/10*(-a*d+3*b*c)/a^4/x^10+1/7*(-a^2*e+3*a*b*d-6*b^2*c)/a^
5/x^7+1/4*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x^4-b*(-3*a^3*f+6*a^2*b
*e-10*a*b^2*d+15*b^3*c)/a^7/x-1/6*b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^
6/(b*x^3+a)^2-1/9*b^2*(-8*a^3*f+11*a^2*b*e-14*a*b^2*d+17*b^3*c)*x^2/a^7/(b*
x^3+a)+1/27*b^(4/3)*(-35*a^3*f+65*a^2*b*e-104*a*b^2*d+152*b^3*c)*ln(a^(1/3)
+b^(1/3)*x)/a^(22/3)-1/54*b^(4/3)*(-35*a^3*f+65*a^2*b*e-104*a*b^2*d+152*b^3
*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(22/3)+1/27*b^(4/3)*(-35*a^
3*f+65*a^2*b*e-104*a*b^2*d+152*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1
/3)*3^(1/2))/a^(22/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1843, 1848, 298, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^3}\,dx$$
$$=\frac{3bc-ad}{10a^4x^{10}}-\frac{c}{13a^3x^{13}}-\frac{a^2e-3abd+6b^2c}{7a^5x^7}$$
$$+\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-35a^3f+65a^2be-104ab^2d+152b^3c)}{9\sqrt{3}a^{22/3}}$$
$$-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-35a^3f+65a^2be-104ab^2d+152b^3c)}{54a^{22/3}}$$
$$+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-35a^3f+65a^2be-104ab^2d+152b^3c)}{27a^{22/3}}$$
$$-\frac{b^2x^2(-8a^3f+11a^2be-14ab^2d+17b^3c)}{9a^7(a+bx^3)}-\frac{b(-3a^3f+6a^2be-10ab^2d+15b^3c)}{a^7x}$$
$$+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{4a^6x^4}-\frac{b^2x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^6(a+bx^3)^2}$$

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x]
```

```
[Out] -1/13*c/(a^3*x^13) + (3*b*c - a*d)/(10*a^4*x^10) - (6*b^2*c - 3*a*b*d + a^2
*e)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(4*a^6*x^4) -
(b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(a^7*x) - (b^2*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^6*(a + b*x^3)^2) - (b^2*(17*b^3*c - 14*
a*b^2*d + 11*a^2*b*e - 8*a^3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^(4/3)*(152*b^
3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(22/3)) + (b^(4/3)*(152*b^3*c - 104*a*b^2*d
 + 65*a^2*b*e - 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(22/3)) - (b^(4/3
)*(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(54*a^(22/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{b^2(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^6\,(a+bx^3)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{6b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}-\frac{6b^4\left(b^3c-ab^2d+a^2be-a^3f\right)x^{12}}{a^4}+\frac{4b^5\left(b^3c-ab^2d+a^2be-a^3f\right)x^{15}}{a^5}}{x^{14}(a+bx^3)^2}\,d\cdots}{6ab^3}$$

$$= -\frac{b^2(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^6\,(a+bx^3)^2} - \frac{b^2(17b^3c - 14ab^2d + 11a^2be - 8a^3f)\,x^2}{9a^7\,(a+bx^3)} + \frac{\int \frac{18b^8c-18b^8\left(\frac{2bc}{a}-d\right)x^3+18b^8\left(\frac{3b^2c}{a^2}-\frac{2bd}{a}+e\right)x^6-18b^8\left(\frac{4b^3c}{a^3}-\frac{3b^2d}{a^2}+\frac{2be}{a}-f\right)x^9+\frac{18b^9\left(5b^3c-4ab^2d+3a^2be-2a^3f\right)x^{12}}{a^4}-\frac{2b^{10}(17b^3\cdots}{\cdots}}{x^{14}(a+bx^3)}\cdots}{18a^2b^8}$$

$$= -\frac{b^2(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^6\,(a+bx^3)^2} - \frac{b^2(17b^3c - 14ab^2d + 11a^2be - 8a^3f)\,x^2}{9a^7\,(a+bx^3)} + \frac{\int \left(\frac{18b^8c}{ax^{14}} + \frac{18b^8(-3bc+ad)}{a^2x^{11}} + \frac{18b^8\left(6b^2c-3abd+a^2e\right)}{a^3x^8} + \frac{18b^8\left(-10b^3c+6ab^2d-3a^2be+a^3f\right)}{a^4x^5} - \frac{18b^9\left(-15b^3c+10ab^2d-6a^2b\cdots\right)}{a^5x^2}\cdots\right)}{18a^2b^8}$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)}{a^7x} - \frac{b^2(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^6\,(a+bx^3)^2} - \frac{b^2(17b^3c - 14ab^2d + 11a^2be - 8a^3f)\,x^2}{9a^7\,(a+bx^3)} - \frac{\left(b^2(152b^3c-104ab^2d+65a^2be-35a^3f)\right)\int \frac{x}{a+bx^3}\,dx}{9a^7}$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)}{a^7x} - \frac{b^2(b^3c - ab^2d + a^2be - a^3f)\,x^2}{6a^6\,(a+bx^3)^2} - \frac{b^2(17b^3c - 14ab^2d + 11a^2be - 8a^3f)\,x^2}{9a^7\,(a+bx^3)} + \frac{\left(b^{5/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{22/3}} - \frac{\left(b^{5/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{22/3}}$$

$$
\begin{aligned}
&= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} \\
&+ \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)}{a^7x} \\
&- \frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^6\,(a+bx^3)^2} - \frac{b^2(17b^3c-14ab^2d+11a^2be-8a^3f)\,x^2}{9a^7\,(a+bx^3)} \\
&+ \frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{22/3}} \\
&- \frac{\left(b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{22/3}} \\
&- \frac{\left(b^{5/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^7}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} \\
&+ \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)}{a^7x} \\
&- \frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^6\,(a+bx^3)^2} - \frac{b^2(17b^3c-14ab^2d+11a^2be-8a^3f)\,x^2}{9a^7\,(a+bx^3)} \\
&+ \frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{22/3}} \\
&- \frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{22/3}} \\
&- \frac{\left(b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{22/3}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} \\
&+ \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)}{a^7x} \\
&- \frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^6\,(a+bx^3)^2} - \frac{b^2(17b^3c-14ab^2d+11a^2be-8a^3f)\,x^2}{9a^7\,(a+bx^3)} \\
&+ \frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{22/3}} \\
&+ \frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{22/3}} \\
&- \frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{22/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^3}\,dx$$
$$= -\frac{c}{13a^3x^{13}}+\frac{3bc-ad}{10a^4x^{10}}-\frac{6b^2c-3abd+a^2e}{7a^5x^7}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4}$$
$$+\frac{b(-15b^3c+10ab^2d-6a^2be+3a^3f)}{a^7x}+\frac{b^2(-b^3c+ab^2d-a^2be+a^3f)\,x^2}{6a^6(a+bx^3)^2}$$
$$+\frac{b^2(-17b^3c+14ab^2d-11a^2be+8a^3f)\,x^2}{9a^7(a+bx^3)}$$
$$+\frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{22/3}}$$
$$+\frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{22/3}}$$
$$+\frac{b^{4/3}(-152b^3c+104ab^2d-65a^2be+35a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{22/3}}$$

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x]
```

```
[Out] -1/13*c/(a^3*x^13) + (3*b*c - a*d)/(10*a^4*x^10) - (6*b^2*c - 3*a*b*d + a^2
*e)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(4*a^6*x^4) +
(b*(-15*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*f))/(a^7*x) + (b^2*(-(b^3*c)
 + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(6*a^6*(a + b*x^3)^2) + (b^2*(-17*b^3*c
+ 14*a*b^2*d - 11*a^2*b*e + 8*a^3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^(4/3)*(1
52*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]])/(9*Sqrt[3]*a^(22/3)) + (b^(4/3)*(152*b^3*c - 104*a*b^2*d
+ 65*a^2*b*e - 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(22/3)) + (b^(4/3)
*(-152*b^3*c + 104*a*b^2*d - 65*a^2*b*e + 35*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(54*a^(22/3))
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.77

method	result
default	$-\frac{c}{13a^3x^{13}}-\frac{ad-3bc}{10a^4x^{10}}-\frac{a^2e-3abd+6b^2c}{7a^5x^7}-\frac{f\,a^3-3a^2be+6a\,b^2d-10b^3c}{4a^6x^4}+\frac{b\left(3f\,a^3-6a^2be+10a\,b^2d-15b^3c\right)}{a^7x}+\frac{b^2\left(\frac{b\left(8f\,a^3-11\right.}{}\right.}{}$
risch	$\frac{-\frac{c}{13a}-\frac{(13ad-19bc)x^3}{130a^2}-\frac{\left(65a^2e-104abd+152b^2c\right)x^6}{455a^3}-\frac{\left(35f\,a^3-65a^2be+104a\,b^2d-152b^3c\right)x^9}{140a^4}+\frac{b\left(35f\,a^3-65a^2be+104a\,b^2d-152b^3c\right)x^{12}}{14a^5}+\frac{7b^2\left(3\right.}{}}{x^{13}\left(b\,x^3+a\right)^2}$

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13*c/a^3/x^13-1/10*(a*d-3*b*c)/a^4/x^10-1/7*(a^2*e-3*a*b*d+6*b^2*c)/a^5/
x^7-1/4*(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6/x^4+b*(3*a^3*f-6*a^2*b*e+1
0*a*b^2*d-15*b^3*c)/a^7/x+b^2/a^7*((1/9*b*(8*a^3*f-11*a^2*b*e+14*a*b^2*d-17
*b^3*c)*x^5+(19/18*a^4*f-25/18*a^3*b*e+31/18*a^2*b^2*d-37/18*a*b^3*c)*x^2)/
(b*x^3+a)^2+(35/9*f*a^3-65/9*a^2*b*e+104/9*a*b^2*d-152/9*b^3*c)*(-1/3/b/(a/
b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.62

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx =$$
$$-\frac{5460\,(152\,b^6c-104\,ab^5d+65\,a^2b^4e-35\,a^3b^3f)x^{18}+9555\,(152\,ab^5c-104\,a^2b^4d+65\,a^3b^3e-35\,a^4b^2f)x}{}$$

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/49140*(5460*(152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^18
 + 9555*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^15 +
3510*(152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^12 - 351
*(152*a^3*b^3*c - 104*a^4*b^2*d + 65*a^5*b*e - 35*a^6*f)*x^9 + 3780*a^6*c +
```

```
108*(152*a^4*b^2*c - 104*a^5*b*d + 65*a^6*e)*x^6 - 378*(19*a^5*b*c - 13*a^
6*d)*x^3 + 1820*sqrt(3)*((152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b
^3*f)*x^19 + 2*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*
x^16 + (152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^13)*(-
b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*((152*b^6
*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^19 + 2*(152*a*b^5*c - 104
*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^16 + (152*a^2*b^4*c - 104*a^3*b
^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^13)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)
^(2/3) - a*(-b/a)^(1/3)) + 1820*((152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e -
35*a^3*b^3*f)*x^19 + 2*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4
*b^2*f)*x^16 + (152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*
x^13)*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^7*b^2*x^19 + 2*a^8*b*x^16
+ a^9*x^13)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx=$$

$$-\frac{1820\,(152\,b^6c-104\,ab^5d+65\,a^2b^4e-35\,a^3b^3f)x^{18}+3185\,(152\,ab^5c-104\,a^2b^4d+65\,a^3b^3e-35\,a^4b^2f)}{}$$

$$-\frac{\sqrt{3}(152\,b^4c-104\,ab^3d+65\,a^2b^2e-35\,a^3bf)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

$$-\frac{(152\,b^4c-104\,ab^3d+65\,a^2b^2e-35\,a^3bf)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

$$+\frac{(152\,b^4c-104\,ab^3d+65\,a^2b^2e-35\,a^3bf)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="maxima")

[Out] -1/16380*(1820*(152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^18 + 3185*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^15 + 1170*(152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^12 - 117*(152*a^3*b^3*c - 104*a^4*b^2*d + 65*a^5*b*e - 35*a^6*f)*x^9 + 1260*a^6*c + 36*(152*a^4*b^2*c - 104*a^5*b*d + 65*a^6*e)*x^6 - 126*(19*a^5*b*c - 13*a^6*d)*x^3)/(a^7*b^2*x^19 + 2*a^8*b*x^16 + a^9*x^13) - 1/27*sqrt(3)*(152*b^4*c - 104*a*b^3*d + 65*a^2*b^2*e - 35*a^3*b*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^7*(a/b)^(1/3)) - 1/54*(152*b^4*c - 104*a*b^3*d + 65*a^2*b^2*e - 35*a^3*b*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^7*(a/b)^(1/3)) + 1/27*(152*b^4*c - 104*a*b^3*d + 65*a^2*b^2*e - 35*a^3*b*f)*log(x + (a/b)^(1/3))/(a^7*(a/b)^(1/3))

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.23

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^3}\,dx$$
$$= \frac{\sqrt{3}\left(152\,(-ab^2)^{\frac{2}{3}}\,b^3c-104\,(-ab^2)^{\frac{2}{3}}\,ab^2d+65\,(-ab^2)^{\frac{2}{3}}\,a^2be-35\,(-ab^2)^{\frac{2}{3}}\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^8}$$
$$+\frac{\left(152\,b^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-104\,ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+65\,a^2b^3e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-35\,a^3b^2f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^8}$$
$$-\frac{\left(152\,(-ab^2)^{\frac{2}{3}}\,b^3c-104\,(-ab^2)^{\frac{2}{3}}\,ab^2d+65\,(-ab^2)^{\frac{2}{3}}\,a^2be-35\,(-ab^2)^{\frac{2}{3}}\,a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^8}$$
$$-\frac{34\,b^6cx^5-28\,ab^5dx^5+22\,a^2b^4ex^5-16\,a^3b^3fx^5+37\,ab^5cx^2-31\,a^2b^4dx^2+25\,a^3b^3ex^2-19\,a^4b^2fx^2}{18\,(bx^3+a)^2a^7}$$
$$-\frac{27300\,b^4cx^{12}-18200\,ab^3dx^{12}+10920\,a^2b^2ex^{12}-5460\,a^3bfx^{12}-4550\,ab^3cx^9+2730\,a^2b^2dx^9-1365\,a^3b}{1820\,a^7x^{13}}$$

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="giac")

[Out] 1/27*sqrt(3)*(152*(-a*b^2)^(2/3)*b^3*c - 104*(-a*b^2)^(2/3)*a*b^2*d + 65*(-a*b^2)^(2/3)*a^2*b*e - 35*(-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^8 + 1/27*(152*b^5*c*(-a/b)^(1/3) - 104*a*b^4*d*(-a/b)^(1/3) + 65*a^2*b^3*e*(-a/b)^(1/3) - 35*a^3*b^2*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^8 - 1/54*(152*(-a*b^2)^(2/3)*b^3*c - 104*(-a*b^2)^(2/3)*a*b^2*d + 65*(-a*b^2)^(2/3)*a^2*b*e - 35*(-a*b^2)^(2/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^8 - 1/18*(34*b^6*c*x^5 - 28*a*b^5*d*x^5 + 22*a^2*b^4*e*x^5 - 16*a^3*b^3*f*x^5 + 37*a*b^5*c*x^2 - 31*

```
a^2*b^4*d*x^2 + 25*a^3*b^3*e*x^2 - 19*a^4*b^2*f*x^2)/((b*x^3 + a)^2*a^7) -
1/1820*(27300*b^4*c*x^12 - 18200*a*b^3*d*x^12 + 10920*a^2*b^2*e*x^12 - 5460
*a^3*b*f*x^12 - 4550*a*b^3*c*x^9 + 2730*a^2*b^2*d*x^9 - 1365*a^3*b*e*x^9 +
455*a^4*f*x^9 + 1560*a^2*b^2*c*x^6 - 780*a^3*b*d*x^6 + 260*a^4*e*x^6 - 546*
a^3*b*c*x^3 + 182*a^4*d*x^3 + 140*a^4*c)/(a^7*x^13)
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx$$
$$= \frac{b^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-35\,f\,a^3+65\,e\,a^2\,b-104\,d\,a\,b^2+152\,c\,b^3\right)}{27\,a^{22/3}}$$
$$-\frac{\frac{c}{13\,a}-\frac{x^9\,\left(-35\,f\,a^3+65\,e\,a^2\,b-104\,d\,a\,b^2+152\,c\,b^3\right)}{140\,a^4}+\frac{x^3\,\left(13\,a\,d-19\,b\,c\right)}{130\,a^2}+\frac{x^6\,\left(65\,e\,a^2-104\,d\,a\,b+152\,c\,b^2\right)}{455\,a^3}+\frac{b\,x^{12}\,\left(-35\,f\,a^3+65\,e\,a^2\,b-\right.}{14\,a^5}}{a^2\,x^{13}+2\,a\,b\,x^{16}-}$$
$$-\frac{b^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-35\,f\,a^3+65\,e\,a^2\,b-104\,d\,a\,b^2+152\,c\,b^3\right)}{27\,a^{22/3}}$$
$$+\frac{b^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-35\,f\,a^3+65\,e\,a^2\,b-104\,d\,a\,b^2+152\,c\,b^3\right)}{27\,a^{22/3}}$$

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x)
```

```
[Out] (b^(4/3)*log(b^(1/3)*x + a^(1/3))*(152*b^3*c - 35*a^3*f - 104*a*b^2*d + 65*
a^2*b*e))/(27*a^(22/3)) - (c/(13*a) - (x^9*(152*b^3*c - 35*a^3*f - 104*a*b^
2*d + 65*a^2*b*e))/(140*a^4) + (x^3*(13*a*d - 19*b*c))/(130*a^2) + (x^6*(15
2*b^2*c + 65*a^2*e - 104*a*b*d))/(455*a^3) + (b*x^12*(152*b^3*c - 35*a^3*f
- 104*a*b^2*d + 65*a^2*b*e))/(14*a^5) + (7*b^2*x^15*(152*b^3*c - 35*a^3*f -
 104*a*b^2*d + 65*a^2*b*e))/(36*a^6) + (b^3*x^18*(152*b^3*c - 35*a^3*f - 10
4*a*b^2*d + 65*a^2*b*e))/(9*a^7))/(a^2*x^13 + b^2*x^19 + 2*a*b*x^16) - (b^(
4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)
*(152*b^3*c - 35*a^3*f - 104*a*b^2*d + 65*a^2*b*e))/(27*a^(22/3)) + (b^(4/3
)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(1
52*b^3*c - 35*a^3*f - 104*a*b^2*d + 65*a^2*b*e))/(27*a^(22/3))
```

3.304 $\int \frac{(1-x)x^4}{1+x^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int \frac{(1-x)x^4}{1+x^3} \, dx = \frac{x^2}{2} - \frac{x^3}{3} + \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right)$$

```
[Out] 1/2*x^2-1/3*x^3+2/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))
*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1901, 1888, 31, 648, 632, 210, 642}

$$\int \frac{(1-x)x^4}{1+x^3} \, dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{x^3}{3} + \frac{x^2}{2} + \frac{1}{6}\log\left(x^2-x+1\right) + \frac{2}{3}\log(x+1)$$

```
[In] Int[((1 - x)*x^4)/(1 + x^3),x]
```

```
[Out] x^2/2 - x^3/3 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x])/3 + Log[
1 - x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1888

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[q*((A - B*q
+ C*q^2)/(3*a)), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(x - x^2 + \frac{(-1+x)x}{1+x^3}\right) dx \\
&= \frac{x^2}{2} - \frac{x^3}{3} + \int \frac{(-1+x)x}{1+x^3} \, dx \\
&= \frac{x^2}{2} - \frac{x^3}{3} + \frac{1}{3} \int \frac{-2+x}{1-x+x^2} \, dx + \frac{2}{3} \int \frac{1}{1+x} \, dx
\end{aligned}$$

$$= \frac{x^2}{2} - \frac{x^3}{3} + \frac{2}{3}\log(1+x) + \frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{2}\int \frac{1}{1-x+x^2}\,dx$$

$$= \frac{x^2}{2} - \frac{x^3}{3} + \frac{2}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right)$$

$$= \frac{x^2}{2} - \frac{x^3}{3} - \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{(1-x)x^4}{1+x^3}\,dx = \frac{1}{6}\left(3x^2 - 2x^3 - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 2\log(1+x) - \log\left(1-x+x^2\right) + 2\log\left(1+x^3\right)\right)$$

```
[In] Integrate[((1 - x)*x^4)/(1 + x^3),x]
[Out] (3*x^2 - 2*x^3 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - Log[
1 - x + x^2] + 2*Log[1 + x^3])/6
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{2\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	45
risch	$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{\ln(4x^2-4x+4)}{6}-\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}+\frac{2\ln(1+x)}{3}$	47
meijerg	$\frac{x^2}{2}-\frac{x^2\left(-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{(x^3)^{\frac{2}{3}}}+\frac{\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2(x^3)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{(x^3)^{\frac{2}{3}}}\right)}{3}-\frac{x^3}{3}+\frac{\ln(x^3+1)}{3}$	94

```
[In] int((1-x)*x^4/(x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/3*x^3+1/2*x^2+2/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(-1+2*x
)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(1-x)x^4}{1+x^3}\,dx = -\frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{6}\,\log\left(x^2-x+1\right) + \frac{2}{3}\,\log\left(x+1\right)$$

```
[In] integrate((1-x)*x^4/(x^3+1),x, algorithm="fricas")
[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^
2 - x + 1) + 2/3*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{(1-x)x^4}{1+x^3}\,dx = -\frac{x^3}{3} + \frac{x^2}{2} + \frac{2\log\left(x+1\right)}{3} + \frac{\log\left(x^2-x+1\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((1-x)*x**4/(x**3+1),x)
[Out] -x**3/3 + x**2/2 + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sq
rt(3)*x/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(1-x)x^4}{1+x^3}\,dx = -\frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{6}\,\log\left(x^2-x+1\right) + \frac{2}{3}\,\log\left(x+1\right)$$

```
[In] integrate((1-x)*x^4/(x^3+1),x, algorithm="maxima")
[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^
2 - x + 1) + 2/3*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{(1-x)x^4}{1+x^3}\,dx = -\frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{6}\,\log\left(x^2-x+1\right)+\frac{2}{3}\,\log\left(|x+1|\right)$$

```
[In] integrate((1-x)*x^4/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^
2 - x + 1) + 2/3*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{(1-x)x^4}{1+x^3}\,dx = \frac{2\,\ln(x+1)}{3}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{x^2}{2}-\frac{x^3}{3}$$

```
[In] int(-(x^4*(x - 1))/(x^3 + 1),x)
```

```
[Out] (2*log(x + 1))/3 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6) - l
og(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) + x^2/2 - x^3/3
```

3.305 $\int \frac{(1-x)x^3}{1+x^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \frac{(1-x)x^3}{1+x^3} dx = x - \frac{x^2}{2} - \frac{2}{3}\log(1+x) + \frac{1}{3}\log\left(1-x+x^2\right)$$

```
[Out] x-1/2*x^2-2/3*ln(1+x)+1/3*ln(x^2-x+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1901, 1874, 31, 642}

$$\int \frac{(1-x)x^3}{1+x^3} dx = -\frac{x^2}{2} + \frac{1}{3}\log\left(x^2-x+1\right) + x - \frac{2}{3}\log(x+1)$$

```
[In] Int[((1 - x)*x^3)/(1 + x^3),x]
```

```
[Out] x - x^2/2 - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 642</u>

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(1-x-\frac{1-x}{1+x^3}\right) dx \\
&= x-\frac{x^2}{2}-\int \frac{1-x}{1+x^3}\,dx \\
&= x-\frac{x^2}{2}-\frac{1}{3}\int \frac{1-2x}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1}{1+x}\,dx \\
&= x-\frac{x^2}{2}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(1-x)x^3}{1+x^3}\,dx = x-\frac{x^2}{2}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)$$

```
[In] Integrate[((1 - x)*x^3)/(1 + x^3),x]
[Out] x - x^2/2 - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result
default	$x-\frac{x^2}{2}-\frac{2\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{3}$
norman	$x-\frac{x^2}{2}-\frac{2\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{3}$
risch	$x-\frac{x^2}{2}-\frac{2\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{3}$
parallelrisch	$x-\frac{x^2}{2}-\frac{2\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{3}$
meijerg	$x-\frac{x\left(\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}\right)}{\left(x^3\right)^{\frac{1}{3}}}-\frac{\ln\left(1-\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2\left(x^3\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2-\left(x^3\right)^{\frac{1}{3}}}\right)}{\left(x^3\right)^{\frac{1}{3}}}\right)}{3}-\frac{x^2}{2}+\frac{x^2\left(-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}\right)}{\left(x^3\right)^{\frac{2}{3}}}+\frac{\ln\left(1-\left(x^3\right)^{\frac{1}{3}}+(x\right.}{2\left(x^3\right)^{\frac{2}{3}}}\right.}{3}$

[In] int((1-x)*x^3/(x^3+1),x,method=_RETURNVERBOSE)

[Out] x-1/2*x^2-2/3*ln(1+x)+1/3*ln(x^2-x+1)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(1-x)x^3}{1+x^3}\,dx=-\frac{1}{2}\,x^2+x+\frac{1}{3}\,\log\left(x^2-x+1\right)-\frac{2}{3}\,\log\left(x+1\right)$$

[In] integrate((1-x)*x^3/(x^3+1),x, algorithm="fricas")

[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(1-x)x^3}{1+x^3}\,dx=-\frac{x^2}{2}+x-\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{3}$$

[In] integrate((1-x)*x**3/(x**3+1),x)

[Out] -x**2/2 + x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(1-x)x^3}{1+x^3} \, dx = -\frac{1}{2} \, x^2 + x + \frac{1}{3} \, \log\left(x^2 - x + 1\right) - \frac{2}{3} \, \log\left(x + 1\right)$$

```
[In] integrate((1-x)*x^3/(x^3+1),x, algorithm="maxima")
[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{(1-x)x^3}{1+x^3} \, dx = -\frac{1}{2} \, x^2 + x + \frac{1}{3} \, \log\left(x^2 - x + 1\right) - \frac{2}{3} \, \log\left(|x + 1|\right)$$

```
[In] integrate((1-x)*x^3/(x^3+1),x, algorithm="giac")
[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(1-x)x^3}{1+x^3} \, dx = x - \frac{2\,\ln\left(x+1\right)}{3} + \frac{\ln\left(x^2-x+1\right)}{3} - \frac{x^2}{2}$$

```
[In] int(-(x^3*(x - 1))/(x^3 + 1),x)
[Out] x - (2*log(x + 1))/3 + log(x^2 - x + 1)/3 - x^2/2
```

3.306 $\int \frac{(1-x)x^2}{1+x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{(1-x)x^2}{1+x^3}\,dx = -x-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)$$

```
[Out] -x+2/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1901, 1888, 31, 648, 632, 210, 642}

$$\int \frac{(1-x)x^2}{1+x^3}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^2-x+1\right)-x+\frac{2}{3}\log(x+1)$$

```
[In] Int[((1 - x)*x^2)/(1 + x^3),x]
```

```
[Out] -x - ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x])/3 + Log[1 - x + x^2
]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1888

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[q*((A - B*q
+ C*q^2)/(3*a)), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-1+\frac{1+x^2}{1+x^3}\right) dx \\
&= -x+\int \frac{1+x^2}{1+x^3}\,dx \\
&= -x+\frac{1}{3}\int \frac{1+x}{1-x+x^2}\,dx+\frac{2}{3}\int \frac{1}{1+x}\,dx \\
&= -x+\frac{2}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx
\end{aligned}$$

$$= -x + \frac{2}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right)$$

$$= -x + \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \frac{(1-x)x^2}{1+x^3}\,dx = -x + \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1+x) - \frac{1}{6}\log\left(1-x+x^2\right) + \frac{1}{3}\log\left(1+x^3\right)$$

```
[In] Integrate[((1 - x)*x^2)/(1 + x^3),x]
```

```
[Out] -x + ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x]/3 - Log[1 - x + x^2]/6
 + Log[1 + x^3]/3
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

method	result	size
risch	$-x+\frac{2\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	36
default	$-x+\frac{2\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	38
meijerg	$\frac{\ln(x^3+1)}{3}-x+\frac{x\left(\frac{\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{(x^3)^{\frac{1}{3}}}-\frac{\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2(x^3)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{(x^3)^{\frac{1}{3}}}\right)}{3}$	84

```
[In] int((1-x)*x^2/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -x+2/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{(1-x)x^2}{1+x^3}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - x + \frac{1}{6}\,\log\left(x^2-x+1\right) + \frac{2}{3}\,\log\left(x+1\right)$$

```
[In] integrate((1-x)*x^2/(x^3+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3*
log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{(1-x)x^2}{1+x^3}\,dx = -x + \frac{2\log\left(x+1\right)}{3} + \frac{\log\left(x^2-x+1\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((1-x)*x**2/(x**3+1),x)
[Out] -x + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sq
rt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{(1-x)x^2}{1+x^3}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - x + \frac{1}{6}\,\log\left(x^2-x+1\right) + \frac{2}{3}\,\log\left(x+1\right)$$

```
[In] integrate((1-x)*x^2/(x^3+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3*
log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(1-x)x^2}{1+x^3}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - x + \frac{1}{6}\,\log\left(x^2-x+1\right) + \frac{2}{3}\,\log\left(|x+1|\right)$$

```
[In] integrate((1-x)*x^2/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3*
log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{(1-x)x^2}{1+x^3}\,dx = \frac{2\,\ln(x+1)}{3} - x - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

```
[In] int(-(x^2*(x - 1))/(x^3 + 1),x)
```

```
[Out] (2*log(x + 1))/3 - x - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6)
 + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6)
```

3.307 $\int \frac{(1-x)x}{1+x^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int \frac{(1-x)x}{1+x^3} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\log(1+x) - \frac{1}{6}\log\left(1-x+x^2\right)$$

```
[Out] -2/3*ln(1+x)-1/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1888, 31, 648, 632, 210, 642}

$$\int \frac{(1-x)x}{1+x^3} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{6}\log\left(x^2-x+1\right) - \frac{2}{3}\log(x+1)$$

```
[In] Int[((1 - x)*x)/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - (2*Log[1 + x])/3 - Log[1 - x + x^2]/
6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1888

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[q*((A - B*q
+ C*q^2)/(3*a)), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\int \frac{2-x}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1}{1+x}\,dx\\ &= -\frac{2}{3}\log(1+x)-\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\ &= -\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{(1-x)x}{1+x^3}\,dx = \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right) - \frac{1}{3}\log\left(1+x^3\right)$$

```
[In] Integrate[((1 - x)*x)/(1 + x^3),x]
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6 - Lo
g[1 + x^3]/3
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{2\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$-\frac{2\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	35
meijerg	$-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}-\frac{\ln(x^3+1)}{3}$	88

```
[In] int((1-x)*x/(x^3+1),x,method=_RETURNVERBOSE)
[Out] -2/3*ln(1+x)-1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{(1-x)x}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log\left(x^2-x+1\right) - \frac{2}{3}\log\left(x+1\right)$$

```
[In] integrate((1-x)*x/(x^3+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log(
x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{(1-x)x}{1+x^3} \, dx = -\frac{2\log(x+1)}{3} - \frac{\log(x^2-x+1)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((1-x)*x/(x**3+1),x)
[Out] -2*log(x + 1)/3 - log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3
)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{(1-x)x}{1+x^3} \, dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log\left(x^2-x+1\right) - \frac{2}{3}\log(x+1)$$

```
[In] integrate((1-x)*x/(x^3+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log(
x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(1-x)x}{1+x^3} \, dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log\left(x^2-x+1\right) - \frac{2}{3}\log(|x+1|)$$

```
[In] integrate((1-x)*x/(x^3+1),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log(
abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

$$\int \frac{(1-x)x}{1+x^3}\,dx = -\frac{\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6}-\frac{\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6}-\frac{2\,\ln\left(x+1\right)}{3} - \frac{\sqrt{3}\,\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6}+\frac{\sqrt{3}\,\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6}$$

```
[In] int(-(x*(x - 1))/(x^3 + 1),x)
```

```
[Out] (3^(1/2)*log(x + (3^(1/2)*1i)/2 - 1/2)*1i)/6 - log(x + (3^(1/2)*1i)/2 - 1/2
)/6 - (2*log(x + 1))/3 - (3^(1/2)*log(x - (3^(1/2)*1i)/2 - 1/2)*1i)/6 - log
(x - (3^(1/2)*1i)/2 - 1/2)/6
```

3.308 $\int \frac{1-x}{x(1+x^3)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int \frac{1-x}{x\left(1+x^3\right)}\,dx=\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)$$

```
[Out] ln(x)-2/3*ln(1+x)-1/6*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1848, 648, 632, 210, 642}

$$\int \frac{1-x}{x\left(1+x^3\right)}\,dx=\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^2-x+1\right)+\log(x)-\frac{2}{3}\log(x+1)$$

```
[In] Int[(1 - x)/(x*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[x] - (2*Log[1 + x])/3 - Log[1 - x +
 x^2]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{x}-\frac{2}{3(1+x)}+\frac{-1-x}{3\left(1-x+x^2\right)}\right) dx \\
&= \log(x)-\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-1-x}{1-x+x^2}\,dx \\
&= \log(x)-\frac{2}{3}\log(1+x)-\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx \\
&= \log(x)-\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right) \\
&= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{1-x}{x(1+x^3)}\,dx = -\frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x) - \frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right) - \frac{1}{3}\log\left(1+x^3\right)$$

```
[In] Integrate[(1 - x)/(x*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 + x]/3 + Log[1 - x +
 x^2]/6 - Log[1 + x^3]/3
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{2\ln(1+x)}{3}+\ln(x)-\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	35
default	$\ln(x)-\frac{2\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	37
meijerg	$\ln(x)-\frac{\ln(x^3+1)}{3}-\frac{x\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{1}{3}}}-\frac{x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{1}{3}}}$	84

```
[In] int((1-x)/x/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*ln(1+x)+ln(x)-1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{1-x}{x(1+x^3)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log\left(x^2-x+1\right) - \frac{2}{3}\log(x+1) + \log(x)$$

```
[In] integrate((1-x)/x/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log
(x + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{1-x}{x\left(1+x^{3}\right)}\,dx = \log\left(x\right) - \frac{2\log\left(x+1\right)}{3} - \frac{\log\left(x^{2}-x+1\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((1-x)/x/(x**3+1),x)
```

```
[Out] log(x) - 2*log(x + 1)/3 - log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3
- sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{1-x}{x\left(1+x^{3}\right)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log\left(x^{2}-x+1\right) - \frac{2}{3}\log\left(x+1\right) + \log\left(x\right)$$

```
[In] integrate((1-x)/x/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log
(x + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{1-x}{x\left(1+x^{3}\right)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log\left(x^{2}-x+1\right) - \frac{2}{3}\log\left(|x+1|\right) + \log\left(|x|\right)$$

```
[In] integrate((1-x)/x/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log
(abs(x + 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{1-x}{x(1+x^3)} \, dx = \ln(x) - \frac{2 \ln(x+1)}{3} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

```
[In] int(-(x - 1)/(x*(x^3 + 1)),x)
[Out] log(x) - (2*log(x + 1))/3 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 -
 1/6) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6)
```

3.309 $\int \frac{1-x}{x^2(1+x^3)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 49

$$\int \frac{1-x}{x^2(1+x^3)}\,dx = -\frac{1}{x} + \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log(x) + \frac{2}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right)$$

```
[Out] -1/x-ln(x)+2/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1
/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1848, 648, 632, 210, 642}

$$\int \frac{1-x}{x^2(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{x} - \log(x) + \frac{2}{3}\log(x+1)$$

```
[In] Int[(1 - x)/(x^2*(1 + x^3)),x]
```

```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + (2*Log[1 + x])/3 + L
og[1 - x + x^2]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{2}{3(1+x)}+\frac{-2+x}{3\left(1-x+x^2\right)}\right) dx \\
&= -\frac{1}{x}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-2+x}{1-x+x^2}\, dx \\
&= -\frac{1}{x}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\, dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\, dx \\
&= -\frac{1}{x}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\, dx, x, -1+2x\right) \\
&= -\frac{1}{x}-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{1-x}{x^2(1+x^3)}\,dx = -\frac{1}{x}-\frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(x)+\frac{1}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)+\frac{1}{3}\log\left(1+x^3\right)$$

```
[In] Integrate[(1 - x)/(x^2*(1 + x^3)),x]
[Out] -x^(-1) - ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + Log[1 + x]/3 - Log[
1 - x + x^2]/6 + Log[1 + x^3]/3
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
risch	$-\frac{1}{x}+\frac{2\ln(1+x)}{3}-\ln(x)+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	42
default	$-\frac{1}{x}-\ln(x)+\frac{2\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	44
meijerg	$-\frac{1}{x}-\frac{x^2\left(-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{(x^3)^{\frac{2}{3}}}+\frac{\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2(x^3)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{(x^3)^{\frac{2}{3}}}\right)}{3}-\ln(x)+\frac{\ln(x^3+1)}{3}$	93

```
[In] int((1-x)/x^2/(x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/x+2/3*ln(1+x)-ln(x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/
2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{1-x}{x^2(1+x^3)}\,dx = -\frac{2\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-x\log\left(x^2-x+1\right)-4x\log(x+1)+6x\log(x)+6}{6x}$$

```
[In] integrate((1-x)/x^2/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x - 1)) - x*log(x^2 - x + 1) - 4*x*
log(x + 1) + 6*x*log(x) + 6)/x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1-x}{x^2\left(1+x^3\right)}\,dx = -\log\left(x\right) + \frac{2\log\left(x+1\right)}{3} + \frac{\log\left(x^2-x+1\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3} - \frac{1}{x}$$

```
[In] integrate((1-x)/x**2/(x**3+1),x)
```

```
[Out] -log(x) + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3
 - sqrt(3)/3)/3 - 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{1-x}{x^2\left(1+x^3\right)}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{x} + \frac{1}{6}\,\log\left(x^2-x+1\right) + \frac{2}{3}\,\log\left(x+1\right) - \log\left(x\right)$$

```
[In] integrate((1-x)/x^2/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/x + 1/6*log(x^2 - x + 1) + 2
/3*log(x + 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{1-x}{x^2\left(1+x^3\right)}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{x} + \frac{1}{6}\,\log\left(x^2-x+1\right) + \frac{2}{3}\,\log\left(|x+1|\right) - \log\left(|x|\right)$$

```
[In] integrate((1-x)/x^2/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/x + 1/6*log(x^2 - x + 1) + 2
/3*log(abs(x + 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{1-x}{x^2(1+x^3)}\,dx = \frac{2\ln(x+1)}{3} - \ln(x) + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \frac{1}{x}$$

```
[In] int(-(x - 1)/(x^2*(x^3 + 1)),x)
[Out] (2*log(x + 1))/3 - log(x) + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 +
 1/6) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) - 1/x
```

3.310 $\int \frac{1-x}{x^3(1+x^3)} dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{1-x}{x^3(1+x^3)} dx = -\frac{1}{2x^2} + \frac{1}{x} - \frac{2}{3}\log(1+x) + \frac{1}{3}\log\left(1-x+x^2\right)$$

```
[Out] -1/2/x^2+1/x-2/3*ln(1+x)+1/3*ln(x^2-x+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1848, 642}

$$\int \frac{1-x}{x^3(1+x^3)} dx = -\frac{1}{2x^2} + \frac{1}{3}\log\left(x^2-x+1\right) + \frac{1}{x} - \frac{2}{3}\log(x+1)$$

```
[In] Int[(1 - x)/(x^3*(1 + x^3)),x]
```

```
[Out] -1/2*1/x^2 + x^(-1) - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{x^3}-\frac{1}{x^2}-\frac{2}{3(1+x)}+\frac{-1+2x}{3\left(1-x+x^2\right)}\right)dx\\
&= -\frac{1}{2x^2}+\frac{1}{x}-\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= -\frac{1}{2x^2}+\frac{1}{x}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$
\int \frac{1-x}{x^3\left(1+x^3\right)}\,dx = -\frac{1}{2x^2}+\frac{1}{x}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)
$$

```
[In] Integrate[(1 - x)/(x^3*(1 + x^3)),x]
[Out] -1/2*1/x^2 + x^(-1) - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result
norman	$\frac{x-\frac{1}{2}}{x^2}-\frac{2\ln(1+x)}{3}+\frac{\ln\left(x^2-x+1\right)}{3}$
risch	$\frac{x-\frac{1}{2}}{x^2}-\frac{2\ln(1+x)}{3}+\frac{\ln\left(x^2-x+1\right)}{3}$
default	$-\frac{1}{2x^2}+\frac{1}{x}-\frac{2\ln(1+x)}{3}+\frac{\ln\left(x^2-x+1\right)}{3}$
parallelrisch	$-\frac{4\ln(1+x)x^2-2\ln\left(x^2-x+1\right)x^2+3-6x}{6x^2}$
meijerg	$-\frac{1}{2x^2}-\frac{x\left(\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}\right)}{\left(x^3\right)^{\frac{1}{3}}}-\frac{\ln\left(1-\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2\left(x^3\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2-\left(x^3\right)^{\frac{1}{3}}}\right)}{\left(x^3\right)^{\frac{1}{3}}}\right)}{3}+\frac{1}{x}+\frac{x^2\left(-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}\right)}{\left(x^3\right)^{\frac{2}{3}}}+\frac{\ln\left(1-\left(x^3\right)^{\frac{1}{3}}+ \right.}{2\left(x^3\right)^{\frac{2}{3}}} \right.}{3}$

```
[In] int((1-x)/x^3/(x^3+1),x,method=_RETURNVERBOSE)
[Out] (x-1/2)/x^2-2/3*ln(1+x)+1/3*ln(x^2-x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1-x}{x^3(1+x^3)}\,dx = \frac{2\,x^2\log\left(x^2-x+1\right)-4\,x^2\log\left(x+1\right)+6\,x-3}{6\,x^2}$$

```
[In] integrate((1-x)/x^3/(x^3+1),x, algorithm="fricas")
[Out] 1/6*(2*x^2*log(x^2 - x + 1) - 4*x^2*log(x + 1) + 6*x - 3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{x^3(1+x^3)}\,dx = -\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{3}-\frac{1-2x}{2x^2}$$

```
[In] integrate((1-x)/x**3/(x**3+1),x)
[Out] -2*log(x + 1)/3 + log(x**2 - x + 1)/3 - (1 - 2*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1-x}{x^3(1+x^3)}\,dx = \frac{2\,x-1}{2\,x^2}+\frac{1}{3}\,\log\left(x^2-x+1\right)-\frac{2}{3}\,\log\left(x+1\right)$$

```
[In] integrate((1-x)/x^3/(x^3+1),x, algorithm="maxima")
[Out] 1/2*(2*x - 1)/x^2 + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{1-x}{x^3(1+x^3)}\,dx = \frac{2\,x-1}{2\,x^2}+\frac{1}{3}\,\log\left(x^2-x+1\right)-\frac{2}{3}\,\log\left(|x+1|\right)$$

```
[In] integrate((1-x)/x^3/(x^3+1),x, algorithm="giac")
[Out] 1/2*(2*x - 1)/x^2 + 1/3*log(x^2 - x + 1) - 2/3*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{1-x}{x^3\,(1+x^3)}\,dx=\frac{\ln\left(x^2-x+1\right)}{3}-\frac{2\,\ln\left(x+1\right)}{3}+\frac{x-\frac{1}{2}}{x^2}$$

```
[In] int(-(x - 1)/(x^3*(x^3 + 1)),x)
[Out] log(x^2 - x + 1)/3 - (2*log(x + 1))/3 + (x - 1/2)/x^2
```

3.311 $\int \frac{x(1+2x)}{1+x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int \frac{x(1+2x)}{1+x^3}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1+x)+\frac{5}{6}\log\left(1-x+x^2\right)$$

```
[Out] 1/3*ln(1+x)+5/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1888, 31, 648, 632, 210, 642}

$$\int \frac{x(1+2x)}{1+x^3}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log(x+1)$$

```
[In] Int[(x*(1 + 2*x))/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x]/3 + (5*Log[1 - x + x^2])/
6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1888

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[q*((A - B*q
+ C*q^2)/(3*a)), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\int \frac{1}{1+x}\,dx + \frac{1}{3}\int \frac{-1+5x}{1-x+x^2}\,dx \\
&= \frac{1}{3}\log(1+x) + \frac{1}{2}\int \frac{1}{1-x+x^2}\,dx + \frac{5}{6}\int \frac{-1+2x}{1-x+x^2}\,dx \\
&= \frac{1}{3}\log(1+x) + \frac{5}{6}\log\left(1-x+x^2\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) \\
&= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1+x) + \frac{5}{6}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{x(1+2x)}{1+x^3}\,dx = \frac{1}{6}\left(2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\log(1+x)+\log\left(1-x+x^2\right)+4\log\left(1+x^3\right)\right)$$

```
[In] Integrate[(x*(1 + 2*x))/(1 + x^3),x]
[Out] (2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + Log[1 - x + x^2] + 4
*Log[1 + x^3])/6
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\ln(1+x)}{3}+\frac{5\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	35
risch	$\frac{5\ln(4x^2-4x+4)}{6}+\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}+\frac{\ln(1+x)}{3}$	37
meijerg	$-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{2\ln(x^3+1)}{3}$	88

```
[In] int(x*(1+2*x)/(x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(1+x)+5/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x(1+2x)}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log\left(x+1\right)$$

```
[In] integrate(x*(1+2*x)/(x^3+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log(
x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{x(1+2x)}{1+x^3}\,dx = \frac{\log{(x+1)}}{3} + \frac{5\log{(x^2-x+1)}}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate(x*(1+2*x)/(x**3+1),x)
[Out] log(x + 1)/3 + 5*log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)
/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x(1+2x)}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{5}{6}\log\left(x^2-x+1\right) + \frac{1}{3}\log\left(x+1\right)$$

```
[In] integrate(x*(1+2*x)/(x^3+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log(
x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x(1+2x)}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{5}{6}\log\left(x^2-x+1\right) + \frac{1}{3}\log\left(|x+1|\right)$$

```
[In] integrate(x*(1+2*x)/(x^3+1),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log(
abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

$$\int \frac{x(1+2x)}{1+x^3}\,dx = \frac{5\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6}+\frac{5\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6}+\frac{\ln(x+1)}{3}-\frac{\sqrt{3}\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{6}+\frac{\sqrt{3}\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{6}$$

```
[In] int((x*(2*x + 1))/(x^3 + 1),x)
```

```
[Out] (5*log(x - (3^(1/2)*1i)/2 - 1/2))/6 + (5*log(x + (3^(1/2)*1i)/2 - 1/2))/6 +
 log(x + 1)/3 - (3^(1/2)*log(x - (3^(1/2)*1i)/2 - 1/2)*1i)/6 + (3^(1/2)*log
(x + (3^(1/2)*1i)/2 - 1/2)*1i)/6
```

3.312 $\int \frac{x(1+2x)}{1-x^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{x(1+2x)}{1-x^3} \, dx = -\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log(1-x) - \frac{1}{2}\log\left(1+x+x^2\right)$$

```
[Out] -ln(1-x)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1889, 31, 648, 632, 210, 642}

$$\int \frac{x(1+2x)}{1-x^3} \, dx = -\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2}\log\left(x^2+x+1\right) - \log(1-x)$$

```
[In] Int[(x*(1 + 2*x))/(1 - x^3),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x] - Log[1 + x + x^2]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1889

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Dist[q*((A + B*q
 + C*q^2)/(3*a)), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q -
 C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b
*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
 && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\int \frac{-3-3x}{1+x+x^2}\,dx + \int \frac{1}{1-x}\,dx \\
&= -\log(1-x) - \frac{1}{2}\int \frac{1}{1+x+x^2}\,dx - \frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx \\
&= -\log(1-x) - \frac{1}{2}\log\left(1+x+x^2\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) \\
&= -\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log(1-x) - \frac{1}{2}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int \frac{x(1+2x)}{1-x^3}\,dx = -\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1-x) + \frac{1}{6}\log\left(1+x+x^2\right) - \frac{2}{3}\log\left(1-x^3\right)$$

[In] Integrate[(x*(1 + 2*x))/(1 - x^3),x]

[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x]/3 + Log[1 + x + x^2]/6 - (2*Log[1 - x^3])/3

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$-\ln\left(-1+x\right)-\frac{\ln\left(x^{2}+x+1\right)}{2}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	33
risch	$-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\ln\left(4x^{2}+4x+4\right)}{2}-\ln\left(-1+x\right)$	37
meijerg	$-\frac{x^{2}\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{3}\right)^{\frac{1}{3}}}{2+\left(x^{3}\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^{3}\right)^{\frac{2}{3}}}-\frac{2\ln\left(-x^{3}+1\right)}{3}$	74

[In] int(x*(1+2*x)/(-x^3+1),x,method=_RETURNVERBOSE)

[Out] -ln(-1+x)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x(1+2x)}{1-x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{2}\log\left(x^2+x+1\right) - \log\left(x-1\right)$$

[In] integrate(x*(1+2*x)/(-x^3+1),x, algorithm="fricas")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(x - 1)

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{x(1+2x)}{1-x^3}\,dx = -\log(x-1) - \frac{\log(x^2+x+1)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate(x*(1+2*x)/(-x**3+1),x)
[Out] -log(x - 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)
/3
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x(1+2x)}{1-x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{2}\log\left(x^2+x+1\right) - \log(x-1)$$

```
[In] integrate(x*(1+2*x)/(-x^3+1),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(x -
 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{x(1+2x)}{1-x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{2}\log\left(x^2+x+1\right) - \log(|x-1|)$$

```
[In] integrate(x*(1+2*x)/(-x^3+1),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(abs
(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \frac{x(1+2x)}{1-x^3}\,dx = -\frac{\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2}-\frac{\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2}-\ln\left(x-1\right)+\frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6}-\frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6}$$

```
[In] int(-(x*(2*x + 1))/(x^3 - 1),x)
```

```
[Out] (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/6 - log(x + (3^(1/2)*1i)/2 + 1/2
)/2 - log(x - 1) - log(x - (3^(1/2)*1i)/2 + 1/2)/2 - (3^(1/2)*log(x + (3^(1
/2)*1i)/2 + 1/2)*1i)/6
```

3.313 $\int x^2(c+dx+ex^2)\left(a+bx^3\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx=\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8$$

```
[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*b*c*x^6+1/7*b*d*x^7+1/8*b*e*x^8
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1642}

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx=\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8$$

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)/7 + (b*e*
x^8)/8
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(acx^2+adx^3+aex^4+bcx^5+bdx^6+bex^7\right)\,dx\\&=\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^2(c+dx+ex^2)(a+bx^3)\,dx=\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8$$

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3),x]
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)/7 + (b*e*
x^8)/8
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{1}{3}ac\,x^3+\frac{1}{4}ad\,x^4+\frac{1}{5}ae\,x^5+\frac{1}{6}bc\,x^6+\frac{1}{7}bd\,x^7+\frac{1}{8}be\,x^8$	44
default	$\frac{1}{3}ac\,x^3+\frac{1}{4}ad\,x^4+\frac{1}{5}ae\,x^5+\frac{1}{6}bc\,x^6+\frac{1}{7}bd\,x^7+\frac{1}{8}be\,x^8$	44
norman	$\frac{1}{3}ac\,x^3+\frac{1}{4}ad\,x^4+\frac{1}{5}ae\,x^5+\frac{1}{6}bc\,x^6+\frac{1}{7}bd\,x^7+\frac{1}{8}be\,x^8$	44
risch	$\frac{1}{3}ac\,x^3+\frac{1}{4}ad\,x^4+\frac{1}{5}ae\,x^5+\frac{1}{6}bc\,x^6+\frac{1}{7}bd\,x^7+\frac{1}{8}be\,x^8$	44
parallelrisch	$\frac{1}{3}ac\,x^3+\frac{1}{4}ad\,x^4+\frac{1}{5}ae\,x^5+\frac{1}{6}bc\,x^6+\frac{1}{7}bd\,x^7+\frac{1}{8}be\,x^8$	44

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*b*c*x^6+1/7*b*d*x^7+1/8*b*e*x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x^2(c+dx+ex^2)(a+bx^3)\,dx=\frac{1}{8}\,bex^8+\frac{1}{7}\,bdx^7+\frac{1}{6}\,bcx^6+\frac{1}{5}\,aex^5+\frac{1}{4}\,adx^4+\frac{1}{3}\,acx^3$$

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")
[Out] 1/8*b*e*x^8 + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*e*x^5 + 1/4*a*d*x^4 + 1/3*a
*c*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{acx^3}{3}+\frac{adx^4}{4}+\frac{aex^5}{5}+\frac{bcx^6}{6}+\frac{bdx^7}{7}+\frac{bex^8}{8}$$

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*c*x**6/6 + b*d*x**7/7 + b*e*x**8/8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{1}{8}\,bex^8+\frac{1}{7}\,bdx^7+\frac{1}{6}\,bcx^6+\frac{1}{5}\,aex^5+\frac{1}{4}\,adx^4+\frac{1}{3}\,acx^3$$

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/8*b*e*x^8 + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*e*x^5 + 1/4*a*d*x^4 + 1/3*a
*c*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{1}{8}\,bex^8+\frac{1}{7}\,bdx^7+\frac{1}{6}\,bcx^6+\frac{1}{5}\,aex^5+\frac{1}{4}\,adx^4+\frac{1}{3}\,acx^3$$

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/8*b*e*x^8 + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*e*x^5 + 1/4*a*d*x^4 + 1/3*a
*c*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{b\,e\,x^8}{8}+\frac{b\,d\,x^7}{7}+\frac{b\,c\,x^6}{6}+\frac{a\,e\,x^5}{5}+\frac{a\,d\,x^4}{4}+\frac{a\,c\,x^3}{3}$$

```
[In] int(x^2*(a + b*x^3)*(c + d*x + e*x^2),x)
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (b*c*x^6)/6 + (a*e*x^5)/5 + (b*d*x^7)/7 + (b*e*
x^8)/8
```

3.314 $\int x(c+dx+ex^2)(a+bx^3)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int x(c+dx+ex^2)(a+bx^3)\,dx=\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

```
[Out] 1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1642}

$$\int x(c+dx+ex^2)(a+bx^3)\,dx=\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e*
x^7)/7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int(acx+adx^2+aex^3+bcx^4+bdx^5+bex^6)\,dx\\&=\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3),x]
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e*
x^7)/7
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{1}{2}ac\,x^2+\frac{1}{3}ad\,x^3+\frac{1}{4}ae\,x^4+\frac{1}{5}bc\,x^5+\frac{1}{6}bd\,x^6+\frac{1}{7}be\,x^7$	44
default	$\frac{1}{2}ac\,x^2+\frac{1}{3}ad\,x^3+\frac{1}{4}ae\,x^4+\frac{1}{5}bc\,x^5+\frac{1}{6}bd\,x^6+\frac{1}{7}be\,x^7$	44
norman	$\frac{1}{2}ac\,x^2+\frac{1}{3}ad\,x^3+\frac{1}{4}ae\,x^4+\frac{1}{5}bc\,x^5+\frac{1}{6}bd\,x^6+\frac{1}{7}be\,x^7$	44
risch	$\frac{1}{2}ac\,x^2+\frac{1}{3}ad\,x^3+\frac{1}{4}ae\,x^4+\frac{1}{5}bc\,x^5+\frac{1}{6}bd\,x^6+\frac{1}{7}be\,x^7$	44
parallelrisch	$\frac{1}{2}ac\,x^2+\frac{1}{3}ad\,x^3+\frac{1}{4}ae\,x^4+\frac{1}{5}bc\,x^5+\frac{1}{6}bd\,x^6+\frac{1}{7}be\,x^7$	44

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{1}{7}\,bex^7+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,aex^4+\frac{1}{3}\,adx^3+\frac{1}{2}\,acx^2$$

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")
[Out] 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*e*x^4 + 1/3*a*d*x^3 + 1/2*a
*c*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int x(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{acx^2}{2}+\frac{adx^3}{3}+\frac{aex^4}{4}+\frac{bcx^5}{5}+\frac{bdx^6}{6}+\frac{bex^7}{7}$$

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a),x)
[Out] a*c*x**2/2 + a*d*x**3/3 + a*e*x**4/4 + b*c*x**5/5 + b*d*x**6/6 + b*e*x**7/7
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{1}{7}\,bex^7+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,aex^4+\frac{1}{3}\,adx^3+\frac{1}{2}\,acx^2$$

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")
[Out] 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*e*x^4 + 1/3*a*d*x^3 + 1/2*a
*c*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{1}{7}\,bex^7+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,aex^4+\frac{1}{3}\,adx^3+\frac{1}{2}\,acx^2$$

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")
[Out] 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*e*x^4 + 1/3*a*d*x^3 + 1/2*a
*c*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{b\,e\,x^7}{7}+\frac{b\,d\,x^6}{6}+\frac{b\,c\,x^5}{5}+\frac{a\,e\,x^4}{4}+\frac{a\,d\,x^3}{3}+\frac{a\,c\,x^2}{2}$$

[In] int(x*(a + b*x^3)*(c + d*x + e*x^2),x)

[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (b*c*x^5)/5 + (a*e*x^4)/4 + (b*d*x^6)/6 + (b*e*
x^7)/7

3.315 $\int (c+dx+ex^2)(a+bx^3)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 50

$$\int (c+dx+ex^2)(a+bx^3)\,dx = acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6$$

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*b*c*x^4+1/5*b*d*x^5+1/6*b*e*x^6
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1671}

$$\int (c+dx+ex^2)(a+bx^3)\,dx = acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6$$

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3),x]
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (b*e*x^6)/6
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int (ac+adx+aex^2+bcx^3+bdx^4+bex^5)\,dx \\ &= acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (c+dx+ex^2)\left(a+bx^3\right)\,dx=acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6$$

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3),x]
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (b*e*x^6)/6
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

method	result	size
gosper	$acx+\frac{1}{2}ad\,x^2+\frac{1}{3}ae\,x^3+\frac{1}{4}bc\,x^4+\frac{1}{5}bd\,x^5+\frac{1}{6}be\,x^6$	41
default	$acx+\frac{1}{2}ad\,x^2+\frac{1}{3}ae\,x^3+\frac{1}{4}bc\,x^4+\frac{1}{5}bd\,x^5+\frac{1}{6}be\,x^6$	41
norman	$acx+\frac{1}{2}ad\,x^2+\frac{1}{3}ae\,x^3+\frac{1}{4}bc\,x^4+\frac{1}{5}bd\,x^5+\frac{1}{6}be\,x^6$	41
risch	$acx+\frac{1}{2}ad\,x^2+\frac{1}{3}ae\,x^3+\frac{1}{4}bc\,x^4+\frac{1}{5}bd\,x^5+\frac{1}{6}be\,x^6$	41
parallelrisch	$acx+\frac{1}{2}ad\,x^2+\frac{1}{3}ae\,x^3+\frac{1}{4}bc\,x^4+\frac{1}{5}bd\,x^5+\frac{1}{6}be\,x^6$	41

```
[In] int((e*x^2+d*x+c)*(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*b*c*x^4+1/5*b*d*x^5+1/6*b*e*x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{1}{6}\,bex^6+\frac{1}{5}\,bdx^5+\frac{1}{4}\,bcx^4+\frac{1}{3}\,aex^3+\frac{1}{2}\,adx^2+acx$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")
[Out] 1/6*b*e*x^6 + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int (c + dx + ex^2) (a + bx^3) dx = acx + \frac{adx^2}{2} + \frac{aex^3}{3} + \frac{bcx^4}{4} + \frac{bdx^5}{5} + \frac{bex^6}{6}$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a),x)
[Out] a*c*x + a*d*x**2/2 + a*e*x**3/3 + b*c*x**4/4 + b*d*x**5/5 + b*e*x**6/6
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (c + dx + ex^2) (a + bx^3) dx = \frac{1}{6} bex^6 + \frac{1}{5} bdx^5 + \frac{1}{4} bcx^4 + \frac{1}{3} aex^3 + \frac{1}{2} adx^2 + acx$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")
[Out] 1/6*b*e*x^6 + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (c + dx + ex^2) (a + bx^3) dx = \frac{1}{6} bex^6 + \frac{1}{5} bdx^5 + \frac{1}{4} bcx^4 + \frac{1}{3} aex^3 + \frac{1}{2} adx^2 + acx$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")
[Out] 1/6*b*e*x^6 + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (c + dx + ex^2) (a + bx^3) dx = \frac{b e x^6}{6} + \frac{b d x^5}{5} + \frac{b c x^4}{4} + \frac{a e x^3}{3} + \frac{a d x^2}{2} + a c x$$

```
[In] int((a + b*x^3)*(c + d*x + e*x^2),x)
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^4)/4 + (a*e*x^3)/3 + (b*d*x^5)/5 + (b*e*x^6)/6
```

3.316 $\int \frac{(c+dx+ex^2)(a+bx^3)}{x} dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x} dx = adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5 + ac\log(x)$$

```
[Out] a*d*x+1/2*a*e*x^2+1/3*b*c*x^3+1/4*b*d*x^4+1/5*b*e*x^5+a*c*ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1642}

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x} dx = ac\log(x) + adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5$$

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3))/x,x]
[Out] a*d*x + (a*e*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4 + (b*e*x^5)/5 + a*c*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(ad + \frac{ac}{x} + aex + bcx^2 + bdx^3 + bex^4\right) dx \\ &= adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5 + ac\log(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x}\,dx = adx+\frac{1}{2}aex^2+\frac{1}{3}bcx^3+\frac{1}{4}bdx^4+\frac{1}{5}bex^5+ac\log(x)$$

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x,x]
[Out] a*d*x + (a*e*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4 + (b*e*x^5)/5 + a*c*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
default	$adx+\frac{ae\,x^2}{2}+\frac{bc\,x^3}{3}+\frac{bd\,x^4}{4}+\frac{be\,x^5}{5}+ac\ln(x)$	39
norman	$adx+\frac{ae\,x^2}{2}+\frac{bc\,x^3}{3}+\frac{bd\,x^4}{4}+\frac{be\,x^5}{5}+ac\ln(x)$	39
risch	$adx+\frac{ae\,x^2}{2}+\frac{bc\,x^3}{3}+\frac{bd\,x^4}{4}+\frac{be\,x^5}{5}+ac\ln(x)$	39
parallelrisch	$adx+\frac{ae\,x^2}{2}+\frac{bc\,x^3}{3}+\frac{bd\,x^4}{4}+\frac{be\,x^5}{5}+ac\ln(x)$	39

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)/x,x,method=_RETURNVERBOSE)
[Out] a*d*x+1/2*a*e*x^2+1/3*b*c*x^3+1/4*b*d*x^4+1/5*b*e*x^5+a*c*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x}\,dx = \frac{1}{5}\,bex^5+\frac{1}{4}\,bdx^4+\frac{1}{3}\,bcx^3+\frac{1}{2}\,aex^2+adx+ac\log(x)$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="fricas")
[Out] 1/5*b*e*x^5 + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*e*x^2 + a*d*x + a*c*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x}\,dx = ac\log(x) + adx + \frac{aex^2}{2} + \frac{bcx^3}{3} + \frac{bdx^4}{4} + \frac{bex^5}{5}$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)/x,x)
[Out] a*c*log(x) + a*d*x + a*e*x**2/2 + b*c*x**3/3 + b*d*x**4/4 + b*e*x**5/5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x}\,dx = \frac{1}{5}\,bex^5 + \frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,aex^2 + adx + ac\log(x)$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="maxima")
[Out] 1/5*b*e*x^5 + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*e*x^2 + a*d*x + a*c*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x}\,dx = \frac{1}{5}\,bex^5 + \frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,aex^2 + adx + ac\log(|x|)$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="giac")
[Out] 1/5*b*e*x^5 + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*e*x^2 + a*d*x + a*c*log(abs
(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x}\,dx = a\,c\,\ln(x) + a\,d\,x + \frac{b\,c\,x^3}{3} + \frac{a\,e\,x^2}{2} + \frac{b\,d\,x^4}{4} + \frac{b\,e\,x^5}{5}$$

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2))/x,x)
[Out] a*c*log(x) + a*d*x + (b*c*x^3)/3 + (a*e*x^2)/2 + (b*d*x^4)/4 + (b*e*x^5)/5
```

3.317 $\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2} dx = -\frac{ac}{x} + aex + \frac{1}{2}bcx^2 + \frac{1}{3}bdx^3 + \frac{1}{4}bex^4 + ad\log(x)$$

```
[Out] -a*c/x+a*e*x+1/2*b*c*x^2+1/3*b*d*x^3+1/4*b*e*x^4+a*d*ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1642}

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2} dx = -\frac{ac}{x} + ad\log(x) + aex + \frac{1}{2}bcx^2 + \frac{1}{3}bdx^3 + \frac{1}{4}bex^4$$

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3))/x^2,x]
```

```
[Out] -((a*c)/x) + a*e*x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + a*d*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(ae + \frac{ac}{x^2} + \frac{ad}{x} + bcx + bdx^2 + bex^3\right) dx \\ &= -\frac{ac}{x} + aex + \frac{1}{2}bcx^2 + \frac{1}{3}bdx^3 + \frac{1}{4}bex^4 + ad\log(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2} \, dx = -\frac{ac}{x} + aex + \frac{1}{2}bcx^2 + \frac{1}{3}bdx^3 + \frac{1}{4}bex^4 + ad\log(x)$$

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x^2,x]
[Out] -((a*c)/x) + a*e*x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + a*d*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{ac}{x}+aex+\frac{cb\,x^{2}}{2}+\frac{bd\,x^{3}}{3}+\frac{be\,x^{4}}{4}+ad\ln\left(x\right)$	39
risch	$-\frac{ac}{x}+aex+\frac{cb\,x^{2}}{2}+\frac{bd\,x^{3}}{3}+\frac{be\,x^{4}}{4}+ad\ln\left(x\right)$	39
norman	$\frac{ae\,x^{2}-ac+\frac{1}{2}bc\,x^{3}+\frac{1}{3}bd\,x^{4}+\frac{1}{4}be\,x^{5}}{x}+ad\ln\left(x\right)$	43
parallelrisch	$\frac{3be\,x^{5}+4bd\,x^{4}+6bc\,x^{3}+12ad\ln\left(x\right)x+12ae\,x^{2}-12ac}{12x}$	46

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)/x^2,x,method=_RETURNVERBOSE)
[Out] -a*c/x+a*e*x+1/2*c*b*x^2+1/3*b*d*x^3+1/4*b*e*x^4+a*d*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2} \, dx = \frac{3\,bex^5 + 4\,bdx^4 + 6\,bcx^3 + 12\,aex^2 + 12\,adx\log(x) - 12\,ac}{12\,x}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="fricas")
[Out] 1/12*(3*b*e*x^5 + 4*b*d*x^4 + 6*b*c*x^3 + 12*a*e*x^2 + 12*a*d*x*log(x) - 12
*a*c)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx = -\frac{ac}{x} + ad\log(x) + aex + \frac{bcx^2}{2} + \frac{bdx^3}{3} + \frac{bex^4}{4}$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)/x**2,x)
[Out] -a*c/x + a*d*log(x) + a*e*x + b*c*x**2/2 + b*d*x**3/3 + b*e*x**4/4
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx = \frac{1}{4}\,bex^4 + \frac{1}{3}\,bdx^3 + \frac{1}{2}\,bcx^2 + aex + ad\log(x) - \frac{ac}{x}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="maxima")
[Out] 1/4*b*e*x^4 + 1/3*b*d*x^3 + 1/2*b*c*x^2 + a*e*x + a*d*log(x) - a*c/x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx = \frac{1}{4}\,bex^4 + \frac{1}{3}\,bdx^3 + \frac{1}{2}\,bcx^2 + aex + ad\log(|x|) - \frac{ac}{x}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="giac")
[Out] 1/4*b*e*x^4 + 1/3*b*d*x^3 + 1/2*b*c*x^2 + a*e*x + a*d*log(abs(x)) - a*c/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx = a\,d\,\ln(x) + a\,e\,x - \frac{a\,c}{x} + \frac{b\,c\,x^2}{2} + \frac{b\,d\,x^3}{3} + \frac{b\,e\,x^4}{4}$$

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2))/x^2,x)
[Out] a*d*log(x) + a*e*x - (a*c)/x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4
```

3.318 $\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx = -\frac{ac}{2x^2}-\frac{ad}{x}+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3+ae\log(x)$$

```
[Out] -1/2*a*c/x^2-a*d/x+b*c*x+1/2*b*d*x^2+1/3*b*e*x^3+a*e*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1642}

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx = -\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3$$

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3))/x^3,x]
[Out] -1/2*(a*c)/x^2 - (a*d)/x + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3 + a*e*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(bc+\frac{ac}{x^3}+\frac{ad}{x^2}+\frac{ae}{x}+bdx+bex^2\right)dx\\ &= -\frac{ac}{2x^2}-\frac{ad}{x}+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3+ae\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx=-\frac{ac}{2x^2}-\frac{ad}{x}+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3+ae\log(x)$$

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x^3,x]
[Out] -1/2*(a*c)/x^2 - (a*d)/x + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3 + a*e*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{ac}{2x^2}-\frac{ad}{x}+bcx+\frac{bd\,x^2}{2}+\frac{be\,x^3}{3}+ae\ln(x)$	39
risch	$\frac{be\,x^3}{3}+\frac{bd\,x^2}{2}+bcx+\frac{-adx-\frac{1}{2}ac}{x^2}+ae\ln(x)$	39
norman	$\frac{bc\,x^3-\frac{1}{2}ac-adx+\frac{1}{2}bd\,x^4+\frac{1}{3}be\,x^5}{x^2}+ae\ln(x)$	41
parallelrisch	$\frac{2be\,x^5+3bd\,x^4+6ae\ln(x)x^2+6bc\,x^3-6adx-3ac}{6x^2}$	46

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a*c/x^2-a*d/x+b*c*x+1/2*b*d*x^2+1/3*b*e*x^3+a*e*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx=\frac{2\,bex^5+3\,bdx^4+6\,bcx^3+6\,aex^2\log(x)-6\,adx-3\,ac}{6\,x^2}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="fricas")
[Out] 1/6*(2*b*e*x^5 + 3*b*d*x^4 + 6*b*c*x^3 + 6*a*e*x^2*log(x) - 6*a*d*x - 3*a*c
)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx = ae\log{(x)} + bcx + \frac{bdx^2}{2} + \frac{bex^3}{3} + \frac{-ac-2adx}{2x^2}$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)/x**3,x)
[Out] a*e*log(x) + b*c*x + b*d*x**2/2 + b*e*x**3/3 + (-a*c - 2*a*d*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx = \frac{1}{3}\,bex^3 + \frac{1}{2}\,bdx^2 + bcx + ae\log{(x)} - \frac{2\,adx + ac}{2\,x^2}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="maxima")
[Out] 1/3*b*e*x^3 + 1/2*b*d*x^2 + b*c*x + a*e*log(x) - 1/2*(2*a*d*x + a*c)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx = \frac{1}{3}\,bex^3 + \frac{1}{2}\,bdx^2 + bcx + ae\log{(|x|)} - \frac{2\,adx + ac}{2\,x^2}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="giac")
[Out] 1/3*b*e*x^3 + 1/2*b*d*x^2 + b*c*x + a*e*log(abs(x)) - 1/2*(2*a*d*x + a*c)/x
^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx = a\,e\,\ln{(x)} - \frac{\frac{a\,c}{2} + a\,d\,x}{x^2} + b\,c\,x + \frac{b\,d\,x^2}{2} + \frac{b\,e\,x^3}{3}$$

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2))/x^3,x)
[Out] a*e*log(x) - ((a*c)/2 + a*d*x)/x^2 + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3
```

3.319 $\int x^2(c+dx+ex^2)\left(a+bx^3\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx=\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{c(a+bx^3)^3}{9b}$$

```
[Out] 1/4*a^2*d*x^4+1/5*a^2*e*x^5+2/7*a*b*d*x^7+1/4*a*b*e*x^8+1/10*b^2*d*x^10+1/1
1*b^2*e*x^11+1/9*c*(b*x^3+a)^3/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1596, 1864}

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx=\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{c(a+bx^3)^3}{9b}+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (2*a*b*d*x^7)/7 + (a*b*e*x^8)/4 + (b^2*d*x^
10)/10 + (b^2*e*x^11)/11 + (c*(a + b*x^3)^3)/(9*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
```

```
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c(a+bx^3)^3}{9b} + \int (a+bx^3)^2 \left(-cx^2 + x^2(c+dx+ex^2)\right) dx \\
&= \frac{c(a+bx^3)^3}{9b} + \int \left(a^2dx^3 + a^2ex^4 + 2abdx^6 + 2abex^7 + b^2dx^9 + b^2ex^{10}\right) dx \\
&= \frac{1}{4}a^2dx^4 + \frac{1}{5}a^2ex^5 + \frac{2}{7}abdx^7 + \frac{1}{4}abex^8 + \frac{1}{10}b^2dx^{10} + \frac{1}{11}b^2ex^{11} + \frac{c(a+bx^3)^3}{9b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^2 dx = \frac{1}{3}a^2cx^3 + \frac{1}{4}a^2dx^4 + \frac{1}{5}a^2ex^5 + \frac{1}{3}abcx^6 + \frac{2}{7}abdx^7 + \frac{1}{4}abex^8 + \frac{1}{9}b^2cx^9 + \frac{1}{10}b^2dx^{10} + \frac{1}{11}b^2ex^{11}$$

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
[Out] (a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a*b*c*x^6)/3 + (2*a*b*d*x^
7)/7 + (a*b*e*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

method	result
gosper	$\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abe\,x^8+\frac{2}{7}ad\,x^7b+\frac{1}{3}abc\,x^6+\frac{1}{5}a^2e\,x^5+\frac{1}{4}a^2d\,x^4+\frac{1}{3}a^2c\,x$
default	$\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abe\,x^8+\frac{2}{7}ad\,x^7b+\frac{1}{3}abc\,x^6+\frac{1}{5}a^2e\,x^5+\frac{1}{4}a^2d\,x^4+\frac{1}{3}a^2c\,x$
norman	$\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abe\,x^8+\frac{2}{7}ad\,x^7b+\frac{1}{3}abc\,x^6+\frac{1}{5}a^2e\,x^5+\frac{1}{4}a^2d\,x^4+\frac{1}{3}a^2c\,x$
risch	$\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abe\,x^8+\frac{2}{7}ad\,x^7b+\frac{1}{3}abc\,x^6+\frac{1}{5}a^2e\,x^5+\frac{1}{4}a^2d\,x^4+\frac{1}{3}a^2c\,x$
parallelrisch	$\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abe\,x^8+\frac{2}{7}ad\,x^7b+\frac{1}{3}abc\,x^6+\frac{1}{5}a^2e\,x^5+\frac{1}{4}a^2d\,x^4+\frac{1}{3}a^2c\,x$

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*a*b*e*x^8+2/7*a*d*x^7*b+1
/3*a*b*c*x^6+1/5*a^2*e*x^5+1/4*a^2*d*x^4+1/3*a^2*c*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx=\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abex^8+\frac{2}{7}\,abdx^7+\frac{1}{3}\,abcx^6+\frac{1}{5}\,a^2ex^5+\frac{1}{4}\,a^2dx^4+\frac{1}{3}\,a^2cx^3$$

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*e*x^8 + 2/7*a*b
*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*e*x^5 + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx=\frac{a^2cx^3}{3}+\frac{a^2dx^4}{4}+\frac{a^2ex^5}{5}+\frac{abcx^6}{3}+\frac{2abdx^7}{7}+\frac{abex^8}{4}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}$$

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*c*x**3/3 + a**2*d*x**4/4 + a**2*e*x**5/5 + a*b*c*x**6/3 + 2*a*b*d*x**7
/7 + a*b*e*x**8/4 + b**2*c*x**9/9 + b**2*d*x**10/10 + b**2*e*x**11/11
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abex^8+\frac{2}{7}\,abdx^7+\frac{1}{3}\,abcx^6+\frac{1}{5}\,a^2ex^5+\frac{1}{4}\,a^2dx^4+\frac{1}{3}\,a^2cx^3$$

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*e*x^8 + 2/7*a*b
*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*e*x^5 + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abex^8+\frac{2}{7}\,abdx^7+\frac{1}{3}\,abcx^6+\frac{1}{5}\,a^2ex^5+\frac{1}{4}\,a^2dx^4+\frac{1}{3}\,a^2cx^3$$

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*e*x^8 + 2/7*a*b
*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*e*x^5 + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{e\,a^2\,x^5}{5}+\frac{d\,a^2\,x^4}{4}+\frac{c\,a^2\,x^3}{3}+\frac{e\,a\,b\,x^8}{4}+\frac{2\,d\,a\,b\,x^7}{7}+\frac{c\,a\,b\,x^6}{3}+\frac{e\,b^2\,x^{11}}{11}+\frac{d\,b^2\,x^{10}}{10}+\frac{c\,b^2\,x^9}{9}$$

```
[In] int(x^2*(a + b*x^3)^2*(c + d*x + e*x^2),x)
```

```
[Out] (a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (b^2*c*x^9)/9 + (a^2*e*x^5)/5 + (b^2*d*x^10
)/10 + (b^2*e*x^11)/11 + (a*b*c*x^6)/3 + (2*a*b*d*x^7)/7 + (a*b*e*x^8)/4
```

3.320 $\int x(c+dx+ex^2)(a+bx^3)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{10}b^2ex^{10}+\frac{d(a+bx^3)^3}{9b}$$

```
[Out] 1/2*a^2*c*x^2+1/4*a^2*e*x^4+2/5*a*b*c*x^5+2/7*a*b*e*x^7+1/8*b^2*c*x^8+1/10*
b^2*e*x^10+1/9*d*(b*x^3+a)^3/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1596, 1864}

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{d(a+bx^3)^3}{9b}+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{10}b^2ex^{10}$$

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (2*a*b*c*x^5)/5 + (2*a*b*e*x^7)/7 + (b^2*c*
x^8)/8 + (b^2*e*x^10)/10 + (d*(a + b*x^3)^3)/(9*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
```

```
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d(a+bx^3)^3}{9b}+\int (a+bx^3)^2\left(-dx^2+x(c+dx+ex^2)\right)\,dx\\ &= \frac{d(a+bx^3)^3}{9b}+\int \left(a^2cx+a^2ex^3+2abcx^4+2abex^6+b^2cx^7+b^2ex^9\right)\,dx\\ &= \frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{10}b^2ex^{10}+\frac{d(a+bx^3)^3}{9b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{1}{2}a^2cx^2+\frac{1}{3}a^2dx^3+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{9}b^2dx^9+\frac{1}{10}b^2ex^{10}$$

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (2*a*b*c*x^5)/5 + (a*b*d*x^
6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^8)/8 + (b^2*d*x^9)/9 + (b^2*e*x^10)/10
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

method	result
gosper	$\frac{1}{10}b^2e\,x^{10}+\frac{1}{9}b^2d\,x^9+\frac{1}{8}b^2c\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2e\,x^4+\frac{1}{3}a^2d\,x^3+\frac{1}{2}a^2c\,x^2$
default	$\frac{1}{10}b^2e\,x^{10}+\frac{1}{9}b^2d\,x^9+\frac{1}{8}b^2c\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2e\,x^4+\frac{1}{3}a^2d\,x^3+\frac{1}{2}a^2c\,x^2$
norman	$\frac{1}{10}b^2e\,x^{10}+\frac{1}{9}b^2d\,x^9+\frac{1}{8}b^2c\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2e\,x^4+\frac{1}{3}a^2d\,x^3+\frac{1}{2}a^2c\,x^2$
risch	$\frac{1}{10}b^2e\,x^{10}+\frac{1}{9}b^2d\,x^9+\frac{1}{8}b^2c\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2e\,x^4+\frac{1}{3}a^2d\,x^3+\frac{1}{2}a^2c\,x^2$
parallelrisch	$\frac{1}{10}b^2e\,x^{10}+\frac{1}{9}b^2d\,x^9+\frac{1}{8}b^2c\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2e\,x^4+\frac{1}{3}a^2d\,x^3+\frac{1}{2}a^2c\,x^2$

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/10*b^2*e*x^10+1/9*b^2*d*x^9+1/8*b^2*c*x^8+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5
*a*b*c*x^5+1/4*a^2*e*x^4+1/3*a^2*d*x^3+1/2*a^2*c*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{1}{10}\,b^2ex^{10}+\frac{1}{9}\,b^2dx^9+\frac{1}{8}\,b^2cx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2ex^4+\frac{1}{3}\,a^2dx^3+\frac{1}{2}\,a^2cx^2$$

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")
[Out] 1/10*b^2*e*x^10 + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d
*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*e*x^4 + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{a^2cx^2}{2}+\frac{a^2dx^3}{3}+\frac{a^2ex^4}{4}+\frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{b^2cx^8}{8}+\frac{b^2dx^9}{9}+\frac{b^2ex^{10}}{10}$$

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**2,x)
[Out] a**2*c*x**2/2 + a**2*d*x**3/3 + a**2*e*x**4/4 + 2*a*b*c*x**5/5 + a*b*d*x**6
/3 + 2*a*b*e*x**7/7 + b**2*c*x**8/8 + b**2*d*x**9/9 + b**2*e*x**10/10
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^2\,dx = \frac{1}{10}\,b^2ex^{10} + \frac{1}{9}\,b^2dx^9 + \frac{1}{8}\,b^2cx^8 + \frac{2}{7}\,abex^7 + \frac{1}{3}\,abdx^6 + \frac{2}{5}\,abcx^5 + \frac{1}{4}\,a^2ex^4 + \frac{1}{3}\,a^2dx^3 + \frac{1}{2}\,a^2cx^2$$

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*e*x^10 + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d
*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*e*x^4 + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^2\,dx = \frac{1}{10}\,b^2ex^{10} + \frac{1}{9}\,b^2dx^9 + \frac{1}{8}\,b^2cx^8 + \frac{2}{7}\,abex^7 + \frac{1}{3}\,abdx^6 + \frac{2}{5}\,abcx^5 + \frac{1}{4}\,a^2ex^4 + \frac{1}{3}\,a^2dx^3 + \frac{1}{2}\,a^2cx^2$$

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^2*e*x^10 + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d
*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*e*x^4 + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^2\,dx = \frac{e\,a^2\,x^4}{4} + \frac{d\,a^2\,x^3}{3} + \frac{c\,a^2\,x^2}{2} + \frac{2\,e\,a\,b\,x^7}{7} + \frac{d\,a\,b\,x^6}{3} + \frac{2\,c\,a\,b\,x^5}{5} + \frac{e\,b^2\,x^{10}}{10} + \frac{d\,b^2\,x^9}{9} + \frac{c\,b^2\,x^8}{8}$$

```
[In] int(x*(a + b*x^3)^2*(c + d*x + e*x^2),x)
```

```
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (b^2*c*x^8)/8 + (a^2*e*x^4)/4 + (b^2*d*x^9)
/9 + (b^2*e*x^10)/10 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7
```

3.321 $\int (c+dx+ex^2) (a+bx^3)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 77

$$\int (c+dx+ex^2) (a+bx^3)^2 \, dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5 +\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8+\frac{e(a+bx^3)^3}{9b}$$

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/2*a*b*c*x^4+2/5*a*b*d*x^5+1/7*b^2*c*x^7+1/8*b^2*d*x
^8+1/9*e*(b*x^3+a)^3/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1596, 1864}

$$\int (c+dx+ex^2) (a+bx^3)^2 \, dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5 +\frac{e(a+bx^3)^3}{9b}+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 +
 (b^2*d*x^8)/8 + (e*(a + b*x^3)^3)/(9*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
```

```
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e(a+bx^3)^3}{9b} + \int (c+dx)\left(a+bx^3\right)^2\,dx\\ &= \frac{e(a+bx^3)^3}{9b} + \int \left(a^2c+a^2dx+2abcx^3+2abdx^4+b^2cx^6+b^2dx^7\right)\,dx\\ &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8+\frac{e(a+bx^3)^3}{9b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5 + \frac{1}{3}abex^6+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8+\frac{1}{9}b^2ex^9$$

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^2,x]
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 +
 (a*b*e*x^6)/3 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8 + (b^2*e*x^9)/9
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{1}{9}b^2e\,x^9+\frac{1}{8}b^2d\,x^8+\frac{1}{7}b^2c\,x^7+\frac{1}{3}aeb\,x^6+\frac{2}{5}x^5dba+\frac{1}{2}abc\,x^4+\frac{1}{3}a^2e\,x^3+\frac{1}{2}a^2d\,x^2+a^2cx$	77
default	$\frac{1}{9}b^2e\,x^9+\frac{1}{8}b^2d\,x^8+\frac{1}{7}b^2c\,x^7+\frac{1}{3}aeb\,x^6+\frac{2}{5}x^5dba+\frac{1}{2}abc\,x^4+\frac{1}{3}a^2e\,x^3+\frac{1}{2}a^2d\,x^2+a^2cx$	77
norman	$\frac{1}{9}b^2e\,x^9+\frac{1}{8}b^2d\,x^8+\frac{1}{7}b^2c\,x^7+\frac{1}{3}aeb\,x^6+\frac{2}{5}x^5dba+\frac{1}{2}abc\,x^4+\frac{1}{3}a^2e\,x^3+\frac{1}{2}a^2d\,x^2+a^2cx$	77
risch	$\frac{1}{9}b^2e\,x^9+\frac{1}{8}b^2d\,x^8+\frac{1}{7}b^2c\,x^7+\frac{1}{3}aeb\,x^6+\frac{2}{5}x^5dba+\frac{1}{2}abc\,x^4+\frac{1}{3}a^2e\,x^3+\frac{1}{2}a^2d\,x^2+a^2cx$	77
parallelrisch	$\frac{1}{9}b^2e\,x^9+\frac{1}{8}b^2d\,x^8+\frac{1}{7}b^2c\,x^7+\frac{1}{3}aeb\,x^6+\frac{2}{5}x^5dba+\frac{1}{2}abc\,x^4+\frac{1}{3}a^2e\,x^3+\frac{1}{2}a^2d\,x^2+a^2cx$	77

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^2*e*x^9+1/8*b^2*d*x^8+1/7*b^2*c*x^7+1/3*a*e*b*x^6+2/5*x^5*d*b*a+1/2*a
*b*c*x^4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{1}{9}\,b^2ex^9+\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{1}{3}\,abex^6+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*b^2*e*x^9 + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*e*x^6 + 2/5*a*b*d*x
^5 + 1/2*a*b*c*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int (c+dx+ex^2)\left(a+bx^3\right)^2\,dx=a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{abcx^4}{2}+\frac{2abdx^5}{5}+\frac{abex^6}{3}+\frac{b^2cx^7}{7}+\frac{b^2dx^8}{8}+\frac{b^2ex^9}{9}$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a*b*c*x**4/2 + 2*a*b*d*x**5/5 +
a*b*e*x**6/3 + b**2*c*x**7/7 + b**2*d*x**8/8 + b**2*e*x**9/9
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2)\,(a+bx^3)^2\,dx = \frac{1}{9}\,b^2ex^9+\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{1}{3}\,abex^6+\frac{2}{5}\,abdx^5 + \frac{1}{2}\,abcx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*e*x^9 + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*e*x^6 + 2/5*a*b*d*x
^5 + 1/2*a*b*c*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2)\,(a+bx^3)^2\,dx = \frac{1}{9}\,b^2ex^9+\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{1}{3}\,abex^6+\frac{2}{5}\,abdx^5 + \frac{1}{2}\,abcx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*e*x^9 + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*e*x^6 + 2/5*a*b*d*x
^5 + 1/2*a*b*c*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2)\,(a+bx^3)^2\,dx = \frac{e\,a^2\,x^3}{3}+\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{e\,a\,b\,x^6}{3}+\frac{2\,d\,a\,b\,x^5}{5} + \frac{c\,a\,b\,x^4}{2}+\frac{e\,b^2\,x^9}{9}+\frac{d\,b^2\,x^8}{8}+\frac{c\,b^2\,x^7}{7}$$

```
[In] int((a + b*x^3)^2*(c + d*x + e*x^2),x)
```

```
[Out] (a^2*d*x^2)/2 + (b^2*c*x^7)/7 + (a^2*e*x^3)/3 + (b^2*d*x^8)/8 + (b^2*e*x^9)
/9 + a^2*c*x + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (a*b*e*x^6)/3
```

3.322 $\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x} \, dx = a^2dx + \frac{1}{2}a^2ex^2 + \frac{2}{3}abcx^3 + \frac{1}{2}abdx^4 + \frac{2}{5}abex^5 + \frac{1}{6}b^2cx^6 + \frac{1}{7}b^2dx^7 + \frac{1}{8}b^2ex^8 + a^2c\log(x)$$

```
[Out] a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*c*x^3+1/2*a*b*d*x^4+2/5*a*b*e*x^5+1/6*b^2*c*x
^6+1/7*b^2*d*x^7+1/8*b^2*e*x^8+a^2*c*ln(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1642}

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x} \, dx = a^2c\log(x) + a^2dx + \frac{1}{2}a^2ex^2 + \frac{2}{3}abcx^3 + \frac{1}{2}abdx^4 + \frac{2}{5}abex^5 + \frac{1}{6}b^2cx^6 + \frac{1}{7}b^2dx^7 + \frac{1}{8}b^2ex^8$$

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5
 + (b^2*c*x^6)/6 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2d+\frac{a^2c}{x}+a^2ex+2abcx^2+2abdx^3+2abex^4+b^2cx^5+b^2dx^6+b^2ex^7\right)\,dx\\ &= a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{2}abdx^4+\frac{2}{5}abex^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b^2dx^7+\frac{1}{8}b^2ex^8+a^2c\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)\,(a+bx^3)^2}{x}\,dx = a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{2}abdx^4+\frac{2}{5}abex^5 +\frac{1}{6}b^2cx^6+\frac{1}{7}b^2dx^7+\frac{1}{8}b^2ex^8+a^2c\log(x)$$

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5
 + (b^2*c*x^6)/6 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*Log[x]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

method	result	size
default	$a^2dx+\frac{a^2e\,x^2}{2}+\frac{2abc\,x^3}{3}+\frac{abd\,x^4}{2}+\frac{2abe\,x^5}{5}+\frac{b^2c\,x^6}{6}+\frac{b^2d\,x^7}{7}+\frac{b^2e\,x^8}{8}+a^2c\ln\left(x\right)$	75
norman	$a^2dx+\frac{a^2e\,x^2}{2}+\frac{2abc\,x^3}{3}+\frac{abd\,x^4}{2}+\frac{2abe\,x^5}{5}+\frac{b^2c\,x^6}{6}+\frac{b^2d\,x^7}{7}+\frac{b^2e\,x^8}{8}+a^2c\ln\left(x\right)$	75
risch	$a^2dx+\frac{a^2e\,x^2}{2}+\frac{2abc\,x^3}{3}+\frac{abd\,x^4}{2}+\frac{2abe\,x^5}{5}+\frac{b^2c\,x^6}{6}+\frac{b^2d\,x^7}{7}+\frac{b^2e\,x^8}{8}+a^2c\ln\left(x\right)$	75
parallelrisch	$a^2dx+\frac{a^2e\,x^2}{2}+\frac{2abc\,x^3}{3}+\frac{abd\,x^4}{2}+\frac{2abe\,x^5}{5}+\frac{b^2c\,x^6}{6}+\frac{b^2d\,x^7}{7}+\frac{b^2e\,x^8}{8}+a^2c\ln\left(x\right)$	75

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*c*x^3+1/2*a*b*d*x^4+2/5*a*b*e*x^5+1/6*b^2*c*x
^6+1/7*b^2*d*x^7+1/8*b^2*e*x^8+a^2*c*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x}\,dx = \frac{1}{8}\,b^2ex^8 + \frac{1}{7}\,b^2dx^7 + \frac{1}{6}\,b^2cx^6 + \frac{2}{5}\,abex^5 + \frac{1}{2}\,abdx^4 + \frac{2}{3}\,abcx^3 + \frac{1}{2}\,a^2ex^2 + a^2dx + a^2c\log\left(x\right)$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*b^2*e*x^8 + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*e*x^5 + 1/2*a*b*d*x
^4 + 2/3*a*b*c*x^3 + 1/2*a^2*e*x^2 + a^2*d*x + a^2*c*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x}\,dx = a^2c\log\left(x\right) + a^2dx + \frac{a^2ex^2}{2} + \frac{2abcx^3}{3} + \frac{abdx^4}{2} + \frac{2abex^5}{5} + \frac{b^2cx^6}{6} + \frac{b^2dx^7}{7} + \frac{b^2ex^8}{8}$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x,x)
```

```
[Out] a**2*c*log(x) + a**2*d*x + a**2*e*x**2/2 + 2*a*b*c*x**3/3 + a*b*d*x**4/2 +
2*a*b*e*x**5/5 + b**2*c*x**6/6 + b**2*d*x**7/7 + b**2*e*x**8/8
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x}\,dx = \frac{1}{8}\,b^2ex^8 + \frac{1}{7}\,b^2dx^7 + \frac{1}{6}\,b^2cx^6 + \frac{2}{5}\,abex^5 + \frac{1}{2}\,abdx^4 + \frac{2}{3}\,abcx^3 + \frac{1}{2}\,a^2ex^2 + a^2dx + a^2c\log\left(x\right)$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*e*x^8 + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*e*x^5 + 1/2*a*b*d*x
^4 + 2/3*a*b*c*x^3 + 1/2*a^2*e*x^2 + a^2*d*x + a^2*c*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x} \, dx = \frac{1}{8} b^2ex^8 + \frac{1}{7} b^2dx^7 + \frac{1}{6} b^2cx^6 + \frac{2}{5} abex^5 + \frac{1}{2} abdx^4 + \frac{2}{3} abcx^3 + \frac{1}{2} a^2ex^2 + a^2dx + a^2c\log(|x|)$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*b^2*e*x^8 + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*e*x^5 + 1/2*a*b*d*x
^4 + 2/3*a*b*c*x^3 + 1/2*a^2*e*x^2 + a^2*d*x + a^2*c*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x} \, dx = \frac{b^2\,c\,x^6}{6} + \frac{a^2\,e\,x^2}{2} + \frac{b^2\,d\,x^7}{7} + \frac{b^2\,e\,x^8}{8} + a^2\,c\,\ln(x) + a^2\,d\,x + \frac{2\,a\,b\,c\,x^3}{3} + \frac{a\,b\,d\,x^4}{2} + \frac{2\,a\,b\,e\,x^5}{5}$$

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2))/x,x)
```

```
[Out] (b^2*c*x^6)/6 + (a^2*e*x^2)/2 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*log(x
) + a^2*d*x + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5
```

3.323 $\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2} \, dx = -\frac{a^2c}{x} + a^2ex + abcx^2 + \frac{2}{3}abdx^3 + \frac{1}{2}abex^4 + \frac{1}{5}b^2cx^5 + \frac{1}{6}b^2dx^6 + \frac{1}{7}b^2ex^7 + a^2d\log(x)$$

```
[Out] -a^2*c/x+a^2*e*x+a*b*c*x^2+2/3*a*b*d*x^3+1/2*a*b*e*x^4+1/5*b^2*c*x^5+1/6*b^
2*d*x^6+1/7*b^2*e*x^7+a^2*d*ln(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1642}

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2} \, dx = -\frac{a^2c}{x} + a^2d\log(x) + a^2ex + abcx^2 + \frac{2}{3}abdx^3 + \frac{1}{2}abex^4 + \frac{1}{5}b^2cx^5 + \frac{1}{6}b^2dx^6 + \frac{1}{7}b^2ex^7$$

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + (b^2
*c*x^5)/5 + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(a^2e + \frac{a^2c}{x^2} + \frac{a^2d}{x} + 2abcx + 2abdx^2 + 2abex^3 + b^2cx^4 + b^2dx^5 + b^2ex^6\right) dx$$
$$= -\frac{a^2c}{x} + a^2ex + abcx^2 + \frac{2}{3}abdx^3 + \frac{1}{2}abex^4 + \frac{1}{5}b^2cx^5 + \frac{1}{6}b^2dx^6 + \frac{1}{7}b^2ex^7 + a^2d\log(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2} dx = -\frac{a^2c}{x} + a^2ex + abcx^2 + \frac{2}{3}abdx^3 + \frac{1}{2}abex^4 + \frac{1}{5}b^2cx^5 + \frac{1}{6}b^2dx^6 + \frac{1}{7}b^2ex^7 + a^2d\log(x)$$

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^2,x]
[Out] -((a^2*c)/x) + a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + (b^2
*c*x^5)/5 + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*Log[x]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^2c}{x}+a^2ex+abc\,x^2+\frac{2x^3abd}{3}+\frac{abe\,x^4}{2}+\frac{b^2c\,x^5}{5}+\frac{b^2d\,x^6}{6}+\frac{b^2e\,x^7}{7}+a^2d\ln\left(x\right)$	74
risch	$-\frac{a^2c}{x}+a^2ex+abc\,x^2+\frac{2x^3abd}{3}+\frac{abe\,x^4}{2}+\frac{b^2c\,x^5}{5}+\frac{b^2d\,x^6}{6}+\frac{b^2e\,x^7}{7}+a^2d\ln\left(x\right)$	74
norman	$\frac{a^2e\,x^2+abc\,x^3-a^2c+\frac{1}{5}b^2c\,x^6+\frac{1}{6}b^2d\,x^7+\frac{1}{7}b^2e\,x^8+\frac{2}{3}abd\,x^4+\frac{1}{2}abe\,x^5}{x}+a^2d\ln\left(x\right)$	78
parallelrisch	$\frac{30b^2e\,x^8+35b^2d\,x^7+42b^2c\,x^6+105abe\,x^5+140abd\,x^4+210abc\,x^3+210a^2d\ln(x)x+210a^2e\,x^2-210a^2c}{210x}$	82

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x,method=_RETURNVERBOSE)
[Out] -a^2*c/x+a^2*e*x+a*b*c*x^2+2/3*x^3*a*b*d+1/2*a*b*e*x^4+1/5*b^2*c*x^5+1/6*b^
2*d*x^6+1/7*b^2*e*x^7+a^2*d*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx+ex^2)\,(a+bx^3)^2}{x^2}\,dx = \frac{30\,b^2ex^8+35\,b^2dx^7+42\,b^2cx^6+105\,abex^5+140\,abdx^4+210\,abcx^3+210\,a^2ex^2+210\,a^2dx\log\left(x\right)-210}{210\,x}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/210*(30*b^2*e*x^8 + 35*b^2*d*x^7 + 42*b^2*c*x^6 + 105*a*b*e*x^5 + 140*a*b
*d*x^4 + 210*a*b*c*x^3 + 210*a^2*e*x^2 + 210*a^2*d*x*log(x) - 210*a^2*c)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx+ex^2)\,(a+bx^3)^2}{x^2}\,dx = -\frac{a^2c}{x}+a^2d\log\left(x\right)+a^2ex+abcx^2+\frac{2abdx^3}{3}+\frac{abex^4}{2}+\frac{b^2cx^5}{5}+\frac{b^2dx^6}{6}+\frac{b^2ex^7}{7}$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x**2,x)
```

```
[Out] -a**2*c/x + a**2*d*log(x) + a**2*e*x + a*b*c*x**2 + 2*a*b*d*x**3/3 + a*b*e*
x**4/2 + b**2*c*x**5/5 + b**2*d*x**6/6 + b**2*e*x**7/7
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)\,(a+bx^3)^2}{x^2}\,dx = \frac{1}{7}\,b^2ex^7+\frac{1}{6}\,b^2dx^6+\frac{1}{5}\,b^2cx^5+\frac{1}{2}\,abex^4+\frac{2}{3}\,abdx^3+abcx^2+a^2ex+a^2d\log\left(x\right)-\frac{a^2c}{x}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*e*x^7 + 1/6*b^2*d*x^6 + 1/5*b^2*c*x^5 + 1/2*a*b*e*x^4 + 2/3*a*b*d*x
^3 + a*b*c*x^2 + a^2*e*x + a^2*d*log(x) - a^2*c/x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2}\,dx = \frac{1}{7}b^2ex^7+\frac{1}{6}b^2dx^6+\frac{1}{5}b^2cx^5+\frac{1}{2}abex^4+\frac{2}{3}abdx^3+abcx^2+a^2ex+a^2d\log(|x|)-\frac{a^2c}{x}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*e*x^7 + 1/6*b^2*d*x^6 + 1/5*b^2*c*x^5 + 1/2*a*b*e*x^4 + 2/3*a*b*d*x
^3 + a*b*c*x^2 + a^2*e*x + a^2*d*log(abs(x)) - a^2*c/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2}\,dx = \frac{b^2\,c\,x^5}{5}-\frac{a^2\,c}{x}+\frac{b^2\,d\,x^6}{6}+\frac{b^2\,e\,x^7}{7}+a^2\,d\,\ln(x)+a^2\,e\,x+a\,b\,c\,x^2+\frac{2\,a\,b\,d\,x^3}{3}+\frac{a\,b\,e\,x^4}{2}$$

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2))/x^2,x)
```

```
[Out] (b^2*c*x^5)/5 - (a^2*c)/x + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*log(x) +
a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2
```

3.324 $\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3} dx = -\frac{a^2c}{2x^2} - \frac{a^2d}{x} + 2abcx + abdx^2 + \frac{2}{3}abex^3 + \frac{1}{4}b^2cx^4 + \frac{1}{5}b^2dx^5 + \frac{1}{6}b^2ex^6 + a^2e\log(x)$$

```
[Out] -1/2*a^2*c/x^2-a^2*d/x+2*a*b*c*x+a*b*d*x^2+2/3*a*b*e*x^3+1/4*b^2*c*x^4+1/5*
b^2*d*x^5+1/6*b^2*e*x^6+a^2*e*ln(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1642}

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3} dx = -\frac{a^2c}{2x^2} - \frac{a^2d}{x} + a^2e\log(x) + 2abcx + abdx^2 + \frac{2}{3}abex^3 + \frac{1}{4}b^2cx^4 + \frac{1}{5}b^2dx^5 + \frac{1}{6}b^2ex^6$$

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^3,x]
```

```
[Out] -1/2*(a^2*c)/x^2 - (a^2*d)/x + 2*a*b*c*x + a*b*d*x^2 + (2*a*b*e*x^3)/3 + (b
^2*c*x^4)/4 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 + a^2*e*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(2abc + \frac{a^2c}{x^3} + \frac{a^2d}{x^2} + \frac{a^2e}{x} + 2abdx + 2abex^2 + b^2cx^3 + b^2dx^4 + b^2ex^5\right) dx$$
$$= -\frac{a^2c}{2x^2} - \frac{a^2d}{x} + 2abcx + abdx^2 + \frac{2}{3}abex^3 + \frac{1}{4}b^2cx^4 + \frac{1}{5}b^2dx^5 + \frac{1}{6}b^2ex^6 + a^2e\log(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3} dx = -\frac{a^2c}{2x^2} - \frac{a^2d}{x} + 2abcx + abdx^2 + \frac{2}{3}abex^3 + \frac{1}{4}b^2cx^4 + \frac{1}{5}b^2dx^5 + \frac{1}{6}b^2ex^6 + a^2e\log(x)$$

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^3,x]
[Out] -1/2*(a^2*c)/x^2 - (a^2*d)/x + 2*a*b*c*x + a*b*d*x^2 + (2*a*b*e*x^3)/3 + (b
^2*c*x^4)/4 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 + a^2*e*Log[x]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+2abcx+x^2abd+\frac{2abe\,x^3}{3}+\frac{b^2c\,x^4}{4}+\frac{b^2d\,x^5}{5}+\frac{b^2e\,x^6}{6}+a^2e\ln\left(x\right)$	75
risch	$\frac{b^2e\,x^6}{6}+\frac{b^2d\,x^5}{5}+\frac{b^2c\,x^4}{4}+\frac{2abe\,x^3}{3}+x^2abd+2abcx+\frac{-a^2dx-\frac{1}{2}a^2c}{x^2}+a^2e\ln\left(x\right)$	75
norman	$\frac{abd\,x^4-\frac{1}{2}a^2c-a^2dx+\frac{1}{4}b^2c\,x^6+\frac{1}{5}b^2d\,x^7+\frac{1}{6}b^2e\,x^8+2abc\,x^3+\frac{2}{3}abe\,x^5}{x^2}+a^2e\ln\left(x\right)$	77
parallelrisch	$\frac{10b^2e\,x^8+12b^2d\,x^7+15b^2c\,x^6+40abe\,x^5+60abd\,x^4+60a^2e\ln(x)x^2+120abc\,x^3-60a^2dx-30a^2c}{60x^2}$	82

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a^2*c/x^2-a^2*d/x+2*a*b*c*x+x^2*a*b*d+2/3*a*b*e*x^3+1/4*b^2*c*x^4+1/5*
b^2*d*x^5+1/6*b^2*e*x^6+a^2*e*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx = \frac{10\,b^2ex^8+12\,b^2dx^7+15\,b^2cx^6+40\,abex^5+60\,abdx^4+120\,abcx^3+60\,a^2ex^2\log(x)-60\,a^2dx-30\,a^2c}{60\,x^2}$$

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="fricas")

[Out] 1/60*(10*b^2*e*x^8 + 12*b^2*d*x^7 + 15*b^2*c*x^6 + 40*a*b*e*x^5 + 60*a*b*d*x^4 + 120*a*b*c*x^3 + 60*a^2*e*x^2*log(x) - 60*a^2*d*x - 30*a^2*c)/x^2

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx = a^2e\log(x)+2abcx+abdx^2+\frac{2abex^3}{3}+\frac{b^2cx^4}{4}+\frac{b^2dx^5}{5}+\frac{b^2ex^6}{6}+\frac{-a^2c-2a^2dx}{2x^2}$$

[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x**3,x)

[Out] a**2*e*log(x) + 2*a*b*c*x + a*b*d*x**2 + 2*a*b*e*x**3/3 + b**2*c*x**4/4 + b**2*d*x**5/5 + b**2*e*x**6/6 + (-a**2*c - 2*a**2*d*x)/(2*x**2)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx = \frac{1}{6}\,b^2ex^6+\frac{1}{5}\,b^2dx^5+\frac{1}{4}\,b^2cx^4+\frac{2}{3}\,abex^3+abdx^2+2\,abcx+a^2e\log(x)-\frac{2\,a^2dx+a^2c}{2\,x^2}$$

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="maxima")

[Out] 1/6*b^2*e*x^6 + 1/5*b^2*d*x^5 + 1/4*b^2*c*x^4 + 2/3*a*b*e*x^3 + a*b*d*x^2 + 2*a*b*c*x + a^2*e*log(x) - 1/2*(2*a^2*d*x + a^2*c)/x^2

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx = \frac{1}{6}\,b^2ex^6 + \frac{1}{5}\,b^2dx^5 + \frac{1}{4}\,b^2cx^4 + \frac{2}{3}\,abex^3 + abdx^2 + 2\,abcx + a^2e\log(|x|) - \frac{2\,a^2dx + a^2c}{2\,x^2}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*b^2*e*x^6 + 1/5*b^2*d*x^5 + 1/4*b^2*c*x^4 + 2/3*a*b*e*x^3 + a*b*d*x^2 +
 2*a*b*c*x + a^2*e*log(abs(x)) - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx = \frac{b^2\,c\,x^4}{4} - \frac{\frac{a^2\,c}{2} + a^2\,d\,x}{x^2} + \frac{b^2\,d\,x^5}{5} + \frac{b^2\,e\,x^6}{6} + a^2\,e\,\ln(x) + a\,b\,d\,x^2 + \frac{2\,a\,b\,e\,x^3}{3} + 2\,a\,b\,c\,x$$

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2))/x^3,x)
```

```
[Out] (b^2*c*x^4)/4 - ((a^2*c)/2 + a^2*d*x)/x^2 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 +
 a^2*e*log(x) + a*b*d*x^2 + (2*a*b*e*x^3)/3 + 2*a*b*c*x
```

3.325 $\int x^2(c+dx+ex^2)\left(a+bx^3\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 110

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx=\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}+\frac{c(a+bx^3)^4}{12b}$$

```
[Out] 1/4*a^3*d*x^4+1/5*a^3*e*x^5+3/7*a^2*b*d*x^7+3/8*a^2*b*e*x^8+3/10*a*b^2*d*x^
10+3/11*a*b^2*e*x^11+1/13*b^3*d*x^13+1/14*b^3*e*x^14+1/12*c*(b*x^3+a)^4/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1596, 1864}

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx=\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{c(a+bx^3)^4}{12b}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}$$

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (3*a^2*b*d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (3*
a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*d*x^13)/13 + (b^3*e*x^14)/14
+ (c*(a + b*x^3)^4)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
```

```
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c(a+bx^3)^4}{12b} + \int (a+bx^3)^3 \left(-cx^2 + x^2(c+dx+ex^2)\right) dx \\
&= \frac{c(a+bx^3)^4}{12b} \\
&\quad + \int \left(a^3dx^3 + a^3ex^4 + 3a^2bdx^6 + 3a^2bex^7 + 3ab^2dx^9 + 3ab^2ex^{10} + b^3dx^{12} + b^3ex^{13}\right) dx \\
&= \frac{1}{4}a^3dx^4 + \frac{1}{5}a^3ex^5 + \frac{3}{7}a^2bdx^7 + \frac{3}{8}a^2bex^8 + \frac{3}{10}ab^2dx^{10} \\
&\quad + \frac{3}{11}ab^2ex^{11} + \frac{1}{13}b^3dx^{13} + \frac{1}{14}b^3ex^{14} + \frac{c(a+bx^3)^4}{12b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.26

$$\begin{aligned}
\int x^2(c+dx+ex^2)\left(a+bx^3\right)^3 dx &= \frac{1}{3}a^3cx^3 + \frac{1}{4}a^3dx^4 + \frac{1}{5}a^3ex^5 + \frac{1}{2}a^2bcx^6 \\
&\quad + \frac{3}{7}a^2bdx^7 + \frac{3}{8}a^2bex^8 + \frac{1}{3}ab^2cx^9 + \frac{3}{10}ab^2dx^{10} \\
&\quad + \frac{3}{11}ab^2ex^{11} + \frac{1}{12}b^3cx^{12} + \frac{1}{13}b^3dx^{13} + \frac{1}{14}b^3ex^{14}
\end{aligned}$$

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*b*c*x^6)/2 + (3*a^2*b*
d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a
*b^2*e*x^11)/11 + (b^3*c*x^12)/12 + (b^3*d*x^13)/13 + (b^3*e*x^14)/14
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

method	result
gosper	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{2}a^2bc\,x^6+\frac{3}{7}a^2bd\,x^7+\frac{3}{8}a^2be\,x^8+\frac{1}{3}a\,b^2c\,x^9+\frac{3}{10}a\,b^2d\,x^{10}+\frac{3}{11}$
default	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{2}a^2bc\,x^6+\frac{3}{7}a^2bd\,x^7+\frac{3}{8}a^2be\,x^8+\frac{1}{3}a\,b^2c\,x^9+\frac{3}{10}a\,b^2d\,x^{10}+\frac{3}{11}$
norman	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{2}a^2bc\,x^6+\frac{3}{7}a^2bd\,x^7+\frac{3}{8}a^2be\,x^8+\frac{1}{3}a\,b^2c\,x^9+\frac{3}{10}a\,b^2d\,x^{10}+\frac{3}{11}$
risch	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{2}a^2bc\,x^6+\frac{3}{7}a^2bd\,x^7+\frac{3}{8}a^2be\,x^8+\frac{1}{3}a\,b^2c\,x^9+\frac{3}{10}a\,b^2d\,x^{10}+\frac{3}{11}$
parallelrisch	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{2}a^2bc\,x^6+\frac{3}{7}a^2bd\,x^7+\frac{3}{8}a^2be\,x^8+\frac{1}{3}a\,b^2c\,x^9+\frac{3}{10}a\,b^2d\,x^{10}+\frac{3}{11}$

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*c*a^3*x^3+1/4*a^3*d*x^4+1/5*a^3*e*x^5+1/2*a^2*b*c*x^6+3/7*a^2*b*d*x^7+3
/8*a^2*b*e*x^8+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/11*a*b^2*e*x^11+1/12*b^3
*c*x^12+1/13*b^3*d*x^13+1/14*b^3*e*x^14
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx=\frac{1}{14}\,b^3ex^{14}+\frac{1}{13}\,b^3dx^{13}+\frac{1}{12}\,b^3cx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bex^8+\frac{3}{7}\,a^2bdx^7+\frac{1}{2}\,a^2bcx^6+\frac{1}{5}\,a^3ex^5+\frac{1}{4}\,a^3dx^4+\frac{1}{3}\,a^3cx^3$$

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/14*b^3*e*x^14 + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*e*x^11 + 3
/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*e*x^8 + 3/7*a^2*b*d*x^7 + 1/
2*a^2*b*c*x^6 + 1/5*a^3*e*x^5 + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.25

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{a^3cx^3}{3}+\frac{a^3dx^4}{4}+\frac{a^3ex^5}{5}+\frac{a^2bcx^6}{2}+\frac{3a^2bdx^7}{7}+\frac{3a^2bex^8}{8}+\frac{ab^2cx^9}{3}+\frac{3ab^2dx^{10}}{10}+\frac{3ab^2ex^{11}}{11}+\frac{b^3cx^{12}}{12}+\frac{b^3dx^{13}}{13}+\frac{b^3ex^{14}}{14}$$

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*x**3/3 + a**3*d*x**4/4 + a**3*e*x**5/5 + a**2*b*c*x**6/2 + 3*a**2*b*
d*x**7/7 + 3*a**2*b*e*x**8/8 + a*b**2*c*x**9/3 + 3*a*b**2*d*x**10/10 + 3*a*
b**2*e*x**11/11 + b**3*c*x**12/12 + b**3*d*x**13/13 + b**3*e*x**14/14
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{1}{14}\,b^3ex^{14}+\frac{1}{13}\,b^3dx^{13}+\frac{1}{12}\,b^3cx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bex^8+\frac{3}{7}\,a^2bdx^7+\frac{1}{2}\,a^2bcx^6+\frac{1}{5}\,a^3ex^5+\frac{1}{4}\,a^3dx^4+\frac{1}{3}\,a^3cx^3$$

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^3*e*x^14 + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*e*x^11 + 3
/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*e*x^8 + 3/7*a^2*b*d*x^7 + 1/
2*a^2*b*c*x^6 + 1/5*a^3*e*x^5 + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{1}{14}\,b^3ex^{14}+\frac{1}{13}\,b^3dx^{13}+\frac{1}{12}\,b^3cx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bex^8+\frac{3}{7}\,a^2bdx^7+\frac{1}{2}\,a^2bcx^6+\frac{1}{5}\,a^3ex^5+\frac{1}{4}\,a^3dx^4+\frac{1}{3}\,a^3cx^3$$

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^3*e*x^14 + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*e*x^11 + 3
/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*e*x^8 + 3/7*a^2*b*d*x^7 + 1/
2*a^2*b*c*x^6 + 1/5*a^3*e*x^5 + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{e\,a^3\,x^5}{5}+\frac{d\,a^3\,x^4}{4}+\frac{c\,a^3\,x^3}{3}+\frac{3\,e\,a^2\,b\,x^8}{8}+\frac{3\,d\,a^2\,b\,x^7}{7}+\frac{c\,a^2\,b\,x^6}{2}+\frac{3\,e\,a\,b^2\,x^{11}}{11}+\frac{3\,d\,a\,b^2\,x^{10}}{10}+\frac{c\,a\,b^2\,x^9}{3}+\frac{e\,b^3\,x^{14}}{14}+\frac{d\,b^3\,x^{13}}{13}+\frac{c\,b^3\,x^{12}}{12}$$

```
[In] int(x^2*(a + b*x^3)^3*(c + d*x + e*x^2),x)
```

```
[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (b^3*c*x^12)/12 + (a^3*e*x^5)/5 + (b^3*d*x^
13)/13 + (b^3*e*x^14)/14 + (a^2*b*c*x^6)/2 + (a*b^2*c*x^9)/3 + (3*a^2*b*d*x
^7)/7 + (3*a*b^2*d*x^10)/10 + (3*a^2*b*e*x^8)/8 + (3*a*b^2*e*x^11)/11
```

3.326 $\int x(c+dx+ex^2)\left(a+bx^3\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{3}{10}ab^2ex^{10}+\frac{1}{11}b^3cx^{11}+\frac{1}{13}b^3ex^{13}+\frac{d\left(a+bx^3\right)^4}{12b}$$

```
[Out] 1/2*a^3*c*x^2+1/4*a^3*e*x^4+3/5*a^2*b*c*x^5+3/7*a^2*b*e*x^7+3/8*a*b^2*c*x^8
+3/10*a*b^2*e*x^10+1/11*b^3*c*x^11+1/13*b^3*e*x^13+1/12*d*(b*x^3+a)^4/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1596, 1864}

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{3}{10}ab^2ex^{10}+\frac{d\left(a+bx^3\right)^4}{12b}+\frac{1}{11}b^3cx^{11}+\frac{1}{13}b^3ex^{13}$$

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*e*x^4)/4 + (3*a^2*b*c*x^5)/5 + (3*a^2*b*e*x^7)/7 + (3*
a*b^2*c*x^8)/8 + (3*a*b^2*e*x^10)/10 + (b^3*c*x^11)/11 + (b^3*e*x^13)/13 +
(d*(a + b*x^3)^4)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
```

```
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d(a+bx^3)^4}{12b} + \int (a+bx^3)^3 \left(-dx^2 + x\left(c+dx+ex^2\right)\right) dx \\ &= \frac{d(a+bx^3)^4}{12b} + \int \left(a^3cx + a^3ex^3 + 3a^2bcx^4 + 3a^2bex^6 + 3ab^2cx^7 + 3ab^2ex^9 + b^3cx^{10} + b^3ex^{12}\right) dx \\ &= \frac{1}{2}a^3cx^2 + \frac{1}{4}a^3ex^4 + \frac{3}{5}a^2bcx^5 + \frac{3}{7}a^2bex^7 + \frac{3}{8}ab^2cx^8 + \frac{3}{10}ab^2ex^{10} + \frac{1}{11}b^3cx^{11} + \frac{1}{13}b^3ex^{13} + \frac{d(a+bx^3)^4}{12b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.26

$$\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^3 dx = \frac{1}{2}a^3cx^2 + \frac{1}{3}a^3dx^3 + \frac{1}{4}a^3ex^4 + \frac{3}{5}a^2bcx^5 + \frac{1}{2}a^2bdx^6 + \frac{3}{7}a^2bex^7 + \frac{3}{8}ab^2cx^8 + \frac{1}{3}ab^2dx^9 + \frac{3}{10}ab^2ex^{10} + \frac{1}{11}b^3cx^{11} + \frac{1}{12}b^3dx^{12} + \frac{1}{13}b^3ex^{13}$$

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (3*a^2*b*c*x^5)/5 + (a^2*b*
d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*c*x^8)/8 + (a*b^2*d*x^9)/3 + (3*a*b
^2*e*x^10)/10 + (b^3*c*x^11)/11 + (b^3*d*x^12)/12 + (b^3*e*x^13)/13
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

method	result
gosper	$\frac{1}{13}b^3e\,x^{13}+\frac{1}{12}b^3d\,x^{12}+\frac{1}{11}b^3c\,x^{11}+\frac{3}{10}a\,b^2e\,x^{10}+\frac{1}{3}x^9a\,b^2d+\frac{3}{8}a\,b^2c\,x^8+\frac{3}{7}a^2be\,x^7+\frac{1}{2}a^2bd\,x^6+$
default	$\frac{1}{13}b^3e\,x^{13}+\frac{1}{12}b^3d\,x^{12}+\frac{1}{11}b^3c\,x^{11}+\frac{3}{10}a\,b^2e\,x^{10}+\frac{1}{3}x^9a\,b^2d+\frac{3}{8}a\,b^2c\,x^8+\frac{3}{7}a^2be\,x^7+\frac{1}{2}a^2bd\,x^6+$
norman	$\frac{1}{13}b^3e\,x^{13}+\frac{1}{12}b^3d\,x^{12}+\frac{1}{11}b^3c\,x^{11}+\frac{3}{10}a\,b^2e\,x^{10}+\frac{1}{3}x^9a\,b^2d+\frac{3}{8}a\,b^2c\,x^8+\frac{3}{7}a^2be\,x^7+\frac{1}{2}a^2bd\,x^6+$
risch	$\frac{1}{13}b^3e\,x^{13}+\frac{1}{12}b^3d\,x^{12}+\frac{1}{11}b^3c\,x^{11}+\frac{3}{10}a\,b^2e\,x^{10}+\frac{1}{3}x^9a\,b^2d+\frac{3}{8}a\,b^2c\,x^8+\frac{3}{7}a^2be\,x^7+\frac{1}{2}a^2bd\,x^6+$
parallelrisch	$\frac{1}{13}b^3e\,x^{13}+\frac{1}{12}b^3d\,x^{12}+\frac{1}{11}b^3c\,x^{11}+\frac{3}{10}a\,b^2e\,x^{10}+\frac{1}{3}x^9a\,b^2d+\frac{3}{8}a\,b^2c\,x^8+\frac{3}{7}a^2be\,x^7+\frac{1}{2}a^2bd\,x^6+$

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^3*e*x^13+1/12*b^3*d*x^12+1/11*b^3*c*x^11+3/10*a*b^2*e*x^10+1/3*x^9*a
*b^2*d+3/8*a*b^2*c*x^8+3/7*a^2*b*e*x^7+1/2*a^2*b*d*x^6+3/5*a^2*b*c*x^5+1/4*
a^3*e*x^4+1/3*a^3*d*x^3+1/2*a^3*c*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x(c+dx+ex^2)\,(a+bx^3)^3\,dx=\frac{1}{13}\,b^3ex^{13}+\frac{1}{12}\,b^3dx^{12}+\frac{1}{11}\,b^3cx^{11}+\frac{3}{10}\,ab^2ex^{10}+\frac{1}{3}\,ab^2dx^9+\frac{3}{8}\,ab^2cx^8+\frac{3}{7}\,a^2bex^7+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3ex^4+\frac{1}{3}\,a^3dx^3+\frac{1}{2}\,a^3cx^2$$

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/13*b^3*e*x^13 + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*e*x^10 + 1
/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*
a^2*b*c*x^5 + 1/4*a^3*e*x^4 + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.25

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^3\,dx = \frac{a^3cx^2}{2}+\frac{a^3dx^3}{3}+\frac{a^3ex^4}{4}+\frac{3a^2bcx^5}{5}+\frac{a^2bdx^6}{2}+\frac{3a^2bex^7}{7}+\frac{3ab^2cx^8}{8}+\frac{ab^2dx^9}{3}+\frac{3ab^2ex^{10}}{10}+\frac{b^3cx^{11}}{11}+\frac{b^3dx^{12}}{12}+\frac{b^3ex^{13}}{13}$$

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*x**2/2 + a**3*d*x**3/3 + a**3*e*x**4/4 + 3*a**2*b*c*x**5/5 + a**2*b*
d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a*b**2*c*x**8/8 + a*b**2*d*x**9/3 + 3*a*b*
*2*e*x**10/10 + b**3*c*x**11/11 + b**3*d*x**12/12 + b**3*e*x**13/13
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^3\,dx = \frac{1}{13}\,b^3ex^{13}+\frac{1}{12}\,b^3dx^{12}+\frac{1}{11}\,b^3cx^{11}+\frac{3}{10}\,ab^2ex^{10}+\frac{1}{3}\,ab^2dx^9+\frac{3}{8}\,ab^2cx^8+\frac{3}{7}\,a^2bex^7+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3ex^4+\frac{1}{3}\,a^3dx^3+\frac{1}{2}\,a^3cx^2$$

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*e*x^13 + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*e*x^10 + 1
/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*
a^2*b*c*x^5 + 1/4*a^3*e*x^4 + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^3\,dx = \frac{1}{13}\,b^3ex^{13}+\frac{1}{12}\,b^3dx^{12}+\frac{1}{11}\,b^3cx^{11}+\frac{3}{10}\,ab^2ex^{10}+\frac{1}{3}\,ab^2dx^9+\frac{3}{8}\,ab^2cx^8+\frac{3}{7}\,a^2bex^7+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3ex^4+\frac{1}{3}\,a^3dx^3+\frac{1}{2}\,a^3cx^2$$

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^3*e*x^13 + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*e*x^10 + 1
/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*
a^2*b*c*x^5 + 1/4*a^3*e*x^4 + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{e\,a^3\,x^4}{4}+\frac{d\,a^3\,x^3}{3}+\frac{c\,a^3\,x^2}{2}+\frac{3\,e\,a^2\,b\,x^7}{7}+\frac{d\,a^2\,b\,x^6}{2}+\frac{3\,c\,a^2\,b\,x^5}{5}+\frac{3\,e\,a\,b^2\,x^{10}}{10}+\frac{d\,a\,b^2\,x^9}{3}+\frac{3\,c\,a\,b^2\,x^8}{8}+\frac{e\,b^3\,x^{13}}{13}+\frac{d\,b^3\,x^{12}}{12}+\frac{c\,b^3\,x^{11}}{11}$$

```
[In] int(x*(a + b*x^3)^3*(c + d*x + e*x^2),x)
```

```
[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (b^3*c*x^11)/11 + (a^3*e*x^4)/4 + (b^3*d*x^
12)/12 + (b^3*e*x^13)/13 + (3*a^2*b*c*x^5)/5 + (3*a*b^2*c*x^8)/8 + (a^2*b*d
*x^6)/2 + (a*b^2*d*x^9)/3 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*e*x^10)/10
```

3.327 $\int (c+dx+ex^2)(a+bx^3)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 105

$$\int (c+dx+ex^2)\left(a+bx^3\right)^3\,dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}+\frac{e(a+bx^3)^4}{12b}$$

```
[Out] a^3*c*x+1/2*a^3*d*x^2+3/4*a^2*b*c*x^4+3/5*a^2*b*d*x^5+3/7*a*b^2*c*x^7+3/8*a
*b^2*d*x^8+1/10*b^3*c*x^10+1/11*b^3*d*x^11+1/12*e*(b*x^3+a)^4/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1596, 1864}

$$\int (c+dx+ex^2)\left(a+bx^3\right)^3\,dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{e(a+bx^3)^4}{12b}+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*
c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + (e*(a +
b*x^3)^4)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
```

```
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e(a+bx^3)^4}{12b} + \int (c+dx)\left(a+bx^3\right)^3\,dx\\ &= \frac{e(a+bx^3)^4}{12b} + \int \left(a^3c+a^3dx+3a^2bcx^3+3a^2bdx^4+3ab^2cx^6+3ab^2dx^7+b^3cx^9+b^3dx^{10}\right)dx\\ &= a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7\\ &\quad+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}+\frac{e(a+bx^3)^4}{12b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx &= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{4}a^2bcx^4\\ &\quad+\frac{3}{5}a^2bdx^5+\frac{1}{2}a^2bex^6+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8\\ &\quad+\frac{1}{3}ab^2ex^9+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}+\frac{1}{12}b^3ex^{12}\end{aligned}$$

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^3,x]

[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^
5)/5 + (a^2*b*e*x^6)/2 + (3*a*b^2*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (a*b^2*e*x
^9)/3 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + (b^3*e*x^12)/12
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

method	result
gosper	$\frac{1}{12}b^3e\,x^{12}+\frac{1}{11}b^3d\,x^{11}+\frac{1}{10}b^3c\,x^{10}+\frac{1}{3}a\,b^2e\,x^9+\frac{3}{8}x^8b^2da+\frac{3}{7}a\,b^2c\,x^7+\frac{1}{2}a^2be\,x^6+\frac{3}{5}x^5bd\,a^2+$
default	$\frac{1}{12}b^3e\,x^{12}+\frac{1}{11}b^3d\,x^{11}+\frac{1}{10}b^3c\,x^{10}+\frac{1}{3}a\,b^2e\,x^9+\frac{3}{8}x^8b^2da+\frac{3}{7}a\,b^2c\,x^7+\frac{1}{2}a^2be\,x^6+\frac{3}{5}x^5bd\,a^2+$
norman	$\frac{1}{12}b^3e\,x^{12}+\frac{1}{11}b^3d\,x^{11}+\frac{1}{10}b^3c\,x^{10}+\frac{1}{3}a\,b^2e\,x^9+\frac{3}{8}x^8b^2da+\frac{3}{7}a\,b^2c\,x^7+\frac{1}{2}a^2be\,x^6+\frac{3}{5}x^5bd\,a^2+$
risch	$\frac{1}{12}b^3e\,x^{12}+\frac{1}{11}b^3d\,x^{11}+\frac{1}{10}b^3c\,x^{10}+\frac{1}{3}a\,b^2e\,x^9+\frac{3}{8}x^8b^2da+\frac{3}{7}a\,b^2c\,x^7+\frac{1}{2}a^2be\,x^6+\frac{3}{5}x^5bd\,a^2+$
parallelrisch	$\frac{1}{12}b^3e\,x^{12}+\frac{1}{11}b^3d\,x^{11}+\frac{1}{10}b^3c\,x^{10}+\frac{1}{3}a\,b^2e\,x^9+\frac{3}{8}x^8b^2da+\frac{3}{7}a\,b^2c\,x^7+\frac{1}{2}a^2be\,x^6+\frac{3}{5}x^5bd\,a^2+$

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^3*e*x^12+1/11*b^3*d*x^11+1/10*b^3*c*x^10+1/3*a*b^2*e*x^9+3/8*x^8*b^2
*d*a+3/7*a*b^2*c*x^7+1/2*a^2*b*e*x^6+3/5*x^5*b*d*a^2+3/4*a^2*b*c*x^4+1/3*a^
3*e*x^3+1/2*a^3*d*x^2+a^3*c*x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx=\frac{1}{12}\,b^3ex^{12}+\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{1}{3}\,ab^2ex^9+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{1}{2}\,a^2bex^6+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{3}\,a^3ex^3+\frac{1}{2}\,a^3dx^2+a^3cx$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*b^3*e*x^12 + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*e*x^9 + 3/8
*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*e*x^6 + 3/5*a^2*b*d*x^5 + 3/4*a^
2*b*c*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.28

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx = a^3cx+\frac{a^3dx^2}{2}+\frac{a^3ex^3}{3}+\frac{3a^2bcx^4}{4}+\frac{3a^2bdx^5}{5}+\frac{a^2bex^6}{2}+\frac{3ab^2cx^7}{7}+\frac{3ab^2dx^8}{8}+\frac{ab^2ex^9}{3}+\frac{b^3cx^{10}}{10}+\frac{b^3dx^{11}}{11}+\frac{b^3ex^{12}}{12}$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + 3*a**2*b*c*x**4/4 + 3*a**2*b*d*x
**5/5 + a**2*b*e*x**6/2 + 3*a*b**2*c*x**7/7 + 3*a*b**2*d*x**8/8 + a*b**2*e*
x**9/3 + b**3*c*x**10/10 + b**3*d*x**11/11 + b**3*e*x**12/12
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx = \frac{1}{12}b^3ex^{12}+\frac{1}{11}b^3dx^{11}+\frac{1}{10}b^3cx^{10}+\frac{1}{3}ab^2ex^9+\frac{3}{8}ab^2dx^8+\frac{3}{7}ab^2cx^7+\frac{1}{2}a^2bex^6+\frac{3}{5}a^2bdx^5+\frac{3}{4}a^2bcx^4+\frac{1}{3}a^3ex^3+\frac{1}{2}a^3dx^2+a^3cx$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^3*e*x^12 + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*e*x^9 + 3/8
*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*e*x^6 + 3/5*a^2*b*d*x^5 + 3/4*a^
2*b*c*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx = \frac{1}{12}b^3ex^{12}+\frac{1}{11}b^3dx^{11}+\frac{1}{10}b^3cx^{10}+\frac{1}{3}ab^2ex^9+\frac{3}{8}ab^2dx^8+\frac{3}{7}ab^2cx^7+\frac{1}{2}a^2bex^6+\frac{3}{5}a^2bdx^5+\frac{3}{4}a^2bcx^4+\frac{1}{3}a^3ex^3+\frac{1}{2}a^3dx^2+a^3cx$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*b^3*e*x^12 + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*e*x^9 + 3/8
*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*e*x^6 + 3/5*a^2*b*d*x^5 + 3/4*a^
2*b*c*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int (c+dx+ex^2)\left(a+bx^3\right)^3 dx = \frac{e\,a^3\,x^3}{3}+\frac{d\,a^3\,x^2}{2}+c\,a^3\,x+\frac{e\,a^2\,b\,x^6}{2} + \frac{3\,d\,a^2\,b\,x^5}{5}+\frac{3\,c\,a^2\,b\,x^4}{4}+\frac{e\,a\,b^2\,x^9}{3}+\frac{3\,d\,a\,b^2\,x^8}{8} + \frac{3\,c\,a\,b^2\,x^7}{7}+\frac{e\,b^3\,x^{12}}{12}+\frac{d\,b^3\,x^{11}}{11}+\frac{c\,b^3\,x^{10}}{10}$$

```
[In] int((a + b*x^3)^3*(c + d*x + e*x^2),x)
```

```
[Out] (a^3*d*x^2)/2 + (b^3*c*x^10)/10 + (a^3*e*x^3)/3 + (b^3*d*x^11)/11 + (b^3*e*
x^12)/12 + a^3*c*x + (3*a^2*b*c*x^4)/4 + (3*a*b^2*c*x^7)/7 + (3*a^2*b*d*x^5
)/5 + (3*a*b^2*d*x^8)/8 + (a^2*b*e*x^6)/2 + (a*b^2*e*x^9)/3
```

3.328 $\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x} dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x} dx = a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9+\frac{1}{10}b^3dx^{10}+\frac{1}{11}b^3ex^{11}+a^3c\log(x)$$

```
[Out] a^3*d*x+1/2*a^3*e*x^2+a^2*b*c*x^3+3/4*a^2*b*d*x^4+3/5*a^2*b*e*x^5+1/2*a*b^2
*c*x^6+3/7*a*b^2*d*x^7+3/8*a*b^2*e*x^8+1/9*b^3*c*x^9+1/10*b^3*d*x^10+1/11*b
^3*e*x^11+a^3*c*ln(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1642}

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x} dx = a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9+\frac{1}{10}b^3dx^{10}+\frac{1}{11}b^3ex^{11}$$

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (3*a^2*b*d*x^4)/4 + (3*a^2*b*e*x^5)
/5 + (a*b^2*c*x^6)/2 + (3*a*b^2*d*x^7)/7 + (3*a*b^2*e*x^8)/8 + (b^3*c*x^9)/
9 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3d+\frac{a^3c}{x}+a^3ex+3a^2bcx^2+3a^2bdx^3+3a^2bex^4+3ab^2cx^5+3ab^2dx^6\right.\\&\qquad\left.+3ab^2ex^7+b^3cx^8+b^3dx^9+b^3ex^{10}\right)\,dx\\&=a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7\\&\quad+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9+\frac{1}{10}b^3dx^{10}+\frac{1}{11}b^3ex^{11}+a^3c\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x}\,dx &= a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5\\&\quad+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9\\&\quad+\frac{1}{10}b^3dx^{10}+\frac{1}{11}b^3ex^{11}+a^3c\log(x)\end{aligned}$$

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (3*a^2*b*d*x^4)/4 + (3*a^2*b*e*x^5)
/5 + (a*b^2*c*x^6)/2 + (3*a*b^2*d*x^7)/7 + (3*a*b^2*e*x^8)/8 + (b^3*c*x^9)/
9 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*Log[x]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

method	result
default	$a^3dx+\frac{a^3e\,x^2}{2}+a^2x^3bc+\frac{3a^2bd\,x^4}{4}+\frac{3a^2be\,x^5}{5}+\frac{a\,b^2c\,x^6}{2}+\frac{3a\,b^2d\,x^7}{7}+\frac{3a\,b^2e\,x^8}{8}+\frac{b^3c\,x^9}{9}+\frac{b^3d\,x^{10}}{10}+\frac{b^3e}{1}$
norman	$a^3dx+\frac{a^3e\,x^2}{2}+a^2x^3bc+\frac{3a^2bd\,x^4}{4}+\frac{3a^2be\,x^5}{5}+\frac{a\,b^2c\,x^6}{2}+\frac{3a\,b^2d\,x^7}{7}+\frac{3a\,b^2e\,x^8}{8}+\frac{b^3c\,x^9}{9}+\frac{b^3d\,x^{10}}{10}+\frac{b^3e}{1}$
risch	$a^3dx+\frac{a^3e\,x^2}{2}+a^2x^3bc+\frac{3a^2bd\,x^4}{4}+\frac{3a^2be\,x^5}{5}+\frac{a\,b^2c\,x^6}{2}+\frac{3a\,b^2d\,x^7}{7}+\frac{3a\,b^2e\,x^8}{8}+\frac{b^3c\,x^9}{9}+\frac{b^3d\,x^{10}}{10}+\frac{b^3e}{1}$
parallelrisch	$a^3dx+\frac{a^3e\,x^2}{2}+a^2x^3bc+\frac{3a^2bd\,x^4}{4}+\frac{3a^2be\,x^5}{5}+\frac{a\,b^2c\,x^6}{2}+\frac{3a\,b^2d\,x^7}{7}+\frac{3a\,b^2e\,x^8}{8}+\frac{b^3c\,x^9}{9}+\frac{b^3d\,x^{10}}{10}+\frac{b^3e}{1}$

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3/x,x,method=_RETURNVERBOSE)
[Out] a^3*d*x+1/2*a^3*e*x^2+a^2*x^3*b*c+3/4*a^2*b*d*x^4+3/5*a^2*b*e*x^5+1/2*a*b^2
*c*x^6+3/7*a*b^2*d*x^7+3/8*a*b^2*e*x^8+1/9*b^3*c*x^9+1/10*b^3*d*x^10+1/11*b
^3*e*x^11+a^3*c*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx+ex^2)\,(a+bx^3)^3}{x}\,dx = \frac{1}{11}\,b^3ex^{11}+\frac{1}{10}\,b^3dx^{10}+\frac{1}{9}\,b^3cx^9+\frac{3}{8}\,ab^2ex^8 + \frac{3}{7}\,ab^2dx^7+\frac{1}{2}\,ab^2cx^6+\frac{3}{5}\,a^2bex^5+\frac{3}{4}\,a^2bdx^4 + a^2bcx^3+\frac{1}{2}\,a^3ex^2+a^3dx+a^3c\log(x)$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="fricas")
[Out] 1/11*b^3*e*x^11 + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*e*x^8 + 3/7*a
*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*e*x^5 + 3/4*a^2*b*d*x^4 + a^2*b*c*
x^3 + 1/2*a^3*e*x^2 + a^3*d*x + a^3*c*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx+ex^2)\,(a+bx^3)^3}{x}\,dx = a^3c\log(x)+a^3dx+\frac{a^3ex^2}{2}+a^2bcx^3+\frac{3a^2bdx^4}{4}+\frac{3a^2bex^5}{5} + \frac{ab^2cx^6}{2}+\frac{3ab^2dx^7}{7}+\frac{3ab^2ex^8}{8}+\frac{b^3cx^9}{9}+\frac{b^3dx^{10}}{10}+\frac{b^3ex^{11}}{11}$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x,x)
```

```
[Out] a**3*c*log(x) + a**3*d*x + a**3*e*x**2/2 + a**2*b*c*x**3 + 3*a**2*b*d*x**4/
4 + 3*a**2*b*e*x**5/5 + a*b**2*c*x**6/2 + 3*a*b**2*d*x**7/7 + 3*a*b**2*e*x*
*8/8 + b**3*c*x**9/9 + b**3*d*x**10/10 + b**3*e*x**11/11
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x}\,dx = \frac{1}{11}\,b^3ex^{11} + \frac{1}{10}\,b^3dx^{10} + \frac{1}{9}\,b^3cx^9 + \frac{3}{8}\,ab^2ex^8 + \frac{3}{7}\,ab^2dx^7 + \frac{1}{2}\,ab^2cx^6 + \frac{3}{5}\,a^2bex^5 + \frac{3}{4}\,a^2bdx^4 + a^2bcx^3 + \frac{1}{2}\,a^3ex^2 + a^3dx + a^3c\log(x)$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*e*x^11 + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*e*x^8 + 3/7*a
*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*e*x^5 + 3/4*a^2*b*d*x^4 + a^2*b*c*
x^3 + 1/2*a^3*e*x^2 + a^3*d*x + a^3*c*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x}\,dx = \frac{1}{11}\,b^3ex^{11} + \frac{1}{10}\,b^3dx^{10} + \frac{1}{9}\,b^3cx^9 + \frac{3}{8}\,ab^2ex^8 + \frac{3}{7}\,ab^2dx^7 + \frac{1}{2}\,ab^2cx^6 + \frac{3}{5}\,a^2bex^5 + \frac{3}{4}\,a^2bdx^4 + a^2bcx^3 + \frac{1}{2}\,a^3ex^2 + a^3dx + a^3c\log(|x|)$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/11*b^3*e*x^11 + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*e*x^8 + 3/7*a
*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*e*x^5 + 3/4*a^2*b*d*x^4 + a^2*b*c*
x^3 + 1/2*a^3*e*x^2 + a^3*d*x + a^3*c*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x}\,dx = \frac{b^3\,c\,x^9}{9}+\frac{a^3\,e\,x^2}{2}+\frac{b^3\,d\,x^{10}}{10}+\frac{b^3\,e\,x^{11}}{11}+a^3\,c\,\ln\left(x\right)+a^3\,d\,x+a^2\,b\,c\,x^3+\frac{a\,b^2\,c\,x^6}{2}+\frac{3\,a^2\,b\,d\,x^4}{4}+\frac{3\,a\,b^2\,d\,x^7}{7}+\frac{3\,a^2\,b\,e\,x^5}{5}+\frac{3\,a\,b^2\,e\,x^8}{8}$$

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2))/x,x)
```

```
[Out] (b^3*c*x^9)/9 + (a^3*e*x^2)/2 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*l
og(x) + a^3*d*x + a^2*b*c*x^3 + (a*b^2*c*x^6)/2 + (3*a^2*b*d*x^4)/4 + (3*a*
b^2*d*x^7)/7 + (3*a^2*b*e*x^5)/5 + (3*a*b^2*e*x^8)/8
```

3.329 $\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2}\,dx = -\frac{a^3c}{x}+a^3ex+\frac{3}{2}a^2bcx^2+a^2bdx^3+\frac{3}{4}a^2bex^4 +\frac{3}{5}ab^2cx^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab^2ex^7+\frac{1}{8}b^3cx^8 +\frac{1}{9}b^3dx^9+\frac{1}{10}b^3ex^{10}+a^3d\log(x)$$

```
[Out] -a^3*c/x+a^3*e*x+3/2*a^2*b*c*x^2+a^2*b*d*x^3+3/4*a^2*b*e*x^4+3/5*a*b^2*c*x^
5+1/2*a*b^2*d*x^6+3/7*a*b^2*e*x^7+1/8*b^3*c*x^8+1/9*b^3*d*x^9+1/10*b^3*e*x^
10+a^3*d*ln(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1642}

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2}\,dx = -\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{3}{2}a^2bcx^2 +a^2bdx^3+\frac{3}{4}a^2bex^4+\frac{3}{5}ab^2cx^5+\frac{1}{2}ab^2dx^6 +\frac{3}{7}ab^2ex^7+\frac{1}{8}b^3cx^8+\frac{1}{9}b^3dx^9+\frac{1}{10}b^3ex^{10}$$

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^2,x]
```

```
[Out] -((a^3*c)/x) + a^3*e*x + (3*a^2*b*c*x^2)/2 + a^2*b*d*x^3 + (3*a^2*b*e*x^4)/
4 + (3*a*b^2*c*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b^2*e*x^7)/7 + (b^3*c*x^8)/8
 + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3e+\frac{a^3c}{x^2}+\frac{a^3d}{x}+3a^2bcx+3a^2bdx^2+3a^2bex^3+3ab^2cx^4+3ab^2dx^5\right.\\&\qquad\left.+3ab^2ex^6+b^3cx^7+b^3dx^8+b^3ex^9\right)\,dx\\&=-\frac{a^3c}{x}+a^3ex+\frac{3}{2}a^2bcx^2+a^2bdx^3+\frac{3}{4}a^2bex^4+\frac{3}{5}ab^2cx^5+\frac{1}{2}ab^2dx^6\\&\quad+\frac{3}{7}ab^2ex^7+\frac{1}{8}b^3cx^8+\frac{1}{9}b^3dx^9+\frac{1}{10}b^3ex^{10}+a^3d\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(c+dx+ex^2)\,(a+bx^3)^3}{x^2}\,dx &= -\frac{a^3c}{x}+a^3ex+\frac{3}{2}a^2bcx^2+a^2bdx^3+\frac{3}{4}a^2bex^4\\&\quad+\frac{3}{5}ab^2cx^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab^2ex^7+\frac{1}{8}b^3cx^8\\&\quad+\frac{1}{9}b^3dx^9+\frac{1}{10}b^3ex^{10}+a^3d\log(x)\end{aligned}$$

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^2,x]
```

```
[Out] -((a^3*c)/x) + a^3*e*x + (3*a^2*b*c*x^2)/2 + a^2*b*d*x^3 + (3*a^2*b*e*x^4)/
4 + (3*a*b^2*c*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b^2*e*x^7)/7 + (b^3*c*x^8)/8
 + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*Log[x]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

method	result
default	$-\frac{a^{3}c}{x}+a^{3}ex+\frac{3a^{2}bc\,x^{2}}{2}+a^{2}bd\,x^{3}+\frac{3a^{2}be\,x^{4}}{4}+\frac{3a\,b^{2}c\,x^{5}}{5}+\frac{a\,b^{2}d\,x^{6}}{2}+\frac{3a\,b^{2}e\,x^{7}}{7}+\frac{b^{3}c\,x^{8}}{8}+\frac{b^{3}d\,x^{9}}{9}+\frac{b^{3}e}{}$
risch	$-\frac{a^{3}c}{x}+a^{3}ex+\frac{3a^{2}bc\,x^{2}}{2}+a^{2}bd\,x^{3}+\frac{3a^{2}be\,x^{4}}{4}+\frac{3a\,b^{2}c\,x^{5}}{5}+\frac{a\,b^{2}d\,x^{6}}{2}+\frac{3a\,b^{2}e\,x^{7}}{7}+\frac{b^{3}c\,x^{8}}{8}+\frac{b^{3}d\,x^{9}}{9}+\frac{b^{3}e}{}$
norman	$\frac{a^{3}e\,x^{2}+a^{2}bd\,x^{4}-c\,a^{3}+\frac{1}{8}b^{3}c\,x^{9}+\frac{1}{9}b^{3}d\,x^{10}+\frac{1}{10}b^{3}e\,x^{11}+\frac{3}{5}a\,b^{2}c\,x^{6}+\frac{1}{2}a\,b^{2}d\,x^{7}+\frac{3}{7}a\,b^{2}e\,x^{8}+\frac{3}{4}a^{2}be\,x^{5}+\frac{3}{2}a^{2}x^{3}bc}{x}+a^{3}d\ln(x)$
parallelrisch	$\frac{252b^{3}e\,x^{11}+280b^{3}d\,x^{10}+315b^{3}c\,x^{9}+1080a\,b^{2}e\,x^{8}+1260a\,b^{2}d\,x^{7}+1512a\,b^{2}c\,x^{6}+1890a^{2}be\,x^{5}+2520a^{2}bd\,x^{4}+3780a^{2}x^{3}bc+2520}{2520x}$

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*c/x+a^3*e*x+3/2*a^2*b*c*x^2+a^2*b*d*x^3+3/4*a^2*b*e*x^4+3/5*a*b^2*c*x^
5+1/2*a*b^2*d*x^6+3/7*a*b^2*e*x^7+1/8*b^3*c*x^8+1/9*b^3*d*x^9+1/10*b^3*e*x^
10+a^3*d*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx+ex^2)\left(a+bx^3\right)^3}{x^2}\,dx$$
$$=\frac{252\,b^3ex^{11}+280\,b^3dx^{10}+315\,b^3cx^9+1080\,ab^2ex^8+1260\,ab^2dx^7+1512\,ab^2cx^6+1890\,a^2bex^5+2520\,a^2b}{2520\,x}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/2520*(252*b^3*e*x^11 + 280*b^3*d*x^10 + 315*b^3*c*x^9 + 1080*a*b^2*e*x^8
+ 1260*a*b^2*d*x^7 + 1512*a*b^2*c*x^6 + 1890*a^2*b*e*x^5 + 2520*a^2*b*d*x^4
 + 3780*a^2*b*c*x^3 + 2520*a^3*e*x^2 + 2520*a^3*d*x*log(x) - 2520*a^3*c)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx+ex^2)\left(a+bx^3\right)^3}{x^2}\,dx=-\frac{a^3c}{x}+a^3d\log{(x)}+a^3ex+\frac{3a^2bcx^2}{2}+a^2bdx^3+\frac{3a^2bex^4}{4}+\frac{3ab^2cx^5}{5}+\frac{ab^2dx^6}{2}+\frac{3ab^2ex^7}{7}+\frac{b^3cx^8}{8}+\frac{b^3dx^9}{9}+\frac{b^3ex^{10}}{10}$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x**2,x)
```

```
[Out] -a**3*c/x + a**3*d*log(x) + a**3*e*x + 3*a**2*b*c*x**2/2 + a**2*b*d*x**3 +
3*a**2*b*e*x**4/4 + 3*a*b**2*c*x**5/5 + a*b**2*d*x**6/2 + 3*a*b**2*e*x**7/7
 + b**3*c*x**8/8 + b**3*d*x**9/9 + b**3*e*x**10/10
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2}\,dx = \frac{1}{10}\,b^3ex^{10}+\frac{1}{9}\,b^3dx^9+\frac{1}{8}\,b^3cx^8+\frac{3}{7}\,ab^2ex^7+\frac{1}{2}\,ab^2dx^6+\frac{3}{5}\,ab^2cx^5+\frac{3}{4}\,a^2bex^4+a^2bdx^3+\frac{3}{2}\,a^2bcx^2+a^3ex+a^3d\log(x)-\frac{a^3c}{x}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*e*x^10 + 1/9*b^3*d*x^9 + 1/8*b^3*c*x^8 + 3/7*a*b^2*e*x^7 + 1/2*a*b
^2*d*x^6 + 3/5*a*b^2*c*x^5 + 3/4*a^2*b*e*x^4 + a^2*b*d*x^3 + 3/2*a^2*b*c*x^
2 + a^3*e*x + a^3*d*log(x) - a^3*c/x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2}\,dx = \frac{1}{10}\,b^3ex^{10}+\frac{1}{9}\,b^3dx^9+\frac{1}{8}\,b^3cx^8+\frac{3}{7}\,ab^2ex^7+\frac{1}{2}\,ab^2dx^6+\frac{3}{5}\,ab^2cx^5+\frac{3}{4}\,a^2bex^4+a^2bdx^3+\frac{3}{2}\,a^2bcx^2+a^3ex+a^3d\log(|x|)-\frac{a^3c}{x}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/10*b^3*e*x^10 + 1/9*b^3*d*x^9 + 1/8*b^3*c*x^8 + 3/7*a*b^2*e*x^7 + 1/2*a*b
^2*d*x^6 + 3/5*a*b^2*c*x^5 + 3/4*a^2*b*e*x^4 + a^2*b*d*x^3 + 3/2*a^2*b*c*x^
2 + a^3*e*x + a^3*d*log(abs(x)) - a^3*c/x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2}\,dx = \frac{b^3\,c\,x^8}{8} - \frac{a^3\,c}{x} + \frac{b^3\,d\,x^9}{9} + \frac{b^3\,e\,x^{10}}{10} + a^3\,d\,\ln(x) + a^3\,e\,x + \frac{3\,a^2\,b\,c\,x^2}{2} + \frac{3\,a\,b^2\,c\,x^5}{5} + a^2\,b\,d\,x^3 + \frac{a\,b^2\,d\,x^6}{2} + \frac{3\,a^2\,b\,e\,x^4}{4} + \frac{3\,a\,b^2\,e\,x^7}{7}$$

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2))/x^2,x)
```

```
[Out] (b^3*c*x^8)/8 - (a^3*c)/x + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*log(x)
+ a^3*e*x + (3*a^2*b*c*x^2)/2 + (3*a*b^2*c*x^5)/5 + a^2*b*d*x^3 + (a*b^2*d*
x^6)/2 + (3*a^2*b*e*x^4)/4 + (3*a*b^2*e*x^7)/7
```

3.330 $\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3} dx = -\frac{a^3c}{2x^2} - \frac{a^3d}{x} + 3a^2bcx + \frac{3}{2}a^2bdx^2 + a^2bex^3 + \frac{3}{4}ab^2cx^4 + \frac{3}{5}ab^2dx^5 + \frac{1}{2}ab^2ex^6 + \frac{1}{7}b^3cx^7 + \frac{1}{8}b^3dx^8 + \frac{1}{9}b^3ex^9 + a^3e\log(x)$$

```
[Out] -1/2*a^3*c/x^2-a^3*d/x+3*a^2*b*c*x+3/2*a^2*b*d*x^2+a^2*b*e*x^3+3/4*a*b^2*c*
x^4+3/5*a*b^2*d*x^5+1/2*a*b^2*e*x^6+1/7*b^3*c*x^7+1/8*b^3*d*x^8+1/9*b^3*e*x
^9+a^3*e*ln(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1642}

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3} dx = -\frac{a^3c}{2x^2} - \frac{a^3d}{x} + a^3e\log(x) + 3a^2bcx + \frac{3}{2}a^2bdx^2 + a^2bex^3 + \frac{3}{4}ab^2cx^4 + \frac{3}{5}ab^2dx^5 + \frac{1}{2}ab^2ex^6 + \frac{1}{7}b^3cx^7 + \frac{1}{8}b^3dx^8 + \frac{1}{9}b^3ex^9$$

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^3,x]
```

```
[Out] -1/2*(a^3*c)/x^2 - (a^3*d)/x + 3*a^2*b*c*x + (3*a^2*b*d*x^2)/2 + a^2*b*e*x^
3 + (3*a*b^2*c*x^4)/4 + (3*a*b^2*d*x^5)/5 + (a*b^2*e*x^6)/2 + (b^3*c*x^7)/7
 + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 + a^3*e*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(3a^2bc + \frac{a^3c}{x^3} + \frac{a^3d}{x^2} + \frac{a^3e}{x} + 3a^2bdx + 3a^2bex^2 + 3ab^2cx^3 + 3ab^2dx^4 + 3ab^2ex^5 + b^3cx^6 + b^3dx^7 + b^3ex^8\right) dx$$

$$= -\frac{a^3c}{2x^2} - \frac{a^3d}{x} + 3a^2bcx + \frac{3}{2}a^2bdx^2 + a^2bex^3 + \frac{3}{4}ab^2cx^4 + \frac{3}{5}ab^2dx^5 + \frac{1}{2}ab^2ex^6 + \frac{1}{7}b^3cx^7 + \frac{1}{8}b^3dx^8 + \frac{1}{9}b^3ex^9 + a^3e\log(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3}\,dx = -\frac{a^3c}{2x^2} - \frac{a^3d}{x} + 3a^2bcx + \frac{3}{2}a^2bdx^2 + a^2bex^3 + \frac{3}{4}ab^2cx^4 + \frac{3}{5}ab^2dx^5 + \frac{1}{2}ab^2ex^6 + \frac{1}{7}b^3cx^7 + \frac{1}{8}b^3dx^8 + \frac{1}{9}b^3ex^9 + a^3e\log(x)$$

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^3,x]
```

```
[Out] -1/2*(a^3*c)/x^2 - (a^3*d)/x + 3*a^2*b*c*x + (3*a^2*b*d*x^2)/2 + a^2*b*e*x^
3 + (3*a*b^2*c*x^4)/4 + (3*a*b^2*d*x^5)/5 + (a*b^2*e*x^6)/2 + (b^3*c*x^7)/7
 + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 + a^3*e*Log[x]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.88

method	result
default	$-\frac{a^{3} c}{2 x^{2}}-\frac{a^{3} d}{x}+3 a^{2} b c x+\frac{3 a^{2} b d \,x^{2}}{2}+a^{2} b e \,x^{3}+\frac{3 a \,b^{2} c \,x^{4}}{4}+\frac{3 a \,b^{2} d \,x^{5}}{5}+\frac{a \,b^{2} e \,x^{6}}{2}+\frac{b^{3} c \,x^{7}}{7}+\frac{b^{3} d \,x^{8}}{8}+\frac{b^{3} e \,x^{9}}{9}$
risch	$\frac{b^{3} e \,x^{9}}{9}+\frac{b^{3} d \,x^{8}}{8}+\frac{b^{3} c \,x^{7}}{7}+\frac{a \,b^{2} e \,x^{6}}{2}+\frac{3 a \,b^{2} d \,x^{5}}{5}+\frac{3 a \,b^{2} c \,x^{4}}{4}+a^{2} b e \,x^{3}+\frac{3 a^{2} b d \,x^{2}}{2}+3 a^{2} b c x+\frac{-a^{3} d x-\frac{1}{2} c \,a^{3}}{x^{2}}$
norman	$\frac{a^{2} b e \,x^{5}-\frac{1}{2} c \,a^{3}-a^{3} d x+\frac{1}{7} b^{3} c \,x^{9}+\frac{1}{8} b^{3} d \,x^{10}+\frac{1}{9} b^{3} e \,x^{11}+\frac{3}{4} a \,b^{2} c \,x^{6}+\frac{3}{5} a \,b^{2} d \,x^{7}+\frac{1}{2} a \,b^{2} e \,x^{8}+\frac{3}{2} a^{2} b d \,x^{4}+3 a^{2} x^{3} b c}{x^{2}}+a^{3} e \ln \left(x \right)$
parallelrisch	$\frac{280 b^{3} e \,x^{11}+315 b^{3} d \,x^{10}+360 b^{3} c \,x^{9}+1260 a \,b^{2} e \,x^{8}+1512 a \,b^{2} d \,x^{7}+1890 a \,b^{2} c \,x^{6}+2520 a^{2} b e \,x^{5}+3780 a^{2} b d \,x^{4}+2520 e \,a^{3} \ln \left(x \right) x^{2}+75}{2520 x^{2}}$

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^3*c/x^2-a^3*d/x+3*a^2*b*c*x+3/2*a^2*b*d*x^2+a^2*b*e*x^3+3/4*a*b^2*c*
x^4+3/5*a*b^2*d*x^5+1/2*a*b^2*e*x^6+1/7*b^3*c*x^7+1/8*b^3*d*x^8+1/9*b^3*e*x
^9+a^3*e*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3}\,dx = \frac{280\,b^3ex^{11}+315\,b^3dx^{10}+360\,b^3cx^9+1260\,ab^2ex^8+1512\,ab^2dx^7+1890\,ab^2cx^6+2520\,a^2bex^5+3780\,a^2bd}{2520\,x^2}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/2520*(280*b^3*e*x^11 + 315*b^3*d*x^10 + 360*b^3*c*x^9 + 1260*a*b^2*e*x^8
+ 1512*a*b^2*d*x^7 + 1890*a*b^2*c*x^6 + 2520*a^2*b*e*x^5 + 3780*a^2*b*d*x^4
 + 7560*a^2*b*c*x^3 + 2520*a^3*e*x^2*log(x) - 2520*a^3*d*x - 1260*a^3*c)/x^
2
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3}\,dx = a^3e\log(x)+3a^2bcx+\frac{3a^2bdx^2}{2}+a^2bex^3+\frac{3ab^2cx^4}{4}+\frac{3ab^2dx^5}{5}+\frac{ab^2ex^6}{2}+\frac{b^3cx^7}{7}+\frac{b^3dx^8}{8}+\frac{b^3ex^9}{9}+\frac{-a^3c-2a^3dx}{2x^2}$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x**3,x)
```

```
[Out] a**3*e*log(x) + 3*a**2*b*c*x + 3*a**2*b*d*x**2/2 + a**2*b*e*x**3 + 3*a*b**2
*c*x**4/4 + 3*a*b**2*d*x**5/5 + a*b**2*e*x**6/2 + b**3*c*x**7/7 + b**3*d*x*
*8/8 + b**3*e*x**9/9 + (-a**3*c - 2*a**3*d*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3}\,dx = \frac{1}{9}\,b^3ex^9+\frac{1}{8}\,b^3dx^8+\frac{1}{7}\,b^3cx^7+\frac{1}{2}\,ab^2ex^6+\frac{3}{5}\,ab^2dx^5+\frac{3}{4}\,ab^2cx^4 + a^2bex^3+\frac{3}{2}\,a^2bdx^2+3\,a^2bcx+a^3e\log(x)-\frac{2\,a^3dx+a^3c}{2\,x^2}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^3*e*x^9 + 1/8*b^3*d*x^8 + 1/7*b^3*c*x^7 + 1/2*a*b^2*e*x^6 + 3/5*a*b^2
*d*x^5 + 3/4*a*b^2*c*x^4 + a^2*b*e*x^3 + 3/2*a^2*b*d*x^2 + 3*a^2*b*c*x + a^
3*e*log(x) - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3}\,dx = \frac{1}{9}\,b^3ex^9+\frac{1}{8}\,b^3dx^8+\frac{1}{7}\,b^3cx^7+\frac{1}{2}\,ab^2ex^6+\frac{3}{5}\,ab^2dx^5+\frac{3}{4}\,ab^2cx^4 + a^2bex^3+\frac{3}{2}\,a^2bdx^2+3\,a^2bcx+a^3e\log(|x|)-\frac{2\,a^3dx+a^3c}{2\,x^2}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/9*b^3*e*x^9 + 1/8*b^3*d*x^8 + 1/7*b^3*c*x^7 + 1/2*a*b^2*e*x^6 + 3/5*a*b^2
*d*x^5 + 3/4*a*b^2*c*x^4 + a^2*b*e*x^3 + 3/2*a^2*b*d*x^2 + 3*a^2*b*c*x + a^
3*e*log(abs(x)) - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3}\,dx = \frac{b^3\,c\,x^7}{7} - \frac{\frac{a^3\,c}{2}+a^3\,d\,x}{x^2} + \frac{b^3\,d\,x^8}{8} + \frac{b^3\,e\,x^9}{9} + a^3\,e\,\ln(x) + 3\,a^2\,b\,c\,x + \frac{3\,a\,b^2\,c\,x^4}{4} + \frac{3\,a^2\,b\,d\,x^2}{2} + \frac{3\,a\,b^2\,d\,x^5}{5} + a^2\,b\,e\,x^3 + \frac{a\,b^2\,e\,x^6}{2}$$

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2))/x^3,x)
```

```
[Out] (b^3*c*x^7)/7 - ((a^3*c)/2 + a^3*d*x)/x^2 + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 +
 a^3*e*log(x) + 3*a^2*b*c*x + (3*a*b^2*c*x^4)/4 + (3*a^2*b*d*x^2)/2 + (3*a*
b^2*d*x^5)/5 + a^2*b*e*x^3 + (a*b^2*e*x^6)/2
```

3.331 $\int x^2(c+dx+ex^2)\left(a+bx^3\right)^4\,dx$

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx=\frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3dx^{13}+\frac{2}{7}ab^3ex^{14}+\frac{1}{16}b^4dx^{16}+\frac{1}{17}b^4ex^{17}+\frac{c\left(a+bx^3\right)^5}{15b}$$

```
[Out] 1/4*a^4*d*x^4+1/5*a^4*e*x^5+4/7*a^3*b*d*x^7+1/2*a^3*b*e*x^8+3/5*a^2*b^2*d*x
^10+6/11*a^2*b^2*e*x^11+4/13*a*b^3*d*x^13+2/7*a*b^3*e*x^14+1/16*b^4*d*x^16+
1/17*b^4*e*x^17+1/15*c*(b*x^3+a)^5/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1596, 1864}

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx=\frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3dx^{13}+\frac{2}{7}ab^3ex^{14}+\frac{c\left(a+bx^3\right)^5}{15b}+\frac{1}{16}b^4dx^{16}+\frac{1}{17}b^4ex^{17}$$

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (4*a^3*b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (3*a^
2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (4*a*b^3*d*x^13)/13 + (2*a*b^3*e*
x^14)/7 + (b^4*d*x^16)/16 + (b^4*e*x^17)/17 + (c*(a + b*x^3)^5)/(15*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{c(a+bx^3)^5}{15b} + \int (a+bx^3)^4 \left(-cx^2 + x^2(c+dx+ex^2)\right) dx \\ &= \frac{c(a+bx^3)^5}{15b} + \int \left(a^4dx^3 + a^4ex^4 + 4a^3bdx^6 + 4a^3bex^7 + 6a^2b^2dx^9 + 6a^2b^2ex^{10} \right.\\ &\qquad \left. + 4ab^3dx^{12} + 4ab^3ex^{13} + b^4dx^{15} + b^4ex^{16}\right) dx \\ &= \frac{1}{4}a^4dx^4 + \frac{1}{5}a^4ex^5 + \frac{4}{7}a^3bdx^7 + \frac{1}{2}a^3bex^8 + \frac{3}{5}a^2b^2dx^{10} + \frac{6}{11}a^2b^2ex^{11} \\ &\quad + \frac{4}{13}ab^3dx^{13} + \frac{2}{7}ab^3ex^{14} + \frac{1}{16}b^4dx^{16} + \frac{1}{17}b^4ex^{17} + \frac{c(a+bx^3)^5}{15b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.31

$$\begin{aligned}\int x^2(c+dx+ex^2)(a+bx^3)^4\,dx &= \frac{1}{3}a^4cx^3 + \frac{1}{4}a^4dx^4 + \frac{1}{5}a^4ex^5 + \frac{2}{3}a^3bcx^6 \\ &\quad + \frac{4}{7}a^3bdx^7 + \frac{1}{2}a^3bex^8 + \frac{2}{3}a^2b^2cx^9 + \frac{3}{5}a^2b^2dx^{10} \\ &\quad + \frac{6}{11}a^2b^2ex^{11} + \frac{1}{3}ab^3cx^{12} + \frac{4}{13}ab^3dx^{13} \\ &\quad + \frac{2}{7}ab^3ex^{14} + \frac{1}{15}b^4cx^{15} + \frac{1}{16}b^4dx^{16} + \frac{1}{17}b^4ex^{17}\end{aligned}$$

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
[Out] (a^4*c*x^3)/3 + (a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (2*a^3*b*c*x^6)/3 + (4*a^3*
b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 +
 (6*a^2*b^2*e*x^11)/11 + (a*b^3*c*x^12)/3 + (4*a*b^3*d*x^13)/13 + (2*a*b^3*
e*x^14)/7 + (b^4*c*x^15)/15 + (b^4*d*x^16)/16 + (b^4*e*x^17)/17
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.10

method	result
gosper	$\frac{1}{3}a^4c\,x^3+\frac{1}{4}a^4d\,x^4+\frac{1}{5}a^4e\,x^5+\frac{2}{3}a^3bc\,x^6+\frac{4}{7}a^3bd\,x^7+\frac{1}{2}a^3be\,x^8+\frac{2}{3}a^2b^2c\,x^9+\frac{3}{5}a^2b^2d\,x^{10}+\frac{6}{11}$
default	$\frac{1}{3}a^4c\,x^3+\frac{1}{4}a^4d\,x^4+\frac{1}{5}a^4e\,x^5+\frac{2}{3}a^3bc\,x^6+\frac{4}{7}a^3bd\,x^7+\frac{1}{2}a^3be\,x^8+\frac{2}{3}a^2b^2c\,x^9+\frac{3}{5}a^2b^2d\,x^{10}+\frac{6}{11}$
norman	$\frac{1}{3}a^4c\,x^3+\frac{1}{4}a^4d\,x^4+\frac{1}{5}a^4e\,x^5+\frac{2}{3}a^3bc\,x^6+\frac{4}{7}a^3bd\,x^7+\frac{1}{2}a^3be\,x^8+\frac{2}{3}a^2b^2c\,x^9+\frac{3}{5}a^2b^2d\,x^{10}+\frac{6}{11}$
risch	$\frac{1}{3}a^4c\,x^3+\frac{1}{4}a^4d\,x^4+\frac{1}{5}a^4e\,x^5+\frac{2}{3}a^3bc\,x^6+\frac{4}{7}a^3bd\,x^7+\frac{1}{2}a^3be\,x^8+\frac{2}{3}a^2b^2c\,x^9+\frac{3}{5}a^2b^2d\,x^{10}+\frac{6}{11}$
parallelrisch	$\frac{1}{3}a^4c\,x^3+\frac{1}{4}a^4d\,x^4+\frac{1}{5}a^4e\,x^5+\frac{2}{3}a^3bc\,x^6+\frac{4}{7}a^3bd\,x^7+\frac{1}{2}a^3be\,x^8+\frac{2}{3}a^2b^2c\,x^9+\frac{3}{5}a^2b^2d\,x^{10}+\frac{6}{11}$

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^4*c*x^3+1/4*a^4*d*x^4+1/5*a^4*e*x^5+2/3*a^3*b*c*x^6+4/7*a^3*b*d*x^7+1
/2*a^3*b*e*x^8+2/3*a^2*b^2*c*x^9+3/5*a^2*b^2*d*x^10+6/11*a^2*b^2*e*x^11+1/3
*a*b^3*c*x^12+4/13*a*b^3*d*x^13+2/7*a*b^3*e*x^14+1/15*b^4*c*x^15+1/16*b^4*d
*x^16+1/17*b^4*e*x^17
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\begin{aligned}\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx=&\ \frac{1}{17}\,b^4ex^{17}+\frac{1}{16}\,b^4dx^{16}+\frac{1}{15}\,b^4cx^{15}+\frac{2}{7}\,ab^3ex^{14}\\&+\frac{4}{13}\,ab^3dx^{13}+\frac{1}{3}\,ab^3cx^{12}+\frac{6}{11}\,a^2b^2ex^{11}\\&+\frac{3}{5}\,a^2b^2dx^{10}+\frac{2}{3}\,a^2b^2cx^9+\frac{1}{2}\,a^3bex^8+\frac{4}{7}\,a^3bdx^7\\&+\frac{2}{3}\,a^3bcx^6+\frac{1}{5}\,a^4ex^5+\frac{1}{4}\,a^4dx^4+\frac{1}{3}\,a^4cx^3\end{aligned}$$

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/17*b^4*e*x^17 + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*e*x^14 + 4/
13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^
10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*e*x^8 + 4/7*a^3*b*d*x^7 + 2/3*a^3*b*c*x^
6 + 1/5*a^4*e*x^5 + 1/4*a^4*d*x^4 + 1/3*a^4*c*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.33

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^4\,dx=\frac{a^4cx^3}{3}+\frac{a^4dx^4}{4}+\frac{a^4ex^5}{5}+\frac{2a^3bcx^6}{3}+\frac{4a^3bdx^7}{7}+\frac{a^3bex^8}{2}+\frac{2a^2b^2cx^9}{3}+\frac{3a^2b^2dx^{10}}{5}+\frac{6a^2b^2ex^{11}}{11}+\frac{ab^3cx^{12}}{3}+\frac{4ab^3dx^{13}}{13}+\frac{2ab^3ex^{14}}{7}+\frac{b^4cx^{15}}{15}+\frac{b^4dx^{16}}{16}+\frac{b^4ex^{17}}{17}$$

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*x**3/3 + a**4*d*x**4/4 + a**4*e*x**5/5 + 2*a**3*b*c*x**6/3 + 4*a**3*
b*d*x**7/7 + a**3*b*e*x**8/2 + 2*a**2*b**2*c*x**9/3 + 3*a**2*b**2*d*x**10/5
 + 6*a**2*b**2*e*x**11/11 + a*b**3*c*x**12/3 + 4*a*b**3*d*x**13/13 + 2*a*b*
*3*e*x**14/7 + b**4*c*x**15/15 + b**4*d*x**16/16 + b**4*e*x**17/17
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^4\,dx=\frac{1}{17}\,b^4ex^{17}+\frac{1}{16}\,b^4dx^{16}+\frac{1}{15}\,b^4cx^{15}+\frac{2}{7}\,ab^3ex^{14}+\frac{4}{13}\,ab^3dx^{13}+\frac{1}{3}\,ab^3cx^{12}+\frac{6}{11}\,a^2b^2ex^{11}+\frac{3}{5}\,a^2b^2dx^{10}+\frac{2}{3}\,a^2b^2cx^9+\frac{1}{2}\,a^3bex^8+\frac{4}{7}\,a^3bdx^7+\frac{2}{3}\,a^3bcx^6+\frac{1}{5}\,a^4ex^5+\frac{1}{4}\,a^4dx^4+\frac{1}{3}\,a^4cx^3$$

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/17*b^4*e*x^17 + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*e*x^14 + 4/
13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^
10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*e*x^8 + 4/7*a^3*b*d*x^7 + 2/3*a^3*b*c*x^
6 + 1/5*a^4*e*x^5 + 1/4*a^4*d*x^4 + 1/3*a^4*c*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^4\,dx=\frac{1}{17}\,b^4ex^{17}+\frac{1}{16}\,b^4dx^{16}+\frac{1}{15}\,b^4cx^{15}+\frac{2}{7}\,ab^3ex^{14}+\frac{4}{13}\,ab^3dx^{13}+\frac{1}{3}\,ab^3cx^{12}+\frac{6}{11}\,a^2b^2ex^{11}+\frac{3}{5}\,a^2b^2dx^{10}+\frac{2}{3}\,a^2b^2cx^9+\frac{1}{2}\,a^3bex^8+\frac{4}{7}\,a^3bdx^7+\frac{2}{3}\,a^3bcx^6+\frac{1}{5}\,a^4ex^5+\frac{1}{4}\,a^4dx^4+\frac{1}{3}\,a^4cx^3$$

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 1/17*b^4*e*x^17 + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*e*x^14 + 4/
13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^
10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*e*x^8 + 4/7*a^3*b*d*x^7 + 2/3*a^3*b*c*x^
6 + 1/5*a^4*e*x^5 + 1/4*a^4*d*x^4 + 1/3*a^4*c*x^3
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^4\,dx=\frac{e\,a^4\,x^5}{5}+\frac{d\,a^4\,x^4}{4}+\frac{c\,a^4\,x^3}{3}+\frac{e\,a^3\,b\,x^8}{2}+\frac{4\,d\,a^3\,b\,x^7}{7}+\frac{2\,c\,a^3\,b\,x^6}{3}+\frac{6\,e\,a^2\,b^2\,x^{11}}{11}+\frac{3\,d\,a^2\,b^2\,x^{10}}{5}+\frac{2\,c\,a^2\,b^2\,x^9}{3}+\frac{2\,e\,a\,b^3\,x^{14}}{7}+\frac{4\,d\,a\,b^3\,x^{13}}{13}+\frac{c\,a\,b^3\,x^{12}}{3}+\frac{e\,b^4\,x^{17}}{17}+\frac{d\,b^4\,x^{16}}{16}+\frac{c\,b^4\,x^{15}}{15}$$

```
[In] int(x^2*(a + b*x^3)^4*(c + d*x + e*x^2),x)
```

```
[Out] (a^4*c*x^3)/3 + (a^4*d*x^4)/4 + (b^4*c*x^15)/15 + (a^4*e*x^5)/5 + (b^4*d*x^
16)/16 + (b^4*e*x^17)/17 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*
a^2*b^2*e*x^11)/11 + (2*a^3*b*c*x^6)/3 + (a*b^3*c*x^12)/3 + (4*a^3*b*d*x^7)
/7 + (4*a*b^3*d*x^13)/13 + (a^3*b*e*x^8)/2 + (2*a*b^3*e*x^14)/7
```

3.332 $\int x(c+dx+ex^2)(a+bx^3)^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int x(c+dx+ex^2)(a+bx^3)^4\,dx = \frac{1}{2}a^4cx^2+\frac{1}{4}a^4ex^4+\frac{4}{5}a^3bcx^5+\frac{4}{7}a^3bex^7 + \frac{3}{4}a^2b^2cx^8+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{11}ab^3cx^{11}+\frac{4}{13}ab^3ex^{13} + \frac{1}{14}b^4cx^{14}+\frac{1}{16}b^4ex^{16}+\frac{d(a+bx^3)^5}{15b}$$

```
[Out] 1/2*a^4*c*x^2+1/4*a^4*e*x^4+4/5*a^3*b*c*x^5+4/7*a^3*b*e*x^7+3/4*a^2*b^2*c*x
^8+3/5*a^2*b^2*e*x^10+4/11*a*b^3*c*x^11+4/13*a*b^3*e*x^13+1/14*b^4*c*x^14+1
/16*b^4*e*x^16+1/15*d*(b*x^3+a)^5/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1596, 1864}

$$\int x(c+dx+ex^2)(a+bx^3)^4\,dx = \frac{1}{2}a^4cx^2+\frac{1}{4}a^4ex^4+\frac{4}{5}a^3bcx^5+\frac{4}{7}a^3bex^7 + \frac{3}{4}a^2b^2cx^8+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{11}ab^3cx^{11}+\frac{4}{13}ab^3ex^{13} + \frac{d(a+bx^3)^5}{15b}+\frac{1}{14}b^4cx^{14}+\frac{1}{16}b^4ex^{16}$$

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*c*x^2)/2 + (a^4*e*x^4)/4 + (4*a^3*b*c*x^5)/5 + (4*a^3*b*e*x^7)/7 + (3*
a^2*b^2*c*x^8)/4 + (3*a^2*b^2*e*x^10)/5 + (4*a*b^3*c*x^11)/11 + (4*a*b^3*e*
x^13)/13 + (b^4*c*x^14)/14 + (b^4*e*x^16)/16 + (d*(a + b*x^3)^5)/(15*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d(a+bx^3)^5}{15b} + \int (a+bx^3)^4 \left(-dx^2 + x(c+dx+ex^2)\right) dx \\ &= \frac{d(a+bx^3)^5}{15b} + \int \left(a^4cx + a^4ex^3 + 4a^3bcx^4 + 4a^3bex^6 + 6a^2b^2cx^7 + 6a^2b^2ex^9 \right. \\ &\qquad \left. + 4ab^3cx^{10} + 4ab^3ex^{12} + b^4cx^{13} + b^4ex^{15}\right) dx \\ &= \frac{1}{2}a^4cx^2 + \frac{1}{4}a^4ex^4 + \frac{4}{5}a^3bcx^5 + \frac{4}{7}a^3bex^7 + \frac{3}{4}a^2b^2cx^8 + \frac{3}{5}a^2b^2ex^{10} \\ &\quad + \frac{4}{11}ab^3cx^{11} + \frac{4}{13}ab^3ex^{13} + \frac{1}{14}b^4cx^{14} + \frac{1}{16}b^4ex^{16} + \frac{d(a+bx^3)^5}{15b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.31

$$\begin{aligned}\int x(c+dx+ex^2)\left(a+bx^3\right)^4 dx &= \frac{1}{2}a^4cx^2 + \frac{1}{3}a^4dx^3 + \frac{1}{4}a^4ex^4 + \frac{4}{5}a^3bcx^5 \\ &\quad + \frac{2}{3}a^3bdx^6 + \frac{4}{7}a^3bex^7 + \frac{3}{4}a^2b^2cx^8 + \frac{2}{3}a^2b^2dx^9 \\ &\quad + \frac{3}{5}a^2b^2ex^{10} + \frac{4}{11}ab^3cx^{11} + \frac{1}{3}ab^3dx^{12} \\ &\quad + \frac{4}{13}ab^3ex^{13} + \frac{1}{14}b^4cx^{14} + \frac{1}{15}b^4dx^{15} + \frac{1}{16}b^4ex^{16}\end{aligned}$$

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*c*x^2)/2 + (a^4*d*x^3)/3 + (a^4*e*x^4)/4 + (4*a^3*b*c*x^5)/5 + (2*a^3*
b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (3*a^2*b^2*c*x^8)/4 + (2*a^2*b^2*d*x^9)/3
+ (3*a^2*b^2*e*x^10)/5 + (4*a*b^3*c*x^11)/11 + (a*b^3*d*x^12)/3 + (4*a*b^3*
e*x^13)/13 + (b^4*c*x^14)/14 + (b^4*d*x^15)/15 + (b^4*e*x^16)/16
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.10

method	result
gosper	$\frac{1}{2}a^4c\,x^2+\frac{1}{3}a^4d\,x^3+\frac{1}{4}a^4e\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{3}{4}a^2b^2c\,x^8+\frac{2}{3}a^2b^2d\,x^9+\frac{3}{5}a^2$
default	$\frac{1}{2}a^4c\,x^2+\frac{1}{3}a^4d\,x^3+\frac{1}{4}a^4e\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{3}{4}a^2b^2c\,x^8+\frac{2}{3}a^2b^2d\,x^9+\frac{3}{5}a^2$
norman	$\frac{1}{2}a^4c\,x^2+\frac{1}{3}a^4d\,x^3+\frac{1}{4}a^4e\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{3}{4}a^2b^2c\,x^8+\frac{2}{3}a^2b^2d\,x^9+\frac{3}{5}a^2$
risch	$\frac{1}{2}a^4c\,x^2+\frac{1}{3}a^4d\,x^3+\frac{1}{4}a^4e\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{3}{4}a^2b^2c\,x^8+\frac{2}{3}a^2b^2d\,x^9+\frac{3}{5}a^2$
parallelrisch	$\frac{1}{2}a^4c\,x^2+\frac{1}{3}a^4d\,x^3+\frac{1}{4}a^4e\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{3}{4}a^2b^2c\,x^8+\frac{2}{3}a^2b^2d\,x^9+\frac{3}{5}a^2$

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^4*c*x^2+1/3*a^4*d*x^3+1/4*a^4*e*x^4+4/5*a^3*b*c*x^5+2/3*a^3*b*d*x^6+4
/7*a^3*b*e*x^7+3/4*a^2*b^2*c*x^8+2/3*a^2*b^2*d*x^9+3/5*a^2*b^2*e*x^10+4/11*
a*b^3*c*x^11+1/3*x^12*a*b^3*d+4/13*a*b^3*e*x^13+1/14*b^4*c*x^14+1/15*d*b^4*
x^15+1/16*b^4*e*x^16
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^4\,dx=\frac{1}{16}\,b^4ex^{16}+\frac{1}{15}\,b^4dx^{15}+\frac{1}{14}\,b^4cx^{14}+\frac{4}{13}\,ab^3ex^{13}+\frac{1}{3}\,ab^3dx^{12}+\frac{4}{11}\,ab^3cx^{11}+\frac{3}{5}\,a^2b^2ex^{10}+\frac{2}{3}\,a^2b^2dx^9+\frac{3}{4}\,a^2b^2cx^8+\frac{4}{7}\,a^3bex^7+\frac{2}{3}\,a^3bdx^6+\frac{4}{5}\,a^3bcx^5+\frac{1}{4}\,a^4ex^4+\frac{1}{3}\,a^4dx^3+\frac{1}{2}\,a^4cx^2$$

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/16*b^4*e*x^16 + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*e*x^13 + 1
/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*e*x^10 + 2/3*a^2*b^2*d*x^
9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5
 + 1/4*a^4*e*x^4 + 1/3*a^4*d*x^3 + 1/2*a^4*c*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.34

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^4\,dx=\frac{a^4cx^2}{2}+\frac{a^4dx^3}{3}+\frac{a^4ex^4}{4}+\frac{4a^3bcx^5}{5}+\frac{2a^3bdx^6}{3}+\frac{4a^3bex^7}{7}+\frac{3a^2b^2cx^8}{4}+\frac{2a^2b^2dx^9}{3}+\frac{3a^2b^2ex^{10}}{5}+\frac{4ab^3cx^{11}}{11}+\frac{ab^3dx^{12}}{3}+\frac{4ab^3ex^{13}}{13}+\frac{b^4cx^{14}}{14}+\frac{b^4dx^{15}}{15}+\frac{b^4ex^{16}}{16}$$

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*x**2/2 + a**4*d*x**3/3 + a**4*e*x**4/4 + 4*a**3*b*c*x**5/5 + 2*a**3*
b*d*x**6/3 + 4*a**3*b*e*x**7/7 + 3*a**2*b**2*c*x**8/4 + 2*a**2*b**2*d*x**9/
3 + 3*a**2*b**2*e*x**10/5 + 4*a*b**3*c*x**11/11 + a*b**3*d*x**12/3 + 4*a*b*
*3*e*x**13/13 + b**4*c*x**14/14 + b**4*d*x**15/15 + b**4*e*x**16/16
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^4\,dx=\frac{1}{16}\,b^4ex^{16}+\frac{1}{15}\,b^4dx^{15}+\frac{1}{14}\,b^4cx^{14}+\frac{4}{13}\,ab^3ex^{13}+\frac{1}{3}\,ab^3dx^{12}+\frac{4}{11}\,ab^3cx^{11}+\frac{3}{5}\,a^2b^2ex^{10}+\frac{2}{3}\,a^2b^2dx^9+\frac{3}{4}\,a^2b^2cx^8+\frac{4}{7}\,a^3bex^7+\frac{2}{3}\,a^3bdx^6+\frac{4}{5}\,a^3bcx^5+\frac{1}{4}\,a^4ex^4+\frac{1}{3}\,a^4dx^3+\frac{1}{2}\,a^4cx^2$$

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/16*b^4*e*x^16 + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*e*x^13 + 1
/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*e*x^10 + 2/3*a^2*b^2*d*x^
9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5
 + 1/4*a^4*e*x^4 + 1/3*a^4*d*x^3 + 1/2*a^4*c*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx=\frac{1}{16}\,b^4ex^{16}+\frac{1}{15}\,b^4dx^{15}+\frac{1}{14}\,b^4cx^{14}+\frac{4}{13}\,ab^3ex^{13}+\frac{1}{3}\,ab^3dx^{12}+\frac{4}{11}\,ab^3cx^{11}+\frac{3}{5}\,a^2b^2ex^{10}+\frac{2}{3}\,a^2b^2dx^9+\frac{3}{4}\,a^2b^2cx^8+\frac{4}{7}\,a^3bex^7+\frac{2}{3}\,a^3bdx^6+\frac{4}{5}\,a^3bcx^5+\frac{1}{4}\,a^4ex^4+\frac{1}{3}\,a^4dx^3+\frac{1}{2}\,a^4cx^2$$

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*b^4*e*x^16 + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*e*x^13 + 1
/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*e*x^10 + 2/3*a^2*b^2*d*x^
9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5
 + 1/4*a^4*e*x^4 + 1/3*a^4*d*x^3 + 1/2*a^4*c*x^2
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx=\frac{e\,a^4\,x^4}{4}+\frac{d\,a^4\,x^3}{3}+\frac{c\,a^4\,x^2}{2}+\frac{4\,e\,a^3\,b\,x^7}{7}+\frac{2\,d\,a^3\,b\,x^6}{3}+\frac{4\,c\,a^3\,b\,x^5}{5}+\frac{3\,e\,a^2\,b^2\,x^{10}}{5}+\frac{2\,d\,a^2\,b^2\,x^9}{3}+\frac{3\,c\,a^2\,b^2\,x^8}{4}+\frac{4\,e\,a\,b^3\,x^{13}}{13}+\frac{d\,a\,b^3\,x^{12}}{3}+\frac{4\,c\,a\,b^3\,x^{11}}{11}+\frac{e\,b^4\,x^{16}}{16}+\frac{d\,b^4\,x^{15}}{15}+\frac{c\,b^4\,x^{14}}{14}$$

```
[In] int(x*(a + b*x^3)^4*(c + d*x + e*x^2),x)
```

```
[Out] (a^4*c*x^2)/2 + (a^4*d*x^3)/3 + (b^4*c*x^14)/14 + (a^4*e*x^4)/4 + (b^4*d*x^
15)/15 + (b^4*e*x^16)/16 + (3*a^2*b^2*c*x^8)/4 + (2*a^2*b^2*d*x^9)/3 + (3*a
^2*b^2*e*x^10)/5 + (4*a^3*b*c*x^5)/5 + (4*a*b^3*c*x^11)/11 + (2*a^3*b*d*x^6
)/3 + (a*b^3*d*x^12)/3 + (4*a^3*b*e*x^7)/7 + (4*a*b^3*e*x^13)/13
```

3.333 $\int (c+dx+ex^2)(a+bx^3)^4 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 130

$$\int (c+dx+ex^2)(a+bx^3)^4 \, dx = a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7 +\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11} +\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}+\frac{e(a+bx^3)^5}{15b}$$

```
[Out] a^4*c*x+1/2*a^4*d*x^2+a^3*b*c*x^4+4/5*a^3*b*d*x^5+6/7*a^2*b^2*c*x^7+3/4*a^2
*b^2*d*x^8+2/5*a*b^3*c*x^10+4/11*a*b^3*d*x^11+1/13*b^4*c*x^13+1/14*b^4*d*x^
14+1/15*e*(b*x^3+a)^5/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1596, 1864}

$$\int (c+dx+ex^2)(a+bx^3)^4 \, dx = a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7 +\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11} +\frac{e(a+bx^3)^5}{15b}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^
7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (b^
4*c*x^13)/13 + (b^4*d*x^14)/14 + (e*(a + b*x^3)^5)/(15*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e(a+bx^3)^5}{15b}+\int (c+dx)\left(a+bx^3\right)^4\,dx\\ &= \frac{e(a+bx^3)^5}{15b}+\int \left(a^4c+a^4dx+4a^3bcx^3+4a^3bdx^4+6a^2b^2cx^6+6a^2b^2dx^7\right.\\ &\qquad \left.+4ab^3cx^9+4ab^3dx^{10}+b^4cx^{12}+b^4dx^{13}\right)\,dx\\ &= a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8\\ &\quad +\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}+\frac{e(a+bx^3)^5}{15b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.33

$$\begin{aligned}\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx &= a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{2}{3}a^3bex^6\\ &\quad +\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{3}a^2b^2ex^9+\frac{2}{5}ab^3cx^{10}\\ &\quad +\frac{4}{11}ab^3dx^{11}+\frac{1}{3}ab^3ex^{12}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}+\frac{1}{15}b^4ex^{15}\end{aligned}$$

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 +
 (2*a^3*b*e*x^6)/3 + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a^2*b^2
*e*x^9)/3 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (a*b^3*e*x^12)/3 + (
b^4*c*x^13)/13 + (b^4*d*x^14)/14 + (b^4*e*x^15)/15
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.14

method	result
gosper	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{2}{3}a^3be\,x^6+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{3}a^2e\,b$
default	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{2}{3}a^3be\,x^6+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{3}a^2e\,b$
norman	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{2}{3}a^3be\,x^6+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{3}a^2e\,b$
risch	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{2}{3}a^3be\,x^6+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{3}a^2e\,b$
parallelrisch	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{2}{3}a^3be\,x^6+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{3}a^2e\,b$

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*c*x+1/2*a^4*d*x^2+1/3*a^4*e*x^3+a^3*b*c*x^4+4/5*d*x^5*b*a^3+2/3*a^3*b*e
*x^6+6/7*a^2*b^2*c*x^7+3/4*x^8*b^2*d*a^2+2/3*a^2*e*b^2*x^9+2/5*a*b^3*c*x^10
+4/11*x^11*d*b^3*a+1/3*a*b^3*e*x^12+1/13*b^4*c*x^13+1/14*b^4*d*x^14+1/15*e*
b^4*x^15
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.13

$$\int (c+dx+ex^2)\left(a+bx^3\right)^4\,dx=\frac{1}{15}\,b^4ex^{15}+\frac{1}{14}\,b^4dx^{14}+\frac{1}{13}\,b^4cx^{13}+\frac{1}{3}\,ab^3ex^{12}+\frac{4}{11}\,ab^3dx^{11}+\frac{2}{5}\,ab^3cx^{10}+\frac{2}{3}\,a^2b^2ex^9+\frac{3}{4}\,a^2b^2dx^8+\frac{6}{7}\,a^2b^2cx^7+\frac{2}{3}\,a^3bex^6+\frac{4}{5}\,a^3bdx^5+a^3bcx^4+\frac{1}{3}\,a^4ex^3+\frac{1}{2}\,a^4dx^2+a^4cx$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/15*b^4*e*x^15 + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*e*x^12 + 4/
11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*e*x^9 + 3/4*a^2*b^2*d*x^8
+ 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*e*x^6 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/3
*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.37

$$\int (c+dx+ex^2)(a+bx^3)^4 dx = a^4cx + \frac{a^4dx^2}{2} + \frac{a^4ex^3}{3} + a^3bcx^4 + \frac{4a^3bdx^5}{5} + \frac{2a^3bex^6}{3} + \frac{6a^2b^2cx^7}{7} + \frac{3a^2b^2dx^8}{4} + \frac{2a^2b^2ex^9}{3} + \frac{2ab^3cx^{10}}{5} + \frac{4ab^3dx^{11}}{11} + \frac{ab^3ex^{12}}{3} + \frac{b^4cx^{13}}{13} + \frac{b^4dx^{14}}{14} + \frac{b^4ex^{15}}{15}$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**4*e*x**3/3 + a**3*b*c*x**4 + 4*a**3*b*d*x**5/
5 + 2*a**3*b*e*x**6/3 + 6*a**2*b**2*c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*a**
2*b**2*e*x**9/3 + 2*a*b**3*c*x**10/5 + 4*a*b**3*d*x**11/11 + a*b**3*e*x**12
/3 + b**4*c*x**13/13 + b**4*d*x**14/14 + b**4*e*x**15/15
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.13

$$\int (c+dx+ex^2)(a+bx^3)^4 dx = \frac{1}{15}b^4ex^{15} + \frac{1}{14}b^4dx^{14} + \frac{1}{13}b^4cx^{13} + \frac{1}{3}ab^3ex^{12} + \frac{4}{11}ab^3dx^{11} + \frac{2}{5}ab^3cx^{10} + \frac{2}{3}a^2b^2ex^9 + \frac{3}{4}a^2b^2dx^8 + \frac{6}{7}a^2b^2cx^7 + \frac{2}{3}a^3bex^6 + \frac{4}{5}a^3bdx^5 + a^3bcx^4 + \frac{1}{3}a^4ex^3 + \frac{1}{2}a^4dx^2 + a^4cx$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/15*b^4*e*x^15 + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*e*x^12 + 4/
11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*e*x^9 + 3/4*a^2*b^2*d*x^8
+ 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*e*x^6 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/3
*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.13

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx=\frac{1}{15}\,b^4ex^{15}+\frac{1}{14}\,b^4dx^{14}+\frac{1}{13}\,b^4cx^{13}+\frac{1}{3}\,ab^3ex^{12}+\frac{4}{11}\,ab^3dx^{11}+\frac{2}{5}\,ab^3cx^{10}+\frac{2}{3}\,a^2b^2ex^9+\frac{3}{4}\,a^2b^2dx^8+\frac{6}{7}\,a^2b^2cx^7+\frac{2}{3}\,a^3bex^6+\frac{4}{5}\,a^3bdx^5+a^3bcx^4+\frac{1}{3}\,a^4ex^3+\frac{1}{2}\,a^4dx^2+a^4cx$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 1/15*b^4*e*x^15 + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*e*x^12 + 4/
11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*e*x^9 + 3/4*a^2*b^2*d*x^8
+ 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*e*x^6 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/3
*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.13

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx=\frac{e\,a^4\,x^3}{3}+\frac{d\,a^4\,x^2}{2}+c\,a^4\,x+\frac{2\,e\,a^3\,b\,x^6}{3}+\frac{4\,d\,a^3\,b\,x^5}{5}+c\,a^3\,b\,x^4+\frac{2\,e\,a^2\,b^2\,x^9}{3}+\frac{3\,d\,a^2\,b^2\,x^8}{4}+\frac{6\,c\,a^2\,b^2\,x^7}{7}+\frac{e\,a\,b^3\,x^{12}}{3}+\frac{4\,d\,a\,b^3\,x^{11}}{11}+\frac{2\,c\,a\,b^3\,x^{10}}{5}+\frac{e\,b^4\,x^{15}}{15}+\frac{d\,b^4\,x^{14}}{14}+\frac{c\,b^4\,x^{13}}{13}$$

```
[In] int((a + b*x^3)^4*(c + d*x + e*x^2),x)
```

```
[Out] (a^4*d*x^2)/2 + (b^4*c*x^13)/13 + (a^4*e*x^3)/3 + (b^4*d*x^14)/14 + (b^4*e*
x^15)/15 + a^4*c*x + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a^2*b^2
*e*x^9)/3 + a^3*b*c*x^4 + (2*a*b^3*c*x^10)/5 + (4*a^3*b*d*x^5)/5 + (4*a*b^3
*d*x^11)/11 + (2*a^3*b*e*x^6)/3 + (a*b^3*e*x^12)/3
```

3.334 $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x} dx$

Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x} dx = a^4dx + \frac{1}{2}a^4ex^2 + \frac{4}{3}a^3bcx^3 + a^3bdx^4 + \frac{4}{5}a^3bex^5 + a^2b^2cx^6 + \frac{6}{7}a^2b^2dx^7 + \frac{3}{4}a^2b^2ex^8 + \frac{4}{9}ab^3cx^9 + \frac{2}{5}ab^3dx^{10} + \frac{4}{11}ab^3ex^{11} + \frac{1}{12}b^4cx^{12} + \frac{1}{13}b^4dx^{13} + \frac{1}{14}b^4ex^{14} + a^4c\log(x)$$

```
[Out] a^4*d*x+1/2*a^4*e*x^2+4/3*a^3*b*c*x^3+a^3*b*d*x^4+4/5*a^3*b*e*x^5+a^2*b^2*c
*x^6+6/7*a^2*b^2*d*x^7+3/4*a^2*b^2*e*x^8+4/9*a*b^3*c*x^9+2/5*a*b^3*d*x^10+4
/11*a*b^3*e*x^11+1/12*b^4*c*x^12+1/13*b^4*d*x^13+1/14*b^4*e*x^14+a^4*c*ln(x
)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1642}

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x} dx = a^4c\log(x) + a^4dx + \frac{1}{2}a^4ex^2 + \frac{4}{3}a^3bcx^3 + a^3bdx^4 + \frac{4}{5}a^3bex^5 + a^2b^2cx^6 + \frac{6}{7}a^2b^2dx^7 + \frac{3}{4}a^2b^2ex^8 + \frac{4}{9}ab^3cx^9 + \frac{2}{5}ab^3dx^{10} + \frac{4}{11}ab^3ex^{11} + \frac{1}{12}b^4cx^{12} + \frac{1}{13}b^4dx^{13} + \frac{1}{14}b^4ex^{14}$$

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x,x]
```

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (4*a^3*b*c*x^3)/3 + a^3*b*d*x^4 + (4*a^3*b*e*x^5)
/5 + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)/4 + (4*a*b^3*c
*x^9)/9 + (2*a*b^3*d*x^10)/5 + (4*a*b^3*e*x^11)/11 + (b^4*c*x^12)/12 + (b^4
*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*c*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^4d+\frac{a^4c}{x}+a^4ex+4a^3bcx^2+4a^3bdx^3+4a^3bex^4+6a^2b^2cx^5+6a^2b^2dx^6\right.\\&\quad\left.+6a^2b^2ex^7+4ab^3cx^8+4ab^3dx^9+4ab^3ex^{10}+b^4cx^{11}+b^4dx^{12}+b^4ex^{13}\right)dx\\&=a^4dx+\frac{1}{2}a^4ex^2+\frac{4}{3}a^3bcx^3+a^3bdx^4+\frac{4}{5}a^3bex^5+a^2b^2cx^6+\frac{6}{7}a^2b^2dx^7+\frac{3}{4}a^2b^2ex^8\\&\quad+\frac{4}{9}ab^3cx^9+\frac{2}{5}ab^3dx^{10}+\frac{4}{11}ab^3ex^{11}+\frac{1}{12}b^4cx^{12}+\frac{1}{13}b^4dx^{13}+\frac{1}{14}b^4ex^{14}+a^4c\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(c+dx+ex^2)\,(a+bx^3)^4}{x}\,dx &= a^4dx+\frac{1}{2}a^4ex^2+\frac{4}{3}a^3bcx^3+a^3bdx^4+\frac{4}{5}a^3bex^5+a^2b^2cx^6\\&\quad+\frac{6}{7}a^2b^2dx^7+\frac{3}{4}a^2b^2ex^8+\frac{4}{9}ab^3cx^9+\frac{2}{5}ab^3dx^{10}+\frac{4}{11}ab^3ex^{11}\\&\quad+\frac{1}{12}b^4cx^{12}+\frac{1}{13}b^4dx^{13}+\frac{1}{14}b^4ex^{14}+a^4c\log(x)\end{aligned}$$

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x,x]
```

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (4*a^3*b*c*x^3)/3 + a^3*b*d*x^4 + (4*a^3*b*e*x^5)
/5 + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)/4 + (4*a*b^3*c
*x^9)/9 + (2*a*b^3*d*x^10)/5 + (4*a*b^3*e*x^11)/11 + (b^4*c*x^12)/12 + (b^4
*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*c*Log[x]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.87

method	result
default	$a^4dx+\frac{a^4e\,x^2}{2}+\frac{4a^3bc\,x^3}{3}+a^3bd\,x^4+\frac{4a^3be\,x^5}{5}+a^2b^2c\,x^6+\frac{6a^2b^2d\,x^7}{7}+\frac{3a^2b^2e\,x^8}{4}+\frac{4a\,b^3c\,x^9}{9}+\frac{2a\,b^3d\,x^1}{5}$
norman	$a^4dx+\frac{a^4e\,x^2}{2}+\frac{4a^3bc\,x^3}{3}+a^3bd\,x^4+\frac{4a^3be\,x^5}{5}+a^2b^2c\,x^6+\frac{6a^2b^2d\,x^7}{7}+\frac{3a^2b^2e\,x^8}{4}+\frac{4a\,b^3c\,x^9}{9}+\frac{2a\,b^3d\,x^1}{5}$
risch	$a^4dx+\frac{a^4e\,x^2}{2}+\frac{4a^3bc\,x^3}{3}+a^3bd\,x^4+\frac{4a^3be\,x^5}{5}+a^2b^2c\,x^6+\frac{6a^2b^2d\,x^7}{7}+\frac{3a^2b^2e\,x^8}{4}+\frac{4a\,b^3c\,x^9}{9}+\frac{2a\,b^3d\,x^1}{5}$
parallelrisch	$a^4dx+\frac{a^4e\,x^2}{2}+\frac{4a^3bc\,x^3}{3}+a^3bd\,x^4+\frac{4a^3be\,x^5}{5}+a^2b^2c\,x^6+\frac{6a^2b^2d\,x^7}{7}+\frac{3a^2b^2e\,x^8}{4}+\frac{4a\,b^3c\,x^9}{9}+\frac{2a\,b^3d\,x^1}{5}$

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*d*x+1/2*a^4*e*x^2+4/3*a^3*b*c*x^3+a^3*b*d*x^4+4/5*a^3*b*e*x^5+a^2*b^2*c
*x^6+6/7*a^2*b^2*d*x^7+3/4*a^2*b^2*e*x^8+4/9*a*b^3*c*x^9+2/5*a*b^3*d*x^10+4
/11*a*b^3*e*x^11+1/12*b^4*c*x^12+1/13*b^4*d*x^13+1/14*b^4*e*x^14+a^4*c*ln(x
)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x}\,dx = \frac{1}{14}\,b^4ex^{14}+\frac{1}{13}\,b^4dx^{13}+\frac{1}{12}\,b^4cx^{12}+\frac{4}{11}\,ab^3ex^{11}+\frac{2}{5}\,ab^3dx^{10} + \frac{4}{9}\,ab^3cx^9+\frac{3}{4}\,a^2b^2ex^8+\frac{6}{7}\,a^2b^2dx^7+a^2b^2cx^6+\frac{4}{5}\,a^3bex^5 + a^3bdx^4+\frac{4}{3}\,a^3bcx^3+\frac{1}{2}\,a^4ex^2+a^4dx+a^4c\log(x)$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="fricas")
```

```
[Out] 1/14*b^4*e*x^14 + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*e*x^11 + 2
/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*e*x^8 + 6/7*a^2*b^2*d*x^7 +
 a^2*b^2*c*x^6 + 4/5*a^3*b*e*x^5 + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*
e*x^2 + a^4*d*x + a^4*c*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x}\,dx = a^4c\log(x)+a^4dx+\frac{a^4ex^2}{2}+\frac{4a^3bcx^3}{3}+a^3bdx^4 + \frac{4a^3bex^5}{5}+a^2b^2cx^6+\frac{6a^2b^2dx^7}{7}+\frac{3a^2b^2ex^8}{4}+\frac{4ab^3cx^9}{9} + \frac{2ab^3dx^{10}}{5}+\frac{4ab^3ex^{11}}{11}+\frac{b^4cx^{12}}{12}+\frac{b^4dx^{13}}{13}+\frac{b^4ex^{14}}{14}$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x,x)
```

```
[Out] a**4*c*log(x) + a**4*d*x + a**4*e*x**2/2 + 4*a**3*b*c*x**3/3 + a**3*b*d*x**
4 + 4*a**3*b*e*x**5/5 + a**2*b**2*c*x**6 + 6*a**2*b**2*d*x**7/7 + 3*a**2*b*
*2*e*x**8/4 + 4*a*b**3*c*x**9/9 + 2*a*b**3*d*x**10/5 + 4*a*b**3*e*x**11/11
+ b**4*c*x**12/12 + b**4*d*x**13/13 + b**4*e*x**14/14
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x}\,dx = \frac{1}{14}\,b^4ex^{14}+\frac{1}{13}\,b^4dx^{13}+\frac{1}{12}\,b^4cx^{12}+\frac{4}{11}\,ab^3ex^{11}+\frac{2}{5}\,ab^3dx^{10} + \frac{4}{9}\,ab^3cx^9+\frac{3}{4}\,a^2b^2ex^8+\frac{6}{7}\,a^2b^2dx^7+a^2b^2cx^6+\frac{4}{5}\,a^3bex^5 + a^3bdx^4+\frac{4}{3}\,a^3bcx^3+\frac{1}{2}\,a^4ex^2+a^4dx+a^4c\log(x)$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="maxima")
```

```
[Out] 1/14*b^4*e*x^14 + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*e*x^11 + 2
/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*e*x^8 + 6/7*a^2*b^2*d*x^7 +
 a^2*b^2*c*x^6 + 4/5*a^3*b*e*x^5 + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*
e*x^2 + a^4*d*x + a^4*c*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x}\,dx = \frac{1}{14}\,b^4ex^{14}+\frac{1}{13}\,b^4dx^{13}+\frac{1}{12}\,b^4cx^{12}+\frac{4}{11}\,ab^3ex^{11}+\frac{2}{5}\,ab^3dx^{10}+\frac{4}{9}\,ab^3cx^9+\frac{3}{4}\,a^2b^2ex^8+\frac{6}{7}\,a^2b^2dx^7+a^2b^2cx^6+\frac{4}{5}\,a^3bex^5+a^3bdx^4+\frac{4}{3}\,a^3bcx^3+\frac{1}{2}\,a^4ex^2+a^4dx+a^4c\log(|x|)$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="giac")
```

```
[Out] 1/14*b^4*e*x^14 + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*e*x^11 + 2
/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*e*x^8 + 6/7*a^2*b^2*d*x^7 +
 a^2*b^2*c*x^6 + 4/5*a^3*b*e*x^5 + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*
e*x^2 + a^4*d*x + a^4*c*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x}\,dx = \frac{b^4\,c\,x^{12}}{12}+\frac{a^4\,e\,x^2}{2}+\frac{b^4\,d\,x^{13}}{13}+\frac{b^4\,e\,x^{14}}{14}+a^4\,c\,\ln(x)+a^4\,d\,x+a^2\,b^2\,c\,x^6+\frac{6\,a^2\,b^2\,d\,x^7}{7}+\frac{3\,a^2\,b^2\,e\,x^8}{4}+\frac{4\,a^3\,b\,c\,x^3}{3}+\frac{4\,a\,b^3\,c\,x^9}{9}+a^3\,b\,d\,x^4+\frac{2\,a\,b^3\,d\,x^{10}}{5}+\frac{4\,a^3\,b\,e\,x^5}{5}+\frac{4\,a\,b^3\,e\,x^{11}}{11}$$

```
[In] int(((a + b*x^3)^4*(c + d*x + e*x^2))/x,x)
```

```
[Out] (b^4*c*x^12)/12 + (a^4*e*x^2)/2 + (b^4*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*c
*log(x) + a^4*d*x + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)
/4 + (4*a^3*b*c*x^3)/3 + (4*a*b^3*c*x^9)/9 + a^3*b*d*x^4 + (2*a*b^3*d*x^10)
/5 + (4*a^3*b*e*x^5)/5 + (4*a*b^3*e*x^11)/11
```

3.335 $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2} dx = -\frac{a^4c}{x} + a^4ex + 2a^3bcx^2 + \frac{4}{3}a^3bdx^3 + a^3bex^4 + \frac{6}{5}a^2b^2cx^5 + a^2b^2dx^6 + \frac{6}{7}a^2b^2ex^7 + \frac{1}{2}ab^3cx^8 + \frac{4}{9}ab^3dx^9 + \frac{2}{5}ab^3ex^{10} + \frac{1}{11}b^4cx^{11} + \frac{1}{12}b^4dx^{12} + \frac{1}{13}b^4ex^{13} + a^4d\log(x)$$

```
[Out] -a^4*c/x+a^4*e*x+2*a^3*b*c*x^2+4/3*a^3*b*d*x^3+a^3*b*e*x^4+6/5*a^2*b^2*c*x^
5+a^2*b^2*d*x^6+6/7*a^2*b^2*e*x^7+1/2*a*b^3*c*x^8+4/9*a*b^3*d*x^9+2/5*a*b^3
*e*x^10+1/11*b^4*c*x^11+1/12*b^4*d*x^12+1/13*b^4*e*x^13+a^4*d*ln(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1642}

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2} dx = -\frac{a^4c}{x} + a^4d\log(x) + a^4ex + 2a^3bcx^2 + \frac{4}{3}a^3bdx^3 + a^3bex^4 + \frac{6}{5}a^2b^2cx^5 + a^2b^2dx^6 + \frac{6}{7}a^2b^2ex^7 + \frac{1}{2}ab^3cx^8 + \frac{4}{9}ab^3dx^9 + \frac{2}{5}ab^3ex^{10} + \frac{1}{11}b^4cx^{11} + \frac{1}{12}b^4dx^{12} + \frac{1}{13}b^4ex^{13}$$

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^2,x]
```

```
[Out] -((a^4*c)/x) + a^4*e*x + 2*a^3*b*c*x^2 + (4*a^3*b*d*x^3)/3 + a^3*b*e*x^4 +
(6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 + (a*b^3*c*x^8)/2
```

```
 + (4*a*b^3*d*x^9)/9 + (2*a*b^3*e*x^10)/5 + (b^4*c*x^11)/11 + (b^4*d*x^12)/
12 + (b^4*e*x^13)/13 + a^4*d*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^4e+\frac{a^4c}{x^2}+\frac{a^4d}{x}+4a^3bcx+4a^3bdx^2+4a^3bex^3+6a^2b^2cx^4+6a^2b^2dx^5+6a^2b^2ex^6\right.\\&\qquad\left.+4ab^3cx^7+4ab^3dx^8+4ab^3ex^9+b^4cx^{10}+b^4dx^{11}+b^4ex^{12}\right)dx\\&= -\frac{a^4c}{x}+a^4ex+2a^3bcx^2+\frac{4}{3}a^3bdx^3+a^3bex^4+\frac{6}{5}a^2b^2cx^5+a^2b^2dx^6+\frac{6}{7}a^2b^2ex^7\\&\quad+\frac{1}{2}ab^3cx^8+\frac{4}{9}ab^3dx^9+\frac{2}{5}ab^3ex^{10}+\frac{1}{11}b^4cx^{11}+\frac{1}{12}b^4dx^{12}+\frac{1}{13}b^4ex^{13}+a^4d\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx &= -\frac{a^4c}{x}+a^4ex+2a^3bcx^2+\frac{4}{3}a^3bdx^3+a^3bex^4+\frac{6}{5}a^2b^2cx^5\\&\quad+a^2b^2dx^6+\frac{6}{7}a^2b^2ex^7+\frac{1}{2}ab^3cx^8+\frac{4}{9}ab^3dx^9+\frac{2}{5}ab^3ex^{10}\\&\quad+\frac{1}{11}b^4cx^{11}+\frac{1}{12}b^4dx^{12}+\frac{1}{13}b^4ex^{13}+a^4d\log(x)\end{aligned}$$

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^2,x]
```

```
[Out] -((a^4*c)/x) + a^4*e*x + 2*a^3*b*c*x^2 + (4*a^3*b*d*x^3)/3 + a^3*b*e*x^4 +
(6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 + (a*b^3*c*x^8)/2
 + (4*a*b^3*d*x^9)/9 + (2*a*b^3*e*x^10)/5 + (b^4*c*x^11)/11 + (b^4*d*x^12)/
12 + (b^4*e*x^13)/13 + a^4*d*Log[x]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a^4c}{x}+a^4ex+2a^3bc\,x^2+\frac{4a^3bd\,x^3}{3}+a^3be\,x^4+\frac{6a^2b^2c\,x^5}{5}+a^2b^2d\,x^6+\frac{6a^2b^2e\,x^7}{7}+\frac{a\,b^3c\,x^8}{2}+\frac{4a\,b^3d}{9}$
risch	$-\frac{a^4c}{x}+a^4ex+2a^3bc\,x^2+\frac{4a^3bd\,x^3}{3}+a^3be\,x^4+\frac{6a^2b^2c\,x^5}{5}+a^2b^2d\,x^6+\frac{6a^2b^2e\,x^7}{7}+\frac{a\,b^3c\,x^8}{2}+\frac{4a\,b^3d}{9}$
norman	$\frac{a^4e\,x^2+a^2b^2d\,x^7+a^3be\,x^5-a^4c+\frac{1}{11}b^4c\,x^{12}+\frac{1}{12}b^4d\,x^{13}+\frac{1}{13}b^4e\,x^{14}+\frac{1}{2}a\,b^3c\,x^9+\frac{4}{9}a\,b^3d\,x^{10}+\frac{2}{5}a\,b^3e\,x^{11}+\frac{6}{5}a^2b^2c\,x^6+\frac{6}{7}a^2b^2e\,x^{}}{x}$
parallelrisch	$\frac{13860b^4e\,x^{14}+15015b^4d\,x^{13}+16380b^4c\,x^{12}+72072a\,b^3e\,x^{11}+80080a\,b^3d\,x^{10}+90090a\,b^3c\,x^9+154440a^2b^2e\,x^8+180180a^2b^2d\,x^{}}{180180x}$

[In] int((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x,method=_RETURNVERBOSE)

[Out] -a^4*c/x+a^4*e*x+2*a^3*b*c*x^2+4/3*a^3*b*d*x^3+a^3*b*e*x^4+6/5*a^2*b^2*c*x^5+a^2*b^2*d*x^6+6/7*a^2*b^2*e*x^7+1/2*a*b^3*c*x^8+4/9*a*b^3*d*x^9+2/5*a*b^3*e*x^10+1/11*b^4*c*x^11+1/12*b^4*d*x^12+1/13*b^4*e*x^13+a^4*d*ln(x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx = \frac{13860\,b^4ex^{14}+15015\,b^4dx^{13}+16380\,b^4cx^{12}+72072\,ab^3ex^{11}+80080\,ab^3dx^{10}+90090\,ab^3cx^9+154440\,a^2\cdots}{}$$

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="fricas")

[Out] 1/180180*(13860*b^4*e*x^14 + 15015*b^4*d*x^13 + 16380*b^4*c*x^12 + 72072*a*b^3*e*x^11 + 80080*a*b^3*d*x^10 + 90090*a*b^3*c*x^9 + 154440*a^2*b^2*e*x^8 + 180180*a^2*b^2*d*x^7 + 216216*a^2*b^2*c*x^6 + 180180*a^3*b*e*x^5 + 240240*a^3*b*d*x^4 + 360360*a^3*b*c*x^3 + 180180*a^4*e*x^2 + 180180*a^4*d*x*log(x) - 180180*a^4*c)/x

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx = -\frac{a^4c}{x}+a^4d\log(x)+a^4ex+2a^3bcx^2+\frac{4a^3bdx^3}{3}+a^3bex^4+\frac{6a^2b^2cx^5}{5}+a^2b^2dx^6+\frac{6a^2b^2ex^7}{7}+\frac{ab^3cx^8}{2}+\frac{4ab^3dx^9}{9}+\frac{2ab^3ex^{10}}{5}+\frac{b^4cx^{11}}{11}+\frac{b^4dx^{12}}{12}+\frac{b^4ex^{13}}{13}$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x**2,x)
```

```
[Out] -a**4*c/x + a**4*d*log(x) + a**4*e*x + 2*a**3*b*c*x**2 + 4*a**3*b*d*x**3/3
+ a**3*b*e*x**4 + 6*a**2*b**2*c*x**5/5 + a**2*b**2*d*x**6 + 6*a**2*b**2*e*x
**7/7 + a*b**3*c*x**8/2 + 4*a*b**3*d*x**9/9 + 2*a*b**3*e*x**10/5 + b**4*c*x
**11/11 + b**4*d*x**12/12 + b**4*e*x**13/13
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx = \frac{1}{13}\,b^4ex^{13} + \frac{1}{12}\,b^4dx^{12} + \frac{1}{11}\,b^4cx^{11} + \frac{2}{5}\,ab^3ex^{10} + \frac{4}{9}\,ab^3dx^9 + \frac{1}{2}\,ab^3cx^8 + \frac{6}{7}\,a^2b^2ex^7 + a^2b^2dx^6 + \frac{6}{5}\,a^2b^2cx^5 + a^3bex^4 + \frac{4}{3}\,a^3bdx^3 + 2\,a^3bcx^2 + a^4ex + a^4d\log(x) - \frac{a^4c}{x}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^4*e*x^13 + 1/12*b^4*d*x^12 + 1/11*b^4*c*x^11 + 2/5*a*b^3*e*x^10 + 4/
9*a*b^3*d*x^9 + 1/2*a*b^3*c*x^8 + 6/7*a^2*b^2*e*x^7 + a^2*b^2*d*x^6 + 6/5*a
^2*b^2*c*x^5 + a^3*b*e*x^4 + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + a^4*e*x + a^
4*d*log(x) - a^4*c/x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx = \frac{1}{13}\,b^4ex^{13} + \frac{1}{12}\,b^4dx^{12} + \frac{1}{11}\,b^4cx^{11} + \frac{2}{5}\,ab^3ex^{10} + \frac{4}{9}\,ab^3dx^9 + \frac{1}{2}\,ab^3cx^8 + \frac{6}{7}\,a^2b^2ex^7 + a^2b^2dx^6 + \frac{6}{5}\,a^2b^2cx^5 + a^3bex^4 + \frac{4}{3}\,a^3bdx^3 + 2\,a^3bcx^2 + a^4ex + a^4d\log(|x|) - \frac{a^4c}{x}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="giac")
```

```
[Out] 1/13*b^4*e*x^13 + 1/12*b^4*d*x^12 + 1/11*b^4*c*x^11 + 2/5*a*b^3*e*x^10 + 4/
9*a*b^3*d*x^9 + 1/2*a*b^3*c*x^8 + 6/7*a^2*b^2*e*x^7 + a^2*b^2*d*x^6 + 6/5*a
^2*b^2*c*x^5 + a^3*b*e*x^4 + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + a^4*e*x + a^
4*d*log(abs(x)) - a^4*c/x
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx = \frac{b^4\,c\,x^{11}}{11}-\frac{a^4\,c}{x}+\frac{b^4\,d\,x^{12}}{12}+\frac{b^4\,e\,x^{13}}{13}+a^4\,d\,\ln\left(x\right)+a^4\,e\,x+\frac{6\,a^2\,b^2\,c\,x^5}{5}+a^2\,b^2\,d\,x^6+\frac{6\,a^2\,b^2\,e\,x^7}{7}+2\,a^3\,b\,c\,x^2+\frac{a\,b^3\,c\,x^8}{2}+\frac{4\,a^3\,b\,d\,x^3}{3}+\frac{4\,a\,b^3\,d\,x^9}{9}+a^3\,b\,e\,x^4+\frac{2\,a\,b^3\,e\,x^{10}}{5}$$

```
[In] int(((a + b*x^3)^4*(c + d*x + e*x^2))/x^2,x)
```

```
[Out] (b^4*c*x^11)/11 - (a^4*c)/x + (b^4*d*x^12)/12 + (b^4*e*x^13)/13 + a^4*d*log
(x) + a^4*e*x + (6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 +
 2*a^3*b*c*x^2 + (a*b^3*c*x^8)/2 + (4*a^3*b*d*x^3)/3 + (4*a*b^3*d*x^9)/9 +
a^3*b*e*x^4 + (2*a*b^3*e*x^10)/5
```

3.336 $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3} dx = -\frac{a^4c}{2x^2} - \frac{a^4d}{x} + 4a^3bcx + 2a^3bdx^2 + \frac{4}{3}a^3bex^3 + \frac{3}{2}a^2b^2cx^4 + \frac{6}{5}a^2b^2dx^5 + a^2b^2ex^6 + \frac{4}{7}ab^3cx^7 + \frac{1}{2}ab^3dx^8 + \frac{4}{9}ab^3ex^9 + \frac{1}{10}b^4cx^{10} + \frac{1}{11}b^4dx^{11} + \frac{1}{12}b^4ex^{12} + a^4e\log(x)$$

```
[Out] -1/2*a^4*c/x^2-a^4*d/x+4*a^3*b*c*x+2*a^3*b*d*x^2+4/3*a^3*b*e*x^3+3/2*a^2*b^
2*c*x^4+6/5*a^2*b^2*d*x^5+a^2*b^2*e*x^6+4/7*a*b^3*c*x^7+1/2*a*b^3*d*x^8+4/9
*a*b^3*e*x^9+1/10*b^4*c*x^10+1/11*b^4*d*x^11+1/12*b^4*e*x^12+a^4*e*ln(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1642}

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3} dx = -\frac{a^4c}{2x^2} - \frac{a^4d}{x} + a^4e\log(x) + 4a^3bcx + 2a^3bdx^2 + \frac{4}{3}a^3bex^3 + \frac{3}{2}a^2b^2cx^4 + \frac{6}{5}a^2b^2dx^5 + a^2b^2ex^6 + \frac{4}{7}ab^3cx^7 + \frac{1}{2}ab^3dx^8 + \frac{4}{9}ab^3ex^9 + \frac{1}{10}b^4cx^{10} + \frac{1}{11}b^4dx^{11} + \frac{1}{12}b^4ex^{12}$$

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^3,x]
```

```
[Out] -1/2*(a^4*c)/x^2 - (a^4*d)/x + 4*a^3*b*c*x + 2*a^3*b*d*x^2 + (4*a^3*b*e*x^3
)/3 + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2*e*x^6 + (4*a*b^3*
```

```
c*x^7)/7 + (a*b^3*d*x^8)/2 + (4*a*b^3*e*x^9)/9 + (b^4*c*x^10)/10 + (b^4*d*x
^11)/11 + (b^4*e*x^12)/12 + a^4*e*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(4a^3bc+\frac{a^4c}{x^3}+\frac{a^4d}{x^2}+\frac{a^4e}{x}+4a^3bdx+4a^3bex^2+6a^2b^2cx^3+6a^2b^2dx^4+6a^2b^2ex^5\right.\\ &\left.\quad+4ab^3cx^6+4ab^3dx^7+4ab^3ex^8+b^4cx^9+b^4dx^{10}+b^4ex^{11}\right)dx\\ &= -\frac{a^4c}{2x^2}-\frac{a^4d}{x}+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3+\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6\\ &\quad+\frac{4}{7}ab^3cx^7+\frac{1}{2}ab^3dx^8+\frac{4}{9}ab^3ex^9+\frac{1}{10}b^4cx^{10}+\frac{1}{11}b^4dx^{11}+\frac{1}{12}b^4ex^{12}+a^4e\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx &= -\frac{a^4c}{2x^2}-\frac{a^4d}{x}+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3+\frac{3}{2}a^2b^2cx^4\\ &\quad+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6+\frac{4}{7}ab^3cx^7+\frac{1}{2}ab^3dx^8+\frac{4}{9}ab^3ex^9\\ &\quad+\frac{1}{10}b^4cx^{10}+\frac{1}{11}b^4dx^{11}+\frac{1}{12}b^4ex^{12}+a^4e\log(x)\end{aligned}$$

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^3,x]
```

```
[Out] -1/2*(a^4*c)/x^2 - (a^4*d)/x + 4*a^3*b*c*x + 2*a^3*b*d*x^2 + (4*a^3*b*e*x^3
)/3 + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2*e*x^6 + (4*a*b^3*
c*x^7)/7 + (a*b^3*d*x^8)/2 + (4*a*b^3*e*x^9)/9 + (b^4*c*x^10)/10 + (b^4*d*x
^11)/11 + (b^4*e*x^12)/12 + a^4*e*Log[x]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^4c}{2x^2}-\frac{a^4d}{x}+4a^3bcx+2a^3bd\,x^2+\frac{4a^3be\,x^3}{3}+\frac{3a^2b^2c\,x^4}{2}+\frac{6a^2b^2d\,x^5}{5}+a^2b^2e\,x^6+\frac{4a\,b^3c\,x^7}{7}+\frac{a\,b^3d\,x^8}{2}$
risch	$\frac{b^4e\,x^{12}}{12}+\frac{b^4d\,x^{11}}{11}+\frac{b^4c\,x^{10}}{10}+\frac{4a\,b^3e\,x^9}{9}+\frac{a\,b^3d\,x^8}{2}+\frac{4a\,b^3c\,x^7}{7}+a^2b^2e\,x^6+\frac{6a^2b^2d\,x^5}{5}+\frac{3a^2b^2c\,x^4}{2}+\frac{4a^3be\,x^3}{3}$
norman	$\frac{a^2b^2e\,x^8-\frac{1}{2}a^4c-a^4dx+\frac{1}{10}b^4c\,x^{12}+\frac{1}{11}b^4d\,x^{13}+\frac{1}{12}b^4e\,x^{14}+\frac{4}{7}a\,b^3c\,x^9+\frac{1}{2}a\,b^3d\,x^{10}+\frac{4}{9}a\,b^3e\,x^{11}+\frac{3}{2}a^2b^2c\,x^6+\frac{6}{5}a^2b^2d\,x^7+4a^3bc\,x^3}{x^2}$
parallelrisch	$\frac{1155b^4e\,x^{14}+1260b^4d\,x^{13}+1386b^4c\,x^{12}+6160a\,b^3e\,x^{11}+6930a\,b^3d\,x^{10}+7920a\,b^3c\,x^9+13860a^2b^2e\,x^8+16632a^2b^2d\,x^7+20790a^2b}{13860x^2}$

[In] int((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x,method=_RETURNVERBOSE)

[Out] -1/2*a^4*c/x^2-a^4*d/x+4*a^3*b*c*x+2*a^3*b*d*x^2+4/3*a^3*b*e*x^3+3/2*a^2*b^2*c*x^4+6/5*a^2*b^2*d*x^5+a^2*b^2*e*x^6+4/7*a*b^3*c*x^7+1/2*a*b^3*d*x^8+4/9*a*b^3*e*x^9+1/10*b^4*c*x^10+1/11*b^4*d*x^11+1/12*b^4*e*x^12+a^4*e*ln(x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx = \frac{1155\,b^4ex^{14}+1260\,b^4dx^{13}+1386\,b^4cx^{12}+6160\,ab^3ex^{11}+6930\,ab^3dx^{10}+7920\,ab^3cx^9+13860\,a^2b^2ex^8+1}{}$$

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="fricas")

[Out] 1/13860*(1155*b^4*e*x^14 + 1260*b^4*d*x^13 + 1386*b^4*c*x^12 + 6160*a*b^3*e*x^11 + 6930*a*b^3*d*x^10 + 7920*a*b^3*c*x^9 + 13860*a^2*b^2*e*x^8 + 16632*a^2*b^2*d*x^7 + 20790*a^2*b^2*c*x^6 + 18480*a^3*b*e*x^5 + 27720*a^3*b*d*x^4 + 55440*a^3*b*c*x^3 + 13860*a^4*e*x^2*log(x) - 13860*a^4*d*x - 6930*a^4*c)/x^2

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx = a^4e\log{(x)} + 4a^3bcx + 2a^3bdx^2 + \frac{4a^3bex^3}{3} + \frac{3a^2b^2cx^4}{2} + \frac{6a^2b^2dx^5}{5} + a^2b^2ex^6 + \frac{4ab^3cx^7}{7} + \frac{ab^3dx^8}{2} + \frac{4ab^3ex^9}{9} + \frac{b^4cx^{10}}{10} + \frac{b^4dx^{11}}{11} + \frac{b^4ex^{12}}{12} + \frac{-a^4c-2a^4dx}{2x^2}$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x**3,x)
```

```
[Out] a**4*e*log(x) + 4*a**3*b*c*x + 2*a**3*b*d*x**2 + 4*a**3*b*e*x**3/3 + 3*a**2
*b**2*c*x**4/2 + 6*a**2*b**2*d*x**5/5 + a**2*b**2*e*x**6 + 4*a*b**3*c*x**7/
7 + a*b**3*d*x**8/2 + 4*a*b**3*e*x**9/9 + b**4*c*x**10/10 + b**4*d*x**11/11
 + b**4*e*x**12/12 + (-a**4*c - 2*a**4*d*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx = \frac{1}{12}\,b^4ex^{12} + \frac{1}{11}\,b^4dx^{11} + \frac{1}{10}\,b^4cx^{10} + \frac{4}{9}\,ab^3ex^9 + \frac{1}{2}\,ab^3dx^8 + \frac{4}{7}\,ab^3cx^7 + a^2b^2ex^6 + \frac{6}{5}\,a^2b^2dx^5 + \frac{3}{2}\,a^2b^2cx^4 + \frac{4}{3}\,a^3bex^3 + 2\,a^3bdx^2 + 4\,a^3bcx + a^4e\log{(x)} - \frac{2\,a^4dx + a^4c}{2\,x^2}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^4*e*x^12 + 1/11*b^4*d*x^11 + 1/10*b^4*c*x^10 + 4/9*a*b^3*e*x^9 + 1/2
*a*b^3*d*x^8 + 4/7*a*b^3*c*x^7 + a^2*b^2*e*x^6 + 6/5*a^2*b^2*d*x^5 + 3/2*a^
2*b^2*c*x^4 + 4/3*a^3*b*e*x^3 + 2*a^3*b*d*x^2 + 4*a^3*b*c*x + a^4*e*log(x)
- 1/2*(2*a^4*d*x + a^4*c)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx = \frac{1}{12}b^4ex^{12} + \frac{1}{11}b^4dx^{11} + \frac{1}{10}b^4cx^{10} + \frac{4}{9}ab^3ex^9 + \frac{1}{2}ab^3dx^8 + \frac{4}{7}ab^3cx^7 + a^2b^2ex^6 + \frac{6}{5}a^2b^2dx^5 + \frac{3}{2}a^2b^2cx^4 + \frac{4}{3}a^3bex^3 + 2a^3bdx^2 + 4a^3bcx + a^4e\log(|x|) - \frac{2a^4dx + a^4c}{2x^2}$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="giac")
```

```
[Out] 1/12*b^4*e*x^12 + 1/11*b^4*d*x^11 + 1/10*b^4*c*x^10 + 4/9*a*b^3*e*x^9 + 1/2
*a*b^3*d*x^8 + 4/7*a*b^3*c*x^7 + a^2*b^2*e*x^6 + 6/5*a^2*b^2*d*x^5 + 3/2*a^
2*b^2*c*x^4 + 4/3*a^3*b*e*x^3 + 2*a^3*b*d*x^2 + 4*a^3*b*c*x + a^4*e*log(abs
(x)) - 1/2*(2*a^4*d*x + a^4*c)/x^2
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx = \frac{b^4\,c\,x^{10}}{10} - \frac{\frac{a^4\,c}{2} + a^4\,d\,x}{x^2} + \frac{b^4\,d\,x^{11}}{11} + \frac{b^4\,e\,x^{12}}{12} + a^4\,e\,\ln(x) + \frac{3\,a^2\,b^2\,c\,x^4}{2} + \frac{6\,a^2\,b^2\,d\,x^5}{5} + a^2\,b^2\,e\,x^6 + 4\,a^3\,b\,c\,x + \frac{4\,a\,b^3\,c\,x^7}{7} + 2\,a^3\,b\,d\,x^2 + \frac{a\,b^3\,d\,x^8}{2} + \frac{4\,a^3\,b\,e\,x^3}{3} + \frac{4\,a\,b^3\,e\,x^9}{9}$$

```
[In] int(((a + b*x^3)^4*(c + d*x + e*x^2))/x^3,x)
```

```
[Out] (b^4*c*x^10)/10 - ((a^4*c)/2 + a^4*d*x)/x^2 + (b^4*d*x^11)/11 + (b^4*e*x^12
)/12 + a^4*e*log(x) + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2*e
*x^6 + 4*a^3*b*c*x + (4*a*b^3*c*x^7)/7 + 2*a^3*b*d*x^2 + (a*b^3*d*x^8)/2 +
(4*a^3*b*e*x^3)/3 + (4*a*b^3*e*x^9)/9
```

3.337 $\int \frac{x^3(c+dx+ex^2)}{a+bx^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 205

$$\int \frac{x^3(c+dx+ex^2)}{a+bx^3} dx = \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}+\frac{\sqrt[3]{a}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}} + \frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}} - \frac{ae\log\left(a+bx^3\right)}{3b^2}$$

```
[Out] c*x/b+1/2*d*x^2/b+1/3*e*x^3/b-1/3*a^(1/3)*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+
b^(1/3)*x)/b^(5/3)+1/6*a^(1/3)*(c-a^(1/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/b^(4/3)-1/3*a*e*ln(b*x^3+a)/b^2+1/3*a^(1/3)*(b^(1/3)*c+
a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)*3^(1/2
)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used

$= \{1901, 1885, 1874, 31, 648, 631, 210, 642, 266\}$

$$\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx = \frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}} + \frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{\sqrt{3}b^{5/3}} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}} - \frac{ae\log\left(a+bx^3\right)}{3b^2} + \frac{cx}{b} + \frac{dx^2}{2b} + \frac{ex^3}{3b}$$

```
[In] Int[(x^3*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (c*x)/b + (d*x^2)/(2*b) + (e*x^3)/(3*b) + (a^(1/3)*(b^(1/3)*c + a^(1/3)*d)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(
1/3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/
3)*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(4/3)) - (a*e*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{b}+\frac{dx}{b}+\frac{ex^2}{b}-\frac{ac+adx+aex^2}{b\,(a+bx^3)}\right)dx \\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\int \frac{ac+adx+aex^2}{a+bx^3}\,dx}{b} \\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\int \frac{ac+adx}{a+bx^3}\,dx}{b}-\frac{(ae)\int \frac{x^2}{a+bx^3}\,dx}{b} \\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{ae\log\left(a+bx^3\right)}{3b^2}-\frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}c+a^{4/3}d\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}c+a^{4/3}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}} \\
&\quad -\frac{\left(\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b}
\end{aligned}$$

$$=\frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}}-\frac{ae\log\left(a+bx^3\right)}{3b^2}-\frac{\left(a^{2/3}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^{4/3}}+\frac{\left(\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{4/3}}$$

$$=\frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{ae\log\left(a+bx^3\right)}{3b^2}-\frac{\left(\sqrt[3]{a}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{5/3}}$$

$$=\frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}+\frac{\sqrt[3]{a}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}-\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{ae\log\left(a+bx^3\right)}{3b^2}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.93

$$\int\frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx$$

$$=\frac{6bcx+3bdx^2+2bex^3+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{b}\left(-\sqrt[3]{a}\sqrt[3]{b}c+a^{2/3}d\right)\log\left(\sqrt[3]{a}+\right.}{6b^2}$$

```
[In] Integrate[(x^3*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (6*b*c*x + 3*b*d*x^2 + 2*b*e*x^3 + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a
^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(-(a^(1/3
```

```
)*b^(1/3)*c) + a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(1/3)*b^(1/
3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*e*Lo
g[a + b*x^3])/(6*b^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.33

method	result
risch	$\frac{e\,x^3}{3b}+\frac{d\,x^2}{2b}+\frac{cx}{b}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-\_R^2e-\_Rd-c\right)\ln\left(x-\_R\right)}{\_R^2}\right)}{3b^2}$
default	$\frac{\frac{1}{3}e\,x^3+\frac{1}{2}d\,x^2+cx}{b}-\frac{\left(\left(c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\right.\right.}{6}\right.\right.\right.}{b}$

```
[In] int(x^3*(e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e*x^3/b+1/2*d*x^2/b+c*x/b+1/3/b^2*a*sum((-_R^2*e-_R*d-c)/_R^2*ln(x-_R),
_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 4798, normalized size of antiderivative = 23.40

$$\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/36*(12*b*e*x^3 + 18*b*d*x^2 - 2*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d
 + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*
b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*
b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)
*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^
3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b^2*log(1/36*((-I*sqrt(3) + 1)*
```

(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4*d + 2*a*b*c*d^2 - a*b*c^2*e + a^2*d*e^2 + 1/6*(b^3*c^2 - 2*a*b^2*d*e)*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2) + (b^2*c^3 + a*b*d^3)*x) + 36*b*c*x + (((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b^2 + 3*sqrt(1/3)*b^2*sqrt(-(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*b*c*d + 36*a^2*e^2)/b^4) - 18*a*e)*log(-1/36*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4*d - 2*a*b*c*d^2 + a*b*c^2*e - a^2*d*e^2 - 1/6*(b^3*c^2 - 2*a*b^2*d*e)*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2) + 2*(b^2*c^3 + a*b*d^3)*x + 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b

^6)^(1/3) + 6*a*e/b^2)*b^4*d - 6*b^3*c^2 - 6*a*b^2*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*b*c*d + 36*a^2*e^2)/b^4)) + (((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b^2 - 3*sqrt(1/3)*b^2*sqrt(-(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*b*c*d + 36*a^2*e^2)/b^4) - 18*a*e)*log(-1/36*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4*d - 2*a*b*c*d^2 + a*b*c^2*e - a^2*d*e^2 - 1/6*(b^3*c^2 - 2*a*b^2*d*e)*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2) + 2*(b^2*c^3 + a*b*d^3)*x - 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d

```
^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b^4*d - 6*b^3*c^2 - 6*a*b^2*d*
e)*sqrt(-(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*
a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6
 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqr
t(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d +
 a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)
^(1/3) + 6*a*e/b^2)^2*b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d +
a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c
*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/
b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/
b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 -
 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*b*c*d + 36*a^2*e^2
)/b^4)))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.87

$$\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx$$
$$= \text{RootSum}\left(27t^3b^6+27t^2ab^4e+t\left(9a^2b^2e^2+9ab^3cd\right)+a^3e^3+3a^2bcde-a^2bd^3+ab^2c^3,\left(t\mapsto t\log\left(x+\frac{9t\right.\right.\right.$$
$$+\frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}$$

```
[In] integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**6 + 27*_t**2*a*b**4*e + _t*(9*a**2*b**2*e**2 + 9*a*b**3
*c*d) + a**3*e**3 + 3*a**2*b*c*d*e - a**2*b*d**3 + a*b**2*c**3, Lambda(_t,
_t*log(x + (9*_t**2*b**4*d + 6*_t*a*b**2*d*e - 3*_t*b**3*c**2 + a**2*d*e**2
 - a*b*c**2*e + 2*a*b*c*d**2)/(a*b*d**3 + b**2*c**3)))) + c*x/b + d*x**2/(2
*b) + e*x**3/(3*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.93

$$\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx = \frac{2\,ex^3+3\,dx^2+6\,cx}{6\,b} - \frac{\sqrt{3}\left(abd\left(\frac{a}{b}\right)^{\frac{2}{3}}+abc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} - \frac{\left(2\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+ad\left(\frac{a}{b}\right)^{\frac{1}{3}}-ac\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(ae\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}+ac\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*e*x^3 + 3*d*x^2 + 6*c*x)/b - 1/3*sqrt(3)*(a*b*d*(a/b)^(2/3) + a*b*c*
(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2) -
1/6*(2*a*e*(a/b)^(2/3) + a*d*(a/b)^(1/3) - a*c)*log(x^2 - x*(a/b)^(1/3) + (
a/b)^(2/3))/(b^2*(a/b)^(2/3)) - 1/3*(a*e*(a/b)^(2/3) - a*d*(a/b)^(1/3) + a*
c)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00

$$\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx = -\frac{ae\log\left(|bx^3+a|\right)}{3\,b^2} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc-\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bc+\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3} + \frac{2\,b^2ex^3+3\,b^2dx^2+6\,b^2cx}{6\,b^3} + \frac{\left(ab^6d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^6c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^7}$$

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*a*e*log(abs(b*x^3 + a))/b^2 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b*c - (-a*b^
2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 - 1/6
*((-a*b^2)^(1/3)*b*c + (-a*b^2)^(2/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^
(2/3))/b^3 + 1/6*(2*b^2*e*x^3 + 3*b^2*d*x^2 + 6*b^2*c*x)/b^3 + 1/3*(a*b^6*d
*(-a/b)^(1/3) + a*b^6*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7)
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.56

$$\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\mathrm{root}\left(27\,b^6\,z^3+27\,a\,b^4\,e\,z^2+9\,a\,b^3\,c\,d\,z+9\,a^2\,b^2\,e^2\,z+3\,a^2\,b\,c\,d\,e+a\,b^2\,c^3+a^3\,e^3-a^2\,b\,d^3,z,k\right)\,\left(6\,a^2\,e\right.\right.\right.$$
$$\left. + \frac{a^3\,e^2+b\,c\,d\,a^2}{b^2} + \frac{x\,(a^2\,d^2-a^2\,c\,e)}{b}\right)\,\mathrm{root}\big(27\,b^6\,z^3+27\,a\,b^4\,e\,z^2+9\,a\,b^3\,c\,d\,z$$
$$\left.\left. +\,9\,a^2\,b^2\,e^2\,z+3\,a^2\,b\,c\,d\,e+a\,b^2\,c^3+a^3\,e^3-a^2\,b\,d^3,z,k\big)\right)\right) + \frac{d\,x^2}{2\,b} + \frac{e\,x^3}{3\,b} + \frac{c\,x}{b}$$

```
[In] int((x^3*(c + d*x + e*x^2))/(a + b*x^3),x)
```

```
[Out] symsum(log(root(27*b^6*z^3 + 27*a*b^4*e*z^2 + 9*a*b^3*c*d*z + 9*a^2*b^2*e^2
*z + 3*a^2*b*c*d*e + a*b^2*c^3 + a^3*e^3 - a^2*b*d^3, z, k)*(6*a^2*e + 9*ro
ot(27*b^6*z^3 + 27*a*b^4*e*z^2 + 9*a*b^3*c*d*z + 9*a^2*b^2*e^2*z + 3*a^2*b*
c*d*e + a*b^2*c^3 + a^3*e^3 - a^2*b*d^3, z, k)*a*b^2 - 3*a*b*c*x) + (a^3*e^
2 + a^2*b*c*d)/b^2 + (x*(a^2*d^2 - a^2*c*e))/b)*root(27*b^6*z^3 + 27*a*b^4*
e*z^2 + 9*a*b^3*c*d*z + 9*a^2*b^2*e^2*z + 3*a^2*b*c*d*e + a*b^2*c^3 + a^3*e
^3 - a^2*b*d^3, z, k), k, 1, 3) + (d*x^2)/(2*b) + (e*x^3)/(3*b) + (c*x)/b
```

3.338 $\int \frac{x^2(c+dx+ex^2)}{a+bx^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 193

$$\int \frac{x^2(c+dx+ex^2)}{a+bx^3} \, dx = \frac{dx}{b} + \frac{ex^2}{2b} + \frac{\sqrt[3]{a}\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}} + \frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}} + \frac{c\log\left(a+bx^3\right)}{3b}$$

```
[Out] d*x/b+1/2*e*x^2/b-1/3*a^(1/3)*(b^(1/3)*d-a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/b
^(5/3)+1/6*a^(1/3)*(d-a^(1/3)*e/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/b^(4/3)+1/3*c*ln(b*x^3+a)/b+1/3*a^(1/3)*(b^(1/3)*d+a^(1/3)*e)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used

$= \{1901, 1885, 1874, 31, 648, 631, 210, 642, 266\}$

$$\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx = \frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}} + \frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)}{\sqrt{3}b^{5/3}} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}} + \frac{c\log\left(a+bx^3\right)}{3b} + \frac{dx}{b} + \frac{ex^2}{2b}$$

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (d*x)/b + (e*x^2)/(2*b) + (a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(1/3)*(b^(1/3)*d
- a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/3)*(d - (a^(1/3)
*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)) +
(c*Log[a + b*x^3])/(3*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d}{b}+\frac{ex}{b}-\frac{ad+aex-bcx^2}{b\left(a+bx^3\right)}\right)dx\\
&= \frac{dx}{b}+\frac{ex^2}{2b}-\frac{\int \frac{ad+aex-bcx^2}{a+bx^3}\,dx}{b}\\
&= \frac{dx}{b}+\frac{ex^2}{2b}-\frac{\int \frac{ad+aex}{a+bx^3}\,dx}{b}+c\int \frac{x^2}{a+bx^3}\,dx\\
&= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{c\log\left(a+bx^3\right)}{3b}-\frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}d+a^{4/3}e\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}d+a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}}\\
&\quad -\frac{\left(\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b}
\end{aligned}
$$

$$= \frac{dx}{b} + \frac{ex^2}{2b} - \frac{\sqrt[3]{a}\left(d - \frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3b^{4/3}} + \frac{c\log(a + bx^3)}{3b} - \frac{\left(a^{2/3}\left(\sqrt[3]{b}d + \sqrt[3]{a}e\right)\right)\int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{2b^{4/3}} + \frac{\left(\sqrt[3]{a}\left(d - \frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{6b^{4/3}}$$

$$= \frac{dx}{b} + \frac{ex^2}{2b} - \frac{\sqrt[3]{a}\left(d - \frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3b^{4/3}} + \frac{\sqrt[3]{a}\left(d - \frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6b^{4/3}} + \frac{c\log(a + bx^3)}{3b} - \frac{\left(\sqrt[3]{a}\left(\sqrt[3]{b}d + \sqrt[3]{a}e\right)\right)\text{Subst}\left(\int \frac{1}{-3 - x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{5/3}}$$

$$= \frac{dx}{b} + \frac{ex^2}{2b} + \frac{\sqrt[3]{a}\left(\sqrt[3]{b}d + \sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}} - \frac{\sqrt[3]{a}\left(d - \frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3b^{4/3}} + \frac{\sqrt[3]{a}\left(d - \frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6b^{4/3}} + \frac{c\log(a + bx^3)}{3b}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.95

$$\int \frac{x^2(c + dx + ex^2)}{a + bx^3}\,dx = \frac{6b^{2/3}dx + 3b^{2/3}ex^2 + 2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{b}d + \sqrt[3]{a}e\right)\arctan\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\left(-\sqrt[3]{a}\sqrt[3]{b}d + a^{2/3}e\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right) -}{6b^{5/3}}$$

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (6*b^(2/3)*d*x + 3*b^(2/3)*e*x^2 + 2*Sqrt[3]*a^(1/3)*(b^(1/3)*d + a^(1/3)*e
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-(a^(1/3)*b^(1/3)*d) + a
```

```
^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x] - (-(a^(1/3)*b^(1/3)*d) + a^(2/3)*e)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c*Log[a + b*x^3])/(
6*b^(5/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.31

method	result
risch	$\frac{e\,x^2}{2b}+\frac{dx}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_R^2bc-\_Rae-ad\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^2}$
default	$\frac{\frac{1}{2}e\,x^2+dx}{b}+\frac{-ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-ae\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x\right.}{6b\left(\frac{a}{b}\right)}\right.}{b}$

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e*x^2/b+d*x/b+1/3/b^2*sum((_R^2*b*c-_R*a*e-a*d)/_R^2*ln(x-_R),_R=RootOf
(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 4261, normalized size of antiderivative = 22.08

$$\int\frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx=\text{Too large to display}$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*e*x^2 - 2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e
)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*
c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) +
1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3
+ a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b*log(1/4*(2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 +
```

a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e
)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d
*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*
b)/b^5)^(1/3) - 2*c/b)^2*b^3*e + b*c*d^2 + b*c^2*e + 2*a*d*e^2 + 1/2*(b^2*d
^2 + 2*b^2*c*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/
b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^
3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)
*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 +
a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b) + (b*d^3 + a*e^3)*x) + 1
2*d*x + ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2
c^3/b^3 - 3(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3
/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3
 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b + 3*sqrt(1/3)*b*sqrt(-((2*(1/
2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b
*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2
 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d
*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2
/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 +
 a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2
)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e
^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b
^2*c + 4*b*c^2 + 16*a*d*e)/b^3) + 6*c)*log(-1/4*(2*(1/2)^(2/3)*(-I*sqrt(3)
+ 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 +
 (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/
3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*
d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) -
 2*c/b)^2*b^3*e - b*c*d^2 - b*c^2*e - 2*a*d*e^2 - 1/2*(b^2*d^2 + 2*b^2*c*e)
(2(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3
 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (
d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3
*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3
- 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b) + 2*(b*d^3 + a*e^3)*x + 3/4*sqrt(1/3)*(
(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3
- 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d
^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*
(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 -
 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b^3*e - 2*b^2*d^2 + 2*b^2*c*e)*sqrt(-((2
(1/2)^(2/3)(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 -
3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3
 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b
*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*
(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d

^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) +
(1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 +
a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/
b)*b^2*c + 4*b*c^2 + 16*a*d*e)/b^3)) + ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^
2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3
+ a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/
2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*
e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*
b - 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2
+ a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5
+ (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqr
t(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b
^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(2*(1
/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(
b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^
2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*
d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b^2*c + 4*b*c^2 + 16*a*d*e)/b^3) + 6*c)*log(-
1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/
b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3
- 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d
^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3*e - b*c*d^2 - b*c^2*e - 2*a*d*
e^2 - 1/2*(b^2*d^2 + 2*b^2*c*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 -
(b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)
*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)
*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b
^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b) + 2*(b*d
^3 + a*e^3)*x - 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (
b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*
a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*
(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^
5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b^3*e - 2
*b^2*d^2 + 2*b^2*c*e)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*
c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/
b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I
sqrt(3) + 1)(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5
+ (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(
2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 -
3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^
3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(
b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b^2*c + 4*b*c^2 + 16*a*d*e)/b^3)))/b

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.78

$$\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx$$
$$= \text{RootSum}\left(27t^3b^5-27t^2b^4c+t\left(9ab^2de+9b^3c^2\right)-a^2e^3-3abcde+abd^3-b^2c^3,\left(t\mapsto t\log\left(x+\frac{9t^2b^3e-\right.\right.\right.$$
$$+\frac{dx}{b}+\frac{ex^2}{2b}$$

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c + _t*(9*a*b**2*d*e + 9*b**3*c**2) -
 a**2*e**3 - 3*a*b*c*d*e + a*b*d**3 - b**2*c**3, Lambda(_t, _t*log(x + (9*_
t**2*b**3*e - 6*_t*b**2*c*e - 3*_t*b**2*d**2 + 2*a*d*e**2 + b*c**2*e + b*c*
d**2)/(a*e**3 + b*d**3)))) + d*x/b + e*x**2/(2*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx = -\frac{\sqrt{3}\left(ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab}+\frac{ex^2+2dx}{2b}$$
$$+\frac{\left(2bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(a*e*(a/b)^(2/3) + a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/2*(e*x^2 + 2*d*x)/b + 1/6*(2*b*c*(a/b)^
(2/3) - a*e*(a/b)^(1/3) + a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*
(a/b)^(2/3)) + 1/3*(b*c*(a/b)^(2/3) + a*e*(a/b)^(1/3) - a*d)*log(x + (a/b)^
(1/3))/(b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.99

$$\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx = \frac{c\log\left(|bx^3+a|\right)}{3\,b} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bd - \left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3} + \frac{bex^2+2\,bdx}{2\,b^2} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bd + \left(-ab^2\right)^{\frac{2}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3} + \frac{\left(ab^4e\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^4d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^5}$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*c*log(abs(b*x^3 + a))/b - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b*d - (-a*b^2)^(2
/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/2*(b*e
*x^2 + 2*b*d*x)/b^2 - 1/6*((-a*b^2)^(1/3)*b*d + (-a*b^2)^(2/3)*e)*log(x^2 +
 x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 + 1/3*(a*b^4*e*(-a/b)^(1/3) + a*b^4*d)*
(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5)
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.76

$$\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(\frac{a\left(b\,c^2+\mathrm{root}(27\,b^5\,z^3-27\,b^4\,c\,z^2+9\,a\,b^2\,d\,e\,z+9\,b^3\,c^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,\right.}{}\right.\right.$$
$$\left.\left.-27\,b^4\,c\,z^2+9\,a\,b^2\,d\,e\,z+9\,b^3\,c^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k\right)\right)+\frac{e\,x^2}{2\,b}+\frac{d\,x}{b}$$

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3),x)
```

```
[Out] symsum(log((a*(b*c^2 + 9*root(27*b^5*z^3 - 27*b^4*c*z^2 + 9*a*b^2*d*e*z + 9
*b^3*c^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)^2*b^3 + a*d*e
```

```
 - 6*root(27*b^5*z^3 - 27*b^4*c*z^2 + 9*a*b^2*d*e*z + 9*b^3*c^2*z - 3*a*b*c
*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*b^2*c + a*e^2*x + b*c*d*x - 3*roo
t(27*b^5*z^3 - 27*b^4*c*z^2 + 9*a*b^2*d*e*z + 9*b^3*c^2*z - 3*a*b*c*d*e + a
*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*b^2*d*x))/b)*root(27*b^5*z^3 - 27*b^4*c*z
^2 + 9*a*b^2*d*e*z + 9*b^3*c^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^
3, z, k), k, 1, 3) + (e*x^2)/(2*b) + (d*x)/b
```

3.339 $\int \frac{x(c+dx+ex^2)}{a+bx^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 183

$$\int \frac{x(c+dx+ex^2)}{a+bx^3} dx = \frac{ex}{b} - \frac{(b^{2/3}c-a^{2/3}e)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}} - \frac{(b^{2/3}c+a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}} + \frac{(b^{2/3}c+a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{4/3}} + \frac{d\log(a+bx^3)}{3b}$$

```
[Out] e*x/b-1/3*(b^(2/3)*c+a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(4/3)+1/6*(
b^(2/3)*c+a^(2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(4
/3)+1/3*d*ln(b*x^3+a)/b-1/3*(b^(2/3)*c-a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used

$= \{1901, 1885, 1874, 31, 648, 631, 210, 642, 266\}$

$$\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(b^{2/3}c-a^{2/3}e\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}} + \frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{4/3}} - \frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}} + \frac{d\log\left(a+bx^3\right)}{3b} + \frac{ex}{b}$$

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (e*x)/b - ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(4/3)) - ((b^(2/3)*c + a^(2/3)*e)*Log[a^(1/3)
 + b^(1/3)*x])/(3*a^(1/3)*b^(4/3)) + ((b^(2/3)*c + a^(2/3)*e)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(4/3)) + (d*Log[a + b*x^3])
/(3*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e}{b} - \frac{ae - bcx - bdx^2}{b\left(a + bx^3\right)}\right) dx \\
&= \frac{ex}{b} - \frac{\int \frac{ae-bcx-bdx^2}{a+bx^3}\,dx}{b} \\
&= \frac{ex}{b} - \frac{\int \frac{ae-bcx}{a+bx^3}\,dx}{b} + d\int \frac{x^2}{a+bx^3}\,dx \\
&= \frac{ex}{b} + \frac{d\log\left(a+bx^3\right)}{3b} - \frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}bc+2a\sqrt[3]{b}e\right)+\sqrt[3]{b}\left(-\sqrt[3]{a}bc-a\sqrt[3]{b}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}} \\
&\quad - \frac{\left(b^{2/3}c+a^{2/3}e\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}b}
\end{aligned}$$

$$= \frac{ex}{b} - \frac{\left(b^{2/3}c + a^{2/3}e\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}} + \frac{d\log\left(a + bx^3\right)}{3b} + \frac{\left(b^{2/3}c - a^{2/3}e\right)\int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{2b} + \frac{\left(b^{2/3}c + a^{2/3}e\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{6\sqrt[3]{a}b^{4/3}}$$

$$= \frac{ex}{b} - \frac{\left(b^{2/3}c + a^{2/3}e\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}} + \frac{\left(b^{2/3}c + a^{2/3}e\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{4/3}} + \frac{d\log\left(a + bx^3\right)}{3b} + \frac{\left(b^{2/3}c - a^{2/3}e\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{4/3}}$$

$$= \frac{ex}{b} - \frac{\left(b^{2/3}c - a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}} - \frac{\left(b^{2/3}c + a^{2/3}e\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}} + \frac{\left(b^{2/3}c + a^{2/3}e\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{4/3}} + \frac{d\log\left(a + bx^3\right)}{3b}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.09

$$\int \frac{x(c + dx + ex^2)}{a + bx^3}\,dx = \frac{ex}{b} + \frac{\left(a^{2/3}bc - a^{4/3}\sqrt[3]{b}e\right)\arctan\left(\frac{-\sqrt[3]{a} + 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}ab^{5/3}} + \frac{\left(-a^{2/3}bc - a^{4/3}\sqrt[3]{b}e\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3ab^{5/3}} - \frac{\left(-a^{2/3}bc - a^{4/3}\sqrt[3]{b}e\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6ab^{5/3}} + \frac{d\log\left(a + bx^3\right)}{3b}$$

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (e*x)/b + ((a^(2/3)*b*c - a^(4/3)*b^(1/3)*e)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a*b^(5/3)) + ((-(a^(2/3)*b*c) - a^(4/3)*b^(1
```

```
/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a*b^(5/3)) - ((-(a^(2/3)*b*c) - a^(4/3)
*b^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a*b^(5/3)) +
 (d*Log[a + b*x^3])/(3*b)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.27

method	result
risch	$\frac{ex}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(\_R^{2}bd+\_Rbc-ae\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3b^{2}}$
default	$\frac{ex}{b}+\frac{-ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+bc\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{b}$

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] e*x/b+1/3/b^2*sum((_R^2*b*d+_R*b*c-a*e)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 4628, normalized size of antiderivative = 25.29

$$\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
[Out] -1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3
/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b
^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d
^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a
*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*log(-1/4*(2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*
d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*
```

e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e
)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^
2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*a*b^3*c - a*b*c*d^2 + 2*a*b*c^2*e + a^2*d*
e^2 - 1/2*(2*a*b^2*c*d - a^2*b*e^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^
2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)
^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^
3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d
/b) - (b^2*c^3 - a^2*e^3)*x) - 12*e*x - ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d
^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) +
(1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a
^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3)
- 2*d/b)*b - 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2
- (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(
1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b
)^2*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*
d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(
a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(
2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)
/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*d + 4*d^2 - 16*c*e
)/b^2) + 6*d)*log(1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e
)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d
*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt
(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*a*b^3*c
+ a*b*c*d^2 - 2*a*b*c^2*e - a^2*d*e^2 + 1/2*(2*a*b^2*c*d - a^2*b*e^2)*(2*(1
/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2
- c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3
 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^
2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c
^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b) - 2*(b^2*c^3 - a^2*e^3)*x + 3/4*sqrt(
1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^
3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4)
 - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/
b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^
4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*a*b^3*c + 2*a*b^2*c*d + 2*
a^2*b*e^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^
2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*
a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e
)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*b^2 + 4*(2*(
1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2

- c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^
3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d
^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*
c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*d + 4*d^2 - 16*c*e)/b^2)) - ((2*(1
/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2
- c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3
 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^
2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c
^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b + 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*
d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*
e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e
)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^
2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d
^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) +
(1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a
^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3)
- 2*d/b)*b*d + 4*d^2 - 16*c*e)/b^2) + 6*d)*log(1/4*(2*(1/2)^(2/3)*(-I*sqrt(
3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2
*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4)
)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b
^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^
4))^(1/3) - 2*d/b)^2*a*b^3*c + a*b*c*d^2 - 2*a*b*c^2*e - a^2*d*e^2 + 1/2*(2
*a*b^2*c*d - a^2*b*e^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c
*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sq
rt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b) - 2*(b^2
*c^3 - a^2*e^3)*x - 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2
 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 -
 (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^
(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3
 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/
b)*a*b^3*c + 2*a*b^2*c*d + 2*a^2*b*e^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) +
 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3
 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1
/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c
^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^
(1/3) - 2*d/b)^2*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 -
c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*
c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*d + 4
*d^2 - 16*c*e)/b^2)))/b

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.87

$$\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx$$
$$=\text{RootSum}\left(27t^3ab^4-27t^2ab^3d+t\left(-9ab^2ce+9ab^2d^2\right)+a^2e^3+3abcde-abd^3+b^2c^3,\left(t\mapsto t\log\left(x+\frac{-9\right.\right.\right.$$
$$+\frac{ex}{b}$$

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a*b**4 - 27*_t**2*a*b**3*d + _t*(-9*a*b**2*c*e + 9*a*b**2*
d**2) + a**2*e**3 + 3*a*b*c*d*e - a*b*d**3 + b**2*c**3, Lambda(_t, _t*log(x
 + (-9*_t**2*a*b**3*c - 3*_t*a**2*b*e**2 + 6*_t*a*b**2*c*d + a**2*d*e**2 +
2*a*b*c**2*e - a*b*c*d**2)/(a**2*e**3 - b**2*c**3)))) + e*x/b
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.95

$$\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx=\frac{ex}{b}+\frac{\sqrt{3}\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ae\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{\left(2\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] e*x/b + 1/3*sqrt(3)*(b*c*(a/b)^(2/3) - a*e*(a/b)^(1/3))*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*b*d*(a/b)^(2/3) + b*c*(a/b)
^(1/3) + a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/
3*(b*d*(a/b)^(2/3) - b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3))/(b^2*(a/b)
^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.95

$$\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx = \frac{\sqrt{3}\left(ae+(-ab^2)^{\frac{1}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}} + \frac{\left(ae-(-ab^2)^{\frac{1}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}} + \frac{ex}{b} + \frac{d\log\left(|bx^3+a|\right)}{3\,b} - \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^3}$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(a*e + (-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/(-a*b^2)^(2/3) + 1/6*(a*e - (-a*b^2)^(1/3)*c)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) + e*x/b + 1/3*d*log(abs(b*x^3 +
a))/b - 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^
(1/3)))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.45

$$\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx = \left(\sum_{k=1}^{3}\ln\left(x\left(b\,c^2+a\,d\,e\right) - \mathrm{root}\left(27\,a\,b^4\,z^3-27\,a\,b^3\,d\,z^2-9\,a\,b^2\,c\,e\,z+9\,a\,b^2\,d^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)\left(6\,a\,b\,d-\mathrm{root}(\right.\right.\right. + a\,d^2-a\,c\,e\Big)\,\mathrm{root}\left(27\,a\,b^4\,z^3-27\,a\,b^3\,d\,z^2-9\,a\,b^2\,c\,e\,z+9\,a\,b^2\,d^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3 + a^2\,e^3+b^2\,c^3,z,k\right)\Bigg) + \frac{e\,x}{b}$$

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3),x)
```

```
[Out] symsum(log(x*(b*c^2 + a*d*e) - root(27*a*b^4*z^3 - 27*a*b^3*d*z^2 - 9*a*b^2
*c*e*z + 9*a*b^2*d^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*(
6*a*b*d - 9*root(27*a*b^4*z^3 - 27*a*b^3*d*z^2 - 9*a*b^2*c*e*z + 9*a*b^2*d^
2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a*b^2 + 3*a*b*e*x) +
 a*d^2 - a*c*e)*root(27*a*b^4*z^3 - 27*a*b^3*d*z^2 - 9*a*b^2*c*e*z + 9*a*b^
2*d^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k), k, 1, 3) + (e*x
)/b
```

3.340 $\int \frac{c+dx+ex^2}{a+bx^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 177

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx = -\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}} - \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{e\log\left(a+bx^3\right)}{3b}$$

```
[Out] 1/3*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(2/3)-1/6*(c-a^(1
/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)+1/
3*e*ln(b*x^3+a)/b-1/3*(b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x
)/a^(1/3)*3^(1/2))/a^(2/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

= {1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{\sqrt{3}a^{2/3}b^{2/3}} - \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}} + \frac{e\log\left(a+bx^3\right)}{3b}$$

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3),x]
```

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)) + (e*Log[a + b*x^3])/(3*b
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= e\int \frac{x^2}{a+bx^3}\,dx+\int \frac{c+dx}{a+bx^3}\,dx\\
&= \frac{e\log\left(a+bx^3\right)}{3b}+\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}c+\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}}\\
&= \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{e\log\left(a+bx^3\right)}{3b}\\
&\quad-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{2/3}}\\
&\quad+\frac{1}{2}\left(\frac{c}{\sqrt[3]{a}}+\frac{d}{\sqrt[3]{b}}\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx
\end{aligned}$$

$$= \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}} + \frac{e\log\left(a+bx^3\right)}{3b} + \frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}b^{2/3}}$$

$$= -\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}} + \frac{e\log\left(a+bx^3\right)}{3b}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.99

$$\int\frac{c+dx+ex^2}{a+bx^3}\,dx$$

$$= \frac{-2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}c\right.}{6ab}$$

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(1/3
) + b^(1/3)*x] - b^(1/3)*(a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*e*Log[a + b*x^3])/(6*a*b)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.21

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{\left(\_R^2 e+\_R d+c\right)\ln\left(x-\_R\right)}{\_R^2}}{3b}$
default	$c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.$

```
[In] int((e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3/b*sum((_R^2*e+_R*d+c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 4671, normalized size of antiderivative = 26.39

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
[Out] -1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)
)/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b
^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2
*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)
*b*log(1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^
2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) +
(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I
*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a
^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/
b)^2*a^2*b^2*d + 2*a*b*c*d^2 - a*b*c^2*e + a^2*d*e^2 - 1/2*(a*b^2*c^2 - 2*a
^2*b*d*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2
))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (
b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*
sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^
2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b
) + (b^2*c^3 + a*b*d^3)*x) - ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b
```

*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 +
a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/
3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) +
(b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2
*b^3))^(1/3) - 2*e/b)*b + 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) +
1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a
*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*
b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a
*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)^2*a*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(
3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*
e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e
)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d
+ a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3
- 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a*b*e + 16*b*c*d + 4*a*e^2)/(a*b
^2)) + 6*e)*log(-1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*
e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a
^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2
)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 +
a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1
/3) - 2*e/b)^2*a^2*b^2*d - 2*a*b*c*d^2 + a*b*c^2*e - a^2*d*e^2 + 1/2*(a*b^2
*c^2 - 2*a^2*b*d*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e
^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^
2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)
^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 +
a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/
3) - 2*e/b) + 2*(b^2*c^3 + a*b*d^3)*x + 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*s
qrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e
^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b
*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a^2*b^2*d + 2*a*b^2*c^2 + 2*
a^2*b*d*e)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2
)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*
b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(
1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*
d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3)
- 2*e/b)^2*a*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a
*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(
a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/
2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3
+ a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(
1/3) - 2*e/b)*a*b*e + 16*b*c*d + 4*a*e^2)/(a*b^2))) - ((2*(1/2)^(2/3)*(-I*s
qrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e
^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c

```
*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b
*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*b - 3*sqrt(1/3)*b*sqrt(-((2*
(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3
 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*
(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2
*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)^2*a*b^2 + 4
*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3
/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 +
 a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) +
 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) +
(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a*b*e +
 16*b*c*d + 4*a*e^2)/(a*b^2)) + 6*e)*log(-1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) +
1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*
b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b
)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*
e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*
c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)^2*a^2*b^2*d - 2*a*b*c*d^2 + a*b*c^2*e
 - a^2*d*e^2 + 1/2*(a*b^2*c^2 - 2*a^2*b*d*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1
)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b
^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)
/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e
^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b) + 2*(b^2*c^3 + a*b*d^3)*x - 3/4*sqrt(1
/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2
*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c
^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(
3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2
) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a^2
*b^2*d + 2*a*b^2*c^2 + 2*a^2*b*d*e)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*
(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3
) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(
a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2
)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d
*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)^2*a*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) +
 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a
*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*
b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a
*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a*b*e + 16*b*c*d + 4*a*e^2)/(a*b^2))
))/b
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.90

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx$$
$$= \text{RootSum}\left(27t^3a^2b^3 - 27t^2a^2b^2e + t\left(9a^2be^2 + 9ab^2cd\right) - a^2e^3 - 3abcde + abd^3 - b^2c^3, \left(t \mapsto t\log\left(x + \frac{9}{}\right.\right.\right.$$

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**3 - 27*_t**2*a**2*b**2*e + _t*(9*a**2*b*e**2 + 9*a
*b**2*c*d) - a**2*e**3 - 3*a*b*c*d*e + a*b*d**3 - b**2*c**3, Lambda(_t, _t*
log(x + (9*_t**2*a**2*b**2*d - 6*_t*a**2*b*d*e + 3*_t*a*b**2*c**2 + a**2*d*
e**2 - a*b*c**2*e + 2*a*b*c*d**2)/(a*b*d**3 + b**2*c**3))))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx = \frac{\sqrt{3}\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}} + bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab} + \frac{\left(2e\left(\frac{a}{b}\right)^{\frac{2}{3}} + d\left(\frac{a}{b}\right)^{\frac{1}{3}} - c\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(e\left(\frac{a}{b}\right)^{\frac{2}{3}} - d\left(\frac{a}{b}\right)^{\frac{1}{3}} + c\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b*d*(a/b)^(2/3) + b*c*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*e*(a/b)^(2/3) + d*(a/b)^(1/3) - c)*
log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(e*(a/b)^(2/3)
 - d*(a/b)^(1/3) + c)*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx = -\frac{\sqrt{3}\left(bc-(-ab^2)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(bc+(-ab^2)^{\frac{1}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}} + \frac{e\log\left(|bx^3+a|\right)}{3\,b} - \frac{\left(bd\left(-\frac{a}{b}\right)^{\frac{1}{3}}+bc\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*c + (-a*b^2)^(1/3)*d)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) + 1/3*e*log(abs(b*x^3 + a))/b -
 1/3*(b*d*(-a/b)^(1/3) + b*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.55

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx = \sum_{k=1}^{3} \ln\left(x\left(b\,d^2-b\,c\,e\right)+\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,a^2\,b^2\,e\,z^2+9\,a\,b^2\,c\,d\,z+9\,a^2\,b\,e^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k\right)\left(-6\,a\,b\,e+\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,a^2\,b^2\,e\,z^2+9\,a\,b^2\,c\,d\,z+9\,a^2\,b\,e^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k\right)\,a\,b^2\,9+3\,b^2\,c\,x\right)+a\,e^2+b\,c\,d\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,a^2\,b^2\,e\,z^2+9\,a\,b^2\,c\,d\,z+9\,a^2\,b\,e^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k\right)$$

```
[In] int((c + d*x + e*x^2)/(a + b*x^3),x)
```

```
[Out] symsum(log(x*(b*d^2 - b*c*e) + root(27*a^2*b^3*z^3 - 27*a^2*b^2*e*z^2 + 9*a
*b^2*c*d*z + 9*a^2*b*e^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z,
k)*(9*root(27*a^2*b^3*z^3 - 27*a^2*b^2*e*z^2 + 9*a*b^2*c*d*z + 9*a^2*b*e^2*
z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a*b^2 - 6*a*b*e + 3*b^
2*c*x) + a*e^2 + b*c*d)*root(27*a^2*b^3*z^3 - 27*a^2*b^2*e*z^2 + 9*a*b^2*c*
d*z + 9*a^2*b*e^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k), k,
1, 3)
```

3.341 $\int \frac{c+dx+ex^2}{x(a+bx^3)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 184

$$\int \frac{c+dx+ex^2}{x(a+bx^3)}\,dx = -\frac{\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{c\log(x)}{a} + \frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}} - \frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} - \frac{c\log(a+bx^3)}{3a}$$

```
[Out] c*ln(x)/a+1/3*(b^(1/3)*d-a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(2/3)-1
/6*(d-a^(1/3)*e/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/
b^(1/3)-1/3*c*ln(b*x^3+a)/a-1/3*(b^(1/3)*d+a^(1/3)*e)*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used

$= \{1848, 1885, 1874, 31, 648, 631, 210, 642, 266\}$

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)}{\sqrt{3}a^{2/3}b^{2/3}} - \frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}} - \frac{c\log\left(a+bx^3\right)}{3a} + \frac{c\log(x)}{a}$$

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)),x]
```

```
[Out] -(((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + (c*Log[x])/a + ((b^(1/3)*d - a^(1/3)*e)*Log
[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((d - (a^(1/3)*e)/b^(1/3))*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)) - (c*Log[a
 + b*x^3])/(3*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{c}{ax} + \frac{ad + aex - bcx^2}{a\left(a + bx^3\right)}\right) dx$$

$$= \frac{c\log(x)}{a} + \frac{\int \frac{ad+aex-bcx^2}{a+bx^3}\, dx}{a}$$

$$= \frac{c\log(x)}{a} + \frac{\int \frac{ad+aex}{a+bx^3}\, dx}{a} - \frac{(bc)\int \frac{x^2}{a+bx^3}\, dx}{a}$$

$$= \frac{c\log(x)}{a} - \frac{c\log\left(a+bx^3\right)}{3a} + \frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}d+a^{4/3}e\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}d+a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\, dx}{3a^{5/3}\sqrt[3]{b}} + \frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\, dx}{3a^{2/3}}$$

$$= \frac{c\log(x)}{a} + \frac{\left(d - \frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{c\log(a+bx^3)}{3a} + \frac{1}{2}\left(\frac{d}{\sqrt[3]{a}} + \frac{e}{\sqrt[3]{b}}\right)\int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx - \frac{\left(d - \frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}\sqrt[3]{b}}$$

$$= \frac{c\log(x)}{a} + \frac{\left(d - \frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(d - \frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} - \frac{c\log(a+bx^3)}{3a} + \frac{\left(\sqrt[3]{b}d + \sqrt[3]{a}e\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}b^{2/3}}$$

$$= -\frac{\left(\sqrt[3]{b}d + \sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{c\log(x)}{a} + \frac{\left(d - \frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(d - \frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} - \frac{c\log(a+bx^3)}{3a}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x(a+bx^3)}\,dx$$

$$= \frac{-2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{b}d + \sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 6b^{2/3}c\log(x) + 2\left(\sqrt[3]{a}\sqrt[3]{b}d - a^{2/3}e\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right) + \left(-\sqrt[3]{}\right.}{6ab^{2/3}}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)),x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] + 6*b^(2/3)*c*Log[x] + 2*(a^(1/3)*b^(1/3)*d - a^(2/3)*e)*Log[a
^(1/3) + b^(1/3)*x] + (-(a^(1/3)*b^(1/3)*d) + a^(2/3)*e)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*b^(2/3)*c*Log[a + b*x^3])/(6*a*b^(2/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.03

method	result
risch	$\frac{c\ln(-x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{3}b^{2}\_Z^{3}+3a^{2}b^{2}c\_Z^{2}+\left(3a^{2}bde+3b^{2}c^{2}a\right)\_Z+a^{2}e^{3}+3abcde-ab\,d^{3}+b^{2}c^{3}\right)}\_R\ln\left(\left(-4\_R^{3}a^{2}b^{2}-8\_R^{2}a\,b \cdots\right.\right.\right.}{3}$
default	$\frac{c\ln(x)}{a}+\frac{ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+ae\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right.\cdots\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\cdots\right.}{a}$

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] c/a*ln(-x)+1/3*sum(_R*ln((-4*_R^3*a^2*b^2-8*_R^2*a*b^2*c+(-10*a*b*d*e-4*b^2
*c^2)*_R-3*a*e^3-6*b*c*d*e+3*b*d^3)*x+a^2*b*e*_R^2+(-2*a*b*c*e-a*b*d^2)*_R-
3*b*c^2*e+3*b*c*d^2),_R=RootOf(a^3*b^2*_Z^3+3*a^2*b^2*c*_Z^2+(3*a^2*b*d*e+3
*a*b^2*c^2)*_Z+a^2*e^3+3*a*b*c*d*e-a*b*d^3+b^2*c^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 4588, normalized size of antiderivative = 24.93

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a
^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54
*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3)
+ 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)
/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3
) + 6*c/a)*a*log(1/36*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))
/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^
2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) +
9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b
```

*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a
^3*b^2))^(1/3) + 6*c/a)^2*a^2*b*e + b*c*d^2 + b*c^2*e + 2*a*d*e^2 - 1/6*(a*
b*d^2 + 2*a*b*c*e)*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-
1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b
^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(
I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^
3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*
b^2))^(1/3) + 6*c/a) + (b*d^3 + a*e^3)*x) - (((-I*sqrt(3) + 1)*(c^2/a^2 - (
b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1
/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a
*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d
*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a + 3*sqrt(1/3)*a*sqrt(-(((-
I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b
*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 +
a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*
c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) -
 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)^2
*a^2*b - 12*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^
3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1
/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(
3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e
^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(
1/3) + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b)) - 18*c)*log(-1/36*((-I
sqrt(3) + 1)(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*
c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a
^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c
^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) -
1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)^2*
a^2*b*e - b*c*d^2 - b*c^2*e - 2*a*d*e^2 + 1/6*(a*b*d^2 + 2*a*b*c*e)*((-I*sq
rt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2
 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*
e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/
a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/5
4*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a) + 2*(
b*d^3 + a*e^3)*x + 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*
d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3
 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b
^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3
b) + 1/54(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a^2*b*e + 6*a*b*d^2 - 6*a*b*c*e)*sqrt(
-(((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/
18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c
^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-
1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b

^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c
/a)^2*a^2*b - 12*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/
27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2
) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*
sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3
+ a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^
2))^(1/3) + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b))) - (((-I*sqrt(3)
+ 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d
*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 +
1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2
*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a - 3*sqrt(
1/3)*a*sqrt(-(((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*
c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) -
1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqr
t(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a
*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))
^(1/3) + 6*c/a)^2*a^2*b - 12*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(
a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e
^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(
1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) +
1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*
a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b)) - 18*
c)*log(-1/36*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c
^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) -
1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt
(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*
e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^
(1/3) + 6*c/a)^2*a^2*b*e - b*c*d^2 - b*c^2*e - 2*a*d*e^2 + 1/6*(a*b*d^2 + 2
*a*b*c*e)*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/
a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/5
4*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3)
+ 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3
)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/
3) + 6*c/a) + 2*(b*d^3 + a*e^3)*x - 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(c^2/
a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3
b) + 1/54(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^
2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a^2*b*e + 6*a*b*d^2 -
6*a*b*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-
1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b
^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(
I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^
3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*

```
b^2))^(1/3) + 6*c/a)^2*a^2*b - 12*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d
*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3
+ a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^
2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*
b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*
d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b)))
 - 36*c*log(x))/a
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)}\,dx = \frac{c\log\left(x\right)}{a} + \frac{\sqrt{3}\left(ae\left(\frac{a}{b}\right)^{\frac{2}{3}} + ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2} - \frac{\left(2\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}} - ae\left(\frac{a}{b}\right)^{\frac{1}{3}} + ad\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}} + ae\left(\frac{a}{b}\right)^{\frac{1}{3}} - ad\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] c*log(x)/a + 1/3*sqrt(3)*(a*e*(a/b)^(2/3) + a*d*(a/b)^(1/3))*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^2 - 1/6*(2*b*c*(a/b)^(2/3) - a*e*(a
/b)^(1/3) + a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/3)) -
 1/3*(b*c*(a/b)^(2/3) + a*e*(a/b)^(1/3) - a*d)*log(x + (a/b)^(1/3))/(a*b*(a
/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x\,(a+bx^3)}\,dx = -\frac{\sqrt{3}\left(bd-\left(-ab^2\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(bd+\left(-ab^2\right)^{\frac{1}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{2}{3}}} - \frac{c\log\left(|bx^3+a|\right)}{3\,a} + \frac{c\log\left(|x|\right)}{a} - \frac{\left(a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b}$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*d - (-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*d + (-a*b^2)^(1/3)*e)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*c*log(abs(b*x^3 + a))/a +
 c*log(abs(x))/a - 1/3*(a^2*b*e*(-a/b)^(1/3) + a^2*b*d)*(-a/b)^(1/3)*log(ab
s(x - (-a/b)^(1/3)))/(a^3*b)
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 716, normalized size of antiderivative = 3.89

$$\int \frac{c+dx+ex^2}{x\,(a+bx^3)}\,dx = \left(\sum_{k=1}^{3}\ln\Big(b^2\,c\,d^2-b^2\,c^2\,e+b^2\,d^3\,x\right.$$
$$-\mathrm{root}\left(27\,a^3\,b^2\,z^3+27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)^3\,a^2\,b^3\,x$$
$$-\,a\,b\,e^3\,x$$
$$-\mathrm{root}\left(27\,a^3\,b^2\,z^3+27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)\,a\,b^2\,d^2$$
$$-\mathrm{root}\left(27\,a^3\,b^2\,z^3+27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)\,b^3\,c^2\,x\,4$$
$$+\mathrm{root}\left(27\,a^3\,b^2\,z^3+27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)^2\,a^2\,b^2\,e$$
$$-\mathrm{root}\left(27\,a^3\,b^2\,z^3+27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)^2\,a\,b^3\,c\,x$$
$$-\mathrm{root}\left(27\,a^3\,b^2\,z^3+27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)\,a\,b^2\,c\,e\,2$$
$$-\,2\,b^2\,c\,d\,e\,x$$
$$\left.-\mathrm{root}\left(27\,a^3\,b^2\,z^3+27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)\,a\,b^2\,d\,e\,x\,10\Big)\right.$$
$$\left.+\,27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)\Bigg)+\frac{c\,\ln\left(x\right)}{a}$$

```
[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)),x)

[Out] symsum(log(b^2*c*d^2 - b^2*c^2*e + b^2*d^3*x - 36*root(27*a^3*b^2*z^3 + 27*
a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2
*e^3 + b^2*c^3, z, k)^3*a^2*b^3*x - a*b*e^3*x - root(27*a^3*b^2*z^3 + 27*a^
2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e
^3 + b^2*c^3, z, k)*a*b^2*d^2 - 4*root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 +
9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3,
z, k)*b^3*c^2*x + 3*root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z
+ 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)^2*a^2*b^
2*e - 24*root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c
^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)^2*a*b^3*c*x - 2*roo
t(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b
*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a*b^2*c*e - 2*b^2*c*d*e*x - 10*
root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*
a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a*b^2*d*e*x)*root(27*a^3*b^2
*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b
*d^3 + a^2*e^3 + b^2*c^3, z, k), k, 1, 3) + (c*log(x))/a
```

3.342 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 192

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx = -\frac{c}{ax} + \frac{\left(b^{2/3}c-a^{2/3}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt[3]{b}} + \frac{d\log(x)}{a} + \frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}} - \frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt[3]{b}} - \frac{d\log\left(a+bx^3\right)}{3a}$$

```
[Out] -c/a/x+d*ln(x)/a+1/3*(b^(2/3)*c+a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^
(1/3)-1/6*(b^(2/3)*c+a^(2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a
^(4/3)/b^(1/3)-1/3*d*ln(b*x^3+a)/a+1/3*(b^(2/3)*c-a^(2/3)*e)*arctan(1/3*(a^
(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used

$= \{1848, 1885, 1874, 31, 648, 631, 210, 642, 266\}$

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(b^{2/3}c-a^{2/3}e\right)}{\sqrt{3}a^{4/3}\sqrt[3]{b}} - \frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt[3]{b}} + \frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}} - \frac{d\log\left(a+bx^3\right)}{3a} - \frac{c}{ax} + \frac{d\log(x)}{a}$$

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(c/(a*x)) + ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(1/3)) + (d*Log[x])/a + ((b^(2/3)*c + a^(2
/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*b^(1/3)) - ((b^(2/3)*c + a^(2/3
)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(1/3)) -
(d*Log[a + b*x^3])/(3*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{ax^2}+\frac{d}{ax}+\frac{ae-bcx-bdx^2}{a\left(a+bx^3\right)}\right)dx\\
&= -\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{\int \frac{ae-bcx-bdx^2}{a+bx^3}\,dx}{a}\\
&= -\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{\int \frac{ae-bcx}{a+bx^3}\,dx}{a}-\frac{(bd)\int \frac{x^2}{a+bx^3}\,dx}{a}\\
&= -\frac{c}{ax}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3\right)}{3a}\\
&\quad+\frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}bc+2a\sqrt[3]{b}e\right)+\sqrt[3]{b}\left(-\sqrt[3]{a}bc-a\sqrt[3]{b}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}\sqrt[3]{b}}+\frac{\left(b^{2/3}c+a^{2/3}e\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}}
\end{aligned}$$

$$= -\frac{c}{ax} + \frac{d\log(x)}{a} + \frac{\left(b^{2/3}c + a^{2/3}e\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}} - \frac{d\log\left(a + bx^3\right)}{3a} - \frac{\left(b^{2/3}c - a^{2/3}e\right)\int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{2a} - \frac{\left(b^{2/3}c + a^{2/3}e\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{6a^{4/3}\sqrt[3]{b}}$$

$$= -\frac{c}{ax} + \frac{d\log(x)}{a} + \frac{\left(b^{2/3}c + a^{2/3}e\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}} - \frac{\left(b^{2/3}c + a^{2/3}e\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6a^{4/3}\sqrt[3]{b}} - \frac{d\log\left(a + bx^3\right)}{3a} - \frac{\left(b^{2/3}c - a^{2/3}e\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{4/3}\sqrt[3]{b}}$$

$$= -\frac{c}{ax} + \frac{\left(b^{2/3}c - a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt[3]{b}} + \frac{d\log(x)}{a} + \frac{\left(b^{2/3}c + a^{2/3}e\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}} - \frac{\left(b^{2/3}c + a^{2/3}e\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6a^{4/3}\sqrt[3]{b}} - \frac{d\log\left(a + bx^3\right)}{3a}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.96

$$\int \frac{c + dx + ex^2}{x^2\left(a + bx^3\right)}\,dx = -\frac{\frac{6ac}{x} + \frac{2\sqrt{3}a^{2/3}\left(-b^{2/3}c + a^{2/3}e\right)\arctan\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} - 6ad\log(x) - \frac{2\left(a^{2/3}b^{2/3}c + a^{4/3}e\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \frac{\left(a^{2/3}b^{2/3}c + a^{4/3}e\right)\log}{}}{6a^2}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)),x]
```

```
[Out] -1/6*((6*a*c)/x + (2*Sqrt[3]*a^(2/3)*(-(b^(2/3)*c) + a^(2/3)*e)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) - 6*a*d*Log[x] - (2*(a^(2/3)*b^(2
```

```
/3)*c + a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((a^(2/3)*b^(2/3)*c
+ a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 2*a*
d*Log[a + b*x^3])/a^2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{c}{ax}+\frac{d\ln(x)}{a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{4}b\_Z^{3}+3a^{3}bd\_Z^{2}+\left(-3a^{2}bce+3a^{2}b\,d^{2}\right)\_Z-a^{2}e^{3}-3abcde+ab\,d^{3}-b^{2}c^{3}\right)}\_R\ln\left(\left(-4\_R^{3}a^{4}b-8\_\right.\right.\right.}{}$
default	$-\frac{c}{ax}+\frac{d\ln(x)}{a}+\frac{ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-bc\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right.\right.}{6b}\right.}{a}$

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -c/a/x+d*ln(x)/a+1/3*sum(_R*ln((-4*_R^3*a^4*b-8*_R^2*a^3*b*d+(10*a^2*b*c*e-
4*a^2*b*d^2)*_R+3*a^2*e^3+6*a*b*c*d*e+3*b^2*c^3)*x-a^3*b*c*_R^2+(-a^3*e^2+2
*a^2*b*c*d)*_R+3*a^2*d*e^2+3*a*b*c*d^2),_R=RootOf(a^4*b*_Z^3+3*a^3*b*d*_Z^2
+(-3*a^2*b*c*e+3*a^2*b*d^2)*_Z-a^2*e^3-3*a*b*c*d*e+a*b*d^3-b^2*c^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 4524, normalized size of antiderivative = 23.56

$$\int \frac{c+dx+ex^2}{x^2\,(a+bx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/1
8*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b
) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/
a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*
```

```
b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*x*log(-1/36
*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 -
 c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54
*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/
18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*
b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a^3*b*c - a*b*c*d^2
 + 2*a*b*c^2*e + a^2*d*e^2 + 1/6*(2*a^2*b*c*d - a^3*e^2)*((-I*sqrt(3) + 1)*
(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*
(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3
)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^
3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3
- a^2*e^3)/(a^4*b))^(1/3) + 6*d/a) - (b^2*c^3 - a^2*e^3)*x) - 36*d*x*log(x)
 - (((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^
2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1
/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 +
 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a
^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*x - 3*sqrt(1/3)*
a*x*sqrt(-(((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1
/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4
*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^
3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*
a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a^2 - 12*
((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 -
c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*
(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/1
8*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b
) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*d + 36*d^2 - 144*c*e
)/a^2) - 18*d*x)*log(1/36*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1
/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3)
 + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (
d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d
/a)^2*a^3*b*c + a*b*c*d^2 - 2*a*b*c^2*e - a^2*d*e^2 - 1/6*(2*a^2*b*c*d - a^
3*e^2)*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*
(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b)
- 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^
3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)
/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a) - 2*(b^2*c^3 -
a^2*e^3)*x + 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/
(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt
(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) +
6*d/a)*a^3*b*c - 6*a^2*b*c*d - 6*a^3*e^2)*sqrt(-(((-I*sqrt(3) + 1)*(d^2/a^2
 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3
```

+ a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b
))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54
*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^
3)/(a^4*b))^(1/3) + 6*d/a)^2*a^2 - 12*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c
*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) +
9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 +
a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))
^(1/3) + 6*d/a)*a*d + 36*d^2 - 144*c*e)/a^2)) - (((-I*sqrt(3) + 1)*(d^2/a^2
- (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3
+ a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b
))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54
*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^
3)/(a^4*b))^(1/3) + 6*d/a)*a*x + 3*sqrt(1/3)*a*x*sqrt(-(((-I*sqrt(3) + 1)*(
d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(
b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)
/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3
+ 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 -
a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a^2 - 12*((-I*sqrt(3) + 1)*(d^2/a^2 - (
d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a
^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(
1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^
2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(
a^4*b))^(1/3) + 6*d/a)*a*d + 36*d^2 - 144*c*e)/a^2) - 18*d*x)*log(1/36*((-I
sqrt(3) + 1)(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)
d/a^3 + 1/54(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2
*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d
^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) -
1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a^3*b*c + a*b*c*d^2 - 2*
a*b*c^2*e - a^2*d*e^2 - 1/6*(2*a^2*b*c*d - a^3*e^2)*((-I*sqrt(3) + 1)*(d^2/
a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*
c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^
4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1
/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2
*e^3)/(a^4*b))^(1/3) + 6*d/a) - 2*(b^2*c^3 - a^2*e^3)*x - 1/12*sqrt(1/3)*((
(-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c
*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(
b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18
*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b)
- 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a^3*b*c - 6*a^2*b*c*d -
6*a^3*e^2)*sqrt(-(((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3
/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a
*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(
-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a

```
^2 - 12*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18
*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b)
 - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a
^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b
)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*d + 36*d^2 -
 144*c*e)/a^2)) + 36*c)/(a*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.97

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx = \frac{d\log(x)}{a} - \frac{\sqrt{3}\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}} - ae\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2} - \frac{\left(2bd\left(\frac{a}{b}\right)^{\frac{2}{3}} + bc\left(\frac{a}{b}\right)^{\frac{1}{3}} + ae\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}} - bc\left(\frac{a}{b}\right)^{\frac{1}{3}} - ae\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{c}{ax}$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] d*log(x)/a - 1/3*sqrt(3)*(b*c*(a/b)^(2/3) - a*e*(a/b)^(1/3))*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^2 - 1/6*(2*b*d*(a/b)^(2/3) + b*c*(a
/b)^(1/3) + a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/3)) -
 1/3*(b*d*(a/b)^(2/3) - b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3))/(a*b*(a
/b)^(2/3)) - c/(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx = -\frac{d\log(|bx^3+a|)}{3a} + \frac{d\log(|x|)}{a} + \frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}ae+(-ab^2)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+(-\frac{a}{b})^{\frac{1}{3}}\right)}{3(-\frac{a}{b})^{\frac{1}{3}}}\right)}{3a^2b} - \frac{c}{ax} + \frac{\left((-ab^2)^{\frac{1}{3}}ae-(-ab^2)^{\frac{2}{3}}c\right)\log\left(x^2+x(-\frac{a}{b})^{\frac{1}{3}}+(-\frac{a}{b})^{\frac{2}{3}}\right)}{6a^2b} + \frac{\left(ab^2c(-\frac{a}{b})^{\frac{1}{3}}-a^2be\right)(-\frac{a}{b})^{\frac{1}{3}}\log\left(\left|x-(-\frac{a}{b})^{\frac{1}{3}}\right|\right)}{3a^3b}$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a + d*log(abs(x))/a + 1/3*sqrt(3)*((-a*b^2)^(1/3
)*a*e + (-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/(a^2*b) - c/(a*x) + 1/6*((-a*b^2)^(1/3)*a*e - (-a*b^2)^(2/3)*c)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) + 1/3*(a*b^2*c*(-a/b)^(1/3) - a^
2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b)
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 723, normalized size of antiderivative = 3.77

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx = \left(\sum_{k=1}^{3}\ln\left(\frac{b^4c^3x+a^2b^2de^2-\mathrm{root}(27a^4bz^3+27a^3bdz^2-9a^2bcez+9a^2bd^2z-3abcde+abd^3-\dots}{\dots}\right.\right.$$

$$\left.+27a^3bdz^2-9a^2bcez+9a^2bd^2z-3abcde+abd^3-a^2e^3-b^2c^3,z,k)\right) - \frac{c}{ax} + \frac{d\ln(x)}{a}$$

```
[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)),x)
```

```
[Out] symsum(log((b^4*c^3*x + a^2*b^2*d*e^2 - 36*root(27*a^4*b*z^3 + 27*a^3*b*d*z
^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*
c^3, z, k)^3*a^4*b^3*x + a^2*b^2*e^3*x + a*b^3*c*d^2 - 3*root(27*a^4*b*z^3
```

```
+ 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 -
a^2*e^3 - b^2*c^3, z, k)^2*a^3*b^3*c - root(27*a^4*b*z^3 + 27*a^3*b*d*z^2 -
 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3,
 z, k)*a^3*b^2*e^2 - 4*root(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z +
 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a^2*b^3*d
^2*x - 24*root(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*
z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)^2*a^3*b^3*d*x + 2*root
(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*
e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a^2*b^3*c*d + 2*a*b^3*c*d*e*x + 10*r
oot(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c
*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a^2*b^3*c*e*x)/a^2)*root(27*a^4*b
*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d
^3 - a^2*e^3 - b^2*c^3, z, k), k, 1, 3) - c/(a*x) + (d*log(x))/a
```

3.343 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 203

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx = -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{\sqrt[3]{b}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}+\frac{e\log(x)}{a}-\frac{\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}}+\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{e\log(a+bx^3)}{3a}$$

```
[Out] -1/2*c/a/x^2-d/a/x+e*ln(x)/a-1/3*b^(1/3)*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b
^(1/3)*x)/a^(5/3)+1/6*b^(2/3)*(c-a^(1/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/a^(5/3)-1/3*e*ln(b*x^3+a)/a+1/3*b^(1/3)*(b^(1/3)*c+a^(1/
3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used

$= \{1848, 1885, 1874, 31, 648, 631, 210, 642, 266\}$

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx = \frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{\sqrt{3}a^{5/3}} + \frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}} - \frac{\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}} - \frac{e\log(a+bx^3)}{3a} - \frac{c}{2ax^2} - \frac{d}{ax} + \frac{e\log(x)}{a}$$

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*c/(a*x^2) - d/(a*x) + (b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (e*Log[x])/a - (b^(
1/3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)) + (b^(2/
3)*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*a^(5/3)) - (e*Log[a + b*x^3])/(3*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{ax^3}+\frac{d}{ax^2}+\frac{e}{ax}-\frac{b(c+dx+ex^2)}{a\,(a+bx^3)}\right)dx\\
&= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{b\int \frac{c+dx+ex^2}{a+bx^3}\,dx}{a}\\
&= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{b\int \frac{c+dx}{a+bx^3}\,dx}{a}-\frac{(be)\int \frac{x^2}{a+bx^3}\,dx}{a}\\
&= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{e\log(a+bx^3)}{3a}\\
&\quad -\frac{b^{2/3}\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}c+\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}}-\frac{\left(b\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{5/3}}
\end{aligned}$$

$$= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e\log(x)}{a} - \frac{b^{2/3}\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3a^{5/3}} - \frac{e\log(a+bx^3)}{3a} + \frac{\left(\sqrt[3]{b}\left(\sqrt[3]{b}c - \sqrt[3]{a}d\right)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{5/3}} - \frac{\left(b^{2/3}\left(\sqrt[3]{b}c + \sqrt[3]{a}d\right)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{4/3}}$$

$$= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e\log(x)}{a} - \frac{b^{2/3}\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3a^{5/3}} + \frac{\sqrt[3]{b}\left(\sqrt[3]{b}c - \sqrt[3]{a}d\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6a^{5/3}} - \frac{e\log(a+bx^3)}{3a} - \frac{\left(\sqrt[3]{b}\left(\sqrt[3]{b}c + \sqrt[3]{a}d\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{5/3}}$$

$$= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{\sqrt[3]{b}\left(\sqrt[3]{b}c + \sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}} + \frac{e\log(x)}{a} - \frac{b^{2/3}\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3a^{5/3}} + \frac{\sqrt[3]{b}\left(\sqrt[3]{b}c - \sqrt[3]{a}d\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6a^{5/3}} - \frac{e\log(a+bx^3)}{3a}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.95

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx = \frac{-\frac{3ac}{x^2} - \frac{6ad}{x} + 2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}c + \sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 6ae\log(x) + 2\sqrt[3]{b}\left(-\sqrt[3]{a}\sqrt[3]{b}c + a^{2/3}d\right)\log\left(\sqrt[3]{\right.}$$

$$6a^2$$

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-3*a*c)/x^2 - (6*a*d)/x + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a^(1/3)*
d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 6*a*e*Log[x] + 2*b^(1/3)*(
-(a^(1/3)*b^(1/3)*c) + a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(1/
3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] -
2*a*e*Log[a + b*x^3])/(6*a^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.68 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.03

method	result
risch	$\frac{-\frac{xd}{a}-\frac{c}{2a}}{x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^3+3a^4e\_Z^2+\left(3a^3e^2+3a^2bcd\right)\_Z+a^2e^3+3abcde-ab\,d^3+b^2c^3\right)}\_R\ln\left(\left(-4\_R^3a^5-8\_R^2a^4e+(-\cdots\right.\right.\right.}{3}$
default	$-\frac{c}{2a\,x^2}-\frac{d}{ax}+\frac{e\ln(x)}{a}-\frac{\left(\left(c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\cdots\right.\right.\right.}{a}$

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/a*x*d-1/2*c/a)/x^2+1/3*sum(_R*ln((-4*_R^3*a^5-8*_R^2*a^4*e+(-4*a^3*e^2-
10*a^2*b*c*d)*_R-6*a*b*c*d*e+3*a*b*d^3-3*b^2*c^3)*x-a^4*d*_R^2+(2*a^3*d*e-a
^2*b*c^2)*_R+3*a^2*d*e^2+3*a*b*c^2*e),_R=RootOf(a^5*_Z^3+3*a^4*e*_Z^2+(3*a^
3*e^2+3*a^2*b*c*d)*_Z+a^2*e^3+3*a*b*c*d*e-a*b*d^3+b^2*c^3))+1/a*e*ln(-x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 4279, normalized size of antiderivative = 21.08

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 +
 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 +
a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^
3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3
 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a*x^2*log(1/36*((-I*s
qrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d +
a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3
- 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d
 + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d
```

^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^4*d + 2*a*b*c*d^2 - a*b*c^2*e +
a^2*d*e^2 + 1/6*(a^2*b*c^2 - 2*a^3*d*e)*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d
 + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 +
a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) +
9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3
 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3)
 + 6*e/a) + (b^2*c^3 + a*b*d^3)*x) - 36*e*x^2*log(x) + 36*d*x - (((-I*sqrt(
3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^
2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*
c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a
*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a*x^2 + 3*sqrt(1/3)*a*x^2*sqrt(-(((-I*sq
rt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a
*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d
+ a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^
3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^3 - 12*((-I*sqrt(3) + 1)*(e^2/a^2
 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*
(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)
^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/
54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a
^5)^(1/3) + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3) - 18*e*x^2)*log(-1/36
*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b
*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3
 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18
*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e
^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^4*d - 2*a*b*c*d^2 + a*b*c
^2*e - a^2*d*e^2 - 1/6*(a^2*b*c^2 - 2*a^3*d*e)*((-I*sqrt(3) + 1)*(e^2/a^2 -
 (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b
*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(
1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54
*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5
)^(1/3) + 6*e/a) + 2*(b^2*c^3 + a*b*d^3)*x + 1/12*sqrt(1/3)*(((-I*sqrt(3) +
 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e
/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*
e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2
)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a^4*d - 6*a^2*b*c^2 - 6*a^3*d*e)*sqrt(-(((-I
sqrt(3) + 1)(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d
+ a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^
3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c
*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^3 - 12*((-I*sqrt(3) + 1)*(e^2/
a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/
54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a

^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3)) - (((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e /a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d* e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c *d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a*x^2 - 3*sqrt(1/3)*a*x^2*sqrt(-(((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c *d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a* e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^3 - 12*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c ^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/ 3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^ (1/3) + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3) - 18*e*x^2)*log(-1/36*((- I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d ^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b* c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^4*d - 2*a*b*c*d^2 + a*b*c^2*e - a^2*d*e^2 - 1/6*(a^2*b*c^2 - 2*a^3*d*e)*((-I*sqrt(3) + 1)*(e^2/a^2 - (b* c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b* c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1 /3) + 6*e/a) + 2*(b^2*c^3 + a*b*d^3)*x - 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)* (e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a *b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/ a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a^4*d - 6*a^2*b*c^2 - 6*a^3*d*e)*sqrt(-(((-I*sqr t(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a* e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^3 - 12*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^ (1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/5 4*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^ 5)^(1/3) + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3)) + 18*c)/(a*x^2)

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx=\text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.87

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx=\frac{e\log(x)}{a}-\frac{\sqrt{3}\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2}-\frac{\left(2\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}+d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(e\left(\frac{a}{b}\right)^{\frac{2}{3}}-d\left(\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\,dx+c}{2\,ax^2}$$

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")
[Out] e*log(x)/a - 1/3*sqrt(3)*(b*d*(a/b)^(2/3) + b*c*(a/b)^(1/3))*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^2 - 1/6*(2*e*(a/b)^(2/3) + d*(a/b)^
(1/3) - c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*(e*
(a/b)^(2/3) - d*(a/b)^(1/3) + c)*log(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 1/2
*(2*d*x + c)/(a*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx = -\frac{e\log(|bx^3+a|)}{3a} + \frac{e\log(|x|)}{a} - \frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}bc-(-ab^2)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b} - \frac{\left((-ab^2)^{\frac{1}{3}}bc+(-ab^2)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b} + \frac{\left(ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b} - \frac{2dx+c}{2ax^2}$$

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a + e*log(abs(x))/a - 1/3*sqrt(3)*((-a*b^2)^(1/3
)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/(a^2*b) - 1/6*((-a*b^2)^(1/3)*b*c + (-a*b^2)^(2/3)*d)*log(x^2 + x*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a^2*b) + 1/3*(a*b^2*d*(-a/b)^(1/3) + a*b^2*c)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) - 1/2*(2*d*x + c)/(a*x^2)
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 701, normalized size of antiderivative = 3.45

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx = \left(\sum_{k=1}^{3}\ln\left(-\frac{b^5c^3x-a^2b^3de^2+\mathrm{root}(27a^5z^3+27a^4ez^2+9a^2bcdz+9a^3e^2z+3abcde-abd^3+a^2\cdots}{\cdots}\right.\right.$$
$$\left.\left. +27a^4ez^2+9a^2bcdz+9a^3e^2z+3abcde-abd^3+a^2e^3+b^2c^3,z,k)\right)\right) - \frac{c}{2ax^2} - \frac{d}{ax} + \frac{e\ln(x)}{a}$$

```
[In] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)),x)
```

```
[Out] symsum(log(-(b^5*c^3*x - a^2*b^3*d*e^2 + 36*root(27*a^5*z^3 + 27*a^4*e*z^2
+ 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3,
z, k)^3*a^5*b^3*x - a*b^4*c^2*e - a*b^4*d^3*x + root(27*a^5*z^3 + 27*a^4*e*
z^2 + 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c
^3, z, k)*a^2*b^4*c^2 + 3*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2*b*c*d*z +
9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)^2*a^4*b^3*d
```

```
+ 4*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*
d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a^3*b^3*e^2*x + 24*root(27*a^5*z^3
 + 27*a^4*e*z^2 + 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2
*e^3 + b^2*c^3, z, k)^2*a^4*b^3*e*x - 2*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*
a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k
)*a^3*b^3*d*e + 2*a*b^4*c*d*e*x + 10*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2
*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a
^2*b^4*c*d*x)/a^3)*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2*b*c*d*z + 9*a^3*e
^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k), k, 1, 3) - c/(2*a*
x^2) - d/(a*x) + (e*log(x))/a
```

3.344 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 190

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx = -\frac{c+dx+ex^2}{3b\,(a+bx^3)} - \frac{\left(\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{5/3}} + \frac{\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{5/3}} - \frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}$$

```
[Out] 1/3*(-e*x^2-d*x-c)/b/(b*x^3+a)+1/9*(b^(1/3)*d-2*a^(1/3)*e)*ln(a^(1/3)+b^(1/
3)*x)/a^(2/3)/b^(5/3)-1/18*(d-2*a^(1/3)*e/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/a^(2/3)/b^(4/3)-1/9*(b^(1/3)*d+2*a^(1/3)*e)*arctan(1/3*(a
^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

$= \{1837, 1874, 31, 648, 631, 210, 642\}$

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)}{3\sqrt{3}a^{2/3}b^{5/3}} - \frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}} + \frac{\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{5/3}} - \frac{c+dx+ex^2}{3b(a+bx^3)}$$

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(c + d*x + e*x^2)/(b*(a + b*x^3)) - ((b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[
(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(5/3)) + (
(b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(5/3)) - (
(d - (2*a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(18*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}+\frac{\int \frac{d+2ex}{a+bx^3}\,dx}{3b}\\
&= -\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}+\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d+2\sqrt[3]{a}e\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}d+2\sqrt[3]{a}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{2/3}b^{4/3}}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{2/3}b}\\
&= -\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{4/3}}\\
&\quad+\frac{\left(\frac{\sqrt[3]{b}d}{\sqrt[3]{a}}+2e\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{4/3}}\\
&\quad-\frac{\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{2/3}b^{5/3}}\\
&= -\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{4/3}}\\
&\quad-\frac{\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{5/3}}\\
&\quad+\frac{\left(\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{2/3}b^{5/3}}
\end{aligned}$$

$$= -\frac{c+dx+ex^2}{3b(a+bx^3)} - \frac{\left(\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{5/3}} + \frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{4/3}} - \frac{\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{5/3}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.92

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{6b^{2/3}(c+x(d+ex))}{a+bx^3} - \frac{2\sqrt{3}\left(\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{\left(-\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\log\left(a^{2/}\right.}{a^{2/3}}}{18b^{5/3}}$$

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*b^(2/3)*(c + x*(d + e*x)))/(a + b*x^3) - (2*Sqrt[3]*(b^(1/3)*d + 2*a^(
1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(b^(1/3)*
d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + ((-(b^(1/3)*d) + 2*a^(
1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(18*b^(5/3
))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.58 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.35

method	result
risch	$\frac{-\frac{e x^2}{3b}-\frac{dx}{3b}-\frac{c}{3b}}{b x^3+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(2e\_R+d\right)\ln\left(x-\_R\right)}{\_R^2}}{9b^2}$
default	$\frac{-\frac{e x^2}{3b}-\frac{dx}{3b}-\frac{c}{3b}}{b x^3+a}+\frac{d\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+2e\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)\right.}{6b\left(\right.}\right.}{3b}$

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*e*x^2/b-1/3*d*x/b-1/3*c/b)/(b*x^3+a)+1/9/b^2*sum((2*_R*e+d)/_R^2*ln(x
-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 2077, normalized size of antiderivative = 10.93

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*e*x^2 + 2*(b^2*x^3 + a*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 +
8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e
*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a
^2*b^5))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a
^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3)
 - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1
/3)))^2*a^2*b^3*e - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^
2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3)
- 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/
3)))*a*b^2*d^2 + 8*a*d*e^2 + (b*d^3 + 8*a*e^3)*x) + 12*d*x - ((b^2*x^3 + a*
```

b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3))) + 3*sqrt(1/3)*(b^2*x^3 + a*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a*b^3 + 32*d*e)/(a*b^3)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a^2*b^3*e + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))*a*b^2*d^2 - 8*a*d*e^2 + 2*(b*d^3 + 8*a*e^3)*x + 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))*a^2*b^3*e + a*b^2*d^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a*b^3 + 32*d*e)/(a*b^3))) - ((b^2*x^3 + a*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3))) - 3*sqrt(1/3)*(b^2*x^3 + a*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a*b^3 + 32*d*e)/(a*b^3)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a^2*b^3*e + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))*a*b^2*d^2 - 8*a*d*e^2 + 2*(b*d^3 + 8*a*e^3)*x - 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))*a^2*b^3*e + a*b^2*d^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a*b^3 + 32*d*e)/(a*b^3))) + 12*c)/(b^2*x^3 + a*b)

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.58

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx$$
$$=\operatorname{RootSum}\left(729t^3a^2b^5+54tab^2de+8ae^3-bd^3,\left(t\mapsto t\log\left(x+\frac{162t^2a^2b^3e+9tab^2d^2+8ade^2}{8ae^3+bd^3}\right)\right)\right)$$
$$+\frac{-c-dx-ex^2}{3ab+3b^2x^3}$$

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**2*b**5 + 54*_t*a*b**2*d*e + 8*a*e**3 - b*d**3, Lambda(
_t, _t*log(x + (162*_t**2*a**2*b**3*e + 9*_t*a*b**2*d**2 + 8*a*d*e**2)/(8*a
*e**3 + b*d**3)))) + (-c - d*x - e*x**2)/(3*a*b + 3*b**2*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.86

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx=-\frac{ex^2+dx+c}{3\,(b^2x^3+ab)}+\frac{\sqrt{3}\left(2\,e\left(\frac{a}{b}\right)^{\frac{1}{3}}+d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(2\,e\left(\frac{a}{b}\right)^{\frac{1}{3}}-d\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\left(2\,e\left(\frac{a}{b}\right)^{\frac{1}{3}}-d\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^2 + d*x + c)/(b^2*x^3 + a*b) + 1/9*sqrt(3)*(2*e*(a/b)^(1/3) + d)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) + 1/1
8*(2*e*(a/b)^(1/3) - d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(
2/3)) - 1/9*(2*e*(a/b)^(1/3) - d)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.93

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(bd-2\left(-ab^2\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}b} - \frac{\left(bd+2\left(-ab^2\right)^{\frac{1}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}b} - \frac{\left(2e\left(-\frac{a}{b}\right)^{\frac{1}{3}}+d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9ab} - \frac{ex^2+dx+c}{3(bx^3+a)b}$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(b*d - 2*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/18*(b*d + 2*(-a*b^2)^(1/3)*e)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/9*(2*e*(-a/b)^
(1/3) + d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 1/3*(e*x^2 + d*x
 + c)/((b*x^3 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.95

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx = \left(\sum_{k=1}^{3}\ln\left(\frac{2\,d\,e+4\,e^2\,x+\mathrm{root}(729\,a^2\,b^5\,z^3+54\,a\,b^2\,d\,e\,z+8\,a\,e^3-b\,d^3,z,k)^2\,a\,b^3\,81+\mathrm{root}(729\,a^2\,b^5\,z^3+5\ldots}{b\,9}\right) + 54\,a\,b^2\,d\,e\,z+8\,a\,e^3-b\,d^3,z,k)\right) - \frac{\frac{c}{3\,b}+\frac{e\,x^2}{3\,b}+\frac{d\,x}{3\,b}}{b\,x^3+a}$$

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((2*d*e + 4*e^2*x + 81*root(729*a^2*b^5*z^3 + 54*a*b^2*d*e*z + 8*
a*e^3 - b*d^3, z, k)^2*a*b^3 + 9*root(729*a^2*b^5*z^3 + 54*a*b^2*d*e*z + 8*
a*e^3 - b*d^3, z, k)*b^2*d*x)/(9*b))*root(729*a^2*b^5*z^3 + 54*a*b^2*d*e*z
+ 8*a*e^3 - b*d^3, z, k), k, 1, 3) - (c/(3*b) + (e*x^2)/(3*b) + (d*x)/(3*b)
)/(a + b*x^3)
```

3.345 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 200

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx = -\frac{x(ae-bcx-bdx^2)}{3ab(a+bx^3)} - \frac{(b^{2/3}c+a^{2/3}e)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{4/3}} - \frac{(b^{2/3}c-a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{4/3}} + \frac{(b^{2/3}c-a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{4/3}}$$

```
[Out] -1/3*x*(-b*d*x^2-b*c*x+a*e)/a/b/(b*x^3+a)-1/9*(b^(2/3)*c-a^(2/3)*e)*ln(a^(1
/3)+b^(1/3)*x)/a^(4/3)/b^(4/3)+1/18*(b^(2/3)*c-a^(2/3)*e)*ln(a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(4/3)-1/9*(b^(2/3)*c+a^(2/3)*e)*arctan(1
/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{1842, 1874, 31, 648, 631, 210, 642\}$

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^{2/3}e+b^{2/3}c)}{3\sqrt{3}a^{4/3}b^{4/3}} + \frac{(b^{2/3}c-a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{4/3}} - \frac{(b^{2/3}c-a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{4/3}} - \frac{x(ae-bcx-bdx^2)}{3ab\,(a+bx^3)}$$

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(x*(a*e - b*c*x - b*d*x^2))/(a*b*(a + b*x^3)) - ((b^(2/3)*c + a^(2/3)*
e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^
(4/3)) - ((b^(2/3)*c - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(4/3)*b^(4
/3)) + ((b^(2/3)*c - a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(18*a^(4/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\text{integral} = -\frac{x(ae-bcx-bdx^2)}{3ab\,(a+bx^3)} - \frac{\int \frac{-ae-bcx}{a+bx^3}\,dx}{3ab}$$

$$= -\frac{x(ae-bcx-bdx^2)}{3ab\,(a+bx^3)} - \frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}bc-2a\sqrt[3]{b}e\right)+\sqrt[3]{b}\left(-\sqrt[3]{a}bc+a\sqrt[3]{b}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b^{4/3}} - \frac{\left(b^{2/3}c-a^{2/3}e\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}b}$$

$$= -\frac{x(ae-bcx-bdx^2)}{3ab\,(a+bx^3)} - \frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{4/3}} + \frac{\left(b^{2/3}c-a^{2/3}e\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{4/3}b^{4/3}} + \frac{\left(b^{2/3}c+a^{2/3}e\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6ab}$$

$$
= -\frac{x(ae-bcx-bdx^2)}{3ab\,(a+bx^3)} - \frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{4/3}} + \frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{4/3}} + \frac{\left(b^{2/3}c+a^{2/3}e\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{4/3}b^{4/3}}
$$

$$
= -\frac{x(ae-bcx-bdx^2)}{3ab\,(a+bx^3)} - \frac{\left(b^{2/3}c+a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{4/3}} - \frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{4/3}} + \frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{4/3}}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.93

$$
\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx
$$

$$
= \frac{-\frac{6ab^{2/3}(-bcx^2+a(d+ex))}{a+bx^3} - 2\sqrt{3}\left(a^{2/3}bc+a^{4/3}\sqrt[3]{b}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\left(-a^{2/3}bc+a^{4/3}\sqrt[3]{b}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{18a^2b^{5/3}}
$$

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a*b^(2/3)*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3) - 2*Sqrt[3]*(a^(2/3)
*b*c + a^(4/3)*b^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(
-(a^(2/3)*b*c) + a^(4/3)*b^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x] - (-(a^(2/3)*b
*c) + a^(4/3)*b^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1
8*a^2*b^(5/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.58 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.36

method	result
risch	$\frac{\frac{c x^2}{3a}-\frac{ex}{3b}-\frac{d}{3b}}{b x^3+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\frac{c\_R}{a}+\frac{e}{b}\right)\ln\left(x-\_R\right)}{\_R^2}}{9b}$
default	$\frac{\frac{c x^2}{3a}-\frac{ex}{3b}-\frac{d}{3b}}{b x^3+a}+\frac{ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+bc\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\cdots\right.}{6b\left(\frac{a}{b}\cdots\right.}\right.}{3ba}$

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/3*c/a*x^2-1/3*e*x/b-1/3*d/b)/(b*x^3+a)+1/9/b*sum((c/a*_R+1/b*e)/_R^2*ln(
x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 2358, normalized size of antiderivative = 11.79

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*b*c*x^2 - 12*a*e*x - 2*(a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)
 - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^
4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)
 - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^
4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^3*b^3*c - 1/2*((1/2)^(1/3)*
(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b
^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^
3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b*e^2 + 2*a*b*c^2
*e + (b^2*c^3 + a^2*e^3)*x) - 12*a*d + ((a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4
))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)
/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3))) + 3*sqrt(1/3)*(a*b^2*x^
3 + a^2*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^
```

```
4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) +
 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4)
)^(1/3)))^2*a^2*b^2 + 16*c*e)/(a^2*b^2)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)
- 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4
) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^3*b^3*c + 1/2*((1/2)^(1/3)*(
I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^
4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3
)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b*e^2 - 2*a*b*c^2*
e + 2*(b^2*c^3 + a^2*e^3)*x + 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(
(b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1
/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b
^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b^3*c + 2*a^3*b*e^2)*sqrt(-(((1/2)
^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)
/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 +
 a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^2*b^2 + 16
*c*e)/(a^2*b^2))) + ((a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2
*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^
(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c
^3 - a^2*e^3)/(a^4*b^4))^(1/3))) - 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-((
(1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2
*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*
c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^2*b^2
 + 16*c*e)/(a^2*b^2)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^
2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e
*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*
e^3)/(a^4*b^4))^(1/3)))^2*a^3*b^3*c + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^
2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)
^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*
c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b*e^2 - 2*a*b*c^2*e + 2*(b^2*c^3 + a^
2*e^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)
/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*s
qrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(
a^4*b^4))^(1/3)))*a^3*b^3*c + 2*a^3*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) +
 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) -
 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4)
 - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^2*b^2 + 16*c*e)/(a^2*b^2))))/
(a*b^2*x^3 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.62

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx$$
$$=\operatorname{RootSum}\left(729t^3a^4b^4+27ta^2b^2ce-a^2e^3+b^2c^3,\left(t\mapsto t\log\left(x+\frac{81t^2a^3b^3c+9ta^3be^2+2abc^2e}{a^2e^3+b^2c^3}\right)\right)\right)$$
$$+\frac{-ad-aex+bcx^2}{3a^2b+3ab^2x^3}$$

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**4*b**4 + 27*_t*a**2*b**2*c*e - a**2*e**3 + b**2*c**3,
Lambda(_t, _t*log(x + (81*_t**2*a**3*b**3*c + 9*_t*a**3*b*e**2 + 2*a*b*c**2
*e)/(a**2*e**3 + b**2*c**3)))) + (-a*d - a*e*x + b*c*x**2)/(3*a**2*b + 3*a*
b**2*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.92

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx=\frac{bcx^2-aex-ad}{3\,(ab^2x^3+a^2b)}+\frac{\sqrt{3}\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*c*x^2 - a*e*x - a*d)/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(b*c*(a/b)^(1
/3) + a*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)
^(2/3)) + 1/18*(b*c*(a/b)^(1/3) - a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
))/(a*b^2*(a/b)^(2/3)) - 1/9*(b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3))/(
a*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.93

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(ae-(-ab^2)^{\frac{1}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+(-\frac{a}{b})^{\frac{1}{3}}\right)}{3(-\frac{a}{b})^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}a} - \frac{\left(ae+(-ab^2)^{\frac{1}{3}}c\right)\log\left(x^2+x(-\frac{a}{b})^{\frac{1}{3}}+(-\frac{a}{b})^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}a} - \frac{\left(bc(-\frac{a}{b})^{\frac{1}{3}}+ae\right)(-\frac{a}{b})^{\frac{1}{3}}\log\left(\left|x-(-\frac{a}{b})^{\frac{1}{3}}\right|\right)}{9\,a^2b} + \frac{bcx^2-aex-ad}{3\,(bx^3+a)ab}$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(a*e - (-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(a*e + (-a*b^2)^(1/3)*c)*log(x^2
 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(b*c*(-a/b)^(1/3
) + a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) + 1/3*(b*c*x^2 - a
*e*x - a*d)/((b*x^3 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx = \left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(729\,a^4\,b^4\,z^3+27\,a^2\,b^2\,c\,e\,z+b^2\,c^3-a^2\,e^3,z,k\right)\,\left(b\,e\,x+\mathrm{root}\left(729\,a^4\,b^4\,z^3+27\,a^2\,b^2\,c\,e\,z+b^2\,c^3-a^2\,e^3,z,k\right)\ldots\right.\right.\right. + \frac{c\,e}{9\,a}+\frac{b\,c^2\,x}{9\,a^2}\Bigg)\,\mathrm{root}\left(729\,a^4\,b^4\,z^3+27\,a^2\,b^2\,c\,e\,z+b^2\,c^3-a^2\,e^3,z,k\right)\Bigg)\Bigg) - \frac{\frac{d}{3\,b}-\frac{c\,x^2}{3\,a}+\frac{e\,x}{3\,b}}{b\,x^3+a}$$

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log(root(729*a^4*b^4*z^3 + 27*a^2*b^2*c*e*z + b^2*c^3 - a^2*e^3, z,
k)*(b*e*x + 9*root(729*a^4*b^4*z^3 + 27*a^2*b^2*c*e*z + b^2*c^3 - a^2*e^3,
z, k)*a*b^2) + (c*e)/(9*a) + (b*c^2*x)/(9*a^2))*root(729*a^4*b^4*z^3 + 27*a
^2*b^2*c*e*z + b^2*c^3 - a^2*e^3, z, k), k, 1, 3) - (d/(3*b) - (c*x^2)/(3*a
) + (e*x)/(3*b))/(a + b*x^3)
```

3.346 $\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 199

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx = -\frac{ae-bx(c+dx)}{3ab(a+bx^3)} - \frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}$$

```
[Out] 1/3*(-a*e+b*x*(d*x+c))/a/b/(b*x^3+a)+1/9*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)
+b^(1/3)*x)/a^(5/3)/b^(2/3)-1/18*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(2/3)-1/9*(2*b^(1/3)*c+a^(1/3)*d)*arctan(
1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used

$= \{1868, 1874, 31, 648, 631, 210, 642\}$

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)}{3\sqrt{3}a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}$$

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(a*e - b*x*(c + d*x))/(a*b*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(2/3
)) + ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3
)) - ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(18*a^(5/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\text{integral} = -\frac{ae - bx(c+dx)}{3ab\,(a+bx^3)} - \frac{\int \frac{-2c-dx}{a+bx^3}\,dx}{3a}$$

$$= -\frac{ae - bx(c+dx)}{3ab\,(a+bx^3)} - \frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}c - \sqrt[3]{a}d\right) + \sqrt[3]{b}\left(2\sqrt[3]{b}c - \sqrt[3]{a}d\right)x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{9a^{5/3}\sqrt[3]{b}} + \frac{\left(2c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$= -\frac{ae - bx(c+dx)}{3ab\,(a+bx^3)} + \frac{\left(2\sqrt[3]{b}c - \sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c - \sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{18a^{5/3}b^{2/3}} + \frac{\left(2c + \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{6a^{4/3}}$$

$$= -\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}b^{2/3}}$$

$$= -\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}-\frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.95

$$\int\frac{c+dx+ex^2}{\left(a+bx^3\right)^2}\,dx$$

$$= \frac{\frac{6a(-ae+bx(c+dx))}{a+bx^3}-2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(4\sqrt[3]{a}b^{2/3}c-2a^{2/3}\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\ldots}{18a^2b}$$

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^2,x]
```

```
[Out] ((6*a*(-(a*e) + b*x*(c + d*x)))/(a + b*x^3) - 2*Sqrt[3]*a^(1/3)*b^(1/3)*(2*
b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (4*a^(
1/3)*b^(2/3)*c - 2*a^(2/3)*b^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*b^
(1/3)*(-2*b^(1/3)*c + a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(18*a^2*b)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.36

method	result
risch	$\frac{\frac{d x^2}{3a}+\frac{cx}{3a}-\frac{e}{3b}}{b x^3+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_Rd+2c\right)\ln\left(x-\_R\right)}{\_R^2}}{9ba}$
default	$c\left(\frac{x}{3a(b x^3+a)}+\frac{\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}\right)+d\left(\frac{x^2}{3a(b x^3+a)}+\frac{-\frac{\ln\left(x+\left(\frac{a}{b}\right.\right.}{3b\left(\frac{a}{b}\right)}\right.$

[In] int((e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] (1/3*d/a*x^2+1/3*c/a*x-1/3/b*e)/(b*x^3+a)+1/9/b/a*sum((_R*d+2*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 2118, normalized size of antiderivative = 10.64

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/36*(12*b*d*x^2 + 12*b*c*x - 2*(a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a

```
^5*b^2))^(1/3)))*a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) - 12*a*e + ((
a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2
) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/
(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))
+ 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b
*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3
)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^
3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*
sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1
/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2)
 + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(
3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) +
 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8
*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d
^3)*x + 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5
*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) -
 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3
)))*a^4*b*d + 8*a^2*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(
I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b))) + ((a*b^2*x^3 + a^2*b)*((1/2)^(1
/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b
^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a
^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) - 3*sqrt(1/3)*(a*b^2*x^3 + a
^2*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8
*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b
*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*
b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d
^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*s
qrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^
2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(
a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3
) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(
1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d^3)*x - 3/4*sqrt(1/3)*(((1/2
)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a
^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3
)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2)*s
qrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3
- a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*
c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*
c*d)/(a^3*b))))/(a*b^2*x^3 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.58

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx = \operatorname{RootSum}\left(729t^3a^5b^2+54ta^2bcd+ad^3-8bc^3, \left(t\mapsto t\log\left(x+\frac{81t^2a^4bd+36ta^2bc^2+4acd^2}{ad^3+8bc^3}\right)\right)\right) + \frac{-ae+bcx+bdx^2}{3a^2b+3ab^2x^3}$$

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda(
_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d**3
 + 8*b*c**3)))) + (-a*e + b*c*x + b*d*x**2)/(3*a**2*b + 3*a*b**2*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.90

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx = \frac{bdx^2+bcx-ae}{3\,(ab^2x^3+a^2b)} + \frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*d*x^2 + b*c*x - a*e)/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(d*(a/b)^(1/3
) + 2*c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/
3)) + 1/18*(d*(a/b)^(1/3) - 2*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*
b*(a/b)^(2/3)) - 1/9*(d*(a/b)^(1/3) - 2*c)*log(x + (a/b)^(1/3))/(a*b*(a/b)^
(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.92

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(2\,bc-\left(-ab^2\right)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}\,a} - \frac{\left(2\,bc+\left(-ab^2\right)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}\,a} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{bdx^2+bcx-ae}{3\,(bx^3+a)ab}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + (-a*b^2)^(1/3)*d)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(d*(-a/b)^(1
/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(b*d*x^2 + b*c
*x - a*e)/((b*x^3 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.88

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx = \left(\sum_{k=1}^{3}\ln\left(\frac{b\left(2\,c\,d+d^2\,x+\mathrm{root}(729\,a^5\,b^2\,z^3+54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k)^2\,a^3\,b\,81+\mathrm{root}(729\,a^5\,b^2\,z^3+\right.}{a^2\,9}\right.\right. \\ \left.\left.+54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k)\right)+\frac{\frac{d\,x^2}{3\,a}-\frac{e}{3\,b}+\frac{c\,x}{3\,a}}{b\,x^3+a}$$

```
[In] int((c + d*x + e*x^2)/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((b*(2*c*d + d^2*x + 81*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z - 8
*b*c^3 + a*d^3, z, k)^2*a^3*b + 18*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z -
8*b*c^3 + a*d^3, z, k)*a*b*c*x))/(9*a^2))*root(729*a^5*b^2*z^3 + 54*a^2*b*c
*d*z - 8*b*c^3 + a*d^3, z, k), k, 1, 3) + ((d*x^2)/(3*a) - e/(3*b) + (c*x)/
(3*a))/(a + b*x^3)
```

3.347 $\int \frac{c+dx+ex^2}{x(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 222

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx = \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\left(2\sqrt[3]{b}d+\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{c\log(x)}{a^2} + \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}} - \frac{c\log\left(a+bx^3\right)}{3a^2}$$

```
[Out] 1/3*x*(-b*c*x^2+a*e*x+a*d)/a^2/(b*x^3+a)+c*ln(x)/a^2+1/9*(2*b^(1/3)*d-a^(1/
3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(2/3)-1/18*(2*b^(1/3)*d-a^(1/3)*e)*ln
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(2/3)-1/3*c*ln(b*x^3+a)/a
^2-1/9*(2*b^(1/3)*d+a^(1/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(
1/2))/a^(5/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules

used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2}{x(a+bx^3)^2}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}e+2\sqrt[3]{b}d\right)}{3\sqrt{3}a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} + \frac{x(ad+aex-bcx^2)}{3a^2(a+bx^3)} - \frac{c\log(a+bx^3)}{3a^2} + \frac{c\log(x)}{a^2}$$

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^2),x]
```

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(3*a^2*(a + b*x^3)) - ((2*b^(1/3)*d + a^(1/3)*e
)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(
2/3)) + (c*Log[x])/a^2 + ((2*b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x
])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*d - a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = \frac{x(ad+aex-bcx^2)}{3a^2(a+bx^3)} - \frac{\int \frac{-3bc-2bdx-bex^2}{x(a+bx^3)}\,dx}{3ab}$$

$$= \frac{x(ad+aex-bcx^2)}{3a^2(a+bx^3)} - \frac{\int \left(-\frac{3bc}{ax} - \frac{b(2ad+aex-3bcx^2)}{a(a+bx^3)}\right)dx}{3ab}$$

$$= \frac{x(ad+aex-bcx^2)}{3a^2(a+bx^3)} + \frac{c\log(x)}{a^2} + \frac{\int \frac{2ad+aex-3bcx^2}{a+bx^3}\,dx}{3a^2}$$

$$= \frac{x(ad+aex-bcx^2)}{3a^2(a+bx^3)} + \frac{c\log(x)}{a^2} + \frac{\int \frac{2ad+aex}{a+bx^3}\,dx}{3a^2} - \frac{(bc)\int \frac{x^2}{a+bx^3}\,dx}{a^2}$$

$$= \frac{x(ad+aex-bcx^2)}{3a^2(a+bx^3)} + \frac{c\log(x)}{a^2} - \frac{c\log(a+bx^3)}{3a^2} + \frac{\int \frac{\sqrt[3]{a}\left(4a\sqrt[3]{b}d+a^{4/3}e\right)+\sqrt[3]{b}\left(-2a\sqrt[3]{b}d+a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{8/3}\sqrt[3]{b}} + \frac{\left(2d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$= \frac{x(ad+aex-bcx^2)}{3a^2(a+bx^3)} + \frac{c\log(x)}{a^2} + \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{c\log(a+bx^3)}{3a^2} - \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{2/3}} + \frac{\left(2d+\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}}$$

$$= \frac{x(ad+aex-bcx^2)}{3a^2(a+bx^3)} + \frac{c\log(x)}{a^2} + \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}} - \frac{c\log(a+bx^3)}{3a^2} + \frac{\left(2\sqrt[3]{b}d+\sqrt[3]{a}e\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}b^{2/3}}$$

$$= \frac{x(ad+aex-bcx^2)}{3a^2(a+bx^3)} - \frac{\left(2\sqrt[3]{b}d+\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{c\log(x)}{a^2} + \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}} - \frac{c\log(a+bx^3)}{3a^2}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.90

$$\int \frac{c+dx+ex^2}{x(a+bx^3)^2}\,dx$$

$$= \frac{\frac{6a(c+x(d+ex))}{a+bx^3} - \frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{b}d+\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + 18c\log(x) + \frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}d-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{(-2}{}}{18a^2}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((6*a*(c + x*(d + e*x)))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*d + a^
(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 18*c*Log[x]
 + (2*(2*a^(1/3)*b^(1/3)*d - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) +
 ((-2*a^(1/3)*b^(1/3)*d + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/b^(2/3) - 6*c*Log[a + b*x^3])/(18*a^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.06

method	result
risch	$\frac{\frac{e\,x^2}{3a}+\frac{xd}{3a}+\frac{c}{3a}}{b\,x^3+a} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^6b^2\_Z^3+9a^4b^2c\_Z^2+\left(6a^3bde+27a^2b^2c^2\right)\_Z+a^2e^3+18abcde-8ab\,d^3+27b^2c^3\right)}\_R\ln\left(\left(-4\_R^3a^5b^2\right.\right.\right.}{}$
default	$\frac{c\ln(x)}{a^2} + \frac{\frac{\frac{1}{3}ae\,x^2+\frac{1}{3}adx+\frac{1}{3}ac}{b\,x^3+a} + \frac{2ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3} + \frac{ae\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.}{}}{a^2}$

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/3/a*e*x^2+1/3/a*x*d+1/3*c/a)/(b*x^3+a)+1/9*sum(_R*ln((-4*_R^3*a^5*b^2-24
*_R^2*a^3*b^2*c+(-20*a^2*b*d*e-36*a*b^2*c^2)*_R-3*a*e^3-36*b*c*d*e+24*b*d^3
)*x+a^4*b*e*_R^2+(-6*a^2*b*c*e-4*a^2*b*d^2)*_R-27*b*c^2*e+36*b*c*d^2),_R=Ro
```

```
otOf(a^6*b^2*_Z^3+9*a^4*b^2*c*_Z^2+(6*a^3*b*d*e+27*a^2*b^2*c^2)*_Z+a^2*e^3+
18*a*b*c*d*e-8*a*b*d^3+27*b^2*c^3))+1/a^2*c*ln(-x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 5018, normalized size of antiderivative = 22.60

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx=&\ \frac{ex^2+dx+c}{3\left(abx^3+a^2\right)}+\frac{c\log\left(x\right)}{a^2}\\&+\frac{\sqrt{3}\left(ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3}\\&-\frac{\left(6\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\&-\frac{\left(3\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(e*x^2 + d*x + c)/(a*b*x^3 + a^2) + c*log(x)/a^2 + 1/9*sqrt(3)*(a*e*(a/
b)^(2/3) + 2*a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/a^3 - 1/18*(6*b*c*(a/b)^(2/3) - a*e*(a/b)^(1/3) + 2*a*d)*log(x^2 - x
*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1/9*(3*b*c*(a/b)^(2/3) +
a*e*(a/b)^(1/3) - 2*a*d)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(2\,bd-\left(-ab^2\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{\left(2\,bd+\left(-ab^2\right)^{\frac{1}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{c\log\left(|bx^3+a|\right)}{3\,a^2} + \frac{c\log\left(|x|\right)}{a^2} + \frac{aex^2+adx+ac}{3\left(bx^3+a\right)a^2} - \frac{\left(a^3be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^3bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b}$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*d - (-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*d + (-a*b^2)^(1/3)*e)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/3*c*log(abs(b*
x^3 + a))/a^2 + c*log(abs(x))/a^2 + 1/3*(a*e*x^2 + a*d*x + a*c)/((b*x^3 + a
)*a^2) - 1/9*(a^3*b*e*(-a/b)^(1/3) + 2*a^3*b*d)*(-a/b)^(1/3)*log(abs(x - (-
a/b)^(1/3)))/(a^5*b)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.21

$$\int \frac{c+dx+ex^2}{x(a+bx^3)^2}\,dx = \frac{\frac{c}{3\,a}+\frac{e\,x^2}{3\,a}+\frac{d\,x}{3\,a}}{b\,x^3+a} + \left(\sum_{k=1}^{3}\ln\left(\frac{4\,b^2\,c\,d^2-3\,b^2\,c^2\,e}{9\,a^3} - \mathrm{root}\left(729\,a^6\,b^2\,z^3+729\,a^4\,b^2\,c\,z^2+54\,a^3\,b\,d\,e\,z+243\,a^2\,b^2\,c^2\,z+18\,a\,b\,c\,d\,e-8\,a\,b\,d^3+27\,b^2\,c^3+a^2\,e^3,z,k\right) - \frac{x\left(-8\,b^2\,d^3+12\,c\,b^2\,d\,e+a\,b\,e^3\right)}{27\,a^3}\right)\mathrm{root}\left(729\,a^6\,b^2\,z^3+729\,a^4\,b^2\,c\,z^2+54\,a^3\,b\,d\,e\,z+243\,a^2\,b^2\,c^2\,z+18\,a\,b\,c\,d\,e-8\,a\,b\,d^3+27\,b^2\,c^3+a^2\,e^3,z,k\right)\right) + \frac{c\,\ln\left(x\right)}{a^2}$$

```
[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)^2),x)
```

```
[Out] (c/(3*a) + (e*x^2)/(3*a) + (d*x)/(3*a))/(a + b*x^3) + symsum(log((4*b^2*c*d
^2 - 3*b^2*c^2*e)/(9*a^3) - root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 + 54*a
^3*b*d*e*z + 243*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3 + a^
2*e^3, z, k)*(root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 + 54*a^3*b*d*e*z + 2
43*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3 + a^2*e^3, z, k)*(
24*b^3*c*x - a*b^2*e + 36*root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 + 54*a^3
*b*d*e*z + 243*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3 + a^2*
e^3, z, k)*a^2*b^3*x) + (4*a^2*b^2*d^2 + 6*a^2*b^2*c*e)/(9*a^3) + (x*(108*a
*b^3*c^2 + 60*a^2*b^2*d*e))/(27*a^3)) - (x*(a*b*e^3 - 8*b^2*d^3 + 12*b^2*c*
d*e))/(27*a^3))*root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 + 54*a^3*b*d*e*z +
 243*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3 + a^2*e^3, z, k)
, k, 1, 3) + (c*log(x))/a^2
```

3.348 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 231

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx=-\frac{c}{a^2x}+\frac{x(ae-bcx-bdx^2)}{3a^2(a+bx^3)}+\frac{2\left(2b^{2/3}c-a^{2/3}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}\sqrt[3]{b}}+\frac{d\log(x)}{a^2}+\frac{2\left(2b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}\sqrt[3]{b}}-\frac{\left(2b^{2/3}c+a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{7/3}\sqrt[3]{b}}-\frac{d\log(a+bx^3)}{3a^2}$$

```
[Out] -c/a^2/x+1/3*x*(-b*d*x^2-b*c*x+a*e)/a^2/(b*x^3+a)+d*ln(x)/a^2+2/9*(2*b^(2/3
)*c+a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(1/3)-1/9*(2*b^(2/3)*c+a^(2/
3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(1/3)-1/3*d*ln(b*
x^3+a)/a^2+2/9*(2*b^(2/3)*c-a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/a^(7/3)/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules

used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^2}\,dx = \frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2b^{2/3}c-a^{2/3}e\right)}{3\sqrt{3}a^{7/3}\sqrt[3]{b}} - \frac{\left(a^{2/3}e+2b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{7/3}\sqrt[3]{b}} + \frac{2\left(a^{2/3}e+2b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}\sqrt[3]{b}} + \frac{x(ae-bcx-bdx^2)}{3a^2\left(a+bx^3\right)} - \frac{d\log\left(a+bx^3\right)}{3a^2} - \frac{c}{a^2x} + \frac{d\log(x)}{a^2}$$

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -(c/(a^2*x)) + (x*(a*e - b*c*x - b*d*x^2))/(3*a^2*(a + b*x^3)) + (2*(2*b^(2
/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sq
rt[3]*a^(7/3)*b^(1/3)) + (d*Log[x])/a^2 + (2*(2*b^(2/3)*c + a^(2/3)*e)*Log[
a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(1/3)) - ((2*b^(2/3)*c + a^(2/3)*e)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(7/3)*b^(1/3)) - (d*Log[a
+ b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = \frac{x(ae-bcx-bdx^2)}{3a^2(a+bx^3)} - \frac{\int \frac{-3bc-3bdx-2bex^2+\frac{b^2cx^3}{a}}{x^2(a+bx^3)}\,dx}{3ab}$$

$$= \frac{x(ae-bcx-bdx^2)}{3a^2(a+bx^3)} - \frac{\int \left(-\frac{3bc}{ax^2}-\frac{3bd}{ax}-\frac{b(2ae-4bcx-3bdx^2)}{a(a+bx^3)}\right)dx}{3ab}$$

$$= -\frac{c}{a^2x} + \frac{x(ae-bcx-bdx^2)}{3a^2(a+bx^3)} + \frac{d\log(x)}{a^2} + \frac{\int \frac{2ae-4bcx-3bdx^2}{a+bx^3}\,dx}{3a^2}$$

$$= -\frac{c}{a^2x} + \frac{x(ae-bcx-bdx^2)}{3a^2(a+bx^3)} + \frac{d\log(x)}{a^2} + \frac{\int \frac{2ae-4bcx}{a+bx^3}\,dx}{3a^2} - \frac{(bd)\int \frac{x^2}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{c}{a^2x} + \frac{x(ae-bcx-bdx^2)}{3a^2(a+bx^3)} + \frac{d\log(x)}{a^2} - \frac{d\log(a+bx^3)}{3a^2} + \frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{a}bc+4a\sqrt[3]{b}e\right)+\sqrt[3]{b}\left(-4\sqrt[3]{a}bc-2a\sqrt[3]{b}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{8/3}\sqrt[3]{b}} + \frac{\left(2\left(2b^{2/3}c+a^{2/3}e\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{7/3}}$$

$$= -\frac{c}{a^2x} + \frac{x(ae-bcx-bdx^2)}{3a^2(a+bx^3)} + \frac{d\log(x)}{a^2} + \frac{2\left(2b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}\sqrt[3]{b}} - \frac{d\log(a+bx^3)}{3a^2} - \frac{\left(2b^{2/3}c-a^{2/3}e\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^2} - \frac{\left(2b^{2/3}c+a^{2/3}e\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{7/3}\sqrt[3]{b}}$$

$$= -\frac{c}{a^2x} + \frac{x(ae-bcx-bdx^2)}{3a^2(a+bx^3)} + \frac{d\log(x)}{a^2} + \frac{2\left(2b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}\sqrt[3]{b}} - \frac{\left(2b^{2/3}c+a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{7/3}\sqrt[3]{b}} - \frac{d\log(a+bx^3)}{3a^2} - \frac{\left(2\left(2b^{2/3}c-a^{2/3}e\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{7/3}\sqrt[3]{b}}$$

$$= -\frac{c}{a^2 x} + \frac{x(ae - bcx - bdx^2)}{3a^2(a + bx^3)} + \frac{2(2b^{2/3}c - a^{2/3}e)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}\sqrt[3]{b}} + \frac{d\log(x)}{a^2} + \frac{2(2b^{2/3}c + a^{2/3}e)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{9a^{7/3}\sqrt[3]{b}} - \frac{(2b^{2/3}c + a^{2/3}e)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{9a^{7/3}\sqrt[3]{b}} - \frac{d\log(a + bx^3)}{3a^2}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.92

$$\int \frac{c + dx + ex^2}{x^2(a + bx^3)^2}\,dx =$$

$$-\frac{\frac{9ac}{x} - \frac{3a(-bcx^2 + a(d + ex))}{a + bx^3} + \frac{2\sqrt{3}a^{2/3}(-2b^{2/3}c + a^{2/3}e)\arctan\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} - 9ad\log(x) - \frac{2(2a^{2/3}b^{2/3}c + a^{4/3}e)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{\sqrt[3]{b}} \cdots}{9a^3}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -1/9*((9*a*c)/x - (3*a*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3) + (2*Sqrt[3]
*a^(2/3)*(-2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]])/b^(1/3) - 9*a*d*Log[x] - (2*(2*a^(2/3)*b^(2/3)*c + a^(4/3)*e)*Log[a^(
1/3) + b^(1/3)*x])/b^(1/3) + ((2*a^(2/3)*b^(2/3)*c + a^(4/3)*e)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 3*a*d*Log[a + b*x^3])/a^3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.06

method	result
risch	$\frac{-\frac{4bc\,x^3}{3a^2}+\frac{e\,x^2}{3a}+\frac{xd}{3a}-\frac{c}{a}}{x(b\,x^3+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^7b\_Z^3+9a^5bd\_Z^2+\left(-24a^3bce+27a^3b\,d^2\right)\_Z-8a^2e^3-72abcde+27ab\,d^3-64b^2c^3\right)}\_R\ln\left(\left(-\_\right.\right.\right.}{}$
default	$-\frac{c}{a^2x}+\frac{d\ln(x)}{a^2}+\frac{\frac{-\frac{1}{3}cb\,x^2+\frac{1}{3}aex+\frac{1}{3}ad}{b\,x^3+a}+\frac{2ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}-\frac{4bc\left(-\frac{\ln(}{3}\right.}{}}{a^2}$

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-4/3*b*c/a^2*x^3+1/3/a*e*x^2+1/3/a*x*d-c/a)/x/(b*x^3+a)+1/9*sum(_R*ln((-_R
^3*a^7*b-6*_R^2*a^5*b*d+(20*a^3*b*c*e-9*a^3*b*d^2)*_R+6*a^2*e^3+36*a*b*c*d*
e+48*b^2*c^3)*x-a^5*b*c*_R^2+(-a^4*e^2+6*a^3*b*c*d)*_R+9*a^2*d*e^2+27*a*b*c
*d^2),_R=RootOf(a^7*b*_Z^3+9*a^5*b*d*_Z^2+(-24*a^3*b*c*e+27*a^3*b*d^2)*_Z-8
*a^2*e^3-72*a*b*c*d*e+27*a*b*d^3-64*b^2*c^3))+d*ln(x)/a^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 4976, normalized size of antiderivative = 21.54

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/324*(432*b*c*x^3 - 108*a*e*x^2 - 108*a*d*x + 2*(a^2*b*x^4 + a^3*x)*((-I*
sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^
2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b
)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)
*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*
e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b
))^(1/3) + 54*d/a^2)*log(-1/324*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c
```

*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3
+ 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3
)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e
)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b)
 - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^5*b*c - 9*a*b
*c*d^2 + 16*a*b*c^2*e + 3*a^2*d*e^2 + 1/18*(6*a^3*b*c*d - a^4*e^2)*((-I*sqr
t(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 -
 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(
a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-
1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3
 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^
(1/3) + 54*d/a^2) - 2*(8*b^2*c^3 - a^2*e^3)*x) + 324*a*c + (162*b*d*x^4 + 1
62*a*d*x - (a^2*b*x^4 + a^3*x)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*
e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 +
 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)
/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)
d/a^6 + 1/1458(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b)
- 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2) + 3*sqrt(1/3)*(a^2
*b*x^4 + a^3*x)*sqrt(-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/
(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e
^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b)
)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 +
 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*
(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) +
 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e
)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b)
 - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d
^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(
3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3)
+ 54*d/a^2)*a^2*d + 2916*d^2 - 10368*c*e)/a^4))*log(1/324*((-I*sqrt(3) + 1)
*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d
/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) -
4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/
a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d
^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 5
4*d/a^2)^2*a^5*b*c + 9*a*b*c*d^2 - 16*a*b*c^2*e - 3*a^2*d*e^2 - 1/18*(6*a^3
*b*c*d - a^4*e^2)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/2
7*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 -
9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/
3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/14
58*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^
2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2) - 4*(8*b^2*c^3 - a^2*e^3)*x + 1
/108*sqrt(1/3)*(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*
d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*
(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3)

+ 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458
*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*
c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)*a^5*b*c - 54*a^3*b*c*d - 18*a^4*e
^2)*sqrt(-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a
^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^
3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81
*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*
b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 -
a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) + 1)*(9*d^2/a
^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/
1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*
b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/1
62*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*
d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)*
a^2*d + 2916*d^2 - 10368*c*e)/a^4)) + (162*b*d*x^4 + 162*a*d*x - (a^2*b*x^4
+ a^3*x)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^
6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3
- 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*
(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b
^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 -
a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2) - 3*sqrt(1/3)*(a^2*b*x^4 + a^3*x)*sqrt(
-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/16
2*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d
*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(
3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 +
8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)
/(a^7*b))^(1/3) + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d
^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*
b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 -
a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2
- 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)
/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)*a^2*d + 2
916*d^2 - 10368*c*e)/a^4))*log(1/324*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2
- 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2
*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^
2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 -
8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a
^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^5*b*c +
9*a*b*c*d^2 - 16*a*b*c^2*e - 3*a^2*d*e^2 - 1/18*(6*a^3*b*c*d - a^4*e^2)*((-
I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*
d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a
*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) +
1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^
2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7
*b))^(1/3) + 54*d/a^2) - 4*(8*b^2*c^3 - a^2*e^3)*x - 1/108*sqrt(1/3)*(((-I*

```
sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^
2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b
)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)
*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*
e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b
))^(1/3) + 54*d/a^2)*a^5*b*c - 54*a^3*b*c*d - 18*a^4*e^2)*sqrt(-(((-I*sqrt(
3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8
*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^
7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/
27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 -
 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1
/3) + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/
a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*
a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a
^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/
a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4
/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)*a^2*d + 2916*d^2 - 10
368*c*e)/a^4)) - 324*(b*d*x^4 + a*d*x)*log(x))/(a^2*b*x^4 + a^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx = -\frac{4\,bcx^3-aex^2-adx+3\,ac}{3\,(a^2bx^4+a^3x)} + \frac{d\log(x)}{a^2} - \frac{2\,\sqrt{3}\Big(2\,bc\big(\frac{a}{b}\big)^{\frac{2}{3}} - ae\big(\frac{a}{b}\big)^{\frac{1}{3}}\Big)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3} - \frac{\Big(3\,bd\big(\frac{a}{b}\big)^{\frac{2}{3}} + 2\,bc\big(\frac{a}{b}\big)^{\frac{1}{3}} + ae\Big)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\Big(3\,bd\big(\frac{a}{b}\big)^{\frac{2}{3}} - 4\,bc\big(\frac{a}{b}\big)^{\frac{1}{3}} - 2\,ae\Big)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
[Out] -1/3*(4*b*c*x^3 - a*e*x^2 - a*d*x + 3*a*c)/(a^2*b*x^4 + a^3*x) + d*log(x)/a
^2 - 2/9*sqrt(3)*(2*b*c*(a/b)^(2/3) - a*e*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^3 - 1/9*(3*b*d*(a/b)^(2/3) + 2*b*c*(a/b)^
(1/3) + a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1
/9*(3*b*d*(a/b)^(2/3) - 4*b*c*(a/b)^(1/3) - 2*a*e)*log(x + (a/b)^(1/3))/(a^
2*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx = -\frac{d\log(|bx^3+a|)}{3\,a^2} + \frac{d\log(|x|)}{a^2} + \frac{2\,\sqrt{3}\Big(\left(-ab^2\right)^{\frac{1}{3}}ae + 2\left(-ab^2\right)^{\frac{2}{3}}c\Big)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} - \frac{4\,bcx^3-aex^2-adx+3\,ac}{3\,(bx^4+ax)a^2} + \frac{\Big(\left(-ab^2\right)^{\frac{1}{3}}ae - 2\left(-ab^2\right)^{\frac{2}{3}}c\Big)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3b} + \frac{2\left(2\,a^2b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b}$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^2 + d*log(abs(x))/a^2 + 2/9*sqrt(3)*((-a*b^2)^
(1/3)*a*e + 2*(-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a
/b)^(1/3))/(a^3*b) - 1/3*(4*b*c*x^3 - a*e*x^2 - a*d*x + 3*a*c)/((b*x^4 + a*
x)*a^2) + 1/9*((-a*b^2)^(1/3)*a*e - 2*(-a*b^2)^(2/3)*c)*log(x^2 + x*(-a/b)^
(1/3) + (-a/b)^(2/3))/(a^3*b) + 2/9*(2*a^2*b^2*c*(-a/b)^(1/3) - a^3*b*e)*(-
a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b)
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.11

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^2}\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(-\mathrm{root}\left(729\,a^7\,b\,z^3+729\,a^5\,b\,d\,z^2-216\,a^3\,b\,c\,e\,z+243\,a^3\,b\,d^2\,z-72\,a\,b\,c\,d\,e+27\,a\,b\,d^3-8\,a^2\,e^3-64\,b\right.\right.\right.$$
$$\left.+\frac{4\left(3\,c\,b^3\,d^2+a\,b^2\,d\,e^2\right)}{9\,a^4}+\frac{4\,x\left(2\,a^2\,b^2\,e^3+12\,d\,a\,b^3\,c\,e+16\,b^4\,c^3\right)}{27\,a^5}\right)\mathrm{root}\left(729\,a^7\,b\,z^3\right.$$
$$\left.\left.+729\,a^5\,b\,d\,z^2-216\,a^3\,b\,c\,e\,z+243\,a^3\,b\,d^2\,z-72\,a\,b\,c\,d\,e+27\,a\,b\,d^3-8\,a^2\,e^3-64\,b^2\,c^3,z,k\right)\right)$$
$$-\frac{\frac{c}{a}-\frac{e\,x^2}{3\,a}-\frac{d\,x}{3\,a}+\frac{4\,b\,c\,x^3}{3\,a^2}}{b\,x^4+a\,x}+\frac{d\,\ln\left(x\right)}{a^2}$$

```
[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] symsum(log((4*(3*b^3*c*d^2 + a*b^2*d*e^2))/(9*a^4) - root(729*a^7*b*z^3 + 7
29*a^5*b*d*z^2 - 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e + 27*a*b*
d^3 - 8*a^2*e^3 - 64*b^2*c^3, z, k)*(root(729*a^7*b*z^3 + 729*a^5*b*d*z^2 -
 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e + 27*a*b*d^3 - 8*a^2*e^3
- 64*b^2*c^3, z, k)*(4*b^3*c + 24*b^3*d*x + 36*root(729*a^7*b*z^3 + 729*a^5
*b*d*z^2 - 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e + 27*a*b*d^3 -
8*a^2*e^3 - 64*b^2*c^3, z, k)*a^2*b^3*x) + (4*(a^3*b^2*e^2 - 6*a^2*b^3*c*d)
)/(9*a^4) + (4*x*(27*a^3*b^3*d^2 - 60*a^3*b^3*c*e))/(27*a^5)) + (4*x*(16*b^
4*c^3 + 2*a^2*b^2*e^3 + 12*a*b^3*c*d*e))/(27*a^5))*root(729*a^7*b*z^3 + 729
*a^5*b*d*z^2 - 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e + 27*a*b*d^
3 - 8*a^2*e^3 - 64*b^2*c^3, z, k), k, 1, 3) - (c/a - (e*x^2)/(3*a) - (d*x)/
(3*a) + (4*b*c*x^3)/(3*a^2))/(a*x + b*x^4) + (d*log(x))/a^2
```

3.349 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 242

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)^2}\,dx = -\frac{c}{2a^2x^2}-\frac{d}{a^2x}-\frac{x(bc+bdx+bex^2)}{3a^2(a+bx^3)} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c+4\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}} + \frac{e\log(x)}{a^2}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{e\log(a+bx^3)}{3a^2}$$

```
[Out] -1/2*c/a^2/x^2-d/a^2/x-1/3*x*(b*e*x^2+b*d*x+b*c)/a^2/(b*x^3+a)+e*ln(x)/a^2-
1/9*b^(1/3)*(5*b^(1/3)*c-4*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)+1/18*b^
(1/3)*(5*b^(1/3)*c-4*a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a
^(8/3)-1/3*e*ln(b*x^3+a)/a^2+1/9*b^(1/3)*(5*b^(1/3)*c+4*a^(1/3)*d)*arctan(1
/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2}{x^3 (a+bx^3)^2} \, dx = \frac{\sqrt[3]{b} \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(4\sqrt[3]{a}d+5\sqrt[3]{b}c\right)}{3\sqrt{3}a^{8/3}} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}} - \frac{x(bc+bdx+bex^2)}{3a^2(a+bx^3)} - \frac{e\log(a+bx^3)}{3a^2} - \frac{c}{2a^2x^2} - \frac{d}{a^2x} + \frac{e\log(x)}{a^2}$$

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/2*c/(a^2*x^2) - d/(a^2*x) - (x*(b*c + b*d*x + b*e*x^2))/(3*a^2*(a + b*x^
3)) + (b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) + (e*Log[x])/a^2 - (b^(1/3)*(5*b^(1/
3)*c - 4*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)) + (b^(1/3)*(5*b^(
1/3)*c - 4*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a
^(8/3)) - (e*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = -\frac{x(bc+bdx+bex^2)}{3a^2(a+bx^3)} - \frac{\int \frac{-3bc-3bdx-3bex^2+\frac{2b^2cx^3}{a}+\frac{b^2dx^4}{a}}{x^3(a+bx^3)}\,dx}{3ab}$$

$$= -\frac{x(bc+bdx+bex^2)}{3a^2(a+bx^3)} - \frac{\int \left(-\frac{3bc}{ax^3}-\frac{3bd}{ax^2}-\frac{3be}{ax}+\frac{b^2(5c+4dx+3ex^2)}{a(a+bx^3)}\right)dx}{3ab}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc+bdx+bex^2)}{3a^2(a+bx^3)} + \frac{e\log(x)}{a^2} - \frac{b\int \frac{5c+4dx+3ex^2}{a+bx^3}\,dx}{3a^2}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc+bdx+bex^2)}{3a^2(a+bx^3)} + \frac{e\log(x)}{a^2} - \frac{b\int \frac{5c+4dx}{a+bx^3}\,dx}{3a^2} - \frac{(be)\int \frac{x^2}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc+bdx+bex^2)}{3a^2(a+bx^3)} + \frac{e\log(x)}{a^2} - \frac{e\log(a+bx^3)}{3a^2} - \frac{b^{2/3}\int \frac{\sqrt[3]{a}\left(10\sqrt[3]{b}c+4\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-5\sqrt[3]{b}c+4\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{8/3}} - \frac{\left(b\left(5c-\frac{4\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{8/3}}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc+bdx+bex^2)}{3a^2(a+bx^3)} + \frac{e\log(x)}{a^2} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}} - \frac{e\log(a+bx^3)}{3a^2} + \frac{\left(\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{8/3}} - \frac{\left(b^{2/3}\left(5\sqrt[3]{b}c+4\sqrt[3]{a}d\right)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{7/3}}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc+bdx+bex^2)}{3a^2(a+bx^3)} + \frac{e\log(x)}{a^2} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}} - \frac{e\log(a+bx^3)}{3a^2} - \frac{\left(\sqrt[3]{b}\left(5\sqrt[3]{b}c+4\sqrt[3]{a}d\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{8/3}}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc+bdx+bex^2)}{3a^2(a+bx^3)} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c+4\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}} + \frac{e\log(x)}{a^2} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}} - \frac{e\log(a+bx^3)}{3a^2}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.91

$$\int\frac{c+dx+ex^2}{x^3(a+bx^3)^2}\,dx = \frac{-\frac{9ac}{x^2}-\frac{18ad}{x}+\frac{6a(ae-bx(c+dx))}{a+bx^3}+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(5\sqrt[3]{b}c+4\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+18ae\log(x)+2\sqrt[3]{b}\left(-5\sqrt[3]{\right.}}{1}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a*c)/x^2 - (18*a*d)/x + (6*a*(a*e - b*x*(c + d*x)))/(a + b*x^3) + 2*Sq
rt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)
/a^(1/3))/Sqrt[3]] + 18*a*e*Log[x] + 2*b^(1/3)*(-5*a^(1/3)*b^(1/3)*c + 4*a^
(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(5*a^(1/3)*b^(1/3)*c - 4*a^(2/3
)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 6*a*e*Log[a + b*x^3])
/(18*a^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.61 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.03

method	result
risch	$\frac{-\frac{4bd\,x^4}{3a^2}-\frac{5bc\,x^3}{6a^2}+\frac{e\,x^2}{3a}-\frac{xd}{a}-\frac{c}{2a}}{x^2(b\,x^3+a)}+\frac{e\ln(-x)}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^8\_Z^3+9a^6e\_Z^2+\left(27a^4e^2+60a^3bcd\right)\_Z+27a^2e^3+180abcde-64ab\,d^3+1\ldots\right.}\right.}{\ldots}$
default	$-\frac{c}{2a^2x^2}-\frac{d}{a^2x}+\frac{e\ln(x)}{a^2}-\frac{b\left(\frac{\frac{d\,x^2}{3}+\frac{cx}{3}-\frac{ae}{3b}}{b\,x^3+a}+\frac{5c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}+\frac{4a\ldots}{\ldots}\right.}{a^2}$

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-4/3*b*d/a^2*x^4-5/6*b*c/a^2*x^3+1/3/a*e*x^2-1/a*x*d-1/2*c/a)/x^2/(b*x^3+a
)+1/a^2*e*ln(-x)+1/9*sum(_R*ln((-4*_R^3*a^8-24*_R^2*a^6*e+(-36*a^4*e^2-200*
a^3*b*c*d)*_R-360*a*b*c*d*e+192*a*b*d^3-375*b^2*c^3)*x-4*a^6*d*_R^2+(24*a^4
*d*e-25*a^3*b*c^2)*_R+108*a^2*d*e^2+225*a*b*c^2*e),_R=RootOf(a^8*_Z^3+9*a^6
*e*_Z^2+(27*a^4*e^2+60*a^3*b*c*d)*_Z+27*a^2*e^3+180*a*b*c*d*e-64*a*b*d^3+12
5*b^2*c^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 4774, normalized size of antiderivative = 19.73

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/324*(432*b*d*x^4 + 270*b*c*x^3 - 108*a*e*x^2 + 324*a*d*x + 2*(a^2*b*x^5
+ a^3*x^2)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*
e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*
b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^
(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^
7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3
- 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*log(1/81*((-I*sqrt(3) +
 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d
 + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c
^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1
)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 6
4*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a
*b)/a^8)^(1/3) + 54*e/a^2)^2*a^6*d + 160*a*b*c*d^2 - 75*a*b*c^2*e + 36*a^2*
d*e^2 + 1/18*(25*a^3*b*c^2 - 24*a^4*d*e)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20
*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 +
1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*
(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1
/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1
458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*
e/a^2) + (125*b^2*c^3 + 64*a*b*d^3)*x) + 162*a*c + (162*b*e*x^5 + 162*a*e*x
^2 - (a^2*b*x^5 + a^3*x^2)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e
^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*
c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c
*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d
 + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c
^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2) - 3*sqr
t(1/3)*(a^2*b*x^5 + a^3*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*
d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/145
8*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d
^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*
(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(
125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2
)^2*a^5 - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/
27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^
3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^
```

8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e
/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e
^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^3*e + 25920*b*c*d
 + 2916*a*e^2)/a^5))*log(-1/81*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*
a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125
*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 4
5*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*
c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^
2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)^2*a^
6*d - 160*a*b*c*d^2 + 75*a*b*c^2*e - 36*a^2*d*e^2 - 1/18*(25*a^3*b*c^2 - 24
*a^4*d*e)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e
^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b
/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(
1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7
 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 -
 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2) + 2*(125*b^2*c^3 + 64*a*
b*d^3)*x + 1/54*sqrt(1/3)*(2*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a
*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*
b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45
*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c
*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2
*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^6*d
 - 225*a^3*b*c^2 - 108*a^4*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b
*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/
1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(1
6*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/1
62*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/145
8*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/
a^2)^2*a^5 - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(
-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a
*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)
/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2
)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^
2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^3*e + 25920*b*c
*d + 2916*a*e^2)/a^5)) + (162*b*e*x^5 + 162*a*e*x^2 - (a^2*b*x^5 + a^3*x^2)
*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 +
1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/
1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81
*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458
*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^
3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2) + 3*sqrt(1/3)*(a^2*b*x^5 + a^3*x^
2)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e
^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b
/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(
1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7

+ 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)^2*a^5 - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^3*e + 25920*b*c*d + 2916*a*e^2)/a^5))*log(-1/81*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)^2*a^6*d - 160*a*b*c*d^2 + 75*a*b*c^2*e - 36*a^2*d*e^2 - 1/18*(25*a^3*b*c^2 - 24*a^4*d*e)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2) + 2*(125*b^2*c^3 + 64*a*b*d^3)*x - 1/54*sqrt(1/3)*(2*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^6*d - 225*a^3*b*c^2 - 108*a^4*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)^2*a^5 - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^3*e + 25920*b*c*d + 2916*a*e^2)/a^5)) - 324*(b*e*x^5 + a*e*x^2)*log(x))/(a^2*b*x^5 + a^3*x^2)

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.91

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{8\,bdx^4+5\,bcx^3-2\,aex^2+6\,adx+3\,ac}{6\,\left(a^2bx^5+a^3x^2\right)}+\frac{e\log\left(x\right)}{a^2} - \frac{\sqrt{3}\left(4\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3} - \frac{\left(6\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(3\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
[Out] -1/6*(8*b*d*x^4 + 5*b*c*x^3 - 2*a*e*x^2 + 6*a*d*x + 3*a*c)/(a^2*b*x^5 + a^3
*x^2) + e*log(x)/a^2 - 1/9*sqrt(3)*(4*b*d*(a/b)^(2/3) + 5*b*c*(a/b)^(1/3))*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^3 - 1/18*(6*e*(a/b)^(
2/3) + 4*d*(a/b)^(1/3) - 5*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*(
a/b)^(2/3)) - 1/9*(3*e*(a/b)^(2/3) - 4*d*(a/b)^(1/3) + 5*c)*log(x + (a/b)^(
1/3))/(a^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{e\log\left(|bx^3+a|\right)}{3\,a^2} + \frac{e\log\left(|x|\right)}{a^2} - \frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}bc - 4\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} - \frac{\left(5\left(-ab^2\right)^{\frac{1}{3}}bc + 4\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b} + \frac{\left(4\,a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2b^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b} - \frac{8\,bdx^4+5\,bcx^3-2\,aex^2+6\,adx+3\,ac}{6\,(bx^3+a)a^2x^2}$$

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^2 + e*log(abs(x))/a^2 - 1/9*sqrt(3)*(5*(-a*b^2
)^(1/3)*b*c - 4*(-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/(a^3*b) - 1/18*(5*(-a*b^2)^(1/3)*b*c + 4*(-a*b^2)^(2/3)*d)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b) + 1/9*(4*a^2*b^2*d*(-a/b)^(1/
3) + 5*a^2*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b) - 1/6*(8*
b*d*x^4 + 5*b*c*x^3 - 2*a*e*x^2 + 6*a*d*x + 3*a*c)/((b*x^3 + a)*a^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 733, normalized size of antiderivative = 3.03

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx = \left(\sum_{k=1}^{3}\ln\left(-\frac{b^3\left(\mathrm{root}(729\,a^8\,z^3+729\,a^6\,e\,z^2+540\,a^3\,b\,c\,d\,z+243\,a^4\,e^2\,z+180\,a\,b\,c\,d\,e-64\,a\,b\,d^3+27\,a^2\,e^3\right.}{\cdots}\right.\right.$$
$$+729\,a^6\,e\,z^2+540\,a^3\,b\,c\,d\,z+243\,a^4\,e^2\,z+180\,a\,b\,c\,d\,e-64\,a\,b\,d^3+27\,a^2\,e^3$$
$$\left.\left.+125\,b^2\,c^3,z,k\right)\right) - \frac{\frac{c}{2\,a}-\frac{e\,x^2}{3\,a}+\frac{d\,x}{a}+\frac{5\,b\,c\,x^3}{6\,a^2}+\frac{4\,b\,d\,x^4}{3\,a^2}}{b\,x^5+a\,x^2} + \frac{e\,\ln\left(x\right)}{a^2}$$

```
[In] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] symsum(log(-(b^3*(108*root(729*a^8*z^3 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z +
243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k
)^2*a^6*d - 36*a^2*d*e^2 + 972*root(729*a^8*z^3 + 729*a^6*e*z^2 + 540*a^3*b
*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 27*a^2*e^3 + 125*b^2*
c^3, z, k)^3*a^8*x + 125*b^2*c^3*x - 72*root(729*a^8*z^3 + 729*a^6*e*z^2 +
540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 27*a^2*e^3 +
 125*b^2*c^3, z, k)*a^4*d*e - 75*a*b*c^2*e - 64*a*b*d^3*x + 75*root(729*a^8
*z^3 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64
*a*b*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k)*a^3*b*c^2 + 108*root(729*a^8*z^3
 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b
*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k)*a^4*e^2*x + 648*root(729*a^8*z^3 + 7
29*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3
 + 27*a^2*e^3 + 125*b^2*c^3, z, k)^2*a^6*e*x + 600*root(729*a^8*z^3 + 729*a
^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 2
7*a^2*e^3 + 125*b^2*c^3, z, k)*a^3*b*c*d*x + 120*a*b*c*d*e*x))/(27*a^6))*ro
ot(729*a^8*z^3 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*
c*d*e - 64*a*b*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k), k, 1, 3) - (c/(2*a) -
 (e*x^2)/(3*a) + (d*x)/a + (5*b*c*x^3)/(6*a^2) + (4*b*d*x^4)/(3*a^2))/(a*x^
2 + b*x^5) + (e*log(x))/a^2
```

3.350 $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 262

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^2}\,dx = -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2(a+bx^3)} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}} - \frac{2bc\log(x)}{a^3}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}} + \frac{2bc\log(a+bx^3)}{3a^3}$$

```
[Out] -1/3*c/a^2/x^3-1/2*d/a^2/x^2-e/a^2/x-1/3*x*(b*d+b*e*x-b^2*c*x^2/a)/a^2/(b*x
^3+a)-2*b*c*ln(x)/a^3-1/9*b^(1/3)*(5*b^(1/3)*d-4*a^(1/3)*e)*ln(a^(1/3)+b^(1
/3)*x)/a^(8/3)+1/18*b^(1/3)*(5*b^(1/3)*d-4*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/a^(8/3)+2/3*b*c*ln(b*x^3+a)/a^3+1/9*b^(1/3)*(5*b^(1/3)
*d+4*a^(1/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)*3
^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2}{x^4 (a+bx^3)^2} \, dx = \frac{\sqrt[3]{b} \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)}{3\sqrt{3}a^{8/3}} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}} + \frac{2bc\log(a+bx^3)}{3a^3} - \frac{2bc\log(x)}{a^3} - \frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{3a^2(a+bx^3)} - \frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x}$$

```
[In] Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -1/3*c/(a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d + b*e*x - (b^2*c*x^2
)/a))/(3*a^2*(a + b*x^3)) + (b^(1/3)*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) - (2*b*c*Log[x
])/a^3 - (b^(1/3)*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*
a^(8/3)) + (b^(1/3)*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(18*a^(8/3)) + (2*b*c*Log[a + b*x^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2(a+bx^3)} - \frac{\int \frac{-3bc-3bdx-3bex^2+\frac{3b^2cx^3}{a}+\frac{2b^2dx^4}{a}+\frac{b^2ex^5}{a}}{x^4(a+bx^3)}\,dx}{3ab}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2(a+bx^3)} - \frac{\int\left(-\frac{3bc}{ax^4}-\frac{3bd}{ax^3}-\frac{3be}{ax^2}+\frac{6b^2c}{a^2x}+\frac{b^2(5ad+4aex-6bcx^2)}{a^2(a+bx^3)}\right)dx}{3ab}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2(a+bx^3)}-\frac{2bc\log(x)}{a^3}-\frac{b\int\frac{5ad+4aex-6bcx^2}{a+bx^3}\,dx}{3a^3}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2(a+bx^3)} - \frac{2bc\log(x)}{a^3}-\frac{b\int\frac{5ad+4aex}{a+bx^3}\,dx}{3a^3}+\frac{(2b^2c)\int\frac{x^2}{a+bx^3}\,dx}{a^3}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2(a+bx^3)}-\frac{2bc\log(x)}{a^3}+\frac{2bc\log(a+bx^3)}{3a^3} - \frac{b^{2/3}\int\frac{\sqrt[3]{a}\left(10a\sqrt[3]{b}d+4a^{4/3}e\right)+\sqrt[3]{b}\left(-5a\sqrt[3]{b}d+4a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{11/3}}-\frac{\left(b\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{8/3}}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2(a+bx^3)}-\frac{2bc\log(x)}{a^3} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}}+\frac{2bc\log(a+bx^3)}{3a^3} + \frac{\left(\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{8/3}} - \frac{\left(b^{2/3}\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{7/3}}$$

$$= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2(a+bx^3)} - \frac{2bc\log(x)}{a^3} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}} + \frac{2bc\log(a+bx^3)}{3a^3} - \frac{\left(\sqrt[3]{b}\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\right)\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{8/3}}$$

$$= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2(a+bx^3)} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}} - \frac{2bc\log(x)}{a^3} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}} + \frac{2bc\log(a+bx^3)}{3a^3}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.86

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{6ac}{x^3} - \frac{9ad}{x^2} - \frac{18ae}{x} - \frac{6ab(c+x(d+ex))}{a+bx^3} + 2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 36bc\log(x) + 2\sqrt[3]{b}\left(\cdots\right.}{\cdots}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a*c)/x^3 - (9*a*d)/x^2 - (18*a*e)/x - (6*a*b*(c + x*(d + e*x)))/(a + b
*x^3) + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 36*b*c*Log[x] + 2*b^(1/3)*(-5*a^(1/3)*b^(1
/3)*d + 4*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(5*a^(1/3)*b^(1/3)*
d - 4*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 12*b*c*Lo
g[a + b*x^3])/(18*a^3)
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.03

method	result
default	$-\frac{e}{a^{2}x}-\frac{c}{3a^{2}x^{3}}-\frac{d}{2a^{2}x^{2}}-\frac{2bc\ln(x)}{a^{3}}-\frac{b\left(\frac{\frac{1}{3}aex^{2}+\frac{1}{3}adx+\frac{1}{3}ac}{bx^{3}+a}+\frac{5ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\cdots\right.}{\cdots}\right.}{3b\left(\frac{a}{b}\cdots\right.}\right.}{3}\cdots\right.}{\cdots}$
risch	$\frac{-\frac{4bex^{5}}{3a^{2}}-\frac{5bdx^{4}}{6a^{2}}-\frac{2bcx^{3}}{3a^{2}}-\frac{ex^{2}}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^{3}(bx^{3}+a)}-\frac{2bc\ln(x)}{a^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{9}\_Z^{3}-18a^{6}bc\_Z^{2}+\left(60a^{4}bde+108a^{3}b^{2}c^{2}\right)\_Z-64a^{2}be^{3}-360a\cdots\right.}\cdots\right.}{\cdots}$

```
[In] int((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -e/a^2/x-1/3*c/a^2/x^3-1/2*d/a^2/x^2-2*b*c*ln(x)/a^3-1/a^3*b*((1/3*a*e*x^2+
1/3*a*d*x+1/3*a*c)/(b*x^3+a)+5/3*a*d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1
/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+4/3*a*e*(-1/3/b/(a/b)^(1/3)*ln(x
+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/
2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-2/3*c*ln(b*x^3+a)
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 5373, normalized size of antiderivative = 20.51

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{c+dx+ex^2}{x^4(a+bx^3)^2}\,dx = &-\frac{8\,bex^5+5\,bdx^4+4\,bcx^3+6\,aex^2+3\,adx+2\,ac}{6\,(a^2bx^6+a^3x^3)}-\frac{2\,bc\log(x)}{a^3}\\&-\frac{\sqrt{3}\left(4\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4}\\&+\frac{\left(12\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\&+\frac{\left(6\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
[Out] -1/6*(8*b*e*x^5 + 5*b*d*x^4 + 4*b*c*x^3 + 6*a*e*x^2 + 3*a*d*x + 2*a*c)/(a^2
*b*x^6 + a^3*x^3) - 2*b*c*log(x)/a^3 - 1/9*sqrt(3)*(4*a*e*(a/b)^(2/3) + 5*a
*d*(a/b)^(1/3))*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^4 +
 1/18*(12*b*c*(a/b)^(2/3) - 4*a*e*(a/b)^(1/3) + 5*a*d)*log(x^2 - x*(a/b)^(1
/3) + (a/b)^(2/3))/(a^3*(a/b)^(2/3)) + 1/9*(6*b*c*(a/b)^(2/3) + 4*a*e*(a/b)
^(1/3) - 5*a*d)*log(x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^2}\,dx = \frac{2\,bc\log\left(|bx^3+a|\right)}{3\,a^3} - \frac{2\,bc\log\left(|x|\right)}{a^3} - \frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}bd - 4\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} - \frac{\left(5\left(-ab^2\right)^{\frac{1}{3}}bd + 4\left(-ab^2\right)^{\frac{2}{3}}e\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b} + \frac{\left(4\,a^4b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5\,a^4b^2d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^7b} - \frac{8\,abex^5 + 5\,abdx^4 + 4\,abcx^3 + 6\,a^2ex^2 + 3\,a^2dx + 2\,a^2c}{6\left(bx^3+a\right)a^3x^3}$$

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*b*c*log(abs(b*x^3 + a))/a^3 - 2*b*c*log(abs(x))/a^3 - 1/9*sqrt(3)*(5*(-
a*b^2)^(1/3)*b*d - 4*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a^3*b) - 1/18*(5*(-a*b^2)^(1/3)*b*d + 4*(-a*b^2)^(2/3)*e
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b) + 1/9*(4*a^4*b^2*e*(-a/b
)^(1/3) + 5*a^4*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b) - 1/
6*(8*a*b*e*x^5 + 5*a*b*d*x^4 + 4*a*b*c*x^3 + 6*a^2*e*x^2 + 3*a^2*d*x + 2*a^
2*c)/((b*x^3 + a)*a^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 537, normalized size of antiderivative = 2.05

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^2}\,dx = \left(\sum_{k=1}^{3}\ln\left(-\frac{50\,b^5\,c\,d^2-48\,b^5\,c^2\,e}{9\,a^6} - \mathrm{root}\left(729\,a^9\,z^3-1458\,a^6\,b\,c\,z^2+540\,a^4\,b\,d\,e\,z+972\,a^3\,b^2\,c^2\,z-360\,a\,b^2\,c\,d\,e-64\,a^2\,b\,e^3+125\,a\,b^2\,d^3-216\,b^3 \right. \right. \right.$$
$$\left. + \frac{x\left(-125\,b^5\,d^3+240\,c\,b^5\,d\,e+64\,a\,b^4\,e^3\right)}{27\,a^6}\right)\mathrm{root}\left(729\,a^9\,z^3-1458\,a^6\,b\,c\,z^2+540\,a^4\,b\,d\,e\,z + 972\,a^3\,b^2\,c^2\,z-360\,a\,b^2\,c\,d\,e-64\,a^2\,b\,e^3+125\,a\,b^2\,d^3-216\,b^3\,c^3,z,k\right)\Bigg) - \frac{\frac{c}{3\,a}+\frac{e\,x^2}{a}+\frac{d\,x}{2\,a}+\frac{2\,b\,c\,x^3}{3\,a^2}+\frac{5\,b\,d\,x^4}{6\,a^2}+\frac{4\,b\,e\,x^5}{3\,a^2}}{b\,x^6+a\,x^3} - \frac{2\,b\,c\,\ln\left(x\right)}{a^3}$$

```
[In] int((c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x)

[Out] symsum(log((x*(64*a*b^4*e^3 - 125*b^5*d^3 + 240*b^5*c*d*e))/(27*a^6) - root
(729*a^9*z^3 - 1458*a^6*b*c*z^2 + 540*a^4*b*d*e*z + 972*a^3*b^2*c^2*z - 360
*a*b^2*c*d*e - 64*a^2*b*e^3 + 125*a*b^2*d^3 - 216*b^3*c^3, z, k)*((25*a^3*b
^4*d^2 + 48*a^3*b^4*c*e)/(9*a^6) + root(729*a^9*z^3 - 1458*a^6*b*c*z^2 + 54
0*a^4*b*d*e*z + 972*a^3*b^2*c^2*z - 360*a*b^2*c*d*e - 64*a^2*b*e^3 + 125*a*
b^2*d^3 - 216*b^3*c^3, z, k)*(4*b^3*e + 36*root(729*a^9*z^3 - 1458*a^6*b*c*
z^2 + 540*a^4*b*d*e*z + 972*a^3*b^2*c^2*z - 360*a*b^2*c*d*e - 64*a^2*b*e^3
+ 125*a*b^2*d^3 - 216*b^3*c^3, z, k)*a^2*b^3*x - (48*b^4*c*x)/a) + (x*(432*
a^2*b^5*c^2 + 600*a^3*b^4*d*e))/(27*a^6)) - (50*b^5*c*d^2 - 48*b^5*c^2*e)/(
9*a^6))*root(729*a^9*z^3 - 1458*a^6*b*c*z^2 + 540*a^4*b*d*e*z + 972*a^3*b^2
*c^2*z - 360*a*b^2*c*d*e - 64*a^2*b*e^3 + 125*a*b^2*d^3 - 216*b^3*c^3, z, k
), k, 1, 3) - (c/(3*a) + (e*x^2)/a + (d*x)/(2*a) + (2*b*c*x^3)/(3*a^2) + (5
*b*d*x^4)/(6*a^2) + (4*b*e*x^5)/(3*a^2))/(a*x^3 + b*x^6) - (2*b*c*log(x))/a
^3
```

3.351 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 215

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx = -\frac{c+dx+ex^2}{6b(a+bx^3)^2}+\frac{x(d+2ex)}{18ab(a+bx^3)} - \frac{\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{5/3}} + \frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{5/3}} - \frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}}$$

```
[Out] 1/6*(-e*x^2-d*x-c)/b/(b*x^3+a)^2+1/18*x*(2*e*x+d)/a/b/(b*x^3+a)+1/27*(b^(1/
3)*d-a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(5/3)-1/54*(d-a^(1/3)*e/b^(
1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(4/3)-1/27*(b^(1/
3)*d+a^(1/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b
^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1837, 1869, 1874, 31, 648, 631, 210, 642}

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)}{9\sqrt{3}a^{5/3}b^{5/3}} - \frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}} + \frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{5/3}} - \frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2} + \frac{x(d+2ex)}{18ab\left(a+bx^3\right)}$$

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(c + d*x + e*x^2)/(b*(a + b*x^3)^2) + (x*(d + 2*e*x))/(18*a*b*(a + b*x
^3)) - ((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(9*Sqrt[3]*a^(5/3)*b^(5/3)) + ((b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3)
+ b^(1/3)*x])/(27*a^(5/3)*b^(5/3)) - ((d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{\int \frac{d+2ex}{\left(a+bx^3\right)^2}\,dx}{6b}\\
&= -\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{x(d+2ex)}{18ab\left(a+bx^3\right)}-\frac{\int \frac{-2d-2ex}{a+bx^3}\,dx}{18ab}\\
&= -\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{x(d+2ex)}{18ab\left(a+bx^3\right)}\\
&\quad -\frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}d-2\sqrt[3]{a}e\right)+\sqrt[3]{b}\left(2\sqrt[3]{b}d-2\sqrt[3]{a}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{5/3}b^{4/3}}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{5/3}b}
\end{aligned}$$

$$= -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{x(d+2ex)}{18ab(a+bx^3)} + \frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}} + \frac{\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{18a^{4/3}b^{4/3}} - \frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{54a^{5/3}b^{4/3}}$$

$$= -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{x(d+2ex)}{18ab(a+bx^3)} + \frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}} - \frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}} + \frac{\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\text{Subst}\left(\int\frac{1}{-3-x^2}dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{5/3}b^{5/3}}$$

$$= -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{x(d+2ex)}{18ab(a+bx^3)} - \frac{\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{5/3}} + \frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}} - \frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.92

$$\int\frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}dx$$

$$= \frac{\frac{3b^{2/3}x(d+2ex)}{a(a+bx^3)} - \frac{9b^{2/3}(c+x(d+ex))}{(a+bx^3)^2} - \frac{2\sqrt{3}\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{2\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}} + \frac{\left(-\sqrt[3]{b}d+\sqrt[3]{\ldots}\right.}{}}{54b^{5/3}}$$

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] ((3*b^(2/3)*x*(d + 2*e*x))/(a*(a + b*x^3)) - (9*b^(2/3)*(c + x*(d + e*x)))/
(a + b*x^3)^2 - (2*Sqrt[3]*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*(b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(
1/3)*x])/a^(5/3) + ((-(b^(1/3)*d) + a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/a^(5/3))/(54*b^(5/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{e\,x^{5}}{9a}+\frac{d\,x^{4}}{18a}-\frac{e\,x^{2}}{18b}-\frac{dx}{9b}-\frac{c}{6b}}{\left(b\,x^{3}+a\right)^{2}}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(e\_R+d\right)\ln\left(x-\_R\right)}{\_R^{2}}}{27a\,b^{2}}$
default	$\frac{\frac{e\,x^{5}}{9a}+\frac{d\,x^{4}}{18a}-\frac{e\,x^{2}}{18b}-\frac{dx}{9b}-\frac{c}{6b}}{\left(b\,x^{3}+a\right)^{2}}+\frac{d\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+e\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln(\ldots}{\ldots}\right.}{9ba}$

[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] (1/9/a*e*x^5+1/18*d/a*x^4-1/18*e*x^2/b-1/9*d*x/b-1/6*c/b)/(b*x^3+a)^2+1/27/a/b^2*sum((_R*e+d)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 2163, normalized size of antiderivative = 10.06

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/108*(12*b*e*x^5 + 6*b*d*x^4 - 6*a*e*x^2 - 12*a*d*x - 2*(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^4*b^3*e - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*a^2*b^2*d^2 + 2*a*d*e^2 + (b

*d^3 + a*e^3)*x) - 18*a*c + ((a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^
(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b
^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3))) + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b
^2*x^3 + a^3*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b
^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)
/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2
*a^3*b^3 + 16*d*e)/(a^3*b^3)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^
3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e
*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^
5*b^5))^(1/3)))^2*a^4*b^3*e + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*
e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*s
qrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5)
)^(1/3)))*a^2*b^2*d^2 - 2*a*d*e^2 + 2*(b*d^3 + a*e^3)*x + 3/4*sqrt(1/3)*(((
1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^
5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3
)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*a^4*b^3*e + 2*a^2*b^2*d^2)
sqrt(-(((1/2)^(1/3)(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 -
a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d
^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^3*b^3 + 16*d
*e)/(a^3*b^3))) + ((a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*((1/2)^(1/3)*(I*sqrt
(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*
(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d
^3 - a*e^3)/(a^5*b^5))^(1/3))) - 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 + a
^3*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d
^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*
((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^3*b^3 +
16*d*e)/(a^3*b^3)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)
/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(
3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1
/3)))^2*a^4*b^3*e + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*
b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1
)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*
a^2*b^2*d^2 - 2*a*d*e^2 + 2*(b*d^3 + a*e^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)
*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1
/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5
) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*a^4*b^3*e + 2*a^2*b^2*d^2)*sqrt(-(((
1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^
5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3
)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^3*b^3 + 16*d*e)/(a^3*b
^3))))/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)

Sympy [A] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.69

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx$$
$$=\mathrm{RootSum}\left(19683t^3a^5b^5+81ta^2b^2de+ae^3-bd^3,\left(t\mapsto t\log\left(x+\frac{729t^2a^4b^3e+27ta^2b^2d^2+2ade^2}{ae^3+bd^3}\right)\right)\right)$$
$$+\frac{-3ac-2adx-aex^2+bdx^4+2bex^5}{18a^3b+36a^2b^2x^3+18ab^3x^6}$$

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**5*b**5 + 81*_t*a**2*b**2*d*e + a*e**3 - b*d**3, Lamb
da(_t, _t*log(x + (729*_t**2*a**4*b**3*e + 27*_t*a**2*b**2*d**2 + 2*a*d*e**
2)/(a*e**3 + b*d**3)))) + (-3*a*c - 2*a*d*x - a*e*x**2 + b*d*x**4 + 2*b*e*x
**5)/(18*a**3*b + 36*a**2*b**2*x**3 + 18*a*b**3*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx=\frac{2\,bex^5+bdx^4-aex^2-2\,adx-3\,ac}{18\,(ab^3x^6+2\,a^2b^2x^3+a^3b)}$$
$$+\frac{\sqrt{3}\left(e\left(\frac{a}{b}\right)^{\frac{1}{3}}+d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(e\left(\frac{a}{b}\right)^{\frac{1}{3}}-d\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\left(e\left(\frac{a}{b}\right)^{\frac{1}{3}}-d\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*b*e*x^5 + b*d*x^4 - a*e*x^2 - 2*a*d*x - 3*a*c)/(a*b^3*x^6 + 2*a^2*b
^2*x^3 + a^3*b) + 1/27*sqrt(3)*(e*(a/b)^(1/3) + d)*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) + 1/54*(e*(a/b)^(1/3) - d)*
log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) - 1/27*(e*(a/b)^
(1/3) - d)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}\left(bd-(-ab^2)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}ab} - \frac{\left(bd+(-ab^2)^{\frac{1}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}ab} - \frac{\left(e\left(-\frac{a}{b}\right)^{\frac{1}{3}}+d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^2b} + \frac{2bex^5+bdx^4-aex^2-2adx-3ac}{18\left(bx^3+a\right)^2ab}$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(b*d - (-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/54*(b*d + (-a*b^2)^(1/3)*e)*log(
x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b) - 1/27*(e*(-a/b)^
(1/3) + d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) + 1/18*(2*b*e*x^
5 + b*d*x^4 - a*e*x^2 - 2*a*d*x - 3*a*c)/((b*x^3 + a)^2*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx = \left(\sum_{k=1}^{3}\ln\left(\frac{d\,e+e^2\,x+\mathrm{root}(19683\,a^5\,b^5\,z^3+81\,a^2\,b^2\,d\,e\,z+a\,e^3-b\,d^3,z,k)^2\,a^3\,b^3\,729+\mathrm{root}(19683\,a^5\,b^5\,z^3 + 81\,a^2\,b^2\,d\,e\,z+a\,e^3-b\,d^3,z,k)\right)\right) - \frac{\frac{c}{6\,b}-\frac{d\,x^4}{18\,a}-\frac{e\,x^5}{9\,a}+\frac{e\,x^2}{18\,b}+\frac{d\,x}{9\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x)
```

```
[Out] symsum(log((d*e + e^2*x + 729*root(19683*a^5*b^5*z^3 + 81*a^2*b^2*d*e*z + a
*e^3 - b*d^3, z, k)^2*a^3*b^3 + 27*root(19683*a^5*b^5*z^3 + 81*a^2*b^2*d*e*
z + a*e^3 - b*d^3, z, k)*a*b^2*d*x)/(81*a^2*b))*root(19683*a^5*b^5*z^3 + 81
*a^2*b^2*d*e*z + a*e^3 - b*d^3, z, k), k, 1, 3) - (c/(6*b) - (d*x^4)/(18*a)
 - (e*x^5)/(9*a) + (e*x^2)/(18*b) + (d*x)/(9*b))/(a^2 + b^2*x^6 + 2*a*b*x^3
)
```

3.352 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 239

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3} dx = -\frac{x(ae-bcx-bdx^2)}{6ab(a+bx^3)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b(a+bx^3)} - \frac{(2b^{2/3}c+a^{2/3}e)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{4/3}} - \frac{(2b^{2/3}c-a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{4/3}} + \frac{(2b^{2/3}c-a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{4/3}}$$

```
[Out] -1/6*x*(-b*d*x^2-b*c*x+a*e)/a/b/(b*x^3+a)^2+1/18*(-3*a*d+x*(4*b*c*x+a*e))/a
^2/b/(b*x^3+a)-1/27*(2*b^(2/3)*c-a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b
^(4/3)+1/54*(2*b^(2/3)*c-a^(2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/a^(7/3)/b^(4/3)-1/27*(2*b^(2/3)*c+a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1842, 1868, 1874, 31, 648, 631, 210, 642}

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{2/3}e+2b^{2/3}c\right)}{9\sqrt{3}a^{7/3}b^{4/3}} + \frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{4/3}} - \frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{4/3}} - \frac{3ad-x(ae+4bcx)}{18a^2b\,(a+bx^3)} - \frac{x(ae-bcx-bdx^2)}{6ab\,(a+bx^3)^2}$$

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(x*(a*e - b*c*x - b*d*x^2))/(a*b*(a + b*x^3)^2) - (3*a*d - x*(a*e + 4*
b*c*x))/(18*a^2*b*(a + b*x^3)) - ((2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(4/3)) - ((2*b^(2/
3)*c - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(4/3)) + ((2*b^(2
/3)*c - a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7
/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x(ae-bcx-bdx^2)}{6ab\,(a+bx^3)^2}-\frac{\int \frac{-ae-4bcx-3bdx^2}{(a+bx^3)^2}\,dx}{6ab}\\ &= -\frac{x(ae-bcx-bdx^2)}{6ab\,(a+bx^3)^2}-\frac{3ad-x(ae+4bcx)}{18a^2b\,(a+bx^3)}+\frac{\int \frac{2ae+4bcx}{a+bx^3}\,dx}{18a^2b}\end{aligned}$$

$$= -\frac{x(ae-bcx-bdx^2)}{6ab\left(a+bx^3\right)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)} + \frac{\int \frac{\sqrt[3]{a}\left(4\sqrt[3]{a}bc+4a\sqrt[3]{b}e\right)+\sqrt[3]{b}\left(4\sqrt[3]{a}bc-2a\sqrt[3]{b}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{54a^{8/3}b^{4/3}} - \frac{\left(2b^{2/3}c-a^{2/3}e\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{27a^{7/3}b}$$

$$= -\frac{x(ae-bcx-bdx^2)}{6ab\left(a+bx^3\right)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)} - \frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{4/3}} + \frac{\left(2b^{2/3}c-a^{2/3}e\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{54a^{7/3}b^{4/3}} + \frac{\left(2b^{2/3}c+a^{2/3}e\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{18a^2b}$$

$$= -\frac{x(ae-bcx-bdx^2)}{6ab\left(a+bx^3\right)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)} - \frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{4/3}} + \frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{4/3}} + \frac{\left(2b^{2/3}c+a^{2/3}e\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{7/3}b^{4/3}}$$

$$= -\frac{x(ae-bcx-bdx^2)}{6ab\left(a+bx^3\right)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)} - \frac{\left(2b^{2/3}c+a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{4/3}} - \frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{4/3}} + \frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{4/3}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.90

$$\int \frac{x(c+dx+ex^2)}{\left(a+bx^3\right)^3}dx$$

$$= \frac{\frac{3ab^{2/3}\left(4b^2cx^5-a^2(3d+2ex)+abx^2(7c+ex^2)\right)}{\left(a+bx^3\right)^2} - 2\sqrt{3}a^{2/3}\sqrt[3]{b}\left(2b^{2/3}c+a^{2/3}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\left(-2a^{2/3}bc+a^{4/3}\sqrt[3]{}\right.}{54a^3b^{5/3}}$$

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
[Out] ((3*a*b^(2/3)*(4*b^2*c*x^5 - a^2*(3*d + 2*e*x) + a*b*x^2*(7*c + e*x^2)))/(a
 + b*x^3)^2 - 2*Sqrt[3]*a^(2/3)*b^(1/3)*(2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-2*a^(2/3)*b*c + a^(4/3)*b^(1/3)*e)
*Log[a^(1/3) + b^(1/3)*x] + (2*a^(2/3)*b*c - a^(4/3)*b^(1/3)*e)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^3*b^(5/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{2bc\,x^{5}}{9a^{2}}+\frac{e\,x^{4}}{18a}+\frac{7c\,x^{2}}{18a}-\frac{ex}{9b}-\frac{d}{6b}}{\left(b\,x^{3}+a\right)^{2}}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(\frac{2c\_R}{a}+\frac{e}{b}\right)\ln\left(x-\_R\right)}{\_R^{2}}}{27ba}$
default	$\frac{\frac{2bc\,x^{5}}{9a^{2}}+\frac{e\,x^{4}}{18a}+\frac{7c\,x^{2}}{18a}-\frac{ex}{9b}-\frac{d}{6b}}{\left(b\,x^{3}+a\right)^{2}}+\frac{ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+2bc\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{9a^{2}b}$

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2/9*b*c/a^2*x^5+1/18/a*e*x^4+7/18*c/a*x^2-1/9*e*x/b-1/6*d/b)/(b*x^3+a)^2+1
/27/b/a*sum((2*c/a*_R+1/b*e)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 2519, normalized size of antiderivative = 10.54

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx=\text{Too large to display}$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(24*b^2*c*x^5 + 6*a*b*e*x^4 + 42*a*b*c*x^2 - 12*a^2*e*x - 18*a^2*d -
2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^
2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/
2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8
*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)
*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)
+ 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^
4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^5*b^3*c - 1/2*((1/2)^(1/3
)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/
(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 +
```

```
a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*a^4*b*e^2 +
8*a*b*c^2*e + (8*b^2*c^3 + a^2*e^3)*x) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^
4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2
*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b
^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3
))) + 3*sqrt(1/3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)
*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(
a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 +
a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2 + 3
2*c*e)/(a^4*b^2)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*
e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e
*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a
^2*e^3)/(a^7*b^4))^(1/3)))^2*a^5*b^3*c + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(
(8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) +
4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4)
 - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*a^4*b*e^2 - 8*a*b*c^2*e + 2*(8*
b^2*c^3 + a^2*e^3)*x + 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*
c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)
^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b
^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*a^5*b^3*c + a^4*b*e^2)*sqrt(-(((1/2)^(
1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^
3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^
3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2
 + 32*c*e)/(a^4*b^2))) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3
)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/
(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 +
 a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3))) - 3*sqrt(1/3
)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)
*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)
+ 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^
4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2 + 32*c*e)/(a^4*b^2)
))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) -
 (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)
/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4
))^(1/3)))^2*a^5*b^3*c + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2
*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*
e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 -
a^2*e^3)/(a^7*b^4))^(1/3)))*a^4*b*e^2 - 8*a*b*c^2*e + 2*(8*b^2*c^3 + a^2*e^
3)*x - 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(
a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sq
rt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3
)/(a^7*b^4))^(1/3)))*a^5*b^3*c + a^4*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1
/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^
7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2 + 32*c*e)/(a^4*
```

```
b^2))))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
```

Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.71

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx$$
$$= \text{RootSum}\left(19683t^3a^7b^4+162ta^3b^2ce-a^2e^3+8b^2c^3,\left(t\mapsto t\log\left(x+\frac{1458t^2a^5b^3c+27ta^4be^2+8abc^2e}{a^2e^3+8b^2c^3}\right)\right.\right.$$
$$+\frac{-3a^2d-2a^2ex+7abcx^2+abex^4+4b^2cx^5}{18a^4b+36a^3b^2x^3+18a^2b^3x^6}$$

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**7*b**4 + 162*_t*a**3*b**2*c*e - a**2*e**3 + 8*b**2*c
**3, Lambda(_t, _t*log(x + (1458*_t**2*a**5*b**3*c + 27*_t*a**4*b*e**2 + 8*
a*b*c**2*e)/(a**2*e**3 + 8*b**2*c**3)))) + (-3*a**2*d - 2*a**2*e*x + 7*a*b*
c*x**2 + a*b*e*x**4 + 4*b**2*c*x**5)/(18*a**4*b + 36*a**3*b**2*x**3 + 18*a*
*2*b**3*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx = \frac{4\,b^2cx^5+abex^4+7\,abcx^2-2\,a^2ex-3\,a^2d}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)} + \frac{\sqrt{3}\left(2\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(2\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(2\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*b^2*c*x^5 + a*b*e*x^4 + 7*a*b*c*x^2 - 2*a^2*e*x - 3*a^2*d)/(a^2*b^3
*x^6 + 2*a^3*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(2*b*c*(a/b)^(1/3) + a*e)*arct
```

```
an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) + 1/5
4*(2*b*c*(a/b)^(1/3) - a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2
*(a/b)^(2/3)) - 1/27*(2*b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3))/(a^2*b^
2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.88

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}\left(ae-2\left(-ab^2\right)^{\frac{1}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^2} - \frac{\left(ae+2\left(-ab^2\right)^{\frac{1}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^2} - \frac{\left(2bc\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^3b} + \frac{4b^2cx^5+abex^4+7abcx^2-2a^2ex-3a^2d}{18(bx^3+a)^2a^2b}$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(a*e - 2*(-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(a*e + 2*(-a*b^2)^(1/3)*c)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(2*b*c
*(-a/b)^(1/3) + a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/18
*(4*b^2*c*x^5 + a*b*e*x^4 + 7*a*b*c*x^2 - 2*a^2*e*x - 3*a^2*d)/((b*x^3 + a)
^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.97

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx = \frac{\frac{7\,c\,x^2}{18\,a}-\frac{d}{6\,b}+\frac{e\,x^4}{18\,a}-\frac{e\,x}{9\,b}+\frac{2\,b\,c\,x^5}{9\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \left(\sum_{k=1}^{3}\ln\left(\frac{2\,a\,c\,e+\mathrm{root}(19683\,a^7\,b^4\,z^3+162\,a^3\,b^2\,c\,e\,z+8\,b^2\,c^3-a^2\,e^3,z,k)^2\,a^5\,b^2\,729+4\,b\,c^2\,x+\mathrm{root}(19\ldots}{a^4\,81} + 162\,a^3\,b^2\,c\,e\,z+8\,b^2\,c^3-a^2\,e^3,z,k)\right)$$

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3)^3,x)
```

```
[Out] ((7*c*x^2)/(18*a) - d/(6*b) + (e*x^4)/(18*a) - (e*x)/(9*b) + (2*b*c*x^5)/(9
*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((2*a*c*e + 729*root(19683*a
^7*b^4*z^3 + 162*a^3*b^2*c*e*z + 8*b^2*c^3 - a^2*e^3, z, k)^2*a^5*b^2 + 4*b
*c^2*x + 27*root(19683*a^7*b^4*z^3 + 162*a^3*b^2*c*e*z + 8*b^2*c^3 - a^2*e^
3, z, k)*a^3*b*e*x)/(81*a^4))*root(19683*a^7*b^4*z^3 + 162*a^3*b^2*c*e*z +
8*b^2*c^3 - a^2*e^3, z, k), k, 1, 3)
```

3.353 $\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 225

$$\begin{aligned}\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx &= \frac{x(5c+4dx)}{18a^2(a+bx^3)}-\frac{ae-bx(c+dx)}{6ab(a+bx^3)^2}\\ &\quad-\frac{\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}\\ &\quad+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}\\ &\quad-\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}\end{aligned}$$

```
[Out] 1/18*x*(4*d*x+5*c)/a^2/(b*x^3+a)+1/6*(-a*e+b*x*(d*x+c))/a/b/(b*x^3+a)^2+1/2
7*(5*b^(1/3)*c-2*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(2/3)-1/54*(5*b
^(1/3)*c-2*a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(
2/3)-1/27*(5*b^(1/3)*c+2*a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3
)*3^(1/2))/a^(8/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1868, 1869, 1874, 31, 648, 631, 210, 642}

$$\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)}{9\sqrt{3}a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} + \frac{x(5c+4dx)}{18a^2(a+bx^3)} - \frac{ae-bx(c+dx)}{6ab(a+bx^3)^2}$$

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^3,x]
```

```
[Out] (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3)) - (a*e - b*x*(c + d*x))/(6*a*b*(a + b*x^3)^2) - ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}-\frac{\int \frac{-5c-4dx}{\left(a+bx^3\right)^2}\,dx}{6a}\\
&= \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}+\frac{\int \frac{10c+4dx}{a+bx^3}\,dx}{18a^2}\\
&= \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}\\
&\quad+\frac{\int \frac{\sqrt[3]{a}\left(20\sqrt[3]{b}c+4\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-10\sqrt[3]{b}c+4\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{8/3}}
\end{aligned}$$

$$= \frac{x(5c+4dx)}{18a^2(a+bx^3)} - \frac{ae-bx(c+dx)}{6ab(a+bx^3)^2} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}$$
$$- \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}b^{2/3}}$$
$$+ \frac{\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{7/3}\sqrt[3]{b}}$$

$$= \frac{x(5c+4dx)}{18a^2(a+bx^3)} - \frac{ae-bx(c+dx)}{6ab(a+bx^3)^2} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}$$
$$- \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}$$
$$+ \frac{\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{8/3}b^{2/3}}$$

$$= \frac{x(5c+4dx)}{18a^2(a+bx^3)} - \frac{ae-bx(c+dx)}{6ab(a+bx^3)^2} - \frac{\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}$$
$$+ \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.95

$$\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx$$
$$= \frac{\frac{3a(-3a^2e+b^2x^4(5c+4dx)+abx(8c+7dx))}{(a+bx^3)^2} - 2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\sqrt[3]{b}\left(5\sqrt[3]{a}\sqrt[3]{b}c-2a^{2/}\right.}{54a^3b}$$

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^3,x]
```

```
[Out] ((3*a*(-3*a^2*e + b^2*x^4*(5*c + 4*d*x) + a*b*x*(8*c + 7*d*x)))/(a + b*x^3)
^2 - 2*Sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(5*a^(1/3)*b^(1/3)*c - 2*a^(2/3)*d)
*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*b^(1/3)*(-5*b^(1/3)*c + 2*a^(1/3)*d)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^3*b)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.41

method	result
risch	$\frac{\frac{2bd\,x^5}{9a^2}+\frac{5bc\,x^4}{18a^2}+\frac{7d\,x^2}{18a}+\frac{4cx}{9a}-\frac{e}{6b}}{\left(b\,x^3+a\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(2\_Rd+5c\right)\ln\left(x-\_R\right)}{\_R^2}}{27a^2b}$
default	$c\left(\frac{x}{6a\left(b\,x^3+a\right)^2}+\frac{\frac{5x}{18a\left(b\,x^3+a\right)}+\frac{5\left(\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{6a}}{a}\right)+d\left(\frac{x^2}{6a\left(b\,x^3+\right.}\right.$

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2/9*b*d/a^2*x^5+5/18*b*c/a^2*x^4+7/18*d/a*x^2+4/9*c/a*x-1/6/b*e)/(b*x^3+a)
^2+1/27/a^2/b*sum((2*_R*d+5*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 2251, normalized size of antiderivative = 10.00

$$\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(24*b^2*d*x^5 + 30*b^2*c*x^4 + 42*a*b*d*x^2 + 48*a*b*c*x - 18*a^2*e -
 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125
```

*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(
1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (
125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)
 - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b
^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d - 25/2*((1/2)^(1/3
)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/
(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 +
 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 +
40*a*c*d^2 + (125*b*c^3 + 8*a*d^3)*x) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4
b)((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*
c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*
b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)
)) + 3*sqrt(1/3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)*
(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a
^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8
*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*
c*d)/(a^5*b)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)
/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-
I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d
^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*
b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1
/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (1
25*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 - 40*a*c*d^2 + 2*(125*b*c^
3 + 8*a*d^3)*x + 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 +
8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3
)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^6*b*d + 25*a^3*b*c^2)*sqrt(-(((1/2)^(1/3)*
(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a
^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8
*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*
c*d)/(a^5*b))) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqr
t(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2
))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3
)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3))) - 3*sqrt(1/3)*(a^2*b
^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b
*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/
2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (12
5*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b)))*log(-1/2
((1/2)^(1/3)(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*(
(125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^
2*a^6*b*d + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b
^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(
3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^

```
8*b^2))^(1/3)))*a^3*b*c^2 - 40*a*c*d^2 + 2*(125*b*c^3 + 8*a*d^3)*x - 3/2*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (1
25*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/
(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^
(1/3)))*a^6*b*d + 25*a^3*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b
*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/
2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (12
5*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b))))/(a^2*b^
3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
```

Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.72

$$\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx$$
$$=\operatorname{RootSum}\left(19683t^3a^8b^2+810ta^3bcd+8ad^3-125bc^3,\left(t\mapsto t\log\left(x+\frac{1458t^2a^6bd+675ta^3bc^2+40acd^2}{8ad^3+125bc^3}\right)\right)\right)+\frac{-3a^2e+8abcx+7abdx^2+5b^2cx^4+4b^2dx^5}{18a^4b+36a^3b^2x^3+18a^2b^3x^6}$$

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**8*b**2 + 810*_t*a**3*b*c*d + 8*a*d**3 - 125*b*c**3,
Lambda(_t, _t*log(x + (1458*_t**2*a**6*b*d + 675*_t*a**3*b*c**2 + 40*a*c*d*
*2)/(8*a*d**3 + 125*b*c**3)))) + (-3*a**2*e + 8*a*b*c*x + 7*a*b*d*x**2 + 5*
b**2*c*x**4 + 4*b**2*d*x**5)/(18*a**4*b + 36*a**3*b**2*x**3 + 18*a**2*b**3*
x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.97

$$\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx = \frac{4\,b^2dx^5+5\,b^2cx^4+7\,abdx^2+8\,abcx-3\,a^2e}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)} + \frac{\sqrt{3}\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*b^2*d*x^5 + 5*b^2*c*x^4 + 7*a*b*d*x^2 + 8*a*b*c*x - 3*a^2*e)/(a^2*b
^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(2*d*(a/b)^(1/3) + 5*c)*arct
an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) + 1/54*
(2*d*(a/b)^(1/3) - 5*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)
^(2/3)) - 1/27*(2*d*(a/b)^(1/3) - 5*c)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2
/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.93

$$\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}\left(5\,bc-2\,(-ab^2)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{\left(5\,bc+2\,(-ab^2)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{\left(2\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3} + \frac{4\,b^2dx^5+5\,b^2cx^4+7\,abdx^2+8\,abcx-3\,a^2e}{18\,(bx^3+a)^2a^2b}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(5*b*c - 2*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(5*b*c + 2*(-a*b^2)^(1/3)
*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(2
*d*(-a/b)^(1/3) + 5*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/18*(
4*b^2*d*x^5 + 5*b^2*c*x^4 + 7*a*b*d*x^2 + 8*a*b*c*x - 3*a^2*e)/((b*x^3 + a)
^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.94

$$\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx = \frac{\frac{7\,d\,x^2}{18\,a}-\frac{e}{6\,b}+\frac{4\,c\,x}{9\,a}+\frac{5\,b\,c\,x^4}{18\,a^2}+\frac{2\,b\,d\,x^5}{9\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \left(\sum_{k=1}^{3}\ln\left(\frac{b\left(10\,c\,d+4\,d^2\,x+\mathrm{root}(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k)^2\,a^5\,b\,729+\mathrm{root}\right.}{a^4\,81} \right.\right. \left.\left. +810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k)\right)$$

```
[In] int((c + d*x + e*x^2)/(a + b*x^3)^3,x)
```

```
[Out] ((7*d*x^2)/(18*a) - e/(6*b) + (4*c*x)/(9*a) + (5*b*c*x^4)/(18*a^2) + (2*b*d
*x^5)/(9*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((b*(10*c*d + 4*d^2*
x + 729*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a*d^3, z,
k)^2*a^5*b + 135*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a
*d^3, z, k)*a^2*b*c*x))/(81*a^4))*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z
- 125*b*c^3 + 8*a*d^3, z, k), k, 1, 3)
```

3.354 $\int \frac{c+dx+ex^2}{x(a+bx^3)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 257

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx=\frac{x(ad+aex-bcx^2)}{6a^2\left(a+bx^3\right)^2}+\frac{x(5ad+4aex-9bcx^2)}{18a^3\left(a+bx^3\right)}-\frac{\left(5\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}+\frac{c\log(x)}{a^3}+\frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}-\frac{c\log\left(a+bx^3\right)}{3a^3}$$

```
[Out] 1/6*x*(-b*c*x^2+a*e*x+a*d)/a^2/(b*x^3+a)^2+1/18*x*(-9*b*c*x^2+4*a*e*x+5*a*d
)/a^3/(b*x^3+a)+c*ln(x)/a^3+1/27*(5*b^(1/3)*d-2*a^(1/3)*e)*ln(a^(1/3)+b^(1/
3)*x)/a^(8/3)/b^(2/3)-1/54*(5*b^(1/3)*d-2*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(2/3)-1/3*c*ln(b*x^3+a)/a^3-1/27*(5*b^(1/3)*d
+2*a^(1/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(
2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2}{x(a+bx^3)^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2\sqrt[3]{a}e+5\sqrt[3]{b}d\right)}{9\sqrt{3}a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}} + \frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} + \frac{x(5ad+4aex-9bcx^2)}{18a^3(a+bx^3)} - \frac{c\log(a+bx^3)}{3a^3} + \frac{c\log(x)}{a^3} + \frac{x(ad+aex-bcx^2)}{6a^2(a+bx^3)^2}$$

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^3),x]
```

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*d + 4*a*e*x - 9
*b*c*x^2))/(18*a^3*(a + b*x^3)) - ((5*b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + (c*Log
[x])/a^3 + ((5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/
3)*b^(2/3)) - ((5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = \frac{x(ad+aex-bcx^2)}{6a^2(a+bx^3)^2} - \frac{\int \frac{-6bc-5bdx-4bex^2+\frac{3b^2cx^3}{a}}{x(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3(a+bx^3)} + \frac{\int \frac{18b^2c+10b^2dx+4b^2ex^2}{x(a+bx^3)}\,dx}{18a^2b^2}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3(a+bx^3)} + \frac{\int \left(\frac{18b^2c}{ax} + \frac{2b^2(5ad+2aex-9bcx^2)}{a(a+bx^3)}\right)dx}{18a^2b^2}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3(a+bx^3)} + \frac{c\log(x)}{a^3} + \frac{\int \frac{5ad+2aex-9bcx^2}{a+bx^3}\,dx}{9a^3}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3(a+bx^3)} + \frac{c\log(x)}{a^3} + \frac{\int \frac{5ad+2aex}{a+bx^3}\,dx}{9a^3} - \frac{(bc)\int \frac{x^2}{a+bx^3}\,dx}{a^3}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3(a+bx^3)} + \frac{c\log(x)}{a^3} - \frac{c\log(a+bx^3)}{3a^3} + \frac{\int \frac{\sqrt[3]{a}\left(10a\sqrt[3]{b}d+2a^{4/3}e\right)+\sqrt[3]{b}\left(-5a\sqrt[3]{b}d+2a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{11/3}\sqrt[3]{b}} + \frac{\left(5d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{8/3}}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3(a+bx^3)} + \frac{c\log(x)}{a^3} + \frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{c\log(a+bx^3)}{3a^3} - \frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}b^{2/3}} + \frac{\left(5\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{7/3}\sqrt[3]{b}}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3(a+bx^3)} + \frac{c\log(x)}{a^3} + \frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}} - \frac{c\log(a+bx^3)}{3a^3} + \frac{\left(5\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{8/3}b^{2/3}}$$

$$= \frac{x(ad+aex-bcx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3(a+bx^3)} - \frac{\left(5\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}} + \frac{c\log(x)}{a^3} + \frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}} - \frac{c\log(a+bx^3)}{3a^3}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.89

$$\int\frac{c+dx+ex^2}{x(a+bx^3)^3}\,dx$$

$$= \frac{\frac{9a^2(c+x(d+ex))}{(a+bx^3)^2} + \frac{3a(6c+x(5d+4ex))}{a+bx^3} - \frac{2\sqrt{3}\sqrt[3]{a}\left(5\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + 54c\log(x) + \frac{2\left(5\sqrt[3]{a}\sqrt[3]{b}d-2a^{2/3}e\right)\text{l}}{b^{2/3}}}{54a^3}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^3),x]
```

```
[Out] ((9*a^2*(c + x*(d + e*x)))/(a + b*x^3)^2 + (3*a*(6*c + x*(5*d + 4*e*x)))/(a
 + b*x^3) - (2*Sqrt[3]*a^(1/3)*(5*b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 54*c*Log[x] + (2*(5*a^(1/3)*b^(1/3)*
d - 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-5*a^(1/3)*b^(1/3)*d
 + 2*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 1
8*c*Log[a + b*x^3])/(54*a^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.05

method	result
risch	$\frac{\frac{2be\,x^5}{9a^2}+\frac{5bd\,x^4}{18a^2}+\frac{bc\,x^3}{3a^2}+\frac{7e\,x^2}{18a}+\frac{4xd}{9a}+\frac{c}{2a}}{(b\,x^3+a)^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(a^9b^2\_Z^3+27a^6b^2c\_Z^2+\left(30a^4bde+243a^3b^2c^2\right)\_Z+8a^2e^3+270abcde-125ab\,d^3+\cdots\right.}\cdots\right.}{\cdots}$
default	$\frac{c\ln(x)}{a^3}+\frac{\frac{\frac{2}{9}abe\,x^5+\frac{5}{18}abd\,x^4+\frac{1}{3}abc\,x^3+\frac{7}{18}a^2e\,x^2+\frac{4}{9}a^2dx+\frac{1}{2}a^2c}{(b\,x^3+a)^2}+\frac{5ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\cdots\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\cdots\right)}{9}\cdots}{a^3}$

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2/9*b*e/a^2*x^5+5/18*b*d/a^2*x^4+1/3*b*c/a^2*x^3+7/18/a*e*x^2+4/9/a*x*d+1/
2*c/a)/(b*x^3+a)^2+1/27*sum(_R*ln((-4*_R^3*a^8*b^2-72*_R^2*a^5*b^2*c+(-100*
a^3*b*d*e-324*a^2*b^2*c^2)*_R-24*a*e^3-540*b*c*d*e+375*b*d^3)*x+2*a^6*b*e*_
R^2+(-36*a^3*b*c*e-25*a^3*b*d^2)*_R-486*b*c^2*e+675*b*c*d^2),_R=RootOf(a^9*
b^2*_Z^3+27*a^6*b^2*c*_Z^2+(30*a^4*b*d*e+243*a^3*b^2*c^2)*_Z+8*a^2*e^3+270*
a*b*c*d*e-125*a*b*d^3+729*b^2*c^3))+c/a^3*ln(-x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 5229, normalized size of antiderivative = 20.35

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx = \frac{4\,bex^5+5\,bdx^4+6\,bcx^3+7\,aex^2+8\,adx+9\,ac}{18\,\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}+\frac{c\log\left(x\right)}{a^3}+\frac{\sqrt{3}\left(2\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4}-\frac{\left(18\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(9\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*b*e*x^5 + 5*b*d*x^4 + 6*b*c*x^3 + 7*a*e*x^2 + 8*a*d*x + 9*a*c)/(a^2
*b^2*x^6 + 2*a^3*b*x^3 + a^4) + c*log(x)/a^3 + 1/27*sqrt(3)*(2*a*e*(a/b)^(2
/3) + 5*a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/a^4 - 1/54*(18*b*c*(a/b)^(2/3) - 2*a*e*(a/b)^(1/3) + 5*a*d)*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) - 1/27*(9*b*c*(a/b)^(2/3) + 2
*a*e*(a/b)^(1/3) - 5*a*d)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx = -\frac{\sqrt{3}\left(5\,bd-2\,\left(-ab^2\right)^{\frac{1}{3}}\,e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}\,a^2} - \frac{\left(5\,bd+2\,\left(-ab^2\right)^{\frac{1}{3}}\,e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{2}{3}}\,a^2} - \frac{c\log\left(\left|bx^3+a\right|\right)}{3\,a^3} + \frac{c\log\left(\left|x\right|\right)}{a^3} + \frac{4\,abex^5+5\,abdx^4+6\,abcx^3+7\,a^2ex^2+8\,a^2dx+9\,a^2c}{18\,\left(bx^3+a\right)^2a^3} - \frac{\left(2\,a^4be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^4bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b}$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(5*b*d - 2*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(5*b*d + 2*(-a*b^2)^(1/3)
*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/3*c*l
og(abs(b*x^3 + a))/a^3 + c*log(abs(x))/a^3 + 1/18*(4*a*b*e*x^5 + 5*a*b*d*x^
4 + 6*a*b*c*x^3 + 7*a^2*e*x^2 + 8*a^2*d*x + 9*a^2*c)/((b*x^3 + a)^2*a^3) -
1/27*(2*a^4*b*e*(-a/b)^(1/3) + 5*a^4*b*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(
1/3)))/(a^7*b)
```

Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 540, normalized size of antiderivative = 2.10

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx$$
$$=\frac{\frac{c}{2\,a}+\frac{7\,e\,x^2}{18\,a}+\frac{4\,d\,x}{9\,a}+\frac{b\,c\,x^3}{3\,a^2}+\frac{5\,b\,d\,x^4}{18\,a^2}+\frac{2\,b\,e\,x^5}{9\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\left(\sum_{k=1}^{3}\ln\left(\frac{25\,b^2\,c\,d^2-18\,b^2\,c^2\,e}{81\,a^6}\right.\right.$$
$$-\mathrm{root}\big(19683\,a^9\,b^2\,z^3+19683\,a^6\,b^2\,c\,z^2+810\,a^4\,b\,d\,e\,z+6561\,a^3\,b^2\,c^2\,z+270\,a\,b\,c\,d\,e-125\,a\,b\,d^3+8\,a^2\,e^3+72$$
$$\left.-\frac{x\left(-125\,b^2\,d^3+180\,c\,b^2\,d\,e+8\,a\,b\,e^3\right)}{729\,a^6}\right)\mathrm{root}\big(19683\,a^9\,b^2\,z^3+19683\,a^6\,b^2\,c\,z^2$$
$$\left.+810\,a^4\,b\,d\,e\,z+6561\,a^3\,b^2\,c^2\,z+270\,a\,b\,c\,d\,e-125\,a\,b\,d^3+8\,a^2\,e^3+729\,b^2\,c^3,z,k\big)\right)$$
$$+\frac{c\,\ln\left(x\right)}{a^3}$$

```
[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)^3),x)
```

```
[Out] (c/(2*a) + (7*e*x^2)/(18*a) + (4*d*x)/(9*a) + (b*c*x^3)/(3*a^2) + (5*b*d*x^
4)/(18*a^2) + (2*b*e*x^5)/(9*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log
((25*b^2*c*d^2 - 18*b^2*c^2*e)/(81*a^6) - root(19683*a^9*b^2*z^3 + 19683*a^
6*b^2*c*z^2 + 810*a^4*b*d*e*z + 6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 125*a*
b*d^3 + 8*a^2*e^3 + 729*b^2*c^3, z, k)*((25*a^3*b^2*d^2 + 36*a^3*b^2*c*e)/(
81*a^6) + root(19683*a^9*b^2*z^3 + 19683*a^6*b^2*c*z^2 + 810*a^4*b*d*e*z +
6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 125*a*b*d^3 + 8*a^2*e^3 + 729*b^2*c^3,
 z, k)*(36*root(19683*a^9*b^2*z^3 + 19683*a^6*b^2*c*z^2 + 810*a^4*b*d*e*z +
 6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 125*a*b*d^3 + 8*a^2*e^3 + 729*b^2*c^3
, z, k)*a^2*b^3*x - (2*b^2*e)/3 + (24*b^3*c*x)/a) + (x*(2916*a^2*b^3*c^2 +
900*a^3*b^2*d*e))/(729*a^6)) - (x*(8*a*b*e^3 - 125*b^2*d^3 + 180*b^2*c*d*e)
)/(729*a^6))*root(19683*a^9*b^2*z^3 + 19683*a^6*b^2*c*z^2 + 810*a^4*b*d*e*z
 + 6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 125*a*b*d^3 + 8*a^2*e^3 + 729*b^2*c
^3, z, k), k, 1, 3) + (c*log(x))/a^3
```

3.355 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 267

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx = -\frac{c}{a^3x}+\frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2}+\frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{(14b^{2/3}c-5a^{2/3}e)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt[3]{b}} + \frac{d\log(x)}{a^3}+\frac{(14b^{2/3}c+5a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt[3]{b}} - \frac{(14b^{2/3}c+5a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}\sqrt[3]{b}}-\frac{d\log(a+bx^3)}{3a^3}$$

```
[Out] -c/a^3/x+1/6*x*(-b*d*x^2-b*c*x+a*e)/a^2/(b*x^3+a)^2+1/18*x*(-9*b*d*x^2-10*b
*c*x+5*a*e)/a^3/(b*x^3+a)+d*ln(x)/a^3+1/27*(14*b^(2/3)*c+5*a^(2/3)*e)*ln(a^
(1/3)+b^(1/3)*x)/a^(10/3)/b^(1/3)-1/54*(14*b^(2/3)*c+5*a^(2/3)*e)*ln(a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(1/3)-1/3*d*ln(b*x^3+a)/a^3+1/2
7*(14*b^(2/3)*c-5*a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/
2))/a^(10/3)/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^3}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(14b^{2/3}c-5a^{2/3}e\right)}{9\sqrt{3}a^{10/3}\sqrt[3]{b}} - \frac{\left(5a^{2/3}e+14b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}\sqrt[3]{b}} + \frac{\left(5a^{2/3}e+14b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt[3]{b}} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3\left(a+bx^3\right)} - \frac{d\log\left(a+bx^3\right)}{3a^3} - \frac{c}{a^3x} + \frac{d\log(x)}{a^3} + \frac{x(ae-bcx-bdx^2)}{6a^2\left(a+bx^3\right)^2}$$

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -(c/(a^3*x)) + (x*(a*e - b*c*x - b*d*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*
e - 10*b*c*x - 9*b*d*x^2))/(18*a^3*(a + b*x^3)) + ((14*b^(2/3)*c - 5*a^(2/3
)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)
*b^(1/3)) + (d*Log[x])/a^3 + ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(1/3) + b^
(1/3)*x])/(27*a^(10/3)*b^(1/3)) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(1/3)) - (d*Log[a + b*x
^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} - \frac{\int \frac{-6bc-6bdx-5bex^2+\frac{4b^2cx^3}{a}+\frac{3b^2dx^4}{a}}{x^2(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{\int \frac{18b^3c+18b^3dx+10b^3ex^2-\frac{10b^4cx^3}{a}}{x^2(a+bx^3)}\,dx}{18a^2b^3}$$

$$= \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{\int \left(\frac{18b^3c}{ax^2}+\frac{18b^3d}{ax}+\frac{2b^3(5ae-14bcx-9bdx^2)}{a(a+bx^3)}\right)dx}{18a^2b^3}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{d\log(x)}{a^3} + \frac{\int \frac{5ae-14bcx-9bdx^2}{a+bx^3}\,dx}{9a^3}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{d\log(x)}{a^3} + \frac{\int \frac{5ae-14bcx}{a+bx^3}\,dx}{9a^3} - \frac{(bd)\int \frac{x^2}{a+bx^3}\,dx}{a^3}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{d\log(x)}{a^3} - \frac{d\log(a+bx^3)}{3a^3} + \frac{\int \frac{\sqrt[3]{a}\left(-14\sqrt[3]{a}bc+10a\sqrt[3]{b}e\right)+\sqrt[3]{b}\left(-14\sqrt[3]{a}bc-5a\sqrt[3]{b}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{11/3}\sqrt[3]{b}} + \frac{(14b^{2/3}c+5a^{2/3}e)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{10/3}}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{d\log(x)}{a^3} + \frac{(14b^{2/3}c+5a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt[3]{b}} - \frac{d\log(a+bx^3)}{3a^3} - \frac{(14b^{2/3}c-5a^{2/3}e)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^3} - \frac{(14b^{2/3}c+5a^{2/3}e)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{10/3}\sqrt[3]{b}}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{d\log(x)}{a^3} + \frac{(14b^{2/3}c+5a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt[3]{b}} - \frac{(14b^{2/3}c+5a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}\sqrt[3]{b}} - \frac{d\log(a+bx^3)}{3a^3} - \frac{(14b^{2/3}c-5a^{2/3}e)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{10/3}\sqrt[3]{b}}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{(14b^{2/3}c-5a^{2/3}e)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt[3]{b}} + \frac{d\log(x)}{a^3} + \frac{(14b^{2/3}c+5a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt[3]{b}} - \frac{(14b^{2/3}c+5a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}\sqrt[3]{b}} - \frac{d\log(a+bx^3)}{3a^3}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.93

$$\int\frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{54ac}{x} + \frac{3a(6ad+5aex-10bcx^2)}{a+bx^3} + \frac{9a^2(-bcx^2+a(d+ex))}{(a+bx^3)^2} - \frac{2\sqrt{3}a^{2/3}(-14b^{2/3}c+5a^{2/3}e)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + 54ad\log(x) + \frac{2(14}{}}{54a^4}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a*c)/x + (3*a*(6*a*d + 5*a*e*x - 10*b*c*x^2))/(a + b*x^3) + (9*a^2*(-
(b*c*x^2) + a*(d + e*x)))/(a + b*x^3)^2 - (2*Sqrt[3]*a^(2/3)*(-14*b^(2/3)*c
 + 5*a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + 54*a
*d*Log[x] + (2*(14*a^(2/3)*b^(2/3)*c + 5*a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x
])/b^(1/3) - ((14*a^(2/3)*b^(2/3)*c + 5*a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/b^(1/3) - 18*a*d*Log[a + b*x^3])/(54*a^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.04

method	result
risch	$\frac{-\frac{14c\,b^{2}x^{6}}{9a^{3}}+\frac{5be\,x^{5}}{18a^{2}}+\frac{bd\,x^{4}}{3a^{2}}-\frac{49bc\,x^{3}}{18a^{2}}+\frac{4e\,x^{2}}{9a}+\frac{xd}{2a}-\frac{c}{a}}{x\left(b\,x^{3}+a\right)^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}b\_Z^{3}+27a^{7}bd\_Z^{2}+\left(-210a^{4}bce+243a^{4}b\,d^{2}\right)\_Z-125a^{2}e^{3}-189\right.}\right.}{}$
default	$-\frac{c}{a^{3}x}+\frac{d\ln(x)}{a^{3}}+\frac{\frac{-\frac{5}{9}b^{2}c\,x^{5}+\frac{5}{18}abe\,x^{4}+\frac{1}{3}x^{3}abd-\frac{13}{18}abc\,x^{2}+\frac{4}{9}a^{2}ex+\frac{1}{2}a^{2}d}{\left(b\,x^{3}+a\right)^{2}}+\frac{5ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan}{}\right.}{9}}{}$

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-14/9*c/a^3*b^2*x^6+5/18*b*e/a^2*x^5+1/3*b*d/a^2*x^4-49/18*b*c/a^2*x^3+4/9
/a*e*x^2+1/2/a*x*d-c/a)/x/(b*x^3+a)^2+1/27*sum(_R*ln((-4*_R^3*a^10*b-72*_R^
2*a^7*b*d+(700*a^4*b*c*e-324*a^4*b*d^2)*_R+375*a^2*e^3+3780*a*b*c*d*e+8232*
b^2*c^3)*x-14*a^7*b*c*_R^2+(-25*a^5*e^2+252*a^4*b*c*d)*_R+675*a^2*d*e^2+340
2*a*b*c*d^2),_R=RootOf(a^10*b*_Z^3+27*a^7*b*d*_Z^2+(-210*a^4*b*c*e+243*a^4*
b*d^2)*_Z-125*a^2*e^3-1890*a*b*c*d*e+729*a*b*d^3-2744*b^2*c^3))+d*ln(x)/a^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 5112, normalized size of antiderivative = 19.15

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^3}\,dx=-\frac{28\,b^2cx^6-5\,abex^5-6\,abdx^4+49\,abcx^3-8\,a^2ex^2-9\,a^2dx+18\,a^2c}{18\,\left(a^3b^2x^7+2\,a^4bx^4+a^5x\right)}+\frac{d\log\left(x\right)}{a^3}-\frac{\sqrt{3}\left(14\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-5\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4}-\frac{\left(18\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(9\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
[Out] -1/18*(28*b^2*c*x^6 - 5*a*b*e*x^5 - 6*a*b*d*x^4 + 49*a*b*c*x^3 - 8*a^2*e*x^
2 - 9*a^2*d*x + 18*a^2*c)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x) + d*log(x)/a^
3 - 1/27*sqrt(3)*(14*b*c*(a/b)^(2/3) - 5*a*e*(a/b)^(1/3))*arctan(1/3*sqrt(3
)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^4 - 1/54*(18*b*d*(a/b)^(2/3) + 14*b*c*
(a/b)^(1/3) + 5*a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2
/3)) - 1/27*(9*b*d*(a/b)^(2/3) - 14*b*c*(a/b)^(1/3) - 5*a*e)*log(x + (a/b)^
(1/3))/(a^3*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx = -\frac{d\log(|bx^3+a|)}{3a^3} + \frac{d\log(|x|)}{a^3} + \frac{\sqrt{3}\left(5(-ab^2)^{\frac{1}{3}}ae + 14(-ab^2)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^4b} + \frac{\left(5(-ab^2)^{\frac{1}{3}}ae - 14(-ab^2)^{\frac{2}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^4b} - \frac{28b^2cx^6 - 5abex^5 - 6abdx^4 + 49abcx^3 - 8a^2ex^2 - 9a^2dx + 18a^2c}{18(bx^3+a)^2a^3x} + \frac{\left(14a^3b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5a^4be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^7b}$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^3 + d*log(abs(x))/a^3 + 1/27*sqrt(3)*(5*(-a*b^
2)^(1/3)*a*e + 14*(-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))/(a^4*b) + 1/54*(5*(-a*b^2)^(1/3)*a*e - 14*(-a*b^2)^(2/3)*c)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/18*(28*b^2*c*x^6 - 5*a
*b*e*x^5 - 6*a*b*d*x^4 + 49*a*b*c*x^3 - 8*a^2*e*x^2 - 9*a^2*d*x + 18*a^2*c)
/((b*x^3 + a)^2*a^3*x) + 1/27*(14*a^3*b^2*c*(-a/b)^(1/3) - 5*a^4*b*e)*(-a/b
)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 793, normalized size of antiderivative = 2.97

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx = \frac{\frac{4ex^2}{9a} - \frac{c}{a} + \frac{dx}{2a} - \frac{14b^2cx^6}{9a^3} - \frac{49bcx^3}{18a^2} + \frac{bdx^4}{3a^2} + \frac{5bex^5}{18a^2}}{a^2x + 2abx^4 + b^2x^7} + \left(\sum_{k=1}^{3}\ln\left(\frac{b^2\left(-\mathrm{root}(19683a^{10}bz^3 + 19683a^7bdz^2 - 5670a^4bcez + 6561a^4bd^2z - 1890abcde + 7\right.}{}\right.\right.$$

$$+ 19683a^7bdz^2 - 5670a^4bcez + 6561a^4bd^2z - 1890abcde + 729abd^3 - 125a^2e^3 - 2744b^2c^3, z, k)\Bigg) + \frac{d\ln(x)}{a^3}$$

```
[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x)
```

```
[Out] ((4*e*x^2)/(9*a) - c/a + (d*x)/(2*a) - (14*b^2*c*x^6)/(9*a^3) - (49*b*c*x^3
)/(18*a^2) + (b*d*x^4)/(3*a^2) + (5*b*e*x^5)/(18*a^2))/(a^2*x + b^2*x^7 + 2
*a*b*x^4) + symsum(log((b^2*(225*a^2*d*e^2 - 225*root(19683*a^10*b*z^3 + 19
683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e + 72
9*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)*a^5*e^2 + 2744*b^2*c^3*x + 12
5*a^2*e^3*x + 1134*a*b*c*d^2 - 3402*root(19683*a^10*b*z^3 + 19683*a^7*b*d*z
^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 - 1
25*a^2*e^3 - 2744*b^2*c^3, z, k)^2*a^7*b*c - 26244*root(19683*a^10*b*z^3 +
19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e +
729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)^3*a^10*b*x - 2916*root(1968
3*a^10*b*z^3 + 19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 18
90*a*b*c*d*e + 729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)*a^4*b*d^2*x
- 17496*root(19683*a^10*b*z^3 + 19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561
*a^4*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z
, k)^2*a^7*b*d*x + 2268*root(19683*a^10*b*z^3 + 19683*a^7*b*d*z^2 - 5670*a^
4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 - 125*a^2*e^3 -
 2744*b^2*c^3, z, k)*a^4*b*c*d + 6300*root(19683*a^10*b*z^3 + 19683*a^7*b*d
*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 -
 125*a^2*e^3 - 2744*b^2*c^3, z, k)*a^4*b*c*e*x + 1260*a*b*c*d*e*x))/(729*a^
8))*root(19683*a^10*b*z^3 + 19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4
*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)
, k, 1, 3) + (d*log(x))/a^3
```

3.356 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 276

$$\begin{aligned}\int \frac{c+dx+ex^2}{x^3(a+bx^3)^3}\,dx = & -\frac{c}{2a^3x^2}-\frac{d}{a^3x}-\frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2}-\frac{x(11bc+10bdx+9bex^2)}{18a^3(a+bx^3)}\\ & +\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}}\\ & +\frac{e\log(x)}{a^3}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}}\\ & +\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}}\\ & -\frac{e\log(a+bx^3)}{3a^3}\end{aligned}$$

```
[Out] -1/2*c/a^3/x^2-d/a^3/x-1/6*x*(b*e*x^2+b*d*x+b*c)/a^2/(b*x^3+a)^2-1/18*x*(9*
b*e*x^2+10*b*d*x+11*b*c)/a^3/(b*x^3+a)+e*ln(x)/a^3-2/27*b^(1/3)*(10*b^(1/3)
*c-7*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)+1/27*b^(1/3)*(10*b^(1/3)*c-7
*a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)-1/3*e*ln(b*x
^3+a)/a^3+2/27*b^(1/3)*(10*b^(1/3)*c+7*a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^3}\,dx = \frac{2\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}d+10\sqrt[3]{b}c\right)}{9\sqrt{3}a^{11/3}} + \frac{\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}} - \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}} - \frac{x(11bc+10bdx+9bex^2)}{18a^3\left(a+bx^3\right)} - \frac{e\log\left(a+bx^3\right)}{3a^3} - \frac{c}{2a^3x^2} - \frac{d}{a^3x} + \frac{e\log(x)}{a^3} - \frac{x(bc+bdx+bex^2)}{6a^2\left(a+bx^3\right)^2}$$

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -1/2*c/(a^3*x^2) - d/(a^3*x) - (x*(b*c + b*d*x + b*e*x^2))/(6*a^2*(a + b*x^
3)^2) - (x*(11*b*c + 10*b*d*x + 9*b*e*x^2))/(18*a^3*(a + b*x^3)) + (2*b^(1/
3)*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(9*Sqrt[3]*a^(11/3)) + (e*Log[x])/a^3 - (2*b^(1/3)*(10*b^(1/3)*c -
7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)) + (b^(1/3)*(10*b^(1/3)
*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(11
/3)) - (e*Log[a + b*x^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = -\frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2} - \frac{\int \frac{-6bc-6bdx-6bex^2+\frac{5b^2cx^3}{a}+\frac{4b^2dx^4}{a}+\frac{3b^2ex^5}{a}}{x^3(a+bx^3)^2}\,dx}{6ab}$$

$$= -\frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3(a+bx^3)} + \frac{\int \frac{18b^3c+18b^3dx+18b^3ex^2-\frac{22b^4cx^3}{a}-\frac{10b^4dx^4}{a}}{x^3(a+bx^3)}\,dx}{18a^2b^3}$$

$$= -\frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3(a+bx^3)} + \frac{\int \left(\frac{18b^3c}{ax^3}+\frac{18b^3d}{ax^2}+\frac{18b^3e}{ax}-\frac{2b^4(20c+14dx+9ex^2)}{a(a+bx^3)}\right)dx}{18a^2b^3}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3(a+bx^3)} + \frac{e\log(x)}{a^3} - \frac{b\int \frac{20c+14dx+9ex^2}{a+bx^3}\,dx}{9a^3}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3(a+bx^3)} + \frac{e\log(x)}{a^3} - \frac{b\int \frac{20c+14dx}{a+bx^3}\,dx}{9a^3} - \frac{(be)\int \frac{x^2}{a+bx^3}\,dx}{a^3}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3(a+bx^3)} + \frac{e\log(x)}{a^3} - \frac{e\log(a+bx^3)}{3a^3} - \frac{b^{2/3}\int \frac{\sqrt[3]{a}\left(40\sqrt[3]{b}c+14\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-20\sqrt[3]{b}c+14\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{11/3}} - \frac{\left(2b\left(10c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{11/3}}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3(a+bx^3)} + \frac{e\log(x)}{a^3} - \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}} - \frac{e\log(a+bx^3)}{3a^3} + \frac{\left(\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{27a^{11/3}} - \frac{\left(b^{2/3}\left(10\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{9a^{10/3}}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3(a+bx^3)} + \frac{e\log(x)}{a^3} - \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}} + \frac{\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}} - \frac{e\log(a+bx^3)}{3a^3} - \frac{\left(2\sqrt[3]{b}\left(10\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{11/3}}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2} - \frac{x(11bc+10bdx+9bex^2)}{18a^3(a+bx^3)} + \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}} + \frac{e\log(x)}{a^3} - \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}} + \frac{\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}} - \frac{e\log(a+bx^3)}{3a^3}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.92

$$\int\frac{c+dx+ex^2}{x^3(a+bx^3)^3}dx$$

$$= \frac{-\frac{27ac}{x^2}-\frac{54ad}{x}+\frac{9a^2(ae-bx(c+dx))}{(a+bx^3)^2}+\frac{3a(6ae-bx(11c+10dx))}{a+bx^3}+4\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(10\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+ \cdots}{\cdots}$$

[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x]

[Out] ((-27*a*c)/x^2 - (54*a*d)/x + (9*a^2*(a*e - b*x*(c + d*x)))/(a + b*x^3)^2 + (3*a*(6*a*e - b*x*(11*c + 10*d*x)))/(a + b*x^3) + 4*Sqrt[3]*a^(1/3)*b^(1/3)*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 54*a*e*Log[x] + 4*b^(1/3)*(-10*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + 2*b^(1/3)*(10*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 18*a*e*Log[a + b*x^3])/(54*a^4)

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.03

method	result
risch	$\frac{-\frac{14 d b^2 x^7}{9a^3} - \frac{10 c b^2 x^6}{9a^3} + \frac{b e x^5}{3a^2} - \frac{49 b d x^4}{18a^2} - \frac{16 b c x^3}{9a^2} + \frac{e x^2}{2a} - \frac{x d}{a} - \frac{c}{2a}}{x^2 (b x^3 + a)^2} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}\_Z^3+27a^8 e\_Z^2+\left(243a^5e^2+840a^4bcd\right)\_Z+729a^2e^3 \cdots\right.}\right.}{\cdots}$
default	$-\frac{c}{2a^3x^2} - \frac{d}{a^3 x} + \frac{e\ln(x)}{a^3} - \frac{b\left(\frac{\frac{5bd x^5}{9} + \frac{11bc x^4}{18} - \frac{ae x^3}{3} + \frac{13ad x^2}{18} + \frac{7acx}{9} - \frac{a^2 e}{2b}}{(bx^3+a)^2} + \frac{20c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\,\mathrm{arc}\cdots}{\cdots}\right)}{9} \cdots\right)}{\cdots}$

[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] (-14/9*d/a^3*b^2*x^7-10/9*c/a^3*b^2*x^6+1/3*b*e/a^2*x^5-49/18*b*d/a^2*x^4-16/9*b*c/a^2*x^3+1/2/a*e*x^2-1/a*x*d-1/2*c/a)/x^2/(b*x^3+a)^2+1/27*sum(_R*ln((-2*_R^3*a^11-36*_R^2*a^8*e+(-162*a^5*e^2-1400*a^4*b*c*d)*_R-7560*a*b*c*d*e+4116*a*b*d^3-12000*b^2*c^3)*x-7*a^8*d*_R^2+(126*a^5*d*e-200*a^4*b*c^2)*_R+1701*a^2*d*e^2+5400*a*b*c^2*e),_R=RootOf(a^11*_Z^3+27*a^8*e*_Z^2+(243*a^5*e^2+840*a^4*b*c*d)*_Z+729*a^2*e^3+7560*a*b*c*d*e-2744*a*b*d^3+8000*b^2*c^3))+e/a^3*ln(-x)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 4911, normalized size of antiderivative = 17.79

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
[Out] -1/2916*(4536*b^2*d*x^7 + 3240*b^2*c*x^6 - 972*a*b*e*x^5 + 7938*a*b*d*x^4 +
 5184*a*b*c*x^3 - 1458*a^2*e*x^2 + 2916*a^2*d*x + 1458*a^2*c + 2*(a^3*b^2*x
^8 + 2*a^4*b*x^5 + a^5*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81
*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/1968
3*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 5
6*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^
9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)
*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b
)/a^11)^(1/3) + 486*e/a^3)*log(7/2916*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*
b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^1
0 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a
^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1
/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 +
343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*
c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)^2*a^8*d + 3920*a*b*c*d^2 - 1800*a*b*c^
2*e + 567*a^2*d*e^2 + 1/27*(100*a^4*b*c^2 - 63*a^5*d*e)*((-I*sqrt(3) + 1)*(
81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d
 + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(80
00*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(
I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19
683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 -
 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3) + 4*(1000*b^2*c^3 +
343*a*b*d^3)*x) + (1458*b^2*e*x^8 + 2916*a*b*e*x^5 + 1458*a^2*e*x^2 - (a^3*
b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d
 + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4
/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^
3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e
^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a
*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e
)*a*b)/a^11)^(1/3) + 486*e/a^3) - 3*sqrt(1/3)*(a^3*b^2*x^8 + 2*a^4*b*x^5 +
a^5*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)
/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^
3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 -
135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(
280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/
```

39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3
) + 486*e/a^3)^2*a^7 - 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*
a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683
*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56
*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9
 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*
b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)
/a^11)^(1/3) + 486*e/a^3)*a^4*e + 3265920*b*c*d + 236196*a*e^2)/a^7))*log(-
7/2916*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e
^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a
*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e
)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d
 + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(80
00*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e
/a^3)^2*a^8*d - 3920*a*b*c*d^2 + 1800*a*b*c^2*e - 567*a^2*d*e^2 - 1/27*(100
*a^4*b*c^2 - 63*a^5*d*e)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*
e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(
1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(
49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 +
 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/
a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a
^11)^(1/3) + 486*e/a^3) + 8*(1000*b^2*c^3 + 343*a*b*d^3)*x + 1/972*sqrt(1/3
)*(7*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3
/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d
^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*
a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d +
 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000
*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a
^3)*a^8*d - 10800*a^4*b*c^2 - 3402*a^5*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(81*e^
2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81
*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^
2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqr
t(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(
1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(
49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)^2*a^7 - 972*((-I*sqrt(3)
+ 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280
*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/393
66*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) +
 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10
+ 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2
e^3 - 56(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)*a^4*e + 326592
0*b*c*d + 236196*a*e^2)/a^7)) + (1458*b^2*e*x^8 + 2916*a*b*e*x^5 + 1458*a^2
*e*x^2 - (a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^
6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e
^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^

3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3)
 + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000
*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d
^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3) + 3*sqrt(1/3)*(a^3*b^2*x^8 +
2*a^4*b*x^5 + a^5*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d +
81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19
683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 -
 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/
a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^
3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a
*b)/a^11)^(1/3) + 486*e/a^3)^2*a^7 - 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (2
80*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/
a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 72
9*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*
(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3
 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 1
35*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)*a^4*e + 3265920*b*c*d + 236196*a*e^
2)/a^7))*log(-7/2916*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)
/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000
*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d
^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1
458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11
 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)
^(1/3) + 486*e/a^3)^2*a^8*d - 3920*a*b*c*d^2 + 1800*a*b*c^2*e - 567*a^2*d*e
^2 - 1/27*(100*a^4*b*c^2 - 63*a^5*d*e)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280
*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^
10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*
a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-
1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 +
 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135
*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3) + 8*(1000*b^2*c^3 + 343*a*b*d^3)*x -
1/972*sqrt(1/3)*(7*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a
^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b
*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3
 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/145
8*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 -
 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(
1/3) + 486*e/a^3)*a^8*d - 10800*a^4*b*c^2 - 3402*a^5*d*e)*sqrt(-(((-I*sqrt(
3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(
280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/
39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3
) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^
10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*
a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)^2*a^7 - 972
*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9

```
 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*
b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)
/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*
a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2
*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)*
a^4*e + 3265920*b*c*d + 236196*a*e^2)/a^7)) - 2916*(b^2*e*x^8 + 2*a*b*e*x^5
 + a^2*e*x^2)*log(x))/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^3}\,dx &= -\frac{28\,b^2dx^7+20\,b^2cx^6-6\,abex^5+49\,abdx^4+32\,abcx^3-9\,a^2ex^2+18\,a^2dx+9\,a^2c}{18\,\left(a^3b^2x^8+2\,a^4bx^5+a^5x^2\right)}\\&\quad+\frac{e\log\left(x\right)}{a^3}-\frac{2\sqrt{3}\left(7\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+10\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4}\\&\quad-\frac{\left(9\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}+7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-10\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\&\quad-\frac{\left(9\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}-14\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(28*b^2*d*x^7 + 20*b^2*c*x^6 - 6*a*b*e*x^5 + 49*a*b*d*x^4 + 32*a*b*c*
x^3 - 9*a^2*e*x^2 + 18*a^2*d*x + 9*a^2*c)/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*
x^2) + e*log(x)/a^3 - 2/27*sqrt(3)*(7*b*d*(a/b)^(2/3) + 10*b*c*(a/b)^(1/3))
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^4 - 1/27*(9*e*(a/b)^
```

```
(2/3) + 7*d*(a/b)^(1/3) - 10*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3
*(a/b)^(2/3)) - 1/27*(9*e*(a/b)^(2/3) - 14*d*(a/b)^(1/3) + 20*c)*log(x + (a
/b)^(1/3))/(a^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)^3}\,dx = -\frac{e\log(|bx^3+a|)}{3\,a^3}+\frac{e\log(|x|)}{a^3} - \frac{2\sqrt{3}\left(10\,(-ab^2)^{\frac{1}{3}}\,bc-7\,(-ab^2)^{\frac{2}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b} - \frac{\left(10\,(-ab^2)^{\frac{1}{3}}\,bc+7\,(-ab^2)^{\frac{2}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4b} - \frac{28\,b^2dx^7+20\,b^2cx^6-6\,abex^5+49\,abdx^4+32\,abcx^3-9\,a^2ex^2+18\,a^2dx+9\,a^2c}{18\,(bx^4+ax)^2a^3} + \frac{2\left(7\,a^3b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+10\,a^3b^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b}$$

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^3 + e*log(abs(x))/a^3 - 2/27*sqrt(3)*(10*(-a*b
^2)^(1/3)*b*c - 7*(-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))/(a^4*b) - 1/27*(10*(-a*b^2)^(1/3)*b*c + 7*(-a*b^2)^(2/3)*d)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/18*(28*b^2*d*x^7 + 20*
b^2*c*x^6 - 6*a*b*e*x^5 + 49*a*b*d*x^4 + 32*a*b*c*x^3 - 9*a^2*e*x^2 + 18*a^
2*d*x + 9*a^2*c)/((b*x^4 + a*x)^2*a^3) + 2/27*(7*a^3*b^2*d*(-a/b)^(1/3) + 1
0*a^3*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 778, normalized size of antiderivative = 2.82

$$\int \frac{c+dx+ex^2}{x^3 (a+bx^3)^3} \, dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(-\frac{b^3 \left(\mathrm{root}(19683\, a^{11}\, z^3+19683\, a^8\, e\, z^2+22680\, a^4\, b\, c\, d\, z+6561\, a^5\, e^2\, z+7560\, a\, b\, c\, d\, e-2744\, a\, b\right.}{}\right.\right.$$
$$+ 19683\, a^8\, e\, z^2 + 22680\, a^4\, b\, c\, d\, z + 6561\, a^5\, e^2\, z + 7560\, a\, b\, c\, d\, e - 2744\, a\, b\, d^3 + 729\, a^2\, e^3$$
$$\left.\left. + 8000\, b^2\, c^3, z, k)\right)\right)$$
$$- \frac{\frac{c}{2\,a} - \frac{e\,x^2}{2\,a} + \frac{d\,x}{a} + \frac{10\,b^2\,c\,x^6}{9\,a^3} + \frac{14\,b^2\,d\,x^7}{9\,a^3} + \frac{16\,b\,c\,x^3}{9\,a^2} + \frac{49\,b\,d\,x^4}{18\,a^2} - \frac{b\,e\,x^5}{3\,a^2}}{a^2\,x^2 + 2\,a\,b\,x^5 + b^2\,x^8} + \frac{e\,\ln(x)}{a^3}$$

```
[In] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x)
```

```
[Out] symsum(log(-(2*b^3*(1701*root(19683*a^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*
b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 80
00*b^2*c^3, z, k)^2*a^8*d - 567*a^2*d*e^2 + 13122*root(19683*a^11*z^3 + 196
83*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 2744*a
*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)^3*a^11*x + 4000*b^2*c^3*x - 1134
*root(19683*a^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z
 + 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)*a^5*d*
e - 1800*a*b*c^2*e - 1372*a*b*d^3*x + 1800*root(19683*a^11*z^3 + 19683*a^8*
e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 2744*a*b*d^3
+ 729*a^2*e^3 + 8000*b^2*c^3, z, k)*a^4*b*c^2 + 1458*root(19683*a^11*z^3 +
19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 274
4*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)*a^5*e^2*x + 8748*root(19683*a
^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c
*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)^2*a^8*e*x + 12600*r
oot(19683*a^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z +
 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)*a^4*b*c*
d*x + 2520*a*b*c*d*e*x))/(729*a^9))*root(19683*a^11*z^3 + 19683*a^8*e*z^2 +
 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a
^2*e^3 + 8000*b^2*c^3, z, k), k, 1, 3) - (c/(2*a) - (e*x^2)/(2*a) + (d*x)/a
 + (10*b^2*c*x^6)/(9*a^3) + (14*b^2*d*x^7)/(9*a^3) + (16*b*c*x^3)/(9*a^2) +
 (49*b*d*x^4)/(18*a^2) - (b*e*x^5)/(3*a^2))/(a^2*x^2 + b^2*x^8 + 2*a*b*x^5)
 + (e*log(x))/a^3
```

3.357 $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 298

$$\begin{aligned}\int \frac{c+dx+ex^2}{x^4(a+bx^3)^3}\,dx =& -\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2(a+bx^3)^2}\\&-\frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3(a+bx^3)}\\&+\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}}\\&-\frac{3bc\log(x)}{a^4}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}}\\&+\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}}\\&+\frac{bc\log(a+bx^3)}{a^4}\end{aligned}$$

```
[Out] -1/3*c/a^3/x^3-1/2*d/a^3/x^2-e/a^3/x-1/6*x*(b*d+b*e*x-b^2*c*x^2/a)/a^2/(b*x
^3+a)^2-1/18*x*(11*b*d+10*b*e*x-15*b^2*c*x^2/a)/a^3/(b*x^3+a)-3*b*c*ln(x)/a
^4-2/27*b^(1/3)*(10*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)+1
/27*b^(1/3)*(10*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/a^(11/3)+b*c*ln(b*x^3+a)/a^4+2/27*b^(1/3)*(10*b^(1/3)*d+7*a^(1/3)*e)*
arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^3}\,dx = \frac{2\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}e+10\sqrt[3]{b}d\right)}{9\sqrt{3}a^{11/3}} + \frac{\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}} - \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}} + \frac{bc\log(a+bx^3)}{a^4} - \frac{3bc\log(x)}{a^4} - \frac{x\left(-\frac{15b^2cx^2}{a}+11bd+10bex\right)}{18a^3(a+bx^3)} - \frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{6a^2(a+bx^3)^2}$$

```
[In] Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -1/3*c/(a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d + b*e*x - (b^2*c*x^2
)/a))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d + 10*b*e*x - (15*b^2*c*x^2)/a))/(1
8*a^3*(a + b*x^3)) + (2*b^(1/3)*(10*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)) - (3*b*c*Log[x])/
a^4 - (2*b^(1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27
*a^(11/3)) + (b^(1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(27*a^(11/3)) + (b*c*Log[a + b*x^3])/a^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2}-\frac{\int\frac{-6bc-6bdx-6bex^2+\frac{6b^2cx^3}{a}+\frac{5b^2dx^4}{a}+\frac{4b^2ex^5}{a}-\frac{3b^3cx^6}{a^2}}{x^4\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2}-\frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)}+\frac{\int\frac{18b^3c+18b^3dx+18b^3ex^2-\frac{36b^4cx^3}{a}-\frac{22b^4dx^4}{a}-\frac{10b^4ex^5}{a}}{x^4\left(a+bx^3\right)}\,dx}{18a^2b^3}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2}-\frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)}+\frac{\int\left(\frac{18b^3c}{ax^4}+\frac{18b^3d}{ax^3}+\frac{18b^3e}{ax^2}-\frac{54b^4c}{a^2x}-\frac{2b^4\left(20ad+14aex-27bcx^2\right)}{a^2\left(a+bx^3\right)}\right)dx}{18a^2b^3}$$

$$= -\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2}-\frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)}-\frac{3bc\log(x)}{a^4}-\frac{b\int\frac{20ad+14aex-27bcx^2}{a+bx^3}\,dx}{9a^4}$$

$$= -\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2}-\frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)}-\frac{3bc\log(x)}{a^4}-\frac{b\int\frac{20ad+14aex}{a+bx^3}\,dx}{9a^4}+\frac{\left(3b^2c\right)\int\frac{x^2}{a+bx^3}\,dx}{a^4}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2(a+bx^3)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3(a+bx^3)} - \frac{3bc\log(x)}{a^4} + \frac{bc\log(a+bx^3)}{a^4} - \frac{b^{2/3}\int \frac{\sqrt[3]{a}\left(40a\sqrt[3]{b}d+14a^{4/3}e\right)+\sqrt[3]{b}\left(-20a\sqrt[3]{b}d+14a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{27a^{14/3}} - \frac{\left(2b\left(10d-\frac{7\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{27a^{11/3}}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2(a+bx^3)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3(a+bx^3)} - \frac{3bc\log(x)}{a^4} - \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}} + \frac{bc\log(a+bx^3)}{a^4} + \frac{\left(\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{27a^{11/3}} - \frac{\left(b^{2/3}\left(10\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{9a^{10/3}}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2(a+bx^3)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3(a+bx^3)} - \frac{3bc\log(x)}{a^4} - \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}} + \frac{\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}} + \frac{bc\log(a+bx^3)}{a^4} - \frac{\left(2\sqrt[3]{b}\left(10\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{11/3}}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^3x} - \frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{6a^2(a+bx^3)^2}$$

$$- \frac{x\left(11bd + 10bex - \frac{15b^2cx^2}{a}\right)}{18a^3(a+bx^3)} + \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}d + 7\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}}$$

$$- \frac{3bc\log(x)}{a^4} - \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}d - 7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{11/3}}$$

$$+ \frac{\sqrt[3]{b}\left(10\sqrt[3]{b}d - 7\sqrt[3]{a}e\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{27a^{11/3}} + \frac{bc\log(a+bx^3)}{a^4}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.86

$$\int \frac{c + dx + ex^2}{x^4(a+bx^3)^3}\,dx =$$

$$-\frac{\frac{18ac}{x^3} + \frac{27ad}{x^2} + \frac{54ae}{x} + \frac{9a^2b(c+x(d+ex))}{(a+bx^3)^2} + \frac{3ab(12c+x(11d+10ex))}{a+bx^3} - 4\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(10\sqrt[3]{b}d + 7\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -1/54*((18*a*c)/x^3 + (27*a*d)/x^2 + (54*a*e)/x + (9*a^2*b*(c + x*(d + e*x)
))/(a + b*x^3)^2 + (3*a*b*(12*c + x*(11*d + 10*e*x)))/(a + b*x^3) - 4*Sqrt[
3]*a^(1/3)*b^(1/3)*(10*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 162*b*c*Log[x] + 4*b^(1/3)*(10*a^(1/3)*b^(1/3)*d - 7*a^(
2/3)*e)*Log[a^(1/3) + b^(1/3)*x] - 2*b^(1/3)*(10*a^(1/3)*b^(1/3)*d - 7*a^(2
/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 54*b*c*Log[a + b*x^
3])/a^4
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.01

method	result
default	$-\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{3bc\ln(x)}{a^4}-\frac{b\left(\frac{\frac{5}{9}abex^5+\frac{11}{18}abdx^4+\frac{2}{3}abcx^3+\frac{13}{18}a^2ex^2+\frac{7}{9}a^2dx+\frac{5}{6}a^2c}{(bx^3+a)^2}+\frac{20ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\ldots\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}$
risch	$\frac{-\frac{14eb^2x^8}{9a^3}-\frac{10db^2x^7}{9a^3}-\frac{cb^2x^6}{a^3}-\frac{49bex^5}{18a^2}-\frac{16bdx^4}{9a^2}-\frac{3bcx^3}{2a^2}-\frac{ex^2}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^3(bx^3+a)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{12}\_Z^3-81a^8bc\_Z^2+\left(840a^5bde+2187a^4b^2c\ldots\right.\right.}\right.}{\ldots}$

```
[In] int((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*c/a^3/x^3-1/2*d/a^3/x^2-e/a^3/x-3*b*c*ln(x)/a^4-1/a^4*b*((5/9*a*b*e*x^
5+11/18*a*b*d*x^4+2/3*a*b*c*x^3+13/18*a^2*e*x^2+7/9*a^2*d*x+5/6*a^2*c)/(b*x
^3+a)^2+20/9*a*d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(
x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1)))+14/9*a*e*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b
/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-c*ln(b*x^3+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 5550, normalized size of antiderivative = 18.62

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.95

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^3}\,dx = -\frac{28\,b^2ex^8+20\,b^2dx^7+18\,b^2cx^6+49\,abex^5+32\,abdx^4+27\,abcx^3+18\,a^2ex^2+9\,a^2dx+6\,a^2c}{18\,\left(a^3b^2x^9+2\,a^4bx^6+a^5x^3\right)} - \frac{3\,bc\log\left(x\right)}{a^4} - \frac{2\,\sqrt{3}\left(7\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+10\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5} + \frac{\left(27\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-7\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+10\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(27\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
[Out] -1/18*(28*b^2*e*x^8 + 20*b^2*d*x^7 + 18*b^2*c*x^6 + 49*a*b*e*x^5 + 32*a*b*d
*x^4 + 27*a*b*c*x^3 + 18*a^2*e*x^2 + 9*a^2*d*x + 6*a^2*c)/(a^3*b^2*x^9 + 2*
a^4*b*x^6 + a^5*x^3) - 3*b*c*log(x)/a^4 - 2/27*sqrt(3)*(7*a*e*(a/b)^(2/3) +
 10*a*d*(a/b)^(1/3))*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/
a^5 + 1/27*(27*b*c*(a/b)^(2/3) - 7*a*e*(a/b)^(1/3) + 10*a*d)*log(x^2 - x*(a
/b)^(1/3) + (a/b)^(2/3))/(a^4*(a/b)^(2/3)) + 1/27*(27*b*c*(a/b)^(2/3) + 14*
a*e*(a/b)^(1/3) - 20*a*d)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^3}\,dx = \frac{bc\log\left(\left|bx^3+a\right|\right)}{a^4} - \frac{3\,bc\log\left(\left|x\right|\right)}{a^4} - \frac{2\,\sqrt{3}\left(10\left(-ab^2\right)^{\frac{1}{3}}bd - 7\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b} - \frac{\left(10\left(-ab^2\right)^{\frac{1}{3}}bd + 7\left(-ab^2\right)^{\frac{2}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4b} + \frac{2\left(7\,a^5b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}}+10\,a^5b^2d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^9b} - \frac{28\,ab^2ex^8+20\,ab^2dx^7+18\,ab^2cx^6+49\,a^2bex^5+32\,a^2bdx^4+27\,a^2bcx^3+18\,a^3ex^2+9\,a^3dx+6\,a^3c}{18\left(bx^3+a\right)^2a^4x^3}$$

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="giac")
[Out] b*c*log(abs(b*x^3 + a))/a^4 - 3*b*c*log(abs(x))/a^4 - 2/27*sqrt(3)*(10*(-a*
b^2)^(1/3)*b*d - 7*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/(a^4*b) - 1/27*(10*(-a*b^2)^(1/3)*b*d + 7*(-a*b^2)^(2/3)*e)
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) + 2/27*(7*a^5*b^2*e*(-a/b
)^(1/3) + 10*a^5*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b) - 1
/18*(28*a*b^2*e*x^8 + 20*a*b^2*d*x^7 + 18*a*b^2*c*x^6 + 49*a^2*b*e*x^5 + 32
*a^2*b*d*x^4 + 27*a^2*b*c*x^3 + 18*a^3*e*x^2 + 9*a^3*d*x + 6*a^3*c)/((b*x^3
 + a)^2*a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 870, normalized size of antiderivative = 2.92

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^3}\,dx = \left(\sum_{k=1}^{3}\ln\left(-\frac{b^3\left(\mathrm{root}(19683\,a^{12}\,z^3-59049\,a^8\,b\,c\,z^2+22680\,a^5\,b\,d\,e\,z+59049\,a^4\,b^2\,c^2\,z-22680\,a\,b^2\,c\,d\,e-\right.}{\ldots}\right.\right. - 59049\,a^8\,b\,c\,z^2+22680\,a^5\,b\,d\,e\,z+59049\,a^4\,b^2\,c^2\,z-22680\,a\,b^2\,c\,d\,e-2744\,a^2\,b\,e^3 \left.+8000\,a\,b^2\,d^3-19683\,b^3\,c^3,z,k)\right) - \frac{\frac{c}{3\,a}+\frac{e\,x^2}{a}+\frac{d\,x}{2\,a}+\frac{b^2\,c\,x^6}{a^3}+\frac{10\,b^2\,d\,x^7}{9\,a^3}+\frac{14\,b^2\,e\,x^8}{9\,a^3}+\frac{3\,b\,c\,x^3}{2\,a^2}+\frac{16\,b\,d\,x^4}{9\,a^2}+\frac{49\,b\,e\,x^5}{18\,a^2}}{a^2\,x^3+2\,a\,b\,x^6+b^2\,x^9} - \frac{3\,b\,c\,\ln\left(x\right)}{a^4}$$

```
[In] int((c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x)
```

```
[Out] symsum(log(-(2*b^3*(1701*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^
5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000
*a*b^2*d^3 - 19683*b^3*c^3, z, k)^2*a^8*e + 5400*b^2*c*d^2 - 5103*b^2*c^2*e
 + 13122*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 5904
9*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 196
83*b^3*c^3, z, k)^3*a^11*x + 4000*b^2*d^3*x - 1372*a*b*e^3*x + 1800*root(19
683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z
- 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k
)*a^4*b*d^2 - 26244*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d
*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^
2*d^3 - 19683*b^3*c^3, z, k)^2*a^7*b*c*x + 13122*root(19683*a^12*z^3 - 5904
9*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e
 - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k)*a^3*b^2*c^2*x + 3
402*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4
*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^
3*c^3, z, k)*a^4*b*c*e - 7560*b^2*c*d*e*x + 12600*root(19683*a^12*z^3 - 590
49*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*
e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k)*a^4*b*d*e*x))/(7
29*a^9))*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 5904
9*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 196
83*b^3*c^3, z, k), k, 1, 3) - (c/(3*a) + (e*x^2)/a + (d*x)/(2*a) + (b^2*c*x
^6)/a^3 + (10*b^2*d*x^7)/(9*a^3) + (14*b^2*e*x^8)/(9*a^3) + (3*b*c*x^3)/(2*
a^2) + (16*b*d*x^4)/(9*a^2) + (49*b*e*x^5)/(18*a^2))/(a^2*x^3 + b^2*x^9 + 2
*a*b*x^6) - (3*b*c*log(x))/a^4
```

3.358 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4} dx$

Optimal result

Integrand size = 23, antiderivative size = 248

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4} dx = -\frac{c+dx+ex^2}{9b(a+bx^3)^3} + \frac{x(d+2ex)}{54ab(a+bx^3)^2} + \frac{x(5d+8ex)}{162a^2b(a+bx^3)} - \frac{\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{8/3}b^{5/3}} + \frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{8/3}b^{5/3}} - \frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{8/3}b^{5/3}}$$

```
[Out] 1/9*(-e*x^2-d*x-c)/b/(b*x^3+a)^3+1/54*x*(2*e*x+d)/a/b/(b*x^3+a)^2+1/162*x*(
8*e*x+5*d)/a^2/b/(b*x^3+a)+1/243*(5*b^(1/3)*d-4*a^(1/3)*e)*ln(a^(1/3)+b^(1/
3)*x)/a^(8/3)/b^(5/3)-1/486*(5*b^(1/3)*d-4*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(5/3)-1/243*(5*b^(1/3)*d+4*a^(1/3)*e)*arctan
(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1837, 1869, 1874, 31, 648, 631, 210, 642}

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)}{81\sqrt{3}a^{8/3}b^{5/3}} - \frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{8/3}b^{5/3}} + \frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{8/3}b^{5/3}} + \frac{x(5d+8ex)}{162a^2b\,(a+bx^3)} - \frac{c+dx+ex^2}{9b\,(a+bx^3)^3} + \frac{x(d+2ex)}{54ab\,(a+bx^3)^2}$$

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] -1/9*(c + d*x + e*x^2)/(b*(a + b*x^3)^3) + (x*(d + 2*e*x))/(54*a*b*(a + b*x
^3)^2) + (x*(5*d + 8*e*x))/(162*a^2*b*(a + b*x^3)) - ((5*b^(1/3)*d + 4*a^(1
/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(8/
3)*b^(5/3)) + ((5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a
^(8/3)*b^(5/3)) - ((5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(486*a^(8/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{\int \frac{d+2ex}{\left(a+bx^3\right)^3}\,dx}{9b}\\
&= -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{x(d+2ex)}{54ab\left(a+bx^3\right)^2}-\frac{\int \frac{-5d-8ex}{\left(a+bx^3\right)^2}\,dx}{54ab}\\
&= -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{x(d+2ex)}{54ab\left(a+bx^3\right)^2}+\frac{x(5d+8ex)}{162a^2b\left(a+bx^3\right)}+\frac{\int \frac{10d+8ex}{a+bx^3}\,dx}{162a^2b}
\end{aligned}$$

$$= -\frac{c+dx+ex^2}{9b(a+bx^3)^3} + \frac{x(d+2ex)}{54ab(a+bx^3)^2} + \frac{x(5d+8ex)}{162a^2b(a+bx^3)} + \frac{\int \frac{\sqrt[3]{a}\left(20\sqrt[3]{b}d+8\sqrt[3]{a}e\right)+\sqrt[3]{b}\left(-10\sqrt[3]{b}d+8\sqrt[3]{a}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{486a^{8/3}b^{4/3}} + \frac{\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{243a^{8/3}b}$$

$$= -\frac{c+dx+ex^2}{9b(a+bx^3)^3} + \frac{x(d+2ex)}{54ab(a+bx^3)^2} + \frac{x(5d+8ex)}{162a^2b(a+bx^3)} + \frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{8/3}b^{5/3}} - \frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{486a^{8/3}b^{5/3}} + \frac{\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{162a^{7/3}b^{4/3}}$$

$$= -\frac{c+dx+ex^2}{9b(a+bx^3)^3} + \frac{x(d+2ex)}{54ab(a+bx^3)^2} + \frac{x(5d+8ex)}{162a^2b(a+bx^3)} + \frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{8/3}b^{5/3}} - \frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{8/3}b^{5/3}} + \frac{\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{81a^{8/3}b^{5/3}}$$

$$= -\frac{c+dx+ex^2}{9b(a+bx^3)^3} + \frac{x(d+2ex)}{54ab(a+bx^3)^2} + \frac{x(5d+8ex)}{162a^2b(a+bx^3)} - \frac{\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{8/3}b^{5/3}} + \frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{8/3}b^{5/3}} - \frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{8/3}b^{5/3}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.93

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}dx$$

$$= \frac{\frac{9b^{2/3}x(d+2ex)}{a(a+bx^3)^2} + \frac{3b^{2/3}x(5d+8ex)}{a^2(a+bx^3)} - \frac{54b^{2/3}(c+x(d+ex))}{(a+bx^3)^3} - \frac{2\sqrt{3}\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}} + \frac{2\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}\right.}{a^{8/3}}}{486b^{5/3}}$$

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] ((9*b^(2/3)*x*(d + 2*e*x))/(a*(a + b*x^3)^2) + (3*b^(2/3)*x*(5*d + 8*e*x))/
(a^2*(a + b*x^3)) - (54*b^(2/3)*(c + x*(d + e*x)))/(a + b*x^3)^3 - (2*Sqrt[
3]*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])
/a^(8/3) + (2*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/a^(8/3)
 + ((-5*b^(1/3)*d + 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/a^(8/3))/(486*b^(5/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\frac{4be\,x^{8}}{81a^{2}}+\frac{5bd\,x^{7}}{162a^{2}}+\frac{11e\,x^{5}}{81a}+\frac{13d\,x^{4}}{162a}-\frac{2e\,x^{2}}{81b}-\frac{5dx}{81b}-\frac{c}{9b}}{(b\,x^{3}+a)^{3}}+\frac{\sum_{\_R=\operatorname{RootOf}(b\_Z^{3}+a)}\frac{(4e\_R+5d)\ln(x-\_R)}{\_R^{2}}}{243a^{2}b^{2}}$
default	$\frac{\frac{4be\,x^{8}}{81a^{2}}+\frac{5bd\,x^{7}}{162a^{2}}+\frac{11e\,x^{5}}{81a}+\frac{13d\,x^{4}}{162a}-\frac{2e\,x^{2}}{81b}-\frac{5dx}{81b}-\frac{c}{9b}}{(b\,x^{3}+a)^{3}}+\frac{5d\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+}{81a}$

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (4/81*b*e/a^2*x^8+5/162*b*d/a^2*x^7+11/81/a*e*x^5+13/162*d/a*x^4-2/81*e*x^2
/b-5/81*d*x/b-1/9*c/b)/(b*x^3+a)^3+1/243/a^2/b^2*sum((4*_R*e+5*d)/_R^2*ln(x
-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 2364, normalized size of antiderivative = 9.53

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(48*b^2*e*x^8 + 30*b^2*d*x^7 + 132*a*b*e*x^5 + 78*a*b*d*x^4 - 24*a^2*
e*x^2 - 60*a^2*d*x - 108*a^2*c - 2*(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2
*x^3 + a^5*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5
) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3
) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/
(a^8*b^5))^(1/3)))*log(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)
/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(
-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 6
4*a*e^3)/(a^8*b^5))^(1/3)))^2*a^6*b^3*e - 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)
*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3
) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a
^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))*a^3*b^2*d^2 + 160*a*d*e
^2 + (125*b*d^3 + 64*a*e^3)*x) + ((a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*
x^3 + a^5*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5)
 + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3)
 + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(
a^8*b^5))^(1/3))) + 3*sqrt(1/3)*(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^
3 + a^5*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8
*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sq
rt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e
^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 320*d*e)/(a^5*b^3)))*log(-((1/2)^(1/3)*(
I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(
a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3
+ 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^6*b^3
*e + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) +
(125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) +
1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8
*b^5))^(1/3)))*a^3*b^2*d^2 - 160*a*d*e^2 + 2*(125*b*d^3 + 64*a*e^3)*x + 3/2
*sqrt(1/3)*(2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5
) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3
) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/
(a^8*b^5))^(1/3)))*a^6*b^3*e + 25*a^3*b^2*d^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(
3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5
))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*
e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 320*
d*e)/(a^5*b^3))) + ((a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 + a^5*b)*(
(1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3
- 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3
*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3
))) - 3*sqrt(1/3)*(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 + a^5*b)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b
*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^
5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))
^(1/3)))^2*a^5*b^3 + 320*d*e)/(a^5*b^3)))*log(-((1/2)^(1/3)*(I*sqrt(3) + 1)
*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3
```

```
) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a
^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^6*b^3*e + 25/2*((1/
2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 6
4*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((
125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))
*a^3*b^2*d^2 - 160*a*d*e^2 + 2*(125*b*d^3 + 64*a*e^3)*x - 3/2*sqrt(1/3)*(2*
((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3
 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^
3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/
3)))*a^6*b^3*e + 25*a^3*b^2*d^2)*sqrt(-((((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*
b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*
(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5)
 + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 320*d*e)/(a^5*b^3)
)))/(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 + a^5*b)
```

Sympy [A] (verification not implemented)

Time = 156.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.81

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx$$
$$= \text{RootSum}\left(14348907t^3a^8b^5 + 14580ta^3b^2de + 64ae^3 - 125bd^3, \left(t \mapsto t\log\left(x + \frac{236196t^2a^6b^3e + 6075ta^3b}{64ae^3 + 125b}\right.\right.\right.$$
$$+ \frac{-18a^2c - 10a^2dx - 4a^2ex^2 + 13abdx^4 + 22abex^5 + 5b^2dx^7 + 8b^2ex^8}{162a^5b + 486a^4b^2x^3 + 486a^3b^3x^6 + 162a^2b^4x^9}$$

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**8*b**5 + 14580*_t*a**3*b**2*d*e + 64*a*e**3 - 125
*b*d**3, Lambda(_t, _t*log(x + (236196*_t**2*a**6*b**3*e + 6075*_t*a**3*b**
2*d**2 + 160*a*d*e**2)/(64*a*e**3 + 125*b*d**3)))) + (-18*a**2*c - 10*a**2*
d*x - 4*a**2*e*x**2 + 13*a*b*d*x**4 + 22*a*b*e*x**5 + 5*b**2*d*x**7 + 8*b**
2*e*x**8)/(162*a**5*b + 486*a**4*b**2*x**3 + 486*a**3*b**3*x**6 + 162*a**2*
b**4*x**9)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx = \frac{8\,b^2ex^8+5\,b^2dx^7+22\,abex^5+13\,abdx^4-4\,a^2ex^2-10\,a^2dx-18\,a^2c}{162\,(a^2b^4x^9+3\,a^3b^3x^6+3\,a^4b^2x^3+a^5b)} + \frac{\sqrt{3}\left(4\,e\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(4\,e\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,d\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(4\,e\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,d\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/162*(8*b^2*e*x^8 + 5*b^2*d*x^7 + 22*a*b*e*x^5 + 13*a*b*d*x^4 - 4*a^2*e*x^
2 - 10*a^2*d*x - 18*a^2*c)/(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 + a
^5*b) + 1/243*sqrt(3)*(4*e*(a/b)^(1/3) + 5*d)*arctan(1/3*sqrt(3)*(2*x - (a/
b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) + 1/486*(4*e*(a/b)^(1/3) - 5*d
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2/3)) - 1/243*(4*e
*(a/b)^(1/3) - 5*d)*log(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.95

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx = -\frac{\sqrt{3}\left(5\,bd-4\,(-ab^2)^{\frac{1}{3}}\,e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,\left(-ab^2\right)^{\frac{2}{3}}\,a^2b} - \frac{\left(5\,bd+4\,(-ab^2)^{\frac{1}{3}}\,e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,\left(-ab^2\right)^{\frac{2}{3}}\,a^2b} - \frac{\left(4\,e\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^3b} + \frac{8\,b^2ex^8+5\,b^2dx^7+22\,abex^5+13\,abdx^4-4\,a^2ex^2-10\,a^2dx-18\,a^2c}{162\,\left(bx^3+a\right)^3a^2b}$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/243*sqrt(3)*(5*b*d - 4*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b) - 1/486*(5*b*d + 4*(-a*b^2)^(
1/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b) - 1
/243*(4*e*(-a/b)^(1/3) + 5*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*
b) + 1/162*(8*b^2*e*x^8 + 5*b^2*d*x^7 + 22*a*b*e*x^5 + 13*a*b*d*x^4 - 4*a^2
*e*x^2 - 10*a^2*d*x - 18*a^2*c)/((b*x^3 + a)^3*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.02

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\frac{20\,d\,e+16\,e^2\,x+\mathrm{root}(14348907\,a^8\,b^5\,z^3+14580\,a^3\,b^2\,d\,e\,z-125\,b\,d^3+64\,a\,e^3,z,k)^2\,a^5\,b^3\,5904\cdots}{a^4\,b\,6561}\right.\right.$$
$$\left.\left.\cdots+14580\,a^3\,b^2\,d\,e\,z-125\,b\,d^3+64\,a\,e^3,z,k\right)\right)$$
$$+\frac{\frac{13\,d\,x^4}{162\,a}-\frac{c}{9\,b}+\frac{11\,e\,x^5}{81\,a}-\frac{2\,e\,x^2}{81\,b}-\frac{5\,d\,x}{81\,b}+\frac{5\,b\,d\,x^7}{162\,a^2}+\frac{4\,b\,e\,x^8}{81\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9}$$

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x)
```

```
[Out] symsum(log((20*d*e + 16*e^2*x + 59049*root(14348907*a^8*b^5*z^3 + 14580*a^3
*b^2*d*e*z - 125*b*d^3 + 64*a*e^3, z, k)^2*a^5*b^3 + 1215*root(14348907*a^8
*b^5*z^3 + 14580*a^3*b^2*d*e*z - 125*b*d^3 + 64*a*e^3, z, k)*a^2*b^2*d*x)/(
6561*a^4*b))*root(14348907*a^8*b^5*z^3 + 14580*a^3*b^2*d*e*z - 125*b*d^3 +
64*a*e^3, z, k), k, 1, 3) + ((13*d*x^4)/(162*a) - c/(9*b) + (11*e*x^5)/(81*
a) - (2*e*x^2)/(81*b) - (5*d*x)/(81*b) + (5*b*d*x^7)/(162*a^2) + (4*b*e*x^8
)/(81*a^2))/(a^3 + b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2*x^6)
```

3.359 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 270

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4} dx = -\frac{x(ae-bcx-bdx^2)}{9ab(a+bx^3)^3} + \frac{x(5ae+28bcx)}{162a^3b(a+bx^3)} - \frac{6ad-x(ae+7bcx)}{54a^2b(a+bx^3)^2} - \frac{(14b^{2/3}c+5a^{2/3}e)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{10/3}b^{4/3}} - \frac{(14b^{2/3}c-5a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{10/3}b^{4/3}} + \frac{(14b^{2/3}c-5a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{10/3}b^{4/3}}$$

```
[Out] -1/9*x*(-b*d*x^2-b*c*x+a*e)/a/b/(b*x^3+a)^3+1/162*x*(28*b*c*x+5*a*e)/a^3/b/
(b*x^3+a)+1/54*(-6*a*d+x*(7*b*c*x+a*e))/a^2/b/(b*x^3+a)^2-1/243*(14*b^(2/3)
*c-5*a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(4/3)+1/486*(14*b^(2/3)*c-
5*a^(2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(4/3)-1/2
43*(14*b^(2/3)*c+5*a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1
/2))/a^(10/3)/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1842, 1868, 1869, 1874, 31, 648, 631, 210, 642}

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(5a^{2/3}e+14b^{2/3}c\right)}{81\sqrt{3}a^{10/3}b^{4/3}} + \frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{10/3}b^{4/3}} - \frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{10/3}b^{4/3}} + \frac{x(5ae+28bcx)}{162a^3b\,(a+bx^3)} - \frac{6ad-x(ae+7bcx)}{54a^2b\,(a+bx^3)^2} - \frac{x(ae-bcx-bdx^2)}{9ab\,(a+bx^3)^3}$$

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] -1/9*(x*(a*e - b*c*x - b*d*x^2))/(a*b*(a + b*x^3)^3) + (x*(5*a*e + 28*b*c*x
))/(162*a^3*b*(a + b*x^3)) - (6*a*d - x*(a*e + 7*b*c*x))/(54*a^2*b*(a + b*x
^3)^2) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(81*Sqrt[3]*a^(10/3)*b^(4/3)) - ((14*b^(2/3)*c - 5*a^(2/3)*e
)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(10/3)*b^(4/3)) + ((14*b^(2/3)*c - 5*a^(
2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(486*a^(10/3)*b^(4/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\text{integral} = -\frac{x(ae-bcx-bdx^2)}{9ab(a+bx^3)^3} - \frac{\int \frac{-ae-7bcx-6bdx^2}{(a+bx^3)^3}\,dx}{9ab}$$

$$= -\frac{x(ae-bcx-bdx^2)}{9ab(a+bx^3)^3} - \frac{6ad-x(ae+7bcx)}{54a^2b(a+bx^3)^2} + \frac{\int \frac{5ae+28bcx}{(a+bx^3)^2}\,dx}{54a^2b}$$

$$= -\frac{x(ae-bcx-bdx^2)}{9ab(a+bx^3)^3} + \frac{x(5ae+28bcx)}{162a^3b(a+bx^3)} - \frac{6ad-x(ae+7bcx)}{54a^2b(a+bx^3)^2} - \frac{\int \frac{-10ae-28bcx}{a+bx^3}\,dx}{162a^3b}$$

$$= -\frac{x(ae-bcx-bdx^2)}{9ab(a+bx^3)^3} + \frac{x(5ae+28bcx)}{162a^3b(a+bx^3)} - \frac{6ad-x(ae+7bcx)}{54a^2b(a+bx^3)^2} - \frac{\int \frac{\sqrt[3]{a}\left(-28\sqrt[3]{a}bc-20a\sqrt[3]{b}e\right)+\sqrt[3]{b}\left(-28\sqrt[3]{a}bc+10a\sqrt[3]{b}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{486a^{11/3}b^{4/3}} - \frac{\left(14b^{2/3}c-5a^{2/3}e\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{243a^{10/3}b}$$

$$= -\frac{x(ae-bcx-bdx^2)}{9ab(a+bx^3)^3} + \frac{x(5ae+28bcx)}{162a^3b(a+bx^3)} - \frac{6ad-x(ae+7bcx)}{54a^2b(a+bx^3)^2} - \frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{10/3}b^{4/3}} + \frac{\left(14b^{2/3}c-5a^{2/3}e\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{486a^{10/3}b^{4/3}} + \frac{\left(14b^{2/3}c+5a^{2/3}e\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{162a^3b}$$

$$= -\frac{x(ae-bcx-bdx^2)}{9ab(a+bx^3)^3} + \frac{x(5ae+28bcx)}{162a^3b(a+bx^3)} - \frac{6ad-x(ae+7bcx)}{54a^2b(a+bx^3)^2} - \frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{10/3}b^{4/3}} + \frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{10/3}b^{4/3}} + \frac{\left(14b^{2/3}c+5a^{2/3}e\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{81a^{10/3}b^{4/3}}$$

$$= -\frac{x(ae - bcx - bdx^2)}{9ab\,(a + bx^3)^3} + \frac{x(5ae + 28bcx)}{162a^3b\,(a + bx^3)} - \frac{6ad - x(ae + 7bcx)}{54a^2b\,(a + bx^3)^2} - \frac{(14b^{2/3}c + 5a^{2/3}e)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{10/3}b^{4/3}} - \frac{(14b^{2/3}c - 5a^{2/3}e)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{243a^{10/3}b^{4/3}} + \frac{(14b^{2/3}c - 5a^{2/3}e)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{486a^{10/3}b^{4/3}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.89

$$\int \frac{x(c + dx + ex^2)}{(a + bx^3)^4}\,dx$$

$$= \frac{\frac{3ab^{2/3}(28b^3cx^8 - 2a^3(9d + 5ex) + ab^2x^5(77c + 5ex^2) + a^2bx^2(67c + 13ex^2))}{(a + bx^3)^3} - 2\sqrt{3}a^{2/3}\sqrt[3]{b}(14b^{2/3}c + 5a^{2/3}e)\arctan\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) +}{486a^4}$$

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] ((3*a*b^(2/3)*(28*b^3*c*x^8 - 2*a^3*(9*d + 5*e*x) + a*b^2*x^5*(77*c + 5*e*x
^2) + a^2*b*x^2*(67*c + 13*e*x^2)))/(a + b*x^3)^3 - 2*Sqrt[3]*a^(2/3)*b^(1/
3)*(14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]]
 + 2*(-14*a^(2/3)*b*c + 5*a^(4/3)*b^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x] + a^(
2/3)*b^(1/3)*(14*b^(2/3)*c - 5*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(486*a^4*b^(5/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\frac{14 c\,b^{2} x^{8}}{81 a^{3}}+\frac{5 b e\,x^{7}}{162 a^{2}}+\frac{77 b c\,x^{5}}{162 a^{2}}+\frac{13 e\,x^{4}}{162 a}+\frac{67 c\,x^{2}}{162 a}-\frac{5 e x}{81 b}-\frac{d}{9 b}}{\left(b\,x^{3}+a \right)^{3}}+\frac{\sum_{\_R =\operatorname{RootOf}\left(b\,\_Z^{3}+a \right)}\frac{\left(\frac{14 c \_R}{a}+\frac{5 e}{b}\right) \ln \left(x -\_R \right)}{\_R^{2}}}{243 a^{2} b}$
default	$\frac{\frac{14 c\,b^{2} x^{8}}{81 a^{3}}+\frac{5 b e\,x^{7}}{162 a^{2}}+\frac{77 b c\,x^{5}}{162 a^{2}}+\frac{13 e\,x^{4}}{162 a}+\frac{67 c\,x^{2}}{162 a}-\frac{5 e x}{81 b}-\frac{d}{9 b}}{\left(b\,x^{3}+a \right)^{3}}+\frac{5 a e \left(\frac{\ln \left(x +\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x +\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}$

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (14/81*c/a^3*b^2*x^8+5/162*b*e/a^2*x^7+77/162*b*c/a^2*x^5+13/162/a*e*x^4+67
/162*c/a*x^2-5/81*e*x/b-1/9*d/b)/(b*x^3+a)^3+1/243/a^2/b*sum((14*c/a*_R+5/b
*e)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 2646, normalized size of antiderivative = 9.80

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(168*b^3*c*x^8 + 30*a*b^2*e*x^7 + 462*a*b^2*c*x^5 + 78*a^2*b*e*x^4 +
402*a^2*b*c*x^2 - 60*a^3*e*x - 108*a^3*d - 2*(a^3*b^4*x^9 + 3*a^4*b^3*x^6 +
 3*a^5*b^2*x^3 + a^6*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a
^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(
1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^1
0*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))*log(7/2*((1/2)^(1
/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^
3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(
a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*
```

```
e^3)/(a^10*b^4))^(1/3)))^2*a^7*b^3*c - 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((
2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10
*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3
 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)
))*a^5*b*e^2 + 1960*a*b*c^2*e + (2744*b^2*c^3 + 125*a^2*e^3)*x) + ((a^3*b^4
*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*
((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^
10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c
^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/
3))) + 3*sqrt(1/3)*(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*sq
rt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4)
- (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*
sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*
c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^6*b^2 + 1120*c*e)/(a^6*b^2)))*lo
g(-7/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4
) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-
I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^
2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^7*b^3*c + 25/2*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 12
5*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^
2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(
a^10*b^4))^(1/3)))*a^5*b*e^2 - 1960*a*b*c^2*e + 2*(2744*b^2*c^3 + 125*a^2*e
^3)*x + 3/2*sqrt(1/3)*(7*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*
a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*
(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^
10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))*a^7*b^3*c + 25*a
^5*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)
/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2
/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4)
- (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^6*b^2 + 1120*c*e)/(a
^6*b^2))) + ((a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*((1/2)^(
1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c
^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/
(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2
*e^3)/(a^10*b^4))^(1/3))) - 3*sqrt(1/3)*(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^
5*b^2*x^3 + a^6*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125
*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140
*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a
^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^6*b^2 + 112
0*c*e)/(a^6*b^2)))*log(-7/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 1
25*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 1
40*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/
(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^7*b^3*c +
 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4)
 - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I
```

```
*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2
*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))*a^5*b*e^2 - 1960*a*b*c^2*e + 2*(274
4*b^2*c^3 + 125*a^2*e^3)*x - 3/2*sqrt(1/3)*(7*((1/2)^(1/3)*(I*sqrt(3) + 1)*
((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^
10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c
^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/
3)))*a^7*b^3*c + 25*a^5*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b
^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))
^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125
*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a
^6*b^2 + 1120*c*e)/(a^6*b^2))))/(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^
3 + a^6*b)
```

Sympy [A] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.79

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx$$
$$=\mathrm{RootSum}\left(14348907t^3a^{10}b^4+51030ta^4b^2ce-125a^2e^3+2744b^2c^3,\left(t\mapsto t\log\left(x+\frac{826686t^2a^7b^3c+607}{125a^2e^3+}\right.\right.\right.$$
$$+\frac{-18a^3d-10a^3ex+67a^2bcx^2+13a^2bex^4+77ab^2cx^5+5ab^2ex^7+28b^3cx^8}{162a^6b+486a^5b^2x^3+486a^4b^3x^6+162a^3b^4x^9}$$

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**10*b**4 + 51030*_t*a**4*b**2*c*e - 125*a**2*e**3
+ 2744*b**2*c**3, Lambda(_t, _t*log(x + (826686*_t**2*a**7*b**3*c + 6075*_t
*a**5*b*e**2 + 1960*a*b*c**2*e)/(125*a**2*e**3 + 2744*b**2*c**3)))) + (-18*
a**3*d - 10*a**3*e*x + 67*a**2*b*c*x**2 + 13*a**2*b*e*x**4 + 77*a*b**2*c*x*
*5 + 5*a*b**2*e*x**7 + 28*b**3*c*x**8)/(162*a**6*b + 486*a**5*b**2*x**3 + 4
86*a**4*b**3*x**6 + 162*a**3*b**4*x**9)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.96

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx = \frac{28\,b^3cx^8+5\,ab^2ex^7+77\,ab^2cx^5+13\,a^2bex^4+67\,a^2bcx^2-10\,a^3ex-18\,a^3d}{162\,(a^3b^4x^9+3\,a^4b^3x^6+3\,a^5b^2x^3+a^6b)} + \frac{\sqrt{3}\left(14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ae\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/162*(28*b^3*c*x^8 + 5*a*b^2*e*x^7 + 77*a*b^2*c*x^5 + 13*a^2*b*e*x^4 + 67*
a^2*b*c*x^2 - 10*a^3*e*x - 18*a^3*d)/(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b
^2*x^3 + a^6*b) + 1/243*sqrt(3)*(14*b*c*(a/b)^(1/3) + 5*a*e)*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3)) + 1/486*(14*b*c
*(a/b)^(1/3) - 5*a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)
^(2/3)) - 1/243*(14*b*c*(a/b)^(1/3) - 5*a*e)*log(x + (a/b)^(1/3))/(a^3*b^2*
(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.88

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx = -\frac{\sqrt{3}\left(5\,ae-14\,(-ab^2)^{\frac{1}{3}}\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,(-ab^2)^{\frac{2}{3}}\,a^3} - \frac{\left(5\,ae+14\,(-ab^2)^{\frac{1}{3}}\,c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,(-ab^2)^{\frac{2}{3}}\,a^3} - \frac{\left(14\,bc\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^4b} + \frac{28\,b^3cx^8+5\,ab^2ex^7+77\,ab^2cx^5+13\,a^2bex^4+67\,a^2bcx^2-10\,a^3ex-18\,a^3d}{162\,(bx^3+a)^3a^3b}$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/243*sqrt(3)*(5*a*e - 14*(-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/486*(5*a*e + 14*(-a*b^2)^(
1/3)*c)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 1/2
43*(14*b*c*(-a/b)^(1/3) + 5*a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a
^4*b) + 1/162*(28*b^3*c*x^8 + 5*a*b^2*e*x^7 + 77*a*b^2*c*x^5 + 13*a^2*b*e*x
^4 + 67*a^2*b*c*x^2 - 10*a^3*e*x - 18*a^3*d)/((b*x^3 + a)^3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.98

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx = \frac{\frac{67\,c\,x^2}{162\,a}-\frac{d}{9\,b}+\frac{13\,e\,x^4}{162\,a}-\frac{5\,e\,x}{81\,b}+\frac{14\,b^2\,c\,x^8}{81\,a^3}+\frac{77\,b\,c\,x^5}{162\,a^2}+\frac{5\,b\,e\,x^7}{162\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9} + \left(\sum_{k=1}^{3}\ln\left(\frac{70\,a\,c\,e+\mathrm{root}(14348907\,a^{10}\,b^4\,z^3+51030\,a^4\,b^2\,c\,e\,z-125\,a^2\,e^3+2744\,b^2\,c^3,z,k)^2\,a^7\,b^2\,59049}{a}\right.\right.$$
$$\left.\left. + 51030\,a^4\,b^2\,c\,e\,z-125\,a^2\,e^3+2744\,b^2\,c^3,z,k)\right)\right.$$

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3)^4,x)
```

```
[Out] ((67*c*x^2)/(162*a) - d/(9*b) + (13*e*x^4)/(162*a) - (5*e*x)/(81*b) + (14*b
^2*c*x^8)/(81*a^3) + (77*b*c*x^5)/(162*a^2) + (5*b*e*x^7)/(162*a^2))/(a^3 +
 b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2*x^6) + symsum(log((70*a*c*e + 59049*root(1
4348907*a^10*b^4*z^3 + 51030*a^4*b^2*c*e*z - 125*a^2*e^3 + 2744*b^2*c^3, z,
 k)^2*a^7*b^2 + 196*b*c^2*x + 1215*root(14348907*a^10*b^4*z^3 + 51030*a^4*b
^2*c*e*z - 125*a^2*e^3 + 2744*b^2*c^3, z, k)*a^4*b*e*x)/(6561*a^6))*root(14
348907*a^10*b^4*z^3 + 51030*a^4*b^2*c*e*z - 125*a^2*e^3 + 2744*b^2*c^3, z,
k), k, 1, 3)
```

3.360 $\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 250

$$\begin{aligned}\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx &= \frac{x(8c+7dx)}{54a^2(a+bx^3)^2}+\frac{2x(10c+7dx)}{81a^3(a+bx^3)}-\frac{ae-bx(c+dx)}{9ab(a+bx^3)^3}\\ &\quad-\frac{2\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}\\ &\quad+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}\\ &\quad-\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}\end{aligned}$$

```
[Out] 1/54*x*(7*d*x+8*c)/a^2/(b*x^3+a)^2+2/81*x*(7*d*x+10*c)/a^3/(b*x^3+a)+1/9*(-
a*e+b*x*(d*x+c))/a/b/(b*x^3+a)^3+2/243*(20*b^(1/3)*c-7*a^(1/3)*d)*ln(a^(1/3
)+b^(1/3)*x)/a^(11/3)/b^(2/3)-1/243*(20*b^(1/3)*c-7*a^(1/3)*d)*ln(a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(2/3)-2/243*(20*b^(1/3)*c+7*a^(1/3
)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(2/3)*3^(
1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1868, 1869, 1874, 31, 648, 631, 210, 642}

$$\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx = -\frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)}{81\sqrt{3}a^{11/3}b^{2/3}} - \frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}} + \frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} + \frac{2x(10c+7dx)}{81a^3(a+bx^3)} + \frac{x(8c+7dx)}{54a^2(a+bx^3)^2} - \frac{ae-bx(c+dx)}{9ab(a+bx^3)^3}$$

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^4,x]
```

```
[Out] (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2) + (2*x*(10*c + 7*d*x))/(81*a^3*(a
 + b*x^3)) - (a*e - b*x*(c + d*x))/(9*a*b*(a + b*x^3)^3) - (2*(20*b^(1/3)*c
 + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[
3]*a^(11/3)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3
)*x])/(243*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}-\frac{\int\frac{-8c-7dx}{\left(a+bx^3\right)^3}\,dx}{9a}\\ &= \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}+\frac{\int\frac{40c+28dx}{\left(a+bx^3\right)^2}\,dx}{54a^2}\\ &= \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}-\frac{\int\frac{-80c-28dx}{a+bx^3}\,dx}{162a^3}\end{aligned}$$

$$= \frac{x(8c+7dx)}{54a^2(a+bx^3)^2} + \frac{2x(10c+7dx)}{81a^3(a+bx^3)} - \frac{ae-bx(c+dx)}{9ab(a+bx^3)^3} - \frac{\int \frac{\sqrt[3]{a}\left(-160\sqrt[3]{b}c-28\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(80\sqrt[3]{b}c-28\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{486a^{11/3}\sqrt[3]{b}} + \frac{\left(2\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{243a^{11/3}}$$

$$= \frac{x(8c+7dx)}{54a^2(a+bx^3)^2} + \frac{2x(10c+7dx)}{81a^3(a+bx^3)} - \frac{ae-bx(c+dx)}{9ab(a+bx^3)^3} + \frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} - \frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{243a^{11/3}b^{2/3}} + \frac{\left(20c+\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{81a^{10/3}}$$

$$= \frac{x(8c+7dx)}{54a^2(a+bx^3)^2} + \frac{2x(10c+7dx)}{81a^3(a+bx^3)} - \frac{ae-bx(c+dx)}{9ab(a+bx^3)^3} + \frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} - \frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}} + \frac{\left(2\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{81a^{11/3}b^{2/3}}$$

$$= \frac{x(8c+7dx)}{54a^2(a+bx^3)^2} + \frac{2x(10c+7dx)}{81a^3(a+bx^3)} - \frac{ae-bx(c+dx)}{9ab(a+bx^3)^3} - \frac{2\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}} + \frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} - \frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx$$

$$= \frac{\frac{9a^2x(8c+7dx)}{(a+bx^3)^2} + \frac{12ax(10c+7dx)}{a+bx^3} - \frac{54a^3(ae-bx(c+dx))}{b(a+bx^3)^3} - \frac{4\sqrt{3}\sqrt[3]{a}\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{4\left(20\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right.}{b^{2/}}}{486a^4}$$

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^4,x]
```

```
[Out] ((9*a^2*x*(8*c + 7*d*x))/(a + b*x^3)^2 + (12*a*x*(10*c + 7*d*x))/(a + b*x^3
) - (54*a^3*(a*e - b*x*(c + d*x)))/(b*(a + b*x^3)^3) - (4*Sqrt[3]*a^(1/3)*(
20*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(2/3) + (4*(20*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b
^(2/3) + (2*(-20*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/b^(2/3))/(486*a^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.51 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.46

method	result
risch	$\frac{\frac{14d\,b^2x^8}{81a^3}+\frac{20c\,b^2x^7}{81a^3}+\frac{77bd\,x^5}{162a^2}+\frac{52bc\,x^4}{81a^2}+\frac{67d\,x^2}{162a}+\frac{41cx}{81a}-\frac{e}{9b}}{(b\,x^3+a)^3}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(7\_Rd+20c)\ln(x-\_R)}{\_R^2}\right)}{243a^3b}$
default	$c\left(\frac{x}{9a(b\,x^3+a)^3}+\frac{\frac{4x}{27a(b\,x^3+a)^2}+\frac{8\left(\frac{5x}{18a(b\,x^3+a)}+\frac{5\left(\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{6a}\right)}{9a}}{a}\right.$

[In] int((e*x^2+d*x+c)/(b*x^3+a)^4,x,method=_RETURNVERBOSE)

[Out] (14/81*d/a^3*b^2*x^8+20/81*c/a^3*b^2*x^7+77/162*b*d/a^2*x^5+52/81*b*c/a^2*x^4+67/162*d/a*x^2+41/81*c/a*x-1/9/b*e)/(b*x^3+a)^3+2/243/a^3/b*sum((7*_R*d+20*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 2344, normalized size of antiderivative = 9.38

$$\int \frac{c+dx+ex^2}{(a+bx^3)^4} dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(168*b^3*d*x^8 + 240*b^3*c*x^7 + 462*a*b^2*d*x^5 + 624*a*b^2*c*x^4 +
402*a^2*b*d*x^2 + 492*a^2*b*c*x - 108*a^3*e - 2*(a^3*b^4*x^9 + 3*a^4*b^3*x^
6 + 3*a^5*b^2*x^3 + a^6*b)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^
3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*
d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*log(7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*
b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)
- 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d - 400*(4^(1/3)
*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a
*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*
c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*
a^4*b*c^2 + 7840*a*c*d^2 + 4*(8000*b*c^3 + 343*a*d^3)*x) + ((a^3*b^4*x^9 +
3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^
3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 14
0*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2)
+ (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3))) + 3*sqrt(1/3)*(a^3*b^4*x^9 +
 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8
000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1
/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^
11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/
(a^7*b))*log(-7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sq
rt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*
a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d + 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*
b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)
- 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2
 + 8*(8000*b*c^3 + 343*a*d^3)*x + 3/4*sqrt(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)
*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2)
)^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)
/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^8*b*d + 1600*a
^4*b*c^2)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt
(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*
```

```
d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b))) + ((a^3*b^4*x^9 + 3*
a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3
+ 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*
4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) +
(8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3))) - 3*sqrt(1/3)*(a^3*b^4*x^9 + 3
*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((800
0*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3
) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a
^7*b)))*log(-7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt
(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*
d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d + 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*
c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) -
140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2
) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2 +
 8*(8000*b*c^3 + 343*a*d^3)*x - 3/4*sqrt(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)*(
(8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^
(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(
a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^8*b*d + 1600*a^4
*b*c^2)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2
) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3
) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^
3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b))))/(a^3*b^4*x^9 + 3*a^4*
b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)
```

Sympy [A] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.81

$$\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx$$
$$= \text{RootSum}\left(14348907t^3a^{11}b^2 + 408240ta^4bcd + 2744ad^3 - 64000bc^3, \left(t \mapsto t\log\left(x + \frac{413343t^2a^8bd + 1944}{1372ad^3 +}\right.\right.\right.$$
$$+ \frac{-18a^3e + 82a^2bcx + 67a^2bdx^2 + 104ab^2cx^4 + 77ab^2dx^5 + 40b^3cx^7 + 28b^3dx^8}{162a^6b + 486a^5b^2x^3 + 486a^4b^3x^6 + 162a^3b^4x^9}$$

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**11*b**2 + 408240*_t*a**4*b*c*d + 2744*a*d**3 - 64
000*b*c**3, Lambda(_t, _t*log(x + (413343*_t**2*a**8*b*d + 194400*_t*a**4*b
*c**2 + 7840*a*c*d**2)/(1372*a*d**3 + 32000*b*c**3)))) + (-18*a**3*e + 82*a
**2*b*c*x + 67*a**2*b*d*x**2 + 104*a*b**2*c*x**4 + 77*a*b**2*d*x**5 + 40*b*
*3*c*x**7 + 28*b**3*d*x**8)/(162*a**6*b + 486*a**5*b**2*x**3 + 486*a**4*b**
3*x**6 + 162*a**3*b**4*x**9)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.02

$$\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx = \frac{28\,b^3dx^8+40\,b^3cx^7+77\,ab^2dx^5+104\,ab^2cx^4+67\,a^2bdx^2+82\,a^2bcx-18\,a^3e}{162\,(a^3b^4x^9+3\,a^4b^3x^6+3\,a^5b^2x^3+a^6b)} + \frac{2\,\sqrt{3}\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{2\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
[Out] 1/162*(28*b^3*d*x^8 + 40*b^3*c*x^7 + 77*a*b^2*d*x^5 + 104*a*b^2*c*x^4 + 67*
a^2*b*d*x^2 + 82*a^2*b*c*x - 18*a^3*e)/(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5
*b^2*x^3 + a^6*b) + 2/243*sqrt(3)*(7*d*(a/b)^(1/3) + 20*c)*arctan(1/3*sqrt(
3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) + 1/243*(7*d*(a/b)^
(1/3) - 20*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) -
2/243*(7*d*(a/b)^(1/3) - 20*c)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.93

$$\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx = -\frac{2\,\sqrt{3}\left(20\,bc-7\,(-ab^2)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,(-ab^2)^{\frac{2}{3}}\,a^3} - \frac{\left(20\,bc+7\,(-ab^2)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,(-ab^2)^{\frac{2}{3}}\,a^3} - \frac{2\left(7\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^4} + \frac{28\,b^3dx^8+40\,b^3cx^7+77\,ab^2dx^5+104\,ab^2cx^4+67\,a^2bdx^2+82\,a^2bcx-18\,a^3e}{162\,(bx^3+a)^3a^3b}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -2/243*sqrt(3)*(20*b*c - 7*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/243*(20*b*c + 7*(-a*b^2)^(
1/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 2/2
43*(7*d*(-a/b)^(1/3) + 20*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 +
1/162*(28*b^3*d*x^8 + 40*b^3*c*x^7 + 77*a*b^2*d*x^5 + 104*a*b^2*c*x^4 + 67*
a^2*b*d*x^2 + 82*a^2*b*c*x - 18*a^3*e)/((b*x^3 + a)^3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx = \frac{\frac{67\,d\,x^2}{162\,a}-\frac{e}{9\,b}+\frac{41\,c\,x}{81\,a}+\frac{20\,b^2\,c\,x^7}{81\,a^3}+\frac{14\,b^2\,d\,x^8}{81\,a^3}+\frac{52\,b\,c\,x^4}{81\,a^2}+\frac{77\,b\,d\,x^5}{162\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9}$$
$$+\left(\sum_{k=1}^{3}\ln\left(\frac{b\left(560\,c\,d+196\,d^2\,x+\mathrm{root}(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k\right.}{}\right.\right.$$
$$\left.\left.+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k)\right)$$

```
[In] int((c + d*x + e*x^2)/(a + b*x^3)^4,x)
```

```
[Out] ((67*d*x^2)/(162*a) - e/(9*b) + (41*c*x)/(81*a) + (20*b^2*c*x^7)/(81*a^3) +
 (14*b^2*d*x^8)/(81*a^3) + (52*b*c*x^4)/(81*a^2) + (77*b*d*x^5)/(162*a^2))/
(a^3 + b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2*x^6) + symsum(log((b*(560*c*d + 196*
d^2*x + 59049*root(14348907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 64000*b*c^3
 + 2744*a*d^3, z, k)^2*a^7*b + 9720*root(14348907*a^11*b^2*z^3 + 408240*a^4
*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k)*a^3*b*c*x))/(6561*a^6))*root(143
48907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k),
k, 1, 3)
```

3.361 $\int \frac{c+dx+ex^2}{x(a+bx^3)^4} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 291

$$\begin{aligned}\int \frac{c+dx+ex^2}{x(a+bx^3)^4}\,dx =& \frac{x(ad+aex-bcx^2)}{9a^2(a+bx^3)^3}+\frac{x(8ad+7aex-15bcx^2)}{54a^3(a+bx^3)^2}\\&+\frac{x(40ad+28aex-99bcx^2)}{162a^4(a+bx^3)}\\&-\frac{2\left(20\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}\\&+\frac{c\log(x)}{a^4}+\frac{2\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}\\&-\frac{\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}-\frac{c\log(a+bx^3)}{3a^4}\end{aligned}$$

```
[Out] 1/9*x*(-b*c*x^2+a*e*x+a*d)/a^2/(b*x^3+a)^3+1/54*x*(-15*b*c*x^2+7*a*e*x+8*a*
d)/a^3/(b*x^3+a)^2+1/162*x*(-99*b*c*x^2+28*a*e*x+40*a*d)/a^4/(b*x^3+a)+c*ln
(x)/a^4+2/243*(20*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(
2/3)-1/243*(20*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/a^(11/3)/b^(2/3)-1/3*c*ln(b*x^3+a)/a^4-2/243*(20*b^(1/3)*d+7*a^(1/3)*e
)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(2/3)*3^(1/2
)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx=-\frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}e+20\sqrt[3]{b}d\right)}{81\sqrt{3}a^{11/3}b^{2/3}}-\frac{\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}+\frac{x(40ad+28aex-99bcx^2)}{162a^4\left(a+bx^3\right)}-\frac{c\log\left(a+bx^3\right)}{3a^4}+\frac{c\log(x)}{a^4}+\frac{x(8ad+7aex-15bcx^2)}{54a^3\left(a+bx^3\right)^2}+\frac{x(ad+aex-bcx^2)}{9a^2\left(a+bx^3\right)^3}$$

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^4),x]
```

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*d + 7*a*e*x - 1
5*b*c*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*d + 28*a*e*x - 99*b*c*x^2))/(
162*a^4*(a + b*x^3)) - (2*(20*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(11/3)*b^(2/3)) + (c*Log[x])/a
^4 + (2*(20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(11/3
)*b^(2/3)) - ((20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
```

```
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = \frac{x(ad+aex-bcx^2)}{9a^2(a+bx^3)^3} - \frac{\int \frac{-9bc-8bdx-7bex^2+\frac{6b^2cx^3}{a}}{x(a+bx^3)^3}\,dx}{9ab}$$

$$= \frac{x(ad+aex-bcx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3(a+bx^3)^2} + \frac{\int \frac{54b^2c+40b^2dx+28b^2ex^2-\frac{45b^3cx^3}{a}}{x(a+bx^3)^2}\,dx}{54a^2b^2}$$

$$= \frac{x(ad+aex-bcx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ad+28aex-99bcx^2)}{162a^4(a+bx^3)} - \frac{\int \frac{-162b^3c-80b^3dx-28b^3ex^2}{x(a+bx^3)}\,dx}{162a^3b^3}$$

$$= \frac{x(ad+aex-bcx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ad+28aex-99bcx^2)}{162a^4(a+bx^3)} - \frac{\int \left(-\frac{162b^3c}{ax} - \frac{2b^3(40ad+14aex-81bcx^2)}{a(a+bx^3)}\right)dx}{162a^3b^3}$$

$$= \frac{x(ad+aex-bcx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ad+28aex-99bcx^2)}{162a^4(a+bx^3)} + \frac{c\log(x)}{a^4} + \frac{\int \frac{40ad+14aex-81bcx^2}{a+bx^3}\,dx}{81a^4}$$

$$= \frac{x(ad+aex-bcx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ad+28aex-99bcx^2)}{162a^4(a+bx^3)} + \frac{c\log(x)}{a^4} + \frac{\int \frac{40ad+14aex}{a+bx^3}\,dx}{81a^4} - \frac{(bc)\int \frac{x^2}{a+bx^3}\,dx}{a^4}$$

$$= \frac{x(ad+aex-bcx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ad+28aex-99bcx^2)}{162a^4(a+bx^3)} + \frac{c\log(x)}{a^4} - \frac{c\log(a+bx^3)}{3a^4} + \frac{\int \frac{\sqrt[3]{a}\left(80a\sqrt[3]{b}d+14a^{4/3}e\right)+\sqrt[3]{b}\left(-40a\sqrt[3]{b}d+14a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{243a^{14/3}\sqrt[3]{b}} + \frac{\left(2\left(20d-\frac{7\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{243a^{11/3}}$$

$$
\begin{aligned}
&= \frac{x(ad+aex-bcx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ad+28aex-99bcx^2)}{162a^4(a+bx^3)} \\
&\quad + \frac{c\log(x)}{a^4} + \frac{2\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} \\
&\quad - \frac{c\log(a+bx^3)}{3a^4} - \frac{\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{243a^{11/3}b^{2/3}} \\
&\quad + \frac{\left(20d+\frac{7\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{81a^{10/3}} \\
&= \frac{x(ad+aex-bcx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ad+28aex-99bcx^2)}{162a^4(a+bx^3)} \\
&\quad + \frac{c\log(x)}{a^4} + \frac{2\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} - \frac{\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right.}{243a^{11/3}b^{2/3}} \\
&\quad - \frac{c\log(a+bx^3)}{3a^4} + \frac{\left(2\left(20\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{81a^{11/3}b^{2/3}} \\
&= \frac{x(ad+aex-bcx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ad+7aex-15bcx^2)}{54a^3(a+bx^3)^2} \\
&\quad + \frac{x(40ad+28aex-99bcx^2)}{162a^4(a+bx^3)} - \frac{2\left(20\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}} \\
&\quad + \frac{c\log(x)}{a^4} + \frac{2\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} \\
&\quad - \frac{\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}} - \frac{c\log(a+bx^3)}{3a^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.89

$$
\int \frac{c+dx+ex^2}{x(a+bx^3)^4}\,dx
$$

$$
= \frac{\frac{54a^3(c+x(d+ex))}{(a+bx^3)^3} + \frac{9a^2(9c+x(8d+7ex))}{(a+bx^3)^2} + \frac{6a(27c+2x(10d+7ex))}{a+bx^3} - \frac{4\sqrt{3}\sqrt[3]{a}\left(20\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + 486c\log(x}{48
$$

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^4),x]
```

```
[Out] ((54*a^3*(c + x*(d + e*x)))/(a + b*x^3)^3 + (9*a^2*(9*c + x*(8*d + 7*e*x)))
/(a + b*x^3)^2 + (6*a*(27*c + 2*x*(10*d + 7*e*x)))/(a + b*x^3) - (4*Sqrt[3]
*a^(1/3)*(20*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(2/3) + 486*c*Log[x] + (4*(20*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Lo
g[a^(1/3) + b^(1/3)*x])/b^(2/3) + (2*(-20*a^(1/3)*b^(1/3)*d + 7*a^(2/3)*e)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 162*c*Log[a + b*x
^3])/(486*a^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.05

method	result
risch	$\frac{\frac{14e\,b^{2}x^{8}}{81a^{3}}+\frac{20d\,b^{2}x^{7}}{81a^{3}}+\frac{c\,b^{2}x^{6}}{3a^{3}}+\frac{77be\,x^{5}}{162a^{2}}+\frac{52bd\,x^{4}}{81a^{2}}+\frac{5bc\,x^{3}}{6a^{2}}+\frac{67e\,x^{2}}{162a}+\frac{41xd}{81a}+\frac{11c}{18a}}{\left(b\,x^{3}+a\right)^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{12}b^{2}\_Z^{3}+243a^{8}b^{2}c\_Z^{2}+\left(1680a^{5}bde+196\right.\right.}}{}$
default	$\frac{c\ln(x)}{a^{4}}+\frac{\frac{\frac{14}{81}a\,b^{2}e\,x^{8}+\frac{20}{81}a\,b^{2}d\,x^{7}+\frac{1}{3}a\,b^{2}c\,x^{6}+\frac{77}{162}a^{2}be\,x^{5}+\frac{52}{81}a^{2}bd\,x^{4}+\frac{5}{6}a^{2}x^{3}bc+\frac{67}{162}a^{3}e\,x^{2}+\frac{41}{81}a^{3}dx+\frac{11}{18}c\,a^{3}}{\left(b\,x^{3}+a\right)^{3}}+\frac{40ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}\right.}{}\right.}{}}{}$

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (14/81*e/a^3*b^2*x^8+20/81*d/a^3*b^2*x^7+1/3*c/a^3*b^2*x^6+77/162*b*e/a^2*x
^5+52/81*b*d/a^2*x^4+5/6*b*c/a^2*x^3+67/162/a*e*x^2+41/81/a*x*d+11/18*c/a)/
(b*x^3+a)^3+1/243*sum(_R*ln((-2*_R^3*a^11*b^2-324*_R^2*a^7*b^2*c+(-2800*a^4
*b*d*e-13122*a^3*b^2*c^2)*_R-4116*a*e^3-136080*b*c*d*e+96000*b*d^3)*x+7*a^8
*b*e*_R^2+(-1134*a^4*b*c*e-800*a^4*b*d^2)*_R-137781*b*c^2*e+194400*b*c*d^2)
,_R=RootOf(a^12*b^2*_Z^3+243*a^8*b^2*c*_Z^2+(1680*a^5*b*d*e+19683*a^4*b^2*c
^2)*_Z+2744*a^2*e^3+136080*a*b*c*d*e-64000*a*b*d^3+531441*b^2*c^3))+c/a^4*l
n(-x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 5370, normalized size of antiderivative = 18.45

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx &= \frac{28\,b^2ex^8+40\,b^2dx^7+54\,b^2cx^6+77\,abex^5+104\,abdx^4+135\,abcx^3+67\,a^2ex^2+82\,a^2dx+99\,a^2c}{162\,\left(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6\right)} \\ &\quad+\frac{c\log\left(x\right)}{a^4}+\frac{2\,\sqrt{3}\left(7\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+20\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5} \\ &\quad-\frac{\left(81\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-7\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &\quad-\frac{\left(81\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-40\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="maxima")
```

[Out] 1/162*(28*b^2*e*x^8 + 40*b^2*d*x^7 + 54*b^2*c*x^6 + 77*a*b*e*x^5 + 104*a*b*d*x^4 + 135*a*b*c*x^3 + 67*a^2*e*x^2 + 82*a^2*d*x + 99*a^2*c)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6) + c*log(x)/a^4 + 2/243*sqrt(3)*(7*a*e*(a/b)^(2/3) + 20*a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^5 - 1/243*(81*b*c*(a/b)^(2/3) - 7*a*e*(a/b)^(1/3) + 20*a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)) - 1/243*(81*b*c*(a/b)^(2/3) + 14*a*e*(a/b)^(1/3) - 40*a*d)*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.98

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx = -\frac{2\sqrt{3}\left(20\,bd-7\left(-ab^2\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\left(-ab^2\right)^{\frac{2}{3}}a^3} - \frac{\left(20\,bd+7\left(-ab^2\right)^{\frac{1}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(-ab^2\right)^{\frac{2}{3}}a^3} - \frac{c\log\left(|bx^3+a|\right)}{3\,a^4} + \frac{c\log\left(|x|\right)}{a^4} + \frac{28\,ab^2ex^8+40\,ab^2dx^7+54\,ab^2cx^6+77\,a^2bex^5+104\,a^2bdx^4+135\,a^2bcx^3+67\,a^3ex^2+82\,a^3dx+99\,a^3c}{162\left(bx^3+a\right)^3a^4} - \frac{2\left(7\,a^5be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,a^5bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^9b}$$

[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="giac")

[Out] -2/243*sqrt(3)*(20*b*d - 7*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/243*(20*b*d + 7*(-a*b^2)^(1/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 1/3*c*log(abs(b*x^3 + a))/a^4 + c*log(abs(x))/a^4 + 1/162*(28*a*b^2*e*x^8 + 40*a*b^2*d*x^7 + 54*a*b^2*c*x^6 + 77*a^2*b*e*x^5 + 104*a^2*b*d*x^4 + 135*a^2*b*c*x^3 + 67*a^3*e*x^2 + 82*a^3*d*x + 99*a^3*c)/((b*x^3 + a)^3*a^4) - 2/243*(7*a^5*b*e*(-a/b)^(1/3) + 20*a^5*b*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b)

Mupad [B] (verification not implemented)

Time = 12.95 (sec) , antiderivative size = 871, normalized size of antiderivative = 2.99

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx$$
$$=\frac{\frac{11\,c}{18\,a}+\frac{67\,e\,x^2}{162\,a}+\frac{41\,d\,x}{81\,a}+\frac{b^2\,c\,x^6}{3\,a^3}+\frac{20\,b^2\,d\,x^7}{81\,a^3}+\frac{14\,b^2\,e\,x^8}{81\,a^3}+\frac{5\,b\,c\,x^3}{6\,a^2}+\frac{52\,b\,d\,x^4}{81\,a^2}+\frac{77\,b\,e\,x^5}{162\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9}$$
$$+\left(\sum_{k=1}^{3}\ln\left(-\frac{b\left(-64800\,b\,c\,d^2+45927\,b\,c^2\,e+1372\,a\,e^3\,x-32000\,b\,d^3\,x+\mathrm{root}(14348907\,a^{12}\,b^2\,z^3+143\right.}{}\right.\right.$$
$$+14348907\,a^8\,b^2\,c\,z^2+408240\,a^5\,b\,d\,e\,z+4782969\,a^4\,b^2\,c^2\,z+136080\,a\,b\,c\,d\,e$$
$$\left.-64000\,a\,b\,d^3+2744\,a^2\,e^3+531441\,b^2\,c^3,z,k)\right)+\frac{c\,\ln\left(x\right)}{a^4}$$

```
[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)^4),x)
```

```
[Out] ((11*c)/(18*a) + (67*e*x^2)/(162*a) + (41*d*x)/(81*a) + (b^2*c*x^6)/(3*a^3)
 + (20*b^2*d*x^7)/(81*a^3) + (14*b^2*e*x^8)/(81*a^3) + (5*b*c*x^3)/(6*a^2)
+ (52*b*d*x^4)/(81*a^2) + (77*b*e*x^5)/(162*a^2))/(a^3 + b^3*x^9 + 3*a^2*b*
x^3 + 3*a*b^2*x^6) + symsum(log(-(2*b*(45927*b*c^2*e - 64800*b*c*d^2 + 1372
*a*e^3*x - 32000*b*d^3*x + 9565938*root(14348907*a^12*b^2*z^3 + 14348907*a^
8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*b*c*d*e
 - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k)^3*a^11*b^2*x + 6480
0*root(14348907*a^12*b^2*z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z
+ 4782969*a^4*b^2*c^2*z + 136080*a*b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 +
 531441*b^2*c^3, z, k)*a^4*b*d^2 - 137781*root(14348907*a^12*b^2*z^3 + 1434
8907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*
b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k)^2*a^8*b*e +
45360*b*c*d*e*x + 1062882*root(14348907*a^12*b^2*z^3 + 14348907*a^8*b^2*c*z
^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*b*c*d*e - 64000*
a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k)*a^3*b^2*c^2*x + 6377292*root
(14348907*a^12*b^2*z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782
969*a^4*b^2*c^2*z + 136080*a*b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 53144
1*b^2*c^3, z, k)^2*a^7*b^2*c*x + 91854*root(14348907*a^12*b^2*z^3 + 1434890
7*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*b*c
*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k)*a^4*b*c*e + 226
800*root(14348907*a^12*b^2*z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*
z + 4782969*a^4*b^2*c^2*z + 136080*a*b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3
 + 531441*b^2*c^3, z, k)*a^4*b*d*e*x))/(531441*a^9))*root(14348907*a^12*b^2
*z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z
+ 136080*a*b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k),
k, 1, 3) + (c*log(x))/a^4
```

3.362 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^4} dx$

Optimal result

Integrand size = 23, antiderivative size = 301

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^4} dx = -\frac{c}{a^4x} + \frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3} + \frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2} + \frac{x(40ae-118bcx-99bdx^2)}{162a^4(a+bx^3)} + \frac{20(7b^{2/3}c-2a^{2/3}e)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{13/3}\sqrt[3]{b}} + \frac{d\log(x)}{a^4} + \frac{20(7b^{2/3}c+2a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{13/3}\sqrt[3]{b}} - \frac{10(7b^{2/3}c+2a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{13/3}\sqrt[3]{b}} - \frac{d\log(a+bx^3)}{3a^4}$$

```
[Out] -c/a^4/x+1/9*x*(-b*d*x^2-b*c*x+a*e)/a^2/(b*x^3+a)^3+1/54*x*(-15*b*d*x^2-16*
b*c*x+8*a*e)/a^3/(b*x^3+a)^2+1/162*x*(-99*b*d*x^2-118*b*c*x+40*a*e)/a^4/(b*
x^3+a)+d*ln(x)/a^4+20/243*(7*b^(2/3)*c+2*a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a
^(13/3)/b^(1/3)-10/243*(7*b^(2/3)*c+2*a^(2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/a^(13/3)/b^(1/3)-1/3*d*ln(b*x^3+a)/a^4+20/243*(7*b^(2/3)*c-
2*a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)/b^(
1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^4}\,dx=\frac{20\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7b^{2/3}c-2a^{2/3}e\right)}{81\sqrt{3}a^{13/3}\sqrt[3]{b}}-\frac{10\left(2a^{2/3}e+7b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{20\left(2a^{2/3}e+7b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{x(40ae-118bcx-99bdx^2)}{162a^4\left(a+bx^3\right)}-\frac{d\log\left(a+bx^3\right)}{3a^4}-\frac{c}{a^4x}+\frac{d\log(x)}{a^4}+\frac{x(8ae-16bcx-15bdx^2)}{54a^3\left(a+bx^3\right)^2}+\frac{x(ae-bcx-bdx^2)}{9a^2\left(a+bx^3\right)^3}$$

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x]
```

```
[Out] -(c/(a^4*x)) + (x*(a*e - b*c*x - b*d*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*
e - 16*b*c*x - 15*b*d*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*e - 118*b*c*x
 - 99*b*d*x^2))/(162*a^4*(a + b*x^3)) + (20*(7*b^(2/3)*c - 2*a^(2/3)*e)*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(13/3)*b^(1/3
)) + (d*Log[x])/a^4 + (20*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)
*x])/(243*a^(13/3)*b^(1/3)) - (10*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(13/3)*b^(1/3)) - (d*Log[a + b*x^
3])/(3*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}-\frac{\int \frac{-9bc-9bdx-8bex^2+\frac{7b^2cx^3}{a}+\frac{6b^2dx^4}{a}}{x^2(a+bx^3)^3}\,dx}{9ab}\\
&=\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}+\frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2}+\frac{\int \frac{54b^3c+54b^3dx+40b^3ex^2-\frac{64b^4cx^3}{a}-\frac{45b^4dx^4}{a}}{x^2(a+bx^3)^2}\,dx}{54a^2b^3}\\
&=\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}+\frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2}\\
&\quad+\frac{x(40ae-118bcx-99bdx^2)}{162a^4(a+bx^3)}-\frac{\int \frac{-162b^5c-162b^5dx-80b^5ex^2+\frac{118b^6cx^3}{a}}{x^2(a+bx^3)}\,dx}{162a^3b^5}\\
&=\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}+\frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2}+\frac{x(40ae-118bcx-99bdx^2)}{162a^4(a+bx^3)}\\
&\quad-\frac{\int \left(-\frac{162b^5c}{ax^2}-\frac{162b^5d}{ax}-\frac{2b^5(40ae-140bcx-81bdx^2)}{a(a+bx^3)}\right)dx}{162a^3b^5}\\
&=-\frac{c}{a^4x}+\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}+\frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2}\\
&\quad+\frac{x(40ae-118bcx-99bdx^2)}{162a^4(a+bx^3)}+\frac{d\log(x)}{a^4}+\frac{\int \frac{40ae-140bcx-81bdx^2}{a+bx^3}\,dx}{81a^4}\\
&=-\frac{c}{a^4x}+\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}+\frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2}\\
&\quad+\frac{x(40ae-118bcx-99bdx^2)}{162a^4(a+bx^3)}+\frac{d\log(x)}{a^4}+\frac{\int \frac{40ae-140bcx}{a+bx^3}\,dx}{81a^4}-\frac{(bd)\int \frac{x^2}{a+bx^3}\,dx}{a^4}\\
&=-\frac{c}{a^4x}+\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}+\frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2}\\
&\quad+\frac{x(40ae-118bcx-99bdx^2)}{162a^4(a+bx^3)}+\frac{d\log(x)}{a^4}-\frac{d\log(a+bx^3)}{3a^4}\\
&\quad+\frac{\int \frac{\sqrt[3]{a}\left(-140\sqrt[3]{a}bc+80a\sqrt[3]{b}e\right)+\sqrt[3]{b}\left(-140\sqrt[3]{a}bc-40a\sqrt[3]{b}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{243a^{14/3}\sqrt[3]{b}}\\
&\quad+\frac{\left(20\left(7b^{2/3}c+2a^{2/3}e\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{243a^{13/3}}
\end{aligned}$$

$$=-\frac{c}{a^4x}+\frac{x(ae-bcx-bdx^2)}{9a^2\left(a+bx^3\right)^3}+\frac{x(8ae-16bcx-15bdx^2)}{54a^3\left(a+bx^3\right)^2}+\frac{x(40ae-118bcx-99bdx^2)}{162a^4\left(a+bx^3\right)}+\frac{d\log(x)}{a^4}+\frac{20\left(7b^{2/3}c+2a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{13/3}\sqrt[3]{b}}-\frac{d\log\left(a+bx^3\right)}{3a^4}-\frac{\left(10\left(7b^{2/3}c-2a^{2/3}e\right)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{81a^4}-\frac{\left(10\left(7b^{2/3}c+2a^{2/3}e\right)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{243a^{13/3}\sqrt[3]{b}}$$

$$=-\frac{c}{a^4x}+\frac{x(ae-bcx-bdx^2)}{9a^2\left(a+bx^3\right)^3}+\frac{x(8ae-16bcx-15bdx^2)}{54a^3\left(a+bx^3\right)^2}+\frac{x(40ae-118bcx-99bdx^2)}{162a^4\left(a+bx^3\right)}+\frac{d\log(x)}{a^4}+\frac{20\left(7b^{2/3}c+2a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{13/3}\sqrt[3]{b}}-\frac{10\left(7b^{2/3}c+2a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{13/3}\sqrt[3]{b}}-\frac{d\log\left(a+bx^3\right)}{3a^4}-\frac{\left(20\left(7b^{2/3}c-2a^{2/3}e\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{81a^{13/3}\sqrt[3]{b}}$$

$$=-\frac{c}{a^4x}+\frac{x(ae-bcx-bdx^2)}{9a^2\left(a+bx^3\right)^3}+\frac{x(8ae-16bcx-15bdx^2)}{54a^3\left(a+bx^3\right)^2}+\frac{x(40ae-118bcx-99bdx^2)}{162a^4\left(a+bx^3\right)}+\frac{20\left(7b^{2/3}c-2a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{13/3}\sqrt[3]{b}}+\frac{d\log(x)}{a^4}+\frac{20\left(7b^{2/3}c+2a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{13/3}\sqrt[3]{b}}-\frac{10\left(7b^{2/3}c+2a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{13/3}\sqrt[3]{b}}-\frac{d\log\left(a+bx^3\right)}{3a^4}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.93

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^4}\,dx$$

$$= \frac{-\frac{486ac}{x} + \frac{9a^2(9ad+8aex-16bcx^2)}{(a+bx^3)^2} + \frac{6a(27ad+20aex-59bcx^2)}{a+bx^3} + \frac{54a^3(-bcx^2+a(d+ex))}{(a+bx^3)^3} - \frac{40\sqrt{3}a^{2/3}(-7b^{2/3}c+2a^{2/3}e)\arctan\left(\frac{1-\frac{2}{\ldots}}{\ldots}\right)}{\sqrt[3]{b}} \ldots}{\ldots}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x]
```

```
[Out] ((-486*a*c)/x + (9*a^2*(9*a*d + 8*a*e*x - 16*b*c*x^2))/(a + b*x^3)^2 + (6*a
*(27*a*d + 20*a*e*x - 59*b*c*x^2))/(a + b*x^3) + (54*a^3*(-(b*c*x^2) + a*(d
 + e*x)))/(a + b*x^3)^3 - (40*Sqrt[3]*a^(2/3)*(-7*b^(2/3)*c + 2*a^(2/3)*e)*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + 486*a*d*Log[x] + (40
*(7*a^(2/3)*b^(2/3)*c + 2*a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (2
0*(7*a^(2/3)*b^(2/3)*c + 2*a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/b^(1/3) - 162*a*d*Log[a + b*x^3])/(486*a^5)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.04

method	result
risch	$\frac{-\frac{140b^3cx^9}{81a^4}+\frac{20eb^2x^8}{81a^3}+\frac{db^2x^7}{3a^3}-\frac{385cb^2x^6}{81a^3}+\frac{52bex^5}{81a^2}+\frac{5bdx^4}{6a^2}-\frac{335bcx^3}{81a^2}+\frac{41ex^2}{81a}+\frac{11xd}{18a}-\frac{c}{a}}{x(bx^3+a)^3} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{13}b\_Z^3+243a^9bd\_Z^2+(\ldots\right.}\ldots\right.}{\ldots}$
default	$-\frac{c}{a^4x}+\frac{d\ln(x)}{a^4}+\frac{-\frac{59}{81}b^3cx^8+\frac{20}{81}ab^2ex^7+\frac{1}{3}ab^2dx^6-\frac{142}{81}ab^2cx^5+\frac{52}{81}a^2bex^4+\frac{5}{6}a^2bdx^3-\frac{92}{81}a^2bcx^2+\frac{41}{81}a^3ex+\frac{11}{18}a^3d}{(bx^3+a)^3}+\frac{40ae\left(\frac{\ln\left(x+(\ldots\right)}{3b\left(\frac{a}{b}\ldots\right.}\ldots\right)}{\ldots}$

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-140/81/a^4*b^3*c*x^9+20/81*e/a^3*b^2*x^8+1/3*d/a^3*b^2*x^7-385/81*c/a^3*b
^2*x^6+52/81*b*e/a^2*x^5+5/6*b*d/a^2*x^4-335/81*b*c/a^2*x^3+41/81/a*e*x^2+1
1/18/a*x*d-c/a)/x/(b*x^3+a)^3+1/243*sum(_R*ln((-_R^3*a^13*b-162*_R^2*a^9*b*
d+(14000*a^5*b*c*e-6561*a^5*b*d^2)*_R+48000*a^2*e^3+680400*a*b*c*d*e+205800
0*b^2*c^3)*x-35*a^9*b*c*_R^2+(-400*a^6*e^2+5670*a^5*b*c*d)*_R+97200*a^2*d*e
^2+688905*a*b*c*d^2),_R=RootOf(a^13*b*_Z^3+243*a^9*b*d*_Z^2+(-16800*a^5*b*c
*e+19683*a^5*b*d^2)*_Z-64000*a^2*e^3-1360800*a*b*c*d*e+531441*a*b*d^3-27440
00*b^2*c^3))+d*ln(x)/a^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 5250, normalized size of antiderivative = 17.44

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^4}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^4}\,dx=\text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^4}\,dx = -\frac{280\,b^3cx^9-40\,ab^2ex^8-54\,ab^2dx^7+770\,ab^2cx^6-104\,a^2bex^5-135\,a^2bdx^4+670\,a^2bcx^3-82\,a^3ex^2-9}{162\,(a^4b^3x^{10}+3\,a^5b^2x^7+3\,a^6bx^4+a^7x)} + \frac{d\log(x)}{a^4} - \frac{20\,\sqrt{3}\left(7\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5} - \frac{\left(81\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+70\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(81\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-140\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-40\,ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] -1/162*(280*b^3*c*x^9 - 40*a*b^2*e*x^8 - 54*a*b^2*d*x^7 + 770*a*b^2*c*x^6 -
 104*a^2*b*e*x^5 - 135*a^2*b*d*x^4 + 670*a^2*b*c*x^3 - 82*a^3*e*x^2 - 99*a^
3*d*x + 162*a^3*c)/(a^4*b^3*x^10 + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x) + d
*log(x)/a^4 - 20/243*sqrt(3)*(7*b*c*(a/b)^(2/3) - 2*a*e*(a/b)^(1/3))*arctan
(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^5 - 1/243*(81*b*d*(a/b)^(2/
3) + 70*b*c*(a/b)^(1/3) + 20*a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
^4*b*(a/b)^(2/3)) - 1/243*(81*b*d*(a/b)^(2/3) - 140*b*c*(a/b)^(1/3) - 40*a*
e)*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^4}\,dx = -\frac{d\log\left(|bx^3+a|\right)}{3\,a^4}+\frac{d\log\left(|x|\right)}{a^4}$$
$$+\frac{20\,\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}ae+7\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5b}$$
$$+\frac{10\left(2\left(-ab^2\right)^{\frac{1}{3}}ae-7\left(-ab^2\right)^{\frac{2}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b}$$
$$-\frac{280\,b^3cx^9-40\,ab^2ex^8-54\,ab^2dx^7+770\,ab^2cx^6-104\,a^2bex^5-135\,a^2bdx^4+670\,a^2bcx^3-82\,a^3ex^2-99}{162\left(bx^3+a\right)^3a^4x}$$
$$+\frac{20\left(7\,a^4b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^5be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^9b}$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^4 + d*log(abs(x))/a^4 + 20/243*sqrt(3)*(2*(-a*
b^2)^(1/3)*a*e + 7*(-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/(a^5*b) + 10/243*(2*(-a*b^2)^(1/3)*a*e - 7*(-a*b^2)^(2/3)*c
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) - 1/162*(280*b^3*c*x^9 -
 40*a*b^2*e*x^8 - 54*a*b^2*d*x^7 + 770*a*b^2*c*x^6 - 104*a^2*b*e*x^5 - 135*
a^2*b*d*x^4 + 670*a^2*b*c*x^3 - 82*a^3*e*x^2 - 99*a^3*d*x + 162*a^3*c)/((b*
x^3 + a)^3*a^4*x) + 20/243*(7*a^4*b^2*c*(-a/b)^(1/3) - 2*a^5*b*e)*(-a/b)^(1
/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b)
```

Mupad [B] (verification not implemented)

Time = 11.85 (sec) , antiderivative size = 840, normalized size of antiderivative = 2.79

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^4}\,dx$$
$$=\frac{\frac{41\,e\,x^2}{81\,a}-\frac{c}{a}+\frac{11\,d\,x}{18\,a}-\frac{385\,b^2\,c\,x^6}{81\,a^3}-\frac{140\,b^3\,c\,x^9}{81\,a^4}+\frac{b^2\,d\,x^7}{3\,a^3}+\frac{20\,b^2\,e\,x^8}{81\,a^3}-\frac{335\,b\,c\,x^3}{81\,a^2}+\frac{5\,b\,d\,x^4}{6\,a^2}+\frac{52\,b\,e\,x^5}{81\,a^2}}{a^3\,x+3\,a^2\,b\,x^4+3\,a\,b^2\,x^7+b^3\,x^{10}}$$
$$+\left(\sum_{k=1}^{3}\ln\left(\frac{b^2\left(-\mathrm{root}(14348907\,a^{13}\,b\,z^3+14348907\,a^9\,b\,d\,z^2-4082400\,a^5\,b\,c\,e\,z+4782969\,a^5\,b\,d^2\,z-136\right.}{}\right.\right.$$
$$+14348907\,a^9\,b\,d\,z^2-4082400\,a^5\,b\,c\,e\,z+4782969\,a^5\,b\,d^2\,z-1360800\,a\,b\,c\,d\,e$$
$$\left.\left.+531441\,a\,b\,d^3-64000\,a^2\,e^3-2744000\,b^2\,c^3,z,k)\right)+\frac{d\,\ln\left(x\right)}{a^4}$$

```
[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x)
```

```
[Out] ((41*e*x^2)/(81*a) - c/a + (11*d*x)/(18*a) - (385*b^2*c*x^6)/(81*a^3) - (14
0*b^3*c*x^9)/(81*a^4) + (b^2*d*x^7)/(3*a^3) + (20*b^2*e*x^8)/(81*a^3) - (33
5*b*c*x^3)/(81*a^2) + (5*b*d*x^4)/(6*a^2) + (52*b*e*x^5)/(81*a^2))/(a^3*x +
 b^3*x^10 + 3*a^2*b*x^4 + 3*a*b^2*x^7) + symsum(log((4*b^2*(32400*a^2*d*e^2
 - 32400*root(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*
e*z + 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*
e^3 - 2744000*b^2*c^3, z, k)*a^6*e^2 + 686000*b^2*c^3*x + 16000*a^2*e^3*x +
 229635*a*b*c*d^2 - 688905*root(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2
- 4082400*a^5*b*c*e*z + 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*
b*d^3 - 64000*a^2*e^3 - 2744000*b^2*c^3, z, k)^2*a^9*b*c - 4782969*root(143
48907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z + 4782969*a^5
*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*e^3 - 2744000*b^2
*c^3, z, k)^3*a^13*b*x - 531441*root(14348907*a^13*b*z^3 + 14348907*a^9*b*d
*z^2 - 4082400*a^5*b*c*e*z + 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 5314
41*a*b*d^3 - 64000*a^2*e^3 - 2744000*b^2*c^3, z, k)*a^5*b*d^2*x - 3188646*r
oot(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z + 4782
969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*e^3 - 2744
000*b^2*c^3, z, k)^2*a^9*b*d*x + 459270*root(14348907*a^13*b*z^3 + 14348907
*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z + 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*
e + 531441*a*b*d^3 - 64000*a^2*e^3 - 2744000*b^2*c^3, z, k)*a^5*b*c*d + 113
4000*root(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z
+ 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*e^3
- 2744000*b^2*c^3, z, k)*a^5*b*c*e*x + 226800*a*b*c*d*e*x))/(531441*a^11))*
root(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z + 478
2969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*e^3 - 274
4000*b^2*c^3, z, k), k, 1, 3) + (d*log(x))/a^4
```

3.363 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 310

$$\begin{aligned}\int \frac{c+dx+ex^2}{x^3(a+bx^3)^4}\,dx =& -\frac{c}{2a^4x^2}-\frac{d}{a^4x}-\frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}\\ &-\frac{x(17bc+16bdx+15bex^2)}{54a^3(a+bx^3)^2}-\frac{x(139bc+118bdx+99bex^2)}{162a^4(a+bx^3)}\\ &+\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{14/3}}\\ &+\frac{e\log(x)}{a^4}-\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}}\\ &+\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}}\\ &-\frac{e\log(a+bx^3)}{3a^4}\end{aligned}$$

```
[Out] -1/2*c/a^4/x^2-d/a^4/x-1/9*x*(b*e*x^2+b*d*x+b*c)/a^2/(b*x^3+a)^3-1/54*x*(15
*b*e*x^2+16*b*d*x+17*b*c)/a^3/(b*x^3+a)^2-1/162*x*(99*b*e*x^2+118*b*d*x+139
*b*c)/a^4/(b*x^3+a)+e*ln(x)/a^4-20/243*b^(1/3)*(11*b^(1/3)*c-7*a^(1/3)*d)*l
n(a^(1/3)+b^(1/3)*x)/a^(14/3)+10/243*b^(1/3)*(11*b^(1/3)*c-7*a^(1/3)*d)*ln(
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)-1/3*e*ln(b*x^3+a)/a^4+20/24
3*b^(1/3)*(11*b^(1/3)*c+7*a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/
3)*3^(1/2))/a^(14/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)^4}\,dx = \frac{20\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}d+11\sqrt[3]{b}c\right)}{81\sqrt{3}a^{14/3}} + \frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}} - \frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}} - \frac{x(139bc+118bdx+99bex^2)}{162a^4(a+bx^3)} - \frac{e\log(a+bx^3)}{3a^4} - \frac{c}{2a^4x^2} - \frac{d}{a^4x} + \frac{e\log(x)}{a^4} - \frac{x(17bc+16bdx+15bex^2)}{54a^3(a+bx^3)^2} - \frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}$$

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x]
```

```
[Out] -1/2*c/(a^4*x^2) - d/(a^4*x) - (x*(b*c + b*d*x + b*e*x^2))/(9*a^2*(a + b*x^
3)^3) - (x*(17*b*c + 16*b*d*x + 15*b*e*x^2))/(54*a^3*(a + b*x^3)^2) - (x*(1
39*b*c + 118*b*d*x + 99*b*e*x^2))/(162*a^4*(a + b*x^3)) + (20*b^(1/3)*(11*b
^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(81*Sqrt[3]*a^(14/3)) + (e*Log[x])/a^4 - (20*b^(1/3)*(11*b^(1/3)*c - 7*a^(1
/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*c
 - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(14/
3)) - (e*Log[a + b*x^3])/(3*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}-\frac{\int \frac{-9bc-9bdx-9bex^2+\frac{8b^2cx^3}{a}+\frac{7b^2dx^4}{a}+\frac{6b^2ex^5}{a}}{x^3(a+bx^3)^3}\,dx}{9ab}\\
&= -\frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}-\frac{x(17bc+16bdx+15bex^2)}{54a^3(a+bx^3)^2}\\
&\quad+\frac{\int \frac{54b^3c+54b^3dx+54b^3ex^2-\frac{85b^4cx^3}{a}-\frac{64b^4dx^4}{a}-\frac{45b^4ex^5}{a}}{x^3(a+bx^3)^2}\,dx}{54a^2b^3}\\
&= -\frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}-\frac{x(17bc+16bdx+15bex^2)}{54a^3(a+bx^3)^2}\\
&\quad-\frac{x(139bc+118bdx+99bex^2)}{162a^4(a+bx^3)}-\frac{\int \frac{-162b^5c-162b^5dx-162b^5ex^2+\frac{278b^6cx^3}{a}+\frac{118b^6dx^4}{a}}{x^3(a+bx^3)}\,dx}{162a^3b^5}\\
&= -\frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}-\frac{x(17bc+16bdx+15bex^2)}{54a^3(a+bx^3)^2}-\frac{x(139bc+118bdx+99bex^2)}{162a^4(a+bx^3)}\\
&\quad-\frac{\int \left(-\frac{162b^5c}{ax^3}-\frac{162b^5d}{ax^2}-\frac{162b^5e}{ax}+\frac{2b^6(220c+140dx+81ex^2)}{a(a+bx^3)}\right)dx}{162a^3b^5}\\
&= -\frac{c}{2a^4x^2}-\frac{d}{a^4x}-\frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}-\frac{x(17bc+16bdx+15bex^2)}{54a^3(a+bx^3)^2}\\
&\quad-\frac{x(139bc+118bdx+99bex^2)}{162a^4(a+bx^3)}+\frac{e\log(x)}{a^4}-\frac{b\int \frac{220c+140dx+81ex^2}{a+bx^3}\,dx}{81a^4}\\
&= -\frac{c}{2a^4x^2}-\frac{d}{a^4x}-\frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}-\frac{x(17bc+16bdx+15bex^2)}{54a^3(a+bx^3)^2}\\
&\quad-\frac{x(139bc+118bdx+99bex^2)}{162a^4(a+bx^3)}+\frac{e\log(x)}{a^4}-\frac{b\int \frac{220c+140dx}{a+bx^3}\,dx}{81a^4}-\frac{(be)\int \frac{x^2}{a+bx^3}\,dx}{a^4}
\end{aligned}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3} - \frac{x(17bc+16bdx+15bex^2)}{54a^3(a+bx^3)^2} - \frac{x(139bc+118bdx+99bex^2)}{162a^4(a+bx^3)} + \frac{e\log(x)}{a^4} - \frac{e\log(a+bx^3)}{3a^4} - \frac{b^{2/3}\int \frac{\sqrt[3]{a}\left(440\sqrt[3]{b}c+140\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-220\sqrt[3]{b}c+140\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{243a^{14/3}} - \frac{\left(20b\left(11c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{243a^{14/3}}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3} - \frac{x(17bc+16bdx+15bex^2)}{54a^3(a+bx^3)^2} - \frac{x(139bc+118bdx+99bex^2)}{162a^4(a+bx^3)} + \frac{e\log(x)}{a^4} - \frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}} - \frac{e\log(a+bx^3)}{3a^4} + \frac{\left(10\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{243a^{14/3}} - \frac{\left(10b^{2/3}\left(11\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{81a^{13/3}}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3} - \frac{x(17bc+16bdx+15bex^2)}{54a^3(a+bx^3)^2} - \frac{x(139bc+118bdx+99bex^2)}{162a^4(a+bx^3)} + \frac{e\log(x)}{a^4} - \frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}} + \frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}} - \frac{e\log(a+bx^3)}{3a^4} - \frac{\left(20\sqrt[3]{b}\left(11\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{81a^{14/3}}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x(bc + bdx + bex^2)}{9a^2(a + bx^3)^3} - \frac{x(17bc + 16bdx + 15bex^2)}{54a^3(a + bx^3)^2} - \frac{x(139bc + 118bdx + 99bex^2)}{162a^4(a + bx^3)} + \frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}c + 7\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{14/3}} + \frac{e\log(x)}{a^4} - \frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}c - 7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{243a^{14/3}} + \frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}c - 7\sqrt[3]{a}d\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{243a^{14/3}} - \frac{e\log(a + bx^3)}{3a^4}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.92

$$\int \frac{c + dx + ex^2}{x^3(a + bx^3)^4}\,dx$$

$$= \frac{-\frac{243ac}{x^2} - \frac{486ad}{x} + \frac{54a^3(ae - bx(c+dx))}{(a+bx^3)^3} + \frac{9a^2(9ae - bx(17c+16dx))}{(a+bx^3)^2} + \frac{3a(54ae - bx(139c+118dx))}{a+bx^3} + 40\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(11\sqrt[3]{b}c + 7\sqrt[3]{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x]
```

```
[Out] ((-243*a*c)/x^2 - (486*a*d)/x + (54*a^3*(a*e - b*x*(c + d*x)))/(a + b*x^3)^
3 + (9*a^2*(9*a*e - b*x*(17*c + 16*d*x)))/(a + b*x^3)^2 + (3*a*(54*a*e - b*
x*(139*c + 118*d*x)))/(a + b*x^3) + 40*Sqrt[3]*a^(1/3)*b^(1/3)*(11*b^(1/3)*
c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 486*a*e*Log[
x] + 40*b^(1/3)*(-11*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)
*x] + 20*b^(1/3)*(11*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2] - 162*a*e*Log[a + b*x^3])/(486*a^5)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.03

method	result
risch	$\frac{-\frac{140 b^3 d x^{10}}{81 a^4}-\frac{110 b^3 c x^9}{81 a^4}+\frac{e b^2 x^8}{3 a^3}-\frac{385 d b^2 x^7}{81 a^3}-\frac{286 c b^2 x^6}{81 a^3}+\frac{5 b e x^5}{6 a^2}-\frac{335 b d x^4}{81 a^2}-\frac{451 b c x^3}{162 a^2}+\frac{11 e x^2}{18 a}-\frac{x d}{a}-\frac{c}{2 a}}{x^2 (b x^3+a)^3}+\frac{e \ln(-x)}{a^4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(a\ldots}\right.}{\ldots}$
default	$-\frac{c}{2a^4x^2}-\frac{d}{a^4x}+\frac{e\ln(x)}{a^4}-\frac{b\left(\frac{\frac{59b^2dx^8}{81}+\frac{139b^2cx^7}{162}-\frac{aebx^6}{3}+\frac{142x^5dba}{81}+\frac{329abcx^4}{162}-\frac{5a^2ex^3}{6}+\frac{92a^2dx^2}{81}+\frac{104a^2cx}{81}-\frac{11ea^3}{18b}}{(bx^3+a)^3}+\frac{220c\left(\frac{\ln(\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}$

[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x,method=_RETURNVERBOSE)

[Out] (-140/81/a^4*b^3*d*x^10-110/81/a^4*b^3*c*x^9+1/3*e/a^3*b^2*x^8-385/81*d/a^3*b^2*x^7-286/81*c/a^3*b^2*x^6+5/6*b*e/a^2*x^5-335/81*b*d/a^2*x^4-451/162*b*c/a^2*x^3+11/18/a*e*x^2-1/a*x*d-1/2*c/a)/x^2/(b*x^3+a)^3+e/a^4*ln(-x)+1/243*sum(_R*ln((-_R^3*a^14-162*_R^2*a^10*e+(-6561*a^6*e^2-77000*a^5*b*c*d)*_R-3742200*a*b*c*d*e+2058000*a*b*d^3-7986000*b^2*c^3)*x-35*a^10*d*_R^2+(5670*a^6*d*e-12100*a^5*b*c^2)*_R+688905*a^2*d*e^2+2940300*a*b*c^2*e),_R=RootOf(a^14*_Z^3+243*a^10*e*_Z^2+(19683*a^6*e^2+92400*a^5*b*c*d)*_Z+531441*a^2*e^3+7484400*a*b*c*d*e-2744000*a*b*d^3+10648000*b^2*c^3))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 5049, normalized size of antiderivative = 16.29

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^4}\,dx=\text{Too large to display}$$

[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^4}\,dx = -\frac{280\,b^3dx^{10}+220\,b^3cx^9-54\,ab^2ex^8+770\,ab^2dx^7+572\,ab^2cx^6-135\,a^2bex^5+670\,a^2bdx^4+451\,a^2bcx^3}{162\,\left(a^4b^3x^{11}+3\,a^5b^2x^8+3\,a^6bx^5+a^7x^2\right)} + \frac{e\log\left(x\right)}{a^4} - \frac{20\,\sqrt{3}\left(7\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+11\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5} - \frac{\left(81\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}+70\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-110\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(81\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}-140\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+220\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="maxima")
[Out] -1/162*(280*b^3*d*x^10 + 220*b^3*c*x^9 - 54*a*b^2*e*x^8 + 770*a*b^2*d*x^7 +
 572*a*b^2*c*x^6 - 135*a^2*b*e*x^5 + 670*a^2*b*d*x^4 + 451*a^2*b*c*x^3 - 99
*a^3*e*x^2 + 162*a^3*d*x + 81*a^3*c)/(a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*
b*x^5 + a^7*x^2) + e*log(x)/a^4 - 20/243*sqrt(3)*(7*b*d*(a/b)^(2/3) + 11*b*
c*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^5 - 1/
243*(81*e*(a/b)^(2/3) + 70*d*(a/b)^(1/3) - 110*c)*log(x^2 - x*(a/b)^(1/3) +
 (a/b)^(2/3))/(a^4*(a/b)^(2/3)) - 1/243*(81*e*(a/b)^(2/3) - 140*d*(a/b)^(1/
3) + 220*c)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.02

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^4}\,dx = -\frac{e\log\left(\left|bx^3+a\right|\right)}{3\,a^4}+\frac{e\log\left(\left|x\right|\right)}{a^4} - \frac{20\,\sqrt{3}\left(11\left(-ab^2\right)^{\frac{1}{3}}bc-7\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5b} - \frac{10\left(11\left(-ab^2\right)^{\frac{1}{3}}bc+7\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b} + \frac{20\left(7\,a^4b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+11\,a^4b^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^9b} - \frac{280\,b^3dx^{10}+220\,b^3cx^9-54\,ab^2ex^8+770\,ab^2dx^7+572\,ab^2cx^6-135\,a^2bex^5+670\,a^2bdx^4+451\,a^2bcx^3-}{162\left(bx^3+a\right)^3a^4x^2}$$

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^4 + e*log(abs(x))/a^4 - 20/243*sqrt(3)*(11*(-a
*b^2)^(1/3)*b*c - 7*(-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(a^5*b) - 10/243*(11*(-a*b^2)^(1/3)*b*c + 7*(-a*b^2)^(2/3)
*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) + 20/243*(7*a^4*b^2*d*
(-a/b)^(1/3) + 11*a^4*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b
) - 1/162*(280*b^3*d*x^10 + 220*b^3*c*x^9 - 54*a*b^2*e*x^8 + 770*a*b^2*d*x^
7 + 572*a*b^2*c*x^6 - 135*a^2*b*e*x^5 + 670*a^2*b*d*x^4 + 451*a^2*b*c*x^3 -
 99*a^3*e*x^2 + 162*a^3*d*x + 81*a^3*c)/((b*x^3 + a)^3*a^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 12.13 (sec) , antiderivative size = 825, normalized size of antiderivative = 2.66

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^4}\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(-\frac{b^3\left(\mathrm{root}(14348907\,a^{14}\,z^3+14348907\,a^{10}\,e\,z^2+22453200\,a^5\,b\,c\,d\,z+4782969\,a^6\,e^2\,z+748440\right.}{}\right.\right.$$
$$+14348907\,a^{10}\,e\,z^2+22453200\,a^5\,b\,c\,d\,z+4782969\,a^6\,e^2\,z+7484400\,a\,b\,c\,d\,e$$
$$\left.\left.-2744000\,a\,b\,d^3+531441\,a^2\,e^3+10648000\,b^2\,c^3,z,k\right)\right)$$
$$-\frac{\frac{c}{2a}-\frac{11\,e\,x^2}{18\,a}+\frac{d\,x}{a}+\frac{286\,b^2\,c\,x^6}{81\,a^3}+\frac{110\,b^3\,c\,x^9}{81\,a^4}+\frac{385\,b^2\,d\,x^7}{81\,a^3}+\frac{140\,b^3\,d\,x^{10}}{81\,a^4}-\frac{b^2\,e\,x^8}{3\,a^3}+\frac{451\,b\,c\,x^3}{162\,a^2}+\frac{335\,b\,d\,x^4}{81\,a^2}-\frac{5\,b\,e\,x^5}{6\,a^2}}{a^3\,x^2+3\,a^2\,b\,x^5+3\,a\,b^2\,x^8+b^3\,x^{11}}$$
$$+\frac{e\,\ln(x)}{a^4}$$

```
[In] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x)
```

```
[Out] symsum(log(-(4*b^3*(688905*root(14348907*a^14*z^3 + 14348907*a^10*e*z^2 + 2
2453200*a^5*b*c*d*z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d
^3 + 531441*a^2*e^3 + 10648000*b^2*c^3, z, k)^2*a^10*d - 229635*a^2*d*e^2 +
 4782969*root(14348907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*c*d*
z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^
3 + 10648000*b^2*c^3, z, k)^3*a^14*x + 2662000*b^2*c^3*x - 459270*root(1434
8907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*c*d*z + 4782969*a^6*e^
2*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^3 + 10648000*b^2*c
^3, z, k)*a^6*d*e - 980100*a*b*c^2*e - 686000*a*b*d^3*x + 980100*root(14348
907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*c*d*z + 4782969*a^6*e^2
*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^3 + 10648000*b^2*c^
3, z, k)*a^5*b*c^2 + 531441*root(14348907*a^14*z^3 + 14348907*a^10*e*z^2 +
22453200*a^5*b*c*d*z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*
d^3 + 531441*a^2*e^3 + 10648000*b^2*c^3, z, k)*a^6*e^2*x + 3188646*root(143
48907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*c*d*z + 4782969*a^6*e
^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^3 + 10648000*b^2*
c^3, z, k)^2*a^10*e*x + 6237000*root(14348907*a^14*z^3 + 14348907*a^10*e*z^
2 + 22453200*a^5*b*c*d*z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*
a*b*d^3 + 531441*a^2*e^3 + 10648000*b^2*c^3, z, k)*a^5*b*c*d*x + 1247400*a*
b*c*d*e*x))/(531441*a^12))*root(14348907*a^14*z^3 + 14348907*a^10*e*z^2 + 2
2453200*a^5*b*c*d*z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d
^3 + 531441*a^2*e^3 + 10648000*b^2*c^3, z, k), k, 1, 3) - (c/(2*a) - (11*e*
x^2)/(18*a) + (d*x)/a + (286*b^2*c*x^6)/(81*a^3) + (110*b^3*c*x^9)/(81*a^4)
 + (385*b^2*d*x^7)/(81*a^3) + (140*b^3*d*x^10)/(81*a^4) - (b^2*e*x^8)/(3*a^
```

```
3) + (451*b*c*x^3)/(162*a^2) + (335*b*d*x^4)/(81*a^2) - (5*b*e*x^5)/(6*a^2)
)/(a^3*x^2 + b^3*x^11 + 3*a^2*b*x^5 + 3*a*b^2*x^8) + (e*log(x))/a^4
```

3.364 $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 340

$$\begin{aligned}\int \frac{c+dx+ex^2}{x^4(a+bx^3)^4}\,dx =& -\frac{c}{3a^4x^3}-\frac{d}{2a^4x^2}-\frac{e}{a^4x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2(a+bx^3)^3}\\ &-\frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3(a+bx^3)^2}-\frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4(a+bx^3)}\\ &+\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{14/3}}\\ &-\frac{4bc\log(x)}{a^5}-\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}}\\ &+\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}}\\ &+\frac{4bc\log(a+bx^3)}{3a^5}\end{aligned}$$

```
[Out] -1/3*c/a^4/x^3-1/2*d/a^4/x^2-e/a^4/x-1/9*x*(b*d+b*e*x-b^2*c*x^2/a)/a^2/(b*x
^3+a)^3-1/54*x*(17*b*d+16*b*e*x-24*b^2*c*x^2/a)/a^3/(b*x^3+a)^2-1/162*x*(13
9*b*d+118*b*e*x-234*b^2*c*x^2/a)/a^4/(b*x^3+a)-4*b*c*ln(x)/a^5-20/243*b^(1/
3)*(11*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)+10/243*b^(1/3)
*(11*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14
/3)+4/3*b*c*ln(b*x^3+a)/a^5+20/243*b^(1/3)*(11*b^(1/3)*d+7*a^(1/3)*e)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^4}\,dx = \frac{20\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}e+11\sqrt[3]{b}d\right)}{81\sqrt{3}a^{14/3}} + \frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}} - \frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}} + \frac{4bc\log(a+bx^3)}{3a^5} - \frac{4bc\log(x)}{a^5} - \frac{x\left(-\frac{234b^2cx^2}{a}+139bd+118bex\right)}{162a^4(a+bx^3)} - \frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(-\frac{24b^2cx^2}{a}+17bd+16bex\right)}{54a^3(a+bx^3)^2} - \frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{9a^2(a+bx^3)^3}$$

```
[In] Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x]
```

```
[Out] -1/3*c/(a^4*x^3) - d/(2*a^4*x^2) - e/(a^4*x) - (x*(b*d + b*e*x - (b^2*c*x^2
)/a))/(9*a^2*(a + b*x^3)^3) - (x*(17*b*d + 16*b*e*x - (24*b^2*c*x^2)/a))/(5
4*a^3*(a + b*x^3)^2) - (x*(139*b*d + 118*b*e*x - (234*b^2*c*x^2)/a))/(162*a
^4*(a + b*x^3)) + (20*b^(1/3)*(11*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(14/3)) - (4*b*c*Log[x])/a
^5 - (20*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(24
3*a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(14/3)) + (4*b*c*Log[a + b*x^3])/(3*a^5)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2(a+bx^3)^3} - \frac{\int \frac{-9bc-9bdx-9bex^2+\frac{9b^2cx^3}{a}+\frac{8b^2dx^4}{a}+\frac{7b^2ex^5}{a}-\frac{6b^3cx^6}{a^2}}{x^4(a+bx^3)^3}\,dx}{9ab}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2(a+bx^3)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3(a+bx^3)^2} + \frac{\int \frac{54b^3c+54b^3dx+54b^3ex^2-\frac{108b^4cx^3}{a}-\frac{85b^4dx^4}{a}-\frac{64b^4ex^5}{a}+\frac{72b^5cx^6}{a^2}}{x^4(a+bx^3)^2}\,dx}{54a^2b^3}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2(a+bx^3)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3(a+bx^3)^2} - \frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4(a+bx^3)} - \frac{\int \frac{-162b^5c-162b^5dx-162b^5ex^2+\frac{486b^6cx^3}{a}+\frac{278b^6dx^4}{a}+\frac{118b^6ex^5}{a}}{x^4(a+bx^3)}\,dx}{162a^3b^5}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2(a+bx^3)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3(a+bx^3)^2} - \frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4(a+bx^3)} - \frac{\int \left(-\frac{162b^5c}{ax^4}-\frac{162b^5d}{ax^3}-\frac{162b^5e}{ax^2}+\frac{648b^6c}{a^2x}+\frac{8b^6(55ad+35aex-81bcx^2)}{a^2(a+bx^3)}\right)dx}{162a^3b^5}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2(a+bx^3)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3(a+bx^3)^2} - \frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4(a+bx^3)} - \frac{4bc\log(x)}{a^5} - \frac{(4b)\int \frac{55ad+35aex-81bcx^2}{a+bx^3}\,dx}{81a^5}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4\left(a+bx^3\right)} - \frac{4bc\log(x)}{a^5} - \frac{(4b)\int\frac{55ad+35aex}{a+bx^3}\,dx}{81a^5} + \frac{\left(4b^2c\right)\int\frac{x^2}{a+bx^3}\,dx}{a^5}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4\left(a+bx^3\right)} - \frac{4bc\log(x)}{a^5} + \frac{4bc\log\left(a+bx^3\right)}{3a^5} - \frac{\left(4b^{2/3}\right)\int\frac{\sqrt[3]{a}\left(110a\sqrt[3]{b}d+35a^{4/3}e\right)+\sqrt[3]{b}\left(-55a\sqrt[3]{b}d+35a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{243a^{17/3}} - \frac{\left(20b\left(11d-\frac{7\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{243a^{14/3}}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4\left(a+bx^3\right)} - \frac{4bc\log(x)}{a^5} - \frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}} + \frac{4bc\log\left(a+bx^3\right)}{3a^5} + \frac{\left(10\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{243a^{14/3}} - \frac{\left(10b^{2/3}\left(11\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{81a^{13/3}}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2(a+bx^3)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3(a+bx^3)^2} - \frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4(a+bx^3)} - \frac{4bc\log(x)}{a^5} - \frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}} + \frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}} + \frac{4bc\log(a+bx^3)}{3a^5} - \frac{\left(20\sqrt[3]{b}\left(11\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{81a^{14/3}}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2(a+bx^3)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3(a+bx^3)^2} - \frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4(a+bx^3)} + \frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{14/3}} - \frac{4bc\log(x)}{a^5} - \frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}} + \frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}} + \frac{4bc\log(a+bx^3)}{3a^5}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.84

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^4}\,dx =$$

$$-\frac{\frac{162ac}{x^3}+\frac{243ad}{x^2}+\frac{486ae}{x}+\frac{54a^3b(c+x(d+ex))}{(a+bx^3)^3}+\frac{9a^2b(18c+x(17d+16ex))}{(a+bx^3)^2}+\frac{3ab(162c+x(139d+118ex))}{a+bx^3}-40\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(11\sqrt[3]{b}\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x]
```

```
[Out] -1/486*((162*a*c)/x^3 + (243*a*d)/x^2 + (486*a*e)/x + (54*a^3*b*(c + x*(d +
 e*x)))/(a + b*x^3)^3 + (9*a^2*b*(18*c + x*(17*d + 16*e*x)))/(a + b*x^3)^2
+ (3*a*b*(162*c + x*(139*d + 118*e*x)))/(a + b*x^3) - 40*Sqrt[3]*a^(1/3)*b^
(1/3)*(11*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] + 1944*b*c*Log[x] + 40*b^(1/3)*(11*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Log
```

```
[a^(1/3) + b^(1/3)*x] - 20*b^(1/3)*(11*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 648*b*c*Log[a + b*x^3])/a^5
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.99

method	result
default	$-\frac{c}{3a^4x^3}-\frac{d}{2a^4x^2}-\frac{e}{a^4x}-\frac{4bc\ln(x)}{a^5}-\frac{b\left(\frac{\frac{59}{81}a\,b^2e\,x^8+\frac{139}{162}a\,b^2d\,x^7+a\,b^2c\,x^6+\frac{142}{81}a^2be\,x^5+\frac{329}{162}a^2bd\,x^4+\frac{7}{3}a^2x^3bc+\frac{92}{81}a^3e\,x^2+\frac{104}{81}a^3\cdots}{\left(b\,x^3+a\right)^3}\cdots\right.}{\cdots}$
risch	$\frac{-\frac{140b^3e\,x^{11}}{81a^4}-\frac{110b^3d\,x^{10}}{81a^4}-\frac{4b^3c\,x^9}{3a^4}-\frac{385e\,b^2x^8}{81a^3}-\frac{286d\,b^2x^7}{81a^3}-\frac{10c\,b^2x^6}{3a^3}-\frac{335be\,x^5}{81a^2}-\frac{451bd\,x^4}{162a^2}-\frac{22bc\,x^3}{9a^2}-\frac{e\,x^2}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^3\left(b\,x^3+a\right)^3}+\frac{4\left(\sum_{\_R=\mathrm{RootOf}(\cdots}\cdots\right.}{\cdots}$

```
[In] int((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*c/a^4/x^3-1/2*d/a^4/x^2-e/a^4/x-4*b*c*ln(x)/a^5-1/a^5*b*((59/81*a*b^2*
e*x^8+139/162*a*b^2*d*x^7+a*b^2*c*x^6+142/81*a^2*b*e*x^5+329/162*a^2*b*d*x^
4+7/3*a^2*x^3*b*c+92/81*a^3*e*x^2+104/81*a^3*d*x+13/9*c*a^3)/(b*x^3+a)^3+22
0/81*a*d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b
)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1)))+140/81*a*e*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)
^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-4/3*c*ln(b*x^3+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 5670, normalized size of antiderivative = 16.68

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx =& -\frac{280\,b^3ex^{11}+220\,b^3dx^{10}+216\,b^3cx^9+770\,ab^2ex^8+572\,ab^2dx^7+540\,ab^2cx^6+670\,a^2bex^5+451\,a^2bdx^4 -}{162\,\left(a^4b^3x^{12}+3\,a^5b^2x^9+3\,a^6bx^6+a^7x^3\right)} \\ &- \frac{4\,bc\log\left(x\right)}{a^5} - \frac{20\,\sqrt{3}\left(7\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+11\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^6} \\ &+ \frac{2\left(162\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-35\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+55\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &+ \frac{4\left(81\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+35\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-55\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] -1/162*(280*b^3*e*x^11 + 220*b^3*d*x^10 + 216*b^3*c*x^9 + 770*a*b^2*e*x^8 +
 572*a*b^2*d*x^7 + 540*a*b^2*c*x^6 + 670*a^2*b*e*x^5 + 451*a^2*b*d*x^4 + 39
6*a^2*b*c*x^3 + 162*a^3*e*x^2 + 81*a^3*d*x + 54*a^3*c)/(a^4*b^3*x^12 + 3*a^
5*b^2*x^9 + 3*a^6*b*x^6 + a^7*x^3) - 4*b*c*log(x)/a^5 - 20/243*sqrt(3)*(7*a
*e*(a/b)^(2/3) + 11*a*d*(a/b)^(1/3))*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3
))/(a/b)^(1/3))/a^6 + 2/243*(162*b*c*(a/b)^(2/3) - 35*a*e*(a/b)^(1/3) + 55*
a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(a/b)^(2/3)) + 4/243*(81*b
*c*(a/b)^(2/3) + 35*a*e*(a/b)^(1/3) - 55*a*d)*log(x + (a/b)^(1/3))/(a^5*(a/
b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx = \frac{4\,bc\log\left(|bx^3+a|\right)}{3\,a^5} - \frac{4\,bc\log\left(|x|\right)}{a^5}$$
$$- \frac{20\,\sqrt{3}\left(11\left(-ab^2\right)^{\frac{1}{3}}bd - 7\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5b}$$
$$- \frac{10\left(11\left(-ab^2\right)^{\frac{1}{3}}bd + 7\left(-ab^2\right)^{\frac{2}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b}$$
$$- \frac{280\,b^3ex^{11}+220\,b^3dx^{10}+216\,b^3cx^9+770\,ab^2ex^8+572\,ab^2dx^7+540\,ab^2cx^6+670\,a^2bex^5+451\,a^2bdx^4}{162\left(bx^4+ax\right)^3a^4}$$
$$+ \frac{20\left(7\,a^6b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}}+11\,a^6b^2d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^{11}b}$$

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 4/3*b*c*log(abs(b*x^3 + a))/a^5 - 4*b*c*log(abs(x))/a^5 - 20/243*sqrt(3)*(1
1*(-a*b^2)^(1/3)*b*d - 7*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/(a^5*b) - 10/243*(11*(-a*b^2)^(1/3)*b*d + 7*(-a*b^2)^
(2/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) - 1/162*(280*b^3*
e*x^11 + 220*b^3*d*x^10 + 216*b^3*c*x^9 + 770*a*b^2*e*x^8 + 572*a*b^2*d*x^7
 + 540*a*b^2*c*x^6 + 670*a^2*b*e*x^5 + 451*a^2*b*d*x^4 + 396*a^2*b*c*x^3 +
162*a^3*e*x^2 + 81*a^3*d*x + 54*a^3*c)/((b*x^4 + a*x)^3*a^4) + 20/243*(7*a^
6*b^2*e*(-a/b)^(1/3) + 11*a^6*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3))
)/(a^11*b)
```

Mupad [B] (verification not implemented)

Time = 12.23 (sec) , antiderivative size = 918, normalized size of antiderivative = 2.70

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx$$
$$=\left(\sum_{k=1}^{3}\ln\left(-\frac{b^3\left(\mathrm{root}(14348907\,a^{15}\,z^3-57395628\,a^{10}\,b\,c\,z^2+22453200\,a^6\,b\,d\,e\,z+76527504\,a^5\,b^2\,c^2\,z-299\right.}{}\right.\right.$$
$$-57395628\,a^{10}\,b\,c\,z^2+22453200\,a^6\,b\,d\,e\,z+76527504\,a^5\,b^2\,c^2\,z-29937600\,a\,b^2\,c\,d\,e$$
$$\left.\left.-2744000\,a^2\,b\,e^3+10648000\,a\,b^2\,d^3-34012224\,b^3\,c^3,z,k)\right)\right)$$
$$-\frac{\frac{c}{3a}+\frac{e\,x^2}{a}+\frac{d\,x}{2a}+\frac{10\,b^2\,c\,x^6}{3a^3}+\frac{4\,b^3\,c\,x^9}{3a^4}+\frac{286\,b^2\,d\,x^7}{81\,a^3}+\frac{110\,b^3\,d\,x^{10}}{81\,a^4}+\frac{385\,b^2\,e\,x^8}{81\,a^3}+\frac{140\,b^3\,e\,x^{11}}{81\,a^4}+\frac{22\,b\,c\,x^3}{9a^2}+\frac{451\,b\,d\,x^4}{162\,a^2}+\frac{335}{8}}{a^3\,x^3+3\,a^2\,b\,x^6+3\,a\,b^2\,x^9+b^3\,x^{12}}$$
$$-\frac{4\,b\,c\,\ln(x)}{a^5}$$

```
[In] int((c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x)
```

```
[Out] symsum(log(-(4*b^3*(688905*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 +
 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 274
4000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)^2*a^10*e + 39
20400*b^2*c*d^2 - 3674160*b^2*c^2*e + 4782969*root(14348907*a^15*z^3 - 5739
5628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 2993760
0*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3,
z, k)^3*a^14*x + 2662000*b^2*d^3*x - 686000*a*b*e^3*x + 980100*root(1434890
7*a^15*z^3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^
2*c^2*z - 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 3
4012224*b^3*c^3, z, k)*a^5*b*d^2 - 12754584*root(14348907*a^15*z^3 - 573956
28*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*
a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z,
 k)^2*a^9*b*c*x + 8503056*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 +
22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2744
000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)*a^4*b^2*c^2*x
+ 1837080*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b*d
*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3 +
10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)*a^5*b*c*e - 4989600*b^2*c*d*e*
x + 6237000*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b
*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3
+ 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)*a^5*b*d*e*x))/(531441*a^12))
*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76
527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*
a*b^2*d^3 - 34012224*b^3*c^3, z, k), k, 1, 3) - (c/(3*a) + (e*x^2)/a + (d*x
```

```
)/(2*a) + (10*b^2*c*x^6)/(3*a^3) + (4*b^3*c*x^9)/(3*a^4) + (286*b^2*d*x^7)/
(81*a^3) + (110*b^3*d*x^10)/(81*a^4) + (385*b^2*e*x^8)/(81*a^3) + (140*b^3*
e*x^11)/(81*a^4) + (22*b*c*x^3)/(9*a^2) + (451*b*d*x^4)/(162*a^2) + (335*b*
e*x^5)/(81*a^2))/(a^3*x^3 + b^3*x^12 + 3*a^2*b*x^6 + 3*a*b^2*x^9) - (4*b*c*
log(x))/a^5
```

3.365 $\int \frac{2ax-x^2}{a^3+x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{2ax-x^2}{a^3+x^3} dx = -\frac{2\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a+x)$$

```
[Out] -ln(a+x)-2/3*arctan(1/3*(a-2*x)/a*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1607, 1882, 31, 631, 210}

$$\int \frac{2ax-x^2}{a^3+x^3} dx = -\frac{2\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a+x)$$

```
[In] Int[(2*a*x - x^2)/(a^3 + x^3),x]
```

```
[Out] (-2*ArcTan[(a - 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1882

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(2a-x)x}{a^3+x^3}\,dx\\ &= a\int \frac{1}{a^2-ax+x^2}\,dx-\int \frac{1}{a+x}\,dx\\ &= -\log(a+x)+2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}-\log(a+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.97

$$\int \frac{2ax-x^2}{a^3+x^3}\,dx=\frac{1}{3}\left(2\sqrt{3}\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right)-2\log(a+x)+\log\left(a^2-ax+x^2\right)-\log\left(a^3+x^3\right)\right)$$

```
[In] Integrate[(2*a*x - x^2)/(a^3 + x^3),x]
```

[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x + x^2] - Log[a^3 + x^3])/3

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3}-\ln\left(a+x\right)$	29
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3}-\ln\left(a+x\right)$	29

[In] int((2*a*x-x^2)/(a^3+x^3),x,method=_RETURNVERBOSE)

[Out] 2/3*3^(1/2)*arctan(1/3*(-a+2*x)*3^(1/2)/a)-ln(a+x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{2ax-x^2}{a^3+x^3}\,dx=\frac{2}{3}\,\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)-\log\left(a+x\right)$$

[In] integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="fricas")

[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{2ax-x^2}{a^3+x^3}\,dx=-\log\left(a+x\right)-\frac{\sqrt{3}i\log\left(-\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

[In] integrate((2*a*x-x**2)/(a**3+x**3),x)

[Out] -log(a + x) - sqrt(3)*I*log(-a/2 - sqrt(3)*I*a/2 + x)/3 + sqrt(3)*I*log(-a/2 + sqrt(3)*I*a/2 + x)/3

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{2ax-x^2}{a^3+x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right) - \log\left(a+x\right)$$

```
[In] integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{2ax-x^2}{a^3+x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right) - \log\left(|a+x|\right)$$

```
[In] integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(abs(a + x))
```

Mupad [B] (verification not implemented)

Time = 11.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{2ax-x^2}{a^3+x^3}\,dx = -\ln\left(a+x\right) - \frac{2\sqrt{3}\,\mathrm{atan}\left(-\frac{\sqrt{3}\,a}{a-2\,x}\right)}{3}$$

```
[In] int((2*a*x - x^2)/(a^3 + x^3),x)
[Out] - log(a + x) - (2*3^(1/2)*atan(-(3^(1/2)*a)/(a - 2*x)))/3
```

3.366 $\int \frac{(2a-x)x}{a^3+x^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{(2a-x)x}{a^3+x^3} \, dx = -\frac{2\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a+x)$$

```
[Out] -ln(a+x)-2/3*arctan(1/3*(a-2*x)/a*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1882, 31, 631, 210}

$$\int \frac{(2a-x)x}{a^3+x^3} \, dx = -\frac{2\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a+x)$$

```
[In] Int[((2*a - x)*x)/(a^3 + x^3),x]
```

```
[Out] (-2*ArcTan[(a - 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1882

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a\int \frac{1}{a^2-ax+x^2}\,dx-\int \frac{1}{a+x}\,dx\\
&= -\log(a+x)+2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)\\
&= -\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}-\log(a+x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.97

$$\int \frac{(2a-x)x}{a^3+x^3}\,dx=\frac{1}{3}\left(2\sqrt{3}\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right)-2\log(a+x)+\log\left(a^2-ax+x^2\right)-\log\left(a^3+x^3\right)\right)$$

```
[In] Integrate[((2*a - x)*x)/(a^3 + x^3),x]
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x +
x^2] - Log[a^3 + x^3])/3
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3}-\ln\left(a+x\right)$	29
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3}-\ln\left(a+x\right)$	29

```
[In] int((2*a-x)*x/(a^3+x^3),x,method=_RETURNVERBOSE)
[Out] 2/3*3^(1/2)*arctan(1/3*(-a+2*x)*3^(1/2)/a)-ln(a+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(2a-x)x}{a^3+x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)-\log\left(a+x\right)$$

```
[In] integrate((2*a-x)*x/(a^3+x^3),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{(2a-x)x}{a^3+x^3}\,dx = -\log\left(a+x\right)-\frac{\sqrt{3}i\log\left(-\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

```
[In] integrate((2*a-x)*x/(a**3+x**3),x)
[Out] -log(a + x) - sqrt(3)*I*log(-a/2 - sqrt(3)*I*a/2 + x)/3 + sqrt(3)*I*log(-a/
2 + sqrt(3)*I*a/2 + x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(2a-x)x}{a^3+x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right) - \log\left(a+x\right)$$

```
[In] integrate((2*a-x)*x/(a^3+x^3),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(2a-x)x}{a^3+x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right) - \log\left(|a+x|\right)$$

```
[In] integrate((2*a-x)*x/(a^3+x^3),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(abs(a + x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(2a-x)x}{a^3+x^3}\,dx = -\ln\left(a+x\right) - \frac{2\sqrt{3}\,\mathrm{atan}\left(-\frac{\sqrt{3}\,a}{a-2\,x}\right)}{3}$$

```
[In] int((x*(2*a - x))/(a^3 + x^3),x)
[Out] - log(a + x) - (2*3^(1/2)*atan(-(3^(1/2)*a)/(a - 2*x)))/3
```

3.367 $\int \frac{2ax+x^2}{a^3-x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 31

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx=-\frac{2\arctan\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}-\log(a-x)$$

```
[Out] -ln(a-x)-2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1607, 1882, 31, 631, 210}

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx=-\frac{2\arctan\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}-\log(a-x)$$

```
[In] Int[(2*a*x + x^2)/(a^3 - x^3),x]
```

```
[Out] (-2*ArcTan[(a + 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1882

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(2a+x)}{a^3-x^3}\,dx\\ &= -\left(a\int \frac{1}{a^2+ax+x^2}\,dx\right)-\int \frac{1}{-a+x}\,dx\\ &= -\log(a-x)+2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{a}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}-\log(a-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx=\frac{1}{3}\left(-2\sqrt{3}\arctan\left(\frac{a+2x}{\sqrt{3}a}\right)-2\log(-a+x)+\log\left(a^2+ax+x^2\right)-\log\left(-a^3+x^3\right)\right)$$

```
[In] Integrate[(2*a*x + x^2)/(a^3 - x^3),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(a + 2*x)/(Sqrt[3]*a)] - 2*Log[-a + x] + Log[a^2 + a*x +
 x^2] - Log[-a^3 + x^3])/3
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
default	$-\ln\left(a-x\right)-\frac{2\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)\sqrt{3}}{3}$	29
risch	$-\frac{2\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)\sqrt{3}}{3}-\ln\left(-a+x\right)$	29

```
[In] int((2*a*x+x^2)/(a^3-x^3),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(a-x)-2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx=-\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2\,x)}{3\,a}\right)-\log\left(-a+x\right)$$

```
[In] integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx=-\log\left(-a+x\right)+\frac{\sqrt{3}i\log\left(\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3}-\frac{\sqrt{3}i\log\left(\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

```
[In] integrate((2*a*x+x**2)/(a**3-x**3),x)
```

```
[Out] -log(-a + x) + sqrt(3)*I*log(a/2 - sqrt(3)*I*a/2 + x)/3 - sqrt(3)*I*log(a/2
 + sqrt(3)*I*a/2 + x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx=-\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2\,x)}{3\,a}\right)-\log\left(-a+x\right)$$

```
[In] integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="maxima")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx=-\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2\,x)}{3\,a}\right)-\log\left(|-a+x|\right)$$

```
[In] integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="giac")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(abs(-a + x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx=\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,a}{a+2\,x}\right)}{3}-\ln\left(x-a\right)$$

```
[In] int((2*a*x + x^2)/(a^3 - x^3),x)
[Out] (2*3^(1/2)*atan((3^(1/2)*a)/(a + 2*x)))/3 - log(x - a)
```

3.368 $\int \frac{x(2a+x)}{a^3-x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{x(2a+x)}{a^3-x^3}\,dx = -\frac{2\arctan\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a-x)$$

```
[Out] -ln(a-x)-2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1882, 31, 631, 210}

$$\int \frac{x(2a+x)}{a^3-x^3}\,dx = -\frac{2\arctan\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a-x)$$

```
[In] Int[(x*(2*a + x))/(a^3 - x^3),x]
```

```
[Out] (-2*ArcTan[(a + 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1882

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(a\int\frac{1}{a^2+ax+x^2}\,dx\right)-\int\frac{1}{-a+x}\,dx\\ &= -\log(a-x)+2\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{a}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}-\log(a-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int\frac{x(2a+x)}{a^3-x^3}\,dx=\frac{1}{3}\left(-2\sqrt{3}\arctan\left(\frac{a+2x}{\sqrt{3}a}\right)-2\log(-a+x)+\log\left(a^2+ax+x^2\right)-\log\left(-a^3+x^3\right)\right)$$

```
[In] Integrate[(x*(2*a + x))/(a^3 - x^3),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(a + 2*x)/(Sqrt[3]*a)] - 2*Log[-a + x] + Log[a^2 + a*x +
 x^2] - Log[-a^3 + x^3])/3
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
default	$-\ln\left(a-x\right)-\frac{2\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)\sqrt{3}}{3}$	29
risch	$-\frac{2\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)\sqrt{3}}{3}-\ln\left(-a+x\right)$	29

```
[In] int(x*(2*a+x)/(a^3-x^3),x,method=_RETURNVERBOSE)
[Out] -ln(a-x)-2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{x(2a+x)}{a^3-x^3}\,dx=-\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2\,x)}{3\,a}\right)-\log\left(-a+x\right)$$

```
[In] integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="fricas")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{x(2a+x)}{a^3-x^3}\,dx=-\log\left(-a+x\right)+\frac{\sqrt{3}i\log\left(\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3}-\frac{\sqrt{3}i\log\left(\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

```
[In] integrate(x*(2*a+x)/(a**3-x**3),x)
[Out] -log(-a + x) + sqrt(3)*I*log(a/2 - sqrt(3)*I*a/2 + x)/3 - sqrt(3)*I*log(a/2
 + sqrt(3)*I*a/2 + x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{x(2a+x)}{a^3-x^3}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right) - \log(-a+x)$$

```
[In] integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="maxima")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x(2a+x)}{a^3-x^3}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right) - \log(|-a+x|)$$

```
[In] integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="giac")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(abs(-a + x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{x(2a+x)}{a^3-x^3}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,a}{a+2x}\right)}{3} - \ln(x-a)$$

```
[In] int((x*(2*a + x))/(a^3 - x^3),x)
[Out] (2*3^(1/2)*atan((3^(1/2)*a)/(a + 2*x)))/3 - log(x - a)
```

3.369 $\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 50

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx=\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

```
[Out] C*ln((a/b)^(1/3)+x)/b+2/3*C*arctan(1/3*(1-2*x/(a/b)^(1/3))*3^(1/2))/b*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1881, 31, 631, 210}

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx=\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

```
[In] Int[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^(1
/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1881

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\text{integral} = \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}+x}\,dx}{b} - \frac{\left(\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}$$

$$= \frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b} - \frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}$$

$$= \frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} + \frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 146 vs. $2(50) = 100$.

Time = 0.05 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.92

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx = \frac{C\left(2\sqrt{3}\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\right.}{3\sqrt[3]{a}b}$$

```
[In] Integrate[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(a/b)^(1/3)*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + 2*(a/b)^(1/3)*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] - (a/b)^(1/3)*b^(1/3)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a + b*x^3]))/(3
*a^(1/3)*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(43) = 86$.

Time = 1.67 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.32

method	result	size
default	$C\left(-2\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+\frac{\ln\left(b\,x^3+a\right)}{3b}\right)$	116

```
[In] int(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-2*(a/b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1)))+1/3*ln(b*x^3+a)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx=-\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-3C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

```
[In] integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="fricas")
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) - 3
*C*log(x + (a/b)^(1/3)))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.00

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$$
$$=\frac{C\left(\log\left(\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

```
[In] integrate(x*(-2*(a/b)**(1/3)*C+C*x)/(b*x**3+a),x)
[Out] C*(log(a/(b*(a/b)**(2/3)) + x) + sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) - sqrt
(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx=-\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}+\frac{C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

```
[In] integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="maxima")
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/b + C*lo
g(x + (a/b)^(1/3))/b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.98

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx=\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{\left(Cb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-2\left(ab^2\right)^{\frac{1}{3}}C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}+\frac{\left(3ab^2-i\sqrt{3}\sqrt{a^2b^4}\right)C\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^3}$$

```
[In] integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1/3
*(C*b*(-a/b)^(2/3) - 2*(a*b^2)^(1/3)*C*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x
 - (-a/b)^(1/3)))/(a*b) + 1/6*(3*a*b^2 - I*sqrt(3)*sqrt(a^2*b^4))*C*log(x^2
 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 11.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.08

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$$
$$=\sum_{k=1}^{3}\ln\left(\frac{C^2\,a+\mathrm{root}(27\,a\,b^3\,z^3-27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z-9\,C^3\,a,z,k)^2\,a\,b^2\,9-C\,\mathrm{root}(27\,a\,b^3\,z^3-27\,C\,a\,b}{b^3}\right.$$
$$\left.-27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z-9\,C^3\,a,z,k\right)$$

```
[In] int((x*(C*x - 2*C*(a/b)^(1/3)))/(a + b*x^3),x)
```

```
[Out] symsum(log((C^2*a + 9*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b*z - 9*
C^3*a, z, k)^2*a*b^2 - 6*C*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b*z
 - 9*C^3*a, z, k)*a*b + 4*C^2*b*x*(a/b)^(2/3))/b^3)*root(27*a*b^3*z^3 - 27*
C*a*b^2*z^2 + 9*C^2*a*b*z - 9*C^3*a, z, k), k, 1, 3)
```

3.370 $\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 53

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx=-\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

```
[Out] -C*ln((-a/b)^(1/3)+x)/b-2/3*C*arctan(1/3*(1-2*x/(-a/b)^(1/3))*3^(1/2))/b*3^
(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1881, 31, 631, 210}

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx=-\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

```
[In] Int[(x*(-2*(-(a/b))^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(-(a
/b))^(1/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1881

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\text{integral} = -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}+x}\,dx}{b} + \frac{\left(\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}-\sqrt[3]{-\frac{a}{b}}x+x^2}\,dx}{b}$$

$$= -\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b} + \frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$

$$= -\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 149 vs. 2(53) = 106.

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.81

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx = -\frac{C\left(-2\sqrt{3}\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)+\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+\right.\right.}{3\sqrt[3]{a}b}$$

```
[In] Integrate[(x*(-2*(-(a/b))^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] -1/3*(C*(-2*Sqrt[3]*(-(a/b))^(1/3)*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]] - 2*(-(a/b))^(1/3)*b^(1/3)*Log[a^(1/3) - b^(1/3)*x] + (-(a/b))^
(1/3)*b^(1/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[
a - b*x^3]))/(a^(1/3)*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(46) = 92.

Time = 1.50 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.25

method	result	s
default	$C\left(-2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-\frac{\ln\left(-bx^3+a\right)}{3b}\right)$	1

```
[In] int(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-2*(-a/b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x-(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*
ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*(1+2
/(a/b)^(1/3)*x)*3^(1/2)))-1/3*ln(-b*x^3+a)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx=-\frac{2\,\sqrt{3}C\arctan\left(\frac{2\,\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right)+3\,C\log\left(x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

```
[In] integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="fricas")
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) + sqrt(3)*a)/a) +
3*C*log(x + (-a/b)^(1/3)))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.08

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx=-\frac{C\left(\log\left(-\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

```
[In] integrate(x*(-2*(-a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)
[Out] -C*(log(-a/(b*(-a/b)**(2/3)) + x) - sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3)) - s
qrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(46) = 92$.

Time = 0.35 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.13

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx = -\frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}+C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\sqrt{3}\left(Ca-\left(3\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$

```
[In] integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*(C*(a/b)^(1/3) + C*(-a/b)^(1/3))*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3)
)/(b*(a/b)^(1/3)) - 1/3*(C*(a/b)^(1/3) - 2*C*(-a/b)^(1/3))*log(x - (a/b)^(1
/3))/(b*(a/b)^(1/3)) - 2/9*sqrt(3)*(C*a - (3*C*(a/b)^(2/3)*(-a/b)^(1/3) + C
*a/b)*b)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.15

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx = -\frac{\left(Cb\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\left(-ab^2\right)^{\frac{1}{3}}C\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab} + \frac{\sqrt{3}\left(ab^2+i\sqrt{3}\sqrt{a^2b^4}\right)C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3}$$

```
[In] integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(C*b*(a/b)^(2/3) - 2*(-a*b^2)^(1/3)*C*(a/b)^(1/3))*(a/b)^(1/3)*log(abs
(x - (a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*(a*b^2 + I*sqrt(3)*sqrt(a^2*b^4))*C*
arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 11.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.94

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$$
$$=\sum_{k=1}^{3}\ln\left(-\frac{C^2\,a+\mathrm{root}(27\,a\,b^3\,z^3+27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z+9\,C^3\,a,z,k)^2\,a\,b^2\,9+C\,\mathrm{root}(27\,a\,b^3\,z^3+27\,C\,a}{b^3}\right.$$
$$\left.+27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z+9\,C^3\,a,z,k\right)$$

```
[In] int((x*(C*x - 2*C*(-a/b)^(1/3)))/(a - b*x^3),x)
```

```
[Out] symsum(log(-(C^2*a + 9*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*b*z + 9
*C^3*a, z, k)^2*a*b^2 + 6*C*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*b*
z + 9*C^3*a, z, k)*a*b - 4*C^2*b*x*(-a/b)^(2/3))/b^3)*root(27*a*b^3*z^3 + 2
7*C*a*b^2*z^2 + 9*C^2*a*b*z + 9*C^3*a, z, k), k, 1, 3)
```

3.371 $\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 54

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx=\frac{2C\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

```
[Out] C*ln((-a/b)^(1/3)-x)/b+2/3*C*arctan(1/3*(1+2*x/(-a/b)^(1/3))*3^(1/2))/b*3^(
1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1883, 31, 631, 210}

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx=\frac{2C\arctan\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

```
[In] Int[(x*(2*(-(a/b))^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(-(a/
b))^(1/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1883

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Dist[-C/b
, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]
] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &&
PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x}\,dx}{b} + \frac{\left(\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}+\sqrt[3]{-\frac{a}{b}}x+x^2}\,dx}{b}$$

$$= \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b} - \frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$

$$= \frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} + \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 148 vs. $2(54) = 108$.

Time = 0.05 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.74

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C + Cx\right)}{a+bx^3}\,dx$$

$$= \frac{C\left(-2\sqrt{3}\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) + \sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b\ldots\right.\right.}{3\sqrt[3]{a}b}$$

```
[In] Integrate[(x*(2*(-(a/b))^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*(-(a/b))^(1/3)*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]] - 2*(-(a/b))^(1/3)*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] + (-(a/b))^(1/3)
*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a + b
*x^3]))/(3*a^(1/3)*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(47) = 94$.

Time = 1.52 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.17

method	result	size
default	$C\left(2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+\frac{\ln\left(b\,x^3+a\right)}{3b}\right)$	117

```
[In] int(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(2*(-a/b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1)))+1/3*ln(b*x^3+a)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx = -\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-3C\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

```
[In] integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) -
3*C*log(x - (-a/b)^(1/3)))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.02

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx = \frac{C\left(\log\left(\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

```
[In] integrate(x*(2*(-a/b)**(1/3)*C+C*x)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(-a/b)**(2/3)) + x) + sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3)) - sq
rt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(47) = 94$.

Time = 0.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.09

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C + Cx\right)}{a + bx^3}\,dx = \frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}} + C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}} - 2\,C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\sqrt{3}\left(Ca - \left(3\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$

```
[In] integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(C*(a/b)^(1/3) + C*(-a/b)^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(b*(a/b)^(1/3)) + 1/3*(C*(a/b)^(1/3) - 2*C*(-a/b)^(1/3))*log(x + (a/b)^(1/
3))/(b*(a/b)^(1/3)) - 2/9*sqrt(3)*(C*a - (3*C*(a/b)^(2/3)*(-a/b)^(1/3) + C*
a/b)*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(47) = 94.

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.80

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C + Cx\right)}{a + bx^3}\,dx = \frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b} - \frac{\left(Cb\left(-\frac{a}{b}\right)^{\frac{2}{3}} + 2\left(-ab^2\right)^{\frac{1}{3}}C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

```
[In] integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1/3
*(C*b*(-a/b)^(2/3) + 2*(-a*b^2)^(1/3)*C*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(
x - (-a/b)^(1/3)))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 11.06 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.87

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$$
$$=\sum_{k=1}^{3}\ln\left(\frac{C^2\,a+\mathrm{root}(27\,a\,b^3\,z^3-27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z-9\,C^3\,a,z,k)^2\,a\,b^2\,9-C\,\mathrm{root}(27\,a\,b^3\,z^3-27\,C\,a\,b}{b^3}\right.$$
$$-27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z-9\,C^3\,a,z,k\Big)$$

```
[In] int((x*(C*x + 2*C*(-a/b)^(1/3)))/(a + b*x^3),x)
```

```
[Out] symsum(log((C^2*a + 9*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b*z - 9*
C^3*a, z, k)^2*a*b^2 - 6*C*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b*z
 - 9*C^3*a, z, k)*a*b + 4*C^2*b*x*(-a/b)^(2/3))/b^3)*root(27*a*b^3*z^3 - 27
*C*a*b^2*z^2 + 9*C^2*a*b*z - 9*C^3*a, z, k), k, 1, 3)
```

3.372 $\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 53

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx=-\frac{2C\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

```
[Out] -C*ln((a/b)^(1/3)-x)/b-2/3*C*arctan(1/3*(1+2*x/(a/b)^(1/3))*3^(1/2))/b*3^(1
/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1883, 31, 631, 210}

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx=-\frac{2C\arctan\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

```
[In] Int[(x*(2*(a/b)^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^(
1/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1883

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Dist[-C/b
, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]
] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &&
PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}-x}\,dx}{b} - \frac{\left(\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}+\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}\\
&= -\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b} + \frac{(2C)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}\\
&= -\frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 147 vs. $2(53) = 106$.

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.77

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C + Cx\right)}{a - bx^3}\,dx = -\frac{C\left(2\sqrt{3}\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\log\left(\sqrt[3]{a} - \sqrt[3]{b}x\right) - \sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\log\left(a^{2/3} + \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)\cdots\right)}{3\sqrt[3]{a}b}$$

```
[In] Integrate[(x*(2*(a/b)^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] -1/3*(C*(2*Sqrt[3]*(a/b)^(1/3)*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]] + 2*(a/b)^(1/3)*b^(1/3)*Log[a^(1/3) - b^(1/3)*x] - (a/b)^(1/3)*b^(1
/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a - b*x^3]
))/(a^(1/3)*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(46) = 92$.

Time = 1.52 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.23

method	result	size
default	$C\left(2\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}\arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - \frac{\ln(-bx^3+a)}{3b}\right)$	118

```
[In] int(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*(2*(a/b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x-(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln
(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*(1+2/(
a/b)^(1/3)*x)*3^(1/2)))-1/3*ln(-b*x^3+a)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx = -\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+3C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

```
[In] integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="fricas")
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) + sqrt(3)*a)/a) + 3
*C*log(x - (a/b)^(1/3)))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.92

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx = -\frac{C\left(\log\left(-\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

```
[In] integrate(x*(2*(a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)
[Out] -C*(log(-a/(b*(a/b)**(2/3)) + x) - sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) - sqr
t(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx = -\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

```
[In] integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="maxima")
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - C*lo
g(x - (a/b)^(1/3))/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx=-\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{\left(Cb\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\left(ab^2\right)^{\frac{1}{3}}C\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}$$

```
[In] integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - 1/3*
(C*b*(a/b)^(2/3) + 2*(a*b^2)^(1/3)*C*(a/b)^(1/3))*(a/b)^(1/3)*log(abs(x - (
a/b)^(1/3)))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 10.96 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.92

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$$
$$=\sum_{k=1}^{3}\ln\left(-\frac{C^2\,a+\mathrm{root}(27\,a\,b^3\,z^3+27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z+9\,C^3\,a,z,k)^2\,a\,b^2\,9+C\,\mathrm{root}(27\,a\,b^3\,z^3+27\,C}{b^3}\right.$$
$$\left.+27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z+9\,C^3\,a,z,k\right)$$

```
[In] int((x*(C*x + 2*C*(a/b)^(1/3)))/(a - b*x^3),x)
```

```
[Out] symsum(log(-(C^2*a + 9*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*b*z + 9
*C^3*a, z, k)^2*a*b^2 + 6*C*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*b*
z + 9*C^3*a, z, k)*a*b - 4*C^2*b*x*(a/b)^(2/3))/b^3)*root(27*a*b^3*z^3 + 27
*C*a*b^2*z^2 + 9*C^2*a*b*z + 9*C^3*a, z, k), k, 1, 3)
```

3.373 $\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal result

Integrand size = 36, antiderivative size = 97

$$\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{8}(bc+af)x^8+\frac{1}{9}(bd+ag)x^9 + \frac{1}{10}(be+ah)x^{10}+\frac{1}{11}bfx^{11}+\frac{1}{12}bgx^{12}+\frac{1}{13}bhx^{13}$$

```
[Out] 1/5*a*c*x^5+1/6*a*d*x^6+1/7*a*e*x^7+1/8*(a*f+b*c)*x^8+1/9*(a*g+b*d)*x^9+1/1
0*(a*h+b*e)*x^10+1/11*b*f*x^11+1/12*b*g*x^12+1/13*b*h*x^13
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1834}

$$\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{8}x^8(af+bc)+\frac{1}{9}x^9(ag+bd)+\frac{1}{10}x^{10}(ah+be)+\frac{1}{5}acx^5 + \frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{11}bfx^{11}+\frac{1}{12}bgx^{12}+\frac{1}{13}bhx^{13}$$

```
[In] Int[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + ((b*d + a*g
)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g*x^12)/12 + (b*h*x^1
3)/13
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(acx^4 + adx^5 + aex^6 + (bc+af)x^7 + (bd+ag)x^8 + (be+ah)x^9 + bfx^{10} \right. \\ &\qquad \left. + bgx^{11} + bhx^{12}\right) dx \\ &= \frac{1}{5}acx^5 + \frac{1}{6}adx^6 + \frac{1}{7}aex^7 + \frac{1}{8}(bc+af)x^8 + \frac{1}{9}(bd+ag)x^9 \\ &\quad + \frac{1}{10}(be+ah)x^{10} + \frac{1}{11}bfx^{11} + \frac{1}{12}bgx^{12} + \frac{1}{13}bhx^{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int x^4\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx \\ &= \frac{1}{5}acx^5 + \frac{1}{6}adx^6 + \frac{1}{7}aex^7 + \frac{1}{8}(bc+af)x^8 + \frac{1}{9}(bd+ag)x^9 \\ &\quad + \frac{1}{10}(be+ah)x^{10} + \frac{1}{11}bfx^{11} + \frac{1}{12}bgx^{12} + \frac{1}{13}bhx^{13}\end{aligned}$$

```
[In] Integrate[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
[Out] (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + ((b*d + a*g
)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g*x^12)/12 + (b*h*x^1
3)/13
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

method	result
default	$\frac{ac\,x^5}{5}+\frac{ad\,x^6}{6}+\frac{ae\,x^7}{7}+\frac{(af+bc)x^8}{8}+\frac{(ag+bd)x^9}{9}+\frac{(ah+be)x^{10}}{10}+\frac{bf\,x^{11}}{11}+\frac{bg\,x^{12}}{12}+\frac{bh\,x^{13}}{13}$
norman	$\frac{bh\,x^{13}}{13}+\frac{bg\,x^{12}}{12}+\frac{bf\,x^{11}}{11}+\left(\frac{ah}{10}+\frac{be}{10}\right)x^{10}+\left(\frac{ag}{9}+\frac{bd}{9}\right)x^9+\left(\frac{af}{8}+\frac{bc}{8}\right)x^8+\frac{ae\,x^7}{7}+\frac{ad\,x^6}{6}+\frac{ac\,x^5}{5}$
gosper	$\frac{1}{13}bh\,x^{13}+\frac{1}{12}bg\,x^{12}+\frac{1}{11}bf\,x^{11}+\frac{1}{10}x^{10}ah+\frac{1}{10}be\,x^{10}+\frac{1}{9}x^9ag+\frac{1}{9}bd\,x^9+\frac{1}{8}x^8af+\frac{1}{8}bc\,x^8+$
risch	$\frac{1}{13}bh\,x^{13}+\frac{1}{12}bg\,x^{12}+\frac{1}{11}bf\,x^{11}+\frac{1}{10}x^{10}ah+\frac{1}{10}be\,x^{10}+\frac{1}{9}x^9ag+\frac{1}{9}bd\,x^9+\frac{1}{8}x^8af+\frac{1}{8}bc\,x^8+$
parallelrisch	$\frac{1}{13}bh\,x^{13}+\frac{1}{12}bg\,x^{12}+\frac{1}{11}bf\,x^{11}+\frac{1}{10}x^{10}ah+\frac{1}{10}be\,x^{10}+\frac{1}{9}x^9ag+\frac{1}{9}bd\,x^9+\frac{1}{8}x^8af+\frac{1}{8}bc\,x^8+$

```
[In] int(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*c*x^5+1/6*a*d*x^6+1/7*a*e*x^7+1/8*(a*f+b*c)*x^8+1/9*(a*g+b*d)*x^9+1/1
0*(a*h+b*e)*x^10+1/11*b*f*x^11+1/12*b*g*x^12+1/13*b*h*x^13
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{13}\,bhx^{13}+\frac{1}{12}\,bgx^{12}+\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,(be+ah)x^{10} + \frac{1}{9}\,(bd+ag)x^9+\frac{1}{7}\,aex^7+\frac{1}{8}\,(bc+af)x^8+\frac{1}{6}\,adx^6+\frac{1}{5}\,acx^5$$

```
[In] integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*(b*e + a*h)*x^10 + 1/9
*(b*d + a*g)*x^9 + 1/7*a*e*x^7 + 1/8*(b*c + a*f)*x^8 + 1/6*a*d*x^6 + 1/5*a*
c*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{acx^5}{5}+\frac{adx^6}{6}+\frac{aex^7}{7}+\frac{bfx^{11}}{11}+\frac{bgx^{12}}{12}+\frac{bhx^{13}}{13} + x^{10}\left(\frac{ah}{10}+\frac{be}{10}\right)+x^9\left(\frac{ag}{9}+\frac{bd}{9}\right)+x^8\left(\frac{af}{8}+\frac{bc}{8}\right)$$

```
[In] integrate(x**4*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**5/5 + a*d*x**6/6 + a*e*x**7/7 + b*f*x**11/11 + b*g*x**12/12 + b*h*x*
*13/13 + x**10*(a*h/10 + b*e/10) + x**9*(a*g/9 + b*d/9) + x**8*(a*f/8 + b*c
/8)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{13}bhx^{13}+\frac{1}{12}bgx^{12}+\frac{1}{11}bfx^{11}+\frac{1}{10}(be+ah)x^{10}+\frac{1}{9}(bd+ag)x^9+\frac{1}{7}aex^7+\frac{1}{8}(bc+af)x^8+\frac{1}{6}adx^6+\frac{1}{5}acx^5$$

```
[In] integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*(b*e + a*h)*x^10 + 1/9
*(b*d + a*g)*x^9 + 1/7*a*e*x^7 + 1/8*(b*c + a*f)*x^8 + 1/6*a*d*x^6 + 1/5*a*
c*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{13}bhx^{13}+\frac{1}{12}bgx^{12}+\frac{1}{11}bfx^{11}+\frac{1}{10}bex^{10}+\frac{1}{10}ahx^{10}+\frac{1}{9}bdx^9+\frac{1}{9}agx^9+\frac{1}{8}bcx^8+\frac{1}{8}afx^8+\frac{1}{7}aex^7+\frac{1}{6}adx^6+\frac{1}{5}acx^5$$

```
[In] integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/10*a*h*x^
10 + 1/9*b*d*x^9 + 1/9*a*g*x^9 + 1/8*b*c*x^8 + 1/8*a*f*x^8 + 1/7*a*e*x^7 +
1/6*a*d*x^6 + 1/5*a*c*x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{b\,h\,x^{13}}{13}+\frac{b\,g\,x^{12}}{12}+\frac{b\,f\,x^{11}}{11}+\left(\frac{b\,e}{10}+\frac{a\,h}{10}\right)\,x^{10}+\left(\frac{b\,d}{9}+\frac{a\,g}{9}\right)\,x^9+\left(\frac{b\,c}{8}+\frac{a\,f}{8}\right)\,x^8+\frac{a\,e\,x^7}{7}+\frac{a\,d\,x^6}{6}+\frac{a\,c\,x^5}{5}$$

```
[In] int(x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^8*((b*c)/8 + (a*f)/8) + x^9*((b*d)/9 + (a*g)/9) + x^10*((b*e)/10 + (a*h)/
10) + (b*h*x^13)/13 + (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + (b*f*x^11)/
11 + (b*g*x^12)/12
```

3.374 $\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal result

Integrand size = 36, antiderivative size = 97

$$\begin{aligned}\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\ &=\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}(bc+af)x^7+\frac{1}{8}(bd+ag)x^8\\ &\quad+\frac{1}{9}(be+ah)x^9+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}\end{aligned}$$

```
[Out] 1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*(a*f+b*c)*x^7+1/8*(a*g+b*d)*x^8+1/9
*(a*h+b*e)*x^9+1/10*b*f*x^10+1/11*b*g*x^11+1/12*b*h*x^12
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1834}

$$\begin{aligned}\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\ &=\frac{1}{7}x^7(af+bc)+\frac{1}{8}x^8(ag+bd)+\frac{1}{9}x^9(ah+be)+\frac{1}{4}acx^4\\ &\quad+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}\end{aligned}$$

```
[In] Int[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + ((b*d + a*g
)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x^11)/11 + (b*h*x^12)
/12
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \int \left(acx^3 + adx^4 + aex^5 + (bc+af)x^6 + (bd+ag)x^7 + (be+ah)x^8 + bfx^9 + bgx^{10} + bhx^{11}\right) dx$$
$$= \frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}(bc+af)x^7 + \frac{1}{8}(bd+ag)x^8 + \frac{1}{9}(be+ah)x^9 + \frac{1}{10}bfx^{10} + \frac{1}{11}bgx^{11} + \frac{1}{12}bhx^{12}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx = \frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}(bc+af)x^7 + \frac{1}{8}(bd+ag)x^8 + \frac{1}{9}(be+ah)x^9 + \frac{1}{10}bfx^{10} + \frac{1}{11}bgx^{11} + \frac{1}{12}bhx^{12}$$

```
[In] Integrate[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + ((b*d + a*g
)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x^11)/11 + (b*h*x^12)
/12
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

method	result
default	$\frac{a\,x^4c}{4}+\frac{ad\,x^5}{5}+\frac{ae\,x^6}{6}+\frac{(af+bc)x^7}{7}+\frac{(ag+bd)x^8}{8}+\frac{(ah+be)x^9}{9}+\frac{bf\,x^{10}}{10}+\frac{bg\,x^{11}}{11}+\frac{bh\,x^{12}}{12}$
norman	$\frac{bh\,x^{12}}{12}+\frac{bg\,x^{11}}{11}+\frac{bf\,x^{10}}{10}+\left(\frac{ah}{9}+\frac{be}{9}\right)x^9+\left(\frac{ag}{8}+\frac{bd}{8}\right)x^8+\left(\frac{af}{7}+\frac{bc}{7}\right)x^7+\frac{ae\,x^6}{6}+\frac{ad\,x^5}{5}+\frac{a\,x^4c}{4}$
gosper	$\frac{1}{12}bh\,x^{12}+\frac{1}{11}bg\,x^{11}+\frac{1}{10}bf\,x^{10}+\frac{1}{9}x^9ah+\frac{1}{9}x^9be+\frac{1}{8}x^8ag+\frac{1}{8}bd\,x^8+\frac{1}{7}af\,x^7+\frac{1}{7}b\,x^7c+\frac{1}{6}ae\,x$
risch	$\frac{1}{12}bh\,x^{12}+\frac{1}{11}bg\,x^{11}+\frac{1}{10}bf\,x^{10}+\frac{1}{9}x^9ah+\frac{1}{9}x^9be+\frac{1}{8}x^8ag+\frac{1}{8}bd\,x^8+\frac{1}{7}af\,x^7+\frac{1}{7}b\,x^7c+\frac{1}{6}ae\,x$
parallelrisch	$\frac{1}{12}bh\,x^{12}+\frac{1}{11}bg\,x^{11}+\frac{1}{10}bf\,x^{10}+\frac{1}{9}x^9ah+\frac{1}{9}x^9be+\frac{1}{8}x^8ag+\frac{1}{8}bd\,x^8+\frac{1}{7}af\,x^7+\frac{1}{7}b\,x^7c+\frac{1}{6}ae\,x$

```
[In] int(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x^4*c+1/5*a*d*x^5+1/6*a*e*x^6+1/7*(a*f+b*c)*x^7+1/8*(a*g+b*d)*x^8+1/9
*(a*h+b*e)*x^9+1/10*b*f*x^10+1/11*b*g*x^11+1/12*b*h*x^12
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\begin{aligned}\int x^3\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{1}{12}\,bhx^{12}+\frac{1}{11}\,bgx^{11}+\frac{1}{10}\,bfx^{10}+\frac{1}{9}\,(be+ah)x^9\\ &\quad+\frac{1}{8}\,(bd+ag)x^8+\frac{1}{6}\,aex^6+\frac{1}{7}\,(bc+af)x^7+\frac{1}{5}\,adx^5+\frac{1}{4}\,acx^4\end{aligned}$$

```
[In] integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*(b*e + a*h)*x^9 + 1/8*(
b*d + a*g)*x^8 + 1/6*a*e*x^6 + 1/7*(b*c + a*f)*x^7 + 1/5*a*d*x^5 + 1/4*a*c*
x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\begin{aligned}\int x^3\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{acx^4}{4}+\frac{adx^5}{5}+\frac{aex^6}{6}+\frac{bfx^{10}}{10}+\frac{bgx^{11}}{11}+\frac{bhx^{12}}{12}\\ &\quad+x^9\left(\frac{ah}{9}+\frac{be}{9}\right)+x^8\left(\frac{ag}{8}+\frac{bd}{8}\right)+x^7\left(\frac{af}{7}+\frac{bc}{7}\right)\end{aligned}$$

```
[In] integrate(x**3*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + b*f*x**10/10 + b*g*x**11/11 + b*h*x*
*12/12 + x**9*(a*h/9 + b*e/9) + x**8*(a*g/8 + b*d/8) + x**7*(a*f/7 + b*c/7)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{12}\,bhx^{12}+\frac{1}{11}\,bgx^{11}+\frac{1}{10}\,bfx^{10}+\frac{1}{9}\,(be+ah)x^9$$
$$+\frac{1}{8}\,(bd+ag)x^8+\frac{1}{6}\,aex^6+\frac{1}{7}\,(bc+af)x^7+\frac{1}{5}\,adx^5+\frac{1}{4}\,acx^4$$

```
[In] integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*(b*e + a*h)*x^9 + 1/8*(
b*d + a*g)*x^8 + 1/6*a*e*x^6 + 1/7*(b*c + a*f)*x^7 + 1/5*a*d*x^5 + 1/4*a*c*
x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{12}\,bhx^{12}+\frac{1}{11}\,bgx^{11}+\frac{1}{10}\,bfx^{10}+\frac{1}{9}\,bex^9+\frac{1}{9}\,ahx^9+\frac{1}{8}\,bdx^8$$
$$+\frac{1}{8}\,agx^8+\frac{1}{7}\,bcx^7+\frac{1}{7}\,afx^7+\frac{1}{6}\,aex^6+\frac{1}{5}\,adx^5+\frac{1}{4}\,acx^4$$

```
[In] integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*b*e*x^9 + 1/9*a*h*x^9 +
 1/8*b*d*x^8 + 1/8*a*g*x^8 + 1/7*b*c*x^7 + 1/7*a*f*x^7 + 1/6*a*e*x^6 + 1/5*
a*d*x^5 + 1/4*a*c*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{b\,h\,x^{12}}{12}+\frac{b\,g\,x^{11}}{11}+\frac{b\,f\,x^{10}}{10}+\left(\frac{b\,e}{9}+\frac{a\,h}{9}\right)\,x^9+\left(\frac{b\,d}{8}+\frac{a\,g}{8}\right)\,x^8+\left(\frac{b\,c}{7}+\frac{a\,f}{7}\right)\,x^7+\frac{a\,e\,x^6}{6}+\frac{a\,d\,x^5}{5}+\frac{a\,c\,x^4}{4}$$

```
[In] int(x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^7*((b*c)/7 + (a*f)/7) + x^8*((b*d)/8 + (a*g)/8) + x^9*((b*e)/9 + (a*h)/9)
 + (b*h*x^12)/12 + (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (b*f*x^10)/10
+ (b*g*x^11)/11
```

3.375 $\int x^2(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal result

Integrand size = 36, antiderivative size = 97

$$\begin{aligned}\int x^2\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\ &= \frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}(bc+af)x^6+\frac{1}{7}(bd+ag)x^7\\ &\quad+\frac{1}{8}(be+ah)x^8+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}\end{aligned}$$

```
[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*(a*f+b*c)*x^6+1/7*(a*g+b*d)*x^7+1/8
*(a*h+b*e)*x^8+1/9*b*f*x^9+1/10*b*g*x^10+1/11*b*h*x^11
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1834}

$$\begin{aligned}\int x^2\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\ &= \frac{1}{6}x^6(af+bc)+\frac{1}{7}x^7(ag+bd)+\frac{1}{8}x^8(ah+be)+\frac{1}{3}acx^3\\ &\quad+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}\end{aligned}$$

```
[In] Int[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + ((b*d + a*g
)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^10)/10 + (b*h*x^11)/1
1
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(acx^2 + adx^3 + aex^4 + (bc+af)x^5 + (bd+ag)x^6 + (be+ah)x^7 + bfx^8 + bgx^9 + bhx^{10}\right) dx \\ &= \frac{1}{3}acx^3 + \frac{1}{4}adx^4 + \frac{1}{5}aex^5 + \frac{1}{6}(bc+af)x^6 + \frac{1}{7}(bd+ag)x^7 + \frac{1}{8}(be+ah)x^8 + \frac{1}{9}bfx^9 + \frac{1}{10}bgx^{10} + \frac{1}{11}bhx^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx = \frac{1}{3}acx^3 + \frac{1}{4}adx^4 + \frac{1}{5}aex^5 + \frac{1}{6}(bc+af)x^6 + \frac{1}{7}(bd+ag)x^7 + \frac{1}{8}(be+ah)x^8 + \frac{1}{9}bfx^9 + \frac{1}{10}bgx^{10} + \frac{1}{11}bhx^{11}$$

```
[In] Integrate[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + ((b*d + a*g
)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^10)/10 + (b*h*x^11)/1
1
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

method	result
default	$\frac{ac\,x^3}{3}+\frac{ad\,x^4}{4}+\frac{ae\,x^5}{5}+\frac{(af+bc)x^6}{6}+\frac{(ag+bd)x^7}{7}+\frac{(ah+be)x^8}{8}+\frac{bf\,x^9}{9}+\frac{bg\,x^{10}}{10}+\frac{bh\,x^{11}}{11}$
norman	$\frac{bh\,x^{11}}{11}+\frac{bg\,x^{10}}{10}+\frac{bf\,x^9}{9}+\left(\frac{ah}{8}+\frac{be}{8}\right)x^8+\left(\frac{ag}{7}+\frac{bd}{7}\right)x^7+\left(\frac{af}{6}+\frac{bc}{6}\right)x^6+\frac{ae\,x^5}{5}+\frac{ad\,x^4}{4}+\frac{ac\,x^3}{3}$
gosper	$\frac{1}{11}bh\,x^{11}+\frac{1}{10}bg\,x^{10}+\frac{1}{9}bf\,x^9+\frac{1}{8}x^8ah+\frac{1}{8}be\,x^8+\frac{1}{7}x^7ag+\frac{1}{7}bd\,x^7+\frac{1}{6}x^6af+\frac{1}{6}bc\,x^6+\frac{1}{5}ae\,x$
risch	$\frac{1}{11}bh\,x^{11}+\frac{1}{10}bg\,x^{10}+\frac{1}{9}bf\,x^9+\frac{1}{8}x^8ah+\frac{1}{8}be\,x^8+\frac{1}{7}x^7ag+\frac{1}{7}bd\,x^7+\frac{1}{6}x^6af+\frac{1}{6}bc\,x^6+\frac{1}{5}ae\,x$
parallelrisch	$\frac{1}{11}bh\,x^{11}+\frac{1}{10}bg\,x^{10}+\frac{1}{9}bf\,x^9+\frac{1}{8}x^8ah+\frac{1}{8}be\,x^8+\frac{1}{7}x^7ag+\frac{1}{7}bd\,x^7+\frac{1}{6}x^6af+\frac{1}{6}bc\,x^6+\frac{1}{5}ae\,x$

```
[In] int(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*(a*f+b*c)*x^6+1/7*(a*g+b*d)*x^7+1/8
*(a*h+b*e)*x^8+1/9*b*f*x^9+1/10*b*g*x^10+1/11*b*h*x^11
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int x^2(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{11}\,bhx^{11}+\frac{1}{10}\,bgx^{10}+\frac{1}{9}\,bfx^9+\frac{1}{8}\,(be+ah)x^8+\frac{1}{7}\,(bd+ag)x^7+\frac{1}{5}\,aex^5+\frac{1}{6}\,(bc+af)x^6+\frac{1}{4}\,adx^4+\frac{1}{3}\,acx^3$$

```
[In] integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*(b*e + a*h)*x^8 + 1/7*(b*
d + a*g)*x^7 + 1/5*a*e*x^5 + 1/6*(b*c + a*f)*x^6 + 1/4*a*d*x^4 + 1/3*a*c*x^
3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{acx^3}{3}+\frac{adx^4}{4}+\frac{aex^5}{5}+\frac{bfx^9}{9}+\frac{bgx^{10}}{10}+\frac{bhx^{11}}{11}+x^8\left(\frac{ah}{8}+\frac{be}{8}\right)+x^7\left(\frac{ag}{7}+\frac{bd}{7}\right)+x^6\left(\frac{af}{6}+\frac{bc}{6}\right)$$

```
[In] integrate(x**2*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*f*x**9/9 + b*g*x**10/10 + b*h*x**1
1/11 + x**8*(a*h/8 + b*e/8) + x**7*(a*g/7 + b*d/7) + x**6*(a*f/6 + b*c/6)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\begin{aligned}\int x^2(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{1}{11}\,bhx^{11}+\frac{1}{10}\,bgx^{10}+\frac{1}{9}\,bfx^9+\frac{1}{8}\,(be+ah)x^8\\ &\quad+\frac{1}{7}\,(bd+ag)x^7+\frac{1}{5}\,aex^5+\frac{1}{6}\,(bc+af)x^6+\frac{1}{4}\,adx^4+\frac{1}{3}\,acx^3\end{aligned}$$

```
[In] integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*(b*e + a*h)*x^8 + 1/7*(b*
d + a*g)*x^7 + 1/5*a*e*x^5 + 1/6*(b*c + a*f)*x^6 + 1/4*a*d*x^4 + 1/3*a*c*x^
3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\begin{aligned}\int x^2(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{1}{11}\,bhx^{11}+\frac{1}{10}\,bgx^{10}+\frac{1}{9}\,bfx^9+\frac{1}{8}\,bex^8+\frac{1}{8}\,ahx^8+\frac{1}{7}\,bdx^7\\ &\quad+\frac{1}{7}\,agx^7+\frac{1}{6}\,bcx^6+\frac{1}{6}\,afx^6+\frac{1}{5}\,aex^5+\frac{1}{4}\,adx^4+\frac{1}{3}\,acx^3\end{aligned}$$

```
[In] integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*b*e*x^8 + 1/8*a*h*x^8 + 1
/7*b*d*x^7 + 1/7*a*g*x^7 + 1/6*b*c*x^6 + 1/6*a*f*x^6 + 1/5*a*e*x^5 + 1/4*a*
d*x^4 + 1/3*a*c*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int x^2(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{b\,h\,x^{11}}{11}+\frac{b\,g\,x^{10}}{10}+\frac{b\,f\,x^9}{9}+\left(\frac{b\,e}{8}+\frac{a\,h}{8}\right)x^8+\left(\frac{b\,d}{7}+\frac{a\,g}{7}\right)x^7$$
$$+\left(\frac{b\,c}{6}+\frac{a\,f}{6}\right)x^6+\frac{a\,e\,x^5}{5}+\frac{a\,d\,x^4}{4}+\frac{a\,c\,x^3}{3}$$

```
[In] int(x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^6*((b*c)/6 + (a*f)/6) + x^7*((b*d)/7 + (a*g)/7) + x^8*((b*e)/8 + (a*h)/8)
 + (b*h*x^11)/11 + (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*f*x^9)/9 +
(b*g*x^10)/10
```

3.376 $\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 97

$$\int x\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}(bc+af)x^5+\frac{1}{6}(bd+ag)x^6 + \frac{1}{7}(be+ah)x^7+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}$$

```
[Out] 1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*(a*f+b*c)*x^5+1/6*(a*g+b*d)*x^6+1/7
*(a*h+b*e)*x^7+1/8*b*f*x^8+1/9*b*g*x^9+1/10*b*h*x^10
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1834}

$$\int x\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{5}x^5(af+bc)+\frac{1}{6}x^6(ag+bd)+\frac{1}{7}x^7(ah+be)+\frac{1}{2}acx^2 + \frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}$$

```
[In] Int[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + ((b*d + a*g
)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9 + (b*h*x^10)/10
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(acx+adx^2+aex^3+(bc+af)x^4+(bd+ag)x^5+(be+ah)x^6+bfx^7+bgx^8+bhx^9\right)\,dx\\ &= \frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}(bc+af)x^5+\frac{1}{6}(bd+ag)x^6\\ &\quad+\frac{1}{7}(be+ah)x^7+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int x\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\ &= \frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}(bc+af)x^5+\frac{1}{6}(bd+ag)x^6\\ &\quad+\frac{1}{7}(be+ah)x^7+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}\end{aligned}$$

```
[In] Integrate[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + ((b*d + a*g
)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9 + (b*h*x^10)/10
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

method	result
default	$\frac{ac\,x^{2}}{2}+\frac{ad\,x^{3}}{3}+\frac{ae\,x^{4}}{4}+\frac{(af+bc)x^{5}}{5}+\frac{(ag+bd)x^{6}}{6}+\frac{(ah+be)x^{7}}{7}+\frac{bf\,x^{8}}{8}+\frac{bg\,x^{9}}{9}+\frac{bh\,x^{10}}{10}$
norman	$\frac{bh\,x^{10}}{10}+\frac{bg\,x^{9}}{9}+\frac{bf\,x^{8}}{8}+\left(\frac{ah}{7}+\frac{be}{7}\right)x^{7}+\left(\frac{ag}{6}+\frac{bd}{6}\right)x^{6}+\left(\frac{af}{5}+\frac{bc}{5}\right)x^{5}+\frac{ae\,x^{4}}{4}+\frac{ad\,x^{3}}{3}+\frac{ac\,x^{2}}{2}$
gosper	$\frac{1}{10}bh\,x^{10}+\frac{1}{9}bg\,x^{9}+\frac{1}{8}bf\,x^{8}+\frac{1}{7}x^{7}ah+\frac{1}{7}be\,x^{7}+\frac{1}{6}x^{6}ag+\frac{1}{6}bd\,x^{6}+\frac{1}{5}x^{5}af+\frac{1}{5}bc\,x^{5}+\frac{1}{4}ae\,x^{4}+$
risch	$\frac{1}{10}bh\,x^{10}+\frac{1}{9}bg\,x^{9}+\frac{1}{8}bf\,x^{8}+\frac{1}{7}x^{7}ah+\frac{1}{7}be\,x^{7}+\frac{1}{6}x^{6}ag+\frac{1}{6}bd\,x^{6}+\frac{1}{5}x^{5}af+\frac{1}{5}bc\,x^{5}+\frac{1}{4}ae\,x^{4}+$
parallelrisch	$\frac{1}{10}bh\,x^{10}+\frac{1}{9}bg\,x^{9}+\frac{1}{8}bf\,x^{8}+\frac{1}{7}x^{7}ah+\frac{1}{7}be\,x^{7}+\frac{1}{6}x^{6}ag+\frac{1}{6}bd\,x^{6}+\frac{1}{5}x^{5}af+\frac{1}{5}bc\,x^{5}+\frac{1}{4}ae\,x^{4}+$

```
[In] int(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*(a*f+b*c)*x^5+1/6*(a*g+b*d)*x^6+1/7
*(a*h+b*e)*x^7+1/8*b*f*x^8+1/9*b*g*x^9+1/10*b*h*x^10
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{10}\,bhx^{10}+\frac{1}{9}\,bgx^9+\frac{1}{8}\,bfx^8+\frac{1}{7}\,(be+ah)x^7+\frac{1}{6}\,(bd+ag)x^6 + \frac{1}{4}\,aex^4+\frac{1}{5}\,(bc+af)x^5+\frac{1}{3}\,adx^3+\frac{1}{2}\,acx^2$$

```
[In] integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] 1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*(b*e + a*h)*x^7 + 1/6*(b*d
+ a*g)*x^6 + 1/4*a*e*x^4 + 1/5*(b*c + a*f)*x^5 + 1/3*a*d*x^3 + 1/2*a*c*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{acx^2}{2}+\frac{adx^3}{3}+\frac{aex^4}{4}+\frac{bfx^8}{8}+\frac{bgx^9}{9}+\frac{bhx^{10}}{10} + x^7\left(\frac{ah}{7}+\frac{be}{7}\right)+x^6\left(\frac{ag}{6}+\frac{bd}{6}\right)+x^5\left(\frac{af}{5}+\frac{bc}{5}\right)$$

```
[In] integrate(x*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**2/2 + a*d*x**3/3 + a*e*x**4/4 + b*f*x**8/8 + b*g*x**9/9 + b*h*x**10/
10 + x**7*(a*h/7 + b*e/7) + x**6*(a*g/6 + b*d/6) + x**5*(a*f/5 + b*c/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{10}\,bhx^{10}+\frac{1}{9}\,bgx^9+\frac{1}{8}\,bfx^8+\frac{1}{7}\,(be+ah)x^7+\frac{1}{6}\,(bd+ag)x^6$$
$$+\frac{1}{4}\,aex^4+\frac{1}{5}\,(bc+af)x^5+\frac{1}{3}\,adx^3+\frac{1}{2}\,acx^2$$

```
[In] integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
```

```
[Out] 1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*(b*e + a*h)*x^7 + 1/6*(b*d
+ a*g)*x^6 + 1/4*a*e*x^4 + 1/5*(b*c + a*f)*x^5 + 1/3*a*d*x^3 + 1/2*a*c*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{10}\,bhx^{10}+\frac{1}{9}\,bgx^9+\frac{1}{8}\,bfx^8+\frac{1}{7}\,bex^7+\frac{1}{7}\,ahx^7+\frac{1}{6}\,bdx^6$$
$$+\frac{1}{6}\,agx^6+\frac{1}{5}\,bcx^5+\frac{1}{5}\,afx^5+\frac{1}{4}\,aex^4+\frac{1}{3}\,adx^3+\frac{1}{2}\,acx^2$$

```
[In] integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/7*a*h*x^7 + 1/6
*b*d*x^6 + 1/6*a*g*x^6 + 1/5*b*c*x^5 + 1/5*a*f*x^5 + 1/4*a*e*x^4 + 1/3*a*d*
x^3 + 1/2*a*c*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{b\,h\,x^{10}}{10}+\frac{b\,g\,x^9}{9}+\frac{b\,f\,x^8}{8}+\left(\frac{b\,e}{7}+\frac{a\,h}{7}\right)\,x^7+\left(\frac{b\,d}{6}+\frac{a\,g}{6}\right)\,x^6$$
$$+\left(\frac{b\,c}{5}+\frac{a\,f}{5}\right)\,x^5+\frac{a\,e\,x^4}{4}+\frac{a\,d\,x^3}{3}+\frac{a\,c\,x^2}{2}$$

```
[In] int(x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^5*((b*c)/5 + (a*f)/5) + x^6*((b*d)/6 + (a*g)/6) + x^7*((b*e)/7 + (a*h)/7)
 + (b*h*x^10)/10 + (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*f*x^8)/8 +
(b*g*x^9)/9
```

3.377 $\int (a+bx^3) (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx$

Optimal result

Integrand size = 33, antiderivative size = 92

$$\int (a+bx^3) (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx = acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}(bc+af)x^4+\frac{1}{5}(bd+ag)x^5 +\frac{1}{6}(be+ah)x^6+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*(a*f+b*c)*x^4+1/5*(a*g+b*d)*x^5+1/6*(a*h+
b*e)*x^6+1/7*b*f*x^7+1/8*b*g*x^8+1/9*b*h*x^9
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {1864}

$$\int (a+bx^3) (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx = \frac{1}{4}x^4(af+bc)+\frac{1}{5}x^5(ag+bd)+\frac{1}{6}x^6(ah+be)+acx +\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

```
[In] Int[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d + a*g)*x^5)
/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 + (b*h*x^9)/9
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ac+adx+aex^2+(bc+af)x^3+(bd+ag)x^4+(be+ah)x^5+bfx^6+bgx^7+bhx^8\right)\,dx\\ &= acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}(bc+af)x^4+\frac{1}{5}(bd+ag)x^5+\frac{1}{6}(be+ah)x^6+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8\\ &\quad+\frac{1}{9}bhx^9\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int \left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\ &=acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}(bc+af)x^4+\frac{1}{5}(bd+ag)x^5\\ &\quad+\frac{1}{6}(be+ah)x^6+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9\end{aligned}$$

```
[In] Integrate[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d + a*g)*x^5)
/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 + (b*h*x^9)/9
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

method	result
default	$acx+\frac{ad\,x^2}{2}+\frac{ae\,x^3}{3}+\frac{(af+bc)x^4}{4}+\frac{(ag+bd)x^5}{5}+\frac{(ah+be)x^6}{6}+\frac{bf\,x^7}{7}+\frac{bg\,x^8}{8}+\frac{bh\,x^9}{9}$
norman	$\frac{bh\,x^9}{9}+\frac{bg\,x^8}{8}+\frac{bf\,x^7}{7}+\left(\frac{ah}{6}+\frac{be}{6}\right)x^6+\left(\frac{ag}{5}+\frac{bd}{5}\right)x^5+\left(\frac{af}{4}+\frac{bc}{4}\right)x^4+\frac{ae\,x^3}{3}+\frac{ad\,x^2}{2}+acx$
gosper	$\frac{1}{9}bh\,x^9+\frac{1}{8}bg\,x^8+\frac{1}{7}bf\,x^7+\frac{1}{6}x^6ah+\frac{1}{6}be\,x^6+\frac{1}{5}x^5ag+\frac{1}{5}bd\,x^5+\frac{1}{4}af\,x^4+\frac{1}{4}bc\,x^4+\frac{1}{3}ae\,x^3+$
risch	$\frac{1}{9}bh\,x^9+\frac{1}{8}bg\,x^8+\frac{1}{7}bf\,x^7+\frac{1}{6}x^6ah+\frac{1}{6}be\,x^6+\frac{1}{5}x^5ag+\frac{1}{5}bd\,x^5+\frac{1}{4}af\,x^4+\frac{1}{4}bc\,x^4+\frac{1}{3}ae\,x^3+$
parallelrisch	$\frac{1}{9}bh\,x^9+\frac{1}{8}bg\,x^8+\frac{1}{7}bf\,x^7+\frac{1}{6}x^6ah+\frac{1}{6}be\,x^6+\frac{1}{5}x^5ag+\frac{1}{5}bd\,x^5+\frac{1}{4}af\,x^4+\frac{1}{4}bc\,x^4+\frac{1}{3}ae\,x^3+$

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*(a*f+b*c)*x^4+1/5*(a*g+b*d)*x^5+1/6*(a*h+
b*e)*x^6+1/7*b*f*x^7+1/8*b*g*x^8+1/9*b*h*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\begin{aligned}\int (a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{1}{9}\,bhx^9+\frac{1}{8}\,bgx^8+\frac{1}{7}\,bfx^7+\frac{1}{6}\,(be+ah)x^6\\ &\quad+\frac{1}{5}\,(bd+ag)x^5+\frac{1}{3}\,aex^3+\frac{1}{4}\,(bc+af)x^4+\frac{1}{2}\,adx^2+acx\end{aligned}$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] 1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*(b*e + a*h)*x^6 + 1/5*(b*d +
a*g)*x^5 + 1/3*a*e*x^3 + 1/4*(b*c + a*f)*x^4 + 1/2*a*d*x^2 + a*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\begin{aligned}&\int (a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\ &= acx+\frac{adx^2}{2}+\frac{aex^3}{3}+\frac{bfx^7}{7}+\frac{bgx^8}{8}+\frac{bhx^9}{9}+x^6\left(\frac{ah}{6}+\frac{be}{6}\right)+x^5\left(\frac{ag}{5}+\frac{bd}{5}\right)+x^4\left(\frac{af}{4}+\frac{bc}{4}\right)\end{aligned}$$

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x + a*d*x**2/2 + a*e*x**3/3 + b*f*x**7/7 + b*g*x**8/8 + b*h*x**9/9 + x*
*6*(a*h/6 + b*e/6) + x**5*(a*g/5 + b*d/5) + x**4*(a*f/4 + b*c/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\begin{aligned}\int (a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{1}{9}\,bhx^9+\frac{1}{8}\,bgx^8+\frac{1}{7}\,bfx^7+\frac{1}{6}\,(be+ah)x^6\\ &\quad+\frac{1}{5}\,(bd+ag)x^5+\frac{1}{3}\,aex^3+\frac{1}{4}\,(bc+af)x^4+\frac{1}{2}\,adx^2+acx\end{aligned}$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] 1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*(b*e + a*h)*x^6 + 1/5*(b*d +
a*g)*x^5 + 1/3*a*e*x^3 + 1/4*(b*c + a*f)*x^4 + 1/2*a*d*x^2 + a*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int (a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{9}bhx^9+\frac{1}{8}bgx^8+\frac{1}{7}bfx^7+\frac{1}{6}bex^6+\frac{1}{6}ahx^6+\frac{1}{5}bdx^5 + \frac{1}{5}agx^5+\frac{1}{4}bcx^4+\frac{1}{4}afx^4+\frac{1}{3}aex^3+\frac{1}{2}adx^2+acx$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*b*e*x^6 + 1/6*a*h*x^6 + 1/5*b
*d*x^5 + 1/5*a*g*x^5 + 1/4*b*c*x^4 + 1/4*a*f*x^4 + 1/3*a*e*x^3 + 1/2*a*d*x^
2 + a*c*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.86

$$\int (a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{b\,h\,x^9}{9}+\frac{b\,g\,x^8}{8}+\frac{b\,f\,x^7}{7}+\left(\frac{b\,e}{6}+\frac{a\,h}{6}\right)\,x^6+\left(\frac{b\,d}{5}+\frac{a\,g}{5}\right)\,x^5 + \left(\frac{b\,c}{4}+\frac{a\,f}{4}\right)\,x^4+\frac{a\,e\,x^3}{3}+\frac{a\,d\,x^2}{2}+a\,c\,x$$

```
[In] int((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^4*((b*c)/4 + (a*f)/4) + x^5*((b*d)/5 + (a*g)/5) + x^6*((b*e)/6 + (a*h)/6)
 + (b*h*x^9)/9 + a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*f*x^7)/7 + (b*g*x^8
)/8
```

3.378 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 88

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = adx+\frac{1}{2}aex^2+\frac{1}{3}(bc+af)x^3+\frac{1}{4}(bd+ag)x^4 +\frac{1}{5}(be+ah)x^5+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8+ac\log(x)$$

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*c)*x^3+1/4*(a*g+b*d)*x^4+1/5*(a*h+b*e)*x^5+1/6
*b*f*x^6+1/7*b*g*x^7+1/8*b*h*x^8+a*c*ln(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1834}

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = \frac{1}{3}x^3(af+bc)+\frac{1}{4}x^4(ag+bd)+\frac{1}{5}x^5(ah+be) +ac\log(x)+adx+\frac{1}{2}aex^2+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8$$

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 + ((b*e + a
*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 + a*c*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ad+\frac{ac}{x}+aex+(bc+af)x^2+(bd+ag)x^3+(be+ah)x^4+bfx^5+bgx^6+bhx^7\right)dx\\ &= adx+\frac{1}{2}aex^2+\frac{1}{3}(bc+af)x^3+\frac{1}{4}(bd+ag)x^4\\ &\quad+\frac{1}{5}(be+ah)x^5+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8+ac\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx\\ &= adx+\frac{1}{2}aex^2+\frac{1}{3}(bc+af)x^3+\frac{1}{4}(bd+ag)x^4\\ &\quad+\frac{1}{5}(be+ah)x^5+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8+ac\log(x)\end{aligned}$$

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
[Out] a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 + ((b*e + a
*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 + a*c*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

method	result
norman	$\left(\frac{af}{3}+\frac{bc}{3}\right)x^3+\left(\frac{ag}{4}+\frac{bd}{4}\right)x^4+\left(\frac{ah}{5}+\frac{be}{5}\right)x^5+adx+\frac{ae\,x^2}{2}+\frac{bf\,x^6}{6}+\frac{bg\,x^7}{7}+\frac{bh\,x^8}{8}+ac\ln\left(x\right)$
default	$\frac{bh\,x^8}{8}+\frac{bg\,x^7}{7}+\frac{bf\,x^6}{6}+\frac{ah\,x^5}{5}+\frac{be\,x^5}{5}+\frac{ag\,x^4}{4}+\frac{bd\,x^4}{4}+\frac{af\,x^3}{3}+\frac{bc\,x^3}{3}+\frac{ae\,x^2}{2}+adx+ac\ln\left(x\right)$
risch	$\frac{bh\,x^8}{8}+\frac{bg\,x^7}{7}+\frac{bf\,x^6}{6}+\frac{ah\,x^5}{5}+\frac{be\,x^5}{5}+\frac{ag\,x^4}{4}+\frac{bd\,x^4}{4}+\frac{af\,x^3}{3}+\frac{bc\,x^3}{3}+\frac{ae\,x^2}{2}+adx+ac\ln\left(x\right)$
parallelrisch	$\frac{bh\,x^8}{8}+\frac{bg\,x^7}{7}+\frac{bf\,x^6}{6}+\frac{ah\,x^5}{5}+\frac{be\,x^5}{5}+\frac{ag\,x^4}{4}+\frac{bd\,x^4}{4}+\frac{af\,x^3}{3}+\frac{bc\,x^3}{3}+\frac{ae\,x^2}{2}+adx+ac\ln\left(x\right)$

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x,method=_RETURNVERBOSE)
[Out] (1/3*a*f+1/3*b*c)*x^3+(1/4*a*g+1/4*b*d)*x^4+(1/5*a*h+1/5*b*e)*x^5+a*d*x+1/2
*a*e*x^2+1/6*b*f*x^6+1/7*b*g*x^7+1/8*b*h*x^8+a*c*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x}\,dx = \frac{1}{8}\,bhx^8+\frac{1}{7}\,bgx^7+\frac{1}{6}\,bfx^6+\frac{1}{5}\,(be+ah)x^5+\frac{1}{4}\,(bd+ag)x^4+\frac{1}{2}\,aex^2+\frac{1}{3}\,(bc+af)x^3+adx+ac\log\left(x\right)$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fricas"
)
```

```
[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*(b*e + a*h)*x^5 + 1/4*(b*d +
a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*c + a*f)*x^3 + a*d*x + a*c*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x}\,dx = ac\log\left(x\right)+adx+\frac{aex^2}{2}+\frac{bfx^6}{6}+\frac{bgx^7}{7}+\frac{bhx^8}{8}+x^5\left(\frac{ah}{5}+\frac{be}{5}\right)+x^4\left(\frac{ag}{4}+\frac{bd}{4}\right)+x^3\left(\frac{af}{3}+\frac{bc}{3}\right)$$

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a*c*log(x) + a*d*x + a*e*x**2/2 + b*f*x**6/6 + b*g*x**7/7 + b*h*x**8/8 + x*
*5*(a*h/5 + b*e/5) + x**4*(a*g/4 + b*d/4) + x**3*(a*f/3 + b*c/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x}\,dx = \frac{1}{8}\,bhx^8+\frac{1}{7}\,bgx^7+\frac{1}{6}\,bfx^6+\frac{1}{5}\,(be+ah)x^5+\frac{1}{4}\,(bd+ag)x^4+\frac{1}{2}\,aex^2+\frac{1}{3}\,(bc+af)x^3+adx+ac\log\left(x\right)$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxima"
)
```

```
[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*(b*e + a*h)*x^5 + 1/4*(b*d +
a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*c + a*f)*x^3 + a*d*x + a*c*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = \frac{1}{8}\,bhx^8+\frac{1}{7}\,bgx^7+\frac{1}{6}\,bfx^6+\frac{1}{5}\,bex^5+\frac{1}{5}\,ahx^5+\frac{1}{4}\,bdx^4+\frac{1}{4}\,agx^4+\frac{1}{3}\,bcx^3+\frac{1}{3}\,afx^3+\frac{1}{2}\,aex^2+adx+ac\log\left(|x|\right)$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*b*e*x^5 + 1/5*a*h*x^5 + 1/4*b
*d*x^4 + 1/4*a*g*x^4 + 1/3*b*c*x^3 + 1/3*a*f*x^3 + 1/2*a*e*x^2 + a*d*x + a*
c*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = x^3\left(\frac{b\,c}{3}+\frac{a\,f}{3}\right)+x^4\left(\frac{b\,d}{4}+\frac{a\,g}{4}\right)+x^5\left(\frac{b\,e}{5}+\frac{a\,h}{5}\right)+\frac{b\,h\,x^8}{8}+a\,c\,\ln\left(x\right)+a\,d\,x+\frac{a\,e\,x^2}{2}+\frac{b\,f\,x^6}{6}+\frac{b\,g\,x^7}{7}$$

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x)
```

```
[Out] x^3*((b*c)/3 + (a*f)/3) + x^4*((b*d)/4 + (a*g)/4) + x^5*((b*e)/5 + (a*h)/5)
 + (b*h*x^8)/8 + a*c*log(x) + a*d*x + (a*e*x^2)/2 + (b*f*x^6)/6 + (b*g*x^7)
/7
```

3.379 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} dx$

Optimal result

Integrand size = 36, antiderivative size = 86

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} dx = -\frac{ac}{x}+aex+\frac{1}{2}(bc+af)x^2+\frac{1}{3}(bd+ag)x^3 + \frac{1}{4}(be+ah)x^4+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7+ad\log(x)$$

```
[Out] -a*c/x+a*e*x+1/2*(a*f+b*c)*x^2+1/3*(a*g+b*d)*x^3+1/4*(a*h+b*e)*x^4+1/5*b*f*
x^5+1/6*b*g*x^6+1/7*b*h*x^7+a*d*ln(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1834}

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} dx = \frac{1}{2}x^2(af+bc)+\frac{1}{3}x^3(ag+bd)+\frac{1}{4}x^4(ah+be)-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7$$

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a*c)/x) + a*e*x + ((b*c + a*f)*x^2)/2 + ((b*d + a*g)*x^3)/3 + ((b*e + a*
h)*x^4)/4 + (b*f*x^5)/5 + (b*g*x^6)/6 + (b*h*x^7)/7 + a*d*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
```

```
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \int \left(ae+\frac{ac}{x^2}+\frac{ad}{x}+(bc+af)x+(bd+ag)x^2+(be+ah)x^3+bfx^4+bgx^5+bhx^6\right)dx$$
$$= -\frac{ac}{x}+aex+\frac{1}{2}(bc+af)x^2+\frac{1}{3}(bd+ag)x^3+\frac{1}{4}(be+ah)x^4+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7 + ad\log(x)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx = -\frac{ac}{x}+aex+\frac{1}{2}(bc+af)x^2+\frac{1}{3}(bd+ag)x^3 + \frac{1}{4}(be+ah)x^4+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7+ad\log(x)$$

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
[Out] -((a*c)/x) + a*e*x + ((b*c + a*f)*x^2)/2 + ((b*d + a*g)*x^3)/3 + ((b*e + a*
h)*x^4)/4 + (b*f*x^5)/5 + (b*g*x^6)/6 + (b*h*x^7)/7 + a*d*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

method	result
default	$\frac{bh\,x^7}{7}+\frac{bg\,x^6}{6}+\frac{bf\,x^5}{5}+\frac{ah\,x^4}{4}+\frac{be\,x^4}{4}+\frac{ag\,x^3}{3}+\frac{bd\,x^3}{3}+\frac{x^2af}{2}+\frac{cb\,x^2}{2}+aex+ad\ln\left(x\right)-\frac{ac}{x}$
risch	$\frac{bh\,x^7}{7}+\frac{bg\,x^6}{6}+\frac{bf\,x^5}{5}+\frac{ah\,x^4}{4}+\frac{be\,x^4}{4}+\frac{ag\,x^3}{3}+\frac{bd\,x^3}{3}+\frac{x^2af}{2}+\frac{cb\,x^2}{2}+aex+ad\ln\left(x\right)-\frac{ac}{x}$
norman	$\frac{\left(\frac{af}{2}+\frac{bc}{2}\right)x^3+\left(\frac{ag}{3}+\frac{bd}{3}\right)x^4+\left(\frac{ah}{4}+\frac{be}{4}\right)x^5+ae\,x^2-ac+\frac{bf\,x^6}{5}+\frac{bg\,x^7}{6}+\frac{bh\,x^8}{7}}{x}+ad\ln\left(x\right)$
parallelrisch	$\frac{60bh\,x^8+70bg\,x^7+84bf\,x^6+105ah\,x^5+105be\,x^5+140ag\,x^4+140bd\,x^4+210af\,x^3+210bc\,x^3+420ad\ln(x)x+420ae\,x^2-420ac}{420x}$

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/7*b*h*x^7+1/6*b*g*x^6+1/5*b*f*x^5+1/4*a*h*x^4+1/4*b*e*x^4+1/3*a*g*x^3+1/3
*b*d*x^3+1/2*x^2*a*f+1/2*c*b*x^2+a*e*x+a*d*ln(x)-a*c/x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= \frac{60\,bhx^8+70\,bgx^7+84\,bfx^6+105\,(be+ah)x^5+140\,(bd+ag)x^4+420\,aex^2+210\,(bc+af)x^3+420\,adx}{420\,x}$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="frica
s")
```

```
[Out] 1/420*(60*b*h*x^8 + 70*b*g*x^7 + 84*b*f*x^6 + 105*(b*e + a*h)*x^5 + 140*(b*
d + a*g)*x^4 + 420*a*e*x^2 + 210*(b*c + a*f)*x^3 + 420*a*d*x*log(x) - 420*a
*c)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= -\frac{ac}{x}+ad\log(x)+aex+\frac{bfx^5}{5}+\frac{bgx^6}{6}+\frac{bhx^7}{7}+x^4\left(\frac{ah}{4}+\frac{be}{4}\right)+x^3\left(\frac{ag}{3}+\frac{bd}{3}\right)+x^2\left(\frac{af}{2}+\frac{bc}{2}\right)$$

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a*c/x + a*d*log(x) + a*e*x + b*f*x**5/5 + b*g*x**6/6 + b*h*x**7/7 + x**4*(
a*h/4 + b*e/4) + x**3*(a*g/3 + b*d/3) + x**2*(a*f/2 + b*c/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= \frac{1}{7}\,bhx^7+\frac{1}{6}\,bgx^6+\frac{1}{5}\,bfx^5+\frac{1}{4}\,(be+ah)x^4+\frac{1}{3}\,(bd+ag)x^3+aex+\frac{1}{2}\,(bc+af)x^2+ad\log(x)-\frac{ac}{x}$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="maxim
a")
```

```
[Out] 1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/5*b*f*x^5 + 1/4*(b*e + a*h)*x^4 + 1/3*(b*d +
a*g)*x^3 + a*e*x + 1/2*(b*c + a*f)*x^2 + a*d*log(x) - a*c/x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx = \frac{1}{7}\,bhx^7+\frac{1}{6}\,bgx^6+\frac{1}{5}\,bfx^5+\frac{1}{4}\,bex^4+\frac{1}{4}\,ahx^4+\frac{1}{3}\,bdx^3+\frac{1}{3}\,agx^3+\frac{1}{2}\,bcx^2+\frac{1}{2}\,afx^2+aex+ad\log\left(|x|\right)-\frac{ac}{x}$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="giac"
)
```

```
[Out] 1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/5*b*f*x^5 + 1/4*b*e*x^4 + 1/4*a*h*x^4 + 1/3*b
*d*x^3 + 1/3*a*g*x^3 + 1/2*b*c*x^2 + 1/2*a*f*x^2 + a*e*x + a*d*log(abs(x))
- a*c/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx = x^2\left(\frac{b\,c}{2}+\frac{a\,f}{2}\right)+x^3\left(\frac{b\,d}{3}+\frac{a\,g}{3}\right)+x^4\left(\frac{b\,e}{4}+\frac{a\,h}{4}\right)+\frac{b\,h\,x^7}{7}+a\,d\,\ln\left(x\right)+a\,e\,x-\frac{a\,c}{x}+\frac{b\,f\,x^5}{5}+\frac{b\,g\,x^6}{6}$$

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x)
```

```
[Out] x^2*((b*c)/2 + (a*f)/2) + x^3*((b*d)/3 + (a*g)/3) + x^4*((b*e)/4 + (a*h)/4)
 + (b*h*x^7)/7 + a*d*log(x) + a*e*x - (a*c)/x + (b*f*x^5)/5 + (b*g*x^6)/6
```

3.380 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3} dx$

Optimal result

Integrand size = 36, antiderivative size = 86

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3} dx = -\frac{ac}{2x^2}-\frac{ad}{x}+(bc+af)x+\frac{1}{2}(bd+ag)x^2+\frac{1}{3}(be+ah)x^3+\frac{1}{4}bfx^4+\frac{1}{5}bgx^5+\frac{1}{6}bhx^6+ae\log(x)$$

```
[Out] -1/2*a*c/x^2-a*d/x+(a*f+b*c)*x+1/2*(a*g+b*d)*x^2+1/3*(a*h+b*e)*x^3+1/4*b*f*
x^4+1/5*b*g*x^5+1/6*b*h*x^6+a*e*ln(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1834}

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3} dx = x(af+bc)+\frac{1}{2}x^2(ag+bd)+\frac{1}{3}x^3(ah+be)-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+\frac{1}{4}bfx^4+\frac{1}{5}bgx^5+\frac{1}{6}bhx^6$$

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] -1/2*(a*c)/x^2 - (a*d)/x + (b*c + a*f)*x + ((b*d + a*g)*x^2)/2 + ((b*e + a*
h)*x^3)/3 + (b*f*x^4)/4 + (b*g*x^5)/5 + (b*h*x^6)/6 + a*e*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \int \left(bc\left(1+\frac{af}{bc}\right)+\frac{ac}{x^3}+\frac{ad}{x^2}+\frac{ae}{x}+(bd+ag)x+(be+ah)x^2+bfx^3+bgx^4+bhx^5\right)\,dx$$
$$= -\frac{ac}{2x^2}-\frac{ad}{x}+(bc+af)x+\frac{1}{2}(bd+ag)x^2+\frac{1}{3}(be+ah)x^3+\frac{1}{4}bfx^4+\frac{1}{5}bgx^5+\frac{1}{6}bhx^6 + ae\log(x)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= bcx+\frac{a(-3c-6dx+6fx^3+3gx^4+2hx^5)}{6x^2} + \frac{1}{60}bx^2\big(30d+x\big(20e+15fx+12gx^2+10hx^3\big)\big)+ae\log(x)$$

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
[Out] b*c*x + (a*(-3*c - 6*d*x + 6*f*x^3 + 3*g*x^4 + 2*h*x^5))/(6*x^2) + (b*x^2*(
30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3)))/60 + a*e*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{bh\,x^6}{6}+\frac{bg\,x^5}{5}+\frac{bf\,x^4}{4}+\frac{ah\,x^3}{3}+\frac{be\,x^3}{3}+\frac{ag\,x^2}{2}+\frac{bd\,x^2}{2}+afx+bcx+ae\ln(x)-\frac{ad}{x}-\frac{ac}{2x^2}$	78
risch	$\frac{bh\,x^6}{6}+\frac{bg\,x^5}{5}+\frac{bf\,x^4}{4}+\frac{ah\,x^3}{3}+\frac{be\,x^3}{3}+\frac{ag\,x^2}{2}+\frac{bd\,x^2}{2}+afx+bcx+\frac{-adx-\frac{1}{2}ac}{x^2}+ae\ln(x)$	78
norman	$\frac{\left(\frac{ag}{2}+\frac{bd}{2}\right)x^4+\left(\frac{ah}{3}+\frac{be}{3}\right)x^5+(af+bc)x^3-\frac{ac}{2}-adx+\frac{bf\,x^6}{4}+\frac{bg\,x^7}{5}+\frac{bh\,x^8}{6}}{x^2}+ae\ln(x)$	79
parallelrisch	$\frac{10bh\,x^8+12bg\,x^7+15bf\,x^6+20ah\,x^5+20be\,x^5+30ag\,x^4+30bd\,x^4+60ae\ln(x)x^2+60af\,x^3+60bc\,x^3-60adx-30ac}{60x^2}$	88

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/6*b*h*x^6+1/5*b*g*x^5+1/4*b*f*x^4+1/3*a*h*x^3+1/3*b*e*x^3+1/2*a*g*x^2+1/2
*b*d*x^2+a*f*x+b*c*x+a*e*ln(x)-a*d/x-1/2*a*c/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= \frac{10\,bhx^8+12\,bgx^7+15\,bfx^6+20\,(be+ah)x^5+30\,(bd+ag)x^4+60\,aex^2\log(x)+60\,(bc+af)x^3-60\,adx}{60\,x^2}$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="frica
s")
```

```
[Out] 1/60*(10*b*h*x^8 + 12*b*g*x^7 + 15*b*f*x^6 + 20*(b*e + a*h)*x^5 + 30*(b*d +
 a*g)*x^4 + 60*a*e*x^2*log(x) + 60*(b*c + a*f)*x^3 - 60*a*d*x - 30*a*c)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= ae\log(x)+\frac{bfx^4}{4}+\frac{bgx^5}{5}+\frac{bhx^6}{6}+x^3\left(\frac{ah}{3}+\frac{be}{3}\right)+x^2\left(\frac{ag}{2}+\frac{bd}{2}\right)+x(af+bc)+\frac{-ac-2adx}{2x^2}$$

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a*e*log(x) + b*f*x**4/4 + b*g*x**5/5 + b*h*x**6/6 + x**3*(a*h/3 + b*e/3) +
x**2*(a*g/2 + b*d/2) + x*(a*f + b*c) + (-a*c - 2*a*d*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= \frac{1}{6}\,bhx^6+\frac{1}{5}\,bgx^5+\frac{1}{4}\,bfx^4+\frac{1}{3}\,(be+ah)x^3+\frac{1}{2}\,(bd+ag)x^2+ae\log(x)+(bc+af)x-\frac{2\,adx+ac}{2\,x^2}$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="maxim
a")
```

```
[Out] 1/6*b*h*x^6 + 1/5*b*g*x^5 + 1/4*b*f*x^4 + 1/3*(b*e + a*h)*x^3 + 1/2*(b*d +
a*g)*x^2 + a*e*log(x) + (b*c + a*f)*x - 1/2*(2*a*d*x + a*c)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx = \frac{1}{6}\,bhx^6+\frac{1}{5}\,bgx^5+\frac{1}{4}\,bfx^4+\frac{1}{3}\,bex^3+\frac{1}{3}\,ahx^3+\frac{1}{2}\,bdx^2 + \frac{1}{2}\,agx^2+bcx+afx+ae\log\left(|x|\right)-\frac{2\,adx+ac}{2\,x^2}$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="giac"
)
```

```
[Out] 1/6*b*h*x^6 + 1/5*b*g*x^5 + 1/4*b*f*x^4 + 1/3*b*e*x^3 + 1/3*a*h*x^3 + 1/2*b
*d*x^2 + 1/2*a*g*x^2 + b*c*x + a*f*x + a*e*log(abs(x)) - 1/2*(2*a*d*x + a*c
)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx = x\,(b\,c+a\,f)-\frac{\frac{a\,c}{2}+a\,d\,x}{x^2}+x^2\left(\frac{b\,d}{2}+\frac{a\,g}{2}\right) + x^3\left(\frac{b\,e}{3}+\frac{a\,h}{3}\right)+\frac{b\,h\,x^6}{6}+a\,e\,\ln\left(x\right)+\frac{b\,f\,x^4}{4}+\frac{b\,g\,x^5}{5}$$

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x)
```

```
[Out] x*(b*c + a*f) - ((a*c)/2 + a*d*x)/x^2 + x^2*((b*d)/2 + (a*g)/2) + x^3*((b*e
)/3 + (a*h)/3) + (b*h*x^6)/6 + a*e*log(x) + (b*f*x^4)/4 + (b*g*x^5)/5
```

3.381 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx$

Optimal result

Integrand size = 36, antiderivative size = 86

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx = -\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+(bd+ag)x+\frac{1}{2}(be+ah)x^2+\frac{1}{3}bfx^3+\frac{1}{4}bgx^4+\frac{1}{5}bhx^5+(bc+af)\log(x)$$

```
[Out] -1/3*a*c/x^3-1/2*a*d/x^2-a*e/x+(a*g+b*d)*x+1/2*(a*h+b*e)*x^2+1/3*b*f*x^3+1/
4*b*g*x^4+1/5*b*h*x^5+(a*f+b*c)*ln(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1834}

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx = \log(x)(af+bc)+x(ag+bd)+\frac{1}{2}x^2(ah+be)-\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+\frac{1}{3}bfx^3+\frac{1}{4}bgx^4+\frac{1}{5}bhx^5$$

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -1/3*(a*c)/x^3 - (a*d)/(2*x^2) - (a*e)/x + (b*d + a*g)*x + ((b*e + a*h)*x^2
)/2 + (b*f*x^3)/3 + (b*g*x^4)/4 + (b*h*x^5)/5 + (b*c + a*f)*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(bd\left(1+\frac{ag}{bd}\right)+\frac{ac}{x^4}+\frac{ad}{x^3}+\frac{ae}{x^2}+\frac{bc+af}{x}+(be+ah)x+bfx^2+bgx^3+bhx^4\right)dx\\&=-\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+(bd+ag)x+\frac{1}{2}(be+ah)x^2+\frac{1}{3}bfx^3+\frac{1}{4}bgx^4+\frac{1}{5}bhx^5+(bc+af)\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\begin{aligned}&\int\frac{(a+bx^3)\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx\\&=-\frac{a(2c+3x(d+2ex-x^3(2g+hx)))}{6x^3}\\&\quad+\frac{1}{60}bx\big(60d+x\big(30e+x\big(20f+15gx+12hx^2\big)\big)\big)+(bc+af)\log(x)\end{aligned}$$

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
[Out] -1/6*(a*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/x^3 + (b*x*(60*d + x*(30
*e + x*(20*f + 15*g*x + 12*h*x^2))))/60 + (b*c + a*f)*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

method	result	siz
default	$\frac{bh\,x^5}{5}+\frac{bg\,x^4}{4}+\frac{f\,x^3b}{3}+\frac{ah\,x^2}{2}+\frac{be\,x^2}{2}+agx+bdx+(af+bc)\ln(x)-\frac{ac}{3x^3}-\frac{ae}{x}-\frac{ad}{2x^2}$	76
risch	$\frac{bh\,x^5}{5}+\frac{bg\,x^4}{4}+\frac{f\,x^3b}{3}+\frac{ah\,x^2}{2}+\frac{be\,x^2}{2}+agx+bdx+\frac{-ae\,x^2-\frac{1}{2}adx-\frac{1}{3}ac}{x^3}+\ln(x)\,af+\ln(x)\,bc$	76
norman	$\frac{\left(\frac{ah}{2}+\frac{be}{2}\right)x^5+(ag+bd)x^4-\frac{ac}{3}-\frac{adx}{2}-ae\,x^2+\frac{bf\,x^6}{3}+\frac{bg\,x^7}{4}+\frac{bh\,x^8}{5}}{x^3}+(af+bc)\ln(x)$	78
parallelrisch	$\frac{12bh\,x^8+15bg\,x^7+20bf\,x^6+30ah\,x^5+30be\,x^5+60\ln(x)x^3af+60\ln(x)x^3bc+60ag\,x^4+60bd\,x^4-60ae\,x^2-30adx-20ac}{60x^3}$	90

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/5*b*h*x^5+1/4*b*g*x^4+1/3*f*x^3*b+1/2*a*h*x^2+1/2*b*e*x^2+a*g*x+b*d*x+(a*
f+b*c)*ln(x)-1/3*a*c/x^3-a*e/x-1/2*a*d/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$= \frac{12\,bhx^8+15\,bgx^7+20\,bfx^6+30\,(be+ah)x^5+60\,(bd+ag)x^4+60\,(bc+af)x^3\log{(x)}-60\,aex^2-30\,adx}{60\,x^3}$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="frica
s")
```

```
[Out] 1/60*(12*b*h*x^8 + 15*b*g*x^7 + 20*b*f*x^6 + 30*(b*e + a*h)*x^5 + 60*(b*d +
 a*g)*x^4 + 60*(b*c + a*f)*x^3*log(x) - 60*a*e*x^2 - 30*a*d*x - 20*a*c)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$= \frac{bfx^3}{3}+\frac{bgx^4}{4}+\frac{bhx^5}{5}+x^2\left(\frac{ah}{2}+\frac{be}{2}\right)+x(ag+bd)$$
$$+(af+bc)\log{(x)}+\frac{-2ac-3adx-6aex^2}{6x^3}$$

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] b*f*x**3/3 + b*g*x**4/4 + b*h*x**5/5 + x**2*(a*h/2 + b*e/2) + x*(a*g + b*d)
 + (a*f + b*c)*log(x) + (-2*a*c - 3*a*d*x - 6*a*e*x**2)/(6*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$= \frac{1}{5}\,bhx^5+\frac{1}{4}\,bgx^4+\frac{1}{3}\,bfx^3+\frac{1}{2}\,(be+ah)x^2+(bd+ag)x$$
$$+(bc+af)\log{(x)}-\frac{6\,aex^2+3\,adx+2\,ac}{6\,x^3}$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="maxim
a")
```

```
[Out] 1/5*b*h*x^5 + 1/4*b*g*x^4 + 1/3*b*f*x^3 + 1/2*(b*e + a*h)*x^2 + (b*d + a*g)
*x + (b*c + a*f)*log(x) - 1/6*(6*a*e*x^2 + 3*a*d*x + 2*a*c)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = \frac{1}{5}\,bhx^5 + \frac{1}{4}\,bgx^4 + \frac{1}{3}\,bfx^3 + \frac{1}{2}\,bex^2 + \frac{1}{2}\,ahx^2 + bdx + agx + (bc+af)\log(|x|) - \frac{6\,aex^2 + 3\,adx + 2\,ac}{6\,x^3}$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="giac"
)
```

```
[Out] 1/5*b*h*x^5 + 1/4*b*g*x^4 + 1/3*b*f*x^3 + 1/2*b*e*x^2 + 1/2*a*h*x^2 + b*d*x
 + a*g*x + (b*c + a*f)*log(abs(x)) - 1/6*(6*a*e*x^2 + 3*a*d*x + 2*a*c)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = x\,(b\,d+a\,g) - \frac{a\,e\,x^2 + \frac{a\,d\,x}{2} + \frac{a\,c}{3}}{x^3} + x^2\left(\frac{b\,e}{2} + \frac{a\,h}{2}\right) + \ln(x)\,\left(b\,c+a\,f\right) + \frac{b\,h\,x^5}{5} + \frac{b\,f\,x^3}{3} + \frac{b\,g\,x^4}{4}$$

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x)
```

```
[Out] x*(b*d + a*g) - ((a*c)/3 + (a*d*x)/2 + a*e*x^2)/x^3 + x^2*((b*e)/2 + (a*h)/
2) + log(x)*(b*c + a*f) + (b*h*x^5)/5 + (b*f*x^3)/3 + (b*g*x^4)/4
```

3.382 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 86

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx = -\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}-\frac{bc+af}{x}+(be+ah)x+\frac{1}{2}bfx^2+\frac{1}{3}bgx^3+\frac{1}{4}bhx^4+(bd+ag)\log(x)$$

```
[Out] -1/4*a*c/x^4-1/3*a*d/x^3-1/2*a*e/x^2+(-a*f-b*c)/x+(a*h+b*e)*x+1/2*b*f*x^2+1
/3*b*g*x^3+1/4*b*h*x^4+(a*g+b*d)*ln(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1834}

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx = -\frac{af+bc}{x}+\log(x)(ag+bd)+x(ah+be)-\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}+\frac{1}{2}bfx^2+\frac{1}{3}bgx^3+\frac{1}{4}bhx^4$$

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] -1/4*(a*c)/x^4 - (a*d)/(3*x^3) - (a*e)/(2*x^2) - (b*c + a*f)/x + (b*e + a*h
)*x + (b*f*x^2)/2 + (b*g*x^3)/3 + (b*h*x^4)/4 + (b*d + a*g)*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \int \left(be\left(1+\frac{ah}{be}\right)+\frac{ac}{x^5}+\frac{ad}{x^4}+\frac{ae}{x^3}+\frac{bc+af}{x^2}+\frac{bd+ag}{x}+bfx+bgx^2+bhx^3\right)\,dx$$
$$= -\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}-\frac{bc+af}{x}+(be+ah)x+\frac{1}{2}bfx^2+\frac{1}{3}bgx^3+\frac{1}{4}bhx^4+(bd+ag)\log(x)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$= b\left(-\frac{c}{x}+ex+\frac{1}{12}x^2\left(6f+4gx+3hx^2\right)\right) - \frac{a(3c+4dx+6x^2(e+2fx-2hx^3))}{12x^4}+(bd+ag)\log(x)$$

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
[Out] b*(-(c/x) + e*x + (x^2*(6*f + 4*g*x + 3*h*x^2))/12) - (a*(3*c + 4*d*x + 6*x
^2*(e + 2*f*x - 2*h*x^3)))/(12*x^4) + (b*d + a*g)*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{bh\,x^4}{4}+\frac{bg\,x^3}{3}+\frac{bf\,x^2}{2}+ahx+bex+(ag+bd)\ln(x)-\frac{ad}{3x^3}-\frac{af+bc}{x}-\frac{ae}{2x^2}-\frac{ac}{4x^4}$	74
risch	$\frac{bh\,x^4}{4}+\frac{bg\,x^3}{3}+\frac{bf\,x^2}{2}+ahx+bex+\frac{(-af-bc)x^3-\frac{ae\,x^2}{2}-\frac{adx}{3}-\frac{ac}{4}}{x^4}+\ln(x)\,ag+\ln(x)\,bd$	75
norman	$\frac{(-af-bc)x^3+(ah+be)x^5-\frac{ac}{4}-\frac{adx}{3}-\frac{ae\,x^2}{2}+\frac{bf\,x^6}{2}+\frac{bg\,x^7}{3}+\frac{bh\,x^8}{4}}{x^4}+(ag+bd)\ln(x)$	78
parallelrisch	$\frac{3bh\,x^8+4bg\,x^7+6bf\,x^6+12\ln(x)x^4ag+12\ln(x)x^4bd+12ah\,x^5+12be\,x^5-12af\,x^3-12bc\,x^3-6ae\,x^2-4adx-3ac}{12x^4}$	90

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x,method=_RETURNVERBOSE)
[Out] 1/4*b*h*x^4+1/3*b*g*x^3+1/2*b*f*x^2+a*h*x+b*e*x+(a*g+b*d)*ln(x)-1/3*a*d/x^3
-(a*f+b*c)/x-1/2*a*e/x^2-1/4*a*c/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx = \frac{3\,bhx^8+4\,bgx^7+6\,bfx^6+12\,(be+ah)x^5+12\,(bd+ag)x^4\log(x)-6\,aex^2-12\,(bc+af)x^3-4\,adx-3\,ac}{12\,x^4}$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="frica
s")
```

```
[Out] 1/12*(3*b*h*x^8 + 4*b*g*x^7 + 6*b*f*x^6 + 12*(b*e + a*h)*x^5 + 12*(b*d + a*
g)*x^4*log(x) - 6*a*e*x^2 - 12*(b*c + a*f)*x^3 - 4*a*d*x - 3*a*c)/x^4
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx = \frac{bfx^2}{2}+\frac{bgx^3}{3}+\frac{bhx^4}{4}+x(ah+be)+(ag+bd)\log(x) + \frac{-3ac-4adx-6aex^2+x^3(-12af-12bc)}{12x^4}$$

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] b*f*x**2/2 + b*g*x**3/3 + b*h*x**4/4 + x*(a*h + b*e) + (a*g + b*d)*log(x) +
 (-3*a*c - 4*a*d*x - 6*a*e*x**2 + x**3*(-12*a*f - 12*b*c))/(12*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx = \frac{1}{4}\,bhx^4+\frac{1}{3}\,bgx^3+\frac{1}{2}\,bfx^2+(be+ah)x+(bd+ag)\log(x) - \frac{6\,aex^2+12\,(bc+af)x^3+4\,adx+3\,ac}{12\,x^4}$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="maxim
a")
```

```
[Out] 1/4*b*h*x^4 + 1/3*b*g*x^3 + 1/2*b*f*x^2 + (b*e + a*h)*x + (b*d + a*g)*log(x
) - 1/12*(6*a*e*x^2 + 12*(b*c + a*f)*x^3 + 4*a*d*x + 3*a*c)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$= \frac{1}{4}\,bhx^4 + \frac{1}{3}\,bgx^3 + \frac{1}{2}\,bfx^2 + bex + ahx + (bd+ag)\log(|x|)$$
$$- \frac{6\,aex^2 + 12\,(bc+af)x^3 + 4\,adx + 3\,ac}{12\,x^4}$$

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="giac"
)
```

```
[Out] 1/4*b*h*x^4 + 1/3*b*g*x^3 + 1/2*b*f*x^2 + b*e*x + a*h*x + (b*d + a*g)*log(a
bs(x)) - 1/12*(6*a*e*x^2 + 12*(b*c + a*f)*x^3 + 4*a*d*x + 3*a*c)/x^4
```

Mupad [B] (verification not implemented)

Time = 10.46 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$= x\,(b\,e+a\,h) - \frac{(b\,c+a\,f)\,x^3 + \frac{a\,e\,x^2}{2} + \frac{a\,d\,x}{3} + \frac{a\,c}{4}}{x^4} + \ln(x)\,(b\,d+a\,g) + \frac{b\,h\,x^4}{4} + \frac{b\,f\,x^2}{2} + \frac{b\,g\,x^3}{3}$$

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x)
```

```
[Out] x*(b*e + a*h) - ((a*c)/4 + x^3*(b*c + a*f) + (a*d*x)/3 + (a*e*x^2)/2)/x^4 +
 log(x)*(b*d + a*g) + (b*h*x^4)/4 + (b*f*x^2)/2 + (b*g*x^3)/3
```

3.383 $\int x^4(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 163

$$\begin{aligned}&\int x^4(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\&=\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{8}a(2bc+af)x^8+\frac{1}{9}a(2bd+ag)x^9\\&\quad+\frac{1}{10}a(2be+ah)x^{10}+\frac{1}{11}b(bc+2af)x^{11}+\frac{1}{12}b(bd+2ag)x^{12}\\&\quad+\frac{1}{13}b(be+2ah)x^{13}+\frac{1}{14}b^2fx^{14}+\frac{1}{15}b^2gx^{15}+\frac{1}{16}b^2hx^{16}\end{aligned}$$

```
[Out] 1/5*a^2*c*x^5+1/6*a^2*d*x^6+1/7*a^2*e*x^7+1/8*a*(a*f+2*b*c)*x^8+1/9*a*(a*g+
2*b*d)*x^9+1/10*a*(a*h+2*b*e)*x^10+1/11*b*(2*a*f+b*c)*x^11+1/12*b*(2*a*g+b*
d)*x^12+1/13*b*(2*a*h+b*e)*x^13+1/14*b^2*f*x^14+1/15*b^2*g*x^15+1/16*b^2*h*
x^16
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1834}

$$\begin{aligned}&\int x^4(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\&=\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{11}bx^{11}(2af+bc)+\frac{1}{8}ax^8(af+2bc)\\&\quad+\frac{1}{12}bx^{12}(2ag+bd)+\frac{1}{9}ax^9(ag+2bd)+\frac{1}{13}bx^{13}(2ah+be)\\&\quad+\frac{1}{10}ax^{10}(ah+2be)+\frac{1}{14}b^2fx^{14}+\frac{1}{15}b^2gx^{15}+\frac{1}{16}b^2hx^{16}\end{aligned}$$

```
[In] Int[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*x^8)/8 + (
a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b*(b*c + 2*a*f)*x^11)
/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2*a*h)*x^13)/13 + (b^2*f*x^14)/
14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2cx^4 + a^2dx^5 + a^2ex^6 + a(2bc+af)x^7 + a(2bd+ag)x^8 + a(2be+ah)x^9 \right.\\ &\quad \left. + b(bc+2af)x^{10} + b(bd+2ag)x^{11} + b(be+2ah)x^{12} + b^2fx^{13} + b^2gx^{14} + b^2hx^{15}\right)\,dx\\ &= \frac{1}{5}a^2cx^5 + \frac{1}{6}a^2dx^6 + \frac{1}{7}a^2ex^7 + \frac{1}{8}a(2bc+af)x^8 + \frac{1}{9}a(2bd+ag)x^9\\ &\quad + \frac{1}{10}a(2be+ah)x^{10} + \frac{1}{11}b(bc+2af)x^{11} + \frac{1}{12}b(bd+2ag)x^{12}\\ &\quad + \frac{1}{13}b(be+2ah)x^{13} + \frac{1}{14}b^2fx^{14} + \frac{1}{15}b^2gx^{15} + \frac{1}{16}b^2hx^{16}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int x^4(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\ &= \frac{1}{5}a^2cx^5 + \frac{1}{6}a^2dx^6 + \frac{1}{7}a^2ex^7 + \frac{1}{8}a(2bc+af)x^8 + \frac{1}{9}a(2bd+ag)x^9\\ &\quad + \frac{1}{10}a(2be+ah)x^{10} + \frac{1}{11}b(bc+2af)x^{11} + \frac{1}{12}b(bd+2ag)x^{12}\\ &\quad + \frac{1}{13}b(be+2ah)x^{13} + \frac{1}{14}b^2fx^{14} + \frac{1}{15}b^2gx^{15} + \frac{1}{16}b^2hx^{16}\end{aligned}$$

```
[In] Integrate[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*x^8)/8 + (
a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b*(b*c + 2*a*f)*x^11)
/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2*a*h)*x^13)/13 + (b^2*f*x^14)/
14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.93

method	result
default	$\frac{b^2 h\,x^{16}}{16}+\frac{b^2 g\,x^{15}}{15}+\frac{b^2 f\,x^{14}}{14}+\frac{(2abh+b^2 e)x^{13}}{13}+\frac{(2abg+b^2 d)x^{12}}{12}+\frac{(2afb+b^2 c)x^{11}}{11}+\frac{(a^2 h+2aeb)x^{10}}{10}+\frac{(a^2 g+2abd}{9}$
norman	$\frac{a^2 c\,x^5}{5}+\frac{a^2 d\,x^6}{6}+\frac{a^2 e\,x^7}{7}+\left(\frac{1}{8}a^2 f+\frac{1}{4}abc\right)x^8+\left(\frac{1}{9}a^2 g+\frac{2}{9}abd\right)x^9+\left(\frac{1}{10}a^2 h+\frac{1}{5}aeb\right)x^{10}+\left(\frac{2}{11}af\right.$
gosper	$\frac{1}{5}a^2 c\,x^5+\frac{1}{6}a^2 d\,x^6+\frac{1}{7}a^2 e\,x^7+\frac{1}{8}x^8 a^2 f+\frac{1}{4}x^8 abc+\frac{1}{9}x^9 a^2 g+\frac{2}{9}abd\,x^9+\frac{1}{10}x^{10} a^2 h+\frac{1}{5}abe\,x^{10}+$
risch	$\frac{1}{5}a^2 c\,x^5+\frac{1}{6}a^2 d\,x^6+\frac{1}{7}a^2 e\,x^7+\frac{1}{8}x^8 a^2 f+\frac{1}{4}x^8 abc+\frac{1}{9}x^9 a^2 g+\frac{2}{9}abd\,x^9+\frac{1}{10}x^{10} a^2 h+\frac{1}{5}abe\,x^{10}+$
parallelrisch	$\frac{1}{5}a^2 c\,x^5+\frac{1}{6}a^2 d\,x^6+\frac{1}{7}a^2 e\,x^7+\frac{1}{8}x^8 a^2 f+\frac{1}{4}x^8 abc+\frac{1}{9}x^9 a^2 g+\frac{2}{9}abd\,x^9+\frac{1}{10}x^{10} a^2 h+\frac{1}{5}abe\,x^{10}+$

```
[In] int(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/16*b^2*h*x^16+1/15*b^2*g*x^15+1/14*b^2*f*x^14+1/13*(2*a*b*h+b^2*e)*x^13+1
/12*(2*a*b*g+b^2*d)*x^12+1/11*(2*a*b*f+b^2*c)*x^11+1/10*(a^2*h+2*a*b*e)*x^1
0+1/9*(a^2*g+2*a*b*d)*x^9+1/8*(a^2*f+2*a*b*c)*x^8+1/7*a^2*e*x^7+1/6*a^2*d*x
^6+1/5*a^2*c*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\begin{aligned}\int x^4\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{1}{16}\,b^2hx^{16}+\frac{1}{15}\,b^2gx^{15}+\frac{1}{14}\,b^2fx^{14}+\frac{1}{13}\,\left(b^2e+2\,abh\right)x^{13}\\ &\quad+\frac{1}{12}\,\left(b^2d+2\,abg\right)x^{12}+\frac{1}{11}\,\left(b^2c+2\,abf\right)x^{11}+\frac{1}{10}\,\left(2\,abe+a^2h\right)x^{10}\\ &\quad+\frac{1}{7}\,a^2ex^7+\frac{1}{9}\,\left(2\,abd+a^2g\right)x^9+\frac{1}{6}\,a^2dx^6+\frac{1}{8}\,\left(2\,abc+a^2f\right)x^8+\frac{1}{5}\,a^2cx^5\end{aligned}$$

```
[In] integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 1/13*(b^2*e + 2*a*b*h
)*x^13 + 1/12*(b^2*d + 2*a*b*g)*x^12 + 1/11*(b^2*c + 2*a*b*f)*x^11 + 1/10*(
2*a*b*e + a^2*h)*x^10 + 1/7*a^2*e*x^7 + 1/9*(2*a*b*d + a^2*g)*x^9 + 1/6*a^2
*d*x^6 + 1/8*(2*a*b*c + a^2*f)*x^8 + 1/5*a^2*c*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02

$$\int x^4(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{a^2cx^5}{5}+\frac{a^2dx^6}{6}+\frac{a^2ex^7}{7}+\frac{b^2fx^{14}}{14}+\frac{b^2gx^{15}}{15}+\frac{b^2hx^{16}}{16}+x^{13}\cdot\left(\frac{2abh}{13}+\frac{b^2e}{13}\right)+x^{12}\left(\frac{abg}{6}+\frac{b^2d}{12}\right)+x^{11}\cdot\left(\frac{2abf}{11}+\frac{b^2c}{11}\right)+x^{10}\left(\frac{a^2h}{10}+\frac{abe}{5}\right)+x^9\left(\frac{a^2g}{9}+\frac{2abd}{9}\right)+x^8\left(\frac{a^2f}{8}+\frac{abc}{4}\right)$$

```
[In] integrate(x**4*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**5/5 + a**2*d*x**6/6 + a**2*e*x**7/7 + b**2*f*x**14/14 + b**2*g*x*
*15/15 + b**2*h*x**16/16 + x**13*(2*a*b*h/13 + b**2*e/13) + x**12*(a*b*g/6
+ b**2*d/12) + x**11*(2*a*b*f/11 + b**2*c/11) + x**10*(a**2*h/10 + a*b*e/5)
 + x**9*(a**2*g/9 + 2*a*b*d/9) + x**8*(a**2*f/8 + a*b*c/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int x^4(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{16}\,b^2hx^{16}+\frac{1}{15}\,b^2gx^{15}+\frac{1}{14}\,b^2fx^{14}+\frac{1}{13}\,(b^2e+2\,abh)x^{13}+\frac{1}{12}\,(b^2d+2\,abg)x^{12}+\frac{1}{11}\,(b^2c+2\,abf)x^{11}+\frac{1}{10}\,(2\,abe+a^2h)x^{10}+\frac{1}{7}\,a^2ex^7+\frac{1}{9}\,(2\,abd+a^2g)x^9+\frac{1}{6}\,a^2dx^6+\frac{1}{8}\,(2\,abc+a^2f)x^8+\frac{1}{5}\,a^2cx^5$$

```
[In] integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 1/13*(b^2*e + 2*a*b*h
)*x^13 + 1/12*(b^2*d + 2*a*b*g)*x^12 + 1/11*(b^2*c + 2*a*b*f)*x^11 + 1/10*(
2*a*b*e + a^2*h)*x^10 + 1/7*a^2*e*x^7 + 1/9*(2*a*b*d + a^2*g)*x^9 + 1/6*a^2
*d*x^6 + 1/8*(2*a*b*c + a^2*f)*x^8 + 1/5*a^2*c*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.96

$$\int x^4(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{16}b^2hx^{16}+\frac{1}{15}b^2gx^{15}+\frac{1}{14}b^2fx^{14}+\frac{1}{13}b^2ex^{13}+\frac{2}{13}abhx^{13}+\frac{1}{12}b^2dx^{12}+\frac{1}{6}abgx^{12}+\frac{1}{11}b^2cx^{11}+\frac{2}{11}abfx^{11}+\frac{1}{5}abex^{10}+\frac{1}{10}a^2hx^{10}+\frac{2}{9}abdx^9+\frac{1}{9}a^2gx^9+\frac{1}{4}abcx^8+\frac{1}{8}a^2fx^8+\frac{1}{7}a^2ex^7+\frac{1}{6}a^2dx^6+\frac{1}{5}a^2cx^5$$

```
[In] integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 1/13*b^2*e*x^13 + 2/1
3*a*b*h*x^13 + 1/12*b^2*d*x^12 + 1/6*a*b*g*x^12 + 1/11*b^2*c*x^11 + 2/11*a*
b*f*x^11 + 1/5*a*b*e*x^10 + 1/10*a^2*h*x^10 + 2/9*a*b*d*x^9 + 1/9*a^2*g*x^9
 + 1/4*a*b*c*x^8 + 1/8*a^2*f*x^8 + 1/7*a^2*e*x^7 + 1/6*a^2*d*x^6 + 1/5*a^2*
c*x^5
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int x^4(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = x^8\left(\frac{f\,a^2}{8}+\frac{b\,c\,a}{4}\right)+x^{11}\left(\frac{c\,b^2}{11}+\frac{2\,a\,f\,b}{11}\right)+x^9\left(\frac{g\,a^2}{9}+\frac{2\,b\,d\,a}{9}\right)+x^{12}\left(\frac{d\,b^2}{12}+\frac{a\,g\,b}{6}\right)+x^{10}\left(\frac{h\,a^2}{10}+\frac{b\,e\,a}{5}\right)+x^{13}\left(\frac{e\,b^2}{13}+\frac{2\,a\,h\,b}{13}\right)+\frac{a^2\,c\,x^5}{5}+\frac{a^2\,d\,x^6}{6}+\frac{a^2\,e\,x^7}{7}+\frac{b^2\,f\,x^{14}}{14}+\frac{b^2\,g\,x^{15}}{15}+\frac{b^2\,h\,x^{16}}{16}$$

```
[In] int(x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^8*((a^2*f)/8 + (a*b*c)/4) + x^11*((b^2*c)/11 + (2*a*b*f)/11) + x^9*((a^2*
g)/9 + (2*a*b*d)/9) + x^12*((b^2*d)/12 + (a*b*g)/6) + x^10*((a^2*h)/10 + (a
*b*e)/5) + x^13*((b^2*e)/13 + (2*a*b*h)/13) + (a^2*c*x^5)/5 + (a^2*d*x^6)/6
 + (a^2*e*x^7)/7 + (b^2*f*x^14)/14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16
```

3.384 $\int x^3(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 163

$$\begin{aligned}\int x^3(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= \frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a(2bc+af)x^7+\frac{1}{8}a(2bd+ag)x^8\\ &+\frac{1}{9}a(2be+ah)x^9+\frac{1}{10}b(bc+2af)x^{10}+\frac{1}{11}b(bd+2ag)x^{11}\\ &+\frac{1}{12}b(be+2ah)x^{12}+\frac{1}{13}b^2fx^{13}+\frac{1}{14}b^2gx^{14}+\frac{1}{15}b^2hx^{15}\end{aligned}$$

```
[Out] 1/4*a^2*c*x^4+1/5*a^2*d*x^5+1/6*a^2*e*x^6+1/7*a*(a*f+2*b*c)*x^7+1/8*a*(a*g+
2*b*d)*x^8+1/9*a*(a*h+2*b*e)*x^9+1/10*b*(2*a*f+b*c)*x^10+1/11*b*(2*a*g+b*d)
*x^11+1/12*b*(2*a*h+b*e)*x^12+1/13*b^2*f*x^13+1/14*b^2*g*x^14+1/15*b^2*h*x^
15
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1834}

$$\begin{aligned}\int x^3(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= \frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{10}bx^{10}(2af+bc)+\frac{1}{7}ax^7(af+2bc)+\frac{1}{11}bx^{11}(2ag+bd)\\ &+\frac{1}{8}ax^8(ag+2bd)+\frac{1}{12}bx^{12}(2ah+be)+\frac{1}{9}ax^9(ah+2be)+\frac{1}{13}b^2fx^{13}+\frac{1}{14}b^2gx^{14}+\frac{1}{15}b^2hx^{15}\end{aligned}$$

```
[In] Int[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*x^7)/7 + (
a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(b*c + 2*a*f)*x^10)/1
0 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a*h)*x^12)/12 + (b^2*f*x^13)/13
 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2cx^3+a^2dx^4+a^2ex^5+a(2bc+af)x^6+a(2bd+ag)x^7+a(2be+ah)x^8\right.\\
&\quad \left.+b(bc+2af)x^9+b(bd+2ag)x^{10}+b(be+2ah)x^{11}+b^2fx^{12}+b^2gx^{13}+b^2hx^{14}\right)\,dx\\
&= \frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a(2bc+af)x^7+\frac{1}{8}a(2bd+ag)x^8\\
&\quad +\frac{1}{9}a(2be+ah)x^9+\frac{1}{10}b(bc+2af)x^{10}+\frac{1}{11}b(bd+2ag)x^{11}\\
&\quad +\frac{1}{12}b(be+2ah)x^{12}+\frac{1}{13}b^2fx^{13}+\frac{1}{14}b^2gx^{14}+\frac{1}{15}b^2hx^{15}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int x^3\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\
&= \frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a(2bc+af)x^7+\frac{1}{8}a(2bd+ag)x^8\\
&\quad +\frac{1}{9}a(2be+ah)x^9+\frac{1}{10}b(bc+2af)x^{10}+\frac{1}{11}b(bd+2ag)x^{11}\\
&\quad +\frac{1}{12}b(be+2ah)x^{12}+\frac{1}{13}b^2fx^{13}+\frac{1}{14}b^2gx^{14}+\frac{1}{15}b^2hx^{15}
\end{aligned}$$

```
[In] Integrate[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*x^7)/7 + (
a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(b*c + 2*a*f)*x^10)/1
0 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a*h)*x^12)/12 + (b^2*f*x^13)/13
 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.93

method	result
default	$\frac{b^2 h\,x^{15}}{15}+\frac{b^2 g\,x^{14}}{14}+\frac{b^2 f\,x^{13}}{13}+\frac{(2abh+b^2e)x^{12}}{12}+\frac{(2abg+b^2d)x^{11}}{11}+\frac{(2afb+b^2c)x^{10}}{10}+\frac{(a^2h+2aeb)x^9}{9}+\frac{(a^2g+2ab}{8}$
norman	$\frac{a^2 c\,x^4}{4}+\frac{a^2 d\,x^5}{5}+\frac{a^2 e\,x^6}{6}+\left(\frac{1}{7}a^2 f+\frac{2}{7}abc\right)x^7+\left(\frac{1}{8}a^2 g+\frac{1}{4}abd\right)x^8+\left(\frac{1}{9}a^2 h+\frac{2}{9}aeb\right)x^9+\left(\frac{1}{5}afb\right.$
gosper	$\frac{1}{4}a^2 c\,x^4+\frac{1}{5}a^2 d\,x^5+\frac{1}{6}a^2 e\,x^6+\frac{1}{7}a^2 f\,x^7+\frac{2}{7}x^7 abc+\frac{1}{8}x^8 a^2 g+\frac{1}{4}x^8 abd+\frac{1}{9}x^9 a^2 h+\frac{2}{9}x^9 aeb+$
risch	$\frac{1}{4}a^2 c\,x^4+\frac{1}{5}a^2 d\,x^5+\frac{1}{6}a^2 e\,x^6+\frac{1}{7}a^2 f\,x^7+\frac{2}{7}x^7 abc+\frac{1}{8}x^8 a^2 g+\frac{1}{4}x^8 abd+\frac{1}{9}x^9 a^2 h+\frac{2}{9}x^9 aeb+$
parallelrisch	$\frac{1}{4}a^2 c\,x^4+\frac{1}{5}a^2 d\,x^5+\frac{1}{6}a^2 e\,x^6+\frac{1}{7}a^2 f\,x^7+\frac{2}{7}x^7 abc+\frac{1}{8}x^8 a^2 g+\frac{1}{4}x^8 abd+\frac{1}{9}x^9 a^2 h+\frac{2}{9}x^9 aeb+$

```
[In] int(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/15*b^2*h*x^15+1/14*b^2*g*x^14+1/13*b^2*f*x^13+1/12*(2*a*b*h+b^2*e)*x^12+1
/11*(2*a*b*g+b^2*d)*x^11+1/10*(2*a*b*f+b^2*c)*x^10+1/9*(a^2*h+2*a*b*e)*x^9+
1/8*(a^2*g+2*a*b*d)*x^8+1/7*(a^2*f+2*a*b*c)*x^7+1/6*a^2*e*x^6+1/5*a^2*d*x^5
+1/4*a^2*c*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\begin{aligned}
&\int x^3\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx \\
&=\frac{1}{15}\,b^2hx^{15}+\frac{1}{14}\,b^2gx^{14}+\frac{1}{13}\,b^2fx^{13}+\frac{1}{12}\,\left(b^2e+2\,abh\right)x^{12} \\
&\quad+\frac{1}{11}\,\left(b^2d+2\,abg\right)x^{11}+\frac{1}{10}\,\left(b^2c+2\,abf\right)x^{10}+\frac{1}{9}\,\left(2\,abe+a^2h\right)x^9 \\
&\quad+\frac{1}{6}\,a^2ex^6+\frac{1}{8}\,\left(2\,abd+a^2g\right)x^8+\frac{1}{5}\,a^2dx^5+\frac{1}{7}\,\left(2\,abc+a^2f\right)x^7+\frac{1}{4}\,a^2cx^4
\end{aligned}$$

```
[In] integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/12*(b^2*e + 2*a*b*h
)*x^12 + 1/11*(b^2*d + 2*a*b*g)*x^11 + 1/10*(b^2*c + 2*a*b*f)*x^10 + 1/9*(2
*a*b*e + a^2*h)*x^9 + 1/6*a^2*e*x^6 + 1/8*(2*a*b*d + a^2*g)*x^8 + 1/5*a^2*d
*x^5 + 1/7*(2*a*b*c + a^2*f)*x^7 + 1/4*a^2*c*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02

$$\int x^3(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{a^2cx^4}{4}+\frac{a^2dx^5}{5}+\frac{a^2ex^6}{6}+\frac{b^2fx^{13}}{13}+\frac{b^2gx^{14}}{14}+\frac{b^2hx^{15}}{15}+x^{12}\left(\frac{abh}{6}+\frac{b^2e}{12}\right)+x^{11}\cdot\left(\frac{2abg}{11}+\frac{b^2d}{11}\right)+x^{10}\left(\frac{abf}{5}+\frac{b^2c}{10}\right)+x^9\left(\frac{a^2h}{9}+\frac{2abe}{9}\right)+x^8\left(\frac{a^2g}{8}+\frac{abd}{4}\right)+x^7\left(\frac{a^2f}{7}+\frac{2abc}{7}\right)$$

```
[In] integrate(x**3*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + b**2*f*x**13/13 + b**2*g*x*
*14/14 + b**2*h*x**15/15 + x**12*(a*b*h/6 + b**2*e/12) + x**11*(2*a*b*g/11
+ b**2*d/11) + x**10*(a*b*f/5 + b**2*c/10) + x**9*(a**2*h/9 + 2*a*b*e/9) +
x**8*(a**2*g/8 + a*b*d/4) + x**7*(a**2*f/7 + 2*a*b*c/7)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int x^3(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{15}b^2hx^{15}+\frac{1}{14}b^2gx^{14}+\frac{1}{13}b^2fx^{13}+\frac{1}{12}(b^2e+2\,abh)x^{12}+\frac{1}{11}(b^2d+2\,abg)x^{11}+\frac{1}{10}(b^2c+2\,abf)x^{10}+\frac{1}{9}(2\,abe+a^2h)x^9+\frac{1}{6}a^2ex^6+\frac{1}{8}(2\,abd+a^2g)x^8+\frac{1}{5}a^2dx^5+\frac{1}{7}(2\,abc+a^2f)x^7+\frac{1}{4}a^2cx^4$$

```
[In] integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/12*(b^2*e + 2*a*b*h
)*x^12 + 1/11*(b^2*d + 2*a*b*g)*x^11 + 1/10*(b^2*c + 2*a*b*f)*x^10 + 1/9*(2
*a*b*e + a^2*h)*x^9 + 1/6*a^2*e*x^6 + 1/8*(2*a*b*d + a^2*g)*x^8 + 1/5*a^2*d
*x^5 + 1/7*(2*a*b*c + a^2*f)*x^7 + 1/4*a^2*c*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.96

$$\int x^3(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{15}\,b^2hx^{15}+\frac{1}{14}\,b^2gx^{14}+\frac{1}{13}\,b^2fx^{13}+\frac{1}{12}\,b^2ex^{12}+\frac{1}{6}\,abhx^{12}+\frac{1}{11}\,b^2dx^{11}+\frac{2}{11}\,abgx^{11}+\frac{1}{10}\,b^2cx^{10}+\frac{1}{5}\,abfx^{10}+\frac{2}{9}\,abex^9+\frac{1}{9}\,a^2hx^9+\frac{1}{4}\,abdx^8+\frac{1}{8}\,a^2gx^8+\frac{2}{7}\,abcx^7+\frac{1}{7}\,a^2fx^7+\frac{1}{6}\,a^2ex^6+\frac{1}{5}\,a^2dx^5+\frac{1}{4}\,a^2cx^4$$

```
[In] integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/12*b^2*e*x^12 + 1/6
*a*b*h*x^12 + 1/11*b^2*d*x^11 + 2/11*a*b*g*x^11 + 1/10*b^2*c*x^10 + 1/5*a*b
*f*x^10 + 2/9*a*b*e*x^9 + 1/9*a^2*h*x^9 + 1/4*a*b*d*x^8 + 1/8*a^2*g*x^8 + 2
/7*a*b*c*x^7 + 1/7*a^2*f*x^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^
4
```

Mupad [B] (verification not implemented)

Time = 10.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int x^3(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = x^7\left(\frac{f\,a^2}{7}+\frac{2\,b\,c\,a}{7}\right)+x^{10}\left(\frac{c\,b^2}{10}+\frac{a\,f\,b}{5}\right)+x^8\left(\frac{g\,a^2}{8}+\frac{b\,d\,a}{4}\right)+x^{11}\left(\frac{d\,b^2}{11}+\frac{2\,a\,g\,b}{11}\right)+x^9\left(\frac{h\,a^2}{9}+\frac{2\,b\,e\,a}{9}\right)+x^{12}\left(\frac{e\,b^2}{12}+\frac{a\,h\,b}{6}\right)+\frac{a^2\,c\,x^4}{4}+\frac{a^2\,d\,x^5}{5}+\frac{a^2\,e\,x^6}{6}+\frac{b^2\,f\,x^{13}}{13}+\frac{b^2\,g\,x^{14}}{14}+\frac{b^2\,h\,x^{15}}{15}$$

```
[In] int(x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^7*((a^2*f)/7 + (2*a*b*c)/7) + x^10*((b^2*c)/10 + (a*b*f)/5) + x^8*((a^2*g
)/8 + (a*b*d)/4) + x^11*((b^2*d)/11 + (2*a*b*g)/11) + x^9*((a^2*h)/9 + (2*a
*b*e)/9) + x^12*((b^2*e)/12 + (a*b*h)/6) + (a^2*c*x^4)/4 + (a^2*d*x^5)/5 +
(a^2*e*x^6)/6 + (b^2*f*x^13)/13 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15
```

3.385 $\int x^2(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx$

Optimal result

Integrand size = 38, antiderivative size = 158

$$\begin{aligned}&\int x^2\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\&=\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{1}{7}a(2bd+ag)x^7+\frac{1}{8}a(2be+ah)x^8+\frac{2}{9}abfx^9\\&+\frac{1}{10}b(bd+2ag)x^{10}+\frac{1}{11}b(be+2ah)x^{11}+\frac{1}{12}b^2fx^{12}+\frac{1}{13}b^2gx^{13}+\frac{1}{14}b^2hx^{14}+\frac{c(a+bx^3)^3}{9b}\end{aligned}$$

```
[Out] 1/4*a^2*d*x^4+1/5*a^2*e*x^5+1/6*a^2*f*x^6+1/7*a*(a*g+2*b*d)*x^7+1/8*a*(a*h+
2*b*e)*x^8+2/9*a*b*f*x^9+1/10*b*(2*a*g+b*d)*x^10+1/11*b*(2*a*h+b*e)*x^11+1/
12*b^2*f*x^12+1/13*b^2*g*x^13+1/14*b^2*h*x^14+1/9*c*(b*x^3+a)^3/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1596, 1864}

$$\begin{aligned}&\int x^2\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\&=\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{c(a+bx^3)^3}{9b}+\frac{1}{10}bx^{10}(2ag+bd)+\frac{1}{7}ax^7(ag+2bd)\\&+\frac{1}{11}bx^{11}(2ah+be)+\frac{1}{8}ax^8(ah+2be)+\frac{2}{9}abfx^9+\frac{1}{12}b^2fx^{12}+\frac{1}{13}b^2gx^{13}+\frac{1}{14}b^2hx^{14}\end{aligned}$$

```
[In] Int[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a^2*f*x^6)/6 + (a*(2*b*d + a*g)*x^7)/7 + (
a*(2*b*e + a*h)*x^8)/8 + (2*a*b*f*x^9)/9 + (b*(b*d + 2*a*g)*x^10)/10 + (b*(
```

```
b*e + 2*a*h)*x^11)/11 + (b^2*f*x^12)/12 + (b^2*g*x^13)/13 + (b^2*h*x^14)/14
 + (c*(a + b*x^3)^3)/(9*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{c(a+bx^3)^3}{9b} + \int \left(a+bx^3\right)^2 \left(-cx^2+x^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\right)\,dx \\ &= \frac{c(a+bx^3)^3}{9b} + \int \left(a^2dx^3+a^2ex^4+a^2fx^5+a(2bd+ag)x^6+a(2be+ah)x^7+2abfx^8\right. \\ &\qquad \left.+b(bd+2ag)x^9+b(be+2ah)x^{10}+b^2fx^{11}+b^2gx^{12}+b^2hx^{13}\right)\,dx \\ &= \frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{1}{7}a(2bd+ag)x^7+\frac{1}{8}a(2be+ah)x^8+\frac{2}{9}abfx^9+\frac{1}{10}b(bd \\ &\qquad +2ag)x^{10}+\frac{1}{11}b(be+2ah)x^{11}+\frac{1}{12}b^2fx^{12}+\frac{1}{13}b^2gx^{13}+\frac{1}{14}b^2hx^{14}+\frac{c(a+bx^3)^3}{9b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.95

$$\begin{aligned}&\int x^2\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx \\ &= a^2\left(\frac{cx^3}{3}+\frac{dx^4}{4}+\frac{ex^5}{5}+\frac{fx^6}{6}+\frac{gx^7}{7}+\frac{hx^8}{8}\right)+ab\left(\frac{cx^6}{3}+\frac{2dx^7}{7}+\frac{ex^8}{4}+\frac{2fx^9}{9}+\frac{gx^{10}}{5}+\frac{2hx^{11}}{11}\right) \\ &\quad + \frac{b^2x^9(20020c+3x(6006d+5460ex+55x^2(91f+84gx+78hx^2)))}{180180}\end{aligned}$$

```
[In] Integrate[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a^2*((c*x^3)/3 + (d*x^4)/4 + (e*x^5)/5 + (f*x^6)/6 + (g*x^7)/7 + (h*x^8)/8)
 + a*b*((c*x^6)/3 + (2*d*x^7)/7 + (e*x^8)/4 + (2*f*x^9)/9 + (g*x^10)/5 + (2
*h*x^11)/11) + (b^2*x^9*(20020*c + 3*x*(6006*d + 5460*e*x + 55*x^2*(91*f +
84*g*x + 78*h*x^2))))/180180
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

method	result
default	$\frac{b^2 h\, x^{14}}{14}+\frac{b^2 g\, x^{13}}{13}+\frac{b^2 f\, x^{12}}{12}+\frac{(2abh+b^2e)x^{11}}{11}+\frac{(2abg+b^2d)x^{10}}{10}+\frac{(2afb+b^2c)x^9}{9}+\frac{(a^2h+2aeb)x^8}{8}+\frac{(a^2g+2abd)}{7}$
norman	$\frac{a^2c\, x^3}{3}+\frac{a^2d\, x^4}{4}+\frac{a^2e\, x^5}{5}+\left(\frac{1}{6}a^2f+\frac{1}{3}abc\right)x^6+\left(\frac{1}{7}a^2g+\frac{2}{7}abd\right)x^7+\left(\frac{1}{8}a^2h+\frac{1}{4}aeb\right)x^8+\left(\frac{2}{9}afb+\right.$
gosper	$\frac{1}{3}a^2c\, x^3+\frac{1}{4}a^2d\, x^4+\frac{1}{5}a^2e\, x^5+\frac{1}{6}a^2f\, x^6+\frac{1}{3}abc\, x^6+\frac{1}{7}x^7a^2g+\frac{2}{7}ad\, x^7b+\frac{1}{8}x^8a^2h+\frac{1}{4}abe\, x^8+$
risch	$\frac{1}{3}a^2c\, x^3+\frac{1}{4}a^2d\, x^4+\frac{1}{5}a^2e\, x^5+\frac{1}{6}a^2f\, x^6+\frac{1}{3}abc\, x^6+\frac{1}{7}x^7a^2g+\frac{2}{7}ad\, x^7b+\frac{1}{8}x^8a^2h+\frac{1}{4}abe\, x^8+$
parallelrisch	$\frac{1}{3}a^2c\, x^3+\frac{1}{4}a^2d\, x^4+\frac{1}{5}a^2e\, x^5+\frac{1}{6}a^2f\, x^6+\frac{1}{3}abc\, x^6+\frac{1}{7}x^7a^2g+\frac{2}{7}ad\, x^7b+\frac{1}{8}x^8a^2h+\frac{1}{4}abe\, x^8+$

```
[In] int(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/14*b^2*h*x^14+1/13*b^2*g*x^13+1/12*b^2*f*x^12+1/11*(2*a*b*h+b^2*e)*x^11+1
/10*(2*a*b*g+b^2*d)*x^10+1/9*(2*a*b*f+b^2*c)*x^9+1/8*(a^2*h+2*a*b*e)*x^8+1/
7*(a^2*g+2*a*b*d)*x^7+1/6*(a^2*f+2*a*b*c)*x^6+1/5*a^2*e*x^5+1/4*a^2*d*x^4+1
/3*a^2*c*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\begin{aligned}\int x^2\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{1}{14}\,b^2hx^{14}+\frac{1}{13}\,b^2gx^{13}+\frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,\left(b^2e+2\,abh\right)x^{11}\\ &\quad+\frac{1}{10}\,\left(b^2d+2\,abg\right)x^{10}+\frac{1}{9}\,\left(b^2c+2\,abf\right)x^9+\frac{1}{8}\,\left(2\,abe+a^2h\right)x^8\\ &\quad+\frac{1}{5}\,a^2ex^5+\frac{1}{7}\,\left(2\,abd+a^2g\right)x^7+\frac{1}{4}\,a^2dx^4+\frac{1}{6}\,\left(2\,abc+a^2f\right)x^6+\frac{1}{3}\,a^2cx^3\end{aligned}$$

```
[In] integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 1/11*(b^2*e + 2*a*b*h
)*x^11 + 1/10*(b^2*d + 2*a*b*g)*x^10 + 1/9*(b^2*c + 2*a*b*f)*x^9 + 1/8*(2*a
*b*e + a^2*h)*x^8 + 1/5*a^2*e*x^5 + 1/7*(2*a*b*d + a^2*g)*x^7 + 1/4*a^2*d*x
^4 + 1/6*(2*a*b*c + a^2*f)*x^6 + 1/3*a^2*c*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06

$$\int x^2\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{a^2cx^3}{3}+\frac{a^2dx^4}{4}+\frac{a^2ex^5}{5}+\frac{b^2fx^{12}}{12}+\frac{b^2gx^{13}}{13}+\frac{b^2hx^{14}}{14}+x^{11}\cdot\left(\frac{2abh}{11}+\frac{b^2e}{11}\right)+x^{10}\left(\frac{abg}{5}+\frac{b^2d}{10}\right)+x^9\cdot\left(\frac{2abf}{9}+\frac{b^2c}{9}\right)+x^8\left(\frac{a^2h}{8}+\frac{abe}{4}\right)+x^7\left(\frac{a^2g}{7}+\frac{2abd}{7}\right)+x^6\left(\frac{a^2f}{6}+\frac{abc}{3}\right)$$

```
[In] integrate(x**2*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**3/3 + a**2*d*x**4/4 + a**2*e*x**5/5 + b**2*f*x**12/12 + b**2*g*x*
*13/13 + b**2*h*x**14/14 + x**11*(2*a*b*h/11 + b**2*e/11) + x**10*(a*b*g/5
+ b**2*d/10) + x**9*(2*a*b*f/9 + b**2*c/9) + x**8*(a**2*h/8 + a*b*e/4) + x*
*7*(a**2*g/7 + 2*a*b*d/7) + x**6*(a**2*f/6 + a*b*c/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int x^2\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{14}\,b^2hx^{14}+\frac{1}{13}\,b^2gx^{13}+\frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,\left(b^2e+2\,abh\right)x^{11}+\frac{1}{10}\,\left(b^2d+2\,abg\right)x^{10}+\frac{1}{9}\,\left(b^2c+2\,abf\right)x^9+\frac{1}{8}\,\left(2\,abe+a^2h\right)x^8+\frac{1}{5}\,a^2ex^5+\frac{1}{7}\,\left(2\,abd+a^2g\right)x^7+\frac{1}{4}\,a^2dx^4+\frac{1}{6}\,\left(2\,abc+a^2f\right)x^6+\frac{1}{3}\,a^2cx^3$$

```
[In] integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 1/11*(b^2*e + 2*a*b*h
)*x^11 + 1/10*(b^2*d + 2*a*b*g)*x^10 + 1/9*(b^2*c + 2*a*b*f)*x^9 + 1/8*(2*a
*b*e + a^2*h)*x^8 + 1/5*a^2*e*x^5 + 1/7*(2*a*b*d + a^2*g)*x^7 + 1/4*a^2*d*x
^4 + 1/6*(2*a*b*c + a^2*f)*x^6 + 1/3*a^2*c*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int x^2(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{14}b^2hx^{14}+\frac{1}{13}b^2gx^{13}+\frac{1}{12}b^2fx^{12}+\frac{1}{11}b^2ex^{11}+\frac{2}{11}abhx^{11}+\frac{1}{10}b^2dx^{10}+\frac{1}{5}abgx^{10}+\frac{1}{9}b^2cx^9+\frac{2}{9}abfx^9+\frac{1}{4}abex^8+\frac{1}{8}a^2hx^8+\frac{2}{7}abdx^7+\frac{1}{7}a^2gx^7+\frac{1}{3}abcx^6+\frac{1}{6}a^2fx^6+\frac{1}{5}a^2ex^5+\frac{1}{4}a^2dx^4+\frac{1}{3}a^2cx^3$$

```
[In] integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 2/1
1*a*b*h*x^11 + 1/10*b^2*d*x^10 + 1/5*a*b*g*x^10 + 1/9*b^2*c*x^9 + 2/9*a*b*f
*x^9 + 1/4*a*b*e*x^8 + 1/8*a^2*h*x^8 + 2/7*a*b*d*x^7 + 1/7*a^2*g*x^7 + 1/3*
a*b*c*x^6 + 1/6*a^2*f*x^6 + 1/5*a^2*e*x^5 + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int x^2(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = x^6\left(\frac{f\,a^2}{6}+\frac{b\,c\,a}{3}\right)+x^9\left(\frac{c\,b^2}{9}+\frac{2\,a\,f\,b}{9}\right)+x^7\left(\frac{g\,a^2}{7}+\frac{2\,b\,d\,a}{7}\right)+x^{10}\left(\frac{d\,b^2}{10}+\frac{a\,g\,b}{5}\right)+x^8\left(\frac{h\,a^2}{8}+\frac{b\,e\,a}{4}\right)+x^{11}\left(\frac{e\,b^2}{11}+\frac{2\,a\,h\,b}{11}\right)+\frac{a^2\,c\,x^3}{3}+\frac{a^2\,d\,x^4}{4}+\frac{a^2\,e\,x^5}{5}+\frac{b^2\,f\,x^{12}}{12}+\frac{b^2\,g\,x^{13}}{13}+\frac{b^2\,h\,x^{14}}{14}$$

```
[In] int(x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^6*((a^2*f)/6 + (a*b*c)/3) + x^9*((b^2*c)/9 + (2*a*b*f)/9) + x^7*((a^2*g)/
7 + (2*a*b*d)/7) + x^10*((b^2*d)/10 + (a*b*g)/5) + x^8*((a^2*h)/8 + (a*b*e)
/4) + x^11*((b^2*e)/11 + (2*a*b*h)/11) + (a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (a
^2*e*x^5)/5 + (b^2*f*x^12)/12 + (b^2*g*x^13)/13 + (b^2*h*x^14)/14
```

3.386 $\int x(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx$

Optimal result

Integrand size = 36, antiderivative size = 158

$$\begin{aligned}&\int x(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx \\ &= \frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{1}{5}a(2bc+af)x^5+\frac{1}{6}a^2gx^6+\frac{1}{7}a(2be+ah)x^7+\frac{1}{8}b(bc+2af)x^8 \\ &\quad+\frac{2}{9}abgx^9+\frac{1}{10}b(be+2ah)x^{10}+\frac{1}{11}b^2fx^{11}+\frac{1}{12}b^2gx^{12}+\frac{1}{13}b^2hx^{13}+\frac{d(a+bx^3)^3}{9b}\end{aligned}$$

```
[Out] 1/2*a^2*c*x^2+1/4*a^2*e*x^4+1/5*a*(a*f+2*b*c)*x^5+1/6*a^2*g*x^6+1/7*a*(a*h+
2*b*e)*x^7+1/8*b*(2*a*f+b*c)*x^8+2/9*a*b*g*x^9+1/10*b*(2*a*h+b*e)*x^10+1/11
*b^2*f*x^11+1/12*b^2*g*x^12+1/13*b^2*h*x^13+1/9*d*(b*x^3+a)^3/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1596, 1864}

$$\begin{aligned}&\int x(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx \\ &= \frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{1}{6}a^2gx^6+\frac{1}{8}bx^8(2af+bc)+\frac{1}{5}ax^5(af+2bc)+\frac{d(a+bx^3)^3}{9b} \\ &\quad+\frac{1}{10}bx^{10}(2ah+be)+\frac{1}{7}ax^7(ah+2be)+\frac{2}{9}abgx^9+\frac{1}{11}b^2fx^{11}+\frac{1}{12}b^2gx^{12}+\frac{1}{13}b^2hx^{13}\end{aligned}$$

```
[In] Int[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 + (a^2*g*x^6)/6 + (
a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/8 + (2*a*b*g*x^9)/9 + (b*(b*
```

```
e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11 + (b^2*g*x^12)/12 + (b^2*h*x^13)/13 +
 (d*(a + b*x^3)^3)/(9*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(a+bx^3)^3}{9b} + \int (a+bx^3)^2 \left(-dx^2 + x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\right) dx \\
&= \frac{d(a+bx^3)^3}{9b} + \int \left(a^2cx + a^2ex^3 + a(2bc+af)x^4 + a^2gx^5 + a(2be+ah)x^6 \right. \\
&\quad \left. + b(bc+2af)x^7 + 2abgx^8 + b(be+2ah)x^9 + b^2fx^{10} + b^2gx^{11} + b^2hx^{12}\right) dx \\
&= \frac{1}{2}a^2cx^2 + \frac{1}{4}a^2ex^4 + \frac{1}{5}a(2bc+af)x^5 + \frac{1}{6}a^2gx^6 + \frac{1}{7}a(2be+ah)x^7 + \frac{1}{8}b(bc+2af)x^8 \\
&\quad + \frac{2}{9}abgx^9 + \frac{1}{10}b(be+2ah)x^{10} + \frac{1}{11}b^2fx^{11} + \frac{1}{12}b^2gx^{12} + \frac{1}{13}b^2hx^{13} + \frac{d(a+bx^3)^3}{9b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

$$\begin{aligned}
&\int x(a+bx^3)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx \\
&= \frac{1}{2}a^2cx^2 + \frac{1}{3}a^2dx^3 + \frac{1}{4}a^2ex^4 + \frac{1}{5}a(2bc+af)x^5 + \frac{1}{6}a(2bd+ag)x^6 + \frac{1}{7}a(2be+ah)x^7 \\
&\quad + \frac{1}{8}b(bc+2af)x^8 + \frac{1}{9}b(bd+2ag)x^9 + \frac{1}{10}b(be+2ah)x^{10} + \frac{1}{11}b^2fx^{11} + \frac{1}{12}b^2gx^{12} + \frac{1}{13}b^2hx^{13}
\end{aligned}$$

```
[In] Integrate[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 + (
a*(2*b*d + a*g)*x^6)/6 + (a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/8
+ (b*(b*d + 2*a*g)*x^9)/9 + (b*(b*e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11 + (
b^2*g*x^12)/12 + (b^2*h*x^13)/13
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

method	result
default	$\frac{b^2 h\,x^{13}}{13}+\frac{b^2 g\,x^{12}}{12}+\frac{b^2 f\,x^{11}}{11}+\frac{(2abh+b^2 e)x^{10}}{10}+\frac{(2abg+b^2 d)x^9}{9}+\frac{(2afb+b^2 c)x^8}{8}+\frac{(a^2 h+2aeb)x^7}{7}+\frac{(a^2 g+2abd}{6}$
norman	$\frac{b^2 h\,x^{13}}{13}+\frac{b^2 g\,x^{12}}{12}+\frac{b^2 f\,x^{11}}{11}+\left(\frac{1}{5}abh+\frac{1}{10}b^2 e\right)x^{10}+\left(\frac{2}{9}abg+\frac{1}{9}b^2 d\right)x^9+\left(\frac{1}{4}afb+\frac{1}{8}b^2 c\right)x^8+\left(\frac{1}{7}a\right.$
gosper	$\frac{1}{13}b^2 h\,x^{13}+\frac{1}{12}b^2 g\,x^{12}+\frac{1}{11}b^2 f\,x^{11}+\frac{1}{5}x^{10}abh+\frac{1}{10}b^2 e\,x^{10}+\frac{2}{9}abg\,x^9+\frac{1}{9}b^2 d\,x^9+\frac{1}{4}abf\,x^8+\frac{1}{8}$
risch	$\frac{1}{13}b^2 h\,x^{13}+\frac{1}{12}b^2 g\,x^{12}+\frac{1}{11}b^2 f\,x^{11}+\frac{1}{5}x^{10}abh+\frac{1}{10}b^2 e\,x^{10}+\frac{2}{9}abg\,x^9+\frac{1}{9}b^2 d\,x^9+\frac{1}{4}abf\,x^8+\frac{1}{8}$
parallelrisch	$\frac{1}{13}b^2 h\,x^{13}+\frac{1}{12}b^2 g\,x^{12}+\frac{1}{11}b^2 f\,x^{11}+\frac{1}{5}x^{10}abh+\frac{1}{10}b^2 e\,x^{10}+\frac{2}{9}abg\,x^9+\frac{1}{9}b^2 d\,x^9+\frac{1}{4}abf\,x^8+\frac{1}{8}$

```
[In] int(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^2*h*x^13+1/12*b^2*g*x^12+1/11*b^2*f*x^11+1/10*(2*a*b*h+b^2*e)*x^10+1
/9*(2*a*b*g+b^2*d)*x^9+1/8*(2*a*b*f+b^2*c)*x^8+1/7*(a^2*h+2*a*b*e)*x^7+1/6*
(a^2*g+2*a*b*d)*x^6+1/5*(a^2*f+2*a*b*c)*x^5+1/4*a^2*e*x^4+1/3*a^2*d*x^3+1/2
*a^2*c*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\begin{aligned}\int x(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{1}{13}\,b^2hx^{13}+\frac{1}{12}\,b^2gx^{12}+\frac{1}{11}\,b^2fx^{11}+\frac{1}{10}\,(b^2e+2\,abh)x^{10}\\&\quad+\frac{1}{9}\,(b^2d+2\,abg)x^9+\frac{1}{8}\,(b^2c+2\,abf)x^8+\frac{1}{7}\,(2\,abe+a^2h)x^7+\frac{1}{4}\,a^2ex^4\\&\quad+\frac{1}{6}\,(2\,abd+a^2g)x^6+\frac{1}{3}\,a^2dx^3+\frac{1}{5}\,(2\,abc+a^2f)x^5+\frac{1}{2}\,a^2cx^2\end{aligned}$$

```
[In] integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/10*(b^2*e + 2*a*b*h
)*x^10 + 1/9*(b^2*d + 2*a*b*g)*x^9 + 1/8*(b^2*c + 2*a*b*f)*x^8 + 1/7*(2*a*b
*e + a^2*h)*x^7 + 1/4*a^2*e*x^4 + 1/6*(2*a*b*d + a^2*g)*x^6 + 1/3*a^2*d*x^3
 + 1/5*(2*a*b*c + a^2*f)*x^5 + 1/2*a^2*c*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06

$$\int x(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{a^2cx^2}{2}+\frac{a^2dx^3}{3}+\frac{a^2ex^4}{4}+\frac{b^2fx^{11}}{11}+\frac{b^2gx^{12}}{12}+\frac{b^2hx^{13}}{13}+x^{10}\left(\frac{abh}{5}+\frac{b^2e}{10}\right)+x^9\cdot\left(\frac{2abg}{9}+\frac{b^2d}{9}\right)$$
$$+x^8\left(\frac{abf}{4}+\frac{b^2c}{8}\right)+x^7\left(\frac{a^2h}{7}+\frac{2abe}{7}\right)+x^6\left(\frac{a^2g}{6}+\frac{abd}{3}\right)+x^5\left(\frac{a^2f}{5}+\frac{2abc}{5}\right)$$

```
[In] integrate(x*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**2/2 + a**2*d*x**3/3 + a**2*e*x**4/4 + b**2*f*x**11/11 + b**2*g*x*
*12/12 + b**2*h*x**13/13 + x**10*(a*b*h/5 + b**2*e/10) + x**9*(2*a*b*g/9 +
b**2*d/9) + x**8*(a*b*f/4 + b**2*c/8) + x**7*(a**2*h/7 + 2*a*b*e/7) + x**6*
(a**2*g/6 + a*b*d/3) + x**5*(a**2*f/5 + 2*a*b*c/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int x(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{13}b^2hx^{13}+\frac{1}{12}b^2gx^{12}+\frac{1}{11}b^2fx^{11}+\frac{1}{10}\left(b^2e+2\,abh\right)x^{10}$$
$$+\frac{1}{9}\left(b^2d+2\,abg\right)x^9+\frac{1}{8}\left(b^2c+2\,abf\right)x^8+\frac{1}{7}\left(2\,abe+a^2h\right)x^7+\frac{1}{4}a^2ex^4$$
$$+\frac{1}{6}\left(2\,abd+a^2g\right)x^6+\frac{1}{3}a^2dx^3+\frac{1}{5}\left(2\,abc+a^2f\right)x^5+\frac{1}{2}a^2cx^2$$

```
[In] integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/10*(b^2*e + 2*a*b*h
)*x^10 + 1/9*(b^2*d + 2*a*b*g)*x^9 + 1/8*(b^2*c + 2*a*b*f)*x^8 + 1/7*(2*a*b
*e + a^2*h)*x^7 + 1/4*a^2*e*x^4 + 1/6*(2*a*b*d + a^2*g)*x^6 + 1/3*a^2*d*x^3
 + 1/5*(2*a*b*c + a^2*f)*x^5 + 1/2*a^2*c*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\begin{aligned}\int x(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= \frac{1}{13}\,b^2hx^{13}+\frac{1}{12}\,b^2gx^{12}+\frac{1}{11}\,b^2fx^{11}+\frac{1}{10}\,b^2ex^{10}+\frac{1}{5}\,abhx^{10}+\frac{1}{9}\,b^2dx^9\\&\quad+\frac{2}{9}\,abgx^9+\frac{1}{8}\,b^2cx^8+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abex^7+\frac{1}{7}\,a^2hx^7+\frac{1}{3}\,abdx^6\\&\quad+\frac{1}{6}\,a^2gx^6+\frac{2}{5}\,abcx^5+\frac{1}{5}\,a^2fx^5+\frac{1}{4}\,a^2ex^4+\frac{1}{3}\,a^2dx^3+\frac{1}{2}\,a^2cx^2\end{aligned}$$

```
[In] integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/10*b^2*e*x^10 + 1/5
*a*b*h*x^10 + 1/9*b^2*d*x^9 + 2/9*a*b*g*x^9 + 1/8*b^2*c*x^8 + 1/4*a*b*f*x^8
 + 2/7*a*b*e*x^7 + 1/7*a^2*h*x^7 + 1/3*a*b*d*x^6 + 1/6*a^2*g*x^6 + 2/5*a*b*
c*x^5 + 1/5*a^2*f*x^5 + 1/4*a^2*e*x^4 + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\begin{aligned}\int x(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= x^5\left(\frac{f\,a^2}{5}+\frac{2\,b\,c\,a}{5}\right)+x^8\left(\frac{c\,b^2}{8}+\frac{a\,f\,b}{4}\right)+x^6\left(\frac{g\,a^2}{6}+\frac{b\,d\,a}{3}\right)\\&\quad+x^9\left(\frac{d\,b^2}{9}+\frac{2\,a\,g\,b}{9}\right)+x^7\left(\frac{h\,a^2}{7}+\frac{2\,b\,e\,a}{7}\right)+x^{10}\left(\frac{e\,b^2}{10}+\frac{a\,h\,b}{5}\right)\\&\quad+\frac{a^2\,c\,x^2}{2}+\frac{a^2\,d\,x^3}{3}+\frac{a^2\,e\,x^4}{4}+\frac{b^2\,f\,x^{11}}{11}+\frac{b^2\,g\,x^{12}}{12}+\frac{b^2\,h\,x^{13}}{13}\end{aligned}$$

```
[In] int(x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^5*((a^2*f)/5 + (2*a*b*c)/5) + x^8*((b^2*c)/8 + (a*b*f)/4) + x^6*((a^2*g)/
6 + (a*b*d)/3) + x^9*((b^2*d)/9 + (2*a*b*g)/9) + x^7*((a^2*h)/7 + (2*a*b*e)
/7) + x^10*((b^2*e)/10 + (a*b*h)/5) + (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*
e*x^4)/4 + (b^2*f*x^11)/11 + (b^2*g*x^12)/12 + (b^2*h*x^13)/13
```

3.387 $\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 153

$$\begin{aligned}\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{4}a(2bc+af)x^4+\frac{1}{5}a(2bd+ag)x^5+\frac{1}{6}a^2hx^6+\frac{1}{7}b(bc+2af)x^7\\ &\quad+\frac{1}{8}b(bd+2ag)x^8+\frac{2}{9}abhx^9+\frac{1}{10}b^2fx^{10}+\frac{1}{11}b^2gx^{11}+\frac{1}{12}b^2hx^{12}+\frac{e(a+bx^3)^3}{9b}\end{aligned}$$

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/4*a*(a*f+2*b*c)*x^4+1/5*a*(a*g+2*b*d)*x^5+1/6*a^2*h
*x^6+1/7*b*(2*a*f+b*c)*x^7+1/8*b*(2*a*g+b*d)*x^8+2/9*a*b*h*x^9+1/10*b^2*f*x
^10+1/11*b^2*g*x^11+1/12*b^2*h*x^12+1/9*e*(b*x^3+a)^3/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1596, 1864}

$$\begin{aligned}\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{6}a^2hx^6+\frac{1}{7}bx^7(2af+bc)+\frac{1}{4}ax^4(af+2bc)+\frac{1}{8}bx^8(2ag+bd)\\ &\quad+\frac{1}{5}ax^5(ag+2bd)+\frac{e(a+bx^3)^3}{9b}+\frac{2}{9}abhx^9+\frac{1}{10}b^2fx^{10}+\frac{1}{11}b^2gx^{11}+\frac{1}{12}b^2hx^{12}\end{aligned}$$

```
[In] Int[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*(2*b*c + a*f)*x^4)/4 + (a*(2*b*d + a*g)*x^5)/5
 + (a^2*h*x^6)/6 + (b*(b*c + 2*a*f)*x^7)/7 + (b*(b*d + 2*a*g)*x^8)/8 + (2*a
```

```
*b*h*x^9)/9 + (b^2*f*x^10)/10 + (b^2*g*x^11)/11 + (b^2*h*x^12)/12 + (e*(a +
 b*x^3)^3)/(9*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e(a+bx^3)^3}{9b} + \int \left(a+bx^3\right)^2 \left(c+dx+fx^3+gx^4+hx^5\right)\, dx \\
&= \frac{e(a+bx^3)^3}{9b} + \int \left(a^2c+a^2dx+a(2bc+af)x^3+a(2bd+ag)x^4+a^2hx^5 \right. \\
&\qquad \left. + b(bc+2af)x^6+b(bd+2ag)x^7+2abhx^8+b^2fx^9+b^2gx^{10}+b^2hx^{11}\right)\, dx \\
&= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{4}a(2bc+af)x^4+\frac{1}{5}a(2bd+ag)x^5+\frac{1}{6}a^2hx^6+\frac{1}{7}b(bc+2af)x^7 \\
&\quad +\frac{1}{8}b(bd+2ag)x^8+\frac{2}{9}abhx^9+\frac{1}{10}b^2fx^{10}+\frac{1}{11}b^2gx^{11}+\frac{1}{12}b^2hx^{12}+\frac{e(a+bx^3)^3}{9b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.82

$$\int \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\, dx$$
$$= \frac{b^2x^7(3960c+7x(495d+440ex+6x^2(66f+60gx+55hx^2)))+462a^2x(60c+x(30d+x(20e+15fx+1}{27720}$$

```
[In] Integrate[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (b^2*x^7*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55*h*x^2))
) + 462*a^2*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))) +
 22*a*b*x^4*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x)))))
)/27720
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^2 h\,x^{12}}{12}+\frac{b^2 g\,x^{11}}{11}+\frac{b^2 f\,x^{10}}{10}+\frac{(2abh+b^2 e)x^9}{9}+\frac{(2abg+b^2 d)x^8}{8}+\frac{(2afb+b^2 c)x^7}{7}+\frac{(a^2 h+2aeb)x^6}{6}+\frac{(a^2 g+2abd)x^5}{5}$
norman	$\frac{b^2 h\,x^{12}}{12}+\frac{b^2 g\,x^{11}}{11}+\frac{b^2 f\,x^{10}}{10}+\left(\frac{2}{9}abh+\frac{1}{9}b^2 e\right)x^9+\left(\frac{1}{4}abg+\frac{1}{8}b^2 d\right)x^8+\left(\frac{2}{7}afb+\frac{1}{7}b^2 c\right)x^7+\left(\frac{1}{6}a^2 h\right.$
gosper	$\frac{1}{12}b^2 h\,x^{12}+\frac{1}{11}b^2 g\,x^{11}+\frac{1}{10}b^2 f\,x^{10}+\frac{2}{9}abh\,x^9+\frac{1}{9}b^2 e\,x^9+\frac{1}{4}x^8 abg+\frac{1}{8}b^2 d\,x^8+\frac{2}{7}x^7 afb+\frac{1}{7}b^2 c\,x$
risch	$\frac{1}{12}b^2 h\,x^{12}+\frac{1}{11}b^2 g\,x^{11}+\frac{1}{10}b^2 f\,x^{10}+\frac{2}{9}abh\,x^9+\frac{1}{9}b^2 e\,x^9+\frac{1}{4}x^8 abg+\frac{1}{8}b^2 d\,x^8+\frac{2}{7}x^7 afb+\frac{1}{7}b^2 c\,x$
parallelrisch	$\frac{1}{12}b^2 h\,x^{12}+\frac{1}{11}b^2 g\,x^{11}+\frac{1}{10}b^2 f\,x^{10}+\frac{2}{9}abh\,x^9+\frac{1}{9}b^2 e\,x^9+\frac{1}{4}x^8 abg+\frac{1}{8}b^2 d\,x^8+\frac{2}{7}x^7 afb+\frac{1}{7}b^2 c\,x$

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^2*h*x^12+1/11*b^2*g*x^11+1/10*b^2*f*x^10+1/9*(2*a*b*h+b^2*e)*x^9+1/8
*(2*a*b*g+b^2*d)*x^8+1/7*(2*a*b*f+b^2*c)*x^7+1/6*(a^2*h+2*a*b*e)*x^6+1/5*(a
^2*g+2*a*b*d)*x^5+1/4*(a^2*f+2*a*b*c)*x^4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c
*x
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\begin{aligned}\int (a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{1}{12}\,b^2hx^{12}+\frac{1}{11}\,b^2gx^{11}+\frac{1}{10}\,b^2fx^{10}+\frac{1}{9}\,(b^2e+2\,abh)x^9\\&\quad+\frac{1}{8}\,(b^2d+2\,abg)x^8+\frac{1}{7}\,(b^2c+2\,abf)x^7+\frac{1}{6}\,(2\,abe+a^2h)x^6\\&\quad+\frac{1}{3}\,a^2ex^3+\frac{1}{5}\,(2\,abd+a^2g)x^5+\frac{1}{2}\,a^2dx^2+\frac{1}{4}\,(2\,abc+a^2f)x^4+a^2cx\end{aligned}$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] 1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 1/9*(b^2*e + 2*a*b*h)
*x^9 + 1/8*(b^2*d + 2*a*b*g)*x^8 + 1/7*(b^2*c + 2*a*b*f)*x^7 + 1/6*(2*a*b*e
 + a^2*h)*x^6 + 1/3*a^2*e*x^3 + 1/5*(2*a*b*d + a^2*g)*x^5 + 1/2*a^2*d*x^2 +
 1/4*(2*a*b*c + a^2*f)*x^4 + a^2*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07

$$\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{b^2fx^{10}}{10}+\frac{b^2gx^{11}}{11}+\frac{b^2hx^{12}}{12}+x^9\cdot\left(\frac{2abh}{9}+\frac{b^2e}{9}\right)+x^8\left(\frac{abg}{4}+\frac{b^2d}{8}\right)+x^7\cdot\left(\frac{2abf}{7}+\frac{b^2c}{7}\right)+x^6\left(\frac{a^2h}{6}+\frac{abe}{3}\right)+x^5\left(\frac{a^2g}{5}+\frac{2abd}{5}\right)+x^4\left(\frac{a^2f}{4}+\frac{abc}{2}\right)$$

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + b**2*f*x**10/10 + b**2*g*x**11/1
1 + b**2*h*x**12/12 + x**9*(2*a*b*h/9 + b**2*e/9) + x**8*(a*b*g/4 + b**2*d/
8) + x**7*(2*a*b*f/7 + b**2*c/7) + x**6*(a**2*h/6 + a*b*e/3) + x**5*(a**2*g
/5 + 2*a*b*d/5) + x**4*(a**2*f/4 + a*b*c/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{12}\,b^2hx^{12}+\frac{1}{11}\,b^2gx^{11}+\frac{1}{10}\,b^2fx^{10}+\frac{1}{9}\,(b^2e+2\,abh)x^9+\frac{1}{8}\,(b^2d+2\,abg)x^8+\frac{1}{7}\,(b^2c+2\,abf)x^7+\frac{1}{6}\,(2\,abe+a^2h)x^6+\frac{1}{3}\,a^2ex^3+\frac{1}{5}\,(2\,abd+a^2g)x^5+\frac{1}{2}\,a^2dx^2+\frac{1}{4}\,(2\,abc+a^2f)x^4+a^2cx$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
```

```
[Out] 1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 1/9*(b^2*e + 2*a*b*h)
*x^9 + 1/8*(b^2*d + 2*a*b*g)*x^8 + 1/7*(b^2*c + 2*a*b*f)*x^7 + 1/6*(2*a*b*e
 + a^2*h)*x^6 + 1/3*a^2*e*x^3 + 1/5*(2*a*b*d + a^2*g)*x^5 + 1/2*a^2*d*x^2 +
 1/4*(2*a*b*c + a^2*f)*x^4 + a^2*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01

$$\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx$$
$$= \frac{1}{12} b^2 h x^{12} + \frac{1}{11} b^2 g x^{11} + \frac{1}{10} b^2 f x^{10} + \frac{1}{9} b^2 e x^9 + \frac{2}{9} abhx^9 + \frac{1}{8} b^2 dx^8 + \frac{1}{4} abgx^8 + \frac{1}{7} b^2 cx^7 + \frac{2}{7} abfx^7 + \frac{1}{3} abex^6 + \frac{1}{6} a^2 hx^6 + \frac{2}{5} abdx^5 + \frac{1}{5} a^2 gx^5 + \frac{1}{2} abcx^4 + \frac{1}{4} a^2 fx^4 + \frac{1}{3} a^2 ex^3 + \frac{1}{2} a^2 dx^2 + a^2 cx$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 1/9*b^2*e*x^9 + 2/9*a
*b*h*x^9 + 1/8*b^2*d*x^8 + 1/4*a*b*g*x^8 + 1/7*b^2*c*x^7 + 2/7*a*b*f*x^7 +
1/3*a*b*e*x^6 + 1/6*a^2*h*x^6 + 2/5*a*b*d*x^5 + 1/5*a^2*g*x^5 + 1/2*a*b*c*x
^4 + 1/4*a^2*f*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx$$
$$= x^4 \left(\frac{f\,a^2}{4} + \frac{b\,c\,a}{2}\right) + x^7 \left(\frac{c\,b^2}{7} + \frac{2\,a\,f\,b}{7}\right) + x^5 \left(\frac{g\,a^2}{5} + \frac{2\,b\,d\,a}{5}\right) + x^8 \left(\frac{d\,b^2}{8} + \frac{a\,g\,b}{4}\right) + x^6 \left(\frac{h\,a^2}{6} + \frac{b\,e\,a}{3}\right) + x^9 \left(\frac{e\,b^2}{9} + \frac{2\,a\,h\,b}{9}\right) + \frac{a^2\,d\,x^2}{2} + \frac{a^2\,e\,x^3}{3} + \frac{b^2\,f\,x^{10}}{10} + \frac{b^2\,g\,x^{11}}{11} + \frac{b^2\,h\,x^{12}}{12} + a^2\,c\,x$$

```
[In] int((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^4*((a^2*f)/4 + (a*b*c)/2) + x^7*((b^2*c)/7 + (2*a*b*f)/7) + x^5*((a^2*g)/
5 + (2*a*b*d)/5) + x^8*((b^2*d)/8 + (a*b*g)/4) + x^6*((a^2*h)/6 + (a*b*e)/3
) + x^9*((b^2*e)/9 + (2*a*b*h)/9) + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (b^2*f*
x^10)/10 + (b^2*g*x^11)/11 + (b^2*h*x^12)/12 + a^2*c*x
```

3.388 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 149

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{4}a(2bd+ag)x^4+\frac{1}{5}a(2be+ah)x^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b(bd+2ag)x^7+\frac{1}{8}b(be+2ah)x^8+\frac{1}{10}b^2gx^{10}+\frac{1}{11}b^2hx^{11}+\frac{f(a+bx^3)^3}{9b}+a^2c\log(x)$$

```
[Out] a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*c*x^3+1/4*a*(a*g+2*b*d)*x^4+1/5*a*(a*h+2*b*e)
*x^5+1/6*b^2*c*x^6+1/7*b*(2*a*g+b*d)*x^7+1/8*b*(2*a*h+b*e)*x^8+1/10*b^2*g*x
^10+1/11*b^2*h*x^11+1/9*f*(b*x^3+a)^3/b+a^2*c*ln(x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1597, 1834}

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{7}bx^7(2ag+bd)+\frac{1}{4}ax^4(ag+2bd)+\frac{1}{8}bx^8(2ah+be)+\frac{1}{5}ax^5(ah+2be)+\frac{f(a+bx^3)^3}{9b}+\frac{1}{6}b^2cx^6+\frac{1}{10}b^2gx^{10}+\frac{1}{11}b^2hx^{11}$$

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*(2*b*d + a*g)*x^4)/4 + (a*(2
*b*e + a*h)*x^5)/5 + (b^2*c*x^6)/6 + (b*(b*d + 2*a*g)*x^7)/7 + (b*(b*e + 2*
a*h)*x^8)/8 + (b^2*g*x^10)/10 + (b^2*h*x^11)/11 + (f*(a + b*x^3)^3)/(9*b) +
 a^2*c*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{f(a+bx^3)^3}{9b} + \int \frac{(a+bx^3)^2(c+dx+ex^2+gx^4+hx^5)}{x}\,dx \\ &= \frac{f(a+bx^3)^3}{9b} + \int \left(a^2d+\frac{a^2c}{x}+a^2ex+2abcx^2+a(2bd+ag)x^3+a(2be+ah)x^4 \right. \\ &\quad \left. +b^2cx^5+b(bd+2ag)x^6+b(be+2ah)x^7+b^2gx^9+b^2hx^{10}\right)dx \\ &= a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{4}a(2bd+ag)x^4+\frac{1}{5}a(2be+ah)x^5+\frac{1}{6}b^2cx^6 \\ &\quad +\frac{1}{7}b(bd+2ag)x^7+\frac{1}{8}b(be+2ah)x^8+\frac{1}{10}b^2gx^{10}+\frac{1}{11}b^2hx^{11}+\frac{f(a+bx^3)^3}{9b}+a^2c\log(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03

$$\begin{aligned} &\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx \\ &= a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bc+af)x^3+\frac{1}{4}a(2bd+ag)x^4+\frac{1}{5}a(2be+ah)x^5+\frac{1}{6}b(bc+2af)x^6 \\ &\quad +\frac{1}{7}b(bd+2ag)x^7+\frac{1}{8}b(be+2ah)x^8+\frac{1}{9}b^2fx^9+\frac{1}{10}b^2gx^{10}+\frac{1}{11}b^2hx^{11}+a^2c\log(x) \end{aligned}$$

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*c + a*f)*x^3)/3 + (a*(2*b*d + a*g)*x^4)/4
 + (a*(2*b*e + a*h)*x^5)/5 + (b*(b*c + 2*a*f)*x^6)/6 + (b*(b*d + 2*a*g)*x^7
)/7 + (b*(b*e + 2*a*h)*x^8)/8 + (b^2*f*x^9)/9 + (b^2*g*x^10)/10 + (b^2*h*x^
11)/11 + a^2*c*Log[x]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99

method	result
norman	$\left(\frac{1}{3}a^2 f+\frac{2}{3}abc\right)x^3+\left(\frac{1}{4}a^2 g+\frac{1}{2}abd\right)x^4+\left(\frac{1}{5}a^2 h+\frac{2}{5}aeb\right)x^5+\left(\frac{2}{7}abg+\frac{1}{7}b^2 d\right)x^7+\left(\frac{1}{4}abh+\frac{1}{8}\right.$
default	$\frac{b^2 h\,x^{11}}{11}+\frac{b^2 g\,x^{10}}{10}+\frac{b^2 f\,x^9}{9}+\frac{abh\,x^8}{4}+\frac{b^2 e\,x^8}{8}+\frac{2abg\,x^7}{7}+\frac{b^2 d\,x^7}{7}+\frac{abf\,x^6}{3}+\frac{b^2 c\,x^6}{6}+\frac{a^2 h\,x^5}{5}+\frac{2abe\,x^5}{5}+$
risch	$\frac{b^2 h\,x^{11}}{11}+\frac{b^2 g\,x^{10}}{10}+\frac{b^2 f\,x^9}{9}+\frac{abh\,x^8}{4}+\frac{b^2 e\,x^8}{8}+\frac{2abg\,x^7}{7}+\frac{b^2 d\,x^7}{7}+\frac{abf\,x^6}{3}+\frac{b^2 c\,x^6}{6}+\frac{a^2 h\,x^5}{5}+\frac{2abe\,x^5}{5}+$
parallelrisch	$\frac{b^2 h\,x^{11}}{11}+\frac{b^2 g\,x^{10}}{10}+\frac{b^2 f\,x^9}{9}+\frac{abh\,x^8}{4}+\frac{b^2 e\,x^8}{8}+\frac{2abg\,x^7}{7}+\frac{b^2 d\,x^7}{7}+\frac{abf\,x^6}{3}+\frac{b^2 c\,x^6}{6}+\frac{a^2 h\,x^5}{5}+\frac{2abe\,x^5}{5}+$

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (1/3*a^2*f+2/3*a*b*c)*x^3+(1/4*a^2*g+1/2*a*b*d)*x^4+(1/5*a^2*h+2/5*a*e*b)*x
^5+(2/7*a*b*g+1/7*b^2*d)*x^7+(1/4*a*b*h+1/8*b^2*e)*x^8+(1/3*a*f*b+1/6*b^2*c
)*x^6+a^2*d*x+1/2*a^2*e*x^2+1/9*b^2*f*x^9+1/10*b^2*g*x^10+1/11*b^2*h*x^11+a
^2*c*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{(a+bx^3)^2\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx &= \frac{1}{11}\,b^2hx^{11}+\frac{1}{10}\,b^2gx^{10}+\frac{1}{9}\,b^2fx^9+\frac{1}{8}\,(b^2e+2\,abh)x^8 \\ &\quad+\frac{1}{7}\,(b^2d+2\,abg)x^7+\frac{1}{6}\,(b^2c+2\,abf)x^6+\frac{1}{5}\,(2\,abe+a^2h)x^5+\frac{1}{2}\,a^2ex^2 \\ &\quad+\frac{1}{4}\,(2\,abd+a^2g)x^4+a^2dx+\frac{1}{3}\,(2\,abc+a^2f)x^3+a^2c\log(x)\end{aligned}$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="frica
s")
```

```
[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*(b^2*e + 2*a*b*h)*x
^8 + 1/7*(b^2*d + 2*a*b*g)*x^7 + 1/6*(b^2*c + 2*a*b*f)*x^6 + 1/5*(2*a*b*e +
 a^2*h)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*d + a^2*g)*x^4 + a^2*d*x + 1/3*(2*
a*b*c + a^2*f)*x^3 + a^2*c*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$$
$$= a^2c\log(x)+a^2dx+\frac{a^2ex^2}{2}+\frac{b^2fx^9}{9}+\frac{b^2gx^{10}}{10}+\frac{b^2hx^{11}}{11}+x^8\left(\frac{abh}{4}+\frac{b^2e}{8}\right)+x^7\cdot\left(\frac{2abg}{7}+\frac{b^2d}{7}\right)+x^6\left(\frac{abf}{3}+\frac{b^2c}{6}\right)+x^5\left(\frac{a^2h}{5}+\frac{2abe}{5}\right)+x^4\left(\frac{a^2g}{4}+\frac{abd}{2}\right)+x^3\left(\frac{a^2f}{3}+\frac{2abc}{3}\right)$$

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a**2*c*log(x) + a**2*d*x + a**2*e*x**2/2 + b**2*f*x**9/9 + b**2*g*x**10/10
+ b**2*h*x**11/11 + x**8*(a*b*h/4 + b**2*e/8) + x**7*(2*a*b*g/7 + b**2*d/7)
 + x**6*(a*b*f/3 + b**2*c/6) + x**5*(a**2*h/5 + 2*a*b*e/5) + x**4*(a**2*g/4
 + a*b*d/2) + x**3*(a**2*f/3 + 2*a*b*c/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$$
$$= \frac{1}{11}b^2hx^{11}+\frac{1}{10}b^2gx^{10}+\frac{1}{9}b^2fx^9+\frac{1}{8}(b^2e+2abh)x^8+\frac{1}{7}(b^2d+2abg)x^7+\frac{1}{6}(b^2c+2abf)x^6+\frac{1}{5}(2abe+a^2h)x^5+\frac{1}{2}a^2ex^2+\frac{1}{4}(2abd+a^2g)x^4+a^2dx+\frac{1}{3}(2abc+a^2f)x^3+a^2c\log(x)$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxim
a")
```

```
[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*(b^2*e + 2*a*b*h)*x
^8 + 1/7*(b^2*d + 2*a*b*g)*x^7 + 1/6*(b^2*c + 2*a*b*f)*x^6 + 1/5*(2*a*b*e +
 a^2*h)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*d + a^2*g)*x^4 + a^2*d*x + 1/3*(2*
a*b*c + a^2*f)*x^3 + a^2*c*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = \frac{1}{11}b^2hx^{11}+\frac{1}{10}b^2gx^{10}+\frac{1}{9}b^2fx^9+\frac{1}{8}b^2ex^8+\frac{1}{4}abhx^8+\frac{1}{7}b^2dx^7+\frac{2}{7}abgx^7+\frac{1}{6}b^2cx^6+\frac{1}{3}abfx^6+\frac{2}{5}abex^5+\frac{1}{5}a^2hx^5+\frac{1}{2}abdx^4+\frac{1}{4}a^2gx^4+\frac{2}{3}abcx^3+\frac{1}{3}a^2fx^3+\frac{1}{2}a^2ex^2+a^2dx+a^2c\log(|x|)$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac"
)
```

```
[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*b^2*e*x^8 + 1/4*a*b
*h*x^8 + 1/7*b^2*d*x^7 + 2/7*a*b*g*x^7 + 1/6*b^2*c*x^6 + 1/3*a*b*f*x^6 + 2/
5*a*b*e*x^5 + 1/5*a^2*h*x^5 + 1/2*a*b*d*x^4 + 1/4*a^2*g*x^4 + 2/3*a*b*c*x^3
 + 1/3*a^2*f*x^3 + 1/2*a^2*e*x^2 + a^2*d*x + a^2*c*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 10.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = x^3\left(\frac{f\,a^2}{3}+\frac{2\,b\,c\,a}{3}\right)+x^6\left(\frac{c\,b^2}{6}+\frac{a\,f\,b}{3}\right)+x^4\left(\frac{g\,a^2}{4}+\frac{b\,d\,a}{2}\right)+x^7\left(\frac{d\,b^2}{7}+\frac{2\,a\,g\,b}{7}\right)+x^5\left(\frac{h\,a^2}{5}+\frac{2\,b\,e\,a}{5}\right)+x^8\left(\frac{e\,b^2}{8}+\frac{a\,h\,b}{4}\right)+\frac{a^2\,e\,x^2}{2}+\frac{b^2\,f\,x^9}{9}+\frac{b^2\,g\,x^{10}}{10}+\frac{b^2\,h\,x^{11}}{11}+a^2\,c\,\ln(x)+a^2\,d\,x$$

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x)
```

```
[Out] x^3*((a^2*f)/3 + (2*a*b*c)/3) + x^6*((b^2*c)/6 + (a*b*f)/3) + x^4*((a^2*g)/
4 + (a*b*d)/2) + x^7*((b^2*d)/7 + (2*a*b*g)/7) + x^5*((a^2*h)/5 + (2*a*b*e)
/5) + x^8*((b^2*e)/8 + (a*b*h)/4) + (a^2*e*x^2)/2 + (b^2*f*x^9)/9 + (b^2*g*
x^10)/10 + (b^2*h*x^11)/11 + a^2*c*log(x) + a^2*d*x
```

3.389 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 147

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx = -\frac{a^2c}{x}+a^2ex+\frac{1}{2}a(2bc+af)x^2+\frac{2}{3}abdx^3+\frac{1}{4}a(2be+ah)x^4+\frac{1}{5}b(bc+2af)x^5 +\frac{1}{6}b^2dx^6+\frac{1}{7}b(be+2ah)x^7+\frac{1}{8}b^2fx^8+\frac{1}{10}b^2hx^{10}+\frac{g(a+bx^3)^3}{9b}+a^2d\log(x)$$

```
[Out] -a^2*c/x+a^2*e*x+1/2*a*(a*f+2*b*c)*x^2+2/3*a*b*d*x^3+1/4*a*(a*h+2*b*e)*x^4+
1/5*b*(2*a*f+b*c)*x^5+1/6*b^2*d*x^6+1/7*b*(2*a*h+b*e)*x^7+1/8*b^2*f*x^8+1/1
0*b^2*h*x^10+1/9*g*(b*x^3+a)^3/b+a^2*d*ln(x)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1597, 1834}

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx = -\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{1}{5}bx^5(2af+bc)+\frac{1}{2}ax^2(af+2bc)+\frac{2}{3}abdx^3 +\frac{1}{7}bx^7(2ah+be)+\frac{1}{4}ax^4(ah+2be)+\frac{g(a+bx^3)^3}{9b}+\frac{1}{6}b^2dx^6+\frac{1}{8}b^2fx^8+\frac{1}{10}b^2hx^{10}$$

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + (a*(2*b*c + a*f)*x^2)/2 + (2*a*b*d*x^3)/3 + (a*(2*
b*e + a*h)*x^4)/4 + (b*(b*c + 2*a*f)*x^5)/5 + (b^2*d*x^6)/6 + (b*(b*e + 2*a
*h)*x^7)/7 + (b^2*f*x^8)/8 + (b^2*h*x^10)/10 + (g*(a + b*x^3)^3)/(9*b) + a^
2*d*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{g(a+bx^3)^3}{9b}+\int \frac{(a+bx^3)^2\left(c+dx+ex^2+fx^3+hx^5\right)}{x^2}\,dx\\ &= \frac{g(a+bx^3)^3}{9b}+\int \left(a^2e+\frac{a^2c}{x^2}+\frac{a^2d}{x}+a(2bc+af)x+2abdx^2+a(2be+ah)x^3\right.\\ &\qquad \left.+b(bc+2af)x^4+b^2dx^5+b(be+2ah)x^6+b^2fx^7+b^2hx^9\right)\,dx\\ &= -\frac{a^2c}{x}+a^2ex+\frac{1}{2}a(2bc+af)x^2+\frac{2}{3}abdx^3+\frac{1}{4}a(2be+ah)x^4+\frac{1}{5}b(bc+2af)x^5\\ &\quad +\frac{1}{6}b^2dx^6+\frac{1}{7}b(be+2ah)x^7+\frac{1}{8}b^2fx^8+\frac{1}{10}b^2hx^{10}+\frac{g(a+bx^3)^3}{9b}+a^2d\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03

$$\begin{aligned}&\int \frac{(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^2}\,dx\\ &= -\frac{a^2c}{x}+a^2ex+\frac{1}{2}a(2bc+af)x^2+\frac{1}{3}a(2bd+ag)x^3+\frac{1}{4}a(2be+ah)x^4+\frac{1}{5}b(bc+2af)x^5\\ &\quad +\frac{1}{6}b(bd+2ag)x^6+\frac{1}{7}b(be+2ah)x^7+\frac{1}{8}b^2fx^8+\frac{1}{9}b^2gx^9+\frac{1}{10}b^2hx^{10}+a^2d\log(x)\end{aligned}$$

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + (a*(2*b*c + a*f)*x^2)/2 + (a*(2*b*d + a*g)*x^3)/3
+ (a*(2*b*e + a*h)*x^4)/4 + (b*(b*c + 2*a*f)*x^5)/5 + (b*(b*d + 2*a*g)*x^6)
/6 + (b*(b*e + 2*a*h)*x^7)/7 + (b^2*f*x^8)/8 + (b^2*g*x^9)/9 + (b^2*h*x^10)
/10 + a^2*d*Log[x]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.02

method	result
norman	$\frac{\left(\frac{1}{2}a^2f+abc\right)x^3+\left(\frac{1}{3}a^2g+\frac{2}{3}abd\right)x^4+\left(\frac{1}{4}a^2h+\frac{1}{2}aeb\right)x^5+\left(\frac{1}{3}abg+\frac{1}{6}b^2d\right)x^7+\left(\frac{2}{7}abh+\frac{1}{7}b^2e\right)x^8+\left(\frac{2}{5}afb+\frac{1}{5}b^2c\right)x^6+a^2e\,x^2-a^2c+\frac{b^2f}{8}}{x}$
default	$\frac{b^2h\,x^{10}}{10}+\frac{b^2g\,x^9}{9}+\frac{b^2f\,x^8}{8}+\frac{2abh\,x^7}{7}+\frac{b^2e\,x^7}{7}+\frac{abg\,x^6}{3}+\frac{b^2d\,x^6}{6}+\frac{2abf\,x^5}{5}+\frac{b^2c\,x^5}{5}+\frac{a^2h\,x^4}{4}+\frac{abe\,x^4}{2}+\frac{a^2g}{3}$
risch	$\frac{b^2h\,x^{10}}{10}+\frac{b^2g\,x^9}{9}+\frac{b^2f\,x^8}{8}+\frac{2abh\,x^7}{7}+\frac{b^2e\,x^7}{7}+\frac{abg\,x^6}{3}+\frac{b^2d\,x^6}{6}+\frac{2abf\,x^5}{5}+\frac{b^2c\,x^5}{5}+\frac{a^2h\,x^4}{4}+\frac{abe\,x^4}{2}+\frac{a^2g}{3}$
parallelrisch	$\frac{252b^2h\,x^{11}+280b^2g\,x^{10}+315b^2f\,x^9+720abh\,x^8+360b^2e\,x^8+840abg\,x^7+420b^2d\,x^7+1008abf\,x^6+504b^2c\,x^6+630a^2h\,x^5+1260ab}{2520x}$

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x,method=_RETURNVERBOSE
)
```

```
[Out] ((1/2*a^2*f+a*b*c)*x^3+(1/3*a^2*g+2/3*a*b*d)*x^4+(1/4*a^2*h+1/2*a*e*b)*x^5+
(1/3*a*b*g+1/6*b^2*d)*x^7+(2/7*a*b*h+1/7*b^2*e)*x^8+(2/5*a*f*b+1/5*b^2*c)*x
^6+a^2*e*x^2-a^2*c+1/8*b^2*f*x^9+1/9*b^2*g*x^10+1/10*b^2*h*x^11)/x+a^2*d*ln
(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^2\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx = \frac{252\,b^2hx^{11}+280\,b^2gx^{10}+315\,b^2fx^9+360\,(b^2e+2\,abh)x^8+420\,(b^2d+2\,abg)x^7+504\,(b^2c+2\,abf)x^6+6}{25}$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fri
cas")
```

```
[Out] 1/2520*(252*b^2*h*x^11 + 280*b^2*g*x^10 + 315*b^2*f*x^9 + 360*(b^2*e + 2*a*
b*h)*x^8 + 420*(b^2*d + 2*a*b*g)*x^7 + 504*(b^2*c + 2*a*b*f)*x^6 + 630*(2*a
*b*e + a^2*h)*x^5 + 2520*a^2*e*x^2 + 840*(2*a*b*d + a^2*g)*x^4 + 2520*a^2*d
*x*log(x) + 1260*(2*a*b*c + a^2*f)*x^3 - 2520*a^2*c)/x
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$=-\frac{a^2c}{x}+a^2d\log{(x)}+a^2ex+\frac{b^2fx^8}{8}+\frac{b^2gx^9}{9}+\frac{b^2hx^{10}}{10}+x^7\cdot\left(\frac{2abh}{7}+\frac{b^2e}{7}\right)+x^6\left(\frac{abg}{3}+\frac{b^2d}{6}\right)$$
$$+x^5\cdot\left(\frac{2abf}{5}+\frac{b^2c}{5}\right)+x^4\left(\frac{a^2h}{4}+\frac{abe}{2}\right)+x^3\left(\frac{a^2g}{3}+\frac{2abd}{3}\right)+x^2\left(\frac{a^2f}{2}+abc\right)$$

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a**2*c/x + a**2*d*log(x) + a**2*e*x + b**2*f*x**8/8 + b**2*g*x**9/9 + b**2
*h*x**10/10 + x**7*(2*a*b*h/7 + b**2*e/7) + x**6*(a*b*g/3 + b**2*d/6) + x**
5*(2*a*b*f/5 + b**2*c/5) + x**4*(a**2*h/4 + a*b*e/2) + x**3*(a**2*g/3 + 2*a
*b*d/3) + x**2*(a**2*f/2 + a*b*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^2\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$=\frac{1}{10}\,b^2hx^{10}+\frac{1}{9}\,b^2gx^9+\frac{1}{8}\,b^2fx^8+\frac{1}{7}\,(b^2e+2\,abh)x^7+\frac{1}{6}\,(b^2d+2\,abg)x^6+\frac{1}{5}\,(b^2c+2\,abf)x^5$$
$$+\frac{1}{4}\,(2\,abe+a^2h)x^4+a^2ex+\frac{1}{3}\,(2\,abd+a^2g)x^3+a^2d\log{(x)}+\frac{1}{2}\,(2\,abc+a^2f)x^2-\frac{a^2c}{x}$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="max
ima")
```

```
[Out] 1/10*b^2*h*x^10 + 1/9*b^2*g*x^9 + 1/8*b^2*f*x^8 + 1/7*(b^2*e + 2*a*b*h)*x^7
 + 1/6*(b^2*d + 2*a*b*g)*x^6 + 1/5*(b^2*c + 2*a*b*f)*x^5 + 1/4*(2*a*b*e + a
^2*h)*x^4 + a^2*e*x + 1/3*(2*a*b*d + a^2*g)*x^3 + a^2*d*log(x) + 1/2*(2*a*b
*c + a^2*f)*x^2 - a^2*c/x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} \, dx$$
$$= \frac{1}{10} b^2hx^{10} + \frac{1}{9} b^2gx^9 + \frac{1}{8} b^2fx^8 + \frac{1}{7} b^2ex^7 + \frac{2}{7} abhx^7 + \frac{1}{6} b^2dx^6 + \frac{1}{3} abgx^6 + \frac{1}{5} b^2cx^5 + \frac{2}{5} abfx^5 + \frac{1}{2} abex^4 + \frac{1}{4} a^2hx^4 + \frac{2}{3} abdx^3 + \frac{1}{3} a^2gx^3 + abcx^2 + \frac{1}{2} a^2fx^2 + a^2ex + a^2d \log(|x|) - \frac{a^2c}{x}$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="gia
c")
```

```
[Out] 1/10*b^2*h*x^10 + 1/9*b^2*g*x^9 + 1/8*b^2*f*x^8 + 1/7*b^2*e*x^7 + 2/7*a*b*h
*x^7 + 1/6*b^2*d*x^6 + 1/3*a*b*g*x^6 + 1/5*b^2*c*x^5 + 2/5*a*b*f*x^5 + 1/2*
a*b*e*x^4 + 1/4*a^2*h*x^4 + 2/3*a*b*d*x^3 + 1/3*a^2*g*x^3 + a*b*c*x^2 + 1/2
*a^2*f*x^2 + a^2*e*x + a^2*d*log(abs(x)) - a^2*c/x
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} \, dx$$
$$= x^2 \left(\frac{f a^2}{2} + b c a\right) + x^5 \left(\frac{c b^2}{5} + \frac{2 a f b}{5}\right) + x^3 \left(\frac{g a^2}{3} + \frac{2 b d a}{3}\right) + x^6 \left(\frac{d b^2}{6} + \frac{a g b}{3}\right) + x^4 \left(\frac{h a^2}{4} + \frac{b e a}{2}\right) + x^7 \left(\frac{e b^2}{7} + \frac{2 a h b}{7}\right) - \frac{a^2 c}{x} + \frac{b^2 f x^8}{8} + \frac{b^2 g x^9}{9} + \frac{b^2 h x^{10}}{10} + a^2 d \ln(x) + a^2 e x$$

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x)
```

```
[Out] x^2*((a^2*f)/2 + a*b*c) + x^5*((b^2*c)/5 + (2*a*b*f)/5) + x^3*((a^2*g)/3 +
(2*a*b*d)/3) + x^6*((b^2*d)/6 + (a*b*g)/3) + x^4*((a^2*h)/4 + (a*b*e)/2) +
x^7*((b^2*e)/7 + (2*a*b*h)/7) - (a^2*c)/x + (b^2*f*x^8)/8 + (b^2*g*x^9)/9 +
 (b^2*h*x^10)/10 + a^2*d*log(x) + a^2*e*x
```

3.390 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 147

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx = -\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a(2bc+af)x+\frac{1}{2}a(2bd+ag)x^2+\frac{2}{3}abex^3+\frac{1}{4}b(bc+2af)x^4+\frac{1}{5}b(bd+2ag)x^5+\frac{1}{6}b^2ex^6+\frac{1}{7}b^2fx^7+\frac{1}{8}b^2gx^8+\frac{h(a+bx^3)^3}{9b}+a^2e\log(x)$$

```
[Out] -1/2*a^2*c/x^2-a^2*d/x+a*(a*f+2*b*c)*x+1/2*a*(a*g+2*b*d)*x^2+2/3*a*b*e*x^3+
1/4*b*(2*a*f+b*c)*x^4+1/5*b*(2*a*g+b*d)*x^5+1/6*b^2*e*x^6+1/7*b^2*f*x^7+1/8
*b^2*g*x^8+1/9*h*(b*x^3+a)^3/b+a^2*e*ln(x)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1597, 1834}

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx = -\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+\frac{1}{4}bx^4(2af+bc)+ax(af+2bc)+\frac{1}{5}bx^5(2ag+bd)+\frac{1}{2}ax^2(ag+2bd)+\frac{2}{3}abex^3+\frac{h(a+bx^3)^3}{9b}+\frac{1}{6}b^2ex^6+\frac{1}{7}b^2fx^7+\frac{1}{8}b^2gx^8$$

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] -1/2*(a^2*c)/x^2 - (a^2*d)/x + a*(2*b*c + a*f)*x + (a*(2*b*d + a*g)*x^2)/2
+ (2*a*b*e*x^3)/3 + (b*(b*c + 2*a*f)*x^4)/4 + (b*(b*d + 2*a*g)*x^5)/5 + (b^
2*e*x^6)/6 + (b^2*f*x^7)/7 + (b^2*g*x^8)/8 + (h*(a + b*x^3)^3)/(9*b) + a^2*
e*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{h(a+bx^3)^3}{9b} + \int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx \\
&= \frac{h(a+bx^3)^3}{9b} + \int \left(a(2bc+af) + \frac{a^2c}{x^3} + \frac{a^2d}{x^2} + \frac{a^2e}{x} + a(2bd+ag)x + 2abex^2 \right. \\
&\qquad \left. + b(bc+2af)x^3 + b(bd+2ag)x^4 + b^2ex^5 + b^2fx^6 + b^2gx^7\right) dx \\
&= -\frac{a^2c}{2x^2} - \frac{a^2d}{x} + a(2bc+af)x + \frac{1}{2}a(2bd+ag)x^2 + \frac{2}{3}abex^3 + \frac{1}{4}b(bc+2af)x^4 \\
&\quad + \frac{1}{5}b(bd+2ag)x^5 + \frac{1}{6}b^2ex^6 + \frac{1}{7}b^2fx^7 + \frac{1}{8}b^2gx^8 + \frac{h(a+bx^3)^3}{9b} + a^2e\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.86

$$\begin{aligned}
&\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx \\
&= \frac{a^2(-3c-6dx+x^3(6f+3gx+2hx^2))}{6x^2} \\
&\quad + \frac{1}{30}abx(60c+x(30d+x(20e+15fx+12gx^2+10hx^3))) \\
&\quad + \frac{b^2x^4(630c+x(504d+5x(84e+x(72f+7x(9g+8hx)))))}{2520} + a^2e\log(x)
\end{aligned}$$

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] (a^2*(-3*c - 6*d*x + x^3*(6*f + 3*g*x + 2*h*x^2)))/(6*x^2) + (a*b*x*(60*c +
 x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))))/30 + (b^2*x^4*(630*c
+ x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x))))))/2520 + a^2*e*Log[
x]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

method	result
norman	$\frac{\left(\frac{1}{2}a^2g+abd\right)x^4+\left(\frac{1}{3}a^2h+\frac{2}{3}aeb\right)x^5+\left(\frac{2}{5}abg+\frac{1}{5}b^2d\right)x^7+\left(\frac{1}{3}abh+\frac{1}{6}b^2e\right)x^8+\left(\frac{1}{2}afb+\frac{1}{4}b^2c\right)x^6+\left(a^2f+2abc\right)x^3-\frac{a^2c}{2}-a^2dx+\frac{b^2f x}{7}}{x^2}$
default	$\frac{b^2h\,x^9}{9}+\frac{b^2g\,x^8}{8}+\frac{b^2f\,x^7}{7}+\frac{abh\,x^6}{3}+\frac{b^2e\,x^6}{6}+\frac{2abg\,x^5}{5}+\frac{b^2d\,x^5}{5}+\frac{abf\,x^4}{2}+\frac{b^2c\,x^4}{4}+\frac{a^2h\,x^3}{3}+\frac{2abe\,x^3}{3}+a^2$
risch	$\frac{b^2h\,x^9}{9}+\frac{b^2g\,x^8}{8}+\frac{b^2f\,x^7}{7}+\frac{abh\,x^6}{3}+\frac{b^2e\,x^6}{6}+\frac{2abg\,x^5}{5}+\frac{b^2d\,x^5}{5}+\frac{abf\,x^4}{2}+\frac{b^2c\,x^4}{4}+\frac{a^2h\,x^3}{3}+\frac{2abe\,x^3}{3}+a^2$
parallelrisch	$\frac{280b^2h\,x^{11}+315b^2g\,x^{10}+360b^2f\,x^9+840abh\,x^8+420b^2e\,x^8+1008abg\,x^7+504b^2d\,x^7+1260abf\,x^6+630b^2c\,x^6+840a^2h\,x^5+1680}{2520x^2}$

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x,method=_RETURNVERBOSE
)
```

```
[Out] ((1/2*a^2*g+a*b*d)*x^4+(1/3*a^2*h+2/3*a*e*b)*x^5+(2/5*a*b*g+1/5*b^2*d)*x^7+
(1/3*a*b*h+1/6*b^2*e)*x^8+(1/2*a*f*b+1/4*b^2*c)*x^6+(a^2*f+2*a*b*c)*x^3-1/2
*a^2*c-a^2*d*x+1/7*b^2*f*x^9+1/8*b^2*g*x^10+1/9*b^2*h*x^11)/x^2+a^2*e*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^2\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$=\frac{280\,b^2hx^{11}+315\,b^2gx^{10}+360\,b^2fx^9+420\,(b^2e+2\,abh)x^8+504\,(b^2d+2\,abg)x^7+630\,(b^2c+2\,abf)x^6+}{2}$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fri
cas")
```

```
[Out] 1/2520*(280*b^2*h*x^11 + 315*b^2*g*x^10 + 360*b^2*f*x^9 + 420*(b^2*e + 2*a*
b*h)*x^8 + 504*(b^2*d + 2*a*b*g)*x^7 + 630*(b^2*c + 2*a*b*f)*x^6 + 840*(2*a
*b*e + a^2*h)*x^5 + 2520*a^2*e*x^2*log(x) + 1260*(2*a*b*d + a^2*g)*x^4 - 25
20*a^2*d*x + 2520*(2*a*b*c + a^2*f)*x^3 - 1260*a^2*c)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= a^2e\log(x)+\frac{b^2fx^7}{7}+\frac{b^2gx^8}{8}+\frac{b^2hx^9}{9}+x^6\left(\frac{abh}{3}+\frac{b^2e}{6}\right)+x^5\cdot\left(\frac{2abg}{5}+\frac{b^2d}{5}\right)$$
$$+x^4\left(\frac{abf}{2}+\frac{b^2c}{4}\right)+x^3\left(\frac{a^2h}{3}+\frac{2abe}{3}\right)+x^2\left(\frac{a^2g}{2}+abd\right)+x(a^2f+2abc)+\frac{-a^2c-2a^2dx}{2x^2}$$

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a**2*e*log(x) + b**2*f*x**7/7 + b**2*g*x**8/8 + b**2*h*x**9/9 + x**6*(a*b*h
/3 + b**2*e/6) + x**5*(2*a*b*g/5 + b**2*d/5) + x**4*(a*b*f/2 + b**2*c/4) +
x**3*(a**2*h/3 + 2*a*b*e/3) + x**2*(a**2*g/2 + a*b*d) + x*(a**2*f + 2*a*b*c
) + (-a**2*c - 2*a**2*d*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= \frac{1}{9}b^2hx^9+\frac{1}{8}b^2gx^8+\frac{1}{7}b^2fx^7+\frac{1}{6}(b^2e+2\,abh)x^6+\frac{1}{5}(b^2d+2\,abg)x^5+\frac{1}{4}(b^2c+2\,abf)x^4$$
$$+\frac{1}{3}(2\,abe+a^2h)x^3+a^2e\log(x)+\frac{1}{2}(2\,abd+a^2g)x^2+(2\,abc+a^2f)x-\frac{2\,a^2dx+a^2c}{2\,x^2}$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="max
ima")
```

```
[Out] 1/9*b^2*h*x^9 + 1/8*b^2*g*x^8 + 1/7*b^2*f*x^7 + 1/6*(b^2*e + 2*a*b*h)*x^6 +
 1/5*(b^2*d + 2*a*b*g)*x^5 + 1/4*(b^2*c + 2*a*b*f)*x^4 + 1/3*(2*a*b*e + a^2
*h)*x^3 + a^2*e*log(x) + 1/2*(2*a*b*d + a^2*g)*x^2 + (2*a*b*c + a^2*f)*x -
1/2*(2*a^2*d*x + a^2*c)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= \frac{1}{9}\,b^2hx^9+\frac{1}{8}\,b^2gx^8+\frac{1}{7}\,b^2fx^7+\frac{1}{6}\,b^2ex^6+\frac{1}{3}\,abhx^6+\frac{1}{5}\,b^2dx^5+\frac{2}{5}\,abgx^5+\frac{1}{4}\,b^2cx^4+\frac{1}{2}\,abfx^4 + \frac{2}{3}\,abex^3+\frac{1}{3}\,a^2hx^3+abdx^2+\frac{1}{2}\,a^2gx^2+2\,abcx+a^2fx+a^2e\log(|x|)-\frac{2\,a^2dx+a^2c}{2\,x^2}$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="gia
c")
```

```
[Out] 1/9*b^2*h*x^9 + 1/8*b^2*g*x^8 + 1/7*b^2*f*x^7 + 1/6*b^2*e*x^6 + 1/3*a*b*h*x
^6 + 1/5*b^2*d*x^5 + 2/5*a*b*g*x^5 + 1/4*b^2*c*x^4 + 1/2*a*b*f*x^4 + 2/3*a*
b*e*x^3 + 1/3*a^2*h*x^3 + a*b*d*x^2 + 1/2*a^2*g*x^2 + 2*a*b*c*x + a^2*f*x +
 a^2*e*log(abs(x)) - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= x\left(f\,a^2+2\,b\,c\,a\right)-\frac{\frac{a^2\,c}{2}+a^2\,d\,x}{x^2}+x^4\left(\frac{c\,b^2}{4}+\frac{a\,f\,b}{2}\right) + x^2\left(\frac{g\,a^2}{2}+b\,d\,a\right)+x^5\left(\frac{d\,b^2}{5}+\frac{2\,a\,g\,b}{5}\right)+x^3\left(\frac{h\,a^2}{3}+\frac{2\,b\,e\,a}{3}\right) + x^6\left(\frac{e\,b^2}{6}+\frac{a\,h\,b}{3}\right)+\frac{b^2\,f\,x^7}{7}+\frac{b^2\,g\,x^8}{8}+\frac{b^2\,h\,x^9}{9}+a^2\,e\,\ln(x)$$

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x)
```

```
[Out] x*(a^2*f + 2*a*b*c) - ((a^2*c)/2 + a^2*d*x)/x^2 + x^4*((b^2*c)/4 + (a*b*f)/
2) + x^2*((a^2*g)/2 + a*b*d) + x^5*((b^2*d)/5 + (2*a*b*g)/5) + x^3*((a^2*h)
/3 + (2*a*b*e)/3) + x^6*((b^2*e)/6 + (a*b*h)/3) + (b^2*f*x^7)/7 + (b^2*g*x^
8)/8 + (b^2*h*x^9)/9 + a^2*e*log(x)
```

3.391 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx$

Optimal result

Integrand size = 38, antiderivative size = 152

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx = -\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+a(2bd+ag)x+\frac{1}{2}a(2be+ah)x^2+\frac{1}{3}b(bc+2af)x^3+\frac{1}{4}b(bd+2ag)x^4+\frac{1}{5}b(be+2ah)x^5+\frac{1}{6}b^2fx^6+\frac{1}{7}b^2gx^7+\frac{1}{8}b^2hx^8+a(2bc+af)\log(x)$$

```
[Out] -1/3*a^2*c/x^3-1/2*a^2*d/x^2-a^2*e/x+a*(a*g+2*b*d)*x+1/2*a*(a*h+2*b*e)*x^2+
1/3*b*(2*a*f+b*c)*x^3+1/4*b*(2*a*g+b*d)*x^4+1/5*b*(2*a*h+b*e)*x^5+1/6*b^2*f
*x^6+1/7*b^2*g*x^7+1/8*b^2*h*x^8+a*(a*f+2*b*c)*ln(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1834}

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx = -\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+\frac{1}{3}bx^3(2af+bc)+a\log(x)(af+2bc)+\frac{1}{4}bx^4(2ag+bd)+ax(ag+2bd)+\frac{1}{5}bx^5(2ah+be)+\frac{1}{2}ax^2(ah+2be)+\frac{1}{6}b^2fx^6+\frac{1}{7}b^2gx^7+\frac{1}{8}b^2hx^8$$

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -1/3*(a^2*c)/x^3 - (a^2*d)/(2*x^2) - (a^2*e)/x + a*(2*b*d + a*g)*x + (a*(2*
b*e + a*h)*x^2)/2 + (b*(b*c + 2*a*f)*x^3)/3 + (b*(b*d + 2*a*g)*x^4)/4 + (b*
```

```
(b*e + 2*a*h)*x^5)/5 + (b^2*f*x^6)/6 + (b^2*g*x^7)/7 + (b^2*h*x^8)/8 + a*(2
*b*c + a*f)*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \int \left(a(2bd+ag)+\frac{a^2c}{x^4}+\frac{a^2d}{x^3}+\frac{a^2e}{x^2}+\frac{a(2bc+af)}{x}+a(2be+ah)x+b(bc+2af)x^2 + b(bd+2ag)x^3+b(be+2ah)x^4+b^2fx^5+b^2gx^6+b^2hx^7\right)dx$$

$$= -\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+a(2bd+ag)x+\frac{1}{2}a(2be+ah)x^2+\frac{1}{3}b(bc+2af)x^3 + \frac{1}{4}b(bd+2ag)x^4+\frac{1}{5}b(be+2ah)x^5+\frac{1}{6}b^2fx^6+\frac{1}{7}b^2gx^7+\frac{1}{8}b^2hx^8+a(2bc+af)\log(x)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}dx = -\frac{a^2(2c+3x(d+2ex-x^3(2g+hx)))}{6x^3}+\frac{1}{30}abx(60d+x(30e+x(20f+15gx+12hx^2))) + \frac{1}{840}b^2x^3(280c+x(210d+x(168e+140fx+120gx^2+105hx^3)))+a(2bc+af)\log(x)$$

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -1/6*(a^2*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/x^3 + (a*b*x*(60*d + x
*(30*e + x*(20*f + 15*g*x + 12*h*x^2))))/30 + (b^2*x^3*(280*c + x*(210*d +
x*(168*e + 140*f*x + 120*g*x^2 + 105*h*x^3))))/840 + a*(2*b*c + a*f)*Log[x]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^2 h\,x^8}{8}+\frac{b^2 g\,x^7}{7}+\frac{b^2 f\,x^6}{6}+\frac{2 a b h\,x^5}{5}+\frac{b^2 e\,x^5}{5}+\frac{a b g\,x^4}{2}+\frac{b^2 d\,x^4}{4}+\frac{2 a b f\,x^3}{3}+\frac{b^2 c\,x^3}{3}+\frac{a^2 h\,x^2}{2}+a b e\,x^2+a^2$
norman	$\frac{\left(\frac{1}{2}a^2 h+a e b\right)x^5+\left(\frac{1}{2}a b g+\frac{1}{4}b^2 d\right)x^7+\left(\frac{2}{5}a b h+\frac{1}{5}b^2 e\right)x^8+\left(\frac{2}{3}a f b+\frac{1}{3}b^2 c\right)x^6+\left(a^2 g+2 a b d\right)x^4-\frac{a^2 c}{3}-\frac{a^2 d x}{2}-a^2 e\,x^2+\frac{b^2 f\,x^9}{6}+\frac{b^2 g\,x^{10}}{7}+}{x^3}$
risch	$\frac{b^2 h\,x^8}{8}+\frac{b^2 g\,x^7}{7}+\frac{b^2 f\,x^6}{6}+\frac{2 a b h\,x^5}{5}+\frac{b^2 e\,x^5}{5}+\frac{a b g\,x^4}{2}+\frac{b^2 d\,x^4}{4}+\frac{2 a b f\,x^3}{3}+\frac{b^2 c\,x^3}{3}+\frac{a^2 h\,x^2}{2}+a b e\,x^2+a^2$
parallelrisch	$\frac{105 b^2 h\,x^{11}+120 b^2 g\,x^{10}+140 b^2 f\,x^9+336 a b h\,x^8+168 b^2 e\,x^8+420 a b g\,x^7+210 b^2 d\,x^7+560 a b f\,x^6+280 b^2 c\,x^6+420 a^2 h\,x^5+840 a b e\,x}{840 x^3}$

[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x,method=_RETURNVERBOSE
)

[Out] 1/8*b^2*h*x^8+1/7*b^2*g*x^7+1/6*b^2*f*x^6+2/5*a*b*h*x^5+1/5*b^2*e*x^5+1/2*a
*b*g*x^4+1/4*b^2*d*x^4+2/3*a*b*f*x^3+1/3*b^2*c*x^3+1/2*a^2*h*x^2+a*b*e*x^2+
a^2*g*x+2*a*x*b*d+a*(a*f+2*b*c)*ln(x)-1/3*a^2*c/x^3-a^2*e/x-1/2/x^2*a^2*d

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^2\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$= \frac{105\,b^2hx^{11}+120\,b^2gx^{10}+140\,b^2fx^9+168\,(b^2e+2\,abh)x^8+210\,(b^2d+2\,abg)x^7+280\,(b^2c+2\,abf)x^6+4}{840\,}$$

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fri
cas")

[Out] 1/840*(105*b^2*h*x^11 + 120*b^2*g*x^10 + 140*b^2*f*x^9 + 168*(b^2*e + 2*a*b
*h)*x^8 + 210*(b^2*d + 2*a*b*g)*x^7 + 280*(b^2*c + 2*a*b*f)*x^6 + 420*(2*a*
b*e + a^2*h)*x^5 - 840*a^2*e*x^2 + 840*(2*a*b*d + a^2*g)*x^4 + 840*(2*a*b*c
 + a^2*f)*x^3*log(x) - 420*a^2*d*x - 280*a^2*c)/x^3

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = a(af+2bc)\log(x)+\frac{b^2fx^6}{6}+\frac{b^2gx^7}{7}+\frac{b^2hx^8}{8}+x^5\cdot\left(\frac{2abh}{5}+\frac{b^2e}{5}\right)+x^4\left(\frac{abg}{2}+\frac{b^2d}{4}\right)+x^3\cdot\left(\frac{2abf}{3}+\frac{b^2c}{3}\right)+x^2\left(\frac{a^2h}{2}+abe\right)+x\left(a^2g+2abd\right)+\frac{-2a^2c-3a^2dx-6a^2ex^2}{6x^3}$$

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] a*(a*f + 2*b*c)*log(x) + b**2*f*x**6/6 + b**2*g*x**7/7 + b**2*h*x**8/8 + x*
*5*(2*a*b*h/5 + b**2*e/5) + x**4*(a*b*g/2 + b**2*d/4) + x**3*(2*a*b*f/3 + b
**2*c/3) + x**2*(a**2*h/2 + a*b*e) + x*(a**2*g + 2*a*b*d) + (-2*a**2*c - 3*
a**2*d*x - 6*a**2*e*x**2)/(6*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = \frac{1}{8}b^2hx^8+\frac{1}{7}b^2gx^7+\frac{1}{6}b^2fx^6+\frac{1}{5}\left(b^2e+2\,abh\right)x^5+\frac{1}{4}\left(b^2d+2\,abg\right)x^4+\frac{1}{3}\left(b^2c+2\,abf\right)x^3+\frac{1}{2}\left(2\,abe+a^2h\right)x^2+\left(2\,abd+a^2g\right)x+\left(2\,abc+a^2f\right)\log(x)-\frac{6\,a^2ex^2+3\,a^2dx+2\,a^2c}{6\,x^3}$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="max
ima")
```

```
[Out] 1/8*b^2*h*x^8 + 1/7*b^2*g*x^7 + 1/6*b^2*f*x^6 + 1/5*(b^2*e + 2*a*b*h)*x^5 +
 1/4*(b^2*d + 2*a*b*g)*x^4 + 1/3*(b^2*c + 2*a*b*f)*x^3 + 1/2*(2*a*b*e + a^2
*h)*x^2 + (2*a*b*d + a^2*g)*x + (2*a*b*c + a^2*f)*log(x) - 1/6*(6*a^2*e*x^2
 + 3*a^2*d*x + 2*a^2*c)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$=\frac{1}{8}b^2hx^8+\frac{1}{7}b^2gx^7+\frac{1}{6}b^2fx^6+\frac{1}{5}b^2ex^5+\frac{2}{5}abhx^5+\frac{1}{4}b^2dx^4+\frac{1}{2}abgx^4+\frac{1}{3}b^2cx^3+\frac{2}{3}abfx^3$$
$$+abex^2+\frac{1}{2}a^2hx^2+2\,abdx+a^2gx+\left(2\,abc+a^2f\right)\log\left(|x|\right)-\frac{6\,a^2ex^2+3\,a^2dx+2\,a^2c}{6\,x^3}$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="gia
c")
```

```
[Out] 1/8*b^2*h*x^8 + 1/7*b^2*g*x^7 + 1/6*b^2*f*x^6 + 1/5*b^2*e*x^5 + 2/5*a*b*h*x
^5 + 1/4*b^2*d*x^4 + 1/2*a*b*g*x^4 + 1/3*b^2*c*x^3 + 2/3*a*b*f*x^3 + a*b*e*
x^2 + 1/2*a^2*h*x^2 + 2*a*b*d*x + a^2*g*x + (2*a*b*c + a^2*f)*log(abs(x)) -
 1/6*(6*a^2*e*x^2 + 3*a^2*d*x + 2*a^2*c)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$=x\left(g\,a^2+2\,b\,d\,a\right)-\frac{e\,a^2\,x^2+\frac{d\,a^2\,x}{2}+\frac{c\,a^2}{3}}{x^3}+x^3\left(\frac{c\,b^2}{3}+\frac{2\,a\,f\,b}{3}\right)$$
$$+x^4\left(\frac{d\,b^2}{4}+\frac{a\,g\,b}{2}\right)+x^2\left(\frac{h\,a^2}{2}+b\,e\,a\right)+x^5\left(\frac{e\,b^2}{5}+\frac{2\,a\,h\,b}{5}\right)$$
$$+\ln\left(x\right)\,\left(f\,a^2+2\,b\,c\,a\right)+\frac{b^2\,f\,x^6}{6}+\frac{b^2\,g\,x^7}{7}+\frac{b^2\,h\,x^8}{8}$$

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x)
```

```
[Out] x*(a^2*g + 2*a*b*d) - ((a^2*c)/3 + a^2*e*x^2 + (a^2*d*x)/2)/x^3 + x^3*((b^2
*c)/3 + (2*a*b*f)/3) + x^4*((b^2*d)/4 + (a*b*g)/2) + x^2*((a^2*h)/2 + a*b*e
) + x^5*((b^2*e)/5 + (2*a*b*h)/5) + log(x)*(a^2*f + 2*a*b*c) + (b^2*f*x^6)/
6 + (b^2*g*x^7)/7 + (b^2*h*x^8)/8
```

3.392 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx$

Optimal result

Integrand size = 38, antiderivative size = 152

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx = -\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}-\frac{a(2bc+af)}{x}+a(2be+ah)x+\frac{1}{2}b(bc+2af)x^2+\frac{1}{3}b(bd+2ag)x^3+\frac{1}{4}b(be+2ah)x^4+\frac{1}{5}b^2fx^5+\frac{1}{6}b^2gx^6+\frac{1}{7}b^2hx^7+a(2bd+ag)\log(x)$$

```
[Out] -1/4*a^2*c/x^4-1/3*a^2*d/x^3-1/2*a^2*e/x^2-a*(a*f+2*b*c)/x+a*(a*h+2*b*e)*x+
1/2*b*(2*a*f+b*c)*x^2+1/3*b*(2*a*g+b*d)*x^3+1/4*b*(2*a*h+b*e)*x^4+1/5*b^2*f
*x^5+1/6*b^2*g*x^6+1/7*b^2*h*x^7+a*(a*g+2*b*d)*ln(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1834}

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx = -\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}+\frac{1}{2}bx^2(2af+bc)-\frac{a(af+2bc)}{x}+\frac{1}{3}bx^3(2ag+bd)+a\log(x)(ag+2bd)+\frac{1}{4}bx^4(2ah+be)+ax(ah+2be)+\frac{1}{5}b^2fx^5+\frac{1}{6}b^2gx^6+\frac{1}{7}b^2hx^7$$

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] -1/4*(a^2*c)/x^4 - (a^2*d)/(3*x^3) - (a^2*e)/(2*x^2) - (a*(2*b*c + a*f))/x
+ a*(2*b*e + a*h)*x + (b*(b*c + 2*a*f)*x^2)/2 + (b*(b*d + 2*a*g)*x^3)/3 + (
```

```
b*(b*e + 2*a*h)*x^4)/4 + (b^2*f*x^5)/5 + (b^2*g*x^6)/6 + (b^2*h*x^7)/7 + a*
(2*b*d + a*g)*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \int \left(a(2be+ah)+\frac{a^2c}{x^5}+\frac{a^2d}{x^4}+\frac{a^2e}{x^3}+\frac{a(2bc+af)}{x^2}+\frac{a(2bd+ag)}{x}+b(bc+2af)x+b(bd+2ag)x^2+b(be+2ah)x^3+b^2fx^4+b^2gx^5+b^2hx^6\right)dx$$
$$=-\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}-\frac{a(2bc+af)}{x}+a(2be+ah)x+\frac{1}{2}b(bc+2af)x^2+\frac{1}{3}b(bd+2ag)x^3+\frac{1}{4}b(be+2ah)x^4+\frac{1}{5}b^2fx^5+\frac{1}{6}b^2gx^6+\frac{1}{7}b^2hx^7+a(2bd+ag)\log(x)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$=-\frac{2abc}{x}-\frac{a^2(3c+4dx+6x^2(e+2fx-2hx^3))}{12x^4}+\frac{1}{6}abx(12e+x(6f+x(4g+3hx)))+\frac{1}{420}b^2x^2\left(210c+x\left(140d+x\left(105e+84fx+70gx^2+60hx^3\right)\right)\right)+a(2bd+ag)\log(x)$$

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
[Out] (-2*a*b*c)/x - (a^2*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)))/(12*x^4) +
 (a*b*x*(12*e + x*(6*f + x*(4*g + 3*h*x))))/6 + (b^2*x^2*(210*c + x*(140*d
+ x*(105*e + 84*f*x + 70*g*x^2 + 60*h*x^3))))/420 + a*(2*b*d + a*g)*Log[x]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95

method	result
default	$\frac{b^2 h\,x^7}{7}+\frac{b^2 g\,x^6}{6}+\frac{f\,x^5 b^2}{5}+\frac{abh\,x^4}{2}+\frac{b^2 e\,x^4}{4}+\frac{2abg\,x^3}{3}+\frac{d\,x^3 b^2}{3}+abf\,x^2+\frac{b^2 c\,x^2}{2}+a^2hx+2abex+a$
norman	$\frac{\left(\frac{2}{3}abg+\frac{1}{3}b^2d\right)x^7+\left(\frac{1}{2}abh+\frac{1}{4}b^2e\right)x^8+\left(afb+\frac{1}{2}b^2c\right)x^6+\left(-a^2f-2abc\right)x^3+\left(a^2h+2aeb\right)x^5-\frac{a^2c}{4}-\frac{a^2dx}{3}-\frac{a^2e\,x^2}{2}+\frac{b^2f\,x^9}{5}+\frac{b^2g\,x^{10}}{6}}{x^4}$
risch	$\frac{b^2 h\,x^7}{7}+\frac{b^2 g\,x^6}{6}+\frac{f\,x^5 b^2}{5}+\frac{abh\,x^4}{2}+\frac{b^2 e\,x^4}{4}+\frac{2abg\,x^3}{3}+\frac{d\,x^3 b^2}{3}+abf\,x^2+\frac{b^2 c\,x^2}{2}+a^2hx+2abex+$
parallelrisch	$\frac{60b^2h\,x^{11}+70b^2g\,x^{10}+84b^2f\,x^9+210abh\,x^8+105b^2e\,x^8+280abg\,x^7+140b^2d\,x^7+420abf\,x^6+210b^2c\,x^6+420\ln(x)x^4a^2g+840\ln}{420x^4}$

[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x,method=_RETURNVERBOSE
)

[Out] 1/7*b^2*h*x^7+1/6*b^2*g*x^6+1/5*f*x^5*b^2+1/2*a*b*h*x^4+1/4*b^2*e*x^4+2/3*a
*b*g*x^3+1/3*d*x^3*b^2+a*b*f*x^2+1/2*b^2*c*x^2+a^2*h*x+2*a*b*e*x+a*(a*g+2*b
*d)*ln(x)-1/3*a^2*d/x^3-a*(a*f+2*b*c)/x-1/2*a^2*e/x^2-1/4*a^2*c/x^4

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^2\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$=\frac{60\,b^2hx^{11}+70\,b^2gx^{10}+84\,b^2fx^9+105\,(b^2e+2\,abh)x^8+140\,(b^2d+2\,abg)x^7+210\,(b^2c+2\,abf)x^6+420}{420\,x}$$

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fri
cas")

[Out] 1/420*(60*b^2*h*x^11 + 70*b^2*g*x^10 + 84*b^2*f*x^9 + 105*(b^2*e + 2*a*b*h)
x^8 + 140(b^2*d + 2*a*b*g)*x^7 + 210*(b^2*c + 2*a*b*f)*x^6 + 420*(2*a*b*e
 + a^2*h)*x^5 + 420*(2*a*b*d + a^2*g)*x^4*log(x) - 210*a^2*e*x^2 - 140*a^2*
d*x - 420*(2*a*b*c + a^2*f)*x^3 - 105*a^2*c)/x^4

Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$= a(ag+2bd)\log(x) + \frac{b^2fx^5}{5} + \frac{b^2gx^6}{6} + \frac{b^2hx^7}{7} + x^4\left(\frac{abh}{2}+\frac{b^2e}{4}\right) + x^3\cdot\left(\frac{2abg}{3}+\frac{b^2d}{3}\right) + x^2\left(abf+\frac{b^2c}{2}\right) + x\left(a^2h+2abe\right) + \frac{-3a^2c-4a^2dx-6a^2ex^2+x^3(-12a^2f-24abc)}{12x^4}$$

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] a*(a*g + 2*b*d)*log(x) + b**2*f*x**5/5 + b**2*g*x**6/6 + b**2*h*x**7/7 + x*
*4*(a*b*h/2 + b**2*e/4) + x**3*(2*a*b*g/3 + b**2*d/3) + x**2*(a*b*f + b**2*
c/2) + x*(a**2*h + 2*a*b*e) + (-3*a**2*c - 4*a**2*d*x - 6*a**2*e*x**2 + x**
3*(-12*a**2*f - 24*a*b*c))/(12*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$= \frac{1}{7}b^2hx^7 + \frac{1}{6}b^2gx^6 + \frac{1}{5}b^2fx^5 + \frac{1}{4}\left(b^2e+2\,abh\right)x^4 + \frac{1}{3}\left(b^2d+2\,abg\right)x^3 + \frac{1}{2}\left(b^2c+2\,abf\right)x^2 + \left(2\,abe+a^2h\right)x + \left(2\,abd+a^2g\right)\log(x) - \frac{6\,a^2ex^2+4\,a^2dx+12\,(2\,abc+a^2f)x^3+3\,a^2c}{12\,x^4}$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="max
ima")
```

```
[Out] 1/7*b^2*h*x^7 + 1/6*b^2*g*x^6 + 1/5*b^2*f*x^5 + 1/4*(b^2*e + 2*a*b*h)*x^4 +
 1/3*(b^2*d + 2*a*b*g)*x^3 + 1/2*(b^2*c + 2*a*b*f)*x^2 + (2*a*b*e + a^2*h)*
x + (2*a*b*d + a^2*g)*log(x) - 1/12*(6*a^2*e*x^2 + 4*a^2*d*x + 12*(2*a*b*c
+ a^2*f)*x^3 + 3*a^2*c)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} \, dx = \frac{1}{7} b^2hx^7 + \frac{1}{6} b^2gx^6 + \frac{1}{5} b^2fx^5 + \frac{1}{4} b^2ex^4 + \frac{1}{2} abhx^4 + \frac{1}{3} b^2dx^3 + \frac{2}{3} abgx^3 + \frac{1}{2} b^2cx^2 + abfx^2 + 2 abex + a^2hx + (2 abd + a^2g) \log(|x|) - \frac{6 a^2ex^2 + 4 a^2dx + 12 (2 abc + a^2f)x^3 + 3 a^2c}{12 x^4}$$

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="gia
c")
```

```
[Out] 1/7*b^2*h*x^7 + 1/6*b^2*g*x^6 + 1/5*b^2*f*x^5 + 1/4*b^2*e*x^4 + 1/2*a*b*h*x
^4 + 1/3*b^2*d*x^3 + 2/3*a*b*g*x^3 + 1/2*b^2*c*x^2 + a*b*f*x^2 + 2*a*b*e*x
+ a^2*h*x + (2*a*b*d + a^2*g)*log(abs(x)) - 1/12*(6*a^2*e*x^2 + 4*a^2*d*x +
 12*(2*a*b*c + a^2*f)*x^3 + 3*a^2*c)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} \, dx = x\,(h\,a^2 + 2\,b\,e\,a) - \frac{\frac{a^2\,c}{4} + x^3\,(f\,a^2 + 2\,b\,c\,a) + \frac{a^2\,e\,x^2}{2} + \frac{a^2\,d\,x}{3}}{x^4} + x^2\left(\frac{c\,b^2}{2} + a\,f\,b\right) + x^3\left(\frac{d\,b^2}{3} + \frac{2\,a\,g\,b}{3}\right) + x^4\left(\frac{e\,b^2}{4} + \frac{a\,h\,b}{2}\right) + \ln(x)\,(g\,a^2 + 2\,b\,d\,a) + \frac{b^2\,f\,x^5}{5} + \frac{b^2\,g\,x^6}{6} + \frac{b^2\,h\,x^7}{7}$$

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x)
```

```
[Out] x*(a^2*h + 2*a*b*e) - ((a^2*c)/4 + x^3*(a^2*f + 2*a*b*c) + (a^2*e*x^2)/2 +
(a^2*d*x)/3)/x^4 + x^2*((b^2*c)/2 + a*b*f) + x^3*((b^2*d)/3 + (2*a*b*g)/3)
+ x^4*((b^2*e)/4 + (a*b*h)/2) + log(x)*(a^2*g + 2*a*b*d) + (b^2*f*x^5)/5 +
(b^2*g*x^6)/6 + (b^2*h*x^7)/7
```

3.393 $\int x^4(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 223

$$\int x^4(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2(3bc+af)x^8+\frac{1}{9}a^2(3bd+ag)x^9+\frac{1}{10}a^2(3be+ah)x^{10}+\frac{3}{11}ab(bc+af)x^{11}+\frac{1}{4}ab(bd+ag)x^{12}+\frac{3}{13}ab(be+ah)x^{13}+\frac{1}{14}b^2(bc+3af)x^{14}+\frac{1}{15}b^2(bd+3ag)x^{15}+\frac{1}{16}b^2(be+3ah)x^{16}+\frac{1}{17}b^3fx^{17}+\frac{1}{18}b^3gx^{18}+\frac{1}{19}b^3hx^{19}$$

```
[Out] 1/5*a^3*c*x^5+1/6*a^3*d*x^6+1/7*a^3*e*x^7+1/8*a^2*(a*f+3*b*c)*x^8+1/9*a^2*(
a*g+3*b*d)*x^9+1/10*a^2*(a*h+3*b*e)*x^10+3/11*a*b*(a*f+b*c)*x^11+1/4*a*b*(a
*g+b*d)*x^12+3/13*a*b*(a*h+b*e)*x^13+1/14*b^2*(3*a*f+b*c)*x^14+1/15*b^2*(3*
a*g+b*d)*x^15+1/16*b^2*(3*a*h+b*e)*x^16+1/17*b^3*f*x^17+1/18*b^3*g*x^18+1/1
9*b^3*h*x^19
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1834}

$$\int x^4(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2x^8(af+3bc)+\frac{1}{9}a^2x^9(ag+3bd)+\frac{1}{10}a^2x^{10}(ah+3be)+\frac{1}{14}b^2x^{14}(3af+bc)+\frac{1}{15}b^2x^{15}(3ag+bd)+\frac{1}{16}b^2x^{16}(3ah+be)+\frac{3}{11}abx^{11}(af+bc)+\frac{1}{4}abx^{12}(ag+bd)+\frac{3}{13}abx^{13}(ah+be)+\frac{1}{17}b^3fx^{17}+\frac{1}{18}b^3gx^{18}+\frac{1}{19}b^3hx^{19}$$

```
[In] Int[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^5)/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (a^2*(3*b*c + a*f)*x^8)/8 +
 (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^10)/10 + (3*a*b*(b*c + a*
f)*x^11)/11 + (a*b*(b*d + a*g)*x^12)/4 + (3*a*b*(b*e + a*h)*x^13)/13 + (b^2
*(b*c + 3*a*f)*x^14)/14 + (b^2*(b*d + 3*a*g)*x^15)/15 + (b^2*(b*e + 3*a*h)*
x^16)/16 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (b^3*h*x^19)/19
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3cx^4+a^3dx^5+a^3ex^6+a^2(3bc+af)x^7+a^2(3bd+ag)x^8+a^2(3be+ah)x^9\right.\\ &\quad +3ab(bc+af)x^{10}+3ab(bd+ag)x^{11}+3ab(be+ah)x^{12}+b^2(bc+3af)x^{13}\\ &\quad \left.+b^2(bd+3ag)x^{14}+b^2(be+3ah)x^{15}+b^3fx^{16}+b^3gx^{17}+b^3hx^{18}\right)\,dx\\ &= \frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2(3bc+af)x^8+\frac{1}{9}a^2(3bd+ag)x^9+\frac{1}{10}a^2(3be+ah)x^{10}\\ &\quad +\frac{3}{11}ab(bc+af)x^{11}+\frac{1}{4}ab(bd+ag)x^{12}+\frac{3}{13}ab(be+ah)x^{13}+\frac{1}{14}b^2(bc+3af)x^{14}\\ &\quad +\frac{1}{15}b^2(bd+3ag)x^{15}+\frac{1}{16}b^2(be+3ah)x^{16}+\frac{1}{17}b^3fx^{17}+\frac{1}{18}b^3gx^{18}+\frac{1}{19}b^3hx^{19}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int x^4\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\ &= \frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2(3bc+af)x^8+\frac{1}{9}a^2(3bd+ag)x^9+\frac{1}{10}a^2(3be+ah)x^{10}\\ &\quad +\frac{3}{11}ab(bc+af)x^{11}+\frac{1}{4}ab(bd+ag)x^{12}+\frac{3}{13}ab(be+ah)x^{13}+\frac{1}{14}b^2(bc+3af)x^{14}\\ &\quad +\frac{1}{15}b^2(bd+3ag)x^{15}+\frac{1}{16}b^2(be+3ah)x^{16}+\frac{1}{17}b^3fx^{17}+\frac{1}{18}b^3gx^{18}+\frac{1}{19}b^3hx^{19}\end{aligned}$$

```
[In] Integrate[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^5)/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (a^2*(3*b*c + a*f)*x^8)/8 +
 (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^10)/10 + (3*a*b*(b*c + a*
f)*x^11)/11 + (a*b*(b*d + a*g)*x^12)/4 + (3*a*b*(b*e + a*h)*x^13)/13 + (b^2
*(b*c + 3*a*f)*x^14)/14 + (b^2*(b*d + 3*a*g)*x^15)/15 + (b^2*(b*e + 3*a*h)*
x^16)/16 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (b^3*h*x^19)/19
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.99

method	result
norman	$\frac{a^3 c x^5}{5}+\frac{a^3 d x^6}{6}+\frac{a^3 e x^7}{7}+\left(\frac{1}{8} f a^3+\frac{3}{8} a^2 b c\right) x^8+\left(\frac{1}{9} a^3 g+\frac{1}{3} d a^2 b\right) x^9+\left(\frac{1}{10} a^3 h+\frac{3}{10} a^2 b e\right) x^{10}+($
default	$\frac{b^3 h x^{19}}{19}+\frac{b^3 g x^{18}}{18}+\frac{b^3 f x^{17}}{17}+\frac{\left(3 a b^2 h+b^3 e\right) x^{16}}{16}+\frac{\left(3 a b^2 g+b^3 d\right) x^{15}}{15}+\frac{\left(3 a b^2 f+b^3 c\right) x^{14}}{14}+\frac{\left(3 a^2 b h+3 a b^2 e\right) x^{13}}{13}+\frac{(3}{}$
gosper	$\frac{1}{5} a^3 c x^5+\frac{1}{6} a^3 d x^6+\frac{1}{7} a^3 e x^7+\frac{1}{8} x^8 f a^3+\frac{3}{8} x^8 a^2 b c+\frac{1}{9} x^9 a^3 g+\frac{1}{3} a^2 b d x^9+\frac{1}{10} x^{10} a^3 h+\frac{3}{10} a^2 b e x$
risch	$\frac{1}{5} a^3 c x^5+\frac{1}{6} a^3 d x^6+\frac{1}{7} a^3 e x^7+\frac{1}{8} x^8 f a^3+\frac{3}{8} x^8 a^2 b c+\frac{1}{9} x^9 a^3 g+\frac{1}{3} a^2 b d x^9+\frac{1}{10} x^{10} a^3 h+\frac{3}{10} a^2 b e x$
parallelrisch	$\frac{1}{5} a^3 c x^5+\frac{1}{6} a^3 d x^6+\frac{1}{7} a^3 e x^7+\frac{1}{8} x^8 f a^3+\frac{3}{8} x^8 a^2 b c+\frac{1}{9} x^9 a^3 g+\frac{1}{3} a^2 b d x^9+\frac{1}{10} x^{10} a^3 h+\frac{3}{10} a^2 b e x$

```
[In] int(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/5*a^3*c*x^5+1/6*a^3*d*x^6+1/7*a^3*e*x^7+(1/8*f*a^3+3/8*a^2*b*c)*x^8+(1/9*
a^3*g+1/3*d*a^2*b)*x^9+(1/10*a^3*h+3/10*a^2*b*e)*x^10+(3/11*f*a^2*b+3/11*a*
b^2*c)*x^11+(1/4*a^2*b*g+1/4*a*b^2*d)*x^12+(3/13*a^2*b*h+3/13*a*b^2*e)*x^13
+(3/14*a*b^2*f+1/14*b^3*c)*x^14+(1/5*a*b^2*g+1/15*b^3*d)*x^15+(3/16*a*b^2*h
+1/16*b^3*e)*x^16+1/17*b^3*f*x^17+1/18*b^3*g*x^18+1/19*b^3*h*x^19
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

$$\begin{aligned}\int x^4 (a+bx^3)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) \, dx &= \frac{1}{19} b^3 h x^{19}+\frac{1}{18} b^3 g x^{18}+\frac{1}{17} b^3 f x^{17}+\frac{1}{16} \left(b^3 e+3 a b^2 h\right) x^{16} \\ &+\frac{1}{15} \left(b^3 d+3 a b^2 g\right) x^{15}+\frac{1}{14} \left(b^3 c+3 a b^2 f\right) x^{14}+\frac{3}{13} \left(a b^2 e+a^2 b h\right) x^{13} \\ &+\frac{1}{4} \left(a b^2 d+a^2 b g\right) x^{12}+\frac{3}{11} \left(a b^2 c+a^2 b f\right) x^{11}+\frac{1}{7} a^3 e x^7+\frac{1}{10} \left(3 a^2 b e+a^3 h\right) x^{10} \\ &+\frac{1}{6} a^3 d x^6+\frac{1}{9} \left(3 a^2 b d+a^3 g\right) x^9+\frac{1}{5} a^3 c x^5+\frac{1}{8} \left(3 a^2 b c+a^3 f\right) x^8\end{aligned}$$

```
[In] integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 1/16*(b^3*e + 3*a*b^2
*h)*x^16 + 1/15*(b^3*d + 3*a*b^2*g)*x^15 + 1/14*(b^3*c + 3*a*b^2*f)*x^14 +
3/13*(a*b^2*e + a^2*b*h)*x^13 + 1/4*(a*b^2*d + a^2*b*g)*x^12 + 3/11*(a*b^2*
c + a^2*b*f)*x^11 + 1/7*a^3*e*x^7 + 1/10*(3*a^2*b*e + a^3*h)*x^10 + 1/6*a^3
*d*x^6 + 1/9*(3*a^2*b*d + a^3*g)*x^9 + 1/5*a^3*c*x^5 + 1/8*(3*a^2*b*c + a^3
*f)*x^8
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.10

$$\int x^4(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)\;dx$$
$$=\frac{a^3cx^5}{5}+\frac{a^3dx^6}{6}+\frac{a^3ex^7}{7}+\frac{b^3fx^{17}}{17}+\frac{b^3gx^{18}}{18}+\frac{b^3hx^{19}}{19}+x^{16}\cdot\left(\frac{3ab^2h}{16}+\frac{b^3e}{16}\right)+x^{15}\left(\frac{ab^2g}{5}+\frac{b^3d}{15}\right)+x^{14}\cdot\left(\frac{3ab^2f}{14}+\frac{b^3c}{14}\right)+x^{13}\cdot\left(\frac{3a^2bh}{13}+\frac{3ab^2e}{13}\right)+x^{12}\left(\frac{a^2bg}{4}+\frac{ab^2d}{4}\right)+x^{11}\cdot\left(\frac{3a^2bf}{11}+\frac{3ab^2c}{11}\right)+x^{10}\left(\frac{a^3h}{10}+\frac{3a^2be}{10}\right)+x^9\left(\frac{a^3g}{9}+\frac{a^2bd}{3}\right)+x^8\left(\frac{a^3f}{8}+\frac{3a^2bc}{8}\right)$$

```
[In] integrate(x**4*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**5/5 + a**3*d*x**6/6 + a**3*e*x**7/7 + b**3*f*x**17/17 + b**3*g*x*
*18/18 + b**3*h*x**19/19 + x**16*(3*a*b**2*h/16 + b**3*e/16) + x**15*(a*b**
2*g/5 + b**3*d/15) + x**14*(3*a*b**2*f/14 + b**3*c/14) + x**13*(3*a**2*b*h/
13 + 3*a*b**2*e/13) + x**12*(a**2*b*g/4 + a*b**2*d/4) + x**11*(3*a**2*b*f/1
1 + 3*a*b**2*c/11) + x**10*(a**3*h/10 + 3*a**2*b*e/10) + x**9*(a**3*g/9 + a
**2*b*d/3) + x**8*(a**3*f/8 + 3*a**2*b*c/8)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

$$\int x^4(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)\;dx$$
$$=\frac{1}{19}\,b^3hx^{19}+\frac{1}{18}\,b^3gx^{18}+\frac{1}{17}\,b^3fx^{17}+\frac{1}{16}\,(b^3e+3\,ab^2h)x^{16}+\frac{1}{15}\,(b^3d+3\,ab^2g)x^{15}+\frac{1}{14}\,(b^3c+3\,ab^2f)x^{14}+\frac{3}{13}\,(ab^2e+a^2bh)x^{13}+\frac{1}{4}\,(ab^2d+a^2bg)x^{12}+\frac{3}{11}\,(ab^2c+a^2bf)x^{11}+\frac{1}{7}\,a^3ex^7+\frac{1}{10}\,(3\,a^2be+a^3h)x^{10}+\frac{1}{6}\,a^3dx^6+\frac{1}{9}\,(3\,a^2bd+a^3g)x^9+\frac{1}{5}\,a^3cx^5+\frac{1}{8}\,(3\,a^2bc+a^3f)x^8$$

```
[In] integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 1/16*(b^3*e + 3*a*b^2
*h)*x^16 + 1/15*(b^3*d + 3*a*b^2*g)*x^15 + 1/14*(b^3*c + 3*a*b^2*f)*x^14 +
3/13*(a*b^2*e + a^2*b*h)*x^13 + 1/4*(a*b^2*d + a^2*b*g)*x^12 + 3/11*(a*b^2*
c + a^2*b*f)*x^11 + 1/7*a^3*e*x^7 + 1/10*(3*a^2*b*e + a^3*h)*x^10 + 1/6*a^3
*d*x^6 + 1/9*(3*a^2*b*d + a^3*g)*x^9 + 1/5*a^3*c*x^5 + 1/8*(3*a^2*b*c + a^3
*f)*x^8
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.03

$$\int x^4(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{19}b^3hx^{19}+\frac{1}{18}b^3gx^{18}+\frac{1}{17}b^3fx^{17}+\frac{1}{16}b^3ex^{16}+\frac{3}{16}ab^2hx^{16}+\frac{1}{15}b^3dx^{15}+\frac{1}{5}ab^2gx^{15}+\frac{1}{14}b^3cx^{14}+\frac{3}{14}ab^2fx^{14}+\frac{3}{13}ab^2ex^{13}+\frac{3}{13}a^2bhx^{13}+\frac{1}{4}ab^2dx^{12}+\frac{1}{4}a^2bgx^{12}+\frac{3}{11}ab^2cx^{11}+\frac{3}{11}a^2bfx^{11}+\frac{3}{10}a^2bex^{10}+\frac{1}{10}a^3hx^{10}+\frac{1}{3}a^2bdx^9+\frac{1}{9}a^3gx^9+\frac{3}{8}a^2bcx^8+\frac{1}{8}a^3fx^8+\frac{1}{7}a^3ex^7+\frac{1}{6}a^3dx^6+\frac{1}{5}a^3cx^5$$

```
[In] integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 1/16*b^3*e*x^16 + 3/1
6*a*b^2*h*x^16 + 1/15*b^3*d*x^15 + 1/5*a*b^2*g*x^15 + 1/14*b^3*c*x^14 + 3/1
4*a*b^2*f*x^14 + 3/13*a*b^2*e*x^13 + 3/13*a^2*b*h*x^13 + 1/4*a*b^2*d*x^12 +
 1/4*a^2*b*g*x^12 + 3/11*a*b^2*c*x^11 + 3/11*a^2*b*f*x^11 + 3/10*a^2*b*e*x^
10 + 1/10*a^3*h*x^10 + 1/3*a^2*b*d*x^9 + 1/9*a^3*g*x^9 + 3/8*a^2*b*c*x^8 +
1/8*a^3*f*x^8 + 1/7*a^3*e*x^7 + 1/6*a^3*d*x^6 + 1/5*a^3*c*x^5
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.92

$$\int x^4(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=x^8\left(\frac{f\,a^3}{8}+\frac{3\,b\,c\,a^2}{8}\right)+x^{14}\left(\frac{c\,b^3}{14}+\frac{3\,a\,f\,b^2}{14}\right)+x^9\left(\frac{g\,a^3}{9}+\frac{b\,d\,a^2}{3}\right)+x^{15}\left(\frac{d\,b^3}{15}+\frac{a\,g\,b^2}{5}\right)+x^{10}\left(\frac{h\,a^3}{10}+\frac{3\,b\,e\,a^2}{10}\right)+x^{16}\left(\frac{e\,b^3}{16}+\frac{3\,a\,h\,b^2}{16}\right)+\frac{a^3\,c\,x^5}{5}+\frac{a^3\,d\,x^6}{6}+\frac{a^3\,e\,x^7}{7}+\frac{b^3\,f\,x^{17}}{17}+\frac{b^3\,g\,x^{18}}{18}+\frac{b^3\,h\,x^{19}}{19}+\frac{3\,a\,b\,x^{11}\,(b\,c+a\,f)}{11}+\frac{a\,b\,x^{12}\,(b\,d+a\,g)}{4}+\frac{3\,a\,b\,x^{13}\,(b\,e+a\,h)}{13}$$

```
[In] int(x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^8*((a^3*f)/8 + (3*a^2*b*c)/8) + x^14*((b^3*c)/14 + (3*a*b^2*f)/14) + x^9*
((a^3*g)/9 + (a^2*b*d)/3) + x^15*((b^3*d)/15 + (a*b^2*g)/5) + x^10*((a^3*h)
/10 + (3*a^2*b*e)/10) + x^16*((b^3*e)/16 + (3*a*b^2*h)/16) + (a^3*c*x^5)/5
+ (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (b^3*
h*x^19)/19 + (3*a*b*x^11*(b*c + a*f))/11 + (a*b*x^12*(b*d + a*g))/4 + (3*a*
b*x^13*(b*e + a*h))/13
```

3.394 $\int x^3(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 223

$$\begin{aligned}\int x^3(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= \frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2(3bc+af)x^7+\frac{1}{8}a^2(3bd+ag)x^8+\frac{1}{9}a^2(3be+ah)x^9\\&\quad+\frac{3}{10}ab(bc+af)x^{10}+\frac{3}{11}ab(bd+ag)x^{11}+\frac{1}{4}ab(be+ah)x^{12}+\frac{1}{13}b^2(bc+3af)x^{13}\\&\quad+\frac{1}{14}b^2(bd+3ag)x^{14}+\frac{1}{15}b^2(be+3ah)x^{15}+\frac{1}{16}b^3fx^{16}+\frac{1}{17}b^3gx^{17}+\frac{1}{18}b^3hx^{18}\end{aligned}$$

```
[Out] 1/4*a^3*c*x^4+1/5*a^3*d*x^5+1/6*a^3*e*x^6+1/7*a^2*(a*f+3*b*c)*x^7+1/8*a^2*(
a*g+3*b*d)*x^8+1/9*a^2*(a*h+3*b*e)*x^9+3/10*a*b*(a*f+b*c)*x^10+3/11*a*b*(a*
g+b*d)*x^11+1/4*a*b*(a*h+b*e)*x^12+1/13*b^2*(3*a*f+b*c)*x^13+1/14*b^2*(3*a*
g+b*d)*x^14+1/15*b^2*(3*a*h+b*e)*x^15+1/16*b^3*f*x^16+1/17*b^3*g*x^17+1/18*
b^3*h*x^18
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1834}

$$\begin{aligned}\int x^3(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= \frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2x^7(af+3bc)+\frac{1}{8}a^2x^8(ag+3bd)+\frac{1}{9}a^2x^9(ah+3be)\\&\quad+\frac{1}{13}b^2x^{13}(3af+bc)+\frac{1}{14}b^2x^{14}(3ag+bd)+\frac{1}{15}b^2x^{15}(3ah+be)+\frac{3}{10}abx^{10}(af+bc)\\&\quad+\frac{3}{11}abx^{11}(ag+bd)+\frac{1}{4}abx^{12}(ah+be)+\frac{1}{16}b^3fx^{16}+\frac{1}{17}b^3gx^{17}+\frac{1}{18}b^3hx^{18}\end{aligned}$$

```
[In] Int[x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^2*(3*b*c + a*f)*x^7)/7 +
 (a^2*(3*b*d + a*g)*x^8)/8 + (a^2*(3*b*e + a*h)*x^9)/9 + (3*a*b*(b*c + a*f)
*x^10)/10 + (3*a*b*(b*d + a*g)*x^11)/11 + (a*b*(b*e + a*h)*x^12)/4 + (b^2*(
b*c + 3*a*f)*x^13)/13 + (b^2*(b*d + 3*a*g)*x^14)/14 + (b^2*(b*e + 3*a*h)*x^
15)/15 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3*h*x^18)/18
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3cx^3+a^3dx^4+a^3ex^5+a^2(3bc+af)x^6+a^2(3bd+ag)x^7+a^2(3be+ah)x^8\right.\\&\quad+3ab(bc+af)x^9+3ab(bd+ag)x^{10}+3ab(be+ah)x^{11}+b^2(bc+3af)x^{12}\\&\quad\left.+b^2(bd+3ag)x^{13}+b^2(be+3ah)x^{14}+b^3fx^{15}+b^3gx^{16}+b^3hx^{17}\right)dx\\&=\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2(3bc+af)x^7+\frac{1}{8}a^2(3bd+ag)x^8+\frac{1}{9}a^2(3be+ah)x^9\\&\quad+\frac{3}{10}ab(bc+af)x^{10}+\frac{3}{11}ab(bd+ag)x^{11}+\frac{1}{4}ab(be+ah)x^{12}+\frac{1}{13}b^2(bc+3af)x^{13}\\&\quad+\frac{1}{14}b^2(bd+3ag)x^{14}+\frac{1}{15}b^2(be+3ah)x^{15}+\frac{1}{16}b^3fx^{16}+\frac{1}{17}b^3gx^{17}+\frac{1}{18}b^3hx^{18}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int x^3\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx\\&=\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2(3bc+af)x^7+\frac{1}{8}a^2(3bd+ag)x^8+\frac{1}{9}a^2(3be+ah)x^9\\&\quad+\frac{3}{10}ab(bc+af)x^{10}+\frac{3}{11}ab(bd+ag)x^{11}+\frac{1}{4}ab(be+ah)x^{12}+\frac{1}{13}b^2(bc+3af)x^{13}\\&\quad+\frac{1}{14}b^2(bd+3ag)x^{14}+\frac{1}{15}b^2(be+3ah)x^{15}+\frac{1}{16}b^3fx^{16}+\frac{1}{17}b^3gx^{17}+\frac{1}{18}b^3hx^{18}\end{aligned}$$

```
[In] Integrate[x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^2*(3*b*c + a*f)*x^7)/7 +
 (a^2*(3*b*d + a*g)*x^8)/8 + (a^2*(3*b*e + a*h)*x^9)/9 + (3*a*b*(b*c + a*f)
*x^10)/10 + (3*a*b*(b*d + a*g)*x^11)/11 + (a*b*(b*e + a*h)*x^12)/4 + (b^2*(
b*c + 3*a*f)*x^13)/13 + (b^2*(b*d + 3*a*g)*x^14)/14 + (b^2*(b*e + 3*a*h)*x^
15)/15 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3*h*x^18)/18
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.99

method	result
norman	$\frac{a^{3} c\,x^{4}}{4}+\frac{a^{3} d\,x^{5}}{5}+\frac{a^{3} e\,x^{6}}{6}+\left(\frac{1}{7} f\,a^{3}+\frac{3}{7} a^{2} b c \right) x^{7}+\left(\frac{1}{8} a^{3} g +\frac{3}{8} d\,a^{2} b \right) x^{8}+\left(\frac{1}{9} a^{3} h +\frac{1}{3} a^{2} b e \right) x^{9}+\left(\frac{3}{10}\right.$
default	$\frac{b^{3} h\,x^{18}}{18}+\frac{b^{3} g\,x^{17}}{17}+\frac{b^{3} f\,x^{16}}{16}+\frac{\left(3 a\,b^{2} h +b^{3} e \right) x^{15}}{15}+\frac{\left(3 a\,b^{2} g +b^{3} d \right) x^{14}}{14}+\frac{\left(3 a\,b^{2} f +b^{3} c \right) x^{13}}{13}+\frac{\left(3 a^{2} b h +3 a\,b^{2} e \right) x^{12}}{12}+$
gosper	$\frac{1}{4} a^{3} c\,x^{4}+\frac{1}{5} a^{3} d\,x^{5}+\frac{1}{6} a^{3} e\,x^{6}+\frac{1}{7} a^{3} f\,x^{7}+\frac{3}{7} x^{7} a^{2} b c +\frac{1}{8} x^{8} a^{3} g +\frac{3}{8} x^{8} d\,a^{2} b +\frac{1}{9} x^{9} a^{3} h +\frac{1}{3} x^{9} a^{2} b e$
risch	$\frac{1}{4} a^{3} c\,x^{4}+\frac{1}{5} a^{3} d\,x^{5}+\frac{1}{6} a^{3} e\,x^{6}+\frac{1}{7} a^{3} f\,x^{7}+\frac{3}{7} x^{7} a^{2} b c +\frac{1}{8} x^{8} a^{3} g +\frac{3}{8} x^{8} d\,a^{2} b +\frac{1}{9} x^{9} a^{3} h +\frac{1}{3} x^{9} a^{2} b e$
parallelrisch	$\frac{1}{4} a^{3} c\,x^{4}+\frac{1}{5} a^{3} d\,x^{5}+\frac{1}{6} a^{3} e\,x^{6}+\frac{1}{7} a^{3} f\,x^{7}+\frac{3}{7} x^{7} a^{2} b c +\frac{1}{8} x^{8} a^{3} g +\frac{3}{8} x^{8} d\,a^{2} b +\frac{1}{9} x^{9} a^{3} h +\frac{1}{3} x^{9} a^{2} b e$

```
[In] int(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/4*a^3*c*x^4+1/5*a^3*d*x^5+1/6*a^3*e*x^6+(1/7*f*a^3+3/7*a^2*b*c)*x^7+(1/8*
a^3*g+3/8*d*a^2*b)*x^8+(1/9*a^3*h+1/3*a^2*b*e)*x^9+(3/10*f*a^2*b+3/10*a*b^2
*c)*x^10+(3/11*a^2*b*g+3/11*a*b^2*d)*x^11+(1/4*a^2*b*h+1/4*a*b^2*e)*x^12+(3
/13*a*b^2*f+1/13*b^3*c)*x^13+(3/14*a*b^2*g+1/14*b^3*d)*x^14+(1/5*a*b^2*h+1/
15*b^3*e)*x^15+1/16*b^3*f*x^16+1/17*b^3*g*x^17+1/18*b^3*h*x^18
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

$$\begin{aligned}\int x^3\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{1}{18}\,b^3hx^{18}+\frac{1}{17}\,b^3gx^{17}+\frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\,(b^3e+3\,ab^2h)x^{15}\\&+\frac{1}{14}\,(b^3d+3\,ab^2g)x^{14}+\frac{1}{13}\,(b^3c+3\,ab^2f)x^{13}+\frac{1}{4}\,(ab^2e+a^2bh)x^{12}\\&+\frac{3}{11}\,(ab^2d+a^2bg)x^{11}+\frac{3}{10}\,(ab^2c+a^2bf)x^{10}+\frac{1}{6}\,a^3ex^6+\frac{1}{9}\,(3\,a^2be+a^3h)x^9\\&+\frac{1}{5}\,a^3dx^5+\frac{1}{8}\,(3\,a^2bd+a^3g)x^8+\frac{1}{4}\,a^3cx^4+\frac{1}{7}\,(3\,a^2bc+a^3f)x^7\end{aligned}$$

```
[In] integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/15*(b^3*e + 3*a*b^2
*h)*x^15 + 1/14*(b^3*d + 3*a*b^2*g)*x^14 + 1/13*(b^3*c + 3*a*b^2*f)*x^13 +
1/4*(a*b^2*e + a^2*b*h)*x^12 + 3/11*(a*b^2*d + a^2*b*g)*x^11 + 3/10*(a*b^2*
c + a^2*b*f)*x^10 + 1/6*a^3*e*x^6 + 1/9*(3*a^2*b*e + a^3*h)*x^9 + 1/5*a^3*d
*x^5 + 1/8*(3*a^2*b*d + a^3*g)*x^8 + 1/4*a^3*c*x^4 + 1/7*(3*a^2*b*c + a^3*f
)*x^7
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.10

$$\int x^3(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{a^3cx^4}{4}+\frac{a^3dx^5}{5}+\frac{a^3ex^6}{6}+\frac{b^3fx^{16}}{16}+\frac{b^3gx^{17}}{17}+\frac{b^3hx^{18}}{18}+x^{15}\left(\frac{ab^2h}{5}+\frac{b^3e}{15}\right)+x^{14}\cdot\left(\frac{3ab^2g}{14}+\frac{b^3d}{14}\right)+x^{13}\cdot\left(\frac{3ab^2f}{13}+\frac{b^3c}{13}\right)+x^{12}\left(\frac{a^2bh}{4}+\frac{ab^2e}{4}\right)+x^{11}\cdot\left(\frac{3a^2bg}{11}+\frac{3ab^2d}{11}\right)+x^{10}\cdot\left(\frac{3a^2bf}{10}+\frac{3ab^2c}{10}\right)+x^9\left(\frac{a^3h}{9}+\frac{a^2be}{3}\right)+x^8\left(\frac{a^3g}{8}+\frac{3a^2bd}{8}\right)+x^7\left(\frac{a^3f}{7}+\frac{3a^2bc}{7}\right)$$

```
[In] integrate(x**3*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + b**3*f*x**16/16 + b**3*g*x*
*17/17 + b**3*h*x**18/18 + x**15*(a*b**2*h/5 + b**3*e/15) + x**14*(3*a*b**2
*g/14 + b**3*d/14) + x**13*(3*a*b**2*f/13 + b**3*c/13) + x**12*(a**2*b*h/4
+ a*b**2*e/4) + x**11*(3*a**2*b*g/11 + 3*a*b**2*d/11) + x**10*(3*a**2*b*f/1
0 + 3*a*b**2*c/10) + x**9*(a**3*h/9 + a**2*b*e/3) + x**8*(a**3*g/8 + 3*a**2
*b*d/8) + x**7*(a**3*f/7 + 3*a**2*b*c/7)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

$$\int x^3(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{18}b^3hx^{18}+\frac{1}{17}b^3gx^{17}+\frac{1}{16}b^3fx^{16}+\frac{1}{15}\left(b^3e+3\,ab^2h\right)x^{15}+\frac{1}{14}\left(b^3d+3\,ab^2g\right)x^{14}+\frac{1}{13}\left(b^3c+3\,ab^2f\right)x^{13}+\frac{1}{4}\left(ab^2e+a^2bh\right)x^{12}+\frac{3}{11}\left(ab^2d+a^2bg\right)x^{11}+\frac{3}{10}\left(ab^2c+a^2bf\right)x^{10}+\frac{1}{6}a^3ex^6+\frac{1}{9}\left(3\,a^2be+a^3h\right)x^9+\frac{1}{5}a^3dx^5+\frac{1}{8}\left(3\,a^2bd+a^3g\right)x^8+\frac{1}{4}a^3cx^4+\frac{1}{7}\left(3\,a^2bc+a^3f\right)x^7$$

```
[In] integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/15*(b^3*e + 3*a*b^2
*h)*x^15 + 1/14*(b^3*d + 3*a*b^2*g)*x^14 + 1/13*(b^3*c + 3*a*b^2*f)*x^13 +
1/4*(a*b^2*e + a^2*b*h)*x^12 + 3/11*(a*b^2*d + a^2*b*g)*x^11 + 3/10*(a*b^2*
c + a^2*b*f)*x^10 + 1/6*a^3*e*x^6 + 1/9*(3*a^2*b*e + a^3*h)*x^9 + 1/5*a^3*d
*x^5 + 1/8*(3*a^2*b*d + a^3*g)*x^8 + 1/4*a^3*c*x^4 + 1/7*(3*a^2*b*c + a^3*f
)*x^7
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.03

$$\int x^3(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$$
$$=\frac{1}{18}b^3hx^{18}+\frac{1}{17}b^3gx^{17}+\frac{1}{16}b^3fx^{16}+\frac{1}{15}b^3ex^{15}+\frac{1}{5}ab^2hx^{15}+\frac{1}{14}b^3dx^{14}+\frac{3}{14}ab^2gx^{14}+\frac{1}{13}b^3cx^{13}+\frac{3}{13}ab^2fx^{13}+\frac{1}{4}ab^2ex^{12}+\frac{1}{4}a^2bhx^{12}+\frac{3}{11}ab^2dx^{11}+\frac{3}{11}a^2bgx^{11}+\frac{3}{10}ab^2cx^{10}+\frac{3}{10}a^2bfx^{10}+\frac{1}{3}a^2bex^9+\frac{1}{9}a^3hx^9+\frac{3}{8}a^2bdx^8+\frac{1}{8}a^3gx^8+\frac{3}{7}a^2bcx^7+\frac{1}{7}a^3fx^7+\frac{1}{6}a^3ex^6+\frac{1}{5}a^3dx^5+\frac{1}{4}a^3cx^4$$

```
[In] integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/5
*a*b^2*h*x^15 + 1/14*b^3*d*x^14 + 3/14*a*b^2*g*x^14 + 1/13*b^3*c*x^13 + 3/1
3*a*b^2*f*x^13 + 1/4*a*b^2*e*x^12 + 1/4*a^2*b*h*x^12 + 3/11*a*b^2*d*x^11 +
3/11*a^2*b*g*x^11 + 3/10*a*b^2*c*x^10 + 3/10*a^2*b*f*x^10 + 1/3*a^2*b*e*x^9
 + 1/9*a^3*h*x^9 + 3/8*a^2*b*d*x^8 + 1/8*a^3*g*x^8 + 3/7*a^2*b*c*x^7 + 1/7*
a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.92

$$\int x^3(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$$
$$=x^7\left(\frac{f\,a^3}{7}+\frac{3\,b\,c\,a^2}{7}\right)+x^{13}\left(\frac{c\,b^3}{13}+\frac{3\,a\,f\,b^2}{13}\right)+x^8\left(\frac{g\,a^3}{8}+\frac{3\,b\,d\,a^2}{8}\right)+x^{14}\left(\frac{d\,b^3}{14}+\frac{3\,a\,g\,b^2}{14}\right)+x^9\left(\frac{h\,a^3}{9}+\frac{b\,e\,a^2}{3}\right)+x^{15}\left(\frac{e\,b^3}{15}+\frac{a\,h\,b^2}{5}\right)+\frac{a^3\,c\,x^4}{4}+\frac{a^3\,d\,x^5}{5}+\frac{a^3\,e\,x^6}{6}+\frac{b^3\,f\,x^{16}}{16}+\frac{b^3\,g\,x^{17}}{17}+\frac{b^3\,h\,x^{18}}{18}+\frac{3\,a\,b\,x^{10}\,(b\,c+a\,f)}{10}+\frac{3\,a\,b\,x^{11}\,(b\,d+a\,g)}{11}+\frac{a\,b\,x^{12}\,(b\,e+a\,h)}{4}$$

```
[In] int(x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^7*((a^3*f)/7 + (3*a^2*b*c)/7) + x^13*((b^3*c)/13 + (3*a*b^2*f)/13) + x^8*
((a^3*g)/8 + (3*a^2*b*d)/8) + x^14*((b^3*d)/14 + (3*a*b^2*g)/14) + x^9*((a^
3*h)/9 + (a^2*b*e)/3) + x^15*((b^3*e)/15 + (a*b^2*h)/5) + (a^3*c*x^4)/4 + (
a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3*h*x
^18)/18 + (3*a*b*x^10*(b*c + a*f))/10 + (3*a*b*x^11*(b*d + a*g))/11 + (a*b*
x^12*(b*e + a*h))/4
```

3.395 $\int x^2(a+bx^3)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal result

Integrand size = 38, antiderivative size = 212

$$\begin{aligned}
&\int x^2\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\
&=\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^3fx^6+\frac{1}{7}a^2(3bd+ag)x^7+\frac{1}{8}a^2(3be+ah)x^8+\frac{1}{3}a^2bfx^9\\
&\quad+\frac{3}{10}ab(bd+ag)x^{10}+\frac{3}{11}ab(be+ah)x^{11}+\frac{1}{4}ab^2fx^{12}+\frac{1}{13}b^2(bd+3ag)x^{13}\\
&\quad+\frac{1}{14}b^2(be+3ah)x^{14}+\frac{1}{15}b^3fx^{15}+\frac{1}{16}b^3gx^{16}+\frac{1}{17}b^3hx^{17}+\frac{c(a+bx^3)^4}{12b}
\end{aligned}$$

```
[Out] 1/4*a^3*d*x^4+1/5*a^3*e*x^5+1/6*a^3*f*x^6+1/7*a^2*(a*g+3*b*d)*x^7+1/8*a^2*(
a*h+3*b*e)*x^8+1/3*a^2*b*f*x^9+3/10*a*b*(a*g+b*d)*x^10+3/11*a*b*(a*h+b*e)*x
^11+1/4*a*b^2*f*x^12+1/13*b^2*(3*a*g+b*d)*x^13+1/14*b^2*(3*a*h+b*e)*x^14+1/
15*b^3*f*x^15+1/16*b^3*g*x^16+1/17*b^3*h*x^17+1/12*c*(b*x^3+a)^4/b
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1596, 1864}

$$\begin{aligned}
&\int x^2\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\
&=\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^3fx^6+\frac{1}{7}a^2x^7(ag+3bd)+\frac{1}{8}a^2x^8(ah+3be)+\frac{1}{3}a^2bfx^9\\
&\quad+\frac{1}{13}b^2x^{13}(3ag+bd)+\frac{1}{14}b^2x^{14}(3ah+be)+\frac{1}{4}ab^2fx^{12}+\frac{c(a+bx^3)^4}{12b}\\
&\quad+\frac{3}{10}abx^{10}(ag+bd)+\frac{3}{11}abx^{11}(ah+be)+\frac{1}{15}b^3fx^{15}+\frac{1}{16}b^3gx^{16}+\frac{1}{17}b^3hx^{17}
\end{aligned}$$

```
[In] Int[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^3*f*x^6)/6 + (a^2*(3*b*d + a*g)*x^7)/7 +
 (a^2*(3*b*e + a*h)*x^8)/8 + (a^2*b*f*x^9)/3 + (3*a*b*(b*d + a*g)*x^10)/10
+ (3*a*b*(b*e + a*h)*x^11)/11 + (a*b^2*f*x^12)/4 + (b^2*(b*d + 3*a*g)*x^13)
/13 + (b^2*(b*e + 3*a*h)*x^14)/14 + (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^
3*h*x^17)/17 + (c*(a + b*x^3)^4)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c(a+bx^3)^4}{12b} + \int (a+bx^3)^3 \left(-cx^2 + x^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\right) dx \\
&= \frac{c(a+bx^3)^4}{12b} + \int \left(a^3dx^3 + a^3ex^4 + a^3fx^5 + a^2(3bd+ag)x^6 + a^2(3be+ah)x^7 \right.\\
&\quad + 3a^2bfx^8 + 3ab(bd+ag)x^9 + 3ab(be+ah)x^{10} + 3ab^2fx^{11} + b^2(bd+3ag)x^{12} \\
&\quad \left. + b^2(be+3ah)x^{13} + b^3fx^{14} + b^3gx^{15} + b^3hx^{16}\right) dx \\
&= \frac{1}{4}a^3dx^4 + \frac{1}{5}a^3ex^5 + \frac{1}{6}a^3fx^6 + \frac{1}{7}a^2(3bd+ag)x^7 + \frac{1}{8}a^2(3be+ah)x^8 + \frac{1}{3}a^2bfx^9 \\
&\quad + \frac{3}{10}ab(bd+ag)x^{10} + \frac{3}{11}ab(be+ah)x^{11} + \frac{1}{4}ab^2fx^{12} + \frac{1}{13}b^2(bd+3ag)x^{13} \\
&\quad + \frac{1}{14}b^2(be+3ah)x^{14} + \frac{1}{15}b^3fx^{15} + \frac{1}{16}b^3gx^{16} + \frac{1}{17}b^3hx^{17} + \frac{c(a+bx^3)^4}{12b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.05

$$\begin{aligned}&\int x^2(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\&=\frac{1}{3}a^3cx^3+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^2(3bc+af)x^6+\frac{1}{7}a^2(3bd+ag)x^7+\frac{1}{8}a^2(3be+ah)x^8\\&+\frac{1}{3}ab(bc+af)x^9+\frac{3}{10}ab(bd+ag)x^{10}+\frac{3}{11}ab(be+ah)x^{11}+\frac{1}{12}b^2(bc+3af)x^{12}\\&+\frac{1}{13}b^2(bd+3ag)x^{13}+\frac{1}{14}b^2(be+3ah)x^{14}+\frac{1}{15}b^3fx^{15}+\frac{1}{16}b^3gx^{16}+\frac{1}{17}b^3hx^{17}\end{aligned}$$

```
[In] Integrate[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*(3*b*c + a*f)*x^6)/6 +
 (a^2*(3*b*d + a*g)*x^7)/7 + (a^2*(3*b*e + a*h)*x^8)/8 + (a*b*(b*c + a*f)*x
^9)/3 + (3*a*b*(b*d + a*g)*x^10)/10 + (3*a*b*(b*e + a*h)*x^11)/11 + (b^2*(b
*c + 3*a*f)*x^12)/12 + (b^2*(b*d + 3*a*g)*x^13)/13 + (b^2*(b*e + 3*a*h)*x^1
4)/14 + (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^3*h*x^17)/17
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.04

method	result
norman	$\frac{c\,a^3x^3}{3}+\frac{a^3d\,x^4}{4}+\frac{a^3e\,x^5}{5}+\left(\frac{1}{6}f\,a^3+\frac{1}{2}a^2bc\right)x^6+\left(\frac{1}{7}a^3g+\frac{3}{7}d\,a^2b\right)x^7+\left(\frac{1}{8}a^3h+\frac{3}{8}a^2be\right)x^8+\Big(\frac{1}{3}f$
default	$\frac{b^3h\,x^{17}}{17}+\frac{b^3g\,x^{16}}{16}+\frac{b^3f\,x^{15}}{15}+\frac{(3a\,b^2h+b^3e)x^{14}}{14}+\frac{(3a\,b^2g+b^3d)x^{13}}{13}+\frac{(3a\,b^2f+b^3c)x^{12}}{12}+\frac{(3a^2bh+3a\,b^2e)x^{11}}{11}+\frac{(3}{}$
gosper	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{6}a^3f\,x^6+\frac{1}{2}a^2bc\,x^6+\frac{1}{7}x^7a^3g+\frac{3}{7}a^2bd\,x^7+\frac{1}{8}x^8a^3h+\frac{3}{8}a^2be\,x^8$
risch	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{6}a^3f\,x^6+\frac{1}{2}a^2bc\,x^6+\frac{1}{7}x^7a^3g+\frac{3}{7}a^2bd\,x^7+\frac{1}{8}x^8a^3h+\frac{3}{8}a^2be\,x^8$
parallelrisch	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{6}a^3f\,x^6+\frac{1}{2}a^2bc\,x^6+\frac{1}{7}x^7a^3g+\frac{3}{7}a^2bd\,x^7+\frac{1}{8}x^8a^3h+\frac{3}{8}a^2be\,x^8$

```
[In] int(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/3*c*a^3*x^3+1/4*a^3*d*x^4+1/5*a^3*e*x^5+(1/6*f*a^3+1/2*a^2*b*c)*x^6+(1/7*
a^3*g+3/7*d*a^2*b)*x^7+(1/8*a^3*h+3/8*a^2*b*e)*x^8+(1/3*f*a^2*b+1/3*a*b^2*c
)*x^9+(3/10*a^2*b*g+3/10*a*b^2*d)*x^10+(3/11*a^2*b*h+3/11*a*b^2*e)*x^11+(1/
4*a*b^2*f+1/12*b^3*c)*x^12+(3/13*a*b^2*g+1/13*b^3*d)*x^13+(3/14*a*b^2*h+1/1
4*b^3*e)*x^14+1/15*b^3*f*x^15+1/16*b^3*g*x^16+1/17*b^3*h*x^17
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02

$$\int x^2(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{17}b^3hx^{17}+\frac{1}{16}b^3gx^{16}+\frac{1}{15}b^3fx^{15}+\frac{1}{14}\left(b^3e+3\,ab^2h\right)x^{14}+\frac{1}{13}\left(b^3d+3\,ab^2g\right)x^{13}+\frac{1}{12}\left(b^3c+3\,ab^2f\right)x^{12}+\frac{3}{11}\left(ab^2e+a^2bh\right)x^{11}+\frac{3}{10}\left(ab^2d+a^2bg\right)x^{10}+\frac{1}{3}\left(ab^2c+a^2bf\right)x^9+\frac{1}{5}a^3ex^5+\frac{1}{8}\left(3\,a^2be+a^3h\right)x^8+\frac{1}{4}a^3dx^4+\frac{1}{7}\left(3\,a^2bd+a^3g\right)x^7+\frac{1}{3}a^3cx^3+\frac{1}{6}\left(3\,a^2bc+a^3f\right)x^6$$

```
[In] integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 1/14*(b^3*e + 3*a*b^2
*h)*x^14 + 1/13*(b^3*d + 3*a*b^2*g)*x^13 + 1/12*(b^3*c + 3*a*b^2*f)*x^12 +
3/11*(a*b^2*e + a^2*b*h)*x^11 + 3/10*(a*b^2*d + a^2*b*g)*x^10 + 1/3*(a*b^2*
c + a^2*b*f)*x^9 + 1/5*a^3*e*x^5 + 1/8*(3*a^2*b*e + a^3*h)*x^8 + 1/4*a^3*d*
x^4 + 1/7*(3*a^2*b*d + a^3*g)*x^7 + 1/3*a^3*c*x^3 + 1/6*(3*a^2*b*c + a^3*f)
*x^6
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.16

$$\int x^2(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{a^3cx^3}{3}+\frac{a^3dx^4}{4}+\frac{a^3ex^5}{5}+\frac{b^3fx^{15}}{15}+\frac{b^3gx^{16}}{16}+\frac{b^3hx^{17}}{17}+x^{14}\cdot\left(\frac{3ab^2h}{14}+\frac{b^3e}{14}\right)+x^{13}\cdot\left(\frac{3ab^2g}{13}+\frac{b^3d}{13}\right)+x^{12}\left(\frac{ab^2f}{4}+\frac{b^3c}{12}\right)+x^{11}\cdot\left(\frac{3a^2bh}{11}+\frac{3ab^2e}{11}\right)+x^{10}\cdot\left(\frac{3a^2bg}{10}+\frac{3ab^2d}{10}\right)+x^9\left(\frac{a^2bf}{3}+\frac{ab^2c}{3}\right)+x^8\left(\frac{a^3h}{8}+\frac{3a^2be}{8}\right)+x^7\left(\frac{a^3g}{7}+\frac{3a^2bd}{7}\right)+x^6\left(\frac{a^3f}{6}+\frac{a^2bc}{2}\right)$$

```
[In] integrate(x**2*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**3/3 + a**3*d*x**4/4 + a**3*e*x**5/5 + b**3*f*x**15/15 + b**3*g*x*
*16/16 + b**3*h*x**17/17 + x**14*(3*a*b**2*h/14 + b**3*e/14) + x**13*(3*a*b
**2*g/13 + b**3*d/13) + x**12*(a*b**2*f/4 + b**3*c/12) + x**11*(3*a**2*b*h/
11 + 3*a*b**2*e/11) + x**10*(3*a**2*b*g/10 + 3*a*b**2*d/10) + x**9*(a**2*b*
f/3 + a*b**2*c/3) + x**8*(a**3*h/8 + 3*a**2*b*e/8) + x**7*(a**3*g/7 + 3*a**
2*b*d/7) + x**6*(a**3*f/6 + a**2*b*c/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02

$$\int x^2(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{17}b^3hx^{17}+\frac{1}{16}b^3gx^{16}+\frac{1}{15}b^3fx^{15}+\frac{1}{14}(b^3e+3ab^2h)x^{14}+\frac{1}{13}(b^3d+3ab^2g)x^{13}+\frac{1}{12}(b^3c+3ab^2f)x^{12}+\frac{3}{11}(ab^2e+a^2bh)x^{11}+\frac{3}{10}(ab^2d+a^2bg)x^{10}+\frac{1}{3}(ab^2c+a^2bf)x^9+\frac{1}{5}a^3ex^5+\frac{1}{8}(3a^2be+a^3h)x^8+\frac{1}{4}a^3dx^4+\frac{1}{7}(3a^2bd+a^3g)x^7+\frac{1}{3}a^3cx^3+\frac{1}{6}(3a^2bc+a^3f)x^6$$

```
[In] integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 1/14*(b^3*e + 3*a*b^2
*h)*x^14 + 1/13*(b^3*d + 3*a*b^2*g)*x^13 + 1/12*(b^3*c + 3*a*b^2*f)*x^12 +
3/11*(a*b^2*e + a^2*b*h)*x^11 + 3/10*(a*b^2*d + a^2*b*g)*x^10 + 1/3*(a*b^2*
c + a^2*b*f)*x^9 + 1/5*a^3*e*x^5 + 1/8*(3*a^2*b*e + a^3*h)*x^8 + 1/4*a^3*d*
x^4 + 1/7*(3*a^2*b*d + a^3*g)*x^7 + 1/3*a^3*c*x^3 + 1/6*(3*a^2*b*c + a^3*f)
*x^6
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.08

$$\int x^2(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{17}b^3hx^{17}+\frac{1}{16}b^3gx^{16}+\frac{1}{15}b^3fx^{15}+\frac{1}{14}b^3ex^{14}+\frac{3}{14}ab^2hx^{14}+\frac{1}{13}b^3dx^{13}+\frac{3}{13}ab^2gx^{13}+\frac{1}{12}b^3cx^{12}+\frac{1}{4}ab^2fx^{12}+\frac{3}{11}ab^2ex^{11}+\frac{3}{11}a^2bhx^{11}+\frac{3}{10}ab^2dx^{10}+\frac{3}{10}a^2bgx^{10}+\frac{1}{3}ab^2cx^9+\frac{1}{3}a^2bfx^9+\frac{3}{8}a^2bex^8+\frac{1}{8}a^3hx^8+\frac{3}{7}a^2bdx^7+\frac{1}{7}a^3gx^7+\frac{1}{2}a^2bcx^6+\frac{1}{6}a^3fx^6+\frac{1}{5}a^3ex^5+\frac{1}{4}a^3dx^4+\frac{1}{3}a^3cx^3$$

```
[In] integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 1/14*b^3*e*x^14 + 3/1
4*a*b^2*h*x^14 + 1/13*b^3*d*x^13 + 3/13*a*b^2*g*x^13 + 1/12*b^3*c*x^12 + 1/
4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/11*a^2*b*h*x^11 + 3/10*a*b^2*d*x^10
+ 3/10*a^2*b*g*x^10 + 1/3*a*b^2*c*x^9 + 1/3*a^2*b*f*x^9 + 3/8*a^2*b*e*x^8 +
 1/8*a^3*h*x^8 + 3/7*a^2*b*d*x^7 + 1/7*a^3*g*x^7 + 1/2*a^2*b*c*x^6 + 1/6*a^
3*f*x^6 + 1/5*a^3*e*x^5 + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.97

$$\int x^2\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=x^6\left(\frac{f\,a^3}{6}+\frac{b\,c\,a^2}{2}\right)+x^{12}\left(\frac{c\,b^3}{12}+\frac{a\,f\,b^2}{4}\right)+x^7\left(\frac{g\,a^3}{7}+\frac{3\,b\,d\,a^2}{7}\right)+x^{13}\left(\frac{d\,b^3}{13}+\frac{3\,a\,g\,b^2}{13}\right)$$
$$+x^8\left(\frac{h\,a^3}{8}+\frac{3\,b\,e\,a^2}{8}\right)+x^{14}\left(\frac{e\,b^3}{14}+\frac{3\,a\,h\,b^2}{14}\right)+\frac{a^3\,c\,x^3}{3}+\frac{a^3\,d\,x^4}{4}+\frac{a^3\,e\,x^5}{5}+\frac{b^3\,f\,x^{15}}{15}$$
$$+\frac{b^3\,g\,x^{16}}{16}+\frac{b^3\,h\,x^{17}}{17}+\frac{a\,b\,x^9\,(b\,c+a\,f)}{3}+\frac{3\,a\,b\,x^{10}\,(b\,d+a\,g)}{10}+\frac{3\,a\,b\,x^{11}\,(b\,e+a\,h)}{11}$$

```
[In] int(x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^6*((a^3*f)/6 + (a^2*b*c)/2) + x^12*((b^3*c)/12 + (a*b^2*f)/4) + x^7*((a^3
*g)/7 + (3*a^2*b*d)/7) + x^13*((b^3*d)/13 + (3*a*b^2*g)/13) + x^8*((a^3*h)/
8 + (3*a^2*b*e)/8) + x^14*((b^3*e)/14 + (3*a*b^2*h)/14) + (a^3*c*x^3)/3 + (
a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^3*h*x
^17)/17 + (a*b*x^9*(b*c + a*f))/3 + (3*a*b*x^10*(b*d + a*g))/10 + (3*a*b*x^
11*(b*e + a*h))/11
```

3.396 $\int x(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx$

Optimal result

Integrand size = 36, antiderivative size = 212

$$\begin{aligned}\int x(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx &= \frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{1}{5}a^2(3bc+af)x^5+\frac{1}{6}a^3gx^6+\frac{1}{7}a^2(3be+ah)x^7 \\ &\quad+\frac{3}{8}ab(bc+af)x^8+\frac{1}{3}a^2bgx^9+\frac{3}{10}ab(be+ah)x^{10}+\frac{1}{11}b^2(bc+3af)x^{11}+\frac{1}{4}ab^2gx^{12} \\ &\quad+\frac{1}{13}b^2(be+3ah)x^{13}+\frac{1}{14}b^3fx^{14}+\frac{1}{15}b^3gx^{15}+\frac{1}{16}b^3hx^{16}+\frac{d(a+bx^3)^4}{12b}\end{aligned}$$

```
[Out] 1/2*a^3*c*x^2+1/4*a^3*e*x^4+1/5*a^2*(a*f+3*b*c)*x^5+1/6*a^3*g*x^6+1/7*a^2*(
a*h+3*b*e)*x^7+3/8*a*b*(a*f+b*c)*x^8+1/3*a^2*b*g*x^9+3/10*a*b*(a*h+b*e)*x^1
0+1/11*b^2*(3*a*f+b*c)*x^11+1/4*a*b^2*g*x^12+1/13*b^2*(3*a*h+b*e)*x^13+1/14
*b^3*f*x^14+1/15*b^3*g*x^15+1/16*b^3*h*x^16+1/12*d*(b*x^3+a)^4/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1596, 1864}

$$\begin{aligned}\int x(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx &= \frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{1}{6}a^3gx^6+\frac{1}{5}a^2x^5(af+3bc)+\frac{1}{7}a^2x^7(ah+3be)+\frac{1}{3}a^2bgx^9 \\ &\quad+\frac{1}{11}b^2x^{11}(3af+bc)+\frac{1}{13}b^2x^{13}(3ah+be)+\frac{1}{4}ab^2gx^{12}+\frac{3}{8}abx^8(af+bc) \\ &\quad+\frac{d(a+bx^3)^4}{12b}+\frac{3}{10}abx^{10}(ah+be)+\frac{1}{14}b^3fx^{14}+\frac{1}{15}b^3gx^{15}+\frac{1}{16}b^3hx^{16}\end{aligned}$$

```
[In] Int[x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*e*x^4)/4 + (a^2*(3*b*c + a*f)*x^5)/5 + (a^3*g*x^6)/6 +
 (a^2*(3*b*e + a*h)*x^7)/7 + (3*a*b*(b*c + a*f)*x^8)/8 + (a^2*b*g*x^9)/3 +
(3*a*b*(b*e + a*h)*x^10)/10 + (b^2*(b*c + 3*a*f)*x^11)/11 + (a*b^2*g*x^12)/
4 + (b^2*(b*e + 3*a*h)*x^13)/13 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*
h*x^16)/16 + (d*(a + b*x^3)^4)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{d(a+bx^3)^4}{12b} + \int \left(a+bx^3\right)^3 \left(-dx^2 + x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\right) dx \\
&= \frac{d(a+bx^3)^4}{12b} + \int \left(a^3cx + a^3ex^3 + a^2(3bc+af)x^4 + a^3gx^5 + a^2(3be+ah)x^6 \right. \\
&\quad + 3ab(bc+af)x^7 + 3a^2bgx^8 + 3ab(be+ah)x^9 + b^2(bc+3af)x^{10} + 3ab^2gx^{11} \\
&\quad \left. + b^2(be+3ah)x^{12} + b^3fx^{13} + b^3gx^{14} + b^3hx^{15}\right) dx \\
&= \frac{1}{2}a^3cx^2 + \frac{1}{4}a^3ex^4 + \frac{1}{5}a^2(3bc+af)x^5 + \frac{1}{6}a^3gx^6 + \frac{1}{7}a^2(3be+ah)x^7 \\
&\quad + \frac{3}{8}ab(bc+af)x^8 + \frac{1}{3}a^2bgx^9 + \frac{3}{10}ab(be+ah)x^{10} + \frac{1}{11}b^2(bc+3af)x^{11} + \frac{1}{4}ab^2gx^{12} \\
&\quad + \frac{1}{13}b^2(be+3ah)x^{13} + \frac{1}{14}b^3fx^{14} + \frac{1}{15}b^3gx^{15} + \frac{1}{16}b^3hx^{16} + \frac{d(a+bx^3)^4}{12b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.05

$$\begin{aligned}&\int x\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\&=\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{1}{5}a^2(3bc+af)x^5+\frac{1}{6}a^2(3bd+ag)x^6+\frac{1}{7}a^2(3be+ah)x^7\\&\quad+\frac{3}{8}ab(bc+af)x^8+\frac{1}{3}ab(bd+ag)x^9+\frac{3}{10}ab(be+ah)x^{10}+\frac{1}{11}b^2(bc+3af)x^{11}\\&\quad+\frac{1}{12}b^2(bd+3ag)x^{12}+\frac{1}{13}b^2(be+3ah)x^{13}+\frac{1}{14}b^3fx^{14}+\frac{1}{15}b^3gx^{15}+\frac{1}{16}b^3hx^{16}\end{aligned}$$

```
[In] Integrate[x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (a^2*(3*b*c + a*f)*x^5)/5 +
 (a^2*(3*b*d + a*g)*x^6)/6 + (a^2*(3*b*e + a*h)*x^7)/7 + (3*a*b*(b*c + a*f)
*x^8)/8 + (a*b*(b*d + a*g)*x^9)/3 + (3*a*b*(b*e + a*h)*x^10)/10 + (b^2*(b*c
 + 3*a*f)*x^11)/11 + (b^2*(b*d + 3*a*g)*x^12)/12 + (b^2*(b*e + 3*a*h)*x^13)
/13 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*h*x^16)/16
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.04

method	result
norman	$\frac{a^3cx^2}{2}+\frac{a^3dx^3}{3}+\frac{a^3ex^4}{4}+\left(\frac{1}{5}fa^3+\frac{3}{5}a^2bc\right)x^5+\left(\frac{1}{6}a^3g+\frac{1}{2}da^2b\right)x^6+\left(\frac{1}{7}a^3h+\frac{3}{7}a^2be\right)x^7+\left(\frac{3}{8}f\right.$
default	$\frac{b^3hx^{16}}{16}+\frac{b^3gx^{15}}{15}+\frac{b^3fx^{14}}{14}+\frac{(3ab^2h+b^3e)x^{13}}{13}+\frac{(3ab^2g+b^3d)x^{12}}{12}+\frac{(3ab^2f+b^3c)x^{11}}{11}+\frac{(3a^2bh+3ab^2e)x^{10}}{10}+\frac{(3}{}$
gosper	$\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{1}{5}x^5fa^3+\frac{3}{5}a^2bcx^5+\frac{1}{6}a^3gx^6+\frac{1}{2}a^2bdx^6+\frac{1}{7}x^7a^3h+\frac{3}{7}a^2bex^7$
risch	$\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{1}{5}x^5fa^3+\frac{3}{5}a^2bcx^5+\frac{1}{6}a^3gx^6+\frac{1}{2}a^2bdx^6+\frac{1}{7}x^7a^3h+\frac{3}{7}a^2bex^7$
parallelrisch	$\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{1}{5}x^5fa^3+\frac{3}{5}a^2bcx^5+\frac{1}{6}a^3gx^6+\frac{1}{2}a^2bdx^6+\frac{1}{7}x^7a^3h+\frac{3}{7}a^2bex^7$

```
[In] int(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^3*c*x^2+1/3*a^3*d*x^3+1/4*a^3*e*x^4+(1/5*f*a^3+3/5*a^2*b*c)*x^5+(1/6*
a^3*g+1/2*d*a^2*b)*x^6+(1/7*a^3*h+3/7*a^2*b*e)*x^7+(3/8*f*a^2*b+3/8*a*b^2*c
)*x^8+(1/3*a^2*b*g+1/3*a*b^2*d)*x^9+(3/10*a^2*b*h+3/10*a*b^2*e)*x^10+(3/11*
a*b^2*f+1/11*b^3*c)*x^11+(1/4*a*b^2*g+1/12*b^3*d)*x^12+(3/13*a*b^2*h+1/13*b
^3*e)*x^13+1/14*b^3*f*x^14+1/15*b^3*g*x^15+1/16*b^3*h*x^16
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02

$$\begin{aligned}\int x(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{1}{16}\,b^3hx^{16}+\frac{1}{15}\,b^3gx^{15}+\frac{1}{14}\,b^3fx^{14}+\frac{1}{13}\,(b^3e+3\,ab^2h)x^{13}\\&\quad+\frac{1}{12}\,(b^3d+3\,ab^2g)x^{12}+\frac{1}{11}\,(b^3c+3\,ab^2f)x^{11}+\frac{3}{10}\,(ab^2e+a^2bh)x^{10}\\&\quad+\frac{1}{3}\,(ab^2d+a^2bg)x^9+\frac{3}{8}\,(ab^2c+a^2bf)x^8+\frac{1}{4}\,a^3ex^4+\frac{1}{7}\,(3\,a^2be+a^3h)x^7\\&\quad+\frac{1}{3}\,a^3dx^3+\frac{1}{6}\,(3\,a^2bd+a^3g)x^6+\frac{1}{2}\,a^3cx^2+\frac{1}{5}\,(3\,a^2bc+a^3f)x^5\end{aligned}$$

```
[In] integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 1/13*(b^3*e + 3*a*b^2
*h)*x^13 + 1/12*(b^3*d + 3*a*b^2*g)*x^12 + 1/11*(b^3*c + 3*a*b^2*f)*x^11 +
3/10*(a*b^2*e + a^2*b*h)*x^10 + 1/3*(a*b^2*d + a^2*b*g)*x^9 + 3/8*(a*b^2*c
+ a^2*b*f)*x^8 + 1/4*a^3*e*x^4 + 1/7*(3*a^2*b*e + a^3*h)*x^7 + 1/3*a^3*d*x^
3 + 1/6*(3*a^2*b*d + a^3*g)*x^6 + 1/2*a^3*c*x^2 + 1/5*(3*a^2*b*c + a^3*f)*x
^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.16

$$\begin{aligned}\int x(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{a^3cx^2}{2}+\frac{a^3dx^3}{3}+\frac{a^3ex^4}{4}+\frac{b^3fx^{14}}{14}+\frac{b^3gx^{15}}{15}+\frac{b^3hx^{16}}{16}+x^{13}\cdot\left(\frac{3ab^2h}{13}+\frac{b^3e}{13}\right)\\&\quad+x^{12}\left(\frac{ab^2g}{4}+\frac{b^3d}{12}\right)+x^{11}\cdot\left(\frac{3ab^2f}{11}+\frac{b^3c}{11}\right)+x^{10}\cdot\left(\frac{3a^2bh}{10}+\frac{3ab^2e}{10}\right)+x^9\left(\frac{a^2bg}{3}+\frac{ab^2d}{3}\right)\\&\quad+x^8\cdot\left(\frac{3a^2bf}{8}+\frac{3ab^2c}{8}\right)+x^7\left(\frac{a^3h}{7}+\frac{3a^2be}{7}\right)+x^6\left(\frac{a^3g}{6}+\frac{a^2bd}{2}\right)+x^5\left(\frac{a^3f}{5}+\frac{3a^2bc}{5}\right)\end{aligned}$$

```
[In] integrate(x*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**2/2 + a**3*d*x**3/3 + a**3*e*x**4/4 + b**3*f*x**14/14 + b**3*g*x*
*15/15 + b**3*h*x**16/16 + x**13*(3*a*b**2*h/13 + b**3*e/13) + x**12*(a*b**
2*g/4 + b**3*d/12) + x**11*(3*a*b**2*f/11 + b**3*c/11) + x**10*(3*a**2*b*h/
10 + 3*a*b**2*e/10) + x**9*(a**2*b*g/3 + a*b**2*d/3) + x**8*(3*a**2*b*f/8 +
 3*a*b**2*c/8) + x**7*(a**3*h/7 + 3*a**2*b*e/7) + x**6*(a**3*g/6 + a**2*b*d
/2) + x**5*(a**3*f/5 + 3*a**2*b*c/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02

$$\int x(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$$
$$=\frac{1}{16}\,b^3hx^{16}+\frac{1}{15}\,b^3gx^{15}+\frac{1}{14}\,b^3fx^{14}+\frac{1}{13}\,(b^3e+3\,ab^2h)x^{13}+\frac{1}{12}\,(b^3d+3\,ab^2g)x^{12}+\frac{1}{11}\,(b^3c+3\,ab^2f)x^{11}+\frac{3}{10}\,(ab^2e+a^2bh)x^{10}+\frac{1}{3}\,(ab^2d+a^2bg)x^9+\frac{3}{8}\,(ab^2c+a^2bf)x^8+\frac{1}{4}\,a^3ex^4+\frac{1}{7}\,(3\,a^2be+a^3h)x^7+\frac{1}{3}\,a^3dx^3+\frac{1}{6}\,(3\,a^2bd+a^3g)x^6+\frac{1}{2}\,a^3cx^2+\frac{1}{5}\,(3\,a^2bc+a^3f)x^5$$

```
[In] integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 1/13*(b^3*e + 3*a*b^2
*h)*x^13 + 1/12*(b^3*d + 3*a*b^2*g)*x^12 + 1/11*(b^3*c + 3*a*b^2*f)*x^11 +
3/10*(a*b^2*e + a^2*b*h)*x^10 + 1/3*(a*b^2*d + a^2*b*g)*x^9 + 3/8*(a*b^2*c
+ a^2*b*f)*x^8 + 1/4*a^3*e*x^4 + 1/7*(3*a^2*b*e + a^3*h)*x^7 + 1/3*a^3*d*x^
3 + 1/6*(3*a^2*b*d + a^3*g)*x^6 + 1/2*a^3*c*x^2 + 1/5*(3*a^2*b*c + a^3*f)*x
^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.08

$$\int x(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$$
$$=\frac{1}{16}\,b^3hx^{16}+\frac{1}{15}\,b^3gx^{15}+\frac{1}{14}\,b^3fx^{14}+\frac{1}{13}\,b^3ex^{13}+\frac{3}{13}\,ab^2hx^{13}+\frac{1}{12}\,b^3dx^{12}+\frac{1}{4}\,ab^2gx^{12}+\frac{1}{11}\,b^3cx^{11}+\frac{3}{11}\,ab^2fx^{11}+\frac{3}{10}\,ab^2ex^{10}+\frac{3}{10}\,a^2bhx^{10}+\frac{1}{3}\,ab^2dx^9+\frac{1}{3}\,a^2bgx^9+\frac{3}{8}\,ab^2cx^8+\frac{3}{8}\,a^2bfx^8+\frac{3}{7}\,a^2bex^7+\frac{1}{7}\,a^3hx^7+\frac{1}{2}\,a^2bdx^6+\frac{1}{6}\,a^3gx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{5}\,a^3fx^5+\frac{1}{4}\,a^3ex^4+\frac{1}{3}\,a^3dx^3+\frac{1}{2}\,a^3cx^2$$

```
[In] integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

[Out] 1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 1/13*b^3*e*x^13 + 3/13*a*b^2*h*x^13 + 1/12*b^3*d*x^12 + 1/4*a*b^2*g*x^12 + 1/11*b^3*c*x^11 + 3/11*a*b^2*f*x^11 + 3/10*a*b^2*e*x^10 + 3/10*a^2*b*h*x^10 + 1/3*a*b^2*d*x^9 + 1/3*a^2*b*g*x^9 + 3/8*a*b^2*c*x^8 + 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/7*a^3*h*x^7 + 1/2*a^2*b*d*x^6 + 1/6*a^3*g*x^6 + 3/5*a^2*b*c*x^5 + 1/5*a^3*f*x^5 + 1/4*a^3*e*x^4 + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.97

$$\int x(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = x^5\left(\frac{f\,a^3}{5}+\frac{3\,b\,c\,a^2}{5}\right)+x^{11}\left(\frac{c\,b^3}{11}+\frac{3\,a\,f\,b^2}{11}\right)+x^6\left(\frac{g\,a^3}{6}+\frac{b\,d\,a^2}{2}\right)+x^{12}\left(\frac{d\,b^3}{12}+\frac{a\,g\,b^2}{4}\right)+x^7\left(\frac{h\,a^3}{7}+\frac{3\,b\,e\,a^2}{7}\right)+x^{13}\left(\frac{e\,b^3}{13}+\frac{3\,a\,h\,b^2}{13}\right)+\frac{a^3\,c\,x^2}{2}+\frac{a^3\,d\,x^3}{3}+\frac{a^3\,e\,x^4}{4}+\frac{b^3\,f\,x^{14}}{14}+\frac{b^3\,g\,x^{15}}{15}+\frac{b^3\,h\,x^{16}}{16}+\frac{3\,a\,b\,x^8\,(b\,c+a\,f)}{8}+\frac{a\,b\,x^9\,(b\,d+a\,g)}{3}+\frac{3\,a\,b\,x^{10}\,(b\,e+a\,h)}{10}$$

[In] int(x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)

[Out] x^5*((a^3*f)/5 + (3*a^2*b*c)/5) + x^11*((b^3*c)/11 + (3*a*b^2*f)/11) + x^6*((a^3*g)/6 + (a^2*b*d)/2) + x^12*((b^3*d)/12 + (a*b^2*g)/4) + x^7*((a^3*h)/7 + (3*a^2*b*e)/7) + x^13*((b^3*e)/13 + (3*a*b^2*h)/13) + (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*h*x^16)/16 + (3*a*b*x^8*(b*c + a*f))/8 + (a*b*x^9*(b*d + a*g))/3 + (3*a*b*x^10*(b*e + a*h))/10

3.397 $\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 207

$$\begin{aligned}\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{4}a^2(3bc+af)x^4+\frac{1}{5}a^2(3bd+ag)x^5+\frac{1}{6}a^3hx^6\\ &+\frac{3}{7}ab(bc+af)x^7+\frac{3}{8}ab(bd+ag)x^8+\frac{1}{3}a^2bhx^9+\frac{1}{10}b^2(bc+3af)x^{10}\\ &+\frac{1}{11}b^2(bd+3ag)x^{11}+\frac{1}{4}ab^2hx^{12}+\frac{1}{13}b^3fx^{13}+\frac{1}{14}b^3gx^{14}+\frac{1}{15}b^3hx^{15}+\frac{e(a+bx^3)^4}{12b}\end{aligned}$$

```
[Out] a^3*c*x+1/2*a^3*d*x^2+1/4*a^2*(a*f+3*b*c)*x^4+1/5*a^2*(a*g+3*b*d)*x^5+1/6*a
^3*h*x^6+3/7*a*b*(a*f+b*c)*x^7+3/8*a*b*(a*g+b*d)*x^8+1/3*a^2*b*h*x^9+1/10*b
^2*(3*a*f+b*c)*x^10+1/11*b^2*(3*a*g+b*d)*x^11+1/4*a*b^2*h*x^12+1/13*b^3*f*x
^13+1/14*b^3*g*x^14+1/15*b^3*h*x^15+1/12*e*(b*x^3+a)^4/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1596, 1864}

$$\begin{aligned}\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{6}a^3hx^6+\frac{1}{4}a^2x^4(af+3bc)+\frac{1}{5}a^2x^5(ag+3bd)+\frac{1}{3}a^2bhx^9\\ &+\frac{1}{10}b^2x^{10}(3af+bc)+\frac{1}{11}b^2x^{11}(3ag+bd)+\frac{1}{4}ab^2hx^{12}+\frac{3}{7}abx^7(af+bc)\\ &+\frac{3}{8}abx^8(ag+bd)+\frac{e(a+bx^3)^4}{12b}+\frac{1}{13}b^3fx^{13}+\frac{1}{14}b^3gx^{14}+\frac{1}{15}b^3hx^{15}\end{aligned}$$

```
[In] Int[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^2*(3*b*c + a*f)*x^4)/4 + (a^2*(3*b*d + a*g)*x^
5)/5 + (a^3*h*x^6)/6 + (3*a*b*(b*c + a*f)*x^7)/7 + (3*a*b*(b*d + a*g)*x^8)/
8 + (a^2*b*h*x^9)/3 + (b^2*(b*c + 3*a*f)*x^10)/10 + (b^2*(b*d + 3*a*g)*x^11
)/11 + (a*b^2*h*x^12)/4 + (b^3*f*x^13)/13 + (b^3*g*x^14)/14 + (b^3*h*x^15)/
15 + (e*(a + b*x^3)^4)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e(a+bx^3)^4}{12b} + \int \left(a+bx^3\right)^3 \left(c+dx+fx^3+gx^4+hx^5\right)\,dx \\
&= \frac{e(a+bx^3)^4}{12b} + \int \left(a^3c+a^3dx+a^2(3bc+af)x^3+a^2(3bd+ag)x^4+a^3hx^5\right. \\
&\quad +3ab(bc+af)x^6+3ab(bd+ag)x^7+3a^2bhx^8+b^2(bc+3af)x^9 \\
&\quad \left.+b^2(bd+3ag)x^{10}+3ab^2hx^{11}+b^3fx^{12}+b^3gx^{13}+b^3hx^{14}\right)\,dx \\
&= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{4}a^2(3bc+af)x^4+\frac{1}{5}a^2(3bd+ag)x^5+\frac{1}{6}a^3hx^6+\frac{3}{7}ab(bc+af)x^7 \\
&\quad +\frac{3}{8}ab(bd+ag)x^8+\frac{1}{3}a^2bhx^9+\frac{1}{10}b^2(bc+3af)x^{10}+\frac{1}{11}b^2(bd+3ag)x^{11} \\
&\quad +\frac{1}{4}ab^2hx^{12}+\frac{1}{13}b^3fx^{13}+\frac{1}{14}b^3gx^{14}+\frac{1}{15}b^3hx^{15}+\frac{e(a+bx^3)^4}{12b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.82

$$\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)\ dx$$
$$= \frac{x(13ab^2x^6(3960c+7x(495d+440ex+6x^2(66f+60gx+55hx^2)))+2002a^3(60c+x(30d+x(20e+15fx$$

```
[In] Integrate[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (x*(13*a*b^2*x^6*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55
*h*x^2))) + 2002*a^3*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x
^3))) + 2*b^3*x^9*(6006*c + x*(5460*d + 11*x*(455*e + 420*f*x + 390*g*x^2 +
 364*h*x^3))) + 143*a^2*b*x^3*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x
*(9*g + 8*h*x)))))))/120120
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.05

method	result
norman	$a^3cx+\frac{a^3d\,x^2}{2}+\frac{a^3e\,x^3}{3}+\left(\frac{1}{4}f\,a^3+\frac{3}{4}a^2bc\right)x^4+\left(\frac{1}{5}a^3g+\frac{3}{5}d\,a^2b\right)x^5+\left(\frac{1}{6}a^3h+\frac{1}{2}a^2be\right)x^6+\left(\frac{3}{7}f\right.$
default	$\frac{b^3h\,x^{15}}{15}+\frac{b^3g\,x^{14}}{14}+\frac{b^3f\,x^{13}}{13}+\frac{(3a\,b^2h+b^3e)x^{12}}{12}+\frac{(3a\,b^2g+b^3d)x^{11}}{11}+\frac{(3a\,b^2f+b^3c)x^{10}}{10}+\frac{(3a^2bh+3a\,b^2e)x^9}{9}+$
gosper	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{4}a^2bc\,x^4+\frac{1}{5}x^5a^3g+\frac{3}{5}x^5bd\,a^2+\frac{1}{6}a^3h\,x^6+\frac{1}{2}a^2be\,x^6+$
risch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{4}a^2bc\,x^4+\frac{1}{5}x^5a^3g+\frac{3}{5}x^5bd\,a^2+\frac{1}{6}a^3h\,x^6+\frac{1}{2}a^2be\,x^6+$
parallelrisch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{4}a^2bc\,x^4+\frac{1}{5}x^5a^3g+\frac{3}{5}x^5bd\,a^2+\frac{1}{6}a^3h\,x^6+\frac{1}{2}a^2be\,x^6+$

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+(1/4*f*a^3+3/4*a^2*b*c)*x^4+(1/5*a^3*g+
3/5*d*a^2*b)*x^5+(1/6*a^3*h+1/2*a^2*b*e)*x^6+(3/7*f*a^2*b+3/7*a*b^2*c)*x^7+
(3/8*a^2*b*g+3/8*a*b^2*d)*x^8+(1/3*a^2*b*h+1/3*a*b^2*e)*x^9+(3/10*a*b^2*f+1
/10*b^3*c)*x^10+(3/11*a*b^2*g+1/11*b^3*d)*x^11+(1/4*a*b^2*h+1/12*b^3*e)*x^1
2+1/13*b^3*f*x^13+1/14*b^3*g*x^14+1/15*b^3*h*x^15
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.03

$$\int \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$= \frac{1}{15}b^3hx^{15}+\frac{1}{14}b^3gx^{14}+\frac{1}{13}b^3fx^{13}+\frac{1}{12}\left(b^3e+3\,ab^2h\right)x^{12}+\frac{1}{11}\left(b^3d+3\,ab^2g\right)x^{11}+\frac{1}{10}\left(b^3c+3\,ab^2f\right)x^{10}+\frac{1}{3}\left(ab^2e+a^2bh\right)x^9+\frac{3}{8}\left(ab^2d+a^2bg\right)x^8+\frac{3}{7}\left(ab^2c+a^2bf\right)x^7+\frac{1}{3}a^3ex^3+\frac{1}{6}\left(3\,a^2be+a^3h\right)x^6+\frac{1}{2}a^3dx^2+\frac{1}{5}\left(3\,a^2bd+a^3g\right)x^5+a^3cx+\frac{1}{4}\left(3\,a^2bc+a^3f\right)x^4$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] 1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/12*(b^3*e + 3*a*b^2
*h)*x^12 + 1/11*(b^3*d + 3*a*b^2*g)*x^11 + 1/10*(b^3*c + 3*a*b^2*f)*x^10 +
1/3*(a*b^2*e + a^2*b*h)*x^9 + 3/8*(a*b^2*d + a^2*b*g)*x^8 + 3/7*(a*b^2*c +
a^2*b*f)*x^7 + 1/3*a^3*e*x^3 + 1/6*(3*a^2*b*e + a^3*h)*x^6 + 1/2*a^3*d*x^2
+ 1/5*(3*a^2*b*d + a^3*g)*x^5 + a^3*c*x + 1/4*(3*a^2*b*c + a^3*f)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.17

$$\int \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$= a^3cx+\frac{a^3dx^2}{2}+\frac{a^3ex^3}{3}+\frac{b^3fx^{13}}{13}+\frac{b^3gx^{14}}{14}+\frac{b^3hx^{15}}{15}+x^{12}\left(\frac{ab^2h}{4}+\frac{b^3e}{12}\right)+x^{11}\cdot\left(\frac{3ab^2g}{11}+\frac{b^3d}{11}\right)+x^{10}\cdot\left(\frac{3ab^2f}{10}+\frac{b^3c}{10}\right)+x^9\left(\frac{a^2bh}{3}+\frac{ab^2e}{3}\right)+x^8\cdot\left(\frac{3a^2bg}{8}+\frac{3ab^2d}{8}\right)+x^7\cdot\left(\frac{3a^2bf}{7}+\frac{3ab^2c}{7}\right)+x^6\left(\frac{a^3h}{6}+\frac{a^2be}{2}\right)+x^5\left(\frac{a^3g}{5}+\frac{3a^2bd}{5}\right)+x^4\left(\frac{a^3f}{4}+\frac{3a^2bc}{4}\right)$$

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + b**3*f*x**13/13 + b**3*g*x**14/1
4 + b**3*h*x**15/15 + x**12*(a*b**2*h/4 + b**3*e/12) + x**11*(3*a*b**2*g/11
 + b**3*d/11) + x**10*(3*a*b**2*f/10 + b**3*c/10) + x**9*(a**2*b*h/3 + a*b*
*2*e/3) + x**8*(3*a**2*b*g/8 + 3*a*b**2*d/8) + x**7*(3*a**2*b*f/7 + 3*a*b**
2*c/7) + x**6*(a**3*h/6 + a**2*b*e/2) + x**5*(a**3*g/5 + 3*a**2*b*d/5) + x*
*4*(a**3*f/4 + 3*a**2*b*c/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.03

$$\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx = \frac{1}{15} b^3hx^{15} + \frac{1}{14} b^3gx^{14} + \frac{1}{13} b^3fx^{13} + \frac{1}{12} (b^3e+3ab^2h)x^{12} + \frac{1}{11} (b^3d+3ab^2g)x^{11} + \frac{1}{10} (b^3c+3ab^2f)x^{10} + \frac{1}{3} (ab^2e+a^2bh)x^9 + \frac{3}{8} (ab^2d+a^2bg)x^8 + \frac{3}{7} (ab^2c+a^2bf)x^7 + \frac{1}{3} a^3ex^3 + \frac{1}{6} (3a^2be+a^3h)x^6 + \frac{1}{2} a^3dx^2 + \frac{1}{5} (3a^2bd+a^3g)x^5 + a^3cx + \frac{1}{4} (3a^2bc+a^3f)x^4$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
```

```
[Out] 1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/12*(b^3*e + 3*a*b^2
*h)*x^12 + 1/11*(b^3*d + 3*a*b^2*g)*x^11 + 1/10*(b^3*c + 3*a*b^2*f)*x^10 +
1/3*(a*b^2*e + a^2*b*h)*x^9 + 3/8*(a*b^2*d + a^2*b*g)*x^8 + 3/7*(a*b^2*c +
a^2*b*f)*x^7 + 1/3*a^3*e*x^3 + 1/6*(3*a^2*b*e + a^3*h)*x^6 + 1/2*a^3*d*x^2
+ 1/5*(3*a^2*b*d + a^3*g)*x^5 + a^3*c*x + 1/4*(3*a^2*b*c + a^3*f)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.09

$$\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx = \frac{1}{15} b^3hx^{15} + \frac{1}{14} b^3gx^{14} + \frac{1}{13} b^3fx^{13} + \frac{1}{12} b^3ex^{12} + \frac{1}{4} ab^2hx^{12} + \frac{1}{11} b^3dx^{11} + \frac{3}{11} ab^2gx^{11} + \frac{1}{10} b^3cx^{10} + \frac{3}{10} ab^2fx^{10} + \frac{1}{3} ab^2ex^9 + \frac{1}{3} a^2bhx^9 + \frac{3}{8} ab^2dx^8 + \frac{3}{8} a^2bgx^8 + \frac{3}{7} ab^2cx^7 + \frac{3}{7} a^2bfx^7 + \frac{1}{2} a^2bex^6 + \frac{1}{6} a^3hx^6 + \frac{3}{5} a^2bdx^5 + \frac{1}{5} a^3gx^5 + \frac{3}{4} a^2bcx^4 + \frac{1}{4} a^3fx^4 + \frac{1}{3} a^3ex^3 + \frac{1}{2} a^3dx^2 + a^3cx$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/12*b^3*e*x^12 + 1/4
*a*b^2*h*x^12 + 1/11*b^3*d*x^11 + 3/11*a*b^2*g*x^11 + 1/10*b^3*c*x^10 + 3/1
0*a*b^2*f*x^10 + 1/3*a*b^2*e*x^9 + 1/3*a^2*b*h*x^9 + 3/8*a*b^2*d*x^8 + 3/8*
```

```
a^2*b*g*x^8 + 3/7*a*b^2*c*x^7 + 3/7*a^2*b*f*x^7 + 1/2*a^2*b*e*x^6 + 1/6*a^3
*h*x^6 + 3/5*a^2*b*d*x^5 + 1/5*a^3*g*x^5 + 3/4*a^2*b*c*x^4 + 1/4*a^3*f*x^4
+ 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.98

$$\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx = x^4 \left(\frac{f\,a^3}{4}+\frac{3\,b\,c\,a^2}{4}\right)+x^{10}\left(\frac{c\,b^3}{10}+\frac{3\,a\,f\,b^2}{10}\right)+x^5\left(\frac{g\,a^3}{5}+\frac{3\,b\,d\,a^2}{5}\right)+x^{11}\left(\frac{d\,b^3}{11}+\frac{3\,a\,g\,b^2}{11}\right)+x^6\left(\frac{h\,a^3}{6}+\frac{b\,e\,a^2}{2}\right)+x^{12}\left(\frac{e\,b^3}{12}+\frac{a\,h\,b^2}{4}\right)+\frac{a^3\,d\,x^2}{2}+\frac{a^3\,e\,x^3}{3}+\frac{b^3\,f\,x^{13}}{13}+\frac{b^3\,g\,x^{14}}{14}+\frac{b^3\,h\,x^{15}}{15}+a^3\,c\,x+\frac{3\,a\,b\,x^7\,(b\,c+a\,f)}{7}+\frac{3\,a\,b\,x^8\,(b\,d+a\,g)}{8}+\frac{a\,b\,x^9\,(b\,e+a\,h)}{3}$$

```
[In] int((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^4*((a^3*f)/4 + (3*a^2*b*c)/4) + x^10*((b^3*c)/10 + (3*a*b^2*f)/10) + x^5*
((a^3*g)/5 + (3*a^2*b*d)/5) + x^11*((b^3*d)/11 + (3*a*b^2*g)/11) + x^6*((a^
3*h)/6 + (a^2*b*e)/2) + x^12*((b^3*e)/12 + (a*b^2*h)/4) + (a^3*d*x^2)/2 + (
a^3*e*x^3)/3 + (b^3*f*x^13)/13 + (b^3*g*x^14)/14 + (b^3*h*x^15)/15 + a^3*c*
x + (3*a*b*x^7*(b*c + a*f))/7 + (3*a*b*x^8*(b*d + a*g))/8 + (a*b*x^9*(b*e +
 a*h))/3
```

3.398 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} \, dx$

Optimal result

Integrand size = 38, antiderivative size = 200

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} \, dx = a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{1}{4}a^2(3bd+ag)x^4+\frac{1}{5}a^2(3be+ah)x^5+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab(bd+ag)x^7+\frac{3}{8}ab(be+ah)x^8+\frac{1}{9}b^3cx^9+\frac{1}{10}b^2(bd+3ag)x^{10}+\frac{1}{11}b^2(be+3ah)x^{11}+\frac{1}{13}b^3gx^{13}+\frac{1}{14}b^3hx^{14}+\frac{f(a+bx^3)^4}{12b}+a^3c\log(x)$$

```
[Out] a^3*d*x+1/2*a^3*e*x^2+a^2*b*c*x^3+1/4*a^2*(a*g+3*b*d)*x^4+1/5*a^2*(a*h+3*b*
e)*x^5+1/2*a*b^2*c*x^6+3/7*a*b*(a*g+b*d)*x^7+3/8*a*b*(a*h+b*e)*x^8+1/9*b^3*
c*x^9+1/10*b^2*(3*a*g+b*d)*x^10+1/11*b^2*(3*a*h+b*e)*x^11+1/13*b^3*g*x^13+1
/14*b^3*h*x^14+1/12*f*(b*x^3+a)^4/b+a^3*c*ln(x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1597, 1834}

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} \, dx = a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{1}{4}a^2x^4(ag+3bd)+\frac{1}{5}a^2x^5(ah+3be)+\frac{1}{2}ab^2cx^6+\frac{1}{10}b^2x^{10}(3ag+bd)+\frac{1}{11}b^2x^{11}(3ah+be)+\frac{3}{7}abx^7(ag+bd)+\frac{3}{8}abx^8(ah+be)+\frac{f(a+bx^3)^4}{12b}+\frac{1}{9}b^3cx^9+\frac{1}{13}b^3gx^{13}+\frac{1}{14}b^3hx^{14}$$

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (a^2*(3*b*d + a*g)*x^4)/4 + (a^2*(3
*b*e + a*h)*x^5)/5 + (a*b^2*c*x^6)/2 + (3*a*b*(b*d + a*g)*x^7)/7 + (3*a*b*(
b*e + a*h)*x^8)/8 + (b^3*c*x^9)/9 + (b^2*(b*d + 3*a*g)*x^10)/10 + (b^2*(b*e
 + 3*a*h)*x^11)/11 + (b^3*g*x^13)/13 + (b^3*h*x^14)/14 + (f*(a + b*x^3)^4)/
(12*b) + a^3*c*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f(a+bx^3)^4}{12b} + \int \frac{(a+bx^3)^3\,(c+dx+ex^2+gx^4+hx^5)}{x}\,dx \\
&= \frac{f(a+bx^3)^4}{12b} + \int \Bigg(a^3d + \frac{a^3c}{x} + a^3ex + 3a^2bcx^2 + a^2(3bd+ag)x^3 + a^2(3be+ah)x^4 \\
&\qquad + 3ab^2cx^5 + 3ab(bd+ag)x^6 + 3ab(be+ah)x^7 + b^3cx^8 + b^2(bd+3ag)x^9 \\
&\qquad + b^2(be+3ah)x^{10} + b^3gx^{12} + b^3hx^{13}\Bigg)\,dx \\
&= a^3dx + \frac{1}{2}a^3ex^2 + a^2bcx^3 + \frac{1}{4}a^2(3bd+ag)x^4 + \frac{1}{5}a^2(3be+ah)x^5 + \frac{1}{2}ab^2cx^6 \\
&\quad + \frac{3}{7}ab(bd+ag)x^7 + \frac{3}{8}ab(be+ah)x^8 + \frac{1}{9}b^3cx^9 + \frac{1}{10}b^2(bd+3ag)x^{10} \\
&\quad + \frac{1}{11}b^2(be+3ah)x^{11} + \frac{1}{13}b^3gx^{13} + \frac{1}{14}b^3hx^{14} + \frac{f(a+bx^3)^4}{12b} + a^3c\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$$
$$= a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{3}a^2(3bc+af)x^3+\frac{1}{4}a^2(3bd+ag)x^4+\frac{1}{5}a^2(3be+ah)x^5 +\frac{1}{2}ab(bc+af)x^6+\frac{3}{7}ab(bd+ag)x^7+\frac{3}{8}ab(be+ah)x^8+\frac{1}{9}b^2(bc+3af)x^9 +\frac{1}{10}b^2(bd+3ag)x^{10}+\frac{1}{11}b^2(be+3ah)x^{11}+\frac{1}{12}b^3fx^{12}+\frac{1}{13}b^3gx^{13}+\frac{1}{14}b^3hx^{14}+a^3c\log(x)$$

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*c + a*f)*x^3)/3 + (a^2*(3*b*d + a*g)*x^
4)/4 + (a^2*(3*b*e + a*h)*x^5)/5 + (a*b*(b*c + a*f)*x^6)/2 + (3*a*b*(b*d +
a*g)*x^7)/7 + (3*a*b*(b*e + a*h)*x^8)/8 + (b^2*(b*c + 3*a*f)*x^9)/9 + (b^2*
(b*d + 3*a*g)*x^10)/10 + (b^2*(b*e + 3*a*h)*x^11)/11 + (b^3*f*x^12)/12 + (b
^3*g*x^13)/13 + (b^3*h*x^14)/14 + a^3*c*Log[x]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.08

method	result
norman	$\left(\frac{1}{4}a^3g+\frac{3}{4}d\,a^2b\right)x^4+\left(\frac{1}{5}a^3h+\frac{3}{5}a^2be\right)x^5+\left(\frac{1}{3}f\,a^3+a^2bc\right)x^3+\left(\frac{1}{3}a\,b^2f+\frac{1}{9}b^3c\right)x^9+\left(\frac{3}{10}a\,b^2g\right.$
default	$\frac{b^3h\,x^{14}}{14}+\frac{b^3g\,x^{13}}{13}+\frac{b^3f\,x^{12}}{12}+\frac{3a\,b^2h\,x^{11}}{11}+\frac{b^3e\,x^{11}}{11}+\frac{3a\,b^2g\,x^{10}}{10}+\frac{b^3d\,x^{10}}{10}+\frac{a\,b^2f\,x^9}{3}+\frac{b^3c\,x^9}{9}+\frac{3a^2bh\,x^8}{8}+3$
risch	$\frac{b^3h\,x^{14}}{14}+\frac{b^3g\,x^{13}}{13}+\frac{b^3f\,x^{12}}{12}+\frac{3a\,b^2h\,x^{11}}{11}+\frac{b^3e\,x^{11}}{11}+\frac{3a\,b^2g\,x^{10}}{10}+\frac{b^3d\,x^{10}}{10}+\frac{a\,b^2f\,x^9}{3}+\frac{b^3c\,x^9}{9}+\frac{3a^2bh\,x^8}{8}+3$
parallelrisch	$\frac{b^3h\,x^{14}}{14}+\frac{b^3g\,x^{13}}{13}+\frac{b^3f\,x^{12}}{12}+\frac{3a\,b^2h\,x^{11}}{11}+\frac{b^3e\,x^{11}}{11}+\frac{3a\,b^2g\,x^{10}}{10}+\frac{b^3d\,x^{10}}{10}+\frac{a\,b^2f\,x^9}{3}+\frac{b^3c\,x^9}{9}+\frac{3a^2bh\,x^8}{8}+3$

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4*a^3*g+3/4*d*a^2*b)*x^4+(1/5*a^3*h+3/5*a^2*b*e)*x^5+(1/3*f*a^3+a^2*b*c)
*x^3+(1/3*a*b^2*f+1/9*b^3*c)*x^9+(3/10*a*b^2*g+1/10*b^3*d)*x^10+(3/11*a*b^2
*h+1/11*b^3*e)*x^11+(3/7*a^2*b*g+3/7*a*b^2*d)*x^7+(3/8*a^2*b*h+3/8*a*b^2*e)
*x^8+(1/2*f*a^2*b+1/2*a*b^2*c)*x^6+a^3*d*x+1/2*a^3*e*x^2+1/12*b^3*f*x^12+1/
13*b^3*g*x^13+1/14*b^3*h*x^14+a^3*c*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = \frac{1}{14}\,b^3hx^{14}+\frac{1}{13}\,b^3gx^{13}+\frac{1}{12}\,b^3fx^{12}+\frac{1}{11}\,(b^3e+3\,ab^2h)x^{11}+\frac{1}{10}\,(b^3d+3\,ab^2g)x^{10}+\frac{1}{9}\,(b^3c+3\,ab^2f)x^9+\frac{3}{8}\,(ab^2e+a^2bh)x^8+\frac{3}{7}\,(ab^2d+a^2bg)x^7+\frac{1}{2}\,(ab^2c+a^2bf)x^6+\frac{1}{2}\,a^3ex^2+\frac{1}{5}\,(3\,a^2be+a^3h)x^5+a^3dx+\frac{1}{4}\,(3\,a^2bd+a^3g)x^4+a^3c\log(x)+\frac{1}{3}\,(3\,a^2bc+a^3f)x^3$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="frica
s")
```

```
[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 1/11*(b^3*e + 3*a*b^2
*h)*x^11 + 1/10*(b^3*d + 3*a*b^2*g)*x^10 + 1/9*(b^3*c + 3*a*b^2*f)*x^9 + 3/
8*(a*b^2*e + a^2*b*h)*x^8 + 3/7*(a*b^2*d + a^2*b*g)*x^7 + 1/2*(a*b^2*c + a^
2*b*f)*x^6 + 1/2*a^3*e*x^2 + 1/5*(3*a^2*b*e + a^3*h)*x^5 + a^3*d*x + 1/4*(3
*a^2*b*d + a^3*g)*x^4 + a^3*c*log(x) + 1/3*(3*a^2*b*c + a^3*f)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = a^3c\log(x)+a^3dx+\frac{a^3ex^2}{2}+\frac{b^3fx^{12}}{12}+\frac{b^3gx^{13}}{13}+\frac{b^3hx^{14}}{14}+x^{11}\cdot\left(\frac{3ab^2h}{11}+\frac{b^3e}{11}\right)+x^{10}\cdot\left(\frac{3ab^2g}{10}+\frac{b^3d}{10}\right)+x^9\left(\frac{ab^2f}{3}+\frac{b^3c}{9}\right)+x^8\cdot\left(\frac{3a^2bh}{8}+\frac{3ab^2e}{8}\right)+x^7\cdot\left(\frac{3a^2bg}{7}+\frac{3ab^2d}{7}\right)+x^6\left(\frac{a^2bf}{2}+\frac{ab^2c}{2}\right)+x^5\left(\frac{a^3h}{5}+\frac{3a^2be}{5}\right)+x^4\left(\frac{a^3g}{4}+\frac{3a^2bd}{4}\right)+x^3\left(\frac{a^3f}{3}+a^2bc\right)$$

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a**3*c*log(x) + a**3*d*x + a**3*e*x**2/2 + b**3*f*x**12/12 + b**3*g*x**13/1
3 + b**3*h*x**14/14 + x**11*(3*a*b**2*h/11 + b**3*e/11) + x**10*(3*a*b**2*g
/10 + b**3*d/10) + x**9*(a*b**2*f/3 + b**3*c/9) + x**8*(3*a**2*b*h/8 + 3*a*
b**2*e/8) + x**7*(3*a**2*b*g/7 + 3*a*b**2*d/7) + x**6*(a**2*b*f/2 + a*b**2*
c/2) + x**5*(a**3*h/5 + 3*a**2*b*e/5) + x**4*(a**3*g/4 + 3*a**2*b*d/4) + x*
*3*(a**3*f/3 + a**2*b*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = \frac{1}{14}\,b^3hx^{14} + \frac{1}{13}\,b^3gx^{13} + \frac{1}{12}\,b^3fx^{12} + \frac{1}{11}\,(b^3e+3\,ab^2h)x^{11} + \frac{1}{10}\,(b^3d+3\,ab^2g)x^{10} + \frac{1}{9}\,(b^3c+3\,ab^2f)x^9 + \frac{3}{8}\,(ab^2e+a^2bh)x^8 + \frac{3}{7}\,(ab^2d+a^2bg)x^7 + \frac{1}{2}\,(ab^2c+a^2bf)x^6 + \frac{1}{2}\,a^3ex^2 + \frac{1}{5}\,(3\,a^2be+a^3h)x^5 + a^3dx + \frac{1}{4}\,(3\,a^2bd+a^3g)x^4 + a^3c\log(x) + \frac{1}{3}\,(3\,a^2bc+a^3f)x^3$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxim
a")
```

```
[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 1/11*(b^3*e + 3*a*b^2
*h)*x^11 + 1/10*(b^3*d + 3*a*b^2*g)*x^10 + 1/9*(b^3*c + 3*a*b^2*f)*x^9 + 3/
8*(a*b^2*e + a^2*b*h)*x^8 + 3/7*(a*b^2*d + a^2*b*g)*x^7 + 1/2*(a*b^2*c + a^
2*b*f)*x^6 + 1/2*a^3*e*x^2 + 1/5*(3*a^2*b*e + a^3*h)*x^5 + a^3*d*x + 1/4*(3
*a^2*b*d + a^3*g)*x^4 + a^3*c*log(x) + 1/3*(3*a^2*b*c + a^3*f)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = \frac{1}{14}\,b^3hx^{14} + \frac{1}{13}\,b^3gx^{13} + \frac{1}{12}\,b^3fx^{12} + \frac{1}{11}\,b^3ex^{11} + \frac{3}{11}\,ab^2hx^{11} + \frac{1}{10}\,b^3dx^{10} + \frac{3}{10}\,ab^2gx^{10} + \frac{1}{9}\,b^3cx^9 + \frac{1}{3}\,ab^2fx^9 + \frac{3}{8}\,ab^2ex^8 + \frac{3}{8}\,a^2bhx^8 + \frac{3}{7}\,ab^2dx^7 + \frac{3}{7}\,a^2bgx^7 + \frac{1}{2}\,ab^2cx^6 + \frac{1}{2}\,a^2bfx^6 + \frac{3}{5}\,a^2bex^5 + \frac{1}{5}\,a^3hx^5 + \frac{3}{4}\,a^2bdx^4 + \frac{1}{4}\,a^3gx^4 + a^2bcx^3 + \frac{1}{3}\,a^3fx^3 + \frac{1}{2}\,a^3ex^2 + a^3dx + a^3c\log(|x|)$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac"
)
```

```
[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 1/11*b^3*e*x^11 + 3/1
1*a*b^2*h*x^11 + 1/10*b^3*d*x^10 + 3/10*a*b^2*g*x^10 + 1/9*b^3*c*x^9 + 1/3*
```

```
a*b^2*f*x^9 + 3/8*a*b^2*e*x^8 + 3/8*a^2*b*h*x^8 + 3/7*a*b^2*d*x^7 + 3/7*a^2
*b*g*x^7 + 1/2*a*b^2*c*x^6 + 1/2*a^2*b*f*x^6 + 3/5*a^2*b*e*x^5 + 1/5*a^3*h*
x^5 + 3/4*a^2*b*d*x^4 + 1/4*a^3*g*x^4 + a^2*b*c*x^3 + 1/3*a^3*f*x^3 + 1/2*a
^3*e*x^2 + a^3*d*x + a^3*c*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$$
$$= x^3\left(\frac{f\,a^3}{3}+b\,c\,a^2\right)+x^9\left(\frac{c\,b^3}{9}+\frac{a\,f\,b^2}{3}\right)+x^4\left(\frac{g\,a^3}{4}+\frac{3\,b\,d\,a^2}{4}\right)+x^{10}\left(\frac{d\,b^3}{10}+\frac{3\,a\,g\,b^2}{10}\right)+x^5\left(\frac{h\,a^3}{5}+\frac{3\,b\,e\,a^2}{5}\right)+x^{11}\left(\frac{e\,b^3}{11}+\frac{3\,a\,h\,b^2}{11}\right)+\frac{a^3\,e\,x^2}{2}+\frac{b^3\,f\,x^{12}}{12}+\frac{b^3\,g\,x^{13}}{13}+\frac{b^3\,h\,x^{14}}{14}+a^3\,c\,\ln(x)+a^3\,d\,x+\frac{a\,b\,x^6\,(b\,c+a\,f)}{2}+\frac{3\,a\,b\,x^7\,(b\,d+a\,g)}{7}+\frac{3\,a\,b\,x^8\,(b\,e+a\,h)}{8}$$

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x)
```

```
[Out] x^3*((a^3*f)/3 + a^2*b*c) + x^9*((b^3*c)/9 + (a*b^2*f)/3) + x^4*((a^3*g)/4
+ (3*a^2*b*d)/4) + x^10*((b^3*d)/10 + (3*a*b^2*g)/10) + x^5*((a^3*h)/5 + (3
*a^2*b*e)/5) + x^11*((b^3*e)/11 + (3*a*b^2*h)/11) + (a^3*e*x^2)/2 + (b^3*f*
x^12)/12 + (b^3*g*x^13)/13 + (b^3*h*x^14)/14 + a^3*c*log(x) + a^3*d*x + (a*
b*x^6*(b*c + a*f))/2 + (3*a*b*x^7*(b*d + a*g))/7 + (3*a*b*x^8*(b*e + a*h))/
8
```

3.399 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 198

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx = -\frac{a^3c}{x}+a^3ex+\frac{1}{2}a^2(3bc+af)x^2+a^2bdx^3+\frac{1}{4}a^2(3be+ah)x^4 +\frac{3}{5}ab(bc+af)x^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab(be+ah)x^7+\frac{1}{8}b^2(bc+3af)x^8+\frac{1}{9}b^3dx^9 +\frac{1}{10}b^2(be+3ah)x^{10}+\frac{1}{11}b^3fx^{11}+\frac{1}{13}b^3hx^{13}+\frac{g(a+bx^3)^4}{12b}+a^3d\log(x)$$

```
[Out] -a^3*c/x+a^3*e*x+1/2*a^2*(a*f+3*b*c)*x^2+a^2*b*d*x^3+1/4*a^2*(a*h+3*b*e)*x^
4+3/5*a*b*(a*f+b*c)*x^5+1/2*a*b^2*d*x^6+3/7*a*b*(a*h+b*e)*x^7+1/8*b^2*(3*a*
f+b*c)*x^8+1/9*b^3*d*x^9+1/10*b^2*(3*a*h+b*e)*x^10+1/11*b^3*f*x^11+1/13*b^3
*h*x^13+1/12*g*(b*x^3+a)^4/b+a^3*d*ln(x)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1597, 1834}

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx = -\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{1}{2}a^2x^2(af+3bc)+a^2bdx^3+\frac{1}{4}a^2x^4(ah+3be) +\frac{1}{8}b^2x^8(3af+bc)+\frac{1}{2}ab^2dx^6+\frac{1}{10}b^2x^{10}(3ah+be)+\frac{3}{5}abx^5(af+bc) +\frac{3}{7}abx^7(ah+be)+\frac{g(a+bx^3)^4}{12b}+\frac{1}{9}b^3dx^9+\frac{1}{11}b^3fx^{11}+\frac{1}{13}b^3hx^{13}$$

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a^3*c)/x) + a^3*e*x + (a^2*(3*b*c + a*f)*x^2)/2 + a^2*b*d*x^3 + (a^2*(3*
b*e + a*h)*x^4)/4 + (3*a*b*(b*c + a*f)*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b*(b
*e + a*h)*x^7)/7 + (b^2*(b*c + 3*a*f)*x^8)/8 + (b^3*d*x^9)/9 + (b^2*(b*e +
3*a*h)*x^10)/10 + (b^3*f*x^11)/11 + (b^3*h*x^13)/13 + (g*(a + b*x^3)^4)/(12
*b) + a^3*d*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g(a+bx^3)^4}{12b} + \int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+hx^5)}{x^2}\,dx \\
&= \frac{g(a+bx^3)^4}{12b} + \int \Bigg(a^3e + \frac{a^3c}{x^2} + \frac{a^3d}{x} + a^2(3bc+af)x + 3a^2bdx^2 + a^2(3be+ah)x^3 \\
&\qquad + 3ab(bc+af)x^4 + 3ab^2dx^5 + 3ab(be+ah)x^6 + b^2(bc+3af)x^7 + b^3dx^8 \\
&\qquad + b^2(be+3ah)x^9 + b^3fx^{10} + b^3hx^{12}\Bigg)\,dx \\
&= -\frac{a^3c}{x} + a^3ex + \frac{1}{2}a^2(3bc+af)x^2 + a^2bdx^3 + \frac{1}{4}a^2(3be+ah)x^4 \\
&\quad + \frac{3}{5}ab(bc+af)x^5 + \frac{1}{2}ab^2dx^6 + \frac{3}{7}ab(be+ah)x^7 + \frac{1}{8}b^2(bc+3af)x^8 + \frac{1}{9}b^3dx^9 \\
&\quad + \frac{1}{10}b^2(be+3ah)x^{10} + \frac{1}{11}b^3fx^{11} + \frac{1}{13}b^3hx^{13} + \frac{g(a+bx^3)^4}{12b} + a^3d\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= a^3\left(-\frac{c}{x}+ex+\frac{1}{12}x^2(6f+4gx+3hx^2)\right) + \frac{b^3x^8(6435c+5720dx+6x^2(858e+780fx+715gx^2+660hx^3))}{51480} + \frac{1}{140}a^2bx^2(210c+x(140d+x(105e+84fx+70gx^2+60hx^3))) + \frac{1}{840}ab^2x^5(504c+x(420d+x(360e+315fx+280gx^2+252hx^3))) + a^3d\log(x)$$

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] a^3*(-(c/x) + e*x + (x^2*(6*f + 4*g*x + 3*h*x^2))/12) + (b^3*x^8*(6435*c +
5720*d*x + 6*x^2*(858*e + 780*f*x + 715*g*x^2 + 660*h*x^3)))/51480 + (a^2*b
*x^2*(210*c + x*(140*d + x*(105*e + 84*f*x + 70*g*x^2 + 60*h*x^3))))/140 +
(a*b^2*x^5*(504*c + x*(420*d + x*(360*e + 315*f*x + 280*g*x^2 + 252*h*x^3))
))/840 + a^3*d*Log[x]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.11

method	result
norman	$\frac{(\frac{1}{3}a^3g+d\,a^2b)x^4+(\frac{1}{4}a^3h+\frac{3}{4}a^2be)x^5+(\frac{1}{2}f\,a^3+\frac{3}{2}a^2bc)x^3+(\frac{3}{8}a\,b^2f+\frac{1}{8}b^3c)x^9+(\frac{1}{3}a\,b^2g+\frac{1}{9}b^3d)x^{10}+(\frac{3}{10}a\,b^2h+\frac{1}{10}b^3e)x^{11}+(\frac{1}{2}a^2}{x}$
default	$\frac{b^3h\,x^{13}}{13}+\frac{b^3g\,x^{12}}{12}+\frac{b^3f\,x^{11}}{11}+\frac{3a\,b^2h\,x^{10}}{10}+\frac{b^3e\,x^{10}}{10}+\frac{a\,b^2g\,x^9}{3}+\frac{b^3d\,x^9}{9}+\frac{3x^8a\,b^2f}{8}+\frac{b^3c\,x^8}{8}+\frac{3a^2bh\,x^7}{7}+3a\,b$
risch	$\frac{b^3h\,x^{13}}{13}+\frac{b^3g\,x^{12}}{12}+\frac{b^3f\,x^{11}}{11}+\frac{3a\,b^2h\,x^{10}}{10}+\frac{b^3e\,x^{10}}{10}+\frac{a\,b^2g\,x^9}{3}+\frac{b^3d\,x^9}{9}+\frac{3x^8a\,b^2f}{8}+\frac{b^3c\,x^8}{8}+\frac{3a^2bh\,x^7}{7}+3a\,b$
parallelrisch	$32760b^3f\,x^{12}+154440a^2bh\,x^8+360360a^2bd\,x^4+120120a\,b^2g\,x^{10}+180180f\,a^3x^3+45045b^3c\,x^9+120120a^3g\,x^4+90090a^3h\,x^5-36$

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x,method=_RETURNVERBOSE
)
```

```
[Out] ((1/3*a^3*g+d*a^2*b)*x^4+(1/4*a^3*h+3/4*a^2*b*e)*x^5+(1/2*f*a^3+3/2*a^2*b*c
)*x^3+(3/8*a*b^2*f+1/8*b^3*c)*x^9+(1/3*a*b^2*g+1/9*b^3*d)*x^10+(3/10*a*b^2*
h+1/10*b^3*e)*x^11+(1/2*a^2*b*g+1/2*a*b^2*d)*x^7+(3/7*a^2*b*h+3/7*a*b^2*e)*
x^8+(3/5*f*a^2*b+3/5*a*b^2*c)*x^6+a^3*e*x^2-c*a^3+1/11*b^3*f*x^12+1/12*b^3*
g*x^13+1/13*b^3*h*x^14)/x+a^3*d*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= \frac{27720\,b^3hx^{14}+30030\,b^3gx^{13}+32760\,b^3fx^{12}+36036\,(b^3e+3\,ab^2h)x^{11}+40040\,(b^3d+3\,ab^2g)x^{10}+45045$$

[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fri cas")

[Out] 1/360360*(27720*b^3*h*x^14 + 30030*b^3*g*x^13 + 32760*b^3*f*x^12 + 36036*(b ^3*e + 3*a*b^2*h)*x^11 + 40040*(b^3*d + 3*a*b^2*g)*x^10 + 45045*(b^3*c + 3* a*b^2*f)*x^9 + 154440*(a*b^2*e + a^2*b*h)*x^8 + 180180*(a*b^2*d + a^2*b*g)* x^7 + 216216*(a*b^2*c + a^2*b*f)*x^6 + 360360*a^3*e*x^2 + 90090*(3*a^2*b*e + a^3*h)*x^5 + 360360*a^3*d*x*log(x) + 120120*(3*a^2*b*d + a^3*g)*x^4 - 360 360*a^3*c + 180180*(3*a^2*b*c + a^3*f)*x^3)/x

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= -\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{b^3fx^{11}}{11}+\frac{b^3gx^{12}}{12}+\frac{b^3hx^{13}}{13}+x^{10}\cdot\left(\frac{3ab^2h}{10}+\frac{b^3e}{10}\right)+x^9\left(\frac{ab^2g}{3}+\frac{b^3d}{9}\right)+x^8\cdot\left(\frac{3ab^2f}{8}+\frac{b^3c}{8}\right)+x^7\cdot\left(\frac{3a^2bh}{7}+\frac{3ab^2e}{7}\right)+x^6\left(\frac{a^2bg}{2}+\frac{ab^2d}{2}\right)+x^5\cdot\left(\frac{3a^2bf}{5}+\frac{3ab^2c}{5}\right)+x^4\left(\frac{a^3h}{4}+\frac{3a^2be}{4}\right)+x^3\left(\frac{a^3g}{3}+a^2bd\right)+x^2\left(\frac{a^3f}{2}+\frac{3a^2bc}{2}\right)$$

[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)

[Out] -a**3*c/x + a**3*d*log(x) + a**3*e*x + b**3*f*x**11/11 + b**3*g*x**12/12 + b**3*h*x**13/13 + x**10*(3*a*b**2*h/10 + b**3*e/10) + x**9*(a*b**2*g/3 + b* *3*d/9) + x**8*(3*a*b**2*f/8 + b**3*c/8) + x**7*(3*a**2*b*h/7 + 3*a*b**2*e/ 7) + x**6*(a**2*b*g/2 + a*b**2*d/2) + x**5*(3*a**2*b*f/5 + 3*a*b**2*c/5) + x**4*(a**3*h/4 + 3*a**2*b*e/4) + x**3*(a**3*g/3 + a**2*b*d) + x**2*(a**3*f/ 2 + 3*a**2*b*c/2)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= \frac{1}{13}\,b^3hx^{13}+\frac{1}{12}\,b^3gx^{12}+\frac{1}{11}\,b^3fx^{11}+\frac{1}{10}\,(b^3e+3\,ab^2h)x^{10}+\frac{1}{9}\,(b^3d+3\,ab^2g)x^9$$
$$+\frac{1}{8}\,(b^3c+3\,ab^2f)x^8+\frac{3}{7}\,(ab^2e+a^2bh)x^7+\frac{1}{2}\,(ab^2d+a^2bg)x^6+\frac{3}{5}\,(ab^2c+a^2bf)x^5$$
$$+a^3ex+\frac{1}{4}\,(3\,a^2be+a^3h)x^4+a^3d\log(x)+\frac{1}{3}\,(3\,a^2bd+a^3g)x^3-\frac{a^3c}{x}+\frac{1}{2}\,(3\,a^2bc+a^3f)x^2$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="max
ima")
```

```
[Out] 1/13*b^3*h*x^13 + 1/12*b^3*g*x^12 + 1/11*b^3*f*x^11 + 1/10*(b^3*e + 3*a*b^2
*h)*x^10 + 1/9*(b^3*d + 3*a*b^2*g)*x^9 + 1/8*(b^3*c + 3*a*b^2*f)*x^8 + 3/7*
(a*b^2*e + a^2*b*h)*x^7 + 1/2*(a*b^2*d + a^2*b*g)*x^6 + 3/5*(a*b^2*c + a^2*
b*f)*x^5 + a^3*e*x + 1/4*(3*a^2*b*e + a^3*h)*x^4 + a^3*d*log(x) + 1/3*(3*a^
2*b*d + a^3*g)*x^3 - a^3*c/x + 1/2*(3*a^2*b*c + a^3*f)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= \frac{1}{13}\,b^3hx^{13}+\frac{1}{12}\,b^3gx^{12}+\frac{1}{11}\,b^3fx^{11}+\frac{1}{10}\,b^3ex^{10}+\frac{3}{10}\,ab^2hx^{10}+\frac{1}{9}\,b^3dx^9+\frac{1}{3}\,ab^2gx^9$$
$$+\frac{1}{8}\,b^3cx^8+\frac{3}{8}\,ab^2fx^8+\frac{3}{7}\,ab^2ex^7+\frac{3}{7}\,a^2bhx^7+\frac{1}{2}\,ab^2dx^6+\frac{1}{2}\,a^2bgx^6+\frac{3}{5}\,ab^2cx^5+\frac{3}{5}\,a^2bfx^5$$
$$+\frac{3}{4}\,a^2bex^4+\frac{1}{4}\,a^3hx^4+a^2bdx^3+\frac{1}{3}\,a^3gx^3+\frac{3}{2}\,a^2bcx^2+\frac{1}{2}\,a^3fx^2+a^3ex+a^3d\log(|x|)-\frac{a^3c}{x}$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="gia
c")
```

```
[Out] 1/13*b^3*h*x^13 + 1/12*b^3*g*x^12 + 1/11*b^3*f*x^11 + 1/10*b^3*e*x^10 + 3/1
0*a*b^2*h*x^10 + 1/9*b^3*d*x^9 + 1/3*a*b^2*g*x^9 + 1/8*b^3*c*x^8 + 3/8*a*b^
2*f*x^8 + 3/7*a*b^2*e*x^7 + 3/7*a^2*b*h*x^7 + 1/2*a*b^2*d*x^6 + 1/2*a^2*b*g
*x^6 + 3/5*a*b^2*c*x^5 + 3/5*a^2*b*f*x^5 + 3/4*a^2*b*e*x^4 + 1/4*a^3*h*x^4
+ a^2*b*d*x^3 + 1/3*a^3*g*x^3 + 3/2*a^2*b*c*x^2 + 1/2*a^3*f*x^2 + a^3*e*x +
 a^3*d*log(abs(x)) - a^3*c/x
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} \, dx$$
$$= x^2 \left(\frac{f\,a^3}{2}+\frac{3\,b\,c\,a^2}{2}\right)+x^8 \left(\frac{c\,b^3}{8}+\frac{3\,a\,f\,b^2}{8}\right)+x^3 \left(\frac{g\,a^3}{3}+b\,d\,a^2\right)+x^9 \left(\frac{d\,b^3}{9}+\frac{a\,g\,b^2}{3}\right)+x^4 \left(\frac{h\,a^3}{4}+\frac{3\,b\,e\,a^2}{4}\right)+x^{10} \left(\frac{e\,b^3}{10}+\frac{3\,a\,h\,b^2}{10}\right)-\frac{a^3\,c}{x}+\frac{b^3\,f\,x^{11}}{11}+\frac{b^3\,g\,x^{12}}{12}+\frac{b^3\,h\,x^{13}}{13}+a^3\,d\,\ln(x)+a^3\,e\,x+\frac{3\,a\,b\,x^5\,(b\,c+a\,f)}{5}+\frac{a\,b\,x^6\,(b\,d+a\,g)}{2}+\frac{3\,a\,b\,x^7\,(b\,e+a\,h)}{7}$$

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x)
```

```
[Out] x^2*((a^3*f)/2 + (3*a^2*b*c)/2) + x^8*((b^3*c)/8 + (3*a*b^2*f)/8) + x^3*((a
^3*g)/3 + a^2*b*d) + x^9*((b^3*d)/9 + (a*b^2*g)/3) + x^4*((a^3*h)/4 + (3*a^
2*b*e)/4) + x^10*((b^3*e)/10 + (3*a*b^2*h)/10) - (a^3*c)/x + (b^3*f*x^11)/1
1 + (b^3*g*x^12)/12 + (b^3*h*x^13)/13 + a^3*d*log(x) + a^3*e*x + (3*a*b*x^5
*(b*c + a*f))/5 + (a*b*x^6*(b*d + a*g))/2 + (3*a*b*x^7*(b*e + a*h))/7
```

3.400 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3} dx$

Optimal result

Integrand size = 38, antiderivative size = 198

$$\begin{aligned}\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx &= -\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^2(3bc+af)x+\frac{1}{2}a^2(3bd+ag)x^2+a^2bex^3+\frac{3}{4}ab(bc+af)x^4\\&\quad+\frac{3}{5}ab(bd+ag)x^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^2(bc+3af)x^7+\frac{1}{8}b^2(bd+3ag)x^8\\&\quad+\frac{1}{9}b^3ex^9+\frac{1}{10}b^3fx^{10}+\frac{1}{11}b^3gx^{11}+\frac{h(a+bx^3)^4}{12b}+a^3e\log(x)\end{aligned}$$

```
[Out] -1/2*a^3*c/x^2-a^3*d/x+a^2*(a*f+3*b*c)*x+1/2*a^2*(a*g+3*b*d)*x^2+a^2*b*e*x^
3+3/4*a*b*(a*f+b*c)*x^4+3/5*a*b*(a*g+b*d)*x^5+1/2*a*b^2*e*x^6+1/7*b^2*(3*a*
f+b*c)*x^7+1/8*b^2*(3*a*g+b*d)*x^8+1/9*b^3*e*x^9+1/10*b^3*f*x^10+1/11*b^3*g
*x^11+1/12*h*(b*x^3+a)^4/b+a^3*e*ln(x)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1597, 1834}

$$\begin{aligned}\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx &= -\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+a^2x(af+3bc)+\frac{1}{2}a^2x^2(ag+3bd)+a^2bex^3\\&\quad+\frac{1}{7}b^2x^7(3af+bc)+\frac{1}{8}b^2x^8(3ag+bd)+\frac{1}{2}ab^2ex^6+\frac{3}{4}abx^4(af+bc)\\&\quad+\frac{3}{5}abx^5(ag+bd)+\frac{h(a+bx^3)^4}{12b}+\frac{1}{9}b^3ex^9+\frac{1}{10}b^3fx^{10}+\frac{1}{11}b^3gx^{11}\end{aligned}$$

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] -1/2*(a^3*c)/x^2 - (a^3*d)/x + a^2*(3*b*c + a*f)*x + (a^2*(3*b*d + a*g)*x^2
)/2 + a^2*b*e*x^3 + (3*a*b*(b*c + a*f)*x^4)/4 + (3*a*b*(b*d + a*g)*x^5)/5 +
 (a*b^2*e*x^6)/2 + (b^2*(b*c + 3*a*f)*x^7)/7 + (b^2*(b*d + 3*a*g)*x^8)/8 +
(b^3*e*x^9)/9 + (b^3*f*x^10)/10 + (b^3*g*x^11)/11 + (h*(a + b*x^3)^4)/(12*b
) + a^3*e*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{h(a+bx^3)^4}{12b} + \int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx \\
&= \frac{h(a+bx^3)^4}{12b} + \int \left(a^2(3bc+af) + \frac{a^3c}{x^3} + \frac{a^3d}{x^2} + \frac{a^3e}{x} + a^2(3bd+ag)x + 3a^2bex^2 + 3ab(bc+af)x^3 + 3ab(bd+ag)x^4 + 3ab^2ex^5 + b^2(bc+3af)x^6 + b^2(bd+3ag)x^7 + b^3ex^8 + b^3fx^9 + b^3gx^{10}\right) dx \\
&= -\frac{a^3c}{2x^2} - \frac{a^3d}{x} + a^2(3bc+af)x + \frac{1}{2}a^2(3bd+ag)x^2 + a^2bex^3 + \frac{3}{4}ab(bc+af)x^4 + \frac{3}{5}ab(bd+ag)x^5 + \frac{1}{2}ab^2ex^6 + \frac{1}{7}b^2(bc+3af)x^7 + \frac{1}{8}b^2(bd+3ag)x^8 + \frac{1}{9}b^3ex^9 + \frac{1}{10}b^3fx^{10} + \frac{1}{11}b^3gx^{11} + \frac{h(a+bx^3)^4}{12b} + a^3e\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.88

$$\begin{aligned}&\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx\\ &=\frac{a^3(-3c-6dx+x^3(6f+3gx+2hx^2))}{6x^2}\\ &\quad+\frac{b^3x^7(3960c+7x(495d+440ex+6x^2(66f+60gx+55hx^2)))}{27720}\\ &\quad+\frac{1}{20}a^2bx\big(60c+x\big(30d+x\big(20e+15fx+12gx^2+10hx^3\big)\big)\big)\\ &\quad+\frac{1}{840}ab^2x^4(630c+x(504d+5x(84e+x(72f+7x(9g+8hx)))))+a^3e\log(x)\end{aligned}$$

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] (a^3*(-3*c - 6*d*x + x^3*(6*f + 3*g*x + 2*h*x^2)))/(6*x^2) + (b^3*x^7*(3960
*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55*h*x^2))))/27720 + (a^
2*b*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))))/20 + (a*
b^2*x^4*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x))))))/84
0 + a^3*e*Log[x]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.10

method	result
norman	$\frac{\left(\frac{1}{2}a^3g+\frac{3}{2}d\,a^2b\right)x^4+\left(\frac{1}{3}a^3h+a^2be\right)x^5+\left(\frac{3}{7}a\,b^2f+\frac{1}{7}b^3c\right)x^9+\left(\frac{3}{8}a\,b^2g+\frac{1}{8}b^3d\right)x^{10}+\left(\frac{1}{3}a\,b^2h+\frac{1}{9}b^3e\right)x^{11}+\left(\frac{3}{5}a^2bg+\frac{3}{5}a\,b^2d\right)x^7+\left(\frac{1}{2}a^2\right.}{x^2}$
default	$\frac{b^3h\,x^{12}}{12}+\frac{b^3g\,x^{11}}{11}+\frac{b^3f\,x^{10}}{10}+\frac{a\,b^2h\,x^9}{3}+\frac{b^3e\,x^9}{9}+\frac{3a\,b^2g\,x^8}{8}+\frac{b^3d\,x^8}{8}+\frac{3x^7a\,b^2f}{7}+\frac{b^3c\,x^7}{7}+\frac{a^2bh\,x^6}{2}+\frac{a\,b^2e\,x}{2}$
risch	$\frac{b^3h\,x^{12}}{12}+\frac{b^3g\,x^{11}}{11}+\frac{b^3f\,x^{10}}{10}+\frac{a\,b^2h\,x^9}{3}+\frac{b^3e\,x^9}{9}+\frac{3a\,b^2g\,x^8}{8}+\frac{b^3d\,x^8}{8}+\frac{3x^7a\,b^2f}{7}+\frac{b^3c\,x^7}{7}+\frac{a^2bh\,x^6}{2}+\frac{a\,b^2e\,x}{2}$
parallelrisch	$2772b^3f\,x^{12}+13860a^2bh\,x^8-27720a^3dx+41580a^2bd\,x^4+10395a\,b^2g\,x^{10}+27720f\,a^3x^3+3960b^3c\,x^9+13860a^3g\,x^4+9240a^3h\,x^5-$

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x,method=_RETURNVERBOSE
)
```

```
[Out] ((1/2*a^3*g+3/2*d*a^2*b)*x^4+(1/3*a^3*h+a^2*b*e)*x^5+(3/7*a*b^2*f+1/7*b^3*c
)*x^9+(3/8*a*b^2*g+1/8*b^3*d)*x^10+(1/3*a*b^2*h+1/9*b^3*e)*x^11+(3/5*a^2*b*
g+3/5*a*b^2*d)*x^7+(1/2*a^2*b*h+1/2*a*b^2*e)*x^8+(3/4*f*a^2*b+3/4*a*b^2*c)*
x^6+(a^3*f+3*a^2*b*c)*x^3-1/2*c*a^3-a^3*d*x+1/10*b^3*f*x^12+1/11*b^3*g*x^13
+1/12*b^3*h*x^14)/x^2+a^3*e*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= \frac{2310\,b^3hx^{14}+2520\,b^3gx^{13}+2772\,b^3fx^{12}+3080\,(b^3e+3\,ab^2h)x^{11}+3465\,(b^3d+3\,ab^2g)x^{10}+3960\,(b^3c+}{}$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fri
cas")
```

```
[Out] 1/27720*(2310*b^3*h*x^14 + 2520*b^3*g*x^13 + 2772*b^3*f*x^12 + 3080*(b^3*e
+ 3*a*b^2*h)*x^11 + 3465*(b^3*d + 3*a*b^2*g)*x^10 + 3960*(b^3*c + 3*a*b^2*f
)*x^9 + 13860*(a*b^2*e + a^2*b*h)*x^8 + 16632*(a*b^2*d + a^2*b*g)*x^7 + 207
90*(a*b^2*c + a^2*b*f)*x^6 + 27720*a^3*e*x^2*log(x) + 9240*(3*a^2*b*e + a^3
*h)*x^5 - 27720*a^3*d*x + 13860*(3*a^2*b*d + a^3*g)*x^4 - 13860*a^3*c + 277
20*(3*a^2*b*c + a^3*f)*x^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= a^3e\log{(x)}+\frac{b^3fx^{10}}{10}+\frac{b^3gx^{11}}{11}+\frac{b^3hx^{12}}{12}+x^9\left(\frac{ab^2h}{3}+\frac{b^3e}{9}\right)+x^8\cdot\left(\frac{3ab^2g}{8}+\frac{b^3d}{8}\right)+x^7\cdot\left(\frac{3ab^2f}{7}+\frac{b^3c}{7}\right)+x^6\left(\frac{a^2bh}{2}+\frac{ab^2e}{2}\right)+x^5\cdot\left(\frac{3a^2bg}{5}+\frac{3ab^2d}{5}\right)+x^4\cdot\left(\frac{3a^2bf}{4}+\frac{3ab^2c}{4}\right)+x^3\left(\frac{a^3h}{3}+a^2be\right)+x^2\left(\frac{a^3g}{2}+\frac{3a^2bd}{2}\right)+x\,(a^3f+3a^2bc)+\frac{-a^3c-2a^3dx}{2x^2}$$

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a**3*e*log(x) + b**3*f*x**10/10 + b**3*g*x**11/11 + b**3*h*x**12/12 + x**9*
(a*b**2*h/3 + b**3*e/9) + x**8*(3*a*b**2*g/8 + b**3*d/8) + x**7*(3*a*b**2*f
/7 + b**3*c/7) + x**6*(a**2*b*h/2 + a*b**2*e/2) + x**5*(3*a**2*b*g/5 + 3*a*
b**2*d/5) + x**4*(3*a**2*b*f/4 + 3*a*b**2*c/4) + x**3*(a**3*h/3 + a**2*b*e)
 + x**2*(a**3*g/2 + 3*a**2*b*d/2) + x*(a**3*f + 3*a**2*b*c) + (-a**3*c - 2*
a**3*d*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3} dx = \frac{1}{12} b^3hx^{12} + \frac{1}{11} b^3gx^{11} + \frac{1}{10} b^3fx^{10} + \frac{1}{9} (b^3e + 3ab^2h)x^9 + \frac{1}{8} (b^3d + 3ab^2g)x^8 + \frac{1}{7} (b^3c + 3ab^2f)x^7 + \frac{1}{2} (ab^2e + a^2bh)x^6 + \frac{3}{5} (ab^2d + a^2bg)x^5 + \frac{3}{4} (ab^2c + a^2bf)x^4 + a^3e\log(x) + \frac{1}{3} (3a^2be + a^3h)x^3 + \frac{1}{2} (3a^2bd + a^3g)x^2 + (3a^2bc + a^3f)x - \frac{2a^3dx + a^3c}{2x^2}$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="max
ima")
```

```
[Out] 1/12*b^3*h*x^12 + 1/11*b^3*g*x^11 + 1/10*b^3*f*x^10 + 1/9*(b^3*e + 3*a*b^2*
h)*x^9 + 1/8*(b^3*d + 3*a*b^2*g)*x^8 + 1/7*(b^3*c + 3*a*b^2*f)*x^7 + 1/2*(a
*b^2*e + a^2*b*h)*x^6 + 3/5*(a*b^2*d + a^2*b*g)*x^5 + 3/4*(a*b^2*c + a^2*b*
f)*x^4 + a^3*e*log(x) + 1/3*(3*a^2*b*e + a^3*h)*x^3 + 1/2*(3*a^2*b*d + a^3*
g)*x^2 + (3*a^2*b*c + a^3*f)*x - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3} dx = \frac{1}{12} b^3hx^{12} + \frac{1}{11} b^3gx^{11} + \frac{1}{10} b^3fx^{10} + \frac{1}{9} b^3ex^9 + \frac{1}{3} ab^2hx^9 + \frac{1}{8} b^3dx^8 + \frac{3}{8} ab^2gx^8 + \frac{1}{7} b^3cx^7 + \frac{3}{7} ab^2fx^7 + \frac{1}{2} ab^2ex^6 + \frac{1}{2} a^2bhx^6 + \frac{3}{5} ab^2dx^5 + \frac{3}{5} a^2bgx^5 + \frac{3}{4} ab^2cx^4 + \frac{3}{4} a^2bfx^4 + a^2bex^3 + \frac{1}{3} a^3hx^3 + \frac{3}{2} a^2bdx^2 + \frac{1}{2} a^3gx^2 + 3a^2bcx + a^3fx + a^3e\log(|x|) - \frac{2a^3dx + a^3c}{2x^2}$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="gia
c")
```

```
[Out] 1/12*b^3*h*x^12 + 1/11*b^3*g*x^11 + 1/10*b^3*f*x^10 + 1/9*b^3*e*x^9 + 1/3*a
*b^2*h*x^9 + 1/8*b^3*d*x^8 + 3/8*a*b^2*g*x^8 + 1/7*b^3*c*x^7 + 3/7*a*b^2*f*
x^7 + 1/2*a*b^2*e*x^6 + 1/2*a^2*b*h*x^6 + 3/5*a*b^2*d*x^5 + 3/5*a^2*b*g*x^5
 + 3/4*a*b^2*c*x^4 + 3/4*a^2*b*f*x^4 + a^2*b*e*x^3 + 1/3*a^3*h*x^3 + 3/2*a^
2*b*d*x^2 + 1/2*a^3*g*x^2 + 3*a^2*b*c*x + a^3*f*x + a^3*e*log(abs(x)) - 1/2
*(2*a^3*d*x + a^3*c)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= x^7\left(\frac{c\,b^3}{7}+\frac{3\,a\,f\,b^2}{7}\right)+x^2\left(\frac{g\,a^3}{2}+\frac{3\,b\,d\,a^2}{2}\right)+x^8\left(\frac{d\,b^3}{8}+\frac{3\,a\,g\,b^2}{8}\right)+x^3\left(\frac{h\,a^3}{3}+b\,e\,a^2\right)+x^9\left(\frac{e\,b^3}{9}+\frac{a\,h\,b^2}{3}\right)-\frac{\frac{a^3\,c}{2}+a^3\,d\,x}{x^2}+x\left(f\,a^3+3\,b\,c\,a^2\right)+\frac{b^3\,f\,x^{10}}{10}+\frac{b^3\,g\,x^{11}}{11}+\frac{b^3\,h\,x^{12}}{12}+a^3\,e\,\ln\left(x\right)+\frac{3\,a\,b\,x^4\,\left(b\,c+a\,f\right)}{4}+\frac{3\,a\,b\,x^5\,\left(b\,d+a\,g\right)}{5}+\frac{a\,b\,x^6\,\left(b\,e+a\,h\right)}{2}$$

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x)
```

```
[Out] x^7*((b^3*c)/7 + (3*a*b^2*f)/7) + x^2*((a^3*g)/2 + (3*a^2*b*d)/2) + x^8*((b
^3*d)/8 + (3*a*b^2*g)/8) + x^3*((a^3*h)/3 + a^2*b*e) + x^9*((b^3*e)/9 + (a*
b^2*h)/3) - ((a^3*c)/2 + a^3*d*x)/x^2 + x*(a^3*f + 3*a^2*b*c) + (b^3*f*x^10
)/10 + (b^3*g*x^11)/11 + (b^3*h*x^12)/12 + a^3*e*log(x) + (3*a*b*x^4*(b*c +
 a*f))/4 + (3*a*b*x^5*(b*d + a*g))/5 + (a*b*x^6*(b*e + a*h))/2
```

3.401 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx$

Optimal result

Integrand size = 38, antiderivative size = 209

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx = -\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+a^2(3bd+ag)x+\frac{1}{2}a^2(3be+ah)x^2+ab(bc+af)x^3 + \frac{3}{4}ab(bd+ag)x^4+\frac{3}{5}ab(be+ah)x^5+\frac{1}{6}b^2(bc+3af)x^6+\frac{1}{7}b^2(bd+3ag)x^7 + \frac{1}{8}b^2(be+3ah)x^8+\frac{1}{9}b^3fx^9+\frac{1}{10}b^3gx^{10}+\frac{1}{11}b^3hx^{11}+a^2(3bc+af)\log(x)$$

```
[Out] -1/3*a^3*c/x^3-1/2*a^3*d/x^2-a^3*e/x+a^2*(a*g+3*b*d)*x+1/2*a^2*(a*h+3*b*e)*
x^2+a*b*(a*f+b*c)*x^3+3/4*a*b*(a*g+b*d)*x^4+3/5*a*b*(a*h+b*e)*x^5+1/6*b^2*(
3*a*f+b*c)*x^6+1/7*b^2*(3*a*g+b*d)*x^7+1/8*b^2*(3*a*h+b*e)*x^8+1/9*b^3*f*x^
9+1/10*b^3*g*x^10+1/11*b^3*h*x^11+a^2*(a*f+3*b*c)*ln(x)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1834}

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx = -\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+a^2\log(x)(af+3bc)+a^2x(ag+3bd)+\frac{1}{2}a^2x^2(ah+3be) + \frac{1}{6}b^2x^6(3af+bc)+\frac{1}{7}b^2x^7(3ag+bd)+\frac{1}{8}b^2x^8(3ah+be)+abx^3(af+bc) + \frac{3}{4}abx^4(ag+bd)+\frac{3}{5}abx^5(ah+be)+\frac{1}{9}b^3fx^9+\frac{1}{10}b^3gx^{10}+\frac{1}{11}b^3hx^{11}$$

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -1/3*(a^3*c)/x^3 - (a^3*d)/(2*x^2) - (a^3*e)/x + a^2*(3*b*d + a*g)*x + (a^2
*(3*b*e + a*h)*x^2)/2 + a*b*(b*c + a*f)*x^3 + (3*a*b*(b*d + a*g)*x^4)/4 + (
3*a*b*(b*e + a*h)*x^5)/5 + (b^2*(b*c + 3*a*f)*x^6)/6 + (b^2*(b*d + 3*a*g)*x
^7)/7 + (b^2*(b*e + 3*a*h)*x^8)/8 + (b^3*f*x^9)/9 + (b^3*g*x^10)/10 + (b^3*
h*x^11)/11 + a^2*(3*b*c + a*f)*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \int \left(a^2(3bd+ag)+\frac{a^3c}{x^4}+\frac{a^3d}{x^3}+\frac{a^3e}{x^2}+\frac{a^2(3bc+af)}{x}+a^2(3be+ah)x+3ab(bc+af)x^2+3ab(bd+ag)x^3+3ab(be+ah)x^4+b^2(bc+3af)x^5+b^2(bd+3ag)x^6+b^2(be+3ah)x^7+b^3fx^8+b^3gx^9+b^3hx^{10}\right)dx$$

$$= -\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+a^2(3bd+ag)x+\frac{1}{2}a^2(3be+ah)x^2+ab(bc+af)x^3+\frac{3}{4}ab(bd+ag)x^4+\frac{3}{5}ab(be+ah)x^5+\frac{1}{6}b^2(bc+3af)x^6+\frac{1}{7}b^2(bd+3ag)x^7+\frac{1}{8}b^2(be+3ah)x^8+\frac{1}{9}b^3fx^9+\frac{1}{10}b^3gx^{10}+\frac{1}{11}b^3hx^{11}+a^2(3bc+af)\log(x)$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}dx = -\frac{a^3(2c+3x(d+2ex-x^3(2g+hx)))}{6x^3}+\frac{1}{20}a^2bx(60d+x(30e+x(20f+15gx+12hx^2)))+\frac{1}{280}ab^2x^3(280c+x(210d+x(168e+140fx+120gx^2+105hx^3)))+\frac{b^3x^6(4620c+x(3960d+7x(495e+4x(110f+99gx+90hx^2))))}{27720}+a^2(3bc+af)\log(x)$$

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -1/6*(a^3*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/x^3 + (a^2*b*x*(60*d +
 x*(30*e + x*(20*f + 15*g*x + 12*h*x^2))))/20 + (a*b^2*x^3*(280*c + x*(210*
d + x*(168*e + 140*f*x + 120*g*x^2 + 105*h*x^3))))/280 + (b^3*x^6*(4620*c +
 x*(3960*d + 7*x*(495*e + 4*x*(110*f + 99*g*x + 90*h*x^2)))))/27720 + a^2*(
3*b*c + a*f)*Log[x]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.03

method	result
norman	$\frac{\left(\frac{1}{2}a^{3}h+\frac{3}{2}a^{2}be\right)x^{5}+\left(\frac{1}{2}a\,b^{2}f+\frac{1}{6}b^{3}c\right)x^{9}+\left(\frac{3}{7}a\,b^{2}g+\frac{1}{7}b^{3}d\right)x^{10}+\left(\frac{3}{8}a\,b^{2}h+\frac{1}{8}b^{3}e\right)x^{11}+\left(\frac{3}{4}a^{2}bg+\frac{3}{4}a\,b^{2}d\right)x^{7}+\left(\frac{3}{5}a^{2}bh+\frac{3}{5}a\,b^{2}e\right)x^{8}+(f}{x^{3}}$
default	$\frac{b^{3}h\,x^{11}}{11}+\frac{b^{3}g\,x^{10}}{10}+\frac{b^{3}f\,x^{9}}{9}+\frac{3a\,b^{2}h\,x^{8}}{8}+\frac{x^{8}b^{3}e}{8}+\frac{3a\,b^{2}g\,x^{7}}{7}+\frac{b^{3}d\,x^{7}}{7}+\frac{a\,b^{2}f\,x^{6}}{2}+\frac{b^{3}c\,x^{6}}{6}+\frac{3a^{2}bh\,x^{5}}{5}+\frac{3a\,b^{2}e}{5}$
risch	$\frac{b^{3}h\,x^{11}}{11}+\frac{b^{3}g\,x^{10}}{10}+\frac{b^{3}f\,x^{9}}{9}+\frac{3a\,b^{2}h\,x^{8}}{8}+\frac{x^{8}b^{3}e}{8}+\frac{3a\,b^{2}g\,x^{7}}{7}+\frac{b^{3}d\,x^{7}}{7}+\frac{a\,b^{2}f\,x^{6}}{2}+\frac{b^{3}c\,x^{6}}{6}+\frac{3a^{2}bh\,x^{5}}{5}+\frac{3a\,b^{2}e}{5}$
parallelrisch	$\frac{3080b^{3}f\,x^{12}+16632a^{2}bh\,x^{8}-13860a^{3}dx+83160a^{2}bd\,x^{4}+11880a\,b^{2}g\,x^{10}+4620b^{3}c\,x^{9}+27720a^{3}g\,x^{4}+13860a^{3}h\,x^{5}-9240c\,a^{3}+10}{}$

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x,method=_RETURNVERBOSE
)
```

```
[Out] ((1/2*a^3*h+3/2*a^2*b*e)*x^5+(1/2*a*b^2*f+1/6*b^3*c)*x^9+(3/7*a*b^2*g+1/7*b
^3*d)*x^10+(3/8*a*b^2*h+1/8*b^3*e)*x^11+(3/4*a^2*b*g+3/4*a*b^2*d)*x^7+(3/5*
a^2*b*h+3/5*a*b^2*e)*x^8+(a^2*b*f+a*b^2*c)*x^6+(a^3*g+3*a^2*b*d)*x^4-1/3*c*
a^3-1/2*a^3*d*x-a^3*e*x^2+1/9*b^3*f*x^12+1/10*b^3*g*x^13+1/11*b^3*h*x^14)/x
^3+(a^3*f+3*a^2*b*c)*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$=\frac{2520\,b^3hx^{14}+2772\,b^3gx^{13}+3080\,b^3fx^{12}+3465\,(b^3e+3\,ab^2h)x^{11}+3960\,(b^3d+3\,ab^2g)x^{10}+4620\,(b^3c+3}{}$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fri
cas")
```

```
[Out] 1/27720*(2520*b^3*h*x^14 + 2772*b^3*g*x^13 + 3080*b^3*f*x^12 + 3465*(b^3*e
+ 3*a*b^2*h)*x^11 + 3960*(b^3*d + 3*a*b^2*g)*x^10 + 4620*(b^3*c + 3*a*b^2*f
)*x^9 + 16632*(a*b^2*e + a^2*b*h)*x^8 + 20790*(a*b^2*d + a^2*b*g)*x^7 + 277
20*(a*b^2*c + a^2*b*f)*x^6 - 27720*a^3*e*x^2 + 13860*(3*a^2*b*e + a^3*h)*x^
5 - 13860*a^3*d*x + 27720*(3*a^2*b*d + a^3*g)*x^4 + 27720*(3*a^2*b*c + a^3*
f)*x^3*log(x) - 9240*a^3*c)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$= a^2(af+3bc)\log(x)+\frac{b^3fx^9}{9}+\frac{b^3gx^{10}}{10}+\frac{b^3hx^{11}}{11}+x^8\cdot\left(\frac{3ab^2h}{8}+\frac{b^3e}{8}\right)+x^7\cdot\left(\frac{3ab^2g}{7}+\frac{b^3d}{7}\right)+x^6\left(\frac{ab^2f}{2}+\frac{b^3c}{6}\right)+x^5\cdot\left(\frac{3a^2bh}{5}+\frac{3ab^2e}{5}\right)+x^4\cdot\left(\frac{3a^2bg}{4}+\frac{3ab^2d}{4}\right)+x^3(a^2bf+ab^2c)+x^2\left(\frac{a^3h}{2}+\frac{3a^2be}{2}\right)+x(a^3g+3a^2bd)+\frac{-2a^3c-3a^3dx-6a^3ex^2}{6x^3}$$

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] a**2*(a*f + 3*b*c)*log(x) + b**3*f*x**9/9 + b**3*g*x**10/10 + b**3*h*x**11/
11 + x**8*(3*a*b**2*h/8 + b**3*e/8) + x**7*(3*a*b**2*g/7 + b**3*d/7) + x**6
*(a*b**2*f/2 + b**3*c/6) + x**5*(3*a**2*b*h/5 + 3*a*b**2*e/5) + x**4*(3*a**
2*b*g/4 + 3*a*b**2*d/4) + x**3*(a**2*b*f + a*b**2*c) + x**2*(a**3*h/2 + 3*a
**2*b*e/2) + x*(a**3*g + 3*a**2*b*d) + (-2*a**3*c - 3*a**3*d*x - 6*a**3*e*x
**2)/(6*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$= \frac{1}{11}b^3hx^{11}+\frac{1}{10}b^3gx^{10}+\frac{1}{9}b^3fx^9+\frac{1}{8}(b^3e+3ab^2h)x^8+\frac{1}{7}(b^3d+3ab^2g)x^7+\frac{1}{6}(b^3c+3ab^2f)x^6+\frac{3}{5}(ab^2e+a^2bh)x^5+\frac{3}{4}(ab^2d+a^2bg)x^4+(ab^2c+a^2bf)x^3+\frac{1}{2}(3a^2be+a^3h)x^2+(3a^2bd+a^3g)x+(3a^2bc+a^3f)\log(x)-\frac{6a^3ex^2+3a^3dx+2a^3c}{6x^3}$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="max
ima")
```

```
[Out] 1/11*b^3*h*x^11 + 1/10*b^3*g*x^10 + 1/9*b^3*f*x^9 + 1/8*(b^3*e + 3*a*b^2*h)
*x^8 + 1/7*(b^3*d + 3*a*b^2*g)*x^7 + 1/6*(b^3*c + 3*a*b^2*f)*x^6 + 3/5*(a*b
^2*e + a^2*b*h)*x^5 + 3/4*(a*b^2*d + a^2*b*g)*x^4 + (a*b^2*c + a^2*b*f)*x^3
 + 1/2*(3*a^2*b*e + a^3*h)*x^2 + (3*a^2*b*d + a^3*g)*x + (3*a^2*b*c + a^3*f
)*log(x) - 1/6*(6*a^3*e*x^2 + 3*a^3*d*x + 2*a^3*c)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$= \frac{1}{11}\,b^3hx^{11} + \frac{1}{10}\,b^3gx^{10} + \frac{1}{9}\,b^3fx^9 + \frac{1}{8}\,b^3ex^8 + \frac{3}{8}\,ab^2hx^8 + \frac{1}{7}\,b^3dx^7 + \frac{3}{7}\,ab^2gx^7 + \frac{1}{6}\,b^3cx^6 + \frac{1}{2}\,ab^2fx^6 + \frac{3}{5}\,ab^2ex^5 + \frac{3}{5}\,a^2bhx^5 + \frac{3}{4}\,ab^2dx^4 + \frac{3}{4}\,a^2bgx^4 + ab^2cx^3 + a^2bfx^3 + \frac{3}{2}\,a^2bex^2 + \frac{1}{2}\,a^3hx^2 + 3\,a^2bdx + a^3gx + \left(3\,a^2bc + a^3f\right)\log\left(|x|\right) - \frac{6\,a^3ex^2 + 3\,a^3dx + 2\,a^3c}{6\,x^3}$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="gia
c")
```

```
[Out] 1/11*b^3*h*x^11 + 1/10*b^3*g*x^10 + 1/9*b^3*f*x^9 + 1/8*b^3*e*x^8 + 3/8*a*b
^2*h*x^8 + 1/7*b^3*d*x^7 + 3/7*a*b^2*g*x^7 + 1/6*b^3*c*x^6 + 1/2*a*b^2*f*x^
6 + 3/5*a*b^2*e*x^5 + 3/5*a^2*b*h*x^5 + 3/4*a*b^2*d*x^4 + 3/4*a^2*b*g*x^4 +
 a*b^2*c*x^3 + a^2*b*f*x^3 + 3/2*a^2*b*e*x^2 + 1/2*a^3*h*x^2 + 3*a^2*b*d*x
+ a^3*g*x + (3*a^2*b*c + a^3*f)*log(abs(x)) - 1/6*(6*a^3*e*x^2 + 3*a^3*d*x
+ 2*a^3*c)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$= x^6\left(\frac{c\,b^3}{6} + \frac{a\,f\,b^2}{2}\right) + x^7\left(\frac{d\,b^3}{7} + \frac{3\,a\,g\,b^2}{7}\right) + x^2\left(\frac{h\,a^3}{2} + \frac{3\,b\,e\,a^2}{2}\right) + x^8\left(\frac{e\,b^3}{8} + \frac{3\,a\,h\,b^2}{8}\right) + \ln(x)\,\left(f\,a^3 + 3\,b\,c\,a^2\right) - \frac{e\,a^3\,x^2 + \frac{d\,a^3\,x}{2} + \frac{c\,a^3}{3}}{x^3} + x\left(g\,a^3 + 3\,b\,d\,a^2\right) + \frac{b^3\,f\,x^9}{9} + \frac{b^3\,g\,x^{10}}{10} + \frac{b^3\,h\,x^{11}}{11} + a\,b\,x^3\,(b\,c + a\,f) + \frac{3\,a\,b\,x^4\,(b\,d + a\,g)}{4} + \frac{3\,a\,b\,x^5\,(b\,e + a\,h)}{5}$$

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x)
```

```
[Out] x^6*((b^3*c)/6 + (a*b^2*f)/2) + x^7*((b^3*d)/7 + (3*a*b^2*g)/7) + x^2*((a^3
*h)/2 + (3*a^2*b*e)/2) + x^8*((b^3*e)/8 + (3*a*b^2*h)/8) + log(x)*(a^3*f +
3*a^2*b*c) - ((a^3*c)/3 + a^3*e*x^2 + (a^3*d*x)/2)/x^3 + x*(a^3*g + 3*a^2*b
*d) + (b^3*f*x^9)/9 + (b^3*g*x^10)/10 + (b^3*h*x^11)/11 + a*b*x^3*(b*c + a*
f) + (3*a*b*x^4*(b*d + a*g))/4 + (3*a*b*x^5*(b*e + a*h))/5
```

3.402 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx$

Optimal result

Integrand size = 38, antiderivative size = 209

$$\begin{aligned}
&\int \frac{(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^5} dx \\
&= -\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}-\frac{a^2(3bc+af)}{x}+a^2(3be+ah)x+\frac{3}{2}ab(bc+af)x^2 \\
&\quad +ab(bd+ag)x^3+\frac{3}{4}ab(be+ah)x^4+\frac{1}{5}b^2(bc+3af)x^5+\frac{1}{6}b^2(bd+3ag)x^6 \\
&\quad +\frac{1}{7}b^2(be+3ah)x^7+\frac{1}{8}b^3fx^8+\frac{1}{9}b^3gx^9+\frac{1}{10}b^3hx^{10}+a^2(3bd+ag)\log(x)
\end{aligned}$$

```
[Out] -1/4*a^3*c/x^4-1/3*a^3*d/x^3-1/2*a^3*e/x^2-a^2*(a*f+3*b*c)/x+a^2*(a*h+3*b*e
)*x+3/2*a*b*(a*f+b*c)*x^2+a*b*(a*g+b*d)*x^3+3/4*a*b*(a*h+b*e)*x^4+1/5*b^2*(
3*a*f+b*c)*x^5+1/6*b^2*(3*a*g+b*d)*x^6+1/7*b^2*(3*a*h+b*e)*x^7+1/8*b^3*f*x^
8+1/9*b^3*g*x^9+1/10*b^3*h*x^10+a^2*(a*g+3*b*d)*ln(x)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1834}

$$\begin{aligned}
&\int \frac{(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^5} dx \\
&= -\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}-\frac{a^2(af+3bc)}{x}+a^2\log(x)(ag+3bd)+a^2x(ah+3be) \\
&\quad +\frac{1}{5}b^2x^5(3af+bc)+\frac{1}{6}b^2x^6(3ag+bd)+\frac{1}{7}b^2x^7(3ah+be)+\frac{3}{2}abx^2(af+bc) \\
&\quad +abx^3(ag+bd)+\frac{3}{4}abx^4(ah+be)+\frac{1}{8}b^3fx^8+\frac{1}{9}b^3gx^9+\frac{1}{10}b^3hx^{10}
\end{aligned}$$

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] -1/4*(a^3*c)/x^4 - (a^3*d)/(3*x^3) - (a^3*e)/(2*x^2) - (a^2*(3*b*c + a*f))/
x + a^2*(3*b*e + a*h)*x + (3*a*b*(b*c + a*f)*x^2)/2 + a*b*(b*d + a*g)*x^3 +
 (3*a*b*(b*e + a*h)*x^4)/4 + (b^2*(b*c + 3*a*f)*x^5)/5 + (b^2*(b*d + 3*a*g)
*x^6)/6 + (b^2*(b*e + 3*a*h)*x^7)/7 + (b^3*f*x^8)/8 + (b^3*g*x^9)/9 + (b^3*
h*x^10)/10 + a^2*(3*b*d + a*g)*Log[x]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2(3be+ah)+\frac{a^3c}{x^5}+\frac{a^3d}{x^4}+\frac{a^3e}{x^3}+\frac{a^2(3bc+af)}{x^2}+\frac{a^2(3bd+ag)}{x}\right.\\&\quad+3ab(bc+af)x+3ab(bd+ag)x^2+3ab(be+ah)x^3+b^2(bc+3af)x^4\\&\quad\left.+b^2(bd+3ag)x^5+b^2(be+3ah)x^6+b^3fx^7+b^3gx^8+b^3hx^9\right)dx\\&=-\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}-\frac{a^2(3bc+af)}{x}+a^2(3be+ah)x+\frac{3}{2}ab(bc+af)x^2\\&\quad+ab(bd+ag)x^3+\frac{3}{4}ab(be+ah)x^4+\frac{1}{5}b^2(bc+3af)x^5+\frac{1}{6}b^2(bd+3ag)x^6\\&\quad+\frac{1}{7}b^2(be+3ah)x^7+\frac{1}{8}b^3fx^8+\frac{1}{9}b^3gx^9+\frac{1}{10}b^3hx^{10}+a^2(3bd+ag)\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.81

$$\begin{aligned}&\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx\\&=\frac{-210a^3(3c+4dx+6x^2(e+2fx-2hx^3))+630a^2bx^3(-12c+x^2(12e+6fx+4gx^2+3hx^3))+18ab^2x^6(2}{}\\&\quad+a^2(3bd+ag)\log(x)\end{aligned}$$

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] (-210*a^3*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)) + 630*a^2*b*x^3*(-12*
c + x^2*(12*e + 6*f*x + 4*g*x^2 + 3*h*x^3)) + 18*a*b^2*x^6*(210*c + x*(140*
d + 105*e*x + 84*f*x^2 + 70*g*x^3 + 60*h*x^4)) + b^3*x^9*(504*c + x*(420*d
+ 360*e*x + 315*f*x^2 + 280*g*x^3 + 252*h*x^4)))/(2520*x^4) + a^2*(3*b*d +
a*g)*Log[x]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.03

method	result
default	$\frac{b^3 h\,x^{10}}{10}+\frac{b^3 g\,x^9}{9}+\frac{b^3 f\,x^8}{8}+\frac{3a\,b^2 h\,x^7}{7}+\frac{x^7 b^3 e}{7}+\frac{a\,b^2 g\,x^6}{2}+\frac{b^3 d\,x^6}{6}+\frac{3a\,b^2 f\,x^5}{5}+\frac{b^3 c\,x^5}{5}+\frac{3a^2 bh\,x^4}{4}+\frac{3a\,b^2 e}{4}$
norman	$\frac{(\frac{3}{5}a\,b^2 f+\frac{1}{5}b^3 c)x^9+(\frac{1}{2}a\,b^2 g+\frac{1}{6}b^3 d)x^{10}+(\frac{3}{7}a\,b^2 h+\frac{1}{7}b^3 e)x^{11}+(\frac{3}{4}a^2 bh+\frac{3}{4}a\,b^2 e)x^8+(\frac{3}{2}f\,a^2 b+\frac{3}{2}a\,b^2 c)x^6+(-f\,a^3-3a^2 bc)x^3+}{x^4}$
risch	$\frac{b^3 h\,x^{10}}{10}+\frac{b^3 g\,x^9}{9}+\frac{b^3 f\,x^8}{8}+\frac{3a\,b^2 h\,x^7}{7}+\frac{x^7 b^3 e}{7}+\frac{a\,b^2 g\,x^6}{2}+\frac{b^3 d\,x^6}{6}+\frac{3a\,b^2 f\,x^5}{5}+\frac{b^3 c\,x^5}{5}+\frac{3a^2 bh\,x^4}{4}+\frac{3a\,b^2 e}{4}$
parallelrisch	$315b^3 f\,x^{12}+1890a^2 bh\,x^8-840a^3 dx+1260a\,b^2 g\,x^{10}+7560\ln(x)x^4 a^2 bd-2520f\,a^3 x^3+504b^3 c\,x^9+2520a^3 h\,x^5-630c\,a^3+1080a$

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/10*b^3*h*x^10+1/9*b^3*g*x^9+1/8*b^3*f*x^8+3/7*a*b^2*h*x^7+1/7*x^7*b^3*e+1
/2*a*b^2*g*x^6+1/6*b^3*d*x^6+3/5*a*b^2*f*x^5+1/5*b^3*c*x^5+3/4*a^2*b*h*x^4+
3/4*a*b^2*e*x^4+a^2*b*g*x^3+a*b^2*d*x^3+3/2*x^2*f*a^2*b+3/2*a*b^2*c*x^2+a^3
*h*x+3*a^2*b*e*x+a^2*(a*g+3*b*d)*ln(x)-1/3*a^3*d/x^3-a^2*(a*f+3*b*c)/x-1/2*
a^3*e/x^2-1/4*a^3*c/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx = \frac{252\,b^3hx^{14}+280\,b^3gx^{13}+315\,b^3fx^{12}+360\,(b^3e+3\,ab^2h)x^{11}+420\,(b^3d+3\,ab^2g)x^{10}+504\,(b^3c+3\,ab^2f)}{}$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fri
cas")
```

```
[Out] 1/2520*(252*b^3*h*x^14 + 280*b^3*g*x^13 + 315*b^3*f*x^12 + 360*(b^3*e + 3*a
*b^2*h)*x^11 + 420*(b^3*d + 3*a*b^2*g)*x^10 + 504*(b^3*c + 3*a*b^2*f)*x^9 +
 1890*(a*b^2*e + a^2*b*h)*x^8 + 2520*(a*b^2*d + a^2*b*g)*x^7 + 3780*(a*b^2*
c + a^2*b*f)*x^6 - 1260*a^3*e*x^2 + 2520*(3*a^2*b*e + a^3*h)*x^5 + 2520*(3*
a^2*b*d + a^3*g)*x^4*log(x) - 840*a^3*d*x - 630*a^3*c - 2520*(3*a^2*b*c + a
^3*f)*x^3)/x^4
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$= a^2(ag+3bd)\log(x)+\frac{b^3fx^8}{8}+\frac{b^3gx^9}{9}+\frac{b^3hx^{10}}{10}+x^7\cdot\left(\frac{3ab^2h}{7}+\frac{b^3e}{7}\right)+x^6\left(\frac{ab^2g}{2}+\frac{b^3d}{6}\right)$$
$$+x^5\cdot\left(\frac{3ab^2f}{5}+\frac{b^3c}{5}\right)+x^4\cdot\left(\frac{3a^2bh}{4}+\frac{3ab^2e}{4}\right)+x^3\left(a^2bg+ab^2d\right)+x^2\cdot\left(\frac{3a^2bf}{2}+\frac{3ab^2c}{2}\right)$$
$$+x\left(a^3h+3a^2be\right)+\frac{-3a^3c-4a^3dx-6a^3ex^2+x^3(-12a^3f-36a^2bc)}{12x^4}$$

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] a**2*(a*g + 3*b*d)*log(x) + b**3*f*x**8/8 + b**3*g*x**9/9 + b**3*h*x**10/10
 + x**7*(3*a*b**2*h/7 + b**3*e/7) + x**6*(a*b**2*g/2 + b**3*d/6) + x**5*(3*
a*b**2*f/5 + b**3*c/5) + x**4*(3*a**2*b*h/4 + 3*a*b**2*e/4) + x**3*(a**2*b*
g + a*b**2*d) + x**2*(3*a**2*b*f/2 + 3*a*b**2*c/2) + x*(a**3*h + 3*a**2*b*e
) + (-3*a**3*c - 4*a**3*d*x - 6*a**3*e*x**2 + x**3*(-12*a**3*f - 36*a**2*b*
c))/(12*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$=\frac{1}{10}b^3hx^{10}+\frac{1}{9}b^3gx^9+\frac{1}{8}b^3fx^8+\frac{1}{7}\left(b^3e+3\,ab^2h\right)x^7$$
$$+\frac{1}{6}\left(b^3d+3\,ab^2g\right)x^6+\frac{1}{5}\left(b^3c+3\,ab^2f\right)x^5+\frac{3}{4}\left(ab^2e+a^2bh\right)x^4$$
$$+\left(ab^2d+a^2bg\right)x^3+\frac{3}{2}\left(ab^2c+a^2bf\right)x^2+\left(3\,a^2be+a^3h\right)x$$
$$+\left(3\,a^2bd+a^3g\right)\log(x)-\frac{6\,a^3ex^2+4\,a^3dx+3\,a^3c+12\,(3\,a^2bc+a^3f)x^3}{12\,x^4}$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="max
ima")
```

```
[Out] 1/10*b^3*h*x^10 + 1/9*b^3*g*x^9 + 1/8*b^3*f*x^8 + 1/7*(b^3*e + 3*a*b^2*h)*x
^7 + 1/6*(b^3*d + 3*a*b^2*g)*x^6 + 1/5*(b^3*c + 3*a*b^2*f)*x^5 + 3/4*(a*b^2
*e + a^2*b*h)*x^4 + (a*b^2*d + a^2*b*g)*x^3 + 3/2*(a*b^2*c + a^2*b*f)*x^2 +
 (3*a^2*b*e + a^3*h)*x + (3*a^2*b*d + a^3*g)*log(x) - 1/12*(6*a^3*e*x^2 + 4
*a^3*d*x + 3*a^3*c + 12*(3*a^2*b*c + a^3*f)*x^3)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$= \frac{1}{10}b^3hx^{10} + \frac{1}{9}b^3gx^9 + \frac{1}{8}b^3fx^8 + \frac{1}{7}b^3ex^7 + \frac{3}{7}ab^2hx^7 + \frac{1}{6}b^3dx^6 + \frac{1}{2}ab^2gx^6 + \frac{1}{5}b^3cx^5 + \frac{3}{5}ab^2fx^5 + \frac{3}{4}ab^2ex^4 + \frac{3}{4}a^2bhx^4 + ab^2dx^3 + a^2bgx^3 + \frac{3}{2}ab^2cx^2 + \frac{3}{2}a^2bfx^2 + 3a^2bex + a^3hx + (3a^2bd+a^3g)\log(|x|) - \frac{6a^3ex^2+4a^3dx+3a^3c+12(3a^2bc+a^3f)x^3}{12x^4}$$

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="gia
c")
```

```
[Out] 1/10*b^3*h*x^10 + 1/9*b^3*g*x^9 + 1/8*b^3*f*x^8 + 1/7*b^3*e*x^7 + 3/7*a*b^2
*h*x^7 + 1/6*b^3*d*x^6 + 1/2*a*b^2*g*x^6 + 1/5*b^3*c*x^5 + 3/5*a*b^2*f*x^5
+ 3/4*a*b^2*e*x^4 + 3/4*a^2*b*h*x^4 + a*b^2*d*x^3 + a^2*b*g*x^3 + 3/2*a*b^2
*c*x^2 + 3/2*a^2*b*f*x^2 + 3*a^2*b*e*x + a^3*h*x + (3*a^2*b*d + a^3*g)*log(
abs(x)) - 1/12*(6*a^3*e*x^2 + 4*a^3*d*x + 3*a^3*c + 12*(3*a^2*b*c + a^3*f)*
x^3)/x^4
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$= x^5\left(\frac{c\,b^3}{5}+\frac{3\,a\,f\,b^2}{5}\right) + x^6\left(\frac{d\,b^3}{6}+\frac{a\,g\,b^2}{2}\right) + x^7\left(\frac{e\,b^3}{7}+\frac{3\,a\,h\,b^2}{7}\right) + \ln(x)\,\left(g\,a^3+3\,b\,d\,a^2\right) - \frac{x^3\,(f\,a^3+3\,b\,c\,a^2)+\frac{a^3\,c}{4}+\frac{a^3\,e\,x^2}{2}+\frac{a^3\,d\,x}{3}}{x^4} + x\,(h\,a^3+3\,b\,e\,a^2) + \frac{b^3\,f\,x^8}{8} + \frac{b^3\,g\,x^9}{9} + \frac{b^3\,h\,x^{10}}{10} + \frac{3\,a\,b\,x^2\,(b\,c+a\,f)}{2} + a\,b\,x^3\,(b\,d+a\,g) + \frac{3\,a\,b\,x^4\,(b\,e+a\,h)}{4}$$

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x)
```

```
[Out] x^5*((b^3*c)/5 + (3*a*b^2*f)/5) + x^6*((b^3*d)/6 + (a*b^2*g)/2) + x^7*((b^3
*e)/7 + (3*a*b^2*h)/7) + log(x)*(a^3*g + 3*a^2*b*d) - (x^3*(a^3*f + 3*a^2*b
*c) + (a^3*c)/4 + (a^3*e*x^2)/2 + (a^3*d*x)/3)/x^4 + x*(a^3*h + 3*a^2*b*e)
+ (b^3*f*x^8)/8 + (b^3*g*x^9)/9 + (b^3*h*x^10)/10 + (3*a*b*x^2*(b*c + a*f))
/2 + a*b*x^3*(b*d + a*g) + (3*a*b*x^4*(b*e + a*h))/4
```

3.403 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 331

$$
\begin{aligned}
&\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx \\
&= -\frac{a(be-ah)x}{b^3}+\frac{(bc-af)x^2}{2b^2}+\frac{(bd-ag)x^3}{3b^2}+\frac{(be-ah)x^4}{4b^2}+\frac{fx^5}{5b}+\frac{gx^6}{6b} \\
&\quad+\frac{hx^7}{7b}+\frac{a^{2/3}\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}} \\
&\quad+\frac{a^{2/3}\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}} \\
&\quad-\frac{a^{2/3}\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}} \\
&\quad-\frac{a(bd-ag)\log\left(a+bx^3\right)}{3b^3}
\end{aligned}
$$

```
[Out] -a*(-a*h+b*e)*x/b^3+1/2*(-a*f+b*c)*x^2/b^2+1/3*(-a*g+b*d)*x^3/b^2+1/4*(-a*h
+b*e)*x^4/b^2+1/5*f*x^5/b+1/6*g*x^6/b+1/7*h*x^7/b+1/3*a^(2/3)*(b^(2/3)*(-a*
f+b*c)+a^(2/3)*(-a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/b^(10/3)-1/6*a^(2/3)*(b^(2
/3)*(-a*f+b*c)+a^(2/3)*(-a*h+b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/b^(10/3)-1/3*a*(-a*g+b*d)*ln(b*x^3+a)/b^3+1/3*a^(2/3)*(b^(5/3)*c-a^(2/3)*
b*e-a*b^(2/3)*f+a^(5/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2)
)/b^(10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1850, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$=\frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}b^{10/3}}$$
$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6b^{10/3}}$$
$$+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3b^{10/3}}-\frac{a(bd-ag)\log\left(a+bx^3\right)}{3b^3}$$
$$-\frac{ax(be-ah)}{b^3}+\frac{x^2(bc-af)}{2b^2}+\frac{x^3(bd-ag)}{3b^2}+\frac{x^4(be-ah)}{4b^2}+\frac{fx^5}{5b}+\frac{gx^6}{6b}+\frac{hx^7}{7b}$$

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] -((a*(b*e - a*h)*x)/b^3) + ((b*c - a*f)*x^2)/(2*b^2) + ((b*d - a*g)*x^3)/(3
*b^2) + ((b*e - a*h)*x^4)/(4*b^2) + (f*x^5)/(5*b) + (g*x^6)/(6*b) + (h*x^7)
/(7*b) + (a^(2/3)*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTa
n[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(10/3)) + (a^(2/3)
*(b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*b
^(10/3)) - (a^(2/3)*(b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(10/3)) - (a*(b*d - a*g)*Log[a +
b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{hx^7}{7b} + \frac{\int \frac{x^4(7bc+7bdx+7(be-ah)x^2+7bfx^3+7bgx^4)}{a+bx^3}\,dx}{7b}$$

$$= \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{\int \frac{x^4(42b^2c+42b(bd-ag)x+42b(be-ah)x^2+42b^2fx^3)}{a+bx^3}\,dx}{42b^2}$$

$$= \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{\int \frac{x^4(210b^2(bc-af)+210b^2(bd-ag)x+210b^2(be-ah)x^2)}{a+bx^3}\,dx}{210b^3}$$

$$= \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{\int \left(-210a(be-ah) + 210b(bc-af)x + 210b(bd-ag)x^2 + 210b(be-ah)x^3 + \frac{210(a^2(be-ah)-ab(b}{a-}\right.}{210b^3}$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{\int \frac{a^2(be-ah)-ab(bc-af)x-ab(bd-ag)x^2}{a+bx^3}\,dx}{b^3}$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{\int \frac{a^2(be-ah)-ab(bc-af)x}{a+bx^3}\,dx}{b^3} - \frac{(a(bd-ag))\int \frac{x^2}{a+bx^3}\,dx}{b^2}$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} - \frac{a(bd-ag)\log(a+bx^3)}{3b^3} + \frac{\int \frac{\sqrt[3]{a}\left(-a^{4/3}b(bc-af)+2a^2\sqrt[3]{b}(be-ah)\right)+\sqrt[3]{b}\left(-a^{4/3}b(bc-af)-a^2\sqrt[3]{b}(be-ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{10/3}} + \frac{\left(a^{2/3}\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^3}$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{a^{2/3}\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}} - \frac{a(bd-ag)\log\left(a+bx^3\right)}{3b^3} - \frac{\left(a\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^3} - \frac{\left(a^{2/3}\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{10/3}}$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{a^{2/3}\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}} - \frac{a^{2/3}\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}} - \frac{a(bd-ag)\log\left(a+bx^3\right)}{3b^3} - \frac{\left(a^{2/3}\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{10/3}}$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{a^{2/3}\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}} + \frac{a^{2/3}\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}} - \frac{a^{2/3}\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}} - \frac{a(bd-ag)\log\left(a+bx^3\right)}{3b^3}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.01

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$= \frac{a(-be+ah)x}{b^3}+\frac{(bc-af)x^2}{2b^2}+\frac{(bd-ag)x^3}{3b^2}+\frac{(be-ah)x^4}{4b^2}+\frac{fx^5}{5b}+\frac{gx^6}{6b}$$
$$+\frac{hx^7}{7b}+\frac{a^{2/3}\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^{10/3}}$$
$$+\frac{a^{2/3}\left(b^{5/3}c+a^{2/3}be-ab^{2/3}f-a^{5/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}}$$
$$+\frac{a^{2/3}\left(-b^{5/3}c-a^{2/3}be+ab^{2/3}f+a^{5/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}}$$
$$+\frac{a(-bd+ag)\log(a+bx^3)}{3b^3}$$

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (a*(-(b*e) + a*h)*x)/b^3 + ((b*c - a*f)*x^2)/(2*b^2) + ((b*d - a*g)*x^3)/(3
*b^2) + ((b*e - a*h)*x^4)/(4*b^2) + (f*x^5)/(5*b) + (g*x^6)/(6*b) + (h*x^7)
/(7*b) + (a^(2/3)*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(10/3)) + (a^(2/3)*(b^(5
/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/(3
*b^(10/3)) + (a^(2/3)*(-(b^(5/3)*c) - a^(2/3)*b*e + a*b^(2/3)*f + a^(5/3)*h
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(10/3)) + (a*(-(b*d)
 + a*g)*Log[a + b*x^3])/(3*b^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.50

method	result
risch	$\frac{hx^7}{7b}+\frac{gx^6}{6b}+\frac{fx^5}{5b}-\frac{ahx^4}{4b^2}+\frac{ex^4}{4b}-\frac{agx^3}{3b^2}+\frac{dx^3}{3b}-\frac{afx^2}{2b^2}+\frac{cx^2}{2b}+\frac{a^2hx}{b^3}-\frac{aex}{b^2}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(b(ag-\right.}{}\right.}{}$
default	$\frac{\frac{1}{7}b^2hx^7+\frac{1}{6}b^2gx^6+\frac{1}{5}fx^5b^2-\frac{1}{4}abhx^4+\frac{1}{4}b^2ex^4-\frac{1}{3}abgx^3+\frac{1}{3}dx^3b^2-\frac{1}{2}abfx^2+\frac{1}{2}b^2cx^2+a^2hx-abex}{b^3}-\frac{\left(\left(a^2h-aeb\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.\right.}{}$

[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 1/7*h*x^7/b+1/6*g*x^6/b+1/5*f*x^5/b-1/4/b^2*a*h*x^4+1/4/b*e*x^4-1/3/b^2*a*g*x^3+1/3*d*x^3/b-1/2/b^2*a*f*x^2+1/2*c*x^2/b+1/b^3*a^2*h*x-1/b^2*a*e*x+1/3/b^4*a*sum((b*(a*g-b*d)*_R^2+b*(a*f-b*c)*_R-a^2*h+a*e*b)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 15635, normalized size of antiderivative = 47.24

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx=\text{Too large to display}$$

[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx=\text{Timed out}$$

[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.14

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$= -\frac{\sqrt{3}\left(ab^2c\left(\frac{a}{b}\right)^{\frac{2}{3}} - a^2bf\left(\frac{a}{b}\right)^{\frac{2}{3}} - a^2be\left(\frac{a}{b}\right)^{\frac{1}{3}} + a^3h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^3} + \frac{60b^2hx^7 + 70b^2gx^6 + 84b^2fx^5 + 105(b^2e - abh)x^4 + 140(b^2d - abg)x^3 + 210(b^2c - abf)x^2 - 420(ab}{420b^3}$$
$$- \frac{\left(2ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2a^2bg\left(\frac{a}{b}\right)^{\frac{2}{3}} + ab^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^2bf\left(\frac{a}{b}\right)^{\frac{1}{3}} + a^2be - a^3h\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$- \frac{\left(ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}} - a^2bg\left(\frac{a}{b}\right)^{\frac{2}{3}} - ab^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} + a^2bf\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^2be + a^3h\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] -1/3*sqrt(3)*(a*b^2*c*(a/b)^(2/3) - a^2*b*f*(a/b)^(2/3) - a^2*b*e*(a/b)^(1/
3) + a^3*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a*b^3) + 1/420*(60*b^2*h*x^7 + 70*b^2*g*x^6 + 84*b^2*f*x^5 + 105*(b^2*e -
 a*b*h)*x^4 + 140*(b^2*d - a*b*g)*x^3 + 210*(b^2*c - a*b*f)*x^2 - 420*(a*b*
e - a^2*h)*x)/b^3 - 1/6*(2*a*b^2*d*(a/b)^(2/3) - 2*a^2*b*g*(a/b)^(2/3) + a*
b^2*c*(a/b)^(1/3) - a^2*b*f*(a/b)^(1/3) + a^2*b*e - a^3*h)*log(x^2 - x*(a/b
)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) - 1/3*(a*b^2*d*(a/b)^(2/3) - a^2*b
*g*(a/b)^(2/3) - a*b^2*c*(a/b)^(1/3) + a^2*b*f*(a/b)^(1/3) - a^2*b*e + a^3*
h)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.13

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = -\frac{(abd-a^2g)\log(|bx^3+a|)}{3\,b^3} + \frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}\,abe-(-ab^2)^{\frac{1}{3}}\,a^2h+(-ab^2)^{\frac{2}{3}}\,bc-(-ab^2)^{\frac{2}{3}}\,af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4} + \frac{\left((-ab^2)^{\frac{1}{3}}\,abe-(-ab^2)^{\frac{1}{3}}\,a^2h-(-ab^2)^{\frac{2}{3}}\,bc+(-ab^2)^{\frac{2}{3}}\,af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4} + \frac{60\,b^6hx^7+70\,b^6gx^6+84\,b^6fx^5+105\,b^6ex^4-105\,ab^5hx^4+140\,b^6dx^3-140\,ab^5gx^3+210\,b^6cx^2-210\,ab^5}{420\,b^7} + \frac{\left(ab^{14}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^{13}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^{13}e+a^3b^{12}h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{15}}$$

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] -1/3*(a*b*d - a^2*g)*log(abs(b*x^3 + a))/b^3 + 1/3*sqrt(3)*((-a*b^2)^(1/3)*
a*b*e - (-a*b^2)^(1/3)*a^2*h + (-a*b^2)^(2/3)*b*c - (-a*b^2)^(2/3)*a*f)*arc
tan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/6*((-a*b^2)^(1/3
)*a*b*e - (-a*b^2)^(1/3)*a^2*h - (-a*b^2)^(2/3)*b*c + (-a*b^2)^(2/3)*a*f)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/420*(60*b^6*h*x^7 + 70*b^6*
g*x^6 + 84*b^6*f*x^5 + 105*b^6*e*x^4 - 105*a*b^5*h*x^4 + 140*b^6*d*x^3 - 14
0*a*b^5*g*x^3 + 210*b^6*c*x^2 - 210*a*b^5*f*x^2 - 420*a*b^5*e*x + 420*a^2*b
^4*h*x)/b^7 + 1/3*(a*b^14*c*(-a/b)^(1/3) - a^2*b^13*f*(-a/b)^(1/3) - a^2*b^
13*e + a^3*b^12*h)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^15)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 1271, normalized size of antiderivative = 3.84

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$= x^2\left(\frac{c}{2\,b}-\frac{a\,f}{2\,b^2}\right)+x^3\left(\frac{d}{3\,b}-\frac{a\,g}{3\,b^2}\right)+x^4\left(\frac{e}{4\,b}-\frac{a\,h}{4\,b^2}\right) + \left(\sum_{k=1}^{3}\ln\left(\mathrm{root}(27\,b^{10}\,z^3+27\,a\,b^8\,d\,z^2-27\,a^2\,b^7\,g\,z^2-9\,a^4\,b^4\,f\,h\,z-18\,a^3\,b^5\,d\,g\,z+9\,a^3\,b^5\,e\,f\,z+9\,a^3\,b^5\,c\,h\,z \right.\right.$$
$$+\frac{a^5\,g^2+a^3\,b^2\,d^2-a^5\,f\,h+a^4\,b\,c\,h-2\,a^4\,b\,d\,g+a^4\,b\,e\,f-a^3\,b^2\,c\,e}{b^4}$$
$$\left.+\frac{x\,(a^4\,b\,f^2+a^2\,b^3\,c^2+a^5\,g\,h-a^4\,b\,d\,h-a^4\,b\,e\,g-2\,a^3\,b^2\,c\,f+a^3\,b^2\,d\,e)}{b^4}\right)\mathrm{root}(27\,b^{10}\,z^3$$
$$+27\,a\,b^8\,d\,z^2-27\,a^2\,b^7\,g\,z^2-9\,a^4\,b^4\,f\,h\,z-18\,a^3\,b^5\,d\,g\,z+9\,a^3\,b^5\,e\,f\,z+9\,a^3\,b^5\,c\,h\,z$$
$$-9\,a^2\,b^6\,c\,e\,z+9\,a^4\,b^4\,g^2\,z+9\,a^2\,b^6\,d^2\,z+3\,a^6\,b\,f\,g\,h-3\,a^5\,b^2\,e\,f\,g-3\,a^5\,b^2\,d\,f\,h$$
$$-3\,a^5\,b^2\,c\,g\,h+3\,a^4\,b^3\,d\,e\,f+3\,a^4\,b^3\,c\,e\,g+3\,a^4\,b^3\,c\,d\,h-3\,a^3\,b^4\,c\,d\,e-3\,a^6\,b\,e\,h^2$$
$$+3\,a^5\,b^2\,e^2\,h+3\,a^5\,b^2\,d\,g^2-3\,a^4\,b^3\,d^2\,g-3\,a^4\,b^3\,c\,f^2+3\,a^3\,b^4\,c^2\,f+a^5\,b^2\,f^3+a^3\,b^4\,d^3$$
$$\left.+a^7\,h^3-a^4\,b^3\,e^3-a^2\,b^5\,c^3-a^6\,b\,g^3,z,k)\right)+\frac{f\,x^5}{5\,b}+\frac{g\,x^6}{6\,b}+\frac{h\,x^7}{7\,b}-\frac{a\,x\,\left(\frac{e}{b}-\frac{a\,h}{b^2}\right)}{b}$$

```
[In] int((x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

```
[Out] x^2*(c/(2*b) - (a*f)/(2*b^2)) + x^3*(d/(3*b) - (a*g)/(3*b^2)) + x^4*(e/(4*b
) - (a*h)/(4*b^2)) + symsum(log(root(27*b^10*z^3 + 27*a*b^8*d*z^2 - 27*a^2*
b^7*g*z^2 - 9*a^4*b^4*f*h*z - 18*a^3*b^5*d*g*z + 9*a^3*b^5*e*f*z + 9*a^3*b^
5*c*h*z - 9*a^2*b^6*c*e*z + 9*a^4*b^4*g^2*z + 9*a^2*b^6*d^2*z + 3*a^6*b*f*g
*h - 3*a^5*b^2*e*f*g - 3*a^5*b^2*d*f*h - 3*a^5*b^2*c*g*h + 3*a^4*b^3*d*e*f
+ 3*a^4*b^3*c*e*g + 3*a^4*b^3*c*d*h - 3*a^3*b^4*c*d*e - 3*a^6*b*e*h^2 + 3*a
^5*b^2*e^2*h + 3*a^5*b^2*d*g^2 - 3*a^4*b^3*d^2*g - 3*a^4*b^3*c*f^2 + 3*a^3*
b^4*c^2*f + a^5*b^2*f^3 + a^3*b^4*d^3 + a^7*h^3 - a^4*b^3*e^3 - a^2*b^5*c^3
 - a^6*b*g^3, z, k)*((6*a^2*b^4*d - 6*a^3*b^3*g)/b^4 + (x*(3*a^2*b^4*e - 3*
a^3*b^3*h))/b^4 + 9*root(27*b^10*z^3 + 27*a*b^8*d*z^2 - 27*a^2*b^7*g*z^2 -
9*a^4*b^4*f*h*z - 18*a^3*b^5*d*g*z + 9*a^3*b^5*e*f*z + 9*a^3*b^5*c*h*z - 9*
a^2*b^6*c*e*z + 9*a^4*b^4*g^2*z + 9*a^2*b^6*d^2*z + 3*a^6*b*f*g*h - 3*a^5*b
^2*e*f*g - 3*a^5*b^2*d*f*h - 3*a^5*b^2*c*g*h + 3*a^4*b^3*d*e*f + 3*a^4*b^3*
c*e*g + 3*a^4*b^3*c*d*h - 3*a^3*b^4*c*d*e - 3*a^6*b*e*h^2 + 3*a^5*b^2*e^2*h
 + 3*a^5*b^2*d*g^2 - 3*a^4*b^3*d^2*g - 3*a^4*b^3*c*f^2 + 3*a^3*b^4*c^2*f +
a^5*b^2*f^3 + a^3*b^4*d^3 + a^7*h^3 - a^4*b^3*e^3 - a^2*b^5*c^3 - a^6*b*g^3
, z, k)*a*b^2) + (a^5*g^2 + a^3*b^2*d^2 - a^5*f*h + a^4*b*c*h - 2*a^4*b*d*g
 + a^4*b*e*f - a^3*b^2*c*e)/b^4 + (x*(a^4*b*f^2 + a^2*b^3*c^2 + a^5*g*h - a
^4*b*d*h - a^4*b*e*g - 2*a^3*b^2*c*f + a^3*b^2*d*e))/b^4)*root(27*b^10*z^3
+ 27*a*b^8*d*z^2 - 27*a^2*b^7*g*z^2 - 9*a^4*b^4*f*h*z - 18*a^3*b^5*d*g*z +
```

```
9*a^3*b^5*e*f*z + 9*a^3*b^5*c*h*z - 9*a^2*b^6*c*e*z + 9*a^4*b^4*g^2*z + 9*a
^2*b^6*d^2*z + 3*a^6*b*f*g*h - 3*a^5*b^2*e*f*g - 3*a^5*b^2*d*f*h - 3*a^5*b^
2*c*g*h + 3*a^4*b^3*d*e*f + 3*a^4*b^3*c*e*g + 3*a^4*b^3*c*d*h - 3*a^3*b^4*c
*d*e - 3*a^6*b*e*h^2 + 3*a^5*b^2*e^2*h + 3*a^5*b^2*d*g^2 - 3*a^4*b^3*d^2*g
- 3*a^4*b^3*c*f^2 + 3*a^3*b^4*c^2*f + a^5*b^2*f^3 + a^3*b^4*d^3 + a^7*h^3 -
 a^4*b^3*e^3 - a^2*b^5*c^3 - a^6*b*g^3, z, k), k, 1, 3) + (f*x^5)/(5*b) + (
g*x^6)/(6*b) + (h*x^7)/(7*b) - (a*x*(e/b - (a*h)/b^2))/b
```

3.404 $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} dx$

Optimal result

Integrand size = 38, antiderivative size = 313

$$\begin{aligned}
&\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx \\
&= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} \\
&\quad + \frac{\sqrt[3]{a}\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}} \\
&\quad - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} \\
&\quad + \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}} \\
&\quad - \frac{a(be-ah)\log(a+bx^3)}{3b^3}
\end{aligned}$$

```
[Out] (-a*f+b*c)*x/b^2+1/2*(-a*g+b*d)*x^2/b^2+1/3*(-a*h+b*e)*x^3/b^2+1/4*f*x^4/b+
1/5*g*x^5/b+1/6*h*x^6/b-1/3*a^(1/3)*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*g+b*d))
*ln(a^(1/3)+b^(1/3)*x)/b^(8/3)+1/6*a^(1/3)*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*
g+b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(8/3)-1/3*a*(-a*h+b*e)*
ln(b*x^3+a)/b^3+1/3*a^(1/3)*(b^(4/3)*c+a^(1/3)*b*d-a*b^(1/3)*f-a^(4/3)*g)*a
rctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1850, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$= \frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}b^{8/3}}$$
$$+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{6b^{8/3}}$$
$$-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3b^{8/3}}-\frac{a(be-ah)\log\left(a+bx^3\right)}{3b^3}$$
$$+\frac{x(bc-af)}{b^2}+\frac{x^2(bd-ag)}{2b^2}+\frac{x^3(be-ah)}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}$$

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] ((b*c - a*f)*x)/b^2 + ((b*d - a*g)*x^2)/(2*b^2) + ((b*e - a*h)*x^3)/(3*b^2)
 + (f*x^4)/(4*b) + (g*x^5)/(5*b) + (h*x^6)/(6*b) + (a^(1/3)*(b^(4/3)*c + a^
(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*b^(8/3)) - (a^(1/3)*(b^(1/3)*(b*c - a*f) - a^(1/3)*(b
*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(3*b^(8/3)) + (a^(1/3)*(b^(1/3)*(b*c -
 a*f) - a^(1/3)*(b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(8/3)) - (a*(b*e - a*h)*Log[a + b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{hx^6}{6b} + \frac{\int \frac{x^3(6bc+6bdx+6(be-ah)x^2+6bfx^3+6bgx^4)}{a+bx^3}\,dx}{6b}$$

$$= \frac{gx^5}{5b} + \frac{hx^6}{6b} + \frac{\int \frac{x^3(30b^2c+30b(bd-ag)x+30b(be-ah)x^2+30b^2fx^3)}{a+bx^3}\,dx}{30b^2}$$

$$= \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} + \frac{\int \frac{x^3(120b^2(bc-af)+120b^2(bd-ag)x+120b^2(be-ah)x^2)}{a+bx^3}\,dx}{120b^3}$$

$$= \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} + \frac{\int \left(120b(bc-af)+120b(bd-ag)x+120b(be-ah)x^2-\frac{120(ab(bc-af)+ab(bd-ag)x+ab(be-ah)x^2)}{a+bx^3}\right)dx}{120b^3}$$

$$= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \frac{\int \frac{ab(bc-af)+ab(bd-ag)x+ab(be-ah)x^2}{a+bx^3}\,dx}{b^3}$$

$$= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \frac{\int \frac{ab(bc-af)+ab(bd-ag)x}{a+bx^3}\,dx}{b^3} - \frac{(a(be-ah))\int \frac{x^2}{a+bx^3}\,dx}{b^2}$$

$$= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \frac{a(be-ah)\log(a+bx^3)}{3b^3} - \frac{\int \frac{\sqrt[3]{a}(2ab^{4/3}(bc-af)+a^{4/3}b(bd-ag))+\sqrt[3]{b}(-ab^{4/3}(bc-af)+a^{4/3}b(bd-ag))x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{10/3}} - \frac{\left(\sqrt[3]{a}\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{7/3}}$$

$$= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bc-af) - \sqrt[3]{a}(bd-ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} - \frac{a(be-ah)\log(a+bx^3)}{3b^3} - \frac{\left(a^{2/3}\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{2b^{7/3}} + \frac{\left(\sqrt[3]{a}\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{6b^{8/3}}$$

$$= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bc-af) - \sqrt[3]{a}(bd-ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} + \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bc-af) - \sqrt[3]{a}(bd-ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}} - \frac{a(be-ah)\log(a+bx^3)}{3b^3} - \frac{\left(\sqrt[3]{a}\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{8/3}}$$

$$= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} + \frac{\sqrt[3]{a}\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bc-af) - \sqrt[3]{a}(bd-ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} + \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bc-af) - \sqrt[3]{a}(bd-ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}} - \frac{a(be-ah)\log(a+bx^3)}{3b^3}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.96

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$

$$= \frac{60b(bc-af)x+30b(bd-ag)x^2+20b(be-ah)x^3+15b^2fx^4+12b^2gx^5+10b^2hx^6-20\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(-b^{4/3}c\right.}{}$$

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (60*b*(b*c - a*f)*x + 30*b*(b*d - a*g)*x^2 + 20*b*(b*e - a*h)*x^3 + 15*b^2*
f*x^4 + 12*b^2*g*x^5 + 10*b^2*h*x^6 - 20*Sqrt[3]*a^(1/3)*b^(1/3)*(-(b^(4/3)
*c) - a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]] - 20*a^(1/3)*b^(1/3)*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f +
 a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x] + 10*a^(1/3)*b^(1/3)*(b^(4/3)*c - a^(1
/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2] + 20*a*(-(b*e) + a*h)*Log[a + b*x^3])/(60*b^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.44

method	result
risch	$\frac{h\,x^{6}}{6b}+\frac{g\,x^{5}}{5b}+\frac{f\,x^{4}}{4b}-\frac{ah\,x^{3}}{3b^{2}}+\frac{e\,x^{3}}{3b}-\frac{ag\,x^{2}}{2b^{2}}+\frac{d\,x^{2}}{2b}-\frac{afx}{b^{2}}+\frac{cx}{b}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left((ah-be)\_R^{2}+(ag-bd)\_R\right.}{\_R^{2}}\right.}{3b^{3}}$
default	$-\frac{-\frac{1}{6}bh\,x^{6}-\frac{1}{5}bg\,x^{5}-\frac{1}{4}bf\,x^{4}+\frac{1}{3}ah\,x^{3}-\frac{1}{3}be\,x^{3}+\frac{1}{2}ag\,x^{2}-\frac{1}{2}bd\,x^{2}+afx-bcx}{b^{2}}+\frac{\left((af-bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\right.\right.}{}$

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*h*x^6/b+1/5*g*x^5/b+1/4*f*x^4/b-1/3/b^2*a*h*x^3+1/3*e*x^3/b-1/2/b^2*a*g
*x^2+1/2*d*x^2/b-1/b^2*a*f*x+c*x/b+1/3/b^3*a*sum(((a*h-b*e)*_R^2+(a*g-b*d)*
```

```
_R+a*f-b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 15451, normalized size of antiderivative = 49.36

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.06

$$\begin{aligned}
&\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx \\
&= \frac{10\,bhx^6+12\,bgx^5+15\,bfx^4+20\,(be-ah)x^3+30\,(bd-ag)x^2+60\,(bc-af)x}{60\,b^2} \\
&\quad - \frac{\sqrt{3}\left(ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2bg\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2bf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3} \\
&\quad - \frac{\left(2\,abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+abd\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}-abc+a^2f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\
&\quad - \frac{\left(abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}+abc-a^2f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}
\end{aligned}$$

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] 1/60*(10*b*h*x^6 + 12*b*g*x^5 + 15*b*f*x^4 + 20*(b*e - a*h)*x^3 + 30*(b*d -
 a*g)*x^2 + 60*(b*c - a*f)*x)/b^2 - 1/3*sqrt(3)*(a*b^2*d*(a/b)^(2/3) - a^2*
b*g*(a/b)^(2/3) + a*b^2*c*(a/b)^(1/3) - a^2*b*f*(a/b)^(1/3))*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) - 1/6*(2*a*b*e*(a/b)^(2/3) -
2*a^2*h*(a/b)^(2/3) + a*b*d*(a/b)^(1/3) - a^2*g*(a/b)^(1/3) - a*b*c + a^2*f
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 1/3*(a*b*e*(a/
b)^(2/3) - a^2*h*(a/b)^(2/3) - a*b*d*(a/b)^(1/3) + a^2*g*(a/b)^(1/3) + a*b*
c - a^2*f)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.12

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = -\frac{(abe-a^2h)\log(|bx^3+a|)}{3\,b^3} - \frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}\,b^2c-(-ab^2)^{\frac{1}{3}}\,abf-(-ab^2)^{\frac{2}{3}}\,bd+(-ab^2)^{\frac{2}{3}}\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4} - \frac{\left((-ab^2)^{\frac{1}{3}}\,b^2c-(-ab^2)^{\frac{1}{3}}\,abf+(-ab^2)^{\frac{2}{3}}\,bd-(-ab^2)^{\frac{2}{3}}\,ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4} + \frac{10\,b^5hx^6+12\,b^5gx^5+15\,b^5fx^4+20\,b^5ex^3-20\,ab^4hx^3+30\,b^5dx^2-30\,ab^4gx^2+60\,b^5cx-60\,ab^4fx}{60\,b^6} + \frac{\left(ab^{12}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^{11}g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^{12}c-a^2b^{11}f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{13}}$$

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] -1/3*(a*b*e - a^2*h)*log(abs(b*x^3 + a))/b^3 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*
b^2*c - (-a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d + (-a*b^2)^(2/3)*a*g)*arc
tan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3
)*b^2*c - (-a*b^2)^(1/3)*a*b*f + (-a*b^2)^(2/3)*b*d - (-a*b^2)^(2/3)*a*g)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/60*(10*b^5*h*x^6 + 12*b^5*g
*x^5 + 15*b^5*f*x^4 + 20*b^5*e*x^3 - 20*a*b^4*h*x^3 + 30*b^5*d*x^2 - 30*a*b
^4*g*x^2 + 60*b^5*c*x - 60*a*b^4*f*x)/b^6 + 1/3*(a*b^12*d*(-a/b)^(1/3) - a^
2*b^11*g*(-a/b)^(1/3) + a*b^12*c - a^2*b^11*f)*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/(a*b^13)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1236, normalized size of antiderivative = 3.95

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = x^2\left(\frac{d}{2\,b}-\frac{a\,g}{2\,b^2}\right)+x^3\left(\frac{e}{3\,b}-\frac{a\,h}{3\,b^2}\right)$$
$$+\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\big(27\,b^9\,z^3+27\,a\,b^7\,e\,z^2-27\,a^2\,b^6\,h\,z^2+9\,a\,b^6\,c\,d\,z-18\,a^3\,b^4\,e\,h\,z+9\,a^3\,b^4\,f\,g\,z-9\,a^2\,b^5\,d\,f\,z-\right.\right.$$
$$+\frac{a^5\,h^2+a^3\,b^2\,e^2-2\,a^4\,b\,e\,h+a^4\,b\,f\,g+a^2\,b^3\,c\,d-a^3\,b^2\,c\,g-a^3\,b^2\,d\,f}{b^4}$$
$$\left.+\frac{x\,(a^4\,g^2+a^2\,b^2\,d^2-a^4\,f\,h+a^3\,b\,c\,h-2\,a^3\,b\,d\,g+a^3\,b\,e\,f-a^2\,b^2\,c\,e)}{b^3}\right)\mathrm{root}\big(27\,b^9\,z^3$$
$$+27\,a\,b^7\,e\,z^2-27\,a^2\,b^6\,h\,z^2+9\,a\,b^6\,c\,d\,z-18\,a^3\,b^4\,e\,h\,z+9\,a^3\,b^4\,f\,g\,z-9\,a^2\,b^5\,d\,f\,z$$
$$-9\,a^2\,b^5\,c\,g\,z+9\,a^4\,b^3\,h^2\,z+9\,a^2\,b^5\,e^2\,z-3\,a^5\,b\,f\,g\,h+3\,a^4\,b^2\,e\,f\,g+3\,a^4\,b^2\,d\,f\,h$$
$$+3\,a^4\,b^2\,c\,g\,h-3\,a^3\,b^3\,d\,e\,f-3\,a^3\,b^3\,c\,e\,g-3\,a^3\,b^3\,c\,d\,h+3\,a^2\,b^4\,c\,d\,e+3\,a^5\,b\,e\,h^2$$
$$-3\,a^4\,b^2\,e^2\,h-3\,a^4\,b^2\,d\,g^2+3\,a^3\,b^3\,d^2\,g+3\,a^3\,b^3\,c\,f^2-3\,a^2\,b^4\,c^2\,f+a^3\,b^3\,e^3+a^5\,b\,g^3$$
$$\left.+a\,b^5\,c^3-a^4\,b^2\,f^3-a^2\,b^4\,d^3-a^6\,h^3,z,k\big)\right)+x\left(\frac{c}{b}-\frac{a\,f}{b^2}\right)+\frac{f\,x^4}{4\,b}+\frac{g\,x^5}{5\,b}+\frac{h\,x^6}{6\,b}$$

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

```
[Out] x^2*(d/(2*b) - (a*g)/(2*b^2)) + x^3*(e/(3*b) - (a*h)/(3*b^2)) + symsum(log(
root(27*b^9*z^3 + 27*a*b^7*e*z^2 - 27*a^2*b^6*h*z^2 + 9*a*b^6*c*d*z - 18*a^
3*b^4*e*h*z + 9*a^3*b^4*f*g*z - 9*a^2*b^5*d*f*z - 9*a^2*b^5*c*g*z + 9*a^4*b
^3*h^2*z + 9*a^2*b^5*e^2*z - 3*a^5*b*f*g*h + 3*a^4*b^2*e*f*g + 3*a^4*b^2*d*
f*h + 3*a^4*b^2*c*g*h - 3*a^3*b^3*d*e*f - 3*a^3*b^3*c*e*g - 3*a^3*b^3*c*d*h
 + 3*a^2*b^4*c*d*e + 3*a^5*b*e*h^2 - 3*a^4*b^2*e^2*h - 3*a^4*b^2*d*g^2 + 3*
a^3*b^3*d^2*g + 3*a^3*b^3*c*f^2 - 3*a^2*b^4*c^2*f + a^3*b^3*e^3 + a^5*b*g^3
 + a*b^5*c^3 - a^4*b^2*f^3 - a^2*b^4*d^3 - a^6*h^3, z, k)*((6*a^2*b^4*e - 6
*a^3*b^3*h)/b^4 + (x*(3*a^2*b^3*f - 3*a*b^4*c))/b^3 + 9*root(27*b^9*z^3 + 2
7*a*b^7*e*z^2 - 27*a^2*b^6*h*z^2 + 9*a*b^6*c*d*z - 18*a^3*b^4*e*h*z + 9*a^3
*b^4*f*g*z - 9*a^2*b^5*d*f*z - 9*a^2*b^5*c*g*z + 9*a^4*b^3*h^2*z + 9*a^2*b^
5*e^2*z - 3*a^5*b*f*g*h + 3*a^4*b^2*e*f*g + 3*a^4*b^2*d*f*h + 3*a^4*b^2*c*g
*h - 3*a^3*b^3*d*e*f - 3*a^3*b^3*c*e*g - 3*a^3*b^3*c*d*h + 3*a^2*b^4*c*d*e
+ 3*a^5*b*e*h^2 - 3*a^4*b^2*e^2*h - 3*a^4*b^2*d*g^2 + 3*a^3*b^3*d^2*g + 3*a
^3*b^3*c*f^2 - 3*a^2*b^4*c^2*f + a^3*b^3*e^3 + a^5*b*g^3 + a*b^5*c^3 - a^4*
b^2*f^3 - a^2*b^4*d^3 - a^6*h^3, z, k)*a*b^2) + (a^5*h^2 + a^3*b^2*e^2 - 2*
a^4*b*e*h + a^4*b*f*g + a^2*b^3*c*d - a^3*b^2*c*g - a^3*b^2*d*f)/b^4 + (x*(
a^4*g^2 + a^2*b^2*d^2 - a^4*f*h + a^3*b*c*h - 2*a^3*b*d*g + a^3*b*e*f - a^2
*b^2*c*e))/b^3)*root(27*b^9*z^3 + 27*a*b^7*e*z^2 - 27*a^2*b^6*h*z^2 + 9*a*b
^6*c*d*z - 18*a^3*b^4*e*h*z + 9*a^3*b^4*f*g*z - 9*a^2*b^5*d*f*z - 9*a^2*b^5
*c*g*z + 9*a^4*b^3*h^2*z + 9*a^2*b^5*e^2*z - 3*a^5*b*f*g*h + 3*a^4*b^2*e*f*
g + 3*a^4*b^2*d*f*h + 3*a^4*b^2*c*g*h - 3*a^3*b^3*d*e*f - 3*a^3*b^3*c*e*g -
```

```
 3*a^3*b^3*c*d*h + 3*a^2*b^4*c*d*e + 3*a^5*b*e*h^2 - 3*a^4*b^2*e^2*h - 3*a^
4*b^2*d*g^2 + 3*a^3*b^3*d^2*g + 3*a^3*b^3*c*f^2 - 3*a^2*b^4*c^2*f + a^3*b^3
*e^3 + a^5*b*g^3 + a*b^5*c^3 - a^4*b^2*f^3 - a^2*b^4*d^3 - a^6*h^3, z, k),
k, 1, 3) + x*(c/b - (a*f)/b^2) + (f*x^4)/(4*b) + (g*x^5)/(5*b) + (h*x^6)/(6
*b)
```

3.405 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 294

$$\begin{aligned}
&\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx \\
&= \frac{(bd-ag)x}{b^2}+\frac{(be-ah)x^2}{2b^2}+\frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b} \\
&\quad+\frac{\sqrt[3]{a}\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}} \\
&\quad-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} \\
&\quad+\frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}} \\
&\quad+\frac{(bc-af)\log\left(a+bx^3\right)}{3b^2}
\end{aligned}$$

```
[Out] (-a*g+b*d)*x/b^2+1/2*(-a*h+b*e)*x^2/b^2+1/3*f*x^3/b+1/4*g*x^4/b+1/5*h*x^5/b
-1/3*a^(1/3)*(b^(1/3)*(-a*g+b*d)-a^(1/3)*(-a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/
b^(8/3)+1/6*a^(1/3)*(b^(1/3)*(-a*g+b*d)-a^(1/3)*(-a*h+b*e))*ln(a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(8/3)+1/3*(-a*f+b*c)*ln(b*x^3+a)/b^2+1/3*a^(1/
3)*(b^(4/3)*d+a^(1/3)*b*e-a*b^(1/3)*g-a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)/a^(1/3)*3^(1/2))/b^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1850, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$= \frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}b^{8/3}}$$
$$+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{6b^{8/3}}$$
$$-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3b^{8/3}}$$
$$+\frac{(bc-af)\log(a+bx^3)}{3b^2}+\frac{x(bd-ag)}{b^2}+\frac{x^2(be-ah)}{2b^2}+\frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b}$$

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] ((b*d - a*g)*x)/b^2 + ((b*e - a*h)*x^2)/(2*b^2) + (f*x^3)/(3*b) + (g*x^4)/(
4*b) + (h*x^5)/(5*b) + (a^(1/3)*(b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g - a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3
)) - (a^(1/3)*(b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(
1/3)*x])/(3*b^(8/3)) + (a^(1/3)*(b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(8/3)) + ((b*c - a*f)
*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{hx^5}{5b} + \frac{\int \frac{x^2\left(5bc+5bdx+5(be-ah)x^2+5bfx^3+5bgx^4\right)}{a+bx^3}\,dx}{5b} \\
&= \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\int \frac{x^2\left(20b^2c+20b(bd-ag)x+20b(be-ah)x^2+20b^2fx^3\right)}{a+bx^3}\,dx}{20b^2} \\
&= \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\int \frac{x^2\left(60b^2(bc-af)+60b^2(bd-ag)x+60b^2(be-ah)x^2\right)}{a+bx^3}\,dx}{60b^3} \\
&= \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\int \left(60b(bd-ag)+60b(be-ah)x-\frac{60\left(ab(bd-ag)+ab(be-ah)x-b^2(bc-af)x^2\right)}{a+bx^3}\right)dx}{60b^3} \\
&= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} - \frac{\int \frac{ab(bd-ag)+ab(be-ah)x-b^2(bc-af)x^2}{a+bx^3}\,dx}{b^3} \\
&= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} \\
&\quad - \frac{\int \frac{ab(bd-ag)+ab(be-ah)x}{a+bx^3}\,dx}{b^3} + \frac{(bc-af)\int \frac{x^2}{a+bx^3}\,dx}{b} \\
&= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{(bc-af)\log\left(a+bx^3\right)}{3b^2} \\
&\quad - \frac{\int \frac{\sqrt[3]{a}\left(2ab^{4/3}(bd-ag)+a^{4/3}b(be-ah)\right)+\sqrt[3]{b}\left(-ab^{4/3}(bd-ag)+a^{4/3}b(be-ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{10/3}} \\
&\quad - \frac{\left(\sqrt[3]{a}\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{7/3}} \\
&= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} \\
&\quad - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} + \frac{(bc-af)\log\left(a+bx^3\right)}{3b^2} \\
&\quad - \frac{\left(a^{2/3}\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^{7/3}} \\
&\quad + \frac{\left(\sqrt[3]{a}\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{8/3}}
\end{aligned}$$

$$= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bd-ag) - \sqrt[3]{a}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} + \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bd-ag) - \sqrt[3]{a}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}} + \frac{(bc-af)\log(a+bx^3)}{3b^2} - \frac{\left(\sqrt[3]{a}\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{8/3}}$$

$$= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\sqrt[3]{a}\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bd-ag) - \sqrt[3]{a}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} + \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bd-ag) - \sqrt[3]{a}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}} + \frac{(bc-af)\log(a+bx^3)}{3b^2}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.99

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$

$$= \frac{60b^{2/3}(bd-ag)x + 30b^{2/3}(be-ah)x^2 + 20b^{5/3}fx^3 + 15b^{5/3}gx^4 + 12b^{5/3}hx^5 - 20\sqrt{3}\sqrt[3]{a}\left(-b^{4/3}d - \sqrt[3]{a}be -\right.}{}$$

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
[Out] (60*b^(2/3)*(b*d - a*g)*x + 30*b^(2/3)*(b*e - a*h)*x^2 + 20*b^(5/3)*f*x^3 +
 15*b^(5/3)*g*x^4 + 12*b^(5/3)*h*x^5 - 20*Sqrt[3]*a^(1/3)*(-(b^(4/3)*d) - a
^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]] + 20*a^(1/3)*(-(b^(4/3)*d) + a^(1/3)*b*e + a*b^(1/3)*g - a^(4/3)*h)*
Log[a^(1/3) + b^(1/3)*x] + 10*a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*
g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 20*b^(2/3)*
(b*c - a*f)*Log[a + b*x^3])/(60*b^(8/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.42

method	result
risch	$\frac{h x^5}{5b}+\frac{g x^4}{4b}+\frac{f x^3}{3b}-\frac{a h x^2}{2b^2}+\frac{e x^2}{2b}-\frac{a g x}{b^2}+\frac{d x}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(b(-af+bc)\_R^2+a(ah-be)\_R+a^2g-abd\right)\ln\left(x-\right.}{\_R^2}}{3b^3}$
default	$-\frac{-\frac{1}{5}bh\,x^5-\frac{1}{4}bg\,x^4-\frac{1}{3}f\,x^3b+\frac{1}{2}ah\,x^2-\frac{1}{2}be\,x^2+agx-bdx}{b^2}+\frac{(a^2g-abd)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-\right.}{\left(\right.}\right.}{3b\left(\frac{a}{b}\right)^{\frac{}{3}}}\right.}{}$

```
[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*h*x^5/b+1/4*g*x^4/b+1/3*f*x^3/b-1/2/b^2*a*h*x^2+1/2*e*x^2/b-1/b^2*a*g*x
+d*x/b+1/3/b^3*sum((b*(-a*f+b*c)*_R^2+a*(a*h-b*e)*_R+a^2*g-a*b*d)/_R^2*ln(x
-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.30 (sec) , antiderivative size = 14746, normalized size of antiderivative = 50.16

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.06

$$\begin{aligned}
&\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx \\
&= -\frac{\sqrt{3}\left(abe\left(\frac{a}{b}\right)^{\frac{2}{3}} - a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}} + abd\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} \\
&\quad + \frac{12\,bhx^5 + 15\,bgx^4 + 20\,bfx^3 + 30\,(be-ah)x^2 + 60\,(bd-ag)x}{60\,b^2} \\
&\quad + \frac{\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}} - abe\left(\frac{a}{b}\right)^{\frac{1}{3}} + a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}} + abd - a^2g\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\
&\quad + \frac{\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}} - abf\left(\frac{a}{b}\right)^{\frac{2}{3}} + abe\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}} - abd + a^2g\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}
\end{aligned}$$

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxim
a")
[Out] -1/3*sqrt(3)*(a*b*e*(a/b)^(2/3) - a^2*h*(a/b)^(2/3) + a*b*d*(a/b)^(1/3) - a
^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^
2) + 1/60*(12*b*h*x^5 + 15*b*g*x^4 + 20*b*f*x^3 + 30*(b*e - a*h)*x^2 + 60*(
b*d - a*g)*x)/b^2 + 1/6*(2*b^2*c*(a/b)^(2/3) - 2*a*b*f*(a/b)^(2/3) - a*b*e*
(a/b)^(1/3) + a^2*h*(a/b)^(1/3) + a*b*d - a^2*g)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(b^2*c*(a/b)^(2/3) - a*b*f*(a/b)^(2/3)
 + a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3) - a*b*d + a^2*g)*log(x + (a/b)^(1/
3))/(b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.12

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \frac{(bc-af)\log(|bx^3+a|)}{3\,b^2} - \frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}\,b^2d-(-ab^2)^{\frac{1}{3}}\,abg-(-ab^2)^{\frac{2}{3}}\,be+(-ab^2)^{\frac{2}{3}}\,ah\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4} - \frac{\left((-ab^2)^{\frac{1}{3}}\,b^2d-(-ab^2)^{\frac{1}{3}}\,abg+(-ab^2)^{\frac{2}{3}}\,be-(-ab^2)^{\frac{2}{3}}\,ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4} + \frac{12\,b^4hx^5+15\,b^4gx^4+20\,b^4fx^3+30\,b^4ex^2-30\,ab^3hx^2+60\,b^4dx-60\,ab^3gx}{60\,b^5} + \frac{\left(ab^{10}e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^9h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^{10}d-a^2b^9g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{11}}$$

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac"
)

[Out] 1/3*(b*c - a*f)*log(abs(b*x^3 + a))/b^2 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*d
 - (-a*b^2)^(1/3)*a*b*g - (-a*b^2)^(2/3)*b*e + (-a*b^2)^(2/3)*a*h)*arctan(1
/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)*b^2
*d - (-a*b^2)^(1/3)*a*b*g + (-a*b^2)^(2/3)*b*e - (-a*b^2)^(2/3)*a*h)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/60*(12*b^4*h*x^5 + 15*b^4*g*x^4
+ 20*b^4*f*x^3 + 30*b^4*e*x^2 - 30*a*b^3*h*x^2 + 60*b^4*d*x - 60*a*b^3*g*x)
/b^5 + 1/3*(a*b^10*e*(-a/b)^(1/3) - a^2*b^9*h*(-a/b)^(1/3) + a*b^10*d - a^2
*b^9*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^11)
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 1170, normalized size of antiderivative = 3.98

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = x^2\left(\frac{e}{2\,b}-\frac{a\,h}{2\,b^2}\right) + \left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,b^8\,z^3+27\,a\,b^6\,f\,z^2-27\,b^7\,c\,z^2-18\,a\,b^5\,c\,f\,z+9\,a\,b^5\,d\,e\,z+9\,a^3\,b^3\,g\,h\,z-9\,a^2\,b^4\,e\,g\,z-9\,a^2 \right.\right.\right.$$
$$+\frac{a\,b^3\,c^2+a^3\,b\,f^2+a^4\,g\,h-a^3\,b\,d\,h-a^3\,b\,e\,g-2\,a^2\,b^2\,c\,f+a^2\,b^2\,d\,e}{b^3}$$
$$+\frac{x\,(a^4\,h^2+a^2\,b^2\,e^2+a\,b^3\,c\,d-2\,a^3\,b\,e\,h+a^3\,b\,f\,g-a^2\,b^2\,c\,g-a^2\,b^2\,d\,f)}{b^3}\Bigg)\,\mathrm{root}\big(27\,b^8\,z^3$$
$$+27\,a\,b^6\,f\,z^2-27\,b^7\,c\,z^2-18\,a\,b^5\,c\,f\,z+9\,a\,b^5\,d\,e\,z+9\,a^3\,b^3\,g\,h\,z-9\,a^2\,b^4\,e\,g\,z-9\,a^2\,b^4\,d\,h\,z$$
$$+9\,a^2\,b^4\,f^2\,z+9\,b^6\,c^2\,z+3\,a^4\,b\,f\,g\,h-3\,a\,b^4\,c\,d\,e-3\,a^3\,b^2\,e\,f\,g-3\,a^3\,b^2\,d\,f\,h-3\,a^3\,b^2\,c\,g\,h$$
$$+3\,a^2\,b^3\,d\,e\,f+3\,a^2\,b^3\,c\,e\,g+3\,a^2\,b^3\,c\,d\,h-3\,a^4\,b\,e\,h^2+3\,a\,b^4\,c^2\,f+3\,a^3\,b^2\,e^2\,h+3\,a^3\,b^2\,d\,g^2$$
$$\left.-3\,a^2\,b^3\,d^2\,g-3\,a^2\,b^3\,c\,f^2+a^3\,b^2\,f^3+a\,b^4\,d^3+a^5\,h^3-a^2\,b^3\,e^3-a^4\,b\,g^3-b^5\,c^3,z,k\big)\right)$$
$$+x\left(\frac{d}{b}-\frac{a\,g}{b^2}\right)+\frac{f\,x^3}{3\,b}+\frac{g\,x^4}{4\,b}+\frac{h\,x^5}{5\,b}$$

```
[In] int((x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

```
[Out] x^2*(e/(2*b) - (a*h)/(2*b^2)) + symsum(log(root(27*b^8*z^3 + 27*a*b^6*f*z^2
 - 27*b^7*c*z^2 - 18*a*b^5*c*f*z + 9*a*b^5*d*e*z + 9*a^3*b^3*g*h*z - 9*a^2*
b^4*e*g*z - 9*a^2*b^4*d*h*z + 9*a^2*b^4*f^2*z + 9*b^6*c^2*z + 3*a^4*b*f*g*h
 - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*
a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4
*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*
f^2 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3
, z, k)*((6*a^2*b^3*f - 6*a*b^4*c)/b^3 + (x*(3*a^2*b^3*g - 3*a*b^4*d))/b^3
+ 9*root(27*b^8*z^3 + 27*a*b^6*f*z^2 - 27*b^7*c*z^2 - 18*a*b^5*c*f*z + 9*a*
b^5*d*e*z + 9*a^3*b^3*g*h*z - 9*a^2*b^4*e*g*z - 9*a^2*b^4*d*h*z + 9*a^2*b^4
*f^2*z + 9*b^6*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*
a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2
*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*
g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3
 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z, k)*a*b^2) + (a*b^3*c^2 + a^3*b*f^2
 + a^4*g*h - a^3*b*d*h - a^3*b*e*g - 2*a^2*b^2*c*f + a^2*b^2*d*e)/b^3 + (x*
(a^4*h^2 + a^2*b^2*e^2 + a*b^3*c*d - 2*a^3*b*e*h + a^3*b*f*g - a^2*b^2*c*g
- a^2*b^2*d*f))/b^3)*root(27*b^8*z^3 + 27*a*b^6*f*z^2 - 27*b^7*c*z^2 - 18*a
*b^5*c*f*z + 9*a*b^5*d*e*z + 9*a^3*b^3*g*h*z - 9*a^2*b^4*e*g*z - 9*a^2*b^4*
d*h*z + 9*a^2*b^4*f^2*z + 9*b^6*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a
^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*
```

```
b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2
*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 + a^3*b^2*f^3 + a*
b^4*d^3 + a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z, k), k, 1, 3) + x*
(d/b - (a*g)/b^2) + (f*x^3)/(3*b) + (g*x^4)/(4*b) + (h*x^5)/(5*b)
```

3.406 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 275

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \frac{(be-ah)x}{b^2}+\frac{fx^2}{2b}+\frac{gx^3}{3b}+\frac{hx^4}{4b}-\frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{7/3}} - \frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{7/3}} + \frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{7/3}} + \frac{(bd-ag)\log\left(a+bx^3\right)}{3b^2}$$

```
[Out] (-a*h+b*e)*x/b^2+1/2*f*x^2/b+1/3*g*x^3/b+1/4*h*x^4/b-1/3*(b^(2/3)*(-a*f+b*c
)+a^(2/3)*(-a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(7/3)+1/6*(b^(2/3)*(-
a*f+b*c)+a^(2/3)*(-a*h+b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1
/3)/b^(7/3)+1/3*(-a*g+b*d)*ln(b*x^3+a)/b^2-1/3*(b^(5/3)*c-a^(2/3)*b*e-a*b^(
2/3)*f+a^(5/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)
/b^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1850, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}\sqrt[3]{a}b^{7/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6\sqrt[3]{a}b^{7/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3\sqrt[3]{a}b^{7/3}} + \frac{(bd-ag)\log(a+bx^3)}{3b^2} + \frac{x(be-ah)}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b}$$

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] ((b*e - a*h)*x)/b^2 + (f*x^2)/(2*b) + (g*x^3)/(3*b) + (h*x^4)/(4*b) - ((b^(
5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(7/3)) - ((b^(2/3)*(b*c - a*f) +
 a^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(7/3)) + ((b^(
2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(6*a^(1/3)*b^(7/3)) + ((b*d - a*g)*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{hx^4}{4b} + \frac{\int \frac{x(4bc+4bdx+4(be-ah)x^2+4bfx^3+4bgx^4)}{a+bx^3}\,dx}{4b} \\
&= \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{\int \frac{x(12b^2c+12b(bd-ag)x+12b(be-ah)x^2+12b^2fx^3)}{a+bx^3}\,dx}{12b^2} \\
&= \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{\int \frac{x(24b^2(bc-af)+24b^2(bd-ag)x+24b^2(be-ah)x^2)}{a+bx^3}\,dx}{24b^3} \\
&= \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{\int \left(24b(be-ah) - \frac{24(ab(be-ah)-b^2(bc-af)x-b^2(bd-ag)x^2)}{a+bx^3}\right)dx}{24b^3} \\
&= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\int \frac{ab(be-ah)-b^2(bc-af)x-b^2(bd-ag)x^2}{a+bx^3}\,dx}{b^3} \\
&= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\int \frac{ab(be-ah)-b^2(bc-af)x}{a+bx^3}\,dx}{b^3} + \frac{(bd-ag)\int \frac{x^2}{a+bx^3}\,dx}{b} \\
&= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{(bd-ag)\log(a+bx^3)}{3b^2} \\
&\quad - \frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}b^2(bc-af)+2ab^{4/3}(be-ah)\right)+\sqrt[3]{b}\left(-\sqrt[3]{a}b^2(bc-af)-ab^{4/3}(be-ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{10/3}} \\
&\quad - \frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}b^2} \\
&= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} \\
&\quad - \frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{7/3}} + \frac{(bd-ag)\log(a+bx^3)}{3b^2} \\
&\quad + \frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^2} \\
&\quad + \frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}b^{7/3}}
\end{aligned}$$

$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{7/3}}$$
$$+ \frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{7/3}}$$
$$+ \frac{(bd-ag)\log\left(a+bx^3\right)}{3b^2} + \frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{7/3}}$$
$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b}$$
$$- \frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{7/3}}$$
$$- \frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{7/3}}$$
$$+ \frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{7/3}}$$
$$+ \frac{(bd-ag)\log\left(a+bx^3\right)}{3b^2}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.99

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$

$$= \frac{12\sqrt[3]{b}(be-ah)x + 6b^{4/3}fx^2 + 4b^{4/3}gx^3 + 3b^{4/3}hx^4 - \frac{4\sqrt{3}\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{4(-b^{5/3}}{}}{}$$

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (12*b^(1/3)*(b*e - a*h)*x + 6*b^(4/3)*f*x^2 + 4*b^(4/3)*g*x^3 + 3*b^(4/3)*h
*x^4 - (4*Sqrt[3]*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (4*(-(b^(5/3)*c) - a^(2/3
)*b*e + a*b^(2/3)*f + a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (2*(b^
(5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/a^(1/3) + 4*b^(1/3)*(b*d - a*g)*Log[a + b*x^3])/(12*b
^(7/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.38

method	result
risch	$\frac{h\,x^4}{4b}+\frac{g\,x^3}{3b}+\frac{f\,x^2}{2b}-\frac{ahx}{b^2}+\frac{ex}{b}+\frac{\sum_{\_R=\text{RootOf}(b\_Z^3+a)}\frac{\left(b(-ag+bd)\_R^2+b(-af+bc)\_R+a^2h-aeb\right)\ln(x-\_R)}{\_R^2}}{3b^3}$
default	$-\frac{-\frac{1}{4}bh\,x^4-\frac{1}{3}bg\,x^3-\frac{1}{2}bf\,x^2+ahx-bex}{b^2}+\frac{(a^2h-aeb)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(\cdot}{}$

[In] int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 1/4*h*x^4/b+1/3*g*x^3/b+1/2*f*x^2/b-1/b^2*a*h*x+e*x/b+1/3/b^3*sum((b*(-a*g+b*d)*_R^2+b*(-a*f+b*c)*_R+a^2*h-a*e*b)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 14875, normalized size of antiderivative = 54.09

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \text{Too large to display}$$

[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.09

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \frac{\sqrt{3}\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}} - abf\left(\frac{a}{b}\right)^{\frac{2}{3}} - abe\left(\frac{a}{b}\right)^{\frac{1}{3}} + a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2} + \frac{3bhx^4+4bgx^3+6bfx^2+12(be-ah)x}{12b^2} + \frac{\left(2b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2abg\left(\frac{a}{b}\right)^{\frac{2}{3}} + b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} - abf\left(\frac{a}{b}\right)^{\frac{1}{3}} + abe - a^2h\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}} - abg\left(\frac{a}{b}\right)^{\frac{2}{3}} - b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} + abf\left(\frac{a}{b}\right)^{\frac{1}{3}} - abe + a^2h\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima"
)
[Out] 1/3*sqrt(3)*(b^2*c*(a/b)^(2/3) - a*b*f*(a/b)^(2/3) - a*b*e*(a/b)^(1/3) + a^
2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2
) + 1/12*(3*b*h*x^4 + 4*b*g*x^3 + 6*b*f*x^2 + 12*(b*e - a*h)*x)/b^2 + 1/6*(
2*b^2*d*(a/b)^(2/3) - 2*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) - a*b*f*(a/b)
^(1/3) + a*b*e - a^2*h)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(
2/3)) + 1/3*(b^2*d*(a/b)^(2/3) - a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^(1/3) + a*
b*f*(a/b)^(1/3) - a*b*e + a^2*h)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.06

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$= \frac{\sqrt{3}\left(abe-a^2h+(-ab^2)^{\frac{1}{3}}bc-(-ab^2)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}b}$$
$$+\frac{\left(abe-a^2h-(-ab^2)^{\frac{1}{3}}bc+(-ab^2)^{\frac{1}{3}}af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}b}$$
$$+\frac{(bd-ag)\log\left(|bx^3+a|\right)}{3\,b^2}+\frac{3\,b^3hx^4+4\,b^3gx^3+6\,b^3fx^2+12\,b^3ex-12\,ab^2hx}{12\,b^4}$$
$$-\frac{\left(b^9c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^8f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^8e+a^2b^7h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^9}$$

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(a*b*e - a^2*h + (-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/3)*a*f)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) + 1/6*(
a*b*e - a^2*h - (-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) + 1/3*(b*d - a*g)*log(abs(b*x^3 +
 a))/b^2 + 1/12*(3*b^3*h*x^4 + 4*b^3*g*x^3 + 6*b^3*f*x^2 + 12*b^3*e*x - 12*
a*b^2*h*x)/b^4 - 1/3*(b^9*c*(-a/b)^(1/3) - a*b^8*f*(-a/b)^(1/3) - a*b^8*e +
 a^2*b^7*h)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^9)
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 1161, normalized size of antiderivative = 4.22

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\mathrm{root}\big(27\,a\,b^7\,z^3-27\,a\,b^6\,d\,z^2+27\,a^2\,b^5\,g\,z^2-9\,a\,b^5\,c\,e\,z-9\,a^3\,b^3\,f\,h\,z-18\,a^2\,b^4\,d\,g\,z+9\,a^2\,b^4\,e\,f\,z+\right.\right.$$
$$+\frac{a^3\,g^2+a\,b^2\,d^2-a^3\,f\,h-a\,b^2\,c\,e+a^2\,b\,c\,h-2\,a^2\,b\,d\,g+a^2\,b\,e\,f}{b^2}$$
$$+\frac{x\,(b^3\,c^2+a^2\,b\,f^2+a^3\,g\,h-2\,a\,b^2\,c\,f+a\,b^2\,d\,e-a^2\,b\,d\,h-a^2\,b\,e\,g)}{b^2}\Bigg)\ \mathrm{root}\big(27\,a\,b^7\,z^3$$
$$-27\,a\,b^6\,d\,z^2+27\,a^2\,b^5\,g\,z^2-9\,a\,b^5\,c\,e\,z-9\,a^3\,b^3\,f\,h\,z-18\,a^2\,b^4\,d\,g\,z+9\,a^2\,b^4\,e\,f\,z$$
$$+9\,a^2\,b^4\,c\,h\,z+9\,a\,b^5\,d^2\,z+9\,a^3\,b^3\,g^2\,z-3\,a^4\,b\,f\,g\,h+3\,a\,b^4\,c\,d\,e+3\,a^3\,b^2\,e\,f\,g$$
$$+3\,a^3\,b^2\,d\,f\,h+3\,a^3\,b^2\,c\,g\,h-3\,a^2\,b^3\,d\,e\,f-3\,a^2\,b^3\,c\,e\,g-3\,a^2\,b^3\,c\,d\,h+3\,a^4\,b\,e\,h^2$$
$$-3\,a\,b^4\,c^2\,f-3\,a^3\,b^2\,e^2\,h-3\,a^3\,b^2\,d\,g^2+3\,a^2\,b^3\,d^2\,g+3\,a^2\,b^3\,c\,f^2+a^2\,b^3\,e^3+a^4\,b\,g^3$$
$$\left.+b^5\,c^3-a^3\,b^2\,f^3-a\,b^4\,d^3-a^5\,h^3,z,k\big)\right)+x\left(\frac{e}{b}-\frac{a\,h}{b^2}\right)+\frac{f\,x^2}{2\,b}+\frac{g\,x^3}{3\,b}+\frac{h\,x^4}{4\,b}$$

```
[In] int((x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

```
[Out] symsum(log(root(27*a*b^7*z^3 - 27*a*b^6*d*z^2 + 27*a^2*b^5*g*z^2 - 9*a*b^5*
c*e*z - 9*a^3*b^3*f*h*z - 18*a^2*b^4*d*g*z + 9*a^2*b^4*e*f*z + 9*a^2*b^4*c*
h*z + 9*a*b^5*d^2*z + 9*a^3*b^3*g^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a
^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*
b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2
*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^3*e^3 + a^
4*b*g^3 + b^5*c^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z, k)*((6*a^2*b^2*g
- 6*a*b^3*d)/b^2 + (x*(3*a^2*b^2*h - 3*a*b^3*e))/b^2 + 9*root(27*a*b^7*z^3
- 27*a*b^6*d*z^2 + 27*a^2*b^5*g*z^2 - 9*a*b^5*c*e*z - 9*a^3*b^3*f*h*z - 18*
a^2*b^4*d*g*z + 9*a^2*b^4*e*f*z + 9*a^2*b^4*c*h*z + 9*a*b^5*d^2*z + 9*a^3*b
^3*g^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*
h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h +
 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*
b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3 - a^3*b^2*f
^3 - a*b^4*d^3 - a^5*h^3, z, k)*a*b^2) + (a^3*g^2 + a*b^2*d^2 - a^3*f*h - a
*b^2*c*e + a^2*b*c*h - 2*a^2*b*d*g + a^2*b*e*f)/b^2 + (x*(b^3*c^2 + a^2*b*f
^2 + a^3*g*h - 2*a*b^2*c*f + a*b^2*d*e - a^2*b*d*h - a^2*b*e*g))/b^2)*root(
27*a*b^7*z^3 - 27*a*b^6*d*z^2 + 27*a^2*b^5*g*z^2 - 9*a*b^5*c*e*z - 9*a^3*b^
3*f*h*z - 18*a^2*b^4*d*g*z + 9*a^2*b^4*e*f*z + 9*a^2*b^4*c*h*z + 9*a*b^5*d^
2*z + 9*a^3*b^3*g^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3
*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^
2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d
```

```
*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^
3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z, k), k, 1, 3) + x*(e/b - (a*h)/b^2
) + (f*x^2)/(2*b) + (g*x^3)/(3*b) + (h*x^4)/(4*b)
```

3.407 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 259

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx = \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}-\frac{\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{5/3}} + \frac{\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{5/3}} - \frac{\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{5/3}} + \frac{(be-ah)\log\left(a+bx^3\right)}{3b^2}$$

```
[Out] f*x/b+1/2*g*x^2/b+1/3*h*x^3/b+1/3*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*g+b*d))*l
n(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(5/3)-1/6*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*g+
b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(5/3)+1/3*(-a*h+b
*e)*ln(b*x^3+a)/b^2-1/3*(b^(4/3)*c+a^(1/3)*b*d-a*b^(1/3)*f-a^(4/3)*g)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx \\
&= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}a^{2/3}b^{5/3}} \\
&\quad-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}-af+bc\right)}{6a^{2/3}b^{4/3}} \\
&\quad+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{2/3}b^{5/3}} \\
&\quad+\frac{(be-ah)\log\left(a+bx^3\right)}{3b^2}+\frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}
\end{aligned}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x]
```

```
[Out] (f*x)/b + (g*x^2)/(2*b) + (h*x^3)/(3*b) - ((b^(4/3)*c + a^(1/3)*b*d - a*b^(
1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqr
t[3]*a^(2/3)*b^(5/3)) + ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^
(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/3)) - ((b*c - a*f - (a^(1/3)*(b*d - a*g
))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4
/3)) + ((b*e - a*h)*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \left(\frac{f}{b}+\frac{gx}{b}+\frac{hx^2}{b}+\frac{bc-af+(bd-ag)x+(be-ah)x^2}{b\,(a+bx^3)}\right)dx$$

$$=\frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{\int \frac{bc-af+(bd-ag)x+(be-ah)x^2}{a+bx^3}\,dx}{b}$$

$$= \frac{fx}{b} + \frac{gx^2}{2b} + \frac{hx^3}{3b} + \frac{\int \frac{bc-af+(bd-ag)x}{a+bx^3}\,dx}{b} + \frac{(be-ah)\int \frac{x^2}{a+bx^3}\,dx}{b}$$

$$= \frac{fx}{b} + \frac{gx^2}{2b} + \frac{hx^3}{3b} + \frac{(be-ah)\log\left(a+bx^3\right)}{3b^2} + \frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}(bc-af)+\sqrt[3]{a}(bd-ag)\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}(bc-af)+\sqrt[3]{a}(bd-ag)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}} + \frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b}$$

$$= \frac{fx}{b} + \frac{gx^2}{2b} + \frac{hx^3}{3b} + \frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} + \frac{(be-ah)\log\left(a+bx^3\right)}{3b^2} + \frac{\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}b^{4/3}} - \frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{4/3}}$$

$$= \frac{fx}{b} + \frac{gx^2}{2b} + \frac{hx^3}{3b} + \frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} + \frac{(be-ah)\log\left(a+bx^3\right)}{3b^2} + \frac{\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}b^{5/3}}$$

$$= \frac{fx}{b} + \frac{gx^2}{2b} + \frac{hx^3}{3b} - \frac{\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{5/3}} + \frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} + \frac{(be-ah)\log\left(a+bx^3\right)}{3b^2}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.98

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx$$
$$= \frac{6b^{2/3}fx+3b^{2/3}gx^2+2b^{2/3}hx^3+\frac{2\sqrt{3}\left(-b^{4/3}c-\sqrt[3]{a}bd+a\sqrt[3]{b}f+a^{4/3}g\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{2\left(b^{4/3}c-\sqrt[3]{a}bd-a\sqrt[3]{b}f+a^{4/3}\right.}{a^{2/3}}}{6b^{5/3}}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x]
```

```
[Out] (6*b^(2/3)*f*x + 3*b^(2/3)*g*x^2 + 2*b^(2/3)*h*x^3 + (2*Sqrt[3]*(-(b^(4/3)*
c) - a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/a^(2/3) + (2*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)
*g)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) - ((b^(4/3)*c - a^(1/3)*b*d - a*b^(1/
3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) +
 (2*(b*e - a*h)*Log[a + b*x^3])/b^(1/3))/(6*b^(5/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.32

method	result
risch	$\frac{h\,x^3}{3b}+\frac{g\,x^2}{2b}+\frac{fx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(bc-af+(-ag+bd)\_R+(-ah+be)\_R^2\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^2}$
default	$\frac{\frac{1}{3}h\,x^3+\frac{1}{2}g\,x^2+fx}{b}+\frac{(-af+bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(-ag+bd)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right.\right.}{3b\left(\frac{a}{b}\right.}\right.}{b}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*h*x^3/b+1/2*g*x^2/b+f*x/b+1/3/b^2*sum((b*c-a*f+(-a*g+b*d)*_R+(-a*h+b*e)
*_R^2)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 15235, normalized size of antiderivative = 58.82

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx &= \frac{2\,hx^3+3\,gx^2+6\,fx}{6\,b} \\ &+ \frac{\sqrt{3}\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} \\ &+ \frac{\left(2\,be\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-bc+af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &+ \frac{\left(be\left(\frac{a}{b}\right)^{\frac{2}{3}}-ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+bc-af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*h*x^3 + 3*g*x^2 + 6*f*x)/b + 1/3*sqrt(3)*(b^2*d*(a/b)^(2/3) - a*b*g*
(a/b)^(2/3) + b^2*c*(a/b)^(1/3) - a*b*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2) + 1/6*(2*b*e*(a/b)^(2/3) - 2*a*h*(a/b
)^(2/3) + b*d*(a/b)^(1/3) - a*g*(a/b)^(1/3) - b*c + a*f)*log(x^2 - x*(a/b)^
(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*(b*e*(a/b)^(2/3) - a*h*(a/b)^(
2/3) - b*d*(a/b)^(1/3) + a*g*(a/b)^(1/3) + b*c - a*f)*log(x + (a/b)^(1/3))/
(b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.05

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx$$
$$= -\frac{\sqrt{3}\left(b^2c-abf-(-ab^2)^{\frac{1}{3}}\,bd+(-ab^2)^{\frac{1}{3}}\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{2}{3}}\,b} - \frac{\left(b^2c-abf+(-ab^2)^{\frac{1}{3}}\,bd-(-ab^2)^{\frac{1}{3}}\,ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{2}{3}}\,b} + \frac{(be-ah)\log\left(|bx^3+a|\right)}{3\,b^2} + \frac{2\,b^2hx^3+3\,b^2gx^2+6\,b^2fx}{6\,b^3} - \frac{\left(b^7d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^6g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b^7c-ab^6f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^7}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^2*c - a*b*f - (-a*b^2)^(1/3)*b*d + (-a*b^2)^(1/3)*a*g)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/6*
(b^2*c - a*b*f + (-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) + 1/3*(b*e - a*h)*log(abs(b*x^3
+ a))/b^2 + 1/6*(2*b^2*h*x^3 + 3*b^2*g*x^2 + 6*b^2*f*x)/b^3 - 1/3*(b^7*d*(-
a/b)^(1/3) - a*b^6*g*(-a/b)^(1/3) + b^7*c - a*b^6*f)*(-a/b)^(1/3)*log(abs(x
 - (-a/b)^(1/3)))/(a*b^7)
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 1150, normalized size of antiderivative = 4.44

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx$$

$$= \left(\sum_{k=1}^{3} \ln\left(\frac{a^3\,h^2+a\,b^2\,e^2+b^3\,c\,d-a\,b^2\,c\,g-a\,b^2\,d\,f-2\,a^2\,b\,e\,h+a^2\,b\,f\,g}{b^2}\right.\right.$$

$$+\mathrm{root}\big(27\,a^2\,b^6\,z^3+27\,a^3\,b^4\,h\,z^2-27\,a^2\,b^5\,e\,z^2+9\,a\,b^5\,c\,d\,z-18\,a^3\,b^3\,e\,h\,z+9\,a^3\,b^3\,f\,g\,z-9\,a^2\,b^4\,d\,f\,z-9\,a^2\,b^4$$

$$\left.+\frac{x\,(b^2\,d^2+a^2\,g^2-b^2\,c\,e-a^2\,f\,h+a\,b\,c\,h-2\,a\,b\,d\,g+a\,b\,e\,f)}{b}\right)\,\mathrm{root}\big(27\,a^2\,b^6\,z^3$$

$$+27\,a^3\,b^4\,h\,z^2-27\,a^2\,b^5\,e\,z^2+9\,a\,b^5\,c\,d\,z-18\,a^3\,b^3\,e\,h\,z+9\,a^3\,b^3\,f\,g\,z-9\,a^2\,b^4\,d\,f\,z$$

$$-9\,a^2\,b^4\,c\,g\,z+9\,a^4\,b^2\,h^2\,z+9\,a^2\,b^4\,e^2\,z+3\,a^4\,b\,f\,g\,h-3\,a\,b^4\,c\,d\,e-3\,a^3\,b^2\,e\,f\,g$$

$$-3\,a^3\,b^2\,d\,f\,h-3\,a^3\,b^2\,c\,g\,h+3\,a^2\,b^3\,d\,e\,f+3\,a^2\,b^3\,c\,e\,g+3\,a^2\,b^3\,c\,d\,h-3\,a^4\,b\,e\,h^2$$

$$+3\,a\,b^4\,c^2\,f+3\,a^3\,b^2\,e^2\,h+3\,a^3\,b^2\,d\,g^2-3\,a^2\,b^3\,d^2\,g-3\,a^2\,b^3\,c\,f^2+a^3\,b^2\,f^3+a\,b^4\,d^3$$

$$\left.+a^5\,h^3-a^2\,b^3\,e^3-a^4\,b\,g^3-b^5\,c^3,z,k\big)\right)+\frac{g\,x^2}{2\,b}+\frac{h\,x^3}{3\,b}+\frac{f\,x}{b}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x)
```

```
[Out] symsum(log((a^3*h^2 + a*b^2*e^2 + b^3*c*d - a*b^2*c*g - a*b^2*d*f - 2*a^2*b
*e*h + a^2*b*f*g)/b^2 + root(27*a^2*b^6*z^3 + 27*a^3*b^4*h*z^2 - 27*a^2*b^5
*e*z^2 + 9*a*b^5*c*d*z - 18*a^3*b^3*e*h*z + 9*a^3*b^3*f*g*z - 9*a^2*b^4*d*f
*z - 9*a^2*b^4*c*g*z + 9*a^4*b^2*h^2*z + 9*a^2*b^4*e^2*z + 3*a^4*b*f*g*h -
3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2
*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^
2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2
 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z
, k)*((6*a^2*b^2*h - 6*a*b^3*e)/b^2 + (x*(3*b^3*c - 3*a*b^2*f))/b + 9*root(
27*a^2*b^6*z^3 + 27*a^3*b^4*h*z^2 - 27*a^2*b^5*e*z^2 + 9*a*b^5*c*d*z - 18*a
^3*b^3*e*h*z + 9*a^3*b^3*f*g*z - 9*a^2*b^4*d*f*z - 9*a^2*b^4*c*g*z + 9*a^4*
b^2*h^2*z + 9*a^2*b^4*e^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f
*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g
+ 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3
*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 + a^3*b^2*f^3 + a*b^4*d^3 +
a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z, k)*a*b^2) + (x*(b^2*d^2 + a
^2*g^2 - b^2*c*e - a^2*f*h + a*b*c*h - 2*a*b*d*g + a*b*e*f))/b)*root(27*a^2
*b^6*z^3 + 27*a^3*b^4*h*z^2 - 27*a^2*b^5*e*z^2 + 9*a*b^5*c*d*z - 18*a^3*b^3
*e*h*z + 9*a^3*b^3*f*g*z - 9*a^2*b^4*d*f*z - 9*a^2*b^4*c*g*z + 9*a^4*b^2*h^
2*z + 9*a^2*b^4*e^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3
*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^
2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d
```

```
*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^
3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z, k), k, 1, 3) + (g*x^2)/(2*b) + (h
*x^3)/(3*b) + (f*x)/b
```

3.408 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 258

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\,(a+bx^3)}\,dx &= \frac{gx}{b}+\frac{hx^2}{2b}-\frac{\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{5/3}}\\ &\quad+\frac{c\log(x)}{a}+\frac{\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{5/3}}\\ &\quad-\frac{\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{5/3}}\\ &\quad-\frac{(bc-af)\log\left(a+bx^3\right)}{3ab}\end{aligned}$$

```
[Out] g*x/b+1/2*h*x^2/b+c*ln(x)/a+1/3*(b^(1/3)*(-a*g+b*d)-a^(1/3)*(-a*h+b*e))*ln(
a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(5/3)-1/6*(b^(1/3)*(-a*g+b*d)-a^(1/3)*(-a*h+b*
e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(5/3)-1/3*(-a*f+b*c
)*ln(b*x^3+a)/a/b-1/3*(b^(4/3)*d+a^(1/3)*b*e-a*b^(1/3)*g-a^(4/3)*h)*arctan(
1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}a^{2/3}b^{5/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}-ag+bd\right)}{6a^{2/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{2/3}b^{5/3}} - \frac{(bc-af)\log\left(a+bx^3\right)}{3ab}+\frac{c\log(x)}{a}+\frac{gx}{b}+\frac{hx^2}{2b}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) - ((b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g - a^(4/3)
*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(
5/3)) + (c*Log[x])/a + ((b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(
1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/3)) - ((b*d - a*g - (a^(1/3)*(b*e - a*h)
)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/
3)) - ((b*c - a*f)*Log[a + b*x^3])/(3*a*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{g}{b}+\frac{c}{ax}+\frac{hx}{b}+\frac{a(bd-ag)+a(be-ah)x-b(bc-af)x^2}{ab\left(a+bx^3\right)}\right)dx\\ &= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}+\frac{\int \frac{a(bd-ag)+a(be-ah)x-b(bc-af)x^2}{a+bx^3}\,dx}{ab}\end{aligned}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{c\log(x)}{a} + \frac{\int \frac{a(bd-ag)+a(be-ah)x}{a+bx^3}\,dx}{ab} - \frac{(bc-af)\int \frac{x^2}{a+bx^3}\,dx}{a}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{c\log(x)}{a} - \frac{(bc-af)\log(a+bx^3)}{3ab} + \frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}(bd-ag)+a^{4/3}(be-ah)\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}(bd-ag)+a^{4/3}(be-ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}b^{4/3}} + \frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{c\log(x)}{a} + \frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-af)\log(a+bx^3)}{3ab} + \frac{\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}b^{4/3}} - \frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{4/3}}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{c\log(x)}{a} + \frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} - \frac{(bc-af)\log(a+bx^3)}{3ab} + \frac{\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}b^{5/3}}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} - \frac{\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{5/3}} + \frac{c\log(x)}{a} + \frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} - \frac{(bc-af)\log(a+bx^3)}{3ab}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)}\,dx$$

$$= \frac{6ab^{2/3}gx+3ab^{2/3}hx^2+2\sqrt{3}\sqrt[3]{a}\left(-b^{4/3}d-\sqrt[3]{a}be+a\sqrt[3]{b}g+a^{4/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6b^{5/3}c\log(x)+2\sqrt[3]{\cdots}}{\cdots}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x]
```

```
[Out] (6*a*b^(2/3)*g*x + 3*a*b^(2/3)*h*x^2 + 2*Sqrt[3]*a^(1/3)*(-(b^(4/3)*d) - a^
(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] + 6*b^(5/3)*c*Log[x] + 2*a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)
*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x] - a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e
 - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
- 2*b^(2/3)*(b*c - a*f)*Log[a + b*x^3])/(6*a*b^(5/3))
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00

method	result
default	$\frac{\frac{1}{2}h\,x^2+gx}{b}+\frac{c\ln(x)}{a}+\frac{\left(-a^2g+abd\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\left(-a^2h+aeb\right)\left(-\cdots\right.}{ba}$
risch	$\frac{h\,x^2}{2b}+\frac{gx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a^3b^2\_Z^3+\left(-3a^3b^2f+3a^2c\,b^3\right)\_Z^2+\left(3a^4bgh-3a^3b^2dh-3a^3b^2eg+3a^3b^2f^2-6a^2b^3cf+3a^2b^3de+3c^2a\,b^4\right)\_Z-a^5\cdots\right)}\cdots}{\cdots}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*h*x^2+g*x)+c*ln(x)/a+((-a^2*g+a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)
^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(-a^2*h+a*b*e)*(-1/3/b/
(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^
```

```
(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1
/3*(a*b*f-b^2*c)*ln(b*x^3+a)/b)/b/a
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 84.14 (sec) , antiderivative size = 15327, normalized size of antiderivative = 59.41

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="fricas"
)
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx &= \frac{c\log(x)}{a} + \frac{hx^2+2\,gx}{2\,b} \\ &\quad + \frac{\sqrt{3}\left(abe\left(\frac{a}{b}\right)^{\frac{2}{3}} - a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}} + abd\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b} \\ &\quad - \frac{\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}} - abe\left(\frac{a}{b}\right)^{\frac{1}{3}} + a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}} + abd - a^2g\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &\quad - \frac{\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}} - abf\left(\frac{a}{b}\right)^{\frac{2}{3}} + abe\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}} - abd + a^2g\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="maxima"
)
```

```
[Out] c*log(x)/a + 1/2*(h*x^2 + 2*g*x)/b + 1/3*sqrt(3)*(a*b*e*(a/b)^(2/3) - a^2*h
*(a/b)^(2/3) + a*b*d*(a/b)^(1/3) - a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b) - 1/6*(2*b^2*c*(a/b)^(2/3) - 2*a*b*f
*(a/b)^(2/3) - a*b*e*(a/b)^(1/3) + a^2*h*(a/b)^(1/3) + a*b*d - a^2*g)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) - 1/3*(b^2*c*(a/b)^(2
/3) - a*b*f*(a/b)^(2/3) + a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3) - a*b*d + a
^2*g)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.08

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)}\,dx$$
$$= \frac{c\log\left(|x|\right)}{a} - \frac{\sqrt{3}\left(b^2d-abg-\left(-ab^2\right)^{\frac{1}{3}}be+\left(-ab^2\right)^{\frac{1}{3}}ah\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}b}$$
$$- \frac{\left(b^2d-abg+\left(-ab^2\right)^{\frac{1}{3}}be-\left(-ab^2\right)^{\frac{1}{3}}ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}b}$$
$$- \frac{(bc-af)\log\left(|bx^3+a|\right)}{3ab} + \frac{bhx^2+2bgx}{2b^2}$$
$$- \frac{\left(a^2b^3e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^2h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^3d-a^3b^2g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b^3}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] c*log(abs(x))/a - 1/3*sqrt(3)*(b^2*d - a*b*g - (-a*b^2)^(1/3)*b*e + (-a*b^2
)^(1/3)*a*h)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2
)^(2/3)*b) - 1/6*(b^2*d - a*b*g + (-a*b^2)^(1/3)*b*e - (-a*b^2)^(1/3)*a*h)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/3*(b*c - a*
f)*log(abs(b*x^3 + a))/(a*b) + 1/2*(b*h*x^2 + 2*b*g*x)/b^2 - 1/3*(a^2*b^3*e
*(-a/b)^(1/3) - a^3*b^2*h*(-a/b)^(1/3) + a^2*b^3*d - a^3*b^2*g)*(-a/b)^(1/3
)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 1731, normalized size of antiderivative = 6.71

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x)
[Out] symsum(log(b^2*c*d^2 - root(27*a^3*b^5*z^3 - 27*a^3*b^4*f*z^2 + 27*a^2*b^5*
c*z^2 + 9*a^4*b^2*g*h*z - 9*a^3*b^3*e*g*z - 9*a^3*b^3*d*h*z - 18*a^2*b^4*c*
f*z + 9*a^2*b^4*d*e*z + 9*a*b^5*c^2*z + 9*a^3*b^3*f^2*z - 3*a^4*b*f*g*h + 3
*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*
b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2
*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2
+ a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z,
 k)*(a^3*g^2 - root(27*a^3*b^5*z^3 - 27*a^3*b^4*f*z^2 + 27*a^2*b^5*c*z^2 +
9*a^4*b^2*g*h*z - 9*a^3*b^3*e*g*z - 9*a^3*b^3*d*h*z - 18*a^2*b^4*c*f*z + 9*
a^2*b^4*d*e*z + 9*a*b^5*c^2*z + 9*a^3*b^3*f^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c
*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*
f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a
^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^
3*e^3 + a^4*b*g^3 + b^5*c^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z, k)*((x*
(33*a^2*b^4*f - 24*a*b^5*c))/b^2 + 3*a^2*b^2*e - 3*a^3*b*h - 36*root(27*a^3
*b^5*z^3 - 27*a^3*b^4*f*z^2 + 27*a^2*b^5*c*z^2 + 9*a^4*b^2*g*h*z - 9*a^3*b^
3*e*g*z - 9*a^3*b^3*d*h*z - 18*a^2*b^4*c*f*z + 9*a^2*b^4*d*e*z + 9*a*b^5*c^
2*z + 9*a^3*b^3*f^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3
*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^
2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d
*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^
3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z, k)*a^2*b^3*x) + (x*(4*b^5*c^2 + 1
0*a^2*b^3*f^2 - 14*a*b^4*c*f + 10*a*b^4*d*e - 10*a^2*b^3*d*h - 10*a^2*b^3*e
*g + 10*a^3*b^2*g*h))/b^2 + a*b^2*d^2 - a^3*f*h + 2*a*b^2*c*e - 2*a^2*b*c*h
 - 2*a^2*b*d*g + a^2*b*e*f) - b^2*c^2*e + a^2*c*g^2 + (x*(b^4*d^3 + a^4*h^3
 - a*b^3*e^3 - a^3*b*g^3 + b^4*c^2*f + a^2*b^2*f^3 + 3*a^2*b^2*d*g^2 + 3*a^
2*b^2*e^2*h - 2*b^4*c*d*e - 2*a*b^3*c*f^2 - 3*a*b^3*d^2*g - 3*a^3*b*e*h^2 -
 2*a^2*b^2*c*g*h - 3*a^2*b^2*d*f*h - 3*a^2*b^2*e*f*g + 2*a*b^3*c*d*h + 2*a*
b^3*c*e*g + 3*a*b^3*d*e*f + 3*a^3*b*f*g*h))/b^2 + a*b*c^2*h - a^2*c*f*h - 2
*a*b*c*d*g + a*b*c*e*f)*root(27*a^3*b^5*z^3 - 27*a^3*b^4*f*z^2 + 27*a^2*b^5
*c*z^2 + 9*a^4*b^2*g*h*z - 9*a^3*b^3*e*g*z - 9*a^3*b^3*d*h*z - 18*a^2*b^4*c
*f*z + 9*a^2*b^4*d*e*z + 9*a*b^5*c^2*z + 9*a^3*b^3*f^2*z - 3*a^4*b*f*g*h +
3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2
*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^
2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2
 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z
, k), k, 1, 3) + (h*x^2)/(2*b) + (c*log(x))/a + (g*x)/b
```

3.409 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 253

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx\\
&= -\frac{c}{ax}+\frac{hx}{b}+\frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{4/3}}\\
&\quad+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{4/3}}\\
&\quad-\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{4/3}}\\
&\quad-\frac{(bd-ag)\log(a+bx^3)}{3ab}
\end{aligned}$$

```
[Out] -c/a/x+h*x/b+d*ln(x)/a+1/3*(b^(2/3)*(-a*f+b*c)+a^(2/3)*(-a*h+b*e))*ln(a^(1/
3)+b^(1/3)*x)/a^(4/3)/b^(4/3)-1/6*(b^(2/3)*(-a*f+b*c)+a^(2/3)*(-a*h+b*e))*l
n(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(4/3)-1/3*(-a*g+b*d)*ln(
b*x^3+a)/a/b+1/3*(b^(5/3)*c-a^(2/3)*b*e-a*b^(2/3)*f+a^(5/3)*h)*arctan(1/3*(
a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx \\
&= \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}a^{4/3}b^{4/3}} \\
&\quad-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6a^{4/3}b^{4/3}} \\
&\quad+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3a^{4/3}b^{4/3}} \\
&\quad-\frac{(bd-ag)\log\left(a+bx^3\right)}{3ab}-\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{hx}{b}
\end{aligned}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(c/(a*x)) + (h*x)/b + ((b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(4/3
)) + (d*Log[x])/a + ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(4/3)*b^(4/3)) - ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e
- a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(4/3))
 - ((b*d - a*g)*Log[a + b*x^3])/(3*a*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{h}{b}+\frac{c}{ax^2}+\frac{d}{ax}+\frac{a(be-ah)-b(bc-af)x-b(bd-ag)x^2}{ab\,(a+bx^3)}\right)\,dx$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\int \frac{a(be-ah)-b(bc-af)x-b(bd-ag)x^2}{a+bx^3}\,dx}{ab}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\int \frac{a(be-ah)-b(bc-af)x}{a+bx^3}\,dx}{ab}-\frac{(bd-ag)\int \frac{x^2}{a+bx^3}\,dx}{a}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}-\frac{(bd-ag)\log(a+bx^3)}{3ab}+\frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}b(bc-af)+2a\sqrt[3]{b}(be-ah)\right)+\sqrt[3]{b}\left(-\sqrt[3]{a}b(bc-af)-a\sqrt[3]{b}(be-ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}b^{4/3}}+\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}b}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{4/3}}-\frac{(bd-ag)\log(a+bx^3)}{3ab}-\frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2ab}-\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}b^{4/3}}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{4/3}}-\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{4/3}}-\frac{(bd-ag)\log(a+bx^3)}{3ab}-\frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{4/3}b^{4/3}}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{4/3}}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{4/3}}-\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{4/3}}-\frac{(bd-ag)\log(a+bx^3)}{3ab}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx = \frac{1}{6}\left(-\frac{6c}{ax}+\frac{6hx}{b}+\frac{2\sqrt{3}\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}b^{4/3}}+\frac{6d\log(x)}{a}+\frac{2\left(b^{5/3}c+a^{2/3}be-ab^{2/3}f-a^{5/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}b^{4/3}}+\frac{\left(-b^{5/3}c-a^{2/3}be+ab^{2/3}f+a^{5/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{4/3}b^{4/3}}+\frac{2(-bd+ag)\log(a+bx^3)}{ab}\right)$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x]
```

```
[Out] ((-6*c)/(a*x) + (6*h*x)/b + (2*Sqrt[3]*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)
*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(4/3)*b^(4/
3)) + (6*d*Log[x])/a + (2*(b^(5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*
h)*Log[a^(1/3) + b^(1/3)*x])/(a^(4/3)*b^(4/3)) + ((-(b^(5/3)*c) - a^(2/3)*b
*e + a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(a^(4/3)*b^(4/3)) + (2*(-(b*d) + a*g)*Log[a + b*x^3])/(a*b))/6
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.03

method	result
default	$\frac{hx}{b}-\frac{c}{ax}+\frac{d\ln(x)}{a}+\frac{(-a^2h+aeb)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(afb-b^2c)\left(-\frac{\ln}{}\right.}{ba}$
risch	$\frac{hx}{b}-\frac{c}{ax}+\frac{d\ln(-x)}{a}+\frac{\_R=\text{RootOf}\left(a^4b\_Z^3+\left(-3a^4bg+3a^3d\,b^2\right)\_Z^2+\left(-3a^4bfh+3a^4b\,g^2+3a^3b^2ch-6a^3b^2dg+3a^3b^2ef-3a^2b^3ce+3a^2\right.\right.}{}$

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] h*x/b-c/a/x+d*ln(x)/a+((-a^2*h+a*b*e)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(a*b*f-b^2*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(a*b*g-b^2*d)*ln(b*x^3+a)/b)/b/a

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 85.94 (sec) , antiderivative size = 15238, normalized size of antiderivative = 60.23

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\,(a+bx^3)}\,dx=\text{Too large to display}$$

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\,(a+bx^3)}\,dx=\text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.15

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\,(a+bx^3)}\,dx\\
&=\frac{hx}{b}+\frac{d\log(x)}{a}\\
&\quad-\frac{\sqrt{3}\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b}-\frac{c}{ax}\\
&\quad-\frac{\left(2\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+abe-a^2h\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\
&\quad-\frac{\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-abe+a^2h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}
\end{aligned}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="maxim
a")
[Out] h*x/b + d*log(x)/a - 1/3*sqrt(3)*(b^2*c*(a/b)^(2/3) - a*b*f*(a/b)^(2/3) - a
*b*e*(a/b)^(1/3) + a^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/(a^2*b) - c/(a*x) - 1/6*(2*b^2*d*(a/b)^(2/3) - 2*a*b*g*(a/b)
^(2/3) + b^2*c*(a/b)^(1/3) - a*b*f*(a/b)^(1/3) + a*b*e - a^2*h)*log(x^2 - x
*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) - 1/3*(b^2*d*(a/b)^(2/3) -
a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^(1/3) + a*b*f*(a/b)^(1/3) - a*b*e + a^2*h)*
log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.08

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx$$
$$= \frac{hx}{b} + \frac{d\log(|x|)}{a} - \frac{\sqrt{3}\left(abe - a^2h + (-ab^2)^{\frac{1}{3}}bc - (-ab^2)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(-ab^2)^{\frac{2}{3}}a} - \frac{\left(abe - a^2h - (-ab^2)^{\frac{1}{3}}bc + (-ab^2)^{\frac{1}{3}}af\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(-ab^2)^{\frac{2}{3}}a} - \frac{(bd-ag)\log(|bx^3+a|)}{3ab} - \frac{c}{ax} + \frac{\left(ab^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2b^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2b^3e + a^3b^2h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b^3}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] h*x/b + d*log(abs(x))/a - 1/3*sqrt(3)*(a*b*e - a^2*h + (-a*b^2)^(1/3)*b*c -
 (-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/
((-a*b^2)^(2/3)*a) - 1/6*(a*b*e - a^2*h - (-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/
3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/3*(
b*d - a*g)*log(abs(b*x^3 + a))/(a*b) - c/(a*x) + 1/3*(a*b^4*c*(-a/b)^(1/3)
- a^2*b^3*f*(-a/b)^(1/3) - a^2*b^3*e + a^3*b^2*h)*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/(a^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 1802, normalized size of antiderivative = 7.12

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x)
```

```
[Out] symsum(log((b^3*c*d^2 + a^3*d*h^2 + a*b^2*d*e^2 - a*b^2*d^2*f - a*b^2*c*d*g
 - 2*a^2*b*d*e*h + a^2*b*d*f*g)/a - root(27*a^4*b^4*z^3 - 27*a^4*b^3*g*z^2
+ 27*a^3*b^4*d*z^2 - 9*a^4*b^2*f*h*z - 18*a^3*b^3*d*g*z + 9*a^3*b^3*e*f*z +
 9*a^3*b^3*c*h*z - 9*a^2*b^4*c*e*z + 9*a^4*b^2*g^2*z + 9*a^2*b^4*d^2*z + 3*
a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2
```

```
*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^
2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3
*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^
3 + a^5*h^3, z, k)*(root(27*a^4*b^4*z^3 - 27*a^4*b^3*g*z^2 + 27*a^3*b^4*d*z
^2 - 9*a^4*b^2*f*h*z - 18*a^3*b^3*d*g*z + 9*a^3*b^3*e*f*z + 9*a^3*b^3*c*h*z
 - 9*a^2*b^4*c*e*z + 9*a^4*b^2*g^2*z + 9*a^2*b^4*d^2*z + 3*a^4*b*f*g*h - 3*
a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b
^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*
f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 -
 a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z,
k)*((3*a^2*b^3*c - 3*a^3*b^2*f)/a + (x*(24*a^3*b^4*d - 33*a^4*b^3*g))/(a^2*
b) + 36*root(27*a^4*b^4*z^3 - 27*a^4*b^3*g*z^2 + 27*a^3*b^4*d*z^2 - 9*a^4*b
^2*f*h*z - 18*a^3*b^3*d*g*z + 9*a^3*b^3*e*f*z + 9*a^3*b^3*c*h*z - 9*a^2*b^4
*c*e*z + 9*a^4*b^2*g^2*z + 9*a^2*b^4*d^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e
- 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3
*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^
2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3
 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z, k)*a^2*b^3*x
) + (a^4*h^2 + a^2*b^2*e^2 - 2*a*b^3*c*d - 2*a^3*b*e*h + a^3*b*f*g - a^2*b^
2*c*g + 2*a^2*b^2*d*f)/a + (x*(4*a^2*b^4*d^2 + 10*a^4*b^2*g^2 - 10*a^2*b^4*
c*e + 10*a^3*b^3*c*h - 14*a^3*b^3*d*g + 10*a^3*b^3*e*f - 10*a^4*b^2*f*h))/(
a^2*b)) + (x*(b^5*c^3 - a^5*h^3 + a^4*b*g^3 + a^2*b^3*e^3 - a^3*b^2*f^3 + 3
*a^2*b^3*c*f^2 + a^2*b^3*d^2*g - 2*a^3*b^2*d*g^2 - 3*a^3*b^2*e^2*h - 3*a*b^
4*c^2*f + 3*a^4*b*e*h^2 - 2*a^2*b^3*c*d*h - 3*a^2*b^3*c*e*g - 2*a^2*b^3*d*e
*f + 3*a^3*b^2*c*g*h + 2*a^3*b^2*d*f*h + 3*a^3*b^2*e*f*g + 2*a*b^4*c*d*e -
3*a^4*b*f*g*h))/(a^2*b))*root(27*a^4*b^4*z^3 - 27*a^4*b^3*g*z^2 + 27*a^3*b^
4*d*z^2 - 9*a^4*b^2*f*h*z - 18*a^3*b^3*d*g*z + 9*a^3*b^3*e*f*z + 9*a^3*b^3*
c*h*z - 9*a^2*b^4*c*e*z + 9*a^4*b^2*g^2*z + 9*a^2*b^4*d^2*z + 3*a^4*b*f*g*h
 - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*
a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4
*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*
f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3
, z, k), k, 1, 3) + (h*x)/b - c/(a*x) + (d*log(x))/a
```

3.410 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 260

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx = -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}b^{2/3}}+\frac{e\log(x)}{a}-\frac{\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}b^{2/3}}+\frac{\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}b^{2/3}}-\frac{(be-ah)\log(a+bx^3)}{3ab}$$

```
[Out] -1/2*c/a/x^2-d/a/x+e*ln(x)/a-1/3*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*g+b*d))*ln
(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(2/3)+1/6*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*g+b
*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(2/3)-1/3*(-a*h+b*
e)*ln(b*x^3+a)/a/b+1/3*(b^(4/3)*c+a^(1/3)*b*d-a*b^(1/3)*f-a^(4/3)*g)*arctan
(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx$$
$$=\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}a^{5/3}b^{2/3}}$$
$$+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}-af+bc\right)}{6a^{5/3}\sqrt[3]{b}}$$
$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{5/3}b^{2/3}}$$
$$-\frac{(be-ah)\log(a+bx^3)}{3ab}-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*c/(a*x^2) - d/(a*x) + ((b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f - a^(4/3
)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*b^
(2/3)) + (e*Log[x])/a - ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^
(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(2/3)) + ((b*c - a*f - (a^(1/3)*(b*d - a*g
))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(1
/3)) - ((b*e - a*h)*Log[a + b*x^3])/(3*a*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{c}{ax^3} + \frac{d}{ax^2} + \frac{e}{ax} + \frac{-bc + af - (bd - ag)x - (be - ah)x^2}{a\,(a + bx^3)}\right) dx$$

$$= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e\log(x)}{a} + \frac{\int \frac{-bc+af-(bd-ag)x-(be-ah)x^2}{a+bx^3}\,dx}{a}$$

$$= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e\log(x)}{a} + \frac{\int \frac{-bc+af+(-bd+ag)x}{a+bx^3}\,dx}{a} + \frac{(-be+ah)\int \frac{x^2}{a+bx^3}\,dx}{a}$$

$$= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e\log(x)}{a} - \frac{(be-ah)\log(a+bx^3)}{3ab} + \frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}(-bc+af)+\sqrt[3]{a}(-bd+ag)\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}(-bc+af)+\sqrt[3]{a}(-bd+ag)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}\sqrt[3]{b}} - \frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{5/3}}$$

$$= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e\log(x)}{a} - \frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}} - \frac{(be-ah)\log(a+bx^3)}{3ab} - \frac{\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{4/3}\sqrt[3]{b}} + \frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{5/3}\sqrt[3]{b}}$$

$$= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e\log(x)}{a} - \frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}} + \frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}} - \frac{(be-ah)\log(a+bx^3)}{3ab} - \frac{\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{5/3}b^{2/3}}$$

$$= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}b^{2/3}} + \frac{e\log(x)}{a} - \frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}} + \frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}} - \frac{(be-ah)\log(a+bx^3)}{3ab}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx$$

$$= \frac{-\frac{3a^{2/3}c}{x^2}-\frac{6a^{2/3}d}{x}+\frac{2\sqrt{3}\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+6a^{2/3}e\log(x)-\frac{2\left(b^{4/3}c-\sqrt[3]{a}bd-a\sqrt[3]{b}f+a^{4/3}\right.}{b^{2/3}}}{6a^{5/3}}$$

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x]

[Out] ((-3*a^(2/3)*c)/x^2 - (6*a^(2/3)*d)/x + (2*Sqrt[3]*(b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 6*a^(2/3)*e*Log[x] - (2*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + (2*a^(2/3)*(-(b*e) + a*h)*Log[a + b*x^3])/b)/(6*a^(5/3))

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.97

method	result
default	$-\frac{c}{2a\,x^2}-\frac{d}{ax}+\frac{e\ln(x)}{a}+\frac{(af-bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(ag-bd)\left(-\frac{\ln(\ldots}{\ldots}\right.}{a}$
risch	$\frac{-\frac{xd}{a}-\frac{c}{2a}}{x^2}+\frac{e\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^5b^3\_Z^3+\left(-3a^5b^2h+3a^4b^3e\right)\_Z^2+\left(3a^5bh^2-6a^4b^2eh+3a^4b^2fg-3a^3b^3cg-3a^3b^3df+3a^3b^3e^2+3a\ldots\right.\right.}\right.}{\ldots}$

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/2*c/a/x^2-d/a/x+e*ln(x)/a+((a*f-b*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(a*g-b*d)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*

```
3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(a*h-b*e
)*ln(b*x^3+a)/b)/a
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 45.13 (sec) , antiderivative size = 15424, normalized size of antiderivative = 59.32

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="frica
s")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.04

$$\begin{aligned}&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx\\&=\frac{e\log(x)}{a}-\frac{\sqrt{3}\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b}\\&\quad-\frac{\left(2be\left(\frac{a}{b}\right)^{\frac{2}{3}}-2ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-bc+af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\&\quad-\frac{\left(be\left(\frac{a}{b}\right)^{\frac{2}{3}}-ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+bc-af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2dx+c}{2ax^2}\end{aligned}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] e*log(x)/a - 1/3*sqrt(3)*(b^2*d*(a/b)^(2/3) - a*b*g*(a/b)^(2/3) + b^2*c*(a/
b)^(1/3) - a*b*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/(a^2*b) - 1/6*(2*b*e*(a/b)^(2/3) - 2*a*h*(a/b)^(2/3) + b*d*(a/b)^(1/
3) - a*g*(a/b)^(1/3) - b*c + a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
*b*(a/b)^(2/3)) - 1/3*(b*e*(a/b)^(2/3) - a*h*(a/b)^(2/3) - b*d*(a/b)^(1/3)
+ a*g*(a/b)^(1/3) + b*c - a*f)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3)) - 1/2
*(2*d*x + c)/(a*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.03

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx = \frac{e\log(|x|)}{a} + \frac{\sqrt{3}\left(b^2c-abf-(-ab^2)^{\frac{1}{3}}bd+(-ab^2)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(-ab^2)^{\frac{2}{3}}a} + \frac{\left(b^2c-abf+(-ab^2)^{\frac{1}{3}}bd-(-ab^2)^{\frac{1}{3}}ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(-ab^2)^{\frac{2}{3}}a} - \frac{(be-ah)\log(|bx^3+a|)}{3ab} + \frac{\left(ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2bg\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^2c-a^2bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b} - \frac{2dx+c}{2ax^2}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] e*log(abs(x))/a + 1/3*sqrt(3)*(b^2*c - a*b*f - (-a*b^2)^(1/3)*b*d + (-a*b^2
)^(1/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2
)^(2/3)*a) + 1/6*(b^2*c - a*b*f + (-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*g)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/3*(b*e - a*
h)*log(abs(b*x^3 + a))/(a*b) + 1/3*(a*b^2*d*(-a/b)^(1/3) - a^2*b*g*(-a/b)^(
1/3) + a*b^2*c - a^2*b*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) -
 1/2*(2*d*x + c)/(a*x^2)
```

Mupad [B] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 6948, normalized size of antiderivative = 26.72

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^3 (a + bx^3)} \, dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x)
[Out] symsum(log(-(b^5*c^3*x - a^5*h^3*x - a^2*b^3*d*e^2 + 36*root(27*a^5*b^3*z^3
 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z
 - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*
a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2
*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*
d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3
*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*
b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)^3*a^5*b^3*x - a^3*b^2*e*f^2 + a^3*b^2*
e^2*g - a^3*b^2*f^3*x - a*b^4*c^2*e - a*b^4*d^3*x + a^4*b*g^3*x + root(27*a
^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4
*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*
h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g +
 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*
a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2
*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*
h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^2*b^4*c^2 + 3*root(27*a^5*
b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^
2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2
*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*
a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2
*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*
g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3
 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)^2*a^4*b^3*d + root(27*a^5*b^3*z
^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g
*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z +
9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b
^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*
c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 +
 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^
2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^4*b^2*f^2 - 3*root(27*a^5*b^3*z^3
- 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z
- 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a
^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*
d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d
*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*
a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b
```

^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)^2*a^5*b^2*g + 2*a^2*b^3*c*e*f + a^3*b^2
*d*e*h + 10*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*
a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2
*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*
e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f -
3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*
b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f
^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^5*b*h
^2*x - 3*a*b^4*c^2*f*x + 2*a^4*b*e*h^2*x + 4*root(27*a^5*b^3*z^3 - 27*a^5*b
^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^
3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2
*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*
a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4
*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^
2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a
^4*b*g^3 + b^5*c^3, z, k)*a^3*b^3*e^2*x + 24*root(27*a^5*b^3*z^3 - 27*a^5*b
^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^
3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2
*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*
a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4
*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^
2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a
^4*b*g^3 + b^5*c^3, z, k)^2*a^4*b^3*e*x - 33*root(27*a^5*b^3*z^3 - 27*a^5*b
^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^
3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2
*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*
a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4
*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^
2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a
^4*b*g^3 + b^5*c^3, z, k)^2*a^5*b^2*h*x + 3*a^2*b^3*c*f^2*x + 3*a^2*b^3*d^2
*g*x - 3*a^3*b^2*d*g^2*x - a^3*b^2*e^2*h*x + root(27*a^5*b^3*z^3 - 27*a^5*b
^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^
3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2
*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*
a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4
*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^
2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a
^4*b*g^3 + b^5*c^3, z, k)*a^5*b*g*h - a^4*b*e*g*h - 2*root(27*a^5*b^3*z^3 -
27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z -
9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^
3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d
*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*
h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a
^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^
3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^3*b^3*c*f - 2*root(27*a^5*b^3*z^3 - 27

*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^3*b^3*d*e - root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^4*b^2*d*h + 2*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^4*b^2*e*g + 2*a*b^4*c*d*e*x - 3*a^4*b*f*g*h*x + 10*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^2*b^4*c*d*x - 10*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^3*b^3*c*g*x - 10*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^3*b^3*d*f*x - 14*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*

```
b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3
*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h
- 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4
*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^4*b^2*e*h*x + 1
0*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*
h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z
 + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*
b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3
*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h
- 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4
*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^4*b^2*f*g*x - 3
*a^2*b^3*c*d*h*x - 2*a^2*b^3*c*e*g*x - 2*a^2*b^3*d*e*f*x + 3*a^3*b^2*c*g*h*
x + 3*a^3*b^2*d*f*h*x + 2*a^3*b^2*e*f*g*x)/a^3)*root(27*a^5*b^3*z^3 - 27*a^
5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3
*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*
e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h +
 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*
a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3
*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3
+ a^4*b*g^3 + b^5*c^3, z, k), k, 1, 3) - c/(2*a*x^2) - d/(a*x) + (e*log(x))
/a
```

3.411 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 276

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx \\
&= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}+\frac{\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}b^{2/3}} \\
&\quad-\frac{(bc-af)\log(x)}{a^2}-\frac{\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}b^{2/3}} \\
&\quad+\frac{\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}b^{2/3}} \\
&\quad+\frac{(bc-af)\log(a+bx^3)}{3a^2}
\end{aligned}$$

```
[Out] -1/3*c/a/x^3-1/2*d/a/x^2-e/a/x-(-a*f+b*c)*ln(x)/a^2-1/3*(b^(1/3)*(-a*g+b*d)
-a^(1/3)*(-a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(2/3)+1/6*(b^(1/3)*(-a
*g+b*d)-a^(1/3)*(-a*h+b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/
3)/b^(2/3)+1/3*(-a*f+b*c)*ln(b*x^3+a)/a^2+1/3*(b^(4/3)*d+a^(1/3)*b*e-a*b^(1
/3)*g-a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/
b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx$$
$$=\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}a^{5/3}b^{2/3}}$$
$$+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}-ag+bd\right)}{6a^{5/3}\sqrt[3]{b}}$$
$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{5/3}b^{2/3}}$$
$$+\frac{(bc-af)\log\left(a+bx^3\right)}{3a^2}-\frac{\log(x)(bc-af)}{a^2}-\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*c/(a*x^3) - d/(2*a*x^2) - e/(a*x) + ((b^(4/3)*d + a^(1/3)*b*e - a*b^(1
/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*a^(5/3)*b^(2/3)) - ((b*c - a*f)*Log[x])/a^2 - ((b^(1/3)*(b*d - a*g) - a
^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(2/3)) + ((b*d -
 a*g - (a^(1/3)*(b*e - a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(6*a^(5/3)*b^(1/3)) + ((b*c - a*f)*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{c}{ax^4}+\frac{d}{ax^3}+\frac{e}{ax^2}+\frac{-bc+af}{a^2x} + \frac{-a(bd-ag)-a(be-ah)x+b(bc-af)x^2}{a^2\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{3ax^3} - \frac{d}{2ax^2} - \frac{e}{ax} - \frac{(bc-af)\log(x)}{a^2} + \frac{\int \frac{-a(bd-ag)-a(be-ah)x+b(bc-af)x^2}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{c}{3ax^3} - \frac{d}{2ax^2} - \frac{e}{ax} - \frac{(bc-af)\log(x)}{a^2} + \frac{\int \frac{-a(bd-ag)-a(be-ah)x}{a+bx^3}\,dx}{a^2} + \frac{(b(bc-af))\int \frac{x^2}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{c}{3ax^3} - \frac{d}{2ax^2} - \frac{e}{ax} - \frac{(bc-af)\log(x)}{a^2} + \frac{(bc-af)\log\left(a+bx^3\right)}{3a^2} + \frac{\int \frac{\sqrt[3]{a}\left(-2a\sqrt[3]{b}(bd-ag)-a^{4/3}(be-ah)\right)+\sqrt[3]{b}\left(a\sqrt[3]{b}(bd-ag)-a^{4/3}(be-ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{8/3}\sqrt[3]{b}} - \frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{5/3}}$$

$$= -\frac{c}{3ax^3} - \frac{d}{2ax^2} - \frac{e}{ax} - \frac{(bc-af)\log(x)}{a^2} - \frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}} + \frac{(bc-af)\log\left(a+bx^3\right)}{3a^2} - \frac{\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{4/3}\sqrt[3]{b}} + \frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{5/3}\sqrt[3]{b}}$$

$$= -\frac{c}{3ax^3} - \frac{d}{2ax^2} - \frac{e}{ax} - \frac{(bc-af)\log(x)}{a^2} - \frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}} + \frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}} + \frac{(bc-af)\log\left(a+bx^3\right)}{3a^2} - \frac{\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{5/3}b^{2/3}}$$

$$= -\frac{c}{3ax^3} - \frac{d}{2ax^2} - \frac{e}{ax} + \frac{\left(b^{4/3}d + \sqrt[3]{a}be - a\sqrt[3]{b}g - a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}b^{2/3}} - \frac{(bc-af)\log(x)}{a^2} - \frac{\left(bd - ag - \frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}} + \frac{\left(bd - ag - \frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}} + \frac{(bc-af)\log\left(a+bx^3\right)}{3a^2}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)}\,dx = -\frac{\frac{2ac}{x^3}+\frac{3ad}{x^2}+\frac{6ae}{x}+\frac{2\sqrt{3}\sqrt[3]{a}\left(-b^{4/3}d-\sqrt[3]{a}be+a\sqrt[3]{b}g+a^{4/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+6(bc-af)\log(x)+\frac{2\sqrt[3]{a}\left(b^{4/3}d-\sqrt[3]{a}\right.}{}}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/6*((2*a*c)/x^3 + (3*a*d)/x^2 + (6*a*e)/x + (2*Sqrt[3]*a^(1/3)*(-(b^(4/3)
*d) - a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(2/3) + 6*(b*c - a*f)*Log[x] + (2*a^(1/3)*(b^(4/3)*d - a^(
1/3)*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^
(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 2*(b*c - a*f)*Log[a + b*x^3])/a^2
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00

method	result
default	$-\frac{e}{ax}-\frac{c}{3a\,x^{3}}-\frac{d}{2a\,x^{2}}+\frac{(af-bc)\ln(x)}{a^{2}}+\frac{(a^{2}g-abd)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}$
risch	$\frac{-\frac{e\,x^{2}}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{6}b^{2}\_Z^{3}+\left(3a^{5}b^{2}f-3a^{4}b^{3}c\right)\_Z^{2}+\left(3a^{5}bgh-3a^{4}b^{2}dh-3a^{4}b^{2}eg+3a^{4}b^{2}f^{2}-6a^{3}b^{3}cf+3a^{3}b^{3}de+3a^{2}b^{4}c^{2}\right)\right.}\right.}{}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -e/a/x-1/3*c/a/x^3-1/2*d/a/x^2+(a*f-b*c)/a^2*ln(x)+((a^2*g-a*b*d)*(1/3/b/(a
/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(a^
2*h-a*b*e)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(
a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1)))+1/3*(-a*b*f+b^2*c)*ln(b*x^3+a)/b)/a^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 74.15 (sec) , antiderivative size = 15204, normalized size of antiderivative = 55.09

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\,(a+bx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)}\,dx = \text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.09

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)}\,dx \\
&= -\frac{(bc-af)\log(x)}{a^2} \\
&\quad -\frac{\sqrt{3}\left(abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+abd\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3} \\
&\quad +\frac{\left(2b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+abd-a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\
&\quad +\frac{\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd+a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\
&\quad -\frac{6ex^2+3dx+2c}{6ax^3}
\end{aligned}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="maxim
a")
[Out] -(b*c - a*f)*log(x)/a^2 - 1/3*sqrt(3)*(a*b*e*(a/b)^(2/3) - a^2*h*(a/b)^(2/3
) + a*b*d*(a/b)^(1/3) - a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/a^3 + 1/6*(2*b^2*c*(a/b)^(2/3) - 2*a*b*f*(a/b)^(2/3) -
a*b*e*(a/b)^(1/3) + a^2*h*(a/b)^(1/3) + a*b*d - a^2*g)*log(x^2 - x*(a/b)^(1
/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) + 1/3*(b^2*c*(a/b)^(2/3) - a*b*f*(a/
b)^(2/3) + a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3) - a*b*d + a^2*g)*log(x + (
a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) - 1/6*(6*e*x^2 + 3*d*x + 2*c)/(a*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx$$
$$= \frac{\sqrt{3}\left(b^2d-abg-(-ab^2)^{\frac{1}{3}}be+(-ab^2)^{\frac{1}{3}}ah\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}a}$$
$$+ \frac{\left(b^2d-abg+(-ab^2)^{\frac{1}{3}}be-(-ab^2)^{\frac{1}{3}}ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}a}$$
$$+ \frac{(bc-af)\log\left(|bx^3+a|\right)}{3a^2} - \frac{(bc-af)\log\left(|x|\right)}{a^2}$$
$$+ \frac{\left(a^3b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^4bh\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^3b^2d-a^4bg\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^5b}$$
$$- \frac{6aex^2+3adx+2ac}{6a^2x^3}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] 1/3*sqrt(3)*(b^2*d - a*b*g - (-a*b^2)^(1/3)*b*e + (-a*b^2)^(1/3)*a*h)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) + 1/6*(
b^2*d - a*b*g + (-a*b^2)^(1/3)*b*e - (-a*b^2)^(1/3)*a*h)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) + 1/3*(b*c - a*f)*log(abs(b*x^3 +
 a))/a^2 - (b*c - a*f)*log(abs(x))/a^2 + 1/3*(a^3*b^2*e*(-a/b)^(1/3) - a^4*
b*h*(-a/b)^(1/3) + a^3*b^2*d - a^4*b*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/
3)))/(a^5*b) - 1/6*(6*a*e*x^2 + 3*a*d*x + 2*a*c)/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 1842, normalized size of antiderivative = 6.67

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x)
```

```
[Out] symsum(log(- (b^5*c*d^2 - b^5*c^2*e + a^2*b^3*c*g^2 - a^2*b^3*e*f^2 - a^3*b
^2*f*g^2 + a^3*b^2*f^2*h - a*b^4*d^2*f + a*b^4*c^2*h - 2*a^2*b^3*c*f*h + 2*
a^2*b^3*d*f*g - 2*a*b^4*c*d*g + 2*a*b^4*c*e*f)/a^3 - root(27*a^6*b^2*z^3 +
```

27*a^5*b^2*f*z^2 - 27*a^4*b^3*c*z^2 + 9*a^5*b*g*h*z - 9*a^4*b^2*e*g*z - 9*a^4*b^2*d*h*z - 18*a^3*b^3*c*f*z + 9*a^3*b^3*d*e*z + 9*a^4*b^2*f^2*z + 9*a^2*b^4*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z, k)*((a^2*b^4*d^2 + a^4*b^2*g^2 + 2*a^2*b^4*c*e - 2*a^3*b^3*c*h - 2*a^3*b^3*d*g - 2*a^3*b^3*e*f + 2*a^4*b^2*f*h)/a^3 + root(27*a^6*b^2*z^3 + 27*a^5*b^2*f*z^2 - 27*a^4*b^3*c*z^2 + 9*a^5*b*g*h*z - 9*a^4*b^2*e*g*z - 9*a^4*b^2*d*h*z - 18*a^3*b^3*c*f*z + 9*a^3*b^3*d*e*z + 9*a^4*b^2*f^2*z + 9*a^2*b^4*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z, k)*((3*a^4*b^3*e - 3*a^5*b^2*h)/a^3 - (x*(24*a^3*b^4*c - 24*a^4*b^3*f))/a^3 + 36*root(27*a^6*b^2*z^3 + 27*a^5*b^2*f*z^2 - 27*a^4*b^3*c*z^2 + 9*a^5*b*g*h*z - 9*a^4*b^2*e*g*z - 9*a^4*b^2*d*h*z - 18*a^3*b^3*c*f*z + 9*a^3*b^3*d*e*z + 9*a^4*b^2*f^2*z + 9*a^2*b^4*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z, k)*a^2*b^3*x) + (x*(4*a*b^5*c^2 + 4*a^3*b^3*f^2 - 8*a^2*b^4*c*f + 10*a^2*b^4*d*e - 10*a^3*b^3*d*h - 10*a^3*b^3*e*g + 10*a^4*b^2*g*h))/a^3) - (x*(b^5*d^3 - a*b^4*e^3 + a^4*b*h^3 - a^3*b^2*g^3 + 3*a^2*b^3*d*g^2 + 3*a^2*b^3*e^2*h - 3*a^3*b^2*e*h^2 - 2*b^5*c*d*e - 3*a*b^4*d^2*g - 2*a^2*b^3*c*g*h - 2*a^2*b^3*d*f*h - 2*a^2*b^3*e*f*g + 2*a^3*b^2*f*g*h + 2*a*b^4*c*d*h + 2*a*b^4*c*e*g + 2*a*b^4*d*e*f))/a^3)*root(27*a^6*b^2*z^3 + 27*a^5*b^2*f*z^2 - 27*a^4*b^3*c*z^2 + 9*a^5*b*g*h*z - 9*a^4*b^2*e*g*z - 9*a^4*b^2*d*h*z - 18*a^3*b^3*c*f*z + 9*a^3*b^3*d*e*z + 9*a^4*b^2*f^2*z + 9*a^2*b^4*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z, k), k, 1, 3) - (c/(3*a) + (e*x^2)/a + (d*x)/(2*a))/x^3 - (log(x)*(b*c - a*f))/a^2

3.412 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 38, antiderivative size = 337

$$\begin{aligned}
&\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2} dx \\
&= \frac{(be-2ah)x}{b^3}+\frac{fx^2}{2b^2}+\frac{gx^3}{3b^2}+\frac{hx^4}{4b^2}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3b^3(a+bx^3)} \\
&-\frac{\left(2b^{5/3}c-4a^{2/3}be-5ab^{2/3}f+7a^{5/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{10/3}} \\
&-\frac{\left(b^{2/3}(2bc-5af)+a^{2/3}(4be-7ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{10/3}} \\
&+\frac{\left(b^{2/3}(2bc-5af)+a^{2/3}(4be-7ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{10/3}} \\
&+\frac{(bd-2ag)\log(a+bx^3)}{3b^3}
\end{aligned}$$

```
[Out] (-2*a*h+b*e)*x/b^3+1/2*f*x^2/b^2+1/3*g*x^3/b^2+1/4*h*x^4/b^2+1/3*x*(a*(-a*h
+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/b^3/(b*x^3+a)-1/9*(b^(2/3)*(-5*a*f+2
*b*c)+a^(2/3)*(-7*a*h+4*b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(10/3)+1/18*(
b^(2/3)*(-5*a*f+2*b*c)+a^(2/3)*(-7*a*h+4*b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/a^(1/3)/b^(10/3)+1/3*(-2*a*g+b*d)*ln(b*x^3+a)/b^3-1/9*(2*b^(5
/3)*c-4*a^(2/3)*b*e-5*a*b^(2/3)*f+7*a^(5/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3
)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1842, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-4a^{2/3}be+7a^{5/3}h-5ab^{2/3}f+2b^{5/3}c\right)}{3\sqrt{3}\sqrt[3]{a}b^{10/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{18\sqrt[3]{a}b^{10/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{9\sqrt[3]{a}b^{10/3}} + \frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{3b^3(a+bx^3)} + \frac{(bd-2ag)\log(a+bx^3)}{3b^3} + \frac{x(be-2ah)}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2}$$

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*h)*x)/b^3 + (f*x^2)/(2*b^2) + (g*x^3)/(3*b^2) + (h*x^4)/(4*b^2)
 + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*b^3*(a + b*
x^3)) - ((2*b^(5/3)*c - 4*a^(2/3)*b*e - 5*a*b^(2/3)*f + 7*a^(5/3)*h)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(10/3)) -
 ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*a*h))*Log[a^(1/3) + b^(1/3)
*x])/(9*a^(1/3)*b^(10/3)) + ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*
a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(10/3))
 + ((b*d - 2*a*g)*Log[a + b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
```

```
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3b^3\left(a+bx^3\right)} \\
&\quad -\frac{\int \frac{a^2(be-ah)-2ab(bc-af)x-3ab(bd-ag)x^2-3ab(be-ah)x^3-3ab^2fx^4-3ab^2gx^5-3ab^2hx^6}{a+bx^3}\,dx}{3ab^3} \\
&= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3b^3\left(a+bx^3\right)} \\
&\quad -\frac{\int \left(-3a(be-2ah)-3abfx-3abgx^2-3abhx^3+\frac{a^2(4be-7ah)-ab(2bc-5af)x-3ab(bd-2ag)x^2}{a+bx^3}\right)dx}{3ab^3} \\
&= \frac{(be-2ah)x}{b^3}+\frac{fx^2}{2b^2}+\frac{gx^3}{3b^2}+\frac{hx^4}{4b^2}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3b^3\left(a+bx^3\right)} \\
&\quad -\frac{\int \frac{a^2(4be-7ah)-ab(2bc-5af)x-3ab(bd-2ag)x^2}{a+bx^3}\,dx}{3ab^3} \\
&= \frac{(be-2ah)x}{b^3}+\frac{fx^2}{2b^2}+\frac{gx^3}{3b^2}+\frac{hx^4}{4b^2}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3b^3\left(a+bx^3\right)} \\
&\quad -\frac{\int \frac{a^2(4be-7ah)-ab(2bc-5af)x}{a+bx^3}\,dx}{3ab^3}+\frac{(bd-2ag)\int \frac{x^2}{a+bx^3}\,dx}{b^2} \\
&= \frac{(be-2ah)x}{b^3}+\frac{fx^2}{2b^2}+\frac{gx^3}{3b^2}+\frac{hx^4}{4b^2} \\
&\quad +\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3b^3\left(a+bx^3\right)}+\frac{(bd-2ag)\log\left(a+bx^3\right)}{3b^3} \\
&\quad -\frac{\int \frac{\sqrt[3]{a}\left(-a^{4/3}b(2bc-5af)+2a^2\sqrt[3]{b}(4be-7ah)\right)+\sqrt[3]{b}\left(-a^{4/3}b(2bc-5af)-a^2\sqrt[3]{b}(4be-7ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b^{10/3}} \\
&\quad -\frac{\left(b^{2/3}(2bc-5af)+a^{2/3}(4be-7ah)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^3}
\end{aligned}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3b^3(a+bx^3)}$$
$$- \frac{\left(b^{2/3}(2bc-5af)+a^{2/3}(4be-7ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{10/3}} + \frac{(bd-2ag)\log(a+bx^3)}{3b^3}$$
$$+ \frac{\left(2b^{5/3}c-4a^{2/3}be-5ab^{2/3}f+7a^{5/3}h\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^3}$$
$$+ \frac{\left(b^{2/3}(2bc-5af)+a^{2/3}(4be-7ah)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18\sqrt[3]{a}b^{10/3}}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3b^3(a+bx^3)}$$
$$- \frac{\left(b^{2/3}(2bc-5af)+a^{2/3}(4be-7ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{10/3}}$$
$$+ \frac{\left(b^{2/3}(2bc-5af)+a^{2/3}(4be-7ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{10/3}}$$
$$+ \frac{(bd-2ag)\log(a+bx^3)}{3b^3}$$
$$+ \frac{\left(2b^{5/3}c-4a^{2/3}be-5ab^{2/3}f+7a^{5/3}h\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}b^{10/3}}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3b^3(a+bx^3)}$$
$$- \frac{\left(2b^{5/3}c-4a^{2/3}be-5ab^{2/3}f+7a^{5/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{10/3}}$$
$$- \frac{\left(b^{2/3}(2bc-5af)+a^{2/3}(4be-7ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{10/3}}$$
$$+ \frac{\left(b^{2/3}(2bc-5af)+a^{2/3}(4be-7ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{10/3}}$$
$$+ \frac{(bd-2ag)\log(a+bx^3)}{3b^3}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.99

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$

$$= \frac{36b^{2/3}(be-2ah)x+18b^{5/3}fx^2+12b^{5/3}gx^3+9b^{5/3}hx^4-\frac{12b^{2/3}(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx)))}{a+bx^3}-\frac{4\sqrt{3}\left(2b^2c-4a^{2/3}b\right.}{}}{}$$

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(2/3)*(b*e - 2*a*h)*x + 18*b^(5/3)*f*x^2 + 12*b^(5/3)*g*x^3 + 9*b^(5/
3)*h*x^4 - (12*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x))))
/(a + b*x^3) - (4*Sqrt[3]*(2*b^2*c - 4*a^(2/3)*b^(4/3)*e - 5*a*b*f + 7*a^(5
/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (4*(-
2*b^2*c - 4*a^(2/3)*b^(4/3)*e + 5*a*b*f + 7*a^(5/3)*b^(1/3)*h)*Log[a^(1/3)
+ b^(1/3)*x])/a^(1/3) + (2*(2*b^2*c + 4*a^(2/3)*b^(4/3)*e - 5*a*b*f - 7*a^(
5/3)*b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3) + 1
2*b^(2/3)*(b*d - 2*a*g)*Log[a + b*x^3])/(36*b^(11/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.48

method	result
risch	$\frac{hx^4}{4b^2}+\frac{gx^3}{3b^2}+\frac{fx^2}{2b^2}-\frac{2ahx}{b^3}+\frac{ex}{b^2}+\frac{\left(\frac{1}{3}afb-\frac{1}{3}b^2c\right)x^2+\left(-\frac{1}{3}a^2h+\frac{1}{3}aeb\right)x-\frac{a(ag-bd)}{3}}{b^3(bx^3+a)}+\frac{\sum_{\_R=\text{RootOf}(b\_Z^3+a)}\left(3b(-2ag+bd)\_\right.}{}$
default	$-\frac{-\frac{1}{4}bhx^4-\frac{1}{3}bgx^3-\frac{1}{2}bfx^2+2ahx-bex}{b^3}+\frac{\left(\frac{1}{3}afb-\frac{1}{3}b^2c\right)x^2+\left(-\frac{1}{3}a^2h+\frac{1}{3}aeb\right)x-\frac{a(ag-bd)}{3}}{bx^3+a}+\frac{(7a^2h-4aeb)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right.\right.}{6b(}\right.}{}$

```
[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/4*h*x^4/b^2+1/3*g*x^3/b^2+1/2*f*x^2/b^2-2/b^3*a*h*x+1/b^2*e*x+((1/3*a*f*b
-1/3*b^2*c)*x^2+(-1/3*a^2*h+1/3*a*e*b)*x-1/3*a*(a*g-b*d))/b^3/(b*x^3+a)+1/9
/b^4*sum((3*b*(-2*a*g+b*d)*_R^2+b*(-5*a*f+2*b*c)*_R+7*a^2*h-4*a*e*b)/_R^2*l
n(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 16147, normalized size of antiderivative = 47.91

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.08

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \frac{abd-a^2g-(b^2c-abf)x^2+(abe-a^2h)x}{3\,(b^4x^3+ab^3)} + \frac{\sqrt{3}\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-5\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+7\,a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3} + \frac{3\,bhx^4+4\,bgx^3+6\,bfx^2+12\,(be-2\,ah)x}{12\,b^3} + \frac{\left(6\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-12\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+4\,abe-7\,a^2h\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(3\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-6\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-4\,abe+7\,a^2h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] 1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2 + (a*b*e - a^2*h)*x)/(b^4*x^3 + a*
b^3) + 1/9*sqrt(3)*(2*b^2*c*(a/b)^(2/3) - 5*a*b*f*(a/b)^(2/3) - 4*a*b*e*(a/
b)^(1/3) + 7*a^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(a*b^3) + 1/12*(3*b*h*x^4 + 4*b*g*x^3 + 6*b*f*x^2 + 12*(b*e - 2*a*
h)*x)/b^3 + 1/18*(6*b^2*d*(a/b)^(2/3) - 12*a*b*g*(a/b)^(2/3) + 2*b^2*c*(a/b
)^(1/3) - 5*a*b*f*(a/b)^(1/3) + 4*a*b*e - 7*a^2*h)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/9*(3*b^2*d*(a/b)^(2/3) - 6*a*b*g*(a/b)
^(2/3) - 2*b^2*c*(a/b)^(1/3) + 5*a*b*f*(a/b)^(1/3) - 4*a*b*e + 7*a^2*h)*log
(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.04

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \frac{\sqrt{3}\left(4\,abe-7\,a^2h+2\,(-ab^2)^{\frac{1}{3}}\,bc-5\,(-ab^2)^{\frac{1}{3}}\,af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,b^2} + \frac{\left(4\,abe-7\,a^2h-2\,(-ab^2)^{\frac{1}{3}}\,bc+5\,(-ab^2)^{\frac{1}{3}}\,af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,b^2} + \frac{(bd-2\,ag)\log\left(|bx^3+a|\right)}{3\,b^3} + \frac{abd-a^2g-(b^2c-abf)x^2+(abe-a^2h)x}{3\,(bx^3+a)b^3} - \frac{\left(2\,b^6c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^5f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,ab^5e+7\,a^2b^4h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^7} + \frac{3\,b^6hx^4+4\,b^6gx^3+6\,b^6fx^2+12\,b^6ex-24\,ab^5hx}{12\,b^8}$$

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] 1/9*sqrt(3)*(4*a*b*e - 7*a^2*h + 2*(-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a*
f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^
2) + 1/18*(4*a*b*e - 7*a^2*h - 2*(-a*b^2)^(1/3)*b*c + 5*(-a*b^2)^(1/3)*a*f)
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) + 1/3*(b*d -
 2*a*g)*log(abs(b*x^3 + a))/b^3 + 1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2
+ (a*b*e - a^2*h)*x)/((b*x^3 + a)*b^3) - 1/9*(2*b^6*c*(-a/b)^(1/3) - 5*a*b^
5*f*(-a/b)^(1/3) - 4*a*b^5*e + 7*a^2*b^4*h)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a*b^7) + 1/12*(3*b^6*h*x^4 + 4*b^6*g*x^3 + 6*b^6*f*x^2 + 12*b^6*e
*x - 24*a*b^5*h*x)/b^8
```

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 1241, normalized size of antiderivative = 3.68

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(729\,a\,b^{10}\,z^3-729\,a\,b^8\,d\,z^2+1458\,a^2\,b^7\,g\,z^2-216\,a\,b^6\,c\,e\,z-945\,a^3\,b^4\,f\,h\,z-972\,a^2\,b^5\,d\,g\,z+540\right.\right.\right.$$
$$+\frac{36\,a^3\,g^2+9\,a\,b^2\,d^2-35\,a^3\,f\,h-8\,a\,b^2\,c\,e+14\,a^2\,b\,c\,h-36\,a^2\,b\,d\,g+20\,a^2\,b\,e\,f}{9\,b^4}$$
$$\left.+\frac{x\,(4\,b^3\,c^2+25\,a^2\,b\,f^2+42\,a^3\,g\,h-20\,a\,b^2\,c\,f+12\,a\,b^2\,d\,e-21\,a^2\,b\,d\,h-24\,a^2\,b\,e\,g)}{9\,b^4}\right)\mathrm{root}\big(729\,a\,b^{10}\,z^3$$
$$-729\,a\,b^8\,d\,z^2+1458\,a^2\,b^7\,g\,z^2-216\,a\,b^6\,c\,e\,z-945\,a^3\,b^4\,f\,h\,z-972\,a^2\,b^5\,d\,g\,z$$
$$+540\,a^2\,b^5\,e\,f\,z+378\,a^2\,b^5\,c\,h\,z+243\,a\,b^6\,d^2\,z+972\,a^3\,b^4\,g^2\,z-630\,a^4\,b\,f\,g\,h$$
$$+72\,a\,b^4\,c\,d\,e+360\,a^3\,b^2\,e\,f\,g+315\,a^3\,b^2\,d\,f\,h+252\,a^3\,b^2\,c\,g\,h-180\,a^2\,b^3\,d\,e\,f$$
$$-144\,a^2\,b^3\,c\,e\,g-126\,a^2\,b^3\,c\,d\,h+588\,a^4\,b\,e\,h^2-60\,a\,b^4\,c^2\,f-336\,a^3\,b^2\,e^2\,h$$
$$-324\,a^3\,b^2\,d\,g^2+162\,a^2\,b^3\,d^2\,g+150\,a^2\,b^3\,c\,f^2-125\,a^3\,b^2\,f^3+64\,a^2\,b^3\,e^3+216\,a^4\,b\,g^3$$
$$\left.\left.-27\,a\,b^4\,d^3-343\,a^5\,h^3+8\,b^5\,c^3,z,k\big)\right)\right)$$
$$+x\left(\frac{e}{b^2}-\frac{2\,a\,h}{b^3}\right)-\frac{x\left(\frac{a^2\,h}{3}-\frac{a\,b\,e}{3}\right)+\frac{a^2\,g}{3}+x^2\left(\frac{b^2\,c}{3}-\frac{a\,b\,f}{3}\right)-\frac{a\,b\,d}{3}}{b^4\,x^3+a\,b^3}+\frac{f\,x^2}{2\,b^2}+\frac{g\,x^3}{3\,b^2}+\frac{h\,x^4}{4\,b^2}$$

```
[In] int((x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log(root(729*a*b^10*z^3 - 729*a*b^8*d*z^2 + 1458*a^2*b^7*g*z^2 - 216
*a*b^6*c*e*z - 945*a^3*b^4*f*h*z - 972*a^2*b^5*d*g*z + 540*a^2*b^5*e*f*z +
378*a^2*b^5*c*h*z + 243*a*b^6*d^2*z + 972*a^3*b^4*g^2*z - 630*a^4*b*f*g*h +
 72*a*b^4*c*d*e + 360*a^3*b^2*e*f*g + 315*a^3*b^2*d*f*h + 252*a^3*b^2*c*g*h
 - 180*a^2*b^3*d*e*f - 144*a^2*b^3*c*e*g - 126*a^2*b^3*c*d*h + 588*a^4*b*e*
h^2 - 60*a*b^4*c^2*f - 336*a^3*b^2*e^2*h - 324*a^3*b^2*d*g^2 + 162*a^2*b^3*
d^2*g + 150*a^2*b^3*c*f^2 - 125*a^3*b^2*f^3 + 64*a^2*b^3*e^3 + 216*a^4*b*g^
3 - 27*a*b^4*d^3 - 343*a^5*h^3 + 8*b^5*c^3, z, k)*((108*a^2*b^3*g - 54*a*b^
4*d)/(9*b^4) + (x*(63*a^2*b^3*h - 36*a*b^4*e))/(9*b^4) + 9*root(729*a*b^10*
z^3 - 729*a*b^8*d*z^2 + 1458*a^2*b^7*g*z^2 - 216*a*b^6*c*e*z - 945*a^3*b^4*
f*h*z - 972*a^2*b^5*d*g*z + 540*a^2*b^5*e*f*z + 378*a^2*b^5*c*h*z + 243*a*b
^6*d^2*z + 972*a^3*b^4*g^2*z - 630*a^4*b*f*g*h + 72*a*b^4*c*d*e + 360*a^3*b
^2*e*f*g + 315*a^3*b^2*d*f*h + 252*a^3*b^2*c*g*h - 180*a^2*b^3*d*e*f - 144*
a^2*b^3*c*e*g - 126*a^2*b^3*c*d*h + 588*a^4*b*e*h^2 - 60*a*b^4*c^2*f - 336*
a^3*b^2*e^2*h - 324*a^3*b^2*d*g^2 + 162*a^2*b^3*d^2*g + 150*a^2*b^3*c*f^2 -
 125*a^3*b^2*f^3 + 64*a^2*b^3*e^3 + 216*a^4*b*g^3 - 27*a*b^4*d^3 - 343*a^5*
h^3 + 8*b^5*c^3, z, k)*a*b^2) + (36*a^3*g^2 + 9*a*b^2*d^2 - 35*a^3*f*h - 8*
a*b^2*c*e + 14*a^2*b*c*h - 36*a^2*b*d*g + 20*a^2*b*e*f)/(9*b^4) + (x*(4*b^3
```

```
*c^2 + 25*a^2*b*f^2 + 42*a^3*g*h - 20*a*b^2*c*f + 12*a*b^2*d*e - 21*a^2*b*d
*h - 24*a^2*b*e*g))/(9*b^4))*root(729*a*b^10*z^3 - 729*a*b^8*d*z^2 + 1458*a
^2*b^7*g*z^2 - 216*a*b^6*c*e*z - 945*a^3*b^4*f*h*z - 972*a^2*b^5*d*g*z + 54
0*a^2*b^5*e*f*z + 378*a^2*b^5*c*h*z + 243*a*b^6*d^2*z + 972*a^3*b^4*g^2*z -
 630*a^4*b*f*g*h + 72*a*b^4*c*d*e + 360*a^3*b^2*e*f*g + 315*a^3*b^2*d*f*h +
 252*a^3*b^2*c*g*h - 180*a^2*b^3*d*e*f - 144*a^2*b^3*c*e*g - 126*a^2*b^3*c*
d*h + 588*a^4*b*e*h^2 - 60*a*b^4*c^2*f - 336*a^3*b^2*e^2*h - 324*a^3*b^2*d*
g^2 + 162*a^2*b^3*d^2*g + 150*a^2*b^3*c*f^2 - 125*a^3*b^2*f^3 + 64*a^2*b^3*
e^3 + 216*a^4*b*g^3 - 27*a*b^4*d^3 - 343*a^5*h^3 + 8*b^5*c^3, z, k), k, 1,
3) + x*(e/b^2 - (2*a*h)/b^3) - (x*((a^2*h)/3 - (a*b*e)/3) + (a^2*g)/3 + x^2
*((b^2*c)/3 - (a*b*f)/3) - (a*b*d)/3)/(a*b^3 + b^4*x^3) + (f*x^2)/(2*b^2) +
 (g*x^3)/(3*b^2) + (h*x^4)/(4*b^2)
```

3.413 $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 38, antiderivative size = 311

$$\begin{aligned}
&\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2} dx \\
&= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)} \\
&-\frac{\left(b^{4/3}c+2\sqrt[3]{a}bd-4a\sqrt[3]{b}f-5a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{8/3}} \\
&+\frac{\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{8/3}} \\
&-\frac{\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{8/3}} \\
&+\frac{(be-2ah)\log(a+bx^3)}{3b^3}
\end{aligned}$$

```
[Out] f*x/b^2+1/2*g*x^2/b^2+1/3*h*x^3/b^2-1/3*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*
x^2)/b^2/(b*x^3+a)+1/9*(b^(1/3)*(-4*a*f+b*c)-a^(1/3)*(-5*a*g+2*b*d))*ln(a^(
1/3)+b^(1/3)*x)/a^(2/3)/b^(8/3)-1/18*(b^(1/3)*(-4*a*f+b*c)-a^(1/3)*(-5*a*g+
2*b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(8/3)+1/3*(-2*a
*h+b*e)*ln(b*x^3+a)/b^3-1/9*(b^(4/3)*c+2*a^(1/3)*b*d-4*a*b^(1/3)*f-5*a^(4/3
)*g)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(8/3)*3^(1
/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1842, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^{4/3}g+2\sqrt[3]{a}bd-4a\sqrt[3]{b}f+b^{4/3}c\right)}{3\sqrt{3}a^{2/3}b^{8/3}}$$
$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)}{18a^{2/3}b^{8/3}}$$
$$+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)}{9a^{2/3}b^{8/3}}+\frac{(be-2ah)\log(a+bx^3)}{3b^3}$$
$$-\frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{3b^2(a+bx^3)}+\frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}$$

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (f*x)/b^2 + (g*x^2)/(2*b^2) + (h*x^3)/(3*b^2) - (x*(b*c - a*f + (b*d - a*g)
*x + (b*e - a*h)*x^2))/(3*b^2*(a + b*x^3)) - ((b^(4/3)*c + 2*a^(1/3)*b*d -
4*a*b^(1/3)*f - 5*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(3*Sqrt[3]*a^(2/3)*b^(8/3)) + ((b^(1/3)*(b*c - 4*a*f) - a^(1/3)*(2*b*d
 - 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(8/3)) - ((b^(1/3)*(b*c -
 4*a*f) - a^(1/3)*(2*b*d - 5*a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*a^(2/3)*b^(8/3)) + ((b*e - 2*a*h)*Log[a + b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)} - \frac{\int \frac{-ab(bc-af)-2ab(bd-ag)x-3ab(be-ah)x^2-3ab^2fx^3-3ab^2gx^4-3ab^2hx^5}{a+bx^3}\,dx}{3ab^3}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)} - \frac{\int \left(-3abf-3abgx-3abhx^2-\frac{ab(bc-4af)+ab(2bd-5ag)x+3ab(be-2ah)x^2}{a+bx^3}\right)dx}{3ab^3}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)} + \frac{\int \frac{ab(bc-4af)+ab(2bd-5ag)x+3ab(be-2ah)x^2}{a+bx^3}\,dx}{3ab^3}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)} + \frac{\int \frac{ab(bc-4af)+ab(2bd-5ag)x}{a+bx^3}\,dx}{3ab^3} + \frac{(be-2ah)\int \frac{x^2}{a+bx^3}\,dx}{b^2}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)}+\frac{(be-2ah)\log(a+bx^3)}{3b^3} + \frac{\int \frac{\sqrt[3]{a}(2ab^{4/3}(bc-4af)+a^{4/3}b(2bd-5ag))+\sqrt[3]{b}(-ab^{4/3}(bc-4af)+a^{4/3}b(2bd-5ag))x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b^{10/3}} + \frac{\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{2/3}b^{7/3}}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)} + \frac{\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{8/3}}+\frac{(be-2ah)\log(a+bx^3)}{3b^3} + \frac{\left(b^{4/3}c+2\sqrt[3]{a}bd-4a\sqrt[3]{b}f-5a^{4/3}g\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}b^{7/3}} - \frac{\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{2/3}b^{8/3}}$$

$$= \frac{fx}{b^2} + \frac{gx^2}{2b^2} + \frac{hx^3}{3b^2} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3b^2(a + bx^3)} + \frac{\left(\sqrt[3]{b}(bc - 4af) - \sqrt[3]{a}(2bd - 5ag)\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{9a^{2/3}b^{8/3}} - \frac{\left(\sqrt[3]{b}(bc - 4af) - \sqrt[3]{a}(2bd - 5ag)\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{18a^{2/3}b^{8/3}} + \frac{(be - 2ah)\log(a + bx^3)}{3b^3} + \frac{\left(b^{4/3}c + 2\sqrt[3]{a}bd - 4a\sqrt[3]{b}f - 5a^{4/3}g\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{2/3}b^{8/3}}$$

$$= \frac{fx}{b^2} + \frac{gx^2}{2b^2} + \frac{hx^3}{3b^2} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3b^2(a + bx^3)} - \frac{\left(b^{4/3}c + 2\sqrt[3]{a}bd - 4a\sqrt[3]{b}f - 5a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{8/3}} + \frac{\left(\sqrt[3]{b}(bc - 4af) - \sqrt[3]{a}(2bd - 5ag)\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{9a^{2/3}b^{8/3}} - \frac{\left(\sqrt[3]{b}(bc - 4af) - \sqrt[3]{a}(2bd - 5ag)\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{18a^{2/3}b^{8/3}} + \frac{(be - 2ah)\log(a + bx^3)}{3b^3}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.95

$$\int \frac{x^3(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^2}\,dx$$

$$= \frac{18bfx + 9bgx^2 + 6bhx^3 - \frac{6(a^2h + b^2x(c + dx) - ab(e + x(f + gx)))}{a + bx^3} + \frac{2\sqrt{3}\sqrt[3]{b}\left(-b^{4/3}c - 2\sqrt[3]{a}bd + 4a\sqrt[3]{b}f + 5a^{4/3}g\right)\arctan\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{a^{2/3}}$$

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (18*b*f*x + 9*b*g*x^2 + 6*b*h*x^3 - (6*(a^2*h + b^2*x*(c + d*x) - a*b*(e +
x*(f + g*x))))/(a + b*x^3) + (2*Sqrt[3]*b^(1/3)*(-(b^(4/3)*c) - 2*a^(1/3)*b
*d + 4*a*b^(1/3)*f + 5*a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]])/a^(2/3) + (2*b^(1/3)*(b^(4/3)*c - 2*a^(1/3)*b*d - 4*a*b^(1/3)*f + 5*a^(
```

```
4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) - (b^(1/3)*(b^(4/3)*c - 2*a^(1/3)
*b*d - 4*a*b^(1/3)*f + 5*a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/a^(2/3) + 6*(b*e - 2*a*h)*Log[a + b*x^3])/(18*b^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.44

method	result
risch	$\frac{h\,x^3}{3b^2}+\frac{g\,x^2}{2b^2}+\frac{fx}{b^2}+\frac{\left(\frac{ag}{3}-\frac{bd}{3}\right)x^2+\left(\frac{af}{3}-\frac{bc}{3}\right)x-\frac{a(ah-be)}{3b}}{b^2(b\,x^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\left(3(-2ah+be)\_R^2+(-5ag+2bd)\_R-4af+\ldots\right.}{\_R^2}}{9b^3}$
default	$\frac{\frac{1}{3}h\,x^3+\frac{1}{2}g\,x^2+fx}{b^2}-\frac{\left(-\frac{ag}{3}+\frac{bd}{3}\right)x^2+\left(-\frac{af}{3}+\frac{bc}{3}\right)x+\frac{a(ah-be)}{3b}}{b\,x^3+a}+\frac{(4af-bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\ldots\right)}{\ldots}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\ldots\right)}{3}$

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/3*h*x^3/b^2+1/2*g*x^2/b^2+f*x/b^2+((1/3*a*g-1/3*b*d)*x^2+(1/3*a*f-1/3*b*c
)*x-1/3*a*(a*h-b*e)/b)/b^2/(b*x^3+a)+1/9/b^3*sum((3*(-2*a*h+b*e)*_R^2+(-5*a
*g+2*b*d)*_R-4*a*f+b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 16285, normalized size of antiderivative = 52.36

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.06

$$\begin{aligned}&\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx\\&=\frac{abe-a^2h-(b^2d-abg)x^2-(b^2c-abf)x}{3\,(b^4x^3+ab^3)}+\frac{2\,hx^3+3\,gx^2+6\,fx}{6\,b^2}\\&\quad+\frac{\sqrt{3}\left(2\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-5\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-4\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3}\\&\quad+\frac{\left(6\,be\left(\frac{a}{b}\right)^{\frac{2}{3}}-12\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-bc+4\,af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\&\quad+\frac{\left(3\,be\left(\frac{a}{b}\right)^{\frac{2}{3}}-6\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+bc-4\,af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="max
ima")
[Out] 1/3*(a*b*e - a^2*h - (b^2*d - a*b*g)*x^2 - (b^2*c - a*b*f)*x)/(b^4*x^3 + a*
b^3) + 1/6*(2*h*x^3 + 3*g*x^2 + 6*f*x)/b^2 + 1/9*sqrt(3)*(2*b^2*d*(a/b)^(2/
3) - 5*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) - 4*a*b*f*(a/b)^(1/3))*arctan(
1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) + 1/18*(6*b*e*(a/b)^(2
/3) - 12*a*h*(a/b)^(2/3) + 2*b*d*(a/b)^(1/3) - 5*a*g*(a/b)^(1/3) - b*c + 4*
a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/9*(3*b*e*
(a/b)^(2/3) - 6*a*h*(a/b)^(2/3) - 2*b*d*(a/b)^(1/3) + 5*a*g*(a/b)^(1/3) + b
*c - 4*a*f)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.06

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= -\frac{\sqrt{3}\left(b^2c-4\,abf-2\,(-ab^2)^{\frac{1}{3}}\,bd+5\,(-ab^2)^{\frac{1}{3}}\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,b^2}$$
$$-\frac{\left(b^2c-4\,abf+2\,(-ab^2)^{\frac{1}{3}}\,bd-5\,(-ab^2)^{\frac{1}{3}}\,ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,b^2}$$
$$+\frac{(be-2\,ah)\log\left(|bx^3+a|\right)}{3\,b^3}+\frac{abe-a^2h-(b^2d-abg)x^2-(b^2c-abf)x}{3\,(bx^3+a)b^3}$$
$$-\frac{\left(2\,b^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^3g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b^4c-4\,ab^3f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^5}$$
$$+\frac{2\,b^4hx^3+3\,b^4gx^2+6\,b^4fx}{6\,b^6}$$

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] -1/9*sqrt(3)*(b^2*c - 4*a*b*f - 2*(-a*b^2)^(1/3)*b*d + 5*(-a*b^2)^(1/3)*a*g
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^2
) - 1/18*(b^2*c - 4*a*b*f + 2*(-a*b^2)^(1/3)*b*d - 5*(-a*b^2)^(1/3)*a*g)*lo
g(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) + 1/3*(b*e - 2*
a*h)*log(abs(b*x^3 + a))/b^3 + 1/3*(a*b*e - a^2*h - (b^2*d - a*b*g)*x^2 - (
b^2*c - a*b*f)*x)/((b*x^3 + a)*b^3) - 1/9*(2*b^4*d*(-a/b)^(1/3) - 5*a*b^3*g
*(-a/b)^(1/3) + b^4*c - 4*a*b^3*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^5) + 1/6*(2*b^4*h*x^3 + 3*b^4*g*x^2 + 6*b^4*f*x)/b^6
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1229, normalized size of antiderivative = 3.95

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(\frac{36\,a^3\,h^2+9\,a\,b^2\,e^2+2\,b^3\,c\,d-5\,a\,b^2\,c\,g-8\,a\,b^2\,d\,f-36\,a^2\,b\,e\,h+20\,a^2\,b\,f\,g}{9\,b^4}\right.\right.$$
$$+\mathrm{root}\big(729\,a^2\,b^9\,z^3+1458\,a^3\,b^6\,h\,z^2-729\,a^2\,b^7\,e\,z^2+54\,a\,b^6\,c\,d\,z-972\,a^3\,b^4\,e\,h\,z+540\,a^3\,b^4\,f\,g\,z-216\,a^2\,b^5\,d$$
$$\left.+\frac{x\,(4\,b^2\,d^2+25\,a^2\,g^2-3\,b^2\,c\,e-24\,a^2\,f\,h+6\,a\,b\,c\,h-20\,a\,b\,d\,g+12\,a\,b\,e\,f)}{9\,b^3}\right)\,\mathrm{root}\big(729\,a^2\,b^9\,z^3$$
$$+1458\,a^3\,b^6\,h\,z^2-729\,a^2\,b^7\,e\,z^2+54\,a\,b^6\,c\,d\,z-972\,a^3\,b^4\,e\,h\,z+540\,a^3\,b^4\,f\,g\,z$$
$$-216\,a^2\,b^5\,d\,f\,z-135\,a^2\,b^5\,c\,g\,z+972\,a^4\,b^3\,h^2\,z+243\,a^2\,b^5\,e^2\,z+360\,a^4\,b\,f\,g\,h$$
$$-18\,a\,b^4\,c\,d\,e-180\,a^3\,b^2\,e\,f\,g-144\,a^3\,b^2\,d\,f\,h-90\,a^3\,b^2\,c\,g\,h+72\,a^2\,b^3\,d\,e\,f+45\,a^2\,b^3\,c\,e\,g$$
$$+36\,a^2\,b^3\,c\,d\,h-324\,a^4\,b\,e\,h^2+12\,a\,b^4\,c^2\,f+162\,a^3\,b^2\,e^2\,h+150\,a^3\,b^2\,d\,g^2-60\,a^2\,b^3\,d^2\,g$$
$$\left.-48\,a^2\,b^3\,c\,f^2+64\,a^3\,b^2\,f^3-27\,a^2\,b^3\,e^3-125\,a^4\,b\,g^3+8\,a\,b^4\,d^3+216\,a^5\,h^3-b^5\,c^3,z,k\big)\right)$$
$$-\frac{x\left(\frac{b\,c}{3}-\frac{a\,f}{3}\right)+\frac{a^2\,h-a\,b\,e}{3\,b}+x^2\left(\frac{b\,d}{3}-\frac{a\,g}{3}\right)}{b^3\,x^3+a\,b^2}+\frac{g\,x^2}{2\,b^2}+\frac{h\,x^3}{3\,b^2}+\frac{f\,x}{b^2}$$

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((36*a^3*h^2 + 9*a*b^2*e^2 + 2*b^3*c*d - 5*a*b^2*c*g - 8*a*b^2*d*
f - 36*a^2*b*e*h + 20*a^2*b*f*g)/(9*b^4) + root(729*a^2*b^9*z^3 + 1458*a^3*
b^6*h*z^2 - 729*a^2*b^7*e*z^2 + 54*a*b^6*c*d*z - 972*a^3*b^4*e*h*z + 540*a^
3*b^4*f*g*z - 216*a^2*b^5*d*f*z - 135*a^2*b^5*c*g*z + 972*a^4*b^3*h^2*z + 2
43*a^2*b^5*e^2*z + 360*a^4*b*f*g*h - 18*a*b^4*c*d*e - 180*a^3*b^2*e*f*g - 1
44*a^3*b^2*d*f*h - 90*a^3*b^2*c*g*h + 72*a^2*b^3*d*e*f + 45*a^2*b^3*c*e*g +
 36*a^2*b^3*c*d*h - 324*a^4*b*e*h^2 + 12*a*b^4*c^2*f + 162*a^3*b^2*e^2*h +
150*a^3*b^2*d*g^2 - 60*a^2*b^3*d^2*g - 48*a^2*b^3*c*f^2 + 64*a^3*b^2*f^3 -
27*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8*a*b^4*d^3 + 216*a^5*h^3 - b^5*c^3, z, k)
*((108*a^2*b^3*h - 54*a*b^4*e)/(9*b^4) + (x*(9*b^4*c - 36*a*b^3*f))/(9*b^3)
 + 9*root(729*a^2*b^9*z^3 + 1458*a^3*b^6*h*z^2 - 729*a^2*b^7*e*z^2 + 54*a*b
^6*c*d*z - 972*a^3*b^4*e*h*z + 540*a^3*b^4*f*g*z - 216*a^2*b^5*d*f*z - 135*
a^2*b^5*c*g*z + 972*a^4*b^3*h^2*z + 243*a^2*b^5*e^2*z + 360*a^4*b*f*g*h - 1
8*a*b^4*c*d*e - 180*a^3*b^2*e*f*g - 144*a^3*b^2*d*f*h - 90*a^3*b^2*c*g*h +
72*a^2*b^3*d*e*f + 45*a^2*b^3*c*e*g + 36*a^2*b^3*c*d*h - 324*a^4*b*e*h^2 +
12*a*b^4*c^2*f + 162*a^3*b^2*e^2*h + 150*a^3*b^2*d*g^2 - 60*a^2*b^3*d^2*g -
 48*a^2*b^3*c*f^2 + 64*a^3*b^2*f^3 - 27*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8*a*b
^4*d^3 + 216*a^5*h^3 - b^5*c^3, z, k)*a*b^2) + (x*(4*b^2*d^2 + 25*a^2*g^2 -
 3*b^2*c*e - 24*a^2*f*h + 6*a*b*c*h - 20*a*b*d*g + 12*a*b*e*f))/(9*b^3))*ro
ot(729*a^2*b^9*z^3 + 1458*a^3*b^6*h*z^2 - 729*a^2*b^7*e*z^2 + 54*a*b^6*c*d*
```

```
z - 972*a^3*b^4*e*h*z + 540*a^3*b^4*f*g*z - 216*a^2*b^5*d*f*z - 135*a^2*b^5
*c*g*z + 972*a^4*b^3*h^2*z + 243*a^2*b^5*e^2*z + 360*a^4*b*f*g*h - 18*a*b^4
*c*d*e - 180*a^3*b^2*e*f*g - 144*a^3*b^2*d*f*h - 90*a^3*b^2*c*g*h + 72*a^2*
b^3*d*e*f + 45*a^2*b^3*c*e*g + 36*a^2*b^3*c*d*h - 324*a^4*b*e*h^2 + 12*a*b^
4*c^2*f + 162*a^3*b^2*e^2*h + 150*a^3*b^2*d*g^2 - 60*a^2*b^3*d^2*g - 48*a^2
*b^3*c*f^2 + 64*a^3*b^2*f^3 - 27*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8*a*b^4*d^3
+ 216*a^5*h^3 - b^5*c^3, z, k), k, 1, 3) - (x*((b*c)/3 - (a*f)/3) + (a^2*h
- a*b*e)/(3*b) + x^2*((b*d)/3 - (a*g)/3))/(a*b^2 + b^3*x^3) + (g*x^2)/(2*b^
2) + (h*x^3)/(3*b^2) + (f*x)/b^2
```

3.414 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 290

$$\begin{aligned}\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx &= \frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b(a+bx^3)} \\ &\quad -\frac{\left(b^{4/3}d+2\sqrt[3]{a}be-4a\sqrt[3]{b}g-5a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{8/3}} \\ &\quad +\frac{\left(\sqrt[3]{b}(bd-4ag)-\sqrt[3]{a}(2be-5ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{8/3}} \\ &\quad -\frac{\left(\sqrt[3]{b}(bd-4ag)-\sqrt[3]{a}(2be-5ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{8/3}}+\frac{f\log(a+bx^3)}{3b^2}\end{aligned}$$

```
[Out] 4/3*g*x/b^2+5/6*h*x^2/b^2+1/3*(-h*x^5-g*x^4-f*x^3-e*x^2-d*x-c)/b/(b*x^3+a)+
1/9*(b^(1/3)*(-4*a*g+b*d)-a^(1/3)*(-5*a*h+2*b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(
2/3)/b^(8/3)-1/18*(b^(1/3)*(-4*a*g+b*d)-a^(1/3)*(-5*a*h+2*b*e))*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(8/3)+1/3*f*ln(b*x^3+a)/b^2-1/9*(b
^(4/3)*d+2*a^(1/3)*b*e-4*a*b^(1/3)*g-5*a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1837, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\begin{aligned}\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx &= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^{4/3}h+2\sqrt[3]{a}be-4a\sqrt[3]{b}g+b^{4/3}d\right)}{3\sqrt{3}a^{2/3}b^{8/3}}\\ &\quad-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}}-4ag+bd\right)}{18a^{2/3}b^{7/3}}\\ &\quad+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-4ag)-\sqrt[3]{a}(2be-5ah)\right)}{9a^{2/3}b^{8/3}}\\ &\quad+\frac{f\log\left(a+bx^3\right)}{3b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}+\frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}\end{aligned}$$

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (4*g*x)/(3*b^2) + (5*h*x^2)/(6*b^2) - (c + d*x + e*x^2 + f*x^3 + g*x^4 + h*
x^5)/(3*b*(a + b*x^3)) - ((b^(4/3)*d + 2*a^(1/3)*b*e - 4*a*b^(1/3)*g - 5*a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2
/3)*b^(8/3)) + ((b^(1/3)*(b*d - 4*a*g) - a^(1/3)*(2*b*e - 5*a*h))*Log[a^(1/
3) + b^(1/3)*x])/(9*a^(2/3)*b^(8/3)) - ((b*d - 4*a*g - (a^(1/3)*(2*b*e - 5*
a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(2/3)*
b^(7/3)) + (f*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b(a+bx^3)}+\frac{\int \frac{d+2ex+3fx^2+4gx^3+5hx^4}{a+bx^3}\,dx}{3b}$$

$$= -\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b(a+bx^3)}+\frac{\int \left(\frac{4g}{b}+\frac{5hx}{b}+\frac{bd-4ag+(2be-5ah)x+3bfx^2}{b(a+bx^3)}\right)dx}{3b}$$

$$= \frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b(a+bx^3)}+\frac{\int \frac{bd-4ag+(2be-5ah)x+3bfx^2}{a+bx^3}\,dx}{3b^2}$$

$$= \frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b(a+bx^3)}+\frac{\int \frac{bd-4ag+(2be-5ah)x}{a+bx^3}\,dx}{3b^2}+\frac{f\int \frac{x^2}{a+bx^3}\,dx}{b}$$

$$= \frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b(a+bx^3)}+\frac{f\log(a+bx^3)}{3b^2}$$
$$+\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}(bd-4ag)+\sqrt[3]{a}(2be-5ah)\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}(bd-4ag)+\sqrt[3]{a}(2be-5ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{2/3}b^{7/3}}$$
$$+\frac{\left(bd-4ag-\frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{2/3}b^2}$$

$$= \frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b(a+bx^3)}$$
$$+\frac{\left(bd-4ag-\frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}}+\frac{f\log(a+bx^3)}{3b^2}$$
$$+\frac{\left(b^{4/3}d+2\sqrt[3]{a}be-4a\sqrt[3]{b}g-5a^{4/3}h\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}b^{7/3}}$$
$$-\frac{\left(bd-4ag-\frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}}\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{2/3}b^{7/3}}$$

$$= \frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b(a+bx^3)}$$
$$+\frac{\left(bd-4ag-\frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}}$$
$$-\frac{\left(bd-4ag-\frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{7/3}}+\frac{f\log(a+bx^3)}{3b^2}$$
$$+\frac{\left(b^{4/3}d+2\sqrt[3]{a}be-4a\sqrt[3]{b}g-5a^{4/3}h\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{2/3}b^{8/3}}$$

$$
= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\,(a+bx^3)} - \frac{\left(b^{4/3}d+2\sqrt[3]{a}be-4a\sqrt[3]{b}g-5a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{8/3}} + \frac{\left(bd-4ag-\frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}} - \frac{\left(bd-4ag-\frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{7/3}} + \frac{f\log\left(a+bx^3\right)}{3b^2}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.97

$$
\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx
$$

$$
= \frac{18b^{2/3}gx + 9b^{2/3}hx^2 - \frac{6b^{2/3}(b(c+x(d+ex))-a(f+x(g+hx)))}{a+bx^3} + \frac{2\sqrt{3}\left(-b^{4/3}d-2\sqrt[3]{a}be+4a\sqrt[3]{b}g+5a^{4/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2
$$

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (18*b^(2/3)*g*x + 9*b^(2/3)*h*x^2 - (6*b^(2/3)*(b*(c + x*(d + e*x)) - a*(f
+ x*(g + h*x))))/(a + b*x^3) + (2*Sqrt[3]*(-(b^(4/3)*d) - 2*a^(1/3)*b*e + 4
*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^
(2/3) + (2*(b^(4/3)*d - 2*a^(1/3)*b*e - 4*a*b^(1/3)*g + 5*a^(4/3)*h)*Log[a^
(1/3) + b^(1/3)*x])/a^(2/3) - ((b^(4/3)*d - 2*a^(1/3)*b*e - 4*a*b^(1/3)*g +
 5*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) + 6*b
^(2/3)*f*Log[a + b*x^3])/(18*b^(8/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.40

method	result
risch	$\frac{hx^2}{2b^2}+\frac{gx}{b^2}+\frac{\left(\frac{ah}{3}-\frac{be}{3}\right)x^2+\left(\frac{ag}{3}-\frac{bd}{3}\right)x+\frac{af}{3}-\frac{bc}{3}}{b^2(bx^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\left(3bf\_R^2+(-5ah+2be)\_R-4ag+bd\right)\ln\left(x-\_R\right)}{\_R^2}}{9b^3}$
default	$\frac{\frac{1}{2}hx^2+gx}{b^2}-\frac{\left(-\frac{ah}{3}+\frac{be}{3}\right)x^2+\left(-\frac{ag}{3}+\frac{bd}{3}\right)x-\frac{af}{3}+\frac{bc}{3}}{bx^3+a}+\frac{(4ag-bd)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{3}$ $\;/\; b^2$

```
[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2*h*x^2/b^2+g*x/b^2+((1/3*a*h-1/3*b*e)*x^2+(1/3*a*g-1/3*b*d)*x+1/3*a*f-1/
3*b*c)/b^2/(b*x^3+a)+1/9/b^3*sum((3*b*f*_R^2+(-5*a*h+2*b*e)*_R-4*a*g+b*d)/_
R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 12153, normalized size of antiderivative = 41.91

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = -\frac{(be-ah)x^2+bc-af+(bd-ag)x}{3\,(b^3x^3+ab^2)} + \frac{\sqrt{3}\left(2\,be\left(\frac{a}{b}\right)^{\frac{2}{3}}-5\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-4\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2} + \frac{hx^2+2\,gx}{2\,b^2} + \frac{\left(6\,bf\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,be\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}}-bd+4\,ag\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(3\,bf\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,be\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}}+bd-4\,ag\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] -1/3*((b*e - a*h)*x^2 + b*c - a*f + (b*d - a*g)*x)/(b^3*x^3 + a*b^2) + 1/9*
sqrt(3)*(2*b*e*(a/b)^(2/3) - 5*a*h*(a/b)^(2/3) + b*d*(a/b)^(1/3) - 4*a*g*(a
/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2) + 1/
2*(h*x^2 + 2*g*x)/b^2 + 1/18*(6*b*f*(a/b)^(2/3) + 2*b*e*(a/b)^(1/3) - 5*a*h
*(a/b)^(1/3) - b*d + 4*a*g)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/
b)^(2/3)) + 1/9*(3*b*f*(a/b)^(2/3) - 2*b*e*(a/b)^(1/3) + 5*a*h*(a/b)^(1/3)
+ b*d - 4*a*g)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.04

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= \frac{f\log(|bx^3+a|)}{3\,b^2} - \frac{\sqrt{3}\Big(b^2d-4\,abg-2\,(-ab^2)^{\frac{1}{3}}\,be+5\,(-ab^2)^{\frac{1}{3}}\,ah\Big)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,b^2} - \frac{\Big(b^2d-4\,abg+2\,(-ab^2)^{\frac{1}{3}}\,be-5\,(-ab^2)^{\frac{1}{3}}\,ah\Big)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,b^2} - \frac{(be-ah)x^2+bc-af+(bd-ag)x}{3\,(bx^3+a)b^2} + \frac{b^2hx^2+2\,b^2gx}{2\,b^4} - \frac{\left(2\,b^4e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^3h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b^4d-4\,ab^3g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^5}$$

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] 1/3*f*log(abs(b*x^3 + a))/b^2 - 1/9*sqrt(3)*(b^2*d - 4*a*b*g - 2*(-a*b^2)^(
1/3)*b*e + 5*(-a*b^2)^(1/3)*a*h)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/((-a*b^2)^(2/3)*b^2) - 1/18*(b^2*d - 4*a*b*g + 2*(-a*b^2)^(1/3)
*b*e - 5*(-a*b^2)^(1/3)*a*h)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*
b^2)^(2/3)*b^2) - 1/3*((b*e - a*h)*x^2 + b*c - a*f + (b*d - a*g)*x)/((b*x^3
 + a)*b^2) + 1/2*(b^2*h*x^2 + 2*b^2*g*x)/b^4 - 1/9*(2*b^4*e*(-a/b)^(1/3) -
5*a*b^3*h*(-a/b)^(1/3) + b^4*d - 4*a*b^3*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 816, normalized size of antiderivative = 2.81

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\frac{9\,a\,b\,f^2+2\,b^2\,d\,e+20\,a^2\,g\,h-5\,a\,b\,d\,h-8\,a\,b\,e\,g}{9\,b^3}\right.\right.$$
$$+\mathrm{root}\big(729\,a^2\,b^8\,z^3-729\,a^2\,b^6\,f\,z^2+54\,a\,b^5\,d\,e\,z+540\,a^3\,b^3\,g\,h\,z-216\,a^2\,b^4\,e\,g\,z-135\,a^2\,b^4\,d\,h\,z+243\,a^2\,b^4\,f^2$$
$$\left.+\frac{x\,(25\,a^2\,h^2-20\,a\,b\,e\,h+12\,f\,g\,a\,b+4\,b^2\,e^2-3\,d\,f\,b^2)}{9\,b^3}\right)\mathrm{root}\big(729\,a^2\,b^8\,z^3$$
$$-729\,a^2\,b^6\,f\,z^2+54\,a\,b^5\,d\,e\,z+540\,a^3\,b^3\,g\,h\,z-216\,a^2\,b^4\,e\,g\,z-135\,a^2\,b^4\,d\,h\,z+243\,a^2\,b^4\,f^2\,z$$
$$-180\,a^3\,b\,f\,g\,h-18\,a\,b^3\,d\,e\,f+72\,a^2\,b^2\,e\,f\,g+45\,a^2\,b^2\,d\,f\,h+150\,a^3\,b\,e\,h^2+12\,a\,b^3\,d^2\,g$$
$$\left.-60\,a^2\,b^2\,e^2\,h-48\,a^2\,b^2\,d\,g^2-27\,a^2\,b^2\,f^3+64\,a^3\,b\,g^3+8\,a\,b^3\,e^3-125\,a^4\,h^3-b^4\,d^3,z,k\big)\right)$$
$$-\frac{\left(\frac{b\,e}{3}-\frac{a\,h}{3}\right)\,x^2+\left(\frac{b\,d}{3}-\frac{a\,g}{3}\right)\,x+\frac{b\,c}{3}-\frac{a\,f}{3}}{b^3\,x^3+a\,b^2}+\frac{h\,x^2}{2\,b^2}+\frac{g\,x}{b^2}$$

```
[In] int((x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((9*a*b*f^2 + 2*b^2*d*e + 20*a^2*g*h - 5*a*b*d*h - 8*a*b*e*g)/(9*
b^3) + root(729*a^2*b^8*z^3 - 729*a^2*b^6*f*z^2 + 54*a*b^5*d*e*z + 540*a^3*
b^3*g*h*z - 216*a^2*b^4*e*g*z - 135*a^2*b^4*d*h*z + 243*a^2*b^4*f^2*z - 180
*a^3*b*f*g*h - 18*a*b^3*d*e*f + 72*a^2*b^2*e*f*g + 45*a^2*b^2*d*f*h + 150*a
^3*b*e*h^2 + 12*a*b^3*d^2*g - 60*a^2*b^2*e^2*h - 48*a^2*b^2*d*g^2 - 27*a^2*
b^2*f^3 + 64*a^3*b*g^3 + 8*a*b^3*e^3 - 125*a^4*h^3 - b^4*d^3, z, k)*((x*(9*
b^4*d - 36*a*b^3*g))/(9*b^3) - 6*a*f + 9*root(729*a^2*b^8*z^3 - 729*a^2*b^6
*f*z^2 + 54*a*b^5*d*e*z + 540*a^3*b^3*g*h*z - 216*a^2*b^4*e*g*z - 135*a^2*b
^4*d*h*z + 243*a^2*b^4*f^2*z - 180*a^3*b*f*g*h - 18*a*b^3*d*e*f + 72*a^2*b^
2*e*f*g + 45*a^2*b^2*d*f*h + 150*a^3*b*e*h^2 + 12*a*b^3*d^2*g - 60*a^2*b^2*
e^2*h - 48*a^2*b^2*d*g^2 - 27*a^2*b^2*f^3 + 64*a^3*b*g^3 + 8*a*b^3*e^3 - 12
5*a^4*h^3 - b^4*d^3, z, k)*a*b^2) + (x*(4*b^2*e^2 + 25*a^2*h^2 - 3*b^2*d*f
- 20*a*b*e*h + 12*a*b*f*g))/(9*b^3))*root(729*a^2*b^8*z^3 - 729*a^2*b^6*f*z
^2 + 54*a*b^5*d*e*z + 540*a^3*b^3*g*h*z - 216*a^2*b^4*e*g*z - 135*a^2*b^4*d
*h*z + 243*a^2*b^4*f^2*z - 180*a^3*b*f*g*h - 18*a*b^3*d*e*f + 72*a^2*b^2*e*
f*g + 45*a^2*b^2*d*f*h + 150*a^3*b*e*h^2 + 12*a*b^3*d^2*g - 60*a^2*b^2*e^2*
h - 48*a^2*b^2*d*g^2 - 27*a^2*b^2*f^3 + 64*a^3*b*g^3 + 8*a*b^3*e^3 - 125*a^
4*h^3 - b^4*d^3, z, k), k, 1, 3) - ((b*c)/3 - (a*f)/3 + x*((b*d)/3 - (a*g)/
3) + x^2*((b*e)/3 - (a*h)/3))/(a*b^2 + b^3*x^3) + (h*x^2)/(2*b^2) + (g*x)/b
^2
```

3.415 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 36, antiderivative size = 289

$$\begin{aligned}&\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx\\
&=\frac{hx}{b^2}-\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)}\\
&\quad-\frac{(b^{5/3}c+a^{2/3}be+2ab^{2/3}f-4a^{5/3}h)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{7/3}}\\
&\quad-\frac{(b^{2/3}(bc+2af)-a^{2/3}(be-4ah))\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{7/3}}\\
&\quad+\frac{(b^{2/3}(bc+2af)-a^{2/3}(be-4ah))\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{7/3}}+\frac{g\log(a+bx^3)}{3b^2}\end{aligned}$$

```
[Out] h*x/b^2-1/3*x*(a*(-a*h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/a/b^2/(b*x^3+a
)-1/9*(b^(2/3)*(2*a*f+b*c)-a^(2/3)*(-4*a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(4
/3)/b^(7/3)+1/18*(b^(2/3)*(2*a*f+b*c)-a^(2/3)*(-4*a*h+b*e))*ln(a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(7/3)+1/3*g*ln(b*x^3+a)/b^2-1/9*(b^(5/
3)*c+a^(2/3)*b*e+2*a*b^(2/3)*f-4*a^(5/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x
)/a^(1/3)*3^(1/2))/a^(4/3)/b^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1842, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{2/3}be-4a^{5/3}h+2ab^{2/3}f+b^{5/3}c\right)}{3\sqrt{3}a^{4/3}b^{7/3}}$$
$$+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(2af+bc)-a^{2/3}(be-4ah)\right)}{18a^{4/3}b^{7/3}}$$
$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(2af+bc)-a^{2/3}(be-4ah)\right)}{9a^{4/3}b^{7/3}}$$
$$-\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{3ab^2(a+bx^3)}+\frac{g\log(a+bx^3)}{3b^2}+\frac{hx}{b^2}$$

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (h*x)/b^2 - (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*a*
b^2*(a + b*x^3)) - ((b^(5/3)*c + a^(2/3)*b*e + 2*a*b^(2/3)*f - 4*a^(5/3)*h)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(7
/3)) - ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h))*Log[a^(1/3) + b^(1/
3)*x])/(9*a^(4/3)*b^(7/3)) + ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h
))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(7/3)) + (
g*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} - \frac{\int \frac{-a(be-ah)-b(bc+2af)x-3abgx^2-3abhx^3}{a+bx^3}\,dx}{3ab^2}$$

$$= -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} - \frac{\int \left(-3ah - \frac{a(be-4ah)+b(bc+2af)x+3abgx^2}{a+bx^3}\right)dx}{3ab^2}$$

$$= \frac{hx}{b^2} - \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} + \frac{\int \frac{a(be-4ah)+b(bc+2af)x+3abgx^2}{a+bx^3}\,dx}{3ab^2}$$

$$= \frac{hx}{b^2} - \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} + \frac{\int \frac{a(be-4ah)+b(bc+2af)x}{a+bx^3}\,dx}{3ab^2} + \frac{g\int \frac{x^2}{a+bx^3}\,dx}{b}$$

$$= \frac{hx}{b^2} - \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} + \frac{g\log(a+bx^3)}{3b^2} + \frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{a}b(bc+2af)+2a\sqrt[3]{b}(be-4ah)\right)+\sqrt[3]{b}\left(\sqrt[3]{a}b(bc+2af)-a\sqrt[3]{b}(be-4ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b^{7/3}} - \frac{\left(b^{2/3}(bc+2af)-a^{2/3}(be-4ah)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}b^2}$$

$$= \frac{hx}{b^2} - \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} - \frac{\left(b^{2/3}(bc+2af)-a^{2/3}(be-4ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{7/3}} + \frac{g\log(a+bx^3)}{3b^2} + \frac{\left(b^{5/3}c+a^{2/3}be+2ab^{2/3}f-4a^{5/3}h\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6ab^2} + \frac{\left(b^{2/3}(bc+2af)-a^{2/3}(be-4ah)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{4/3}b^{7/3}}$$

$$= \frac{hx}{b^2} - \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} - \frac{(b^{2/3}(bc+2af)-a^{2/3}(be-4ah))\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{7/3}} + \frac{(b^{2/3}(bc+2af)-a^{2/3}(be-4ah))\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{7/3}} + \frac{g\log(a+bx^3)}{3b^2} + \frac{(b^{5/3}c+a^{2/3}be+2ab^{2/3}f-4a^{5/3}h)\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{4/3}b^{7/3}}$$

$$= \frac{hx}{b^2} - \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)} - \frac{(b^{5/3}c+a^{2/3}be+2ab^{2/3}f-4a^{5/3}h)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{7/3}} - \frac{(b^{2/3}(bc+2af)-a^{2/3}(be-4ah))\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{7/3}} + \frac{(b^{2/3}(bc+2af)-a^{2/3}(be-4ah))\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{7/3}} + \frac{g\log(a+bx^3)}{3b^2}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.99

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$

$$= \frac{18b^{2/3}hx + \frac{6b^{2/3}(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx)))}{a(a+bx^3)} - \frac{2\sqrt{3}\left(b^2c+a^{2/3}b^{4/3}e+2abf-4a^{5/3}\sqrt[3]{b}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} - \frac{2\left(b^2c-a^{2/3}\right.}{}}{}$$

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (18*b^(2/3)*h*x + (6*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f
*x))))/(a*(a + b*x^3)) - (2*Sqrt[3]*(b^2*c + a^(2/3)*b^(4/3)*e + 2*a*b*f -
4*a^(5/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(4/3) -
 (2*(b^2*c - a^(2/3)*b^(4/3)*e + 2*a*b*f + 4*a^(5/3)*b^(1/3)*h)*Log[a^(1/3)
```

```
+ b^(1/3)*x])/a^(4/3) + ((b^2*c - a^(2/3)*b^(4/3)*e + 2*a*b*f + 4*a^(5/3)*
b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3) + 6*b^(2
/3)*g*Log[a + b*x^3])/(18*b^(8/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.39

method	result
risch	$\frac{hx}{b^2}+\frac{-\frac{b(af-bc)x^2}{3a}+\left(\frac{ah}{3}-\frac{be}{3}\right)x+\frac{ag}{3}-\frac{bd}{3}}{b^2(bx^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\left(3gb\_R^2+\frac{b(2af+bc)\_R}{a}-4ah+be\right)\ln(x-\_R)}{\_R^2}}{9b^3}$
default	$\frac{hx}{b^2}-\frac{\frac{\frac{b(af-bc)x^2}{3a}+\left(-\frac{ah}{3}+\frac{be}{3}\right)x-\frac{ag}{3}+\frac{bd}{3}}{bx^3+a}+\left(4a^2h-aeb\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(-2}{b^2}$

```
[In] int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] h*x/b^2+(-1/3*b*(a*f-b*c)/a*x^2+(1/3*a*h-1/3*b*e)*x+1/3*a*g-1/3*b*d)/b^2/(b
*x^3+a)+1/9/b^3*sum((3*g*b*_R^2+b*(2*a*f+b*c)/a*_R-4*a*h+b*e)/_R^2*ln(x-_R)
,_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 12617, normalized size of antiderivative = 43.66

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx=\text{Too large to display}$$

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="frica
s")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.08

$$\begin{aligned}&\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx\\&=-\frac{abd-a^2g-(b^2c-abf)x^2+(abe-a^2h)x}{3\,(ab^3x^3+a^2b^2)}+\frac{hx}{b^2}\\&\quad+\frac{\sqrt{3}\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2}\\&\quad+\frac{\left(6\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-abe+4\,a^2h\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\&\quad+\frac{\left(3\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+abe-4\,a^2h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxim
a")
[Out] -1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2 + (a*b*e - a^2*h)*x)/(a*b^3*x^3 +
 a^2*b^2) + h*x/b^2 + 1/9*sqrt(3)*(b^2*c*(a/b)^(2/3) + 2*a*b*f*(a/b)^(2/3)
+ a*b*e*(a/b)^(1/3) - 4*a^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/(a^2*b^2) + 1/18*(6*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/
3) + 2*a*b*f*(a/b)^(1/3) - a*b*e + 4*a^2*h)*log(x^2 - x*(a/b)^(1/3) + (a/b)
^(2/3))/(a*b^3*(a/b)^(2/3)) + 1/9*(3*a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^(1/3)
- 2*a*b*f*(a/b)^(1/3) + a*b*e - 4*a^2*h)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^
(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.08

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= \frac{hx}{b^2} + \frac{g\log(|bx^3+a|)}{3\,b^2}$$
$$- \frac{\sqrt{3}\Big(abe-4\,a^2h-(-ab^2)^{\frac{1}{3}}\,bc-2\,(-ab^2)^{\frac{1}{3}}\,af\Big)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab}$$
$$- \frac{\Big(abe-4\,a^2h+(-ab^2)^{\frac{1}{3}}\,bc+2\,(-ab^2)^{\frac{1}{3}}\,af\Big)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,ab}$$
$$- \frac{abd-a^2g-(b^2c-abf)x^2+(abe-a^2h)x}{3\,(bx^3+a)ab^2}$$
$$- \frac{\left(ab^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2b^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^4e-4\,a^3b^3h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^5}$$

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac"
)
```

```
[Out] h*x/b^2 + 1/3*g*log(abs(b*x^3 + a))/b^2 - 1/9*sqrt(3)*(a*b*e - 4*a^2*h - (-
a*b^2)^(1/3)*b*c - 2*(-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/18*(a*b*e - 4*a^2*h + (-a*b^2)
^(1/3)*b*c + 2*(-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/((-a*b^2)^(2/3)*a*b) - 1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2 + (a*b*e -
 a^2*h)*x)/((b*x^3 + a)*a*b^2) - 1/9*(a*b^5*c*(-a/b)^(1/3) + 2*a^2*b^4*f*(-
a/b)^(1/3) + a^2*b^4*e - 4*a^3*b^3*h)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)
))/(a^3*b^5)
```

Mupad [B] (verification not implemented)

Time = 10.61 (sec) , antiderivative size = 827, normalized size of antiderivative = 2.86

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(\frac{9\,a^2\,g^2+b^2\,c\,e-8\,a^2\,f\,h-4\,a\,b\,c\,h+2\,a\,b\,e\,f}{9\,a\,b^2} - \mathrm{root}\big(729\,a^4\,b^7\,z^3-729\,a^4\,b^5\,g\,z^2-216\,a^4\,b^3\,f\,h\,z-108\,a^3\,b^4\,c\,h\,z+54\,a^3\,b^4\,e\,f\,z+27\,a^2\,b^5\,c\,e\,z+243\,a^4\,b^3\,g \right.\right.$$
$$\left.\left. + \frac{x\,(12\,g\,h\,a^3+4\,a^2\,b\,f^2-3\,e\,g\,a^2\,b+4\,a\,b^2\,c\,f+b^3\,c^2)}{9\,a^2\,b^2}\right)\mathrm{root}\big(729\,a^4\,b^7\,z^3-729\,a^4\,b^5\,g\,z^2 - 216\,a^4\,b^3\,f\,h\,z-108\,a^3\,b^4\,c\,h\,z+54\,a^3\,b^4\,e\,f\,z+27\,a^2\,b^5\,c\,e\,z+243\,a^4\,b^3\,g^2\,z + 72\,a^4\,b\,f\,g\,h+36\,a^3\,b^2\,c\,g\,h-18\,a^3\,b^2\,e\,f\,g-9\,a^2\,b^3\,c\,e\,g-48\,a^4\,b\,e\,h^2+6\,a\,b^4\,c^2\,f + 12\,a^3\,b^2\,e^2\,h+12\,a^2\,b^3\,c\,f^2+8\,a^3\,b^2\,f^3-27\,a^4\,b\,g^3+64\,a^5\,h^3+b^5\,c^3-a^2\,b^3\,e^3,z,k\big)\right)$$
$$- \frac{\frac{b\,d}{3}-\frac{a\,g}{3}+x\left(\frac{b\,e}{3}-\frac{a\,h}{3}\right)-\frac{b\,x^2\,(b\,c-a\,f)}{3\,a}}{b^3\,x^3+a\,b^2}+\frac{h\,x}{b^2}$$

```
[In] int((x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((9*a^2*g^2 + b^2*c*e - 8*a^2*f*h - 4*a*b*c*h + 2*a*b*e*f)/(9*a*b
^2) - root(729*a^4*b^7*z^3 - 729*a^4*b^5*g*z^2 - 216*a^4*b^3*f*h*z - 108*a^
3*b^4*c*h*z + 54*a^3*b^4*e*f*z + 27*a^2*b^5*c*e*z + 243*a^4*b^3*g^2*z + 72*
a^4*b*f*g*h + 36*a^3*b^2*c*g*h - 18*a^3*b^2*e*f*g - 9*a^2*b^3*c*e*g - 48*a^
4*b*e*h^2 + 6*a*b^4*c^2*f + 12*a^3*b^2*e^2*h + 12*a^2*b^3*c*f^2 + 8*a^3*b^2
*f^3 - 27*a^4*b*g^3 + 64*a^5*h^3 + b^5*c^3 - a^2*b^3*e^3, z, k)*(6*a*g - b*
e*x + 4*a*h*x - 9*root(729*a^4*b^7*z^3 - 729*a^4*b^5*g*z^2 - 216*a^4*b^3*f*
h*z - 108*a^3*b^4*c*h*z + 54*a^3*b^4*e*f*z + 27*a^2*b^5*c*e*z + 243*a^4*b^3
*g^2*z + 72*a^4*b*f*g*h + 36*a^3*b^2*c*g*h - 18*a^3*b^2*e*f*g - 9*a^2*b^3*c
*e*g - 48*a^4*b*e*h^2 + 6*a*b^4*c^2*f + 12*a^3*b^2*e^2*h + 12*a^2*b^3*c*f^2
 + 8*a^3*b^2*f^3 - 27*a^4*b*g^3 + 64*a^5*h^3 + b^5*c^3 - a^2*b^3*e^3, z, k)
*a*b^2) + (x*(b^3*c^2 + 4*a^2*b*f^2 + 12*a^3*g*h + 4*a*b^2*c*f - 3*a^2*b*e*
g))/(9*a^2*b^2))*root(729*a^4*b^7*z^3 - 729*a^4*b^5*g*z^2 - 216*a^4*b^3*f*h
*z - 108*a^3*b^4*c*h*z + 54*a^3*b^4*e*f*z + 27*a^2*b^5*c*e*z + 243*a^4*b^3*
g^2*z + 72*a^4*b*f*g*h + 36*a^3*b^2*c*g*h - 18*a^3*b^2*e*f*g - 9*a^2*b^3*c*
e*g - 48*a^4*b*e*h^2 + 6*a*b^4*c^2*f + 12*a^3*b^2*e^2*h + 12*a^2*b^3*c*f^2
+ 8*a^3*b^2*f^3 - 27*a^4*b*g^3 + 64*a^5*h^3 + b^5*c^3 - a^2*b^3*e^3, z, k),
 k, 1, 3) - ((b*d)/3 - (a*g)/3 + x*((b*e)/3 - (a*h)/3) - (b*x^2*(b*c - a*f)
)/(3*a))/(a*b^2 + b^3*x^3) + (h*x)/b^2
```

3.416 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 35, antiderivative size = 276

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2} dx \\
&= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3ab(a+bx^3)} \\
&\quad - \frac{\left(2b^{4/3}c+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{5/3}} \\
&\quad + \frac{\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{5/3}} \\
&\quad - \frac{\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{5/3}} + \frac{h\log(a+bx^3)}{3b^2}
\end{aligned}$$

```
[Out] 1/3*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*x^2)/a/b/(b*x^3+a)+1/9*(b^(1/3)*(a*f
+2*b*c)-a^(1/3)*(2*a*g+b*d))*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(5/3)-1/18*(b^
(1/3)*(a*f+2*b*c)-a^(1/3)*(2*a*g+b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/a^(5/3)/b^(5/3)+1/3*h*ln(b*x^3+a)/b^2-1/9*(2*b^(4/3)*c+a^(1/3)*b*d+a*
b^(1/3)*f+2*a^(4/3)*g)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^
(5/3)/b^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1872, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^{4/3}g+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2b^{4/3}c\right)}{3\sqrt{3}a^{5/3}b^{5/3}}$$
$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)}{18a^{5/3}b^{5/3}}$$
$$+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)}{9a^{5/3}b^{5/3}}$$
$$+\frac{h\log\left(a+bx^3\right)}{3b^2}+\frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{3ab\left(a+bx^3\right)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x]
```

```
[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(3*a*b*(a + b*x^3)) - ((2
*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)*f + 2*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(5/3)) + ((b^(1/3)*(2*b*
c + a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(5
/3)) - ((b^(1/3)*(2*b*c + a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(5/3)) + (h*Log[a + b*x^3])/(3*b
^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3ab\left(a+bx^3\right)} - \frac{\int \frac{-b(2bc+af)-b(bd+2ag)x-3abhx^2}{a+bx^3}\,dx}{3ab^2}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3ab\left(a+bx^3\right)} - \frac{\int \frac{-b(2bc+af)-b(bd+2ag)x}{a+bx^3}\,dx}{3ab^2} + \frac{h\int \frac{x^2}{a+bx^3}\,dx}{b}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3ab\left(a+bx^3\right)} + \frac{h\log\left(a+bx^3\right)}{3b^2} - \frac{\int \frac{\sqrt[3]{a}\left(-2b^{4/3}(2bc+af)-\sqrt[3]{a}b(bd+2ag)\right)+\sqrt[3]{b}\left(b^{4/3}(2bc+af)-\sqrt[3]{a}b(bd+2ag)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b^{7/3}} + \frac{\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b^{4/3}}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3ab\left(a+bx^3\right)} + \frac{\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{5/3}} + \frac{h\log\left(a+bx^3\right)}{3b^2} + \frac{\left(2b^{4/3}c+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2a^{4/3}g\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}b^{4/3}} - \frac{\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{5/3}}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3ab\left(a+bx^3\right)} + \frac{\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{5/3}} - \frac{\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{5/3}} + \frac{h\log\left(a+bx^3\right)}{3b^2} + \frac{\left(2b^{4/3}c+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2a^{4/3}g\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}b^{5/3}}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3ab\left(a+bx^3\right)} - \frac{\left(2b^{4/3}c+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{5/3}} + \frac{\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{5/3}} - \frac{\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{5/3}} + \frac{h\log\left(a+bx^3\right)}{3b^2}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.97

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^3\right)^2}\,dx$$

$$= \frac{\frac{6(a^2h+b^2x(c+dx)-ab(e+x(f+gx)))}{a(a+bx^3)} - \frac{2\sqrt{3}\sqrt[3]{b}\left(2b^{4/3}c+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2a^{4/3}g\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{2\sqrt[3]{b}\left(2b^{4/3}c-\sqrt[3]{a}bd+a\sqrt[3]{b}f-\right.}{a^{5/3}}}{18b^2}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x]
```

```
[Out] ((6*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a*(a + b*x^3)) - (2
*Sqrt[3]*b^(1/3)*(2*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)*f + 2*a^(4/3)*g)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*b^(1/3)*(2*b^(4/3)*
c - a^(1/3)*b*d + a*b^(1/3)*f - 2*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/a^(5
/3) + (b^(1/3)*(-2*b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f + 2*a^(4/3)*g)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + 6*h*Log[a + b*x^3])/
(18*b^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.43

method	result
risch	$\frac{-\frac{(ag-bd)x^{2}}{3ab}-\frac{(af-bc)x}{3ab}+\frac{ah-be}{3b^{2}}}{b\,x^{3}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(3h\_R^{2}+\frac{(2ag+bd)\_R}{a}+\frac{af+2bc}{a}\right)\ln\left(x-\_R\right)}{\_R^{2}}}{9b^{2}}$
default	$\frac{-\frac{(ag-bd)x^{2}}{3ab}-\frac{(af-bc)x}{3ab}+\frac{ah-be}{3b^{2}}}{b\,x^{3}+a}+\frac{(af+2bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(2ag+bd)}{3ba}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*(a*g-b*d)/a/b*x^2-1/3*(a*f-b*c)/a/b*x+1/3*(a*h-b*e)/b^2)/(b*x^3+a)+1/
9/b^2*sum((3*h*_R^2+1/a*(2*a*g+b*d)*_R+(a*f+2*b*c)/a)/_R^2*ln(x-_R),_R=Root
Of(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 12636, normalized size of antiderivative = 45.78

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^3\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas"
)
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.06

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx$$
$$= -\frac{abe-a^2h-(b^2d-abg)x^2-(b^2c-abf)x}{3\,(ab^3x^3+a^2b^2)} + \frac{\sqrt{3}\Big(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\Big)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2} + \frac{\Big(6\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,bc-af\Big)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\Big(3\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,bc+af\Big)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima"
)
```

```
[Out] -1/3*(a*b*e - a^2*h - (b^2*d - a*b*g)*x^2 - (b^2*c - a*b*f)*x)/(a*b^3*x^3 +
 a^2*b^2) + 1/9*sqrt(3)*(b^2*d*(a/b)^(2/3) + 2*a*b*g*(a/b)^(2/3) + 2*b^2*c*
(a/b)^(1/3) + a*b*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/
b)^(1/3))/(a^2*b^2) + 1/18*(6*a*h*(a/b)^(2/3) + b*d*(a/b)^(1/3) + 2*a*g*(a/
b)^(1/3) - 2*b*c - a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)
^(2/3)) + 1/9*(3*a*h*(a/b)^(2/3) - b*d*(a/b)^(1/3) - 2*a*g*(a/b)^(1/3) + 2*
b*c + a*f)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.09

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx$$
$$= \frac{h\log\left(|bx^3+a|\right)}{3\,b^2} - \frac{\sqrt{3}\left(2\,b^2c+abf-\left(-ab^2\right)^{\frac{1}{3}}bd-2\left(-ab^2\right)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab} - \frac{\left(2\,b^2c+abf+\left(-ab^2\right)^{\frac{1}{3}}bd+2\left(-ab^2\right)^{\frac{1}{3}}ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}ab} + \frac{(bd-ag)x^2+(bc-af)x-\frac{abe-a^2h}{b}}{3\,(bx^3+a)ab} - \frac{\left(ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2b^2g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,ab^3c+a^2b^2f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^3}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*h*log(abs(b*x^3 + a))/b^2 - 1/9*sqrt(3)*(2*b^2*c + a*b*f - (-a*b^2)^(1/
3)*b*d - 2*(-a*b^2)^(1/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/18*(2*b^2*c + a*b*f + (-a*b^2)^(1/3)*b*d
 + 2*(-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)
^(2/3)*a*b) + 1/3*((b*d - a*g)*x^2 + (b*c - a*f)*x - (a*b*e - a^2*h)/b)/((b
*x^3 + a)*a*b) - 1/9*(a*b^3*d*(-a/b)^(1/3) + 2*a^2*b^2*g*(-a/b)^(1/3) + 2*a
*b^3*c + a^2*b^2*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.77 (sec) , antiderivative size = 835, normalized size of antiderivative = 3.03

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(\frac{\mathrm{root}(729\,a^5\,b^6\,z^3-729\,a^5\,b^4\,h\,z^2+54\,a^4\,b^3\,f\,g\,z+108\,a^3\,b^4\,c\,g\,z+27\,a^3\,b^4\,d\,f\,z+54\,a^2\,b^5\,c\,d\,z+\cdots}{\cdots}\right.\right.$$
$$+\frac{9\,a^3\,h^2+2\,b^3\,c\,d+4\,a\,b^2\,c\,g+a\,b^2\,d\,f+2\,a^2\,b\,f\,g}{9\,a^2\,b^2}$$
$$\left.+\frac{x\,(4\,a^2\,g^2-3\,f\,h\,a^2+4\,a\,b\,d\,g-6\,c\,h\,a\,b+b^2\,d^2)}{9\,a^2\,b}\right)\mathrm{root}\big(729\,a^5\,b^6\,z^3-729\,a^5\,b^4\,h\,z^2$$
$$+54\,a^4\,b^3\,f\,g\,z+108\,a^3\,b^4\,c\,g\,z+27\,a^3\,b^4\,d\,f\,z+54\,a^2\,b^5\,c\,d\,z+243\,a^5\,b^2\,h^2\,z$$
$$-18\,a^4\,b\,f\,g\,h-36\,a^3\,b^2\,c\,g\,h-9\,a^3\,b^2\,d\,f\,h-18\,a^2\,b^3\,c\,d\,h-12\,a\,b^4\,c^2\,f+12\,a^3\,b^2\,d\,g^2$$
$$\left.+6\,a^2\,b^3\,d^2\,g-6\,a^2\,b^3\,c\,f^2+8\,a^4\,b\,g^3+a\,b^4\,d^3-27\,a^5\,h^3-8\,b^5\,c^3-a^3\,b^2\,f^3,z,k\big)\right)$$
$$+\frac{\frac{x\,(b\,c-a\,f)}{3\,a\,b}-\frac{b\,e-a\,h}{3\,b^2}+\frac{x^2\,(b\,d-a\,g)}{3\,a\,b}}{b\,x^3+a}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((root(729*a^5*b^6*z^3 - 729*a^5*b^4*h*z^2 + 54*a^4*b^3*f*g*z + 1
08*a^3*b^4*c*g*z + 27*a^3*b^4*d*f*z + 54*a^2*b^5*c*d*z + 243*a^5*b^2*h^2*z
- 18*a^4*b*f*g*h - 36*a^3*b^2*c*g*h - 9*a^3*b^2*d*f*h - 18*a^2*b^3*c*d*h -
12*a*b^4*c^2*f + 12*a^3*b^2*d*g^2 + 6*a^2*b^3*d^2*g - 6*a^2*b^3*c*f^2 + 8*a
^4*b*g^3 + a*b^4*d^3 - 27*a^5*h^3 - 8*b^5*c^3 - a^3*b^2*f^3, z, k)*(9*root(
729*a^5*b^6*z^3 - 729*a^5*b^4*h*z^2 + 54*a^4*b^3*f*g*z + 108*a^3*b^4*c*g*z
+ 27*a^3*b^4*d*f*z + 54*a^2*b^5*c*d*z + 243*a^5*b^2*h^2*z - 18*a^4*b*f*g*h
- 36*a^3*b^2*c*g*h - 9*a^3*b^2*d*f*h - 18*a^2*b^3*c*d*h - 12*a*b^4*c^2*f +
12*a^3*b^2*d*g^2 + 6*a^2*b^3*d^2*g - 6*a^2*b^3*c*f^2 + 8*a^4*b*g^3 + a*b^4*
d^3 - 27*a^5*h^3 - 8*b^5*c^3 - a^3*b^2*f^3, z, k)*a^2*b^2 - 6*a^2*h + 2*b^2
*c*x + a*b*f*x))/a + (9*a^3*h^2 + 2*b^3*c*d + 4*a*b^2*c*g + a*b^2*d*f + 2*a
^2*b*f*g)/(9*a^2*b^2) + (x*(b^2*d^2 + 4*a^2*g^2 - 3*a^2*f*h - 6*a*b*c*h + 4
*a*b*d*g))/(9*a^2*b))*root(729*a^5*b^6*z^3 - 729*a^5*b^4*h*z^2 + 54*a^4*b^3
*f*g*z + 108*a^3*b^4*c*g*z + 27*a^3*b^4*d*f*z + 54*a^2*b^5*c*d*z + 243*a^5*
b^2*h^2*z - 18*a^4*b*f*g*h - 36*a^3*b^2*c*g*h - 9*a^3*b^2*d*f*h - 18*a^2*b^
3*c*d*h - 12*a*b^4*c^2*f + 12*a^3*b^2*d*g^2 + 6*a^2*b^3*d^2*g - 6*a^2*b^3*c
*f^2 + 8*a^4*b*g^3 + a*b^4*d^3 - 27*a^5*h^3 - 8*b^5*c^3 - a^3*b^2*f^3, z, k
), k, 1, 3) + ((x*(b*c - a*f))/(3*a*b) - (b*e - a*h)/(3*b^2) + (x^2*(b*d -
a*g))/(3*a*b))/(a + b*x^3)
```

3.417 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 289

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2}\,dx \\
&= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b(a+bx^3)} \\
&\quad - \frac{\left(2b^{4/3}d+\sqrt[3]{a}be+a\sqrt[3]{b}g+2a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{5/3}} \\
&\quad + \frac{c\log(x)}{a^2} + \frac{\left(\sqrt[3]{b}(2bd+ag)-\sqrt[3]{a}(be+2ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{5/3}} \\
&\quad - \frac{\left(\sqrt[3]{b}(2bd+ag)-\sqrt[3]{a}(be+2ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{5/3}} - \frac{c\log(a+bx^3)}{3a^2}
\end{aligned}$$

```
[Out] 1/3*x*(a*(-a*g+b*d)+a*(-a*h+b*e)*x-b*(-a*f+b*c)*x^2)/a^2/b/(b*x^3+a)+c*ln(x
)/a^2+1/9*(b^(1/3)*(a*g+2*b*d)-a^(1/3)*(2*a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a
^(5/3)/b^(5/3)-1/18*(b^(1/3)*(a*g+2*b*d)-a^(1/3)*(2*a*h+b*e))*ln(a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(5/3)-1/3*c*ln(b*x^3+a)/a^2-1/9*(2*b
^(4/3)*d+a^(1/3)*b*e+a*b^(1/3)*g+2*a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)
*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^{4/3}h+\sqrt[3]{a}be+a\sqrt[3]{b}g+2b^{4/3}d\right)}{3\sqrt{3}a^{5/3}b^{5/3}}$$
$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(2ah+be)}{\sqrt[3]{b}}+ag+2bd\right)}{18a^{5/3}b^{4/3}}$$
$$+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(ag+2bd)-\sqrt[3]{a}(2ah+be)\right)}{9a^{5/3}b^{5/3}}$$
$$+\frac{x(-bx^2(bc-af)+a(bd-ag)+ax(be-ah))}{3a^2b(a+bx^3)}-\frac{c\log(a+bx^3)}{3a^2}+\frac{c\log(x)}{a^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x]
```

```
[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(3*a^2*b*(a + b*x
^3)) - ((2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^(4/3)*h)*ArcTan[(a^(
1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(5/3)) + (c*Lo
g[x])/a^2 + ((b^(1/3)*(2*b*d + a*g) - a^(1/3)*(b*e + 2*a*h))*Log[a^(1/3) +
b^(1/3)*x])/(9*a^(5/3)*b^(5/3)) - ((2*b*d + a*g - (a^(1/3)*(b*e + 2*a*h))/b
^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3)
) - (c*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\,(a+bx^3)} - \frac{\int \frac{-3b^2c-b(2bd+ag)x-b(be+2ah)x^2}{x(a+bx^3)}\,dx}{3ab^2}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\,(a+bx^3)} - \frac{\int \left(-\frac{3b^2c}{ax}+\frac{b(-a(2bd+ag)-a(be+2ah)x+3b^2cx^2)}{a(a+bx^3)}\right)dx}{3ab^2}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\,(a+bx^3)} + \frac{c\log(x)}{a^2} - \frac{\int \frac{-a(2bd+ag)-a(be+2ah)x+3b^2cx^2}{a+bx^3}\,dx}{3a^2b}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\,(a+bx^3)} + \frac{c\log(x)}{a^2} - \frac{\int \frac{-a(2bd+ag)-a(be+2ah)x}{a+bx^3}\,dx}{3a^2b} - \frac{(bc)\int \frac{x^2}{a+bx^3}\,dx}{a^2}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\,(a+bx^3)} + \frac{c\log(x)}{a^2} - \frac{c\log\left(a+bx^3\right)}{3a^2} - \frac{\int \frac{\sqrt[3]{a}\left(-2a\sqrt[3]{b}(2bd+ag)-a^{4/3}(be+2ah)\right)+\sqrt[3]{b}\left(a\sqrt[3]{b}(2bd+ag)-a^{4/3}(be+2ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{8/3}b^{4/3}} + \frac{\left(2bd+ag-\frac{\sqrt[3]{a}(be+2ah)}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\,(a+bx^3)} + \frac{c\log(x)}{a^2} + \frac{\left(2bd+ag-\frac{\sqrt[3]{a}(be+2ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{c\log\left(a+bx^3\right)}{3a^2} + \frac{\left(2b^{4/3}d+\sqrt[3]{a}be+a\sqrt[3]{b}g+2a^{4/3}h\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}b^{4/3}} - \frac{\left(2bd+ag-\frac{\sqrt[3]{a}(be+2ah)}{\sqrt[3]{b}}\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{4/3}}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\,(a+bx^3)}+\frac{c\log(x)}{a^2} + \frac{\left(2bd+ag-\frac{\sqrt[3]{a}(be+2ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{\left(2bd+ag-\frac{\sqrt[3]{a}(be+2ah)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}} - \frac{c\log\left(a+bx^3\right)}{3a^2} + \frac{\left(2b^{4/3}d+\sqrt[3]{a}be+a\sqrt[3]{b}g+2a^{4/3}h\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}b^{5/3}}$$

$$= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b\,(a+bx^3)} - \frac{\left(2b^{4/3}d+\sqrt[3]{a}be+a\sqrt[3]{b}g+2a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{5/3}} + \frac{c\log(x)}{a^2}+\frac{\left(2bd+ag-\frac{\sqrt[3]{a}(be+2ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{\left(2bd+ag-\frac{\sqrt[3]{a}(be+2ah)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}} - \frac{c\log\left(a+bx^3\right)}{3a^2}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.93

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{6a(-b(c+x(d+ex))+a(f+x(g+hx)))}{b(a+bx^3)} - \frac{2\sqrt{3}\sqrt[3]{a}\left(2b^{4/3}d+\sqrt[3]{a}be+a\sqrt[3]{b}g+2a^{4/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}} + 18c\log(x) + \frac{2\sqrt[3]{a}\left(2b\right.}{}}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a*(-(b*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(b*(a + b*x^3)) - (2
*Sqrt[3]*a^(1/3)*(2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^(4/3)*h)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(5/3) + 18*c*Log[x] + (2*a^(1/
3)*(2*b^(4/3)*d - a^(1/3)*b*e + a*b^(1/3)*g - 2*a^(4/3)*h)*Log[a^(1/3) + b^
(1/3)*x])/b^(5/3) + (a^(1/3)*(-2*b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g + 2*
a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - 6*c*Lo
g[a + b*x^3])/(18*a^2)
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.01

method	result
default	$\frac{c\ln(x)}{a^2}+\frac{\frac{-\frac{a(ah-be)x^2}{3b}-\frac{a(ag-bd)x}{3b}-\frac{a(af-bc)}{3b}}{b\,x^3+a}+\frac{\left(a^2g+2abd\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{}}{a^2}$
risch	$\frac{-\frac{(ah-be)x^2}{3ab}-\frac{(ag-bd)x}{3ab}-\frac{af-bc}{3ab}}{b\,x^3+a}+\frac{c\ln(-x)}{a^2}+\left(\sum_{\_R=\mathrm{RootOf}\left(a^6b^5\_Z^3+9a^4b^5c\_Z^2+\left(6a^5b^2gh+12a^4b^3dh+3a^4b^3eg+6a^3b^4de+27a^2b^5c^2\right.\right.}\right.$

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] c*ln(x)/a^2+1/a^2*((-1/3*a*(a*h-b*e)/b*x^2-1/3*a*(a*g-b*d)/b*x-1/3*a*(a*f-b *c)/b)/(b*x^3+a)+1/3/b*((a^2*g+2*a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(2*a^2*h+a*b*e)*(-1/3/b/(a/b) ^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-b*c*ln (b*x^3+a)))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.33 (sec) , antiderivative size = 12541, normalized size of antiderivative = 43.39

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^2}\,dx=\text{Too large to display}$$

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="frica s")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.04

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^2}\,dx \\
&= \frac{(be-ah)x^2+bc-af+(bd-ag)x}{3\left(ab^2x^3+a^2b\right)}+\frac{c\log(x)}{a^2} \\
&+ \frac{\sqrt{3}\left(abe\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} \\
&- \frac{\left(6\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abd+a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\
&- \frac{\left(3\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}+abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abd-a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}
\end{aligned}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxim
a")
[Out] 1/3*((b*e - a*h)*x^2 + b*c - a*f + (b*d - a*g)*x)/(a*b^2*x^3 + a^2*b) + c*l
og(x)/a^2 + 1/9*sqrt(3)*(a*b*e*(a/b)^(2/3) + 2*a^2*h*(a/b)^(2/3) + 2*a*b*d*
(a/b)^(1/3) + a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/
b)^(1/3))/(a^3*b) - 1/18*(6*b^2*c*(a/b)^(2/3) - a*b*e*(a/b)^(1/3) - 2*a^2*h
*(a/b)^(1/3) + 2*a*b*d + a^2*g)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2
*b^2*(a/b)^(2/3)) - 1/9*(3*b^2*c*(a/b)^(2/3) + a*b*e*(a/b)^(1/3) + 2*a^2*h*
(a/b)^(1/3) - 2*a*b*d - a^2*g)*log(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.09

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2}\,dx$$
$$= -\frac{c\log(|bx^3+a|)}{3a^2}+\frac{c\log(|x|)}{a^2}$$
$$-\frac{\sqrt{3}\left(2b^2d+abg-(-ab^2)^{\frac{1}{3}}be-2(-ab^2)^{\frac{1}{3}}ah\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9(-ab^2)^{\frac{2}{3}}ab}$$
$$-\frac{\left(2b^2d+abg+(-ab^2)^{\frac{1}{3}}be+2(-ab^2)^{\frac{1}{3}}ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18(-ab^2)^{\frac{2}{3}}ab}$$
$$+\frac{abc-a^2f+(abe-a^2h)x^2+(abd-a^2g)x}{3(bx^3+a)a^2b}$$
$$-\frac{\left(a^3b^3e\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2a^4b^2h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2a^3b^3d+a^4b^2g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^5b^3}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac"
)
```

```
[Out] -1/3*c*log(abs(b*x^3 + a))/a^2 + c*log(abs(x))/a^2 - 1/9*sqrt(3)*(2*b^2*d +
 a*b*g - (-a*b^2)^(1/3)*b*e - 2*(-a*b^2)^(1/3)*a*h)*arctan(1/3*sqrt(3)*(2*x
 + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/18*(2*b^2*d + a*b*g
 + (-a*b^2)^(1/3)*b*e + 2*(-a*b^2)^(1/3)*a*h)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/((-a*b^2)^(2/3)*a*b) + 1/3*(a*b*c - a^2*f + (a*b*e - a^2*h)*x^2
 + (a*b*d - a^2*g)*x)/((b*x^3 + a)*a^2*b) - 1/9*(a^3*b^3*e*(-a/b)^(1/3) + 2
*a^4*b^2*h*(-a/b)^(1/3) + 2*a^3*b^3*d + a^4*b^2*g)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a^5*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.78 (sec) , antiderivative size = 1660, normalized size of antiderivative = 5.74

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x)
```

```
[Out] ((b*c - a*f)/(3*a*b) + (x*(b*d - a*g))/(3*a*b) + (x^2*(b*e - a*h))/(3*a*b))
/(a + b*x^3) + symsum(log((c*(4*b^2*d^2 + a^2*g^2 - 3*b^2*c*e - 6*a*b*c*h +
```

4*a*b*d*g))/(9*a^3) - (root(729*a^6*b^5*z^3 + 729*a^4*b^5*c*z^2 + 54*a^5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a^3*b^4*d*e*z + 243*a^2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 36*a^2*b^3*c*d*h + 9*a^2*b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a^2*b^3*d^2*g - 6*a^3*b^2*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b^5*c^3 + a^2*b^3*e^3, z, k)*(a^3*g^2 + 4*a*b^2*d^2 + 36*b^3*c^2*x + 324*root(729*a^6*b^5*z^3 + 729*a^4*b^5*c*z^2 + 54*a^5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a^3*b^4*d*e*z + 243*a^2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 36*a^2*b^3*c*d*h + 9*a^2*b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a^2*b^3*d^2*g - 6*a^3*b^2*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b^5*c^3 + a^2*b^3*e^3, z, k)^2*a^4*b^3*x - 18*root(729*a^6*b^5*z^3 + 729*a^4*b^5*c*z^2 + 54*a^5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a^3*b^4*d*e*z + 243*a^2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 36*a^2*b^3*c*d*h + 9*a^2*b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a^2*b^3*d^2*g - 6*a^3*b^2*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b^5*c^3 + a^2*b^3*e^3, z, k)*a^4*b*h + 6*a*b^2*c*e + 12*a^2*b*c*h + 4*a^2*b*d*g + 20*a^3*g*h*x - 9*root(729*a^6*b^5*z^3 + 729*a^4*b^5*c*z^2 + 54*a^5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a^3*b^4*d*e*z + 243*a^2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 36*a^2*b^3*c*d*h + 9*a^2*b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a^2*b^3*d^2*g - 6*a^3*b^2*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b^5*c^3 + a^2*b^3*e^3, z, k)*a^3*b^2*e + 216*root(729*a^6*b^5*z^3 + 729*a^4*b^5*c*z^2 + 54*a^5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a^3*b^4*d*e*z + 243*a^2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 36*a^2*b^3*c*d*h + 9*a^2*b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a^2*b^3*d^2*g - 6*a^3*b^2*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b^5*c^3 + a^2*b^3*e^3, z, k)*a^2*b^3*c*x + 20*a*b^2*d*e*x + 40*a^2*b*d*h*x + 10*a^2*b*e*g*x))/(9*a^2) - (x*(8*a^4*h^3 - 8*b^4*d^3 + a*b^3*e^3 - a^3*b*g^3 - 6*a^2*b^2*d*g^2 + 6*a^2*b^2*e^2*h + 12*b^4*c*d*e - 12*a*b^3*d^2*g + 12*a^3*b*e*h^2 + 12*a^2*b^2*c*g*h + 24*a*b^3*c*d*h + 6*a*b^3*c*e*g))/(27*a^3*b^2))*root(729*a^6*b^5*z^3 + 729*a^4*b^5*c*z^2 + 54*a^5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a^3*b^4*d*e*z + 243*a^2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 36*a^2*b^3*c*d*h + 9*a^2*b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a^2*b^3*d^2*g - 6*a^3*b^2*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b^5*c^3 + a^2*b^3*e^3, z, k), k, 1, 3) + (c*log(x))/a^2

3.418 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^2} dx$

Optimal result

Integrand size = 38, antiderivative size = 301

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^2}\,dx &= -\frac{c}{a^2x}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b(a+bx^3)} \\ &\quad+\frac{\left(4b^{5/3}c-2a^{2/3}be-ab^{2/3}f-a^{5/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{4/3}} \\ &\quad+\frac{d\log(x)}{a^2}+\frac{\left(b^{2/3}(4bc-af)+a^{2/3}(2be+ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{4/3}} \\ &\quad-\frac{\left(b^{2/3}(4bc-af)+a^{2/3}(2be+ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{4/3}}-\frac{d\log(a+bx^3)}{3a^2}\end{aligned}$$

```
[Out] -c/a^2/x+1/3*x*(a*(-a*h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/a^2/b/(b*x^3+
a)+d*ln(x)/a^2+1/9*(b^(2/3)*(-a*f+4*b*c)+a^(2/3)*(a*h+2*b*e))*ln(a^(1/3)+b^
(1/3)*x)/a^(7/3)/b^(4/3)-1/18*(b^(2/3)*(-a*f+4*b*c)+a^(2/3)*(a*h+2*b*e))*ln
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(4/3)-1/3*d*ln(b*x^3+a)/a
^2+1/9*(4*b^(5/3)*c-2*a^(2/3)*b*e-a*b^(2/3)*f-a^(5/3)*h)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^2}\,dx$$
$$=\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{2/3}be+a^{5/3}(-h)-ab^{2/3}f+4b^{5/3}c\right)}{3\sqrt{3}a^{7/3}b^{4/3}}$$
$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+2be)+b^{2/3}(4bc-af)\right)}{18a^{7/3}b^{4/3}}$$
$$+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(ah+2be)+b^{2/3}(4bc-af)\right)}{9a^{7/3}b^{4/3}}$$
$$+\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{3a^2b(a+bx^3)}-\frac{d\log(a+bx^3)}{3a^2}-\frac{c}{a^2x}+\frac{d\log(x)}{a^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -(c/(a^2*x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3
*a^2*b*(a + b*x^3)) + ((4*b^(5/3)*c - 2*a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)
*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b
^(4/3)) + (d*Log[x])/a^2 + ((b^(2/3)*(4*b*c - a*f) + a^(2/3)*(2*b*e + a*h))
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(4/3)) - ((b^(2/3)*(4*b*c - a*f) +
a^(2/3)*(2*b*e + a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*
a^(7/3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
```

```
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

integral

$$= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\,(a+bx^3)} - \frac{\int \frac{-3b^2c-3b^2dx-b(2be+ah)x^2+b^2\left(\frac{bc}{a}-f\right)x^3}{x^2(a+bx^3)}\,dx}{3ab^2}$$

$$= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\,(a+bx^3)} - \frac{\int \left(-\frac{3b^2c}{ax^2}-\frac{3b^2d}{ax}+\frac{b(-a(2be+ah)+b(4bc-af)x+3b^2dx^2)}{a(a+bx^3)}\right)dx}{3ab^2}$$

$$= -\frac{c}{a^2x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\,(a+bx^3)} + \frac{d\log(x)}{a^2} - \frac{\int \frac{-a(2be+ah)+b(4bc-af)x+3b^2dx^2}{a+bx^3}\,dx}{3a^2b}$$

$$= -\frac{c}{a^2x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\,(a+bx^3)} + \frac{d\log(x)}{a^2} - \frac{\int \frac{-a(2be+ah)+b(4bc-af)x}{a+bx^3}\,dx}{3a^2b} - \frac{(bd)\int \frac{x^2}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{c}{a^2x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\,(a+bx^3)} + \frac{d\log(x)}{a^2} - \frac{d\log(a+bx^3)}{3a^2} - \frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{a}b(4bc-af)-2a\sqrt[3]{b}(2be+ah)\right)+\sqrt[3]{b}\left(\sqrt[3]{a}b(4bc-af)+a\sqrt[3]{b}(2be+ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{8/3}b^{4/3}} + \frac{\left(b^{2/3}(4bc-af)+a^{2/3}(2be+ah)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{7/3}b}$$

$$= -\frac{c}{a^2x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\,(a+bx^3)} + \frac{d\log(x)}{a^2} + \frac{\left(b^{2/3}(4bc-af)+a^{2/3}(2be+ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{4/3}} - \frac{d\log(a+bx^3)}{3a^2} - \frac{\left(4b^{5/3}c-2a^{2/3}be-ab^{2/3}f-a^{5/3}h\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^2b} - \frac{\left(b^{2/3}(4bc-af)+a^{2/3}(2be+ah)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{7/3}b^{4/3}}$$

$$= -\frac{c}{a^2x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\,(a+bx^3)} + \frac{d\log(x)}{a^2} + \frac{\left(b^{2/3}(4bc-af)+a^{2/3}(2be+ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{4/3}} - \frac{\left(b^{2/3}(4bc-af)+a^{2/3}(2be+ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{4/3}} - \frac{d\log\left(a+bx^3\right)}{3a^2} - \frac{\left(4b^{5/3}c-2a^{2/3}be-ab^{2/3}f-a^{5/3}h\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{7/3}b^{4/3}}$$

$$= -\frac{c}{a^2x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b\,(a+bx^3)} + \frac{\left(4b^{5/3}c-2a^{2/3}be-ab^{2/3}f-a^{5/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{4/3}} + \frac{d\log(x)}{a^2} + \frac{\left(b^{2/3}(4bc-af)+a^{2/3}(2be+ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{4/3}} - \frac{\left(b^{2/3}(4bc-af)+a^{2/3}(2be+ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{4/3}} - \frac{d\log\left(a+bx^3\right)}{3a^2}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.95

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^2}\,dx =$$

$$-\frac{\frac{18ac}{x}+\frac{6a\left(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx))\right)}{b(a+bx^3)}+\frac{2\sqrt{3}a^{2/3}\left(-4b^{5/3}c+2a^{2/3}be+ab^{2/3}f+a^{5/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}}-18ad\log(x)-}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -1/18*((18*a*c)/x + (6*a*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x))
))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(2/3)*(-4*b^(5/3)*c + 2*a^(2/3)*b*e + a*b
^(2/3)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(4/3)
- 18*a*d*Log[x] - (2*a^(2/3)*(4*b^(5/3)*c + 2*a^(2/3)*b*e - a*b^(2/3)*f + a
^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) + (a^(2/3)*(4*b^(5/3)*c + 2*a^(
2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/b^(4/3) + 6*a*d*Log[a + b*x^3])/a^3
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.99

method	result
default	$-\frac{c}{a^{2}x}+\frac{d\ln(x)}{a^{2}}+\frac{\frac{\left(\frac{af}{3}-\frac{bc}{3}\right)x^{2}-\frac{a(ah-be)x}{3b}-\frac{a(ag-bd)}{3b}}{b\,x^{3}+a}+\frac{\left(a^{2}h+2aeb\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\cdots\right.}{\cdots}\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}\cdots}\right.}{\cdots}}{\cdots}$
risch	$\frac{\frac{(af-4bc)x^{3}}{3a^{2}}-\frac{(ah-be)x^{2}}{3ab}-\frac{(ag-bd)x}{3ab}-\frac{c}{a}}{x(b\,x^{3}+a)}+\frac{d\ln(x)}{a^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{7}b^{4}\_Z^{3}+9a^{5}b^{4}d\_Z^{2}+\left(3a^{5}b^{2}fh-12a^{4}b^{3}ch+6a^{4}b^{3}ef-24a^{3}b^{4}ce+\cdots\right.\right.}\cdots\right.}{\cdots}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -c/a^2/x+d*ln(x)/a^2+1/a^2*(((1/3*a*f-1/3*b*c)*x^2-1/3*a*(a*h-b*e)/b*x-1/3*
a*(a*g-b*d)/b)/(b*x^3+a)+1/3/b*((a^2*h+2*a*b*e)*(1/3/b/(a/b)^(2/3)*ln(x+(a/
b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(a*b*f-4*b^2*c)*(-1/3
/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/
b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
)-b*d*ln(b*x^3+a)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 22.13 (sec) , antiderivative size = 12556, normalized size of antiderivative = 41.71

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^2 (a + bx^3)^2} \, dx = \text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.09

$$\begin{aligned}
&\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^2 (a + bx^3)^2} \, dx \\
&= -\frac{(4\,b^2c - abf)x^3 + 3\,abc - (abe - a^2h)x^2 - (abd - a^2g)x}{3\,(a^2b^2x^4 + a^3bx)} + \frac{d\log(x)}{a^2} \\
&\quad - \frac{\sqrt{3}\Big(4\,b^2c\big(\frac{a}{b}\big)^{\frac{2}{3}} - abf\big(\frac{a}{b}\big)^{\frac{2}{3}} - 2\,abe\big(\frac{a}{b}\big)^{\frac{1}{3}} - a^2h\big(\frac{a}{b}\big)^{\frac{1}{3}}\Big)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} \\
&\quad - \frac{\Big(6\,b^2d\big(\frac{a}{b}\big)^{\frac{2}{3}} + 4\,b^2c\big(\frac{a}{b}\big)^{\frac{1}{3}} - abf\big(\frac{a}{b}\big)^{\frac{1}{3}} + 2\,abe + a^2h\Big)\log\Big(x^2 - x\big(\frac{a}{b}\big)^{\frac{1}{3}} + \big(\frac{a}{b}\big)^{\frac{2}{3}}\Big)}{18\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\
&\quad - \frac{\Big(3\,b^2d\big(\frac{a}{b}\big)^{\frac{2}{3}} - 4\,b^2c\big(\frac{a}{b}\big)^{\frac{1}{3}} + abf\big(\frac{a}{b}\big)^{\frac{1}{3}} - 2\,abe - a^2h\Big)\log\Big(x + \big(\frac{a}{b}\big)^{\frac{1}{3}}\Big)}{9\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}
\end{aligned}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="max
ima")
[Out] -1/3*((4*b^2*c - a*b*f)*x^3 + 3*a*b*c - (a*b*e - a^2*h)*x^2 - (a*b*d - a^2*
g)*x)/(a^2*b^2*x^4 + a^3*b*x) + d*log(x)/a^2 - 1/9*sqrt(3)*(4*b^2*c*(a/b)^(
2/3) - a*b*f*(a/b)^(2/3) - 2*a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3))*arctan(
1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b) - 1/18*(6*b^2*d*(a/b)^
(2/3) + 4*b^2*c*(a/b)^(1/3) - a*b*f*(a/b)^(1/3) + 2*a*b*e + a^2*h)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2/3)) - 1/9*(3*b^2*d*(a/b)^(
2/3) - 4*b^2*c*(a/b)^(1/3) + a*b*f*(a/b)^(1/3) - 2*a*b*e - a^2*h)*log(x + (
a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.08

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^2}\,dx$$
$$= -\frac{d\log\left(|bx^3+a|\right)}{3\,a^2} + \frac{d\log\left(|x|\right)}{a^2} - \frac{\sqrt{3}\left(2\,abe+a^2h+4\,(-ab^2)^{\frac{1}{3}}\,bc-(-ab^2)^{\frac{1}{3}}\,af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{\left(2\,abe+a^2h-4\,(-ab^2)^{\frac{1}{3}}\,bc+(-ab^2)^{\frac{1}{3}}\,af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{4\,b^2cx^3-abfx^3-abex^2+a^2hx^2-abdx+a^2gx+3\,abc}{3\,(bx^4+ax)a^2b} + \frac{\left(4\,a^2b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^3b^3e-a^4b^2h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b^3}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^2 + d*log(abs(x))/a^2 - 1/9*sqrt(3)*(2*a*b*e +
 a^2*h + 4*(-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2*x
 + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/18*(2*a*b*e + a^2*h
 - 4*(-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/3*(4*b^2*c*x^3 - a*b*f*x^3 - a*b*e*x^2
 + a^2*h*x^2 - a*b*d*x + a^2*g*x + 3*a*b*c)/((b*x^4 + a*x)*a^2*b) + 1/9*(4*
a^2*b^4*c*(-a/b)^(1/3) - a^3*b^3*f*(-a/b)^(1/3) - 2*a^3*b^3*e - a^4*b^2*h)*
(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b^3)
```

Mupad [B] (verification not implemented)

Time = 10.01 (sec) , antiderivative size = 1684, normalized size of antiderivative = 5.59

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] symsum(log((d*(a^3*h^2 + 4*a*b^2*e^2 + 12*b^3*c*d - 3*a*b^2*d*f + 4*a^2*b*e
*h))/(9*a^4) - (root(729*a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*f*h*z
```

```
 - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a^3*b^4*d
^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^3*d*e
*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*f^2 -
 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z, k)*(
a^3*h^2 + 4*a*b^2*e^2 + 36*b^3*d^2*x - 24*b^3*c*d + 324*root(729*a^7*b^4*z^
3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*f*h*z - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e
*f*z - 216*a^3*b^4*c*e*z + 243*a^3*b^4*d^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d
*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^3*d*e*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f
 - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*f^2 - 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5
*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z, k)^2*a^4*b^3*x + 6*a*b^2*d*f + 4*a^2*b*
e*h - 80*b^3*c*e*x + 36*root(729*a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b
^2*f*h*z - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a
^3*b^4*d^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2
*b^3*d*e*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3
*c*f^2 - 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3,
 z, k)*a^2*b^3*c - 9*root(729*a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*
f*h*z - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a^3*
b^4*d^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^
3*d*e*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*
f^2 - 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z,
 k)*a^3*b^2*f + 216*root(729*a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*f
*h*z - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a^3*b
^4*d^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^3
*d*e*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*f
^2 - 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z,
k)*a^2*b^3*d*x - 40*a*b^2*c*h*x + 20*a*b^2*e*f*x + 10*a^2*b*f*h*x))/(9*a^2)
 + (x*(64*b^5*c^3 + a^5*h^3 + 8*a^2*b^3*e^3 - a^3*b^2*f^3 + 12*a^2*b^3*c*f^
2 + 12*a^3*b^2*e^2*h - 48*a*b^4*c^2*f + 6*a^4*b*e*h^2 + 24*a^2*b^3*c*d*h -
12*a^2*b^3*d*e*f - 6*a^3*b^2*d*f*h + 48*a*b^4*c*d*e))/(27*a^5*b))*root(729*
a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*f*h*z - 108*a^4*b^3*c*h*z + 54
*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a^3*b^4*d^2*z - 72*a*b^4*c*d*e + 9
*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^3*d*e*f - 6*a^4*b*e*h^2 + 48*a
*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*f^2 - 8*a^2*b^3*e^3 + 27*a*b^4
*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z, k), k, 1, 3) - (c/a + (x^3*(4
*b*c - a*f))/(3*a^2) - (x*(b*d - a*g))/(3*a*b) - (x^2*(b*e - a*h))/(3*a*b))
/(a*x + b*x^4) + (d*log(x))/a^2
```

3.419 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 306

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2}\,dx \\
&= -\frac{c}{2a^2x^2}-\frac{d}{a^2x}-\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2(a+bx^3)} \\
&\quad+\frac{\left(5b^{4/3}c+4\sqrt[3]{a}bd-2a\sqrt[3]{b}f-a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}b^{2/3}} \\
&\quad+\frac{e\log(x)}{a^2}-\frac{\left(\sqrt[3]{b}(5bc-2af)-\sqrt[3]{a}(4bd-ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}b^{2/3}} \\
&\quad+\frac{\left(\sqrt[3]{b}(5bc-2af)-\sqrt[3]{a}(4bd-ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}b^{2/3}}-\frac{e\log(a+bx^3)}{3a^2}
\end{aligned}$$

```
[Out] -1/2*c/a^2/x^2-d/a^2/x-1/3*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*x^2)/a^2/(b*x
^3+a)+e*ln(x)/a^2-1/9*(b^(1/3)*(-2*a*f+5*b*c)-a^(1/3)*(-a*g+4*b*d))*ln(a^(1
/3)+b^(1/3)*x)/a^(8/3)/b^(2/3)+1/18*(b^(1/3)*(-2*a*f+5*b*c)-a^(1/3)*(-a*g+4
*b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(2/3)-1/3*e*ln(b
*x^3+a)/a^2+1/9*(5*b^(4/3)*c+4*a^(1/3)*b*d-2*a*b^(1/3)*f-a^(4/3)*g)*arctan(
1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2}\,dx$$
$$=\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+4\sqrt[3]{a}bd-2a\sqrt[3]{b}f+5b^{4/3}c\right)}{3\sqrt{3}a^{8/3}b^{2/3}}$$
$$+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(4bd-ag)}{\sqrt[3]{b}}-2af+5bc\right)}{18a^{8/3}\sqrt[3]{b}}$$
$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(5bc-2af)-\sqrt[3]{a}(4bd-ag)\right)}{9a^{8/3}b^{2/3}}$$
$$-\frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{3a^2(a+bx^3)}-\frac{e\log(a+bx^3)}{3a^2}-\frac{c}{2a^2x^2}-\frac{d}{a^2x}+\frac{e\log(x)}{a^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/2*c/(a^2*x^2) - d/(a^2*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*
x^2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*c + 4*a^(1/3)*b*d - 2*a*b^(1/3)*f -
 a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a
^(8/3)*b^(2/3)) + (e*Log[x])/a^2 - ((b^(1/3)*(5*b*c - 2*a*f) - a^(1/3)*(4*b
*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b^(2/3)) + ((5*b*c - 2*a*f
- (a^(1/3)*(4*b*d - a*g))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*a^(8/3)*b^(1/3)) - (e*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2(a+bx^3)} - \frac{\int \frac{-3b^2c-3b^2dx-3b^2ex^2+2b^2\left(\frac{bc}{a}-f\right)x^3+b^2\left(\frac{bd}{a}-g\right)x^4}{x^3(a+bx^3)}\,dx}{3ab^2}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2(a+bx^3)} - \frac{\int \left(-\frac{3b^2c}{ax^3}-\frac{3b^2d}{ax^2}-\frac{3b^2e}{ax}+\frac{b^2(5bc-2af+(4bd-ag)x+3bex^2)}{a(a+bx^3)}\right)dx}{3ab^2}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2(a+bx^3)} + \frac{e\log(x)}{a^2} - \frac{\int \frac{5bc-2af+(4bd-ag)x+3bex^2}{a+bx^3}\,dx}{3a^2}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2(a+bx^3)} + \frac{e\log(x)}{a^2} - \frac{\int \frac{5bc-2af+(4bd-ag)x}{a+bx^3}\,dx}{3a^2} - \frac{(be)\int \frac{x^2}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2(a+bx^3)} + \frac{e\log(x)}{a^2} - \frac{e\log(a+bx^3)}{3a^2} - \frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}(5bc-2af)+\sqrt[3]{a}(4bd-ag)\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}(5bc-2af)+\sqrt[3]{a}(4bd-ag)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{8/3}\sqrt[3]{b}} - \frac{\left(5bc-2af-\frac{\sqrt[3]{a}(4bd-ag)}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{8/3}}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2(a+bx^3)} + \frac{e\log(x)}{a^2} - \frac{\left(5bc-2af-\frac{\sqrt[3]{a}(4bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}} - \frac{e\log(a+bx^3)}{3a^2} - \frac{\left(5b^{4/3}c+4\sqrt[3]{a}bd-2a\sqrt[3]{b}f-a^{4/3}g\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{6a^{7/3}\sqrt[3]{b}} + \frac{\left(5bc-2af-\frac{\sqrt[3]{a}(4bd-ag)}{\sqrt[3]{b}}\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{18a^{8/3}\sqrt[3]{b}}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2(a+bx^3)} + \frac{e\log(x)}{a^2} - \frac{\left(5bc-2af-\frac{\sqrt[3]{a}(4bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}} + \frac{\left(5bc-2af-\frac{\sqrt[3]{a}(4bd-ag)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}} - \frac{e\log(a+bx^3)}{3a^2} - \frac{\left(5b^{4/3}c+4\sqrt[3]{a}bd-2a\sqrt[3]{b}f-a^{4/3}g\right)\text{Subst}\left(\int\frac{1}{-3-x^2}dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{8/3}b^{2/3}}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2(a+bx^3)} + \frac{\left(5b^{4/3}c+4\sqrt[3]{a}bd-2a\sqrt[3]{b}f-a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}b^{2/3}} + \frac{e\log(x)}{a^2} - \frac{\left(5bc-2af-\frac{\sqrt[3]{a}(4bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}} + \frac{\left(5bc-2af-\frac{\sqrt[3]{a}(4bd-ag)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}} - \frac{e\log(a+bx^3)}{3a^2}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.95

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2}\,dx = -\frac{\frac{9ac}{x^2}+\frac{18ad}{x}+\frac{6a(a^2h+b^2x(c+dx)-ab(e+x(f+gx)))}{b(a+bx^3)}+\frac{2\sqrt{3}\sqrt[3]{a}\left(-5b^{4/3}c-4\sqrt[3]{a}bd+2a\sqrt[3]{b}f+a^{4/3}g\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}-18ae\,\cdots}{\cdots}$$

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x]

[Out] -1/18*((9*a*c)/x^2 + (18*a*d)/x + (6*a*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(1/3)*(-5*b^(4/3)*c - 4*a^(1/3)*b*d + 2*a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) - 18*a*e*Log[x] + (2*a^(1/3)*(5*b^(4/3)*c - 4*a^(1/3)*b*d - 2*a*b^(1/3)*f + a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^(1/3)*(5*b^(4/3)*c - 4*a^(1/3)*b*d - 2*a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 6*a*e*Log[a + b*x^3])/a^3

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.96

method	result
default	$-\frac{c}{2a^2x^2}-\frac{d}{a^2x}+\frac{e\ln(x)}{a^2}+\frac{\left(\frac{ag}{3}-\frac{bd}{3}\right)x^2+\left(\frac{af}{3}-\frac{bc}{3}\right)x-\frac{a(ah-be)}{3b}}{b\,x^3+a}+\frac{(2af-5bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\cdots\right)}{3\cdots}\right)}{3}\cdots$
risch	$\frac{\frac{(ag-4bd)x^4}{3a^2}+\frac{(2af-5bc)x^3}{6a^2}-\frac{(ah-be)x^2}{3ab}-\frac{xd}{a}-\frac{c}{2a}}{x^2(b\,x^3+a)}+\left(\sum_{\_R=\mathrm{RootOf}\left(a^8b^2\_Z^3+9a^6b^2e\_Z^2+\left(6a^5bfg-15a^4b^2cg-24a^4b^2df+27a^4b^2e^2+60a^3b\cdots\right.\right)}\cdots\right)$

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE
)

```
[Out] -1/2*c/a^2/x^2-d/a^2/x+e*ln(x)/a^2+1/a^2*(((1/3*a*g-1/3*b*d)*x^2+(1/3*a*f-1
/3*b*c)*x-1/3*a*(a*h-b*e)/b)/(b*x^3+a)+1/3*(2*a*f-5*b*c)*(1/3/b/(a/b)^(2/3)
*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/
b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(a*g-4*b
*d)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1)))-1/3*e*ln(b*x^3+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.92 (sec) , antiderivative size = 12231, normalized size of antiderivative = 39.97

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.03

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2}\,dx$$
$$= -\frac{2(4b^2d-abg)x^4+6abdx+(5b^2c-2abf)x^3+3abc-2(abe-a^2h)x^2}{6(a^2b^2x^5+a^3bx^2)}+\frac{e\log(x)}{a^2}$$
$$-\frac{\sqrt{3}\left(4bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-ag\left(\frac{a}{b}\right)^{\frac{2}{3}}+5bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-2af\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^3}$$
$$-\frac{\left(6be\left(\frac{a}{b}\right)^{\frac{2}{3}}+4bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-5bc+2af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\left(3be\left(\frac{a}{b}\right)^{\frac{2}{3}}-4bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+5bc-2af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] -1/6*(2*(4*b^2*d - a*b*g)*x^4 + 6*a*b*d*x + (5*b^2*c - 2*a*b*f)*x^3 + 3*a*b
*c - 2*(a*b*e - a^2*h)*x^2)/(a^2*b^2*x^5 + a^3*b*x^2) + e*log(x)/a^2 - 1/9*
sqrt(3)*(4*b*d*(a/b)^(2/3) - a*g*(a/b)^(2/3) + 5*b*c*(a/b)^(1/3) - 2*a*f*(a
/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^3 - 1/18*(
6*b*e*(a/b)^(2/3) + 4*b*d*(a/b)^(1/3) - a*g*(a/b)^(1/3) - 5*b*c + 2*a*f)*lo
g(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1/9*(3*b*e*(a/b)
^(2/3) - 4*b*d*(a/b)^(1/3) + a*g*(a/b)^(1/3) + 5*b*c - 2*a*f)*log(x + (a/b)
^(1/3))/(a^2*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.09

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2}\,dx$$
$$= -\frac{e\log(|bx^3+a|)}{3a^2} + \frac{e\log(|x|)}{a^2} + \frac{\sqrt{3}\left(5b^2c-2abf-4(-ab^2)^{\frac{1}{3}}bd+(-ab^2)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9(-ab^2)^{\frac{2}{3}}a^2} + \frac{\left(5b^2c-2abf+4(-ab^2)^{\frac{1}{3}}bd-(-ab^2)^{\frac{1}{3}}ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18(-ab^2)^{\frac{2}{3}}a^2} + \frac{\left(4a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3bg\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5a^2b^2c-2a^3bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^5b} - \frac{2(4b^2d-abg)x^4+6abdx+(5b^2c-2abf)x^3+3abc-2(abe-a^2h)x^2}{6(bx^3+a)a^2bx^2}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^2 + e*log(abs(x))/a^2 + 1/9*sqrt(3)*(5*b^2*c -
 2*a*b*f - 4*(-a*b^2)^(1/3)*b*d + (-a*b^2)^(1/3)*a*g)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) + 1/18*(5*b^2*c - 2*a
*b*f + 4*(-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) + 1/9*(4*a^2*b^2*d*(-a/b)^(1/3) - a^3*
b*g*(-a/b)^(1/3) + 5*a^2*b^2*c - 2*a^3*b*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^5*b) - 1/6*(2*(4*b^2*d - a*b*g)*x^4 + 6*a*b*d*x + (5*b^2*c - 2*
a*b*f)*x^3 + 3*a*b*c - 2*(a*b*e - a^2*h)*x^2)/((b*x^3 + a)*a^2*b*x^2)
```

Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 1632, normalized size of antiderivative = 5.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] symsum(log((b^2*e*(25*b^2*c^2 + 4*a^2*f^2 - 3*a^2*e*g - 20*a*b*c*f + 12*a*b
*d*e))/(9*a^5) - (root(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g*z
```

- 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g^3, z, k)*b^2*(25*b^2*c^2 + 4*a^2*f^2 - 9*root(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g^3, z, k)*a^4*g + 6*a^2*e*g + 36*root(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g^3, z, k)*a^3*b*d + 36*a*b*e^2*x + 200*b^2*c*d*x + 20*a^2*f*g*x + 324*root(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g^3, z, k)^2*a^5*b*x - 20*a*b*c*f - 24*a*b*d*e - 50*a*b*c*g*x - 80*a*b*d*f*x + 216*root(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g^3, z, k)*a^3*b*e*x))/(9*a^3) - (b*x*(125*b^4*c^3 + a^4*g^3 - 64*a*b^3*d^3 - 8*a^3*b*f^3 + 60*a^2*b^2*c*f^2 + 48*a^2*b^2*d^2*g - 150*a*b^3*c^2*f - 12*a^3*b*d*g^2 - 30*a^2*b^2*c*e*g - 48*a^2*b^2*d*e*f + 120*a*b^3*c*d*e + 12*a^3*b*e*f*g))/(27*a^6))*root(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g^3, z, k), k, 1, 3) - (c/(2*a) + (x^3*(5*b*c - 2*a*f))/(6*a^2) + (x^4*(4*b*d - a*g))/(3*a^2) + (d*x)/a - (x^2*(b*e - a*h))/(3*a*b))/(a*x^2 + b*x^5) + (e*log(x))/a^2

3.420 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 338

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^2}\,dx \\
&= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2(a+bx^3)} \\
&\quad+\frac{\left(5b^{4/3}d+4\sqrt[3]{a}be-2a\sqrt[3]{b}g-a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}b^{2/3}} \\
&\quad-\frac{(2bc-af)\log(x)}{a^3}-\frac{\left(\sqrt[3]{b}(5bd-2ag)-\sqrt[3]{a}(4be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}b^{2/3}} \\
&\quad+\frac{\left(\sqrt[3]{b}(5bd-2ag)-\sqrt[3]{a}(4be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}b^{2/3}} \\
&\quad+\frac{(2bc-af)\log(a+bx^3)}{3a^3}
\end{aligned}$$

```
[Out] -1/3*c/a^2/x^3-1/2*d/a^2/x^2-e/a^2/x-1/3*x*(b*d-a*g+(-a*h+b*e)*x-b*(b*c/a-f
)*x^2)/a^2/(b*x^3+a)-(-a*f+2*b*c)*ln(x)/a^3-1/9*(b^(1/3)*(-2*a*g+5*b*d)-a^(
1/3)*(-a*h+4*b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(2/3)+1/18*(b^(1/3)*(-2*
a*g+5*b*d)-a^(1/3)*(-a*h+4*b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
a^(8/3)/b^(2/3)+1/3*(-a*f+2*b*c)*ln(b*x^3+a)/a^3+1/9*(5*b^(4/3)*d+4*a^(1/3)
*b*e-2*a*b^(1/3)*g-a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1
/2))/a^(8/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^2}\,dx$$
$$= \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+4\sqrt[3]{a}be-2a\sqrt[3]{b}g+5b^{4/3}d\right)}{3\sqrt{3}a^{8/3}b^{2/3}}$$
$$+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(4be-ah)}{\sqrt[3]{b}}-2ag+5bd\right)}{18a^{8/3}\sqrt[3]{b}}$$
$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(5bd-2ag)-\sqrt[3]{a}(4be-ah)\right)}{9a^{8/3}b^{2/3}}+\frac{(2bc-af)\log(a+bx^3)}{3a^3}$$
$$-\frac{\log(x)(2bc-af)}{a^3}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{3a^2(a+bx^3)}-\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -1/3*c/(a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d - a*g + (b*e - a*h)*
x - b*((b*c)/a - f)*x^2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*d + 4*a^(1/3)*b
*e - 2*a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(3*Sqrt[3]*a^(8/3)*b^(2/3)) - ((2*b*c - a*f)*Log[x])/a^3 - ((b^(1/3
)*(5*b*d - 2*a*g) - a^(1/3)*(4*b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(
8/3)*b^(2/3)) + ((5*b*d - 2*a*g - (a^(1/3)*(4*b*e - a*h))/b^(1/3))*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3)*b^(1/3)) + ((2*b*c - a*
f)*Log[a + b*x^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3b^2c-3b^2dx-3b^2ex^2+3b^2\left(\frac{bc}{a}-f\right)x^3+2b^2\left(\frac{bd}{a}-g\right)x^4+b^2\left(\frac{be}{a}-h\right)x^5}{x^4(a+bx^3)}\,dx}{3ab^2} \\
&= -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^2c}{ax^4}-\frac{3b^2d}{ax^3}-\frac{3b^2e}{ax^2}-\frac{3b^2(-2bc+af)}{a^2x}+\frac{b^2\left(a(5bd-2ag)+a(4be-ah)x-3b(2bc-af)x^2\right)}{a^2(a+bx^3)}\right)dx}{3ab^2} \\
&= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{(2bc-af)\log(x)}{a^3} - \frac{\int \frac{a(5bd-2ag)+a(4be-ah)x-3b(2bc-af)x^2}{a+bx^3}\,dx}{3a^3} \\
&= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{(2bc-af)\log(x)}{a^3} - \frac{\int \frac{a(5bd-2ag)+a(4be-ah)x}{a+bx^3}\,dx}{3a^3} + \frac{(b(2bc-af))\int \frac{x^2}{a+bx^3}\,dx}{a^3} \\
&= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{(2bc-af)\log(x)}{a^3} + \frac{(2bc-af)\log\left(a+bx^3\right)}{3a^3} \\
&\quad - \frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}(5bd-2ag)+a^{4/3}(4be-ah)\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}(5bd-2ag)+a^{4/3}(4be-ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{11/3}\sqrt[3]{b}} - \frac{\left(5bd-2ag-\frac{\sqrt[3]{a}(4be-ah)}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{8/3}}
\end{aligned}$$

$$= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{3a^2\left(a + bx^3\right)} - \frac{(2bc - af)\log(x)}{a^3} - \frac{\left(5bd - 2ag - \frac{\sqrt[3]{a}(4be - ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}} + \frac{(2bc - af)\log\left(a + bx^3\right)}{3a^3} - \frac{\left(5b^{4/3}d + 4\sqrt[3]{a}be - 2a\sqrt[3]{b}g - a^{4/3}h\right)\int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{6a^{7/3}\sqrt[3]{b}} + \frac{\left(5bd - 2ag - \frac{\sqrt[3]{a}(4be - ah)}{\sqrt[3]{b}}\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{18a^{8/3}\sqrt[3]{b}}$$

$$= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{3a^2\left(a + bx^3\right)} - \frac{(2bc - af)\log(x)}{a^3} - \frac{\left(5bd - 2ag - \frac{\sqrt[3]{a}(4be - ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}} + \frac{\left(5bd - 2ag - \frac{\sqrt[3]{a}(4be - ah)}{\sqrt[3]{b}}\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}} + \frac{(2bc - af)\log\left(a + bx^3\right)}{3a^3} - \frac{\left(5b^{4/3}d + 4\sqrt[3]{a}be - 2a\sqrt[3]{b}g - a^{4/3}h\right)\text{Subst}\left(\int \frac{1}{-3 - x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{8/3}b^{2/3}}$$

$$= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{3a^2\left(a + bx^3\right)} + \frac{\left(5b^{4/3}d + 4\sqrt[3]{a}be - 2a\sqrt[3]{b}g - a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}b^{2/3}} - \frac{(2bc - af)\log(x)}{a^3} - \frac{\left(5bd - 2ag - \frac{\sqrt[3]{a}(4be - ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}} + \frac{\left(5bd - 2ag - \frac{\sqrt[3]{a}(4be - ah)}{\sqrt[3]{b}}\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}} + \frac{(2bc - af)\log\left(a + bx^3\right)}{3a^3}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.90

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{6ac}{x^3}-\frac{9ad}{x^2}-\frac{18ae}{x}+\frac{a(-6b(c+x(d+ex))+6a(f+x(g+hx)))}{a+bx^3}-\frac{2\sqrt{3}\sqrt[3]{a}\left(-5b^{4/3}d-4\sqrt[3]{a}be+2a\sqrt[3]{b}g+a^{4/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a*c)/x^3 - (9*a*d)/x^2 - (18*a*e)/x + (a*(-6*b*(c + x*(d + e*x)) + 6*a
*(f + x*(g + h*x))))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(-5*b^(4/3)*d - 4*a^(
1/3)*b*e + 2*a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(2/3) + 18*(-2*b*c + a*f)*Log[x] - (2*a^(1/3)*(5*b^(4/3)*d - 4*a^
(1/3)*b*e - 2*a*b^(1/3)*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) +
(a^(1/3)*(5*b^(4/3)*d - 4*a^(1/3)*b*e - 2*a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 6*(2*b*c - a*f)*Log[a + b
*x^3])/(18*a^3)
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.98

method	result
default	$-\frac{e}{a^2x}-\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}+\frac{(af-2bc)\ln(x)}{a^3}+\frac{\left(\frac{1}{3}a^2h-\frac{1}{3}aeb\right)x^2+\left(\frac{1}{3}a^2g-\frac{1}{3}abd\right)x+\frac{a(af-bc)}{3}}{b\,x^3+a}+\frac{(2a^2g-5abd)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2\right.}{}\right.}{}$
risch	$\frac{\frac{(ah-4be)x^5}{3a^2}+\frac{(2ag-5bd)x^4}{6a^2}+\frac{(af-2bc)x^3}{3a^2}-\frac{e\,x^2}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^3(b\,x^3+a)}+\frac{\left(\_R=\text{RootOf}\left(a^9b^2\_Z^3+\left(9a^7b^2f-18a^6b^3c\right)\_Z^2+\left(6a^6bgh-15a^5b^2dh-24a^5b^2\right.\right.\right.}{}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -e/a^2/x-1/3*c/a^2/x^3-1/2*d/a^2/x^2+(a*f-2*b*c)/a^3*ln(x)+1/a^3*(((1/3*a^2
*h-1/3*a*e*b)*x^2+(1/3*a^2*g-1/3*a*b*d)*x+1/3*a*(a*f-b*c))/(b*x^3+a)+1/3*(2
*a^2*g-5*a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x
^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1)))+1/3*(a^2*h-4*a*b*e)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1
/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b
)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/9*(-3*a*b*f+6*b^2*c)*ln(
b*x^3+a)/b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 71.98 (sec) , antiderivative size = 16568, normalized size of antiderivative = 49.02

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.08

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^2}\,dx$$
$$= -\frac{2(4be-ah)x^5+(5bd-2ag)x^4+6aex^2+2(2bc-af)x^3+3adx+2ac}{6(a^2bx^6+a^3x^3)}$$
$$-\frac{(2bc-af)\log(x)}{a^3}$$
$$-\frac{\sqrt{3}\left(4abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+5abd\left(\frac{a}{b}\right)^{\frac{1}{3}}-2a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^4}$$
$$+\frac{\left(12b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-6abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-4abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+5abd-2a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(6b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-3abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+4abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-5abd+2a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] -1/6*(2*(4*b*e - a*h)*x^5 + (5*b*d - 2*a*g)*x^4 + 6*a*e*x^2 + 2*(2*b*c - a*
f)*x^3 + 3*a*d*x + 2*a*c)/(a^2*b*x^6 + a^3*x^3) - (2*b*c - a*f)*log(x)/a^3
- 1/9*sqrt(3)*(4*a*b*e*(a/b)^(2/3) - a^2*h*(a/b)^(2/3) + 5*a*b*d*(a/b)^(1/3
) - 2*a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/a^4 + 1/18*(12*b^2*c*(a/b)^(2/3) - 6*a*b*f*(a/b)^(2/3) - 4*a*b*e*(a/b)^(1
/3) + a^2*h*(a/b)^(1/3) + 5*a*b*d - 2*a^2*g)*log(x^2 - x*(a/b)^(1/3) + (a/b
)^(2/3))/(a^3*b*(a/b)^(2/3)) + 1/9*(6*b^2*c*(a/b)^(2/3) - 3*a*b*f*(a/b)^(2/
3) + 4*a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3) - 5*a*b*d + 2*a^2*g)*log(x + (
a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.06

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^2}\,dx$$
$$= \frac{\sqrt{3}\left(5\,b^2d-2\,abg-4\,\left(-ab^2\right)^{\frac{1}{3}}\,be+\left(-ab^2\right)^{\frac{1}{3}}\,ah\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}\,a^2}$$
$$+\frac{\left(5\,b^2d-2\,abg+4\,\left(-ab^2\right)^{\frac{1}{3}}\,be-\left(-ab^2\right)^{\frac{1}{3}}\,ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}\,a^2}$$
$$+\frac{(2\,bc-af)\log\left(\left|bx^3+a\right|\right)}{3\,a^3}-\frac{(2\,bc-af)\log\left(|x|\right)}{a^3}$$
$$+\frac{\left(4\,a^4b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^5bh\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^4b^2d-2\,a^5bg\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^7b}$$
$$-\frac{2\,(4\,abe-a^2h)x^5+6\,a^2ex^2+(5\,abd-2\,a^2g)x^4+3\,a^2dx+2\,(2\,abc-a^2f)x^3+2\,a^2c}{6\,(bx^3+a)a^3x^3}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] 1/9*sqrt(3)*(5*b^2*d - 2*a*b*g - 4*(-a*b^2)^(1/3)*b*e + (-a*b^2)^(1/3)*a*h)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2)
 + 1/18*(5*b^2*d - 2*a*b*g + 4*(-a*b^2)^(1/3)*b*e - (-a*b^2)^(1/3)*a*h)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) + 1/3*(2*b*c - a
*f)*log(abs(b*x^3 + a))/a^3 - (2*b*c - a*f)*log(abs(x))/a^3 + 1/9*(4*a^4*b^
2*e*(-a/b)^(1/3) - a^5*b*h*(-a/b)^(1/3) + 5*a^4*b^2*d - 2*a^5*b*g)*(-a/b)^(
1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b) - 1/6*(2*(4*a*b*e - a^2*h)*x^5 + 6*
a^2*e*x^2 + (5*a*b*d - 2*a^2*g)*x^4 + 3*a^2*d*x + 2*(2*a*b*c - a^2*f)*x^3 +
 2*a^2*c)/((b*x^3 + a)*a^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 1924, normalized size of antiderivative = 5.69

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x)
```

[Out] symsum(log(- (50*b^5*c*d^2 - 48*b^5*c^2*e + 8*a^2*b^3*c*g^2 - 12*a^2*b^3*e*f^2 - 4*a^3*b^2*f*g^2 + 3*a^3*b^2*f^2*h - 25*a*b^4*d^2*f + 12*a*b^4*c^2*h - 12*a^2*b^3*c*f*h + 20*a^2*b^3*d*f*g - 40*a*b^4*c*d*g + 48*a*b^4*c*e*f)/(9*a^6) - root(729*a^9*b^2*z^3 + 729*a^7*b^2*f*z^2 - 1458*a^6*b^3*c*z^2 + 54*a^6*b*g*h*z - 216*a^5*b^2*e*g*z - 135*a^5*b^2*d*h*z - 972*a^4*b^3*c*f*z + 540*a^4*b^3*d*e*z + 243*a^5*b^2*f^2*z + 972*a^3*b^4*c^2*z + 18*a^4*b*f*g*h - 360*a*b^4*c*d*e - 72*a^3*b^2*e*f*g - 45*a^3*b^2*d*f*h - 36*a^3*b^2*c*g*h + 180*a^2*b^3*d*e*f + 144*a^2*b^3*c*e*g + 90*a^2*b^3*c*d*h - 12*a^4*b*e*h^2 + 324*a*b^4*c^2*f + 48*a^3*b^2*e^2*h - 150*a^2*b^3*d^2*g + 60*a^3*b^2*d*g^2 - 162*a^2*b^3*c*f^2 + 27*a^3*b^2*f^3 - 64*a^2*b^3*e^3 - 8*a^4*b*g^3 + 125*a*b^4*d^3 - 216*b^5*c^3 + a^5*h^3, z, k)*((25*a^3*b^4*d^2 + 4*a^5*b^2*g^2 + 48*a^3*b^4*c*e - 12*a^4*b^3*c*h - 20*a^4*b^3*d*g - 24*a^4*b^3*e*f + 6*a^5*b^2*f*h)/(9*a^6) + root(729*a^9*b^2*z^3 + 729*a^7*b^2*f*z^2 - 1458*a^6*b^3*c*z^2 + 54*a^6*b*g*h*z - 216*a^5*b^2*e*g*z - 135*a^5*b^2*d*h*z - 972*a^4*b^3*c*f*z + 540*a^4*b^3*d*e*z + 243*a^5*b^2*f^2*z + 972*a^3*b^4*c^2*z + 18*a^4*b*f*g*h - 360*a*b^4*c*d*e - 72*a^3*b^2*e*f*g - 45*a^3*b^2*d*f*h - 36*a^3*b^2*c*g*h + 180*a^2*b^3*d*e*f + 144*a^2*b^3*c*e*g + 90*a^2*b^3*c*d*h - 12*a^4*b*e*h^2 + 324*a*b^4*c^2*f + 48*a^3*b^2*e^2*h - 150*a^2*b^3*d^2*g + 60*a^3*b^2*d*g^2 - 162*a^2*b^3*c*f^2 + 27*a^3*b^2*f^3 - 64*a^2*b^3*e^3 - 8*a^4*b*g^3 + 125*a*b^4*d^3 - 216*b^5*c^3 + a^5*h^3, z, k)*((36*a^6*b^3*e - 9*a^7*b^2*h)/(9*a^6) - (x*(1296*a^5*b^4*c - 648*a^6*b^3*f))/(27*a^6) + 36*root(729*a^9*b^2*z^3 + 729*a^7*b^2*f*z^2 - 1458*a^6*b^3*c*z^2 + 54*a^6*b*g*h*z - 216*a^5*b^2*e*g*z - 135*a^5*b^2*d*h*z - 972*a^4*b^3*c*f*z + 540*a^4*b^3*d*e*z + 243*a^5*b^2*f^2*z + 972*a^3*b^4*c^2*z + 18*a^4*b*f*g*h - 360*a*b^4*c*d*e - 72*a^3*b^2*e*f*g - 45*a^3*b^2*d*f*h - 36*a^3*b^2*c*g*h + 180*a^2*b^3*d*e*f + 144*a^2*b^3*c*e*g + 90*a^2*b^3*c*d*h - 12*a^4*b*e*h^2 + 324*a*b^4*c^2*f + 48*a^3*b^2*e^2*h - 150*a^2*b^3*d^2*g + 60*a^3*b^2*d*g^2 - 162*a^2*b^3*c*f^2 + 27*a^3*b^2*f^3 - 64*a^2*b^3*e^3 - 8*a^4*b*g^3 + 125*a*b^4*d^3 - 216*b^5*c^3 + a^5*h^3, z, k)*a^2*b^3*x) + (x*(432*a^2*b^5*c^2 + 108*a^4*b^3*f^2 - 432*a^3*b^4*c*f + 600*a^3*b^4*d*e - 150*a^4*b^3*d*h - 240*a^4*b^3*e*g + 60*a^5*b^2*g*h))/(27*a^6)) - (x*(125*b^5*d^3 - 64*a*b^4*e^3 + a^4*b*h^3 - 8*a^3*b^2*g^3 + 60*a^2*b^3*d*g^2 + 48*a^2*b^3*e^2*h - 12*a^3*b^2*e*h^2 - 240*b^5*c*d*e - 150*a*b^4*d^2*g - 24*a^2*b^3*c*g*h - 30*a^2*b^3*d*f*h - 48*a^2*b^3*e*f*g + 12*a^3*b^2*f*g*h + 60*a*b^4*c*d*h + 96*a*b^4*c*e*g + 120*a*b^4*d*e*f))/(27*a^6))*root(729*a^9*b^2*z^3 + 729*a^7*b^2*f*z^2 - 1458*a^6*b^3*c*z^2 + 54*a^6*b*g*h*z - 216*a^5*b^2*e*g*z - 135*a^5*b^2*d*h*z - 972*a^4*b^3*c*f*z + 540*a^4*b^3*d*e*z + 243*a^5*b^2*f^2*z + 972*a^3*b^4*c^2*z + 18*a^4*b*f*g*h - 360*a*b^4*c*d*e - 72*a^3*b^2*e*f*g - 45*a^3*b^2*d*f*h - 36*a^3*b^2*c*g*h + 180*a^2*b^3*d*e*f + 144*a^2*b^3*c*e*g + 90*a^2*b^3*c*d*h - 12*a^4*b*e*h^2 + 324*a*b^4*c^2*f + 48*a^3*b^2*e^2*h - 150*a^2*b^3*d^2*g + 60*a^3*b^2*d*g^2 - 162*a^2*b^3*c*f^2 + 27*a^3*b^2*f^3 - 64*a^2*b^3*e^3 - 8*a^4*b*g^3 + 125*a*b^4*d^3 - 216*b^5*c^3 + a^5*h^3, z, k), k, 1, 3) - (c/(3*a) + (e*x^2)/a + (x^3*(2*b*c - a*f))/(3*a^2) + (x^4*(5*b*d - 2*a*g))/(6*a^2) + (x^5*(4*b*e - a*h))/(3*a^2) + (d*x)/(2*a))/(a*x^3 + b*x^6) - (log(x)*(2*b*c - a*f))/a^3

3.421 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 345

$$\begin{aligned}
&\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx \\
&= \frac{hx}{b^3}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} \\
&\quad-\frac{x(a(7be-13ah)-2b(bc-4af)x-3b(bd-3ag)x^2)}{18ab^3(a+bx^3)} \\
&\quad-\frac{(b^{5/3}c+2a^{2/3}be+5ab^{2/3}f-14a^{5/3}h)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{10/3}} \\
&\quad-\frac{(b^{2/3}(bc+5af)-2a^{2/3}(be-7ah))\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{10/3}} \\
&\quad+\frac{(b^{2/3}(bc+5af)-2a^{2/3}(be-7ah))\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{10/3}}+\frac{g\log(a+bx^3)}{3b^3}
\end{aligned}$$

```
[Out] h*x/b^3+1/6*x*(a*(-a*h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/b^3/(b*x^3+a)^
2-1/18*x*(a*(-13*a*h+7*b*e)-2*b*(-4*a*f+b*c)*x-3*b*(-3*a*g+b*d)*x^2)/a/b^3/
(b*x^3+a)-1/27*(b^(2/3)*(5*a*f+b*c)-2*a^(2/3)*(-7*a*h+b*e))*ln(a^(1/3)+b^(1
/3)*x)/a^(4/3)/b^(10/3)+1/54*(b^(2/3)*(5*a*f+b*c)-2*a^(2/3)*(-7*a*h+b*e))*l
n(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(10/3)+1/3*g*ln(b*x^3+a)
/b^3-1/27*(b^(5/3)*c+2*a^(2/3)*b*e+5*a*b^(2/3)*f-14*a^(5/3)*h)*arctan(1/3*(
a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {1842, 1872, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^{2/3}be-14a^{5/3}h+5ab^{2/3}f+b^{5/3}c\right)}{9\sqrt{3}a^{4/3}b^{10/3}}$$
$$+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-7ah)\right)}{54a^{4/3}b^{10/3}}$$
$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-7ah)\right)}{27a^{4/3}b^{10/3}}$$
$$-\frac{x(-2bx(bc-4af)-3bx^2(bd-3ag)+a(7be-13ah))}{18ab^3(a+bx^3)}$$
$$+\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{6b^3(a+bx^3)^2}+\frac{g\log(a+bx^3)}{3b^3}+\frac{hx}{b^3}$$

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] (h*x)/b^3 + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*b^
3*(a + b*x^3)^2) - (x*(a*(7*b*e - 13*a*h) - 2*b*(b*c - 4*a*f)*x - 3*b*(b*d
- 3*a*g)*x^2))/(18*a*b^3*(a + b*x^3)) - ((b^(5/3)*c + 2*a^(2/3)*b*e + 5*a*b
^(2/3)*f - 14*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/(9*Sqrt[3]*a^(4/3)*b^(10/3)) - ((b^(2/3)*(b*c + 5*a*f) - 2*a^(2/3)*(b*e -
7*a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(10/3)) + ((b^(2/3)*(b*c +
5*a*f) - 2*a^(2/3)*(b*e - 7*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(54*a^(4/3)*b^(10/3)) + (g*Log[a + b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} - \frac{\int \frac{a^2(be-ah)-2ab(bc-af)x-3ab(bd-ag)x^2-6ab(be-ah)x^3-6ab^2fx^4-6ab^2gx^5-6ab^2hx^6}{(a+bx^3)^2}\,dx}{6ab^3} \\
&= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} - \frac{x(a(7be-13ah)-2b(bc-4af)x-3b(bd-3ag)x^2)}{18ab^3(a+bx^3)} + \frac{\int \frac{2a^2b^2(2be-5ah)+2ab^3(bc+5af)x+18a^2b^3gx^2+18a^2b^3hx^3}{a+bx^3}\,dx}{18a^2b^5} \\
&= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} - \frac{x(a(7be-13ah)-2b(bc-4af)x-3b(bd-3ag)x^2)}{18ab^3(a+bx^3)} + \frac{\int \left(18a^2b^2h+\frac{2(2a^2b^2(be-7ah)+ab^3(bc+5af)x+9a^2b^3gx^2)}{a+bx^3}\right)dx}{18a^2b^5} \\
&= \frac{hx}{b^3} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} - \frac{x(a(7be-13ah)-2b(bc-4af)x-3b(bd-3ag)x^2)}{18ab^3(a+bx^3)} + \frac{\int \frac{2a^2b^2(be-7ah)+ab^3(bc+5af)x+9a^2b^3gx^2}{a+bx^3}\,dx}{9a^2b^5}
\end{aligned}$$

$$=\frac{hx}{b^3}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3\left(a+bx^3\right)^2}-\frac{x(a(7be-13ah)-2b(bc-4af)x-3b(bd-3ag)x^2)}{18ab^3\left(a+bx^3\right)}+\frac{\int\frac{2a^2b^2(be-7ah)+ab^3(bc+5af)x}{a+bx^3}dx}{9a^2b^5}+\frac{g\int\frac{x^2}{a+bx^3}dx}{b^2}$$

$$=\frac{hx}{b^3}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3\left(a+bx^3\right)^2}-\frac{x(a(7be-13ah)-2b(bc-4af)x-3b(bd-3ag)x^2)}{18ab^3\left(a+bx^3\right)}+\frac{g\log\left(a+bx^3\right)}{3b^3}+\frac{\int\frac{\sqrt[3]{a}\left(a^{4/3}b^3(bc+5af)+4a^2b^{7/3}(be-7ah)\right)+\sqrt[3]{b}\left(a^{4/3}b^3(bc+5af)-2a^2b^{7/3}(be-7ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{27a^{8/3}b^{16/3}}-\frac{\left(b^{2/3}(bc+5af)-2a^{2/3}(be-7ah)\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{27a^{4/3}b^3}$$

$$=\frac{hx}{b^3}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3\left(a+bx^3\right)^2}-\frac{x(a(7be-13ah)-2b(bc-4af)x-3b(bd-3ag)x^2)}{18ab^3\left(a+bx^3\right)}-\frac{\left(b^{2/3}(bc+5af)-2a^{2/3}(be-7ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{10/3}}+\frac{g\log\left(a+bx^3\right)}{3b^3}+\frac{\left(b^{5/3}c+2a^{2/3}be+5ab^{2/3}f-14a^{5/3}h\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{18ab^3}+\frac{\left(b^{2/3}(bc+5af)-2a^{2/3}(be-7ah)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{54a^{4/3}b^{10/3}}$$

$$= \frac{hx}{b^3} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} - \frac{x(a(7be-13ah)-2b(bc-4af)x-3b(bd-3ag)x^2)}{18ab^3(a+bx^3)} - \frac{\left(b^{2/3}(bc+5af)-2a^{2/3}(be-7ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{10/3}} + \frac{\left(b^{2/3}(bc+5af)-2a^{2/3}(be-7ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{10/3}} + \frac{g\log(a+bx^3)}{3b^3} + \frac{\left(b^{5/3}c+2a^{2/3}be+5ab^{2/3}f-14a^{5/3}h\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{4/3}b^{10/3}}$$

$$= \frac{hx}{b^3} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} - \frac{x(a(7be-13ah)-2b(bc-4af)x-3b(bd-3ag)x^2)}{18ab^3(a+bx^3)} - \frac{\left(b^{5/3}c+2a^{2/3}be+5ab^{2/3}f-14a^{5/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{10/3}} - \frac{\left(b^{2/3}(bc+5af)-2a^{2/3}(be-7ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{10/3}} + \frac{\left(b^{2/3}(bc+5af)-2a^{2/3}(be-7ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{10/3}} + \frac{g\log(a+bx^3)}{3b^3}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.99

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$

$$= \frac{54b^{2/3}hx - \frac{9b^{2/3}(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx)))}{(a+bx^3)^2} + \frac{3b^{2/3}(2b^2cx^2+a^2(12g+13hx)-ab(6d+x(7e+8fx)))}{a(a+bx^3)} - \frac{2\sqrt{3}\left(b^2c+2a^{2/3}b^{4/3}e+5a\right.}{}}{}$$

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

[Out] (54*b^(2/3)*h*x - (9*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f *x))))/(a + b*x^3)^2 + (3*b^(2/3)*(2*b^2*c*x^2 + a^2*(12*g + 13*h*x) - a*b* (6*d + x*(7*e + 8*f*x))))/(a*(a + b*x^3)) - (2*Sqrt[3]*(b^2*c + 2*a^(2/3)*b ^(4/3)*e + 5*a*b*f - 14*a^(5/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(4/3) - (2*(b^2*c - 2*a^(2/3)*b^(4/3)*e + 5*a*b*f + 14*a^(5/ 3)*b^(1/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) + ((b^2*c - 2*a^(2/3)*b^(4/ 3)*e + 5*a*b*f + 14*a^(5/3)*b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^ (2/3)*x^2])/a^(4/3) + 18*b^(2/3)*g*Log[a + b*x^3])/(54*b^(11/3))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.49

method	result
risch	$\frac{hx}{b^3}+\frac{-\frac{b^2(4af-bc)x^5}{9a}+(\frac{13}{18}abh-\frac{7}{18}b^2e)x^4+(\frac{2}{3}abg-\frac{1}{3}b^2d)x^3-\frac{b(5af+bc)x^2}{18}+\frac{a(5ah-2be)x}{9}+\frac{a^2g}{2}-\frac{abd}{6}}{b^3(bx^3+a)^2}+\frac{\sum_{\_R=\text{RootOf}(b\_Z^3+a)}\left(9\right.}{}$
default	$\frac{hx}{b^3}-\frac{\frac{\frac{b^2(4af-bc)x^5}{9a}+\left(-\frac{13}{18}abh+\frac{7}{18}b^2e\right)x^4+\left(-\frac{2}{3}abg+\frac{1}{3}b^2d\right)x^3+\frac{b(5af+bc)x^2}{18}-\frac{a(5ah-2be)x}{9}-\frac{a^2g}{2}+\frac{abd}{6}}{\left(bx^3+a\right)^2}+\frac{\left(14a^2h-2aeb\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}}{}$

[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] h*x/b^3+(-1/9*b^2*(4*a*f-b*c)/a*x^5+(13/18*a*b*h-7/18*b^2*e)*x^4+(2/3*a*b*g -1/3*b^2*d)*x^3-1/18*b*(5*a*f+b*c)*x^2+1/9*a*(5*a*h-2*b*e)*x+1/2*a^2*g-1/6* a*b*d)/b^3/(b*x^3+a)^2+1/27/b^4*sum((9*g*b*_R^2+b*(5*a*f+b*c)/a*_R-14*a*h+2 *b*e)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 12967, normalized size of antiderivative = 37.59

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fri
cas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx &= \frac{2\,(b^3c-4\,ab^2f)x^5-(7\,ab^2e-13\,a^2bh)x^4-3\,a^2bd+9\,a^3g-6\,(ab^2d-2\,a^2bg)x^3-(ab^2c+5\,a^2bf)x^2-2\,(2}{18\,(ab^5x^6+2\,a^2b^4x^3+a^3b^3)}\\&+\frac{hx}{b^3}+\frac{\sqrt{3}\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-14\,a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^3}\\&+\frac{\left(18\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abe+14\,a^2h\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\&+\frac{\left(9\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abe-14\,a^2h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] 1/18*(2*(b^3*c - 4*a*b^2*f)*x^5 - (7*a*b^2*e - 13*a^2*b*h)*x^4 - 3*a^2*b*d
+ 9*a^3*g - 6*(a*b^2*d - 2*a^2*b*g)*x^3 - (a*b^2*c + 5*a^2*b*f)*x^2 - 2*(2*
a^2*b*e - 5*a^3*h)*x)/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3) + h*x/b^3 + 1/2
7*sqrt(3)*(b^2*c*(a/b)^(2/3) + 5*a*b*f*(a/b)^(2/3) + 2*a*b*e*(a/b)^(1/3) -
14*a^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(
a^2*b^3) + 1/54*(18*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) + 5*a*b*f*(a/b)^(
1/3) - 2*a*b*e + 14*a^2*h)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^4*(a
/b)^(2/3)) + 1/27*(9*a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^(1/3) - 5*a*b*f*(a/b)^
(1/3) + 2*a*b*e - 14*a^2*h)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.10

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \frac{hx}{b^3} + \frac{g\log\left(|bx^3+a|\right)}{3\,b^3} - \frac{\sqrt{3}\left(2\,abe-14\,a^2h-\left(-ab^2\right)^{\frac{1}{3}}bc-5\left(-ab^2\right)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}ab^2} - \frac{\left(2\,abe-14\,a^2h+\left(-ab^2\right)^{\frac{1}{3}}bc+5\left(-ab^2\right)^{\frac{1}{3}}af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}ab^2} + \frac{2\,(b^3c-4\,ab^2f)x^5-(7\,ab^2e-13\,a^2bh)x^4-3\,a^2bd+9\,a^3g-6\,(ab^2d-2\,a^2bg)x^3-(ab^2c+5\,a^2bf)x^2-}{18\,(bx^3+a)^2ab^3} - \frac{\left(ab^6c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2b^5f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2b^5e-14\,a^3b^4h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^7}$$

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] h*x/b^3 + 1/3*g*log(abs(b*x^3 + a))/b^3 - 1/27*sqrt(3)*(2*a*b*e - 14*a^2*h
- (-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) - 1/54*(2*a*b*e - 14*a^2*h +
 (-a*b^2)^(1/3)*b*c + 5*(-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3))/((-a*b^2)^(2/3)*a*b^2) + 1/18*(2*(b^3*c - 4*a*b^2*f)*x^5 - (7*a*b
^2*e - 13*a^2*b*h)*x^4 - 3*a^2*b*d + 9*a^3*g - 6*(a*b^2*d - 2*a^2*b*g)*x^3
- (a*b^2*c + 5*a^2*b*f)*x^2 - 2*(2*a^2*b*e - 5*a^3*h)*x)/((b*x^3 + a)^2*a*b
^3) - 1/27*(a*b^6*c*(-a/b)^(1/3) + 5*a^2*b^5*f*(-a/b)^(1/3) + 2*a^2*b^5*e -
 14*a^3*b^4*h)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^7)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 916, normalized size of antiderivative = 2.66

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(19683\,a^4\,b^{10}\,z^3-19683\,a^4\,b^7\,g\,z^2-5670\,a^4\,b^4\,f\,h\,z-1134\,a^3\,b^5\,c\,h\,z+810\,a^3\,b^5\,e\,f\,z+162\,a^2\,b^6\right.\right.\right.$$
$$+\frac{81\,a^2\,g^2+2\,b^2\,c\,e-70\,a^2\,f\,h-14\,a\,b\,c\,h+10\,a\,b\,e\,f}{81\,a\,b^4}$$
$$\left.+\frac{x\,(126\,g\,h\,a^3+25\,a^2\,b\,f^2-18\,e\,g\,a^2\,b+10\,a\,b^2\,c\,f+b^3\,c^2)}{81\,a^2\,b^4}\right)\mathrm{root}\big(19683\,a^4\,b^{10}\,z^3$$
$$-19683\,a^4\,b^7\,g\,z^2-5670\,a^4\,b^4\,f\,h\,z-1134\,a^3\,b^5\,c\,h\,z+810\,a^3\,b^5\,e\,f\,z+162\,a^2\,b^6\,c\,e\,z$$
$$+6561\,a^4\,b^4\,g^2\,z+1890\,a^4\,b\,f\,g\,h+378\,a^3\,b^2\,c\,g\,h-270\,a^3\,b^2\,e\,f\,g-54\,a^2\,b^3\,c\,e\,g$$
$$-1176\,a^4\,b\,e\,h^2+15\,a\,b^4\,c^2\,f+168\,a^3\,b^2\,e^2\,h+75\,a^2\,b^3\,c\,f^2+125\,a^3\,b^2\,f^3-8\,a^2\,b^3\,e^3$$
$$\left.\left.-729\,a^4\,b\,g^3+2744\,a^5\,h^3+b^5\,c^3,z,k\big)\right)\right)$$
$$-\frac{x^2\left(\frac{c\,b^2}{18}+\frac{5\,a\,f\,b}{18}\right)-\frac{a^2\,g}{2}-x\left(\frac{5\,a^2\,h}{9}-\frac{2\,a\,b\,e}{9}\right)+x^3\left(\frac{b^2\,d}{3}-\frac{2\,a\,b\,g}{3}\right)+\frac{b\,x^4\,(7\,b\,e-13\,a\,h)}{18}+\frac{a\,b\,d}{6}-\frac{b\,x^5\,(b^2\,c-4\,a\,b\,f)}{9\,a}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}$$
$$+\frac{h\,x}{b^3}$$

```
[In] int((x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

```
[Out] symsum(log(root(19683*a^4*b^10*z^3 - 19683*a^4*b^7*g*z^2 - 5670*a^4*b^4*f*h
*z - 1134*a^3*b^5*c*h*z + 810*a^3*b^5*e*f*z + 162*a^2*b^6*c*e*z + 6561*a^4*
b^4*g^2*z + 1890*a^4*b*f*g*h + 378*a^3*b^2*c*g*h - 270*a^3*b^2*e*f*g - 54*a
^2*b^3*c*e*g - 1176*a^4*b*e*h^2 + 15*a*b^4*c^2*f + 168*a^3*b^2*e^2*h + 75*a
^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 8*a^2*b^3*e^3 - 729*a^4*b*g^3 + 2744*a^5*h
^3 + b^5*c^3, z, k)*(9*root(19683*a^4*b^10*z^3 - 19683*a^4*b^7*g*z^2 - 5670
*a^4*b^4*f*h*z - 1134*a^3*b^5*c*h*z + 810*a^3*b^5*e*f*z + 162*a^2*b^6*c*e*z
 + 6561*a^4*b^4*g^2*z + 1890*a^4*b*f*g*h + 378*a^3*b^2*c*g*h - 270*a^3*b^2*
e*f*g - 54*a^2*b^3*c*e*g - 1176*a^4*b*e*h^2 + 15*a*b^4*c^2*f + 168*a^3*b^2*
e^2*h + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 8*a^2*b^3*e^3 - 729*a^4*b*g^3
+ 2744*a^5*h^3 + b^5*c^3, z, k)*a*b^2 - (6*a*g)/b + (x*(54*a^2*b^4*e - 378*
a^3*b^3*h))/(81*a^2*b^4)) + (81*a^2*g^2 + 2*b^2*c*e - 70*a^2*f*h - 14*a*b*c
*h + 10*a*b*e*f)/(81*a*b^4) + (x*(b^3*c^2 + 25*a^2*b*f^2 + 126*a^3*g*h + 10
*a*b^2*c*f - 18*a^2*b*e*g))/(81*a^2*b^4))*root(19683*a^4*b^10*z^3 - 19683*a
^4*b^7*g*z^2 - 5670*a^4*b^4*f*h*z - 1134*a^3*b^5*c*h*z + 810*a^3*b^5*e*f*z
+ 162*a^2*b^6*c*e*z + 6561*a^4*b^4*g^2*z + 1890*a^4*b*f*g*h + 378*a^3*b^2*c
*g*h - 270*a^3*b^2*e*f*g - 54*a^2*b^3*c*e*g - 1176*a^4*b*e*h^2 + 15*a*b^4*c
^2*f + 168*a^3*b^2*e^2*h + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 8*a^2*b^3*e
^3 - 729*a^4*b*g^3 + 2744*a^5*h^3 + b^5*c^3, z, k), k, 1, 3) - (x^2*((b^2*c
```

```
)/18 + (5*a*b*f)/18) - (a^2*g)/2 - x*((5*a^2*h)/9 - (2*a*b*e)/9) + x^3*((b^
2*d)/3 - (2*a*b*g)/3) + (b*x^4*(7*b*e - 13*a*h))/18 + (a*b*d)/6 - (b*x^5*(b
^2*c - 4*a*b*f))/(9*a))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (h*x)/b^3
```

3.422 $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 325

$$\begin{aligned}\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx &= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} \\ &\quad+\frac{x(bc-7af+2(bd-4ag)x+3(be-3ah)x^2)}{18ab^2(a+bx^3)} \\ &\quad-\frac{\left(b^{4/3}c+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+5a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{8/3}} \\ &\quad+\frac{\left(\sqrt[3]{b}(bc+2af)-\sqrt[3]{a}(bd+5ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{8/3}} \\ &\quad-\frac{\left(\sqrt[3]{b}(bc+2af)-\sqrt[3]{a}(bd+5ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{8/3}}+\frac{h\log(a+bx^3)}{3b^3}\end{aligned}$$

```
[Out] -1/6*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*x^2)/b^2/(b*x^3+a)^2+1/18*x*(b*c-7*
a*f+2*(-4*a*g+b*d)*x+3*(-3*a*h+b*e)*x^2)/a/b^2/(b*x^3+a)+1/27*(b^(1/3)*(2*a
*f+b*c)-a^(1/3)*(5*a*g+b*d))*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(8/3)-1/54*(b^
(1/3)*(2*a*f+b*c)-a^(1/3)*(5*a*g+b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/a^(5/3)/b^(8/3)+1/3*h*ln(b*x^3+a)/b^3-1/27*(b^(4/3)*c+a^(1/3)*b*d+2*a
*b^(1/3)*f+5*a^(4/3)*g)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a
^(5/3)/b^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1842, 1872, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(5a^{4/3}g+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+b^{4/3}c\right)}{9\sqrt{3}a^{5/3}b^{8/3}}$$
$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)}{54a^{5/3}b^{8/3}}$$
$$+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)}{27a^{5/3}b^{8/3}}$$
$$+\frac{h\log(a+bx^3)}{3b^3}+\frac{x(2x(bd-4ag)+3x^2(be-3ah)-7af+bc)}{18ab^2(a+bx^3)}$$
$$-\frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{6b^2(a+bx^3)^2}$$

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]

[Out] -1/6*(x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(b^2*(a + b*x^3)^2)
+ (x*(b*c - 7*a*f + 2*(b*d - 4*a*g)*x + 3*(b*e - 3*a*h)*x^2))/(18*a*b^2*(a
+ b*x^3)) - ((b^(4/3)*c + a^(1/3)*b*d + 2*a*b^(1/3)*f + 5*a^(4/3)*g)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(8/3)) +
((b^(1/3)*(b*c + 2*a*f) - a^(1/3)*(b*d + 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/
(27*a^(5/3)*b^(8/3)) - ((b^(1/3)*(b*c + 2*a*f) - a^(1/3)*(b*d + 5*a*g))*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3)) + (h*Log[
a + b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} \\
&\quad -\frac{\int \frac{-ab(bc-af)-2ab(bd-ag)x-3ab(be-ah)x^2-6ab^2fx^3-6ab^2gx^4-6ab^2hx^5}{(a+bx^3)^2}\,dx}{6ab^3} \\
&= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} \\
&\quad +\frac{x(bc-7af+2(bd-4ag)x+3(be-3ah)x^2)}{18ab^2(a+bx^3)} \\
&\quad +\frac{\int \frac{2ab^3(bc+2af)+2ab^3(bd+5ag)x+18a^2b^3hx^2}{a+bx^3}\,dx}{18a^2b^5} \\
&= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} \\
&\quad +\frac{x(bc-7af+2(bd-4ag)x+3(be-3ah)x^2)}{18ab^2(a+bx^3)} \\
&\quad +\frac{\int \frac{2ab^3(bc+2af)+2ab^3(bd+5ag)x}{a+bx^3}\,dx}{18a^2b^5}+\frac{h\int \frac{x^2}{a+bx^3}\,dx}{b^2} \\
&= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} \\
&\quad +\frac{x(bc-7af+2(bd-4ag)x+3(be-3ah)x^2)}{18ab^2(a+bx^3)}+\frac{h\log(a+bx^3)}{3b^3} \\
&\quad +\frac{\int \frac{\sqrt[3]{a}(4ab^{10/3}(bc+2af)+2a^{4/3}b^3(bd+5ag))+\sqrt[3]{b}(-2ab^{10/3}(bc+2af)+2a^{4/3}b^3(bd+5ag))x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}b^{16/3}} \\
&\quad +\frac{\left(\sqrt[3]{b}(bc+2af)-\sqrt[3]{a}(bd+5ag)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{5/3}b^{7/3}}
\end{aligned}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} + \frac{x(bc-7af+2(bd-4ag)x+3(be-3ah)x^2)}{18ab^2(a+bx^3)} + \frac{\left(\sqrt[3]{b}(bc+2af)-\sqrt[3]{a}(bd+5ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{8/3}} + \frac{h\log(a+bx^3)}{3b^3} + \frac{\left(b^{4/3}c+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+5a^{4/3}g\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{4/3}b^{7/3}} - \frac{\left(\sqrt[3]{b}(bc+2af)-\sqrt[3]{a}(bd+5ag)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{5/3}b^{8/3}}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} + \frac{x(bc-7af+2(bd-4ag)x+3(be-3ah)x^2)}{18ab^2(a+bx^3)} + \frac{\left(\sqrt[3]{b}(bc+2af)-\sqrt[3]{a}(bd+5ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{8/3}} - \frac{\left(\sqrt[3]{b}(bc+2af)-\sqrt[3]{a}(bd+5ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{8/3}} + \frac{h\log(a+bx^3)}{3b^3} + \frac{\left(b^{4/3}c+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+5a^{4/3}g\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{5/3}b^{8/3}}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} + \frac{x(bc-7af+2(bd-4ag)x+3(be-3ah)x^2)}{18ab^2(a+bx^3)} - \frac{\left(b^{4/3}c+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+5a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{8/3}} + \frac{\left(\sqrt[3]{b}(bc+2af)-\sqrt[3]{a}(bd+5ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{8/3}} - \frac{\left(\sqrt[3]{b}(bc+2af)-\sqrt[3]{a}(bd+5ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{8/3}} + \frac{h\log(a+bx^3)}{3b^3}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.97

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{9(a^2h+b^2x(c+dx)-ab(e+x(f+gx)))}{(a+bx^3)^2}+\frac{36a^2h+3b^2x(c+2dx)-3ab(6e+x(7f+8gx))}{a(a+bx^3)}-\frac{2\sqrt{3}\sqrt[3]{b}\left(b^{4/3}c+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+5a^{4/3}g\right)\arctan}{a^{5/3}}}{}$$

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a + b*x^3)^2 + (36
*a^2*h + 3*b^2*x*(c + 2*d*x) - 3*a*b*(6*e + x*(7*f + 8*g*x)))/(a*(a + b*x^3
)) - (2*Sqrt[3]*b^(1/3)*(b^(4/3)*c + a^(1/3)*b*d + 2*a*b^(1/3)*f + 5*a^(4/3
)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*b^(1/3)*(b^(
4/3)*c - a^(1/3)*b*d + 2*a*b^(1/3)*f - 5*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x
])/a^(5/3) + (b^(1/3)*(-(b^(4/3)*c) + a^(1/3)*b*d - 2*a*b^(1/3)*f + 5*a^(4/
3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + 18*h*Log[a
+ b*x^3])/(54*b^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.53

method	result
risch	$\frac{-\frac{(4ag-bd)x^5}{9ab}-\frac{(7af-bc)x^4}{18ab}+\frac{(2ah-be)x^3}{3b^2}-\frac{(5ag+bd)x^2}{18b^2}-\frac{(2af+bc)x}{9b^2}+\frac{a(3ah-be)}{6b^3}}{(bx^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\left(9h\_R^2+\frac{(5ag+bd)}{a}\_R+\right.}{\_R^2}}{27b^3}$
default	$\frac{-\frac{(4ag-bd)x^5}{9ab}-\frac{(7af-bc)x^4}{18ab}+\frac{(2ah-be)x^3}{3b^2}-\frac{(5ag+bd)x^2}{18b^2}-\frac{(2af+bc)x}{9b^2}+\frac{a(3ah-be)}{6b^3}}{(bx^3+a)^2}+\frac{(2af+bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}$

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] (-1/9*(4*a*g-b*d)/a/b*x^5-1/18*(7*a*f-b*c)/a/b*x^4+1/3*(2*a*h-b*e)/b^2*x^3-
1/18*(5*a*g+b*d)/b^2*x^2-1/9*(2*a*f+b*c)/b^2*x+1/6*a*(3*a*h-b*e)/b^3)/(b*x^
3+a)^2+1/27/b^3*sum((9*h*_R^2+1/a*(5*a*g+b*d)*_R+(2*a*f+b*c)/a)/_R^2*ln(x-_
R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 12939, normalized size of antiderivative = 39.81

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.13

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$
$$= \frac{2\,(b^3d-4\,ab^2g)x^5+(b^3c-7\,ab^2f)x^4-3\,a^2be+9\,a^3h-6\,(ab^2e-2\,a^2bh)x^3-(ab^2d+5\,a^2bg)x^2-2\,(ab^2}{18\,(ab^5x^6+2\,a^2b^4x^3+a^3b^3)}$$
$$+\frac{\sqrt{3}\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^3}$$
$$+\frac{\left(18\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-bc-2\,af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(9\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+bc+2\,af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] 1/18*(2*(b^3*d - 4*a*b^2*g)*x^5 + (b^3*c - 7*a*b^2*f)*x^4 - 3*a^2*b*e + 9*a
^3*h - 6*(a*b^2*e - 2*a^2*b*h)*x^3 - (a*b^2*d + 5*a^2*b*g)*x^2 - 2*(a*b^2*c
 + 2*a^2*b*f)*x)/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3) + 1/27*sqrt(3)*(b^2*
d*(a/b)^(2/3) + 5*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) + 2*a*b*f*(a/b)^(1/
3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3) + 1/54*(1
8*a*h*(a/b)^(2/3) + b*d*(a/b)^(1/3) + 5*a*g*(a/b)^(1/3) - b*c - 2*a*f)*log(
x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 1/27*(9*a*h*(a/b)^
(2/3) - b*d*(a/b)^(1/3) - 5*a*g*(a/b)^(1/3) + b*c + 2*a*f)*log(x + (a/b)^(1
/3))/(a*b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.10

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \frac{h\log\left(|bx^3+a|\right)}{3\,b^3}$$
$$-\frac{\sqrt{3}\left(b^2c+2\,abf-(-ab^2)^{\frac{1}{3}}\,bd-5\,(-ab^2)^{\frac{1}{3}}\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,ab^2}$$
$$-\frac{\left(b^2c+2\,abf+(-ab^2)^{\frac{1}{3}}\,bd+5\,(-ab^2)^{\frac{1}{3}}\,ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,ab^2}$$
$$+\frac{2\,(b^2d-4\,abg)x^5+(b^2c-7\,abf)x^4-6\,(abe-2\,a^2h)x^3-(abd+5\,a^2g)x^2-2\,(abc+2\,a^2f)x-\frac{3\,(a^2be-3\,a}{b}}{18\,(bx^3+a)^2ab^2}$$
$$-\frac{\left(ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2b^3g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^4c+2\,a^2b^3f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^5}$$

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] 1/3*h*log(abs(b*x^3 + a))/b^3 - 1/27*sqrt(3)*(b^2*c + 2*a*b*f - (-a*b^2)^(1
/3)*b*d - 5*(-a*b^2)^(1/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a
/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) - 1/54*(b^2*c + 2*a*b*f + (-a*b^2)^(1/3)*
b*d + 5*(-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b
^2)^(2/3)*a*b^2) + 1/18*(2*(b^2*d - 4*a*b*g)*x^5 + (b^2*c - 7*a*b*f)*x^4 -
6*(a*b*e - 2*a^2*h)*x^3 - (a*b*d + 5*a^2*g)*x^2 - 2*(a*b*c + 2*a^2*f)*x - 3
*(a^2*b*e - 3*a^3*h)/b)/((b*x^3 + a)^2*a*b^2) - 1/27*(a*b^4*d*(-a/b)^(1/3)
+ 5*a^2*b^3*g*(-a/b)^(1/3) + a*b^4*c + 2*a^2*b^3*f)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a^3*b^5)
```

Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 908, normalized size of antiderivative = 2.79

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \frac{\frac{3\,a^2\,h-a\,b\,e}{6\,b^3}-\frac{x\,(b\,c+2\,a\,f)}{9\,b^2}-\frac{x^2\,(b\,d+5\,a\,g)}{18\,b^2}-\frac{x^3\,(b\,e-2\,a\,h)}{3\,b^2}+\frac{x^4\,(b\,c-7\,a\,f)}{18\,a\,b}+\frac{x^5\,(b\,d-4\,a\,g)}{9\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(19683\,a^5\,b^9\,z^3-19683\,a^5\,b^6\,h\,z^2+810\,a^4\,b^4\,f\,g\,z+405\,a^3\,b^5\,c\,g\,z+162\,a^3\,b^5\,d\,f\,z+81\,a^2\,b^6\,c\right.\right.\right. + \frac{81\,a^3\,h^2+b^3\,c\,d+5\,a\,b^2\,c\,g+2\,a\,b^2\,d\,f+10\,a^2\,b\,f\,g}{81\,a^2\,b^4} + \left.\frac{x\,(25\,a^2\,g^2-18\,f\,h\,a^2+10\,a\,b\,d\,g-9\,c\,h\,a\,b+b^2\,d^2)}{81\,a^2\,b^3}\right)\mathrm{root}\big(19683\,a^5\,b^9\,z^3 - 19683\,a^5\,b^6\,h\,z^2+810\,a^4\,b^4\,f\,g\,z+405\,a^3\,b^5\,c\,g\,z+162\,a^3\,b^5\,d\,f\,z+81\,a^2\,b^6\,c\,d\,z + 6561\,a^5\,b^3\,h^2\,z-270\,a^4\,b\,f\,g\,h-135\,a^3\,b^2\,c\,g\,h-54\,a^3\,b^2\,d\,f\,h-27\,a^2\,b^3\,c\,d\,h - 6\,a\,b^4\,c^2\,f+75\,a^3\,b^2\,d\,g^2+15\,a^2\,b^3\,d^2\,g-12\,a^2\,b^3\,c\,f^2-8\,a^3\,b^2\,f^3+125\,a^4\,b\,g^3+a\,b^4\,d^3 \left. - 729\,a^5\,h^3-b^5\,c^3,z,k\big)\right)$$

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

```
[Out] ((3*a^2*h - a*b*e)/(6*b^3) - (x*(b*c + 2*a*f))/(9*b^2) - (x^2*(b*d + 5*a*g)
)/(18*b^2) - (x^3*(b*e - 2*a*h))/(3*b^2) + (x^4*(b*c - 7*a*f))/(18*a*b) + (
x^5*(b*d - 4*a*g))/(9*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log(root(1
9683*a^5*b^9*z^3 - 19683*a^5*b^6*h*z^2 + 810*a^4*b^4*f*g*z + 405*a^3*b^5*c*
g*z + 162*a^3*b^5*d*f*z + 81*a^2*b^6*c*d*z + 6561*a^5*b^3*h^2*z - 270*a^4*b
*f*g*h - 135*a^3*b^2*c*g*h - 54*a^3*b^2*d*f*h - 27*a^2*b^3*c*d*h - 6*a*b^4*
c^2*f + 75*a^3*b^2*d*g^2 + 15*a^2*b^3*d^2*g - 12*a^2*b^3*c*f^2 - 8*a^3*b^2*
f^3 + 125*a^4*b*g^3 + a*b^4*d^3 - 729*a^5*h^3 - b^5*c^3, z, k)*(9*root(1968
3*a^5*b^9*z^3 - 19683*a^5*b^6*h*z^2 + 810*a^4*b^4*f*g*z + 405*a^3*b^5*c*g*z
 + 162*a^3*b^5*d*f*z + 81*a^2*b^6*c*d*z + 6561*a^5*b^3*h^2*z - 270*a^4*b*f*
g*h - 135*a^3*b^2*c*g*h - 54*a^3*b^2*d*f*h - 27*a^2*b^3*c*d*h - 6*a*b^4*c^2
*f + 75*a^3*b^2*d*g^2 + 15*a^2*b^3*d^2*g - 12*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3
 + 125*a^4*b*g^3 + a*b^4*d^3 - 729*a^5*h^3 - b^5*c^3, z, k)*a*b^2 - (6*a*h)
/b + (x*(54*a^2*b^3*f + 27*a*b^4*c))/(81*a^2*b^3)) + (81*a^3*h^2 + b^3*c*d
+ 5*a*b^2*c*g + 2*a*b^2*d*f + 10*a^2*b*f*g)/(81*a^2*b^4) + (x*(b^2*d^2 + 25
*a^2*g^2 - 18*a^2*f*h - 9*a*b*c*h + 10*a*b*d*g))/(81*a^2*b^3))*root(19683*a
^5*b^9*z^3 - 19683*a^5*b^6*h*z^2 + 810*a^4*b^4*f*g*z + 405*a^3*b^5*c*g*z +
162*a^3*b^5*d*f*z + 81*a^2*b^6*c*d*z + 6561*a^5*b^3*h^2*z - 270*a^4*b*f*g*h
 - 135*a^3*b^2*c*g*h - 54*a^3*b^2*d*f*h - 27*a^2*b^3*c*d*h - 6*a*b^4*c^2*f
+ 75*a^3*b^2*d*g^2 + 15*a^2*b^3*d^2*g - 12*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 +
125*a^4*b*g^3 + a*b^4*d^3 - 729*a^5*h^3 - b^5*c^3, z, k), k, 1, 3)
```

3.423 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 297

$$\begin{aligned}
\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx &= \frac{x(bd-4ag+(2be-5ah)x+3bfx^2)}{18ab^2(a+bx^3)} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b(a+bx^3)^2} \\
&\quad - \frac{\left(b^{4/3}d+\sqrt[3]{a}be+2a\sqrt[3]{b}g+5a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{8/3}} \\
&\quad + \frac{\left(\sqrt[3]{b}(bd+2ag)-\sqrt[3]{a}(be+5ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{8/3}} \\
&\quad - \frac{\left(\sqrt[3]{b}(bd+2ag)-\sqrt[3]{a}(be+5ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{8/3}}
\end{aligned}$$

```
[Out] 1/18*x*(b*d-4*a*g+(-5*a*h+2*b*e)*x+3*b*f*x^2)/a/b^2/(b*x^3+a)+1/6*(-h*x^5-g
*x^4-f*x^3-e*x^2-d*x-c)/b/(b*x^3+a)^2+1/27*(b^(1/3)*(2*a*g+b*d)-a^(1/3)*(5*
a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(8/3)-1/54*(b^(1/3)*(2*a*g+b*d)-a
^(1/3)*(5*a*h+b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(8/
3)-1/27*(b^(4/3)*d+a^(1/3)*b*e+2*a*b^(1/3)*g+5*a^(4/3)*h)*arctan(1/3*(a^(1/
3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1837, 1872, 1874, 31, 648, 631, 210, 642}

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(5a^{4/3}h+\sqrt[3]{a}be+2a\sqrt[3]{b}g+b^{4/3}d\right)}{9\sqrt{3}a^{5/3}b^{8/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(2ag+bd)-\sqrt[3]{a}(5ah+be)\right)}{54a^{5/3}b^{8/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(2ag+bd)-\sqrt[3]{a}(5ah+be)\right)}{27a^{5/3}b^{8/3}} + \frac{x(x(2be-5ah)-4ag+bd+3bfx^2)}{18ab^2(a+bx^3)} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b(a+bx^3)^2}$$

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] (x*(b*d - 4*a*g + (2*b*e - 5*a*h)*x + 3*b*f*x^2))/(18*a*b^2*(a + b*x^3)) -
(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(6*b*(a + b*x^3)^2) - ((b^(4/3)*d
 + a^(1/3)*b*e + 2*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(8/3)) + ((b^(1/3)*(b*d + 2*a*g)
 - a^(1/3)*(b*e + 5*a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(8/3)) -
((b^(1/3)*(b*d + 2*a*g) - a^(1/3)*(b*e + 5*a*h))*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\text{integral} = -\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2}+\frac{\int \frac{d+2ex+3fx^2+4gx^3+5hx^4}{\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= \frac{x(bd-4ag+(2be-5ah)x+3bfx^2)}{18ab^2(a+bx^3)} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b(a+bx^3)^2} - \frac{\int \frac{-2b(bd+2ag)-2b(be+5ah)x}{a+bx^3}\,dx}{18ab^3}$$

$$= \frac{x(bd-4ag+(2be-5ah)x+3bfx^2)}{18ab^2(a+bx^3)} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b(a+bx^3)^2} - \frac{\int \frac{\sqrt[3]{a}\left(-4b^{4/3}(bd+2ag)-2\sqrt[3]{a}b(be+5ah)\right)+\sqrt[3]{b}\left(2b^{4/3}(bd+2ag)-2\sqrt[3]{a}b(be+5ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{5/3}b^{10/3}} + \frac{\left(\sqrt[3]{b}(bd+2ag)-\sqrt[3]{a}(be+5ah)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{5/3}b^{7/3}}$$

$$= \frac{x(bd-4ag+(2be-5ah)x+3bfx^2)}{18ab^2(a+bx^3)} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b(a+bx^3)^2} + \frac{\left(\sqrt[3]{b}(bd+2ag)-\sqrt[3]{a}(be+5ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{8/3}} + \frac{\left(b^{4/3}d+\sqrt[3]{a}be+2a\sqrt[3]{b}g+5a^{4/3}h\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{4/3}b^{7/3}} - \frac{\left(\sqrt[3]{b}(bd+2ag)-\sqrt[3]{a}(be+5ah)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{5/3}b^{8/3}}$$

$$= \frac{x(bd-4ag+(2be-5ah)x+3bfx^2)}{18ab^2(a+bx^3)} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b(a+bx^3)^2} + \frac{\left(\sqrt[3]{b}(bd+2ag)-\sqrt[3]{a}(be+5ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{8/3}} - \frac{\left(\sqrt[3]{b}(bd+2ag)-\sqrt[3]{a}(be+5ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{8/3}} + \frac{\left(b^{4/3}d+\sqrt[3]{a}be+2a\sqrt[3]{b}g+5a^{4/3}h\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{5/3}b^{8/3}}$$

$$= \frac{x(bd - 4ag + (2be - 5ah)x + 3bfx^2)}{18ab^2(a + bx^3)} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{6b(a + bx^3)^2} - \frac{\left(b^{4/3}d + \sqrt[3]{a}be + 2a\sqrt[3]{b}g + 5a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{8/3}} + \frac{\left(\sqrt[3]{b}(bd + 2ag) - \sqrt[3]{a}(be + 5ah)\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{5/3}b^{8/3}} - \frac{\left(\sqrt[3]{b}(bd + 2ag) - \sqrt[3]{a}(be + 5ah)\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54a^{5/3}b^{8/3}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.97

$$\int \frac{x^2(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^3}\,dx$$

$$= \frac{-\frac{9b^{2/3}(b(c+x(d+ex))-a(f+x(g+hx)))}{(a+bx^3)^2} + \frac{3b^{2/3}(bx(d+2ex)-a(6f+x(7g+8hx)))}{a(a+bx^3)} - \frac{2\sqrt{3}\left(b^{4/3}d+\sqrt[3]{a}be+2a\sqrt[3]{b}g+5a^{4/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{}}{\sqrt[3]{}}}{\sqrt{3}}\right)}{a^{5/3}}}{54b^{8/}}$$

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(2/3)*(b*(c + x*(d + e*x)) - a*(f + x*(g + h*x))))/(a + b*x^3)^2 + (
3*b^(2/3)*(b*x*(d + 2*e*x) - a*(6*f + x*(7*g + 8*h*x))))/(a*(a + b*x^3)) -
(2*Sqrt[3]*(b^(4/3)*d + a^(1/3)*b*e + 2*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*(b^(4/3)*d - a^(1/3)*b*e
+ 2*a*b^(1/3)*g - 5*a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) + ((-(b^(4
/3)*d) + a^(1/3)*b*e - 2*a*b^(1/3)*g + 5*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(54*b^(8/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.51

method	result
risch	$\frac{-\frac{(4ah-be)x^5}{9ab}-\frac{(7ag-bd)x^4}{18ab}-\frac{f\,x^3}{3b}-\frac{(5ah+be)x^2}{18b^2}-\frac{(2ag+bd)x}{9b^2}-\frac{af+bc}{6b^2}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{((5ah+be)\_R+2ag+bd)\ln(x-\_R)}{\_R^2}}{27a\,b^3}$
default	$\frac{-\frac{(4ah-be)x^5}{9ab}-\frac{(7ag-bd)x^4}{18ab}-\frac{f\,x^3}{3b}-\frac{(5ah+be)x^2}{18b^2}-\frac{(2ag+bd)x}{9b^2}-\frac{af+bc}{6b^2}}{(b\,x^3+a)^2}+\frac{(2ag+bd)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\mathrm{arcta}}{}\right.}{}$

```
[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] (-1/9*(4*a*h-b*e)/a/b*x^5-1/18*(7*a*g-b*d)/a/b*x^4-1/3*f*x^3/b-1/18*(5*a*h+
b*e)/b^2*x^2-1/9*(2*a*g+b*d)/b^2*x-1/6*(a*f+b*c)/b^2)/(b*x^3+a)^2+1/27/a/b^
3*sum(((5*a*h+b*e)*_R+2*a*g+b*d)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 6926, normalized size of antiderivative = 23.32

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.04

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = -\frac{6\,abfx^3-2\,(b^2e-4\,abh)x^5-(b^2d-7\,abg)x^4+3\,abc+3\,a^2f+(abe+5\,a^2h)x^2+2\,(abd+2\,a^2g)x}{18\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)} + \frac{\sqrt{3}\left(be\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}}+bd+2\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(be\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}}-bd-2\,ag\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(be\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}}-bd-2\,ag\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] -1/18*(6*a*b*f*x^3 - 2*(b^2*e - 4*a*b*h)*x^5 - (b^2*d - 7*a*b*g)*x^4 + 3*a*
b*c + 3*a^2*f + (a*b*e + 5*a^2*h)*x^2 + 2*(a*b*d + 2*a^2*g)*x)/(a*b^4*x^6 +
 2*a^2*b^3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(b*e*(a/b)^(1/3) + 5*a*h*(a/b)^(1/
3) + b*d + 2*a*g)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^
3*(a/b)^(2/3)) + 1/54*(b*e*(a/b)^(1/3) + 5*a*h*(a/b)^(1/3) - b*d - 2*a*g)*l
og(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) - 1/27*(b*e*(a/b)
^(1/3) + 5*a*h*(a/b)^(1/3) - b*d - 2*a*g)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)
^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.06

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$
$$= -\frac{\sqrt{3}\left(b^2d+2\,abg-\left(-ab^2\right)^{\frac{1}{3}}be-5\left(-ab^2\right)^{\frac{1}{3}}ah\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}ab^2}$$
$$-\frac{\left(b^2d+2\,abg+\left(-ab^2\right)^{\frac{1}{3}}be+5\left(-ab^2\right)^{\frac{1}{3}}ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}ab^2}$$
$$-\frac{\left(be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,ah\left(-\frac{a}{b}\right)^{\frac{1}{3}}+bd+2\,ag\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^2}$$
$$+\frac{2\,b^2ex^5-8\,abhx^5+b^2dx^4-7\,abgx^4-6\,abfx^3-abex^2-5\,a^2hx^2-2\,abdx-4\,a^2gx-3\,abc-3\,a^2f}{18\left(bx^3+a\right)^2ab^2}$$

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] -1/27*sqrt(3)*(b^2*d + 2*a*b*g - (-a*b^2)^(1/3)*b*e - 5*(-a*b^2)^(1/3)*a*h)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^
2) - 1/54*(b^2*d + 2*a*b*g + (-a*b^2)^(1/3)*b*e + 5*(-a*b^2)^(1/3)*a*h)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^2) - 1/27*(b*e*(-
a/b)^(1/3) + 5*a*h*(-a/b)^(1/3) + b*d + 2*a*g)*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/(a^2*b^2) + 1/18*(2*b^2*e*x^5 - 8*a*b*h*x^5 + b^2*d*x^4 - 7*a*b
*g*x^4 - 6*a*b*f*x^3 - a*b*e*x^2 - 5*a^2*h*x^2 - 2*a*b*d*x - 4*a^2*g*x - 3*
a*b*c - 3*a^2*f)/((b*x^3 + a)^2*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 627, normalized size of antiderivative = 2.11

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\operatorname{root}\left(19683\,a^5\,b^8\,z^3+810\,a^4\,b^3\,g\,h\,z+405\,a^3\,b^4\,d\,h\,z+162\,a^3\,b^4\,e\,g\,z+81\,a^2\,b^5\,d\,e\,z+75\,a^3\,b\,e\,h^2-6\,a\right.\right.\right.$$
$$+\frac{b^2\,d\,e+10\,a^2\,g\,h+5\,a\,b\,d\,h+2\,a\,b\,e\,g}{81\,a^2\,b^3}$$
$$\left.+\frac{x\,(25\,a^2\,h^2+10\,a\,b\,e\,h+b^2\,e^2)}{81\,a^2\,b^3}\right)\operatorname{root}\big(19683\,a^5\,b^8\,z^3+810\,a^4\,b^3\,g\,h\,z+405\,a^3\,b^4\,d\,h\,z$$
$$+162\,a^3\,b^4\,e\,g\,z+81\,a^2\,b^5\,d\,e\,z+75\,a^3\,b\,e\,h^2-6\,a\,b^3\,d^2\,g+15\,a^2\,b^2\,e^2\,h-12\,a^2\,b^2\,d\,g^2$$
$$\left.\left.-8\,a^3\,b\,g^3+a\,b^3\,e^3+125\,a^4\,h^3-b^4\,d^3,z,k\big)\right)\right)$$
$$-\frac{\frac{b\,c+a\,f}{6\,b^2}+\frac{x\,(b\,d+2\,a\,g)}{9\,b^2}+\frac{f\,x^3}{3\,b}+\frac{x^2\,(b\,e+5\,a\,h)}{18\,b^2}-\frac{x^4\,(b\,d-7\,a\,g)}{18\,a\,b}-\frac{x^5\,(b\,e-4\,a\,h)}{9\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

```
[In] int((x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

```
[Out] symsum(log(root(19683*a^5*b^8*z^3 + 810*a^4*b^3*g*h*z + 405*a^3*b^4*d*h*z +
 162*a^3*b^4*e*g*z + 81*a^2*b^5*d*e*z + 75*a^3*b*e*h^2 - 6*a*b^3*d^2*g + 15
*a^2*b^2*e^2*h - 12*a^2*b^2*d*g^2 - 8*a^3*b*g^3 + a*b^3*e^3 + 125*a^4*h^3 -
 b^4*d^3, z, k)*(9*root(19683*a^5*b^8*z^3 + 810*a^4*b^3*g*h*z + 405*a^3*b^4
*d*h*z + 162*a^3*b^4*e*g*z + 81*a^2*b^5*d*e*z + 75*a^3*b*e*h^2 - 6*a*b^3*d^
2*g + 15*a^2*b^2*e^2*h - 12*a^2*b^2*d*g^2 - 8*a^3*b*g^3 + a*b^3*e^3 + 125*a
^4*h^3 - b^4*d^3, z, k)*a*b^2 + (x*(54*a^2*b^3*g + 27*a*b^4*d))/(81*a^2*b^3
)) + (b^2*d*e + 10*a^2*g*h + 5*a*b*d*h + 2*a*b*e*g)/(81*a^2*b^3) + (x*(b^2*
e^2 + 25*a^2*h^2 + 10*a*b*e*h))/(81*a^2*b^3))*root(19683*a^5*b^8*z^3 + 810*
a^4*b^3*g*h*z + 405*a^3*b^4*d*h*z + 162*a^3*b^4*e*g*z + 81*a^2*b^5*d*e*z +
75*a^3*b*e*h^2 - 6*a*b^3*d^2*g + 15*a^2*b^2*e^2*h - 12*a^2*b^2*d*g^2 - 8*a^
3*b*g^3 + a*b^3*e^3 + 125*a^4*h^3 - b^4*d^3, z, k), k, 1, 3) - ((b*c + a*f)
/(6*b^2) + (x*(b*d + 2*a*g))/(9*b^2) + (f*x^3)/(3*b) + (x^2*(b*e + 5*a*h))/
(18*b^2) - (x^4*(b*d - 7*a*g))/(18*a*b) - (x^5*(b*e - 4*a*h))/(9*a*b))/(a^2
 + b^2*x^6 + 2*a*b*x^3)
```

3.424 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 323

$$\begin{aligned}
&\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx \\
&= -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6ab^2(a+bx^3)^2} \\
&\quad+\frac{x(a(be-7ah)+2b(2bc+af)x+3b(bd+ag)x^2)}{18a^2b^2(a+bx^3)} \\
&\quad-\frac{\left(2b^{5/3}c+a^{2/3}be+ab^{2/3}f+2a^{5/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{7/3}} \\
&\quad-\frac{\left(b^{2/3}(2bc+af)-a^{2/3}(be+2ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{7/3}} \\
&\quad+\frac{\left(b^{2/3}(2bc+af)-a^{2/3}(be+2ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{7/3}}
\end{aligned}$$

```
[Out] -1/6*x*(a*(-a*h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/a/b^2/(b*x^3+a)^2+1/1
8*x*(a*(-7*a*h+b*e)+2*b*(a*f+2*b*c)*x+3*b*(a*g+b*d)*x^2)/a^2/b^2/(b*x^3+a)-
1/27*(b^(2/3)*(a*f+2*b*c)-a^(2/3)*(2*a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(7/3
)/b^(7/3)+1/54*(b^(2/3)*(a*f+2*b*c)-a^(2/3)*(2*a*h+b*e))*ln(a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(7/3)-1/27*(2*b^(5/3)*c+a^(2/3)*b*e+a*b^(
2/3)*f+2*a^(5/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/
3)/b^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1842, 1872, 1874, 31, 648, 631, 210, 642}

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{2/3}be+2a^{5/3}h+ab^{2/3}f+2b^{5/3}c\right)}{9\sqrt{3}a^{7/3}b^{7/3}}$$
$$+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{54a^{7/3}b^{7/3}}$$
$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{27a^{7/3}b^{7/3}}$$
$$+\frac{x(2bx(af+2bc)+3bx^2(ag+bd)+a(be-7ah))}{18a^2b^2(a+bx^3)}$$
$$-\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{6ab^2(a+bx^3)^2}$$

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(a*b^2*(a +
b*x^3)^2) + (x*(a*(b*e - 7*a*h) + 2*b*(2*b*c + a*f)*x + 3*b*(b*d + a*g)*x^2
))/(18*a^2*b^2*(a + b*x^3)) - ((2*b^(5/3)*c + a^(2/3)*b*e + a*b^(2/3)*f + 2
*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a
^(7/3)*b^(7/3)) - ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b*e + 2*a*h))*Log[a^(1
/3) + b^(1/3)*x])/(27*a^(7/3)*b^(7/3)) + ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*
(b*e + 2*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*
b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
```

```
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\text{integral} = -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6ab^2(a+bx^3)^2} - \frac{\int \frac{-a(be-ah)-2b(2bc+af)x-3b(bd+ag)x^2-6abhx^3}{(a+bx^3)^2}\,dx}{6ab^2}$$

$$= -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6ab^2(a+bx^3)^2} + \frac{x(a(be-7ah)+2b(2bc+af)x+3b(bd+ag)x^2)}{18a^2b^2(a+bx^3)} + \frac{\int \frac{2ab(be+2ah)+2b^2(2bc+af)x}{a+bx^3}\,dx}{18a^2b^3}$$

$$= -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6ab^2(a+bx^3)^2} + \frac{x(a(be-7ah)+2b(2bc+af)x+3b(bd+ag)x^2)}{18a^2b^2(a+bx^3)} + \frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{a}b^2(2bc+af)+4ab^{4/3}(be+2ah)\right)+\sqrt[3]{b}\left(2\sqrt[3]{a}b^2(2bc+af)-2ab^{4/3}(be+2ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}b^{10/3}} - \frac{\left(b^{2/3}(2bc+af)-a^{2/3}(be+2ah)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{7/3}b^2}$$

$$= -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6ab^2(a+bx^3)^2} + \frac{x(a(be-7ah)+2b(2bc+af)x+3b(bd+ag)x^2)}{18a^2b^2(a+bx^3)} - \frac{\left(b^{2/3}(2bc+af)-a^{2/3}(be+2ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{7/3}} + \frac{\left(2b^{5/3}c+a^{2/3}be+ab^{2/3}f+2a^{5/3}h\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^2b^2} + \frac{\left(b^{2/3}(2bc+af)-a^{2/3}(be+2ah)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{7/3}b^{7/3}}$$

$$= -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6ab^2(a+bx^3)^2} + \frac{x(a(be-7ah)+2b(2bc+af)x+3b(bd+ag)x^2)}{18a^2b^2(a+bx^3)} - \frac{(b^{2/3}(2bc+af)-a^{2/3}(be+2ah))\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{7/3}} + \frac{(b^{2/3}(2bc+af)-a^{2/3}(be+2ah))\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{7/3}} + \frac{(2b^{5/3}c+a^{2/3}be+ab^{2/3}f+2a^{5/3}h)\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{7/3}b^{7/3}}$$

$$= -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6ab^2(a+bx^3)^2} + \frac{x(a(be-7ah)+2b(2bc+af)x+3b(bd+ag)x^2)}{18a^2b^2(a+bx^3)} - \frac{(2b^{5/3}c+a^{2/3}be+ab^{2/3}f+2a^{5/3}h)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{7/3}} - \frac{(b^{2/3}(2bc+af)-a^{2/3}(be+2ah))\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{7/3}} + \frac{(b^{2/3}(2bc+af)-a^{2/3}(be+2ah))\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{7/3}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.92

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{3\sqrt[3]{a}\sqrt[3]{b}(-4b^2cx^2-abx(e+2fx)+a^2(6g+7hx))}{a+bx^3}+\frac{9a^{4/3}\sqrt[3]{b}(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx)))}{(a+bx^3)^2}-2\sqrt{3}(2b^{5/3}c+a^{2/3}be+ab^{2}}{}$$

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
[Out] ((-3*a^(1/3)*b^(1/3)*(-4*b^2*c*x^2 - a*b*x*(e + 2*f*x) + a^2*(6*g + 7*h*x))
)/(a + b*x^3) + (9*a^(4/3)*b^(1/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*
(e + f*x))))/(a + b*x^3)^2 - 2*Sqrt[3]*(2*b^(5/3)*c + a^(2/3)*b*e + a*b^(2/
3)*f + 2*a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-2*b^(
```

```
5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f + 2*a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x]
+ (2*b^(5/3)*c - a^(2/3)*b*e + a*b^(2/3)*f - 2*a^(5/3)*h)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(7/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.67 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.49

method	result
risch	$\frac{\frac{(af+2bc)x^5}{9a^2}-\frac{(7ah-be)x^4}{18ab}-\frac{g x^3}{3b}-\frac{(af-7bc)x^2}{18ab}-\frac{(2ah+be)x}{9b^2}-\frac{ag+bd}{6b^2}}{(b x^3+a)^2}+\frac{\sum_{\_R=\operatorname{RootOf}(b\_Z^3+a)}\frac{\left(\frac{(af+2bc)\_R}{a}+\frac{2ah+be}{b}\right)\ln(x-\_R)}{\_R^2}}{27a b^2}$
default	$\frac{\frac{(af+2bc)x^5}{9a^2}-\frac{(7ah-be)x^4}{18ab}-\frac{g x^3}{3b}-\frac{(af-7bc)x^2}{18ab}-\frac{(2ah+be)x}{9b^2}-\frac{ag+bd}{6b^2}}{(b x^3+a)^2}+\frac{(2a^2h+aeb)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan}{}\right.}{}$

```
[In] int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/9*(a*f+2*b*c)/a^2*x^5-1/18*(7*a*h-b*e)/a/b*x^4-1/3*g*x^3/b-1/18*(a*f-7*b
*c)/a/b*x^2-1/9*(2*a*h+b*e)/b^2*x-1/6*(a*g+b*d)/b^2)/(b*x^3+a)^2+1/27/a/b^2
*sum((1/a*(a*f+2*b*c)*_R+1/b*(2*a*h+b*e))/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a)
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 7190, normalized size of antiderivative = 22.26

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="frica
s")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^3}\, dx = \text{Timed out}$$

```
[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.07

$$\int \frac{x(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^3}\, dx =$$

$$-\frac{6\,a^2bgx^3 - 2\,(2\,b^3c + ab^2f)x^5 - (ab^2e - 7\,a^2bh)x^4 + 3\,a^2bd + 3\,a^3g - (7\,ab^2c - a^2bf)x^2 + 2\,(a^2be + 2\,a^3}{18\,(a^2b^4x^6 + 2\,a^3b^3x^3 + a^4b^2)}$$

$$+ \frac{\sqrt{3}\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} + abf\left(\frac{a}{b}\right)^{\frac{1}{3}} + abe + 2\,a^2h\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$$+ \frac{\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} + abf\left(\frac{a}{b}\right)^{\frac{1}{3}} - abe - 2\,a^2h\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$$- \frac{\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} + abf\left(\frac{a}{b}\right)^{\frac{1}{3}} - abe - 2\,a^2h\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxim
a")
```

```
[Out] -1/18*(6*a^2*b*g*x^3 - 2*(2*b^3*c + a*b^2*f)*x^5 - (a*b^2*e - 7*a^2*b*h)*x^
4 + 3*a^2*b*d + 3*a^3*g - (7*a*b^2*c - a^2*b*f)*x^2 + 2*(a^2*b*e + 2*a^3*h)
*x)/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2) + 1/27*sqrt(3)*(2*b^2*c*(a/b)^(
1/3) + a*b*f*(a/b)^(1/3) + a*b*e + 2*a^2*h)*arctan(1/3*sqrt(3)*(2*x - (a/b)
^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3)) + 1/54*(2*b^2*c*(a/b)^(1/3) + a*
b*f*(a/b)^(1/3) - a*b*e - 2*a^2*h)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(
a^2*b^3*(a/b)^(2/3)) - 1/27*(2*b^2*c*(a/b)^(1/3) + a*b*f*(a/b)^(1/3) - a*b*
e - 2*a^2*h)*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.04

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$

$$= -\frac{\sqrt{3}\left(abe+2\,a^2h-2\left(-ab^2\right)^{\frac{1}{3}}bc-\left(-ab^2\right)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^2b}$$
$$-\frac{\left(abe+2\,a^2h+2\left(-ab^2\right)^{\frac{1}{3}}bc+\left(-ab^2\right)^{\frac{1}{3}}af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^2b}$$
$$-\frac{\left(2\,b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(-\frac{a}{b}\right)^{\frac{1}{3}}+abe+2\,a^2h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^2}$$
$$+\frac{4\,b^3cx^5+2\,ab^2fx^5+ab^2ex^4-7\,a^2bhx^4-6\,a^2bgx^3+7\,ab^2cx^2-a^2bfx^2-2\,a^2bex-4\,a^3hx-3\,a^2bd-3}{18\left(bx^3+a\right)^2a^2b^2}$$

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac"
)
```

```
[Out] -1/27*sqrt(3)*(a*b*e + 2*a^2*h - 2*(-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/3)*a*f)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*
b) - 1/54*(a*b*e + 2*a^2*h + 2*(-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/3)*a*f)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b) - 1/27*(2*b^2*
c*(-a/b)^(1/3) + a*b*f*(-a/b)^(1/3) + a*b*e + 2*a^2*h)*(-a/b)^(1/3)*log(abs
(x - (-a/b)^(1/3)))/(a^3*b^2) + 1/18*(4*b^3*c*x^5 + 2*a*b^2*f*x^5 + a*b^2*e
*x^4 - 7*a^2*b*h*x^4 - 6*a^2*b*g*x^3 + 7*a*b^2*c*x^2 - a^2*b*f*x^2 - 2*a^2*
b*e*x - 4*a^3*h*x - 3*a^2*b*d - 3*a^3*g)/((b*x^3 + a)^2*a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.98

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(19683\,a^7\,b^7\,z^3+162\,a^5\,b^3\,f\,h\,z+324\,a^4\,b^4\,c\,h\,z+81\,a^4\,b^4\,e\,f\,z+162\,a^3\,b^5\,c\,e\,z-12\,a^4\,b\,e\,h^2+1\right.\right.\right.$$
$$+\frac{2\,b^2\,c\,e+2\,a^2\,f\,h+4\,a\,b\,c\,h+a\,b\,e\,f}{81\,a^3\,b^2}$$
$$\left.+\frac{x\,(a^2\,f^2+4\,a\,b\,c\,f+4\,b^2\,c^2)}{81\,a^4\,b}\right)\mathrm{root}\big(19683\,a^7\,b^7\,z^3+162\,a^5\,b^3\,f\,h\,z+324\,a^4\,b^4\,c\,h\,z$$
$$+81\,a^4\,b^4\,e\,f\,z+162\,a^3\,b^5\,c\,e\,z-12\,a^4\,b\,e\,h^2+12\,a\,b^4\,c^2\,f-6\,a^3\,b^2\,e^2\,h+6\,a^2\,b^3\,c\,f^2$$
$$\left.+a^3\,b^2\,f^3-8\,a^5\,h^3+8\,b^5\,c^3-a^2\,b^3\,e^3,z,k\big)\right)$$
$$-\frac{\frac{b\,d+a\,g}{6\,b^2}+\frac{x\,(b\,e+2\,a\,h)}{9\,b^2}+\frac{g\,x^3}{3\,b}-\frac{x^5\,(2\,b\,c+a\,f)}{9\,a^2}-\frac{x^2\,(7\,b\,c-a\,f)}{18\,a\,b}-\frac{x^4\,(b\,e-7\,a\,h)}{18\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

```
[In] int((x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

```
[Out] symsum(log(root(19683*a^7*b^7*z^3 + 162*a^5*b^3*f*h*z + 324*a^4*b^4*c*h*z +
 81*a^4*b^4*e*f*z + 162*a^3*b^5*c*e*z - 12*a^4*b*e*h^2 + 12*a*b^4*c^2*f - 6
*a^3*b^2*e^2*h + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 - 8*a^5*h^3 + 8*b^5*c^3 - a^
2*b^3*e^3, z, k)*(9*root(19683*a^7*b^7*z^3 + 162*a^5*b^3*f*h*z + 324*a^4*b^
4*c*h*z + 81*a^4*b^4*e*f*z + 162*a^3*b^5*c*e*z - 12*a^4*b*e*h^2 + 12*a*b^4*
c^2*f - 6*a^3*b^2*e^2*h + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 - 8*a^5*h^3 + 8*b^5
*c^3 - a^2*b^3*e^3, z, k)*a*b^2 + (x*(27*a^3*b^2*e + 54*a^4*b*h))/(81*a^4*b
)) + (2*b^2*c*e + 2*a^2*f*h + 4*a*b*c*h + a*b*e*f)/(81*a^3*b^2) + (x*(4*b^2
*c^2 + a^2*f^2 + 4*a*b*c*f))/(81*a^4*b))*root(19683*a^7*b^7*z^3 + 162*a^5*b
^3*f*h*z + 324*a^4*b^4*c*h*z + 81*a^4*b^4*e*f*z + 162*a^3*b^5*c*e*z - 12*a^
4*b*e*h^2 + 12*a*b^4*c^2*f - 6*a^3*b^2*e^2*h + 6*a^2*b^3*c*f^2 + a^3*b^2*f^
3 - 8*a^5*h^3 + 8*b^5*c^3 - a^2*b^3*e^3, z, k), k, 1, 3) - ((b*d + a*g)/(6*
b^2) + (x*(b*e + 2*a*h))/(9*b^2) + (g*x^3)/(3*b) - (x^5*(2*b*c + a*f))/(9*a
^2) - (x^2*(7*b*c - a*f))/(18*a*b) - (x^4*(b*e - 7*a*h))/(18*a*b))/(a^2 + b
^2*x^6 + 2*a*b*x^3)
```

3.425 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 313

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx &= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6ab\,(a+bx^3)^2} - \frac{3a(be+ah)-bx(5bc+af+2(2bd+ag)x)}{18a^2b^2\,(a+bx^3)} \\ &\quad - \frac{\left(5b^{4/3}c+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{5/3}} \\ &\quad + \frac{\left(\sqrt[3]{b}(5bc+af)-\sqrt[3]{a}(2bd+ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{5/3}} \\ &\quad - \frac{\left(\sqrt[3]{b}(5bc+af)-\sqrt[3]{a}(2bd+ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{5/3}}\end{aligned}$$

```
[Out] 1/6*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*x^2)/a/b/(b*x^3+a)^2+1/18*(-3*a*(a*h
+b*e)+b*x*(5*b*c+a*f+2*(a*g+2*b*d)*x))/a^2/b^2/(b*x^3+a)+1/27*(b^(1/3)*(a*f
+5*b*c)-a^(1/3)*(a*g+2*b*d))*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(5/3)-1/54*(b^
(1/3)*(a*f+5*b*c)-a^(1/3)*(a*g+2*b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/a^(8/3)/b^(5/3)-1/27*(5*b^(4/3)*c+2*a^(1/3)*b*d+a*b^(1/3)*f+a^(4/3)*g
)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1872, 1868, 1874, 31, 648, 631, 210, 642}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}g+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+5b^{4/3}c\right)}{9\sqrt{3}a^{8/3}b^{5/3}}$$
$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(af+5bc)-\sqrt[3]{a}(ag+2bd)\right)}{54a^{8/3}b^{5/3}}$$
$$+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(af+5bc)-\sqrt[3]{a}(ag+2bd)\right)}{27a^{8/3}b^{5/3}}$$
$$-\frac{3a(ah+be)-bx(2x(ag+2bd)+af+5bc)}{18a^2b^2(a+bx^3)}+\frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{6ab(a+bx^3)^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x]
```

```
[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*a*b*(a + b*x^3)^2) - (
3*a*(b*e + a*h) - b*x*(5*b*c + a*f + 2*(2*b*d + a*g)*x))/(18*a^2*b^2*(a + b
*x^3)) - ((5*b^(4/3)*c + 2*a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(5/3)) + ((b
^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^(1/3) + b^(1/3)*x])/(27
*a^(8/3)*b^(5/3)) - ((b^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\text{integral} = \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6ab\,(a+bx^3)^2} - \frac{\int \frac{-b(5bc+af)-2b(2bd+ag)x-3b(be+ah)x^2}{(a+bx^3)^2}\,dx}{6ab^2}$$

$$=\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6ab\left(a+bx^3\right)^2}-\frac{3a(be+ah)-bx(5bc+af+2(2bd+ag)x)}{18a^2b^2\left(a+bx^3\right)}+\frac{\int\frac{2b(5bc+af)+2b(2bd+ag)x}{a+bx^3}\,dx}{18a^2b^2}$$

$$=\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6ab\left(a+bx^3\right)^2}-\frac{3a(be+ah)-bx(5bc+af+2(2bd+ag)x)}{18a^2b^2\left(a+bx^3\right)}+\frac{\int\frac{\sqrt[3]{a}\left(4b^{4/3}(5bc+af)+2\sqrt[3]{a}b(2bd+ag)\right)+\sqrt[3]{b}\left(-2b^{4/3}(5bc+af)+2\sqrt[3]{a}b(2bd+ag)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}b^{7/3}}+\frac{\left(\sqrt[3]{b}(5bc+af)-\sqrt[3]{a}(2bd+ag)\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{8/3}b^{4/3}}$$

$$=\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6ab\left(a+bx^3\right)^2}-\frac{3a(be+ah)-bx(5bc+af+2(2bd+ag)x)}{18a^2b^2\left(a+bx^3\right)}+\frac{\left(\sqrt[3]{b}(5bc+af)-\sqrt[3]{a}(2bd+ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{5/3}}+\frac{\left(5b^{4/3}c+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+a^{4/3}g\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{7/3}b^{4/3}}-\frac{\left(\sqrt[3]{b}(5bc+af)-\sqrt[3]{a}(2bd+ag)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}b^{5/3}}$$

$$=\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6ab\left(a+bx^3\right)^2}-\frac{3a(be+ah)-bx(5bc+af+2(2bd+ag)x)}{18a^2b^2\left(a+bx^3\right)}+\frac{\left(\sqrt[3]{b}(5bc+af)-\sqrt[3]{a}(2bd+ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{5/3}}-\frac{\left(\sqrt[3]{b}(5bc+af)-\sqrt[3]{a}(2bd+ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{5/3}}+\frac{\left(5b^{4/3}c+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+a^{4/3}g\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{8/3}b^{5/3}}$$

$$= \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6ab\left(a+bx^3\right)^2} - \frac{3a(be+ah)-bx(5bc+af+2(2bd+ag)x)}{18a^2b^2\left(a+bx^3\right)} - \frac{\left(5b^{4/3}c+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{5/3}} + \frac{\left(\sqrt[3]{b}(5bc+af)-\sqrt[3]{a}(2bd+ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{5/3}} - \frac{\left(\sqrt[3]{b}(5bc+af)-\sqrt[3]{a}(2bd+ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{5/3}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.94

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^3\right)^3}\,dx$$

$$= \frac{\frac{3a^{2/3}\left(-6a^2h+b^2x(5c+4dx)+abx(f+2gx)\right)}{a+bx^3}+\frac{9a^{5/3}\left(a^2h+b^2x(c+dx)-ab(e+x(f+gx))\right)}{\left(a+bx^3\right)^2}-2\sqrt{3}\sqrt[3]{b}\left(5b^{4/3}c+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+a^{4/3}g\right)\cdots}{\cdots}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x]
[Out] ((3*a^(2/3)*(-6*a^2*h + b^2*x*(5*c + 4*d*x) + a*b*x*(f + 2*g*x)))/(a + b*x^
3) + (9*a^(5/3)*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a + b*x
^3)^2 - 2*Sqrt[3]*b^(1/3)*(5*b^(4/3)*c + 2*a^(1/3)*b*d + a*b^(1/3)*f + a^(4
/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(5*b^(4/3)*c
 - 2*a^(1/3)*b*d + a*b^(1/3)*f - a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x] + b^(1
/3)*(-5*b^(4/3)*c + 2*a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.71 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.48

method	result
risch	$\frac{\frac{(ag+2bd)x^5}{9a^2}+\frac{(af+5bc)x^4}{18a^2}-\frac{h\,x^3}{3b}-\frac{(ag-7bd)x^2}{18ab}-\frac{(af-4bc)x}{9ab}-\frac{ah+be}{6b^2}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{((ag+2bd)\_R+af+5bc)\ln(x-\_R)}{\_R^2}}{27a^2b^2}$
default	$\frac{\frac{(ag+2bd)x^5}{9a^2}+\frac{(af+5bc)x^4}{18a^2}-\frac{h\,x^3}{3b}-\frac{(ag-7bd)x^2}{18ab}-\frac{(af-4bc)x}{9ab}-\frac{ah+be}{6b^2}}{(b\,x^3+a)^2}+\frac{(af+5bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan}{}\right.}{}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/9*(a*g+2*b*d)/a^2*x^5+1/18*(a*f+5*b*c)/a^2*x^4-1/3*h*x^3/b-1/18*(a*g-7*b
*d)/a/b*x^2-1/9*(a*f-4*b*c)/a/b*x-1/6*(a*h+b*e)/b^2)/(b*x^3+a)^2+1/27/a^2/b
^2*sum(((a*g+2*b*d)*_R+a*f+5*b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 6984, normalized size of antiderivative = 22.31

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas"
)
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx =$$

$$-\frac{6\,a^2bhx^3-2\,(2\,b^3d+ab^2g)x^5-(5\,b^3c+ab^2f)x^4+3\,a^2be+3\,a^3h-(7\,ab^2d-a^2bg)x^2-2\,(4\,ab^2c-a^2bf}{18\,(a^2b^4x^6+2\,a^3b^3x^3+a^4b^2)}$$

$$+\frac{\sqrt{3}\left(2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,bc+af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$$+\frac{\left(2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,bc-af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$$-\frac{\left(2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,bc-af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima"
)
```

```
[Out] -1/18*(6*a^2*b*h*x^3 - 2*(2*b^3*d + a*b^2*g)*x^5 - (5*b^3*c + a*b^2*f)*x^4
+ 3*a^2*b*e + 3*a^3*h - (7*a*b^2*d - a^2*b*g)*x^2 - 2*(4*a*b^2*c - a^2*b*f)
*x)/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2) + 1/27*sqrt(3)*(2*b*d*(a/b)^(1/
3) + a*g*(a/b)^(1/3) + 5*b*c + a*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/
(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) + 1/54*(2*b*d*(a/b)^(1/3) + a*g*(a/b)^(1
/3) - 5*b*c - a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2
/3)) - 1/27*(2*b*d*(a/b)^(1/3) + a*g*(a/b)^(1/3) - 5*b*c - a*f)*log(x + (a/
b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.05

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx$$
$$= -\frac{\sqrt{3}\left(5\,b^2c+abf-2\,(-ab^2)^{\frac{1}{3}}\,bd-(-ab^2)^{\frac{1}{3}}\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^2b}$$
$$-\frac{\left(5\,b^2c+abf+2\,(-ab^2)^{\frac{1}{3}}\,bd+(-ab^2)^{\frac{1}{3}}\,ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^2b}$$
$$-\frac{\left(2\,bd\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,bc+af\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b}$$
$$+\frac{4\,b^3dx^5+2\,ab^2gx^5+5\,b^3cx^4+ab^2fx^4-6\,a^2bhx^3+7\,ab^2dx^2-a^2bgx^2+8\,ab^2cx-2\,a^2bfx-3\,a^2be-3}{18\,(bx^3+a)^2a^2b^2}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(5*b^2*c + a*b*f - 2*(-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*g)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*
b) - 1/54*(5*b^2*c + a*b*f + 2*(-a*b^2)^(1/3)*b*d + (-a*b^2)^(1/3)*a*g)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b) - 1/27*(2*b*d*
(-a/b)^(1/3) + a*g*(-a/b)^(1/3) + 5*b*c + a*f)*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/(a^3*b) + 1/18*(4*b^3*d*x^5 + 2*a*b^2*g*x^5 + 5*b^3*c*x^4 + a*b
^2*f*x^4 - 6*a^2*b*h*x^3 + 7*a*b^2*d*x^2 - a^2*b*g*x^2 + 8*a*b^2*c*x - 2*a^
2*b*f*x - 3*a^2*b*e - 3*a^3*h)/((b*x^3 + a)^2*a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 630, normalized size of antiderivative = 2.01

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx$$
$$= \frac{\frac{x^4(5bc+af)}{18a^2}-\frac{hx^3}{3b}-\frac{be+ah}{6b^2}+\frac{x^5(2bd+ag)}{9a^2}+\frac{x(4bc-af)}{9ab}+\frac{x^2(7bd-ag)}{18ab}}{a^2+2abx^3+b^2x^6}$$
$$+\left(\sum_{k=1}^{3}\ln\left(\operatorname{root}\left(19683a^8b^5z^3+81a^5b^2fgz+405a^4b^3cgz+162a^4b^3dfz+810a^3b^4cdz+6a^3bdg^2-75\right.\right.\right.$$
$$+\frac{10b^2cd+a^2fg+5abcg+2abdf}{81a^4b}$$
$$\left.+\frac{x(a^2g^2+4abdg+4b^2d^2)}{81a^4b}\right)\operatorname{root}\left(19683a^8b^5z^3+81a^5b^2fgz+405a^4b^3cgz\right.$$
$$+162a^4b^3dfz+810a^3b^4cdz+6a^3bdg^2-75ab^3c^2f+12a^2b^2d^2g-15a^2b^2cf^2$$
$$\left.\left.+8ab^3d^3+a^4g^3-125b^4c^3-a^3bf^3,z,k\right)\right)$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x)
```

```
[Out] ((x^4*(5*b*c + a*f))/(18*a^2) - (h*x^3)/(3*b) - (b*e + a*h)/(6*b^2) + (x^5*
(2*b*d + a*g))/(9*a^2) + (x*(4*b*c - a*f))/(9*a*b) + (x^2*(7*b*d - a*g))/(1
8*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log(root(19683*a^8*b^5*z^3 + 8
1*a^5*b^2*f*g*z + 405*a^4*b^3*c*g*z + 162*a^4*b^3*d*f*z + 810*a^3*b^4*c*d*z
 + 6*a^3*b*d*g^2 - 75*a*b^3*c^2*f + 12*a^2*b^2*d^2*g - 15*a^2*b^2*c*f^2 + 8
*a*b^3*d^3 + a^4*g^3 - 125*b^4*c^3 - a^3*b*f^3, z, k)*(9*root(19683*a^8*b^5
*z^3 + 81*a^5*b^2*f*g*z + 405*a^4*b^3*c*g*z + 162*a^4*b^3*d*f*z + 810*a^3*b
^4*c*d*z + 6*a^3*b*d*g^2 - 75*a*b^3*c^2*f + 12*a^2*b^2*d^2*g - 15*a^2*b^2*c
*f^2 + 8*a*b^3*d^3 + a^4*g^3 - 125*b^4*c^3 - a^3*b*f^3, z, k)*a*b^2 + (x*(1
35*a^2*b^3*c + 27*a^3*b^2*f))/(81*a^4*b)) + (10*b^2*c*d + a^2*f*g + 5*a*b*c
*g + 2*a*b*d*f)/(81*a^4*b) + (x*(4*b^2*d^2 + a^2*g^2 + 4*a*b*d*g))/(81*a^4*
b))*root(19683*a^8*b^5*z^3 + 81*a^5*b^2*f*g*z + 405*a^4*b^3*c*g*z + 162*a^4
*b^3*d*f*z + 810*a^3*b^4*c*d*z + 6*a^3*b*d*g^2 - 75*a*b^3*c^2*f + 12*a^2*b^
2*d^2*g - 15*a^2*b^2*c*f^2 + 8*a*b^3*d^3 + a^4*g^3 - 125*b^4*c^3 - a^3*b*f^
3, z, k), k, 1, 3)
```

3.426 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3} dx$

Optimal result

Integrand size = 38, antiderivative size = 347

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3} dx \\
&= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} \\
&\quad + \frac{x(a(5bd+ag)+2a(2be+ah)x-3b(3bc-af)x^2)}{18a^3b\left(a+bx^3\right)} \\
&\quad - \frac{\left(5b^{4/3}d+2\sqrt[3]{a}be+a\sqrt[3]{b}g+a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{5/3}} \\
&\quad + \frac{c\log(x)}{a^3} + \frac{\left(\sqrt[3]{b}(5bd+ag)-\sqrt[3]{a}(2be+ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{5/3}} \\
&\quad - \frac{\left(\sqrt[3]{b}(5bd+ag)-\sqrt[3]{a}(2be+ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{5/3}} - \frac{c\log\left(a+bx^3\right)}{3a^3}
\end{aligned}$$

```
[Out] 1/6*x*(a*(-a*g+b*d)+a*(-a*h+b*e)*x-b*(-a*f+b*c)*x^2)/a^2/b/(b*x^3+a)^2+1/18
*x*(a*(a*g+5*b*d)+2*a*(a*h+2*b*e)*x-3*b*(-a*f+3*b*c)*x^2)/a^3/b/(b*x^3+a)+c
*ln(x)/a^3+1/27*(b^(1/3)*(a*g+5*b*d)-a^(1/3)*(a*h+2*b*e))*ln(a^(1/3)+b^(1/3
)*x)/a^(8/3)/b^(5/3)-1/54*(b^(1/3)*(a*g+5*b*d)-a^(1/3)*(a*h+2*b*e))*ln(a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(5/3)-1/3*c*ln(b*x^3+a)/a^3-1/
27*(5*b^(4/3)*d+2*a^(1/3)*b*e+a*b^(1/3)*g+a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*
b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}h+2\sqrt[3]{a}be+a\sqrt[3]{b}g+5b^{4/3}d\right)}{9\sqrt{3}a^{8/3}b^{5/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(ah+2be)}{\sqrt[3]{b}}+ag+5bd\right)}{54a^{8/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(ag+5bd)-\sqrt[3]{a}(ah+2be)\right)}{27a^{8/3}b^{5/3}} + \frac{x(-3bx^2(3bc-af)+a(ag+5bd)+2ax(ah+2be))}{18a^3b\left(a+bx^3\right)} - \frac{c\log\left(a+bx^3\right)}{3a^3} + \frac{c\log(x)}{a^3} + \frac{x(-bx^2(bc-af)+a(bd-ag)+ax(be-ah))}{6a^2b\left(a+bx^3\right)^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x]
```

```
[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(6*a^2*b*(a + b*x
^3)^2) + (x*(a*(5*b*d + a*g) + 2*a*(2*b*e + a*h)*x - 3*b*(3*b*c - a*f)*x^2)
)/(18*a^3*b*(a + b*x^3)) - ((5*b^(4/3)*d + 2*a^(1/3)*b*e + a*b^(1/3)*g + a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8
/3)*b^(5/3)) + (c*Log[x])/a^3 + ((b^(1/3)*(5*b*d + a*g) - a^(1/3)*(2*b*e +
a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(5/3)) - ((5*b*d + a*g - (a^(
1/3)*(2*b*e + a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(54*a^(8/3)*b^(4/3)) - (c*Log[a + b*x^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
```

```
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^2c-b(5bd+ag)x-2b(2be+ah)x^2+3b^2\left(\frac{bc}{a}-f\right)x^3}{x\left(a+bx^3\right)^2}\,dx}{6ab^2} \\
&= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5bd+ag)+2a(2be+ah)x-3b(3bc-af)x^2)}{18a^3b\left(a+bx^3\right)} + \frac{\int \frac{18b^3c+2b^2(5bd+ag)x+2b^2(2be+ah)x^2}{x\left(a+bx^3\right)}\,dx}{18a^2b^3} \\
&= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5bd+ag)+2a(2be+ah)x-3b(3bc-af)x^2)}{18a^3b\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^3c}{ax}+\frac{2b^2\left(a(5bd+ag)+a(2be+ah)x-9b^2cx^2\right)}{a\left(a+bx^3\right)}\right)dx}{18a^2b^3} \\
&= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5bd+ag)+2a(2be+ah)x-3b(3bc-af)x^2)}{18a^3b\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} + \frac{\int \frac{a(5bd+ag)+a(2be+ah)x-9b^2cx^2}{a+bx^3}\,dx}{9a^3b} \\
&= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} + \frac{x(a(5bd+ag)+2a(2be+ah)x-3b(3bc-af)x^2)}{18a^3b\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} + \frac{\int \frac{a(5bd+ag)+a(2be+ah)x}{a+bx^3}\,dx}{9a^3b} - \frac{(bc)\int \frac{x^2}{a+bx^3}\,dx}{a^3}
\end{aligned}$$

$$=\frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2}+\frac{x(a(5bd+ag)+2a(2be+ah)x-3b(3bc-af)x^2)}{18a^3b\left(a+bx^3\right)}+\frac{c\log(x)}{a^3}-\frac{c\log\left(a+bx^3\right)}{3a^3}+\frac{\int\frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}(5bd+ag)+a^{4/3}(2be+ah)\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}(5bd+ag)+a^{4/3}(2be+ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{27a^{11/3}b^{4/3}}+\frac{\left(5bd+ag-\frac{\sqrt[3]{a}(2be+ah)}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{27a^{8/3}b}$$

$$=\frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2}+\frac{x(a(5bd+ag)+2a(2be+ah)x-3b(3bc-af)x^2)}{18a^3b\left(a+bx^3\right)}+\frac{c\log(x)}{a^3}+\frac{\left(5bd+ag-\frac{\sqrt[3]{a}(2be+ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}}-\frac{c\log\left(a+bx^3\right)}{3a^3}+\frac{\left(5b^{4/3}d+2\sqrt[3]{a}be+a\sqrt[3]{b}g+a^{4/3}h\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{18a^{7/3}b^{4/3}}-\frac{\left(5bd+ag-\frac{\sqrt[3]{a}(2be+ah)}{\sqrt[3]{b}}\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{54a^{8/3}b^{4/3}}$$

$$=\frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2}+\frac{x(a(5bd+ag)+2a(2be+ah)x-3b(3bc-af)x^2)}{18a^3b\left(a+bx^3\right)}+\frac{c\log(x)}{a^3}+\frac{\left(5bd+ag-\frac{\sqrt[3]{a}(2be+ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}}-\frac{\left(5bd+ag-\frac{\sqrt[3]{a}(2be+ah)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}}-\frac{c\log\left(a+bx^3\right)}{3a^3}+\frac{\left(5b^{4/3}d+2\sqrt[3]{a}be+a\sqrt[3]{b}g+a^{4/3}h\right)\text{Subst}\left(\int\frac{1}{-3-x^2}dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{8/3}b^{5/3}}$$

$$
\begin{aligned}
&= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b\left(a+bx^3\right)^2} \\
&\quad + \frac{x(a(5bd+ag)+2a(2be+ah)x-3b(3bc-af)x^2)}{18a^3b\left(a+bx^3\right)} \\
&\quad - \frac{\left(5b^{4/3}d+2\sqrt[3]{a}be+a\sqrt[3]{b}g+a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{5/3}} \\
&\quad + \frac{c\log(x)}{a^3} + \frac{\left(5bd+ag-\frac{\sqrt[3]{a}(2be+ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}} \\
&\quad - \frac{\left(5bd+ag-\frac{\sqrt[3]{a}(2be+ah)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}} - \frac{c\log\left(a+bx^3\right)}{3a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.90

$$
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx
$$

$$
= \frac{\frac{3a(6bc+bx(5d+4ex)+ax(g+2hx))}{b(a+bx^3)} - \frac{9a^2(-b(c+x(d+ex))+a(f+x(g+hx)))}{b(a+bx^3)^2} - \frac{2\sqrt{3}\sqrt[3]{a}\left(5b^{4/3}d+2\sqrt[3]{a}be+a\sqrt[3]{b}g+a^{4/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}} \cdots}{\cdots}
$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x]
```

```
[Out] ((3*a*(6*b*c + b*x*(5*d + 4*e*x) + a*x*(g + 2*h*x)))/(b*(a + b*x^3)) - (9*a
^2*(-(b*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(b*(a + b*x^3)^2) - (2*S
qrt[3]*a^(1/3)*(5*b^(4/3)*d + 2*a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcT
an[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(5/3) + 54*c*Log[x] + (2*a^(1/3)
*(5*b^(4/3)*d - 2*a^(1/3)*b*e + a*b^(1/3)*g - a^(4/3)*h)*Log[a^(1/3) + b^(1
/3)*x])/b^(5/3) + (a^(1/3)*(-5*b^(4/3)*d + 2*a^(1/3)*b*e - a*b^(1/3)*g + a^
(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - 18*c*Log
[a + b*x^3])/(54*a^3)
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.98

method	result
default	$\frac{c\ln(x)}{a^3}+\frac{\frac{\left(\frac{1}{9}a^2h+\frac{2}{9}aeb\right)x^5+\left(\frac{1}{18}a^2g+\frac{5}{18}abd\right)x^4+\frac{abc\,x^3}{3}-\frac{a^2(ah-7be)x^2}{18b}-\frac{a^2(ag-4bd)x}{9b}-\frac{a^2(af-3bc)}{6b}}{\left(b\,x^3+a\right)^2}+\frac{\left(a^2g+5abd\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\right.}{}\right.}{}}{}$
risch	$\frac{\frac{(ah+2be)x^5}{9a^2}+\frac{(ag+5bd)x^4}{18a^2}+\frac{bc\,x^3}{3a^2}-\frac{(ah-7be)x^2}{18ab}-\frac{(ag-4bd)x}{9ab}-\frac{af-3bc}{6ab}}{\left(b\,x^3+a\right)^2}+\frac{c\ln(-x)}{a^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^9b^5\_Z^3+27a^6b^5c\_Z^2+\left(3a^6b^2gh\right.\right.}\right.}{}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*ln(x)/a^3+1/a^3*(((1/9*a^2*h+2/9*a*e*b)*x^5+(1/18*a^2*g+5/18*a*b*d)*x^4+1
/3*a*b*c*x^3-1/18*a^2*(a*h-7*b*e)/b*x^2-1/9*a^2*(a*g-4*b*d)/b*x-1/6*a^2*(a*
f-3*b*c)/b)/(b*x^3+a)^2+1/9/b*((a^2*g+5*a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b
)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(a^2*h+2*a*b*e)*(-1/3/
b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))
-3*b*c*ln(b*x^3+a)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.99 (sec) , antiderivative size = 12815, normalized size of antiderivative = 36.93

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="frica
s")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.06

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx \\
&=\frac{6\,b^2cx^3+2\,(2\,b^2e+abh)x^5+(5\,b^2d+abg)x^4+9\,abc-3\,a^2f+(7\,abe-a^2h)x^2+2\,(4\,abd-a^2g)x}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)} \\
&\quad+\frac{c\log(x)}{a^3}+\frac{\sqrt{3}\left(2\,abe\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b} \\
&\quad-\frac{\left(18\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,abd+a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\
&\quad-\frac{\left(9\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,abd-a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}
\end{aligned}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxim
a")
[Out] 1/18*(6*b^2*c*x^3 + 2*(2*b^2*e + a*b*h)*x^5 + (5*b^2*d + a*b*g)*x^4 + 9*a*b
*c - 3*a^2*f + (7*a*b*e - a^2*h)*x^2 + 2*(4*a*b*d - a^2*g)*x)/(a^2*b^3*x^6
+ 2*a^3*b^2*x^3 + a^4*b) + c*log(x)/a^3 + 1/27*sqrt(3)*(2*a*b*e*(a/b)^(2/3)
 + a^2*h*(a/b)^(2/3) + 5*a*b*d*(a/b)^(1/3) + a^2*g*(a/b)^(1/3))*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b) - 1/54*(18*b^2*c*(a/b)^(2/
3) - 2*a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3) + 5*a*b*d + a^2*g)*log(x^2 - x
*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(2/3)) - 1/27*(9*b^2*c*(a/b)^(2/
3) + 2*a*b*e*(a/b)^(1/3) + a^2*h*(a/b)^(1/3) - 5*a*b*d - a^2*g)*log(x + (a/
b)^(1/3))/(a^3*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.07

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx = -\frac{c\log\left(|bx^3+a|\right)}{3\,a^3}+\frac{c\log\left(|x|\right)}{a^3}$$
$$-\frac{\sqrt{3}\left(5\,b^2d+abg-2\left(-ab^2\right)^{\frac{1}{3}}be-\left(-ab^2\right)^{\frac{1}{3}}ah\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^2b}$$
$$-\frac{\left(5\,b^2d+abg+2\left(-ab^2\right)^{\frac{1}{3}}be+\left(-ab^2\right)^{\frac{1}{3}}ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^2b}$$
$$+\frac{6\,ab^2cx^3+2\left(2\,ab^2e+a^2bh\right)x^5+\left(5\,ab^2d+a^2bg\right)x^4+9\,a^2bc-3\,a^3f+\left(7\,a^2be-a^3h\right)x^2+2\left(4\,a^2bd-a^3\right.}{18\left(bx^3+a\right)^2a^3b}$$
$$-\frac{\left(2\,a^4b^3e\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^5b^2h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^4b^3d+a^5b^2g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b^3}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac"
)
```

```
[Out] -1/3*c*log(abs(b*x^3 + a))/a^3 + c*log(abs(x))/a^3 - 1/27*sqrt(3)*(5*b^2*d
+ a*b*g - 2*(-a*b^2)^(1/3)*b*e - (-a*b^2)^(1/3)*a*h)*arctan(1/3*sqrt(3)*(2*
x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b) - 1/54*(5*b^2*d + a*
b*g + 2*(-a*b^2)^(1/3)*b*e + (-a*b^2)^(1/3)*a*h)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b) + 1/18*(6*a*b^2*c*x^3 + 2*(2*a*b^2*e
+ a^2*b*h)*x^5 + (5*a*b^2*d + a^2*b*g)*x^4 + 9*a^2*b*c - 3*a^3*f + (7*a^2*b
*e - a^3*h)*x^2 + 2*(4*a^2*b*d - a^3*g)*x)/((b*x^3 + a)^2*a^3*b) - 1/27*(2*
a^4*b^3*e*(-a/b)^(1/3) + a^5*b^2*h*(-a/b)^(1/3) + 5*a^4*b^3*d + a^5*b^2*g)*
(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 1716, normalized size of antiderivative = 4.95

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x)
```

```
[Out] ((3*b*c - a*f)/(6*a*b) + (x^4*(5*b*d + a*g))/(18*a^2) + (x^5*(2*b*e + a*h))
/(9*a^2) + (x*(4*b*d - a*g))/(9*a*b) + (x^2*(7*b*e - a*h))/(18*a*b) + (b*c*
```

x^3)/(3*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((c*(25*b^2*d^2 + a^2 *g^2 - 18*b^2*c*e - 9*a*b*c*h + 10*a*b*d*g))/(81*a^6) - (root(19683*a^9*b^5 *z^3 + 19683*a^6*b^5*c*z^2 + 81*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5 *b^3*e*g*z + 810*a^4*b^4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27* a^3*b^2*c*g*h + 135*a^2*b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a ^3*b^2*e^2*h - 75*a^2*b^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b* g^3 - 125*a*b^4*d^3 + 729*b^5*c^3 + a^5*h^3, z, k)*(a^3*g^2 + 25*a*b^2*d^2 + 324*b^3*c^2*x + 2916*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5*c*z^2 + 81*a^ 6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4*b^4*d*e*z + 6 561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135*a^2*b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^2*b^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3 + 729*b^5*c^3 + a^5*h^3, z, k)^2*a^6*b^3*x - 27*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5*c* z^2 + 81*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4*b^ 4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135*a^2 *b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^2*b ^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3 + 7 29*b^5*c^3 + a^5*h^3, z, k)*a^5*b*h + 36*a*b^2*c*e + 18*a^2*b*c*h + 10*a^2* b*d*g + 10*a^3*g*h*x - 54*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5*c*z^2 + 81 *a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4*b^4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135*a^2*b^3*c*d *h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^2*b^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3 + 729*b^5*c ^3 + a^5*h^3, z, k)*a^4*b^2*e + 1944*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5 *c*z^2 + 81*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4 *b^4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135* a^2*b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^ 2*b^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3 + 729*b^5*c^3 + a^5*h^3, z, k)*a^3*b^3*c*x + 100*a*b^2*d*e*x + 50*a^2*b*d*h *x + 20*a^2*b*e*g*x))/(81*a^4) - (x*(a^4*h^3 - 125*b^4*d^3 + 8*a*b^3*e^3 - a^3*b*g^3 - 15*a^2*b^2*d*g^2 + 12*a^2*b^2*e^2*h + 180*b^4*c*d*e - 75*a*b^3* d^2*g + 6*a^3*b*e*h^2 + 18*a^2*b^2*c*g*h + 90*a*b^3*c*d*h + 36*a*b^3*c*e*g))/(729*a^6*b^2))*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5*c*z^2 + 81*a^6*b^2* g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4*b^4*d*e*z + 6561*a^ 3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135*a^2*b^3*c*d*h + 54*a ^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^2*b^3*d^2*g - 15*a^3 *b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3 + 729*b^5*c^3 + a^5* h^3, z, k), k, 1, 3) + (c*log(x))/a^3

3.427 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 362

$$\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^3}\,dx
&= -\frac{c}{a^3x}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b(a+bx^3)^2}\\
&\quad+\frac{x(a(5be+ah)-2b(5bc-2af)x-3b(3bd-ag)x^2)}{18a^3b(a+bx^3)}\\
&\quad+\frac{(14b^{5/3}c-5a^{2/3}be-2ab^{2/3}f-a^{5/3}h)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{4/3}}\\
&\quad+\frac{d\log(x)}{a^3}+\frac{(2b^{2/3}(7bc-af)+a^{2/3}(5be+ah))\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{4/3}}\\
&\quad-\frac{(2b^{2/3}(7bc-af)+a^{2/3}(5be+ah))\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}b^{4/3}}-\frac{d\log(a+bx^3)}{3a^3}
\end{aligned}$$

```
[Out] -c/a^3/x+1/6*x*(a*(-a*h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/a^2/b/(b*x^3+
a)^2+1/18*x*(a*(a*h+5*b*e)-2*b*(-2*a*f+5*b*c)*x-3*b*(-a*g+3*b*d)*x^2)/a^3/b
/(b*x^3+a)+d*ln(x)/a^3+1/27*(2*b^(2/3)*(-a*f+7*b*c)+a^(2/3)*(a*h+5*b*e))*ln
(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(4/3)-1/54*(2*b^(2/3)*(-a*f+7*b*c)+a^(2/3)*(
a*h+5*b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(4/3)-1/3*
d*ln(b*x^3+a)/a^3+1/27*(14*b^(5/3)*c-5*a^(2/3)*b*e-2*a*b^(2/3)*f-a^(5/3)*h)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^3}\,dx$$
$$=\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^{2/3}be+a^{5/3}(-h)-2ab^{2/3}f+14b^{5/3}c\right)}{9\sqrt{3}a^{10/3}b^{4/3}}$$
$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{54a^{10/3}b^{4/3}}$$
$$+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{27a^{10/3}b^{4/3}}$$
$$+\frac{x(-2bx(5bc-2af)-3bx^2(3bd-ag)+a(ah+5be))}{18a^3b(a+bx^3)}-\frac{d\log(a+bx^3)}{3a^3}$$
$$-\frac{c}{a^3x}+\frac{d\log(x)}{a^3}+\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{6a^2b(a+bx^3)^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -(c/(a^3*x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6
*a^2*b*(a + b*x^3)^2) + (x*(a*(5*b*e + a*h) - 2*b*(5*b*c - 2*a*f)*x - 3*b*(
3*b*d - a*g)*x^2))/(18*a^3*b*(a + b*x^3)) + ((14*b^(5/3)*c - 5*a^(2/3)*b*e
- 2*a*b^(2/3)*f - a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(9*Sqrt[3]*a^(10/3)*b^(4/3)) + (d*Log[x])/a^3 + ((2*b^(2/3)*(7*b*c - a
*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(4/3)
) - ((2*b^(2/3)*(7*b*c - a*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a
^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
```

```
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\left(a+bx^3\right)^2} \\
&\quad -\frac{\int \frac{-6b^2c-6b^2dx-b(5be+ah)x^2+4b^2\left(\frac{bc}{a}-f\right)x^3+3b^2\left(\frac{bd}{a}-g\right)x^4}{x^2\left(a+bx^3\right)^2}\,dx}{6ab^2} \\
&= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\left(a+bx^3\right)^2} \\
&\quad +\frac{x(a(5be+ah)-2b(5bc-2af)x-3b(3bd-ag)x^2)}{18a^3b\left(a+bx^3\right)} \\
&\quad +\frac{\int \frac{18b^4c+18b^4dx+2b^3(5be+ah)x^2-2b^4\left(\frac{5bc}{a}-2f\right)x^3}{x^2\left(a+bx^3\right)}\,dx}{18a^2b^4} \\
&= \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\left(a+bx^3\right)^2} \\
&\quad +\frac{x(a(5be+ah)-2b(5bc-2af)x-3b(3bd-ag)x^2)}{18a^3b\left(a+bx^3\right)} \\
&\quad +\frac{\int \left(\frac{18b^4c}{ax^2}+\frac{18b^4d}{ax}+\frac{2b^3\left(a(5be+ah)-2b(7bc-af)x-9b^2dx^2\right)}{a\left(a+bx^3\right)}\right)dx}{18a^2b^4} \\
&= -\frac{c}{a^3x}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\left(a+bx^3\right)^2} \\
&\quad +\frac{x(a(5be+ah)-2b(5bc-2af)x-3b(3bd-ag)x^2)}{18a^3b\left(a+bx^3\right)} \\
&\quad +\frac{d\log(x)}{a^3}+\frac{\int \frac{a(5be+ah)-2b(7bc-af)x-9b^2dx^2}{a+bx^3}\,dx}{9a^3b} \\
&= -\frac{c}{a^3x}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\left(a+bx^3\right)^2} \\
&\quad +\frac{x(a(5be+ah)-2b(5bc-2af)x-3b(3bd-ag)x^2)}{18a^3b\left(a+bx^3\right)} \\
&\quad +\frac{d\log(x)}{a^3}+\frac{\int \frac{a(5be+ah)-2b(7bc-af)x}{a+bx^3}\,dx}{9a^3b}-\frac{(bd)\int \frac{x^2}{a+bx^3}\,dx}{a^3}
\end{aligned}
$$

$$= -\frac{c}{a^3x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\,(a+bx^3)^2} + \frac{x(a(5be+ah)-2b(5bc-2af)x-3b(3bd-ag)x^2)}{18a^3b\,(a+bx^3)} + \frac{d\log(x)}{a^3} - \frac{d\log\left(a+bx^3\right)}{3a^3} + \frac{\int \frac{\sqrt[3]{a}\left(-2\sqrt[3]{a}b(7bc-af)+2a\sqrt[3]{b}(5be+ah)\right)+\sqrt[3]{b}\left(-2\sqrt[3]{a}b(7bc-af)-a\sqrt[3]{b}(5be+ah)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{11/3}b^{4/3}} + \frac{\left(2b^{2/3}(7bc-af)+a^{2/3}(5be+ah)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{10/3}b}$$

$$= -\frac{c}{a^3x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\,(a+bx^3)^2} + \frac{x(a(5be+ah)-2b(5bc-2af)x-3b(3bd-ag)x^2)}{18a^3b\,(a+bx^3)} + \frac{d\log(x)}{a^3} + \frac{\left(2b^{2/3}(7bc-af)+a^{2/3}(5be+ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{4/3}} - \frac{d\log\left(a+bx^3\right)}{3a^3} - \frac{\left(14b^{5/3}c-5a^{2/3}be-2ab^{2/3}f-a^{5/3}h\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^3b} - \frac{\left(2b^{2/3}(7bc-af)+a^{2/3}(5be+ah)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{10/3}b^{4/3}}$$

$$= -\frac{c}{a^3x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\,(a+bx^3)^2} + \frac{x(a(5be+ah)-2b(5bc-2af)x-3b(3bd-ag)x^2)}{18a^3b\,(a+bx^3)} + \frac{d\log(x)}{a^3} + \frac{\left(2b^{2/3}(7bc-af)+a^{2/3}(5be+ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{4/3}} - \frac{\left(2b^{2/3}(7bc-af)+a^{2/3}(5be+ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}b^{4/3}} - \frac{d\log\left(a+bx^3\right)}{3a^3} - \frac{\left(14b^{5/3}c-5a^{2/3}be-2ab^{2/3}f-a^{5/3}h\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{10/3}b^{4/3}}$$

$$
\begin{aligned}
&= -\frac{c}{a^3x} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b\,(a+bx^3)^2} \\
&\quad + \frac{x(a(5be+ah)-2b(5bc-2af)x-3b(3bd-ag)x^2)}{18a^3b\,(a+bx^3)} \\
&\quad + \frac{\left(14b^{5/3}c-5a^{2/3}be-2ab^{2/3}f-a^{5/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{4/3}} \\
&\quad + \frac{d\log(x)}{a^3} + \frac{\left(2b^{2/3}(7bc-af)+a^{2/3}(5be+ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{4/3}} \\
&\quad - \frac{\left(2b^{2/3}(7bc-af)+a^{2/3}(5be+ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}b^{4/3}} \\
&\quad - \frac{d\log\left(a+bx^3\right)}{3a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.93

$$
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\,(a+bx^3)^3}\,dx =
$$

$$
-\frac{\frac{54ac}{x} + \frac{9a^2(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx)))}{b(a+bx^3)^2} - \frac{3a(a^2hx-10b^2cx^2+ab(6d+x(5e+4fx)))}{b(a+bx^3)} + \frac{2\sqrt{3}a^{2/3}(-14b^{5/3}c+5a^{2/3}be+2ab^{2/3}f+a^{5}}{b^{4/3}}}{}
$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -1/54*((54*a*c)/x + (9*a^2*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x
))))/(b*(a + b*x^3)^2) - (3*a*(a^2*h*x - 10*b^2*c*x^2 + a*b*(6*d + x*(5*e +
 4*f*x))))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(2/3)*(-14*b^(5/3)*c + 5*a^(2/3)*
b*e + 2*a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
])/b^(4/3) - 54*a*d*Log[x] - (2*a^(2/3)*(14*b^(5/3)*c + 5*a^(2/3)*b*e - 2*a
*b^(2/3)*f + a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) + (a^(2/3)*(14*b^
(5/3)*c + 5*a^(2/3)*b*e - 2*a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/b^(4/3) + 18*a*d*Log[a + b*x^3])/a^4
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.95

method	result
default	$-\frac{c}{a^{3}x}+\frac{d\ln\left(x\right)}{a^{3}}+\frac{\frac{\left(\frac{2}{9}afb-\frac{5}{9}b^{2}c\right)x^{5}+\left(\frac{1}{18}a^{2}h+\frac{5}{18}aeb\right)x^{4}+\frac{x^{3}abd}{3}+\frac{a(7af-13bc)x^{2}}{18}-\frac{a^{2}(ah-4be)x}{9b}-\frac{a^{2}(ag-3bd)}{6b}}{\left(bx^{3}+a\right)^{2}}+\frac{\left(a^{2}h+5aeb\right)\left(\frac{\ln\left(x+\left(\cdots\right.\right.}{3b\left(\frac{a}{b}\cdots\right.}\cdots\right.}{\cdots}$
risch	$\frac{\frac{2b(af-7bc)x^{6}}{9a^{3}}+\frac{(ah+5be)x^{5}}{18a^{2}}+\frac{bd\,x^{4}}{3a^{2}}+\frac{7(af-7bc)x^{3}}{18a^{2}}-\frac{(ah-4be)x^{2}}{9ab}-\frac{(ag-3bd)x}{6ab}-\frac{c}{a}}{x\left(bx^{3}+a\right)^{2}}+\frac{d\ln\left(x\right)}{a^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}b^{4}\_Z^{3}+27a^{7}b^{4}d\_Z^{2}\cdots\right.}\cdots\right.}{\cdots}$

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x,method=_RETURNVERBOSE
)

[Out] -c/a^3/x+d*ln(x)/a^3+1/a^3*(((2/9*a*f*b-5/9*b^2*c)*x^5+(1/18*a^2*h+5/18*a*e
*b)*x^4+1/3*x^3*a*b*d+1/18*a*(7*a*f-13*b*c)*x^2-1/9*a^2*(a*h-4*b*e)/b*x-1/6
a^2(a*g-3*b*d)/b)/(b*x^3+a)^2+1/9/b*((a^2*h+5*a*b*e)*(1/3/b/(a/b)^(2/3)*l
n(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/
(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(2*a*b*f-14*b^
2*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1)))-3*b*d*ln(b*x^3+a)))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 22.72 (sec) , antiderivative size = 12951, normalized size of antiderivative = 35.78

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="fri
cas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.10

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}\,dx$$
$$= \frac{6\,ab^2dx^4-4\,(7\,b^3c-ab^2f)x^6+(5\,ab^2e+a^2bh)x^5-18\,a^2bc-7\,(7\,ab^2c-a^2bf)x^3+2\,(4\,a^2be-a^3h)x^2+3}{18\,(a^3b^3x^7+2\,a^4b^2x^4+a^5bx)}$$
$$+\frac{d\log(x)}{a^3}$$
$$-\frac{\sqrt{3}\Big(14\,b^2c\big(\frac{a}{b}\big)^{\frac{2}{3}}-2\,abf\big(\frac{a}{b}\big)^{\frac{2}{3}}-5\,abe\big(\frac{a}{b}\big)^{\frac{1}{3}}-a^2h\big(\frac{a}{b}\big)^{\frac{1}{3}}\Big)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b}$$
$$-\frac{\Big(18\,b^2d\big(\frac{a}{b}\big)^{\frac{2}{3}}+14\,b^2c\big(\frac{a}{b}\big)^{\frac{1}{3}}-2\,abf\big(\frac{a}{b}\big)^{\frac{1}{3}}+5\,abe+a^2h\Big)\log\Big(x^2-x\big(\frac{a}{b}\big)^{\frac{1}{3}}+\big(\frac{a}{b}\big)^{\frac{2}{3}}\Big)}{54\,a^3b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\Big(9\,b^2d\big(\frac{a}{b}\big)^{\frac{2}{3}}-14\,b^2c\big(\frac{a}{b}\big)^{\frac{1}{3}}+2\,abf\big(\frac{a}{b}\big)^{\frac{1}{3}}-5\,abe-a^2h\Big)\log\Big(x+\big(\frac{a}{b}\big)^{\frac{1}{3}}\Big)}{27\,a^3b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="max
ima")
[Out] 1/18*(6*a*b^2*d*x^4 - 4*(7*b^3*c - a*b^2*f)*x^6 + (5*a*b^2*e + a^2*b*h)*x^5
 - 18*a^2*b*c - 7*(7*a*b^2*c - a^2*b*f)*x^3 + 2*(4*a^2*b*e - a^3*h)*x^2 + 3
*(3*a^2*b*d - a^3*g)*x)/(a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^5*b*x) + d*log(x)/
a^3 - 1/27*sqrt(3)*(14*b^2*c*(a/b)^(2/3) - 2*a*b*f*(a/b)^(2/3) - 5*a*b*e*(a
/b)^(1/3) - a^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)
^(1/3))/(a^4*b) - 1/54*(18*b^2*d*(a/b)^(2/3) + 14*b^2*c*(a/b)^(1/3) - 2*a*b
*f*(a/b)^(1/3) + 5*a*b*e + a^2*h)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
^3*b^2*(a/b)^(2/3)) - 1/27*(9*b^2*d*(a/b)^(2/3) - 14*b^2*c*(a/b)^(1/3) + 2*
a*b*f*(a/b)^(1/3) - 5*a*b*e - a^2*h)*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(2
/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.07

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}\,dx = -\frac{d\log\left(|bx^3+a|\right)}{3\,a^3} + \frac{d\log\left(|x|\right)}{a^3} - \frac{\sqrt{3}\left(5\,abe+a^2h+14\,(-ab^2)^{\frac{1}{3}}\,bc-2\,(-ab^2)^{\frac{1}{3}}\,af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}\,a^3} - \frac{\left(5\,abe+a^2h-14\,(-ab^2)^{\frac{1}{3}}\,bc+2\,(-ab^2)^{\frac{1}{3}}\,af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{2}{3}}\,a^3} + \frac{6\,ab^2dx^4-4\,(7\,b^3c-ab^2f)x^6+(5\,ab^2e+a^2bh)x^5-18\,a^2bc-7\,(7\,ab^2c-a^2bf)x^3+2\,(4\,a^2be-a^3h)x^2}{18\,(bx^3+a)^2a^3bx} + \frac{\left(14\,a^3b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^4b^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^4b^3e-a^5b^2h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b^3}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^3 + d*log(abs(x))/a^3 - 1/27*sqrt(3)*(5*a*b*e
+ a^2*h + 14*(-a*b^2)^(1/3)*b*c - 2*(-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/54*(5*a*b*e + a
^2*h - 14*(-a*b^2)^(1/3)*b*c + 2*(-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) + 1/18*(6*a*b^2*d*x^4 - 4*(7*b^3*c
- a*b^2*f)*x^6 + (5*a*b^2*e + a^2*b*h)*x^5 - 18*a^2*b*c - 7*(7*a*b^2*c - a^
2*b*f)*x^3 + 2*(4*a^2*b*e - a^3*h)*x^2 + 3*(3*a^2*b*d - a^3*g)*x)/((b*x^3 +
 a)^2*a^3*b*x) + 1/27*(14*a^3*b^4*c*(-a/b)^(1/3) - 2*a^4*b^3*f*(-a/b)^(1/3)
 - 5*a^4*b^3*e - a^5*b^2*h)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b^
3)
```

Mupad [B] (verification not implemented)

Time = 9.87 (sec) , antiderivative size = 1747, normalized size of antiderivative = 4.83

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x)
```

```
[Out] symsum(log((d*(a^3*h^2 + 25*a*b^2*e^2 + 126*b^3*c*d - 18*a*b^2*d*f + 10*a^2
*b*e*h))/(81*a^7) - (root(19683*a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^
```

6*b^2*f*h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*z + 6561*a^4*b^4*d^2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d *h + 270*a^2*b^3*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2 *h - 168*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 - a^5*h^3 - 2744*b^5*c^3, z, k)*(a^3*h^2 + 25*a*b^2*e^2 + 324*b^3*d^2*x - 252 *b^3*c*d + 2916*root(19683*a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^6*b^2 *f*h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*z + 6561 *a^4*b^4*d^2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d*h + 270*a^2*b^3*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2*h - 168*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 - a^5*h ^3 - 2744*b^5*c^3, z, k)^2*a^6*b^3*x + 36*a*b^2*d*f + 10*a^2*b*e*h - 700*b^ 3*c*e*x + 378*root(19683*a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^6*b^2*f *h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*z + 6561*a ^4*b^4*d^2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d*h + 27 0*a^2*b^3*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2*h - 16 8*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 - a^5*h^3 - 2744*b^5*c^3, z, k)*a^3*b^3*c - 54*root(19683*a^10*b^4*z^3 + 19683*a^7*b ^4*d*z^2 + 162*a^6*b^2*f*h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 567 0*a^4*b^4*c*e*z + 6561*a^4*b^4*d^2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d*h + 270*a^2*b^3*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2*h - 168*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 - a^5*h^3 - 2744*b^5*c^3, z, k)*a^4*b^2*f + 1944*root(19683* a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^6*b^2*f*h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*z + 6561*a^4*b^4*d^2*z - 1890*a*b^4 *c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d*h + 270*a^2*b^3*d*e*f - 15*a^4* b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2*h - 168*a^2*b^3*c*f^2 + 8*a^3*b ^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 - a^5*h^3 - 2744*b^5*c^3, z, k)*a^ 3*b^3*d*x - 140*a*b^2*c*h*x + 100*a*b^2*e*f*x + 20*a^2*b*f*h*x))/(81*a^4) + (x*(2744*b^5*c^3 + a^5*h^3 + 125*a^2*b^3*e^3 - 8*a^3*b^2*f^3 + 168*a^2*b^3 *c*f^2 + 75*a^3*b^2*e^2*h - 1176*a*b^4*c^2*f + 15*a^4*b*e*h^2 + 252*a^2*b^3 *c*d*h - 180*a^2*b^3*d*e*f - 36*a^3*b^2*d*f*h + 1260*a*b^4*c*d*e))/(729*a^8 *b))*root(19683*a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^6*b^2*f*h*z - 11 34*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*z + 6561*a^4*b^4*d^ 2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d*h + 270*a^2*b^3 *d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2*h - 168*a^2*b^3 *c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 - a^5*h^3 - 2744*b ^5*c^3, z, k), k, 1, 3) + ((x^5*(5*b*e + a*h))/(18*a^2) - (7*x^3*(7*b*c - a *f))/(18*a^2) - c/a - (2*b*x^6*(7*b*c - a*f))/(9*a^3) + (x*(3*b*d - a*g))/(6*a*b) + (x^2*(4*b*e - a*h))/(9*a*b) + (b*d*x^4)/(3*a^2))/(a^2*x + b^2*x^7 + 2*a*b*x^4) + (d*log(x))/a^3

3.428 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 360

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^3}\,dx \\
&= -\frac{c}{2a^3x^2}-\frac{d}{a^3x}-\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2(a+bx^3)^2} \\
&\quad -\frac{x(11bc-5af+2(5bd-2ag)x+3(3be-ah)x^2)}{18a^3(a+bx^3)} \\
&\quad +\frac{\left(20b^{4/3}c+14\sqrt[3]{a}bd-5a\sqrt[3]{b}f-2a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}b^{2/3}} \\
&\quad +\frac{e\log(x)}{a^3}-\frac{\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}b^{2/3}} \\
&\quad +\frac{\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}b^{2/3}}-\frac{e\log(a+bx^3)}{3a^3}
\end{aligned}$$

```
[Out] -1/2*c/a^3/x^2-d/a^3/x-1/6*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*x^2)/a^2/(b*x
^3+a)^2-1/18*x*(11*b*c-5*a*f+2*(-2*a*g+5*b*d)*x+3*(-a*h+3*b*e)*x^2)/a^3/(b*
x^3+a)+e*ln(x)/a^3-1/27*(5*b^(1/3)*(-a*f+4*b*c)-2*a^(1/3)*(-a*g+7*b*d))*ln(
a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(2/3)+1/54*(5*b^(1/3)*(-a*f+4*b*c)-2*a^(1/3)*
(-a*g+7*b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(2/3)-1/
3*e*ln(b*x^3+a)/a^3+1/27*(20*b^(4/3)*c+14*a^(1/3)*b*d-5*a*b^(1/3)*f-2*a^(4/
3)*g)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(2/3)*3^
(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^3}\,dx$$
$$= \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{4/3}g+14\sqrt[3]{a}bd-5a\sqrt[3]{b}f+20b^{4/3}c\right)}{9\sqrt{3}a^{11/3}b^{2/3}}$$
$$+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{2\sqrt[3]{a}(7bd-ag)}{\sqrt[3]{b}}-5af+20bc\right)}{54a^{11/3}\sqrt[3]{b}}$$
$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)}{27a^{11/3}b^{2/3}}$$
$$-\frac{x(2x(5bd-2ag)+3x^2(3be-ah)-5af+11bc)}{18a^3\left(a+bx^3\right)}-\frac{e\log\left(a+bx^3\right)}{3a^3}$$
$$-\frac{c}{2a^3x^2}-\frac{d}{a^3x}+\frac{e\log(x)}{a^3}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6a^2\left(a+bx^3\right)^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -1/2*c/(a^3*x^2) - d/(a^3*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*
x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*c - 5*a*f + 2*(5*b*d - 2*a*g)*x + 3*
(3*b*e - a*h)*x^2))/(18*a^3*(a + b*x^3)) + ((20*b^(4/3)*c + 14*a^(1/3)*b*d
- 5*a*b^(1/3)*f - 2*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(9*Sqrt[3]*a^(11/3)*b^(2/3)) + (e*Log[x])/a^3 - ((5*b^(1/3)*(4*b*c -
 a*f) - 2*a^(1/3)*(7*b*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(
2/3)) + ((20*b*c - 5*a*f - (2*a^(1/3)*(7*b*d - a*g))/b^(1/3))*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(1/3)) - (e*Log[a + b*x^3
])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
```

```
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2(a+bx^3)^2} - \frac{\int \frac{-6b^2c-6b^2dx-6b^2ex^2+5b^2\left(\frac{bc}{a}-f\right)x^3+4b^2\left(\frac{bd}{a}-g\right)x^4+3b^2\left(\frac{be}{a}-h\right)x^5}{x^3(a+bx^3)^2}\,dx}{6ab^2}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2(a+bx^3)^2} - \frac{x(11bc-5af+2(5bd-2ag)x+3(3be-ah)x^2)}{18a^3(a+bx^3)} + \frac{\int \frac{18b^4c+18b^4dx+18b^4ex^2-2b^4\left(\frac{11bc}{a}-5f\right)x^3-2b^4\left(\frac{5bd}{a}-2g\right)x^4}{x^3(a+bx^3)}\,dx}{18a^2b^4}$$

$$= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2(a+bx^3)^2} - \frac{x(11bc-5af+2(5bd-2ag)x+3(3be-ah)x^2)}{18a^3(a+bx^3)} + \frac{\int \left(\frac{18b^4c}{ax^3}+\frac{18b^4d}{ax^2}+\frac{18b^4e}{ax}+\frac{2b^4(-5(4bc-af)-2(7bd-ag)x-9bex^2)}{a(a+bx^3)}\right)dx}{18a^2b^4}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2(a+bx^3)^2} - \frac{x(11bc-5af+2(5bd-2ag)x+3(3be-ah)x^2)}{18a^3(a+bx^3)} + \frac{e\log(x)}{a^3} + \frac{\int \frac{-5(4bc-af)-2(7bd-ag)x-9bex^2}{a+bx^3}\,dx}{9a^3}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2(a+bx^3)^2} - \frac{x(11bc-5af+2(5bd-2ag)x+3(3be-ah)x^2)}{18a^3(a+bx^3)} + \frac{e\log(x)}{a^3} + \frac{\int \frac{-5(4bc-af)-2(7bd-ag)x}{a+bx^3}\,dx}{9a^3} - \frac{(be)\int \frac{x^2}{a+bx^3}\,dx}{a^3}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2(a+bx^3)^2} - \frac{x(11bc-5af+2(5bd-2ag)x+3(3be-ah)x^2)}{18a^3(a+bx^3)} + \frac{e\log(x)}{a^3} - \frac{e\log(a+bx^3)}{3a^3} + \frac{\int \frac{\sqrt[3]{a}\left(-10\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)+\sqrt[3]{b}\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{27a^{11/3}\sqrt[3]{b}} - \frac{\left(20bc-5af-\frac{2\sqrt[3]{a}(7bd-ag)}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{27a^{11/3}}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2(a+bx^3)^2} - \frac{x(11bc-5af+2(5bd-2ag)x+3(3be-ah)x^2)}{18a^3(a+bx^3)} + \frac{e\log(x)}{a^3} - \frac{\left(20bc-5af-\frac{2\sqrt[3]{a}(7bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}} - \frac{e\log(a+bx^3)}{3a^3} - \frac{\left(20b^{4/3}c+14\sqrt[3]{a}bd-5a\sqrt[3]{b}f-2a^{4/3}g\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{18a^{10/3}\sqrt[3]{b}} + \frac{\left(20bc-5af-\frac{2\sqrt[3]{a}(7bd-ag)}{\sqrt[3]{b}}\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{54a^{11/3}\sqrt[3]{b}}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2(a+bx^3)^2} - \frac{x(11bc-5af+2(5bd-2ag)x+3(3be-ah)x^2)}{18a^3(a+bx^3)} + \frac{e\log(x)}{a^3} - \frac{\left(20bc-5af-\frac{2\sqrt[3]{a}(7bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}} + \frac{\left(20bc-5af-\frac{2\sqrt[3]{a}(7bd-ag)}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}} - \frac{e\log(a+bx^3)}{3a^3} - \frac{\left(20b^{4/3}c+14\sqrt[3]{a}bd-5a\sqrt[3]{b}f-2a^{4/3}g\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{11/3}b^{2/3}}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6a^2(a + bx^3)^2} - \frac{x(11bc - 5af + 2(5bd - 2ag)x + 3(3be - ah)x^2)}{18a^3(a + bx^3)} + \frac{\left(20b^{4/3}c + 14\sqrt[3]{a}bd - 5a\sqrt[3]{b}f - 2a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}b^{2/3}} + \frac{e\log(x)}{a^3} - \frac{\left(20bc - 5af - \frac{2\sqrt[3]{a}(7bd - ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}} + \frac{\left(20bc - 5af - \frac{2\sqrt[3]{a}(7bd - ag)}{\sqrt[3]{b}}\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}} - \frac{e\log(a + bx^3)}{3a^3}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.94

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^3(a + bx^3)^3}\,dx =$$

$$-\frac{\frac{27ac}{x^2} + \frac{54ad}{x} - \frac{3a(6ae - bx(11c + 10dx) + ax(5f + 4gx))}{a + bx^3} + \frac{9a^2(a^2h + b^2x(c + dx) - ab(e + x(f + gx)))}{b(a + bx^3)^2} + \frac{2\sqrt{3}\sqrt[3]{a}\left(-20b^{4/3}c - 14\sqrt[3]{a}bd + 5a\cdots\right.}{b^2\cdots}}{\cdots}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -1/54*((27*a*c)/x^2 + (54*a*d)/x - (3*a*(6*a*e - b*x*(11*c + 10*d*x) + a*x*
(5*f + 4*g*x)))/(a + b*x^3) + (9*a^2*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*
(f + g*x))))/(b*(a + b*x^3)^2) + (2*Sqrt[3]*a^(1/3)*(-20*b^(4/3)*c - 14*a^(
1/3)*b*d + 5*a*b^(1/3)*f + 2*a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(2/3) - 54*a*e*Log[x] + (2*a^(1/3)*(20*b^(4/3)*c - 14*a^(1/3)*b
*d - 5*a*b^(1/3)*f + 2*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^(1
/3)*(20*b^(4/3)*c - 14*a^(1/3)*b*d - 5*a*b^(1/3)*f + 2*a^(4/3)*g)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 18*a*e*Log[a + b*x^3])/a^4
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.94

method	result
default	$-\frac{c}{2a^3x^2}-\frac{d}{a^3x}+\frac{e\ln(x)}{a^3}+\frac{\frac{\left(\frac{2}{9}abg-\frac{5}{9}b^2d\right)x^5+\left(\frac{5}{18}afb-\frac{11}{18}b^2c\right)x^4+\frac{abe\,x^3}{3}+\frac{a(7ag-13bd)x^2}{18}+\frac{a(4af-7bc)x}{9}-\frac{a^2(ah-3be)}{6b}}{\left(b\,x^3+a\right)^2}+\frac{(5af-20bc)}{}}{}$
risch	$\frac{\frac{2b(ag-7bd)x^7}{9a^3}+\frac{5b(af-4bc)x^6}{18a^3}+\frac{be\,x^5}{3a^2}+\frac{7(ag-7bd)x^4}{18a^2}+\frac{4(af-4bc)x^3}{9a^2}-\frac{(ah-3be)x^2}{6ab}-\frac{xd}{a}-\frac{c}{2a}}{x^2\left(b\,x^3+a\right)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}b^2\_Z^3+27a^8b^2e\_Z^2+\left(\right.\right.}\right.}{}$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/2*c/a^3/x^2-d/a^3/x+e*ln(x)/a^3+1/a^3*(((2/9*a*b*g-5/9*b^2*d)*x^5+(5/18*
a*f*b-11/18*b^2*c)*x^4+1/3*a*b*e*x^3+1/18*a*(7*a*g-13*b*d)*x^2+1/9*a*(4*a*f
-7*b*c)*x-1/6*a^2*(a*h-3*b*e)/b)/(b*x^3+a)^2+1/9*(5*a*f-20*b*c)*(1/3/b/(a/b
)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/9*(
2*a*g-14*b*d)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1)))-1/3*e*ln(b*x^3+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.28 (sec) , antiderivative size = 12435, normalized size of antiderivative = 34.54

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.08

$$\begin{aligned}&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^3}\,dx\\&=\frac{6\,ab^2ex^5-4\,(7\,b^3d-ab^2g)x^7-5\,(4\,b^3c-ab^2f)x^6-18\,a^2bdx-7\,(7\,ab^2d-a^2bg)x^4-9\,a^2bc-8\,(4\,ab^2c-a}{18\,(a^3b^3x^8+2\,a^4b^2x^5+a^5bx^2)}\\&\quad+\frac{e\log\left(x\right)}{a^3}-\frac{\sqrt{3}\left(14\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,ag\left(\frac{a}{b}\right)^{\frac{2}{3}}+20\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,af\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4}\\&\quad-\frac{\left(18\,be\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,bc+5\,af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\&\quad-\frac{\left(9\,be\left(\frac{a}{b}\right)^{\frac{2}{3}}-14\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,bc-5\,af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="max
ima")
[Out] 1/18*(6*a*b^2*e*x^5 - 4*(7*b^3*d - a*b^2*g)*x^7 - 5*(4*b^3*c - a*b^2*f)*x^6
 - 18*a^2*b*d*x - 7*(7*a*b^2*d - a^2*b*g)*x^4 - 9*a^2*b*c - 8*(4*a*b^2*c -
a^2*b*f)*x^3 + 3*(3*a^2*b*e - a^3*h)*x^2)/(a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^
5*b*x^2) + e*log(x)/a^3 - 1/27*sqrt(3)*(14*b*d*(a/b)^(2/3) - 2*a*g*(a/b)^(2
/3) + 20*b*c*(a/b)^(1/3) - 5*a*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/
b)^(1/3))/(a/b)^(1/3))/a^4 - 1/54*(18*b*e*(a/b)^(2/3) + 14*b*d*(a/b)^(1/3)
- 2*a*g*(a/b)^(1/3) - 20*b*c + 5*a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(a^3*b*(a/b)^(2/3)) - 1/27*(9*b*e*(a/b)^(2/3) - 14*b*d*(a/b)^(1/3) + 2*a*
g*(a/b)^(1/3) + 20*b*c - 5*a*f)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.10

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^3}\,dx = -\frac{e\log\left(|bx^3+a|\right)}{3\,a^3}+\frac{e\log\left(|x|\right)}{a^3}+\frac{\sqrt{3}\left(20\,b^2c-5\,abf-14\,(-ab^2)^{\frac{1}{3}}\,bd+2\,(-ab^2)^{\frac{1}{3}}\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^3}+\frac{\left(20\,b^2c-5\,abf+14\,(-ab^2)^{\frac{1}{3}}\,bd-2\,(-ab^2)^{\frac{1}{3}}\,ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^3}-\frac{28\,b^3dx^7-4\,ab^2gx^7+20\,b^3cx^6-5\,ab^2fx^6-6\,ab^2ex^5+49\,ab^2dx^4-7\,a^2bgx^4+32\,ab^2cx^3-8\,a^2bfx^3-}{18\,(bx^4+ax)^2a^3b}+\frac{\left(14\,a^3b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^4bg\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,a^3b^2c-5\,a^4bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^3 + e*log(abs(x))/a^3 + 1/27*sqrt(3)*(20*b^2*c
 - 5*a*b*f - 14*(-a*b^2)^(1/3)*b*d + 2*(-a*b^2)^(1/3)*a*g)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) + 1/54*(20*b^2*c
 - 5*a*b*f + 14*(-a*b^2)^(1/3)*b*d - 2*(-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/
b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 1/18*(28*b^3*d*x^7 - 4*a*b^
2*g*x^7 + 20*b^3*c*x^6 - 5*a*b^2*f*x^6 - 6*a*b^2*e*x^5 + 49*a*b^2*d*x^4 - 7
*a^2*b*g*x^4 + 32*a*b^2*c*x^3 - 8*a^2*b*f*x^3 - 9*a^2*b*e*x^2 + 3*a^3*h*x^2
 + 18*a^2*b*d*x + 9*a^2*b*c)/((b*x^4 + a*x)^2*a^3*b) + 1/27*(14*a^3*b^2*d*(
-a/b)^(1/3) - 2*a^4*b*g*(-a/b)^(1/3) + 20*a^3*b^2*c - 5*a^4*b*f)*(-a/b)^(1/
3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 1697, normalized size of antiderivative = 4.71

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x)
```

```
[Out] symsum(log((b^2*e*(400*b^2*c^2 + 25*a^2*f^2 - 18*a^2*e*g - 200*a*b*c*f + 12
6*a*b*d*e))/(81*a^8) - (root(19683*a^11*b^2*z^3 + 19683*a^8*b^2*e*z^2 + 810
```

*a^6*b*f*g*z - 5670*a^5*b^2*d*f*z - 3240*a^5*b^2*c*g*z + 22680*a^4*b^3*c*d*
z + 6561*a^5*b^2*e^2*z + 270*a^3*b*e*f*g + 7560*a*b^3*c*d*e - 1890*a^2*b^2*
d*e*f - 1080*a^2*b^2*c*e*g - 168*a^3*b*d*g^2 - 6000*a*b^3*c^2*f + 1176*a^2*
b^2*d^2*g + 1500*a^2*b^2*c*f^2 + 729*a^2*b^2*e^3 - 125*a^3*b*f^3 - 2744*a*b
^3*d^3 + 8*a^4*g^3 + 8000*b^4*c^3, z, k)*b^2*(400*b^2*c^2 + 25*a^2*f^2 - 54
*root(19683*a^11*b^2*z^3 + 19683*a^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5
*b^2*d*f*z - 3240*a^5*b^2*c*g*z + 22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z
+ 270*a^3*b*e*f*g + 7560*a*b^3*c*d*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*
e*g - 168*a^3*b*d*g^2 - 6000*a*b^3*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^
2*c*f^2 + 729*a^2*b^2*e^3 - 125*a^3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 80
00*b^4*c^3, z, k)*a^5*g + 36*a^2*e*g + 378*root(19683*a^11*b^2*z^3 + 19683*
a^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5*b^2*d*f*z - 3240*a^5*b^2*c*g*z +
22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z + 270*a^3*b*e*f*g + 7560*a*b^3*c*
d*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*e*g - 168*a^3*b*d*g^2 - 6000*a*b^
3*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^2*c*f^2 + 729*a^2*b^2*e^3 - 125*a
^3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 8000*b^4*c^3, z, k)*a^4*b*d + 324*a
*b*e^2*x + 2800*b^2*c*d*x + 100*a^2*f*g*x + 2916*root(19683*a^11*b^2*z^3 +
19683*a^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5*b^2*d*f*z - 3240*a^5*b^2*c
*g*z + 22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z + 270*a^3*b*e*f*g + 7560*a*
b^3*c*d*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*e*g - 168*a^3*b*d*g^2 - 600
0*a*b^3*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^2*c*f^2 + 729*a^2*b^2*e^3 -
125*a^3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 8000*b^4*c^3, z, k)^2*a^7*b*x
- 200*a*b*c*f - 252*a*b*d*e - 400*a*b*c*g*x - 700*a*b*d*f*x + 1944*root(19
683*a^11*b^2*z^3 + 19683*a^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5*b^2*d*f
*z - 3240*a^5*b^2*c*g*z + 22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z + 270*a^
3*b*e*f*g + 7560*a*b^3*c*d*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*e*g - 16
8*a^3*b*d*g^2 - 6000*a*b^3*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^2*c*f^2
+ 729*a^2*b^2*e^3 - 125*a^3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 8000*b^4*c
^3, z, k)*a^4*b*e*x))/(81*a^5) - (b*x*(8000*b^4*c^3 + 8*a^4*g^3 - 2744*a*b^
3*d^3 - 125*a^3*b*f^3 + 1500*a^2*b^2*c*f^2 + 1176*a^2*b^2*d^2*g - 6000*a*b^
3*c^2*f - 168*a^3*b*d*g^2 - 720*a^2*b^2*c*e*g - 1260*a^2*b^2*d*e*f + 5040*a
*b^3*c*d*e + 180*a^3*b*e*f*g))/(729*a^9))*root(19683*a^11*b^2*z^3 + 19683*a
^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5*b^2*d*f*z - 3240*a^5*b^2*c*g*z +
22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z + 270*a^3*b*e*f*g + 7560*a*b^3*c*d
*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*e*g - 168*a^3*b*d*g^2 - 6000*a*b^3
*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^2*c*f^2 + 729*a^2*b^2*e^3 - 125*a^
3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 8000*b^4*c^3, z, k), k, 1, 3) - (c/(
2*a) + (4*x^3*(4*b*c - a*f))/(9*a^2) + (7*x^4*(7*b*d - a*g))/(18*a^2) + (d*
x)/a + (5*b*x^6*(4*b*c - a*f))/(18*a^3) + (2*b*x^7*(7*b*d - a*g))/(9*a^3) -
(x^2*(3*b*e - a*h))/(6*a*b) - (b*e*x^5)/(3*a^2))/(a^2*x^2 + b^2*x^8 + 2*a*
b*x^5) + (e*log(x))/a^3

3.429 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 395

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^3}\,dx \\
&= -\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2(a+bx^3)^2} \\
&\quad-\frac{x\left(11bd-5ag+2(5be-2ah)x-3b\left(\frac{5bc}{a}-3f\right)x^2\right)}{18a^3(a+bx^3)} \\
&\quad+\frac{\left(20b^{4/3}d+14\sqrt[3]{a}be-5a\sqrt[3]{b}g-2a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}b^{2/3}} \\
&\quad-\frac{(3bc-af)\log(x)}{a^4}-\frac{\left(5\sqrt[3]{b}(4bd-ag)-2\sqrt[3]{a}(7be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}b^{2/3}} \\
&\quad+\frac{\left(5\sqrt[3]{b}(4bd-ag)-2\sqrt[3]{a}(7be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}b^{2/3}} \\
&\quad+\frac{(3bc-af)\log(a+bx^3)}{3a^4}
\end{aligned}$$

```
[Out] -1/3*c/a^3/x^3-1/2*d/a^3/x^2-e/a^3/x-1/6*x*(b*d-a*g+(-a*h+b*e)*x-b*(b*c/a-f
)*x^2)/a^2/(b*x^3+a)^2-1/18*x*(11*b*d-5*a*g+2*(-2*a*h+5*b*e)*x-3*b*(5*b*c/a
-3*f)*x^2)/a^3/(b*x^3+a)-(-a*f+3*b*c)*ln(x)/a^4-1/27*(5*b^(1/3)*(-a*g+4*b*d
)-2*a^(1/3)*(-a*h+7*b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(2/3)+1/54*(5*b^
(1/3)*(-a*g+4*b*d)-2*a^(1/3)*(-a*h+7*b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/a^(11/3)/b^(2/3)+1/3*(-a*f+3*b*c)*ln(b*x^3+a)/a^4+1/27*(20*b^(4/3
)*d+14*a^(1/3)*b*e-5*a*b^(1/3)*g-2*a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)
*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1843, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^3}\,dx \\
&= \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{4/3}h+14\sqrt[3]{a}be-5a\sqrt[3]{b}g+20b^{4/3}d\right)}{9\sqrt{3}a^{11/3}b^{2/3}} \\
&+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{2\sqrt[3]{a}(7be-ah)}{\sqrt[3]{b}}-5ag+20bd\right)}{54a^{11/3}\sqrt[3]{b}} \\
&-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bd-ag)-2\sqrt[3]{a}(7be-ah)\right)}{27a^{11/3}b^{2/3}}+\frac{(3bc-af)\log(a+bx^3)}{3a^4} \\
&-\frac{\log(x)(3bc-af)}{a^4}-\frac{x\left(-3bx^2\left(\frac{5bc}{a}-3f\right)+2x(5be-2ah)-5ag+11bd\right)}{18a^3(a+bx^3)} \\
&-\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{6a^2(a+bx^3)^2}
\end{aligned}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -1/3*c/(a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d - a*g + (b*e - a*h)*
x - b*((b*c)/a - f)*x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d - 5*a*g + 2*(5
*b*e - 2*a*h)*x - 3*b*((5*b*c)/a - 3*f)*x^2))/(18*a^3*(a + b*x^3)) + ((20*b
^(4/3)*d + 14*a^(1/3)*b*e - 5*a*b^(1/3)*g - 2*a^(4/3)*h)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(2/3)) - ((3*b*c - a
*f)*Log[x])/a^4 - ((5*b^(1/3)*(4*b*d - a*g) - 2*a^(1/3)*(7*b*e - a*h))*Log[
a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(2/3)) + ((20*b*d - 5*a*g - (2*a^(1/3)
*(7*b*e - a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(5
4*a^(11/3)*b^(1/3)) + ((3*b*c - a*f)*Log[a + b*x^3])/(3*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
```

```
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\text{integral} = -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^2c-6b^2dx-6b^2ex^2+6b^2\left(\frac{bc}{a}-f\right)x^3+5b^2\left(\frac{bd}{a}-g\right)x^4+4b^2\left(\frac{be}{a}-h\right)x^5-\frac{3b^3(bc-af)x^6}{a^2}}{x^4\left(a+bx^3\right)^2}\,dx}{6ab^2}$$

$$= -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd-5ag+2(5be-2ah)x-3b\left(\frac{5bc}{a}-3f\right)x^2\right)}{18a^3\left(a+bx^3\right)} + \frac{\int \frac{18b^4c+18b^4dx+18b^4ex^2-18b^4\left(\frac{2bc}{a}-f\right)x^3-2b^4\left(\frac{11bd}{a}-5g\right)x^4-2b^4\left(\frac{5be}{a}-2h\right)x^5}{x^4\left(a+bx^3\right)}\,dx}{18a^2b^4}$$

$$= -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd-5ag+2(5be-2ah)x-3b\left(\frac{5bc}{a}-3f\right)x^2\right)}{18a^3\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^4c}{ax^4}+\frac{18b^4d}{ax^3}+\frac{18b^4e}{ax^2}+\frac{18b^4(-3bc+af)}{a^2x}+\frac{2b^4\left(-5a(4bd-ag)-2a(7be-ah)x+9b(3bc-af)x^2\right)}{a^2\left(a+bx^3\right)}\right)dx}{18a^2b^4}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd-5ag+2(5be-2ah)x-3b\left(\frac{5bc}{a}-3f\right)x^2\right)}{18a^3\left(a+bx^3\right)} - \frac{(3bc-af)\log(x)}{a^4} + \frac{\int \frac{-5a(4bd-ag)-2a(7be-ah)x+9b(3bc-af)x^2}{a+bx^3}\,dx}{9a^4}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{6a^2\left(a + bx^3\right)^2}$$
$$- \frac{x\left(11bd - 5ag + 2(5be - 2ah)x - 3b\left(\frac{5bc}{a} - 3f\right)x^2\right)}{18a^3\left(a + bx^3\right)} - \frac{(3bc - af)\log(x)}{a^4}$$
$$+ \frac{\int \frac{-5a(4bd - ag) - 2a(7be - ah)x}{a + bx^3}\,dx}{9a^4} + \frac{(b(3bc - af))\int \frac{x^2}{a + bx^3}\,dx}{a^4}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{6a^2\left(a + bx^3\right)^2}$$
$$- \frac{x\left(11bd - 5ag + 2(5be - 2ah)x - 3b\left(\frac{5bc}{a} - 3f\right)x^2\right)}{18a^3\left(a + bx^3\right)}$$
$$- \frac{(3bc - af)\log(x)}{a^4} + \frac{(3bc - af)\log\left(a + bx^3\right)}{3a^4}$$
$$+ \frac{\int \frac{\sqrt[3]{a}\left(-10a\sqrt[3]{b}(4bd - ag) - 2a^{4/3}(7be - ah)\right) + \sqrt[3]{b}\left(5a\sqrt[3]{b}(4bd - ag) - 2a^{4/3}(7be - ah)\right)x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{27a^{14/3}\sqrt[3]{b}}$$
$$- \frac{\left(20bd - 5ag - \frac{2\sqrt[3]{a}(7be - ah)}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x}\,dx}{27a^{11/3}}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{6a^2\left(a + bx^3\right)^2}$$
$$- \frac{x\left(11bd - 5ag + 2(5be - 2ah)x - 3b\left(\frac{5bc}{a} - 3f\right)x^2\right)}{18a^3\left(a + bx^3\right)} - \frac{(3bc - af)\log(x)}{a^4}$$
$$- \frac{\left(20bd - 5ag - \frac{2\sqrt[3]{a}(7be - ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}} + \frac{(3bc - af)\log\left(a + bx^3\right)}{3a^4}$$
$$- \frac{\left(20b^{4/3}d + 14\sqrt[3]{a}be - 5a\sqrt[3]{b}g - 2a^{4/3}h\right)\int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{18a^{10/3}\sqrt[3]{b}}$$
$$+ \frac{\left(20bd - 5ag - \frac{2\sqrt[3]{a}(7be - ah)}{\sqrt[3]{b}}\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{54a^{11/3}\sqrt[3]{b}}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{6a^2(a + bx^3)^2} - \frac{x\left(11bd - 5ag + 2(5be - 2ah)x - 3b\left(\frac{5bc}{a} - 3f\right)x^2\right)}{18a^3(a + bx^3)} - \frac{(3bc - af)\log(x)}{a^4} - \frac{\left(20bd - 5ag - \frac{2\sqrt[3]{a}(7be - ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}} + \frac{\left(20bd - 5ag - \frac{2\sqrt[3]{a}(7be - ah)}{\sqrt[3]{b}}\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}} + \frac{(3bc - af)\log(a + bx^3)}{3a^4} - \frac{\left(20b^{4/3}d + 14\sqrt[3]{a}be - 5a\sqrt[3]{b}g - 2a^{4/3}h\right)\text{Subst}\left(\int \frac{1}{-3 - x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{11/3}b^{2/3}}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{6a^2(a + bx^3)^2} - \frac{x\left(11bd - 5ag + 2(5be - 2ah)x - 3b\left(\frac{5bc}{a} - 3f\right)x^2\right)}{18a^3(a + bx^3)} + \frac{\left(20b^{4/3}d + 14\sqrt[3]{a}be - 5a\sqrt[3]{b}g - 2a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}b^{2/3}} - \frac{(3bc - af)\log(x)}{a^4} - \frac{\left(20bd - 5ag - \frac{2\sqrt[3]{a}(7be - ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}} + \frac{\left(20bd - 5ag - \frac{2\sqrt[3]{a}(7be - ah)}{\sqrt[3]{b}}\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}} + \frac{(3bc - af)\log(a + bx^3)}{3a^4}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.89

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^4(a + bx^3)^3}\,dx$$

$$= \frac{-\frac{18ac}{x^3} - \frac{27ad}{x^2} - \frac{54ae}{x} + \frac{3a(-12bc + 6af - bx(11d + 10ex) + ax(5g + 4hx))}{a + bx^3} + \frac{a^2(-9b(c + x(d + ex)) + 9a(f + x(g + hx)))}{(a + bx^3)^2} + \frac{2\sqrt{3}\sqrt[3]{a}\left(20b^{4/3}d + \cdots\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] ((-18*a*c)/x^3 - (27*a*d)/x^2 - (54*a*e)/x + (3*a*(-12*b*c + 6*a*f - b*x*(1
1*d + 10*e*x) + a*x*(5*g + 4*h*x)))/(a + b*x^3) + (a^2*(-9*b*(c + x*(d + e*
x)) + 9*a*(f + x*(g + h*x))))/(a + b*x^3)^2 + (2*Sqrt[3]*a^(1/3)*(20*b^(4/3
)*d + 14*a^(1/3)*b*e - 5*a*b^(1/3)*g - 2*a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 54*(-3*b*c + a*f)*Log[x] - (2*a^(1/3)*(20*b
^(4/3)*d - 14*a^(1/3)*b*e - 5*a*b^(1/3)*g + 2*a^(4/3)*h)*Log[a^(1/3) + b^(1
/3)*x])/b^(2/3) + (a^(1/3)*(20*b^(4/3)*d - 14*a^(1/3)*b*e - 5*a*b^(1/3)*g +
 2*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 18*
(3*b*c - a*f)*Log[a + b*x^3])/(54*a^4)
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00

method	result
default	$-\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}+\frac{(af-3bc)\ln(x)}{a^4}+\frac{\frac{\left(\frac{2}{9}a^2bh-\frac{5}{9}ab^2e\right)x^5+\left(\frac{5}{18}a^2bg-\frac{11}{18}ab^2d\right)x^4+\left(\frac{1}{3}fa^2b-\frac{2}{3}ab^2c\right)x^3+\frac{a^2(7ah-13be)x^2}{18}+}{\left(bx^3+a\right)^2}}{}$
risch	$\frac{\frac{2b(ah-7be)x^8}{9a^3}+\frac{5b(ag-4bd)x^7}{18a^3}+\frac{b(af-3bc)x^6}{3a^3}+\frac{7(ah-7be)x^5}{18a^2}+\frac{4(ag-4bd)x^4}{9a^2}+\frac{(af-3bc)x^3}{2a^2}-\frac{ex^2}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^3(bx^3+a)^2}+\frac{\ln(-x)f}{a^3}-\frac{3\ln(-x)bc}{a^4}+$

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/3*c/a^3/x^3-1/2*d/a^3/x^2-e/a^3/x+(a*f-3*b*c)/a^4*ln(x)+1/a^4*(((2/9*a^2
*b*h-5/9*a*b^2*e)*x^5+(5/18*a^2*b*g-11/18*a*b^2*d)*x^4+(1/3*f*a^2*b-2/3*a*b
^2*c)*x^3+1/18*a^2*(7*a*h-13*b*e)*x^2+1/9*a^2*(4*a*g-7*b*d)*x+1/2*f*a^3-5/6
*a^2*b*c)/(b*x^3+a)^2+1/9*(5*a^2*g-20*a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^
(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/9*(2*a^2*h-14*a*b*e)*(
-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))+1/27*(-9*a*b*f+27*b^2*c)*ln(b*x^3+a)/b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 101.30 (sec) , antiderivative size = 16697, normalized size of antiderivative = 42.27

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\,(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\,(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\,(a+bx^3)^3}\,dx =& -\frac{4\,(7\,b^2e-abh)x^8+5\,(4\,b^2d-abg)x^7+6\,(3\,b^2c-abf)x^6+7\,(7\,abe-a^2h)x^5+18\,a^2ex^2+8\,(4\,abd-a^2g}{18\,(a^3b^2x^9+2\,a^4bx^6+a^5x^3)} \\ & -\frac{(3\,bc-af)\log(x)}{a^4} \\ & -\frac{\sqrt{3}\left(14\,abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+20\,abd\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5} \\ & +\frac{\left(54\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-18\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-14\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,abd-5\,a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ & +\frac{\left(27\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-9\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,abd+5\,a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] -1/18*(4*(7*b^2*e - a*b*h)*x^8 + 5*(4*b^2*d - a*b*g)*x^7 + 6*(3*b^2*c - a*b
*f)*x^6 + 7*(7*a*b*e - a^2*h)*x^5 + 18*a^2*e*x^2 + 8*(4*a*b*d - a^2*g)*x^4
+ 9*a^2*d*x + 9*(3*a*b*c - a^2*f)*x^3 + 6*a^2*c)/(a^3*b^2*x^9 + 2*a^4*b*x^6
 + a^5*x^3) - (3*b*c - a*f)*log(x)/a^4 - 1/27*sqrt(3)*(14*a*b*e*(a/b)^(2/3)
 - 2*a^2*h*(a/b)^(2/3) + 20*a*b*d*(a/b)^(1/3) - 5*a^2*g*(a/b)^(1/3))*arctan
(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^5 + 1/54*(54*b^2*c*(a/b)^(2
/3) - 18*a*b*f*(a/b)^(2/3) - 14*a*b*e*(a/b)^(1/3) + 2*a^2*h*(a/b)^(1/3) + 2
0*a*b*d - 5*a^2*g)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3
)) + 1/27*(27*b^2*c*(a/b)^(2/3) - 9*a*b*f*(a/b)^(2/3) + 14*a*b*e*(a/b)^(1/3
) - 2*a^2*h*(a/b)^(1/3) - 20*a*b*d + 5*a^2*g)*log(x + (a/b)^(1/3))/(a^4*b*(
a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.08

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^3}\,dx
&= \frac{\sqrt{3}\left(20\,b^2d-5\,abg-14\left(-ab^2\right)^{\frac{1}{3}}be+2\left(-ab^2\right)^{\frac{1}{3}}ah\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}a^3} \\
&\quad+\frac{\left(20\,b^2d-5\,abg+14\left(-ab^2\right)^{\frac{1}{3}}be-2\left(-ab^2\right)^{\frac{1}{3}}ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{2}{3}}a^3} \\
&\quad+\frac{(3\,bc-af)\log\left(|bx^3+a|\right)}{3\,a^4}-\frac{(3\,bc-af)\log\left(|x|\right)}{a^4} \\
&\quad+\frac{\left(14\,a^5b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^6bh\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,a^5b^2d-5\,a^6bg\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^9b} \\
&\quad-\frac{4\,(7\,ab^2e-a^2bh)x^8+5\,(4\,ab^2d-a^2bg)x^7+6\,(3\,ab^2c-a^2bf)x^6+18\,a^3ex^2+7\,(7\,a^2be-a^3h)x^5+9\,a^3}{18\,\left(bx^3+a\right)^2a^4x^3}
\end{aligned}
$$

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] 1/27*sqrt(3)*(20*b^2*d - 5*a*b*g - 14*(-a*b^2)^(1/3)*b*e + 2*(-a*b^2)^(1/3)
*a*h)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)
*a^3) + 1/54*(20*b^2*d - 5*a*b*g + 14*(-a*b^2)^(1/3)*b*e - 2*(-a*b^2)^(1/3)
*a*h)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) + 1/3*(
3*b*c - a*f)*log(abs(b*x^3 + a))/a^4 - (3*b*c - a*f)*log(abs(x))/a^4 + 1/27
```

```
*(14*a^5*b^2*e*(-a/b)^(1/3) - 2*a^6*b*h*(-a/b)^(1/3) + 20*a^5*b^2*d - 5*a^6
*b*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b) - 1/18*(4*(7*a*b^2*e
- a^2*b*h)*x^8 + 5*(4*a*b^2*d - a^2*b*g)*x^7 + 6*(3*a*b^2*c - a^2*b*f)*x^6
+ 18*a^3*e*x^2 + 7*(7*a^2*b*e - a^3*h)*x^5 + 9*a^3*d*x + 8*(4*a^2*b*d - a^3
*g)*x^4 + 6*a^3*c + 9*(3*a^2*b*c - a^3*f)*x^3)/((b*x^3 + a)^2*a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 10.35 (sec) , antiderivative size = 1994, normalized size of antiderivative = 5.05

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x)
```

```
[Out] symsum(log(- (1200*b^5*c*d^2 - 1134*b^5*c^2*e + 75*a^2*b^3*c*g^2 - 126*a^2*
b^3*e*f^2 - 25*a^3*b^2*f*g^2 + 18*a^3*b^2*f^2*h - 400*a*b^4*d^2*f + 162*a*b
^4*c^2*h - 108*a^2*b^3*c*f*h + 200*a^2*b^3*d*f*g - 600*a*b^4*c*d*g + 756*a*
b^4*c*e*f)/(81*a^9) - root(19683*a^12*b^2*z^3 + 19683*a^9*b^2*f*z^2 - 59049
*a^8*b^3*c*z^2 + 810*a^7*b*g*h*z - 5670*a^6*b^2*e*g*z - 3240*a^6*b^2*d*h*z
- 39366*a^5*b^3*c*f*z + 22680*a^5*b^3*d*e*z + 6561*a^6*b^2*f^2*z + 59049*a^
4*b^4*c^2*z + 270*a^4*b*f*g*h - 22680*a*b^4*c*d*e - 1890*a^3*b^2*e*f*g - 10
80*a^3*b^2*d*f*h - 810*a^3*b^2*c*g*h + 7560*a^2*b^3*d*e*f + 5670*a^2*b^3*c*
e*g + 3240*a^2*b^3*c*d*h - 168*a^4*b*e*h^2 + 19683*a*b^4*c^2*f + 1176*a^3*b
^2*e^2*h - 6000*a^2*b^3*d^2*g + 1500*a^3*b^2*d*g^2 - 6561*a^2*b^3*c*f^2 + 7
29*a^3*b^2*f^3 - 2744*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8000*a*b^4*d^3 + 8*a^5*
h^3 - 19683*b^5*c^3, z, k)*((400*a^4*b^4*d^2 + 25*a^6*b^2*g^2 + 756*a^4*b^4
*c*e - 108*a^5*b^3*c*h - 200*a^5*b^3*d*g - 252*a^5*b^3*e*f + 36*a^6*b^2*f*h
)/(81*a^9) + root(19683*a^12*b^2*z^3 + 19683*a^9*b^2*f*z^2 - 59049*a^8*b^3*
c*z^2 + 810*a^7*b*g*h*z - 5670*a^6*b^2*e*g*z - 3240*a^6*b^2*d*h*z - 39366*a
^5*b^3*c*f*z + 22680*a^5*b^3*d*e*z + 6561*a^6*b^2*f^2*z + 59049*a^4*b^4*c^2
*z + 270*a^4*b*f*g*h - 22680*a*b^4*c*d*e - 1890*a^3*b^2*e*f*g - 1080*a^3*b^
2*d*f*h - 810*a^3*b^2*c*g*h + 7560*a^2*b^3*d*e*f + 5670*a^2*b^3*c*e*g + 324
0*a^2*b^3*c*d*h - 168*a^4*b*e*h^2 + 19683*a*b^4*c^2*f + 1176*a^3*b^2*e^2*h
- 6000*a^2*b^3*d^2*g + 1500*a^3*b^2*d*g^2 - 6561*a^2*b^3*c*f^2 + 729*a^3*b^
2*f^3 - 2744*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8000*a*b^4*d^3 + 8*a^5*h^3 - 196
83*b^5*c^3, z, k)*((378*a^8*b^3*e - 54*a^9*b^2*h)/(81*a^9) - (x*(52488*a^7*
b^4*c - 17496*a^8*b^3*f))/(729*a^9) + 36*root(19683*a^12*b^2*z^3 + 19683*a^
9*b^2*f*z^2 - 59049*a^8*b^3*c*z^2 + 810*a^7*b*g*h*z - 5670*a^6*b^2*e*g*z -
3240*a^6*b^2*d*h*z - 39366*a^5*b^3*c*f*z + 22680*a^5*b^3*d*e*z + 6561*a^6*b
^2*f^2*z + 59049*a^4*b^4*c^2*z + 270*a^4*b*f*g*h - 22680*a*b^4*c*d*e - 1890
*a^3*b^2*e*f*g - 1080*a^3*b^2*d*f*h - 810*a^3*b^2*c*g*h + 7560*a^2*b^3*d*e*
f + 5670*a^2*b^3*c*e*g + 3240*a^2*b^3*c*d*h - 168*a^4*b*e*h^2 + 19683*a*b^4
*c^2*f + 1176*a^3*b^2*e^2*h - 6000*a^2*b^3*d^2*g + 1500*a^3*b^2*d*g^2 - 656
1*a^2*b^3*c*f^2 + 729*a^3*b^2*f^3 - 2744*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8000
```

*a*b^4*d^3 + 8*a^5*h^3 - 19683*b^5*c^3, z, k)*a^2*b^3*x) + (x*(26244*a^3*b^5*c^2 + 2916*a^5*b^3*f^2 - 17496*a^4*b^4*c*f + 25200*a^4*b^4*d*e - 3600*a^5*b^3*d*h - 6300*a^5*b^3*e*g + 900*a^6*b^2*g*h))/(729*a^9)) - (x*(8000*b^5*d^3 - 2744*a*b^4*e^3 + 8*a^4*b*h^3 - 125*a^3*b^2*g^3 + 1500*a^2*b^3*d*g^2 + 1176*a^2*b^3*e^2*h - 168*a^3*b^2*e*h^2 - 15120*b^5*c*d*e - 6000*a*b^4*d^2*g - 540*a^2*b^3*c*g*h - 720*a^2*b^3*d*f*h - 1260*a^2*b^3*e*f*g + 180*a^3*b^2*f*g*h + 2160*a*b^4*c*d*h + 3780*a*b^4*c*e*g + 5040*a*b^4*d*e*f))/(729*a^9))*root(19683*a^12*b^2*z^3 + 19683*a^9*b^2*f*z^2 - 59049*a^8*b^3*c*z^2 + 810*a^7*b*g*h*z - 5670*a^6*b^2*e*g*z - 3240*a^6*b^2*d*h*z - 39366*a^5*b^3*c*f*z + 22680*a^5*b^3*d*e*z + 6561*a^6*b^2*f^2*z + 59049*a^4*b^4*c^2*z + 270*a^4*b*f*g*h - 22680*a*b^4*c*d*e - 1890*a^3*b^2*e*f*g - 1080*a^3*b^2*d*f*h - 810*a^3*b^2*c*g*h + 7560*a^2*b^3*d*e*f + 5670*a^2*b^3*c*e*g + 3240*a^2*b^3*c*d*h - 168*a^4*b*e*h^2 + 19683*a*b^4*c^2*f + 1176*a^3*b^2*e^2*h - 6000*a^2*b^3*d^2*g + 1500*a^3*b^2*d*g^2 - 6561*a^2*b^3*c*f^2 + 729*a^3*b^2*f^3 - 2744*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8000*a*b^4*d^3 + 8*a^5*h^3 - 19683*b^5*c^3, z, k), k, 1, 3) - (c/(3*a) + (e*x^2)/a + (x^3*(3*b*c - a*f))/(2*a^2) + (4*x^4*(4*b*d - a*g))/(9*a^2) + (7*x^5*(7*b*e - a*h))/(18*a^2) + (d*x)/(2*a) + (b*x^6*(3*b*c - a*f))/(3*a^3) + (5*b*x^7*(4*b*d - a*g))/(18*a^3) + (2*b*x^8*(7*b*e - a*h))/(9*a^3))/(a^2*x^3 + b^2*x^9 + 2*a*b*x^6) - (log(x)*(3*b*c - a*f))/a^4

3.430 $\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 583

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = -\frac{4ae\sqrt{a+bx^3}}{9b^2}+\frac{2cx\sqrt{a+bx^3}}{5b}+\frac{2dx^2\sqrt{a+bx^3}}{7b}+\frac{2ex^3\sqrt{a+bx^3}}{9b}-\frac{8ad\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{2+\sqrt{3}}a\left(7\sqrt[3]{b}c-10\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -4/9*a*e*(b*x^3+a)^(1/2)/b^2+2/5*c*x*(b*x^3+a)^(1/2)/b+2/7*d*x^2*(b*x^3+a)^
(1/2)/b+2/9*e*x^3*(b*x^3+a)^(1/2)/b-8/7*a*d*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))+4/7*3^(1/4)*a^(4/3)*d*(a^(1/3)+b^(1/3)*x)*Elliptic
E((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-4/105*a*(a^(1/3)+
b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+
```

```
3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*c-10*a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2
)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1902, 1608, 1900, 267, 1892, 224, 1891}

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx =$$

$$-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}c-10\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{8ad\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{4ae\sqrt{a+bx^3}}{9b^2}+\frac{2cx\sqrt{a+bx^3}}{5b}$$

$$+\frac{2dx^2\sqrt{a+bx^3}}{7b}+\frac{2ex^3\sqrt{a+bx^3}}{9b}$$

```
[In] Int[(x^3*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (-4*a*e*Sqrt[a + b*x^3])/(9*b^2) + (2*c*x*Sqrt[a + b*x^3])/(5*b) + (2*d*x^2
*Sqrt[a + b*x^3])/(7*b) + (2*e*x^3*Sqrt[a + b*x^3])/(9*b) - (8*a*d*Sqrt[a +
 b*x^3])/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[
2 - Sqrt[3]]*a^(4/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*Sqrt[2 + Sqrt[3]]*a*(7*b^
(1/3)*c - 10*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
```

```
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
```

```
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{2\int \frac{-3aex^2+\frac{9}{2}bcx^3+\frac{9}{2}bdx^4}{\sqrt{a+bx^3}}\,dx}{9b}$$

$$= \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{2\int \frac{x^2\left(-3ae+\frac{9bcx}{2}+\frac{9}{2}bdx^2\right)}{\sqrt{a+bx^3}}\,dx}{9b}$$

$$= \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{4\int \frac{-9abdx-\frac{21}{2}abex^2+\frac{63}{4}b^2cx^3}{\sqrt{a+bx^3}}\,dx}{63b^2}$$

$$= \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{4\int \frac{x\left(-9abd-\frac{21}{2}abex+\frac{63}{4}b^2cx^2\right)}{\sqrt{a+bx^3}}\,dx}{63b^2}$$

$$= \frac{2cx\sqrt{a+bx^3}}{5b} + \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{8\int \frac{-\frac{63}{4}ab^2c-\frac{45}{2}ab^2dx-\frac{105}{4}ab^2ex^2}{\sqrt{a+bx^3}}\,dx}{315b^3}$$

$$= \frac{2cx\sqrt{a+bx^3}}{5b} + \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{8\int \frac{-\frac{63}{4}ab^2c-\frac{45}{2}ab^2dx}{\sqrt{a+bx^3}}\,dx}{315b^3} - \frac{(2ae)\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{3b}$$

$$= -\frac{4ae\sqrt{a+bx^3}}{9b^2} + \frac{2cx\sqrt{a+bx^3}}{5b} + \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} - \frac{(4ad)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}} - \frac{\left(2a\left(7\sqrt[3]{b}c-10\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{35b^{4/3}}$$

$$= -\frac{4ae\sqrt{a+bx^3}}{9b^2} + \frac{2cx\sqrt{a+bx^3}}{5b} + \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} - \frac{8ad\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+ \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{4\sqrt{2+\sqrt{3}}a\left(7\sqrt[3]{b}c-10\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{}\ldots}{\left(1+\sqrt{3}\right)\sqrt[3]{}\ldots}\right.\right.}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.23

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{-2(a+bx^3)\left(70ae-bx(63c+5x(9d+7ex))\right)-126abcx\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)-9\ldots}{315b^2\sqrt{a+bx^3}}$$

```
[In] Integrate[(x^3*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (-2*(a + b*x^3)*(70*a*e - b*x*(63*c + 5*x*(9*d + 7*e*x))) - 126*a*b*c*x*Sqr
t[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 90*a*b*d*
x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(31
5*b^2*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.31

method	result
risch	$-\frac{2\left(-35be\,x^{3}-45bd\,x^{2}-63bcx+70ae\right)\sqrt{b\,x^{3}+a}}{315b^{2}}-\frac{2a\left(-\frac{14ic\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)\cdots}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\cdots}{\cdots}}}\cdots}{\cdots}\cdots\right)}{\cdots}$ $\cdots\ 4iac\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\cdots}\cdots$
elliptic	$\frac{2e\,x^{3}\sqrt{b\,x^{3}+a}}{9b}+\frac{2d\,x^{2}\sqrt{b\,x^{3}+a}}{7b}+\frac{2cx\sqrt{b\,x^{3}+a}}{5b}-\frac{4ae\sqrt{b\,x^{3}+a}}{9b^{2}}+\frac{\cdots}{\cdots}$
default	Expression too large to display

```
[In] int(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*(-35*b*e*x^3-45*b*d*x^2-63*b*c*x+70*a*e)/b^2*(b*x^3+a)^(1/2)-2/35*a/
b*(-14/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)-20/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.14

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = -\frac{2\left(126\,a\sqrt{b}c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-180\,a\sqrt{b}d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0\right.\right.\right.}{315\,b^2}$$

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(126*a*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 180*a*sqrt(b)*d
*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (35*b*e*x^
3 + 45*b*d*x^2 + 63*b*c*x - 70*a*e)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.22

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = e\left(\begin{cases} -\frac{4a\sqrt{a+bx^3}}{9b^2}+\frac{2x^3\sqrt{a+bx^3}}{9b} & \text{for } b\neq 0 \\ \frac{x^6}{6\sqrt{a}} & \text{otherwise}\end{cases}\right) + \frac{cx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{dx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] e*Piecewise((-4*a*sqrt(a + b*x**3)/(9*b**2) + 2*x**3*sqrt(a + b*x**3)/(9*b)
, Ne(b, 0)), (x**6/(6*sqrt(a)), True)) + c*x**4*gamma(4/3)*hyper((1/2, 4/3)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + d*x**5*gamma(5
/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/
3))
```

Maxima [F]

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \int \frac{(ex^2+dx+c)x^3}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^3/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \int \frac{(ex^2+dx+c)x^3}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^3/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \int \frac{x^3\,(e\,x^2+d\,x+c)}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((x^3*(c + d*x + e*x^2))/(a + b*x^3)^(1/2),x)
[Out] int((x^3*(c + d*x + e*x^2))/(a + b*x^3)^(1/2), x)
```

3.431 $\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 560

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \frac{2c\sqrt{a+bx^3}}{3b}+\frac{2dx\sqrt{a+bx^3}}{5b}+\frac{2ex^2\sqrt{a+bx^3}}{7b}-\frac{8ae\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{4\sqrt{2+\sqrt{3}}a\left(7\sqrt[3]{b}d-10\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\ldots\right)}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/3*c*(b*x^3+a)^(1/2)/b+2/5*d*x*(b*x^3+a)^(1/2)/b+2/7*e*x^2*(b*x^3+a)^(1/2)
/b-8/7*a*e*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+4/7*3^(1
/4)*a^(4/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)-4/105*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/
```

```
3)*d-10*a^(1/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(
5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1902, 1608, 1900, 267, 1892, 224, 1891}

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx =$$

$$-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}d-10\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(}{\sqrt[3]{b}x+(}\right.\right.}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{8ae\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2c\sqrt{a+bx^3}}{3b}+\frac{2dx\sqrt{a+bx^3}}{5b}+\frac{2ex^2\sqrt{a+bx^3}}{7b}$$

```
[In] Int[(x^2*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*c*Sqrt[a + b*x^3])/(3*b) + (2*d*x*Sqrt[a + b*x^3])/(5*b) + (2*e*x^2*Sqrt
[a + b*x^3])/(7*b) - (8*a*e*Sqrt[a + b*x^3])/(7*b^(5/3)*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*e*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3]) - (4*Sqrt[2 + Sqrt[3]]*a*(7*b^(1/3)*d - 10*(1 - Sqrt[3])*a^(1/3)*e)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1
/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{2ex^2\sqrt{a+bx^3}}{7b} + \frac{2\int \frac{-2aex+\frac{7}{2}bcx^2+\frac{7}{2}bdx^3}{\sqrt{a+bx^3}}\,dx}{7b}$$

$$= \frac{2ex^2\sqrt{a+bx^3}}{7b} + \frac{2\int \frac{x\left(-2ae+\frac{7bcx}{2}+\frac{7}{2}bdx^2\right)}{\sqrt{a+bx^3}}\,dx}{7b}$$

$$= \frac{2dx\sqrt{a+bx^3}}{5b} + \frac{2ex^2\sqrt{a+bx^3}}{7b} + \frac{4\int \frac{-\frac{7}{2}abd-5abex+\frac{35}{4}b^2cx^2}{\sqrt{a+bx^3}}\,dx}{35b^2}$$

$$= \frac{2dx\sqrt{a+bx^3}}{5b} + \frac{2ex^2\sqrt{a+bx^3}}{7b} + \frac{4\int \frac{-\frac{7}{2}abd-5abex}{\sqrt{a+bx^3}}\,dx}{35b^2} + c\int \frac{x^2}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2c\sqrt{a+bx^3}}{3b} + \frac{2dx\sqrt{a+bx^3}}{5b} + \frac{2ex^2\sqrt{a+bx^3}}{7b} - \frac{(4ae)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}} - \frac{\left(2a\left(7\sqrt[3]{b}d-10\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{35b^{4/3}}$$

$$= \frac{2c\sqrt{a+bx^3}}{3b} + \frac{2dx\sqrt{a+bx^3}}{5b} + \frac{2ex^2\sqrt{a+bx^3}}{7b} - \frac{8ae\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+ \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right.}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$- \frac{4\sqrt{2+\sqrt{3}}a\left(7\sqrt[3]{b}d-10\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.22

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{2(a+bx^3)\left(35c+3x(7d+5ex)\right)-42adx\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)-30aex^2\sqrt{1+\frac{bx^3}{a}}}{105b\sqrt{a+bx^3}}$$

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(a + b*x^3)*(35*c + 3*x*(7*d + 5*e*x)) - 42*a*d*x*Sqrt[1 + (b*x^3)/a]*Hy
pergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 30*a*e*x^2*Sqrt[1 + (b*x^3)/
a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(105*b*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.35

method	result
risch	$\frac{2(15e\,x^2+21dx+35c)\sqrt{b\,x^3+a}}{105b} - \frac{2a\left(-\frac{14id\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\cdots}}{\cdots}\right.}{\cdots}$
elliptic	$\frac{2e\,x^2\sqrt{b\,x^3+a}}{7b}+\frac{2dx\sqrt{b\,x^3+a}}{5b}+\frac{2c\sqrt{b\,x^3+a}}{3b}+\frac{4iad\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-\cdots\right.}{\cdots}}}\cdots}{\cdots}$ $\left(\frac{8ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\cdots}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{\cdots}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\cdots\right)}{\cdots}}}{\cdots}\right.$

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(15*e*x^2+21*d*x+35*c)/b*(b*x^3+a)^(1/2)-2/35*a/b*(-14/3*I*d*3^(1/2)/
b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-20/3*I*e*3^(1/2)/b*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.13

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = -\frac{2\left(42\,a\sqrt{b}d\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-60\,a\sqrt{b}e\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-\left(15\,bex^2+21\,bdx+35\,bc\right)\sqrt{bx^3+a}\right)}{105\,b^2}$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(42*a*sqrt(b)*d*weierstrassPInverse(0, -4*a/b, x) - 60*a*sqrt(b)*e*w
eierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (15*b*e*x^2
+ 21*b*d*x + 35*b*c)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.19

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = c\left(\begin{cases} \frac{x^3}{3\sqrt{a}} & \text{for } b=0 \\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise} \end{cases}\right) + \frac{dx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{ex^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
[Out] c*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True))
 + d*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3
*sqrt(a)*gamma(7/3)) + e*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \int \frac{(ex^2+dx+c)x^2}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(b*x^3 + a)*c/b + integrate((e*x^4 + d*x^3)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \int \frac{(ex^2+dx+c)x^2}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d*x + c)*x^2/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx=\int \frac{x^2\,\left(e\,x^2+d\,x+c\right)}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^(1/2),x)
[Out] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^(1/2), x)
```

3.432 $\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 537

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \frac{2d\sqrt{a+bx^3}}{3b}+\frac{2ex\sqrt{a+bx^3}}{5b}+\frac{2c\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(5\left(1-\sqrt{3}\right)b^{2/3}c+2a^{2/3}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{(1-\sqrt{}}{(1+\sqrt{}}\right.\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/3*d*(b*x^3+a)^(1/2)/b+2/5*e*x*(b*x^3+a)^(1/2)/b+2*c*(b*x^3+a)^(1/2)/b^(2/
3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)*c*(a^(1/3)+b^(1/3)*x)*El
lipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-2/15*3^(3/4
)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(2*a^(2/3)*e+5*b^(2/3)*c*(1-3^(
1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1902, 1900, 267, 1892, 224, 1891}

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx =$$

$$-\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(}{\sqrt[3]{b}x+(}\right.\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2c\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2d\sqrt{a+bx^3}}{3b}+\frac{2ex\sqrt{a+bx^3}}{5b}$$

```
[In] Int[(x*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*d*Sqrt[a + b*x^3])/(3*b) + (2*e*x*Sqrt[a + b*x^3])/(5*b) + (2*c*Sqrt[a +
 b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 -
Sqrt[3]]*a^(1/3)*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2 + Sqrt[3]]*a^(1/3)*(5*(1
 - Sqrt[3])*b^(2/3)*c + 2*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{2ex\sqrt{a+bx^3}}{5b} + \frac{2\int \frac{-ae+\frac{5bcx}{2}+\frac{5}{2}bdx^2}{\sqrt{a+bx^3}}\,dx}{5b}$$

$$= \frac{2ex\sqrt{a+bx^3}}{5b} + \frac{2\int \frac{-ae+\frac{5bcx}{2}}{\sqrt{a+bx^3}}\,dx}{5b} + d\int \frac{x^2}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2d\sqrt{a+bx^3}}{3b} + \frac{2ex\sqrt{a+bx^3}}{5b} + \frac{c\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} - \frac{\left(\sqrt[3]{a}\left(5\left(1-\sqrt{3}\right)b^{2/3}c+2a^{2/3}e\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{5b}$$

$$= \frac{2d\sqrt{a+bx^3}}{3b} + \frac{2ex\sqrt{a+bx^3}}{5b} + \frac{2c\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(5\left(1-\sqrt{3}\right)b^{2/3}c+2a^{2/3}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.21

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \frac{4(5d+3ex)\left(a+bx^3\right)-12aex\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+15bcx^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeom}}{30b\sqrt{a+bx^3}}$$

```
[In] Integrate[(x*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (4*(5*d + 3*e*x)*(a + b*x^3) - 12*a*e*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 15*b*c*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeome
tric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(30*b*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.39

method	result
risch	$\frac{2(3ex+5d)\sqrt{b\,x^3+a}}{15b}-\frac{-\frac{4iae\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\cdots\right.}{\left(-a\cdots\right.}}}{\cdots}}{3b\sqrt{b\,x^3\cdots}}$
elliptic	$\frac{2ex\sqrt{b\,x^3+a}}{5b}+\frac{2d\sqrt{b\,x^3+a}}{3b}+\frac{4iae\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-\cdots\right.}{\cdots}\right.}{\cdots}}\cdots}{1\cdots}$
default	$e\left(\frac{2x\sqrt{b\,x^3+a}}{5b}+\frac{4ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(\cdots\right.}{\cdots}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\cdots}{15b^2\sqrt{b\,x^3+a}\cdots}\right.$

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(3*e*x+5*d)/b*(b*x^3+a)^(1/2)-1/5/b*(-4/3*I*a*e*3^(1/2)/b*(-a*b^2)^(1/
3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+10/3*I*c*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.12

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = -\frac{2\left(6\,a\sqrt{b}e\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+15\,b^{\frac{3}{2}}c\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b}\right.\right.\right.}{15\,b^2}$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(6*a*sqrt(b)*e*weierstrassPInverse(0, -4*a/b, x) + 15*b^(3/2)*c*weier
strassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - sqrt(b*x^3 + a)*
(3*b*e*x + 5*b*d))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.20

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = d\left(\begin{cases}\frac{x^3}{3\sqrt{a}} & \text{for } b=0\\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise}\end{cases}\right) + \frac{cx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] d*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True))
 + c*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3
*sqrt(a)*gamma(5/3)) + e*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3))
```

Maxima [F]

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \int \frac{(ex^2+dx+c)x}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*x/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \int \frac{(ex^2+dx+c)x}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \int \frac{x\,(e\,x^2+d\,x+c)}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3)^(1/2),x)
[Out] int((x*(c + d*x + e*x^2))/(a + b*x^3)^(1/2), x)
```

3.433 $\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 509

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx=\frac{2e\sqrt{a+bx^3}}{3b}+\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/3*e*(b*x^3+a)^(1/2)/b+2*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))-3^(1/4)*a^(1/3)*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^
(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)
*(b^(1/3)*c-a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)
/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1900, 267, 1892, 224, 1891}

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2e\sqrt{a+bx^3}}{3b}$$

```
[In] Int[(c + d*x + e*x^2)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b
*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$\text{integral} = e\int \frac{x^2}{\sqrt{a+bx^3}}\,dx + \int \frac{c+dx}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2e\sqrt{a+bx^3}}{3b} + \frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} + \left(c - \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2e\sqrt{a+bx^3}}{3b} + \frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right.}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.21

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{4e(a+bx^3)+6bcx\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+3bdx^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\right.}{6b\sqrt{a+bx^3}}$$

```
[In] Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x^3],x]
```

```
[Out] (4*e*(a + b*x^3) + 6*b*c*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2,
4/3, -((b*x^3)/a)] + 3*b*d*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2
/3, 5/3, -((b*x^3)/a)])/(6*b*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.44

method	result
default	$-\frac{2ic\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,F}{3b\sqrt{b\,x^{3}+a}}$
risch	$-\frac{2ic\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,F}{3b\sqrt{b\,x^{3}+a}}$
elliptic	$-\frac{2ic\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,F}{3b\sqrt{b\,x^{3}+a}}$

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/
3*e*(b*x^3+a)^(1/2)/b-2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-
1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+
1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx = \frac{2\left(3\sqrt{b}c\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-3\sqrt{b}d\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right.}{3b}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 3*sqrt(b)*d*weierstras
sZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + sqrt(b*x^3 + a)*e)/b
```

Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.21

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx = e\left(\begin{cases}\frac{x^3}{3\sqrt{a}} & \text{for } b=0\\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise}\end{cases}\right)+\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
[Out] e*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True))
 + c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx=\int \frac{e\,x^2+d\,x+c}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((c + d*x + e*x^2)/(a + b*x^3)^(1/2),x)
[Out] int((c + d*x + e*x^2)/(a + b*x^3)^(1/2), x)
```

3.434 $\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 518

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx = \frac{2e\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$
$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}d-\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/3*c*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+2*e*(b*x^3+a)^(1/2)/b^(2/3)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)*e*(a^(1/3)+b^(1/3)*x)*Elli
pticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/3*(a^(1/3)+
b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*d-a^(1/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2
*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*
x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx$$
$$= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}d-\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{2c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{2e\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[(c + d*x + e*x^2)/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*e*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*c*
ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*
a^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]
])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d - (1 - Sqrt
[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
```

```
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = c\int \frac{1}{x\sqrt{a+bx^3}}\,dx + \int \frac{d+ex}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{1}{3}c\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right) + \frac{e\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} + \left(d - \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2e\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}d-\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{(2c)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}$$

$$= \frac{2e\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}d-\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\ }}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\ }}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.25

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx = -\frac{2c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} + \frac{dx\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{a+bx^3}} + \frac{ex^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{a+bx^3}}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) + (d*x*Sqrt[1 + (b*x^3)
/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/Sqrt[a + b*x^3] + (e*x^
2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sq
rt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 740, normalized size of antiderivative = 1.43

method	result
default	$-\frac{2 i d \sqrt{3}\, \left(-a b^2\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\, \left(-a b^2\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a b^2\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3 \left(-a b^2\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a b^2\right)^{\frac{1}{3}}}{2 b}}}\, \sqrt{-\frac{i \left(x+\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a b^2\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a b^2\right)^{\frac{1}{3}}}}\, F}{3 b \sqrt{b x^3+a}}$
elliptic	$-\frac{2 i d \sqrt{3}\, \left(-a b^2\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\, \left(-a b^2\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a b^2\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3 \left(-a b^2\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a b^2\right)^{\frac{1}{3}}}{2 b}}}\, \sqrt{-\frac{i \left(x+\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a b^2\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a b^2\right)^{\frac{1}{3}}}}\, F}{3 b \sqrt{b x^3+a}}$

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/
3*I*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
```

```
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*c*arctanh((b*x^3+a)^(
1/2)/a^(1/2))/a^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.37

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx = \left[\frac{\sqrt{a}bc\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+12\,a\sqrt{b}d\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-12\,a\sqrt{b}e\text{weier}}{6\,ab}\right.$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*b*c*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 +
a)*sqrt(a) + 8*a^2)/x^6) + 12*a*sqrt(b)*d*weierstrassPInverse(0, -4*a/b, x)
 - 12*a*sqrt(b)*e*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b,
 x)))/(a*b), 1/3*(sqrt(-a)*b*c*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2
*a)) + 6*a*sqrt(b)*d*weierstrassPInverse(0, -4*a/b, x) - 6*a*sqrt(b)*e*weie
rstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/(a*b)]
```

Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.20

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx = -\frac{2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}+\frac{dx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{ex^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**(1/2),x)
```

```
[Out] -2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)) + d*x*gamma(1/3)*hyper((
1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + e*x**
2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)
*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^3+ax}}\,dx$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^3+ax}}\,dx$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx = \int \frac{e\,x^2+d\,x+c}{x\,\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((c + d*x + e*x^2)/(x*(a + b*x^3)^(1/2)), x)
```

3.435 $\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 547

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx = -\frac{c\sqrt{a+bx^3}}{ax}+\frac{\sqrt[3]{b}c\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt{2+\sqrt{3}}\left(\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{\sqrt[4]{3}a^{2/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-c*(b*x^3+a)^(1/2)/a/x+b^(1/
3)*c*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-1/2*3^(1/4)*b^(1/3)*
c*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3
)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)-1/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2
)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(-2*a^(2/3)*e+b^(2/3)*c*
(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b^(1/3)/(b*x^
```

```
3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx =$$

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right.}\right.\right.}{\sqrt[4]{3}a^{2/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2d\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{c\sqrt{a+bx^3}}{ax}+\frac{\sqrt[3]{b}c\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[(c + d*x + e*x^2)/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] -((c*Sqrt[a + b*x^3])/(a*x)) + (b^(1/3)*c*Sqrt[a + b*x^3])/(a*((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)) - (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]
) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (Sqrt[2 +
Sqrt[3]]*((1 - Sqrt[3])*b^(2/3)*c - 2*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*b^(1/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{c\sqrt{a+bx^3}}{ax} - \frac{\int \frac{-2ad-2aex-bcx^2}{x\sqrt{a+bx^3}}\,dx}{2a}$$

$$= -\frac{c\sqrt{a+bx^3}}{ax} - \frac{\int \frac{-2ae-bcx}{\sqrt{a+bx^3}}\,dx}{2a} + d\int \frac{1}{x\sqrt{a+bx^3}}\,dx$$

$$= -\frac{c\sqrt{a+bx^3}}{ax} + \frac{\left(b^{2/3}c\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{2a} + \frac{1}{3}d\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right) - \frac{1}{2}\left(\frac{\left(1-\sqrt{3}\right)b^{2/3}c}{a^{2/3}} - 2e\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= -\frac{c\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}c\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\sqrt{2+\sqrt{3}}\left(\frac{\left(1-\sqrt{3}\right)b^{2/3}c}{a^{2/3}}-2e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|\cdots}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{(2d)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3b}$$

$$= -\frac{c\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}c\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\sqrt{2+\sqrt{3}}\left(\frac{\left(1-\sqrt{3}\right)b^{2/3}c}{a^{2/3}}-2e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|\cdots}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.23

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx = -\frac{2d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{c\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\sqrt{a+bx^3}} + \frac{ex\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{a+bx^3}}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (c*Sqrt[1 + (b*x^3)/a
]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/(x*Sqrt[a + b*x^3]) + (e
*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/Sqrt
[a + b*x^3]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 758, normalized size of antiderivative = 1.39

method	result
elliptic	$-\frac{c\sqrt{b\,x^{3}+a}}{a x}-\frac{2 i e\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\right.}{2 b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{3 b\sqrt{b\,x^{3}+a}}$
default	$-\frac{2 i e\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,F\left(\right.}{3 b\sqrt{b\,x^{3}+a}}$

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -c*(b*x^3+a)^(1/2)/a/x-2/3*I*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2))-1/3*I*c/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/
b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3
*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.43

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx = \left[\frac{\sqrt{a}bdx\log\left(\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+12\,a\sqrt{b}ex\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-6\,b^{\frac{3}{2}}cx\text{weier}}{6\,abx}\right.$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*b*d*x*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 +
 a)*sqrt(a) + 8*a^2)/x^6) + 12*a*sqrt(b)*e*x*weierstrassPInverse(0, -4*a/b,
 x) - 6*b^(3/2)*c*x*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/
b, x)) - 6*sqrt(b*x^3 + a)*b*c)/(a*b*x), 1/3*(sqrt(-a)*b*d*x*arctan(1/2*(b*
x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) + 6*a*sqrt(b)*e*x*weie
rstrassPInverse(0, -4*a/b, x) - 3*b^(3/2)*c*x*weierstrassZeta(0, -4*a/b, we
ierstrassPInverse(0, -4*a/b, x)) - 3*sqrt(b*x^3 + a)*b*c)/(a*b*x)]
```

Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.20

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx = \frac{c\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\Gamma\left(\frac{2}{3}\right)} - \frac{2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}} + \frac{ex\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**(1/2),x)
[Out] c*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x*gamma(2/3)) - 2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)) + e*x*
gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*g
amma(4/3))
```

Maxima [F]

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^3+a}x^2}\,dx$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^3+a}x^2}\,dx$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.22

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx = \frac{d\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,\sqrt{a}} - \frac{2\,c\,\sqrt{\frac{a}{b\,x^3}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{6};\ \frac{11}{6};\ -\frac{a}{b\,x^3}\right)}{5\,x\,\sqrt{b\,x^3+a}} + \frac{e\,x\,\sqrt{\frac{b\,x^3}{a}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\sqrt{b\,x^3+a}}$$

```
[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^(1/2)),x)
```

```
[Out] (d*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)
)/(3*a^(1/2)) - (2*c*(a/(b*x^3) + 1)^(1/2)*hypergeom([1/2, 5/6], 11/6, -a/(
b*x^3)))/(5*x*(a + b*x^3)^(1/2)) + (e*x*((b*x^3)/a + 1)^(1/2)*hypergeom([1/
3, 1/2], 4/3, -(b*x^3)/a))/(a + b*x^3)^(1/2)
```

3.436 $\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 569

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx = -\frac{c\sqrt{a+bx^3}}{2ax^2}-\frac{d\sqrt{a+bx^3}}{ax}+\frac{\sqrt[3]{b}d\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2e\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{b}c+2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots}{\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots}\right.\right.}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/3*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/2*c*(b*x^3+a)^(1/2)/a/x^2
-d*(b*x^3+a)^(1/2)/a/x+b^(1/3)*d*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))-1/2*3^(1/4)*b^(1/3)*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1
/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2
)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/6*3^(3/4)*b^(1/3)*(a^(1/3)+b^(1/
3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/
```

```
2))),I*3^(1/2)+2*I)*(b^(1/3)*c+2*a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\right.}{\sqrt[3]{b}x+\left(1-\right.}\right.\right.}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{2e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}d\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[(c + d*x + e*x^2)/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/2*(c*Sqrt[a + b*x^3])/(a*x^2) - (d*Sqrt[a + b*x^3])/(a*x) + (b^(1/3)*d*S
qrt[a + b*x^3])/(a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*e*ArcTanh[Sqrt
[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*d*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a^(2/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3]) - (Sqrt[2 + Sqrt[3]]*b^(1/3)*(b^(1/3)*c + 2*(1 - Sqrt[3
])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(2*3^(1/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{\int \frac{-4ad-4aex+bcx^2}{x^2\sqrt{a+bx^3}}\,dx}{4a} \\
&= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\int \frac{8a^2e-2abcx+4abdx^2}{x\sqrt{a+bx^3}}\,dx}{8a^2} \\
&= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\int \frac{-2abc+4abdx}{\sqrt{a+bx^3}}\,dx}{8a^2} + e\int \frac{1}{x\sqrt{a+bx^3}}\,dx \\
&= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\left(b^{2/3}d\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{2a} \\
&\quad - \frac{\left(b^{2/3}\left(\sqrt[3]{b}c+2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{4a} + \frac{1}{3}e\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)
\end{aligned}$$

$$= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}d\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{b}c+2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{(2e)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}$$

$$= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}d\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{b}c+2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.23

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx = -\frac{2e\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{c\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2\sqrt{a+bx^3}} - \frac{d\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\sqrt{a+bx^3}}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (c*Sqrt[1 + (b*x^3)/a
]*Hypergeometric2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])/(2*x^2*Sqrt[a + b*x^3])
- (d*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/(
x*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 763, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(2dx+c)}{2a\,x^2}+\frac{2ic\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}\right.}{\left(-a\,b^2\right)}}}{3\sqrt{b\,x^3+a}}$
elliptic	$-\frac{c\sqrt{b\,x^3+a}}{2a\,x^2}-\frac{d\sqrt{b\,x^3+a}}{ax}+\frac{ic\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right.}{2b}\right.}{}}}{6a}$
default	Expression too large to display

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*x^3+a)^(1/2)*(2*d*x+c)/a/x^2+1/4/a*(2/3*I*c*3^(1/2)*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-4/3*I*d*3^(1/2)*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
```

```
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)))-8/3*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.43

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx = \left[\frac{\sqrt{a}ex^2\log\left(\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)-3\,\sqrt{b}cx^2\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-6\,\sqrt{b}dx^2\text{weie}}{6\,ax^2}\right.$$

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*e*x^2*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 +
 a)*sqrt(a) + 8*a^2)/x^6) - 3*sqrt(b)*c*x^2*weierstrassPInverse(0, -4*a/b,
x) - 6*sqrt(b)*d*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a
/b, x)) - 3*sqrt(b*x^3 + a)*(2*d*x + c))/(a*x^2), 1/6*(2*sqrt(-a)*e*x^2*arc
tan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) - 3*sqrt(b)
*c*x^2*weierstrassPInverse(0, -4*a/b, x) - 6*sqrt(b)*d*x^2*weierstrassZeta(
0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - 3*sqrt(b*x^3 + a)*(2*d*x +
c))/(a*x^2)]
```

Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.20

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx = \frac{c\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^2\Gamma\left(\frac{1}{3}\right)} + \frac{d\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\Gamma\left(\frac{2}{3}\right)} - \frac{2e\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**(1/2),x)
```

```
[Out] c*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x**2*gamma(1/3)) + d*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*x*gamma(2/3)) - 2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2
)))/(3*sqrt(a))
```

Maxima [F]

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^3+a}x^3}\,dx$$

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^3), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^3+a}x^3}\,dx$$

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx = \int \frac{e\,x^2+d\,x+c}{x^3\,\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^(1/2)), x)
```

3.437 $\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 594

$$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}} + \frac{4c\sqrt{a+bx^3}}{3b^2} + \frac{2dx\sqrt{a+bx^3}}{5b^2} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2} - \frac{80ae\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+ \frac{40\sqrt{2-\sqrt{3}}a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7\ 3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$- \frac{16\sqrt{2+\sqrt{3}}a\left(14\sqrt[3]{b}d-25\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{(1-\sqrt{3})}{(1+\sqrt{3})}\right.\right.}{105\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/3*x*(-b*c*x^2+a*e*x+a*d)/b^2/(b*x^3+a)^(1/2)+4/3*c*(b*x^3+a)^(1/2)/b^2+2/
5*d*x*(b*x^3+a)^(1/2)/b^2+2/7*e*x^2*(b*x^3+a)^(1/2)/b^2-80/21*a*e*(b*x^3+a)
^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+40/21*a^(4/3)*e*(a^(1/3)+b^(
1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(8/3)/(b*x
^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
```

```
^(1/2)-16/315*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(14*b^(1/3)*d-25*a^(1/3)*
e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(8/3)/(b*x^3+a)^(
1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1842, 1902, 1900, 267, 1892, 224, 1891}

$$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = -\frac{16\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(14\sqrt[3]{b}d-25\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{105\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{40\sqrt{2-\sqrt{3}}a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\ 3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{80ae\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}} + \frac{4c\sqrt{a+bx^3}}{3b^2} + \frac{2dx\sqrt{a+bx^3}}{5b^2} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2}$$

```
[In] Int[(x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*c*Sqrt[a + b*x^3
])/(3*b^2) + (2*d*x*Sqrt[a + b*x^3])/(5*b^2) + (2*e*x^2*Sqrt[a + b*x^3])/(7
*b^2) - (80*a*e*Sqrt[a + b*x^3])/(21*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) + (40*Sqrt[2 - Sqrt[3]]*a^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (1
6*Sqrt[2 + Sqrt[3]]*a*(14*b^(1/3)*d - 25*(1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
```

```
)*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(105*3^(1/4)*b^(
8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
```

```
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}} - \frac{2\int \frac{a^2bd+2a^2bex-3ab^2cx^2-\frac{3}{2}ab^2dx^3-\frac{3}{2}ab^2ex^4}{\sqrt{a+bx^3}}\,dx}{3ab^3}\\
&= \frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2} - \frac{4\int \frac{\frac{7}{2}a^2b^2d+10a^2b^2ex-\frac{21}{2}ab^3cx^2-\frac{21}{4}ab^3dx^3}{\sqrt{a+bx^3}}\,dx}{21ab^4}\\
&= \frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}} + \frac{2dx\sqrt{a+bx^3}}{5b^2} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2} - \frac{8\int \frac{14a^2b^3d+25a^2b^3ex-\frac{105}{4}ab^4cx^2}{\sqrt{a+bx^3}}\,dx}{105ab^5}\\
&= \frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}} + \frac{2dx\sqrt{a+bx^3}}{5b^2} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2}\\
&\quad - \frac{8\int \frac{14a^2b^3d+25a^2b^3ex}{\sqrt{a+bx^3}}\,dx}{105ab^5} + \frac{(2c)\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{b}\\
&= \frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}} + \frac{4c\sqrt{a+bx^3}}{3b^2} + \frac{2dx\sqrt{a+bx^3}}{5b^2} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2}\\
&\quad - \frac{(40ae)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{21b^{7/3}} - \frac{\left(8a\left(14\sqrt[3]{b}d-25\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{105b^{7/3}}
\end{aligned}$$

$$= \frac{2x(ad + aex - bcx^2)}{3b^2\sqrt{a + bx^3}} + \frac{4c\sqrt{a + bx^3}}{3b^2} + \frac{2dx\sqrt{a + bx^3}}{5b^2} + \frac{2ex^2\sqrt{a + bx^3}}{7b^2} - \frac{80ae\sqrt{a + bx^3}}{21b^{8/3}\left(\left(1 + \sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x\right)} + \frac{40\sqrt{2 - \sqrt{3}}a^{4/3}e\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x}\right)|-7 - 4\sqrt{3}\right)}{7\ 3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x\right)^2}}\sqrt{a + bx^3}} - \frac{16\sqrt{2 + \sqrt{3}}a\left(14\sqrt[3]{b}d - 25\left(1 - \sqrt{3}\right)\sqrt[3]{a}e\right)\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x}\right)\right.}{105\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x\right)^2}}\sqrt{a + bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.23

$$\int \frac{x^5(c + dx + ex^2)}{(a + bx^3)^{3/2}}\,dx = \frac{2\left(70ac + 56adx - 150aex^2 + 35bcx^3 + 21bdx^4 + 15bex^5 - 56adx\sqrt{1 + \frac{bx^3}{a}}\,\text{Hyper}\right.}{105b}$$

```
[In] Integrate[(x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(70*a*c + 56*a*d*x - 150*a*e*x^2 + 35*b*c*x^3 + 21*b*d*x^4 + 15*b*e*x^5
- 56*a*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a
)] + 150*a*e*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*
x^3)/a)]))/(105*b^2*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 813, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	813
default	Expression too large to display	836
risch	Expression too large to display	1587

```
[In] int(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(-1/3/b^3*a*e*x^2-1/3*a*d/b^3*x-1/3*a*c/b^3)/((x^3+a/b)*b)^(1/2)+2/7*e
*x^2*(b*x^3+a)^(1/2)/b^2+2/5*d*x*(b*x^3+a)^(1/2)/b^2+2/3*c*(b*x^3+a)^(1/2)/
b^2+32/45*I*a*d/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)
^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2))+80/63*I*a*e/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b
^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-
I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.23

$$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = -\frac{2\left(112\,(abdx^3+a^2d)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-200\,(abex^3+a^2e)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{wei}\right.\right.}{105\,(b^4x^3}$$

```
[In] integrate(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/105*(112*(a*b*d*x^3 + a^2*d)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) -
 200*(a*b*e*x^3 + a^2*e)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInv
erse(0, -4*a/b, x)) - (15*b^2*e*x^5 + 21*b^2*d*x^4 + 35*b^2*c*x^3 + 50*a*b*
e*x^2 + 56*a*b*d*x + 70*a*b*c)*sqrt(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 6.88 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.22

$$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = c\left(\begin{cases} \frac{4a}{3b^2\sqrt{a+bx^3}} + \frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{dx^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)} + \frac{ex^8\Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{3}\right)}$$

```
[In] integrate(x**5*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
[Out] c*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3)),
 Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + d*x**7*gamma(7/3)*hyper((3/2, 7/3)
, (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(10/3)) + e*x**8*gamm
a(8/3)*hyper((3/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gam
ma(11/3))
```

Maxima [F]

$$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x^5}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] 2/3*c*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2)) + integrate((e*x^7 +
d*x^6)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Giac [F]

$$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x^5}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d*x + c)*x^5/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^5\,(e\,x^2+d\,x+c)}{(b\,x^3+a)^{3/2}}\,dx$$

```
[In] int((x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x)
[Out] int((x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x)
```

3.438 $\int \frac{x^4\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 574

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx=\frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}}+\frac{4d\sqrt{a+bx^3}}{3b^2}$$
$$+\frac{2ex\sqrt{a+bx^3}}{5b^2}+\frac{8c\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{8\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(5\left(1-\sqrt{3}\right)b^{2/3}c+4a^{2/3}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/3*x*(-b*d*x^2-b*c*x+a*e)/b^2/(b*x^3+a)^(1/2)+4/3*d*(b*x^3+a)^(1/2)/b^2+2/
5*e*x*(b*x^3+a)^(1/2)/b^2+8/3*c*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))-4/3*a^(1/3)*c*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)*3^(1/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-8/45*a^(1/3)*(a^(1/3)+b^(1/3)
```

```
*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(4*a^(2/3)*e+5*b^(2/3)*c*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)*3^(3/4)/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1842, 1902, 1900, 267, 1892, 224, 1891}

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = -\frac{8\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(4a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{8c\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}} + \frac{4d\sqrt{a+bx^3}}{3b^2} + \frac{2ex\sqrt{a+bx^3}}{5b^2}$$

```
[In] Int[(x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*d*Sqrt[a + b*x^3
])/(3*b^2) + (2*e*x*Sqrt[a + b*x^3])/(5*b^2) + (8*c*Sqrt[a + b*x^3])/(3*b^(
5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*c*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4
)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2 + Sqrt[3]]*a^(1/3)*(5*(1 - Sqrt[3])
*b^(2/3)*c + 4*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
```

```
, -7 - 4*Sqrt[3]])/(15*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
```

```
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}} - \frac{2\int \frac{a^2e-2abcx-3abdx^2-\frac{3}{2}abex^3}{\sqrt{a+bx^3}}\,dx}{3ab^2} \\
&= \frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}} + \frac{2ex\sqrt{a+bx^3}}{5b^2} - \frac{4\int \frac{4a^2be-5ab^2cx-\frac{15}{2}ab^2dx^2}{\sqrt{a+bx^3}}\,dx}{15ab^3} \\
&= \frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}} + \frac{2ex\sqrt{a+bx^3}}{5b^2} - \frac{4\int \frac{4a^2be-5ab^2cx}{\sqrt{a+bx^3}}\,dx}{15ab^3} + \frac{(2d)\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{b} \\
&= \frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}} + \frac{4d\sqrt{a+bx^3}}{3b^2} + \frac{2ex\sqrt{a+bx^3}}{5b^2} \\
&\quad + \frac{(4c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{3b^{4/3}} - \frac{\left(4\sqrt[3]{a}\left(5\left(1-\sqrt{3}\right)b^{2/3}c+4a^{2/3}e\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{15b^2}
\end{aligned}$$

$$= \frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}} + \frac{4d\sqrt{a+bx^3}}{3b^2} + \frac{2ex\sqrt{a+bx^3}}{5b^2} + \frac{8c\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$- \frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{8\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(5\left(1-\sqrt{3}\right)b^{2/3}c+4a^{2/3}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\cdots}{\left(1+\sqrt{3}\right)\cdots}\right.\right.}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.22

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(10ad+8aex+15bcx^2+5bdx^3+3bex^4-8aex\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\cdots\right.\right.}{15b^2\sqrt{a+bx^3}}$$

```
[In] Integrate[(x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(10*a*d + 8*a*e*x + 15*b*c*x^2 + 5*b*d*x^3 + 3*b*e*x^4 - 8*a*e*x*Sqrt[1
+ (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 15*b*c*x^2*Sq
rt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(15*b^2*
Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 793, normalized size of antiderivative = 1.38

method	result	size
elliptic	Expression too large to display	793
default	Expression too large to display	817
risch	Expression too large to display	1123

```
[In] int(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(1/3*c/b^2*x^2-1/3/b^3*a*e*x-1/3*a*d/b^3)/((x^3+a/b)*b)^(1/2)+2/5*e*x*
(b*x^3+a)^(1/2)/b^2+2/3*d*(b*x^3+a)^(1/2)/b^2+32/45*I*a*e/b^3*3^(1/2)*(-a*b
^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-8/9*I*c/b^2*3^(1/2)*(-a*b^
2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.23

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = -\frac{2\left(16\,(abex^3+a^2e)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+20\,(b^2cx^3+abc)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierst}\right.\right.}{15\,(b^4x^3+ab^3)}$$

```
[In] integrate(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(16*(a*b*e*x^3 + a^2*e)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + 2
0*(b^2*c*x^3 + a*b*c)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInvers
e(0, -4*a/b, x)) - (3*b^2*e*x^4 + 5*b^2*d*x^3 - 5*b^2*c*x^2 + 8*a*b*e*x + 1
0*a*b*d)*sqrt(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 5.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.22

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = d\left(\begin{cases} \frac{4a}{3b^2\sqrt{a+bx^3}}+\frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)} + \frac{ex^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate(x**4*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
[Out] d*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3)),
 Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + c*x**5*gamma(5/3)*hyper((3/2, 5/3)
, (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3)) + e*x**7*gamma(
7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma
(10/3))
```

Maxima [F]

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x^4}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d*x + c)*x^4/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x^4}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d*x + c)*x^4/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx=\int \frac{x^4\,(e\,x^2+d\,x+c)}{(b\,x^3+a)^{3/2}}\,dx$$

```
[In] int((x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x)
[Out] int((x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x)
```

3.439 $\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 542

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = -\frac{2x(c+dx+ex^2)}{3b\sqrt{a+bx^3}}+\frac{4e\sqrt{a+bx^3}}{3b^2}+\frac{8d\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c-2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/3*x*(e*x^2+d*x+c)/b/(b*x^3+a)^(1/2)+4/3*e*(b*x^3+a)^(1/2)/b^2+8/3*d*(b*x
^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-4/3*a^(1/3)*d*(a^(1/3)+
b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(5/3)/(
b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)+4/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c-2*a^(1/3)*d*(1-3
^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
```

```
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(
a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1842, 1900, 267, 1892, 224, 1891}

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\right.}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{8d\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{4e\sqrt{a+bx^3}}{3b^2}-\frac{2x(c+dx+ex^2)}{3b\sqrt{a+bx^3}}$$

```
[In] Int[(x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*x*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (4*e*Sqrt[a + b*x^3])/(3*b
^2) + (8*d*Sqrt[a + b*x^3])/(3*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*Sqrt[2 +
 Sqrt[3]]*(b^(1/3)*c - 2*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
```

```
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$\text{integral} = -\frac{2x(c+dx+ex^2)}{3b\sqrt{a+bx^3}} - \frac{2\int \frac{-abc-2abdx-3abex^2}{\sqrt{a+bx^3}}\,dx}{3ab^2}$$

$$= -\frac{2x(c+dx+ex^2)}{3b\sqrt{a+bx^3}} - \frac{2\int \frac{-abc-2abdx}{\sqrt{a+bx^3}}\,dx}{3ab^2} + \frac{(2e)\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{b}$$

$$= -\frac{2x(c+dx+ex^2)}{3b\sqrt{a+bx^3}} + \frac{4e\sqrt{a+bx^3}}{3b^2} + \frac{(4d)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{3b^{4/3}} + \frac{\left(2\left(\sqrt[3]{b}c-2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3b^{4/3}}$$

$$= -\frac{2x(c+dx+ex^2)}{3b\sqrt{a+bx^3}} + \frac{4e\sqrt{a+bx^3}}{3b^2} + \frac{8d\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c-2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.22

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(2ae-bcx+3bdx^2+bex^3+bcx\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)-\right.}{3b^2\sqrt{a+bx^3}}$$

```
[In] Integrate[(x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
[Out] (2*(2*a*e - b*c*x + 3*b*d*x^2 + b*e*x^3 + b*c*x*Sqrt[1 + (b*x^3)/a]*Hyperge
ometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 3*b*d*x^2*Sqrt[1 + (b*x^3)/a]*Hyp
ergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(3*b^2*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.43

method	result
elliptic	$-\frac{2b\left(\frac{d\,x^2}{3b^2}+\frac{cx}{3b^2}-\frac{ae}{3b^3}\right)}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2e\sqrt{b\,x^3+a}}{3b^2}-\frac{4ic\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{\cdots}}{\cdots}$
default	Expression too large to display
risch	Expression too large to display

```
[In] int(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(1/3*d*x^2/b^2+1/3*c/b^2*x-1/3*a*e/b^3)/((x^3+a/b)*b)^(1/2)+2/3*e*(b*x
^3+a)^(1/2)/b^2-4/9*I*c/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*
(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2))-8/9*I*d/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(
-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(
-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.20

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(2\left(bcx^3+ac\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-4\left(bdx^3+ad\right)\sqrt{b}\text{weierstrassZe}\right.}{3\left(b^3x^3\right.}$$

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*(b*c*x^3 + a*c)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - 4*(b*d*x
^3 + a*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b,
 x)) + (b*e*x^3 - b*d*x^2 - b*c*x + 2*a*e)*sqrt(b*x^3 + a))/(b^3*x^3 + a*b^
2)
```

Sympy [A] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.24

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = e\left(\begin{cases}\frac{4a}{3b^2\sqrt{a+bx^3}}+\frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0\\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + \frac{cx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{dx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] e*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3)),
 Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + c*x**4*gamma(4/3)*hyper((4/3, 3/2)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3)) + d*x**5*gamma(
5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(
8/3))
```

Maxima [F]

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x^3}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x^3}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^3\,(e\,x^2+d\,x+c)}{(b\,x^3+a)^{3/2}}\,dx$$

```
[In] int((x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x)
[Out] int((x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x)
```

3.440 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 522

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = -\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}}+\frac{8e\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}d-2\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/3*(e*x^2+d*x+c)/b/(b*x^3+a)^(1/2)+8/3*e*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))-4/3*a^(1/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/9*(a^(1/3)+b^(1/
3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*(b^(1/3)*d-2*a^(1/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
```

```
2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1837, 1892, 224, 1891}

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}d-2\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\text{EllipticF}}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{8e\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}}$$

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (8*e*Sqrt[a + b*x^3])/(3*b^(
5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*e*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4
)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d - 2*(1 - Sqrt
[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}}+\frac{2\int \frac{d+2ex}{\sqrt{a+bx^3}}\,dx}{3b}$$

$$= -\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}}+\frac{(4e)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{3b^{4/3}}+\frac{\left(2\left(d-\frac{2\left(1-\sqrt{3}\right)\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3b}$$

$$= -\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}} + \frac{8e\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$- \frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}d-2\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-}\right.\right.}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.20

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2dx\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)-2\left(c+x(d-3ex)+3ex^2\sqrt{1\right.}{3b\sqrt{a+bx^3}}$$

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] -
 2*(c + x*(d - 3*e*x) + 3*e*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3,
3/2, 5/3, -((b*x^3)/a)]))/(3*b*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 759, normalized size of antiderivative = 1.45

method	result
elliptic	$-\frac{2b\left(\frac{e\,x^2}{3b^2}+\frac{dx}{3b^2}+\frac{c}{3b^2}\right)}{\sqrt{\left(x^3+\frac{a}{b}\right)b}} - \frac{4id\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{(}}}{9b^2\sqrt{b}}$
default	Expression too large to display

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(1/3*e/b^2*x^2+1/3*d*x/b^2+1/3*c/b^2)/((x^3+a/b)*b)^(1/2)-4/9*I*d/b^2*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-8/9*I/b^2*e*3
^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.19

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(2\left(bdx^3+ad\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-4\left(bex^3+ae\right)\sqrt{b}\text{weierstrassZ}\right.}{3\left(b^3x^3+\right.}$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*(b*d*x^3 + a*d)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - 4*(b*e*x
^3 + a*e)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b,
 x)) - (b*e*x^2 + b*d*x + b*c)*sqrt(b*x^3 + a))/(b^3*x^3 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.21

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = c\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{ex^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] c*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True
)) + d*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/
(3*a**(3/2)*gamma(7/3)) + e*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**
3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x^2}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*c/(sqrt(b*x^3 + a)*b) + integrate((e*x^4 + d*x^3)*sqrt(b*x^3 + a)/(b^2
*x^6 + 2*a*b*x^3 + a^2), x)
```

Giac [F]

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x^2}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^2\,(e\,x^2+d\,x+c)}{(b\,x^3+a)^{3/2}}\,dx$$

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x)
[Out] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x)
```

3.441 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 561

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = -\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}} - \frac{2d\sqrt{a+bx^3}}{3ab} - \frac{2c\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{\sqrt{2-\sqrt{3}}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(b^{2/3}\left(c-\sqrt{3}c\right)+2a^{2/3}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}\right.\right.}{3\sqrt[4]{3}a^{2/3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/3*x*(-b*d*x^2-b*c*x+a*e)/a/b/(b*x^3+a)^(1/2)-2/3*d*(b*x^3+a)^(1/2)/a/b-2
/3*c*(b*x^3+a)^(1/2)/a/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/3*c*(a^(1/
3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3
)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)+2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(
```

```
1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(2*a^(2/3)*e+b^(
2/3)*(c-c*3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b^(4/3
)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1842, 1900, 267, 1892, 224, 1891}

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2a^{2/3}e+b^{2/3}(c-\sqrt{3}c)\right)\text{EllipticF}\left(\right.}{3\sqrt[4]{3}a^{2/3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\sqrt{2-\sqrt{3}}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2c\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}}-\frac{2d\sqrt{a+bx^3}}{3ab}$$

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*x*(a*e - b*c*x - b*d*x^2))/(3*a*b*Sqrt[a + b*x^3]) - (2*d*Sqrt[a + b*x^
3])/(3*a*b) - (2*c*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*
Sqrt[2 + Sqrt[3]]*(b^(2/3)*(c - Sqrt[3]*c) + 2*a^(2/3)*e)*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(2/3)*b^(4/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
```

```
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$\text{integral} = -\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}} - \frac{2\int \frac{-ae+\frac{bcx}{2}+\frac{3}{2}bdx^2}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= -\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}} - \frac{2\int \frac{-ae+\frac{bcx}{2}}{\sqrt{a+bx^3}}\,dx}{3ab} - \frac{d\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{a}$$

$$= -\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}} - \frac{2d\sqrt{a+bx^3}}{3ab} - \frac{c\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{3a\sqrt[3]{b}} + \frac{\left(\frac{(1-\sqrt{3})b^{2/3}c}{a^{2/3}}+2e\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3b}$$

$$= -\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}} - \frac{2d\sqrt{a+bx^3}}{3ab} - \frac{2c\sqrt{a+bx^3}}{3ab^{2/3}\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt{2-\sqrt{3}}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\frac{(1-\sqrt{3})b^{2/3}c}{a^{2/3}}+2e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{3\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.19

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{-4a(d+ex)+4aex\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+3bcx^2\sqrt{1+\frac{bx^3}{a}}}{6ab\sqrt{a+bx^3}}$$

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-4*a*(d + e*x) + 4*a*e*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4
/3, -((b*x^3)/a)] + 3*b*c*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/
2, 5/3, -((b*x^3)/a)])/(6*a*b*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.36

method	result
elliptic	$-\frac{2b\left(-\frac{c\,x^2}{3ba}+\frac{ex}{3b^2}+\frac{d}{3b^2}\right)}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{4ie\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b\right.}{2b}\right.}{}}}{9b}$
default	Expression too large to display

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(-1/3/b/a*c*x^2+1/3*e/b^2*x+1/3*d/b^2)/((x^3+a/b)*b)^(1/2)-4/9*I/b^2*e
*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/9*I*c/a*3^
(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
```

```
*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.20

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(2\left(abex^3+a^2e\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\left(b^2cx^3+abc\right)\sqrt{b}\text{weierstrassZ}\right.}{3\left(ab^3x^3+\right.}$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*(a*b*e*x^3 + a^2*e)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (b^2
*c*x^3 + a*b*c)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -
4*a/b, x)) + (b^2*c*x^2 - a*b*e*x - a*b*d)*sqrt(b*x^3 + a))/(a*b^3*x^3 + a^
2*b^2)
```

Sympy [A] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.19

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = d\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix} \frac{2}{3},\frac{3}{2} \\ \frac{5}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix} \frac{4}{3},\frac{3}{2} \\ \frac{7}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] d*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True
)) + c*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/
(3*a**(3/2)*gamma(5/3)) + e*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**
3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3))
```

Maxima [F]

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x\,(e\,x^2+d\,x+c)}{(b\,x^3+a)^{3/2}}\,dx$$

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x)
[Out] int((x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x)
```

3.442 $\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 532

$$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx = -\frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}}$$
$$+\frac{\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c+\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/3*(a*e-b*x*(d*x+c))/a/b/(b*x^3+a)^(1/2)-2/3*d*(b*x^3+a)^(1/2)/a/b^(2/3)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/3*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/9*(a^(1/
3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c+a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+
1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1868, 1892, 224, 1891}

$$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)\text{EllipticF}\left(\text{arc}\right.}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}}$$

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*d*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
(2*(a*e - b*x*(c + d*x)))/(3*a*b*Sqrt[a + b*x^3]) + (Sqrt[2 - Sqrt[3]]*d*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*
a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c + (1 -
Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{c}{2}+\frac{dx}{2}}{\sqrt{a+bx^3}}\,dx}{3a}$$

$$= -\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}} - \frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{3a\sqrt[3]{b}} + \frac{\left(c+\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3a}$$

$$= -\frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}}$$

$$+ \frac{\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{2\sqrt{2+\sqrt{3}}\left(c+\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.20

$$\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{-4ae+4bcx+2bcx\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+3bdx^2\sqrt{1+\frac{bx^3}{a}}\,\text{H}}{6ab\sqrt{a+bx^3}}$$

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^(3/2),x]
```

```
[Out] (-4*a*e + 4*b*c*x + 2*b*c*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2,
 4/3, -((b*x^3)/a)] + 3*b*d*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3,
3/2, 5/3, -((b*x^3)/a)])/(6*a*b*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.45

method	result
elliptic	$-\frac{2b\left(-\frac{d\,x^{2}}{3ab}-\frac{cx}{3ba}+\frac{e}{3b^{2}}\right)}{\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2ic\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b\right.}{2b}\right.}{}}}{9a}$
default	$c\left(\frac{2x}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\cdots\right.}{\left(-a\,b\right.}}}{9ab\sqrt{b\,x^{3}}}\right.$

[In] int((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)

```
[Out] -2*b*(-1/3/a/b*d*x^2-1/3/b/a*c*x+1/3*e/b^2)/((x^3+a/b)*b)^(1/2)-2/9*I*c/a*3
^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/9*I*d/a*3^
(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.18

$$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx = \frac{2\left((bcx^3+ac)\sqrt{b}\,\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+(bdx^3+ad)\sqrt{b}\,\text{weierstrassZeta}\left(0,-\right.\right.}{3\left(ab^2x^3+a^2b\right)}$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*((b*c*x^3 + a*c)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (b*d*x^3 +
 a*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))
 + (b*d*x^2 + b*c*x - a*e)*sqrt(b*x^3 + a))/(a*b^2*x^3 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 3.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.20

$$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx = e\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] e*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True
)) + c*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx = \int \frac{ex^2+dx+c}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx = \int \frac{ex^2+dx+c}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx = \int \frac{e\,x^2+d\,x+c}{(b\,x^3+a)^{3/2}}\,dx$$

```
[In] int((c + d*x + e*x^2)/(a + b*x^3)^(3/2),x)
```

```
[Out] int((c + d*x + e*x^2)/(a + b*x^3)^(3/2), x)
```

3.443 $\int \frac{c+dx+ex^2}{x(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 579

$$\int \frac{c+dx+ex^2}{x(a+bx^3)^{3/2}}\,dx = \frac{2x(ad+aex-bcx^2)}{3a^2\sqrt{a+bx^3}}+\frac{2c\sqrt{a+bx^3}}{3a^2}$$

$$-\frac{2e\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

$$+\frac{\sqrt{2-\sqrt{3}}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}d+\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/3*c*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)+2/3*x*(-b*c*x^2+a*e*x+a*d)/
a^2/(b*x^3+a)^(1/2)+2/3*c*(b*x^3+a)^(1/2)/a^2-2/3*e*(b*x^3+a)^(1/2)/a/b^(2/
3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/3*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(
1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/9*(a^
```

```
(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*d+a^(1/3)*e*(1-3^(1/2)))*(1/2*6^(1/
2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1843, 1846, 272, 65, 214, 1900, 267, 1892, 224, 1891}

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}e+\sqrt[3]{b}d\right)\text{EllipticF}\left(\text{ar}\right.}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{\sqrt{2-\sqrt{3}}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2c\,\text{arctanh}\left(\frac{\surd}{}\right.}{3a^{3/2}}$$
$$+\frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}}+\frac{2c\sqrt{a+bx^3}}{3a^2}-\frac{2e\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*c*Sqrt[a + b*x^3
])/(3*a^2) - (2*e*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)) - (2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)) + (Sqrt[2 -
Sqrt[3]]*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3
]])/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1
/3)*d + (1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
```

```
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$\text{integral} = \frac{2x(ad+aex-bcx^2)}{3a^2\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{3bc}{2}-\frac{bdx}{2}+\frac{1}{2}bex^2-\frac{3b^2cx^3}{2a}}{x\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2x(ad+aex-bcx^2)}{3a^2\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{bd}{2}+\frac{bex}{2}-\frac{3b^2cx^2}{2a}}{\sqrt{a+bx^3}}\,dx}{3ab} + \frac{c\int \frac{1}{x\sqrt{a+bx^3}}\,dx}{a}$$

$$= \frac{2x(ad+aex-bcx^2)}{3a^2\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{bd}{2}+\frac{bex}{2}}{\sqrt{a+bx^3}}\,dx}{3ab} + \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{3a} + \frac{(bc)\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{a^2}$$

$$= \frac{2x(ad+aex-bcx^2)}{3a^2\sqrt{a+bx^3}} + \frac{2c\sqrt{a+bx^3}}{3a^2} + \frac{(2c)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3ab} - \frac{e\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{3a\sqrt[3]{b}} + \frac{\left(d+\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3a}$$

$$= \frac{2x(ad+aex-bcx^2)}{3a^2\sqrt{a+bx^3}} + \frac{2c\sqrt{a+bx^3}}{3a^2} - \frac{2e\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}} + \frac{\sqrt{2-\sqrt{3}}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(d+\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|\right.}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.21

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx = \frac{4c\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{bx^3}{a}\right)+x\left(4d+2d\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\right.}{6a\sqrt{a+bx^3}}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (4*c*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^3)/a] + x*(4*d + 2*d*Sqrt[1 +
 (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 3*e*x*Sqrt[1 +
 (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(6*a*Sqrt[a +
b*x^3])
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	794
default	Expression too large to display	810

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(-1/3/a/b*x^2*e-1/3/a/b*d*x-1/3/b/a*c)/((x^3+a/b)*b)^(1/2)-2/9*I*d/a*3
^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/9*I/a*e*3^
(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*c*arctanh((b*x^3+a)^(1/2)/a^(
1/2))/a^(3/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.60

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx=\left[\frac{(b^2cx^3+abc)\sqrt{a}\log\left(\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+4\,(abdx^3+a^2d)\sqrt{b}\text{weiers}}{}\right.$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*((b^2*c*x^3 + a*b*c)*sqrt(a)*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a
)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 4*(a*b*d*x^3 + a^2*d)*sqrt(b)*wei
erstrassPInverse(0, -4*a/b, x) + 4*(a*b*e*x^3 + a^2*e)*sqrt(b)*weierstrassZ
```

```
eta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + 4*(a*b*e*x^2 + a*b*d*x
+ a*b*c)*sqrt(b*x^3 + a))/(a^2*b^2*x^3 + a^3*b), 1/3*((b^2*c*x^3 + a*b*c)*s
qrt(-a)*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2))
+ 2*(a*b*d*x^3 + a^2*d)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + 2*(a*b*
e*x^3 + a^2*e)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4
*a/b, x)) + 2*(a*b*e*x^2 + a*b*d*x + a*b*c)*sqrt(b*x^3 + a))/(a^2*b^2*x^3 +
 a^3*b)]
```

Sympy [A] (verification not implemented)

Time = 5.70 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.46

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx = c\left(\frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^2bx^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^2bx^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}\right)+\frac{dx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{ex^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**(3/2),x)
```

```
[Out] c*(2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*
x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) +
1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/2)
 + 3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/
2) + 3*a**(7/2)*b*x**3)) + d*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3)) + e*x**2*gamma(2/3)*hyper((2/3,
3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{ex^2+dx+c}{\left(bx^3+a\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{ex^2+dx+c}{\left(bx^3+a\right)^{\frac{3}{2}}x}\,dx$$

[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{e\,x^2+d\,x+c}{x\,{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)),x)

[Out] int((c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)), x)

3.444 $\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 607

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^{3/2}}\,dx = \frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}}+\frac{2d\sqrt{a+bx^3}}{3a^2} - \frac{c\sqrt{a+bx^3}}{a^2x}+\frac{5\sqrt[3]{b}c\sqrt{a+bx^3}}{3a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2d\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\sqrt{2+\sqrt{3}}\left(5\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{3\sqrt[4]{3}a^{5/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)+2/3*x*(-b*d*x^2-b*c*x+a*e)/
a^2/(b*x^3+a)^(1/2)+2/3*d*(b*x^3+a)^(1/2)/a^2-c*(b*x^3+a)^(1/2)/a^2/x+5/3*b
^(1/3)*c*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-5/6*b^(1/3)*c*
(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/
a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+
```

```
3^(1/2)))^2)^(1/2)-1/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(-2*a^(2/3)*e+5*b^
(2/3)*c*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b^(1/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1843, 1849, 1846, 272, 65, 214, 1900, 267, 1892, 224, 1891}

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^{3/2}}\,dx =$$

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{3\sqrt[4]{3}a^{5/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2d\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}+\frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}}-\frac{c\sqrt{a+bx^3}}{a^2x}$$
$$+\frac{5\sqrt[3]{b}c\sqrt{a+bx^3}}{3a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2d\sqrt{a+bx^3}}{3a^2}$$

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*d*Sqrt[a + b*x^3
])/(3*a^2) - (c*Sqrt[a + b*x^3])/(a^2*x) + (5*b^(1/3)*c*Sqrt[a + b*x^3])/(3
*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*d*ArcTanh[Sqrt[a + b*x^3]/Sq
rt[a]])/(3*a^(3/2)) - (5*Sqrt[2 - Sqrt[3]]*b^(1/3)*c*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*a^(5/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
```

```
b*x^3]) - (Sqrt[2 + Sqrt[3]]*(5*(1 - Sqrt[3])*b^(2/3)*c - 2*a^(2/3)*e)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*
a^(5/3)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
```

```
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
```

```
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$\text{integral} = \frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{3bc}{2}-\frac{3bdx}{2}-\frac{1}{2}bex^2-\frac{b^2cx^3}{2a}-\frac{3b^2dx^4}{2a}}{x^2\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}} - \frac{c\sqrt{a+bx^3}}{a^2x} + \frac{\int \frac{3abd+abex+\frac{5}{2}b^2cx^2+3b^2dx^3}{x\sqrt{a+bx^3}}\,dx}{3a^2b}$$

$$= \frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}} - \frac{c\sqrt{a+bx^3}}{a^2x} + \frac{\int \frac{abe+\frac{5}{2}b^2cx+3b^2dx^2}{\sqrt{a+bx^3}}\,dx}{3a^2b} + \frac{d\int \frac{1}{x\sqrt{a+bx^3}}\,dx}{a}$$

$$= \frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}} - \frac{c\sqrt{a+bx^3}}{a^2x} + \frac{\int \frac{abe+\frac{5}{2}b^2cx}{\sqrt{a+bx^3}}\,dx}{3a^2b} + \frac{d\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{3a} + \frac{(bd)\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{a^2}$$

$$= \frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}} + \frac{2d\sqrt{a+bx^3}}{3a^2} - \frac{c\sqrt{a+bx^3}}{a^2x} + \frac{(5b^{2/3}c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{6a^2} + \frac{(2d)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3ab} - \frac{\left(5\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{6a^{5/3}}$$

$$= \frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}} + \frac{2d\sqrt{a+bx^3}}{3a^2} - \frac{c\sqrt{a+bx^3}}{a^2x} + \frac{5\sqrt[3]{b}c\sqrt{a+bx^3}}{3a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$
$$- \frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$- \frac{\sqrt{2+\sqrt{3}}\left(5\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{3\sqrt[4]{3}a^{5/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.20

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \frac{2dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{bx^3}{a}\right)-3c\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3}\right.}{3ax\sqrt{a+bx^3}}$$

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*d*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^3)/a] - 3*c*Sqrt[1 + (b*x^3
)/a]*Hypergeometric2F1[-1/3, 3/2, 2/3, -((b*x^3)/a)] + e*x^2*(2 + Sqrt[1 +
(b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(3*a*x*Sqrt[a +
 b*x^3])
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	806
default	Expression too large to display	825
risch	Expression too large to display	1306

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -c*(b*x^3+a)^(1/2)/a^2/x-2*b*(1/3/a^2*c*x^2-1/3/a/b*x*e-1/3/a/b*d)/((x^3+a/
b)*b)^(1/2)-2/9*I/a*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*
b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(
-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))-5/9*I*c/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b
^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-
I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
```

```
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*d*arct
anh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.62

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^{3/2}}\,dx = \left[\frac{(b^2dx^4+abdx)\sqrt{a}\log\left(\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+4\,(abex^4+a^2ex)\sqrt{b}\text{wei}}{}\right.$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*((b^2*d*x^4 + a*b*d*x)*sqrt(a)*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2
*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 4*(a*b*e*x^4 + a^2*e*x)*sqrt(b)
*weierstrassPInverse(0, -4*a/b, x) - 10*(b^2*c*x^4 + a*b*c*x)*sqrt(b)*weier
strassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - 2*(5*b^2*c*x^3 -
 2*a*b*e*x^2 - 2*a*b*d*x + 3*a*b*c)*sqrt(b*x^3 + a))/(a^2*b^2*x^4 + a^3*b*x
), 1/3*((b^2*d*x^4 + a*b*d*x)*sqrt(-a)*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3
+ a)*sqrt(-a)/(a*b*x^3 + a^2)) + 2*(a*b*e*x^4 + a^2*e*x)*sqrt(b)*weierstras
sPInverse(0, -4*a/b, x) - 5*(b^2*c*x^4 + a*b*c*x)*sqrt(b)*weierstrassZeta(0
, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (5*b^2*c*x^3 - 2*a*b*e*x^2 -
 2*a*b*d*x + 3*a*b*c)*sqrt(b*x^3 + a))/(a^2*b^2*x^4 + a^3*b*x)]
```

Sympy [A] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.44

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^{3/2}}\,dx = d\left(\frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^2bx^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^2bx^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}\right)+\frac{c\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{2}{3}\right)}+\frac{ex\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**(3/2),x)
```

```
[Out] d*(2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*
x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) +
1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/2)
 + 3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/
2) + 3*a**(7/2)*b*x**3)) + c*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(3/2)*x*gamma(2/3)) + e*x*gamma(1/3)*hyper((1/3, 3
/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3))
```

Maxima [F]

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{ex^2+dx+c}{\left(bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{ex^2+dx+c}{\left(bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 10.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.22

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^{3/2}}\,dx=\frac{2\,d}{3\,a\,\sqrt{b\,x^3+a}}+\frac{d\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{3/2}}-\frac{2\,c\,{\left(\frac{a}{b\,x^3}+1\right)}^{3/2}\,{{}}_2{\mathrm{F}}_1\left(\frac{3}{2},\frac{11}{6};\ \frac{17}{6};\ -\frac{a}{b\,x^3}\right)}{11\,x\,{\left(b\,x^3+a\right)}^{3/2}}+\frac{e\,x\,{\left(\frac{b\,x^3}{a}+1\right)}^{3/2}\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{3},\frac{3}{2};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{{\left(b\,x^3+a\right)}^{3/2}}$$

```
[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^(3/2)),x)
[Out] (2*d)/(3*a*(a + b*x^3)^(1/2)) + (d*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a
 + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(3/2)) - (2*c*(a/(b*x^3) + 1)^(3/2)*
hypergeom([3/2, 11/6], 17/6, -a/(b*x^3)))/(11*x*(a + b*x^3)^(3/2)) + (e*x*(
(b*x^3)/a + 1)^(3/2)*hypergeom([1/3, 3/2], 4/3, -(b*x^3)/a))/(a + b*x^3)^(3
/2)
```

3.445 $\int x^3\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 733

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$= -\frac{4a^2e\sqrt{a+bx^3}}{45b^2}+\frac{6a(17bc-8af)x\sqrt{a+bx^3}}{935b^2}+\frac{6a(19bd-10ag)x^2\sqrt{a+bx^3}}{1729b^2}$$
$$+\frac{2aex^3\sqrt{a+bx^3}}{45b}+\frac{6afx^4\sqrt{a+bx^3}}{187b}+\frac{6agx^5\sqrt{a+bx^3}}{247b}-\frac{24a^2(19bd-10ag)\sqrt{a+bx^3}}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{2x^3\sqrt{a+bx^3}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835}$$
$$+\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19bd-10ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7\right.}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(1729\sqrt[3]{b}(17bc-8af)-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}(19bd-10ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}}}{1616615b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -4/45*a^2*e*(b*x^3+a)^(1/2)/b^2+6/935*a*(-8*a*f+17*b*c)*x*(b*x^3+a)^(1/2)/b
^2+6/1729*a*(-10*a*g+19*b*d)*x^2*(b*x^3+a)^(1/2)/b^2+2/45*a*e*x^3*(b*x^3+a)
^(1/2)/b+6/187*a*f*x^4*(b*x^3+a)^(1/2)/b+6/247*a*g*x^5*(b*x^3+a)^(1/2)/b+2/
692835*x^3*(36465*g*x^5+40755*f*x^4+46189*e*x^3+53295*d*x^2+62985*c*x)*(b*x
```

```
^3+a)^(1/2)-24/1729*a^2*(-10*a*g+19*b*d)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))+12/1729*3^(1/4)*a^(7/3)*(-10*a*g+19*b*d)*(a^(1/3)+b^(
1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1
/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-4
/1616615*3^(3/4)*a^2*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1729*b^(1/3)*(-8*a*
f+17*b*c)-1870*a^(1/3)*(-10*a*g+19*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/
2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 733, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1850, 1902, 1608, 1900, 267, 1892, 224, 1891}

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$

$$= \frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19bd-10ag)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{24a^2\sqrt{a+bx^3}(19bd-10ag)}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{4a^2e\sqrt{a+bx^3}}{45b^2}$$

$$-\frac{4\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right.}{1616615b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx}}$$

$$+\frac{6ax\sqrt{a+bx^3}(17bc-8af)}{935b^2}+\frac{6ax^2\sqrt{a+bx^3}(19bd-10ag)}{1729b^2}$$
$$+\frac{2x^3\sqrt{a+bx^3}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835}$$
$$+\frac{2aex^3\sqrt{a+bx^3}}{45b}+\frac{6afx^4\sqrt{a+bx^3}}{187b}+\frac{6agx^5\sqrt{a+bx^3}}{247b}$$

```
[In] Int[x^3*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (-4*a^2*e*Sqrt[a + b*x^3])/(45*b^2) + (6*a*(17*b*c - 8*a*f)*x*Sqrt[a + b*x^
3])/(935*b^2) + (6*a*(19*b*d - 10*a*g)*x^2*Sqrt[a + b*x^3])/(1729*b^2) + (2
*a*e*x^3*Sqrt[a + b*x^3])/(45*b) + (6*a*f*x^4*Sqrt[a + b*x^3])/(187*b) + (6
*a*g*x^5*Sqrt[a + b*x^3])/(247*b) - (24*a^2*(19*b*d - 10*a*g)*Sqrt[a + b*x^
3])/(1729*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^3*Sqrt[a + b*
x^3]*(62985*c*x + 53295*d*x^2 + 46189*e*x^3 + 40755*f*x^4 + 36465*g*x^5))/6
92835 + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*b*d - 10*a*g)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(8/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3]) - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1729*b^(1/3)*(17*b*c - 8*
a*f) - 1870*(1 - Sqrt[3])*a^(1/3)*(19*b*d - 10*a*g))*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1616615*b^(8/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
```

```
GtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
```

```
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x^3\sqrt{a+bx^3}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835} \\
&\quad + \frac{1}{2}(3a)\int \frac{x^3\left(\frac{2c}{11}+\frac{2dx}{13}+\frac{2ex^2}{15}+\frac{2fx^3}{17}+\frac{2gx^4}{19}\right)}{\sqrt{a+bx^3}}\,dx \\
&= \frac{6agx^5\sqrt{a+bx^3}}{247b} \\
&\quad + \frac{2x^3\sqrt{a+bx^3}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835} \\
&\quad + \frac{(3a)\int \frac{x^3\left(\frac{13bc}{11}+\frac{1}{19}(19bd-10ag)x+\frac{13}{15}bex^2+\frac{13}{17}bfx^3\right)}{\sqrt{a+bx^3}}\,dx}{13b} \\
&= \frac{6afx^4\sqrt{a+bx^3}}{187b}+\frac{6agx^5\sqrt{a+bx^3}}{247b} \\
&\quad + \frac{2x^3\sqrt{a+bx^3}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835} \\
&\quad + \frac{(6a)\int \frac{x^3\left(\frac{13}{34}b(17bc-8af)+\frac{11}{38}b(19bd-10ag)x+\frac{143}{30}b^2ex^2\right)}{\sqrt{a+bx^3}}\,dx}{143b^2} \\
&= \frac{2aex^3\sqrt{a+bx^3}}{45b}+\frac{6afx^4\sqrt{a+bx^3}}{187b}+\frac{6agx^5\sqrt{a+bx^3}}{247b} \\
&\quad + \frac{2x^3\sqrt{a+bx^3}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835} \\
&\quad + \frac{(4a)\int \frac{-\frac{143}{10}ab^2ex^2+\frac{117}{68}b^2(17bc-8af)x^3+\frac{99}{76}b^2(19bd-10ag)x^4}{\sqrt{a+bx^3}}\,dx}{429b^3} \\
&= \frac{2aex^3\sqrt{a+bx^3}}{45b}+\frac{6afx^4\sqrt{a+bx^3}}{187b}+\frac{6agx^5\sqrt{a+bx^3}}{247b} \\
&\quad + \frac{2x^3\sqrt{a+bx^3}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835} \\
&\quad + \frac{(4a)\int \frac{x^2\left(-\frac{143}{10}ab^2e+\frac{117}{68}b^2(17bc-8af)x+\frac{99}{76}b^2(19bd-10ag)x^2\right)}{\sqrt{a+bx^3}}\,dx}{429b^3} \\
&= \frac{6a(19bd-10ag)x^2\sqrt{a+bx^3}}{1729b^2}+\frac{2aex^3\sqrt{a+bx^3}}{45b}+\frac{6afx^4\sqrt{a+bx^3}}{187b}+\frac{6agx^5\sqrt{a+bx^3}}{247b} \\
&\quad + \frac{2x^3\sqrt{a+bx^3}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835} \\
&\quad + \frac{(8a)\int \frac{-\frac{99}{38}ab^2(19bd-10ag)x-\frac{1001}{20}ab^3ex^2+\frac{819}{136}b^3(17bc-8af)x^3}{\sqrt{a+bx^3}}\,dx}{3003b^4}
\end{aligned}
$$

$$= \frac{6a(19bd-10ag)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2aex^3\sqrt{a+bx^3}}{45b} + \frac{6afx^4\sqrt{a+bx^3}}{187b} + \frac{6agx^5\sqrt{a+bx^3}}{247b} + \frac{2x^3\sqrt{a+bx^3}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835} + \frac{(8a)\int \frac{x\left(-\frac{99}{38}ab^2(19bd-10ag)-\frac{1001}{20}ab^3ex+\frac{819}{136}b^3(17bc-8af)x^2\right)}{\sqrt{a+bx^3}}\,dx}{3003b^4}$$

$$= \frac{6a(17bc-8af)x\sqrt{a+bx^3}}{935b^2} + \frac{6a(19bd-10ag)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2aex^3\sqrt{a+bx^3}}{45b} + \frac{6afx^4\sqrt{a+bx^3}}{187b} + \frac{6agx^5\sqrt{a+bx^3}}{247b} + \frac{2x^3\sqrt{a+bx^3}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835} + \frac{(16a)\int \frac{-\frac{819}{136}ab^3(17bc-8af)-\frac{495}{76}ab^3(19bd-10ag)x-\frac{1001}{8}ab^4ex^2}{\sqrt{a+bx^3}}\,dx}{15015b^5}$$

$$= \frac{6a(17bc-8af)x\sqrt{a+bx^3}}{935b^2} + \frac{6a(19bd-10ag)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2aex^3\sqrt{a+bx^3}}{45b} + \frac{6afx^4\sqrt{a+bx^3}}{187b} + \frac{6agx^5\sqrt{a+bx^3}}{247b} + \frac{2x^3\sqrt{a+bx^3}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835} + \frac{(16a)\int \frac{-\frac{819}{136}ab^3(17bc-8af)-\frac{495}{76}ab^3(19bd-10ag)x}{\sqrt{a+bx^3}}\,dx}{15015b^5} - \frac{(2a^2e)\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{15b}$$

$$= -\frac{4a^2e\sqrt{a+bx^3}}{45b^2} + \frac{6a(17bc-8af)x\sqrt{a+bx^3}}{935b^2} + \frac{6a(19bd-10ag)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2aex^3\sqrt{a+bx^3}}{45b} + \frac{6afx^4\sqrt{a+bx^3}}{187b} + \frac{6agx^5\sqrt{a+bx^3}}{247b} + \frac{2x^3\sqrt{a+bx^3}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835} - \frac{(12a^2(19bd-10ag))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{1729b^{7/3}} - \frac{\left(6a^2\left(1729\sqrt[3]{b}(17bc-8af)-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}(19bd-10ag)\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{1616615b^{7/3}}$$

$$= -\frac{4a^2e\sqrt{a+bx^3}}{45b^2} + \frac{6a(17bc-8af)x\sqrt{a+bx^3}}{935b^2} + \frac{6a(19bd-10ag)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2aex^3\sqrt{a+bx^3}}{45b} + \frac{6afx^4\sqrt{a+bx^3}}{187b} + \frac{6agx^5\sqrt{a+bx^3}}{247b} - \frac{24a^2(19bd-10ag)\sqrt{a+bx^3}}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2x^3\sqrt{a+bx^3}(62985cx+53295dx^2+46189ex^3+40755fx^4+}{692835}$$

$$+ \frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19bd-10ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(1729\sqrt[3]{b}(17bc-8af)-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}(19bd-10ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/}}{((}}}{1616615b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.91 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.23

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{2\sqrt{a+bx^3}\left(-\left(\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}(a(92378e+90x(988f+935gx))-3bx(62985c+11x(4845d+13x(32\right.\right.}{}$$

```
[In] Integrate[x^3*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-((a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(a*(92378*e + 90*x*(9
88*f + 935*g*x)) - 3*b*x*(62985*c + 11*x*(4845*d + 13*x*(323*e + 285*f*x +
255*g*x^2))))) + 11115*a*(-17*b*c + 8*a*f)*x*Hypergeometric2F1[-1/2, 1/3, 4
/3, -((b*x^3)/a)] + 8415*a*(-19*b*d + 10*a*g)*x^2*Hypergeometric2F1[-1/2, 2
/3, 5/3, -((b*x^3)/a)]))/(2078505*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 956, normalized size of antiderivative = 1.30

method	result	size
elliptic	Expression too large to display	956
risch	Expression too large to display	1138
default	Expression too large to display	1674

```
[In] int(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/19*g*x^8*(b*x^3+a)^(1/2)+2/17*f*x^7*(b*x^3+a)^(1/2)+2/15*e*x^6*(b*x^3+a)^
(1/2)+2/13*(3/19*a*g+b*d)/b*x^5*(b*x^3+a)^(1/2)+2/11*(3/17*a*f+b*c)/b*x^4*(
b*x^3+a)^(1/2)+2/45*a*e*x^3*(b*x^3+a)^(1/2)/b+2/7*(a*d-10/13*a/b*(3/19*a*g+
b*d))/b*x^2*(b*x^3+a)^(1/2)+2/5*(a*c-8/11*a/b*(3/17*a*f+b*c))/b*x*(b*x^3+a)
^(1/2)-4/45*a^2*e*(b*x^3+a)^(1/2)/b^2+4/15*I*a/b^2*(a*c-8/11*a/b*(3/17*a*f+
b*c))*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+8/21*I*
a/b^2*(a*d-10/13*a/b*(3/19*a*g+b*d))*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.28

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx =$$

$$-\frac{2\left(93366\,(17\,a^2bc-8\,a^3f)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-100980\,(19\,a^2bd-10\,a^3g)\sqrt{b}\text{weierstrass}\right.}{}$$

```
[In] integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] -2/14549535*(93366*(17*a^2*b*c - 8*a^3*f)*sqrt(b)*weierstrassPInverse(0, -4
*a/b, x) - 100980*(19*a^2*b*d - 10*a^3*g)*sqrt(b)*weierstrassZeta(0, -4*a/b
, weierstrassPInverse(0, -4*a/b, x)) - (765765*b^3*g*x^8 + 855855*b^3*f*x^7
 + 969969*b^3*e*x^6 + 323323*a*b^2*e*x^3 + 58905*(19*b^3*d + 3*a*b^2*g)*x^5
 + 77805*(17*b^3*c + 3*a*b^2*f)*x^4 - 646646*a^2*b*e + 25245*(19*a*b^2*d -
10*a^2*b*g)*x^2 + 46683*(17*a*b^2*c - 8*a^2*b*f)*x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.32

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$=\frac{\sqrt{a}cx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}dx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$
$$+\frac{\sqrt{a}fx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt{a}gx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$
$$+e\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**3*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(7/3)) + sqrt(a)*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*f*x**7*gamma(7/3)*hyper((
-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*g*
x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*ga
mma(11/3)) + e*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqr
t(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6
/6, True))
```

Maxima [F]

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}x^3\,dx$$

```
[In] integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}x^3\,dx$$

```
[In] integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$=\int x^3\,\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

```
[In] int(x^3*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
```

```
[Out] int(x^3*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.446 $\int x^2\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 681

$$\int x^2\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx = \frac{2a(5bc-2af)\sqrt{a+bx^3}}{45b^2}+\frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2}+\frac{6aex^2\sqrt{a+bx^3}}{91b}+\frac{2afx^3\sqrt{a+bx^3}}{45b}+\frac{6agx^4\sqrt{a+bx^3}}{187b}-\frac{24a^2e\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x^2\sqrt{a+bx^3}(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5)}{109395}+\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(1547bd-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-728ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{Ellipt}}{85085b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/45*a*(-2*a*f+5*b*c)*(b*x^3+a)^(1/2)/b^2+6/935*a*(-8*a*g+17*b*d)*x*(b*x^3+
a)^(1/2)/b^2+6/91*a*e*x^2*(b*x^3+a)^(1/2)/b+2/45*a*f*x^3*(b*x^3+a)^(1/2)/b+
6/187*a*g*x^4*(b*x^3+a)^(1/2)/b+2/109395*x^2*(6435*g*x^5+7293*f*x^4+8415*e*
x^3+9945*d*x^2+12155*c*x)*(b*x^3+a)^(1/2)-24/91*a^2*e*(b*x^3+a)^(1/2)/b^(5/
```

```
3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+12/91*3^(1/4)*a^(7/3)*e*(a^(1/3)+b^(1/3)
*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/
(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-4/850
85*3^(3/4)*a^2*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2))
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1547*b*d-728*a*g-1870*a^(
1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(7/3)/(b*x^3
+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1850, 1902, 1608, 1900, 267, 1892, 224, 1891}

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$

$$= \frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{24a^2e\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{}\right.}{85085b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2a\sqrt{a+bx^3}(5bc-2af)}{45b^2}+\frac{6ax\sqrt{a+bx^3}(17bd-8ag)}{935b^2}$$

$$+\frac{2x^2\sqrt{a+bx^3}(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5)}{109395}$$

$$+\frac{6aex^2\sqrt{a+bx^3}}{91b}+\frac{2afx^3\sqrt{a+bx^3}}{45b}+\frac{6agx^4\sqrt{a+bx^3}}{187b}$$

```
[In] Int[x^2*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a*(5*b*c - 2*a*f)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*(17*b*d - 8*a*g)*x*Sq
rt[a + b*x^3])/(935*b^2) + (6*a*e*x^2*Sqrt[a + b*x^3])/(91*b) + (2*a*f*x^3*
Sqrt[a + b*x^3])/(45*b) + (6*a*g*x^4*Sqrt[a + b*x^3])/(187*b) - (24*a^2*e*S
qrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^2*S
qrt[a + b*x^3]*(12155*c*x + 9945*d*x^2 + 8415*e*x^3 + 7293*f*x^4 + 6435*g*x
^5))/109395 + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*e*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
) - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1547*b*d - 1870*(1 - Sqrt[3])*a^(1/3)
*b^(2/3)*e - 728*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(85085*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

<u>Rule 224</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

<u>Rule 267</u>

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

<u>Rule 1608</u>

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

<u>Rule 1840</u>

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{2x^2\sqrt{a+bx^3}(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5)}{109395} + \frac{1}{2}(3a)\int \frac{x^2\left(\frac{2c}{9}+\frac{2dx}{11}+\frac{2ex^2}{13}+\frac{2fx^3}{15}+\frac{2gx^4}{17}\right)}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{6agx^4\sqrt{a+bx^3}}{187b} + \frac{2x^2\sqrt{a+bx^3}(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5)}{109395} + \frac{(3a)\int \frac{x^2\left(\frac{11bc}{9}+\frac{1}{17}(17bd-8ag)x+\frac{11}{13}bex^2+\frac{11}{15}bfx^3\right)}{\sqrt{a+bx^3}}\,dx}{11b}$$

$$= \frac{2afx^3\sqrt{a+bx^3}}{45b} + \frac{6agx^4\sqrt{a+bx^3}}{187b} + \frac{2x^2\sqrt{a+bx^3}(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5)}{109395} + \frac{(2a)\int \frac{x^2\left(\frac{11}{10}b(5bc-2af)+\frac{9}{34}b(17bd-8ag)x+\frac{99}{26}b^2ex^2\right)}{\sqrt{a+bx^3}}\,dx}{33b^2}$$

$$= \frac{6aex^2\sqrt{a+bx^3}}{91b} + \frac{2afx^3\sqrt{a+bx^3}}{45b} + \frac{6agx^4\sqrt{a+bx^3}}{187b} + \frac{2x^2\sqrt{a+bx^3}(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5)}{109395} + \frac{(4a)\int \frac{-\frac{99}{13}ab^2ex+\frac{77}{20}b^2(5bc-2af)x^2+\frac{63}{68}b^2(17bd-8ag)x^3}{\sqrt{a+bx^3}}\,dx}{231b^3}$$

$$= \frac{6aex^2\sqrt{a+bx^3}}{91b} + \frac{2afx^3\sqrt{a+bx^3}}{45b} + \frac{6agx^4\sqrt{a+bx^3}}{187b} + \frac{2x^2\sqrt{a+bx^3}(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5)}{109395} + \frac{(4a)\int \frac{x\left(-\frac{99}{13}ab^2e+\frac{77}{20}b^2(5bc-2af)x+\frac{63}{68}b^2(17bd-8ag)x^2\right)}{\sqrt{a+bx^3}}\,dx}{231b^3}$$

$$= \frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2} + \frac{6aex^2\sqrt{a+bx^3}}{91b} + \frac{2afx^3\sqrt{a+bx^3}}{45b} + \frac{6agx^4\sqrt{a+bx^3}}{187b} + \frac{2x^2\sqrt{a+bx^3}(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5)}{109395} + \frac{(8a)\int \frac{-\frac{63}{68}ab^2(17bd-8ag)-\frac{495}{26}ab^3ex+\frac{77}{8}b^3(5bc-2af)x^2}{\sqrt{a+bx^3}}\,dx}{1155b^4}$$

$$= \frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2} + \frac{6aex^2\sqrt{a+bx^3}}{91b} + \frac{2afx^3\sqrt{a+bx^3}}{45b} + \frac{6agx^4\sqrt{a+bx^3}}{187b} + \frac{2x^2\sqrt{a+bx^3}(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5)}{109395} + \frac{(8a)\int \frac{-\frac{63}{68}ab^2(17bd-8ag)-\frac{495}{26}ab^3ex}{\sqrt{a+bx^3}}\,dx}{1155b^4} + \frac{(a(5bc-2af))\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{15b}$$

$$= \frac{2a(5bc-2af)\sqrt{a+bx^3}}{45b^2} + \frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2} + \frac{6aex^2\sqrt{a+bx^3}}{91b} + \frac{2afx^3\sqrt{a+bx^3}}{45b} + \frac{6agx^4\sqrt{a+bx^3}}{187b} + \frac{2x^2\sqrt{a+bx^3}(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5)}{109395} - \frac{(12a^2e)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{91b^{4/3}} - \frac{\left(6a^2\left(1547bd-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-728ag\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{85085b^2}$$

$$= \frac{2a(5bc-2af)\sqrt{a+bx^3}}{45b^2} + \frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2} + \frac{6aex^2\sqrt{a+bx^3}}{91b} + \frac{2afx^3\sqrt{a+bx^3}}{45b} + \frac{6agx^4\sqrt{a+bx^3}}{187b} - \frac{24a^2e\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2x^2\sqrt{a+bx^3}(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5)}{109395} + \frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{\ldots}\right.}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{4\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(1547bd-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-728ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\ldots\right.}}}{85085b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.23

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$= \frac{2\sqrt{a+bx^3}\left(-\left(\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}(26a(187f+180gx)-b(12155c+9945dx+33x^2(255e+13x(17f+15\right.\right.\right.}{1}$$

```
[In] Integrate[x^2*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-((a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(26*a*(187*f + 180*g*
x) - b*(12155*c + 9945*d*x + 33*x^2*(255*e + 13*x*(17*f + 15*g*x))))) + 585
*a*(-17*b*d + 8*a*g)*x*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)] - 84
15*a*b*e*x^2*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)]))/(109395*b^2*
Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	920
risch	Expression too large to display	1115
default	Expression too large to display	1197

```
[In] int(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/17*g*x^7*(b*x^3+a)^(1/2)+2/15*f*x^6*(b*x^3+a)^(1/2)+2/13*e*x^5*(b*x^3+a)^
(1/2)+2/11*(3/17*a*g+b*d)/b*x^4*(b*x^3+a)^(1/2)+2/9*(1/5*a*f+b*c)/b*x^3*(b*
x^3+a)^(1/2)+6/91*a*e*x^2*(b*x^3+a)^(1/2)/b+2/5*(a*d-8/11*a/b*(3/17*a*g+b*d
))/b*x*(b*x^3+a)^(1/2)+2/3*(a*c-2/3*a/b*(1/5*a*f+b*c))/b*(b*x^3+a)^(1/2)+4/
15*I*a/b^2*(a*d-8/11*a/b*(3/17*a*g+b*d))*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2))+8/91*I/b^2*a^2*e*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
```

```
*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.26

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$=\frac{2\left(100980\,a^2b^{\frac{3}{2}}e\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-4914\left(17\,a^2bd-8\,a^3g\right)\sqrt{b}\mathrm{weie}\right.}{}$$

```
[In] integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] 2/765765*(100980*a^2*b^(3/2)*e*weierstrassZeta(0, -4*a/b, weierstrassPInver
se(0, -4*a/b, x)) - 4914*(17*a^2*b*d - 8*a^3*g)*sqrt(b)*weierstrassPInverse
(0, -4*a/b, x) + (45045*b^3*g*x^7 + 51051*b^3*f*x^6 + 58905*b^3*e*x^5 + 252
45*a*b^2*e*x^2 + 4095*(17*b^3*d + 3*a*b^2*g)*x^4 + 85085*a*b^2*c - 34034*a^
2*b*f + 17017*(5*b^3*c + a*b^2*f)*x^3 + 2457*(17*a*b^2*d - 8*a^2*b*g)*x)*sq
rt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.33

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{\sqrt{a}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}ex^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}gx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + c\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for}\ b=0\\ \frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right) + f\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**2*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(7/3)) + sqrt(a)*e*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*g*x**7*gamma(7/3)*hyper((
-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + c*Piecewis
e((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + f*Piec
ewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b
) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Maxima [F]

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \int \left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}x^2\,dx$$

```
[In] integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)*c/b + integrate((g*x^6 + f*x^5 + e*x^4 + d*x^3)*sqrt(
b*x^3 + a), x)
```

Giac [F]

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}x^2\,dx$$

```
[In] integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac"
)
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$=\int x^2\,\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

```
[In] int(x^2*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
[Out] int(x^2*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.447 $\int x\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 667

$$\int x\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx = \frac{2a(5bd-2ag)\sqrt{a+bx^3}}{45b^2}+\frac{6aex\sqrt{a+bx^3}}{55b}+\frac{6afx^2\sqrt{a+bx^3}}{91b}+\frac{2agx^3\sqrt{a+bx^3}}{45b}+\frac{6a(13bc-4af)\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x\sqrt{a+bx^3}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13bc-4af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\ldots\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^{4/3}\left(182a^{2/3}\sqrt[3]{b}e+55\left(1-\sqrt{3}\right)(13bc-4af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{Ellipt}\ldots}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/45*a*(-2*a*g+5*b*d)*(b*x^3+a)^(1/2)/b^2+6/55*a*e*x*(b*x^3+a)^(1/2)/b+6/91
*a*f*x^2*(b*x^3+a)^(1/2)/b+2/45*a*g*x^3*(b*x^3+a)^(1/2)/b+2/45045*x*(3003*g
*x^5+3465*f*x^4+4095*e*x^3+5005*d*x^2+6435*c*x)*(b*x^3+a)^(1/2)+6/91*a*(-4*
a*f+13*b*c)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3/91*3^
(1/4)*a^(4/3)*(-4*a*f+13*b*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1
/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2
```

```
)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-2/5005*3^(3/4)*a^(4/3)*(a^(1/3)+b^
(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(182*a^(2/3)*b^(1/3)*e+55*(-4*a*f+13*b*c)*(1-3^(1/2)
))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/
3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1840, 1850, 1902, 1900, 267, 1892, 224, 1891}

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = -\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\cdots\right)}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13bc-4af)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\cdots\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{6a\sqrt{a+bx^3}(13bc-4af)}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2a\sqrt{a+bx^3}(5bd-2ag)}{45b^2} + \frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045} + \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b}$$

```
[In] Int[x*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a*(5*b*d - 2*a*g)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*e*x*Sqrt[a + b*x^3])/
(55*b) + (6*a*f*x^2*Sqrt[a + b*x^3])/(91*b) + (2*a*g*x^3*Sqrt[a + b*x^3])/(
45*b) + (6*a*(13*b*c - 4*a*f)*Sqrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)) + (2*x*Sqrt[a + b*x^3]*(6435*c*x + 5005*d*x^2 + 4095*e
*x^3 + 3465*f*x^4 + 3003*g*x^5))/45045 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/
```

```
3)*(13*b*c - 4*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a
^(4/3)*(182*a^(2/3)*b^(1/3)*e + 55*(1 - Sqrt[3])*(13*b*c - 4*a*f))*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005*b^(5/3)*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{2x\sqrt{a+bx^3}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045} + \frac{1}{2}(3a)\int \frac{x\left(\frac{2c}{7}+\frac{2dx}{9}+\frac{2ex^2}{11}+\frac{2fx^3}{13}+\frac{2gx^4}{15}\right)}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{2x\sqrt{a+bx^3}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045} + \frac{a\int \frac{x\left(\frac{9bc}{7}+\frac{1}{5}(5bd-2ag)x+\frac{9}{11}bex^2+\frac{9}{13}bfx^3\right)}{\sqrt{a+bx^3}}dx}{3b}$$

$$= \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{2x\sqrt{a+bx^3}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045} + \frac{(2a)\int \frac{x\left(\frac{9}{26}b(13bc-4af)+\frac{7}{10}b(5bd-2ag)x+\frac{63}{22}b^2ex^2\right)}{\sqrt{a+bx^3}}dx}{21b^2}$$

$$= \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{2x\sqrt{a+bx^3}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045} + \frac{(4a)\int \frac{-\frac{63}{22}ab^2e+\frac{45}{52}b^2(13bc-4af)x+\frac{7}{4}b^2(5bd-2ag)x^2}{\sqrt{a+bx^3}}dx}{105b^3}$$

$$= \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{2x\sqrt{a+bx^3}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045} + \frac{(4a)\int \frac{-\frac{63}{22}ab^2e+\frac{45}{52}b^2(13bc-4af)x}{\sqrt{a+bx^3}}dx}{105b^3} + \frac{(a(5bd-2ag))\int \frac{x^2}{\sqrt{a+bx^3}}dx}{15b}$$

$$= \frac{2a(5bd-2ag)\sqrt{a+bx^3}}{45b^2} + \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{2x\sqrt{a+bx^3}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045} + \frac{(3a(13bc-4af))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}dx}{91b^{4/3}} - \frac{\left(3a^{4/3}\left(182a^{2/3}\sqrt[3]{b}e+55\left(1-\sqrt{3}\right)(13bc-4af)\right)\right)\int \frac{1}{\sqrt{a+bx^3}}dx}{5005b^{4/3}}$$

$$= \frac{2a(5bd-2ag)\sqrt{a+bx^3}}{45b^2} + \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{6a(13bc-4af)\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2x\sqrt{a+bx^3}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13bc-4af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{4/3}\left(182a^{2/3}\sqrt[3]{b}e+55\left(1-\sqrt{3}\right)(13bc-4af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.21

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$

$$= \frac{\sqrt{a+bx^3}\left(-4(a+bx^3)\sqrt{1+\frac{bx^3}{a}}(286ag-b(715d+585ex+495fx^2+429gx^3))-2340abex\,\text{Hypergeom}\right.}{12870b^2\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[x*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (Sqrt[a + b*x^3]*(-4*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(286*a*g - b*(715*d +
585*e*x + 495*f*x^2 + 429*g*x^3)) - 2340*a*b*e*x*Hypergeometric2F1[-1/2, 1/
3, 4/3, -((b*x^3)/a)] + 495*b*(13*b*c - 4*a*f)*x^2*Hypergeometric2F1[-1/2,
2/3, 5/3, -((b*x^3)/a)]))/(12870*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.24

method	result	size
risch	Expression too large to display	829
elliptic	Expression too large to display	884
default	Expression too large to display	1311

```
[In] int(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/45045*(-3003*b^2*g*x^6-3465*b^2*f*x^5-4095*b^2*e*x^4-1001*a*b*g*x^3-5005
*b^2*d*x^3-1485*a*b*f*x^2-6435*b^2*c*x^2-2457*a*b*e*x+2002*a^2*g-5005*a*b*d
)/b^2*(b*x^3+a)^(1/2)-3/5005*a/b*(-364/3*I*a*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(220*a*f-715*b*c)*3^(1/2)/b*(-a*b
^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.22

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx =$$

$$-\frac{2\left(4914\,a^2\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+1485\,(13\,abc-4\,a^2f)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierst}\right.\right.}{}$$

```
[In] integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/45045*(4914*a^2*sqrt(b)*e*weierstrassPInverse(0, -4*a/b, x) + 1485*(13*a
*b*c - 4*a^2*f)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -
4*a/b, x)) - (3003*b^2*g*x^6 + 3465*b^2*f*x^5 + 4095*b^2*e*x^4 + 2457*a*b*e
*x + 1001*(5*b^2*d + a*b*g)*x^3 + 5005*a*b*d - 2002*a^2*g + 495*(13*b^2*c +
 3*a*b*f)*x^2)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.33

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{\sqrt{a}cx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}fx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + d\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for}\ b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right) + g\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(5/3)) + sqrt(a)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*f*x**5*gamma(5/3)*hyper((
-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + d*Piecewise(
(sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + g*Piecew
ise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b)
+ 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Maxima [F]

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \int \left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}\,x\,dx$$

```
[In] integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x, x)
```

Giac [F]

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}x\,dx$$

```
[In] integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int x\,\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

```
[In] int(x*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
[Out] int(x*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.448 $\int \sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 639

$$\int \sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx = \frac{2ae\sqrt{a+bx^3}}{9b}+\frac{6afx\sqrt{a+bx^3}}{55b}+\frac{6agx^2\sqrt{a+bx^3}}{91b}+\frac{6a(13bd-4ag)\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{2\sqrt{a+bx^3}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7\ldots\right.}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(91\sqrt[3]{b}(11bc-2af)-55\left(1-\sqrt{3}\right)\sqrt[3]{a}(13bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/\ldots}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\ldots\right.}}}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/9*a*e*(b*x^3+a)^(1/2)/b+6/55*a*f*x*(b*x^3+a)^(1/2)/b+6/91*a*g*x^2*(b*x^3+
a)^(1/2)/b+2/45045*(3465*g*x^5+4095*f*x^4+5005*e*x^3+6435*d*x^2+9009*c*x)*(
b*x^3+a)^(1/2)+6/91*a*(-4*a*g+13*b*d)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))-3/91*3^(1/4)*a^(4/3)*(-4*a*g+13*b*d)*(a^(1/3)+b^(1/3)*x)
*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^
```

```
(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/5005*3
^(3/4)*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(91*b^(1/3)*(-2*a*f+11*b*c)-55*a
^(1/3)*(-4*a*g+13*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3
)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1867, 1902, 1900, 267, 1892, 224, 1891}

$$\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$

$$= \frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\cdots\right.}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13bd-4ag)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\cdots\right.}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{6a\sqrt{a+bx^3}(13bd-4ag)}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{2\sqrt{a+bx^3}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045}$$

$$+\frac{2ae\sqrt{a+bx^3}}{9b}+\frac{6afx\sqrt{a+bx^3}}{55b}+\frac{6agx^2\sqrt{a+bx^3}}{91b}$$

```
[In] Int[Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a*e*Sqrt[a + b*x^3])/(9*b) + (6*a*f*x*Sqrt[a + b*x^3])/(55*b) + (6*a*g*x
^2*Sqrt[a + b*x^3])/(91*b) + (6*a*(13*b*d - 4*a*g)*Sqrt[a + b*x^3])/(91*b^(
5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqrt[a + b*x^3]*(9009*c*x +
6435*d*x^2 + 5005*e*x^3 + 4095*f*x^4 + 3465*g*x^5))/45045 - (3*3^(1/4)*Sqrt
[2 - Sqrt[3]]*a^(4/3)*(13*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
```

```
llipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*
Sqrt[2 + Sqrt[3]]*a*(91*b^(1/3)*(11*b*c - 2*a*f) - 55*(1 - Sqrt[3])*a^(1/3)
*(13*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(5005*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{a+bx^3}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045} \\
&\quad+\frac{1}{2}(3a)\int\frac{\frac{2c}{5}+\frac{2dx}{7}+\frac{2ex^2}{9}+\frac{2fx^3}{11}+\frac{2gx^4}{13}}{\sqrt{a+bx^3}}\,dx \\
&= \frac{6agx^2\sqrt{a+bx^3}}{91b}+\frac{2\sqrt{a+bx^3}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045} \\
&\quad+\frac{(3a)\int\frac{\frac{7bc}{5}+\frac{1}{13}(13bd-4ag)x+\frac{7}{9}bex^2+\frac{7}{11}bfx^3}{\sqrt{a+bx^3}}\,dx}{7b} \\
&= \frac{6afx\sqrt{a+bx^3}}{55b}+\frac{6agx^2\sqrt{a+bx^3}}{91b} \\
&\quad+\frac{2\sqrt{a+bx^3}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045} \\
&\quad+\frac{(6a)\int\frac{\frac{7}{22}b(11bc-2af)+\frac{5}{26}b(13bd-4ag)x+\frac{35}{18}b^2ex^2}{\sqrt{a+bx^3}}\,dx}{35b^2} \\
&= \frac{6afx\sqrt{a+bx^3}}{55b}+\frac{6agx^2\sqrt{a+bx^3}}{91b} \\
&\quad+\frac{2\sqrt{a+bx^3}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045} \\
&\quad+\frac{(6a)\int\frac{\frac{7}{22}b(11bc-2af)+\frac{5}{26}b(13bd-4ag)x}{\sqrt{a+bx^3}}\,dx}{35b^2}+\frac{1}{3}(ae)\int\frac{x^2}{\sqrt{a+bx^3}}\,dx
\end{aligned}$$

$$= \frac{2ae\sqrt{a+bx^3}}{9b} + \frac{6afx\sqrt{a+bx^3}}{55b} + \frac{6agx^2\sqrt{a+bx^3}}{91b} + \frac{2\sqrt{a+bx^3}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045} + \frac{(3a(13bd-4ag))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{91b^{4/3}} + \frac{\left(3a\left(91\sqrt[3]{b}(11bc-2af)-55\left(1-\sqrt{3}\right)\sqrt[3]{a}(13bd-4ag)\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{5005b^{4/3}}$$

$$= \frac{2ae\sqrt{a+bx^3}}{9b} + \frac{6afx\sqrt{a+bx^3}}{55b} + \frac{6agx^2\sqrt{a+bx^3}}{91b} + \frac{6a(13bd-4ag)\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2\sqrt{a+bx^3}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(91\sqrt[3]{b}(11bc-2af)-55\left(1-\sqrt{3}\right)\sqrt[3]{a}(13bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}}{\left(\left(1+\sqrt{3}\right)\right.}}}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.21

$$\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{\sqrt{a+bx^3}\left(4(a+bx^3)\sqrt{1+\frac{bx^3}{a}}(143e+9x(13f+11gx))+234(11bc-2af)x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\right.\right.}{2574b\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
[Out] (Sqrt[a + b*x^3]*(4*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(143*e + 9*x*(13*f + 11
*g*x)) + 234*(11*b*c - 2*a*f)*x*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)
/a)] + 99*(13*b*d - 4*a*g)*x^2*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/
a)]))/(2574*b*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 863, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	863
risch	Expression too large to display	1080
default	Expression too large to display	1557

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/13*g*x^5*(b*x^3+a)^(1/2)+2/11*f*x^4*(b*x^3+a)^(1/2)+2/9*e*x^3*(b*x^3+a)^(
1/2)+2/7*(3/13*a*g+b*d)/b*x^2*(b*x^3+a)^(1/2)+2/5*(3/11*a*f+b*c)/b*x*(b*x^3
+a)^(1/2)+2/9*a*e*(b*x^3+a)^(1/2)/b-2/3*I*(a*c-2/5*a/b*(3/11*a*f+b*c))*3^(1
/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(a*d-4/7*
a/b*(3/13*a*g+b*d))*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^
2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^
2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.22

$$\int \sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\ dx$$
$$= \frac{2\left(2457\,(11\,abc-2\,a^2f)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-1485\,(13\,abd-4\,a^2g)\sqrt{b}\text{weierstrassZeta}\left(0,-\right.\right.}{}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(2457*(11*a*b*c - 2*a^2*f)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x
) - 1485*(13*a*b*d - 4*a^2*g)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstras
sPInverse(0, -4*a/b, x)) + (3465*b^2*g*x^5 + 4095*b^2*f*x^4 + 5005*b^2*e*x^
3 + 5005*a*b*e + 495*(13*b^2*d + 3*a*b*g)*x^2 + 819*(11*b^2*c + 3*a*b*f)*x)
*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.30

$$\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{\sqrt{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + e\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) + sqrt(a)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*f*x**4*gamma(4/3)*hyper((-1/
2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*g*x**5*
gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/
3)) + e*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b),
 True))
```

Maxima [F]

$$\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

```
[In] int((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
[Out] int((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.449 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 620

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx \\
&= \frac{2af\sqrt{a+bx^3}}{9b}+\frac{6agx\sqrt{a+bx^3}}{55b}+\frac{6ae\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} \\
&\quad+\frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x} \\
&\quad-\frac{2}{3}\sqrt{a}c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) \\
&\quad-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} \\
&\quad+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(77bd-55\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-14ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\right.}{385b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}$$

```
[Out] -2/3*c*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/9*a*f*(b*x^3+a)^(1/2)/b+6
/55*a*g*x*(b*x^3+a)^(1/2)/b+2/3465*(315*g*x^5+385*f*x^4+495*e*x^3+693*d*x^2
+1155*c*x)*(b*x^3+a)^(1/2)/x+6/7*a*e*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(
```

```
1/3)*(1+3^(1/2)))-3/7*3^(1/4)*a^(4/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1
/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(
1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b
^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/385*3^(3/4)*a*(a^(1/3)
+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(77*b*d-14*a*g-55*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*
(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {1840, 1846, 272, 65, 214, 1902, 1900, 267, 1892, 224, 1891}

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$$

$$= \frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\cdots\right.}{385b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2}{3}\sqrt{a}c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{6ae\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+38\cdots}{3465x}$$

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]

[Out] (2*a*f*Sqrt[a + b*x^3])/(9*b) + (6*a*g*x*Sqrt[a + b*x^3])/(55*b) + (6*a*e*S
qrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqrt[a
 + b*x^3]*(1155*c*x + 693*d*x^2 + 495*e*x^3 + 385*f*x^4 + 315*g*x^5))/(3465
*x) - (2*Sqrt[a]*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
```

```
- 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a
*(77*b*d - 55*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 14*a*g)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(385*b^(4/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
```

```
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
```

```
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x} + \frac{1}{2}(3a)\int \frac{\frac{2c}{3}+\frac{2dx}{5}+\frac{2ex^2}{7}+\frac{2fx^3}{9}+\frac{2gx^4}{11}}{x\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x} + \frac{1}{2}(3a)\int \frac{\frac{2d}{5}+\frac{2ex}{7}+\frac{2fx^2}{9}+\frac{2gx^3}{11}}{\sqrt{a+bx^3}}\,dx + (ac)\int \frac{1}{x\sqrt{a+bx^3}}\,dx$$

$$= \frac{6agx\sqrt{a+bx^3}}{55b} + \frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x} + \frac{(3a)\int \frac{\frac{1}{11}(11bd-2ag)+\frac{5bex}{7}+\frac{5}{9}bfx^2}{\sqrt{a+bx^3}}\,dx}{5b} + \frac{1}{3}(ac)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)$$

$$= \frac{6agx\sqrt{a+bx^3}}{55b} + \frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x} + \frac{(3a)\int \frac{\frac{1}{11}(11bd-2ag)+\frac{5bex}{7}}{\sqrt{a+bx^3}}\,dx}{5b} + \frac{(2ac)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b} + \frac{1}{3}(af)\int \frac{x^2}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2af\sqrt{a+bx^3}}{9b} + \frac{6agx\sqrt{a+bx^3}}{55b} + \frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x} - \frac{2}{3}\sqrt{a}c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) + \frac{(3ae)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{7\sqrt[3]{b}} + \frac{\left(3a\left(77bd-55\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-14ag\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{385b}$$

$$= \frac{2af\sqrt{a+bx^3}}{9b} + \frac{6agx\sqrt{a+bx^3}}{55b} + \frac{6ae\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+ \frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x}$$
$$- \frac{2}{3}\sqrt{a}c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$
$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(77bd-55\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-14ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\mathrm{si}\right.}{385b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$$
$$= \frac{4\sqrt{1+\frac{bx^3}{a}}\left(\sqrt{a+bx^3}(33bc+11af+9agx+11bfx^3+9bgx^4)-33\sqrt{a}bc\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)+18(11bd-2}{198b\sqrt{}}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

```
[Out] (4*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3]*(33*b*c + 11*a*f + 9*a*g*x + 11*b*f
*x^3 + 9*b*g*x^4) - 33*Sqrt[a]*b*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]) + 18*(
11*b*d - 2*a*g)*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^
3)/a)] + 99*b*e*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*
x^3)/a)])/(198*b*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	848
default	Expression too large to display	1118

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/11*g*x^4*(b*x^3+a)^(1/2)+2/9*f*x^3*(b*x^3+a)^(1/2)+2/7*e*x^2*(b*x^3+a)^(1
/2)+2/5*(3/11*a*g+b*d)/b*x*(b*x^3+a)^(1/2)+2/3*(1/3*a*f+b*c)/b*(b*x^3+a)^(1
/2)-2/3*I*(a*d-2/5*(3/11*a*g+b*d)/b*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2))-2/7*I*a*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)))-2/3*c*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.23 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$$
$$= \left[\frac{1155\,\sqrt{a}b^2c\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)-5940\,ab^{\frac{3}{2}}e\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPI}\right.}{}\right.$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] [1/6930*(1155*sqrt(a)*b^2*c*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqr
t(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 5940*a*b^(3/2)*e*weierstrassZeta(0, -4
*a/b, weierstrassPInverse(0, -4*a/b, x)) + 756*(11*a*b*d - 2*a^2*g)*sqrt(b)
*weierstrassPInverse(0, -4*a/b, x) + 4*(315*b^2*g*x^4 + 385*b^2*f*x^3 + 495
*b^2*e*x^2 + 1155*b^2*c + 385*a*b*f + 63*(11*b^2*d + 3*a*b*g)*x)*sqrt(b*x^3
 + a))/b^2, 1/3465*(1155*sqrt(-a)*b^2*c*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(
b*x^3 + 2*a)) - 2970*a*b^(3/2)*e*weierstrassZeta(0, -4*a/b, weierstrassPInv
erse(0, -4*a/b, x)) + 378*(11*a*b*d - 2*a^2*g)*sqrt(b)*weierstrassPInverse(
0, -4*a/b, x) + 2*(315*b^2*g*x^4 + 385*b^2*f*x^3 + 495*b^2*e*x^2 + 1155*b^2
*c + 385*a*b*f + 63*(11*b^2*d + 3*a*b*g)*x)*sqrt(b*x^3 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = -\frac{2\sqrt{a}c\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}dx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}ex^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}gx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2ac}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2\sqrt{b}cx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + f\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(a)*d*x*gamma(1/3)*h
yper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a
)*e*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3
*gamma(5/3)) + sqrt(a)*g*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a*c/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**
```

```
3) + 1)) + 2*sqrt(b)*c*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + f*Piecewise((sqr
t(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="maxima"
)
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="giac")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = \int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x}\,dx$$

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x)
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x, x)
```

3.450 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 638

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \frac{2ag\sqrt{a+bx^3}}{9b}-\frac{3c\sqrt{a+bx^3}}{x}+\frac{3(7bc+2af)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2\sqrt{a+bx^3}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2}-\frac{2}{3}\sqrt{a}d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7bc+2af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(14a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(7bc+2af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\right.}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/9*a*g*(b*x^3+a)^(1/2)/b-3
*c*(b*x^3+a)^(1/2)/x+2/315*(35*g*x^5+45*f*x^4+63*e*x^3+105*d*x^2+315*c*x)*(
b*x^3+a)^(1/2)/x^2+3/7*(2*a*f+7*b*c)*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))-3/14*3^(1/4)*a^(1/3)*(2*a*f+7*b*c)*(a^(1/3)+b^(1/3)*x)*El
lipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3
```

```
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/35*3^(3/4
)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(14*a^(2/3)*b^(1/3)*e-5*(2*a*f+
7*b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2
)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {1840, 1849, 1846, 272, 65, 214, 1900, 267, 1892, 224, 1891}

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$$

$$= \frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2af+7bc)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2}{3}\sqrt{a}d\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{3\sqrt{a+bx^3}(2af+7bc)}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2\sqrt{a+bx^3}(315cx+105dx^2+63ex^3+45\ldots}{315x^2}$$

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

```
[Out] (2*a*g*Sqrt[a + b*x^3])/(9*b) - (3*c*Sqrt[a + b*x^3])/x + (3*(7*b*c + 2*a*f
)*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqr
t[a + b*x^3]*(315*c*x + 105*d*x^2 + 63*e*x^3 + 45*f*x^4 + 35*g*x^5))/(315*x
^2) - (2*Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(1/3)*(7*b*c + 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2
```

```
 + Sqrt[3]]*a^(1/3)*(14*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(7*b*c + 2*a*f)
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*b
^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
```

```
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\sqrt{a+bx^3}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2} \\
&\quad + \frac{1}{2}(3a)\int \frac{2c+\frac{2dx}{3}+\frac{2ex^2}{5}+\frac{2fx^3}{7}+\frac{2gx^4}{9}}{x^2\sqrt{a+bx^3}}\,dx \\
&= -\frac{3c\sqrt{a+bx^3}}{x}+\frac{2\sqrt{a+bx^3}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2} \\
&\quad -\frac{3}{4}\int \frac{-\frac{4ad}{3}-\frac{4aex}{5}-\frac{2}{7}(7bc+2af)x^2-\frac{4}{9}agx^3}{x\sqrt{a+bx^3}}\,dx \\
&= -\frac{3c\sqrt{a+bx^3}}{x}+\frac{2\sqrt{a+bx^3}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2} \\
&\quad -\frac{3}{4}\int \frac{-\frac{4ae}{5}-\frac{2}{7}(7bc+2af)x-\frac{4}{9}agx^2}{\sqrt{a+bx^3}}\,dx+(ad)\int \frac{1}{x\sqrt{a+bx^3}}\,dx \\
&= -\frac{3c\sqrt{a+bx^3}}{x}+\frac{2\sqrt{a+bx^3}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2} \\
&\quad -\frac{3}{4}\int \frac{-\frac{4ae}{5}-\frac{2}{7}(7bc+2af)x}{\sqrt{a+bx^3}}\,dx \\
&\quad +\frac{1}{3}(ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)+\frac{1}{3}(ag)\int \frac{x^2}{\sqrt{a+bx^3}}\,dx \\
&= \frac{2ag\sqrt{a+bx^3}}{9b}-\frac{3c\sqrt{a+bx^3}}{x} \\
&\quad +\frac{2\sqrt{a+bx^3}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2} \\
&\quad +\frac{(2ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3b}+\frac{(3(7bc+2af))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{14\sqrt[3]{b}} \\
&\quad +\frac{1}{70}\left(3\sqrt[3]{a}\left(14a^{2/3}e-\frac{5\left(1-\sqrt{3}\right)(7bc+2af)}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx
\end{aligned}
$$

$$= \frac{2ag\sqrt{a+bx^3}}{9b} - \frac{3c\sqrt{a+bx^3}}{x} + \frac{3(7bc+2af)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2\sqrt{a+bx^3}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2} - \frac{2}{3}\sqrt{a}d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7bc+2af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(14a^{2/3}e-\frac{5\left(1-\sqrt{3}\right)(7bc+2af)}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\right.\right.}{35\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \frac{2}{3}d\sqrt{a+bx^3} + \frac{2g(a+bx^3)^{3/2}}{9b} - \frac{2}{3}\sqrt{a}d\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{c\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\sqrt{1+\frac{bx^3}{a}}} + \frac{ex\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}} + \frac{fx^2\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

[Out] (2*d*Sqrt[a + b*x^3])/3 + (2*g*(a + b*x^3)^(3/2))/(9*b) - (2*Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (c*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, -1/3, 2/3, -((b*x^3)/a)])/(x*Sqrt[1 + (b*x^3)/a]) + (e*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a] + (f*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.30

method	result	size
elliptic	Expression too large to display	829
default	Expression too large to display	1248
risch	Expression too large to display	2011

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)

[Out] -c*(b*x^3+a)^(1/2)/x+2/9*g*x^3*(b*x^3+a)^(1/2)+2/7*f*x^2*(b*x^3+a)^(1/2)+2/5*e*x*(b*x^3+a)^(1/2)+2/3*(1/3*a*g+b*d)/b*(b*x^3+a)^(1/2)-2/5*I*a*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(3/7*a*f+3/2*b*c)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.23 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$$
$$= \left[\frac{105\sqrt{a}bdx\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+756\,a\sqrt{b}ex\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-270}{}\right.$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] [1/630*(105*sqrt(a)*b*d*x*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(
b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 756*a*sqrt(b)*e*x*weierstrassPInverse(0,
 -4*a/b, x) - 270*(7*b*c + 2*a*f)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weie
rstrassPInverse(0, -4*a/b, x)) + 2*(70*b*g*x^4 + 90*b*f*x^3 + 126*b*e*x^2 -
 315*b*c + 70*(3*b*d + a*g)*x)*sqrt(b*x^3 + a))/(b*x), 1/315*(105*sqrt(-a)*
b*d*x*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) + 378*a*sqrt(b)*e*x*
weierstrassPInverse(0, -4*a/b, x) - 135*(7*b*c + 2*a*f)*sqrt(b)*x*weierstra
ssZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (70*b*g*x^4 + 90*b*f
*x^3 + 126*b*e*x^2 - 315*b*c + 70*(3*b*d + a*g)*x)*sqrt(b*x^3 + a))/(b*x)]
```

Sympy [A] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}ex\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}fx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{2ad}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2\sqrt{b}dx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + g\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**2,x)
```

```
[Out] sqrt(a)*c*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x*gamma(2/3)) - 2*sqrt(a)*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(
a)*e*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(4/3)) + sqrt(a)*f*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a*d/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3)
 + 1)) + 2*sqrt(b)*d*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + g*Piecewise((sqrt(
a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^2}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="maxim
a")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^2}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="giac"
)
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx=\int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^2}\,dx$$

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x)
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2, x)
```

3.451 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 640

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$$
$$= \frac{3c\sqrt{a+bx^3}}{2x^2} - \frac{3d\sqrt{a+bx^3}}{x} + \frac{3(7bd+2ag)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$- \frac{2\sqrt{a+bx^3}(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3} - \frac{2}{3}\sqrt{a}e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$
$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7bd+2ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7-4\right.}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}(5bc+4af)-10\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd+2ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{Ellip}}{70b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/3*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+3/2*c*(b*x^3+a)^(1/2)/x^2-3
*d*(b*x^3+a)^(1/2)/x-2/105*(-15*g*x^5-21*f*x^4-35*e*x^3-105*d*x^2+105*c*x)*
(b*x^3+a)^(1/2)/x^3+3/7*(2*a*g+7*b*d)*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))-3/14*3^(1/4)*a^(1/3)*(2*a*g+7*b*d)*(a^(1/3)+b^(1/3)*x)*E
llipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*
```

```
3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/70*3^(3/
4)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*(4*a*f+5*b*c)-10*a^(1/3)*(2
*a*g+7*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)
^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$$
$$= \frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(7\cdots\right.}{70b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2ag+7bd)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\cdots\right.}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{2}{3}\sqrt{a}e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{3\sqrt{a+bx^3}(2ag+7bd)}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2\sqrt{a+bx^3}(105cx-105dx^2-35ex^3-21\cdots}{105x^3}$$

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

```
[Out] (3*c*Sqrt[a + b*x^3])/(2*x^2) - (3*d*Sqrt[a + b*x^3])/x + (3*(7*b*d + 2*a*g
)*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*Sqr
t[a + b*x^3]*(105*c*x - 105*d*x^2 - 35*e*x^3 - 21*f*x^4 - 15*g*x^5))/(105*x
^3) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(1/3)*(7*b*d + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
```

```
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2
 + Sqrt[3]]*(7*b^(1/3)*(5*b*c + 4*a*f) - 10*(1 - Sqrt[3])*a^(1/3)*(7*b*d +
2*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(70*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{a+bx^3}(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3} \\
&\quad+\frac{1}{2}(3a)\int\frac{-2c+2dx+\frac{2ex^2}{3}+\frac{2fx^3}{5}+\frac{2gx^4}{7}}{x^3\sqrt{a+bx^3}}\,dx \\
&= \frac{3c\sqrt{a+bx^3}}{2x^2}-\frac{2\sqrt{a+bx^3}(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3} \\
&\quad-\frac{3}{8}\int\frac{-8ad-\frac{8aex}{3}-\frac{2}{5}(5bc+4af)x^2-\frac{8}{7}agx^3}{x^2\sqrt{a+bx^3}}\,dx
\end{aligned}$$

$$=\frac{3c\sqrt{a+bx^3}}{2x^2}-\frac{3d\sqrt{a+bx^3}}{x}-\frac{2\sqrt{a+bx^3}(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3}+\frac{3\int\frac{\frac{16a^2e}{3}+\frac{4}{5}a(5bc+4af)x+\frac{8}{7}a(7bd+2ag)x^2}{x\sqrt{a+bx^3}}\,dx}{16a}$$

$$=\frac{3c\sqrt{a+bx^3}}{2x^2}-\frac{3d\sqrt{a+bx^3}}{x}-\frac{2\sqrt{a+bx^3}(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3}+\frac{3\int\frac{\frac{4}{5}a(5bc+4af)+\frac{8}{7}a(7bd+2ag)x}{\sqrt{a+bx^3}}\,dx}{16a}+(ae)\int\frac{1}{x\sqrt{a+bx^3}}\,dx$$

$$=\frac{3c\sqrt{a+bx^3}}{2x^2}-\frac{3d\sqrt{a+bx^3}}{x}-\frac{2\sqrt{a+bx^3}(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3}+\frac{1}{3}(ae)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)+\frac{(3(7bd+2ag))\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{14\sqrt[3]{b}}+\frac{1}{140}\left(3\left(7(5bc+4af)-\frac{10\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd+2ag)}{\sqrt[3]{b}}\right)\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx$$

$$=\frac{3c\sqrt{a+bx^3}}{2x^2}-\frac{3d\sqrt{a+bx^3}}{x}+\frac{3(7bd+2ag)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2\sqrt{a+bx^3}(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7bd+2ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|\right.}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(7(5bc+4af)-\frac{10\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd+2ag)}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin\right.}{70\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{(2ae)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3b}$$

$$= \frac{3c\sqrt{a+bx^3}}{2x^2} - \frac{3d\sqrt{a+bx^3}}{x} + \frac{3(7bd+2ag)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$- \frac{2\sqrt{a+bx^3}(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3}$$
$$- \frac{2}{3}\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$
$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7bd+2ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(7(5bc+4af)-\frac{10\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd+2ag)}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\mathrm{si}\right.}{70\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.78 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$$
$$= \frac{-3c\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a}\right)+x\left(-6d\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3}\right.\right.}{}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

```
[Out] (-3*c*Sqrt[a + b*x^3]*Hypergeometric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)] + x*
(-6*d*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, -1/3, 2/3, -((b*x^3)/a)] + x*
(4*e*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]
/Sqrt[a]]) + 6*f*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x
^3)/a)] + 3*g*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^
3)/a)])))/(6*x^2*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.29

method	result	size
elliptic	Expression too large to display	826
default	Expression too large to display	1529
risch	Expression too large to display	2244

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c*(b*x^3+a)^(1/2)/x^2-d*(b*x^3+a)^(1/2)/x+2/7*g*x^2*(b*x^3+a)^(1/2)+2/
5*f*x*(b*x^3+a)^(1/2)+2/3*e*(b*x^3+a)^(1/2)-2/3*I*(3/5*a*f+3/4*b*c)*3^(1/2)
/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(3/7*a*g+3/2
*b*d)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*e*arctanh((b*x^3+a)^(
1/2)/a^(1/2))*a^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$$

$$= \left[\frac{35\sqrt{a}bex^2\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+63\,(5\,bc+4\,af)\sqrt{b}x^2\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right.}{}\right.$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="frica
s")
```

```
[Out] [1/210*(35*sqrt(a)*b*e*x^2*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt
(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 63*(5*b*c + 4*a*f)*sqrt(b)*x^2*weierstr
assPInverse(0, -4*a/b, x) - 90*(7*b*d + 2*a*g)*sqrt(b)*x^2*weierstrassZeta(
0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (60*b*g*x^4 + 84*b*f*x^3 +
140*b*e*x^2 - 210*b*d*x - 105*b*c)*sqrt(b*x^3 + a))/(b*x^2), 1/210*(70*sqrt
(-a)*b*e*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) + 63*(5*b*c +
 4*a*f)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) - 90*(7*b*d + 2*a*g)*
sqrt(b)*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) +
 (60*b*g*x^4 + 84*b*f*x^3 + 140*b*e*x^2 - 210*b*d*x - 105*b*c)*sqrt(b*x^3 +
 a))/(b*x^2)]
```

Sympy [A] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}d\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}fx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}gx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{2ae}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2\sqrt{b}ex^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] sqrt(a)*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**2*gamma(1/3)) + sqrt(a)*d*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*
```

```
x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(a)*e*asinh(sqrt(a)/(sqrt(
b)*x**(3/2)))/3 + sqrt(a)*f*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*g*x**2*gamma(2/3)*hyper((-1/2,
2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a*e/(3*sqrt(b)*x
**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*sqrt(b)*e*x**(3/2)/(3*sqrt(a/(b*x**3) + 1
))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^3}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="maxim
a")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^3}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="giac"
)
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^3}\,dx$$

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x)
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3, x)
```

3.452 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 637

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}+\frac{3\sqrt[3]{b}e\sqrt{a+bx^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}$$
$$-\frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4}-\frac{(bc+2af)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(5bd-10\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\right.}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/3*(2*a*f+b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+1/3*c*(b*x^3+a)^(
1/2)/x^3+3/2*d*(b*x^3+a)^(1/2)/x^2-3*e*(b*x^3+a)^(1/2)/x-2/15*(-3*g*x^5-5*f
*x^4-15*e*x^3+15*d*x^2+5*c*x)*(b*x^3+a)^(1/2)/x^4+3*b^(1/3)*e*(b*x^3+a)^(1/
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3/2*3^(1/4)*a^(1/3)*b^(1/3)*e*(a^(1/3)+b
^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3
```

```
^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/10*3^
(3/4)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(5*b*d+4*a*g-10*a^(1/3)*b^(2/3)*e*(
1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$$

$$= \frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)(-10}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(2af+bc)}{3\sqrt{a}}-\frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4}$$

$$+\frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}+\frac{3\sqrt[3]{b}e\sqrt{a+bx^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}$$

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

```
[Out] (c*Sqrt[a + b*x^3])/(3*x^3) + (3*d*Sqrt[a + b*x^3])/(2*x^2) - (3*e*Sqrt[a +
 b*x^3])/x + (3*b^(1/3)*e*Sqrt[a + b*x^3])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x) - (2*Sqrt[a + b*x^3]*(5*c*x + 15*d*x^2 - 15*e*x^3 - 5*f*x^4 - 3*g*x^5))
/(15*x^4) - ((b*c + 2*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) -
(3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
```

```
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqr
t[2 + Sqrt[3]]*(5*b*d - 10*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10*b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4} \\ &\quad +\frac{1}{2}(3a)\int\frac{-\frac{2c}{3}-2dx+2ex^2+\frac{2fx^3}{3}+\frac{2gx^4}{5}}{x^4\sqrt{a+bx^3}}\,dx \\ &= \frac{c\sqrt{a+bx^3}}{3x^3}-\frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4} \\ &\quad -\frac{1}{4}\int\frac{12ad-12aex-2(bc+2af)x^2-\frac{12}{5}agx^3}{x^3\sqrt{a+bx^3}}\,dx\end{aligned}$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3} + \frac{3d\sqrt{a+bx^3}}{2x^2} - \frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4} + \frac{\int \frac{48a^2e+8a(bc+2af)x+\frac{12}{5}a(5bd+4ag)x^2}{x^2\sqrt{a+bx^3}}\,dx}{16a}$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3} + \frac{3d\sqrt{a+bx^3}}{2x^2} - \frac{3e\sqrt{a+bx^3}}{x} - \frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4} - \frac{\int \frac{-16a^2(bc+2af)-\frac{24}{5}a^2(5bd+4ag)x-48a^2bex^2}{x\sqrt{a+bx^3}}\,dx}{32a^2}$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3} + \frac{3d\sqrt{a+bx^3}}{2x^2} - \frac{3e\sqrt{a+bx^3}}{x} - \frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4} - \frac{\int \frac{-\frac{24}{5}a^2(5bd+4ag)-48a^2bex}{\sqrt{a+bx^3}}\,dx}{32a^2} - \frac{1}{2}(-bc-2af)\int \frac{1}{x\sqrt{a+bx^3}}\,dx$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3} + \frac{3d\sqrt{a+bx^3}}{2x^2} - \frac{3e\sqrt{a+bx^3}}{x} - \frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4} + \frac{1}{2}\left(3b^{2/3}e\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx - \frac{1}{6}(-bc-2af)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right) + \frac{1}{20}\left(3\left(5bd-10\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3} + \frac{3d\sqrt{a+bx^3}}{2x^2} - \frac{3e\sqrt{a+bx^3}}{x} + \frac{3\sqrt[3]{b}e\sqrt{a+bx^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}$$

$$- \frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{}\right.}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(5bd-10\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\right.\right.}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{(bc+2af)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3} + \frac{3d\sqrt{a+bx^3}}{2x^2} - \frac{3e\sqrt{a+bx^3}}{x} + \frac{3\sqrt[3]{b}e\sqrt{a+bx^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}$$

$$- \frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4} - \frac{(bc+2af)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{}\right.}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(5bd-10\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\right.\right.}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$$

$$= \frac{-2ac-2bcx^3+4afx^3+4bfx^6-4\sqrt{a}fx^3\sqrt{a+bx^3}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-2bcx^3\sqrt{1+\frac{bx^3}{a}}\operatorname{arctanh}\left(\sqrt{1+\frac{b}{}}\right.}{}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

```
[Out] (-2*a*c - 2*b*c*x^3 + 4*a*f*x^3 + 4*b*f*x^6 - 4*Sqrt[a]*f*x^3*Sqrt[a + b*x^
3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - 2*b*c*x^3*Sqrt[1 + (b*x^3)/a]*ArcTanh
[Sqrt[1 + (b*x^3)/a]] - 3*a*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3,
 -1/2, 1/3, -((b*x^3)/a)] - 6*a*e*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1
[-1/2, -1/3, 2/3, -((b*x^3)/a)] + 6*a*g*x^4*Sqrt[1 + (b*x^3)/a]*Hypergeomet
ric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/(6*x^3*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 822, normalized size of antiderivative = 1.29

method	result	size
elliptic	Expression too large to display	822
default	Expression too large to display	1114
risch	Expression too large to display	1368

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*c*(b*x^3+a)^(1/2)/x^3-1/2*d*(b*x^3+a)^(1/2)/x^2-e*(b*x^3+a)^(1/2)/x+2/
5*g*x*(b*x^3+a)^(1/2)+2/3*f*(b*x^3+a)^(1/2)-2/3*I*(3/5*a*g+3/4*b*d)*3^(1/2)
/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-I*e*3^(1/2)*(-a*b^
2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2
```

/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*(a*f+1/2*b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$$

$$= \left[-\frac{180\,ab^{\frac{3}{2}}ex^3\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-5\,(b^2c+2\,abf)\sqrt{a}x^3\log\left(-\frac{b^2x^6+\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{90\,ab^{\frac{3}{2}}ex^3\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-5\,(b^2c+2\,abf)\sqrt{-a}x^3\arctan\left(\frac{2\sqrt{b\cdots}}{b\cdots}\right.}{\cdots}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/60*(180*a*b^(3/2)*e*x^3*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - 5*(b^2*c + 2*a*b*f)*sqrt(a)*x^3*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 18*(5*a*b*d + 4*a^2*g)*sqrt(b)*x^3*weierstrassPInverse(0, -4*a/b, x) - 2*(12*a*b*g*x^4 + 20*a*b*f*x^3 - 30*a*b*e*x^2 - 15*a*b*d*x - 10*a*b*c)*sqrt(b*x^3 + a))/(a*b*x^3), -1/30*(90*a*b^(3/2)*e*x^3*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - 5*(b^2*c + 2*a*b*f)*sqrt(-a)*x^3*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 9*(5*a*b*d + 4*a^2*g)*sqrt(b)*x^3*weierstrassPInverse(0, -4*a/b, x) - (12*a*b*g*x^4 + 20*a*b*f*x^3 - 30*a*b*e*x^2 - 15*a*b*d*x - 10*a*b*c)*sqrt(b*x^3 + a))/(a*b*x^3)]
```

Sympy [A] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \frac{\sqrt{a}d\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}gx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2af}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}c\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2\sqrt{b}fx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} - \frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] sqrt(a)*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**2*gamma(1/3)) + sqrt(a)*e*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*
x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(a)*f*asinh(sqrt(a)/(sqrt(
b)*x**(3/2)))/3 + sqrt(a)*g*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(4/3)) + 2*a*f/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**
3) + 1)) - sqrt(b)*c*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*sqrt(b)*f*x**(3/
2)/(3*sqrt(a/(b*x**3) + 1)) - b*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt
(a))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^4}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^4}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^4}\,dx$$

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x)
```

```
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4, x)
```

3.453 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 694

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}$$
$$+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{3(bc+8af)\sqrt{a+bx^3}}{8ax}+\frac{3\sqrt[3]{b}(bc+8af)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5}-\frac{(bd+2ag)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(bc+8af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(4a^{2/3}\sqrt[3]{b}e-\left(1-\sqrt{3}\right)(bc+8af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{ar}\right.}{8a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/3*(2*a*g+b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+3/20*c*(b*x^3+a)^
(1/2)/x^4+1/3*d*(b*x^3+a)^(1/2)/x^3+3/2*e*(b*x^3+a)^(1/2)/x^2-3/8*(8*a*f+b*
c)*(b*x^3+a)^(1/2)/a/x-2/15*(-5*g*x^5-15*f*x^4+15*e*x^3+5*d*x^2+3*c*x)*(b*x
^3+a)^(1/2)/x^5+3/8*b^(1/3)*(8*a*f+b*c)*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))-3/16*3^(1/4)*b^(1/3)*(8*a*f+b*c)*(a^(1/3)+b^(1/3)*x)*Ellipti
```

```
cE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/8*3^(3/4)*b^(1
/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(4*a^(2/3)*b^(1/3)*e-(8*a*f+b*c)*(1-3
^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*
(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx$$

$$= \frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(4\right.}{8a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8af+bc)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{\right.}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(2ag+bd)}{3\sqrt{a}} - \frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5}$$

$$- \frac{3\sqrt{a+bx^3}(8af+bc)}{8ax} + \frac{3\sqrt[3]{b}\sqrt{a+bx^3}(8af+bc)}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+ \frac{3c\sqrt{a+bx^3}}{20x^4} + \frac{d\sqrt{a+bx^3}}{3x^3} + \frac{3e\sqrt{a+bx^3}}{2x^2}$$

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

```
[Out] (3*c*Sqrt[a + b*x^3])/(20*x^4) + (d*Sqrt[a + b*x^3])/(3*x^3) + (3*e*Sqrt[a
+ b*x^3])/(2*x^2) - (3*(b*c + 8*a*f)*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(1/3)*
(b*c + 8*a*f)*Sqrt[a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
```

```
(2*Sqrt[a + b*x^3]*(3*c*x + 5*d*x^2 + 15*e*x^3 - 15*f*x^4 - 5*g*x^5))/(15*x
^5) - ((b*d + 2*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(
1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3
/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(4*a^(2/3)*b^(1/3)*e - (1 - Sqrt[3])*(b*c + 8
*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(8*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
```

```
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5} + \frac{1}{2}(3a)\int \frac{-\frac{2c}{5}-\frac{2dx}{3}-2ex^2+2fx^3+\frac{2gx^4}{3}}{x^5\sqrt{a+bx^3}}\,dx$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4} - \frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5} - \frac{3}{16}\int \frac{\frac{16ad}{3}+16aex-2(bc+8af)x^2-\frac{16}{3}agx^3}{x^4\sqrt{a+bx^3}}\,dx$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4} + \frac{d\sqrt{a+bx^3}}{3x^3} - \frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5} + \frac{\int \frac{-96a^2e+12a(bc+8af)x+16a(bd+2ag)x^2}{x^3\sqrt{a+bx^3}}\,dx}{32a}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4} + \frac{d\sqrt{a+bx^3}}{3x^3} + \frac{3e\sqrt{a+bx^3}}{2x^2} - \frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5} - \frac{\int \frac{-48a^2(bc+8af)-64a^2(bd+2ag)x-96a^2bex^2}{x^2\sqrt{a+bx^3}}\,dx}{128a^2}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4} + \frac{d\sqrt{a+bx^3}}{3x^3} + \frac{3e\sqrt{a+bx^3}}{2x^2} - \frac{3(bc+8af)\sqrt{a+bx^3}}{8ax} - \frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5} + \frac{\int \frac{128a^3(bd+2ag)+192a^3bex+48a^2b(bc+8af)x^2}{x\sqrt{a+bx^3}}\,dx}{256a^3}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4} + \frac{d\sqrt{a+bx^3}}{3x^3} + \frac{3e\sqrt{a+bx^3}}{2x^2} - \frac{3(bc+8af)\sqrt{a+bx^3}}{8ax} - \frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5} + \frac{\int \frac{192a^3be+48a^2b(bc+8af)x}{\sqrt{a+bx^3}}\,dx}{256a^3} + \frac{1}{2}(bd+2ag)\int \frac{1}{x\sqrt{a+bx^3}}\,dx$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4} + \frac{d\sqrt{a+bx^3}}{3x^3} + \frac{3e\sqrt{a+bx^3}}{2x^2} - \frac{3(bc+8af)\sqrt{a+bx^3}}{8ax} - \frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5} + \frac{\left(3b^{2/3}(bc+8af)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{16a} + \frac{1}{16}\left(3b^{2/3}\left(4\sqrt[3]{b}e-\frac{\left(1-\sqrt{3}\right)(bc+8af)}{a^{2/3}}\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx+\frac{1}{6}(bd+2ag)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4} + \frac{d\sqrt{a+bx^3}}{3x^3} + \frac{3e\sqrt{a+bx^3}}{2x^2} - \frac{3(bc+8af)\sqrt{a+bx^3}}{8ax} + \frac{3\sqrt[3]{b}(bc+8af)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(bc+8af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(4\sqrt[3]{b}e-\frac{\left(1-\sqrt{3}\right)(bc+8af)}{a^{2/3}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{8\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{(bd+2ag)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4} + \frac{d\sqrt{a+bx^3}}{3x^3} + \frac{3e\sqrt{a+bx^3}}{2x^2} - \frac{3(bc+8af)\sqrt{a+bx^3}}{8ax} + \frac{3\sqrt[3]{b}(bc+8af)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5} - \frac{(bd+2ag)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(bc+8af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(4\sqrt[3]{b}e-\frac{\left(1-\sqrt{3}\right)(bc+8af)}{a^{2/3}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{8\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.45 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx =$$

$$-\frac{4adx+4bdx^4-8agx^4-8bgx^7+8\sqrt{a}gx^4\sqrt{a+bx^3}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+4bdx^4\sqrt{1+\frac{bx^3}{a}}\operatorname{arctanh}\left(\sqrt{1+\right.}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

```
[Out] -1/12*(4*a*d*x + 4*b*d*x^4 - 8*a*g*x^4 - 8*b*g*x^7 + 8*Sqrt[a]*g*x^4*Sqrt[a
 + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + 4*b*d*x^4*Sqrt[1 + (b*x^3)/a]*
ArcTanh[Sqrt[1 + (b*x^3)/a]] + 3*a*c*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[
-4/3, -1/2, -1/3, -((b*x^3)/a)] + 6*a*e*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeomet
ric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)] + 12*a*f*x^3*Sqrt[1 + (b*x^3)/a]*Hype
rgeometric2F1[-1/2, -1/3, 2/3, -((b*x^3)/a)])/(x^4*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 845, normalized size of antiderivative = 1.22

method	result	size
elliptic	Expression too large to display	845
risch	Expression too large to display	1243
default	Expression too large to display	1286

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*c*(b*x^3+a)^(1/2)/x^4-1/3*d*(b*x^3+a)^(1/2)/x^3-1/2*e*(b*x^3+a)^(1/2)/
x^2-1/8*(8*a*f+3*b*c)/a*(b*x^3+a)^(1/2)/x+2/3*g*(b*x^3+a)^(1/2)-1/2*I*e*3^(
1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(b*f+1/16*
b*(8*a*f+3*b*c)/a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2
)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
```

```
(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2
)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*(a*g+1/2
*b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx$$
$$= \left[\frac{36\,a\sqrt{b}ex^4\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+2\,(bd+2\,ag)\sqrt{a}x^4\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)\right.$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="frica
s")
```

```
[Out] [1/24*(36*a*sqrt(b)*e*x^4*weierstrassPInverse(0, -4*a/b, x) + 2*(b*d + 2*a*
g)*sqrt(a)*x^4*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*
sqrt(a) + 8*a^2)/x^6) - 9*(b*c + 8*a*f)*sqrt(b)*x^4*weierstrassZeta(0, -4*a
/b, weierstrassPInverse(0, -4*a/b, x)) + (16*a*g*x^4 - 12*a*e*x^2 - 3*(3*b*
c + 8*a*f)*x^3 - 8*a*d*x - 6*a*c)*sqrt(b*x^3 + a))/(a*x^4), 1/24*(36*a*sqrt
(b)*e*x^4*weierstrassPInverse(0, -4*a/b, x) + 4*(b*d + 2*a*g)*sqrt(-a)*x^4*
arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 9*(b*c + 8*a*f)*sqrt(b)*
x^4*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (16*a*g
*x^4 - 12*a*e*x^2 - 3*(3*b*c + 8*a*f)*x^3 - 8*a*d*x - 6*a*c)*sqrt(b*x^3 + a
))/(a*x^4)]
```

Sympy [A] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}g\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{2ag}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}d\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2\sqrt{b}gx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} - \frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**5,x)
```

```
[Out] sqrt(a)*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**4*gamma(-1/3)) + sqrt(a)*e*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*f*gamma(-1/3)*hyper
((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(
a)*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + 2*a*g/(3*sqrt(b)*x**(3/2)*sqrt(a
/(b*x**3) + 1)) - sqrt(b)*d*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*sqrt(b)*g
*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) - b*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/
(3*sqrt(a))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^5}\,dx$$

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5, x)

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^5}\,dx$$

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^5}\,dx$$

[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x)

[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5, x)

3.454 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 652

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{20ax^2} - \frac{3bd\sqrt{a+bx^3}}{8ax} + \frac{3\sqrt[3]{b}(bd+8ag)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{be\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(bd+8ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(2\sqrt[3]{b}(bc-10af)+5\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd+8ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{El}\ldots}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/3*b*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/60*(12*c/x^5+15*d/x^4+2
0*e/x^3+30*f/x^2+60*g/x)*(b*x^3+a)^(1/2)-3/20*b*c*(b*x^3+a)^(1/2)/a/x^2-3/8
*b*d*(b*x^3+a)^(1/2)/a/x+3/8*b^(1/3)*(8*a*g+b*d)*(b*x^3+a)^(1/2)/a/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))-3/16*3^(1/4)*b^(1/3)*(8*a*g+b*d)*(a^(1/3)+b^(1/3)*x
)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a
```

```
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/40*3^
(3/4)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(2*b^(1/3)*(-10*a*f+b*c)+5*
a^(1/3)*(8*a*g+b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a/(b*x^3+
a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx =$$

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8ag+bd)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{}\right.}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{be\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)-\frac{3bc\sqrt{a+bx^3}}{20ax^2}+\frac{3\sqrt[3]{b}\sqrt{a+bx^3}(8ag+bd)}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{3bd\sqrt{a+bx^3}}{8ax}$$

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

```
[Out] -1/60*(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*Sqrt[
a + b*x^3]) - (3*b*c*Sqrt[a + b*x^3])/(20*a*x^2) - (3*b*d*Sqrt[a + b*x^3])/
(8*a*x) + (3*b^(1/3)*(b*d + 8*a*g)*Sqrt[a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)) - (b*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (
3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*d + 8*a*g)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
```

```
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) -
(3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(2*b^(1/3)*(b*c - 10*a*f) + 5*(1 - Sqrt[
3])*a^(1/3)*(b*d + 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Arc
Sin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
], -7 - 4*Sqrt[3]])/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
```

```
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3}$$
$$-\frac{1}{2}(3b)\int\frac{-\frac{c}{5}-\frac{dx}{4}-\frac{ex^2}{3}-\frac{fx^3}{2}-gx^4}{x^3\sqrt{a+bx^3}}\,dx$$

$$=-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3}$$
$$-\frac{3bc\sqrt{a+bx^3}}{20ax^2}+\frac{(3b)\int\frac{ad+\frac{4aex}{3}-\frac{1}{5}(bc-10af)x^2+4agx^3}{x^2\sqrt{a+bx^3}}\,dx}{8a}$$
$$=-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{20ax^2}$$
$$-\frac{3bd\sqrt{a+bx^3}}{8ax}-\frac{(3b)\int\frac{-\frac{8a^2e}{3}+\frac{2}{5}a(bc-10af)x-a(bd+8ag)x^2}{x\sqrt{a+bx^3}}\,dx}{16a^2}$$
$$=-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{20ax^2}$$
$$-\frac{3bd\sqrt{a+bx^3}}{8ax}-\frac{(3b)\int\frac{\frac{2}{5}a(bc-10af)-a(bd+8ag)x}{\sqrt{a+bx^3}}\,dx}{16a^2}+\frac{1}{2}(be)\int\frac{1}{x\sqrt{a+bx^3}}\,dx$$
$$=-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{20ax^2}-\frac{3bd\sqrt{a+bx^3}}{8ax}$$
$$+\frac{1}{6}(be)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)+\frac{\left(3b^{2/3}(bd+8ag)\right)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{16a}$$
$$-\frac{\left(3b\left(2bc-20af+\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd+8ag)}{\sqrt[3]{b}}\right)\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{80a}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{20ax^2} - \frac{3bd\sqrt{a+bx^3}}{8ax} + \frac{3\sqrt[3]{b}(bd+8ag)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(bd+8ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(2bc-20af+\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd+8ag)}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin\right.}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{1}{3}e\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{20ax^2} - \frac{3bd\sqrt{a+bx^3}}{8ax} + \frac{3\sqrt[3]{b}(bd+8ag)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{be\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(bd+8ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(2bc-20af+\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd+8ag)}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin\right.}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = -\frac{\sqrt{a+bx^3}\left(12ac\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{2},-\frac{2}{3},-\frac{bx^3}{a}\right)+5x\left(3ad\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{1}{2},-\frac{1}{3}\right.\right.\right.}{}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

```
[Out] -1/60*(Sqrt[a + b*x^3]*(12*a*c*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*x^3
)/a)] + 5*x*(3*a*d*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)] + 2*x*
(2*a*e*Sqrt[1 + (b*x^3)/a] + 2*b*e*x^3*ArcTanh[Sqrt[1 + (b*x^3)/a]] + 3*a*f
*x*Hypergeometric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)] + 6*a*g*x^2*Hypergeomet
ric2F1[-1/2, -1/3, 2/3, -((b*x^3)/a)]))))/(a*x^5*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	874
risch	Expression too large to display	1523
default	Expression too large to display	1571

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*c*(b*x^3+a)^(1/2)/x^5-1/4*d*(b*x^3+a)^(1/2)/x^4-1/3*e*(b*x^3+a)^(1/2)/
x^3-1/20*(10*a*f+3*b*c)/a*(b*x^3+a)^(1/2)/x^2-1/8*(8*a*g+3*b*d)/a*(b*x^3+a)
^(1/2)/x-2/3*I*(b*f-1/40*b*(10*a*f+3*b*c)/a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(g*b+1/16*b/a*(8*a*g+3*b*d))*3^(1/2
)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
```

```
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-1/3*b*e*arctanh((b*x^3+a)^(1/2)/a^(1/
2))/a^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx$$
$$= \left[\frac{10\sqrt{a}bex^5\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)-18\,(bc-10\,af)\sqrt{b}x^5\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right.}{}\right.$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="frica
s")
```

```
[Out] [1/120*(10*sqrt(a)*b*e*x^5*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt
(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 18*(b*c - 10*a*f)*sqrt(b)*x^5*weierstra
ssPInverse(0, -4*a/b, x) - 45*(b*d + 8*a*g)*sqrt(b)*x^5*weierstrassZeta(0,
-4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (15*(3*b*d + 8*a*g)*x^4 + 40*a
*e*x^2 + 6*(3*b*c + 10*a*f)*x^3 + 30*a*d*x + 24*a*c)*sqrt(b*x^3 + a))/(a*x^
5), 1/120*(20*sqrt(-a)*b*e*x^5*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2
*a)) - 18*(b*c - 10*a*f)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) - 45
*(b*d + 8*a*g)*sqrt(b)*x^5*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0
, -4*a/b, x)) - (15*(3*b*d + 8*a*g)*x^4 + 40*a*e*x^2 + 6*(3*b*c + 10*a*f)*x
^3 + 30*a*d*x + 24*a*c)*sqrt(b*x^3 + a))/(a*x^5)]
```

Sympy [A] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}d\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}g\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**6,x)
```

```
[Out] sqrt(a)*c*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**5*gamma(-2/3)) + sqrt(a)*d*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*f*gamma(-2/3)*hyp
er((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sq
rt(a)*g*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(
3*x*gamma(2/3)) - sqrt(b)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b*e*asinh(s
qrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^6}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^6}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^6}\,dx$$

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x)
```

```
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6, x)
```

3.455 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 659

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx$$
$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3}-\frac{3bd\sqrt{a+bx^3}}{20ax^2}$$
$$-\frac{3be\sqrt{a+bx^3}}{8ax}+\frac{3b^{4/3}e\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{b(bc-4af)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(2bd+5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-20ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{ar}\right.}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/12*b*(-4*a*f+b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/60*(10*c/x^6
+12*d/x^5+15*e/x^4+20*f/x^3+30*g/x^2)*(b*x^3+a)^(1/2)-1/12*b*c*(b*x^3+a)^(1
/2)/a/x^3-3/20*b*d*(b*x^3+a)^(1/2)/a/x^2-3/8*b*e*(b*x^3+a)^(1/2)/a/x+3/8*b^
(4/3)*e*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3/16*3^(1/4)*b^(4
/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3
```

```
)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^
(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)-1/40*3^(3/4)*b^(2/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(2*
b*d-20*a*g+5*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2
)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx =$$

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(bc-4af)}{12a^{3/2}}+\frac{3b^{4/3}e\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)-\frac{bc\sqrt{a+bx^3}}{12ax^3}-\frac{3bd\sqrt{a+bx^3}}{20ax^2}-\frac{3be\sqrt{a+bx^3}}{8ax}$$

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] -1/60*(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3 + (30*g)/x^2)*Sqr
t[a + b*x^3]) - (b*c*Sqrt[a + b*x^3])/(12*a*x^3) - (3*b*d*Sqrt[a + b*x^3])/
(20*a*x^2) - (3*b*e*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(4/3)*e*Sqrt[a + b*x^3]
)/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (b*(b*c - 4*a*f)*ArcTanh[Sqrt
[a + b*x^3]/Sqrt[a]])/(12*a^(3/2)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*e
```

```
*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3)
 + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^
(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(2*b*d + 5*(1
- Sqrt[3])*a^(1/3)*b^(2/3)*e - 20*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3} \\
&\quad -\frac{1}{2}(3b)\int \frac{-\frac{c}{6}-\frac{dx}{5}-\frac{ex^2}{4}-\frac{fx^3}{3}-\frac{gx^4}{2}}{x^4\sqrt{a+bx^3}}\,dx \\
&= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3} \\
&\quad -\frac{bc\sqrt{a+bx^3}}{12ax^3}+\frac{b\int \frac{\frac{6ad}{5}+\frac{3aex}{2}-\frac{1}{2}(bc-4af)x^2+3agx^3}{x^3\sqrt{a+bx^3}}\,dx}{4a} \\
&= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3} \\
&\quad -\frac{3bd\sqrt{a+bx^3}}{20ax^2}-\frac{b\int \frac{-6a^2e+2a(bc-4af)x+\frac{6}{5}a(bd-10ag)x^2}{x^2\sqrt{a+bx^3}}\,dx}{16a^2} \\
&= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3} \\
&\quad -\frac{3bd\sqrt{a+bx^3}}{20ax^2}-\frac{3be\sqrt{a+bx^3}}{8ax}+\frac{b\int \frac{-4a^2(bc-4af)-\frac{12}{5}a^2(bd-10ag)x+6a^2bex^2}{x\sqrt{a+bx^3}}\,dx}{32a^3} \\
&= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3}-\frac{3bd\sqrt{a+bx^3}}{20ax^2} \\
&\quad -\frac{3be\sqrt{a+bx^3}}{8ax}+\frac{b\int \frac{-\frac{12}{5}a^2(bd-10ag)+6a^2bex}{\sqrt{a+bx^3}}\,dx}{32a^3}-\frac{(b(bc-4af))\int \frac{1}{x\sqrt{a+bx^3}}\,dx}{8a} \\
&= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3} \\
&\quad -\frac{3bd\sqrt{a+bx^3}}{20ax^2}-\frac{3be\sqrt{a+bx^3}}{8ax}+\frac{\left(3b^{5/3}e\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{16a} \\
&\quad -\frac{(b(bc-4af))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{24a} \\
&\quad -\frac{\left(3b\left(2bd+5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-20ag\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{80a}
\end{aligned}
$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3} - \frac{3bd\sqrt{a+bx^3}}{20ax^2}-\frac{3be\sqrt{a+bx^3}}{8ax}+\frac{3b^{4/3}e\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right.}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(2bd+5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-20ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\right.}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{(bc-4af)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{12a}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3}-\frac{3bd\sqrt{a+bx^3}}{20ax^2} - \frac{3be\sqrt{a+bx^3}}{8ax}+\frac{3b^{4/3}e\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{b(bc-4af)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right.}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(2bd+5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-20ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\right.}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.43 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx =$$

$$-\frac{\sqrt{a+bx^3}\left(36a^3d\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{2},-\frac{2}{3},-\frac{bx^3}{a}\right)+5x\left(9a^3e\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{1}{2},-\right.\right.\right.}{}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] -1/180*(Sqrt[a + b*x^3]*(36*a^3*d*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*
x^3)/a)] + 5*x*(9*a^3*e*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)] +
 2*x*(6*a^2*f*(a*Sqrt[1 + (b*x^3)/a] + b*x^3*ArcTanh[Sqrt[1 + (b*x^3)/a]])
+ 9*a^3*g*x*Hypergeometric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)] + 4*b^2*c*x^3*
(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x^3)/
a]))))/(a^3*x^5*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 883, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	883
risch	Expression too large to display	1102
default	Expression too large to display	1180

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*c*(b*x^3+a)^(1/2)/x^6-1/5*d*(b*x^3+a)^(1/2)/x^5-1/4*e*(b*x^3+a)^(1/2)/
x^4-1/12*(4*a*f+b*c)/a*(b*x^3+a)^(1/2)/x^3-1/20/a*(10*a*g+3*b*d)*(b*x^3+a)^
(1/2)/x^2-3/8*b*e*(b*x^3+a)^(1/2)/a/x-2/3*I*(g*b-1/40*b/a*(10*a*g+3*b*d))*3
^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-1/8*I/a*b*e*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
```

```
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-1/12*(4*a*f-b*c)*b/a^(3/2)*arctan
h((b*x^3+a)^(1/2)/a^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx$$
$$= \left[-\frac{90\,ab^{\frac{3}{2}}ex^6\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+5\,(b^2c-4\,abf)\sqrt{a}x^6\log\left(\frac{b^2x^6+8\,ab\cdots}{\cdots}\right.}{\cdots}\right.$$
$$-\frac{45\,ab^{\frac{3}{2}}ex^6\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+5\,(b^2c-4\,abf)\sqrt{-a}x^6\arctan\left(\frac{(bx^3\cdots}{\cdots}\right.}{\cdots}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="frica
s")
```

```
[Out] [-1/240*(90*a*b^(3/2)*e*x^6*weierstrassZeta(0, -4*a/b, weierstrassPInverse(
0, -4*a/b, x)) + 5*(b^2*c - 4*a*b*f)*sqrt(a)*x^6*log((b^2*x^6 + 8*a*b*x^3 -
 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 36*(a*b*d - 10*a^2
*g)*sqrt(b)*x^6*weierstrassPInverse(0, -4*a/b, x) + 2*(45*a*b*e*x^5 + 30*a^
2*e*x^2 + 6*(3*a*b*d + 10*a^2*g)*x^4 + 24*a^2*d*x + 10*(a*b*c + 4*a^2*f)*x^
3 + 20*a^2*c)*sqrt(b*x^3 + a))/(a^2*x^6), -1/120*(45*a*b^(3/2)*e*x^6*weiers
trassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + 5*(b^2*c - 4*a*b*
f)*sqrt(-a)*x^6*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3
+ a^2)) + 18*(a*b*d - 10*a^2*g)*sqrt(b)*x^6*weierstrassPInverse(0, -4*a/b,
x) + (45*a*b*e*x^5 + 30*a^2*e*x^2 + 6*(3*a*b*d + 10*a^2*g)*x^4 + 24*a^2*d*x
 + 10*(a*b*c + 4*a^2*f)*x^3 + 20*a^2*c)*sqrt(b*x^3 + a))/(a^2*x^6)]
```

Sympy [A] (verification not implemented)

Time = 4.76 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \frac{\sqrt{a}d\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}g\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} - \frac{ac}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}c}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}}c}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{bf\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}} + \frac{b^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] sqrt(a)*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**5*gamma(-2/3)) + sqrt(a)*e*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*g*gamma(-2/3)*hyp
er((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) - a*
c/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)*c/(4*x**(9/2)*sqrt(a
/(b*x**3) + 1)) - sqrt(b)*f*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*c/
(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b*f*asinh(sqrt(a)/(sqrt(b)*x**(3/2))
)/(3*sqrt(a)) + b**2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^7}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="maxim
a")
```

```
[Out] -1/24*(b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(
3/2) + 2*((b*x^3 + a)^(3/2)*b^2 + sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a -
 2*(b*x^3 + a)*a^2 + a^3))*c + integrate(sqrt(b*x^3 + a)*(g*x^3 + f*x^2 + e
*x + d)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^7}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^7}\,dx$$

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x)
```

```
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7, x)
```

3.456 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 711

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$$
$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}$$
$$-\frac{3bc\sqrt{a+bx^3}}{56ax^4}-\frac{bd\sqrt{a+bx^3}}{12ax^3}-\frac{3be\sqrt{a+bx^3}}{20ax^2}+\frac{3b(5bc-14af)\sqrt{a+bx^3}}{112a^2x}$$
$$-\frac{3b^{4/3}(5bc-14af)\sqrt{a+bx^3}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{b(bd-4ag)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$
$$+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(5bc-14af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(5bc-14af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}}{560a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/12*b*(-4*a*g+b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/420*(60*c/x^
7+70*d/x^6+84*e/x^5+105*f/x^4+140*g/x^3)*(b*x^3+a)^(1/2)-3/56*b*c*(b*x^3+a)
^(1/2)/a/x^4-1/12*b*d*(b*x^3+a)^(1/2)/a/x^3-3/20*b*e*(b*x^3+a)^(1/2)/a/x^2+
```

```
3/112*b*(-14*a*f+5*b*c)*(b*x^3+a)^(1/2)/a^2/x-3/112*b^(4/3)*(-14*a*f+5*b*c)
*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+3/224*3^(1/4)*b^(4/3)*
(-14*a*f+5*b*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)-1/560*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)*x)*Ellipt
icF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/
2)+2*I)*(28*a^(2/3)*b^(1/3)*e-5*(-14*a*f+5*b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx =$$

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{560a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5bc-14af)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(bd-4ag)}{12a^{3/2}}$$

$$-\frac{3b^{4/3}\sqrt{a+bx^3}(5bc-14af)}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{3b\sqrt{a+bx^3}(5bc-14af)}{112a^2x}$$

$$-\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)-\frac{3bc\sqrt{a+bx^3}}{56ax^4}-\frac{bd\sqrt{a+bx^3}}{12ax^3}-\frac{3be\sqrt{a+bx^3}}{20ax^2}$$

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

```
[Out] -1/420*(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 + (140*g)/x^3)*
Sqrt[a + b*x^3]) - (3*b*c*Sqrt[a + b*x^3])/(56*a*x^4) - (b*d*Sqrt[a + b*x^3
])/(12*a*x^3) - (3*b*e*Sqrt[a + b*x^3])/(20*a*x^2) + (3*b*(5*b*c - 14*a*f)*
Sqrt[a + b*x^3])/(112*a^2*x) - (3*b^(4/3)*(5*b*c - 14*a*f)*Sqrt[a + b*x^3])
/(112*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (b*(b*d - 4*a*g)*ArcTanh[S
qrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3
)*(5*b*c - 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(224*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(
4/3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(5*b*c - 14*a*f))*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(560*a^(5/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
```

```
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3} \\
&\quad -\frac{1}{2}(3b)\int\frac{-\frac{c}{7}-\frac{dx}{6}-\frac{ex^2}{5}-\frac{fx^3}{4}-\frac{gx^4}{3}}{x^5\sqrt{a+bx^3}}\,dx \\
&= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3} \\
&\quad -\frac{3bc\sqrt{a+bx^3}}{56ax^4}+\frac{(3b)\int\frac{\frac{4ad}{3}+\frac{8aex}{5}-\frac{1}{7}(5bc-14af)x^2+\frac{8}{3}agx^3}{x^4\sqrt{a+bx^3}}\,dx}{16a} \\
&= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{56ax^4} \\
&\quad -\frac{bd\sqrt{a+bx^3}}{12ax^3}-\frac{b\int\frac{-\frac{48a^2e}{5}+\frac{6}{7}a(5bc-14af)x+4a(bd-4ag)x^2}{x^3\sqrt{a+bx^3}}\,dx}{32a^2} \\
&= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{56ax^4} \\
&\quad -\frac{bd\sqrt{a+bx^3}}{12ax^3}-\frac{3be\sqrt{a+bx^3}}{20ax^2}+\frac{b\int\frac{-\frac{24}{7}a^2(5bc-14af)-16a^2(bd-4ag)x-\frac{48}{5}a^2bex^2}{x^2\sqrt{a+bx^3}}\,dx}{128a^3} \\
&= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{56ax^4}-\frac{bd\sqrt{a+bx^3}}{12ax^3} \\
&\quad -\frac{3be\sqrt{a+bx^3}}{20ax^2}+\frac{3b(5bc-14af)\sqrt{a+bx^3}}{112a^2x}-\frac{b\int\frac{32a^3(bd-4ag)+\frac{96}{5}a^3bex+\frac{24}{7}a^2b(5bc-14af)x^2}{x\sqrt{a+bx^3}}\,dx}{256a^4} \\
&= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{56ax^4} \\
&\quad -\frac{bd\sqrt{a+bx^3}}{12ax^3}-\frac{3be\sqrt{a+bx^3}}{20ax^2}+\frac{3b(5bc-14af)\sqrt{a+bx^3}}{112a^2x} \\
&\quad -\frac{b\int\frac{\frac{96}{5}a^3be+\frac{24}{7}a^2b(5bc-14af)x}{\sqrt{a+bx^3}}\,dx}{256a^4}-\frac{(b(bd-4ag))\int\frac{1}{x\sqrt{a+bx^3}}\,dx}{8a}
\end{aligned}
$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{56ax^4} - \frac{bd\sqrt{a+bx^3}}{12ax^3} - \frac{3be\sqrt{a+bx^3}}{20ax^2} + \frac{3b(5bc-14af)\sqrt{a+bx^3}}{112a^2x} - \frac{\left(3b^{5/3}(5bc-14af)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{224a^2} - \frac{\left(3b^{5/3}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(5bc-14af)\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{1120a^{5/3}} - \frac{(b(bd-4ag))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{24a}$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{56ax^4} - \frac{bd\sqrt{a+bx^3}}{12ax^3} - \frac{3be\sqrt{a+bx^3}}{20ax^2} + \frac{3b(5bc-14af)\sqrt{a+bx^3}}{112a^2x} - \frac{3b^{4/3}(5bc-14af)\sqrt{a+bx^3}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(5bc-14af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(5bc-14af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F(}{560a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{(bd-4ag)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{12a}$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{56ax^4} - \frac{bd\sqrt{a+bx^3}}{12ax^3}-\frac{3be\sqrt{a+bx^3}}{20ax^2}+\frac{3b(5bc-14af)\sqrt{a+bx^3}}{112a^2x} - \frac{3b^{4/3}(5bc-14af)\sqrt{a+bx^3}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{b(bd-4ag)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(5bc-14af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(5bc-14af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F}{560a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.49 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = -\frac{\sqrt{a+bx^3}\left(180a^3c\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{1}{2},-\frac{4}{3},-\frac{bx^3}{a}\right)+7x^2\left(36a^3e\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\right.\right.\right.}{}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

```
[Out] -1/1260*(Sqrt[a + b*x^3]*(180*a^3*c*Hypergeometric2F1[-7/3, -1/2, -4/3, -((
b*x^3)/a)] + 7*x^2*(36*a^3*e*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*x^3)/
a)] + 5*x*(12*a^2*g*x*(a*Sqrt[1 + (b*x^3)/a] + b*x^3*ArcTanh[Sqrt[1 + (b*x^
3)/a]]) + 9*a^3*f*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)] + 8*b^2
*d*x^4*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2, 3, 5/2, 1 + (
b*x^3)/a]))))/(a^3*x^7*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 901, normalized size of antiderivative = 1.27

method	result	size
elliptic	Expression too large to display	901
risch	Expression too large to display	1283
default	Expression too large to display	1376

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*c*(b*x^3+a)^(1/2)/x^7-1/6*d*(b*x^3+a)^(1/2)/x^6-1/5*e*(b*x^3+a)^(1/2)/
x^5-1/56*(14*a*f+3*b*c)/a*(b*x^3+a)^(1/2)/x^4-1/12/a*(4*a*g+b*d)*(b*x^3+a)^
(1/2)/x^3-3/20*b*e*(b*x^3+a)^(1/2)/a/x^2-3/112*(14*a*f-5*b*c)*b/a^2*(b*x^3+
a)^(1/2)/x+1/20*I/a*b*e*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*
b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(
-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))-1/112*I*(14*a*f-5*b*c)*b/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)))-1/12*(4*a*g-b*d)*b/a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$$

$$= \left[-\frac{252\,ab^{\frac{3}{2}}ex^7\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+35\,(b^2d-4\,abg)\sqrt{a}x^7\log\left(\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8}{x^6}\right.}{} \right.$$

$$-\frac{252\,ab^{\frac{3}{2}}ex^7\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+70\,(b^2d-4\,abg)\sqrt{-a}x^7\arctan\left(\frac{(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{-a}}{2\,(abx^3+a^2)}\right)-45\,(}{}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="frica
s")
```

```
[Out] [-1/1680*(252*a*b^(3/2)*e*x^7*weierstrassPInverse(0, -4*a/b, x) + 35*(b^2*d
 - 4*a*b*g)*sqrt(a)*x^7*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x
^3 + a)*sqrt(a) + 8*a^2)/x^6) - 45*(5*b^2*c - 14*a*b*f)*sqrt(b)*x^7*weierst
rassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (252*a*b*e*x^5 - 4
5*(5*b^2*c - 14*a*b*f)*x^6 + 336*a^2*e*x^2 + 140*(a*b*d + 4*a^2*g)*x^4 + 28
0*a^2*d*x + 30*(3*a*b*c + 14*a^2*f)*x^3 + 240*a^2*c)*sqrt(b*x^3 + a))/(a^2*
x^7), -1/1680*(252*a*b^(3/2)*e*x^7*weierstrassPInverse(0, -4*a/b, x) + 70*(
b^2*d - 4*a*b*g)*sqrt(-a)*x^7*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt
(-a)/(a*b*x^3 + a^2)) - 45*(5*b^2*c - 14*a*b*f)*sqrt(b)*x^7*weierstrassZeta
(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (252*a*b*e*x^5 - 45*(5*b^2
*c - 14*a*b*f)*x^6 + 336*a^2*e*x^2 + 140*(a*b*d + 4*a^2*g)*x^4 + 280*a^2*d*
x + 30*(3*a*b*c + 14*a^2*f)*x^3 + 240*a^2*c)*sqrt(b*x^3 + a))/(a^2*x^7)]
```

Sympy [A] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} - \frac{ad}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}d}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}g\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}}d}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{bg\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}} + \frac{b^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**8,x)
```

```
[Out] sqrt(a)*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**7*gamma(-4/3)) + sqrt(a)*e*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*f*gamma(-4/3)*hyp
er((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) -
a*d/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)*d/(4*x**(9/2)*sqrt
(a/(b*x**3) + 1)) - sqrt(b)*g*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*
d/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2
)))/(3*sqrt(a)) + b**2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^8}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^8}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx=\int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^8}\,dx$$

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x)
```

```
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8, x)
```

3.457 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 743

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$$
$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3}$$
$$-\frac{3bc\sqrt{a+bx^3}}{80ax^5}-\frac{3bd\sqrt{a+bx^3}}{56ax^4}-\frac{be\sqrt{a+bx^3}}{12ax^3}+\frac{3b(7bc-16af)\sqrt{a+bx^3}}{320a^2x^2}$$
$$+\frac{3b(5bd-14ag)\sqrt{a+bx^3}}{112a^2x}-\frac{3b^{4/3}(5bd-14ag)\sqrt{a+bx^3}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{b^2e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$
$$+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(5bd-14ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(7\sqrt[3]{b}(7bc-16af)+20\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd-14ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}}}{2240a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/12*b^2*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/840*(105*c/x^8+120*d/
x^7+140*e/x^6+168*f/x^5+210*g/x^4)*(b*x^3+a)^(1/2)-3/80*b*c*(b*x^3+a)^(1/2)
/a/x^5-3/56*b*d*(b*x^3+a)^(1/2)/a/x^4-1/12*b*e*(b*x^3+a)^(1/2)/a/x^3+3/320*
```

```
b*(-16*a*f+7*b*c)*(b*x^3+a)^(1/2)/a^2/x^2+3/112*b*(-14*a*g+5*b*d)*(b*x^3+a)
^(1/2)/a^2/x-3/112*b^(4/3)*(-14*a*g+5*b*d)*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))+3/224*3^(1/4)*b^(4/3)*(-14*a*g+5*b*d)*(a^(1/3)+b^(1/3)*
x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(
a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/2240
*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2
)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*(-16*a*f+7*b*
c)+20*a^(1/3)*(-14*a*g+5*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
/a^2/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$$

$$= \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5bd-14ag)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\right.}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{b^2e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}-\frac{3b^{4/3}\sqrt{a+bx^3}(5bd-14ag)}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{3b\sqrt{a+bx^3}(7bc-16af)}{320a^2x^2}+\frac{3b\sqrt{a+bx^3}(5bd-14ag)}{112a^2x}$$

$$+\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right.}{2240a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)-\frac{3bc\sqrt{a+bx^3}}{80ax^5}-\frac{3bd\sqrt{a+bx^3}}{56ax^4}-\frac{be\sqrt{a+bx^3}}{12ax^3}$$

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

```
[Out] -1/840*(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5 + (210*g)/x^
4)*Sqrt[a + b*x^3]) - (3*b*c*Sqrt[a + b*x^3])/(80*a*x^5) - (3*b*d*Sqrt[a +
b*x^3])/(56*a*x^4) - (b*e*Sqrt[a + b*x^3])/(12*a*x^3) + (3*b*(7*b*c - 16*a*
f)*Sqrt[a + b*x^3])/(320*a^2*x^2) + (3*b*(5*b*d - 14*a*g)*Sqrt[a + b*x^3])/
(112*a^2*x) - (3*b^(4/3)*(5*b*d - 14*a*g)*Sqrt[a + b*x^3])/(112*a^2*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) + (b^2*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(1
2*a^(3/2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*b*d - 14*a*g)*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(5/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(7*b^(1/3)*(7*b*c - 16
*a*f) + 20*(1 - Sqrt[3])*a^(1/3)*(5*b*d - 14*a*g))*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2240*a^2*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
```

```
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} - \frac{1}{2}(3b)\int\frac{-\frac{c}{8}-\frac{dx}{7}-\frac{ex^2}{6}-\frac{fx^3}{5}-\frac{gx^4}{4}}{x^6\sqrt{a+bx^3}}\,dx$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} + \frac{(3b)\int\frac{\frac{10ad}{7}+\frac{5aex}{3}-\frac{1}{8}(7bc-16af)x^2+\frac{5}{2}agx^3}{x^5\sqrt{a+bx^3}}\,dx}{20a}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bd\sqrt{a+bx^3}}{56ax^4} - \frac{(3b)\int\frac{-\frac{40a^2e}{3}+a(7bc-16af)x+\frac{10}{7}a(5bd-14ag)x^2}{x^4\sqrt{a+bx^3}}\,dx}{160a^2}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bd\sqrt{a+bx^3}}{56ax^4} - \frac{be\sqrt{a+bx^3}}{12ax^3} + \frac{b\int\frac{-6a^2(7bc-16af)-\frac{60}{7}a^2(5bd-14ag)x-40a^2bex^2}{x^3\sqrt{a+bx^3}}\,dx}{320a^3}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bd\sqrt{a+bx^3}}{56ax^4} - \frac{be\sqrt{a+bx^3}}{12ax^3} + \frac{3b(7bc-16af)\sqrt{a+bx^3}}{320a^2x^2} - \frac{b\int\frac{\frac{240}{7}a^3(5bd-14ag)+160a^3bex-6a^2b(7bc-16af)x^2}{x^2\sqrt{a+bx^3}}\,dx}{1280a^4}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bd\sqrt{a+bx^3}}{56ax^4} - \frac{be\sqrt{a+bx^3}}{12ax^3} + \frac{3b(7bc-16af)\sqrt{a+bx^3}}{320a^2x^2} + \frac{3b(5bd-14ag)\sqrt{a+bx^3}}{112a^2x} + \frac{b\int\frac{-320a^4be+12a^3b(7bc-16af)x-\frac{240}{7}a^3b(5bd-14ag)x^2}{x\sqrt{a+bx^3}}\,dx}{2560a^5}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bd\sqrt{a+bx^3}}{56ax^4} - \frac{be\sqrt{a+bx^3}}{12ax^3} + \frac{3b(7bc-16af)\sqrt{a+bx^3}}{320a^2x^2} + \frac{3b(5bd-14ag)\sqrt{a+bx^3}}{112a^2x} + \frac{b\int\frac{12a^3b(7bc-16af)-\frac{240}{7}a^3b(5bd-14ag)x}{\sqrt{a+bx^3}}\,dx}{2560a^5} - \frac{(b^2e)\int\frac{1}{x\sqrt{a+bx^3}}\,dx}{8a}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bd\sqrt{a+bx^3}}{56ax^4} - \frac{be\sqrt{a+bx^3}}{12ax^3} + \frac{3b(7bc-16af)\sqrt{a+bx^3}}{320a^2x^2} + \frac{3b(5bd-14ag)\sqrt{a+bx^3}}{112a^2x} - \frac{(b^2e)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{24a} - \frac{\left(3b^{5/3}(5bd-14ag)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{224a^2} + \frac{\left(3b^{5/3}\left(7\sqrt[3]{b}(7bc-16af)+20\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd-14ag)\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{4480a^2}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bd\sqrt{a+bx^3}}{56ax^4} - \frac{be\sqrt{a+bx^3}}{12ax^3} + \frac{3b(7bc-16af)\sqrt{a+bx^3}}{320a^2x^2} + \frac{3b(5bd-14ag)\sqrt{a+bx^3}}{112a^2x} - \frac{3b^{4/3}(5bd-14ag)\sqrt{a+bx^3}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}(5bd-14ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{4/3}\left(7\sqrt[3]{b}(7bc-16af)+20\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd-14ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}{2240a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{(be)\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{12a}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bd\sqrt{a+bx^3}}{56ax^4}-\frac{be\sqrt{a+bx^3}}{12ax^3}+\frac{3b(7bc-16af)\sqrt{a+bx^3}}{320a^2x^2} + \frac{3b(5bd-14ag)\sqrt{a+bx^3}}{112a^2x}-\frac{3b^{4/3}(5bd-14ag)\sqrt{a+bx^3}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{b^2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}} + \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(5bd-14ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(7\sqrt[3]{b}(7bc-16af)+20\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd-14ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}}}{2240a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = -\frac{\sqrt{a+bx^3}\left(315a^3c\,\text{Hypergeometric2F1}\left(-\frac{8}{3},-\frac{1}{2},-\frac{5}{3},-\frac{bx^3}{a}\right)+360a^3dx\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{1}{2},-\right.\right.}{}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

```
[Out] -1/2520*(Sqrt[a + b*x^3]*(315*a^3*c*Hypergeometric2F1[-8/3, -1/2, -5/3, -((
b*x^3)/a)] + 360*a^3*d*x*Hypergeometric2F1[-7/3, -1/2, -4/3, -((b*x^3)/a)]
+ 14*x^3*(36*a^3*f*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*x^3)/a)] + 45*a
^3*g*x*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)] + 40*b^2*e*x^5*(a
+ b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x^3)/a])
))/(a^3*x^8*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 931, normalized size of antiderivative = 1.25

method	result	size
elliptic	Expression too large to display	931
risch	Expression too large to display	1579
default	Expression too large to display	1679

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*c*(b*x^3+a)^(1/2)/x^8-1/7*d*(b*x^3+a)^(1/2)/x^7-1/6*e*(b*x^3+a)^(1/2)/
x^6-1/80*(16*a*f+3*b*c)/a*(b*x^3+a)^(1/2)/x^5-1/56/a*(14*a*g+3*b*d)*(b*x^3+
a)^(1/2)/x^4-1/12*b*e*(b*x^3+a)^(1/2)/a/x^3-3/320*b*(16*a*f-7*b*c)/a^2*(b*x
^3+a)^(1/2)/x^2-3/112*(14*a*g-5*b*d)*b/a^2*(b*x^3+a)^(1/2)/x+1/320*I*(16*a*
f-7*b*c)*b/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)-1/112*I*(14*a*g-5*b*d)*b/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1
/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/
12*b^2*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 482, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$$

$$= \left[\frac{140\sqrt{a}b^2ex^8\log\left(\frac{b^2x^6+8\,abx^3+4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+63\,(7\,b^2c-16\,abf)\sqrt{b}x^8\text{weierstrassPInverse}\left(0,-\frac{4}{b}\right.}{}\right.$$

$$-\frac{280\sqrt{-a}b^2ex^8\arctan\left(\frac{(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{-a}}{2\,(abx^3+a^2)}\right)-63\,(7\,b^2c-16\,abf)\sqrt{b}x^8\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-}{}$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="frica
s")
```

```
[Out] [1/6720*(140*sqrt(a)*b^2*e*x^8*log((b^2*x^6 + 8*a*b*x^3 + 4*(b*x^3 + 2*a)*s
qrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 63*(7*b^2*c - 16*a*b*f)*sqrt(b)*x^8*
weierstrassPInverse(0, -4*a/b, x) + 180*(5*b^2*d - 14*a*b*g)*sqrt(b)*x^8*we
ierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (560*a*b*e*x^
5 - 180*(5*b^2*d - 14*a*b*g)*x^7 - 63*(7*b^2*c - 16*a*b*f)*x^6 + 1120*a^2*e
*x^2 + 120*(3*a*b*d + 14*a^2*g)*x^4 + 960*a^2*d*x + 84*(3*a*b*c + 16*a^2*f)
*x^3 + 840*a^2*c)*sqrt(b*x^3 + a))/(a^2*x^8), -1/6720*(280*sqrt(-a)*b^2*e*x
^8*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) - 63*
(7*b^2*c - 16*a*b*f)*sqrt(b)*x^8*weierstrassPInverse(0, -4*a/b, x) - 180*(5
*b^2*d - 14*a*b*g)*sqrt(b)*x^8*weierstrassZeta(0, -4*a/b, weierstrassPInver
se(0, -4*a/b, x)) + (560*a*b*e*x^5 - 180*(5*b^2*d - 14*a*b*g)*x^7 - 63*(7*b
^2*c - 16*a*b*f)*x^6 + 1120*a^2*e*x^2 + 120*(3*a*b*d + 14*a^2*g)*x^4 + 960*
a^2*d*x + 84*(3*a*b*c + 16*a^2*f)*x^3 + 840*a^2*c)*sqrt(b*x^3 + a))/(a^2*x^
8)]
```

Sympy [A] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)} + \frac{\sqrt{a}d\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}g\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} - \frac{ae}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}e}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{3}{2}}e}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{b^2e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}$$

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**9,x)
```

```
[Out] sqrt(a)*c*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**8*gamma(-5/3)) + sqrt(a)*d*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*f*gamma(-5/3)*hyp
er((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) +
sqrt(a)*g*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**4*gamma(-1/3)) - a*e/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - s
qrt(b)*e/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*e/(12*a*x**(3/2)*sqrt
(a/(b*x**3) + 1)) + b**2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^9}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="maxim
a")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^9}\,dx$$

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="giac"
)
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = \int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^9}\,dx$$

```
[In] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x)
[Out] int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9, x)
```

3.458 $\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 791

$$\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = -\frac{4a^3e\sqrt{a+bx^3}}{105b^2} + \frac{54a^2(23bc-8af)x\sqrt{a+bx^3}}{21505b^2} + \frac{54a^2(5bd-2ag)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{2a^2ex^3\sqrt{a+bx^3}}{105b} + \frac{54a^2fx^4\sqrt{a+bx^3}}{4301b} + \frac{54a^2gx^5\sqrt{a+bx^3}}{6175b} - \frac{216a^3(5bd-2ag)\sqrt{a+bx^3}}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2x^3(a+bx^3)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225} + \frac{2ax^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5)}{185910725} + \frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}(5bd-2ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(1729\sqrt[3]{b}(23bc-8af)-8602\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd-2ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots\right.}}}{371821450b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/3900225*x^3*(b*x^3+a)^(3/2)*(156009*g*x^5+169575*f*x^4+185725*e*x^3+20527
5*d*x^2+229425*c*x)-4/105*a^3*e*(b*x^3+a)^(1/2)/b^2+54/21505*a^2*(-8*a*f+23
```

```
*b*c)*x*(b*x^3+a)^(1/2)/b^2+54/8645*a^2*(-2*a*g+5*b*d)*x^2*(b*x^3+a)^(1/2)/
b^2+2/105*a^2*e*x^3*(b*x^3+a)^(1/2)/b+54/4301*a^2*f*x^4*(b*x^3+a)^(1/2)/b+5
4/6175*a^2*g*x^5*(b*x^3+a)^(1/2)/b+2/185910725*a*x^3*(3522519*g*x^5+4279275
*f*x^4+5311735*e*x^3+6774075*d*x^2+8947575*c*x)*(b*x^3+a)^(1/2)-216/8645*a^
3*(-2*a*g+5*b*d)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+10
8/8645*3^(1/4)*a^(10/3)*(-2*a*g+5*b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/
3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1
/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^
(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-36/37182145*3^(3/4)*a^3*(
a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1729*b^(1/3)*(-8*a*f+23*b*c)-8602*a^(1/3)
*(-2*a*g+5*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^
3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 791, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used

$= \{1840, 1850, 1902, 1608, 1900, 267, 1892, 224, 1891\}$

$$\int x^3(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)\,dx = \frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5bd-2ag)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{216a^3\sqrt{a+bx^3}(5bd-2ag)}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{4a^3e\sqrt{a+bx^3}}{105b^2}+\frac{54a^2x\sqrt{a+bx^3}(23bc-8af)}{21505b^2}$$

$$+\frac{54a^2x^2\sqrt{a+bx^3}(5bd-2ag)}{8645b^2}+\frac{2a^2ex^3\sqrt{a+bx^3}}{105b}+\frac{54a^2fx^4\sqrt{a+bx^3}}{4301b}+\frac{54a^2gx^5\sqrt{a+bx^3}}{6175b}$$

$$-\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right.}{371821145b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2x^3(a+bx^3)^{3/2}(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5)}{3900225}$$

$$+\frac{2ax^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5)}{185910725}$$

```
[In] Int[x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (-4*a^3*e*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*(23*b*c - 8*a*f)*x*Sqrt[a +
b*x^3])/(21505*b^2) + (54*a^2*(5*b*d - 2*a*g)*x^2*Sqrt[a + b*x^3])/(8645*b^
2) + (2*a^2*e*x^3*Sqrt[a + b*x^3])/(105*b) + (54*a^2*f*x^4*Sqrt[a + b*x^3])
/(4301*b) + (54*a^2*g*x^5*Sqrt[a + b*x^3])/(6175*b) - (216*a^3*(5*b*d - 2*a
*g)*Sqrt[a + b*x^3])/(8645*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (
2*x^3*(a + b*x^3)^(3/2)*(229425*c*x + 205275*d*x^2 + 185725*e*x^3 + 169575*
f*x^4 + 156009*g*x^5))/3900225 + (2*a*x^3*Sqrt[a + b*x^3]*(8947575*c*x + 67
74075*d*x^2 + 5311735*e*x^3 + 4279275*f*x^4 + 3522519*g*x^5))/185910725 + (
108*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*(5*b*d - 2*a*g)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3]) - (36*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(1729*b^(1/3)*(23*b*c - 8*a*f) - 8
602*(1 - Sqrt[3])*a^(1/3)*(5*b*d - 2*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
```

```
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(37182145*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2x^3(a+bx^3)^{3/2}(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5)}{3900225}\\ &\quad+\frac{1}{2}(9a)\int x^3\sqrt{a+bx^3}\left(\frac{2c}{17}+\frac{2dx}{19}+\frac{2ex^2}{21}+\frac{2fx^3}{23}+\frac{2gx^4}{25}\right)dx\\ &= \frac{2x^3(a+bx^3)^{3/2}(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5)}{3900225}\\ &\quad+\frac{2ax^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5)}{185910725}\\ &\quad+\frac{1}{4}(27a^2)\int\frac{x^3\left(\frac{4c}{187}+\frac{4dx}{247}+\frac{4ex^2}{315}+\frac{4fx^3}{391}+\frac{4gx^4}{475}\right)}{\sqrt{a+bx^3}}dx\end{aligned}$$

$$= \frac{54a^2gx^5\sqrt{a+bx^3}}{6175b} + \frac{2x^3(a+bx^3)^{3/2}(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5)}{3900225} + \frac{2ax^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5)}{185910725} + \frac{(27a^2)\int \frac{x^3\left(\frac{26bc}{187}+\frac{2}{95}(5bd-2ag)x+\frac{26}{315}bex^2+\frac{26}{391}bfx^3\right)}{\sqrt{a+bx^3}}dx}{26b}$$

$$= \frac{54a^2fx^4\sqrt{a+bx^3}}{4301b} + \frac{54a^2gx^5\sqrt{a+bx^3}}{6175b} + \frac{2x^3(a+bx^3)^{3/2}(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5)}{3900225} + \frac{2ax^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5)}{185910725} + \frac{(27a^2)\int \frac{x^3\left(\frac{13}{391}b(23bc-8af)+\frac{11}{95}b(5bd-2ag)x+\frac{143}{315}b^2ex^2\right)}{\sqrt{a+bx^3}}dx}{143b^2}$$

$$= \frac{2a^2ex^3\sqrt{a+bx^3}}{105b} + \frac{54a^2fx^4\sqrt{a+bx^3}}{4301b} + \frac{54a^2gx^5\sqrt{a+bx^3}}{6175b} + \frac{2x^3(a+bx^3)^{3/2}(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5)}{3900225} + \frac{2ax^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5)}{185910725} + \frac{(6a^2)\int \frac{-\frac{143}{105}ab^2ex^2+\frac{117}{782}b^2(23bc-8af)x^3+\frac{99}{190}b^2(5bd-2ag)x^4}{\sqrt{a+bx^3}}dx}{143b^3}$$

$$= \frac{2a^2ex^3\sqrt{a+bx^3}}{105b} + \frac{54a^2fx^4\sqrt{a+bx^3}}{4301b} + \frac{54a^2gx^5\sqrt{a+bx^3}}{6175b} + \frac{2x^3(a+bx^3)^{3/2}(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5)}{3900225} + \frac{2ax^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5)}{185910725} + \frac{(6a^2)\int \frac{x^2\left(-\frac{143}{105}ab^2e+\frac{117}{782}b^2(23bc-8af)x+\frac{99}{190}b^2(5bd-2ag)x^2\right)}{\sqrt{a+bx^3}}dx}{143b^3}$$

$$
\begin{aligned}
&= \frac{54a^2(5bd-2ag)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{2a^2ex^3\sqrt{a+bx^3}}{105b} \\
&+ \frac{54a^2fx^4\sqrt{a+bx^3}}{4301b} + \frac{54a^2gx^5\sqrt{a+bx^3}}{6175b} \\
&+ \frac{2x^3(a+bx^3)^{3/2}(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5)}{3900225} \\
&+ \frac{2ax^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5)}{185910725} \\
&+ \frac{(12a^2)\int \frac{-\frac{99}{95}ab^2(5bd-2ag)x-\frac{143}{30}ab^3ex^2+\frac{819b^3(23bc-8af)x^3}{1564}}{\sqrt{a+bx^3}}\,dx}{1001b^4} \\
&= \frac{54a^2(5bd-2ag)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{2a^2ex^3\sqrt{a+bx^3}}{105b} \\
&+ \frac{54a^2fx^4\sqrt{a+bx^3}}{4301b} + \frac{54a^2gx^5\sqrt{a+bx^3}}{6175b} \\
&+ \frac{2x^3(a+bx^3)^{3/2}(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5)}{3900225} \\
&+ \frac{2ax^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5)}{185910725} \\
&+ \frac{(12a^2)\int \frac{x\left(-\frac{99}{95}ab^2(5bd-2ag)-\frac{143}{30}ab^3ex+\frac{819b^3(23bc-8af)x^2}{1564}\right)}{\sqrt{a+bx^3}}\,dx}{1001b^4} \\
&= \frac{54a^2(23bc-8af)x\sqrt{a+bx^3}}{21505b^2} + \frac{54a^2(5bd-2ag)x^2\sqrt{a+bx^3}}{8645b^2} \\
&+ \frac{2a^2ex^3\sqrt{a+bx^3}}{105b} + \frac{54a^2fx^4\sqrt{a+bx^3}}{4301b} + \frac{54a^2gx^5\sqrt{a+bx^3}}{6175b} \\
&+ \frac{2x^3(a+bx^3)^{3/2}(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5)}{3900225} \\
&+ \frac{2ax^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5)}{185910725} \\
&+ \frac{(24a^2)\int \frac{-\frac{819ab^3(23bc-8af)}{1564}-\frac{99}{38}ab^3(5bd-2ag)x-\frac{143}{12}ab^4ex^2}{\sqrt{a+bx^3}}\,dx}{5005b^5}
\end{aligned}
$$

$$= \frac{54a^2(23bc-8af)x\sqrt{a+bx^3}}{21505b^2} + \frac{54a^2(5bd-2ag)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{2a^2ex^3\sqrt{a+bx^3}}{105b} + \frac{54a^2fx^4\sqrt{a+bx^3}}{4301b} + \frac{54a^2gx^5\sqrt{a+bx^3}}{6175b} + \frac{2x^3(a+bx^3)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225} + \frac{2ax^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5)}{185910725} + \frac{(24a^2)\int \frac{-\frac{819ab^3(23bc-8af)}{1564}-\frac{99}{38}ab^3(5bd-2ag)x}{\sqrt{a+bx^3}}\,dx}{5005b^5} - \frac{(2a^3e)\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{35b}$$

$$= -\frac{4a^3e\sqrt{a+bx^3}}{105b^2} + \frac{54a^2(23bc-8af)x\sqrt{a+bx^3}}{21505b^2} + \frac{54a^2(5bd-2ag)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{2a^2ex^3\sqrt{a+bx^3}}{105b} + \frac{54a^2fx^4\sqrt{a+bx^3}}{4301b} + \frac{54a^2gx^5\sqrt{a+bx^3}}{6175b} + \frac{2x^3(a+bx^3)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225} + \frac{2ax^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5)}{185910725} - \frac{(108a^3(5bd-2ag))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{8645b^{7/3}} - \frac{\left(54a^3\left(1729\sqrt[3]{b}(23bc-8af)-8602\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd-2ag)\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{37182145b^{7/3}}$$

$$
\begin{aligned}
&= -\frac{4a^3e\sqrt{a+bx^3}}{105b^2} + \frac{54a^2(23bc-8af)x\sqrt{a+bx^3}}{21505b^2} \\
&+ \frac{54a^2(5bd-2ag)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{2a^2ex^3\sqrt{a+bx^3}}{105b} + \frac{54a^2fx^4\sqrt{a+bx^3}}{4301b} \\
&+ \frac{54a^2gx^5\sqrt{a+bx^3}}{6175b} - \frac{216a^3(5bd-2ag)\sqrt{a+bx^3}}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} \\
&+ \frac{2x^3(a+bx^3)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225} \\
&+ \frac{2ax^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5)}{185910725} \\
&+ \frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}(5bd-2ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} \\
&- \frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(1729\sqrt[3]{b}(23bc-8af)-8602\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd-2ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/}}{((}}}{371821145b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.56 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.23

$$
\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{2\sqrt{a+bx^3}\left(-(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}(10a(7429e+21x(380f+391gx))-bx(229425c+17x(1207\right.}{}
$$

```
[In] Integrate[x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(10*a*(7429*e + 21*
x*(380*f + 391*g*x)) - b*x*(229425*c + 17*x*(12075*d + 19*x*(575*e + 525*f*
x + 483*g*x^2))))) + 9975*a^2*(-23*b*c + 8*a*f)*x*Hypergeometric2F1[-3/2, 1
/3, 4/3, -((b*x^3)/a)] + 41055*a^2*(-5*b*d + 2*a*g)*x^2*Hypergeometric2F1[-
3/2, 2/3, 5/3, -((b*x^3)/a)]))/(3900225*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 1161, normalized size of antiderivative = 1.47

method	result	size
elliptic	Expression too large to display	1161
risch	Expression too large to display	1198
default	Expression too large to display	1764

```
[In] int(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/25*g*b*x^11*(b*x^3+a)^(1/2)+2/23*b*f*x^10*(b*x^3+a)^(1/2)+2/21*b*e*x^9*(b
*x^3+a)^(1/2)+2/19*(28/25*a*b*g+b^2*d)/b*x^8*(b*x^3+a)^(1/2)+2/17*(26/23*a*
f*b+b^2*c)/b*x^7*(b*x^3+a)^(1/2)+16/105*a*e*x^6*(b*x^3+a)^(1/2)+2/13*(a^2*g
+2*a*b*d-16/19*a/b*(28/25*a*b*g+b^2*d))/b*x^5*(b*x^3+a)^(1/2)+2/11*(a^2*f+2
*a*b*c-14/17*a/b*(26/23*a*f*b+b^2*c))/b*x^4*(b*x^3+a)^(1/2)+2/105*a^2*e*x^3
*(b*x^3+a)^(1/2)/b+2/7*(a^2*d-10/13*a/b*(a^2*g+2*a*b*d-16/19*a/b*(28/25*a*b
*g+b^2*d)))/b*x^2*(b*x^3+a)^(1/2)+2/5*(a^2*c-8/11*a/b*(a^2*f+2*a*b*c-14/17*
a/b*(26/23*a*f*b+b^2*c)))/b*x*(b*x^3+a)^(1/2)-4/105*a^3*e*(b*x^3+a)^(1/2)/b
^2+4/15*I*a/b^2*(a^2*c-8/11*a/b*(a^2*f+2*a*b*c-14/17*a/b*(26/23*a*f*b+b^2*c
)))*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+8/21*I*a/
b^2*(a^2*d-10/13*a/b*(a^2*g+2*a*b*d-16/19*a/b*(28/25*a*b*g+b^2*d)))*3^(1/2)
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.33

$$\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx =$$

$$-\frac{2\left(1400490\,(23\,a^3bc-8\,a^4f)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-6967620\,(5\,a^3bd-2\,a^4g)\sqrt{b}\text{weierstrassZ}\right.}{}$$

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] -2/557732175*(1400490*(23*a^3*b*c - 8*a^4*f)*sqrt(b)*weierstrassPInverse(0,
 -4*a/b, x) - 6967620*(5*a^3*b*d - 2*a^4*g)*sqrt(b)*weierstrassZeta(0, -4*a
/b, weierstrassPInverse(0, -4*a/b, x)) - (22309287*b^4*g*x^11 + 24249225*b^
4*f*x^10 + 26558675*b^4*e*x^9 + 42493880*a*b^3*e*x^6 + 1174173*(25*b^4*d +
28*a*b^3*g)*x^8 + 5311735*a^2*b^2*e*x^3 + 1426425*(23*b^4*c + 26*a*b^3*f)*x
^7 + 90321*(550*a*b^3*d + 27*a^2*b^2*g)*x^5 - 10623470*a^3*b*e + 129675*(46
0*a*b^3*c + 27*a^2*b^2*f)*x^4 + 1741905*(5*a^2*b^2*d - 2*a^3*b*g)*x^2 + 700
245*(23*a^2*b^2*c - 8*a^3*b*f)*x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 512, normalized size of antiderivative = 0.65

$$\int x^3\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\frac{a^{\frac{3}{2}}cx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{3}{2}}dx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{a^{\frac{3}{2}}fx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{a^{\frac{3}{2}}gx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}+\frac{\sqrt{a}bcx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt{a}bdx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}+\frac{\sqrt{a}bfx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}+\frac{\sqrt{a}bgx^{11}\Gamma\left(\frac{11}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{3}\\ \frac{14}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{14}{3}\right)}+ae\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)+be\left(\begin{cases}\frac{16a^3\sqrt{a+bx^3}}{315b^3}-\frac{8a^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2ax^6\sqrt{a+bx^3}}{105b}+\frac{2x^9\sqrt{a+bx^3}}{21} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^9}{9} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**3*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(7/3)) + a**(3/2)*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + a**(3/2)*f*x**7*gamma(7/3)*hype
r((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(3/2
)*g*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(11/3)) + sqrt(a)*b*c*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*d*x**8*gamma(8/3)*hyper
((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + sqrt(a)*
b*f*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(13/3)) + sqrt(a)*b*g*x**11*gamma(11/3)*hyper((-1/2, 11/3), (14/3
```

```
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(14/3)) + a*e*Piecewise((-4*a**2*sqrt
(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a +
 b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b*e*Piecewise((16*a**3*sq
rt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*a*x
**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt
(a)*x**9/9, True))
```

Maxima [F]

$$\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int\left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}x^3\,dx$$

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^3, x)
```

Giac [F]

$$\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int\left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}x^3\,dx$$

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int x^3\,{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

```
[In] int(x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
[Out] int(x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.459 $\int x^2(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 742

$$\int x^2\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\frac{2a^2(7bc-2af)\sqrt{a+bx^3}}{105b^2}$$
$$+\frac{54a^2(23bd-8ag)x\sqrt{a+bx^3}}{21505b^2}+\frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2fx^3\sqrt{a+bx^3}}{105b}$$
$$+\frac{54a^2gx^4\sqrt{a+bx^3}}{4301b}-\frac{216a^3e\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$
$$+\frac{2ax^2\sqrt{a+bx^3}(7436429cx+5368545dx^2+4064445ex^3+3187041fx^4+2567565gx^5)}{111546435}$$
$$+\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(43010\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-1729(23bd-8ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{Ellip}}{37182145b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/780045*x^2*(b*x^3+a)^(3/2)*(33915*g*x^5+37145*f*x^4+41055*e*x^3+45885*d*x
^2+52003*c*x)+2/105*a^2*(-2*a*f+7*b*c)*(b*x^3+a)^(1/2)/b^2+54/21505*a^2*(-8
```

```
*a*g+23*b*d)*x*(b*x^3+a)^(1/2)/b^2+54/1729*a^2*e*x^2*(b*x^3+a)^(1/2)/b+2/10
5*a^2*f*x^3*(b*x^3+a)^(1/2)/b+54/4301*a^2*g*x^4*(b*x^3+a)^(1/2)/b+2/1115464
35*a*x^2*(2567565*g*x^5+3187041*f*x^4+4064445*e*x^3+5368545*d*x^2+7436429*c
*x)*(b*x^3+a)^(1/2)-216/1729*a^3*e*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))+108/1729*3^(1/4)*a^(10/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((
b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*
I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/
3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+36/37182145*3^(3/4)*
a^3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(13832*a*g-39767*b*d+43010*a^(1/3)*b^
(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(7/3)/(b*x^3+a)^(1/
2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1850, 1902, 1608, 1900, 267, 1892, 224, 1891}

$$\int x^2\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{216a^3e\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a^2\sqrt{a+bx^3}(7bc-2af)}{105b^2}$$

$$+\frac{54a^2x\sqrt{a+bx^3}(23bd-8ag)}{21505b^2}+\frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2fx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2gx^4\sqrt{a+bx^3}}{4301b}$$

$$+\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{371821455b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$

$$+\frac{2ax^2\sqrt{a+bx^3}\left(7436429cx+5368545dx^2+4064445ex^3+3187041fx^4+2567565gx^5\right)}{111546435}$$

```
[In] Int[x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a^2*(7*b*c - 2*a*f)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*(23*b*d - 8*a*g
)*x*Sqrt[a + b*x^3])/(21505*b^2) + (54*a^2*e*x^2*Sqrt[a + b*x^3])/(1729*b)
+ (2*a^2*f*x^3*Sqrt[a + b*x^3])/(105*b) + (54*a^2*g*x^4*Sqrt[a + b*x^3])/(4
301*b) - (216*a^3*e*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) + (2*x^2*(a + b*x^3)^(3/2)*(52003*c*x + 45885*d*x^2 + 41055*e*
x^3 + 37145*f*x^4 + 33915*g*x^5))/780045 + (2*a*x^2*Sqrt[a + b*x^3]*(743642
9*c*x + 5368545*d*x^2 + 4064445*e*x^3 + 3187041*f*x^4 + 2567565*g*x^5))/111
546435 + (108*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*e*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
 + (36*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(43010*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e
 - 1729*(23*b*d - 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(37182145*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
```

```
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
```

```
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{2x^2(a+bx^3)^{3/2}(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5)}{780045} + \frac{1}{2}(9a)\int x^2\sqrt{a+bx^3}\left(\frac{2c}{15}+\frac{2dx}{17}+\frac{2ex^2}{19}+\frac{2fx^3}{21}+\frac{2gx^4}{23}\right)dx$$

$$= \frac{2x^2(a+bx^3)^{3/2}(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5)}{780045} + \frac{2ax^2\sqrt{a+bx^3}(7436429cx+5368545dx^2+4064445ex^3+3187041fx^4+2567565gx^5)}{111546435} + \frac{1}{4}(27a^2)\int \frac{x^2\left(\frac{4c}{135}+\frac{4dx}{187}+\frac{4ex^2}{247}+\frac{4fx^3}{315}+\frac{4gx^4}{391}\right)}{\sqrt{a+bx^3}}dx$$

$$= \frac{54a^2gx^4\sqrt{a+bx^3}}{4301b} + \frac{2x^2(a+bx^3)^{3/2}(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5)}{780045} + \frac{2ax^2\sqrt{a+bx^3}(7436429cx+5368545dx^2+4064445ex^3+3187041fx^4+2567565gx^5)}{111546435} + \frac{(27a^2)\int \frac{x^2\left(\frac{22bc}{135}+\frac{2}{391}(23bd-8ag)x+\frac{22}{247}bex^2+\frac{22}{315}bfx^3\right)}{\sqrt{a+bx^3}}dx}{22b}$$

$$= \frac{2a^2fx^3\sqrt{a+bx^3}}{105b} + \frac{54a^2gx^4\sqrt{a+bx^3}}{4301b} + \frac{2x^2(a+bx^3)^{3/2}(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5)}{780045} + \frac{2ax^2\sqrt{a+bx^3}(7436429cx+5368545dx^2+4064445ex^3+3187041fx^4+2567565gx^5)}{111546435} + \frac{(3a^2)\int \frac{x^2\left(\frac{11}{105}b(7bc-2af)+\frac{9}{391}b(23bd-8ag)x+\frac{99}{247}b^2ex^2\right)}{\sqrt{a+bx^3}}dx}{11b^2}$$

$$= \frac{54a^2ex^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2fx^3\sqrt{a+bx^3}}{105b} + \frac{54a^2gx^4\sqrt{a+bx^3}}{4301b} + \frac{2x^2(a+bx^3)^{3/2}(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5)}{780045} + \frac{2ax^2\sqrt{a+bx^3}(7436429cx+5368545dx^2+4064445ex^3+3187041fx^4+2567565gx^5)}{111546435} + \frac{(6a^2)\int \frac{-\frac{198}{247}ab^2ex+\frac{11}{30}b^2(7bc-2af)x^2+\frac{63}{782}b^2(23bd-8ag)x^3}{\sqrt{a+bx^3}}dx}{77b^3}$$

$$=\frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2fx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2gx^4\sqrt{a+bx^3}}{4301b}$$
$$+\frac{2x^2(a+bx^3)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$
$$+\frac{2ax^2\sqrt{a+bx^3}(7436429cx+5368545dx^2+4064445ex^3+3187041fx^4+2567565gx^5)}{111546435}$$
$$+\frac{(6a^2)\int\frac{x\left(-\frac{198}{247}ab^2e+\frac{11}{30}b^2(7bc-2af)x+\frac{63}{782}b^2(23bd-8ag)x^2\right)}{\sqrt{a+bx^3}}\,dx}{77b^3}$$

$$=\frac{54a^2(23bd-8ag)x\sqrt{a+bx^3}}{21505b^2}+\frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}$$
$$+\frac{2a^2fx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2gx^4\sqrt{a+bx^3}}{4301b}$$
$$+\frac{2x^2(a+bx^3)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$
$$+\frac{2ax^2\sqrt{a+bx^3}(7436429cx+5368545dx^2+4064445ex^3+3187041fx^4+2567565gx^5)}{111546435}$$
$$+\frac{(12a^2)\int\frac{-\frac{63}{782}ab^2(23bd-8ag)-\frac{495}{247}ab^3ex+\frac{11}{12}b^3(7bc-2af)x^2}{\sqrt{a+bx^3}}\,dx}{385b^4}$$

$$=\frac{54a^2(23bd-8ag)x\sqrt{a+bx^3}}{21505b^2}+\frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}$$
$$+\frac{2a^2fx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2gx^4\sqrt{a+bx^3}}{4301b}$$
$$+\frac{2x^2(a+bx^3)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$
$$+\frac{2ax^2\sqrt{a+bx^3}(7436429cx+5368545dx^2+4064445ex^3+3187041fx^4+2567565gx^5)}{111546435}$$
$$+\frac{(12a^2)\int\frac{-\frac{63}{782}ab^2(23bd-8ag)-\frac{495}{247}ab^3ex}{\sqrt{a+bx^3}}\,dx}{385b^4}+\frac{\left(a^2(7bc-2af)\right)\int\frac{x^2}{\sqrt{a+bx^3}}\,dx}{35b}$$

$$=\frac{2a^2(7bc-2af)\sqrt{a+bx^3}}{105b^2}+\frac{54a^2(23bd-8ag)x\sqrt{a+bx^3}}{21505b^2}$$
$$+\frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2fx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2gx^4\sqrt{a+bx^3}}{4301b}$$
$$+\frac{2x^2(a+bx^3)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$
$$+\frac{2ax^2\sqrt{a+bx^3}(7436429cx+5368545dx^2+4064445ex^3+3187041fx^4+2567565gx^5)}{111546435}$$
$$-\frac{(108a^3e)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{1729b^{4/3}}$$
$$-\frac{\left(54a^3\left(39767bd-43010\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-13832ag\right)\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{371821455b^2}$$

$$
= \frac{2a^2(7bc-2af)\sqrt{a+bx^3}}{105b^2} + \frac{54a^2(23bd-8ag)x\sqrt{a+bx^3}}{21505b^2} + \frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}
+ \frac{2a^2fx^3\sqrt{a+bx^3}}{105b} + \frac{54a^2gx^4\sqrt{a+bx^3}}{4301b} - \frac{216a^3e\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}
+ \frac{2x^2(a+bx^3)^{3/2}(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5)}{780045}
+ \frac{2ax^2\sqrt{a+bx^3}(7436429cx+5368545dx^2+4064445ex^3+3187041fx^4+2567565gx^5)}{111546435}
+ \frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\cdots}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
- \frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(39767bd-43010\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-13832ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}}{37182145b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.36 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.22

$$
\int x^2\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{2\left(\left(a+bx^3\right)^3\left(52003bc-38a(391f+420gx)+5bx(9177d+17x(483e+19x(23f+21gx)))\right)+1\cdots\right.}{\cdots}
$$

```
[In] Integrate[x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*((a + b*x^3)^3*(52003*b*c - 38*a*(391*f + 420*g*x) + 5*b*x*(9177*d + 17*
x*(483*e + 19*x*(23*f + 21*g*x)))) + 1995*a^3*(-23*b*d + 8*a*g)*x*Sqrt[1 +
(b*x^3)/a]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] - 41055*a^3*b*e*
x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]))/(
780045*b^2*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 1103, normalized size of antiderivative = 1.49

method	result	size
elliptic	Expression too large to display	1103
risch	Expression too large to display	1175
default	Expression too large to display	1269

```
[In] int(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/23*g*b*x^10*(b*x^3+a)^(1/2)+2/21*b*f*x^9*(b*x^3+a)^(1/2)+2/19*b*e*x^8*(b*
x^3+a)^(1/2)+2/17*(26/23*a*b*g+b^2*d)/b*x^7*(b*x^3+a)^(1/2)+2/15*(8/7*a*f*b
+b^2*c)/b*x^6*(b*x^3+a)^(1/2)+44/247*a*e*x^5*(b*x^3+a)^(1/2)+2/11*(a^2*g+2*
a*b*d-14/17*a/b*(26/23*a*b*g+b^2*d))/b*x^4*(b*x^3+a)^(1/2)+2/9*(a^2*f+2*a*b
*c-4/5*a/b*(8/7*a*f*b+b^2*c))/b*x^3*(b*x^3+a)^(1/2)+54/1729*a^2*e*x^2*(b*x^
3+a)^(1/2)/b+2/5*(a^2*d-8/11*a/b*(a^2*g+2*a*b*d-14/17*a/b*(26/23*a*b*g+b^2*
d)))/b*x*(b*x^3+a)^(1/2)+2/3*(a^2*c-2/3*a/b*(a^2*f+2*a*b*c-4/5*a/b*(8/7*a*f
*b+b^2*c)))/b*(b*x^3+a)^(1/2)+4/15*I*a/b^2*(a^2*d-8/11*a/b*(a^2*g+2*a*b*d-1
4/17*a/b*(26/23*a*b*g+b^2*d)))*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2))+72/1729*I*e*a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.32

$$\int x^2(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)\,dx = \frac{2\left(6967620\,a^3b^{\frac{3}{2}}e\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-280098\,(23\,a^3bd-8\right.}{}$$

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 2/111546435*(6967620*a^3*b^(3/2)*e*weierstrassZeta(0, -4*a/b, weierstrassPI
nverse(0, -4*a/b, x)) - 280098*(23*a^3*b*d - 8*a^4*g)*sqrt(b)*weierstrassPI
nverse(0, -4*a/b, x) + (4849845*b^4*g*x^10 + 5311735*b^4*f*x^9 + 5870865*b^
4*e*x^8 + 9935310*a*b^3*e*x^5 + 285285*(23*b^4*d + 26*a*b^3*g)*x^7 + 174190
5*a^2*b^2*e*x^2 + 1062347*(7*b^4*c + 8*a*b^3*f)*x^6 + 7436429*a^2*b^2*c - 2
124694*a^3*b*f + 25935*(460*a*b^3*d + 27*a^2*b^2*g)*x^4 + 1062347*(14*a*b^3
*c + a^2*b^2*f)*x^3 + 140049*(23*a^2*b^2*d - 8*a^3*b*g)*x)*sqrt(b*x^3 + a))
/b^3
```

Sympy [A] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.71

$$\int x^2(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)dx = \frac{a^{\frac{3}{2}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{3}{2}}ex^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{a^{\frac{3}{2}}gx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{a}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{a}bex^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)} + \frac{\sqrt{a}bgx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)} + ac\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right) + af\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right) + bc\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right) + bf\left(\begin{cases}\frac{16a^3\sqrt{a+bx^3}}{315b^3}-\frac{8a^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2ax^6\sqrt{a+bx^3}}{105b}+\frac{2x^9\sqrt{a+bx^3}}{21} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^9}{9} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**2*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*d*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(7/3)) + a**(3/2)*e*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + a**(3/2)*g*x**7*gamma(7/3)*hype
r((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)
*b*d*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(10/3)) + sqrt(a)*b*e*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + sqrt(a)*b*g*x**10*gamma(10/3)*hy
per((-1/2, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(13/3)) + a*c*
Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
 + a*f*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x
```

```
**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)
) + b*c*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b
*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, Tru
e)) + b*f*Piecewise((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt
(a + b*x**3)/(315*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a
 + b*x**3)/21, Ne(b, 0)), (sqrt(a)*x**9/9, True))
```

Maxima [F]

$$\int x^2(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int\left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}x^2\,dx$$

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
[Out] 2/15*(b*x^3 + a)^(5/2)*c/b + integrate((b*g*x^9 + b*f*x^8 + b*e*x^7 + a*f*x
^5 + (b*d + a*g)*x^6 + a*e*x^4 + a*d*x^3)*sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int x^2(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int\left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}x^2\,dx$$

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int x^2\,{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

```
[In] int(x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
[Out] int(x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.460 $\int x(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 723

$$\int x(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)\,dx = \frac{2a^2(7bd-2ag)\sqrt{a+bx^3}}{105b^2}+\frac{54a^2ex\sqrt{a+bx^3}}{935b}+\frac{54a^2fx^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2gx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2(19bc-4af)\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x(a+bx^3)^{3/2}(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5)}{440895}+\frac{2ax\sqrt{a+bx^3}(479655cx+323323dx^2+233415ex^3+176715fx^4+138567gx^5)}{4849845}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19bc-4af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\left(3458a^{2/3}\sqrt[3]{b}e+935\left(1-\sqrt{3}\right)(19bc-4af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{Ell}}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/440895*x*(b*x^3+a)^(3/2)*(20995*g*x^5+23205*f*x^4+25935*e*x^3+29393*d*x^2
+33915*c*x)+2/105*a^2*(-2*a*g+7*b*d)*(b*x^3+a)^(1/2)/b^2+54/935*a^2*e*x*(b*
x^3+a)^(1/2)/b+54/1729*a^2*f*x^2*(b*x^3+a)^(1/2)/b+2/105*a^2*g*x^3*(b*x^3+a
)^(1/2)/b+2/4849845*a*x*(138567*g*x^5+176715*f*x^4+233415*e*x^3+323323*d*x^
```

```
2+479655*c*x)*(b*x^3+a)^(1/2)+54/1729*a^2*(-4*a*f+19*b*c)*(b*x^3+a)^(1/2)/b
^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/1729*3^(1/4)*a^(7/3)*(-4*a*f+19*b
*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5
/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)-18/1616615*3^(3/4)*a^(7/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^
(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)
*(3458*a^(2/3)*b^(1/3)*e+935*(-4*a*f+19*b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*
2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1840, 1850, 1902, 1900, 267, 1892, 224, 1891}

$$\int x\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx =$$

$$-\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right.}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19bc-4af)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{54a^2\sqrt{a+bx^3}(19bc-4af)}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a^2\sqrt{a+bx^3}(7bd-2ag)}{105b^2}$$

$$+\frac{54a^2ex\sqrt{a+bx^3}}{935b}+\frac{54a^2fx^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2gx^3\sqrt{a+bx^3}}{105b}$$

$$+\frac{2x(a+bx^3)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$$+\frac{2ax\sqrt{a+bx^3}(479655cx+323323dx^2+233415ex^3+176715fx^4+138567gx^5)}{4849845}$$

```
[In] Int[x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a^2*(7*b*d - 2*a*g)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*e*x*Sqrt[a + b*
x^3])/(935*b) + (54*a^2*f*x^2*Sqrt[a + b*x^3])/(1729*b) + (2*a^2*g*x^3*Sqrt
[a + b*x^3])/(105*b) + (54*a^2*(19*b*c - 4*a*f)*Sqrt[a + b*x^3])/(1729*b^(5
/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x*(a + b*x^3)^(3/2)*(33915*c*
x + 29393*d*x^2 + 25935*e*x^3 + 23205*f*x^4 + 20995*g*x^5))/440895 + (2*a*x
*Sqrt[a + b*x^3]*(479655*c*x + 323323*d*x^2 + 233415*e*x^3 + 176715*f*x^4 +
 138567*g*x^5))/4849845 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*b*c - 4
*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
1729*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(345
8*a^(2/3)*b^(1/3)*e + 935*(1 - Sqrt[3])*(19*b*c - 4*a*f))*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1616615*b^(5/3)*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
```

```
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{2x(a+bx^3)^{3/2}(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5)}{440895} + \frac{1}{2}(9a)\int x\sqrt{a+bx^3}\left(\frac{2c}{13}+\frac{2dx}{15}+\frac{2ex^2}{17}+\frac{2fx^3}{19}+\frac{2gx^4}{21}\right)dx$$

$$= \frac{2x(a+bx^3)^{3/2}(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5)}{440895} + \frac{2ax\sqrt{a+bx^3}(479655cx+323323dx^2+233415ex^3+176715fx^4+138567gx^5)}{4849845} + \frac{1}{4}(27a^2)\int \frac{x\left(\frac{4c}{91}+\frac{4dx}{135}+\frac{4ex^2}{187}+\frac{4fx^3}{247}+\frac{4gx^4}{315}\right)}{\sqrt{a+bx^3}}dx$$

$$= \frac{2a^2gx^3\sqrt{a+bx^3}}{105b} + \frac{2x(a+bx^3)^{3/2}(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5)}{440895} + \frac{2ax\sqrt{a+bx^3}(479655cx+323323dx^2+233415ex^3+176715fx^4+138567gx^5)}{4849845} + \frac{(3a^2)\int \frac{x\left(\frac{18bc}{91}+\frac{2}{105}(7bd-2ag)x+\frac{18}{187}bex^2+\frac{18}{247}bfx^3\right)}{\sqrt{a+bx^3}}dx}{2b}$$

$$= \frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2gx^3\sqrt{a+bx^3}}{105b} + \frac{2x(a+bx^3)^{3/2}(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5)}{440895} + \frac{2ax\sqrt{a+bx^3}(479655cx+323323dx^2+233415ex^3+176715fx^4+138567gx^5)}{4849845} + \frac{(3a^2)\int \frac{x\left(\frac{9}{247}b(19bc-4af)+\frac{1}{15}b(7bd-2ag)x+\frac{63}{187}b^2ex^2\right)}{\sqrt{a+bx^3}}dx}{7b^2}$$

$$= \frac{54a^2ex\sqrt{a+bx^3}}{935b} + \frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2gx^3\sqrt{a+bx^3}}{105b} + \frac{2x(a+bx^3)^{3/2}(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5)}{440895} + \frac{2ax\sqrt{a+bx^3}(479655cx+323323dx^2+233415ex^3+176715fx^4+138567gx^5)}{4849845} + \frac{(6a^2)\int \frac{-\frac{63}{187}ab^2e+\frac{45}{494}b^2(19bc-4af)x+\frac{1}{6}b^2(7bd-2ag)x^2}{\sqrt{a+bx^3}}dx}{35b^3}$$

$$
\begin{aligned}
&= \frac{54a^2ex\sqrt{a+bx^3}}{935b}+\frac{54a^2fx^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2gx^3\sqrt{a+bx^3}}{105b} \\
&\quad+\frac{2x(a+bx^3)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895} \\
&\quad+\frac{2ax\sqrt{a+bx^3}(479655cx+323323dx^2+233415ex^3+176715fx^4+138567gx^5)}{4849845} \\
&\quad+\frac{(6a^2)\int\frac{-\frac{63}{187}ab^2e+\frac{45}{494}b^2(19bc-4af)x}{\sqrt{a+bx^3}}\,dx}{35b^3}+\frac{(a^2(7bd-2ag))\int\frac{x^2}{\sqrt{a+bx^3}}\,dx}{35b}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2a^2(7bd-2ag)\sqrt{a+bx^3}}{105b^2}+\frac{54a^2ex\sqrt{a+bx^3}}{935b}+\frac{54a^2fx^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2gx^3\sqrt{a+bx^3}}{105b} \\
&\quad+\frac{2x(a+bx^3)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895} \\
&\quad+\frac{2ax\sqrt{a+bx^3}(479655cx+323323dx^2+233415ex^3+176715fx^4+138567gx^5)}{4849845} \\
&\quad+\frac{(27a^2(19bc-4af))\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{1729b^{4/3}} \\
&\quad-\frac{\left(27a^{7/3}\left(3458a^{2/3}\sqrt[3]{b}e+935\left(1-\sqrt{3}\right)(19bc-4af)\right)\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{1616615b^{4/3}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2a^2(7bd-2ag)\sqrt{a+bx^3}}{105b^2}+\frac{54a^2ex\sqrt{a+bx^3}}{935b}+\frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} \\
&\quad+\frac{2a^2gx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2(19bc-4af)\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} \\
&\quad+\frac{2x(a+bx^3)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895} \\
&\quad+\frac{2ax\sqrt{a+bx^3}(479655cx+323323dx^2+233415ex^3+176715fx^4+138567gx^5)}{4849845} \\
&\quad-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19bc-4af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} \\
&\quad-\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\left(3458a^{2/3}\sqrt[3]{b}e+935\left(1-\sqrt{3}\right)(19bc-4af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.20

$$\int x\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx =$$

$$-\frac{\sqrt{a+bx^3}\left(4(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}(-2261bd+646ag-5bx(399e+17x(21f+19gx)))+7980a^2bex\,\text{Hyper}\right.}{67830b^2\sqrt{1-}}$$

```
[In] Integrate[x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] -1/67830*(Sqrt[a + b*x^3]*(4*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(-2261*b*d +
 646*a*g - 5*b*x*(399*e + 17*x*(21*f + 19*g*x))) + 7980*a^2*b*e*x*Hypergeom
etric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] + 1785*a*b*(-19*b*c + 4*a*f)*x^2*Hyp
ergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]))/(b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 889, normalized size of antiderivative = 1.23

method	result	size
risch	Expression too large to display	889
elliptic	Expression too large to display	1045
default	Expression too large to display	1383

```
[In] int(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/4849845/b^2*(-230945*b^3*g*x^9-255255*b^3*f*x^8-285285*b^3*e*x^7-369512*
a*b^2*g*x^6-323323*b^3*d*x^6-431970*a*b^2*f*x^5-373065*b^3*c*x^5-518700*a*b
^2*e*x^4-46189*a^2*b*g*x^3-646646*a*b^2*d*x^3-75735*a^2*b*f*x^2-852720*a*b^
2*c*x^2-140049*a^2*b*e*x+92378*a^3*g-323323*a^2*b*d)*(b*x^3+a)^(1/2)-27/161
6615*a^2/b*(-6916/3*I*a*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*
(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2))-2/3*I*(3740*a*f-17765*b*c)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
```

```
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.29

$$\int x(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx =$$

$$-\frac{2\left(280098\,a^3\sqrt{b}e\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+75735\left(19\,a^2bc-4\,a^3f\right)\sqrt{b}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weier}\right.\right.}{}$$

```
[In] integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] -2/4849845*(280098*a^3*sqrt(b)*e*weierstrassPInverse(0, -4*a/b, x) + 75735*
(19*a^2*b*c - 4*a^3*f)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInver
se(0, -4*a/b, x)) - (230945*b^3*g*x^9 + 255255*b^3*f*x^8 + 285285*b^3*e*x^7
 + 518700*a*b^2*e*x^4 + 46189*(7*b^3*d + 8*a*b^2*g)*x^6 + 19635*(19*b^3*c +
 22*a*b^2*f)*x^5 + 140049*a^2*b*e*x + 323323*a^2*b*d - 92378*a^3*g + 46189*
(14*a*b^2*d + a^2*b*g)*x^3 + 2805*(304*a*b^2*c + 27*a^2*b*f)*x^2)*sqrt(b*x^
3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.73

$$\int x\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\frac{a^{\frac{3}{2}}cx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{a^{\frac{3}{2}}ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{3}{2}}fx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{\sqrt{a}bcx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{\sqrt{a}bex^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt{a}bfx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}+ad\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for}\ b=0\\ \frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)+ag\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)+bd\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)+bg\left(\begin{cases}\frac{16a^3\sqrt{a+bx^3}}{315b^3}-\frac{8a^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2ax^6\sqrt{a+bx^3}}{105b}+\frac{2x^9\sqrt{a+bx^3}}{21} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^9}{9} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*c*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(5/3)) + a**(3/2)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(3/2)*f*x**5*gamma(5/3)*hype
r((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*b
*c*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(8/3)) + sqrt(a)*b*e*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**
3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*f*x**8*gamma(8/3)*hyper((-
1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + a*d*Piecewi
se((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + a*g*P
iecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(4
```

```
5*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b*d
*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/
(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b
*g*Piecewise((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt(a + b*
x**3)/(315*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a + b*x*
*3)/21, Ne(b, 0)), (sqrt(a)*x**9/9, True))
```

Maxima [F]

$$\int x(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int\left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x, x)
```

Giac [F]

$$\int x(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int\left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int x\left(b\,x^3+a\right)^{3/2}\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

```
[In] int(x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
[Out] int(x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.461 $\int (a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 694

$$\int (a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)\,dx = \frac{2a^2e\sqrt{a+bx^3}}{15b} + \frac{54a^2fx\sqrt{a+bx^3}}{935b} + \frac{54a^2gx^2\sqrt{a+bx^3}}{1729b} + \frac{54a^2(19bd-4ag)\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2(a+bx^3)^{3/2}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835} + \frac{2a\sqrt{a+bx^3}(793611cx+479655dx^2+323323ex^3+233415fx^4+176715gx^5)}{4849845} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(1729\sqrt[3]{b}(17bc-2af)-935\left(1-\sqrt{3}\right)\sqrt[3]{a}(19bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/692835*(b*x^3+a)^(3/2)*(36465*g*x^5+40755*f*x^4+46189*e*x^3+53295*d*x^2+6
2985*c*x)+2/15*a^2*e*(b*x^3+a)^(1/2)/b+54/935*a^2*f*x*(b*x^3+a)^(1/2)/b+54/
1729*a^2*g*x^2*(b*x^3+a)^(1/2)/b+2/4849845*a*(176715*g*x^5+233415*f*x^4+323
323*e*x^3+479655*d*x^2+793611*c*x)*(b*x^3+a)^(1/2)+54/1729*a^2*(-4*a*g+19*b
```

```
*d)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/1729*3^(1/4)
*a^(7/3)*(-4*a*g+19*b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(
1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2
*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+18/1616615*3^(3/4)*a^2*(a^(1/3)+b^(1/3)
*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(1729*b^(1/3)*(-2*a*f+17*b*c)-935*a^(1/3)*(-4*a*g+19*b*d)
*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1867, 1902, 1900, 267, 1892, 224, 1891}

$$\int (a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx =$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19bd-4ag)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{54a^2\sqrt{a+bx^3}(19bd-4ag)}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a^2e\sqrt{a+bx^3}}{15b}+\frac{54a^2fx\sqrt{a+bx^3}}{935b}+\frac{54a^2gx^2\sqrt{a+bx^3}}{1729b}$$

$$+\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2a\sqrt{a+bx^3}(793611cx+479655dx^2+323323ex^3+233415fx^4+176715gx^5)}{4849845}$$

$$+\frac{2(a+bx^3)^{3/2}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835}$$

```
[In] Int[(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]

[Out] (2*a^2*e*Sqrt[a + b*x^3])/(15*b) + (54*a^2*f*x*Sqrt[a + b*x^3])/(935*b) + (
54*a^2*g*x^2*Sqrt[a + b*x^3])/(1729*b) + (54*a^2*(19*b*d - 4*a*g)*Sqrt[a +
```

```
b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*(a + b*x^3)
^(3/2)*(62985*c*x + 53295*d*x^2 + 46189*e*x^3 + 40755*f*x^4 + 36465*g*x^5))
/692835 + (2*a*Sqrt[a + b*x^3]*(793611*c*x + 479655*d*x^2 + 323323*e*x^3 +
233415*f*x^4 + 176715*g*x^5))/4849845 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/
3)*(19*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]
]*a^2*(1729*b^(1/3)*(17*b*c - 2*a*f) - 935*(1 - Sqrt[3])*a^(1/3)*(19*b*d -
4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(1616615*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
```

```
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(a+bx^3)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835} \\
&\quad+\frac{1}{2}(9a)\int \sqrt{a+bx^3}\left(\frac{2c}{11}+\frac{2dx}{13}+\frac{2ex^2}{15}+\frac{2fx^3}{17}+\frac{2gx^4}{19}\right)\,dx \\
&= \frac{2(a+bx^3)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835} \\
&\quad+\frac{2a\sqrt{a+bx^3}(793611cx+479655dx^2+323323ex^3+233415fx^4+176715gx^5)}{4849845} \\
&\quad+\frac{1}{4}\left(27a^2\right)\int \frac{\frac{4c}{55}+\frac{4dx}{91}+\frac{4ex^2}{135}+\frac{4fx^3}{187}+\frac{4gx^4}{247}}{\sqrt{a+bx^3}}\,dx
\end{aligned}$$

$$=\frac{54a^2gx^2\sqrt{a+bx^3}}{1729b}+\frac{2(a+bx^3)^{3/2}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835}+\frac{2a\sqrt{a+bx^3}(793611cx+479655dx^2+323323ex^3+233415fx^4+176715gx^5)}{4849845}+\frac{(27a^2)\int\frac{\frac{14bc}{55}+\frac{2}{247}(19bd-4ag)x+\frac{14}{135}bex^2+\frac{14}{187}bfx^3}{\sqrt{a+bx^3}}dx}{14b}$$

$$=\frac{54a^2fx\sqrt{a+bx^3}}{935b}+\frac{54a^2gx^2\sqrt{a+bx^3}}{1729b}+\frac{2(a+bx^3)^{3/2}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835}+\frac{2a\sqrt{a+bx^3}(793611cx+479655dx^2+323323ex^3+233415fx^4+176715gx^5)}{4849845}+\frac{(27a^2)\int\frac{\frac{7}{187}b(17bc-2af)+\frac{5}{247}b(19bd-4ag)x+\frac{7}{27}b^2ex^2}{\sqrt{a+bx^3}}dx}{35b^2}$$

$$=\frac{54a^2fx\sqrt{a+bx^3}}{935b}+\frac{54a^2gx^2\sqrt{a+bx^3}}{1729b}+\frac{2(a+bx^3)^{3/2}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835}+\frac{2a\sqrt{a+bx^3}(793611cx+479655dx^2+323323ex^3+233415fx^4+176715gx^5)}{4849845}+\frac{(27a^2)\int\frac{\frac{7}{187}b(17bc-2af)+\frac{5}{247}b(19bd-4ag)x}{\sqrt{a+bx^3}}dx}{35b^2}+\frac{1}{5}(a^2e)\int\frac{x^2}{\sqrt{a+bx^3}}dx$$

$$=\frac{2a^2e\sqrt{a+bx^3}}{15b}+\frac{54a^2fx\sqrt{a+bx^3}}{935b}+\frac{54a^2gx^2\sqrt{a+bx^3}}{1729b}+\frac{2(a+bx^3)^{3/2}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835}+\frac{2a\sqrt{a+bx^3}(793611cx+479655dx^2+323323ex^3+233415fx^4+176715gx^5)}{4849845}+\frac{(27a^2(19bd-4ag))\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}dx}{1729b^{4/3}}+\frac{\left(27a^2\left(1729\sqrt[3]{b}(17bc-2af)-935\left(1-\sqrt{3}\right)\sqrt[3]{a}(19bd-4ag)\right)\right)\int\frac{1}{\sqrt{a+bx^3}}dx}{1616615b^{4/3}}$$

$$= \frac{2a^2e\sqrt{a+bx^3}}{15b} + \frac{54a^2fx\sqrt{a+bx^3}}{935b} + \frac{54a^2gx^2\sqrt{a+bx^3}}{1729b} + \frac{54a^2(19bd-4ag)\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2(a+bx^3)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+\right.}{692835}$$

$$+ \frac{2a\sqrt{a+bx^3}(793611cx+479655dx^2+323323ex^3+233415fx^4+176715gx^5)}{4849845}$$

$$- \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(1729\sqrt[3]{b}(17bc-2af)-935\left(1-\sqrt{3}\right)\sqrt[3]{a}(19bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-}{\left(\left(1+\right.\right.}}}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+b}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.20

$$\int \left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{\sqrt{a+bx^3}\left(4(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}(323e+15x(19f+17gx))-570a(-17bc+2af)x\,\text{Hypergeomet}\right.}{9690b\sqrt{1+}}$$

```
[In] Integrate[(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (Sqrt[a + b*x^3]*(4*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(323*e + 15*x*(19*f +
 17*g*x)) - 570*a*(-17*b*c + 2*a*f)*x*Hypergeometric2F1[-3/2, 1/3, 4/3, -((
b*x^3)/a)] - 255*a*(-19*b*d + 4*a*g)*x^2*Hypergeometric2F1[-3/2, 2/3, 5/3,
-((b*x^3)/a)]))/(9690*b*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 1024, normalized size of antiderivative = 1.48

method	result	size
elliptic	Expression too large to display	1024
risch	Expression too large to display	1138
default	Expression too large to display	1629

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/19*g*b*x^8*(b*x^3+a)^(1/2)+2/17*b*f*x^7*(b*x^3+a)^(1/2)+2/15*b*e*x^6*(b*x
^3+a)^(1/2)+2/13*(22/19*a*b*g+b^2*d)/b*x^5*(b*x^3+a)^(1/2)+2/11*(20/17*a*f*
b+b^2*c)/b*x^4*(b*x^3+a)^(1/2)+4/15*a*e*x^3*(b*x^3+a)^(1/2)+2/7*(a^2*g+2*a*
b*d-10/13*a/b*(22/19*a*b*g+b^2*d))/b*x^2*(b*x^3+a)^(1/2)+2/5*(a^2*f+2*a*b*c
-8/11*a/b*(20/17*a*f*b+b^2*c))/b*x*(b*x^3+a)^(1/2)+2/15*a^2*e*(b*x^3+a)^(1/
2)/b-2/3*I*(a^2*c-2/5*a/b*(a^2*f+2*a*b*c-8/11*a/b*(20/17*a*f*b+b^2*c)))*3^(
1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(a^2*d-4
/7*a/b*(a^2*g+2*a*b*d-10/13*a/b*(22/19*a*b*g+b^2*d)))*3^(1/2)/b*(-a*b^2)^(1
/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.29

$$\int (a+bx^3)^{3/2} (c+dx+ex^2+fx^3+gx^4) \, dx = \frac{2 \left(140049 (17 a^2bc-2 a^3 f)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4 a}{b},x\right)-75735 (19 a^2bd-4 a^3 g)\sqrt{b}\text{weiers}\right.}{}$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] 2/4849845*(140049*(17*a^2*b*c - 2*a^3*f)*sqrt(b)*weierstrassPInverse(0, -4*
a/b, x) - 75735*(19*a^2*b*d - 4*a^3*g)*sqrt(b)*weierstrassZeta(0, -4*a/b, w
eierstrassPInverse(0, -4*a/b, x)) + (255255*b^3*g*x^8 + 285285*b^3*f*x^7 +
323323*b^3*e*x^6 + 646646*a*b^2*e*x^3 + 19635*(19*b^3*d + 22*a*b^2*g)*x^5 +
 25935*(17*b^3*c + 20*a*b^2*f)*x^4 + 323323*a^2*b*e + 2805*(304*a*b^2*d + 2
7*a^2*b*g)*x^2 + 5187*(238*a*b^2*c + 27*a^2*b*f)*x)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.64

$$\int (a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)\,dx = \frac{a^{\frac{3}{2}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{a^{\frac{3}{2}}fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{3}{2}}gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}bfx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{a}bgx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)} + ae\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right) + be\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(3/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + a**(3/2)*f*x**4*gamma(4/3)*hyper((
-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(3/2)*g*x
**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(8/3)) + sqrt(a)*b*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp
_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*d*x**5*gamma(5/3)*hyper((-1/2, 5
/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*b*f*x**7*ga
mma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/
3)) + sqrt(a)*b*g*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*gamma(11/3)) + a*e*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2
*(a + b*x**3)**(3/2)/(9*b), True)) + b*e*Piecewise((-4*a**2*sqrt(a + b*x**3
```

```
)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15
, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Maxima [F]

$$\int \left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right) \, dx = \int \left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}} \, dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right) \, dx = \int \left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}} \, dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right) \, dx = \int \left(b\,x^3+a\right)^{3/2} \left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right) \, dx$$

```
[In] int((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
[Out] int((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.462 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 676

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = \frac{2a^2f\sqrt{a+bx^3}}{15b} + \frac{54a^2gx\sqrt{a+bx^3}}{935b} + \frac{54a^2e\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2(a+bx^3)^{3/2}(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5)}{109395x} + \frac{2a\sqrt{a+bx^3}(85085cx+41769dx^2+25245ex^3+17017fx^4+12285gx^5)}{255255x} - \frac{2}{3}a^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/109395*(b*x^3+a)^(3/2)*(6435*g*x^5+7293*f*x^4+8415*e*x^3+9945*d*x^2+12155
*c*x)/x-2/3*a^(3/2)*c*arctanh((b*x^3+a)^(1/2)/a^(1/2))+2/15*a^2*f*(b*x^3+a)
^(1/2)/b+54/935*a^2*g*x*(b*x^3+a)^(1/2)/b+2/255255*a*(12285*g*x^5+17017*f*x
^4+25245*e*x^3+41769*d*x^2+85085*c*x)*(b*x^3+a)^(1/2)/x+54/91*a^2*e*(b*x^3+
a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/91*3^(1/4)*a^(7/3)*e*(a
^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b
*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)+18/85085*3^(3/4)*a^2*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1
```

```
/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1547*b*d-1
82*a*g-935*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/
b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {1840, 1846, 272, 65, 214, 1902, 1900, 267, 1892, 224, 1891}

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx =$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2}{3}a^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{54a^2e\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a^2f\sqrt{a+bx^3}}{15b}+\frac{54a^2gx\sqrt{a+bx^3}}{935b}+\frac{18\ 3^{3/4}\cdots}{\cdots}$$

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

```
[Out] (2*a^2*f*Sqrt[a + b*x^3])/(15*b) + (54*a^2*g*x*Sqrt[a + b*x^3])/(935*b) + (
54*a^2*e*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 + (2*(a + b*x^3)^(3/2)*(12155*c*x + 9945*d*x^2 + 8415*e*x^3 + 7293*f*x^4 +
6435*g*x^5))/(109395*x) + (2*a*Sqrt[a + b*x^3]*(85085*c*x + 41769*d*x^2 + 2
5245*e*x^3 + 17017*f*x^4 + 12285*g*x^5))/(255255*x) - (2*a^(3/2)*c*ArcTanh[
Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*e*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1547*b*d - 935*(1 -
Sqrt[3])*a^(1/3)*b^(2/3)*e - 182*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
```

```
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(85085*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{2(a+bx^3)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x} + \frac{1}{2}(9a)\int \frac{\sqrt{a+bx^3}\left(\frac{2c}{9}+\frac{2dx}{11}+\frac{2ex^2}{13}+\frac{2fx^3}{15}+\frac{2gx^4}{17}\right)}{x}\,dx$$

$$=\frac{2(a+bx^3)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$
$$+\frac{2a\sqrt{a+bx^3}(85085cx+41769dx^2+25245ex^3+17017fx^4+12285gx^5)}{255255x}$$
$$+\frac{1}{4}\left(27a^2\right)\int\frac{\frac{4c}{27}+\frac{4dx}{55}+\frac{4ex^2}{91}+\frac{4fx^3}{135}+\frac{4gx^4}{187}}{x\sqrt{a+bx^3}}\,dx$$

$$=\frac{2(a+bx^3)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$
$$+\frac{2a\sqrt{a+bx^3}(85085cx+41769dx^2+25245ex^3+17017fx^4+12285gx^5)}{255255x}$$
$$+\frac{1}{4}\left(27a^2\right)\int\frac{\frac{4d}{55}+\frac{4ex}{91}+\frac{4fx^2}{135}+\frac{4gx^3}{187}}{\sqrt{a+bx^3}}\,dx+\left(a^2c\right)\int\frac{1}{x\sqrt{a+bx^3}}\,dx$$

$$=\frac{54a^2gx\sqrt{a+bx^3}}{935b}+\frac{2(a+bx^3)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$
$$+\frac{2a\sqrt{a+bx^3}(85085cx+41769dx^2+25245ex^3+17017fx^4+12285gx^5)}{255255x}$$
$$+\frac{\left(27a^2\right)\int\frac{\frac{2}{187}(17bd-2ag)+\frac{10bex}{91}+\frac{2}{27}bfx^2}{\sqrt{a+bx^3}}\,dx}{10b}+\frac{1}{3}\left(a^2c\right)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)$$

$$=\frac{54a^2gx\sqrt{a+bx^3}}{935b}$$
$$+\frac{2(a+bx^3)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$
$$+\frac{2a\sqrt{a+bx^3}(85085cx+41769dx^2+25245ex^3+17017fx^4+12285gx^5)}{255255x}$$
$$+\frac{\left(27a^2\right)\int\frac{\frac{2}{187}(17bd-2ag)+\frac{10bex}{91}}{\sqrt{a+bx^3}}\,dx}{10b}$$
$$+\frac{\left(2a^2c\right)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3b}+\frac{1}{5}\left(a^2f\right)\int\frac{x^2}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2a^2f\sqrt{a+bx^3}}{15b} + \frac{54a^2gx\sqrt{a+bx^3}}{935b} + \frac{2(a+bx^3)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x} + \frac{2a\sqrt{a+bx^3}(85085cx+41769dx^2+25245ex^3+17017fx^4+12285gx^5)}{255255x} - \frac{2}{3}a^{3/2}c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) + \frac{(27a^2e)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}dx}{91\sqrt[3]{b}} + \frac{\left(27a^2\left(1547bd-935\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-182ag\right)\right)\int\frac{1}{\sqrt{a+bx^3}}dx}{85085b}$$

$$= \frac{2a^2f\sqrt{a+bx^3}}{15b} + \frac{54a^2gx\sqrt{a+bx^3}}{935b} + \frac{54a^2e\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2(a+bx^3)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x} + \frac{2a\sqrt{a+bx^3}(85085cx+41769dx^2+25245ex^3+17017fx^4+12285gx^5)}{255255x} - \frac{2}{3}a^{3/2}c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\right.}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+b}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.45 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x}\,dx = \frac{4\sqrt{1+\frac{bx^3}{a}}\left(\sqrt{a+bx^3}(a^2(51f+45gx)+b^2x^3(85c+51fx^3\right.}{}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

```
[Out] (4*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3]*(a^2*(51*f + 45*g*x) + b^2*x^3*(85*
c + 51*f*x^3 + 45*g*x^4) + 2*a*b*(170*c + 51*f*x^3 + 45*g*x^4)) - 255*a^(3/
2)*b*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]) - 90*a*(-17*b*d + 2*a*g)*x*Sqrt[a
+ b*x^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] + 765*a*b*e*x^2*Sq
rt[a + b*x^3]*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(1530*b*Sqrt
[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 987, normalized size of antiderivative = 1.46

method	result	size
elliptic	Expression too large to display	987
default	Expression too large to display	1188

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x,method=_RETURNVERBOSE)
[Out] 2/17*g*b*x^7*(b*x^3+a)^(1/2)+2/15*b*f*x^6*(b*x^3+a)^(1/2)+2/13*b*e*x^5*(b*x
^3+a)^(1/2)+2/11*(20/17*a*b*g+b^2*d)/b*x^4*(b*x^3+a)^(1/2)+2/9*(6/5*a*f*b+b
^2*c)/b*x^3*(b*x^3+a)^(1/2)+32/91*a*e*x^2*(b*x^3+a)^(1/2)+2/5*(a^2*g+2*a*b*
d-8/11*(20/17*a*b*g+b^2*d)/b*a)/b*x*(b*x^3+a)^(1/2)+2/3*(a^2*f+2*a*b*c-2/3*
(6/5*a*f*b+b^2*c)/b*a)/b*(b*x^3+a)^(1/2)-2/3*I*(a^2*d-2/5*(a^2*g+2*a*b*d-8/
11*(20/17*a*b*g+b^2*d)/b*a)/b*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*(
(x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2))-18/91*I*a^2*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)))-2/3*a^(3/2)*c*arctanh((b*x^3+a)^(1/2)/a^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = \left[\frac{255255\,a^{\frac{3}{2}}b^2c\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right) -\right.$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fricas"
```

)

```
[Out] [1/1531530*(255255*a^(3/2)*b^2*c*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a
)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 908820*a^2*b^(3/2)*e*weierstrassZ
eta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + 88452*(17*a^2*b*d - 2*a
^3*g)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + 4*(45045*b^3*g*x^7 + 5105
1*b^3*f*x^6 + 58905*b^3*e*x^5 + 134640*a*b^2*e*x^2 + 4095*(17*b^3*d + 20*a*
b^2*g)*x^4 + 340340*a*b^2*c + 51051*a^2*b*f + 17017*(5*b^3*c + 6*a*b^2*f)*x
^3 + 819*(238*a*b^2*d + 27*a^2*b*g)*x)*sqrt(b*x^3 + a))/b^2, 1/765765*(2552
55*sqrt(-a)*a*b^2*c*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 4544
10*a^2*b^(3/2)*e*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b,
x)) + 44226*(17*a^2*b*d - 2*a^3*g)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x
) + 2*(45045*b^3*g*x^7 + 51051*b^3*f*x^6 + 58905*b^3*e*x^5 + 134640*a*b^2*e
*x^2 + 4095*(17*b^3*d + 20*a*b^2*g)*x^4 + 340340*a*b^2*c + 51051*a^2*b*f +
17017*(5*b^3*c + 6*a*b^2*f)*x^3 + 819*(238*a*b^2*d + 27*a^2*b*g)*x)*sqrt(b*
x^3 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 473, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = -\frac{2a^{\frac{3}{2}}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{a^{\frac{3}{2}}dx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}ex^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{a^{\frac{3}{2}}gx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}bdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}bex^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}bgx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{2a^2c}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2a\sqrt{b}cx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + af\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right) + bc\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right) + bf\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] -2*a**(3/2)*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + a**(3/2)*d*x*gamma(1/3)
*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + a**(
3/2)*e*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(5/3)) + a**(3/2)*g*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*d*x**4*gamma(4/3)*hyper((
-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*e*
x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gam
ma(8/3)) + sqrt(a)*b*g*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**2*c/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*
x**3) + 1)) + 2*a*sqrt(b)*c*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + a*f*Piecewi
se((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*c*P
```

```
iecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) +
 b*f*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x*
*3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxima"
)
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x}\,dx=\int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x}\,dx$$

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x)
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x, x)
```

3.463 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 692

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \frac{2a^2g\sqrt{a+bx^3}}{15b} - \frac{27ac\sqrt{a+bx^3}}{7x} + \frac{27a(13bc+2af)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2a\sqrt{a+bx^3}(19305cx+5005dx^2+2457ex^3+1485fx^4+1001gx^5)}{15015x^2} + \frac{2(a+bx^3)^{3/2}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045x^2} - \frac{2}{3}a^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13bc+2af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\text{ar}\right.}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/45045*(b*x^3+a)^(3/2)*(3003*g*x^5+3465*f*x^4+4095*e*x^3+5005*d*x^2+6435*c
*x)/x^2-2/3*a^(3/2)*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))+2/15*a^2*g*(b*x^3+a)
^(1/2)/b-27/7*a*c*(b*x^3+a)^(1/2)/x+2/15015*a*(1001*g*x^5+1485*f*x^4+2457*e
*x^3+5005*d*x^2+19305*c*x)*(b*x^3+a)^(1/2)/x^2+27/91*a*(2*a*f+13*b*c)*(b*x^
3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/182*3^(1/4)*a^(4/3)*(
2*a*f+13*b*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)+9/5005*3^(3/4)*a^(4/3)*(a^(1/3)+b^(1/3)*x)*Ellipti
```

```
cF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(182*a^(2/3)*b^(1/3)*e-55*(2*a*f+13*b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {1840, 1849, 1846, 272, 65, 214, 1900, 267, 1892, 224, 1891}

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{Ellip}}{5005}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2af+13bc)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2}{3}a^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2a^2g\sqrt{a+bx^3}}{15b}+\frac{27a\sqrt{a+bx^3}(2af+13bc)}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2a\sqrt{a+bx^3}(19305cx+500}{}$$

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

```
[Out] (2*a^2*g*Sqrt[a + b*x^3])/(15*b) - (27*a*c*Sqrt[a + b*x^3])/(7*x) + (27*a*(
13*b*c + 2*a*f)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) + (2*a*Sqrt[a + b*x^3]*(19305*c*x + 5005*d*x^2 + 2457*e*x^3 + 1485*f
*x^4 + 1001*g*x^5))/(15015*x^2) + (2*(a + b*x^3)^(3/2)*(6435*c*x + 5005*d*x
^2 + 4095*e*x^3 + 3465*f*x^4 + 3003*g*x^5))/(45045*x^2) - (2*a^(3/2)*d*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13
*b*c + 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(4/3
)*(182*a^(2/3)*b^(1/3)*e - 55*(1 - Sqrt[3])*(13*b*c + 2*a*f))*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
```

```
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005*b^(2/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rubi steps

$$\text{integral} = \frac{2(a+bx^3)^{3/2}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045x^2} + \frac{1}{2}(9a)\int \frac{\sqrt{a+bx^3}\left(\frac{2c}{7}+\frac{2dx}{9}+\frac{2ex^2}{11}+\frac{2fx^3}{13}+\frac{2gx^4}{15}\right)}{x^2}\,dx$$

$$= \frac{2a\sqrt{a+bx^3}(19305cx+5005dx^2+2457ex^3+1485fx^4+1001gx^5)}{15015x^2} + \frac{2(a+bx^3)^{3/2}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045x^2} + \frac{1}{4}(27a^2)\int \frac{\frac{4c}{7}+\frac{4dx}{27}+\frac{4ex^2}{55}+\frac{4fx^3}{91}+\frac{4gx^4}{135}}{x^2\sqrt{a+bx^3}}\,dx$$

$$= -\frac{27ac\sqrt{a+bx^3}}{7x} + \frac{2a\sqrt{a+bx^3}(19305cx+5005dx^2+2457ex^3+1485fx^4+1001gx^5)}{15015x^2} + \frac{2(a+bx^3)^{3/2}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045x^2} - \frac{1}{8}(27a)\int \frac{-\frac{8ad}{27}-\frac{8aex}{55}-\frac{4}{91}(13bc+2af)x^2-\frac{8}{135}agx^3}{x\sqrt{a+bx^3}}\,dx$$

$$= -\frac{27ac\sqrt{a+bx^3}}{7x} + \frac{2a\sqrt{a+bx^3}(19305cx+5005dx^2+2457ex^3+1485fx^4+1001gx^5)}{15015x^2} + \frac{2(a+bx^3)^{3/2}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045x^2} - \frac{1}{8}(27a)\int \frac{-\frac{8ae}{55}-\frac{4}{91}(13bc+2af)x-\frac{8}{135}agx^2}{\sqrt{a+bx^3}}\,dx + (a^2d)\int \frac{1}{x\sqrt{a+bx^3}}\,dx$$

$$= -\frac{27ac\sqrt{a+bx^3}}{7x} + \frac{2a\sqrt{a+bx^3}(19305cx+5005dx^2+2457ex^3+1485fx^4+1001gx^5)}{15015x^2} + \frac{2(a+bx^3)^{3/2}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045x^2} - \frac{1}{8}(27a)\int \frac{-\frac{8ae}{55}-\frac{4}{91}(13bc+2af)x}{\sqrt{a+bx^3}}\,dx + \frac{1}{3}(a^2d)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right) + \frac{1}{5}(a^2g)\int \frac{x^2}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2a^2g\sqrt{a+bx^3}}{15b} - \frac{27ac\sqrt{a+bx^3}}{7x} + \frac{2a\sqrt{a+bx^3}(19305cx+5005dx^2+2457ex^3+1485fx^4+1001gx^5)}{15015x^2} + \frac{2(a+bx^3)^{3/2}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045x^2} + \frac{(2a^2d)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b} + \frac{(27a(13bc+2af))\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{182\sqrt[3]{b}} + \frac{\left(27a^{4/3}\left(182a^{2/3}e - \frac{55(1-\sqrt{3})(13bc+2af)}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{10010}$$

$$= \frac{2a^2g\sqrt{a+bx^3}}{15b} - \frac{27ac\sqrt{a+bx^3}}{7x} + \frac{27a(13bc+2af)\sqrt{a+bx^3}}{91b^{2/3}\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2a\sqrt{a+bx^3}(19305cx+5005dx^2+2457ex^3+1485fx^4+1001gx^5)}{15015x^2} + \frac{2(a+bx^3)^{3/2}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045x^2} - \frac{2}{3}a^{3/2}d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13bc+2af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right.}}}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.37 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \frac{2g(a+bx^3)^{5/2}}{15b} + \frac{2}{9}d\left(\sqrt{a+bx^3}(4a+bx^3) - 3a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right) - \frac{ac\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right.}{x\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

```
[Out] (2*g*(a + b*x^3)^(5/2))/(15*b) + (2*d*(Sqrt[a + b*x^3]*(4*a + b*x^3) - 3*a^
(3/2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]))/9 - (a*c*Sqrt[a + b*x^3]*Hypergeom
etric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)])/(x*Sqrt[1 + (b*x^3)/a]) + (a*e*x*S
qrt[a + b*x^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (b
*x^3)/a] + (a*f*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*
x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 946, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	946
default	Expression too large to display	1317
risch	Expression too large to display	3382

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a*c*(b*x^3+a)^(1/2)/x+2/15*g*b*x^6*(b*x^3+a)^(1/2)+2/13*b*f*x^5*(b*x^3+a)^
(1/2)+2/11*b*e*x^4*(b*x^3+a)^(1/2)+2/9*(6/5*a*b*g+b^2*d)/b*x^3*(b*x^3+a)^(1
/2)+2/7*(16/13*a*f*b+b^2*c)/b*x^2*(b*x^3+a)^(1/2)+28/55*a*e*x*(b*x^3+a)^(1/
2)+2/3*(a^2*g+2*a*b*d-2/3*(6/5*a*b*g+b^2*d)/b*a)/b*(b*x^3+a)^(1/2)-18/55*I*
a^2*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I
*(a^2*f+5/2*a*b*c-4/7*(16/13*a*f*b+b^2*c)/b*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)))-2/3*a^(3/2)*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^2}\,dx = \left[\frac{15015\,a^{\frac{3}{2}}bdx\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+88}{}\right.$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="frica
s")
```

```
[Out] [1/90090*(15015*a^(3/2)*b*d*x*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*s
qrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 88452*a^2*sqrt(b)*e*x*weierstrassPIn
verse(0, -4*a/b, x) - 26730*(13*a*b*c + 2*a^2*f)*sqrt(b)*x*weierstrassZeta(
0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + 2*(6006*b^2*g*x^7 + 6930*b^
2*f*x^6 + 8190*b^2*e*x^5 + 22932*a*b*e*x^2 + 2002*(5*b^2*d + 6*a*b*g)*x^4 +
 990*(13*b^2*c + 16*a*b*f)*x^3 - 45045*a*b*c + 2002*(20*a*b*d + 3*a^2*g)*x)
*sqrt(b*x^3 + a))/(b*x), 1/45045*(15015*sqrt(-a)*a*b*d*x*arctan(2*sqrt(b*x^
3 + a)*sqrt(-a)/(b*x^3 + 2*a)) + 44226*a^2*sqrt(b)*e*x*weierstrassPInverse(
0, -4*a/b, x) - 13365*(13*a*b*c + 2*a^2*f)*sqrt(b)*x*weierstrassZeta(0, -4*
a/b, weierstrassPInverse(0, -4*a/b, x)) + (6006*b^2*g*x^7 + 6930*b^2*f*x^6
+ 8190*b^2*e*x^5 + 22932*a*b*e*x^2 + 2002*(5*b^2*d + 6*a*b*g)*x^4 + 990*(13
*b^2*c + 16*a*b*f)*x^3 - 45045*a*b*c + 2002*(20*a*b*d + 3*a^2*g)*x)*sqrt(b*
x^3 + a))/(b*x)]
```

Sympy [A] (verification not implemented)

Time = 5.03 (sec) , antiderivative size = 474, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2a^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{a^{\frac{3}{2}}ex\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}fx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}bcx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}bex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}bfx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{2a^2d}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2a\sqrt{b}dx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + ag\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right) + bd\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right) + bg\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2} + \frac{2ax^3\sqrt{a+bx^3}}{45b} + \frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] a**(3/2)*c*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x*gamma(2/3)) - 2*a**(3/2)*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + a**
(3/2)*e*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) + a**(3/2)*f*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**
3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x**2*gamma(2/3)*hyper((-1
/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*e*x*
*4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma
(7/3)) + sqrt(a)*b*f*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(8/3)) + 2*a**2*d/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3
```

```
) + 1)) + 2*a*sqrt(b)*d*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + a*g*Piecewise((
sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*d*Piece
wise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*g
*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/
(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="maxim
a")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="giac"
)
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx=\int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^2}\,dx$$

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x)
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2, x)
```

3.464 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 694

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} + \frac{27a(13bd+2ag)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2a\sqrt{a+bx^3}(27027cx-19305dx^2-5005ex^3-2457fx^4-1485gx^5)}{15015x^3} + \frac{2(a+bx^3)^{3/2}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045x^3} - \frac{2}{3}a^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13bd+2ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\Big(\text{arc}}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/45045*(b*x^3+a)^(3/2)*(3465*g*x^5+4095*f*x^4+5005*e*x^3+6435*d*x^2+9009*c
*x)/x^3-2/3*a^(3/2)*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))+27/10*a*c*(b*x^3+a)^
(1/2)/x^2-27/7*a*d*(b*x^3+a)^(1/2)/x-2/15015*a*(-1485*g*x^5-2457*f*x^4-5005
*e*x^3-19305*d*x^2+27027*c*x)*(b*x^3+a)^(1/2)/x^3+27/91*a*(2*a*g+13*b*d)*(b
*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/182*3^(1/4)*a^(4/3
)*(2*a*g+13*b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2
)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2)
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)+9/10010*3^(3/4)*a*(a^(1/3)+b^(1/3)*x)*EllipticF
```

```
((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*(91*b^(1/3)*(4*a*f+11*b*c)-110*a^(1/3)*(2*a*g+13*b*d)*(1-3^(1/2)))*(1/
2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{Ellipti}}{1}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2ag+13bd)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\right.}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2}{3}a^{3/2}e\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{27a\sqrt{a+bx^3}(2ag+13bd)}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2a\sqrt{a+bx^3}(27027cx-19305dx^2-5005ex^3}{15015x^3}$$

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

```
[Out] (27*a*c*Sqrt[a + b*x^3])/(10*x^2) - (27*a*d*Sqrt[a + b*x^3])/(7*x) + (27*a*
(13*b*d + 2*a*g)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)) - (2*a*Sqrt[a + b*x^3]*(27027*c*x - 19305*d*x^2 - 5005*e*x^3 - 2457
*f*x^4 - 1485*g*x^5))/(15015*x^3) + (2*(a + b*x^3)^(3/2)*(9009*c*x + 6435*d
*x^2 + 5005*e*x^3 + 4095*f*x^4 + 3465*g*x^5))/(45045*x^3) - (2*a^(3/2)*e*Ar
cTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(
13*b*d + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(9
1*b^(1/3)*(11*b*c + 4*a*f) - 110*(1 - Sqrt[3])*a^(1/3)*(13*b*d + 2*a*g))*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
```

```
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10010*b^
(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(a+bx^3)^{3/2}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045x^3} \\
&\quad+\frac{1}{2}(9a)\int\frac{\sqrt{a+bx^3}\left(\frac{2c}{5}+\frac{2dx}{7}+\frac{2ex^2}{9}+\frac{2fx^3}{11}+\frac{2gx^4}{13}\right)}{x^3}\,dx \\
&= -\frac{2a\sqrt{a+bx^3}(27027cx-19305dx^2-5005ex^3-2457fx^4-1485gx^5)}{15015x^3} \\
&\quad+\frac{2(a+bx^3)^{3/2}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045x^3} \\
&\quad+\frac{1}{4}\left(27a^2\right)\int\frac{-\frac{4c}{5}+\frac{4dx}{7}+\frac{4ex^2}{27}+\frac{4fx^3}{55}+\frac{4gx^4}{91}}{x^3\sqrt{a+bx^3}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{27ac\sqrt{a+bx^3}}{10x^2} \\
&\quad - \frac{2a\sqrt{a+bx^3}(27027cx-19305dx^2-5005ex^3-2457fx^4-1485gx^5)}{15015x^3} \\
&\quad + \frac{2(a+bx^3)^{3/2}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045x^3} \\
&\quad - \frac{1}{16}(27a)\int \frac{-\frac{16ad}{7}-\frac{16aex}{27}-\frac{4}{55}(11bc+4af)x^2-\frac{16}{91}agx^3}{x^2\sqrt{a+bx^3}}\,dx \\
&= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} \\
&\quad - \frac{2a\sqrt{a+bx^3}(27027cx-19305dx^2-5005ex^3-2457fx^4-1485gx^5)}{15015x^3} \\
&\quad + \frac{2(a+bx^3)^{3/2}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045x^3} \\
&\quad + \frac{27}{32}\int \frac{\frac{32a^2e}{27}+\frac{8}{55}a(11bc+4af)x+\frac{16}{91}a(13bd+2ag)x^2}{x\sqrt{a+bx^3}}\,dx \\
&= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} \\
&\quad - \frac{2a\sqrt{a+bx^3}(27027cx-19305dx^2-5005ex^3-2457fx^4-1485gx^5)}{15015x^3} \\
&\quad + \frac{2(a+bx^3)^{3/2}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045x^3} \\
&\quad + \frac{27}{32}\int \frac{\frac{8}{55}a(11bc+4af)+\frac{16}{91}a(13bd+2ag)x}{\sqrt{a+bx^3}}\,dx + \left(a^2e\right)\int \frac{1}{x\sqrt{a+bx^3}}\,dx \\
&= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} \\
&\quad - \frac{2a\sqrt{a+bx^3}(27027cx-19305dx^2-5005ex^3-2457fx^4-1485gx^5)}{15015x^3} \\
&\quad + \frac{2(a+bx^3)^{3/2}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045x^3} \\
&\quad + \frac{1}{3}\left(a^2e\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right) \\
&\quad + \frac{(27a(13bd+2ag))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{182\sqrt[3]{b}} \\
&\quad + \frac{\left(27a\left(91(11bc+4af)-\frac{110\left(1-\sqrt{3}\right)\sqrt[3]{a}(13bd+2ag)}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{20020}
\end{aligned}
$$

$$= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} + \frac{27a(13bd+2ag)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$- \frac{2a\sqrt{a+bx^3}(27027cx-19305dx^2-5005ex^3-2457fx^4-1485gx^5)}{15015x^3}$$
$$+ \frac{2(a+bx^3)^{3/2}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045x^3}$$
$$- \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13bd+2ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(91(11bc+4af)-\frac{110\left(1-\sqrt{3}\right)\sqrt[3]{a}(13bd+2ag)}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}{10010\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{(2a^2e)\,\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3b}$$
$$= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} + \frac{27a(13bd+2ag)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$- \frac{2a\sqrt{a+bx^3}(27027cx-19305dx^2-5005ex^3-2457fx^4-1485gx^5)}{15015x^3}$$
$$+ \frac{2(a+bx^3)^{3/2}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045x^3}$$
$$- \frac{2}{3}a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13bd+2ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.35 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.33

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \frac{4ex^2\sqrt{1+\frac{bx^3}{a}}\left(\sqrt{a+bx^3}(4a+bx^3)-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right.\right.}{}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

```
[Out] (4*e*x^2*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3]*(4*a + b*x^3) - 3*a^(3/2)*Arc
Tanh[Sqrt[a + b*x^3]/Sqrt[a]]) - 9*a*c*Sqrt[a + b*x^3]*Hypergeometric2F1[-3
/2, -2/3, 1/3, -((b*x^3)/a)] - 18*a*d*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-
3/2, -1/3, 2/3, -((b*x^3)/a)] + 18*a*f*x^3*Sqrt[a + b*x^3]*Hypergeometric2F
1[-3/2, 1/3, 4/3, -((b*x^3)/a)] + 9*a*g*x^4*Sqrt[a + b*x^3]*Hypergeometric2
F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(18*x^2*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 941, normalized size of antiderivative = 1.36

method	result	size
elliptic	Expression too large to display	941
default	Expression too large to display	1613
risch	Expression too large to display	3858

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*c*(b*x^3+a)^(1/2)/x^2-a*d*(b*x^3+a)^(1/2)/x+2/13*g*b*x^5*(b*x^3+a)^(
1/2)+2/11*b*f*x^4*(b*x^3+a)^(1/2)+2/9*b*e*x^3*(b*x^3+a)^(1/2)+2/7*(16/13*a*
b*g+b^2*d)/b*x^2*(b*x^3+a)^(1/2)+2/5*(14/11*a*f*b+b^2*c)/b*x*(b*x^3+a)^(1/2
)+8/9*a*e*(b*x^3+a)^(1/2)-2/3*I*(a^2*f+7/4*a*b*c-2/5*(14/11*a*f*b+b^2*c)/b*
a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(a
^2*g+5/2*a*b*d-4/7*(16/13*a*b*g+b^2*d)/b*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
```

```
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)))-2/3*a^(3/2)*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \left[\frac{15015\,a^{\frac{3}{2}}bex^2\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+2\cdots}{\cdots}\right.$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="frica
s")
```

```
[Out] [1/90090*(15015*a^(3/2)*b*e*x^2*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)
*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 22113*(11*a*b*c + 4*a^2*f)*sqrt(b)
*x^2*weierstrassPInverse(0, -4*a/b, x) - 26730*(13*a*b*d + 2*a^2*g)*sqrt(b)
*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (13860
*b^2*g*x^7 + 16380*b^2*f*x^6 + 20020*b^2*e*x^5 + 80080*a*b*e*x^2 + 1980*(13
*b^2*d + 16*a*b*g)*x^4 - 90090*a*b*d*x + 3276*(11*b^2*c + 14*a*b*f)*x^3 - 4
5045*a*b*c)*sqrt(b*x^3 + a))/(b*x^2), 1/90090*(30030*sqrt(-a)*a*b*e*x^2*arc
tan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) + 22113*(11*a*b*c + 4*a^2*f)*
sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) - 26730*(13*a*b*d + 2*a^2*g)*
sqrt(b)*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) +
 (13860*b^2*g*x^7 + 16380*b^2*f*x^6 + 20020*b^2*e*x^5 + 80080*a*b*e*x^2 + 1
980*(13*b^2*d + 16*a*b*g)*x^4 - 90090*a*b*d*x + 3276*(11*b^2*c + 14*a*b*f)*
x^3 - 45045*a*b*c)*sqrt(b*x^3 + a))/(b*x^2)]
```

Sympy [A] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 462, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2a^{\frac{3}{2}}e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{a^{\frac{3}{2}}fx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}gx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}bcx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}bdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}bfx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}bgx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{2a^2e}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2a\sqrt{b}ex^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + be\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a**(3/2)*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**2*gamma(1/3)) + a**(3/2)*d*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a**(3/2)*e*asinh(sqrt(a)/(sq
rt(b)*x**(3/2)))/3 + a**(3/2)*f*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + a**(3/2)*g*x**2*gamma(2/3)*hyper((-
1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x
*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4
/3)) + sqrt(a)*b*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*f*x**4*gamma(4/3)*hyper((-1/2, 4/3)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*g*x**5*gamma
(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) +
 2*a**2*e/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*a*sqrt(b)*e*x**(3/2
)/(3*sqrt(a/(b*x**3) + 1)) + b*e*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(
a + b*x**3)**(3/2)/(9*b), True))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="maxim
a")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="giac"
)
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^3}\,dx$$

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x)
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3, x)
```

3.465 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4} dx$

Optimal result

Integrand size = 35, antiderivative size = 692

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4} dx = \frac{ac\sqrt{a+bx^3}}{x^3} + \frac{27ad\sqrt{a+bx^3}}{10x^2} - \frac{27ae\sqrt{a+bx^3}}{7x} + \frac{27a\sqrt[3]{b}e\sqrt{a+bx^3}}{7\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2a\sqrt{a+bx^3}(1155cx+2079dx^2-1485ex^3-385fx^4-189gx^5)}{1155x^4} + \frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x^4} - \frac{1}{3}\sqrt{a}(3bc+2af)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\sqrt[3]{b}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{14\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(77bd-110\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+28ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{ar}\right.}{770\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/3465*(b*x^3+a)^(3/2)*(315*g*x^5+385*f*x^4+495*e*x^3+693*d*x^2+1155*c*x)/x
```

```
^4-1/3*(2*a*f+3*b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+a*c*(b*x^3+a)
^(1/2)/x^3+27/10*a*d*(b*x^3+a)^(1/2)/x^2-27/7*a*e*(b*x^3+a)^(1/2)/x-2/1155*
a*(-189*g*x^5-385*f*x^4-1485*e*x^3+2079*d*x^2+1155*c*x)*(b*x^3+a)^(1/2)/x^4
+27/7*a*b^(1/3)*e*(b*x^3+a)^(1/2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/14*3^(
1/4)*a^(4/3)*b^(1/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)+9/770*3^(3/4)*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^
(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)
*(77*b*d+28*a*g-110*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2)
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{Ellipti}}{770\sqrt[3]{}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\sqrt[3]{b}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{14\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{1}{3}\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(2af+3bc)+\frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x^4}-\frac{2a}{}$$

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

```
[Out] (a*c*Sqrt[a + b*x^3])/x^3 + (27*a*d*Sqrt[a + b*x^3])/(10*x^2) - (27*a*e*Sqr
t[a + b*x^3])/(7*x) + (27*a*b^(1/3)*e*Sqrt[a + b*x^3])/(7*((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(1155*c*x + 2079*d*x^2 - 1485*e*
x^3 - 385*f*x^4 - 189*g*x^5))/(1155*x^4) + (2*(a + b*x^3)^(3/2)*(1155*c*x +
 693*d*x^2 + 495*e*x^3 + 385*f*x^4 + 315*g*x^5))/(3465*x^4) - (Sqrt[a]*(3*b
```

```
*c + 2*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt
[3]]*a^(4/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(14*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(77*
b*d - 110*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 28*a*g)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(770*b^(1/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
```

```
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2(a+bx^3)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4} + \frac{1}{2}(9a)\int \frac{\sqrt{a+bx^3}\left(\frac{2c}{3}+\frac{2dx}{5}+\frac{2ex^2}{7}+\frac{2fx^3}{9}+\frac{2gx^4}{11}\right)}{x^4}\,dx$$

$$= -\frac{2a\sqrt{a+bx^3}(1155cx+2079dx^2-1485ex^3-385fx^4-189gx^5)}{1155x^4} + \frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x^4} + \frac{1}{4}(27a^2)\int \frac{-\frac{4c}{9}-\frac{4dx}{5}+\frac{4ex^2}{7}+\frac{4fx^3}{27}+\frac{4gx^4}{55}}{x^4\sqrt{a+bx^3}}\,dx$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3} - \frac{2a\sqrt{a+bx^3}(1155cx+2079dx^2-1485ex^3-385fx^4-189gx^5)}{1155x^4} + \frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x^4} - \frac{1}{8}(9a)\int \frac{\frac{24ad}{5}-\frac{24aex}{7}-\frac{4}{9}(3bc+2af)x^2-\frac{24}{55}agx^3}{x^3\sqrt{a+bx^3}}\,dx$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3} + \frac{27ad\sqrt{a+bx^3}}{10x^2} - \frac{2a\sqrt{a+bx^3}(1155cx+2079dx^2-1485ex^3-385fx^4-189gx^5)}{1155x^4} + \frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x^4} + \frac{9}{32}\int \frac{\frac{96a^2e}{7}+\frac{16}{9}a(3bc+2af)x+\frac{24}{55}a(11bd+4ag)x^2}{x^2\sqrt{a+bx^3}}\,dx$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3} + \frac{27ad\sqrt{a+bx^3}}{10x^2} - \frac{27ae\sqrt{a+bx^3}}{7x} - \frac{2a\sqrt{a+bx^3}(1155cx+2079dx^2-1485ex^3-385fx^4-189gx^5)}{1155x^4} + \frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x^4} - \frac{9\int \frac{-\frac{32}{9}a^2(3bc+2af)-\frac{48}{55}a^2(11bd+4ag)x-\frac{96}{7}a^2bex^2}{x\sqrt{a+bx^3}}\,dx}{64a}$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3} + \frac{27ad\sqrt{a+bx^3}}{10x^2} - \frac{27ae\sqrt{a+bx^3}}{7x} - \frac{2a\sqrt{a+bx^3}(1155cx+2079dx^2-1485ex^3-385fx^4-189gx^5)}{1155x^4} + \frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x^4} - \frac{9\int \frac{-\frac{48}{55}a^2(11bd+4ag)-\frac{96}{7}a^2bex}{\sqrt{a+bx^3}}\,dx}{64a} + \frac{1}{2}(a(3bc+2af))\int \frac{1}{x\sqrt{a+bx^3}}\,dx$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3} + \frac{27ad\sqrt{a+bx^3}}{10x^2} - \frac{27ae\sqrt{a+bx^3}}{7x} - \frac{2a\sqrt{a+bx^3}(1155cx+2079dx^2-1485ex^3-385fx^4-189gx^5)}{1155x^4} + \frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x^4} + \frac{1}{14}(27ab^{2/3}e)\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx + \frac{1}{6}(a(3bc+2af))\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right) + \frac{\left(27a\left(77bd-110(1-\sqrt{3})\sqrt[3]{a}b^{2/3}e+28ag\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{1540}$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3} + \frac{27ad\sqrt{a+bx^3}}{10x^2} - \frac{27ae\sqrt{a+bx^3}}{7x} + \frac{27a\sqrt[3]{b}e\sqrt{a+bx^3}}{7\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2a\sqrt{a+bx^3}(1155cx+2079dx^2-1485ex^3-385fx^4-189gx^5)}{1155x^4} + \frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x^4} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\sqrt[3]{b}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\right.}{14\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(77bd-110(1-\sqrt{3})\sqrt[3]{a}b^{2/3}e+28ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\mathrm{s}\right.}{770\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{(a(3bc+2af))\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3} + \frac{27ad\sqrt{a+bx^3}}{10x^2} - \frac{27ae\sqrt{a+bx^3}}{7x} + \frac{27a\sqrt[3]{b}e\sqrt{a+bx^3}}{7\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$- \frac{2a\sqrt{a+bx^3}(1155cx + 2079dx^2 - 1485ex^3 - 385fx^4 - 189gx^5)}{1155x^4}$$

$$+ \frac{2(a+bx^3)^{3/2}(1155cx + 693dx^2 + 495ex^3 + 385fx^4 + 315gx^5)}{3465x^4}$$

$$- \frac{1}{3}\sqrt{a}(3bc+2af)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

$$- \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\sqrt[3]{b}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-}{14\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(77bd-110\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+28ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F}{770\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.60 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.35

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \frac{-45a^3d\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2}{3},\frac{1}{3},-\frac{bx^3}{a}\right) -}{}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

```
[Out] (-45*a^3*d*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)]
 - 90*a^3*e*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/
a)] + 90*a^3*g*x^3*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x
^3)/a)] + 4*x^2*Sqrt[1 + (b*x^3)/a]*(5*a^2*f*(Sqrt[a + b*x^3]*(4*a + b*x^3)
 - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]) + 3*b*c*(a + b*x^3)^(5/2)*Hy
pergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a]))/(90*a^2*x^2*Sqrt[1 + (b*x^3)/
a])
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	920
default	Expression too large to display	1193
risch	Expression too large to display	2513

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a*c*(b*x^3+a)^(1/2)/x^3-1/2*a*d*(b*x^3+a)^(1/2)/x^2-a*e*(b*x^3+a)^(1/2
)/x+2/11*g*b*x^4*(b*x^3+a)^(1/2)+2/9*b*f*x^3*(b*x^3+a)^(1/2)+2/7*b*e*x^2*(b
*x^3+a)^(1/2)+2/5*(14/11*a*b*g+b^2*d)/b*x*(b*x^3+a)^(1/2)+2/3*(4/3*a*f*b+b^
2*c)/b*(b*x^3+a)^(1/2)-2/3*I*(a^2*g+7/4*a*b*d-2/5*(14/11*a*b*g+b^2*d)/b*a)*
3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-9/7*I*a*e*3
^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-1/3*(2*a*f+3*b*c)*arctanh((b*x^3+a
)^(1/2)/a^(1/2))*a^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.67 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \left[-\frac{53460\,ab^{\frac{3}{2}}ex^3\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInve}\right.}{}\right.$$

$$-\frac{26730\,ab^{\frac{3}{2}}ex^3\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-1155\,(3\,b^2c+2\,abf)\sqrt{-a}x^3\,\text{arcta}}{}$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="frica
s")
```

```
[Out] [-1/13860*(53460*a*b^(3/2)*e*x^3*weierstrassZeta(0, -4*a/b, weierstrassPInv
erse(0, -4*a/b, x)) - 1155*(3*b^2*c + 2*a*b*f)*sqrt(a)*x^3*log(-(b^2*x^6 +
8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 3402*(1
1*a*b*d + 4*a^2*g)*sqrt(b)*x^3*weierstrassPInverse(0, -4*a/b, x) - 2*(1260*
b^2*g*x^7 + 1540*b^2*f*x^6 + 1980*b^2*e*x^5 - 6930*a*b*e*x^2 + 252*(11*b^2*
d + 14*a*b*g)*x^4 - 3465*a*b*d*x + 1540*(3*b^2*c + 4*a*b*f)*x^3 - 2310*a*b*
c)*sqrt(b*x^3 + a))/(b*x^3), -1/6930*(26730*a*b^(3/2)*e*x^3*weierstrassZeta
(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - 1155*(3*b^2*c + 2*a*b*f)*s
qrt(-a)*x^3*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 1701*(11*a*b
*d + 4*a^2*g)*sqrt(b)*x^3*weierstrassPInverse(0, -4*a/b, x) - (1260*b^2*g*x
^7 + 1540*b^2*f*x^6 + 1980*b^2*e*x^5 - 6930*a*b*e*x^2 + 252*(11*b^2*d + 14*
a*b*g)*x^4 - 3465*a*b*d*x + 1540*(3*b^2*c + 4*a*b*f)*x^3 - 2310*a*b*c)*sqrt
(b*x^3 + a))/(b*x^3)]
```

Sympy [A] (verification not implemented)

Time = 6.19 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2a^{\frac{3}{2}}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{a^{\frac{3}{2}}gx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \sqrt{a}bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) + \frac{\sqrt{a}bdx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}bex^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}bgx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2a^2f}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}c\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2a\sqrt{b}c}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2a\sqrt{b}fx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + \frac{2b^{\frac{3}{2}}cx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + bf\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] a**(3/2)*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**2*gamma(1/3)) + a**(3/2)*e*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a**(3/2)*f*asinh(sqrt(a)/(sq
rt(b)*x**(3/2)))/3 + a**(3/2)*g*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) - sqrt(a)*b*c*asinh(sqrt(a)/(sqrt(b)*
x**(3/2))) + sqrt(a)*b*d*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp
_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*b*e*x**2*gamma(2/3)*hyper((-1/2, 2
/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*g*x**4*ga
mma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)
) + 2*a**2*f/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*c*sqrt(a
/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*c/(3*x**(3/2)*sqrt(a/(b*x**3) + 1
)) + 2*a*sqrt(b)*f*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*c*x**(3/2
```

```
)/(3*sqrt(a/(b*x**3) + 1)) + b*f*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(
a + b*x**3)**(3/2)/(9*b), True))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="maxim
a")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="giac"
)
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^4}\,dx$$

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x)
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4, x)
```

3.466 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 741

$$
\begin{aligned}
\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx &= \frac{27ac\sqrt{a+bx^3}}{20x^4}+\frac{ad\sqrt{a+bx^3}}{x^3} \\
&+\frac{27ae\sqrt{a+bx^3}}{10x^2}-\frac{27(7bc+8af)\sqrt{a+bx^3}}{56x}+\frac{27\sqrt[3]{b}(7bc+8af)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} \\
&-\frac{2a\sqrt{a+bx^3}(189cx+105dx^2+189ex^3-135fx^4-35gx^5)}{105x^5} \\
&+\frac{2(a+bx^3)^{3/2}\,(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^5} \\
&-\frac{1}{3}\sqrt{a}(3bd+2ag)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) \\
&-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}(7bc+8af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} \\
&+\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(7bc+8af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}
$$

```
[Out] 2/315*(b*x^3+a)^(3/2)*(35*g*x^5+45*f*x^4+63*e*x^3+105*d*x^2+315*c*x)/x^5-1/
```

```
3*(2*a*g+3*b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+27/20*a*c*(b*x^3+a
)^(1/2)/x^4+a*d*(b*x^3+a)^(1/2)/x^3+27/10*a*e*(b*x^3+a)^(1/2)/x^2-27/56*(8*
a*f+7*b*c)*(b*x^3+a)^(1/2)/x-2/105*a*(-35*g*x^5-135*f*x^4+189*e*x^3+105*d*x
^2+189*c*x)*(b*x^3+a)^(1/2)/x^5+27/56*b^(1/3)*(8*a*f+7*b*c)*(b*x^3+a)^(1/2)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/112*3^(1/4)*a^(1/3)*b^(1/3)*(8*a*f+7*b*
c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^
3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)+9/280*3^(3/4)*a^(1/3)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(28*a
^(2/3)*b^(1/3)*e-5*(8*a*f+7*b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 741, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\ \cdots}{2\cdots}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8af+7bc)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\cdots\right.}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{1}{3}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(2ag+3bd)+\frac{2(a+bx^3)^{3/2}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^5}-\frac{2a\sqrt{\cdots}}{\cdots}$$

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

```
[Out] (27*a*c*Sqrt[a + b*x^3])/(20*x^4) + (a*d*Sqrt[a + b*x^3])/x^3 + (27*a*e*Sqr
t[a + b*x^3])/(10*x^2) - (27*(7*b*c + 8*a*f)*Sqrt[a + b*x^3])/(56*x) + (27*
b^(1/3)*(7*b*c + 8*a*f)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) - (2*a*Sqrt[a + b*x^3]*(189*c*x + 105*d*x^2 + 189*e*x^3 - 135*f*x^4
```

```
- 35*g*x^5))/(105*x^5) + (2*(a + b*x^3)^(3/2)*(315*c*x + 105*d*x^2 + 63*e*x
^3 + 45*f*x^4 + 35*g*x^5))/(315*x^5) - (Sqrt[a]*(3*b*d + 2*a*g)*ArcTanh[Sqr
t[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7
*b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(112*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*b^(1/3
)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(7*b*c + 8*a*f))*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(280*Sqrt[(a^(1/3)*(a^(
1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
```

```
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2(a+bx^3)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5} + \frac{1}{2}(9a)\int\frac{\sqrt{a+bx^3}\left(2c+\frac{2dx}{3}+\frac{2ex^2}{5}+\frac{2fx^3}{7}+\frac{2gx^4}{9}\right)}{x^5}\,dx$$

$$=-\frac{2a\sqrt{a+bx^3}(189cx+105dx^2+189ex^3-135fx^4-35gx^5)}{105x^5}+\frac{2(a+bx^3)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}+\frac{1}{4}\left(27a^2\right)\int\frac{-\frac{4c}{5}-\frac{4dx}{9}-\frac{4ex^2}{5}+\frac{4fx^3}{7}+\frac{4gx^4}{27}}{x^5\sqrt{a+bx^3}}\,dx$$

$$=\frac{27ac\sqrt{a+bx^3}}{20x^4}-\frac{2a\sqrt{a+bx^3}(189cx+105dx^2+189ex^3-135fx^4-35gx^5)}{105x^5}+\frac{2(a+bx^3)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}-\frac{1}{32}(27a)\int\frac{\frac{32ad}{9}+\frac{32aex}{5}-\frac{4}{7}(7bc+8af)x^2-\frac{32}{27}agx^3}{x^4\sqrt{a+bx^3}}\,dx$$

$$=\frac{27ac\sqrt{a+bx^3}}{20x^4}+\frac{ad\sqrt{a+bx^3}}{x^3}-\frac{2a\sqrt{a+bx^3}(189cx+105dx^2+189ex^3-135fx^4-35gx^5)}{105x^5}+\frac{2(a+bx^3)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}+\frac{9}{64}\int\frac{-\frac{192a^2e}{5}+\frac{24}{7}a(7bc+8af)x+\frac{32}{9}a(3bd+2ag)x^2}{x^3\sqrt{a+bx^3}}\,dx$$

$$=\frac{27ac\sqrt{a+bx^3}}{20x^4}+\frac{ad\sqrt{a+bx^3}}{x^3}+\frac{27ae\sqrt{a+bx^3}}{10x^2}-\frac{2a\sqrt{a+bx^3}(189cx+105dx^2+189ex^3-135fx^4-35gx^5)}{105x^5}+\frac{2(a+bx^3)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}-\frac{9\int\frac{-\frac{96}{7}a^2(7bc+8af)-\frac{128}{9}a^2(3bd+2ag)x-\frac{192}{5}a^2bex^2}{x^2\sqrt{a+bx^3}}\,dx}{256a}$$

$$=\frac{27ac\sqrt{a+bx^3}}{20x^4}+\frac{ad\sqrt{a+bx^3}}{x^3}+\frac{27ae\sqrt{a+bx^3}}{10x^2}-\frac{27(7bc+8af)\sqrt{a+bx^3}}{56x}-\frac{2a\sqrt{a+bx^3}(189cx+105dx^2+189ex^3-135fx^4-35gx^5)}{105x^5}+\frac{2(a+bx^3)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}+\frac{9\int\frac{\frac{256}{9}a^3(3bd+2ag)+\frac{384}{5}a^3bex+\frac{96}{7}a^2b(7bc+8af)x^2}{x\sqrt{a+bx^3}}\,dx}{512a^2}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4} + \frac{ad\sqrt{a+bx^3}}{x^3} + \frac{27ae\sqrt{a+bx^3}}{10x^2} - \frac{27(7bc+8af)\sqrt{a+bx^3}}{56x}$$
$$- \frac{2a\sqrt{a+bx^3}(189cx+105dx^2+189ex^3-135fx^4-35gx^5)}{105x^5}$$
$$+ \frac{2(a+bx^3)^{3/2}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^5}$$
$$+ \frac{9\int \frac{\frac{384}{5}a^3be+\frac{96}{7}a^2b(7bc+8af)x}{\sqrt{a+bx^3}}\,dx}{512a^2} + \frac{1}{2}(a(3bd+2ag))\int \frac{1}{x\sqrt{a+bx^3}}\,dx$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4} + \frac{ad\sqrt{a+bx^3}}{x^3} + \frac{27ae\sqrt{a+bx^3}}{10x^2} - \frac{27(7bc+8af)\sqrt{a+bx^3}}{56x}$$
$$- \frac{2a\sqrt{a+bx^3}(189cx+105dx^2+189ex^3-135fx^4-35gx^5)}{105x^5}$$
$$+ \frac{2(a+bx^3)^{3/2}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^5}$$
$$+ \frac{1}{112}(27b^{2/3}(7bc+8af))\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$$
$$+ \frac{1}{560}\left(27\sqrt[3]{a}b^{2/3}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(7bc+8af)\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx+\frac{1}{6}(a(3bd+2ag))\text{Subst}\Bigg($$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4} + \frac{ad\sqrt{a+bx^3}}{x^3} + \frac{27ae\sqrt{a+bx^3}}{10x^2} - \frac{27(7bc+8af)\sqrt{a+bx^3}}{56x} + \frac{27\sqrt[3]{b}(7bc+8af)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2a\sqrt{a+bx^3}(189cx+105dx^2+189ex^3-135fx^4-35gx^5)}{105x^5} + \frac{2(a+bx^3)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}(7bc+8af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(7bc+8af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}}}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{(a(3bd+2ag))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4} + \frac{ad\sqrt{a+bx^3}}{x^3} + \frac{27ae\sqrt{a+bx^3}}{10x^2} - \frac{27(7bc+8af)\sqrt{a+bx^3}}{56x} + \frac{27\sqrt[3]{b}(7bc+8af)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2a\sqrt{a+bx^3}(189cx+105dx^2+189ex^3-135fx^4-35gx^5)}{105x^5} + \frac{2(a+bx^3)^{3/2}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^5} - \frac{1}{3}\sqrt{a}(3bd+2ag)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}(7bc+8af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(7bc+8af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.60 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.33

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \frac{-45a^3c\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{4}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

```
[Out] (-45*a^3*c*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)
] - 90*a^3*e*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^
3)/a)] + 4*x^3*(-45*a^3*f*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -1/3, 2/3
, -((b*x^3)/a)] + 2*x*Sqrt[1 + (b*x^3)/a]*(5*a^2*g*(Sqrt[a + b*x^3]*(4*a +
b*x^3) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]) + 3*b*d*(a + b*x^3)^(5
/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a])))/(180*a^2*x^4*Sqrt[1 +
(b*x^3)/a])
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 900, normalized size of antiderivative = 1.21

method	result	size
elliptic	Expression too large to display	900
default	Expression too large to display	1342
risch	Expression too large to display	2048

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a*c*(b*x^3+a)^(1/2)/x^4-1/3*a*d*(b*x^3+a)^(1/2)/x^3-1/2*a*e*(b*x^3+a)^
(1/2)/x^2-(a*f+11/8*b*c)*(b*x^3+a)^(1/2)/x+2/9*g*b*x^3*(b*x^3+a)^(1/2)+2/7*
b*f*x^2*(b*x^3+a)^(1/2)+2/5*b*e*x*(b*x^3+a)^(1/2)+2/3*(4/3*a*b*g+b^2*d)/b*(
b*x^3+a)^(1/2)-9/10*I*a*e*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-
a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))-2/3*I*(27/14*a*f*b+27/16*b^2*c)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)))-2/3*(a^2*g+3/2*a*b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2
)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.82 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \left[\frac{6804\,a\sqrt{b}ex^4\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+210\,(3\,bd+\right.$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="frica
s")
```

```
[Out] [1/2520*(6804*a*sqrt(b)*e*x^4*weierstrassPInverse(0, -4*a/b, x) + 210*(3*b*
d + 2*a*g)*sqrt(a)*x^4*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x
^3 + a)*sqrt(a) + 8*a^2)/x^6) - 1215*(7*b*c + 8*a*f)*sqrt(b)*x^4*weierstras
sZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (560*b*g*x^7 + 720*b*
f*x^6 + 1008*b*e*x^5 + 560*(3*b*d + 4*a*g)*x^4 - 1260*a*e*x^2 - 315*(11*b*c
 + 8*a*f)*x^3 - 840*a*d*x - 630*a*c)*sqrt(b*x^3 + a))/x^4, 1/2520*(6804*a*s
qrt(b)*e*x^4*weierstrassPInverse(0, -4*a/b, x) + 420*(3*b*d + 2*a*g)*sqrt(-
a)*x^4*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 1215*(7*b*c + 8*a
*f)*sqrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x
)) + (560*b*g*x^7 + 720*b*f*x^6 + 1008*b*e*x^5 + 560*(3*b*d + 4*a*g)*x^4 -
1260*a*e*x^2 - 315*(11*b*c + 8*a*f)*x^3 - 840*a*d*x - 630*a*c)*sqrt(b*x^3 +
 a))/x^4]
```

Sympy [A] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 495, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2a^{\frac{3}{2}}g\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}bc\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \sqrt{a}bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) + \frac{\sqrt{a}bex\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}bfx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{2a^2g}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}d\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2a\sqrt{b}d}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2a\sqrt{b}gx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + \frac{2b^{\frac{3}{2}}dx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + bg\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\\frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] a**(3/2)*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**4*gamma(-1/3)) + a**(3/2)*e*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + a**(3/2)*f*gamma(-1/3)*hy
per((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a*
*(3/2)*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(a)*b*c*gamma(-1/3)*hype
r((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a
)*b*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) + sqrt(a)*b*e*x*gamma(1/3)*hyper((-
1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*b*f*x
**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(5/3)) + 2*a**2*g/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*d*
sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*d/(3*x**(3/2)*sqrt(a/(b*x**
3) + 1)) + 2*a*sqrt(b)*g*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*d*x
**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + b*g*Piecewise((sqrt(a)*x**3/3, Eq(b, 0))
, (2*(a + b*x**3)**(3/2)/(9*b), True))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^5, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx=\int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^5}\,dx$$

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x)
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5, x)
```

3.467 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 689

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \frac{27bc\sqrt{a+bx^3}}{20x^2}-\frac{27bd\sqrt{a+bx^3}}{8x}+\frac{27\sqrt[3]{b}(7bd+8ag)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)(a+bx^3)^{3/2}-\frac{b\sqrt{a+bx^3}(252cx-315dx^2-140ex^3-126fx^4-180gx^5)}{140x^3}-\sqrt{a}be\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}(7bd+8ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(14\sqrt[3]{b}(bc+2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd+8ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{Elli}}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/60*(12*c/x^5+15*d/x^4+20*e/x^3+30*f/x^2+60*g/x)*(b*x^3+a)^(3/2)-b*e*arct
anh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+27/20*b*c*(b*x^3+a)^(1/2)/x^2-27/8*b*d
*(b*x^3+a)^(1/2)/x-1/140*b*(-180*g*x^5-126*f*x^4-140*e*x^3-315*d*x^2+252*c*
x)*(b*x^3+a)^(1/2)/x^3+27/56*b^(1/3)*(8*a*g+7*b*d)*(b*x^3+a)^(1/2)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))-27/112*3^(1/4)*a^(1/3)*b^(1/3)*(8*a*g+7*b*d)*(a^(1/
```

```
3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2
)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+9/2
80*3^(3/4)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1
/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(14*b^(1/3)*(2*a*f+b*c
)-5*a^(1/3)*(8*a*g+7*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*
x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {14, 1839, 1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{Elli}}{}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8ag+7bd)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\sqrt{a}be\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-\frac{1}{60}(a+bx^3)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)-\frac{b\sqrt{a+bx^3}(252cx-315dx^2}{1}$$

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

```
[Out] (27*b*c*Sqrt[a + b*x^3])/(20*x^2) - (27*b*d*Sqrt[a + b*x^3])/(8*x) + (27*b^
(1/3)*(7*b*d + 8*a*g)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)) - (((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*(a +
 b*x^3)^(3/2))/60 - (b*Sqrt[a + b*x^3]*(252*c*x - 315*d*x^2 - 140*e*x^3 - 1
26*f*x^4 - 180*g*x^5))/(140*x^3) - Sqrt[a]*b*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]] - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7*b*d + 8*a*g)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(112*Sqrt[(a^
```

```
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(14*b^(1/3)*(b*c + 2*a*f)
- 5*(1 - Sqrt[3])*a^(1/3)*(7*b*d + 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(280*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
```

```
x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
```

```
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\left(a+bx^3\right)^{3/2} - \frac{1}{2}(9b)\int \frac{\sqrt{a+bx^3}\left(-\frac{c}{5}-\frac{dx}{4}-\frac{ex^2}{3}-\frac{fx^3}{2}-gx^4\right)}{x^3}\,dx$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\left(a+bx^3\right)^{3/2} - \frac{b\sqrt{a+bx^3}(252cx-315dx^2-140ex^3-126fx^4-180gx^5)}{140x^3} - \frac{1}{4}(27ab)\int \frac{\frac{2c}{5}-\frac{dx}{2}-\frac{2ex^2}{9}-\frac{fx^3}{5}-\frac{2gx^4}{7}}{x^3\sqrt{a+bx^3}}\,dx$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\left(a+bx^3\right)^{3/2} - \frac{b\sqrt{a+bx^3}(252cx-315dx^2-140ex^3-126fx^4-180gx^5)}{140x^3} + \frac{1}{16}(27b)\int \frac{2ad+\frac{8aex}{9}+\frac{2}{5}(bc+2af)x^2+\frac{8}{7}agx^3}{x^2\sqrt{a+bx^3}}\,dx$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2} - \frac{27bd\sqrt{a+bx^3}}{8x} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\left(a+bx^3\right)^{3/2} - \frac{b\sqrt{a+bx^3}(252cx-315dx^2-140ex^3-126fx^4-180gx^5)}{140x^3} - \frac{(27b)\int \frac{-\frac{16a^2e}{9}-\frac{4}{5}a(bc+2af)x-\frac{2}{7}a(7bd+8ag)x^2}{x\sqrt{a+bx^3}}\,dx}{32a}$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2} - \frac{27bd\sqrt{a+bx^3}}{8x} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\left(a+bx^3\right)^{3/2} - \frac{b\sqrt{a+bx^3}(252cx-315dx^2-140ex^3-126fx^4-180gx^5)}{140x^3} - \frac{(27b)\int \frac{-\frac{4}{5}a(bc+2af)-\frac{2}{7}a(7bd+8ag)x}{\sqrt{a+bx^3}}\,dx}{32a} + \frac{1}{2}(3abe)\int \frac{1}{x\sqrt{a+bx^3}}\,dx$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2} - \frac{27bd\sqrt{a+bx^3}}{8x} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\left(a+bx^3\right)^{3/2}$$
$$- \frac{b\sqrt{a+bx^3}(252cx-315dx^2-140ex^3-126fx^4-180gx^5)}{140x^3}$$
$$+ \frac{1}{2}(abe)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)$$
$$+ \frac{1}{112}\left(27b^{2/3}(7bd+8ag)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$$
$$+ \frac{1}{560}\left(27b\left(14(bc+2af)-\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd+8ag)}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2} - \frac{27bd\sqrt{a+bx^3}}{8x} + \frac{27\sqrt[3]{b}(7bd+8ag)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$- \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\left(a+bx^3\right)^{3/2}$$
$$- \frac{b\sqrt{a+bx^3}(252cx-315dx^2-140ex^3-126fx^4-180gx^5)}{140x^3}$$
$$- \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}(7bd+8ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}\right.\right.}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(14(bc+2af)-\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd+8ag)}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ (ae)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2} - \frac{27bd\sqrt{a+bx^3}}{8x} + \frac{27\sqrt[3]{b}(7bd+8ag)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$- \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\left(a+bx^3\right)^{3/2}$$
$$- \frac{b\sqrt{a+bx^3}(252cx-315dx^2-140ex^3-126fx^4-180gx^5)}{140x^3}$$
$$- \sqrt{a}be\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$
$$- \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}(7bd+8ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(14(bc+2af)-\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd+8ag)}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.28

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^6}\,dx = \frac{\sqrt{a+bx^3}\left(-12a^3c\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{3}{2},-\frac{2}{3},-\frac{bx^3}{a}\right)\right.}{}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-12*a^3*c*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a
)] - 15*a^3*d*x*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)] - 30*a^3*
f*x^3*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)] - 60*a^3*g*x^4*Hyper
geometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)] + 8*b*e*x^5*(a + b*x^3)^2*Sqrt[
1 + (b*x^3)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a]))/(60*a^2*x^5*
Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	920
default	Expression too large to display	1606
risch	Expression too large to display	2289

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x,method=_RETURNVERBOSE)
[Out] -1/5*a*c*(b*x^3+a)^(1/2)/x^5-1/4*a*d*(b*x^3+a)^(1/2)/x^4-1/3*a*e*(b*x^3+a)^
(1/2)/x^3-1/2*(a*f+13/10*b*c)*(b*x^3+a)^(1/2)/x^2-(a*g+11/8*b*d)*(b*x^3+a)^
(1/2)/x+2/7*g*b*x^2*(b*x^3+a)^(1/2)+2/5*b*f*x*(b*x^3+a)^(1/2)+2/3*b*e*(b*x^
3+a)^(1/2)-2/3*I*(8/5*a*f*b+b^2*c-1/40*b*(10*a*f+13*b*c))*3^(1/2)/b*(-a*b^2
)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(10/7*a*b*g+b^2*d+1/16
*b*(8*a*g+11*b*d))*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2
)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2
)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-b*e*arctanh(
(b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.55

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \left[\frac{210\sqrt{a}bex^5\log\left(-\frac{b^2x^6+8abx^3-4(bx^3+2a)\sqrt{bx^3+a}\sqrt{a}+8a^2}{x^6}\right)+1\cdots}{\cdots}\right.$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="frica
s")
```

```
[Out] [1/840*(210*sqrt(a)*b*e*x^5*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqr
t(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 1134*(b*c + 2*a*f)*sqrt(b)*x^5*weierst
rassPInverse(0, -4*a/b, x) - 405*(7*b*d + 8*a*g)*sqrt(b)*x^5*weierstrassZet
a(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (240*b*g*x^7 + 336*b*f*x^
6 + 560*b*e*x^5 - 105*(11*b*d + 8*a*g)*x^4 - 280*a*e*x^2 - 42*(13*b*c + 10*
a*f)*x^3 - 210*a*d*x - 168*a*c)*sqrt(b*x^3 + a))/x^5, 1/840*(420*sqrt(-a)*b
*e*x^5*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) + 1134*(b*c + 2*a*f
)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) - 405*(7*b*d + 8*a*g)*sqrt(
b)*x^5*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (240
*b*g*x^7 + 336*b*f*x^6 + 560*b*e*x^5 - 105*(11*b*d + 8*a*g)*x^4 - 280*a*e*x
^2 - 42*(13*b*c + 10*a*f)*x^3 - 210*a*d*x - 168*a*c)*sqrt(b*x^3 + a))/x^5]
```

Sympy [A] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 476, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}}g\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \sqrt{a}be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) + \frac{\sqrt{a}bfx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}bgx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{a\sqrt{b}e\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2a\sqrt{b}e}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2b^{\frac{3}{2}}ex^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**6,x)
```

```
[Out] a**(3/2)*c*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**5*gamma(-2/3)) + a**(3/2)*d*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + a**(3/2)*f*gamma(-2/3)*
hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) +
 a**(3/2)*g*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x*gamma(2/3)) + sqrt(a)*b*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b
*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*d*gamma(-1/3)*hype
r((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a
)*b*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) + sqrt(a)*b*f*x*gamma(1/3)*hyper((-
1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*b*g*x
**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(5/3)) - a*sqrt(b)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*e/(3*
x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*e*x**(3/2)/(3*sqrt(a/(b*x**3) +
 1))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^6}\,dx$$

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x)
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6, x)
```

3.468 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 692

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} + \frac{27b^{4/3}e\sqrt{a+bx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)(a+bx^3)^{3/2} - \frac{b\sqrt{a+bx^3}(10cx+36dx^2-45ex^3-20fx^4-18gx^5)}{20x^4} - \frac{b(bc+4af)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{16\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(2bd-5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{arcs}\right.}{40\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/60*(10*c/x^6+12*d/x^5+15*e/x^4+20*f/x^3+30*g/x^2)*(b*x^3+a)^(3/2)-1/4*b*
(4*a*f+b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+1/4*b*c*(b*x^3+a)^(1/2
)/x^3+27/20*b*d*(b*x^3+a)^(1/2)/x^2-27/8*b*e*(b*x^3+a)^(1/2)/x-1/20*b*(-18*
```

```
g*x^5-20*f*x^4-45*e*x^3+36*d*x^2+10*c*x)*(b*x^3+a)^(1/2)/x^4+27/8*b^(4/3)*e
*(b*x^3+a)^(1/2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/16*3^(1/4)*a^(1/3)*b^(4
/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b
*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)+9/40*3^(3/4)*b^(2/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1
/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(2*b*d+4*a*
g-5*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+
a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {14, 1839, 1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{Ellip}}{40\sqrt{}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(4af+bc)}{4\sqrt{a}}+\frac{27b^{4/3}e\sqrt{a+bx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{b\sqrt{a+bx^3}(10cx+36dx^2-45ex^3-20fx^4-18gx^5)}{20x^4}$$

$$-\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)+\frac{bc\sqrt{a+bx^3}}{4x^3}+\frac{27bd\sqrt{a+bx^3}}{20x^2}-\frac{27be\sqrt{a+bx^3}}{8x}$$

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] (b*c*Sqrt[a + b*x^3])/(4*x^3) + (27*b*d*Sqrt[a + b*x^3])/(20*x^2) - (27*b*e
*Sqrt[a + b*x^3])/(8*x) + (27*b^(4/3)*e*Sqrt[a + b*x^3])/(8*((1 + Sqrt[3])*
```

```
a^(1/3) + b^(1/3)*x)) - (((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3
 + (30*g)/x^2)*(a + b*x^3)^(3/2))/60 - (b*Sqrt[a + b*x^3]*(10*c*x + 36*d*x^
2 - 45*e*x^3 - 20*f*x^4 - 18*g*x^5))/(20*x^4) - (b*(b*c + 4*a*f)*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)
*b^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3
]])/(16*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(2*b*d - 5
*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(40*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\left(a+bx^3\right)^{3/2} \\
&\quad -\frac{1}{2}(9b)\int\frac{\sqrt{a+bx^3}\left(-\frac{c}{6}-\frac{dx}{5}-\frac{ex^2}{4}-\frac{fx^3}{3}-\frac{gx^4}{2}\right)}{x^4}\,dx \\
&= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\left(a+bx^3\right)^{3/2} \\
&\quad -\frac{b\sqrt{a+bx^3}(10cx+36dx^2-45ex^3-20fx^4-18gx^5)}{20x^4} \\
&\quad -\frac{1}{4}(27ab)\int\frac{\frac{c}{9}+\frac{2dx}{5}-\frac{ex^2}{2}-\frac{2fx^3}{9}-\frac{gx^4}{5}}{x^4\sqrt{a+bx^3}}\,dx \\
&= \frac{bc\sqrt{a+bx^3}}{4x^3}-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\left(a+bx^3\right)^{3/2} \\
&\quad -\frac{b\sqrt{a+bx^3}(10cx+36dx^2-45ex^3-20fx^4-18gx^5)}{20x^4} \\
&\quad +\frac{1}{8}(9b)\int\frac{-\frac{12ad}{5}+3aex+\frac{1}{3}(bc+4af)x^2+\frac{6}{5}agx^3}{x^3\sqrt{a+bx^3}}\,dx \\
&= \frac{bc\sqrt{a+bx^3}}{4x^3}+\frac{27bd\sqrt{a+bx^3}}{20x^2}-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\left(a+bx^3\right)^{3/2} \\
&\quad -\frac{b\sqrt{a+bx^3}(10cx+36dx^2-45ex^3-20fx^4-18gx^5)}{20x^4} \\
&\quad -\frac{(9b)\int\frac{-12a^2e-\frac{4}{3}a(bc+4af)x-\frac{12}{5}a(bd+2ag)x^2}{x^2\sqrt{a+bx^3}}\,dx}{32a} \\
&= \frac{bc\sqrt{a+bx^3}}{4x^3}+\frac{27bd\sqrt{a+bx^3}}{20x^2}-\frac{27be\sqrt{a+bx^3}}{8x} \\
&\quad -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\left(a+bx^3\right)^{3/2} \\
&\quad -\frac{b\sqrt{a+bx^3}(10cx+36dx^2-45ex^3-20fx^4-18gx^5)}{20x^4} \\
&\quad +\frac{(9b)\int\frac{\frac{8}{3}a^2(bc+4af)+\frac{24}{5}a^2(bd+2ag)x+12a^2bex^2}{x\sqrt{a+bx^3}}\,dx}{64a^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} \\
&\quad - \frac{1}{60}\left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2}\right)\left(a+bx^3\right)^{3/2} \\
&\quad - \frac{b\sqrt{a+bx^3}(10cx + 36dx^2 - 45ex^3 - 20fx^4 - 18gx^5)}{20x^4} \\
&\quad + \frac{(9b)\int \frac{\frac{24}{5}a^2(bd+2ag)+12a^2bex}{\sqrt{a+bx^3}}\,dx}{64a^2} + \frac{1}{8}(3b(bc+4af))\int \frac{1}{x\sqrt{a+bx^3}}\,dx \\
&= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} \\
&\quad - \frac{1}{60}\left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2}\right)\left(a+bx^3\right)^{3/2} \\
&\quad - \frac{b\sqrt{a+bx^3}(10cx + 36dx^2 - 45ex^3 - 20fx^4 - 18gx^5)}{20x^4} \\
&\quad + \frac{1}{16}\left(27b^{5/3}e\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx \\
&\quad + \frac{1}{8}(b(bc+4af))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right) \\
&\quad + \frac{1}{80}\left(27b\left(2bd - 5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e + 4ag\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx
\end{aligned}
$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} + \frac{27b^{4/3}e\sqrt{a+bx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$- \frac{1}{60}\left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2}\right)\left(a+bx^3\right)^{3/2}$$

$$- \frac{b\sqrt{a+bx^3}(10cx + 36dx^2 - 45ex^3 - 20fx^4 - 18gx^5)}{20x^4}$$

$$- \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-}{16\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(2bd-5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\mathrm{si}\right.}{40\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{1}{4}(bc+4af)\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} + \frac{27b^{4/3}e\sqrt{a+bx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$- \frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\left(a+bx^3\right)^{3/2}$$

$$- \frac{b\sqrt{a+bx^3}(10cx+36dx^2-45ex^3-20fx^4-18gx^5)}{20x^4}$$

$$- \frac{b(bc+4af)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

$$- \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{16\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(2bd-5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin\right.}{40\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.56 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.35

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^7}\,dx = \frac{-\frac{15bc(a+bx^3)}{x^3} - \frac{10c(a+bx^3)^2}{x^6} - 15b^2c\sqrt{1+\frac{bx^3}{a}}\operatorname{arctanh}\left(\sqrt{1+\right.}{}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] ((-15*b*c*(a + b*x^3))/x^3 - (10*c*(a + b*x^3)^2)/x^6 - 15*b^2*c*Sqrt[1 + (
b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]] - (12*a^2*d*Sqrt[1 + (b*x^3)/a]*Hype
rgeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)])/x^5 - (15*a^2*e*Sqrt[1 + (b*
x^3)/a]*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)])/x^4 - (30*a^2*g*
Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)])/x^2 +
 (8*b*f*(a + b*x^3)^3*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a])/a^2)/(
60*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 903, normalized size of antiderivative = 1.30

method	result	size
elliptic	Expression too large to display	903
default	Expression too large to display	1196
risch	Expression too large to display	1411

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a*c*(b*x^3+a)^(1/2)/x^6-1/5*a*d*(b*x^3+a)^(1/2)/x^5-1/4*a*e*(b*x^3+a)^
(1/2)/x^4-1/3*(a*f+5/4*b*c)*(b*x^3+a)^(1/2)/x^3-1/2*(a*g+13/10*b*d)*(b*x^3+
a)^(1/2)/x^2-11/8*b*e*(b*x^3+a)^(1/2)/x+2/5*g*b*x*(b*x^3+a)^(1/2)+2/3*b*f*(
b*x^3+a)^(1/2)-2/3*I*(8/5*a*b*g+b^2*d-1/40*b*(10*a*g+13*b*d))*3^(1/2)/b*(-a
*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-9/8*I*b*e*3^(1/2)*(-a*b^
2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))-1/4*b*(4*a*f+b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/
2))/a^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \left[-\frac{810\,ab^{\frac{3}{2}}ex^6\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(\right.\right.}{}\right.$$

$$-\frac{405\,ab^{\frac{3}{2}}ex^6\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-15\,(b^2c+4\,abf)\sqrt{-a}x^6\arctan\left(\frac{2\,\sqrt{ba}}{ba}\right.}{}$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="frica
s")
```

```
[Out] [-1/240*(810*a*b^(3/2)*e*x^6*weierstrassZeta(0, -4*a/b, weierstrassPInverse
(0, -4*a/b, x)) - 15*(b^2*c + 4*a*b*f)*sqrt(a)*x^6*log(-(b^2*x^6 + 8*a*b*x^
3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 324*(a*b*d + 2*
a^2*g)*sqrt(b)*x^6*weierstrassPInverse(0, -4*a/b, x) - 2*(48*a*b*g*x^7 + 80
*a*b*f*x^6 - 165*a*b*e*x^5 - 30*a^2*e*x^2 - 6*(13*a*b*d + 10*a^2*g)*x^4 - 2
4*a^2*d*x - 10*(5*a*b*c + 4*a^2*f)*x^3 - 20*a^2*c)*sqrt(b*x^3 + a))/(a*x^6)
, -1/120*(405*a*b^(3/2)*e*x^6*weierstrassZeta(0, -4*a/b, weierstrassPInvers
e(0, -4*a/b, x)) - 15*(b^2*c + 4*a*b*f)*sqrt(-a)*x^6*arctan(2*sqrt(b*x^3 +
a)*sqrt(-a)/(b*x^3 + 2*a)) - 162*(a*b*d + 2*a^2*g)*sqrt(b)*x^6*weierstrassP
Inverse(0, -4*a/b, x) - (48*a*b*g*x^7 + 80*a*b*f*x^6 - 165*a*b*e*x^5 - 30*a
^2*e*x^2 - 6*(13*a*b*d + 10*a^2*g)*x^4 - 24*a^2*d*x - 10*(5*a*b*c + 4*a^2*f
)*x^3 - 20*a^2*c)*sqrt(b*x^3 + a))/(a*x^6)]
```

Sympy [A] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 524, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}}g\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \sqrt{a}bf\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) + \frac{\sqrt{a}bgx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{a^2c}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}c}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}f\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2a\sqrt{b}f}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}}c}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2b^{\frac{3}{2}}fx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} - \frac{b^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}}$$

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**7,x)
```

```
[Out] a**(3/2)*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**5*gamma(-2/3)) + a**(3/2)*e*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + a**(3/2)*g*gamma(-2/3)*
hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) +
 sqrt(a)*b*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*e*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a)*b*f*asinh(sqrt(a)/(
sqrt(b)*x**(3/2))) + sqrt(a)*b*g*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) - a**2*c/(6*sqrt(b)*x**(15/2)*sqrt(a
/(b*x**3) + 1)) - a*sqrt(b)*c/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)
*f*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*f/(3*x**(3/2)*sqrt(a/(b*
x**3) + 1)) - b**(3/2)*c*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*c/(12
*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*f*x**(3/2)/(3*sqrt(a/(b*x**3)
+ 1)) - b**2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="maxim
a")
```

```
[Out] 1/24*(3*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sq
rt(a) - 2*(5*(b*x^3 + a)^(3/2)*b^2 - 3*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^
2 - 2*(b*x^3 + a)*a + a^2))*c + integrate((b*g*x^6 + b*f*x^5 + b*e*x^4 + a*
f*x^2 + (b*d + a*g)*x^3 + a*e*x + a*d)*sqrt(b*x^3 + a)/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx=\int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^7}\,dx$$

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x)
```

```
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7, x)
```

3.469 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 746

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \frac{27bc\sqrt{a+bx^3}}{280x^4}+\frac{bd\sqrt{a+bx^3}}{4x^3}$$
$$+\frac{27be\sqrt{a+bx^3}}{20x^2}-\frac{27b(bc+14af)\sqrt{a+bx^3}}{112ax}+\frac{27b^{4/3}(bc+14af)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\left(a+bx^3\right)^{3/2}$$
$$-\frac{b\sqrt{a+bx^3}(36cx+70dx^2+252ex^3-315fx^4-140gx^5)}{140x^5}-\frac{b(bd+4ag)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}$$
$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(bc+14af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(bc+14af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\right.}{560a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/420*(60*c/x^7+70*d/x^6+84*e/x^5+105*f/x^4+140*g/x^3)*(b*x^3+a)^(3/2)-1/4
*b*(4*a*g+b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+27/280*b*c*(b*x^3+a
)^(1/2)/x^4+1/4*b*d*(b*x^3+a)^(1/2)/x^3+27/20*b*e*(b*x^3+a)^(1/2)/x^2-27/11
```

```
2*b*(14*a*f+b*c)*(b*x^3+a)^(1/2)/a/x-1/140*b*(-140*g*x^5-315*f*x^4+252*e*x^
3+70*d*x^2+36*c*x)*(b*x^3+a)^(1/2)/x^5+27/112*b^(4/3)*(14*a*f+b*c)*(b*x^3+a
)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/224*3^(1/4)*b^(4/3)*(14*a*f+b*
c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)+9/560*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(28*a
^(2/3)*b^(1/3)*e-5*(14*a*f+b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/
2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {14, 1839, 1840, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{Ellip}}{560a^{2/}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(14af+bc)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{}\right.}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(4ag+bd)}{4\sqrt{a}}+\frac{27b^{4/3}\sqrt{a+bx^3}(14af+bc)}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{b\sqrt{a+bx^3}(36cx+70dx^2+252ex^3-315fx^4-140gx^5)}{140x^5}$$

$$-\frac{1}{420}(a+bx^3)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)-\frac{27b\sqrt{a+bx^3}(14af+bc)}{112ax}+\frac{27bc\sqrt{a+bx^3}}{280x^4}+\frac{bd\sqrt{a+bx}}{4x^3}$$

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]

[Out] (27*b*c*Sqrt[a + b*x^3])/(280*x^4) + (b*d*Sqrt[a + b*x^3])/(4*x^3) + (27*b*
e*Sqrt[a + b*x^3])/(20*x^2) - (27*b*(b*c + 14*a*f)*Sqrt[a + b*x^3])/(112*a*
```

```
x) + (27*b^(4/3)*(b*c + 14*a*f)*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) - (((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 +
(140*g)/x^3)*(a + b*x^3)^(3/2))/420 - (b*Sqrt[a + b*x^3]*(36*c*x + 70*d*x^2
 + 252*e*x^3 - 315*f*x^4 - 140*g*x^5))/(140*x^5) - (b*(b*d + 4*a*g)*ArcTanh
[Sqrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4
/3)*(b*c + 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b
^(4/3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(b*c + 14*a*f))*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(560*a^(2/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1840

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\left(a+bx^3\right)^{3/2} \\
&\quad -\frac{1}{2}(9b)\int\frac{\sqrt{a+bx^3}\left(-\frac{c}{7}-\frac{dx}{6}-\frac{ex^2}{5}-\frac{fx^3}{4}-\frac{gx^4}{3}\right)}{x^5}\,dx \\
&= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\left(a+bx^3\right)^{3/2} \\
&\quad -\frac{b\sqrt{a+bx^3}(36cx+70dx^2+252ex^3-315fx^4-140gx^5)}{140x^5} \\
&\quad -\frac{1}{4}(27ab)\int\frac{\frac{2c}{35}+\frac{dx}{9}+\frac{2ex^2}{5}-\frac{fx^3}{2}-\frac{2gx^4}{9}}{x^5\sqrt{a+bx^3}}\,dx \\
&= \frac{27bc\sqrt{a+bx^3}}{280x^4}-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\left(a+bx^3\right)^{3/2} \\
&\quad -\frac{b\sqrt{a+bx^3}(36cx+70dx^2+252ex^3-315fx^4-140gx^5)}{140x^5} \\
&\quad +\frac{1}{32}(27b)\int\frac{-\frac{8ad}{9}-\frac{16aex}{5}+\frac{2}{7}(bc+14af)x^2+\frac{16}{9}agx^3}{x^4\sqrt{a+bx^3}}\,dx \\
&= \frac{27bc\sqrt{a+bx^3}}{280x^4}+\frac{bd\sqrt{a+bx^3}}{4x^3}-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\left(a+bx^3\right)^{3/2} \\
&\quad -\frac{b\sqrt{a+bx^3}(36cx+70dx^2+252ex^3-315fx^4-140gx^5)}{140x^5} \\
&\quad -\frac{(9b)\int\frac{\frac{96a^2e}{5}-\frac{12}{7}a(bc+14af)x-\frac{8}{3}a(bd+4ag)x^2}{x^3\sqrt{a+bx^3}}\,dx}{64a}
\end{aligned}$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4} + \frac{bd\sqrt{a+bx^3}}{4x^3} + \frac{27be\sqrt{a+bx^3}}{20x^2} - \frac{1}{420}\left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3}\right)\left(a+bx^3\right)^{3/2} - \frac{b\sqrt{a+bx^3}(36cx+70dx^2+252ex^3-315fx^4-140gx^5)}{140x^5} + \frac{(9b)\int \frac{\frac{48}{7}a^2(bc+14af)+\frac{32}{3}a^2(bd+4ag)x+\frac{96}{5}a^2bex^2}{x^2\sqrt{a+bx^3}}\,dx}{256a^2}$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4} + \frac{bd\sqrt{a+bx^3}}{4x^3} + \frac{27be\sqrt{a+bx^3}}{20x^2} - \frac{27b(bc+14af)\sqrt{a+bx^3}}{112ax} - \frac{1}{420}\left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3}\right)\left(a+bx^3\right)^{3/2} - \frac{b\sqrt{a+bx^3}(36cx+70dx^2+252ex^3-315fx^4-140gx^5)}{140x^5} - \frac{(9b)\int \frac{-\frac{64}{3}a^3(bd+4ag)-\frac{192}{5}a^3bex-\frac{48}{7}a^2b(bc+14af)x^2}{x\sqrt{a+bx^3}}\,dx}{512a^3}$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4} + \frac{bd\sqrt{a+bx^3}}{4x^3} + \frac{27be\sqrt{a+bx^3}}{20x^2} - \frac{27b(bc+14af)\sqrt{a+bx^3}}{112ax} - \frac{1}{420}\left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3}\right)\left(a+bx^3\right)^{3/2} - \frac{b\sqrt{a+bx^3}(36cx+70dx^2+252ex^3-315fx^4-140gx^5)}{140x^5} - \frac{(9b)\int \frac{-\frac{192}{5}a^3be-\frac{48}{7}a^2b(bc+14af)x}{\sqrt{a+bx^3}}\,dx}{512a^3} + \frac{1}{8}(3b(bd+4ag))\int \frac{1}{x\sqrt{a+bx^3}}\,dx$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4} + \frac{bd\sqrt{a+bx^3}}{4x^3} + \frac{27be\sqrt{a+bx^3}}{20x^2} - \frac{27b(bc+14af)\sqrt{a+bx^3}}{112ax} - \frac{1}{420}\left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3}\right)\left(a+bx^3\right)^{3/2} - \frac{b\sqrt{a+bx^3}(36cx+70dx^2+252ex^3-315fx^4-140gx^5)}{140x^5} + \frac{\left(27b^{5/3}(bc+14af)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{224a} + \frac{\left(27b^{5/3}\left(28\sqrt[3]{b}e - \frac{5\left(1-\sqrt{3}\right)(bc+14af)}{a^{2/3}}\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{1120} + \frac{1}{8}(b(bd+4ag))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4} + \frac{bd\sqrt{a+bx^3}}{4x^3} + \frac{27be\sqrt{a+bx^3}}{20x^2} - \frac{27b(bc+14af)\sqrt{a+bx^3}}{112ax} + \frac{27b^{4/3}(bc+14af)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\left(a+bx^3\right)^{3/2} - \frac{b\sqrt{a+bx^3}(36cx+70dx^2+252ex^3-315fx^4-140gx^5)}{140x^5} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(bc+14af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(28\sqrt[3]{b}e-\frac{5\left(1-\sqrt{3}\right)(bc+14af)}{a^{2/3}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\right.\right.}{560\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{1}{4}(bd+4ag)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4} + \frac{bd\sqrt{a+bx^3}}{4x^3} + \frac{27be\sqrt{a+bx^3}}{20x^2} - \frac{27b(bc+14af)\sqrt{a+bx^3}}{112ax} + \frac{27b^{4/3}(bc+14af)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\left(a+bx^3\right)^{3/2} - \frac{b\sqrt{a+bx^3}(36cx+70dx^2+252ex^3-315fx^4-140gx^5)}{140x^5} - \frac{b(bd+4ag)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(bc+14af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}}}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}}} + \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(28\sqrt[3]{b}e-\frac{5\left(1-\sqrt{3}\right)(bc+14af)}{a^{2/3}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\right.\right.}{560\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.88 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \frac{-\frac{105bd(a+bx^3)}{x^3}-\frac{70d(a+bx^3)^2}{x^6}-105b^2d\sqrt{1+\frac{bx^3}{a}}\operatorname{arctanh}\left(\sqrt{1-\right.}{}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

```
[Out] ((-105*b*d*(a + b*x^3))/x^3 - (70*d*(a + b*x^3)^2)/x^6 - 105*b^2*d*Sqrt[1 +
 (b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]] - (60*a^2*c*Sqrt[1 + (b*x^3)/a]*Hy
pergeometric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)])/x^7 - (84*a^2*e*Sqrt[1 + (
b*x^3)/a]*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)])/x^5 - (105*a^2
*f*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)])/x
^4 + (56*b*g*(a + b*x^3)^3*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a])/a
^2)/(420*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 916, normalized size of antiderivative = 1.23

method	result	size
elliptic	Expression too large to display	916
risch	Expression too large to display	1295
default	Expression too large to display	1375

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a*c*(b*x^3+a)^(1/2)/x^7-1/6*a*d*(b*x^3+a)^(1/2)/x^6-1/5*a*e*(b*x^3+a)^
(1/2)/x^5-1/4*(a*f+17/14*b*c)*(b*x^3+a)^(1/2)/x^4-1/3*(a*g+5/4*b*d)*(b*x^3+
a)^(1/2)/x^3-13/20*b*e*(b*x^3+a)^(1/2)/x^2-1/112*b*(154*a*f+27*b*c)/a*(b*x^
3+a)^(1/2)/x+2/3*g*b*(b*x^3+a)^(1/2)-9/20*I*b*e*3^(1/2)*(-a*b^2)^(1/3)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(b^2*f+1/224*b^2*(154*a*f+27*b*c)/
a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
```

```
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-1/4*b*(4*a*g+b*d)*arctanh((b
*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.60

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \left[\frac{2268\,ab^{\frac{3}{2}}ex^7\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+105\,(b^2d+\right.$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="frica
s")
```

```
[Out] [1/1680*(2268*a*b^(3/2)*e*x^7*weierstrassPInverse(0, -4*a/b, x) + 105*(b^2*
d + 4*a*b*g)*sqrt(a)*x^7*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b
*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 405*(b^2*c + 14*a*b*f)*sqrt(b)*x^7*weiers
trassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (1120*a*b*g*x^7 -
 1092*a*b*e*x^5 - 15*(27*b^2*c + 154*a*b*f)*x^6 - 336*a^2*e*x^2 - 140*(5*a*
b*d + 4*a^2*g)*x^4 - 280*a^2*d*x - 30*(17*a*b*c + 14*a^2*f)*x^3 - 240*a^2*c
)*sqrt(b*x^3 + a))/(a*x^7), 1/1680*(2268*a*b^(3/2)*e*x^7*weierstrassPInvers
e(0, -4*a/b, x) + 210*(b^2*d + 4*a*b*g)*sqrt(-a)*x^7*arctan(2*sqrt(b*x^3 +
a)*sqrt(-a)/(b*x^3 + 2*a)) - 405*(b^2*c + 14*a*b*f)*sqrt(b)*x^7*weierstrass
Zeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (1120*a*b*g*x^7 - 1092
*a*b*e*x^5 - 15*(27*b^2*c + 154*a*b*f)*x^6 - 336*a^2*e*x^2 - 140*(5*a*b*d +
 4*a^2*g)*x^4 - 280*a^2*d*x - 30*(17*a*b*c + 14*a^2*f)*x^3 - 240*a^2*c)*sqr
t(b*x^3 + a))/(a*x^7)]
```

Sympy [A] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}bf\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \sqrt{a}bg\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) - \frac{a^2d}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}d}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}g\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2a\sqrt{b}g}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}}d}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2b^{\frac{3}{2}}gx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} - \frac{b^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}}$$

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**8,x)
```

```
[Out] a**(3/2)*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**7*gamma(-4/3)) + a**(3/2)*e*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + a**(3/2)*f*gamma(-4/3)*
hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3))
 + sqrt(a)*b*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*
pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*e*gamma(-2/3)*hyper((-2/3, -1/2), (
1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*f*gamma(-1
/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3))
- sqrt(a)*b*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - a**2*d/(6*sqrt(b)*x**(15/
2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*d/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) -
a*sqrt(b)*g*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*g/(3*x**(3/2)*s
qrt(a/(b*x**3) + 1)) - b**(3/2)*d*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3
/2)*d/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*g*x**(3/2)/(3*sqrt(a/
(b*x**3) + 1)) - b**2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^8}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="maxim
a")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^8, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^8}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="giac"
)
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^8}\,dx$$

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x)
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8, x)
```

3.470 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 705

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3} - \frac{27b^2c\sqrt{a+bx^3}}{320ax^2} - \frac{27b^2d\sqrt{a+bx^3}}{112ax} + \frac{27b^{4/3}(bd+14ag)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)(a+bx^3)^{3/2} - \frac{b^2e\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(bd+14ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{}\right.}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(7\sqrt[3]{b}(bc-16af)+20\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd+14ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{E}}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/840*(105*c/x^8+120*d/x^7+140*e/x^6+168*f/x^5+210*g/x^4)*(b*x^3+a)^(3/2)-
1/4*b^2*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/560*b*(63*c/x^5+90*d/x
^4+140*e/x^3+252*f/x^2+630*g/x)*(b*x^3+a)^(1/2)-27/320*b^2*c*(b*x^3+a)^(1/2
```

```
)/a/x^2-27/112*b^2*d*(b*x^3+a)^(1/2)/a/x+27/112*b^(4/3)*(14*a*g+b*d)*(b*x^3
+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/224*3^(1/4)*b^(4/3)*(14*a*g+
b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(
2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)-9/2240*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/
3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7
*b^(1/3)*(-16*a*f+b*c)+20*a^(1/3)*(14*a*g+b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/
2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx =$$

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right.}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(14ag+bd)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\right.}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{b^2e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{27b^{4/3}\sqrt{a+bx^3}(14ag+bd)}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{27b^2c\sqrt{a+bx^3}}{320ax^2}-\frac{27b^2d\sqrt{a+bx^3}}{112ax}$$

$$-\frac{1}{560}b\sqrt{a+bx^3}\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)-\frac{1}{840}(a+bx^3)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right.$$

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]

[Out] -1/560*(b*((63*c)/x^5 + (90*d)/x^4 + (140*e)/x^3 + (252*f)/x^2 + (630*g)/x)
*Sqrt[a + b*x^3]) - (27*b^2*c*Sqrt[a + b*x^3])/(320*a*x^2) - (27*b^2*d*Sqrt
```

```
[a + b*x^3])/(112*a*x) + (27*b^(4/3)*(b*d + 14*a*g)*Sqrt[a + b*x^3])/(112*a
*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((105*c)/x^8 + (120*d)/x^7 + (140*
e)/x^6 + (168*f)/x^5 + (210*g)/x^4)*(a + b*x^3)^(3/2))/840 - (b^2*e*ArcTanh
[Sqrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4
/3)*(b*d + 14*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b
^(4/3)*(7*b^(1/3)*(b*c - 16*a*f) + 20*(1 - Sqrt[3])*a^(1/3)*(b*d + 14*a*g))
*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
 + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2240*
a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\left(a+bx^3\right)^{3/2} \\
&\quad -\frac{1}{2}(9b)\int \frac{\sqrt{a+bx^3}\left(-\frac{c}{8}-\frac{dx}{7}-\frac{ex^2}{6}-\frac{fx^3}{5}-\frac{gx^4}{4}\right)}{x^6}\,dx \\
&= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3} \\
&\quad -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\left(a+bx^3\right)^{3/2} \\
&\quad +\frac{1}{4}\left(27b^2\right)\int \frac{\frac{c}{40}+\frac{dx}{28}+\frac{ex^2}{18}+\frac{fx^3}{10}+\frac{gx^4}{4}}{x^3\sqrt{a+bx^3}}\,dx \\
&= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3}-\frac{27b^2c\sqrt{a+bx^3}}{320ax^2} \\
&\quad -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\left(a+bx^3\right)^{3/2} \\
&\quad -\frac{\left(27b^2\right)\int \frac{-\frac{ad}{7}-\frac{2aex}{9}+\frac{1}{40}(bc-16af)x^2-agx^3}{x^2\sqrt{a+bx^3}}\,dx}{16a} \\
&= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3}-\frac{27b^2c\sqrt{a+bx^3}}{320ax^2} \\
&\quad -\frac{27b^2d\sqrt{a+bx^3}}{112ax}-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\left(a+bx^3\right)^{3/2} \\
&\quad +\frac{\left(27b^2\right)\int \frac{\frac{4a^2e}{9}-\frac{1}{20}a(bc-16af)x+\frac{1}{7}a(bd+14ag)x^2}{x\sqrt{a+bx^3}}\,dx}{32a^2} \\
&= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3}-\frac{27b^2c\sqrt{a+bx^3}}{320ax^2} \\
&\quad -\frac{27b^2d\sqrt{a+bx^3}}{112ax}-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\left(a+bx^3\right)^{3/2} \\
&\quad +\frac{\left(27b^2\right)\int \frac{-\frac{1}{20}a(bc-16af)+\frac{1}{7}a(bd+14ag)x}{\sqrt{a+bx^3}}\,dx}{32a^2}+\frac{1}{8}\left(3b^2e\right)\int \frac{1}{x\sqrt{a+bx^3}}\,dx
\end{aligned}$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3}-\frac{27b^2c\sqrt{a+bx^3}}{320ax^2}$$
$$-\frac{27b^2d\sqrt{a+bx^3}}{112ax}-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\left(a+bx^3\right)^{3/2}$$
$$+\frac{1}{8}\left(b^2e\right)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)$$
$$+\frac{\left(27b^{5/3}(bd+14ag)\right)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{224a}$$
$$-\frac{\left(27b^2\left(7(bc-16af)+\frac{20\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd+14ag)}{\sqrt[3]{b}}\right)\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{4480a}$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3}$$
$$-\frac{27b^2c\sqrt{a+bx^3}}{320ax^2}-\frac{27b^2d\sqrt{a+bx^3}}{112ax}+\frac{27b^{4/3}(bd+14ag)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\left(a+bx^3\right)^{3/2}$$
$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(bd+14ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}\left(7(bc-16af)+\frac{20\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd+14ag)}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{1}{4}(be)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3} - \frac{27b^2c\sqrt{a+bx^3}}{320ax^2} - \frac{27b^2d\sqrt{a+bx^3}}{112ax} + \frac{27b^{4/3}(bd+14ag)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\left(a+bx^3\right)^{3/2} - \frac{b^2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(bd+14ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}\left(7(bc-16af)+\frac{20\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd+14ag)}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.44 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.29

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^9}\,dx = -\frac{\sqrt{a+bx^3}\left(105a^2c\,\text{Hypergeometric2F1}\left(-\frac{8}{3},-\frac{3}{2},-\frac{5}{3},-\frac{bx^3}{a}\right)+2x\left(60a^2d\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{3}{2},-\frac{4}{3}\right.\right.\right.}{}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

```
[Out] -1/840*(Sqrt[a + b*x^3]*(105*a^2*c*Hypergeometric2F1[-8/3, -3/2, -5/3, -((b
*x^3)/a)] + 2*x*(60*a^2*d*Hypergeometric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)]
 + 7*x*(12*a^2*f*x*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)] + 5*(a
*e*(2*a + 5*b*x^3)*Sqrt[1 + (b*x^3)/a] + 3*b^2*e*x^6*ArcTanh[Sqrt[1 + (b*x^
3)/a]] + 3*a^2*g*x^2*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)]))))
/(a*x^8*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 949, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	949
risch	Expression too large to display	1579
default	Expression too large to display	1663

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*a*c*(b*x^3+a)^(1/2)/x^8-1/7*a*d*(b*x^3+a)^(1/2)/x^7-1/6*a*e*(b*x^3+a)^
(1/2)/x^6-1/5*(a*f+19/16*b*c)*(b*x^3+a)^(1/2)/x^5-1/4*(a*g+17/14*b*d)*(b*x^
3+a)^(1/2)/x^4-5/12*b*e*(b*x^3+a)^(1/2)/x^3-1/320*b*(208*a*f+27*b*c)/a*(b*x
^3+a)^(1/2)/x^2-1/112*(154*a*g+27*b*d)*b/a*(b*x^3+a)^(1/2)/x-2/3*I*(b^2*f-1
/640*b^2*(208*a*f+27*b*c)/a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2))-2/3*I*(b^2*g+1/224*b^2/a*(154*a*g+27*b*d))*3^(1/2)/b*(-a*
b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)))-1/4*b^2*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^
(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = \left[\frac{420\sqrt{a}b^2ex^8\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)-\cdots}{\cdots}\right.$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x, algorithm="frica
s")
```

```
[Out] [1/6720*(420*sqrt(a)*b^2*e*x^8*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*
sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 567*(b^2*c - 16*a*b*f)*sqrt(b)*x^8*
weierstrassPInverse(0, -4*a/b, x) - 1620*(b^2*d + 14*a*b*g)*sqrt(b)*x^8*wei
erstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (2800*a*b*e*x^
5 + 60*(27*b^2*d + 154*a*b*g)*x^7 + 21*(27*b^2*c + 208*a*b*f)*x^6 + 1120*a^
2*e*x^2 + 120*(17*a*b*d + 14*a^2*g)*x^4 + 960*a^2*d*x + 84*(19*a*b*c + 16*a
^2*f)*x^3 + 840*a^2*c)*sqrt(b*x^3 + a))/(a*x^8), 1/6720*(840*sqrt(-a)*b^2*e
*x^8*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 567*(b^2*c - 16*a*b
*f)*sqrt(b)*x^8*weierstrassPInverse(0, -4*a/b, x) - 1620*(b^2*d + 14*a*b*g)
*sqrt(b)*x^8*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))
- (2800*a*b*e*x^5 + 60*(27*b^2*d + 154*a*b*g)*x^7 + 21*(27*b^2*c + 208*a*b*
f)*x^6 + 1120*a^2*e*x^2 + 120*(17*a*b*d + 14*a^2*g)*x^4 + 960*a^2*d*x + 84*
(19*a*b*c + 16*a^2*f)*x^3 + 840*a^2*c)*sqrt(b*x^3 + a))/(a*x^8)]
```

Sympy [A] (verification not implemented)

Time = 7.32 (sec) , antiderivative size = 527, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)} + \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{a^{\frac{3}{2}}g\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}bf\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}bg\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{a^2e}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}e}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}}e}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^2e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}}$$

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**9,x)
```

```
[Out] a**(3/2)*c*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**8*gamma(-5/3)) + a**(3/2)*d*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + a**(3/2)*f*gamma(-5/3)*
hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3))
 + a**(3/2)*g*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*p
i)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*c*gamma(-5/3)*hyper((-5/3, -1/2), (-
2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*d*gamma(-
4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-
1/3)) + sqrt(a)*b*f*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*g*gamma(-1/3)*hyper((-1/2, -1/3)
, (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - a**2*e/(6*sqrt(b)*x*
*(15/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*e/(4*x**(9/2)*sqrt(a/(b*x**3) + 1
)) - b**(3/2)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*e/(12*x**(3/2)
*sqrt(a/(b*x**3) + 1)) - b**2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a
))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^9}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^9, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^9}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = \int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^9}\,dx$$

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x)
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9, x)
```

3.471 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 714

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx = -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680}-\frac{b^2c\sqrt{a+bx^3}}{24ax^3}$$
$$-\frac{27b^2d\sqrt{a+bx^3}}{320ax^2}-\frac{27b^2e\sqrt{a+bx^3}}{112ax}+\frac{27b^{7/3}e\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)(a+bx^3)^{3/2}}{2520}+\frac{b^2(bc-6af)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}}$$
$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}\left(7bd+20\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-112ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Big(}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/2520*(280*c/x^9+315*d/x^8+360*e/x^7+420*f/x^6+504*g/x^5)*(b*x^3+a)^(3/2)
+1/24*b^2*(-6*a*f+b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/1680*b*(1
40*c/x^6+189*d/x^5+270*e/x^4+420*f/x^3+756*g/x^2)*(b*x^3+a)^(1/2)-1/24*b^2*
```

```
c*(b*x^3+a)^(1/2)/a/x^3-27/320*b^2*d*(b*x^3+a)^(1/2)/a/x^2-27/112*b^2*e*(b*
x^3+a)^(1/2)/a/x+27/112*b^(7/3)*e*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))-27/224*3^(1/4)*b^(7/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^
(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)
*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-9/2240*3^(3/4)*b^(5/3)*(a^(1/3
)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2))),I*3^(1/2)+2*I)*(7*b*d-112*a*g+20*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))
*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{10}}\,dx =$$

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(bc-6af)}{24a^{3/2}}+\frac{27b^{7/3}e\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{b^2c\sqrt{a+bx^3}}{24ax^3}$$

$$-\frac{27b^2d\sqrt{a+bx^3}}{320ax^2}-\frac{27b^2e\sqrt{a+bx^3}}{112ax}-\frac{b\sqrt{a+bx^3}\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)}{1680}$$

$$-\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10,x]
```

```
[Out] -1/1680*(b*((140*c)/x^6 + (189*d)/x^5 + (270*e)/x^4 + (420*f)/x^3 + (756*g)
/x^2)*Sqrt[a + b*x^3]) - (b^2*c*Sqrt[a + b*x^3])/(24*a*x^3) - (27*b^2*d*Sqr
t[a + b*x^3])/(320*a*x^2) - (27*b^2*e*Sqrt[a + b*x^3])/(112*a*x) + (27*b^(7
/3)*e*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((280
*c)/x^9 + (315*d)/x^8 + (360*e)/x^7 + (420*f)/x^6 + (504*g)/x^5)*(a + b*x^3
)^(3/2))/2520 + (b^2*(b*c - 6*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^
(3/2)) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) -
(9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(5/3)*(7*b*d + 20*(1 - Sqrt[3])*a^(1/3)*b^(2
/3)*e - 112*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(2240*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
```

```
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)(a+bx^3)^{3/2}}{2520} - \frac{1}{2}(9b)\int \frac{\sqrt{a+bx^3}\left(-\frac{c}{9}-\frac{dx}{8}-\frac{ex^2}{7}-\frac{fx^3}{6}-\frac{gx^4}{5}\right)}{x^7}\,dx$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)(a+bx^3)^{3/2}}{2520} + \frac{1}{4}(27b^2)\int \frac{\frac{c}{54}+\frac{dx}{40}+\frac{ex^2}{28}+\frac{fx^3}{18}+\frac{gx^4}{10}}{x^4\sqrt{a+bx^3}}\,dx$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+bx^3}}{24ax^3} - \frac{\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)(a+bx^3)^{3/2}}{2520} - \frac{(9b^2)\int \frac{-\frac{3ad}{20}-\frac{3aex}{14}+\frac{1}{18}(bc-6af)x^2-\frac{3}{5}agx^3}{x^3\sqrt{a+bx^3}}\,dx}{8a}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2d\sqrt{a+bx^3}}{320ax^2} - \frac{\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)(a+bx^3)^{3/2}}{2520} + \frac{(9b^2)\int \frac{\frac{6a^2e}{7}-\frac{2}{9}a(bc-6af)x-\frac{3}{20}a(bd-16ag)x^2}{x^2\sqrt{a+bx^3}}\,dx}{32a^2}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2d\sqrt{a+bx^3}}{320ax^2} - \frac{27b^2e\sqrt{a+bx^3}}{112ax} - \frac{\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)(a+bx^3)^{3/2}}{2520} - \frac{(9b^2)\int \frac{\frac{4}{9}a^2(bc-6af)+\frac{3}{10}a^2(bd-16ag)x-\frac{6}{7}a^2bex^2}{x\sqrt{a+bx^3}}\,dx}{64a^3}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2d\sqrt{a+bx^3}}{320ax^2} - \frac{27b^2e\sqrt{a+bx^3}}{112ax} - \frac{\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)(a+bx^3)^{3/2}}{2520} - \frac{(9b^2)\int \frac{\frac{3}{10}a^2(bd-16ag)-\frac{6}{7}a^2bex}{\sqrt{a+bx^3}}\,dx}{64a^3} - \frac{(b^2(bc-6af))\int \frac{1}{x\sqrt{a+bx^3}}\,dx}{16a}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2d\sqrt{a+bx^3}}{320ax^2} - \frac{27b^2e\sqrt{a+bx^3}}{112ax} - \frac{\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)\left(a+bx^3\right)^{3/2}}{2520} + \frac{\left(27b^{8/3}e\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{224a} - \frac{\left(b^2(bc-6af)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{48a} - \frac{\left(27b^2\left(7bd+20\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-112ag\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{4480a}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2d\sqrt{a+bx^3}}{320ax^2} - \frac{27b^2e\sqrt{a+bx^3}}{112ax} + \frac{27b^{7/3}e\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)\left(a+bx^3\right)^{3/2}}{2520} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right.}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}\left(7bd+20\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-112ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\right.}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{(b(bc-6af))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{24a}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2d\sqrt{a+bx^3}}{320ax^2} - \frac{27b^2e\sqrt{a+bx^3}}{112ax} + \frac{27b^{7/3}e\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)\left(a+bx^3\right)^{3/2}}{2520} + \frac{b^2(bc-6af)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{}\right.}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}\left(7bd+20\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-112ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.70 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.32

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{10}}\,dx = -\frac{\sqrt{a+bx^3}\left(105a^5d\,\text{Hypergeometric2F1}\left(-\frac{8}{3},-\frac{3}{2},-\frac{5}{3},-\frac{bx^3}{a}\right)+2x\left(60a^5e\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{3}{2},-\right.\right.\right.}{}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10,x]
```

```
[Out] -1/840*(Sqrt[a + b*x^3]*(105*a^5*d*Hypergeometric2F1[-8/3, -3/2, -5/3, -((b
*x^3)/a)] + 2*x*(60*a^5*e*Hypergeometric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)]
 + 7*x*(5*a^3*f*(a*(2*a + 5*b*x^3)*Sqrt[1 + (b*x^3)/a] + 3*b^2*x^6*ArcTanh[
Sqrt[1 + (b*x^3)/a]]) + 12*a^5*g*x*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b
*x^3)/a)] - 8*b^3*c*x^6*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1
[5/2, 4, 7/2, 1 + (b*x^3)/a])))/(a^4*x^8*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 958, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	958
risch	Expression too large to display	1160
default	Expression too large to display	1273

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a*c*(b*x^3+a)^(1/2)/x^9-1/8*a*d*(b*x^3+a)^(1/2)/x^8-1/7*a*e*(b*x^3+a)^
(1/2)/x^7-1/6*(a*f+7/6*b*c)*(b*x^3+a)^(1/2)/x^6-1/5*(a*g+19/16*b*d)*(b*x^3+
a)^(1/2)/x^5-17/56*b*e*(b*x^3+a)^(1/2)/x^4-1/24*b*(10*a*f+b*c)/a*(b*x^3+a)^
(1/2)/x^3-1/320*b/a*(208*a*g+27*b*d)*(b*x^3+a)^(1/2)/x^2-27/112*b^2*e*(b*x^
3+a)^(1/2)/a/x-2/3*I*(b^2*g-1/640*b^2/a*(208*a*g+27*b*d))*3^(1/2)/b*(-a*b^2
)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-9/112*I/a*b^2*e*3^(1/2)*(-a*
b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)))-1/24*(6*a*f-b*c)*b^2/a^(3/2)*arctanh((b*x^3+a
)^(1/2)/a^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx = \left[-\frac{4860\,ab^{\frac{5}{2}}ex^9\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInvers}\right.}{}\right.$$

$$-\frac{4860\,ab^{\frac{5}{2}}ex^9\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+420\,(b^3c-6\,ab^2f)\sqrt{-a}x^9\arctan\left(\right.}{}$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="fric
as")
```

```
[Out] [-1/20160*(4860*a*b^(5/2)*e*x^9*weierstrassZeta(0, -4*a/b, weierstrassPInve
rse(0, -4*a/b, x)) + 210*(b^3*c - 6*a*b^2*f)*sqrt(a)*x^9*log((b^2*x^6 + 8*a
*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 1701*(a*b^
2*d - 16*a^2*b*g)*sqrt(b)*x^9*weierstrassPInverse(0, -4*a/b, x) + (4860*a*b
^2*e*x^8 + 6120*a^2*b*e*x^5 + 63*(27*a*b^2*d + 208*a^2*b*g)*x^7 + 840*(a*b^
2*c + 10*a^2*b*f)*x^6 + 2880*a^3*e*x^2 + 2520*a^3*d*x + 252*(19*a^2*b*d + 1
6*a^3*g)*x^4 + 2240*a^3*c + 560*(7*a^2*b*c + 6*a^3*f)*x^3)*sqrt(b*x^3 + a))
/(a^2*x^9), -1/20160*(4860*a*b^(5/2)*e*x^9*weierstrassZeta(0, -4*a/b, weier
strassPInverse(0, -4*a/b, x)) + 420*(b^3*c - 6*a*b^2*f)*sqrt(-a)*x^9*arctan
(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) + 1701*(a*b^2*
d - 16*a^2*b*g)*sqrt(b)*x^9*weierstrassPInverse(0, -4*a/b, x) + (4860*a*b^2
*e*x^8 + 6120*a^2*b*e*x^5 + 63*(27*a*b^2*d + 208*a^2*b*g)*x^7 + 840*(a*b^2*
c + 10*a^2*b*f)*x^6 + 2880*a^3*e*x^2 + 2520*a^3*d*x + 252*(19*a^2*b*d + 16*
a^3*g)*x^4 + 2240*a^3*c + 560*(7*a^2*b*c + 6*a^3*f)*x^3)*sqrt(b*x^3 + a))/(
a^2*x^9)]
```

Sympy [A] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 573, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix} -\frac{8}{3},-\frac{1}{2} \\ -\frac{5}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix} -\frac{7}{3},-\frac{1}{2} \\ -\frac{4}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}g\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix} -\frac{5}{3},-\frac{1}{2} \\ -\frac{2}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix} -\frac{5}{3},-\frac{1}{2} \\ -\frac{2}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix} -\frac{4}{3},-\frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}bg\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix} -\frac{2}{3},-\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} - \frac{a^2c}{9\sqrt{b}x^{\frac{21}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a^2f}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{11a\sqrt{b}c}{36x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}f}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{17b^{\frac{3}{2}}c}{72x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}}f}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{5}{2}}c}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^2f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}} + \frac{b^3c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{24a^{\frac{3}{2}}}$$

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**10,x)
```

```
[Out] a**(3/2)*d*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**8*gamma(-5/3)) + a**(3/2)*e*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + a**(3/2)*g*gamma(-5/3)*
hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3))
 + sqrt(a)*b*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*
pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*e*gamma(-4/3)*hyper((-4/3, -1/2), (
-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*g*gamma(
-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1
/3)) - a**2*c/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - a**2*f/(6*sqrt(b
)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*c/(36*x**(15/2)*sqrt(a/(b*
x**3) + 1)) - a*sqrt(b)*f/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - 17*b**(3/2)*c
/(72*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*f*sqrt(a/(b*x**3) + 1)/(3*x*
*(3/2)) - b**(3/2)*f/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**(5/2)*c/(24*a*
x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**2*f*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(
4*sqrt(a)) + b**3*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(24*a**(3/2))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^{10}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="maxi
ma")
```

```
[Out] -1/144*(3*b^3*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/
a^(3/2) + 2*(3*(b*x^3 + a)^(5/2)*b^3 + 8*(b*x^3 + a)^(3/2)*a*b^3 - 3*sqrt(b
*x^3 + a)*a^2*b^3)/((b*x^3 + a)^3*a - 3*(b*x^3 + a)^2*a^2 + 3*(b*x^3 + a)*a
^3 - a^4))*c + integrate((b*g*x^6 + b*f*x^5 + b*e*x^4 + a*f*x^2 + (b*d + a*
g)*x^3 + a*e*x + a*d)*sqrt(b*x^3 + a)/x^9, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^{10}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="giac
")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx = \int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^{10}}\,dx$$

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10,x)
```

```
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10, x)
```

3.472 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 764

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx = -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680}-\frac{27b^2c\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2d\sqrt{a+bx^3}}{24ax^3}$$
$$-\frac{27b^2e\sqrt{a+bx^3}}{320ax^2}+\frac{27b^2(bc-4af)\sqrt{a+bx^3}}{448a^2x}-\frac{27b^{7/3}(bc-4af)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)(a+bx^3)^{3/2}}{2520}+\frac{b^2(bd-6ag)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}}$$
$$+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}(bc-4af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right.}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{7/3}\left(7a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(bc-4af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{arc}\right.}{2240a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/2520*(252*c/x^10+280*d/x^9+315*e/x^8+360*f/x^7+420*g/x^6)*(b*x^3+a)^(3/2
)+1/24*b^2*(-6*a*g+b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/1680*b*(
108*c/x^7+140*d/x^6+189*e/x^5+270*f/x^4+420*g/x^3)*(b*x^3+a)^(1/2)-27/1120*
```

```
b^2*c*(b*x^3+a)^(1/2)/a/x^4-1/24*b^2*d*(b*x^3+a)^(1/2)/a/x^3-27/320*b^2*e*(
b*x^3+a)^(1/2)/a/x^2+27/448*b^2*(-4*a*f+b*c)*(b*x^3+a)^(1/2)/a^2/x-27/448*b
^(7/3)*(-4*a*f+b*c)*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+27/
896*3^(1/4)*b^(7/3)*(-4*a*f+b*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(
1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*
x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-9/2240*3^(3/4)*b^(7/3)*(a^(1/3)
+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(7*a^(2/3)*b^(1/3)*e-5*(-4*a*f+b*c)*(1-3^(1/2)))*
(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx =$$

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right.}{2240a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(bc-4af)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{\right.}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(bd-6ag)}{24a^{3/2}}-\frac{27b^{7/3}\sqrt{a+bx^3}(bc-4af)}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{27b^2\sqrt{a+bx^3}(bc-4af)}{448a^2x}-\frac{27b^2c\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2d\sqrt{a+bx^3}}{24ax^3}$$

$$-\frac{27b^2e\sqrt{a+bx^3}}{320ax^2}-\frac{b\sqrt{a+bx^3}\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)}{1680}$$

$$-\frac{(a+bx^3)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11,x]
```

```
[Out] -1/1680*(b*((108*c)/x^7 + (140*d)/x^6 + (189*e)/x^5 + (270*f)/x^4 + (420*g)
/x^3)*Sqrt[a + b*x^3]) - (27*b^2*c*Sqrt[a + b*x^3])/(1120*a*x^4) - (b^2*d*S
qrt[a + b*x^3])/(24*a*x^3) - (27*b^2*e*Sqrt[a + b*x^3])/(320*a*x^2) + (27*b
^2*(b*c - 4*a*f)*Sqrt[a + b*x^3])/(448*a^2*x) - (27*b^(7/3)*(b*c - 4*a*f)*S
qrt[a + b*x^3])/(448*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((252*c)/x
^10 + (280*d)/x^9 + (315*e)/x^8 + (360*f)/x^7 + (420*g)/x^6)*(a + b*x^3)^(3
/2))/2520 + (b^2*(b*d - 6*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(3/2
)) + (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(b*c - 4*a*f)*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3]) - (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(7/3)*(7*a^(2/3)*b^(1/3)*e - 5*(1 -
Sqrt[3])*(b*c - 4*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
-7 - 4*Sqrt[3]])/(2240*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)(a+bx^3)^{3/2}}{2520} - \frac{1}{2}(9b)\int \frac{\sqrt{a+bx^3}\left(-\frac{c}{10}-\frac{dx}{9}-\frac{ex^2}{8}-\frac{fx^3}{7}-\frac{gx^4}{6}\right)}{x^8}\,dx$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)(a+bx^3)^{3/2}}{2520} + \frac{1}{4}(27b^2)\int \frac{\frac{c}{70}+\frac{dx}{54}+\frac{ex^2}{40}+\frac{fx^3}{28}+\frac{gx^4}{18}}{x^5\sqrt{a+bx^3}}\,dx$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)(a+bx^3)^{3/2}}{2520} - \frac{(27b^2)\int \frac{-\frac{4ad}{27}-\frac{aex}{5}+\frac{1}{14}(bc-4af)x^2-\frac{4}{9}agx^3}{x^4\sqrt{a+bx^3}}\,dx}{32a}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2d\sqrt{a+bx^3}}{24ax^3} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)(a+bx^3)^{3/2}}{2520} + \frac{(9b^2)\int \frac{\frac{6a^2e}{5}-\frac{3}{7}a(bc-4af)x-\frac{4}{9}a(bd-6ag)x^2}{x^3\sqrt{a+bx^3}}\,dx}{64a^2}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2d\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2e\sqrt{a+bx^3}}{320ax^2} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)(a+bx^3)^{3/2}}{2520} - \frac{(9b^2)\int \frac{\frac{12}{7}a^2(bc-4af)+\frac{16}{9}a^2(bd-6ag)x+\frac{6}{5}a^2bex^2}{x^2\sqrt{a+bx^3}}\,dx}{256a^3}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2d\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2e\sqrt{a+bx^3}}{320ax^2} + \frac{27b^2(bc-4af)\sqrt{a+bx^3}}{448a^2x} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)(a+bx^3)^{3/2}}{2520} + \frac{(9b^2)\int \frac{-\frac{32}{9}a^3(bd-6ag)-\frac{12}{5}a^3bex-\frac{12}{7}a^2b(bc-4af)x^2}{x\sqrt{a+bx^3}}\,dx}{512a^4}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2d\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2e\sqrt{a+bx^3}}{320ax^2} + \frac{27b^2(bc-4af)\sqrt{a+bx^3}}{448a^2x} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)(a+bx^3)^{3/2}}{2520} + \frac{(9b^2)\int \frac{-\frac{12}{5}a^3be-\frac{12}{7}a^2b(bc-4af)x}{\sqrt{a+bx^3}}\,dx}{512a^4} - \frac{(b^2(bd-6ag))\int \frac{1}{x\sqrt{a+bx^3}}\,dx}{16a}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2d\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2e\sqrt{a+bx^3}}{320ax^2} + \frac{27b^2(bc-4af)\sqrt{a+bx^3}}{448a^2x} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)(a+bx^3)^{3/2}}{2520} - \frac{\left(27b^{8/3}(bc-4af)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{896a^2} - \frac{\left(27b^{8/3}\left(7a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(bc-4af)\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{4480a^{5/3}} - \frac{(b^2(bd-6ag))\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{48a}$$

$$=-\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680}-\frac{27b^2c\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2d\sqrt{a+bx^3}}{24ax^3}-\frac{27b^2e\sqrt{a+bx^3}}{320ax^2}+\frac{27b^2(bc-4af)\sqrt{a+bx^3}}{448a^2x}-\frac{27b^{7/3}(bc-4af)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)\left(a+bx^3\right)^{3/2}}{2520}+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}(bc-4af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|\right.}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{7/3}\left(7a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(bc-4af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\right.}{2240a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{(b(bd-6ag))\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{24a}$$

$$=-\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680}-\frac{27b^2c\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2d\sqrt{a+bx^3}}{24ax^3}-\frac{27b^2e\sqrt{a+bx^3}}{320ax^2}+\frac{27b^2(bc-4af)\sqrt{a+bx^3}}{448a^2x}-\frac{27b^{7/3}(bc-4af)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)\left(a+bx^3\right)^{3/2}}{2520}+\frac{b^2(bd-6ag)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}}+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}(bc-4af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|\right.}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{7/3}\left(7a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(bc-4af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\right.}{2240a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.54 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx =$$

$$-\frac{\sqrt{a+bx^3}\left(84a^5c\,\text{Hypergeometric2F1}\left(-\frac{10}{3},-\frac{3}{2},-\frac{7}{3},-\frac{bx^3}{a}\right)+105a^5ex^2\,\text{Hypergeometric2F1}\left(-\frac{8}{3},-\frac{3}{2},-\frac{5}{3}\right.\right.}{}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11,x]
```

```
[Out] -1/840*(Sqrt[a + b*x^3]*(84*a^5*c*Hypergeometric2F1[-10/3, -3/2, -7/3, -((b
*x^3)/a)] + 105*a^5*e*x^2*Hypergeometric2F1[-8/3, -3/2, -5/3, -((b*x^3)/a)]
 + 2*x^3*(35*a^3*g*x*(a*(2*a + 5*b*x^3)*Sqrt[1 + (b*x^3)/a] + 3*b^2*x^6*Arc
Tanh[Sqrt[1 + (b*x^3)/a]]) + 60*a^5*f*Hypergeometric2F1[-7/3, -3/2, -4/3, -
((b*x^3)/a)] - 56*b^3*d*x^7*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometri
c2F1[5/2, 4, 7/2, 1 + (b*x^3)/a])))/(a^4*x^10*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.28

method	result	size
elliptic	Expression too large to display	976
risch	Expression too large to display	1343
default	Expression too large to display	1470

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*a*c*(b*x^3+a)^(1/2)/x^10-1/9*a*d*(b*x^3+a)^(1/2)/x^9-1/8*a*e*(b*x^3+a
)^(1/2)/x^8-1/7*(a*f+23/20*b*c)*(b*x^3+a)^(1/2)/x^7-1/6*(a*g+7/6*b*d)*(b*x^
3+a)^(1/2)/x^6-19/80*b*e*(b*x^3+a)^(1/2)/x^5-1/1120*b*(340*a*f+27*b*c)/a*(b
*x^3+a)^(1/2)/x^4-1/24*b/a*(10*a*g+b*d)*(b*x^3+a)^(1/2)/x^3-27/320*b^2*e*(b
*x^3+a)^(1/2)/a/x^2-27/448*(4*a*f-b*c)*b^2/a^2*(b*x^3+a)^(1/2)/x+9/320*I/a*
b^2*e*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-9/448*I
```

```
*b^2*(4*a*f-b*c)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2
)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2
)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-1/24*(6*a*g-
b*d)*b^2/a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.23 (sec) , antiderivative size = 559, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx = \left[-\frac{1701\,ab^{\frac{5}{2}}ex^{10}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+210\,(b^3d\,\cdots}{\cdots}\right.$$

$$-\frac{1701\,ab^{\frac{5}{2}}ex^{10}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+420\,(b^3d-6\,ab^2g)\sqrt{-a}x^{10}\arctan\left(\frac{(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{-a}}{2\,(abx^3+a^2)}\right)-12\cdots}{\cdots}$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="fric
as")
```

```
[Out] [-1/20160*(1701*a*b^(5/2)*e*x^10*weierstrassPInverse(0, -4*a/b, x) + 210*(b
^3*d - 6*a*b^2*g)*sqrt(a)*x^10*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*s
qrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 1215*(b^3*c - 4*a*b^2*f)*sqrt(b)*x^1
0*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (1701*a*b
^2*e*x^8 - 1215*(b^3*c - 4*a*b^2*f)*x^9 + 4788*a^2*b*e*x^5 + 840*(a*b^2*d +
 10*a^2*b*g)*x^7 + 18*(27*a*b^2*c + 340*a^2*b*f)*x^6 + 2520*a^3*e*x^2 + 224
0*a^3*d*x + 560*(7*a^2*b*d + 6*a^3*g)*x^4 + 2016*a^3*c + 144*(23*a^2*b*c +
20*a^3*f)*x^3)*sqrt(b*x^3 + a))/(a^2*x^10), -1/20160*(1701*a*b^(5/2)*e*x^10
*weierstrassPInverse(0, -4*a/b, x) + 420*(b^3*d - 6*a*b^2*g)*sqrt(-a)*x^10*
arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) - 1215*(
b^3*c - 4*a*b^2*f)*sqrt(b)*x^10*weierstrassZeta(0, -4*a/b, weierstrassPInve
rse(0, -4*a/b, x)) + (1701*a*b^2*e*x^8 - 1215*(b^3*c - 4*a*b^2*f)*x^9 + 478
8*a^2*b*e*x^5 + 840*(a*b^2*d + 10*a^2*b*g)*x^7 + 18*(27*a*b^2*c + 340*a^2*b
*f)*x^6 + 2520*a^3*e*x^2 + 2240*a^3*d*x + 560*(7*a^2*b*d + 6*a^3*g)*x^4 + 2
016*a^3*c + 144*(23*a^2*b*c + 20*a^3*f)*x^3)*sqrt(b*x^3 + a))/(a^2*x^10)]
```

Sympy [A] (verification not implemented)

Time = 14.17 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{11}}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{10}{3},-\frac{1}{2}\\-\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{8}\Gamma\left(-\frac{5}{3}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{5}\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}bf\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{4}\Gamma\left(-\frac{1}{3}\right)} - \frac{a^2d}{9\sqrt{b}x^{\frac{21}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a^2g}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{11a\sqrt{b}d}{36x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}g}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{17b^{\frac{3}{2}}d}{72x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{3}{2}}g\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}}g}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{5}{2}}d}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^2g\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}} + \frac{b^3d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{24a^{\frac{3}{2}}}$$

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**11,x)
```

```
[Out] a**(3/2)*c*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*x**10*gamma(-7/3)) + a**(3/2)*e*gamma(-8/3)*hyper((-8/3, -1/2), (-5
/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + a**(3/2)*f*gamma(-7/
3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/
3)) + sqrt(a)*b*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar
(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*b*e*gamma(-5/3)*hyper((-5/3, -1/2)
, (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*f*gam
ma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gam
ma(-1/3)) - a**2*d/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - a**2*g/(6*s
qrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*d/(36*x**(15/2)*sqrt(
a/(b*x**3) + 1)) - a*sqrt(b)*g/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - 17*b**(3
/2)*d/(72*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*g*sqrt(a/(b*x**3) + 1)/
(3*x**(3/2)) - b**(3/2)*g/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**(5/2)*d/(
24*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**2*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2
)))/(4*sqrt(a)) + b**3*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(24*a**(3/2))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^{11}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="maxi
ma")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^11, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^{11}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="giac
")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx = \int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^{11}}\,dx$$

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11,x)
```

```
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11, x)
```

3.473 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 796

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx = -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480}-\frac{27b^2c\sqrt{a+bx^3}}{1760ax^5}-\frac{27b^2d\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2e\sqrt{a+bx^3}}{24ax^3}+\frac{27b^2(7bc-22af)\sqrt{a+bx^3}}{7040a^2x^2}+\frac{27b^2(bd-4ag)\sqrt{a+bx^3}}{448a^2x}-\frac{27b^{7/3}(bd-4ag)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)(a+bx^3)^{3/2}}{27720}+\frac{b^3e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}}+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}(bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{\ldots}}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}\,b^{7/3}\left(7\sqrt[3]{b}(7bc-22af)+110\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\ldots}{49280a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/27720*(2520*c/x^11+2772*d/x^10+3080*e/x^9+3465*f/x^8+3960*g/x^7)*(b*x^3+
a)^(3/2)+1/24*b^3*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/18480*b*(945
```

```
*c/x^8+1188*d/x^7+1540*e/x^6+2079*f/x^5+2970*g/x^4)*(b*x^3+a)^(1/2)-27/1760
*b^2*c*(b*x^3+a)^(1/2)/a/x^5-27/1120*b^2*d*(b*x^3+a)^(1/2)/a/x^4-1/24*b^2*e
*(b*x^3+a)^(1/2)/a/x^3+27/7040*b^2*(-22*a*f+7*b*c)*(b*x^3+a)^(1/2)/a^2/x^2+
27/448*b^2*(-4*a*g+b*d)*(b*x^3+a)^(1/2)/a^2/x-27/448*b^(7/3)*(-4*a*g+b*d)*(
b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+27/896*3^(1/4)*b^(7/3)*(
-4*a*g+b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)+9/49280*3^(3/4)*b^(7/3)*(a^(1/3)+b^(1/3)*x)*Elliptic
F((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*(7*b^(1/3)*(-22*a*f+7*b*c)+110*a^(1/3)*(-4*a*g+b*d)*(1-3^(1/2)))*(1/2
*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^2/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*
x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 796, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules

used = {14, 1839, 1849, 1846, 272, 65, 214, 1892, 224, 1891}

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx = \frac{e\text{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)b^3}{24a^{3/2}}$$

$$+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(bd-4ag)\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}}$$

$$+\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}(7bc-22af)+110\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\text{Ell}}{49280a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}}$$

$$-\frac{27(bd-4ag)\sqrt{bx^3+a}b^{7/3}}{448a^2\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)}+\frac{27(bd-4ag)\sqrt{bx^3+a}b^2}{448a^2x}$$

$$+\frac{27(7bc-22af)\sqrt{bx^3+a}b^2}{7040a^2x^2}-\frac{e\sqrt{bx^3+a}b^2}{24ax^3}-\frac{27d\sqrt{bx^3+a}b^2}{1120ax^4}$$

$$-\frac{27c\sqrt{bx^3+a}b^2}{1760ax^5}-\frac{\left(\frac{945c}{x^8}+\frac{2970g}{x^4}+\frac{2079f}{x^5}+\frac{1540e}{x^6}+\frac{1188d}{x^7}\right)\sqrt{bx^3+a}b}{18480}$$

$$-\frac{\left(\frac{2520c}{x^{11}}+\frac{3960g}{x^7}+\frac{3465f}{x^8}+\frac{3080e}{x^9}+\frac{2772d}{x^{10}}\right)(bx^3+a)^{3/2}}{27720}$$

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12,x]
```

```
[Out] -1/18480*(b*((945*c)/x^8 + (1188*d)/x^7 + (1540*e)/x^6 + (2079*f)/x^5 + (29
70*g)/x^4)*Sqrt[a + b*x^3]) - (27*b^2*c*Sqrt[a + b*x^3])/(1760*a*x^5) - (27
*b^2*d*Sqrt[a + b*x^3])/(1120*a*x^4) - (b^2*e*Sqrt[a + b*x^3])/(24*a*x^3) +
 (27*b^2*(7*b*c - 22*a*f)*Sqrt[a + b*x^3])/(7040*a^2*x^2) + (27*b^2*(b*d -
4*a*g)*Sqrt[a + b*x^3])/(448*a^2*x) - (27*b^(7/3)*(b*d - 4*a*g)*Sqrt[a + b*
x^3])/(448*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((2520*c)/x^11 + (27
72*d)/x^10 + (3080*e)/x^9 + (3465*f)/x^8 + (3960*g)/x^7)*(a + b*x^3)^(3/2))
/27720 + (b^3*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(3/2)) + (27*3^(1/4
)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3
/4)*Sqrt[2 + Sqrt[3]]*b^(7/3)*(7*b^(1/3)*(7*b*c - 22*a*f) + 110*(1 - Sqrt[3
])*a^(1/3)*(b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
```

```
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(49280*a^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)(a+bx^3)^{3/2}}{27720} - \frac{1}{2}(9b)\int \frac{\sqrt{a+bx^3}\left(-\frac{c}{11}-\frac{dx}{10}-\frac{ex^2}{9}-\frac{fx^3}{8}-\frac{gx^4}{7}\right)}{x^9}\,dx$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)\left(a+bx^3\right)^{3/2}}{27720} + \frac{1}{4}\left(27b^2\right)\int \frac{\frac{c}{88}+\frac{dx}{70}+\frac{ex^2}{54}+\frac{fx^3}{40}+\frac{gx^4}{28}}{x^6\sqrt{a+bx^3}}\,dx$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} - \frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)\left(a+bx^3\right)^{3/2}}{27720} - \frac{\left(27b^2\right)\int \frac{-\frac{ad}{7}-\frac{5aex}{27}+\frac{1}{88}(7bc-22af)x^2-\frac{5}{14}agx^3}{x^5\sqrt{a+bx^3}}\,dx}{40a}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} - \frac{27b^2d\sqrt{a+bx^3}}{1120ax^4} - \frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)\left(a+bx^3\right)^{3/2}}{27720} + \frac{\left(27b^2\right)\int \frac{\frac{40a^2e}{27}-\frac{1}{11}a(7bc-22af)x-\frac{5}{7}a(bd-4ag)x^2}{x^4\sqrt{a+bx^3}}\,dx}{320a^2}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} - \frac{27b^2d\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2e\sqrt{a+bx^3}}{24ax^3} - \frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)\left(a+bx^3\right)^{3/2}}{27720} - \frac{\left(9b^2\right)\int \frac{\frac{6}{11}a^2(7bc-22af)+\frac{30}{7}a^2(bd-4ag)x+\frac{40}{9}a^2bex^2}{x^3\sqrt{a+bx^3}}\,dx}{640a^3}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} - \frac{27b^2d\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2e\sqrt{a+bx^3}}{24ax^3} + \frac{27b^2(7bc-22af)\sqrt{a+bx^3}}{7040a^2x^2} - \frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)\left(a+bx^3\right)^{3/2}}{27720} + \frac{\left(9b^2\right)\int \frac{-\frac{120}{7}a^3(bd-4ag)-\frac{160}{9}a^3bex+\frac{6}{11}a^2b(7bc-22af)x^2}{x^2\sqrt{a+bx^3}}\,dx}{2560a^4}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480}-\frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} -\frac{27b^2d\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2e\sqrt{a+bx^3}}{24ax^3}+\frac{27b^2(7bc-22af)\sqrt{a+bx^3}}{7040a^2x^2} +\frac{27b^2(bd-4ag)\sqrt{a+bx^3}}{448a^2x}-\frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)(a+bx^3)^{3/2}}{27720} -\frac{(9b^2)\int\frac{\frac{320}{9}a^4be-\frac{12}{11}a^3b(7bc-22af)x+\frac{120}{7}a^3b(bd-4ag)x^2}{x\sqrt{a+bx^3}}\,dx}{5120a^5}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480}-\frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} -\frac{27b^2d\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2e\sqrt{a+bx^3}}{24ax^3}+\frac{27b^2(7bc-22af)\sqrt{a+bx^3}}{7040a^2x^2} +\frac{27b^2(bd-4ag)\sqrt{a+bx^3}}{448a^2x}-\frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)(a+bx^3)^{3/2}}{27720} -\frac{(9b^2)\int\frac{-\frac{12}{11}a^3b(7bc-22af)+\frac{120}{7}a^3b(bd-4ag)x}{\sqrt{a+bx^3}}\,dx}{5120a^5}-\frac{(b^3e)\int\frac{1}{x\sqrt{a+bx^3}}\,dx}{16a}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480}-\frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} -\frac{27b^2d\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2e\sqrt{a+bx^3}}{24ax^3}+\frac{27b^2(7bc-22af)\sqrt{a+bx^3}}{7040a^2x^2} +\frac{27b^2(bd-4ag)\sqrt{a+bx^3}}{448a^2x}-\frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)(a+bx^3)^{3/2}}{27720} -\frac{(b^3e)\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{48a}-\frac{\left(27b^{8/3}(bd-4ag)\right)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{896a^2} +\frac{\left(27b^{8/3}\left(7\sqrt[3]{b}(7bc-22af)+110\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{98560a^2}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} - \frac{27b^2d\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2e\sqrt{a+bx^3}}{24ax^3} + \frac{27b^2(7bc-22af)\sqrt{a+bx^3}}{7040a^2x^2} + \frac{27b^2(bd-4ag)\sqrt{a+bx^3}}{448a^2x} - \frac{27b^{7/3}(bd-4ag)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)\left(a+bx^3\right)^{3/2}}{27720} + \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}(bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|\right.}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{7/3}\left(7\sqrt[3]{b}(7bc-22af)+110\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}}}{49280a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\left(b^2e\right)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{24a}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} - \frac{27b^2d\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2e\sqrt{a+bx^3}}{24ax^3} + \frac{27b^2(7bc-22af)\sqrt{a+bx^3}}{7040a^2x^2} + \frac{27b^2(bd-4ag)\sqrt{a+bx^3}}{448a^2x} - \frac{27b^{7/3}(bd-4ag)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)(a+bx^3)^{3/2}}{27720} + \frac{b^3e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}} + \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}(bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{7/3}\left(7\sqrt[3]{b}(7bc-22af)+110\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}}{\left(\left(1+\sqrt{3}\right)\right.}}}{49280a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.43 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.24

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx = \frac{\sqrt{a+bx^3}\left(-840a^5c\,\text{Hypergeometric2F1}\left(-\frac{11}{3},-\frac{3}{2},-\frac{8}{3},-\right.\right.}{}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-840*a^5*c*Hypergeometric2F1[-11/3, -3/2, -8/3, -((b*x^3)
/a)] - 924*a^5*d*x*Hypergeometric2F1[-10/3, -3/2, -7/3, -((b*x^3)/a)] + 11*
x^3*(-105*a^5*f*Hypergeometric2F1[-8/3, -3/2, -5/3, -((b*x^3)/a)] - 120*a^5
*g*x*Hypergeometric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)] + 112*b^3*e*x^8*(a +
 b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[5/2, 4, 7/2, 1 + (b*x^3)/a]
)))/(9240*a^4*x^11*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 1006, normalized size of antiderivative = 1.26

method	result	size
elliptic	Expression too large to display	1006
risch	Expression too large to display	1639
default	Expression too large to display	1773

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/11*a*c*(b*x^3+a)^(1/2)/x^11-1/10*a*d*(b*x^3+a)^(1/2)/x^10-1/9*a*e*(b*x^3
+a)^(1/2)/x^9-1/8*(a*f+25/22*b*c)*(b*x^3+a)^(1/2)/x^8-1/7*(a*g+23/20*b*d)*(
b*x^3+a)^(1/2)/x^7-7/36*b*e*(b*x^3+a)^(1/2)/x^6-1/1760*b*(418*a*f+27*b*c)/a
*(b*x^3+a)^(1/2)/x^5-1/1120*b/a*(340*a*g+27*b*d)*(b*x^3+a)^(1/2)/x^4-1/24*b
^2*e*(b*x^3+a)^(1/2)/a/x^3-27/7040*b^2*(22*a*f-7*b*c)/a^2*(b*x^3+a)^(1/2)/x
^2-27/448*(4*a*g-b*d)*b^2/a^2*(b*x^3+a)^(1/2)/x+9/7040*I*(22*a*f-7*b*c)*b^2
/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-9/448*I*
(4*a*g-b*d)*b^2/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)
^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/24*b^3*e*ar
ctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 606, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx = \left[\frac{4620\sqrt{a}b^3ex^{11}\log\left(\frac{b^2x^6+8\,abx^3+4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+}{}\right.$$

$$-\frac{9240\sqrt{-a}b^3ex^{11}\arctan\left(\frac{(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{-a}}{2\,(abx^3+a^2)}\right)-1701\,(7\,b^3c-22\,ab^2f)\sqrt{b}x^{11}\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right.}{}$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="fric
as")
```

```
[Out] [1/443520*(4620*sqrt(a)*b^3*e*x^11*log((b^2*x^6 + 8*a*b*x^3 + 4*(b*x^3 + 2*
a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 1701*(7*b^3*c - 22*a*b^2*f)*sqrt
(b)*x^11*weierstrassPInverse(0, -4*a/b, x) + 26730*(b^3*d - 4*a*b^2*g)*sqrt
(b)*x^11*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (1
8480*a*b^2*e*x^8 - 26730*(b^3*d - 4*a*b^2*g)*x^10 - 1701*(7*b^3*c - 22*a*b^
2*f)*x^9 + 86240*a^2*b*e*x^5 + 396*(27*a*b^2*d + 340*a^2*b*g)*x^7 + 252*(27
*a*b^2*c + 418*a^2*b*f)*x^6 + 49280*a^3*e*x^2 + 44352*a^3*d*x + 3168*(23*a^
2*b*d + 20*a^3*g)*x^4 + 40320*a^3*c + 2520*(25*a^2*b*c + 22*a^3*f)*x^3)*sqr
t(b*x^3 + a))/(a^2*x^11), -1/443520*(9240*sqrt(-a)*b^3*e*x^11*arctan(1/2*(b
*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) - 1701*(7*b^3*c - 22*
a*b^2*f)*sqrt(b)*x^11*weierstrassPInverse(0, -4*a/b, x) - 26730*(b^3*d - 4*
a*b^2*g)*sqrt(b)*x^11*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*
a/b, x)) + (18480*a*b^2*e*x^8 - 26730*(b^3*d - 4*a*b^2*g)*x^10 - 1701*(7*b^
3*c - 22*a*b^2*f)*x^9 + 86240*a^2*b*e*x^5 + 396*(27*a*b^2*d + 340*a^2*b*g)*
x^7 + 252*(27*a*b^2*c + 418*a^2*b*f)*x^6 + 49280*a^3*e*x^2 + 44352*a^3*d*x
+ 3168*(23*a^2*b*d + 20*a^3*g)*x^4 + 40320*a^3*c + 2520*(25*a^2*b*c + 22*a^
3*f)*x^3)*sqrt(b*x^3 + a))/(a^2*x^11)]
```

Sympy [A] (verification not implemented)

Time = 12.15 (sec) , antiderivative size = 541, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{11}{3}\right){}_2F_1\left(\begin{matrix}-\frac{11}{3},-\frac{1}{2}\\-\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{11}\Gamma\left(-\frac{8}{3}\right)} + \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{10}{3},-\frac{1}{2}\\-\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{8}\Gamma\left(-\frac{5}{3}\right)} + \frac{a^{\frac{3}{2}}g\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{8}\Gamma\left(-\frac{5}{3}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{a}bf\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{5}\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}bg\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{4}\Gamma\left(-\frac{1}{3}\right)} - \frac{a^2e}{9\sqrt{b}x^{\frac{21}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{11a\sqrt{b}e}{36x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{17b^{\frac{3}{2}}e}{72x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{5}{2}}e}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{b^3e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{24a^{\frac{3}{2}}}$$

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**12,x)
```

```
[Out] a**(3/2)*c*gamma(-11/3)*hyper((-11/3, -1/2), (-8/3,), b*x**3*exp_polar(I*pi
)/a)/(3*x**11*gamma(-8/3)) + a**(3/2)*d*gamma(-10/3)*hyper((-10/3, -1/2), (
-7/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**10*gamma(-7/3)) + a**(3/2)*f*gamma(
-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(
-5/3)) + a**(3/2)*g*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*b*c*gamma(-8/3)*hyper((-8/3, -1/
2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + sqrt(a)*b*d*g
amma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*g
amma(-4/3)) + sqrt(a)*b*f*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*g*gamma(-4/3)*hyper((-4/
3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) - a**2*e/
(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*e/(36*x**(15/2)*s
qrt(a/(b*x**3) + 1)) - 17*b**(3/2)*e/(72*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b
```

```
**(5/2)*e/(24*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + b**3*e*asinh(sqrt(a)/(sqrt
(b)*x**(3/2)))/(24*a**(3/2))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^{12}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="maxi
ma")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^12, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^{12}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="giac
")
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^12, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx = \int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^{12}}\,dx$$

```
[In] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12,x)
[Out] int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12, x)
```

3.474 $\int (c+dx+ex^2)(a+bx^3)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 102

$$\int (c+dx+ex^2)(a+bx^3)^p\,dx=\frac{e(a+bx^3)^{1+p}}{3b(1+p)}+\frac{cx(a+bx^3)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{4}{3}+p,\frac{4}{3},-\frac{bx^3}{a}\right)}{a}+\frac{dx^2(a+bx^3)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{5}{3}+p,\frac{5}{3},-\frac{bx^3}{a}\right)}{2a}$$

```
[Out] 1/3*e*(b*x^3+a)^(p+1)/b/(p+1)+c*x*(b*x^3+a)^(p+1)*hypergeom([1, 4/3+p],[4/3
],-b*x^3/a)/a+1/2*d*x^2*(b*x^3+a)^(p+1)*hypergeom([1, 5/3+p],[5/3],-b*x^3/a
)/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1900, 267, 1907, 252, 251, 372, 371}

$$\int (c+dx+ex^2)(a+bx^3)^p\,dx=cx(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},-\frac{bx^3}{a}\right)+\frac{1}{2}dx^2(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{bx^3}{a}\right)+\frac{e(a+bx^3)^{p+1}}{3b(p+1)}$$

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] (e*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x*(a + b*x^3)^p*Hypergeometric2F
1[1/3, -p, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^p + (d*x^2*(a + b*x^3)^p*Hyp
ergeometric2F1[2/3, -p, 5/3, -((b*x^3)/a)])/(2*(1 + (b*x^3)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= e\int x^2\left(a+bx^3\right)^p\,dx+\int (c+dx)\left(a+bx^3\right)^p\,dx\\
&= \frac{e\left(a+bx^3\right)^{1+p}}{3b(1+p)}+\int \left(c\left(a+bx^3\right)^p+dx\left(a+bx^3\right)^p\right)\,dx\\
&= \frac{e\left(a+bx^3\right)^{1+p}}{3b(1+p)}+c\int \left(a+bx^3\right)^p\,dx+d\int x\left(a+bx^3\right)^p\,dx\\
&= \frac{e\left(a+bx^3\right)^{1+p}}{3b(1+p)}+\left(c\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}\right)\int \left(1+\frac{bx^3}{a}\right)^p\,dx\\
&\quad+\left(d\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}\right)\int x\left(1+\frac{bx^3}{a}\right)^p\,dx\\
&= \frac{e\left(a+bx^3\right)^{1+p}}{3b(1+p)}+cx\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}{}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};-\frac{bx^3}{a}\right)\\
&\quad+\frac{1}{2}dx^2\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$
\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx
= \frac{\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(2e\left(a+bx^3\right)\left(1+\frac{bx^3}{a}\right)^p+6bc(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},-\frac{bx^3}{a}\right)+3bd\right.}{6b(1+p)}
$$

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^p,x]
[Out] ((a + b*x^3)^p*(2*e*(a + b*x^3)*(1 + (b*x^3)/a)^p + 6*b*c*(1 + p)*x*Hyperge
ometric2F1[1/3, -p, 4/3, -((b*x^3)/a)] + 3*b*d*(1 + p)*x^2*Hypergeometric2F
1[2/3, -p, 5/3, -((b*x^3)/a)]))/(6*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

Maple [F]

$$\int \left(e\,x^{2}+d x +c \right) \left(b\,x^{3}+a \right)^{p}d x$$

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

```
[Out] int((e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

Fricas [F]

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx=\int \left(ex^2+dx+c\right)\left(bx^3+a\right)^p\,dx$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 28.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx=\frac{a^pcx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},-p\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^pdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},-p\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+e\left(\begin{cases}\frac{a^px^3}{3} & \text{for}\ b=0\\ \frac{\begin{cases}\frac{(a+bx^3)^{p+1}}{p+1} & \text{for}\ p\neq -1\\ \log\left(a+bx^3\right) & \text{otherwise}\end{cases}}{3b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] a**p*c*x*gamma(1/3)*hyper((1/3, -p), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(4/3)) + a**p*d*x**2*gamma(2/3)*hyper((2/3, -p), (5/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(5/3)) + e*Piecewise((a**p*x**3/3, Eq(b, 0)), (Piecewis
e(((a + b*x**3)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**3), True))/(3*b
), True))
```

Maxima [F]

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx=\int \left(ex^2+dx+c\right)\left(bx^3+a\right)^p\,dx$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="maxima")
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)
```

Giac [F]

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx=\int \left(ex^2+dx+c\right)\left(bx^3+a\right)^p\,dx$$

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="giac")
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx=\int {\left(b\,x^3+a\right)}^p\,\left(e\,x^2+d\,x+c\right)\,dx$$

```
[In] int((a + b*x^3)^p*(c + d*x + e*x^2),x)
[Out] int((a + b*x^3)^p*(c + d*x + e*x^2), x)
```

3.475 $\int x(c+dx+ex^2)(a+bx^3)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^p\,dx = \frac{d(a+bx^3)^{1+p}}{3b(1+p)}+\frac{cx^2(a+bx^3)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{5}{3}+p,\frac{5}{3},-\frac{bx^3}{a}\right)}{2a}+\frac{ex^4(a+bx^3)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{7}{3}+p,\frac{7}{3},-\frac{bx^3}{a}\right)}{4a}$$

```
[Out] 1/3*d*(b*x^3+a)^(p+1)/b/(p+1)+1/2*c*x^2*(b*x^3+a)^(p+1)*hypergeom([1, 5/3+p
],[5/3],-b*x^3/a)/a+1/4*e*x^4*(b*x^3+a)^(p+1)*hypergeom([1, 7/3+p],[7/3],-b
*x^3/a)/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1907, 372, 371, 267}

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^p\,dx = \frac{1}{2}cx^2\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{bx^3}{a}\right)+\frac{d(a+bx^3)^{p+1}}{3b(p+1)}+\frac{1}{4}ex^4\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{4}{3},-p,\frac{7}{3},-\frac{bx^3}{a}\right)$$

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] (d*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x^2*(a + b*x^3)^p*Hypergeometric
2F1[2/3, -p, 5/3, -((b*x^3)/a)])/(2*(1 + (b*x^3)/a)^p) + (e*x^4*(a + b*x^3)
^p*Hypergeometric2F1[4/3, -p, 7/3, -((b*x^3)/a)])/(4*(1 + (b*x^3)/a)^p)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(cx(a+bx^3)^p + dx^2(a+bx^3)^p + ex^3(a+bx^3)^p\right)\,dx \\
&= c\int x(a+bx^3)^p\,dx + d\int x^2(a+bx^3)^p\,dx + e\int x^3(a+bx^3)^p\,dx \\
&= \frac{d(a+bx^3)^{1+p}}{3b(1+p)} + \left(c(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\right)\int x\left(1+\frac{bx^3}{a}\right)^p\,dx \\
&\quad + \left(e(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\right)\int x^3\left(1+\frac{bx^3}{a}\right)^p\,dx \\
&= \frac{d(a+bx^3)^{1+p}}{3b(1+p)} + \frac{1}{2}cx^2(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right) \\
&\quad + \frac{1}{4}ex^4(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}{}_2F_1\left(\frac{4}{3},-p;\frac{7}{3};-\frac{bx^3}{a}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^p\,dx$$
$$=\frac{\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(4d(a+bx^3)\left(1+\frac{bx^3}{a}\right)^p+6bc(1+p)x^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{bx^3}{a}\right)+3\right.}{12b(1+p)}$$

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] ((a + b*x^3)^p*(4*d*(a + b*x^3)*(1 + (b*x^3)/a)^p + 6*b*c*(1 + p)*x^2*Hyper
geometric2F1[2/3, -p, 5/3, -((b*x^3)/a)] + 3*b*e*(1 + p)*x^4*Hypergeometric
2F1[4/3, -p, 7/3, -((b*x^3)/a)]))/(12*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

Maple [F]

$$\int x\left(e\,x^2+dx+c\right)\left(b\,x^3+a\right)^p dx$$

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

```
[Out] int(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

Fricas [F]

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\int\left(ex^2+dx+c\right)\left(bx^3+a\right)^p x\,dx$$

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^3 + d*x^2 + c*x)*(b*x^3 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 43.56 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07

$$\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx=\frac{a^pcx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},-p\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{a^pex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},-p\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+d\left(\begin{cases}\frac{a^px^3}{3} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^3)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+bx^3\right) & \text{otherwise}\end{cases}}{3b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] a**p*c*x**2*gamma(2/3)*hyper((2/3, -p), (5/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(5/3)) + a**p*e*x**4*gamma(4/3)*hyper((4/3, -p), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(7/3)) + d*Piecewise((a**p*x**3/3, Eq(b, 0)), (Piece
wise(((a + b*x**3)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**3), True))/(
3*b), True))
```

Maxima [F]

$$\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx=\int\left(ex^2+dx+c\right)\left(bx^3+a\right)^px\,dx$$

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x, x)
```

Giac [F]

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\int \left(ex^2+dx+c\right)\left(bx^3+a\right)^p x\,dx$$

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="giac")
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\int x\,\left(b\,x^3+a\right)^p\,\left(e\,x^2+d\,x+c\right)\,dx$$

```
[In] int(x*(a + b*x^3)^p*(c + d*x + e*x^2),x)
[Out] int(x*(a + b*x^3)^p*(c + d*x + e*x^2), x)
```

3.476 $\int x^2(c+dx+ex^2)\left(a+bx^3\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx = \frac{c(a+bx^3)^{1+p}}{3b(1+p)}+\frac{dx^4(a+bx^3)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{7}{3}+p,\frac{7}{3},-\frac{bx^3}{a}\right)}{4a}+\frac{ex^5(a+bx^3)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{8}{3}+p,\frac{8}{3},-\frac{bx^3}{a}\right)}{5a}$$

```
[Out] 1/3*c*(b*x^3+a)^(p+1)/b/(p+1)+1/4*d*x^4*(b*x^3+a)^(p+1)*hypergeom([1, 7/3+p
],[7/3],-b*x^3/a)/a+1/5*e*x^5*(b*x^3+a)^(p+1)*hypergeom([1, 8/3+p],[8/3],-b
*x^3/a)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1907, 267, 372, 371}

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx = \frac{c(a+bx^3)^{p+1}}{3b(p+1)}+\frac{1}{4}dx^4\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{4}{3},-p,\frac{7}{3},-\frac{bx^3}{a}\right)+\frac{1}{5}ex^5\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{3},-p,\frac{8}{3},-\frac{bx^3}{a}\right)$$

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] (c*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (d*x^4*(a + b*x^3)^p*Hypergeometric
2F1[4/3, -p, 7/3, -((b*x^3)/a)])/(4*(1 + (b*x^3)/a)^p) + (e*x^5*(a + b*x^3)
^p*Hypergeometric2F1[5/3, -p, 8/3, -((b*x^3)/a)])/(5*(1 + (b*x^3)/a)^p)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(cx^2(a+bx^3)^p + dx^3(a+bx^3)^p + ex^4(a+bx^3)^p\right)\,dx \\
&= c\int x^2(a+bx^3)^p\,dx + d\int x^3(a+bx^3)^p\,dx + e\int x^4(a+bx^3)^p\,dx \\
&= \frac{c(a+bx^3)^{1+p}}{3b(1+p)} + \left(d(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\right)\int x^3\left(1+\frac{bx^3}{a}\right)^p\,dx \\
&\quad + \left(e(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^3}{a}\right)^p\,dx
\end{aligned}$$

$$= \frac{c(a+bx^3)^{1+p}}{3b(1+p)} + \frac{1}{4}dx^4(a+bx^3)^p \left(1+\frac{bx^3}{a}\right)^{-p} {}_2F_1\left(\frac{4}{3},-p;\frac{7}{3};-\frac{bx^3}{a}\right) + \frac{1}{5}ex^5(a+bx^3)^p \left(1+\frac{bx^3}{a}\right)^{-p} {}_2F_1\left(\frac{5}{3},-p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

$$\int x^2(c+dx+ex^2)\,(a+bx^3)^p\,dx$$
$$= \frac{(a+bx^3)^p \left(1+\frac{bx^3}{a}\right)^{-p} \left(20c(a+bx^3)\left(1+\frac{bx^3}{a}\right)^p + 15bd(1+p)x^4 \operatorname{Hypergeometric2F1}\left(\frac{4}{3},-p,\frac{7}{3},-\frac{bx^3}{a}\right) + \right.}{60b(1+p)}$$

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] ((a + b*x^3)^p*(20*c*(a + b*x^3)*(1 + (b*x^3)/a)^p + 15*b*d*(1 + p)*x^4*Hyp
ergeometric2F1[4/3, -p, 7/3, -((b*x^3)/a)] + 12*b*e*(1 + p)*x^5*Hypergeomet
ric2F1[5/3, -p, 8/3, -((b*x^3)/a)]))/(60*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

Maple [F]

$$\int x^2\left(e\,x^2+dx+c\right)\left(b\,x^3+a\right)^p dx$$

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

```
[Out] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^2(c+dx+ex^2)\,(a+bx^3)^p\,dx = \int (ex^2+dx+c)(bx^3+a)^p x^2\,dx$$

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^4 + d*x^3 + c*x^2)*(b*x^3 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 62.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\frac{a^p dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},-p\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^p ex^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},-p\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+c\left(\begin{cases}\frac{a^px^3}{3} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^3)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+bx^3\right) & \text{otherwise}\end{cases}}{3b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] a**p*d*x**4*gamma(4/3)*hyper((4/3, -p), (7/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(7/3)) + a**p*e*x**5*gamma(5/3)*hyper((5/3, -p), (8/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(8/3)) + c*Piecewise((a**p*x**3/3, Eq(b, 0)), (Piece
wise(((a + b*x**3)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**3), True))/(
3*b), True))
```

Maxima [F]

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\int\left(ex^2+dx+c\right)\left(bx^3+a\right)^px^2\,dx$$

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] 1/3*(b*x^3 + a)^(p + 1)*c/(b*(p + 1)) + integrate((e*x^4 + d*x^3)*(b*x^3 +
a)^p, x)
```

Giac [F]

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\int\left(ex^2+dx+c\right)\left(bx^3+a\right)^p x^2\,dx$$

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="giac")
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\int x^2\left(b\,x^3+a\right)^p\left(e\,x^2+d\,x+c\right)\,dx$$

```
[In] int(x^2*(a + b*x^3)^p*(c + d*x + e*x^2),x)
[Out] int(x^2*(a + b*x^3)^p*(c + d*x + e*x^2), x)
```

3.477 $\int (c + dx + ex^2 + fx^3) (a + bx^4) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 68

$$\int (c + dx + ex^2 + fx^3) (a + bx^4) \, dx = acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}afx^4 + \frac{1}{5}bcx^5 + \frac{1}{6}bdx^6 + \frac{1}{7}bex^7 + \frac{1}{8}bfx^8$$

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*a*f*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x
^7+1/8*b*f*x^8
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1864}

$$\int (c + dx + ex^2 + fx^3) (a + bx^4) \, dx = acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}afx^4 + \frac{1}{5}bcx^5 + \frac{1}{6}bdx^6 + \frac{1}{7}bex^7 + \frac{1}{8}bfx^8$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6
 + (b*e*x^7)/7 + (b*f*x^8)/8
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ac + adx + aex^2 + afx^3 + bcx^4 + bdx^5 + bex^6 + bfx^7\right) \, dx \\
&= acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}afx^4 + \frac{1}{5}bcx^5 + \frac{1}{6}bdx^6 + \frac{1}{7}bex^7 + \frac{1}{8}bfx^8
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$
\int \left(c + dx + ex^2 + fx^3\right)\left(a + bx^4\right) \, dx = acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}afx^4 + \frac{1}{5}bcx^5 + \frac{1}{6}bdx^6 + \frac{1}{7}bex^7 + \frac{1}{8}bfx^8
$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6
 + (b*e*x^7)/7 + (b*f*x^8)/8
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

method	result	size
gosper	$acx + \frac{1}{2}ad\,x^2 + \frac{1}{3}ae\,x^3 + \frac{1}{4}af\,x^4 + \frac{1}{5}bc\,x^5 + \frac{1}{6}bd\,x^6 + \frac{1}{7}be\,x^7 + \frac{1}{8}bf\,x^8$	55
default	$acx + \frac{1}{2}ad\,x^2 + \frac{1}{3}ae\,x^3 + \frac{1}{4}af\,x^4 + \frac{1}{5}bc\,x^5 + \frac{1}{6}bd\,x^6 + \frac{1}{7}be\,x^7 + \frac{1}{8}bf\,x^8$	55
norman	$acx + \frac{1}{2}ad\,x^2 + \frac{1}{3}ae\,x^3 + \frac{1}{4}af\,x^4 + \frac{1}{5}bc\,x^5 + \frac{1}{6}bd\,x^6 + \frac{1}{7}be\,x^7 + \frac{1}{8}bf\,x^8$	55
risch	$acx + \frac{1}{2}ad\,x^2 + \frac{1}{3}ae\,x^3 + \frac{1}{4}af\,x^4 + \frac{1}{5}bc\,x^5 + \frac{1}{6}bd\,x^6 + \frac{1}{7}be\,x^7 + \frac{1}{8}bf\,x^8$	55
parallelrisch	$acx + \frac{1}{2}ad\,x^2 + \frac{1}{3}ae\,x^3 + \frac{1}{4}af\,x^4 + \frac{1}{5}bc\,x^5 + \frac{1}{6}bd\,x^6 + \frac{1}{7}be\,x^7 + \frac{1}{8}bf\,x^8$	55

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*a*f*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x
^7+1/8*b*f*x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx=\frac{1}{8}\,bfx^8+\frac{1}{7}\,bex^7+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,afx^4+\frac{1}{3}\,aex^3+\frac{1}{2}\,adx^2+acx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*a
*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx=acx+\frac{adx^2}{2}+\frac{aex^3}{3}+\frac{afx^4}{4}+\frac{bcx^5}{5}+\frac{bdx^6}{6}+\frac{bex^7}{7}+\frac{bfx^8}{8}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
```

```
[Out] a*c*x + a*d*x**2/2 + a*e*x**3/3 + a*f*x**4/4 + b*c*x**5/5 + b*d*x**6/6 + b*
e*x**7/7 + b*f*x**8/8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx=\frac{1}{8}\,bfx^8+\frac{1}{7}\,bex^7+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,afx^4+\frac{1}{3}\,aex^3+\frac{1}{2}\,adx^2+acx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*a
*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx=\frac{1}{8}\,bfx^8+\frac{1}{7}\,bex^7+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,afx^4+\frac{1}{3}\,aex^3+\frac{1}{2}\,adx^2+acx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="giac")
[Out] 1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*a
*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx=\frac{b\,f\,x^8}{8}+\frac{b\,e\,x^7}{7}+\frac{b\,d\,x^6}{6}+\frac{b\,c\,x^5}{5}+\frac{a\,f\,x^4}{4}+\frac{a\,e\,x^3}{3}+\frac{a\,d\,x^2}{2}+a\,c\,x$$

```
[In] int((a + b*x^4)*(c + d*x + e*x^2 + f*x^3),x)
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^5)/5 + (a*e*x^3)/3 + (b*d*x^6)/6 + (a*f*x^4)/4
 + (b*e*x^7)/7 + (b*f*x^8)/8
```

3.478 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 73

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx=\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}afx^7+\frac{1}{8}bcx^8+\frac{1}{9}bdx^9+\frac{1}{10}bex^{10}+\frac{1}{11}bfx^{11}$$

```
[Out] 1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*a*f*x^7+1/8*b*c*x^8+1/9*b*d*x^9+1/1
0*b*e*x^10+1/11*b*f*x^11
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1834}

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx=\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}afx^7+\frac{1}{8}bcx^8+\frac{1}{9}bdx^9+\frac{1}{10}bex^{10}+\frac{1}{11}bfx^{11}$$

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d*
x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \int \left(acx^3 + adx^4 + aex^5 + afx^6 + bcx^7 + bdx^8 + bex^9 + bfx^{10}\right)\,dx$$
$$= \frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^3\left(c + dx + ex^2 + fx^3\right)\left(a + bx^4\right)\,dx = \frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11}$$

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d*
x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{1}{4}a\,x^4c+\frac{1}{5}ad\,x^5+\frac{1}{6}ae\,x^6+\frac{1}{7}af\,x^7+\frac{1}{8}bc\,x^8+\frac{1}{9}bd\,x^9+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
default	$\frac{1}{4}a\,x^4c+\frac{1}{5}ad\,x^5+\frac{1}{6}ae\,x^6+\frac{1}{7}af\,x^7+\frac{1}{8}bc\,x^8+\frac{1}{9}bd\,x^9+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
norman	$\frac{1}{4}a\,x^4c+\frac{1}{5}ad\,x^5+\frac{1}{6}ae\,x^6+\frac{1}{7}af\,x^7+\frac{1}{8}bc\,x^8+\frac{1}{9}bd\,x^9+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
risch	$\frac{1}{4}a\,x^4c+\frac{1}{5}ad\,x^5+\frac{1}{6}ae\,x^6+\frac{1}{7}af\,x^7+\frac{1}{8}bc\,x^8+\frac{1}{9}bd\,x^9+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
parallelrisch	$\frac{1}{4}a\,x^4c+\frac{1}{5}ad\,x^5+\frac{1}{6}ae\,x^6+\frac{1}{7}af\,x^7+\frac{1}{8}bc\,x^8+\frac{1}{9}bd\,x^9+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4*a*x^4*c+1/5*a*d*x^5+1/6*a*e*x^6+1/7*a*f*x^7+1/8*b*c*x^8+1/9*b*d*x^9+1/1
0*b*e*x^10+1/11*b*f*x^11
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)\,dx = \frac{1}{11}\,bfx^{11}+\frac{1}{10}\,bex^{10}+\frac{1}{9}\,bdx^9+\frac{1}{8}\,bcx^8+\frac{1}{7}\,afx^7+\frac{1}{6}\,aex^6+\frac{1}{5}\,adx^5+\frac{1}{4}\,acx^4$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="fricas")
[Out] 1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 + 1
/6*a*e*x^6 + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)\,dx = \frac{acx^4}{4}+\frac{adx^5}{5}+\frac{aex^6}{6}+\frac{afx^7}{7}+\frac{bcx^8}{8}+\frac{bdx^9}{9}+\frac{bex^{10}}{10}+\frac{bfx^{11}}{11}$$

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
[Out] a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + a*f*x**7/7 + b*c*x**8/8 + b*d*x**9/9
 + b*e*x**10/10 + b*f*x**11/11
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)\,dx = \frac{1}{11}\,bfx^{11}+\frac{1}{10}\,bex^{10}+\frac{1}{9}\,bdx^9+\frac{1}{8}\,bcx^8+\frac{1}{7}\,afx^7+\frac{1}{6}\,aex^6+\frac{1}{5}\,adx^5+\frac{1}{4}\,acx^4$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="maxima")
[Out] 1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 + 1
/6*a*e*x^6 + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx=\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,bex^{10}+\frac{1}{9}\,bdx^9+\frac{1}{8}\,bcx^8+\frac{1}{7}\,afx^7+\frac{1}{6}\,aex^6+\frac{1}{5}\,adx^5+\frac{1}{4}\,acx^4$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 + 1
/6*a*e*x^6 + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx=\frac{b\,f\,x^{11}}{11}+\frac{b\,e\,x^{10}}{10}+\frac{b\,d\,x^9}{9}+\frac{b\,c\,x^8}{8}+\frac{a\,f\,x^7}{7}+\frac{a\,e\,x^6}{6}+\frac{a\,d\,x^5}{5}+\frac{a\,c\,x^4}{4}$$

```
[In] int(x^3*(a + b*x^4)*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (b*c*x^8)/8 + (a*e*x^6)/6 + (b*d*x^9)/9 + (a*f*
x^7)/7 + (b*e*x^10)/10 + (b*f*x^11)/11
```

3.479 $\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^2 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 109

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^2 \, dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7 + \frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{f(a+bx^4)^3}{12b}$$

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/3*a^2*e*x^3+2/5*a*b*c*x^5+1/3*a*b*d*x^6+2/7*a*b*e*x
^7+1/9*b^2*c*x^9+1/10*b^2*d*x^10+1/11*b^2*e*x^11+1/12*f*(b*x^4+a)^3/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1596, 1671}

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^2 \, dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7 + \frac{f(a+bx^4)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 +
 (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f*(
a + b*x^4)^3)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
```

```
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{f(a+bx^4)^3}{12b}+\int (c+dx+ex^2)\left(a+bx^4\right)^2\,dx\\ &= \frac{f(a+bx^4)^3}{12b}+\int \left(a^2c+a^2dx+a^2ex^2+2abcx^4+2abdx^5+2abex^6+b^2cx^8+b^2dx^9+b^2ex^{10}\right)dx\\ &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{f(a+bx^4)^3}{12b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{4}a^2fx^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{4}abfx^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{12}b^2fx^{12}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*c*x^5)/5 +
 (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x
^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2 f$
default	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2 f$
norman	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2 f$
risch	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2 f$
parallelrisch	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2 f$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*a*b*f*x^8
+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*a*b*c*x^5+1/4*a^2*f*x^4+1/3*a^2*e*x^3+1/2*
a^2*d*x^2+a^2*c*x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx=\frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{a^2fx^4}{4}+\frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{abfx^8}{4}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}+\frac{b^2fx^{12}}{12}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*c*x**5/5 +
 a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*x**9/9 + b**2*d*x**1
0/10 + b**2*e*x**11/11 + b**2*f*x**12/12
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = \frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = \frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx=\frac{f\,a^2\,x^4}{4}+\frac{e\,a^2\,x^3}{3}+\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{f\,a\,b\,x^8}{4}+\frac{2\,e\,a\,b\,x^7}{7}+\frac{d\,a\,b\,x^6}{3}+\frac{2\,c\,a\,b\,x^5}{5}+\frac{f\,b^2\,x^{12}}{12}+\frac{e\,b^2\,x^{11}}{11}+\frac{d\,b^2\,x^{10}}{10}+\frac{c\,b^2\,x^9}{9}$$

```
[In] int((a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^2*d*x^2)/2 + (b^2*c*x^9)/9 + (a^2*e*x^3)/3 + (b^2*d*x^10)/10 + (a^2*f*x^
4)/4 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12 + a^2*c*x + (2*a*b*c*x^5)/5 + (a*b
*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4
```

3.480 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 114

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx=\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}+\frac{1}{15}b^2fx^{15}+\frac{c\left(a+bx^4\right)^3}{12b}$$

```
[Out] 1/5*a^2*d*x^5+1/6*a^2*e*x^6+1/7*a^2*f*x^7+2/9*a*b*d*x^9+1/5*a*b*e*x^10+2/11
*a*b*f*x^11+1/13*b^2*d*x^13+1/14*b^2*e*x^14+1/15*b^2*f*x^15+1/12*c*(b*x^4+a
)^3/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1596, 1864}

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx=\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{c\left(a+bx^4\right)^3}{12b}+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}+\frac{1}{15}b^2fx^{15}$$

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (2*a*b*d*x^9)/9 + (a*b*e*x^
10)/5 + (2*a*b*f*x^11)/11 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^15
)/15 + (c*(a + b*x^4)^3)/(12*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n - 
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c(a+bx^4)^3}{12b} + \int \left(a+bx^4\right)^2 \left(-cx^3 + x^3\left(c+dx+ex^2+fx^3\right)\right)\,dx \\
&= \frac{c(a+bx^4)^3}{12b} + \int \left(a^2dx^4 + a^2ex^5 + a^2fx^6 + 2abdx^8 + 2abex^9 + 2abfx^{10} + b^2dx^{12} + b^2ex^{13} + b^2fx^{14}\right)\,dx \\
&= \frac{1}{5}a^2dx^5 + \frac{1}{6}a^2ex^6 + \frac{1}{7}a^2fx^7 + \frac{2}{9}abdx^9 + \frac{1}{5}abex^{10} + \frac{2}{11}abfx^{11} + \frac{1}{13}b^2dx^{13} + \frac{1}{14}b^2ex^{14} + \frac{1}{15}b^2fx^{15} + \frac{c(a+bx^4)^3}{12b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = \frac{1}{4}a^2cx^4 + \frac{1}{5}a^2dx^5 + \frac{1}{6}a^2ex^6 + \frac{1}{7}a^2fx^7 + \frac{1}{4}abcx^8 + \frac{2}{9}abdx^9 + \frac{1}{5}abex^{10} + \frac{2}{11}abfx^{11} + \frac{1}{12}b^2cx^{12} + \frac{1}{13}b^2dx^{13} + \frac{1}{14}b^2ex^{14} + \frac{1}{15}b^2fx^{15}$$

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (a*b*c*x^8)
/4 + (2*a*b*d*x^9)/9 + (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11 + (b^2*c*x^12)/12
 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^15)/15
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

method	result
gosper	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}a^2d\,x^5+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}abd\,x^9+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c\,x^{1}$
default	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}a^2d\,x^5+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}abd\,x^9+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c\,x^{1}$
norman	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}a^2d\,x^5+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}abd\,x^9+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c\,x^{1}$
risch	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}a^2d\,x^5+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}abd\,x^9+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c\,x^{1}$
parallelrisch	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}a^2d\,x^5+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}abd\,x^9+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c\,x^{1}$

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*c*x^4+1/5*a^2*d*x^5+1/6*a^2*e*x^6+1/7*a^2*f*x^7+1/4*a*b*c*x^8+2/9*a
*b*d*x^9+1/5*a*b*e*x^10+2/11*a*b*f*x^11+1/12*b^2*c*x^12+1/13*b^2*d*x^13+1/1
4*b^2*e*x^14+1/15*b^2*f*x^15
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx=\frac{1}{15}\,b^2fx^{15}+\frac{1}{14}\,b^2ex^{14}+\frac{1}{13}\,b^2dx^{13}+\frac{1}{12}\,b^2cx^{12}+\frac{2}{11}\,abfx^{11}+\frac{1}{5}\,abex^{10}+\frac{2}{9}\,abdx^9+\frac{1}{4}\,abcx^8+\frac{1}{7}\,a^2fx^7+\frac{1}{6}\,a^2ex^6+\frac{1}{5}\,a^2dx^5+\frac{1}{4}\,a^2cx^4$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/15*b^2*f*x^15 + 1/14*b^2*e*x^14 + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/1
1*a*b*f*x^11 + 1/5*a*b*e*x^10 + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f*x
^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

$$\int x^3 \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^2 \, dx = \frac{a^2cx^4}{4} + \frac{a^2dx^5}{5} + \frac{a^2ex^6}{6} + \frac{a^2fx^7}{7} + \frac{abcx^8}{4} + \frac{2abdx^9}{9} + \frac{abex^{10}}{5} + \frac{2abfx^{11}}{11} + \frac{b^2cx^{12}}{12} + \frac{b^2dx^{13}}{13} + \frac{b^2ex^{14}}{14} + \frac{b^2fx^{15}}{15}$$

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + a**2*f*x**7/7 + a*b*c*x**8/
4 + 2*a*b*d*x**9/9 + a*b*e*x**10/5 + 2*a*b*f*x**11/11 + b**2*c*x**12/12 + b
**2*d*x**13/13 + b**2*e*x**14/14 + b**2*f*x**15/15
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int x^3 \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^2 \, dx = \frac{1}{15}\,b^2fx^{15} + \frac{1}{14}\,b^2ex^{14} + \frac{1}{13}\,b^2dx^{13} + \frac{1}{12}\,b^2cx^{12} + \frac{2}{11}\,abfx^{11} + \frac{1}{5}\,abex^{10} + \frac{2}{9}\,abdx^9 + \frac{1}{4}\,abcx^8 + \frac{1}{7}\,a^2fx^7 + \frac{1}{6}\,a^2ex^6 + \frac{1}{5}\,a^2dx^5 + \frac{1}{4}\,a^2cx^4$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^2*f*x^15 + 1/14*b^2*e*x^14 + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/1
1*a*b*f*x^11 + 1/5*a*b*e*x^10 + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f*x
^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int x^3 \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^2 \, dx = \frac{1}{15}\,b^2fx^{15} + \frac{1}{14}\,b^2ex^{14} + \frac{1}{13}\,b^2dx^{13} + \frac{1}{12}\,b^2cx^{12} + \frac{2}{11}\,abfx^{11} + \frac{1}{5}\,abex^{10} + \frac{2}{9}\,abdx^9 + \frac{1}{4}\,abcx^8 + \frac{1}{7}\,a^2fx^7 + \frac{1}{6}\,a^2ex^6 + \frac{1}{5}\,a^2dx^5 + \frac{1}{4}\,a^2cx^4$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/15*b^2*f*x^15 + 1/14*b^2*e*x^14 + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/1
1*a*b*f*x^11 + 1/5*a*b*e*x^10 + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f*x
^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx=\frac{f\,a^2\,x^7}{7}+\frac{e\,a^2\,x^6}{6}+\frac{d\,a^2\,x^5}{5}+\frac{c\,a^2\,x^4}{4}+\frac{2\,f\,a\,b\,x^{11}}{11}+\frac{e\,a\,b\,x^{10}}{5}+\frac{2\,d\,a\,b\,x^9}{9}+\frac{c\,a\,b\,x^8}{4}+\frac{f\,b^2\,x^{15}}{15}+\frac{e\,b^2\,x^{14}}{14}+\frac{d\,b^2\,x^{13}}{13}+\frac{c\,b^2\,x^{12}}{12}$$

```
[In] int(x^3*(a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (b^2*c*x^12)/12 + (a^2*e*x^6)/6 + (b^2*d*x^
13)/13 + (a^2*f*x^7)/7 + (b^2*e*x^14)/14 + (b^2*f*x^15)/15 + (a*b*c*x^8)/4
+ (2*a*b*d*x^9)/9 + (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11
```

3.481 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 151

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx=a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{f(a+bx^4)^4}{16b}$$

```
[Out] a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+3/5*a^2*b*c*x^5+1/2*a^2*b*d*x^6+3/7*a^2
*b*e*x^7+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/11*a*b^2*e*x^11+1/13*b^3*c*x^1
3+1/14*b^3*d*x^14+1/15*b^3*e*x^15+1/16*f*(b*x^4+a)^4/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1596, 1671}

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx=a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f(a+bx^4)^4}{16b}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)
/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e
```

```
*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a + b
*x^4)^4)/(16*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f(a+bx^4)^4}{16b} + \int (c+dx+ex^2)\left(a+bx^4\right)^3\,dx\\
&= \frac{f(a+bx^4)^4}{16b} + \int \left(a^3c+a^3dx+a^3ex^2+3a^2bcx^4+3a^2bdx^5+3a^2bex^6+3ab^2cx^8\right.\\
&\qquad \left.+3ab^2dx^9+3ab^2ex^{10}+b^3cx^{12}+b^3dx^{13}+b^3ex^{14}\right)\,dx\\
&= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9\\
&\quad +\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{f(a+bx^4)^4}{16b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\begin{aligned}
\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx &= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{4}a^3fx^4+\frac{3}{5}a^2bcx^5\\
&\quad +\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}a^2bfx^8+\frac{1}{3}ab^2cx^9\\
&\quad +\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{4}ab^2fx^{12}\\
&\quad +\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{1}{16}b^3fx^{16}
\end{aligned}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*b*c*x^5)/5
 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8)/8 + (a*b^2*c*x^9)/
3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (a*b^2*f*x^12)/4 + (b^3*c*x
^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

method	result
gosper	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x$
default	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x$
norman	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x$
risch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x$
parallelrisch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+1/4*f*a^3*x^4+3/5*a^2*b*c*x^5+1/2*a^2*b
*d*x^6+3/7*a^2*b*e*x^7+3/8*f*a^2*b*x^8+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/
11*a*b^2*e*x^11+1/4*a*b^2*f*x^12+1/13*b^3*c*x^13+1/14*b^3*d*x^14+1/15*b^3*e
*x^15+1/16*b^3*f*x^16
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx=\frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\,b^3ex^{15}+\frac{1}{14}\,b^3dx^{14}+\frac{1}{13}\,b^3cx^{13}+\frac{1}{4}\,ab^2fx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bfx^8+\frac{3}{7}\,a^2bex^7+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3fx^4+\frac{1}{3}\,a^3ex^3+\frac{1}{2}\,a^3dx^2+a^3cx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = a^3cx + \frac{a^3dx^2}{2} + \frac{a^3ex^3}{3} + \frac{a^3fx^4}{4} + \frac{3a^2bcx^5}{5} + \frac{a^2bdx^6}{2} + \frac{3a^2bex^7}{7} + \frac{3a^2bfx^8}{8} + \frac{ab^2cx^9}{3} + \frac{3ab^2dx^{10}}{10} + \frac{3ab^2ex^{11}}{11} + \frac{ab^2fx^{12}}{4} + \frac{b^3cx^{13}}{13} + \frac{b^3dx^{14}}{14} + \frac{b^3ex^{15}}{15} + \frac{b^3fx^{16}}{16}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2*b*c*x**5/
5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x**8/8 + a*b**2*c*x**9
/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + a*b**2*f*x**12/4 + b**3*c*
x**13/13 + b**3*d*x**14/14 + b**3*e*x**15/15 + b**3*f*x**16/16
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = \frac{1}{16}b^3fx^{16} + \frac{1}{15}b^3ex^{15} + \frac{1}{14}b^3dx^{14} + \frac{1}{13}b^3cx^{13} + \frac{1}{4}ab^2fx^{12} + \frac{3}{11}ab^2ex^{11} + \frac{3}{10}ab^2dx^{10} + \frac{1}{3}ab^2cx^9 + \frac{3}{8}a^2bfx^8 + \frac{3}{7}a^2bex^7 + \frac{1}{2}a^2bdx^6 + \frac{3}{5}a^2bcx^5 + \frac{1}{4}a^3fx^4 + \frac{1}{3}a^3ex^3 + \frac{1}{2}a^3dx^2 + a^3cx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx=\frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\,b^3ex^{15}+\frac{1}{14}\,b^3dx^{14}+\frac{1}{13}\,b^3cx^{13}+\frac{1}{4}\,ab^2fx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bfx^8+\frac{3}{7}\,a^2bex^7+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3fx^4+\frac{1}{3}\,a^3ex^3+\frac{1}{2}\,a^3dx^2+a^3cx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx=\frac{f\,a^3\,x^4}{4}+\frac{e\,a^3\,x^3}{3}+\frac{d\,a^3\,x^2}{2}+c\,a^3\,x+\frac{3\,f\,a^2\,b\,x^8}{8}+\frac{3\,e\,a^2\,b\,x^7}{7}+\frac{d\,a^2\,b\,x^6}{2}+\frac{3\,c\,a^2\,b\,x^5}{5}+\frac{f\,a\,b^2\,x^{12}}{4}+\frac{3\,e\,a\,b^2\,x^{11}}{11}+\frac{3\,d\,a\,b^2\,x^{10}}{10}+\frac{c\,a\,b^2\,x^9}{3}+\frac{f\,b^3\,x^{16}}{16}+\frac{e\,b^3\,x^{15}}{15}+\frac{d\,b^3\,x^{14}}{14}+\frac{c\,b^3\,x^{13}}{13}$$

```
[In] int((a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^3*d*x^2)/2 + (b^3*c*x^13)/13 + (a^3*e*x^3)/3 + (b^3*d*x^14)/14 + (a^3*f*
x^4)/4 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16 + a^3*c*x + (3*a^2*b*c*x^5)/5 +
(a*b^2*c*x^9)/3 + (a^2*b*d*x^6)/2 + (3*a*b^2*d*x^10)/10 + (3*a^2*b*e*x^7)/7
 + (3*a*b^2*e*x^11)/11 + (3*a^2*b*f*x^8)/8 + (a*b^2*f*x^12)/4
```

3.482 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx=\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{1}{17}b^3dx^{17}+\frac{1}{18}b^3ex^{18}+\frac{1}{19}b^3fx^{19}+\frac{c(a+bx^4)^4}{16b}$$

```
[Out] 1/5*a^3*d*x^5+1/6*a^3*e*x^6+1/7*a^3*f*x^7+1/3*a^2*b*d*x^9+3/10*a^2*b*e*x^10
+3/11*a^2*b*f*x^11+3/13*a*b^2*d*x^13+3/14*a*b^2*e*x^14+1/5*a*b^2*f*x^15+1/1
7*b^3*d*x^17+1/18*b^3*e*x^18+1/19*b^3*f*x^19+1/16*c*(b*x^4+a)^4/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1596, 1864}

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx=\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{c(a+bx^4)^4}{16b}+\frac{1}{17}b^3dx^{17}+\frac{1}{18}b^3ex^{18}+\frac{1}{19}b^3fx^{19}$$

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (a^2*b*d*x^9)/3 + (3*a^2*b*
e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)/1
```

```
4 + (a*b^2*f*x^15)/5 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19
+ (c*(a + b*x^4)^4)/(16*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{c(a+bx^4)^4}{16b} + \int \left(a+bx^4\right)^3 \left(-cx^3 + x^3\left(c+dx+ex^2+fx^3\right)\right)\,dx \\
&= \frac{c(a+bx^4)^4}{16b} + \int \left(a^3dx^4 + a^3ex^5 + a^3fx^6 + 3a^2bdx^8 + 3a^2bex^9 + 3a^2bfx^{10} \right. \\
&\qquad \left. + 3ab^2dx^{12} + 3ab^2ex^{13} + 3ab^2fx^{14} + b^3dx^{16} + b^3ex^{17} + b^3fx^{18}\right)\,dx \\
&= \frac{1}{5}a^3dx^5 + \frac{1}{6}a^3ex^6 + \frac{1}{7}a^3fx^7 + \frac{1}{3}a^2bdx^9 + \frac{3}{10}a^2bex^{10} + \frac{3}{11}a^2bfx^{11} + \frac{3}{13}ab^2dx^{13} \\
&\quad + \frac{3}{14}ab^2ex^{14} + \frac{1}{5}ab^2fx^{15} + \frac{1}{17}b^3dx^{17} + \frac{1}{18}b^3ex^{18} + \frac{1}{19}b^3fx^{19} + \frac{c(a+bx^4)^4}{16b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19

$$
\begin{aligned}
\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx &= \frac{1}{4}a^3cx^4 + \frac{1}{5}a^3dx^5 + \frac{1}{6}a^3ex^6 + \frac{1}{7}a^3fx^7 + \frac{3}{8}a^2bcx^8 \\
&\quad + \frac{1}{3}a^2bdx^9 + \frac{3}{10}a^2bex^{10} + \frac{3}{11}a^2bfx^{11} + \frac{1}{4}ab^2cx^{12} \\
&\quad + \frac{3}{13}ab^2dx^{13} + \frac{3}{14}ab^2ex^{14} + \frac{1}{5}ab^2fx^{15} \\
&\quad + \frac{1}{16}b^3cx^{16} + \frac{1}{17}b^3dx^{17} + \frac{1}{18}b^3ex^{18} + \frac{1}{19}b^3fx^{19}
\end{aligned}
$$

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (3*a^2*b*c*
x^8)/8 + (a^2*b*d*x^9)/3 + (3*a^2*b*e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (a*b
^2*c*x^12)/4 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)/14 + (a*b^2*f*x^15)/5
 + (b^3*c*x^16)/16 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

method	result
gosper	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}a^2bc\,x^8+\frac{1}{3}a^2bd\,x^9+\frac{3}{10}a^2be\,x^{10}+\frac{3}{11}a^2bf\,x^{11}+\frac{1}{4}a$
default	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}a^2bc\,x^8+\frac{1}{3}a^2bd\,x^9+\frac{3}{10}a^2be\,x^{10}+\frac{3}{11}a^2bf\,x^{11}+\frac{1}{4}a$
norman	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}a^2bc\,x^8+\frac{1}{3}a^2bd\,x^9+\frac{3}{10}a^2be\,x^{10}+\frac{3}{11}a^2bf\,x^{11}+\frac{1}{4}a$
risch	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}a^2bc\,x^8+\frac{1}{3}a^2bd\,x^9+\frac{3}{10}a^2be\,x^{10}+\frac{3}{11}a^2bf\,x^{11}+\frac{1}{4}a$
parallelrisch	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}a^2bc\,x^8+\frac{1}{3}a^2bd\,x^9+\frac{3}{10}a^2be\,x^{10}+\frac{3}{11}a^2bf\,x^{11}+\frac{1}{4}a$

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^3*c*x^4+1/5*a^3*d*x^5+1/6*a^3*e*x^6+1/7*a^3*f*x^7+3/8*a^2*b*c*x^8+1/3
*a^2*b*d*x^9+3/10*a^2*b*e*x^10+3/11*a^2*b*f*x^11+1/4*a*b^2*c*x^12+3/13*a*b^
2*d*x^13+3/14*a*b^2*e*x^14+1/5*a*b^2*f*x^15+1/16*b^3*c*x^16+1/17*b^3*d*x^17
+1/18*b^3*e*x^18+1/19*b^3*f*x^19
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx=\frac{1}{19}\,b^3fx^{19}+\frac{1}{18}\,b^3ex^{18}+\frac{1}{17}\,b^3dx^{17}+\frac{1}{16}\,b^3cx^{16}+\frac{1}{5}\,ab^2fx^{15}+\frac{3}{14}\,ab^2ex^{14}+\frac{3}{13}\,ab^2dx^{13}+\frac{1}{4}\,ab^2cx^{12}+\frac{3}{11}\,a^2bfx^{11}+\frac{3}{10}\,a^2bex^{10}+\frac{1}{3}\,a^2bdx^9+\frac{3}{8}\,a^2bcx^8+\frac{1}{7}\,a^3fx^7+\frac{1}{6}\,a^3ex^6+\frac{1}{5}\,a^3dx^5+\frac{1}{4}\,a^3cx^4$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/19*b^3*f*x^19 + 1/18*b^3*e*x^18 + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/5
*a*b^2*f*x^15 + 3/14*a*b^2*e*x^14 + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12 +
3/11*a^2*b*f*x^11 + 3/10*a^2*b*e*x^10 + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^8 +
 1/7*a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.18

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx=\frac{a^3cx^4}{4}+\frac{a^3dx^5}{5}+\frac{a^3ex^6}{6}+\frac{a^3fx^7}{7}+\frac{3a^2bcx^8}{8}+\frac{a^2bdx^9}{3}+\frac{3a^2bex^{10}}{10}+\frac{3a^2bfx^{11}}{11}+\frac{ab^2cx^{12}}{4}+\frac{3ab^2dx^{13}}{13}+\frac{3ab^2ex^{14}}{14}+\frac{ab^2fx^{15}}{5}+\frac{b^3cx^{16}}{16}+\frac{b^3dx^{17}}{17}+\frac{b^3ex^{18}}{18}+\frac{b^3fx^{19}}{19}$$

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + a**3*f*x**7/7 + 3*a**2*b*c*
x**8/8 + a**2*b*d*x**9/3 + 3*a**2*b*e*x**10/10 + 3*a**2*b*f*x**11/11 + a*b*
*2*c*x**12/4 + 3*a*b**2*d*x**13/13 + 3*a*b**2*e*x**14/14 + a*b**2*f*x**15/5
 + b**3*c*x**16/16 + b**3*d*x**17/17 + b**3*e*x**18/18 + b**3*f*x**19/19
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx=\frac{1}{19}b^3fx^{19}+\frac{1}{18}b^3ex^{18}+\frac{1}{17}b^3dx^{17}+\frac{1}{16}b^3cx^{16}+\frac{1}{5}ab^2fx^{15}+\frac{3}{14}ab^2ex^{14}+\frac{3}{13}ab^2dx^{13}+\frac{1}{4}ab^2cx^{12}+\frac{3}{11}a^2bfx^{11}+\frac{3}{10}a^2bex^{10}+\frac{1}{3}a^2bdx^9+\frac{3}{8}a^2bcx^8+\frac{1}{7}a^3fx^7+\frac{1}{6}a^3ex^6+\frac{1}{5}a^3dx^5+\frac{1}{4}a^3cx^4$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/19*b^3*f*x^19 + 1/18*b^3*e*x^18 + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/5
*a*b^2*f*x^15 + 3/14*a*b^2*e*x^14 + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12 +
3/11*a^2*b*f*x^11 + 3/10*a^2*b*e*x^10 + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^8 +
 1/7*a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx = \frac{1}{19}b^3fx^{19}+\frac{1}{18}b^3ex^{18}+\frac{1}{17}b^3dx^{17}+\frac{1}{16}b^3cx^{16}+\frac{1}{5}ab^2fx^{15}+\frac{3}{14}ab^2ex^{14}+\frac{3}{13}ab^2dx^{13}+\frac{1}{4}ab^2cx^{12}+\frac{3}{11}a^2bfx^{11}+\frac{3}{10}a^2bex^{10}+\frac{1}{3}a^2bdx^9+\frac{3}{8}a^2bcx^8+\frac{1}{7}a^3fx^7+\frac{1}{6}a^3ex^6+\frac{1}{5}a^3dx^5+\frac{1}{4}a^3cx^4$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/19*b^3*f*x^19 + 1/18*b^3*e*x^18 + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/5
*a*b^2*f*x^15 + 3/14*a*b^2*e*x^14 + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12 +
3/11*a^2*b*f*x^11 + 3/10*a^2*b*e*x^10 + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^8 +
 1/7*a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx = \frac{f\,a^3\,x^7}{7}+\frac{e\,a^3\,x^6}{6}+\frac{d\,a^3\,x^5}{5}+\frac{c\,a^3\,x^4}{4}+\frac{3\,f\,a^2\,b\,x^{11}}{11}+\frac{3\,e\,a^2\,b\,x^{10}}{10}+\frac{d\,a^2\,b\,x^9}{3}+\frac{3\,c\,a^2\,b\,x^8}{8}+\frac{f\,a\,b^2\,x^{15}}{5}+\frac{3\,e\,a\,b^2\,x^{14}}{14}+\frac{3\,d\,a\,b^2\,x^{13}}{13}+\frac{c\,a\,b^2\,x^{12}}{4}+\frac{f\,b^3\,x^{19}}{19}+\frac{e\,b^3\,x^{18}}{18}+\frac{d\,b^3\,x^{17}}{17}+\frac{c\,b^3\,x^{16}}{16}$$

```
[In] int(x^3*(a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (b^3*c*x^16)/16 + (a^3*e*x^6)/6 + (b^3*d*x^
17)/17 + (a^3*f*x^7)/7 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19 + (3*a^2*b*c*x^8
)/8 + (a*b^2*c*x^12)/4 + (a^2*b*d*x^9)/3 + (3*a*b^2*d*x^13)/13 + (3*a^2*b*e
*x^10)/10 + (3*a*b^2*e*x^14)/14 + (3*a^2*b*f*x^11)/11 + (a*b^2*f*x^15)/5
```

3.483 $\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3dx^{14}+\frac{4}{15}ab^3ex^{15}+\frac{1}{17}b^4cx^{17}+\frac{1}{18}b^4dx^{18}+\frac{1}{19}b^4ex^{19}+\frac{f\left(a+bx^4\right)^5}{20b}$$

```
[Out] a^4*c*x+1/2*a^4*d*x^2+1/3*a^4*e*x^3+4/5*a^3*b*c*x^5+2/3*a^3*b*d*x^6+4/7*a^3
*b*e*x^7+2/3*a^2*b^2*c*x^9+3/5*a^2*b^2*d*x^10+6/11*a^2*b^2*e*x^11+4/13*a*b^
3*c*x^13+2/7*a*b^3*d*x^14+4/15*a*b^3*e*x^15+1/17*b^4*c*x^17+1/18*b^4*d*x^18
+1/19*b^4*e*x^19+1/20*f*(b*x^4+a)^5/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1596, 1671}

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3dx^{14}+\frac{4}{15}ab^3ex^{15}+\frac{f\left(a+bx^4\right)^5}{20b}+\frac{1}{17}b^4cx^{17}+\frac{1}{18}b^4dx^{18}+\frac{1}{19}b^4ex^{19}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (4*a^3*b*c*x^5)/5 + (2*a^3*b*d*x^
6)/3 + (4*a^3*b*e*x^7)/7 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*
a^2*b^2*e*x^11)/11 + (4*a*b^3*c*x^13)/13 + (2*a*b^3*d*x^14)/7 + (4*a*b^3*e*
x^15)/15 + (b^4*c*x^17)/17 + (b^4*d*x^18)/18 + (b^4*e*x^19)/19 + (f*(a + b*
x^4)^5)/(20*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f(a+bx^4)^5}{20b} + \int \left(c+dx+ex^2\right)\left(a+bx^4\right)^4\,dx \\
&= \frac{f(a+bx^4)^5}{20b} + \int \left(a^4c+a^4dx+a^4ex^2+4a^3bcx^4+4a^3bdx^5+4a^3bex^6+6a^2b^2cx^8\right. \\
&\quad +6a^2b^2dx^9+6a^2b^2ex^{10}+4ab^3cx^{12}+4ab^3dx^{13}+4ab^3ex^{14}+b^4cx^{16}+b^4dx^{17} \\
&\quad \left.+b^4ex^{18}\right)dx \\
&= a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10} \\
&\quad +\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3dx^{14}+\frac{4}{15}ab^3ex^{15}+\frac{1}{17}b^4cx^{17}+\frac{1}{18}b^4dx^{18}+\frac{1}{19}b^4ex^{19} \\
&\quad +\frac{f(a+bx^4)^5}{20b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.22

$$\int \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^4 \, dx = a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{1}{4}a^4fx^4+\frac{4}{5}a^3bcx^5 +\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{1}{2}a^3bfx^8+\frac{2}{3}a^2b^2cx^9 +\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{1}{2}a^2b^2fx^{12} +\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3dx^{14}+\frac{4}{15}ab^3ex^{15}+\frac{1}{4}ab^3fx^{16} +\frac{1}{17}b^4cx^{17}+\frac{1}{18}b^4dx^{18}+\frac{1}{19}b^4ex^{19}+\frac{1}{20}b^4fx^{20}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (a^4*f*x^4)/4 + (4*a^3*b*c*x^5)/5
 + (2*a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (a^3*b*f*x^8)/2 + (2*a^2*b^2*c*x
^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a^2*b^2*f*x^12)/2 +
 (4*a*b^3*c*x^13)/13 + (2*a*b^3*d*x^14)/7 + (4*a*b^3*e*x^15)/15 + (a*b^3*f*
x^16)/4 + (b^4*c*x^17)/17 + (b^4*d*x^18)/18 + (b^4*e*x^19)/19 + (b^4*f*x^20
)/20
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.03

method	result
gosper	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+\frac{1}{4}a^4f\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{1}{2}a^3bf\,x^8+\frac{2}{3}a^2b^2c$
default	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+\frac{1}{4}a^4f\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{1}{2}a^3bf\,x^8+\frac{2}{3}a^2b^2c$
norman	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+\frac{1}{4}a^4f\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{1}{2}a^3bf\,x^8+\frac{2}{3}a^2b^2c$
risch	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+\frac{1}{4}a^4f\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{1}{2}a^3bf\,x^8+\frac{2}{3}a^2b^2c$
parallelrisch	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+\frac{1}{4}a^4f\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{1}{2}a^3bf\,x^8+\frac{2}{3}a^2b^2c$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*c*x+1/2*a^4*d*x^2+1/3*a^4*e*x^3+1/4*a^4*f*x^4+4/5*a^3*b*c*x^5+2/3*a^3*b
*d*x^6+4/7*a^3*b*e*x^7+1/2*a^3*b*f*x^8+2/3*a^2*b^2*c*x^9+3/5*a^2*b^2*d*x^10
+6/11*a^2*b^2*e*x^11+1/2*f*a^2*b^2*x^12+4/13*a*b^3*c*x^13+2/7*a*b^3*d*x^14+
4/15*a*b^3*e*x^15+1/4*f*a*b^3*x^16+1/17*b^4*c*x^17+1/18*b^4*d*x^18+1/19*b^4
*e*x^19+1/20*f*b^4*x^20
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx=\frac{1}{20}\,b^4fx^{20}+\frac{1}{19}\,b^4ex^{19}+\frac{1}{18}\,b^4dx^{18}+\frac{1}{17}\,b^4cx^{17}+\frac{1}{4}\,ab^3fx^{16}+\frac{4}{15}\,ab^3ex^{15}+\frac{2}{7}\,ab^3dx^{14}+\frac{4}{13}\,ab^3cx^{13}+\frac{1}{2}\,a^2b^2fx^{12}+\frac{6}{11}\,a^2b^2ex^{11}+\frac{3}{5}\,a^2b^2dx^{10}+\frac{2}{3}\,a^2b^2cx^9+\frac{1}{2}\,a^3bfx^8+\frac{4}{7}\,a^3bex^7+\frac{2}{3}\,a^3bdx^6+\frac{4}{5}\,a^3bcx^5+\frac{1}{4}\,a^4fx^4+\frac{1}{3}\,a^4ex^3+\frac{1}{2}\,a^4dx^2+a^4cx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] 1/20*b^4*f*x^20 + 1/19*b^4*e*x^19 + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/4
*a*b^3*f*x^16 + 4/15*a*b^3*e*x^15 + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13 +
1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*b^2
*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*
x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.25

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx=a^4cx+\frac{a^4dx^2}{2}+\frac{a^4ex^3}{3}+\frac{a^4fx^4}{4}+\frac{4a^3bcx^5}{5}+\frac{2a^3bdx^6}{3}+\frac{4a^3bex^7}{7}+\frac{a^3bfx^8}{2}+\frac{2a^2b^2cx^9}{3}+\frac{3a^2b^2dx^{10}}{5}+\frac{6a^2b^2ex^{11}}{11}+\frac{a^2b^2fx^{12}}{2}+\frac{4ab^3cx^{13}}{13}+\frac{2ab^3dx^{14}}{7}+\frac{4ab^3ex^{15}}{15}+\frac{ab^3fx^{16}}{4}+\frac{b^4cx^{17}}{17}+\frac{b^4dx^{18}}{18}+\frac{b^4ex^{19}}{19}+\frac{b^4fx^{20}}{20}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**4*e*x**3/3 + a**4*f*x**4/4 + 4*a**3*b*c*x**5/
5 + 2*a**3*b*d*x**6/3 + 4*a**3*b*e*x**7/7 + a**3*b*f*x**8/2 + 2*a**2*b**2*c
*x**9/3 + 3*a**2*b**2*d*x**10/5 + 6*a**2*b**2*e*x**11/11 + a**2*b**2*f*x**1
2/2 + 4*a*b**3*c*x**13/13 + 2*a*b**3*d*x**14/7 + 4*a*b**3*e*x**15/15 + a*b*
*3*f*x**16/4 + b**4*c*x**17/17 + b**4*d*x**18/18 + b**4*e*x**19/19 + b**4*f
*x**20/20
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{1}{20}\,b^4fx^{20}+\frac{1}{19}\,b^4ex^{19}+\frac{1}{18}\,b^4dx^{18}+\frac{1}{17}\,b^4cx^{17}+\frac{1}{4}\,ab^3fx^{16}+\frac{4}{15}\,ab^3ex^{15}+\frac{2}{7}\,ab^3dx^{14}+\frac{4}{13}\,ab^3cx^{13}+\frac{1}{2}\,a^2b^2fx^{12}+\frac{6}{11}\,a^2b^2ex^{11}+\frac{3}{5}\,a^2b^2dx^{10}+\frac{2}{3}\,a^2b^2cx^9+\frac{1}{2}\,a^3bfx^8+\frac{4}{7}\,a^3bex^7+\frac{2}{3}\,a^3bdx^6+\frac{4}{5}\,a^3bcx^5+\frac{1}{4}\,a^4fx^4+\frac{1}{3}\,a^4ex^3+\frac{1}{2}\,a^4dx^2+a^4cx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/20*b^4*f*x^20 + 1/19*b^4*e*x^19 + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/4
*a*b^3*f*x^16 + 4/15*a*b^3*e*x^15 + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13 +
1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*b^2
*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*
x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{1}{20}\,b^4fx^{20}+\frac{1}{19}\,b^4ex^{19}+\frac{1}{18}\,b^4dx^{18}+\frac{1}{17}\,b^4cx^{17}+\frac{1}{4}\,ab^3fx^{16}+\frac{4}{15}\,ab^3ex^{15}+\frac{2}{7}\,ab^3dx^{14}+\frac{4}{13}\,ab^3cx^{13}+\frac{1}{2}\,a^2b^2fx^{12}+\frac{6}{11}\,a^2b^2ex^{11}+\frac{3}{5}\,a^2b^2dx^{10}+\frac{2}{3}\,a^2b^2cx^9+\frac{1}{2}\,a^3bfx^8+\frac{4}{7}\,a^3bex^7+\frac{2}{3}\,a^3bdx^6+\frac{4}{5}\,a^3bcx^5+\frac{1}{4}\,a^4fx^4+\frac{1}{3}\,a^4ex^3+\frac{1}{2}\,a^4dx^2+a^4cx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/20*b^4*f*x^20 + 1/19*b^4*e*x^19 + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/4
*a*b^3*f*x^16 + 4/15*a*b^3*e*x^15 + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13 +
1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*b^2
*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*
x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx=\frac{f\,a^4\,x^4}{4}+\frac{e\,a^4\,x^3}{3}+\frac{d\,a^4\,x^2}{2}+c\,a^4\,x+\frac{f\,a^3\,b\,x^8}{2}+\frac{4\,e\,a^3\,b\,x^7}{7}+\frac{2\,d\,a^3\,b\,x^6}{3}+\frac{4\,c\,a^3\,b\,x^5}{5}+\frac{f\,a^2\,b^2\,x^{12}}{2}+\frac{6\,e\,a^2\,b^2\,x^{11}}{11}+\frac{3\,d\,a^2\,b^2\,x^{10}}{5}+\frac{2\,c\,a^2\,b^2\,x^9}{3}+\frac{f\,a\,b^3\,x^{16}}{4}+\frac{4\,e\,a\,b^3\,x^{15}}{15}+\frac{2\,d\,a\,b^3\,x^{14}}{7}+\frac{4\,c\,a\,b^3\,x^{13}}{13}+\frac{f\,b^4\,x^{20}}{20}+\frac{e\,b^4\,x^{19}}{19}+\frac{d\,b^4\,x^{18}}{18}+\frac{c\,b^4\,x^{17}}{17}$$

```
[In] int((a + b*x^4)^4*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^4*d*x^2)/2 + (b^4*c*x^17)/17 + (a^4*e*x^3)/3 + (b^4*d*x^18)/18 + (a^4*f*
x^4)/4 + (b^4*e*x^19)/19 + (b^4*f*x^20)/20 + a^4*c*x + (2*a^2*b^2*c*x^9)/3
+ (3*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a^2*b^2*f*x^12)/2 + (4*a^
3*b*c*x^5)/5 + (4*a*b^3*c*x^13)/13 + (2*a^3*b*d*x^6)/3 + (2*a*b^3*d*x^14)/7
 + (4*a^3*b*e*x^7)/7 + (4*a*b^3*e*x^15)/15 + (a^3*b*f*x^8)/2 + (a*b^3*f*x^1
6)/4
```

3.484 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx$

Optimal result

Integrand size = 28, antiderivative size = 198

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx = \frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{4}{17}ab^3dx^{17}+\frac{2}{9}ab^3ex^{18}+\frac{4}{19}ab^3fx^{19}+\frac{1}{21}b^4dx^{21}+\frac{1}{22}b^4ex^{22}+\frac{1}{23}b^4fx^{23}+\frac{c\left(a+bx^4\right)^5}{20b}$$

```
[Out] 1/5*a^4*d*x^5+1/6*a^4*e*x^6+1/7*a^4*f*x^7+4/9*a^3*b*d*x^9+2/5*a^3*b*e*x^10+
4/11*a^3*b*f*x^11+6/13*a^2*b^2*d*x^13+3/7*a^2*b^2*e*x^14+2/5*a^2*b^2*f*x^15
+4/17*a*b^3*d*x^17+2/9*a*b^3*e*x^18+4/19*a*b^3*f*x^19+1/21*b^4*d*x^21+1/22*
b^4*e*x^22+1/23*b^4*f*x^23+1/20*c*(b*x^4+a)^5/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used

$= \{1596, 1864\}$

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx=\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{4}{17}ab^3dx^{17}+\frac{2}{9}ab^3ex^{18}+\frac{4}{19}ab^3fx^{19}+\frac{c(a+bx^4)^5}{20b}+\frac{1}{21}b^4dx^{21}+\frac{1}{22}b^4ex^{22}+\frac{1}{23}b^4fx^{23}$$

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (4*a^3*b*d*x^9)/9 + (2*a^3*
b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^
14)/7 + (2*a^2*b^2*f*x^15)/5 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9 + (
4*a*b^3*f*x^19)/19 + (b^4*d*x^21)/21 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23 +
(c*(a + b*x^4)^5)/(20*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral}=\frac{c(a+bx^4)^5}{20b}+\int\left(a+bx^4\right)^4\left(-cx^3+x^3\left(c+dx+ex^2+fx^3\right)\right)\,dx$$

$$=\frac{c(a+bx^4)^5}{20b}+\int\left(a^4dx^4+a^4ex^5+a^4fx^6+4a^3bdx^8+4a^3bex^9+4a^3bfx^{10}+6a^2b^2dx^{12}+6a^2b^2ex^{13}+6a^2b^2fx^{14}+4ab^3dx^{16}+4ab^3ex^{17}+4ab^3fx^{18}+b^4dx^{20}+b^4ex^{21}+b^4fx^{22}\right)\,dx$$

$$= \frac{1}{5}a^4dx^5 + \frac{1}{6}a^4ex^6 + \frac{1}{7}a^4fx^7 + \frac{4}{9}a^3bdx^9 + \frac{2}{5}a^3bex^{10} + \frac{4}{11}a^3bfx^{11} + \frac{6}{13}a^2b^2dx^{13} + \frac{3}{7}a^2b^2ex^{14} + \frac{2}{5}a^2b^2fx^{15} + \frac{4}{17}ab^3dx^{17} + \frac{2}{9}ab^3ex^{18} + \frac{4}{19}ab^3fx^{19} + \frac{1}{21}b^4dx^{21} + \frac{1}{22}b^4ex^{22} + \frac{1}{23}b^4fx^{23} + \frac{c(a+bx^4)^5}{20b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.22

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{1}{4}a^4cx^4 + \frac{1}{5}a^4dx^5 + \frac{1}{6}a^4ex^6 + \frac{1}{7}a^4fx^7 + \frac{1}{2}a^3bcx^8 + \frac{4}{9}a^3bdx^9 + \frac{2}{5}a^3bex^{10} + \frac{4}{11}a^3bfx^{11} + \frac{1}{2}a^2b^2cx^{12} + \frac{6}{13}a^2b^2dx^{13} + \frac{3}{7}a^2b^2ex^{14} + \frac{2}{5}a^2b^2fx^{15} + \frac{1}{4}ab^3cx^{16} + \frac{4}{17}ab^3dx^{17} + \frac{2}{9}ab^3ex^{18} + \frac{4}{19}ab^3fx^{19} + \frac{1}{20}b^4cx^{20} + \frac{1}{21}b^4dx^{21} + \frac{1}{22}b^4ex^{22} + \frac{1}{23}b^4fx^{23}$$

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] (a^4*c*x^4)/4 + (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (a^3*b*c*x^
8)/2 + (4*a^3*b*d*x^9)/9 + (2*a^3*b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (a^2*
b^2*c*x^12)/2 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^2*f
*x^15)/5 + (a*b^3*c*x^16)/4 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9 + (4
*a*b^3*f*x^19)/19 + (b^4*c*x^20)/20 + (b^4*d*x^21)/21 + (b^4*e*x^22)/22 + (
b^4*f*x^23)/23
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.02

method	result
gosper	$\frac{1}{4}a^4c\,x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}a^3bc\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+\frac{1}{2}a$
default	$\frac{1}{4}a^4c\,x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}a^3bc\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+\frac{1}{2}a$
norman	$\frac{1}{4}a^4c\,x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}a^3bc\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+\frac{1}{2}a$
risch	$\frac{1}{4}a^4c\,x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}a^3bc\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+\frac{1}{2}a$
parallelrisch	$\frac{1}{4}a^4c\,x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}a^3bc\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+\frac{1}{2}a$

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^4*c*x^4+1/5*a^4*d*x^5+1/6*a^4*e*x^6+1/7*a^4*f*x^7+1/2*a^3*b*c*x^8+4/9
*a^3*b*d*x^9+2/5*a^3*b*e*x^10+4/11*a^3*b*f*x^11+1/2*a^2*b^2*c*x^12+6/13*a^2
*b^2*d*x^13+3/7*a^2*b^2*e*x^14+2/5*a^2*b^2*f*x^15+1/4*a*b^3*c*x^16+4/17*a*b
^3*d*x^17+2/9*a*b^3*e*x^18+4/19*a*b^3*f*x^19+1/20*b^4*c*x^20+1/21*b^4*d*x^2
1+1/22*b^4*e*x^22+1/23*b^4*f*x^23
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{1}{23}\,b^4fx^{23}+\frac{1}{22}\,b^4ex^{22}+\frac{1}{21}\,b^4dx^{21}+\frac{1}{20}\,b^4cx^{20} + \frac{4}{19}\,ab^3fx^{19}+\frac{2}{9}\,ab^3ex^{18}+\frac{4}{17}\,ab^3dx^{17} + \frac{1}{4}\,ab^3cx^{16}+\frac{2}{5}\,a^2b^2fx^{15}+\frac{3}{7}\,a^2b^2ex^{14} + \frac{6}{13}\,a^2b^2dx^{13}+\frac{1}{2}\,a^2b^2cx^{12}+\frac{4}{11}\,a^3bfx^{11} + \frac{2}{5}\,a^3bex^{10}+\frac{4}{9}\,a^3bdx^9+\frac{1}{2}\,a^3bcx^8 + \frac{1}{7}\,a^4fx^7+\frac{1}{6}\,a^4ex^6+\frac{1}{5}\,a^4dx^5+\frac{1}{4}\,a^4cx^4$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] 1/23*b^4*f*x^23 + 1/22*b^4*e*x^22 + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/1
9*a*b^3*f*x^19 + 2/9*a*b^3*e*x^18 + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16 +
2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*e*x^14 + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*b^2
*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*e*x^10 + 4/9*a^3*b*d*x^9 + 1/2*a^3*
b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*e*x^6 + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.24

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{a^4cx^4}{4}+\frac{a^4dx^5}{5}+\frac{a^4ex^6}{6}+\frac{a^4fx^7}{7}+\frac{a^3bcx^8}{2} + \frac{4a^3bdx^9}{9}+\frac{2a^3bex^{10}}{5}+\frac{4a^3bfx^{11}}{11}+\frac{a^2b^2cx^{12}}{2} + \frac{6a^2b^2dx^{13}}{13}+\frac{3a^2b^2ex^{14}}{7}+\frac{2a^2b^2fx^{15}}{5} + \frac{ab^3cx^{16}}{4}+\frac{4ab^3dx^{17}}{17}+\frac{2ab^3ex^{18}}{9}+\frac{4ab^3fx^{19}}{19} + \frac{b^4cx^{20}}{20}+\frac{b^4dx^{21}}{21}+\frac{b^4ex^{22}}{22}+\frac{b^4fx^{23}}{23}$$

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

```
[Out] a**4*c*x**4/4 + a**4*d*x**5/5 + a**4*e*x**6/6 + a**4*f*x**7/7 + a**3*b*c*x*
*8/2 + 4*a**3*b*d*x**9/9 + 2*a**3*b*e*x**10/5 + 4*a**3*b*f*x**11/11 + a**2*
b**2*c*x**12/2 + 6*a**2*b**2*d*x**13/13 + 3*a**2*b**2*e*x**14/7 + 2*a**2*b*
*2*f*x**15/5 + a*b**3*c*x**16/4 + 4*a*b**3*d*x**17/17 + 2*a*b**3*e*x**18/9
+ 4*a*b**3*f*x**19/19 + b**4*c*x**20/20 + b**4*d*x**21/21 + b**4*e*x**22/22
 + b**4*f*x**23/23
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{1}{23}\,b^4fx^{23} + \frac{1}{22}\,b^4ex^{22} + \frac{1}{21}\,b^4dx^{21} + \frac{1}{20}\,b^4cx^{20} + \frac{4}{19}\,ab^3fx^{19} + \frac{2}{9}\,ab^3ex^{18} + \frac{4}{17}\,ab^3dx^{17} + \frac{1}{4}\,ab^3cx^{16} + \frac{2}{5}\,a^2b^2fx^{15} + \frac{3}{7}\,a^2b^2ex^{14} + \frac{6}{13}\,a^2b^2dx^{13} + \frac{1}{2}\,a^2b^2cx^{12} + \frac{4}{11}\,a^3bfx^{11} + \frac{2}{5}\,a^3bex^{10} + \frac{4}{9}\,a^3bdx^9 + \frac{1}{2}\,a^3bcx^8 + \frac{1}{7}\,a^4fx^7 + \frac{1}{6}\,a^4ex^6 + \frac{1}{5}\,a^4dx^5 + \frac{1}{4}\,a^4cx^4$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/23*b^4*f*x^23 + 1/22*b^4*e*x^22 + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/1
9*a*b^3*f*x^19 + 2/9*a*b^3*e*x^18 + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16 +
2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*e*x^14 + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*b^2
*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*e*x^10 + 4/9*a^3*b*d*x^9 + 1/2*a^3*
b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*e*x^6 + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{1}{23}b^4fx^{23}+\frac{1}{22}b^4ex^{22}+\frac{1}{21}b^4dx^{21}+\frac{1}{20}b^4cx^{20}+\frac{4}{19}ab^3fx^{19}+\frac{2}{9}ab^3ex^{18}+\frac{4}{17}ab^3dx^{17}+\frac{1}{4}ab^3cx^{16}+\frac{2}{5}a^2b^2fx^{15}+\frac{3}{7}a^2b^2ex^{14}+\frac{6}{13}a^2b^2dx^{13}+\frac{1}{2}a^2b^2cx^{12}+\frac{4}{11}a^3bfx^{11}+\frac{2}{5}a^3bex^{10}+\frac{4}{9}a^3bdx^9+\frac{1}{2}a^3bcx^8+\frac{1}{7}a^4fx^7+\frac{1}{6}a^4ex^6+\frac{1}{5}a^4dx^5+\frac{1}{4}a^4cx^4$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/23*b^4*f*x^23 + 1/22*b^4*e*x^22 + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/1
9*a*b^3*f*x^19 + 2/9*a*b^3*e*x^18 + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16 +
2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*e*x^14 + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*b^2
*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*e*x^10 + 4/9*a^3*b*d*x^9 + 1/2*a^3*
b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*e*x^6 + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^4
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{f\,a^4\,x^7}{7}+\frac{e\,a^4\,x^6}{6}+\frac{d\,a^4\,x^5}{5}+\frac{c\,a^4\,x^4}{4}+\frac{4\,f\,a^3\,b\,x^{11}}{11}+\frac{2\,e\,a^3\,b\,x^{10}}{5}+\frac{4\,d\,a^3\,b\,x^9}{9}+\frac{c\,a^3\,b\,x^8}{2}+\frac{2\,f\,a^2\,b^2\,x^{15}}{5}+\frac{3\,e\,a^2\,b^2\,x^{14}}{7}+\frac{6\,d\,a^2\,b^2\,x^{13}}{13}+\frac{c\,a^2\,b^2\,x^{12}}{2}+\frac{4\,f\,a\,b^3\,x^{19}}{19}+\frac{2\,e\,a\,b^3\,x^{18}}{9}+\frac{4\,d\,a\,b^3\,x^{17}}{17}+\frac{c\,a\,b^3\,x^{16}}{4}+\frac{f\,b^4\,x^{23}}{23}+\frac{e\,b^4\,x^{22}}{22}+\frac{d\,b^4\,x^{21}}{21}+\frac{c\,b^4\,x^{20}}{20}$$

```
[In] int(x^3*(a + b*x^4)^4*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^4*c*x^4)/4 + (a^4*d*x^5)/5 + (b^4*c*x^20)/20 + (a^4*e*x^6)/6 + (b^4*d*x^
21)/21 + (a^4*f*x^7)/7 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23 + (a^2*b^2*c*x^1
2)/2 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^2*f*x^15)/5
+ (a^3*b*c*x^8)/2 + (a*b^3*c*x^16)/4 + (4*a^3*b*d*x^9)/9 + (4*a*b^3*d*x^17)
/17 + (2*a^3*b*e*x^10)/5 + (2*a*b^3*e*x^18)/9 + (4*a^3*b*f*x^11)/11 + (4*a*
b^3*f*x^19)/19
```

3.485 $\int \frac{c+dx+ex^2+fx^3}{a-bx^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx = \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

```
[Out] -1/4*f*ln(-b*x^4+a)/b+1/2*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)+1/
2*arctan(b^(1/4)*x/a^(1/4))*(-e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)+1/2*arct
anh(b^(1/4)*x/a^(1/4))*(e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1890, 1181, 211, 214, 1262, 649, 266}

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}c-\sqrt{a}e\right)}{2a^{3/4}b^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2a^{3/4}b^{3/4}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
```

```
 + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^
4])/(4*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c+ex^2}{a-bx^4}+\frac{x(d+fx^2)}{a-bx^4}\right)dx \\
&= \int \frac{c+ex^2}{a-bx^4}\,dx+\int \frac{x(d+fx^2)}{a-bx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}\left(-\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx \\
&\quad +\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx \\
&= \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} \\
&\quad +\frac{1}{2}d\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}f\text{Subst}\left(\int \frac{x}{a-bx^2}\,dx,x,x^2\right) \\
&= \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} \\
&\quad +\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.61

$$\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx &= \frac{\left(\sqrt[4]{a}\sqrt{b}c-a^{3/4}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2ab^{3/4}} \\
&\quad -\frac{\left(\sqrt[4]{a}\sqrt{b}c+\sqrt{a}\sqrt[4]{b}d+a^{3/4}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{4ab^{3/4}} \\
&\quad -\frac{\left(-\sqrt[4]{a}\sqrt{b}c+\sqrt{a}\sqrt[4]{b}d-a^{3/4}e\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{4ab^{3/4}} \\
&\quad +\frac{d\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}
\end{aligned}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x]
[Out] ((a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a*b^(3/4))
 - ((a^(1/4)*Sqrt[b]*c + Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/
```

```
4)*x])/(4*a*b^(3/4)) - ((-(a^(1/4)*Sqrt[b]*c) + Sqrt[a]*b^(1/4)*d - a^(3/4)
*e)*Log[a^(1/4) + b^(1/4)*x])/(4*a*b^(3/4)) + (d*Log[Sqrt[a] + Sqrt[b]*x^2]
)/(4*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.33

method	result
risch	$-\frac{\sum_{\_R=\operatorname{RootOf}(\_Z^4 b-a)}\frac{\left(\_R^3 f+\_R^2 e+\_R d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{f\ln\left(-b\,x^4+a\right)}{4b}$

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
[Out] -1/4/b*sum((_R^3*f+_R^2*e+_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.08 (sec) , antiderivative size = 241149, normalized size of antiderivative = 1813.15

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.31

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx = \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}b} - \frac{\left(\sqrt{b}d+\sqrt{a}f\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}b} - \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 1/4*(sqrt(b)*d - sqrt(a)*f)*log(sqrt(b)*
x^2 + sqrt(a))/(sqrt(a)*b) - 1/4*(sqrt(b)*d + sqrt(a)*f)*log(sqrt(b)*x^2 -
sqrt(a))/(sqrt(a)*b) - 1/4*(sqrt(b)*c + sqrt(a)*e)*log((sqrt(b)*x - sqrt(sq
rt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(b))*sqrt(b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(93) = 186.

Time = 0.28 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.08

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx$$
$$= -\frac{\sqrt{2}\left(b^2c-\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,\left(-ab^3\right)^{\frac{3}{4}}}$$
$$-\frac{\sqrt{2}\left(b^2c+\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,\left(-ab^3\right)^{\frac{3}{4}}}$$
$$-\frac{\sqrt{2}(b^2c-\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,\left(-ab^3\right)^{\frac{3}{4}}}$$
$$+\frac{\sqrt{2}(b^2c-\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,\left(-ab^3\right)^{\frac{3}{4}}}-\frac{f\log\left(|bx^4-a|\right)}{4\,b}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c - sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*
sqrt(2)*(b^2*c + sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*sqrt(
2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/
(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) - 1/4*f*log(abs(b*x^4 - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 1970, normalized size of antiderivative = 14.81

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x)
```

```
[Out] symsum(log(b^2*c^2*e - b^2*c*d^2 - b^2*d^3*x - a*b*e^3 - a*b*c*f^2 - 16*roo
t(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2
*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^
```

2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)^2*a*b^3*c - 4*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)*b^3*c^2*x - b^2*c^2*f*x + 16*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)^2*a*b^3*d*x - 4*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)*a*b^2*e^2*x + 2*a*b*d*e*f - 8*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)*a*b^2*c*f + 8*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)*a*b^2*d*e + a*b*d*f^2*x - a*b*e^2*f*x + 2*b^2*c*d*e*x + 8*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)*a*b^2*d*f*x)*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k), k, 1, 4)

3.486 $\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 162

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx = -\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}+\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{b}d+\sqrt{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt{a}e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}-\frac{c\log\left(a-bx^4\right)}{4b}$$

```
[Out] -d*x/b-1/2*e*x^2/b-1/3*f*x^3/b-1/4*c*ln(-b*x^4+a)/b+1/2*e*arctanh(x^2*b^(1/
2)/a^(1/2))*a^(1/2)/b^(3/2)+1/2*a^(1/4)*arctan(b^(1/4)*x/a^(1/4))*(-f*a^(1/
2)+d*b^(1/2))/b^(7/4)+1/2*a^(1/4)*arctanh(b^(1/4)*x/a^(1/4))*(f*a^(1/2)+d*b
^(1/2))/b^(7/4)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used

$= \{1845, 1266, 788, 649, 214, 266, 1294, 1181, 211\}$

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx = \frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}d-\sqrt{a}f\right)}{2b^{7/4}} + \frac{\sqrt[4]{a}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}f+\sqrt{b}d\right)}{2b^{7/4}} + \frac{\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}} - \frac{c\log\left(a-bx^4\right)}{4b} - \frac{dx}{b} - \frac{ex^2}{2b} - \frac{fx^3}{3b}$$

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x]
```

```
[Out] -((d*x)/b) - (e*x^2)/(2*b) - (f*x^3)/(3*b) + (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*
f)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]
*f)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (Sqrt[a]*e*ArcTanh[(Sqrt[b]
*x^2)/Sqrt[a]])/(2*b^(3/2)) - (c*Log[a - b*x^4])/(4*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{x^3(c+ex^2)}{a-bx^4}+\frac{x^4(d+fx^2)}{a-bx^4}\right) dx \\
&= \int \frac{x^3(c+ex^2)}{a-bx^4}\,dx+\int \frac{x^4(d+fx^2)}{a-bx^4}\,dx \\
&= -\frac{fx^3}{3b}+\frac{1}{2}\text{Subst}\left(\int \frac{x(c+ex)}{a-bx^2}\,dx,x,x^2\right)+\frac{\int \frac{x^2(3af+3bdx^2)}{a-bx^4}\,dx}{3b} \\
&= -\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}+\frac{\int \frac{3abd+3abfx^2}{a-bx^4}\,dx}{3b^2}-\frac{\text{Subst}\left(\int \frac{-ae-bcx}{a-bx^2}\,dx,x,x^2\right)}{2b} \\
&= -\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}+\frac{1}{2}c\text{Subst}\left(\int \frac{x}{a-bx^2}\,dx,x,x^2\right)+\frac{(ae)\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{2b} \\
&\quad -\frac{\left(\sqrt{a}\left(\sqrt{b}d-\sqrt{a}f\right)\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{2b}+\frac{\left(\sqrt{a}\left(\sqrt{b}d+\sqrt{a}f\right)\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx}{2b}
\end{aligned}
$$

$$= -\frac{dx}{b} - \frac{ex^2}{2b} - \frac{fx^3}{3b} + \frac{\sqrt[4]{a}\left(\sqrt{b}d - \sqrt{a}f\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}} + \frac{\sqrt[4]{a}\left(\sqrt{b}d + \sqrt{a}f\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}} + \frac{\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}} - \frac{c\log\left(a-bx^4\right)}{4b}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.36

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx = \frac{-12b^{3/4}dx - 6b^{3/4}ex^2 - 4b^{3/4}fx^3 + 6\left(\sqrt[4]{a}\sqrt{b}d - a^{3/4}f\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - 3\left(\sqrt[4]{a}\sqrt{b}d + \sqrt{a}\sqrt[4]{b}e + a^{3/4}f\right)}{}$$

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x]
```

```
[Out] (-12*b^(3/4)*d*x - 6*b^(3/4)*e*x^2 - 4*b^(3/4)*f*x^3 + 6*(a^(1/4)*Sqrt[b]*d
 - a^(3/4)*f)*ArcTan[(b^(1/4)*x)/a^(1/4)] - 3*(a^(1/4)*Sqrt[b]*d + Sqrt[a]*
b^(1/4)*e + a^(3/4)*f)*Log[a^(1/4) - b^(1/4)*x] + 3*(a^(1/4)*Sqrt[b]*d - Sq
rt[a]*b^(1/4)*e + a^(3/4)*f)*Log[a^(1/4) + b^(1/4)*x] + 3*Sqrt[a]*b^(1/4)*e
*Log[Sqrt[a] + Sqrt[b]*x^2] - 3*b^(3/4)*c*Log[a - b*x^4])/(12*b^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.49

method	result
risch	$-\frac{f\,x^3}{3b}-\frac{e\,x^2}{2b}-\frac{dx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(-\_R^3 bc-\_R^2 af-\_R ae-ad\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$-\frac{\frac{1}{3}f\,x^3+\frac{1}{2}e\,x^2+dx}{b}+\frac{\frac{d\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4}+\frac{ae\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{af\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-c}{b}$

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*f*x^3/b-1/2*e*x^2/b-d*x/b+1/4/b^2*sum((-_R^3*b*c-_R^2*a*f-_R*a*e-a*d)/
_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.17 (sec) , antiderivative size = 220680, normalized size of antiderivative = 1362.22

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.28

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx = -\frac{2\,fx^3+3\,ex^2+6\,dx}{6\,b} + \frac{\frac{2\left(a\sqrt{b}d-a^{\frac{3}{2}}f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(\sqrt{a}bc-a\sqrt{b}e\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(\sqrt{a}bc+a\sqrt{b}e\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(a\sqrt{b}d+a^{\frac{3}{2}}f\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\cdots}}{\sqrt{b}x+\sqrt{\sqrt{a}\cdots}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{4\,b}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/6*(2*f*x^3 + 3*e*x^2 + 6*d*x)/b + 1/4*(2*(a*sqrt(b)*d - a^(3/2)*f)*arcta
n(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
- (sqrt(a)*b*c - a*sqrt(b)*e)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (sqr
t(a)*b*c + a*sqrt(b)*e)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (a*sqrt(b)
*d + a^(3/2)*f)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(s
qrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(118) = 236$.

Time = 0.28 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.01

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$$
$$= -\frac{c\log(|bx^4-a|)}{4\,b} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2e-(-ab^3)^{\frac{1}{4}}\,b^2d-(-ab^3)^{\frac{3}{4}}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2e-(-ab^3)^{\frac{1}{4}}\,b^2d-(-ab^3)^{\frac{3}{4}}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4} + \frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}\,b^2d-(-ab^3)^{\frac{3}{4}}\,f\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,b^4} - \frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}\,b^2d-(-ab^3)^{\frac{3}{4}}\,f\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,b^4} - \frac{2\,b^2fx^3+3\,b^2ex^2+6\,b^2dx}{6\,b^3}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*c*log(abs(b*x^4 - a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*e - (-a*
b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/
b)^(1/4))/(-a/b)^(1/4))/b^4 - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*e - (-a*b
^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b
)^(1/4))/(-a/b)^(1/4))/b^4 + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(
3/4)*f)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4 - 1/8*sqrt(2)*((
-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) +
sqrt(-a/b))/b^4 - 1/6*(2*b^2*f*x^3 + 3*b^2*e*x^2 + 6*b^2*d*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 846, normalized size of antiderivative = 5.22

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$$

$$= \left(\sum_{k=1}^{4} \ln\left(-\frac{a^4 f^3 - 2a^3 b c e f - a^3 b d^2 f + a^3 b d e^2 + a^2 b^2 c^2 d}{b^2}\right.\right.$$

$$-\mathrm{root}\big(256 b^7 z^4+256 b^6 c z^3-64 a b^4 d f z^2-32 a b^4 e^2 z^2+96 b^5 c^2 z^2-32 a b^3 c d f z+16 a^2 b^2 e f^2 z+16 a b^3$$

$$\left.- \frac{x\,(a^3 c f^2 - 2a^3 d e f + a^3 e^3 - b a^2 c^2 e + b a^2 c d^2)}{b}\right) \mathrm{root}\big(256 b^7 z^4 + 256 b^6 c z^3$$

$$- 64 a b^4 d f z^2 - 32 a b^4 e^2 z^2 + 96 b^5 c^2 z^2 - 32 a b^3 c d f z + 16 a^2 b^2 e f^2 z + 16 a b^3 d^2 e z$$

$$- 16 a b^3 c e^2 z + 16 b^4 c^3 z - 4a^2 b d e^2 f + 4a^2 b c e f^2 - 4 a b^2 c^2 d f + 4 a b^2 c d^2 e$$

$$\left. + 2a^2 b d^2 f^2 - 2 a b^2 c^2 e^2 + a^2 b e^4 + b^3 c^4 - a b^2 d^4 - a^3 f^4, z, k\big)\right) - \frac{e\,x^2}{2\,b} - \frac{f\,x^3}{3\,b} - \frac{d\,x}{b}$$

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x)
```

```
[Out] symsum(log(- (a^4*f^3 + a^2*b^2*c^2*d + a^3*b*d*e^2 - a^3*b*d^2*f - 2*a^3*b
*c*e*f)/b^2 - root(256*b^7*z^4 + 256*b^6*c*z^3 - 64*a*b^4*d*f*z^2 - 32*a*b^
4*e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z + 16*a^2*b^2*e*f^2*z + 16*a*b
^3*d^2*e*z - 16*a*b^3*c*e^2*z + 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*
e*f^2 - 4*a*b^2*c^2*d*f + 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 - 2*a*b^2*c^2*e
^2 + a^2*b*e^4 + b^3*c^4 - a*b^2*d^4 - a^3*f^4, z, k)*(root(256*b^7*z^4 + 2
56*b^6*c*z^3 - 64*a*b^4*d*f*z^2 - 32*a*b^4*e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*
b^3*c*d*f*z + 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3*c*e^2*z + 16
*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 - 4*a*b^2*c^2*d*f + 4*a*b^2*
c*d^2*e + 2*a^2*b*d^2*f^2 - 2*a*b^2*c^2*e^2 + a^2*b*e^4 + b^3*c^4 - a*b^2*d
^4 - a^3*f^4, z, k)*(16*a^2*b^2*d - 16*a^2*b^2*e*x) + (8*a^2*b^3*c*d - 8*a^
3*b^2*e*f)/b^2 + (x*(4*a^3*b*f^2 + 4*a^2*b^2*d^2 - 8*a^2*b^2*c*e))/b) - (x*
(a^3*e^3 + a^3*c*f^2 - 2*a^3*d*e*f + a^2*b*c*d^2 - a^2*b*c^2*e))/b)*root(25
6*b^7*z^4 + 256*b^6*c*z^3 - 64*a*b^4*d*f*z^2 - 32*a*b^4*e^2*z^2 + 96*b^5*c^
2*z^2 - 32*a*b^3*c*d*f*z + 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3
*c*e^2*z + 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 - 4*a*b^2*c^2*d
*f + 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 - 2*a*b^2*c^2*e^2 + a^2*b*e^4 + b^3*
c^4 - a*b^2*d^4 - a^3*f^4, z, k), k, 1, 4) - (e*x^2)/(2*b) - (f*x^3)/(3*b)
- (d*x)/b
```

3.487 $\int \frac{c+dx+ex^2+fx^3}{a+bx^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 293

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4} dx = \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{f\log\left(a+bx^4\right)}{4b}$$

```
[Out] 1/4*f*ln(b*x^4+a)/b+1/2*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)-1/8*l
n(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-e*a^(1/2)+c*b^(1/2))/a^
(3/4)/b^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))
*(-e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(-1+b^(1/4)*x*2^(
1/2)/a^(1/4))*(e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(1+b^
(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 211, 266}

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} + \frac{f\log\left(a+bx^4\right)}{4b}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4))
+ ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sq
rt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(3/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c+ex^2}{a+bx^4}+\frac{x(d+fx^2)}{a+bx^4}\right)dx \\
&= \int \frac{c+ex^2}{a+bx^4}\,dx+\int \frac{x(d+fx^2)}{a+bx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{a+bx^2}\,dx,x,x^2\right)+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b} \\
&= \frac{1}{2}d\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b} \\
&\quad+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b}-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{3/4}} \\
&\quad-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{1}{2}f\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx,x,x^2\right) \\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} \\
&\quad+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{f\log\left(a+bx^4\right)}{4b} \\
&\quad+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} \\
&\quad-\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}
\end{aligned}$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{f\log\left(a+bx^4\right)}{4b}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx = \frac{-2\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{2}\sqrt{b}c+2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{2}\sqrt{b}c-2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x]
```

```
[Out] (-2*a^(1/4)*b^(1/4)*(Sqrt[2]*Sqrt[b]*c + 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt
[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*b^(1/4)*(Sqrt[2]
*Sqrt[b]*c - 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*x)/a^(1/4)] - Sqrt[2]*b^(1/4)*(a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(a^(1/4)
*Sqrt[b]*c - a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2] + 2*a*f*Log[a + b*x^4])/(8*a*b)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.14

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{(\_R^3f+\_R^2e+\_Rd+c)\ln(x-\_R)}{\_R^3}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{d\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)}{x^2+\left(\frac{a}{b}\right)}\right.\right.}{}$

[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)

[Out] 1/4/b*sum((_R^3*f+_R^2*e+_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.41 (sec) , antiderivative size = 254687, normalized size of antiderivative = 869.24

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx = \text{Too large to display}$$

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx = \text{Timed out}$$

[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.95

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f+bc-\sqrt{a}\sqrt{b}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f-bc+\sqrt{a}\sqrt{b}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{5}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{3}{4}}e-2\,\sqrt{a}bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{5}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{5}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{3}{4}}e+2\,\sqrt{a}bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{5}{4}}}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(sqrt(2)*a^(3/4)*b^(1/4)*f + b*c - sqrt(a)*sqrt(b)*e)*log(sqrt(
b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/8*sqrt(
2)*(sqrt(2)*a^(3/4)*b^(1/4)*f - b*c + sqrt(a)*sqrt(b)*e)*log(sqrt(b)*x^2 -
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/4*(sqrt(2)*a^(1/
4)*b^(5/4)*c + sqrt(2)*a^(3/4)*b^(3/4)*e - 2*sqrt(a)*b*d)*arctan(1/2*sqrt(2
)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*s
qrt(sqrt(a)*sqrt(b))*b^(5/4)) + 1/4*(sqrt(2)*a^(1/4)*b^(5/4)*c + sqrt(2)*a^
(3/4)*b^(3/4)*e + 2*sqrt(a)*b*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*
a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5
/4))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.98

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx = \frac{f\log\left(|bx^4+a|\right)}{4\,b} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*f*log(abs(b*x^4 + a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3
)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^
(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4
))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*
e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b
^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 1952, normalized size of antiderivative = 6.66

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x)
```

```
[Out] symsum(log(b^2*c*d^2 - b^2*c^2*e + b^2*d^3*x - a*b*e^3 - a*b*c*f^2 - 16*roo
t(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2
*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^
2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4*a^2
```

*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*
c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)^2*a*b^3*c - 4*ro
ot(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^
2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a
^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4*a^
2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2
*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)*b^3*c^2*x + b^2
*c^2*f*x + 16*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2
 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^
2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*
b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d
^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)
^2*a*b^3*d*x + 4*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*
z^2 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2
*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a
^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*
b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z,
 k)*a*b^2*e^2*x + 2*a*b*d*e*f + 8*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3
+ 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2
*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*
a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2
*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*
f^4 + b^3*c^4, z, k)*a*b^2*c*f - 8*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3
 + 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^
2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16
*a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^
2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3
*f^4 + b^3*c^4, z, k)*a*b^2*d*e + a*b*d*f^2*x - a*b*e^2*f*x - 2*b^2*c*d*e*x
 - 8*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3
*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z
 + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2*f
 + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 +
2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)*a*b^2*d*
f*x)*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3
*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z
 + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2*f
 + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 +
2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k), k, 1, 4
)

3.488 $\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 321

$$\begin{aligned}\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx &= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}-\frac{\sqrt{a}e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}\\
&+\frac{\sqrt[4]{a}\left(\sqrt{b}d+\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{7/4}}\\
&-\frac{\sqrt[4]{a}\left(\sqrt{b}d+\sqrt{a}f\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{7/4}}\\
&+\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}\\
&-\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}\\
&+\frac{c\log\left(a+bx^4\right)}{4b}\end{aligned}$$

```
[Out] d*x/b+1/2*e*x^2/b+1/3*f*x^3/b+1/4*c*ln(b*x^4+a)/b-1/2*e*arctan(x^2*b^(1/2)/
a^(1/2))*a^(1/2)/b^(3/2)+1/8*a^(1/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+
x^2*b^(1/2))*(-f*a^(1/2)+d*b^(1/2))/b^(7/4)*2^(1/2)-1/8*a^(1/4)*ln(a^(1/4)*
b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-f*a^(1/2)+d*b^(1/2))/b^(7/4)*2^(1/
2)-1/4*a^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(f*a^(1/2)+d*b^(1/2))/b
^(7/4)*2^(1/2)-1/4*a^(1/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(f*a^(1/2)+d
*b^(1/2))/b^(7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {1845, 1266, 788, 649, 211, 266, 1294, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx = \frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}f+\sqrt{b}d\right)}{2\sqrt{2}b^{7/4}} - \frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}f+\sqrt{b}d\right)}{2\sqrt{2}b^{7/4}} - \frac{\sqrt{a}e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}} + \frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}} - \frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}} + \frac{c\log\left(a+bx^4\right)}{4b} + \frac{dx}{b} + \frac{ex^2}{2b} + \frac{fx^3}{3b}$$

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x]
```

```
[Out] (d*x)/b + (e*x^2)/(2*b) + (f*x^3)/(3*b) - (Sqrt[a]*e*ArcTan[(Sqrt[b]*x^2)/S
qrt[a]])/(2*b^(3/2)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f
)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) + (a^(1/4)*(
Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/(4*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(7/4)) + (c*Log[a + b*
x^4])/(4*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x^3(c+ex^2)}{a+bx^4}+\frac{x^4(d+fx^2)}{a+bx^4}\right) dx \\
&= \int \frac{x^3(c+ex^2)}{a+bx^4}\,dx+\int \frac{x^4(d+fx^2)}{a+bx^4}\,dx \\
&= \frac{fx^3}{3b}+\frac{1}{2}\text{Subst}\left(\int \frac{x(c+ex)}{a+bx^2}\,dx,x,x^2\right)-\frac{\int \frac{x^2(3af-3bdx^2)}{a+bx^4}\,dx}{3b} \\
&= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}+\frac{\int \frac{-3abd-3abfx^2}{a+bx^4}\,dx}{3b^2}+\frac{\text{Subst}\left(\int \frac{-ae+bcx}{a+bx^2}\,dx,x,x^2\right)}{2b} \\
&= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}+\frac{1}{2}c\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx,x,x^2\right)-\frac{(ae)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{2b} \\
&\quad -\frac{\left(\sqrt{a}\left(\sqrt{b}d-\sqrt{a}f\right)\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b^2}-\frac{\left(\sqrt{a}\left(\sqrt{b}d+\sqrt{a}f\right)\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b^2}
\end{aligned}$$

$$=\frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}-\frac{\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}+\frac{c\log\left(a+bx^4\right)}{4b}$$
$$+\frac{\left(\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}b^{7/4}}$$
$$+\frac{\left(\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}b^{7/4}}$$
$$-\frac{\left(\sqrt{a}\left(\sqrt{b}d+\sqrt{a}f\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b^2}$$
$$-\frac{\left(\sqrt{a}\left(\sqrt{b}d+\sqrt{a}f\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b^2}$$

$$=\frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}-\frac{\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}$$
$$+\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}$$
$$-\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}+\frac{c\log\left(a+bx^4\right)}{4b}$$
$$-\frac{\left(\sqrt[4]{a}\left(\sqrt{b}d+\sqrt{a}f\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{7/4}}$$
$$+\frac{\left(\sqrt[4]{a}\left(\sqrt{b}d+\sqrt{a}f\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{7/4}}$$

$$= \frac{dx}{b} + \frac{ex^2}{2b} + \frac{fx^3}{3b} - \frac{\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}} + \frac{\sqrt[4]{a}\left(\sqrt{b}d+\sqrt{a}f\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{7/4}}$$
$$- \frac{\sqrt[4]{a}\left(\sqrt{b}d+\sqrt{a}f\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{7/4}}$$
$$+ \frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}$$
$$- \frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}} + \frac{c\log(a+bx^4)}{4b}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.97

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx$$
$$= \frac{24b^{3/4}dx + 12b^{3/4}ex^2 + 8b^{3/4}fx^3 + 6\sqrt[4]{a}\left(\sqrt{2}\sqrt{b}d + 2\sqrt[4]{a}\sqrt[4]{b}e + \sqrt{2}\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - 6\sqrt[4]{a}\left(\sqrt{\right.}}{}$$

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x]
```

```
[Out] (24*b^(3/4)*d*x + 12*b^(3/4)*e*x^2 + 8*b^(3/4)*f*x^3 + 6*a^(1/4)*(Sqrt[2]*S
qrt[b]*d + 2*a^(1/4)*b^(1/4)*e + Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)] - 6*a^(1/4)*(Sqrt[2]*Sqrt[b]*d - 2*a^(1/4)*b^(1/4)*e + Sqr
t[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(-(a^(1
/4)*Sqrt[b]*d) + a^(3/4)*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2] + 3*Sqrt[2]*(-(a^(1/4)*Sqrt[b]*d) + a^(3/4)*f)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 6*b^(3/4)*c*Log[a + b*x^4])/(24*b^(7/4
))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.23

method	result
risch	$\frac{f\,x^{3}}{3b}+\frac{e\,x^{2}}{2b}+\frac{dx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}\frac{\left(\_R^{3}bc-\_R^{2}af-\_Rae-ad\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b^{2}}$
default	$\frac{\frac{1}{3}f\,x^{3}+\frac{1}{2}e\,x^{2}+dx}{b}+\frac{-\frac{d\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8}-\frac{ae\arctan\left(x^{2}\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}-\frac{af\sqrt{2}\,(\ldots}{\ldots}}{b}$

[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)

[Out] 1/3*f*x^3/b+1/2*e*x^2/b+d*x/b+1/4/b^2*sum((_R^3*b*c-_R^2*a*f-_R*a*e-a*d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.10 (sec) , antiderivative size = 219615, normalized size of antiderivative = 684.16

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx = \text{Too large to display}$$

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx = \text{Timed out}$$

[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.95

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx = \frac{2\,fx^3+3\,ex^2+6\,dx}{6\,b} + \frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}c-abd+a^{\frac{3}{2}}\sqrt{b}f\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}c+abd-a^{\frac{3}{2}}\sqrt{b}f\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} - \frac{2\left(\sqrt{2}a^{\frac{5}{4}}b^{\frac{5}{4}}d \cdots\right.}{\cdots}}{8\,b}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*f*x^3 + 3*e*x^2 + 6*d*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*c
 - a*b*d + a^(3/2)*sqrt(b)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x +
 sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*c + a*b*d -
a^(3/2)*sqrt(b)*f)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(
a^(3/4)*b^(5/4)) - 2*(sqrt(2)*a^(5/4)*b^(5/4)*d + sqrt(2)*a^(7/4)*b^(3/4)*f
 - 2*a^(3/2)*b*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) - 2*(sqrt(
2)*a^(5/4)*b^(5/4)*d + sqrt(2)*a^(7/4)*b^(3/4)*f + 2*a^(3/2)*b*e)*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx &= \frac{c\log\left(|bx^4+a|\right)}{4\,b} + \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2e-(ab^3)^{\frac{1}{4}}b^2d-(ab^3)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4} \\ &+ \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2e-(ab^3)^{\frac{1}{4}}b^2d-(ab^3)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4} \\ &- \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2d-(ab^3)^{\frac{3}{4}}f\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,b^4} \\ &+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2d-(ab^3)^{\frac{3}{4}}f\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,b^4} + \frac{2\,b^2fx^3+3\,b^2ex^2+6\,b^2dx}{6\,b^3}\end{aligned}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*c*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*e - (a*b^3
)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/b^4 + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*e - (a*b^3)^(1/4
)*b^2*d - (a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/b^4 - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*log(x
^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^
2*d - (a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 + 1
/6*(2*b^2*f*x^3 + 3*b^2*e*x^2 + 6*b^2*d*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 838, normalized size of antiderivative = 2.61

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx = \left(\sum_{k=1}^{4} \ln\left(\frac{a^4\,f^3+2\,a^3\,b\,c\,e\,f+a^3\,b\,d^2\,f-a^3\,b\,d\,e^2+a^2\,b^2\,c^2\,d}{b^2}\right.\right.$$
$$+\mathrm{root}\big(256\,b^7\,z^4-256\,b^6\,c\,z^3+64\,a\,b^4\,d\,f\,z^2+32\,a\,b^4\,e^2\,z^2+96\,b^5\,c^2\,z^2-32\,a\,b^3\,c\,d\,f\,z-16\,a^2\,b^2\,e\,f^2\,z+16\,a\,b^3$$
$$\left.-\frac{x\,(a^3\,c\,f^2-2\,a^3\,d\,e\,f+a^3\,e^3+b\,a^2\,c^2\,e-b\,a^2\,c\,d^2)}{b}\right)\mathrm{root}\big(256\,b^7\,z^4-256\,b^6\,c\,z^3$$
$$+64\,a\,b^4\,d\,f\,z^2+32\,a\,b^4\,e^2\,z^2+96\,b^5\,c^2\,z^2-32\,a\,b^3\,c\,d\,f\,z-16\,a^2\,b^2\,e\,f^2\,z+16\,a\,b^3\,d^2\,e\,z$$
$$-16\,a\,b^3\,c\,e^2\,z-16\,b^4\,c^3\,z-4\,a^2\,b\,d\,e^2\,f+4\,a^2\,b\,c\,e\,f^2+4\,a\,b^2\,c^2\,d\,f-4\,a\,b^2\,c\,d^2\,e$$
$$\left.+2\,a^2\,b\,d^2\,f^2+2\,a\,b^2\,c^2\,e^2+a^2\,b\,e^4+a\,b^2\,d^4+a^3\,f^4+b^3\,c^4,z,k\big)\right)+\frac{e\,x^2}{2\,b}+\frac{f\,x^3}{3\,b}+\frac{d\,x}{b}$$

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x)
```

```
[Out] symsum(log((a^4*f^3 + a^2*b^2*c^2*d - a^3*b*d*e^2 + a^3*b*d^2*f + 2*a^3*b*c
*e*f)/b^2 + root(256*b^7*z^4 - 256*b^6*c*z^3 + 64*a*b^4*d*f*z^2 + 32*a*b^4*
e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z - 16*a^2*b^2*e*f^2*z + 16*a*b^3
*d^2*e*z - 16*a*b^3*c*e^2*z - 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*
f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2
 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)*(root(256*b^7*z^4 - 256
*b^6*c*z^3 + 64*a*b^4*d*f*z^2 + 32*a*b^4*e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*b^
3*c*d*f*z - 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3*c*e^2*z - 16*b
^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*
d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4
 + b^3*c^4, z, k)*(16*a^2*b^2*d - 16*a^2*b^2*e*x) - (8*a^2*b^3*c*d + 8*a^3*
b^2*e*f)/b^2 + (x*(4*a^3*b*f^2 - 4*a^2*b^2*d^2 + 8*a^2*b^2*c*e))/b) - (x*(a
^3*e^3 + a^3*c*f^2 - 2*a^3*d*e*f - a^2*b*c*d^2 + a^2*b*c^2*e))/b)*root(256*
b^7*z^4 - 256*b^6*c*z^3 + 64*a*b^4*d*f*z^2 + 32*a*b^4*e^2*z^2 + 96*b^5*c^2*
z^2 - 32*a*b^3*c*d*f*z - 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3*c
*e^2*z - 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f
```

```
 - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*
d^4 + a^3*f^4 + b^3*c^4, z, k), k, 1, 4) + (e*x^2)/(2*b) + (f*x^3)/(3*b) +
(d*x)/b
```

3.489 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 318

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2} dx = -\frac{af-bx(c+dx+ex^2)}{4ab(a+bx^4)} + \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{(3\sqrt{b}c+\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} + \frac{(3\sqrt{b}c+\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} - \frac{(3\sqrt{b}c-\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{(3\sqrt{b}c-\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}$$

```
[Out] 1/4*(-a*f+b*x*(e*x^2+d*x+c))/a/b/(b*x^4+a)+1/4*d*arctan(x^2*b^(1/2)/a^(1/2)
)/a^(3/2)/b^(1/2)-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(
-e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(3/4)*2^(1/2)+1/32*ln(a^(1/4)*b^(1/4)*x*2
^(1/2)+a^(1/2)+x^2*b^(1/2))*(-e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(3/4)*2^(1/2
)+1/16*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+3*c*b^(1/2))/a^(7/4)
/b^(3/4)*2^(1/2)+1/16*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+3*c*b^
(1/2))/a^(7/4)/b^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{8\sqrt{2}a^{7/4}b^{3/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{8\sqrt{2}a^{7/4}b^{3/4}} + \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} - \frac{af-bx(c+dx+ex^2)}{4ab\,(a+bx^4)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]
```

```
[Out] -1/4*(a*f - b*x*(c + d*x + e*x^2))/(a*b*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x
^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c +
Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3
/4)) - ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
 Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b
^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{af-bx(c+dx+ex^2)}{4ab\,(a+bx^4)}-\frac{\int\frac{-3c-2dx-ex^2}{a+bx^4}\,dx}{4a}\\
&= -\frac{af-bx(c+dx+ex^2)}{4ab\,(a+bx^4)}-\frac{\int\left(-\frac{2dx}{a+bx^4}+\frac{-3c-ex^2}{a+bx^4}\right)dx}{4a}\\
&= -\frac{af-bx(c+dx+ex^2)}{4ab\,(a+bx^4)}-\frac{\int\frac{-3c-ex^2}{a+bx^4}\,dx}{4a}+\frac{d\int\frac{x}{a+bx^4}\,dx}{2a}\\
&= -\frac{af-bx(c+dx+ex^2)}{4ab\,(a+bx^4)}+\frac{d\text{Subst}\left(\int\frac{1}{a+bx^2}\,dx,x,x^2\right)}{4a}\\
&\quad+\frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}-e\right)\int\frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{8ab}+\frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}+e\right)\int\frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{8ab}\\
&= -\frac{af-bx(c+dx+ex^2)}{4ab\,(a+bx^4)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}\\
&\quad+\frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}+e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16ab}+\frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}+e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16ab}\\
&\quad-\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}b^{3/4}}-\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}b^{3/4}}
\end{aligned}$$

$$= -\frac{af-bx(c+dx+ex^2)}{4ab\,(a+bx^4)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} - \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}}$$

$$= -\frac{af-bx(c+dx+ex^2)}{4ab\,(a+bx^4)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} - \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.99

$$\int\frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx$$

$$= \frac{-\frac{8a(af-bx(c+x(d+ex)))}{a+bx^4}-2\sqrt[4]{a}\sqrt[4]{b}\left(3\sqrt{2}\sqrt{b}c+4\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\sqrt[4]{b}\left(3\sqrt{2}\sqrt{b}c\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a*(a*f - b*x*(c + x*(d + e*x))))/(a + b*x^4) - 2*a^(1/4)*b^(1/4)*(3*Sq
rt[2]*Sqrt[b]*c + 4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*b^(1/4)*(3*Sqrt[2]*Sqrt[b]*c - 4*a^(1/4
)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] +
Sqrt[2]*b^(1/4)*(-3*a^(1/4)*Sqrt[b]*c + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(3*a^(1/4)*Sqrt[b]*c - a^(
3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^2*b)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.27

method	result
risch	$\frac{\frac{e x^3}{4a}+\frac{d x^2}{4a}+\frac{cx}{4a}-\frac{f}{4b}}{b x^4+a}+\frac{\sum_{\_R=\operatorname{RootOf}(\_Z^4 b+a)} \frac{(\_R^2 e+2\_R d+3c)\ln(x-\_R)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(b x^4+a)}+\frac{3(\frac{a}{b})^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+(\frac{a}{b})^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-(\frac{a}{b})^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(\frac{a}{b})^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(\frac{a}{b})^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+d\left(\frac{x^2}{4a(b x^4+a)}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
[Out] (1/4/a*e*x^3+1/4*d/a*x^2+1/4*c/a*x-1/4*f/b)/(b*x^4+a)+1/16/b/a*sum((_R^2*e+
2*_R*d+3*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.02 (sec) , antiderivative size = 124301, normalized size of antiderivative = 390.88

$$\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 43.91 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.63

$$\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^2}\,dx$$

$$= \operatorname{RootSum}\Big(65536t^4a^7b^3+t^2\cdot\left(3072a^4b^2ce+2048a^4b^2d^2\right)+t\left(128a^3bde^2-1152a^2b^2c^2d\right)+a^2e^4+18abc^2$$

$$+\frac{-af+bcx+bdx^2+bex^3}{4a^2b+4ab^2x^4}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*a**4*b**2*d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d) + a**2*e**4 + 18*a*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 + 81*b**2*c**4, Lambda(_t, _t*log(x + (4096*_t**3*a**7*b**2*e**3 - 36864*_t**3*a**6*b**3*c**2*e + 98304*_t**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2*d**3*e + 13824*_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t*a**4*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e + 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 + 120*a**2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c**3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4 + 96*a**2*b*c*d**2*e**3 - 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**2*d**4 + 729*b**3*c**6)))) + (-a*f + b*c*x + b*d*x**2 + b*e*x**3)/(4*a**2*b + 4*a*b**2*x**4)

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx=\frac{bex^3+bdx^2+bcx-af}{4\,(ab^2x^4+a^2b)}$$

$$+\frac{\frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-4\sqrt{a}\sqrt{b}d\right)\text{ar}}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b}}{32\,a}$$

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")

[Out] 1/4*(b*e*x^3 + b*d*x^2 + b*c*x - a*f)/(a*b^2*x^4 + a^2*b) + 1/32*(sqrt(2)*(3*sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(3*sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e - 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e + 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/a

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.98

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx$$
$$= \frac{bex^3+bdx^2+bcx-af}{4\,(bx^4+a)ab}$$
$$+ \frac{\sqrt{2}\Big(2\,\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\Big(2\,\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$
$$- \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*e*x^3 + b*d*x^2 + b*c*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(2*s
qrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)
*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arct
an(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sq
rt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)
^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.50

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx$$

$$= \left(\sum_{k=1}^{4} \ln\left(-\operatorname{root}\left(65536\,a^7\,b^3\,z^4+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\right.\right.\right.$$
$$\left. - \frac{9\,b^2\,c^2\,e-12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3} + \frac{x\,(2\,b^2\,d^3-3\,b^2\,c\,d\,e)}{16\,a^3}\right)\operatorname{root}\left(65536\,a^7\,b^3\,z^4\right.$$
$$+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e$$
$$\left.\left.+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4+81\,b^2\,c^4+a^2\,e^4,z,k\right)\right) + \frac{\frac{d\,x^2}{4\,a}-\frac{f}{4\,b}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{b\,x^4+a}$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x)
```

```
[Out] symsum(log((x*(2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3) - (9*b^2*c^2*e - 12*b^2*c
*d^2 + a*b*e^3)/(64*a^3) - root(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 +
2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*
d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k)*(root(655
36*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2
*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4
 + 81*b^2*c^4 + a^2*e^4, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 -
4*a^2*b^2*e^2))/(16*a^3) + (b^2*d*e)/a))*root(65536*a^7*b^3*z^4 + 3072*a^4*
b^2*c*e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2
*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z
, k), k, 1, 4) + ((d*x^2)/(4*a) - f/(4*b) + (e*x^3)/(4*a) + (c*x)/(4*a))/(a
 + b*x^4)
```

3.490 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 310

$$\begin{aligned}\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = & -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)}+\frac{e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{a}b^{3/2}} \\ & -\frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{7/4}} \\ & +\frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{7/4}} \\ & -\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}} \\ & +\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}}\end{aligned}$$

```
[Out] 1/4*(-f*x^3-e*x^2-d*x-c)/b/(b*x^4+a)+1/4*e*arctan(x^2*b^(1/2)/a^(1/2))/b^(3
/2)/a^(1/2)-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-3*f*a
^(1/2)+d*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)+1/32*ln(a^(1/4)*b^(1/4)*x*2^(1/2)
+a^(1/2)+x^2*b^(1/2))*(-3*f*a^(1/2)+d*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)+1/16
*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(3*f*a^(1/2)+d*b^(1/2))/a^(3/4)/b^(7/
4)*2^(1/2)+1/16*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(3*f*a^(1/2)+d*b^(1/2))
/a^(3/4)/b^(7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1837, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{a}f+\sqrt{b}d\right)}{8\sqrt{2}a^{3/4}b^{7/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{a}f+\sqrt{b}d\right)}{8\sqrt{2}a^{3/4}b^{7/4}} - \frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}} + \frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}} + \frac{e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{a}b^{3/2}} - \frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}$$

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]
```

```
[Out] -1/4*(c + d*x + e*x^2 + f*x^3)/(b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sq
rt[a]])/(4*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*d + 3*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*d + 3*Sqrt[a
]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*b^(7/4)) -
 ((Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/(16*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(3/4)*b^(7/4)
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)}+\frac{\int \frac{d+2ex+3fx^2}{a+bx^4}\,dx}{4b}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)}+\frac{\int \left(\frac{2ex}{a+bx^4}+\frac{d+3fx^2}{a+bx^4}\right)dx}{4b}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)}+\frac{\int \frac{d+3fx^2}{a+bx^4}\,dx}{4b}+\frac{e\int \frac{x}{a+bx^4}\,dx}{2b}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)}+\frac{e\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{4b}+\frac{\left(\frac{\sqrt{b}d}{\sqrt{a}}-3f\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{8b^2}+\frac{\left(\frac{\sqrt{b}d}{\sqrt{a}}+3f\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{8b^2}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{a}b^{3/2}}+\frac{\left(\frac{\sqrt{b}d}{\sqrt{a}}+3f\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16b^2}+\frac{\left(\frac{\sqrt{b}d}{\sqrt{a}}+3f\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16b^2}-\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{3/4}b^{7/4}}-\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{3/4}b^{7/4}}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)} + \frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{a}b^{3/2}} - \frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}} + \frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}} + \frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{7/4}} - \frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{7/4}}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)} + \frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{a}b^{3/2}} - \frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{7/4}} + \frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{7/4}} - \frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}} + \frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.95

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx$$

$$= \frac{-\frac{8b^{3/4}(c+x(d+x(e+fx)))}{a+bx^4} - \frac{2\left(\sqrt{2}\sqrt{b}d+4\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{2\left(\sqrt{2}\sqrt{b}d-4\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f\right)\arctan\left(1+\right.}{a^{3/4}}}{32b^{7/4}}$$

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]
```

```
[Out] ((-8*b^(3/4)*(c + x*(d + x*(e + f*x))))/(a + b*x^4) - (2*(Sqrt[2]*Sqrt[b]*d
 + 4*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/a^(3/4) + (2*(Sqrt[2]*Sqrt[b]*d - 4*a^(1/4)*b^(1/4)*e + 3*Sqrt[
2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (Sqrt[2]*(
```

```
-(Sqrt[b]*d) + 3*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/a^(3/4) + (Sqrt[2]*(Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4))/(32*b^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.26

method	result
risch	$\frac{-\frac{f x^3}{4b}-\frac{e x^2}{4b}-\frac{dx}{4b}-\frac{c}{4b}}{b x^4+a}+\frac{\sum_{\_R=\text{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(3f\_R^2+2e\_R+d\right)\ln\left(x-\_R\right)}{\_R^3}}{16b^2}$
default	$\frac{-\frac{f x^3}{4b}-\frac{e x^2}{4b}-\frac{dx}{4b}-\frac{c}{4b}}{b x^4+a}+\frac{\frac{d\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{e\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{\sqrt{ab}}+\frac{3f\sqrt{2}\,\left(\ldots\right.}{\ldots}}{4b}$

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*f*x^3/b-1/4*e*x^2/b-1/4*d*x/b-1/4*c/b)/(b*x^4+a)+1/16/b^2*sum((3*_R^2
*f+2*_R*e+d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.58 (sec) , antiderivative size = 122993, normalized size of antiderivative = 396.75

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.95

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = -\frac{fx^3+ex^2+dx+c}{4\,(b^2x^4+ab)}$$

$$+\frac{\frac{\sqrt{2}\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}d+3\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f-4\sqrt{a}\sqrt{b}e\right.}{a^{\frac{3}{4}}\sqrt{\sqrt{a}}}}{32\,b}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(f*x^3 + e*x^2 + d*x + c)/(b^2*x^4 + a*b) + 1/32*(sqrt(2)*(sqrt(b)*d -
 3*sqrt(a)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(3/4)) - sqrt(2)*(sqrt(b)*d - 3*sqrt(a)*f)*log(sqrt(b)*x^2 - sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(sqrt(2)*a^(1/4)*b^(3/4)*
d + 3*sqrt(2)*a^(3/4)*b^(1/4)*f - 4*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(
2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt
(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(sqrt(2)*a^(1/4)*b^(3/4)*d + 3*sqrt(2)*a^(3/
4)*b^(1/4)*f + 4*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b
^(3/4)))/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.97

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx$$
$$= -\frac{fx^3+ex^2+dx+c}{4\,(bx^4+a)b}$$
$$+ \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2e+(ab^3)^{\frac{1}{4}}b^2d+3\,(ab^3)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^4}$$
$$+ \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2e+(ab^3)^{\frac{1}{4}}b^2d+3\,(ab^3)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^4}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2d-3\,(ab^3)^{\frac{3}{4}}f\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,ab^4}$$
$$- \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2d-3\,(ab^3)^{\frac{3}{4}}f\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,ab^4}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)*b) + 1/16*sqrt(2)*(2*sqrt(2)*sq
rt(a*b)*b^2*e + (a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)
*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/16*sqrt(2)*(2*sqrt(2)
*sqrt(a*b)*b^2*e + (a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/32*sqrt(2)*((a*b^3
)^(1/4)*b^2*d - 3*(a*b^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(a*b^4) - 1/32*sqrt(2)*((a*b^3)^(1/4)*b^2*d - 3*(a*b^3)^(3/4)*f)*log(x
^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)
```

Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.80

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = \left(\sum_{k=1}^{4}\ln\left(\frac{x\,(2\,e^3-3\,d\,e\,f)}{16\,b}-\frac{3\,b\,d^2\,f-4\,b\,d\,e^2+27\,a\,f^3}{64\,b^2}\right.\right.$$
$$-\mathrm{root}\big(65536\,a^3\,b^7\,z^4+3072\,a^2\,b^4\,d\,f\,z^2+2048\,a^2\,b^4\,e^2\,z^2+1152\,a^2\,b^2\,e\,f^2\,z-128\,a\,b^3\,d^2\,e\,z-48\,a\,b\,d\,e^2\,f+18$$
$$+3072\,a^2\,b^4\,d\,f\,z^2+2048\,a^2\,b^4\,e^2\,z^2+1152\,a^2\,b^2\,e\,f^2\,z-128\,a\,b^3\,d^2\,e\,z-48\,a\,b\,d\,e^2\,f$$
$$\left.\left.+18\,a\,b\,d^2\,f^2+16\,a\,b\,e^4+81\,a^2\,f^4+b^2\,d^4,z,k\big)\right)\right)-\frac{\frac{c}{4\,b}+\frac{e\,x^2}{4\,b}+\frac{f\,x^3}{4\,b}+\frac{d\,x}{4\,b}}{b\,x^4+a}$$

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x)
```

```
[Out] symsum(log((x*(2*e^3 - 3*d*e*f))/(16*b) - (27*a*f^3 - 4*b*d*e^2 + 3*b*d^2*f
)/(64*b^2) - root(65536*a^3*b^7*z^4 + 3072*a^2*b^4*d*f*z^2 + 2048*a^2*b^4*e
^2*z^2 + 1152*a^2*b^2*e*f^2*z - 128*a*b^3*d^2*e*z - 48*a*b*d*e^2*f + 18*a*b
*d^2*f^2 + 16*a*b*e^4 + 81*a^2*f^4 + b^2*d^4, z, k)*(3*a*e*f + (b*d^2*x)/4
- (9*a*f^2*x)/4 + 4*root(65536*a^3*b^7*z^4 + 3072*a^2*b^4*d*f*z^2 + 2048*a^
2*b^4*e^2*z^2 + 1152*a^2*b^2*e*f^2*z - 128*a*b^3*d^2*e*z - 48*a*b*d*e^2*f +
 18*a*b*d^2*f^2 + 16*a*b*e^4 + 81*a^2*f^4 + b^2*d^4, z, k)*a*b^2*d - 8*root
(65536*a^3*b^7*z^4 + 3072*a^2*b^4*d*f*z^2 + 2048*a^2*b^4*e^2*z^2 + 1152*a^2
*b^2*e*f^2*z - 128*a*b^3*d^2*e*z - 48*a*b*d*e^2*f + 18*a*b*d^2*f^2 + 16*a*b
*e^4 + 81*a^2*f^4 + b^2*d^4, z, k)*a*b^2*e*x))*root(65536*a^3*b^7*z^4 + 307
2*a^2*b^4*d*f*z^2 + 2048*a^2*b^4*e^2*z^2 + 1152*a^2*b^2*e*f^2*z - 128*a*b^3
*d^2*e*z - 48*a*b*d*e^2*f + 18*a*b*d^2*f^2 + 16*a*b*e^4 + 81*a^2*f^4 + b^2*
d^4, z, k), k, 1, 4) - (c/(4*b) + (e*x^2)/(4*b) + (f*x^3)/(4*b) + (d*x)/(4*
b))/(a + b*x^4)
```

3.491 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 351

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx = \frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)} - \frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2} + \frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}} + \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}} - \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}} + \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}$$

```
[Out] 1/32*x*(5*e*x^2+6*d*x+7*c)/a^2/(b*x^4+a)+1/8*(-a*f+b*x*(e*x^2+d*x+c))/a/b/(
b*x^4+a)^2+3/16*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)-1/256*ln(-a^(
1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*e*a^(1/2)+21*c*b^(1/2))/a^(
11/4)/b^(3/4)*2^(1/2)+1/256*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2
))*(-5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)*2^(1/2)+1/128*arctan(-1+b^(
1/4)*x*2^(1/2)/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)*2^(1/2)
+1/128*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11
/4)/b^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}e+21\sqrt{b}c\right)}{64\sqrt{2}a^{11/4}b^{3/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}e+21\sqrt{b}c\right)}{64\sqrt{2}a^{11/4}b^{3/4}} + \frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}} + \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}} + \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)} - \frac{af-bx(c+dx+ex^2)}{8ab\left(a+bx^4\right)^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x]
```

```
[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a + b*x^4)) - (a*f - b*x*(c + d*x + e*
x^2))/(8*a*b*(a + b*x^4)^2) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/
2)*Sqrt[b]) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*
Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(
3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
```

```
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{af-bx(c+dx+ex^2)}{8ab\left(a+bx^4\right)^2}-\frac{\int\frac{-7c-6dx-5ex^2}{\left(a+bx^4\right)^2}\,dx}{8a}\\
&= \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)}-\frac{af-bx(c+dx+ex^2)}{8ab\left(a+bx^4\right)^2}+\frac{\int\frac{21c+12dx+5ex^2}{a+bx^4}\,dx}{32a^2}\\
&= \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)}-\frac{af-bx(c+dx+ex^2)}{8ab\left(a+bx^4\right)^2}+\frac{\int\left(\frac{12dx}{a+bx^4}+\frac{21c+5ex^2}{a+bx^4}\right)dx}{32a^2}\\
&= \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)}-\frac{af-bx(c+dx+ex^2)}{8ab\left(a+bx^4\right)^2}+\frac{\int\frac{21c+5ex^2}{a+bx^4}\,dx}{32a^2}+\frac{(3d)\int\frac{x}{a+bx^4}\,dx}{8a^2}\\
&= \frac{x(7c+6dx+5ex^2)}{32a^2\left(a+bx^4\right)}-\frac{af-bx(c+dx+ex^2)}{8ab\left(a+bx^4\right)^2}+\frac{(3d)\text{Subst}\left(\int\frac{1}{a+bx^2}\,dx,x,x^2\right)}{16a^2}\\
&\quad+\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\int\frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{64a^2b}+\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}+5e\right)\int\frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{64a^2b}
\end{aligned}$$

$$=\frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)}-\frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{128\sqrt{2}a^{9/4}b^{3/4}}-\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{128\sqrt{2}a^{9/4}b^{3/4}}+\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}+5e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{128a^2b}+\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}+5e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{128a^2b}$$

$$=\frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)}-\frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{3/4}}+\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{3/4}}+\frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}}-\frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}}$$

$$=\frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)}-\frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}}-\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{3/4}}+\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{9/4}b^{3/4}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx$$

$$= \frac{\frac{8ax(7c+x(6d+5ex))}{a+bx^4}-\frac{32a^2(af-bx(c+x(d+ex)))}{b(a+bx^4)^2}-\frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{b}c+24\sqrt[4]{a}\sqrt[4]{b}d+5\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}+\frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{b}\right.}{}}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a*x*(7*c + x*(6*d + 5*e*x)))/(a + b*x^4) - (32*a^2*(a*f - b*x*(c + x*(d
 + e*x))))/(b*(a + b*x^4)^2) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c + 24*a^(1/4
)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/b^(3/4) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c - 24*a^(1/4)*b^(1/4)*d + 5*Sqrt
[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2]*
(-21*a^(1/4)*Sqrt[b]*c + 5*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*x + Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3/4)*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(256*a^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.71 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.33

method	result
risch	$\frac{\frac{5be\,x^7}{32a^2}+\frac{3bd\,x^6}{16a^2}+\frac{7bc\,x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\operatorname{RootOf}(\_Z^4b+a)}\frac{\left(5\_R^2e+12\_Rd+21c\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2b}$
default	$c\left(\frac{x}{8a(b\,x^4+a)^2}+\frac{\frac{7x}{32a(b\,x^4+a)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)+d\left(\frac{}{8a}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (5/32*b*e/a^2*x^7+3/16*b*d/a^2*x^6+7/32*b*c/a^2*x^5+9/32/a*e*x^3+5/16*d/a*x
^2+11/32*c/a*x-1/8*f/b)/(b*x^4+a)^2+1/128/a^2/b*sum((5*_R^2*e+12*_R*d+21*c)
/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.94 (sec) , antiderivative size = 124838, normalized size of antiderivative = 355.66

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx = \frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5+9\,abex^3+10\,abdx^2+11\,abcx-4\,a^2f}{32\,(a^2b^3x^8+2\,a^3b^2x^4+a^4b)} + \frac{\frac{\sqrt{2}\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(21\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+5\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-24\,\sqrt{}\right.}{a^{\frac{3}{4}}\sqrt{}}}{256\,a^2}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
[Out] 1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*c*x^5 + 9*a*b*e*x^3 + 10*a*b*d*x^2
+ 11*a*b*c*x - 4*a^2*f)/(a^2*b^3*x^8 + 2*a^3*b^2*x^4 + a^4*b) + 1/256*(sqrt
(2)*(21*sqrt(b)*c - 5*sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*
x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*sqrt(b)*c - 5*sqrt(a)*e)*log(s
qrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21
```

```
*sqrt(2)*a^(1/4)*b^(3/4)*c + 5*sqrt(2)*a^(3/4)*b^(1/4)*e - 24*sqrt(a)*sqrt(
b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(
a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4
)*b^(3/4)*c + 5*sqrt(2)*a^(3/4)*b^(1/4)*e + 24*sqrt(a)*sqrt(b)*d)*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx$$
$$= \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+21\left(ab^3\right)^{\frac{1}{4}}b^2c+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$
$$+ \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+21\left(ab^3\right)^{\frac{1}{4}}b^2c+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$
$$+ \frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c-5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$
$$- \frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c-5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$
$$+ \frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5+9\,abex^3+10\,abdx^2+11\,abcx-4\,a^2f}{32\left(bx^4+a\right)^2a^2b}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b
^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*
c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)
^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)
*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*
(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4
) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*c*x^5 +
9*a*b*e*x^3 + 10*a*b*d*x^2 + 11*a*b*c*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 832, normalized size of antiderivative = 2.37

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx$$
$$= \left(\sum_{k=1}^{4}\ln\left(-\frac{b\left(125\,a\,e^3-3024\,b\,c\,d^2+2205\,b\,c^2\,e-1728\,b\,d^3\,x+\mathrm{root}(268435456\,a^{11}\,b^3\,z^4+6881280\,a^6\,b^2\,c\right.}{}\right.\right.$$
$$+6881280\,a^6\,b^2\,c\,e\,z^2+4718592\,a^6\,b^2\,d^2\,z^2-2709504\,a^3\,b^2\,c^2\,d\,z+153600\,a^4\,b\,d\,e^2\,z$$
$$\left.\left.-60480\,a\,b\,c\,d^2\,e+22050\,a\,b\,c^2\,e^2+20736\,a\,b\,d^4+625\,a^2\,e^4+194481\,b^2\,c^4,z,k)\right)\right)$$
$$+\frac{\frac{5\,d\,x^2}{16\,a}-\frac{f}{8\,b}+\frac{9\,e\,x^3}{32\,a}+\frac{11\,c\,x}{32\,a}+\frac{7\,b\,c\,x^5}{32\,a^2}+\frac{3\,b\,d\,x^6}{16\,a^2}+\frac{5\,b\,e\,x^7}{32\,a^2}}{a^2+2\,a\,b\,x^4+b^2\,x^8}$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x)
```

```
[Out] symsum(log(-(b*(125*a*e^3 - 3024*b*c*d^2 + 2205*b*c^2*e - 1728*b*d^3*x + 34
4064*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^
2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^
2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z,
k)^2*a^5*b^2*c - 3200*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2
 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z
 - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 19
4481*b^2*c^4, z, k)*a^3*b*e^2*x + 2520*b*c*d*e*x + 56448*root(268435456*a^1
1*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3
*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2
 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k)*a^2*b^2*c^2*x - 1966
08*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*
d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*
e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k)
^2*a^5*b^2*d*x + 15360*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^
2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*
z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 1
94481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435456*a^11*b^3*z^4 +
 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*
z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*
b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k), k, 1, 4) + ((5*d*x^2)/(16*a) -
 f/(8*b) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) + (7*b*c*x^5)/(32*a^2) + (3*b
*d*x^6)/(16*a^2) + (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4)
```

3.492 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 340

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = -\frac{c+dx+ex^2+fx^3}{8b(a+bx^4)^2}+\frac{x(d+2ex+3fx^2)}{32ab(a+bx^4)} + \frac{e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}} - \frac{3\left(\sqrt{b}d+\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{7/4}b^{7/4}} + \frac{3\left(\sqrt{b}d+\sqrt{a}f\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{7/4}b^{7/4}} - \frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}} + \frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}}$$

```
[Out] 1/8*(-f*x^3-e*x^2-d*x-c)/b/(b*x^4+a)^2+1/32*x*(3*f*x^2+2*e*x+d)/a/b/(b*x^4+
a)+1/16*e*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-3/256*ln(-a^(1/4)*b^(
1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-f*a^(1/2)+d*b^(1/2))/a^(7/4)/b^(7/4)*
2^(1/2)+3/256*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-f*a^(1/2)
+d*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+3/128*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/
4))*(f*a^(1/2)+d*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+3/128*arctan(1+b^(1/4)*x*
2^(1/2)/a^(1/4))*(f*a^(1/2)+d*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1837, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}f+\sqrt{b}d\right)}{64\sqrt{2}a^{7/4}b^{7/4}} + \frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}f+\sqrt{b}d\right)}{64\sqrt{2}a^{7/4}b^{7/4}} + \frac{e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}} - \frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}} + \frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}} - \frac{c+dx+ex^2+fx^3}{8b(a+bx^4)^2} + \frac{x(d+2ex+3fx^2)}{32ab(a+bx^4)}$$

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]
```

```
[Out] -1/8*(c + d*x + e*x^2 + f*x^3)/(b*(a + b*x^4)^2) + (x*(d + 2*e*x + 3*f*x^2)
)/(32*a*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(3/2)*b^(3
/2)) - (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/
(64*Sqrt[2]*a^(7/4)*b^(7/4)) + (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(7/4)*b^(7/4)) - (3*(Sqrt[b]*d - Sqrt
[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]
*a^(7/4)*b^(7/4)) + (3*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(7/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{\int \frac{d+2ex+3fx^2}{\left(a+bx^4\right)^2}\,dx}{8b}\\
&= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x(d+2ex+3fx^2)}{32ab\left(a+bx^4\right)}-\frac{\int \frac{-3d-4ex-3fx^2}{a+bx^4}\,dx}{32ab}\\
&= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x(d+2ex+3fx^2)}{32ab\left(a+bx^4\right)}-\frac{\int \left(-\frac{4ex}{a+bx^4}+\frac{-3d-3fx^2}{a+bx^4}\right)\,dx}{32ab}\\
&= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x(d+2ex+3fx^2)}{32ab\left(a+bx^4\right)}-\frac{\int \frac{-3d-3fx^2}{a+bx^4}\,dx}{32ab}+\frac{e\int \frac{x}{a+bx^4}\,dx}{8ab}\\
&= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x(d+2ex+3fx^2)}{32ab\left(a+bx^4\right)}+\frac{e\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{16ab}\\
&\quad+\frac{\left(3\left(\frac{\sqrt{b}d}{\sqrt{a}}-f\right)\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{64ab^2}+\frac{\left(3\left(\frac{\sqrt{b}d}{\sqrt{a}}+f\right)\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{64ab^2}
\end{aligned}
$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2} + \frac{x(d+2ex+3fx^2)}{32ab\left(a+bx^4\right)} + \frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}} + \frac{\left(3\left(\frac{\sqrt{b}d}{\sqrt{a}}+f\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{128ab^2} + \frac{\left(3\left(\frac{\sqrt{b}d}{\sqrt{a}}+f\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{128ab^2} - \frac{\left(3\left(\sqrt{b}d-\sqrt{a}f\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{128\sqrt{2}a^{7/4}b^{7/4}} - \frac{\left(3\left(\sqrt{b}d-\sqrt{a}f\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{128\sqrt{2}a^{7/4}b^{7/4}}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2} + \frac{x(d+2ex+3fx^2)}{32ab\left(a+bx^4\right)} + \frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}} - \frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}} + \frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}} + \frac{\left(3\left(\sqrt{b}d+\sqrt{a}f\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{7/4}b^{7/4}} - \frac{\left(3\left(\sqrt{b}d+\sqrt{a}f\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{7/4}b^{7/4}}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2} + \frac{x(d+2ex+3fx^2)}{32ab\left(a+bx^4\right)} + \frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}} - \frac{3\left(\sqrt{b}d+\sqrt{a}f\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{7/4}b^{7/4}} + \frac{3\left(\sqrt{b}d+\sqrt{a}f\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{7/4}b^{7/4}} - \frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}} + \frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.97

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx$$

$$= \frac{\frac{8b^{3/4}x(d+x(2e+3fx))}{a(a+bx^4)} - \frac{32b^{3/4}(c+x(d+x(e+fx)))}{(a+bx^4)^2} - \frac{2\left(3\sqrt{2}\sqrt{b}d+8\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{2\left(3\sqrt{2}\sqrt{b}d-8\sqrt[4]{a}\sqrt[4]{b}\right.}{}}{25}$$

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]
```

```
[Out] ((8*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)) - (32*b^(3/4)*(c + x*(
d + x*(e + f*x))))/(a + b*x^4)^2 - (2*(3*Sqrt[2]*Sqrt[b]*d + 8*a^(1/4)*b^(1
/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/
4) + (2*(3*Sqrt[2]*Sqrt[b]*d - 8*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (3*Sqrt[2]*(-(Sqrt[b]*d)
+ Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4
) + (3*Sqrt[2]*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/a^(7/4))/(256*b^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.63 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\frac{3f\,x^7}{32a}+\frac{e\,x^6}{16a}+\frac{d\,x^5}{32a}-\frac{f\,x^3}{32b}-\frac{e\,x^2}{16b}-\frac{3dx}{32b}-\frac{c}{8b}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{\left(3f\,\_R^2+4e\,\_R+3d\right)\ln\left(x-\_R\right)}{\_R^3}}{128a\,b^2}$
default	$\frac{\frac{3f\,x^7}{32a}+\frac{e\,x^6}{16a}+\frac{d\,x^5}{32a}-\frac{f\,x^3}{32b}-\frac{e\,x^2}{16b}-\frac{3dx}{32b}-\frac{c}{8b}}{(b\,x^4+a)^2}+\frac{\frac{3d\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{2e\,\mathrm{arc}}{}}{32}$

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (3/32*f/a*x^7+1/16/a*e*x^6+1/32*d/a*x^5-1/32*f*x^3/b-1/16*e*x^2/b-3/32*d*x/
b-1/8*c/b)/(b*x^4+a)^2+1/128/a/b^2*sum((3*_R^2*f+4*_R*e+3*d)/_R^3*ln(x-_R),
_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.28 (sec) , antiderivative size = 124542, normalized size of antiderivative = 366.30

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.01

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = \frac{3\,bfx^7+2\,bex^6+bdx^5-afx^3-2\,aex^2-3\,adx-4\,ac}{32\,(ab^3x^8+2\,a^2b^2x^4+a^3b)}$$

$$+\frac{\frac{3\sqrt{2}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{3\sqrt{2}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}d+3\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f-8\sqrt{a}\sqrt{b}\cdots\right.}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\cdots}}}{256\,ab}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(3*b*f*x^7 + 2*b*e*x^6 + b*d*x^5 - a*f*x^3 - 2*a*e*x^2 - 3*a*d*x - 4*a
*c)/(a*b^3*x^8 + 2*a^2*b^2*x^4 + a^3*b) + 1/256*(3*sqrt(2)*(sqrt(b)*d - sqr
t(a)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(
3/4)) - 3*sqrt(2)*(sqrt(b)*d - sqrt(a)*f)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*d +
3*sqrt(2)*a^(3/4)*b^(1/4)*f - 8*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(2*sq
rt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqr
t(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*d + 3*sqrt(2)*a^(3/4)
*b^(1/4)*f + 8*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)
*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(
3/4)))/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.98

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx$$
$$= \frac{3\,bfx^7+2\,bex^6+bdx^5-afx^3-2\,aex^2-3\,adx-4\,ac}{32\,(bx^4+a)^2ab}$$
$$+ \frac{\sqrt{2}\Big(4\,\sqrt{2}\sqrt{ab}b^2e+3\,(ab^3)^{\frac{1}{4}}\,b^2d+3\,(ab^3)^{\frac{3}{4}}\,f\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4}$$
$$+ \frac{\sqrt{2}\Big(4\,\sqrt{2}\sqrt{ab}b^2e+3\,(ab^3)^{\frac{1}{4}}\,b^2d+3\,(ab^3)^{\frac{3}{4}}\,f\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4}$$
$$+ \frac{3\,\sqrt{2}\Big((ab^3)^{\frac{1}{4}}\,b^2d-(ab^3)^{\frac{3}{4}}\,f\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4}$$
$$- \frac{3\,\sqrt{2}\Big((ab^3)^{\frac{1}{4}}\,b^2d-(ab^3)^{\frac{3}{4}}\,f\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
[Out] 1/32*(3*b*f*x^7 + 2*b*e*x^6 + b*d*x^5 - a*f*x^3 - 2*a*e*x^2 - 3*a*d*x - 4*a
*c)/((b*x^4 + a)^2*a*b) + 1/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b^2*e + 3*(a*b
^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 1/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b^2*e
+ 3*(a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/256*sqrt(2)*((a*b^3)^(1/4)*b^2*
d - (a*b^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)
 - 3/256*sqrt(2)*((a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.53

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(268435456\,a^7\,b^7\,z^4+589824\,a^4\,b^4\,d\,f\,z^2+524288\,a^4\,b^4\,e^2\,z^2+18432\,a^3\,b^2\,e\,f^2\,z-18432\,a^2\,b^3\,d^2\,e\,z - 576\,a\,b\,d\,e^2\,f+162\,a\,b\,d^2\,f^2+256\,a\,b\,e^4+81\,a^2\,f^4+81\,b^2\,d^4, z, k\right)\left(\ldots\right) - \frac{3\,(9\,b\,d^2\,f-16\,b\,d\,e^2+9\,a\,f^3)}{32768\,a^3\,b^2}+\frac{x\,(8\,e^3-9\,d\,e\,f)}{4096\,a^3\,b}\right)\mathrm{root}\left(268435456\,a^7\,b^7\,z^4 + 589824\,a^4\,b^4\,d\,f\,z^2+524288\,a^4\,b^4\,e^2\,z^2+18432\,a^3\,b^2\,e\,f^2\,z-18432\,a^2\,b^3\,d^2\,e\,z - 576\,a\,b\,d\,e^2\,f+162\,a\,b\,d^2\,f^2+256\,a\,b\,e^4+81\,a^2\,f^4+81\,b^2\,d^4, z, k\right)\right) - \frac{\frac{c}{8\,b}-\frac{d\,x^5}{32\,a}-\frac{e\,x^6}{16\,a}+\frac{e\,x^2}{16\,b}-\frac{3\,f\,x^7}{32\,a}+\frac{f\,x^3}{32\,b}+\frac{3\,d\,x}{32\,b}}{a^2+2\,a\,b\,x^4+b^2\,x^8}$$

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x)
```

```
[Out] symsum(log((x*(8*e^3 - 9*d*e*f))/(4096*a^3*b) - (3*(9*a*f^3 - 16*b*d*e^2 +
9*b*d^2*f))/(32768*a^3*b^2) - root(268435456*a^7*b^7*z^4 + 589824*a^4*b^4*d
*f*z^2 + 524288*a^4*b^4*e^2*z^2 + 18432*a^3*b^2*e*f^2*z - 18432*a^2*b^3*d^2
*e*z - 576*a*b*d*e^2*f + 162*a*b*d^2*f^2 + 256*a*b*e^4 + 81*a^2*f^4 + 81*b^
2*d^4, z, k)*(root(268435456*a^7*b^7*z^4 + 589824*a^4*b^4*d*f*z^2 + 524288*
a^4*b^4*e^2*z^2 + 18432*a^3*b^2*e*f^2*z - 18432*a^2*b^3*d^2*e*z - 576*a*b*d
*e^2*f + 162*a*b*d^2*f^2 + 256*a*b*e^4 + 81*a^2*f^4 + 81*b^2*d^4, z, k)*((3
*b^2*d)/2 - 2*b^2*e*x) + (3*e*f)/(32*a) + (x*(144*a*b^2*d^2 - 144*a^2*b*f^2
))/(4096*a^3*b)))*root(268435456*a^7*b^7*z^4 + 589824*a^4*b^4*d*f*z^2 + 524
288*a^4*b^4*e^2*z^2 + 18432*a^3*b^2*e*f^2*z - 18432*a^2*b^3*d^2*e*z - 576*a
*b*d*e^2*f + 162*a*b*d^2*f^2 + 256*a*b*e^4 + 81*a^2*f^4 + 81*b^2*d^4, z, k)
, k, 1, 4) - (c/(8*b) - (d*x^5)/(32*a) - (e*x^6)/(16*a) + (e*x^2)/(16*b) -
(3*f*x^7)/(32*a) + (f*x^3)/(32*b) + (3*d*x)/(32*b))/(a^2 + b^2*x^8 + 2*a*b*
x^4)
```

3.493 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 382

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4} dx = \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} - \frac{af-bx(c+dx+ex^2)}{12ab(a+bx^4)^3} + \frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}} + \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}} - \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}} + \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}$$

```
[Out] 1/96*x*(9*e*x^2+10*d*x+11*c)/a^2/(b*x^4+a)^2+1/384*x*(45*e*x^2+60*d*x+77*c)
/a^3/(b*x^4+a)+1/12*(-a*f+b*x*(e*x^2+d*x+c))/a/b/(b*x^4+a)^3+5/32*d*arctan(
x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*b^(1/2))*(-15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)+1
/1024*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-15*e*a^(1/2)+77*c
*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4
))*(15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)+1/512*arctan(1+b^(1
/4)*x*2^(1/2)/a^(1/4))*(15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}e+77\sqrt{b}c\right)}{256\sqrt{2}a^{15/4}b^{3/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}e+77\sqrt{b}c\right)}{256\sqrt{2}a^{15/4}b^{3/4}} + \frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}} + \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} - \frac{af-bx(c+dx+ex^2)}{12ab(a+bx^4)^3}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]
```

```
[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x +
45*e*x^2))/(384*a^3*(a + b*x^4)) - (a*f - b*x*(c + d*x + e*x^2))/(12*a*b*(a
 + b*x^4)^3) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - (
(77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256
*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c -
 15*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512
*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
```

```
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{af-bx(c+dx+ex^2)}{12ab\left(a+bx^4\right)^3}-\frac{\int\frac{-11c-10dx-9ex^2}{\left(a+bx^4\right)^3}\,dx}{12a}\\
&=\frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2}-\frac{af-bx(c+dx+ex^2)}{12ab\left(a+bx^4\right)^3}+\frac{\int\frac{77c+60dx+45ex^2}{\left(a+bx^4\right)^2}\,dx}{96a^2}\\
&=\frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a+bx^4\right)}\\
&\quad-\frac{af-bx(c+dx+ex^2)}{12ab\left(a+bx^4\right)^3}-\frac{\int\frac{-231c-120dx-45ex^2}{a+bx^4}\,dx}{384a^3}\\
&=\frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a+bx^4\right)}\\
&\quad-\frac{af-bx(c+dx+ex^2)}{12ab\left(a+bx^4\right)^3}-\frac{\int\left(-\frac{120dx}{a+bx^4}+\frac{-231c-45ex^2}{a+bx^4}\right)dx}{384a^3}\\
&=\frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3\left(a+bx^4\right)}\\
&\quad-\frac{af-bx(c+dx+ex^2)}{12ab\left(a+bx^4\right)^3}-\frac{\int\frac{-231c-45ex^2}{a+bx^4}\,dx}{384a^3}+\frac{(5d)\int\frac{x}{a+bx^4}\,dx}{16a^3}
\end{aligned}
$$

$$= \frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\left(a+bx^4\right)} - \frac{af-bx(c+dx+ex^2)}{12ab\left(a+bx^4\right)^3} + \frac{(5d)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{32a^3} + \frac{\left(\frac{77\sqrt{b}c}{\sqrt{a}}-15e\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{256a^3b} + \frac{\left(\frac{77\sqrt{b}c}{\sqrt{a}}+15e\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{256a^3b}$$

$$= \frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\left(a+bx^4\right)} - \frac{af-bx(c+dx+ex^2)}{12ab\left(a+bx^4\right)^3} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} + \frac{\left(\frac{77\sqrt{b}c}{\sqrt{a}}+15e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{512a^3b} + \frac{\left(\frac{77\sqrt{b}c}{\sqrt{a}}+15e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{512a^3b} - \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{512\sqrt{2}a^{15/4}b^{3/4}} - \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{512\sqrt{2}a^{15/4}b^{3/4}}$$

$$= \frac{x(11c+10dx+9ex^2)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3\left(a+bx^4\right)} - \frac{af-bx(c+dx+ex^2)}{12ab\left(a+bx^4\right)^3} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}} + \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}} + \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}} - \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}}$$

$$=\frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)}-\frac{af-bx(c+dx+ex^2)}{12ab(a+bx^4)^3}$$
$$+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}-\frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}}$$
$$+\frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}}$$
$$-\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}$$
$$+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.99

$$\int\frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$$

$$=\frac{\frac{8ax(77c+15x(4d+3ex))}{a+bx^4}+\frac{32a^2x(11c+x(10d+9ex))}{(a+bx^4)^2}-\frac{256a^3(af-bx(c+x(d+ex)))}{b(a+bx^4)^3}-\frac{6\sqrt[4]{a}\left(77\sqrt{2}\sqrt{b}c+80\sqrt[4]{a}\sqrt[4]{b}d+15\sqrt{2}\sqrt{a}e\right)\arctan\left(\right.}{b^{3/4}}}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]
```

```
[Out] ((8*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a + b*x^4) + (32*a^2*x*(11*c + x*(10*
d + 9*e*x)))/(a + b*x^4)^2 - (256*a^3*(a*f - b*x*(c + x*(d + e*x))))/(b*(a
+ b*x^4)^3) - (6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c + 80*a^(1/4)*b^(1/4)*d + 15*
Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (6*a^
(1/4)*(77*Sqrt[2]*Sqrt[b]*c - 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (3*Sqrt[2]*(-77*a^(1/4)*
Sqrt[b]*c + 15*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/b^(3/4) + (3*Sqrt[2]*(77*a^(1/4)*Sqrt[b]*c - 15*a^(3/4)*e)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(3072*a^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.64 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}-\frac{f}{12b}}{(b\,x^4+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{(15\_R^2e+40\_R d+}{\_R}}{512a^3b}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}-\frac{f}{12b}}{(b\,x^4+a)^3}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{ar}\right.}{8a}}{}$

[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)

[Out] (15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9+21/64*b*e/a^2*x^7+5/12*b*d/a^2*x^6+33/64*b*c/a^2*x^5+113/384/a*e*x^3+11/32*d/a*x^2+51/128*c/a*x-1/12*f/b)/(b*x^4+a)^3+1/512/a^3/b*sum((15*_R^2*e+40*_R*d+77*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.78 (sec) , antiderivative size = 125011, normalized size of antiderivative = 327.25

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx = \text{Timed out}$$

[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.05

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx = \frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9+126\,ab^2ex^7+160\,ab^2dx^6+198\,ab^2cx^5+113\,a^2bex^3+132\,a^2bdx^2+153}{384\,(a^3b^4x^{12}+3\,a^4b^3x^8+3\,a^5b^2x^4+a^6b)} + \frac{\frac{\sqrt{2}\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+15\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e\right.}{}}{1024\,a^3}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 77*b^3*c*x^9 + 126*a*b^2*e*x^7 + 160
*a*b^2*d*x^6 + 198*a*b^2*c*x^5 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 153*a^
2*b*c*x - 32*a^3*f)/(a^3*b^4*x^12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*b)
+ 1/1024*(sqrt(2)*(77*sqrt(b)*c - 15*sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*sqrt(b)*c - 15*
sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*
b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e -
 80*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^
(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(
77*sqrt(2)*a^(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e + 80*sqrt(a)*sq
rt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sq
rt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$$
$$= \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77\,(ab^3)^{\frac{1}{4}}\,b^2c+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$
$$+ \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77\,(ab^3)^{\frac{1}{4}}\,b^2c+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$
$$+ \frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$
$$- \frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$
$$+ \frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9+126\,ab^2ex^7+160\,ab^2dx^6+198\,ab^2cx^5+113\,a^2bex^3+132\,a^2bdx^2+15}{384\,(bx^4+a)^3a^3b}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 15*(a*
b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2
*c + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/
b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(
3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqr
t(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b
)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 77*
b^3*c*x^9 + 126*a*b^2*e*x^7 + 160*a*b^2*d*x^6 + 198*a*b^2*c*x^5 + 113*a^2*b
*e*x^3 + 132*a^2*b*d*x^2 + 153*a^2*b*c*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 879, normalized size of antiderivative = 2.30

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$$
$$= \left(\sum_{k=1}^{4}\ln\left(-\frac{b\left(3375\,a\,e^3-123200\,b\,c\,d^2+88935\,b\,c^2\,e-64000\,b\,d^3\,x+\mathrm{root}(68719476736\,a^{15}\,b^3\,z^4+1211\right.}{}\right.\right.$$
$$+1211105280\,a^8\,b^2\,c\,e\,z^2+838860800\,a^8\,b^2\,d^2\,z^2-485703680\,a^4\,b^2\,c^2\,d\,z+18432000\,a^5\,b\,d\,e^2\,z-7392000\,a\,b\,c\,d^2\,e+2668050\,a\,b\,c^2\,e^2+2560000\,a\,b\,d^4$$
$$\left.\left.+35153041\,b^2\,c^4+50625\,a^2\,e^4,z,k)\right)\right)$$
$$+\frac{\frac{11\,d\,x^2}{32\,a}-\frac{f}{12\,b}+\frac{113\,e\,x^3}{384\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}+\frac{15\,b^2\,e\,x^{11}}{128\,a^3}+\frac{33\,b\,c\,x^5}{64\,a^2}+\frac{5\,b\,d\,x^6}{12\,a^2}+\frac{21\,b\,e\,x^7}{64\,a^2}}{a^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^8+b^3\,x^{12}}$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x)
```

```
[Out] symsum(log(-(b*(3375*a*e^3 - 123200*b*c*d^2 + 88935*b*c^2*e - 64000*b*d^3*x
 + 20185088*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 83
8860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*
z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*
b^2*c^4 + 50625*a^2*e^4, z, k)^2*a^7*b^2*c - 115200*root(68719476736*a^15*b
^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680
*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a
*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)*a^4*
b*e^2*x + 92400*b*c*d*e*x + 3035648*root(68719476736*a^15*b^3*z^4 + 1211105
280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z
 + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 256
0000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)*a^3*b^2*c^2*x - 1048
5760*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800
*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 739
2000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4
 + 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x + 614400*root(68719476736*a^15*b^3*z^
4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*
b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)*a^4*b*d*e
))/(2097152*a^9))*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^
2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*
d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 351
53041*b^2*c^4 + 50625*a^2*e^4, z, k), k, 1, 4) + ((11*d*x^2)/(32*a) - f/(12
*b) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9)/(384*a^3) + (
5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) + (33*b*c*x^5)/(64*a^2)
```

```
+ (5*b*d*x^6)/(12*a^2) + (21*b*e*x^7)/(64*a^2))/(a^3 + b^3*x^12 + 3*a^2*b*x
^4 + 3*a*b^2*x^8)
```

3.494 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 380

$$\begin{aligned}\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx &= -\frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3}+\frac{x(d+2ex+3fx^2)}{96ab(a+bx^4)^2}\\ &\quad+\frac{x(7d+12ex+15fx^2)}{384a^2b(a+bx^4)}+\frac{e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{5/2}b^{3/2}}\\ &\quad-\frac{\left(7\sqrt{b}d+5\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{11/4}b^{7/4}}\\ &\quad+\frac{\left(7\sqrt{b}d+5\sqrt{a}f\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{11/4}b^{7/4}}\\ &\quad-\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{11/4}b^{7/4}}\\ &\quad+\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{11/4}b^{7/4}}\end{aligned}$$

```
[Out] 1/12*(-f*x^3-e*x^2-d*x-c)/b/(b*x^4+a)^3+1/96*x*(3*f*x^2+2*e*x+d)/a/b/(b*x^4
+a)^2+1/384*x*(15*f*x^2+12*e*x+7*d)/a^2/b/(b*x^4+a)+1/32*e*arctan(x^2*b^(1/
2)/a^(1/2))/a^(5/2)/b^(3/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*b^(1/2))*(-5*f*a^(1/2)+7*d*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/1024*ln(a^
(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*f*a^(1/2)+7*d*b^(1/2))/a^(
11/4)/b^(7/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*f*a^(1/
2)+7*d*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/512*arctan(1+b^(1/4)*x*2^(1/2)/a
^(1/4))*(5*f*a^(1/2)+7*d*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1837, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}f+7\sqrt{b}d\right)}{256\sqrt{2}a^{11/4}b^{7/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}f+7\sqrt{b}d\right)}{256\sqrt{2}a^{11/4}b^{7/4}} + \frac{e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{5/2}b^{3/2}} - \frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{11/4}b^{7/4}} + \frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{11/4}b^{7/4}} + \frac{x(7d+12ex+15fx^2)}{384a^2b(a+bx^4)} - \frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab(a+bx^4)^2}$$

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]
```

```
[Out] -1/12*(c + d*x + e*x^2 + f*x^3)/(b*(a + b*x^4)^3) + (x*(d + 2*e*x + 3*f*x^2
))/(96*a*b*(a + b*x^4)^2) + (x*(7*d + 12*e*x + 15*f*x^2))/(384*a^2*b*(a + b
*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(5/2)*b^(3/2)) - ((7*Sqrt[
b]*d + 5*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a
^(11/4)*b^(7/4)) + ((7*Sqrt[b]*d + 5*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(256*Sqrt[2]*a^(11/4)*b^(7/4)) - ((7*Sqrt[b]*d - 5*Sqrt[a]*f)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(11
/4)*b^(7/4)) + ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
```

```
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{\int\frac{d+2ex+3fx^2}{\left(a+bx^4\right)^3}\,dx}{12b}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x(d+2ex+3fx^2)}{96ab\left(a+bx^4\right)^2}-\frac{\int\frac{-7d-12ex-15fx^2}{\left(a+bx^4\right)^2}\,dx}{96ab}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x(d+2ex+3fx^2)}{96ab\left(a+bx^4\right)^2}+\frac{x(7d+12ex+15fx^2)}{384a^2b\left(a+bx^4\right)}+\frac{\int\frac{21d+24ex+15fx^2}{a+bx^4}\,dx}{384a^2b}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x(d+2ex+3fx^2)}{96ab\left(a+bx^4\right)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b\left(a+bx^4\right)}+\frac{\int\left(\frac{24ex}{a+bx^4}+\frac{21d+15fx^2}{a+bx^4}\right)\,dx}{384a^2b}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x(d+2ex+3fx^2)}{96ab\left(a+bx^4\right)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b\left(a+bx^4\right)}+\frac{\int\frac{21d+15fx^2}{a+bx^4}\,dx}{384a^2b}+\frac{e\int\frac{x}{a+bx^4}\,dx}{16a^2b}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x(d+2ex+3fx^2)}{96ab\left(a+bx^4\right)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b\left(a+bx^4\right)}+\frac{e\text{Subst}\left(\int\frac{1}{a+bx^2}\,dx,x,x^2\right)}{32a^2b} + \frac{\left(\frac{7\sqrt{b}d}{\sqrt{a}}-5f\right)\int\frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{256a^2b^2}+\frac{\left(\frac{7\sqrt{b}d}{\sqrt{a}}+5f\right)\int\frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{256a^2b^2}$$

$$
\begin{aligned}
&= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x(d+2ex+3fx^2)}{96ab\left(a+bx^4\right)^2} \\
&+\frac{x(7d+12ex+15fx^2)}{384a^2b\left(a+bx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{5/2}b^{3/2}} \\
&-\frac{\left(\frac{7\sqrt{b}d}{\sqrt{a}}-5f\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{512\sqrt{2}a^{9/4}b^{7/4}}-\frac{\left(\frac{7\sqrt{b}d}{\sqrt{a}}-5f\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{512\sqrt{2}a^{9/4}b^{7/4}} \\
&+\frac{\left(\frac{7\sqrt{b}d}{\sqrt{a}}+5f\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{512a^2b^2}+\frac{\left(\frac{7\sqrt{b}d}{\sqrt{a}}+5f\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{512a^2b^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x(d+2ex+3fx^2)}{96ab\left(a+bx^4\right)^2}+\frac{x(7d+12ex+15fx^2)}{384a^2b\left(a+bx^4\right)} \\
&+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{5/2}b^{3/2}}-\frac{\left(\frac{7\sqrt{b}d}{\sqrt{a}}-5f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{9/4}b^{7/4}} \\
&+\frac{\left(\frac{7\sqrt{b}d}{\sqrt{a}}-5f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{9/4}b^{7/4}} \\
&+\frac{\left(7\sqrt{b}d+5\sqrt{a}f\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{11/4}b^{7/4}} \\
&-\frac{\left(7\sqrt{b}d+5\sqrt{a}f\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{11/4}b^{7/4}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x(d+2ex+3fx^2)}{96ab\left(a+bx^4\right)^2}+\frac{x(7d+12ex+15fx^2)}{384a^2b\left(a+bx^4\right)} \\
&+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{5/2}b^{3/2}}-\frac{\left(7\sqrt{b}d+5\sqrt{a}f\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{11/4}b^{7/4}} \\
&+\frac{\left(7\sqrt{b}d+5\sqrt{a}f\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{11/4}b^{7/4}} \\
&-\frac{\left(\frac{7\sqrt{b}d}{\sqrt{a}}-5f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{9/4}b^{7/4}} \\
&+\frac{\left(\frac{7\sqrt{b}d}{\sqrt{a}}-5f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{9/4}b^{7/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.96

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$$

$$= \frac{\frac{32b^{3/4}x(d+x(2e+3fx))}{a(a+bx^4)^2}+\frac{8b^{3/4}x(7d+3x(4e+5fx))}{a^2(a+bx^4)}-\frac{256b^{3/4}(c+x(d+x(e+fx)))}{(a+bx^4)^3}-\frac{6\left(7\sqrt{2}\sqrt{b}d+16\sqrt[4]{a}\sqrt[4]{b}e+5\sqrt{2}\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.}{a^{11/4}}}{\cdots}$$

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]
```

```
[Out] ((32*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)^2) + (8*b^(3/4)*x*(7*d
 + 3*x*(4*e + 5*f*x)))/(a^2*(a + b*x^4)) - (256*b^(3/4)*(c + x*(d + x*(e +
f*x))))/(a + b*x^4)^3 - (6*(7*Sqrt[2]*Sqrt[b]*d + 16*a^(1/4)*b^(1/4)*e + 5*
Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*(
7*Sqrt[2]*Sqrt[b]*d - 16*a^(1/4)*b^(1/4)*e + 5*Sqrt[2]*Sqrt[a]*f)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(11/4) + (3*Sqrt[2]*(-7*Sqrt[b]*d + 5*Sqr
t[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(11/4) +
(3*Sqrt[2]*(7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/a^(11/4))/(3072*b^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.38

method	result
risch	$\frac{\frac{5bf\,x^{11}}{128a^2}+\frac{be\,x^{10}}{32a^2}+\frac{7bd\,x^9}{384a^2}+\frac{7f\,x^7}{64a}+\frac{e\,x^6}{12a}+\frac{3d\,x^5}{64a}-\frac{5f\,x^3}{384b}-\frac{e\,x^2}{32b}-\frac{7dx}{128b}-\frac{c}{12b}}{(b\,x^4+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{\left(5f\,\_R^2+8e\,\_R+7d\right)\ln\left(x-\_R\right)}{\_R^3}}{512a^2b^2}$
default	$\frac{\frac{5bf\,x^{11}}{128a^2}+\frac{be\,x^{10}}{32a^2}+\frac{7bd\,x^9}{384a^2}+\frac{7f\,x^7}{64a}+\frac{e\,x^6}{12a}+\frac{3d\,x^5}{64a}-\frac{5f\,x^3}{384b}-\frac{e\,x^2}{32b}-\frac{7dx}{128b}-\frac{c}{12b}}{(b\,x^4+a)^3}+\frac{\frac{7d\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)\cdots\right.}{8a}\cdots}{\cdots}$

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (5/128/a^2*b*f*x^11+1/32*b*e/a^2*x^10+7/384*b*d/a^2*x^9+7/64*f/a*x^7+1/12/a
*e*x^6+3/64*d/a*x^5-5/384*f*x^3/b-1/32*e*x^2/b-7/128*d*x/b-1/12*c/b)/(b*x^4
+a)^3+1/512/a^2/b^2*sum((5*_R^2*f+8*_R*e+7*d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*
b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.92 (sec) , antiderivative size = 125996, normalized size of antiderivative = 331.57

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.04

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx = \frac{15\,b^2fx^{11}+12\,b^2ex^{10}+7\,b^2dx^9+42\,abfx^7+32\,abex^6+18\,abdx^5-5\,a^2fx^3-12\,a^2ex^2-21\,a^2dx-32\,a^2}{384\,(a^2b^4x^{12}+3\,a^3b^3x^8+3\,a^4b^2x^4+a^5b)} + \frac{\frac{\sqrt{2}\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(7\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}d+5\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f-16\sqrt{}\right.}{a^{\frac{3}{4}}\sqrt{}}}{1024\,a^2b}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
[Out] 1/384*(15*b^2*f*x^11 + 12*b^2*e*x^10 + 7*b^2*d*x^9 + 42*a*b*f*x^7 + 32*a*b*
e*x^6 + 18*a*b*d*x^5 - 5*a^2*f*x^3 - 12*a^2*e*x^2 - 21*a^2*d*x - 32*a^2*c)/
(a^2*b^4*x^12 + 3*a^3*b^3*x^8 + 3*a^4*b^2*x^4 + a^5*b) + 1/1024*(sqrt(2)*(7
*sqrt(b)*d - 5*sqrt(a)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqr
t(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(7*sqrt(b)*d - 5*sqrt(a)*f)*log(sqrt(b)*x
```

```
^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(7*sqrt(2)*
a^(1/4)*b^(3/4)*d + 5*sqrt(2)*a^(3/4)*b^(1/4)*f - 16*sqrt(a)*sqrt(b)*e)*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(7*sqrt(2)*a^(1/4)*b^(3/4)*
d + 5*sqrt(2)*a^(3/4)*b^(1/4)*f + 16*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*
(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqr
t(sqrt(a)*sqrt(b))*b^(3/4)))/(a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.99

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$$
$$= \frac{\sqrt{2}\left(8\sqrt{2}\sqrt{ab}b^2e+7\left(ab^3\right)^{\frac{1}{4}}b^2d+5\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^3b^4}$$
$$+\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{ab}b^2e+7\left(ab^3\right)^{\frac{1}{4}}b^2d+5\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^3b^4}$$
$$+\frac{\sqrt{2}\left(7\left(ab^3\right)^{\frac{1}{4}}b^2d-5\left(ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^3b^4}$$
$$-\frac{\sqrt{2}\left(7\left(ab^3\right)^{\frac{1}{4}}b^2d-5\left(ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^3b^4}$$
$$+\frac{15\,b^2fx^{11}+12\,b^2ex^{10}+7\,b^2dx^9+42\,abfx^7+32\,abex^6+18\,abdx^5-5\,a^2fx^3-12\,a^2ex^2-21\,a^2dx-32\,a}{384\left(bx^4+a\right)^3a^2b}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^2*e + 7*(a*b^3)^(1/4)*b^2*d + 5*(a*b^3
)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3
*b^4) + 1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^2*e + 7*(a*b^3)^(1/4)*b^2*d +
5*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(a^3*b^4) + 1/1024*sqrt(2)*(7*(a*b^3)^(1/4)*b^2*d - 5*(a*b^3)^(3/4)*f)*
log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) - 1/1024*sqrt(2)*(7*
(a*b^3)^(1/4)*b^2*d - 5*(a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
sqrt(a/b))/(a^3*b^4) + 1/384*(15*b^2*f*x^11 + 12*b^2*e*x^10 + 7*b^2*d*x^9 +
 42*a*b*f*x^7 + 32*a*b*e*x^6 + 18*a*b*d*x^5 - 5*a^2*f*x^3 - 12*a^2*e*x^2 -
21*a^2*d*x - 32*a^2*c)/((b*x^4 + a)^3*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 888, normalized size of antiderivative = 2.34

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$$
$$= \frac{\frac{3\,d\,x^5}{64\,a}-\frac{c}{12\,b}+\frac{e\,x^6}{12\,a}-\frac{e\,x^2}{32\,b}+\frac{7\,f\,x^7}{64\,a}-\frac{5\,f\,x^3}{384\,b}-\frac{7\,d\,x}{128\,b}+\frac{7\,b\,d\,x^9}{384\,a^2}+\frac{b\,e\,x^{10}}{32\,a^2}+\frac{5\,b\,f\,x^{11}}{128\,a^2}}{a^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^8+b^3\,x^{12}}$$
$$+\left(\sum_{k=1}^{4}\ln\left(-\frac{125\,a\,f^3-448\,b\,d\,e^2+245\,b\,d^2\,f-512\,b\,e^3\,x+\mathrm{root}(68719476736\,a^{11}\,b^7\,z^4+36700160\,a^6\,b^4\right.\right.$$
$$+\,36700160\,a^6\,b^4\,d\,f\,z^2+33554432\,a^6\,b^4\,e^2\,z^2+409600\,a^4\,b^2\,e\,f^2\,z-802816\,a^3\,b^3\,d^2\,e\,z$$
$$\left.\left.-\,8960\,a\,b\,d\,e^2\,f+2450\,a\,b\,d^2\,f^2+4096\,a\,b\,e^4+625\,a^2\,f^4+2401\,b^2\,d^4,z,k\right)\right)$$

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x)
```

```
[Out] ((3*d*x^5)/(64*a) - c/(12*b) + (e*x^6)/(12*a) - (e*x^2)/(32*b) + (7*f*x^7)/
(64*a) - (5*f*x^3)/(384*b) - (7*d*x)/(128*b) + (7*b*d*x^9)/(384*a^2) + (b*e
*x^10)/(32*a^2) + (5*b*f*x^11)/(128*a^2))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3
*a*b^2*x^8) + symsum(log(-(125*a*f^3 - 448*b*d*e^2 + 245*b*d^2*f - 512*b*e^
3*x + 1835008*root(68719476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 33
554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z -
8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^2
*d^4, z, k)^2*a^5*b^4*d + 560*b*d*e*f*x + 25088*root(68719476736*a^11*b^7*z
^4 + 36700160*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e
*f^2*z - 802816*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 409
6*a*b*e^4 + 625*a^2*f^4 + 2401*b^2*d^4, z, k)*a^2*b^3*d^2*x - 2097152*root(
68719476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*
z^2 + 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f +
2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^2*d^4, z, k)^2*a^5*b
^4*e*x - 12800*root(68719476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 3
3554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z -
 8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^
2*d^4, z, k)*a^3*b^2*f^2*x + 40960*root(68719476736*a^11*b^7*z^4 + 36700160
*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e*f^2*z - 8028
16*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 6
25*a^2*f^4 + 2401*b^2*d^4, z, k)*a^3*b^2*e*f)/(2097152*a^6*b^2))*root(68719
476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*z^2 +
 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f + 2450*
a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^2*d^4, z, k), k, 1, 4)
```

3.495 $\int x^4(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 418

$$\begin{aligned}
&\int x^4(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx\\
&=\frac{2acx\sqrt{a+bx^4}}{21b}-\frac{adx^2\sqrt{a+bx^4}}{16b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}-\frac{2a^2ex\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}\\
&\quad+\frac{1}{63}x^5(9c+7ex^2)\sqrt{a+bx^4}+\frac{fx^4(a+bx^4)^{3/2}}{10b}-\frac{(8af-15bdx^2)(a+bx^4)^{3/2}}{120b^2}\\
&\quad-\frac{a^2d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}+\frac{2a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}}\\
&\quad-\frac{a^{7/4}\left(5\sqrt{b}c+7\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}
\end{aligned}$$

```
[Out] 1/10*f*x^4*(b*x^4+a)^(3/2)/b-1/120*(-15*b*d*x^2+8*a*f)*(b*x^4+a)^(3/2)/b^2-
1/16*a^2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+2/21*a*c*x*(b*x^4+a
)^(1/2)/b-1/16*a*d*x^2*(b*x^4+a)^(1/2)/b+2/45*a*e*x^3*(b*x^4+a)^(1/2)/b+1/6
3*x^5*(7*e*x^2+9*c)*(b*x^4+a)^(1/2)-2/15*a^2*e*x*(b*x^4+a)^(1/2)/b^(3/2)/(a
^(1/2)+x^2*b^(1/2))+2/15*a^(9/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/
2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4
))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^
(1/2)/b^(7/4)/(b*x^4+a)^(1/2)-1/105*a^(7/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)
*x/a^(1/4))),1/2*2^(1/2))*(7*e*a^(1/2)+5*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*(
(b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1847, 1288, 1294, 1212, 226, 1210, 1266, 847, 794, 201, 223, 212}

$$\int x^4\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$$
$$=-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}$$
$$+\frac{2a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}}$$
$$-\frac{a^2d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^2ex\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\left(a+bx^4\right)^{3/2}\left(8af-15bdx^2\right)}{120b^2}$$
$$+\frac{1}{63}x^5\sqrt{a+bx^4}\left(9c+7ex^2\right)+\frac{2acx\sqrt{a+bx^4}}{21b}-\frac{adx^2\sqrt{a+bx^4}}{16b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}+\frac{fx^4\left(a+bx^4\right)^{3/2}}{10b}$$

```
[In] Int[x^4*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (2*a*c*x*Sqrt[a + b*x^4])/(21*b) - (a*d*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a*
e*x^3*Sqrt[a + b*x^4])/(45*b) - (2*a^2*e*x*Sqrt[a + b*x^4])/(15*b^(3/2)*(Sq
rt[a] + Sqrt[b]*x^2)) + (x^5*(9*c + 7*e*x^2)*Sqrt[a + b*x^4])/63 + (f*x^4*(
a + b*x^4)^(3/2))/(10*b) - ((8*a*f - 15*b*d*x^2)*(a + b*x^4)^(3/2))/(120*b^
2) - (a^2*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3/2)) + (2*a^(9/
4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^4]) -
 (a^(7/4)*(5*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(105*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(x^4\left(c+ex^2\right)\sqrt{a+bx^4}+x^5\left(d+fx^2\right)\sqrt{a+bx^4}\right)\,dx\\
&= \int x^4\left(c+ex^2\right)\sqrt{a+bx^4}\,dx+\int x^5\left(d+fx^2\right)\sqrt{a+bx^4}\,dx\\
&= \frac{1}{63}x^5\left(9c+7ex^2\right)\sqrt{a+bx^4}+\frac{1}{2}\text{Subst}\left(\int x^2(d+fx)\sqrt{a+bx^2}\,dx,x,x^2\right)\\
&\quad+\frac{1}{63}(2a)\int \frac{x^4(9c+7ex^2)}{\sqrt{a+bx^4}}\,dx
\end{aligned}$$

$$= \frac{2aex^3\sqrt{a+bx^4}}{45b} + \frac{1}{63}x^5\left(9c+7ex^2\right)\sqrt{a+bx^4} + \frac{fx^4\left(a+bx^4\right)^{3/2}}{10b} + \frac{\text{Subst}\left(\int x(-2af+5bdx)\sqrt{a+bx^2}\,dx,x,x^2\right)}{10b} - \frac{(2a)\int \frac{x^2\left(21ae-45bcx^2\right)}{\sqrt{a+bx^4}}\,dx}{315b}$$

$$= \frac{2acx\sqrt{a+bx^4}}{21b} + \frac{2aex^3\sqrt{a+bx^4}}{45b} + \frac{1}{63}x^5\left(9c+7ex^2\right)\sqrt{a+bx^4} + \frac{fx^4\left(a+bx^4\right)^{3/2}}{10b} - \frac{\left(8af-15bdx^2\right)\left(a+bx^4\right)^{3/2}}{120b^2} + \frac{(2a)\int \frac{-45abc-63abex^2}{\sqrt{a+bx^4}}\,dx}{945b^2} - \frac{(ad)\text{Subst}\left(\int \sqrt{a+bx^2}\,dx,x,x^2\right)}{8b}$$

$$= \frac{2acx\sqrt{a+bx^4}}{21b} - \frac{adx^2\sqrt{a+bx^4}}{16b} + \frac{2aex^3\sqrt{a+bx^4}}{45b} + \frac{1}{63}x^5\left(9c+7ex^2\right)\sqrt{a+bx^4} + \frac{fx^4\left(a+bx^4\right)^{3/2}}{10b} - \frac{\left(8af-15bdx^2\right)\left(a+bx^4\right)^{3/2}}{120b^2} - \frac{\left(a^2d\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)}{16b} + \frac{\left(2a^{5/2}e\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{15b^{3/2}} - \frac{\left(2a^2\left(5\sqrt{b}c+7\sqrt{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{105b^{3/2}}$$

$$= \frac{2acx\sqrt{a+bx^4}}{21b} - \frac{adx^2\sqrt{a+bx^4}}{16b} + \frac{2aex^3\sqrt{a+bx^4}}{45b} - \frac{2a^2ex\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{63}x^5\left(9c+7ex^2\right)\sqrt{a+bx^4} + \frac{fx^4\left(a+bx^4\right)^{3/2}}{10b} - \frac{\left(8af-15bdx^2\right)\left(a+bx^4\right)^{3/2}}{120b^2} + \frac{2a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}} - \frac{a^{7/4}\left(5\sqrt{b}c+7\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}} - \frac{\left(a^2d\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)}{16b}$$

$$= \frac{2acx\sqrt{a+bx^4}}{21b} - \frac{adx^2\sqrt{a+bx^4}}{16b} + \frac{2aex^3\sqrt{a+bx^4}}{45b} - \frac{2a^2ex\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{63}x^5\left(9c+7ex^2\right)\sqrt{a+bx^4} + \frac{fx^4\left(a+bx^4\right)^{3/2}}{10b} - \frac{\left(8af-15bdx^2\right)\left(a+bx^4\right)^{3/2}}{120b^2} - \frac{a^2d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}} + \frac{2a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}} - \frac{a^{7/4}\left(5\sqrt{b}c+7\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.66 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.48

$$\int x^4\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$$

$$= \frac{\sqrt{a+bx^4}\Big(720bcx(a+bx^4)+560bex^3(a+bx^4)+315bdx^2(a+2bx^4)+168f(a+bx^4)\left(-2a+3bx^4\right)- \frac{31}{}}{5040b^2}$$

```
[In] Integrate[x^4*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[a + b*x^4]*(720*b*c*x*(a + b*x^4) + 560*b*e*x^3*(a + b*x^4) + 315*b*d
*x^2*(a + 2*b*x^4) + 168*f*(a + b*x^4)*(-2*a + 3*b*x^4) - (315*a^(3/2)*Sqrt
[b]*d*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a] - (720*a*b*c*x*Hy
pergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a] - (560*a
*b*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/
a]))/(5040*b^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\left(-504 b^{2} f \,x^{8}-560 b^{2} e \,x^{7}-630 b^{2} d \,x^{6}-720 b^{2} c \,x^{5}-168 a b f \,x^{4}-224 a b e \,x^{3}-315 x^{2} a b d -480 a b c x +336 a^{2} f \right) \sqrt{b \,x^{4}+a}}{5040 b^{2}}-\frac{a^{2} \left(\frac{80 c \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}}{\ldots}\right.}{\ldots}$
default	$-\frac{f \left(b \,x^{4}+a \right)^{\frac{3}{2}} \left(-3 b \,x^{4}+2 a \right)}{30 b^{2}}+e \left(\frac{x^{7} \sqrt{b \,x^{4}+a}}{9}+\frac{2 a \,x^{3} \sqrt{b \,x^{4}+a}}{45 b}-\frac{2 i a^{\frac{5}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(F\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-E\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right.}{15 b^{\frac{3}{2}} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}\right.$
elliptic	$\frac{f \,x^{8} \sqrt{b \,x^{4}+a}}{10}+\frac{e \,x^{7} \sqrt{b \,x^{4}+a}}{9}+\frac{d \,x^{6} \sqrt{b \,x^{4}+a}}{8}+\frac{c \,x^{5} \sqrt{b \,x^{4}+a}}{7}+\frac{a f \,x^{4} \sqrt{b \,x^{4}+a}}{30 b}+\frac{2 a e \,x^{3} \sqrt{b \,x^{4}+a}}{45 b}+\frac{a d \,x^{2} \sqrt{b \,x^{4}+a}}{16 b}+\frac{2 a c x}{\ldots}$

```
[In] int(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5040*(-504*b^2*f*x^8-560*b^2*e*x^7-630*b^2*d*x^6-720*b^2*c*x^5-168*a*b*f
*x^4-224*a*b*e*x^3-315*a*b*d*x^2-480*a*b*c*x+336*a^2*f)/b^2*(b*x^4+a)^(1/2)
-1/840*a^2/b*(80*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^
(1/2))^(1/2),I)+112*I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2)
,I))+105/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.51

$$\int x^4\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx = -\frac{1344\,a^2\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-315\,a^2\sqrt{b}dx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+192\,(\ldots}{\ldots}$$

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/10080*(1344*a^2*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/
x), -1) - 315*a^2*sqrt(b)*d*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2
- a) + 192*(5*a*b*c - 7*a^2*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a
/b)^(1/4)/x), -1) - 2*(504*b^2*f*x^9 + 560*b^2*e*x^8 + 630*b^2*d*x^7 + 720*
b^2*c*x^6 + 168*a*b*f*x^5 + 224*a*b*e*x^4 + 315*a*b*d*x^3 + 480*a*b*c*x^2 -
 336*a^2*f*x - 672*a^2*e)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.60

$$\int x^4(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \frac{a^{\frac{3}{2}}dx^2}{16b\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}dx^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} - \frac{a^2d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} + f\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right) + \frac{bdx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] a**(3/2)*d*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*c*x**5*gamma(5/4)*hyper
((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*
d*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*e*x**7*gamma(7/4)*hyper((-1/2, 7/4
), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) - a**2*d*asinh(sqrt(b
)*x**2/sqrt(a))/(16*b**(3/2)) + f*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**
2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)),
(sqrt(a)*x**8/8, True)) + b*d*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int x^4(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \int \sqrt{bx^4+a}(fx^3+ex^2+dx+c)x^4\,dx$$

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

Giac [F]

$$\int x^4(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int \sqrt{bx^4+a}(fx^3+ex^2+dx+c)x^4\,dx$$

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int x^4\,\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

```
[In] int(x^4*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
[Out] int(x^4*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.496 $\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 394

$$\begin{aligned}
&\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx\\
&=\frac{2adx\sqrt{a+bx^4}}{21b}-\frac{aex^2\sqrt{a+bx^4}}{16b}+\frac{2afx^3\sqrt{a+bx^4}}{45b}-\frac{2a^2fx\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}\\
&+\frac{1}{63}x^5(9d+7fx^2)\sqrt{a+bx^4}+\frac{(4c+3ex^2)(a+bx^4)^{3/2}}{24b}-\frac{a^2e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}\\
&+\frac{2a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}}\\
&-\frac{a^{7/4}\left(5\sqrt{b}d+7\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}
\end{aligned}$$

```
[Out] 1/24*(3*e*x^2+4*c)*(b*x^4+a)^(3/2)/b-1/16*a^2*e*arctanh(x^2*b^(1/2)/(b*x^4+
a)^(1/2))/b^(3/2)+2/21*a*d*x*(b*x^4+a)^(1/2)/b-1/16*a*e*x^2*(b*x^4+a)^(1/2)
/b+2/45*a*f*x^3*(b*x^4+a)^(1/2)/b+1/63*x^5*(7*f*x^2+9*d)*(b*x^4+a)^(1/2)-2/
15*a^2*f*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))+2/15*a^(9/4)*f*(co
s(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*El
lipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))
*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)-1/105*a^
(7/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(
1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(7*f*a^(1/2)
+5*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/
2)/b^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1847, 1266, 794, 201, 223, 212, 1288, 1294, 1212, 226, 1210}

$$\int x^3\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$$
$$=-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}$$
$$+\frac{2a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}}$$
$$-\frac{a^2e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^2fx\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\left(a+bx^4\right)^{3/2}\left(4c+3ex^2\right)}{24b}$$
$$+\frac{1}{63}x^5\sqrt{a+bx^4}\left(9d+7fx^2\right)+\frac{2adx\sqrt{a+bx^4}}{21b}-\frac{aex^2\sqrt{a+bx^4}}{16b}+\frac{2afx^3\sqrt{a+bx^4}}{45b}$$

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (2*a*d*x*Sqrt[a + b*x^4])/(21*b) - (a*e*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a*
f*x^3*Sqrt[a + b*x^4])/(45*b) - (2*a^2*f*x*Sqrt[a + b*x^4])/(15*b^(3/2)*(Sq
rt[a] + Sqrt[b]*x^2)) + (x^5*(9*d + 7*f*x^2)*Sqrt[a + b*x^4])/63 + ((4*c +
3*e*x^2)*(a + b*x^4)^(3/2))/(24*b) - (a^2*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
b*x^4]])/(16*b^(3/2)) + (2*a^(9/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(15*b^(7/4)*Sqrt[a + b*x^4]) - (a^(7/4)*(5*Sqrt[b]*d + 7*Sqrt[a]*f)*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
```

```
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(x^3\left(c+ex^2\right)\sqrt{a+bx^4}+x^4\left(d+fx^2\right)\sqrt{a+bx^4}\right)dx\\
&= \int x^3\left(c+ex^2\right)\sqrt{a+bx^4}\,dx+\int x^4\left(d+fx^2\right)\sqrt{a+bx^4}\,dx\\
&= \frac{1}{63}x^5\left(9d+7fx^2\right)\sqrt{a+bx^4}+\frac{1}{2}\text{Subst}\left(\int x(c+ex)\sqrt{a+bx^2}\,dx,x,x^2\right)\\
&\quad+\frac{1}{63}(2a)\int \frac{x^4(9d+7fx^2)}{\sqrt{a+bx^4}}\,dx\\
&= \frac{2afx^3\sqrt{a+bx^4}}{45b}+\frac{1}{63}x^5\left(9d+7fx^2\right)\sqrt{a+bx^4}+\frac{\left(4c+3ex^2\right)\left(a+bx^4\right)^{3/2}}{24b}\\
&\quad-\frac{(2a)\int \frac{x^2\left(21af-45bdx^2\right)}{\sqrt{a+bx^4}}\,dx}{315b}-\frac{(ae)\text{Subst}\left(\int \sqrt{a+bx^2}\,dx,x,x^2\right)}{8b}\\
&= \frac{2adx\sqrt{a+bx^4}}{21b}-\frac{aex^2\sqrt{a+bx^4}}{16b}+\frac{2afx^3\sqrt{a+bx^4}}{45b}+\frac{1}{63}x^5\left(9d+7fx^2\right)\sqrt{a+bx^4}\\
&\quad+\frac{\left(4c+3ex^2\right)\left(a+bx^4\right)^{3/2}}{24b}+\frac{(2a)\int \frac{-45abd-63abfx^2}{\sqrt{a+bx^4}}\,dx}{945b^2}-\frac{\left(a^2e\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)}{16b}
\end{aligned}
$$

$$= \frac{2adx\sqrt{a+bx^4}}{21b} - \frac{aex^2\sqrt{a+bx^4}}{16b} + \frac{2afx^3\sqrt{a+bx^4}}{45b} + \frac{1}{63}x^5(9d+7fx^2)\sqrt{a+bx^4} + \frac{(4c+3ex^2)(a+bx^4)^{3/2}}{24b} - \frac{(a^2e)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)}{16b} + \frac{(2a^{5/2}f)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{15b^{3/2}} - \frac{\left(2a^2\left(5\sqrt{b}d+7\sqrt{a}f\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{105b^{3/2}}$$

$$= \frac{2adx\sqrt{a+bx^4}}{21b} - \frac{aex^2\sqrt{a+bx^4}}{16b} + \frac{2afx^3\sqrt{a+bx^4}}{45b} - \frac{2a^2fx\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{63}x^5(9d+7fx^2)\sqrt{a+bx^4} + \frac{(4c+3ex^2)(a+bx^4)^{3/2}}{24b} - \frac{a^2e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}} + \frac{2a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}} - \frac{a^{7/4}\left(5\sqrt{b}d+7\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.59 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.55

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \frac{\sqrt{a+bx^4}\left(168\sqrt{b}c(a+bx^4)+144\sqrt{b}dx(a+bx^4)+112\sqrt{b}fx^3(a+bx^4)+63e\left(\sqrt{b}x^2(a+2bx^4)-\frac{a^{3/2}\text{arc}}{\sqrt{}}\right.\right.}{1008b^{3/2}}$$

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[a + b*x^4]*(168*Sqrt[b]*c*(a + b*x^4) + 144*Sqrt[b]*d*x*(a + b*x^4) +
 112*Sqrt[b]*f*x^3*(a + b*x^4) + 63*e*(Sqrt[b]*x^2*(a + 2*b*x^4) - (a^(3/2)
*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a]) - (144*a*Sqrt[b]*d*x*
Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a] - (112
*a*Sqrt[b]*f*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 +
(b*x^4)/a]))/(1008*b^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.03 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.68

method	result
risch	$\frac{(560bf\,x^7+630be\,x^6+720bd\,x^5+840bc\,x^4+224af\,x^3+315ae\,x^2+480adx+840ac)\sqrt{b\,x^4+a}}{5040b} - \frac{a^2\left(\frac{80d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right.}{}$
default	$f\left(\frac{x^7\sqrt{b\,x^4+a}}{9}+\frac{2a\,x^3\sqrt{b\,x^4+a}}{45b}-\frac{2ia^{\frac{5}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{15b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+e\left(\frac{x^2(b\,x^4+a)^{\frac{3}{2}}}{8b}-\right.$
elliptic	$\frac{f\,x^7\sqrt{b\,x^4+a}}{9}+\frac{e\,x^6\sqrt{b\,x^4+a}}{8}+\frac{d\,x^5\sqrt{b\,x^4+a}}{7}+\frac{c\,x^4\sqrt{b\,x^4+a}}{6}+\frac{2af\,x^3\sqrt{b\,x^4+a}}{45b}+\frac{ae\,x^2\sqrt{b\,x^4+a}}{16b}+\frac{2adx\sqrt{b\,x^4+a}}{21b}+\frac{ac\sqrt{b}}{6}$

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5040*(560*b*f*x^7+630*b*e*x^6+720*b*d*x^5+840*b*c*x^4+224*a*f*x^3+315*a*e
*x^2+480*a*d*x+840*a*c)/b*(b*x^4+a)^(1/2)-1/840*a^2/b*(80*d/(I/a^(1/2)*b^(1
/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/
(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+112*I*f*a^(1/2)/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2
)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2)
,I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+105/2*e*ln(x^2*b^(1/2)+(b*x^4
+a)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.52

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx =$$

$$-\frac{1344\,a^2\sqrt{b}fx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-315\,a^2\sqrt{b}ex\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+192\,(}{}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/10080*(1344*a^2*sqrt(b)*f*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/
x), -1) - 315*a^2*sqrt(b)*e*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2
- a) + 192*(5*a*b*d - 7*a^2*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a
/b)^(1/4)/x), -1) - 2*(560*b^2*f*x^8 + 630*b^2*e*x^7 + 720*b^2*d*x^6 + 840*
b^2*c*x^5 + 224*a*b*f*x^4 + 315*a*b*e*x^3 + 480*a*b*d*x^2 + 840*a*b*c*x - 6
72*a^2*f)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 3.35 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.54

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \frac{a^{\frac{3}{2}}ex^2}{16b\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}ex^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} - \frac{a^2e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} + c\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + \frac{bex^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] a**(3/2)*e*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*d*x**5*gamma(5/4)*hyper
((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*
e*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*f*x**7*gamma(7/4)*hyper((-1/2, 7/4
), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) - a**2*e*asinh(sqrt(b
)*x**2/sqrt(a))/(16*b**(3/2)) + c*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a
 + b*x**4)**(3/2)/(6*b), True)) + b*e*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \int \sqrt{bx^4+a}(fx^3+ex^2+dx+c)x^3\,dx$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^4 + a)^(3/2)*c/b + integrate((f*x^6 + e*x^5 + d*x^4)*sqrt(b*x^4 +
a), x)
```

Giac [F]

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int\sqrt{bx^4+a}(fx^3+ex^2+dx+c)x^3\,dx$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int x^3\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

```
[In] int(x^3*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
[Out] int(x^3*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.497 $\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 369

$$\begin{aligned}
&\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx\\
&=\frac{2aex\sqrt{a+bx^4}}{21b}-\frac{afx^2\sqrt{a+bx^4}}{16b}+\frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}\\
&\quad+\frac{1}{35}x^3(7c+5ex^2)\sqrt{a+bx^4}+\frac{(4d+3fx^2)(a+bx^4)^{3/2}}{24b}-\frac{a^2f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}\\
&\quad-\frac{2a^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}\\
&\quad+\frac{a^{5/4}\left(21\sqrt{b}c-5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}
\end{aligned}$$

```
[Out] 1/24*(3*f*x^2+4*d)*(b*x^4+a)^(3/2)/b-1/16*a^2*f*arctanh(x^2*b^(1/2)/(b*x^4+
a)^(1/2))/b^(3/2)+2/21*a*e*x*(b*x^4+a)^(1/2)/b-1/16*a*f*x^2*(b*x^4+a)^(1/2)
/b+1/35*x^3*(5*e*x^2+7*c)*(b*x^4+a)^(1/2)+2/5*a*c*x*(b*x^4+a)^(1/2)/b^(1/2)
/(a^(1/2)+x^2*b^(1/2))-2/5*a^(5/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(
1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1
/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2
)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+1/105*a^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/
4)*x/a^(1/4))),1/2*2^(1/2))*(-5*e*a^(1/2)+21*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2
))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1847, 1288, 1294, 1212, 226, 1210, 1266, 794, 201, 223, 212}

$$\int x^2\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$$
$$=\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{b}c-5\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}$$
$$-\frac{2a^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}-\frac{a^2f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}$$
$$+\frac{1}{35}x^3\sqrt{a+bx^4}\left(7c+5ex^2\right)+\frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\left(a+bx^4\right)^{3/2}\left(4d+3fx^2\right)}{24b}+\frac{2aex\sqrt{a+bx^4}}{21b}-\frac{afx^2\sqrt{a+}}{16b}$$

```
[In] Int[x^2*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (2*a*e*x*Sqrt[a + b*x^4])/(21*b) - (a*f*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a*
c*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x^3*(7*c + 5*e*
x^2)*Sqrt[a + b*x^4])/35 + ((4*d + 3*f*x^2)*(a + b*x^4)^(3/2))/(24*b) - (a^
2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3/2)) - (2*a^(5/4)*c*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(5/4)
*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqr
t[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*
b^(5/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
```

```
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(x^2\left(c+ex^2\right)\sqrt{a+bx^4}+x^3\left(d+fx^2\right)\sqrt{a+bx^4}\right)dx\\
&= \int x^2\left(c+ex^2\right)\sqrt{a+bx^4}\,dx+\int x^3\left(d+fx^2\right)\sqrt{a+bx^4}\,dx\\
&= \frac{1}{35}x^3\left(7c+5ex^2\right)\sqrt{a+bx^4}+\frac{1}{2}\text{Subst}\left(\int x(d+fx)\sqrt{a+bx^2}\,dx,x,x^2\right)\\
&\quad+\frac{1}{35}(2a)\int\frac{x^2(7c+5ex^2)}{\sqrt{a+bx^4}}\,dx\\
&= \frac{2aex\sqrt{a+bx^4}}{21b}+\frac{1}{35}x^3\left(7c+5ex^2\right)\sqrt{a+bx^4}+\frac{\left(4d+3fx^2\right)\left(a+bx^4\right)^{3/2}}{24b}\\
&\quad-\frac{(2a)\int\frac{5ae-21bcx^2}{\sqrt{a+bx^4}}\,dx}{105b}-\frac{(af)\text{Subst}\left(\int\sqrt{a+bx^2}\,dx,x,x^2\right)}{8b}\\
&= \frac{2aex\sqrt{a+bx^4}}{21b}-\frac{afx^2\sqrt{a+bx^4}}{16b}+\frac{1}{35}x^3\left(7c+5ex^2\right)\sqrt{a+bx^4}\\
&\quad+\frac{\left(4d+3fx^2\right)\left(a+bx^4\right)^{3/2}}{24b}-\frac{\left(2a^{3/2}c\right)\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{5\sqrt{b}}\\
&\quad+\frac{\left(2a^{3/2}\left(21\sqrt{b}c-5\sqrt{a}e\right)\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx}{105b}-\frac{\left(a^2f\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)}{16b}
\end{aligned}$$

$$= \frac{2aex\sqrt{a+bx^4}}{21b} - \frac{afx^2\sqrt{a+bx^4}}{16b} + \frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

$$+ \frac{1}{35}x^3\left(7c+5ex^2\right)\sqrt{a+bx^4} + \frac{(4d+3fx^2)\left(a+bx^4\right)^{3/2}}{24b}$$

$$- \frac{2a^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}$$

$$+ \frac{a^{5/4}\left(21\sqrt{b}c-5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}$$

$$- \frac{(a^2f)\,\mathrm{Subst}\left(\int\frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)}{16b}$$

$$= \frac{2aex\sqrt{a+bx^4}}{21b} - \frac{afx^2\sqrt{a+bx^4}}{16b} + \frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

$$+ \frac{1}{35}x^3\left(7c+5ex^2\right)\sqrt{a+bx^4} + \frac{(4d+3fx^2)\left(a+bx^4\right)^{3/2}}{24b} - \frac{a^2f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}$$

$$- \frac{2a^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}$$

$$+ \frac{a^{5/4}\left(21\sqrt{b}c-5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.69 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.49

$$\int x^2\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\frac{1}{336}\sqrt{a+bx^4}\left(\frac{56d(a+bx^4)}{b}+\frac{48ex(a+bx^4)}{b}+\frac{21fx^2(a+2bx^4)}{b}-\frac{21a^{3/2}f\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{b^{3/2}\sqrt{1+\frac{bx^4}{a}}}-\frac{48aex\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{b\sqrt{1+\frac{bx^4}{a}}}+\frac{112cx^3\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)$$

```
[In] Integrate[x^2*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
[Out] (Sqrt[a + b*x^4]*((56*d*(a + b*x^4))/b + (48*e*x*(a + b*x^4))/b + (21*f*x^2
*(a + 2*b*x^4))/b - (21*a^(3/2)*f*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/(b^(3/2)*
Sqrt[1 + (b*x^4)/a]) - (48*a*e*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4
)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (112*c*x^3*Hypergeometric2F1[-1/2, 3/4, 7/
4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/336
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\left(210bf\,x^6+240be\,x^5+280bd\,x^4+336bc\,x^3+105x^2af+160aex+280ad\right)\sqrt{b\,x^4+a}}{1680b}-\frac{a\left(\frac{80ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{336i\sqrt{}}{}\right.}{}$
default	$f\left(\frac{x^2\left(b\,x^4+a\right)^{\frac{3}{2}}}{8b}-\frac{a\,x^2\sqrt{b\,x^4+a}}{16b}-\frac{a^2\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{16b^{\frac{3}{2}}}\right)+e\left(\frac{x^5\sqrt{b\,x^4+a}}{7}+\frac{2ax\sqrt{b\,x^4+a}}{21b}-\frac{2a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}}{}}}{21b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b}}\right.$
elliptic	$\frac{f\,x^6\sqrt{b\,x^4+a}}{8}+\frac{e\,x^5\sqrt{b\,x^4+a}}{7}+\frac{d\,x^4\sqrt{b\,x^4+a}}{6}+\frac{c\,x^3\sqrt{b\,x^4+a}}{5}+\frac{af\,x^2\sqrt{b\,x^4+a}}{16b}+\frac{2aex\sqrt{b\,x^4+a}}{21b}+\frac{ad\sqrt{b\,x^4+a}}{6b}-\frac{2a^2e\sqrt{1-}}{}$

```
[In] int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1680*(210*b*f*x^6+240*b*e*x^5+280*b*d*x^4+336*b*c*x^3+105*a*f*x^2+160*a*e
*x+280*a*d)/b*(b*x^4+a)^(1/2)-1/840*a/b*(80*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-336*I*b^(1/2)*c*a^(1/2)/(I/a^(
1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^
2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Elliptic
E(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+105/2*a*f*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))
/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.52

$$\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$$
$$= \frac{1344\,ab^{\frac{3}{2}}cx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+105\,a^2\sqrt{b}fx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)-64\,(21\,a}{}$$

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3360*(1344*a*b^(3/2)*c*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x),
-1) + 105*a^2*sqrt(b)*f*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a)
 - 64*(21*a*b*c + 5*a*b*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^
(1/4)/x), -1) + 2*(210*b^2*f*x^7 + 240*b^2*e*x^6 + 280*b^2*d*x^5 + 336*b^2*
c*x^4 + 105*a*b*f*x^3 + 160*a*b*e*x^2 + 280*a*b*d*x + 672*a*b*c)*sqrt(b*x^4
 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.57

$$\int x^2\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\frac{a^{\frac{3}{2}}fx^2}{16b\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{a}ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}fx^6}{16\sqrt{1+\frac{bx^4}{a}}}-\frac{a^2f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+d\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\\frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)+\frac{bfx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] a**(3/2)*f*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*c*x**3*gamma(3/4)*hyper
((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*e*
x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gam
ma(9/4)) + 3*sqrt(a)*f*x**6/(16*sqrt(1 + b*x**4/a)) - a**2*f*asinh(sqrt(b)*
x**2/sqrt(a))/(16*b**(3/2)) + d*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a +
 b*x**4)**(3/2)/(6*b), True)) + b*f*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int x^2\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\int\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)x^2\,dx$$

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

Giac [F]

$$\int x^2 (c + dx + ex^2 + fx^3) \sqrt{a + bx^4} \, dx = \int \sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)x^2 \, dx$$

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (c + dx + ex^2 + fx^3) \sqrt{a + bx^4} \, dx = \int x^2 \sqrt{b\,x^4 + a} \,(f\,x^3 + e\,x^2 + d\,x + c) \, dx$$

```
[In] int(x^2*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
[Out] int(x^2*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.498 $\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 354

$$\begin{aligned}\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx &= \frac{2afx\sqrt{a+bx^4}}{21b}+\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}\\ &\quad+\frac{1}{35}x^3\left(7d+5fx^2\right)\sqrt{a+bx^4}+\frac{e(a+bx^4)^{3/2}}{6b}+\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}\\ &\quad-\frac{2a^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}\\ &\quad+\frac{a^{5/4}\left(21\sqrt{b}d-5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}\end{aligned}$$

```
[Out] 1/6*e*(b*x^4+a)^(3/2)/b+1/4*a*c*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2
)+2/21*a*f*x*(b*x^4+a)^(1/2)/b+1/4*c*x^2*(b*x^4+a)^(1/2)+1/35*x^3*(5*f*x^2+
7*d)*(b*x^4+a)^(1/2)+2/5*a*d*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2)
)-2/5*a^(5/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^
4+a)^(1/2)+1/105*a^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^
(1/2))*(-5*f*a^(1/2)+21*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2
)+x^2*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1847, 1262, 655, 201, 223, 212, 1288, 1294, 1212, 226, 1210}

$$\int x\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$$
$$=\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{b}d-5\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}$$
$$-\frac{2a^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{a\text{carctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{1}{4}cx^2\sqrt{a+bx^4}$$
$$+\frac{1}{35}x^3\sqrt{a+bx^4}\left(7d+5fx^2\right)+\frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\left(a+bx^4\right)^{3/2}}{6b}+\frac{2afx\sqrt{a+bx^4}}{21b}$$

```
[In] Int[x*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (2*a*f*x*Sqrt[a + b*x^4])/(21*b) + (c*x^2*Sqrt[a + b*x^4])/4 + (2*a*d*x*Sqr
t[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x^3*(7*d + 5*f*x^2)*Sq
rt[a + b*x^4])/35 + (e*(a + b*x^4)^(3/2))/(6*b) + (a*c*ArcTanh[(Sqrt[b]*x^2
)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (2*a^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(5/4)*(21*Sqrt[b]*d - 5*Sqrt[a
]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
```

```
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(x\left(c+ex^2\right)\sqrt{a+bx^4}+x^2\left(d+fx^2\right)\sqrt{a+bx^4}\right)dx \\
&= \int x\left(c+ex^2\right)\sqrt{a+bx^4}\,dx+\int x^2\left(d+fx^2\right)\sqrt{a+bx^4}\,dx \\
&= \frac{1}{35}x^3\left(7d+5fx^2\right)\sqrt{a+bx^4}+\frac{1}{2}\text{Subst}\left(\int (c+ex)\sqrt{a+bx^2}\,dx,x,x^2\right) \\
&\quad +\frac{1}{35}(2a)\int \frac{x^2(7d+5fx^2)}{\sqrt{a+bx^4}}\,dx \\
&= \frac{2afx\sqrt{a+bx^4}}{21b}+\frac{1}{35}x^3\left(7d+5fx^2\right)\sqrt{a+bx^4}+\frac{e(a+bx^4)^{3/2}}{6b} \\
&\quad -\frac{(2a)\int \frac{5af-21bdx^2}{\sqrt{a+bx^4}}\,dx}{105b}+\frac{1}{2}c\text{Subst}\left(\int \sqrt{a+bx^2}\,dx,x,x^2\right) \\
&= \frac{2afx\sqrt{a+bx^4}}{21b}+\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{1}{35}x^3\left(7d+5fx^2\right)\sqrt{a+bx^4}+\frac{e(a+bx^4)^{3/2}}{6b} \\
&\quad +\frac{1}{4}(ac)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\left(2a^{3/2}d\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{5\sqrt{b}} \\
&\quad +\frac{\left(2a^{3/2}\left(21\sqrt{b}d-5\sqrt{a}f\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{105b}
\end{aligned}
$$

$$= \frac{2afx\sqrt{a+bx^4}}{21b} + \frac{1}{4}cx^2\sqrt{a+bx^4} + \frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{35}x^3\left(7d+5fx^2\right)\sqrt{a+bx^4}$$
$$+ \frac{e(a+bx^4)^{3/2}}{6b} - \frac{2a^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}$$
$$+ \frac{a^{5/4}\left(21\sqrt{b}d-5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}$$
$$+ \frac{1}{4}(ac)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)$$
$$= \frac{2afx\sqrt{a+bx^4}}{21b} + \frac{1}{4}cx^2\sqrt{a+bx^4} + \frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$+ \frac{1}{35}x^3\left(7d+5fx^2\right)\sqrt{a+bx^4} + \frac{e(a+bx^4)^{3/2}}{6b} + \frac{ac\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}$$
$$- \frac{2a^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}$$
$$+ \frac{a^{5/4}\left(21\sqrt{b}d-5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.60

$$\int x\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$$
$$= \frac{\sqrt{a+bx^4}\left(14ae\sqrt{1+\frac{bx^4}{a}}+12afx\sqrt{1+\frac{bx^4}{a}}+21bcx^2\sqrt{1+\frac{bx^4}{a}}+14bex^4\sqrt{1+\frac{bx^4}{a}}+12bfx^5\sqrt{1+\frac{bx^4}{a}}+2\right.}{}$$

```
[In] Integrate[x*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[a + b*x^4]*(14*a*e*Sqrt[1 + (b*x^4)/a] + 12*a*f*x*Sqrt[1 + (b*x^4)/a]
 + 21*b*c*x^2*Sqrt[1 + (b*x^4)/a] + 14*b*e*x^4*Sqrt[1 + (b*x^4)/a] + 12*b*f
*x^5*Sqrt[1 + (b*x^4)/a] + 21*Sqrt[a]*Sqrt[b]*c*ArcSinh[(Sqrt[b]*x^2)/Sqrt[
a]] - 12*a*f*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)] + 28*b*d*x^3
*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)]))/(84*b*Sqrt[1 + (b*x^4)/a
])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.72

method	result
risch	$\frac{\left(60 b f \,x^{5}+70 b e \,x^{4}+84 b d \,x^{3}+105 c b \,x^{2}+40 a f x +70 a e \right) \sqrt{b \,x^{4}+a}}{420 b}-\frac{a \left(\frac{20 a f \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}-\frac{84 i \sqrt{b}\, d \sqrt{a}\, \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}}{}\right.}{}$
default	$f \left(\frac{x^{5} \sqrt{b \,x^{4}+a}}{7}+\frac{2 a x \sqrt{b \,x^{4}+a}}{21 b}-\frac{2 a^{2} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{21 b \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}\right)+\frac{e \left(b \,x^{4}+a \right)^{\frac{3}{2}}}{6 b}+d \left(\frac{x^{3} \sqrt{b \,x^{4}+a}}{5}+\frac{2 i a^{\frac{3}{2}}}{}\right.$
elliptic	$\frac{f \,x^{5} \sqrt{b \,x^{4}+a}}{7}+\frac{e \,x^{4} \sqrt{b \,x^{4}+a}}{6}+\frac{d \,x^{3} \sqrt{b \,x^{4}+a}}{5}+\frac{c \,x^{2} \sqrt{b \,x^{4}+a}}{4}+\frac{2 a f x \sqrt{b \,x^{4}+a}}{21 b}+\frac{a e \sqrt{b \,x^{4}+a}}{6 b}-\frac{2 a^{2} f \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}}{}}}{21 b \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b}}$

```
[In] int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/420*(60*b*f*x^5+70*b*e*x^4+84*b*d*x^3+105*b*c*x^2+40*a*f*x+70*a*e)/b*(b*x
^4+a)^(1/2)-1/210*a/b*(20*a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I
/a^(1/2)*b^(1/2))^(1/2),I)-84*I*b^(1/2)*d*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^
(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1
/2))^(1/2),I))-105/2*b^(1/2)*c*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.48

$$\int x\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx = \frac{336\,a\sqrt{b}dx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+105\,a\sqrt{b}cx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)-16\,(21\,ad}{}$$

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/840*(336*a*sqrt(b)*d*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1
) + 105*a*sqrt(b)*c*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) - 1
6*(21*a*d + 5*a*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x)
, -1) + 2*(60*b*f*x^6 + 70*b*e*x^5 + 84*b*d*x^4 + 105*b*c*x^3 + 40*a*f*x^2
+ 70*a*e*x + 168*a*d)*sqrt(b*x^4 + a))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.45

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \frac{\sqrt{a}cx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{\sqrt{a}dx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}fx^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{ac\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}} + e\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*d*x**3*gamma(3/4)*hyper((-1/2
, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*f*x**5*g
amma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4
)) + a*c*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + e*Piecewise((sqrt(a)*x**
4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))
```

Maxima [F]

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \int \sqrt{bx^4+a}(fx^3+ex^2+dx+c)x\,dx$$

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(a*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 + a)/x^2
))/sqrt(b) + 2*sqrt(b*x^4 + a)*a/((b - (b*x^4 + a)/x^4)*x^2))*c + integrate
(sqrt(b*x^4 + a)*(f*x^4 + e*x^3 + d*x^2), x)
```

Giac [F]

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \int \sqrt{bx^4+a}(fx^3+ex^2+dx+c)x\,dx$$

[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x, x)

Mupad [F(-1)]

Timed out.

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \int x\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

[In] int(x*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)

[Out] int(x*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)

3.499 $\int (c + dx + ex^2 + fx^3) \sqrt{a + bx^4} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 331

$$\int (c + dx + ex^2 + fx^3) \sqrt{a + bx^4} \, dx = \frac{1}{4} dx^2 \sqrt{a + bx^4} + \frac{2aex\sqrt{a + bx^4}}{5\sqrt{b} \left(\sqrt{a} + \sqrt{b}x^2\right)} + \frac{1}{15} x (5c + 3ex^2) \sqrt{a + bx^4} + \frac{f(a + bx^4)^{3/2}}{6b} + \frac{ad \operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}} - \frac{2a^{5/4} e \left(\sqrt{a} + \sqrt{b}x^2\right) \sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}} E\left(2 \arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) | \frac{1}{2}\right)}{5b^{3/4}\sqrt{a + bx^4}} + \frac{a^{3/4} \left(5\sqrt{b}c + 3\sqrt{a}e\right) \left(\sqrt{a} + \sqrt{b}x^2\right) \sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}} \operatorname{EllipticF}\left(2 \arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15b^{3/4}\sqrt{a + bx^4}}$$

```
[Out] 1/6*f*(b*x^4+a)^(3/2)/b+1/4*a*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2
)+1/4*d*x^2*(b*x^4+a)^(1/2)+1/15*x*(3*e*x^2+5*c)*(b*x^4+a)^(1/2)+2/5*a*e*x*
(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))-2/5*a^(5/4)*e*(cos(2*arctan(b
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(
2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/
(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+1/15*a^(3/4)*(cos(2*
arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ellipt
icF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*e*a^(1/2)+5*c*b^(1/2))
*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b
*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {1899, 1191, 1212, 226, 1210, 1262, 655, 201, 223, 212}

$$\begin{aligned}&\int \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx\\&=\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}\\&\quad-\frac{2a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}+\frac{ad\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}\\&\quad+\frac{1}{15}x\sqrt{a+bx^4}\left(5c+3ex^2\right)+\frac{1}{4}dx^2\sqrt{a+bx^4}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\left(a+bx^4\right)^{3/2}}{6b}\end{aligned}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (d*x^2*Sqrt[a + b*x^4])/4 + (2*a*e*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] +
 Sqrt[b]*x^2)) + (x*(5*c + 3*e*x^2)*Sqrt[a + b*x^4])/15 + (f*(a + b*x^4)^(3
/2))/(6*b) + (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (2*
a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4
]) + (a^(3/4)*(5*Sqrt[b]*c + 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\left(c+ex^2\right)\sqrt{a+bx^4}+x\left(d+fx^2\right)\sqrt{a+bx^4}\right)dx\\
&= \int \left(c+ex^2\right)\sqrt{a+bx^4}\,dx+\int x\left(d+fx^2\right)\sqrt{a+bx^4}\,dx\\
&= \frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+bx^4}+\frac{1}{15}\int \frac{10ac+6aex^2}{\sqrt{a+bx^4}}\,dx+\frac{1}{2}\text{Subst}\left(\int (d+fx)\sqrt{a+bx^2}\,dx,x,x^2\right)\\
&= \frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+bx^4}+\frac{f\left(a+bx^4\right)^{3/2}}{6b}+\frac{1}{2}d\text{Subst}\left(\int \sqrt{a+bx^2}\,dx,x,x^2\right)\\
&\quad -\frac{\left(2a^{3/2}e\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{5\sqrt{b}}+\frac{1}{15}\left(2a\left(5c+\frac{3\sqrt{a}e}{\sqrt{b}}\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx\\
&= \frac{1}{4}dx^2\sqrt{a+bx^4}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+bx^4}\\
&\quad +\frac{f\left(a+bx^4\right)^{3/2}}{6b}-\frac{2a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}\\
&\quad +\frac{a^{3/4}\left(5\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}\\
&\quad +\frac{1}{4}(ad)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)\\
&= \frac{1}{4}dx^2\sqrt{a+bx^4}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+bx^4}\\
&\quad +\frac{f\left(a+bx^4\right)^{3/2}}{6b}-\frac{2a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}\\
&\quad +\frac{a^{3/4}\left(5\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}\\
&\quad +\frac{1}{4}(ad)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)
\end{aligned}
$$

$$= \frac{1}{4}dx^2\sqrt{a+bx^4} + \frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+bx^4} + \frac{f(a+bx^4)^{3/2}}{6b} + \frac{ad\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}} - \frac{2a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}} + \frac{a^{3/4}\left(5\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.52

$$\int \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$$

$$= \frac{\sqrt{a+bx^4}\left(2af\sqrt{1+\frac{bx^4}{a}} + 3bdx^2\sqrt{1+\frac{bx^4}{a}} + 2bfx^4\sqrt{1+\frac{bx^4}{a}} + 3\sqrt{a}\sqrt{b}d\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right) + 12bcx\,\text{Hypergeon}\right.}{12b\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[a + b*x^4]*(2*a*f*Sqrt[1 + (b*x^4)/a] + 3*b*d*x^2*Sqrt[1 + (b*x^4)/a]
 + 2*b*f*x^4*Sqrt[1 + (b*x^4)/a] + 3*Sqrt[a]*Sqrt[b]*d*ArcSinh[(Sqrt[b]*x^2
)/Sqrt[a]] + 12*b*c*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)] + 4*b
*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)]))/(12*b*Sqrt[1 + (b*
x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.73

method	result
risch	$\frac{(10bf\,x^4+12be\,x^3+15bd\,x^2+20bcx+10af)\sqrt{b\,x^4+a}}{60b}+\frac{a\left(\frac{20c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{12ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,(}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,}}\right.}{30}$
default	$c\left(\frac{x\sqrt{b\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+\frac{f\,(b\,x^4+a)^{\frac{3}{2}}}{6b}+e\left(\frac{x^3\sqrt{b\,x^4+a}}{5}+\frac{2ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{}}{}}}{5\sqrt{}}\right.$
elliptic	$\frac{f\,x^4\sqrt{b\,x^4+a}}{6}+\frac{e\,x^3\sqrt{b\,x^4+a}}{5}+\frac{d\,x^2\sqrt{b\,x^4+a}}{4}+\frac{cx\sqrt{b\,x^4+a}}{3}+\frac{af\sqrt{b\,x^4+a}}{6b}+\frac{2ac\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(10*b*f*x^4+12*b*e*x^3+15*b*d*x^2+20*b*c*x+10*a*f)/b*(b*x^4+a)^(1/2)+1
/30*a*(20*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^
(1/2),I)+12*I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+15/
2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.49

$$\int \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$$
$$=\frac{48\,a\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid-1)+15\,a\sqrt{b}dx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+16\,(5\,bc-3}{}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/120*(48*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1)
 + 15*a*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 16*
(5*b*c - 3*a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -
1) + 2*(10*b*f*x^5 + 12*b*e*x^4 + 15*b*d*x^3 + 20*b*c*x^2 + 10*a*f*x + 24*a
*e)*sqrt(b*x^4 + a))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.47

$$\int \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx = \frac{\sqrt{a}cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}dx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{\sqrt{a}ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{ad\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}} + f\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)
/(4*gamma(5/4)) + sqrt(a)*d*x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*e*x**3*gamm
a(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4))
+ a*d*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + f*Piecewise((sqrt(a)*x**4/4
, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))
```

Maxima [F]

$$\int \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx = \int \sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c), x)
```

Giac [F]

$$\int \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx = \int \sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\int \sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

```
[In] int((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
[Out] int((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.500 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 345

$$\begin{aligned}
&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x} \, dx \\
&= \frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{4}(2c+ex^2)\sqrt{a+bx^4} + \frac{1}{15}x(5d+3fx^2)\sqrt{a+bx^4} \\
&\quad + \frac{a e \operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}} - \frac{1}{2}\sqrt{a} c \operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) \\
&\quad - \frac{2a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}} \\
&\quad + \frac{a^{3/4}\left(5\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}
\end{aligned}$$

```
[Out] -1/2*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+1/4*a*e*arctanh(x^2*b^(1/2)
/(b*x^4+a)^(1/2))/b^(1/2)+1/4*(e*x^2+2*c)*(b*x^4+a)^(1/2)+1/15*x*(3*f*x^2+5
*d)*(b*x^4+a)^(1/2)+2/5*a*f*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))
-2/5*a^(5/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^
(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4
+a)^(1/2)+1/15*a^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1
/2))*(3*f*a^(1/2)+5*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^
2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {1847, 1266, 829, 858, 223, 212, 272, 65, 214, 1191, 1212, 226, 1210}

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx$$
$$=\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}$$
$$-\frac{2a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}$$
$$-\frac{1}{2}\sqrt{a}c\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{ae\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{1}{4}\sqrt{a+bx^4}(2c+ex^2)+\frac{1}{15}x\sqrt{a+bx^4}(5d+3fx^2)+$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x,x]
```

```
[Out] (2*a*f*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*c + e*x
^2)*Sqrt[a + b*x^4])/4 + (x*(5*d + 3*f*x^2)*Sqrt[a + b*x^4])/15 + (a*e*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*c*ArcTanh[Sqrt[a
 + b*x^4]/Sqrt[a]])/2 - (2*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(3/4)*(5*Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*A
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{(c+ex^2)\sqrt{a+bx^4}}{x}+(d+fx^2)\sqrt{a+bx^4}\right)dx \\
&= \int \frac{(c+ex^2)\sqrt{a+bx^4}}{x}\,dx+\int (d+fx^2)\sqrt{a+bx^4}\,dx \\
&= \frac{1}{15}x(5d+3fx^2)\sqrt{a+bx^4}+\frac{1}{15}\int \frac{10ad+6afx^2}{\sqrt{a+bx^4}}\,dx \\
&\quad +\frac{1}{2}\text{Subst}\left(\int \frac{(c+ex)\sqrt{a+bx^2}}{x}\,dx,x,x^2\right) \\
&= \frac{1}{4}(2c+ex^2)\sqrt{a+bx^4}+\frac{1}{15}x(5d+3fx^2)\sqrt{a+bx^4}+\frac{\text{Subst}\left(\int \frac{2abc+abex}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)}{4b} \\
&\quad -\frac{(2a^{3/2}f)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{5\sqrt{b}}+\frac{1}{15}\left(2a\left(5d+\frac{3\sqrt{a}f}{\sqrt{b}}\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx
\end{aligned}
$$

$$=\frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{4}\left(2c+ex^2\right)\sqrt{a+bx^4}+\frac{1}{15}x\left(5d+3fx^2\right)\sqrt{a+bx^4}$$
$$-\frac{2a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{a^{3/4}\left(5\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{1}{2}(ac)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)+\frac{1}{4}(ae)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)$$

$$=\frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{4}\left(2c+ex^2\right)\sqrt{a+bx^4}+\frac{1}{15}x\left(5d+3fx^2\right)\sqrt{a+bx^4}$$
$$-\frac{2a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{a^{3/4}\left(5\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{1}{4}(ac)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)+\frac{1}{4}(ae)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)$$

$$=\frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{4}\left(2c+ex^2\right)\sqrt{a+bx^4}$$
$$+\frac{1}{15}x\left(5d+3fx^2\right)\sqrt{a+bx^4}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}$$
$$-\frac{2a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{a^{3/4}\left(5\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{(ac)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)}{2b}$$

$$
\begin{aligned}
&= \frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{4}(2c+ex^2)\sqrt{a+bx^4} + \frac{1}{15}x(5d+3fx^2)\sqrt{a+bx^4} \\
&\quad + \frac{ae\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}} - \frac{1}{2}\sqrt{a}c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) \\
&\quad - \frac{2a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}} \\
&\quad + \frac{a^{3/4}\left(5\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.34 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.60

$$
\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx
$$

$$
= \frac{3a^{3/2}e\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right) + 3\sqrt{b}\left((2c+ex^2)(a+bx^4) - 2\sqrt{a}c\sqrt{a+bx^4}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right) + 12a\sqrt{b}}{12\sqrt{b}}
$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x,x]
```

```
[Out] (3*a^(3/2)*e*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 3*Sqrt[b]
*((2*c + e*x^2)*(a + b*x^4) - 2*Sqrt[a]*c*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a +
b*x^4]/Sqrt[a]]) + 12*a*Sqrt[b]*d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-
1/2, 1/4, 5/4, -((b*x^4)/a)] + 4*a*Sqrt[b]*f*x^3*Sqrt[1 + (b*x^4)/a]*Hyperg
eometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)])/(12*Sqrt[b]*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.79

method	result
elliptic	$\frac{f\,x^{3}\sqrt{b\,x^{4}+a}}{5}+\frac{e\,x^{2}\sqrt{b\,x^{4}+a}}{4}+\frac{dx\sqrt{b\,x^{4}+a}}{3}+\frac{c\sqrt{b\,x^{4}+a}}{2}+\frac{2ad\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{ae\ln\left(2x^{2}\sqrt{b}+2\sqrt{b\,x}\right.}{4\sqrt{b}}$
default	$d\left(\frac{x\sqrt{b\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+f\left(\frac{x^{3}\sqrt{b\,x^{4}+a}}{5}+\frac{2ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right.\right.}{5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*f*x^3*(b*x^4+a)^(1/2)+1/4*e*x^2*(b*x^4+a)^(1/2)+1/3*d*x*(b*x^4+a)^(1/2)
+1/2*c*(b*x^4+a)^(1/2)+2/3*a*d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(
I/a^(1/2)*b^(1/2))^(1/2),I)+1/4*a*e*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))/b^(
1/2)+2/5*I*a^(3/2)*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I
/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*a^
(1/2)*c*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.06 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.59

$$\begin{aligned}\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx=&-\frac{\sqrt{a}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{\sqrt{a}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}\\&+\frac{\sqrt{a}ex^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{\sqrt{a}fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}\\&+\frac{ac}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{ae\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}}+\frac{\sqrt{b}cx^2}{2\sqrt{\frac{a}{bx^4}+1}}\end{aligned}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x,x)
```

```
[Out] -sqrt(a)*c*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*d*x*gamma(1/4)*hyper((
-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*e*x*
*2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4
,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a*c/(2*sqrt(b)*x**2*sqrt(a/(
b*x**4) + 1)) + a*e*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + sqrt(b)*c*x**
2/(2*sqrt(a/(b*x**4) + 1))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx=\int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x}\,dx$$

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x,x)
```

```
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x, x)
```

3.501 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 341

$$\begin{aligned}
&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx \\
&= \frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{(3c-ex^2)\sqrt{a+bx^4}}{3x} + \frac{1}{4}(2d+fx^2)\sqrt{a+bx^4} \\
&\quad + \frac{af\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}} - \frac{1}{2}\sqrt{a}d\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) \\
&\quad - \frac{2\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} \\
&\quad + \frac{\sqrt[4]{a}\left(3\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}
\end{aligned}$$

```
[Out] -1/2*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+1/4*a*f*arctanh(x^2*b^(1/2)
/(b*x^4+a)^(1/2))/b^(1/2)-1/3*(-e*x^2+3*c)*(b*x^4+a)^(1/2)/x+1/4*(f*x^2+2*d
)*(b*x^4+a)^(1/2)+2*c*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-2*a^(
1/4)*b^(1/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^
(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2
)+1/3*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/
4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(e*a
^(1/2)+3*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^
2)^(1/2)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {1847, 1286, 1212, 226, 1210, 1266, 829, 858, 223, 212, 272, 65, 214}

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx$$
$$=\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+3\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}-\frac{1}{2}\sqrt{a}d\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{af\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}-\frac{\sqrt{a+bx^4}(3c-ex^2)}{3x}+\frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}+\frac{1}{4}\sqrt{a+bx^4}(2d+fx^2)$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^2,x]
```

```
[Out] (2*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((3*c - e*x^2)*Sq
rt[a + b*x^4])/(3*x) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/4 + (a*f*ArcTanh[(Sq
rt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^
4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(c+ex^2)\sqrt{a+bx^4}}{x^2}+\frac{(d+fx^2)\sqrt{a+bx^4}}{x}\right)dx\\
&= \int \frac{(c+ex^2)\sqrt{a+bx^4}}{x^2}\,dx+\int \frac{(d+fx^2)\sqrt{a+bx^4}}{x}\,dx\\
&= -\frac{(3c-ex^2)\sqrt{a+bx^4}}{3x}+\frac{1}{2}\text{Subst}\left(\int \frac{(d+fx)\sqrt{a+bx^2}}{x}\,dx,x,x^2\right)-\frac{2}{3}\int \frac{-ae-3bcx^2}{\sqrt{a+bx^4}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&=-\frac{(3c-ex^2)\sqrt{a+bx^4}}{3x}+\frac{1}{4}(2d+fx^2)\sqrt{a+bx^4}+\frac{\text{Subst}\left(\int\frac{2abd+abfx}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)}{4b}\\
&-\left(2\sqrt{a}\sqrt{b}c\right)\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx+\frac{1}{3}\left(2\sqrt{a}\left(3\sqrt{b}c+\sqrt{a}e\right)\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx\\
&=\frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{(3c-ex^2)\sqrt{a+bx^4}}{3x}+\frac{1}{4}(2d+fx^2)\sqrt{a+bx^4}\\
&-\frac{2\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}\\
&+\frac{\sqrt[4]{a}\left(3\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}\\
&+\frac{1}{2}(ad)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)+\frac{1}{4}(af)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)\\
&=\frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{(3c-ex^2)\sqrt{a+bx^4}}{3x}+\frac{1}{4}(2d+fx^2)\sqrt{a+bx^4}\\
&-\frac{2\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}\\
&+\frac{\sqrt[4]{a}\left(3\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}\\
&+\frac{1}{4}(ad)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)\\
&+\frac{1}{4}(af)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{(3c-ex^2)\sqrt{a+bx^4}}{3x} \\
&\quad + \frac{1}{4}(2d+fx^2)\sqrt{a+bx^4} + \frac{af\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}} \\
&\quad - \frac{2\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} \\
&\quad + \frac{\sqrt[4]{a}\left(3\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}} \\
&\quad + \frac{(ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)}{2b} \\
&= \frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{(3c-ex^2)\sqrt{a+bx^4}}{3x} + \frac{1}{4}(2d+fx^2)\sqrt{a+bx^4} \\
&\quad + \frac{af\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}} - \frac{1}{2}\sqrt{a}d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) \\
&\quad - \frac{2\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} \\
&\quad + \frac{\sqrt[4]{a}\left(3\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.36 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.61

$$
\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx
$$

$$
= \frac{-4\sqrt{b}c\sqrt{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{bx^4}{a}\right)+x\left(\sqrt{a}f\sqrt{a+bx^4}\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+\sqrt{b}\sqrt{1+\frac{bx}{a}}\right.}{4\sqrt{b}x}
$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^2,x]
```

```
[Out] (-4*Sqrt[b]*c*Sqrt[a + b*x^4]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^4)/
a)] + x*(Sqrt[a]*f*Sqrt[a + b*x^4]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + Sqrt[b]
```

```
*Sqrt[1 + (b*x^4)/a]*((2*d + f*x^2)*Sqrt[a + b*x^4] - 2*Sqrt[a]*d*ArcTanh[S
qrt[a + b*x^4]/Sqrt[a]]) + 4*Sqrt[b]*e*x*Sqrt[a + b*x^4]*Hypergeometric2F1[
-1/2, 1/4, 5/4, -((b*x^4)/a)]))/(4*Sqrt[b]*x*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.80

method	result
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{x}+\frac{f\,x^{2}\sqrt{b\,x^{4}+a}}{4}+\frac{ex\sqrt{b\,x^{4}+a}}{3}+\frac{d\sqrt{b\,x^{4}+a}}{2}+\frac{2ae\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{af\ln\left(2x^{2}\sqrt{b}+2\sqrt{b\,x}\right.}{4\sqrt{b}}$
risch	$-\frac{c\sqrt{b\,x^{4}+a}}{x}+\frac{2ae\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{f\,x^{2}\sqrt{b\,x^{4}+a}}{4}+\frac{af\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{4\sqrt{b}}+\frac{ex\sqrt{b\,x^{4}+a}}{3}+\frac{d\sqrt{b\,x^{4}}}{2}$
default	$e\left(\frac{x\sqrt{b\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+f\left(\frac{x^{2}\sqrt{b\,x^{4}+a}}{4}+\frac{a\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{4\sqrt{b}}\right)+d\left(\frac{\sqrt{b\,x^{4}+a}}{2}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -c*(b*x^4+a)^(1/2)/x+1/4*f*x^2*(b*x^4+a)^(1/2)+1/3*e*x*(b*x^4+a)^(1/2)+1/2*
d*(b*x^4+a)^(1/2)+2/3*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(
1/2)*b^(1/2))^(1/2),I)+1/4*a*f*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))/b^(1/2)+
2*I*b^(1/2)*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2
)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*a^(1/2)*d
*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.92 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{a}d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{\sqrt{a}ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}fx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{ad}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{af\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}} + \frac{\sqrt{b}dx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**2,x)
```

```
[Out] sqrt(a)*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)
/(4*x*gamma(3/4)) - sqrt(a)*d*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*e*x
*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5
/4)) + sqrt(a)*f*x**2*sqrt(1 + b*x**4/a)/4 + a*d/(2*sqrt(b)*x**2*sqrt(a/(b*
x**4) + 1)) + a*f*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + sqrt(b)*d*x**2/
(2*sqrt(a/(b*x**4) + 1))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^2}\,dx$$

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^2,x)
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^2, x)
```

3.502 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 342

$$
\begin{aligned}
&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx \\
&= \frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{(c-ex^2)\sqrt{a+bx^4}}{2x^2}-\frac{(3d-fx^2)\sqrt{a+bx^4}}{3x} \\
&\quad+\frac{1}{2}\sqrt{b}c\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{2}\sqrt{a}e\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) \\
&\quad-\frac{2\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} \\
&\quad+\frac{\sqrt[4]{a}\left(3\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}
\end{aligned}
$$

```
[Out] -1/2*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+1/2*c*arctanh(x^2*b^(1/2)/(
b*x^4+a)^(1/2))*b^(1/2)-1/2*(-e*x^2+c)*(b*x^4+a)^(1/2)/x^2-1/3*(-f*x^2+3*d)
*(b*x^4+a)^(1/2)/x+2*d*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-2*a^
(1/4)*b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/
2)+1/3*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(f*
a^(1/2)+3*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))
^2)^(1/2)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {1847, 1266, 827, 858, 223, 212, 272, 65, 214, 1286, 1212, 226, 1210}

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx = \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+3\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{2\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} + \frac{1}{2}\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{1}{2}\sqrt{a}e\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{\sqrt{a+bx^4}(c-ex^2)}{2x^2} - \frac{\sqrt{a+bx^4}(3d-fx^2)}{3x} + \frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^3,x]
```

```
[Out] (2*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((c - e*x^2)*Sqrt
[a + b*x^4])/(2*x^2) - ((3*d - f*x^2)*Sqrt[a + b*x^4])/(3*x) + (Sqrt[b]*c*A
rcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (Sqrt[a]*e*ArcTanh[Sqrt[a + b*x^
4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(c+ex^2)\sqrt{a+bx^4}}{x^3}+\frac{(d+fx^2)\sqrt{a+bx^4}}{x^2}\right)dx \\
&= \int \frac{(c+ex^2)\sqrt{a+bx^4}}{x^3}\,dx+\int \frac{(d+fx^2)\sqrt{a+bx^4}}{x^2}\,dx \\
&= -\frac{(3d-fx^2)\sqrt{a+bx^4}}{3x}+\frac{1}{2}\text{Subst}\left(\int \frac{(c+ex)\sqrt{a+bx^2}}{x^2}\,dx,x,x^2\right)-\frac{2}{3}\int \frac{-af-3bdx^2}{\sqrt{a+bx^4}}\,dx
\end{aligned}$$

$$= -\frac{(c-ex^2)\sqrt{a+bx^4}}{2x^2} - \frac{(3d-fx^2)\sqrt{a+bx^4}}{3x} - \frac{1}{4}\text{Subst}\left(\int \frac{-2ae-2bcx}{x\sqrt{a+bx^2}}\,dx, x, x^2\right)$$

$$-\left(2\sqrt{a}\sqrt{b}d\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx + \frac{1}{3}\left(2\sqrt{a}\left(3\sqrt{b}d+\sqrt{a}f\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{(c-ex^2)\sqrt{a+bx^4}}{2x^2} - \frac{(3d-fx^2)\sqrt{a+bx^4}}{3x}$$

$$- \frac{2\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}$$

$$+ \frac{\sqrt[4]{a}\left(3\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}$$

$$+ \frac{1}{2}(bc)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^2\right) + \frac{1}{2}(ae)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, x^2\right)$$

$$= \frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{(c-ex^2)\sqrt{a+bx^4}}{2x^2} - \frac{(3d-fx^2)\sqrt{a+bx^4}}{3x}$$

$$- \frac{2\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}$$

$$+ \frac{\sqrt[4]{a}\left(3\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}$$

$$+ \frac{1}{2}(bc)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)$$

$$+ \frac{1}{4}(ae)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^4\right)$$

$$= \frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{(c-ex^2)\sqrt{a+bx^4}}{2x^2} - \frac{(3d-fx^2)\sqrt{a+bx^4}}{3x} + \frac{1}{2}\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{2\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} + \frac{\sqrt[4]{a}\left(3\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}} + \frac{(ae)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)}{2b}$$

$$= \frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{(c-ex^2)\sqrt{a+bx^4}}{2x^2} - \frac{(3d-fx^2)\sqrt{a+bx^4}}{3x} + \frac{1}{2}\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{1}{2}\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{2\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} + \frac{\sqrt[4]{a}\left(3\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx$$

$$= \frac{-ac+aex^2-bcx^4+bex^6+\sqrt{a}\sqrt{b}cx^2\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)-\sqrt{a}ex^2\sqrt{a+bx^4}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-2adx\ldots}{2x^2\sqrt{a}\ldots}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^3,x]
[Out] (-(a*c) + a*e*x^2 - b*c*x^4 + b*e*x^6 + Sqrt[a]*Sqrt[b]*c*x^2*Sqrt[1 + (b*x
^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - Sqrt[a]*e*x^2*Sqrt[a + b*x^4]*ArcTa
nh[Sqrt[a + b*x^4]/Sqrt[a]] - 2*a*d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1
[-1/2, -1/4, 3/4, -((b*x^4)/a)] + 2*a*f*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeomet
ric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)])/(2*x^2*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(2dx+c)}{2x^{2}}+\frac{2af\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{fx\sqrt{b\,x^{4}+a}}{3}+\frac{e\sqrt{b\,x^{4}+a}}{2}+\frac{\sqrt{b}\,c\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{2}$ -
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{2x^{2}}-\frac{d\sqrt{b\,x^{4}+a}}{x}+\frac{fx\sqrt{b\,x^{4}+a}}{3}+\frac{e\sqrt{b\,x^{4}+a}}{2}+\frac{2af\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{\sqrt{b}\,c\ln\left(2x^{2}\sqrt{b}+2\sqrt{b}\right.}{2}$
default	$f\left(\frac{x\sqrt{b\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+e\left(\frac{\sqrt{b\,x^{4}+a}}{2}-\frac{\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{2}\right)+d\left(-\frac{\sqrt{b\,x}}{x}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*x^4+a)^(1/2)*(2*d*x+c)/x^2+2/3*a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a
^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*E
llipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/3*f*x*(b*x^4+a)^(1/2)+1/2*e*(b*x^
4+a)^(1/2)+1/2*b^(1/2)*c*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/2*a^(1/2)*e*ln((
2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+2*I*b^(1/2)*d*a^(1/2)/(I/a^(1/2)*b^(1/2
))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b
*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1
/2)*b^(1/2))^(1/2),I))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^3}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx = -\frac{\sqrt{a}c}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{a}e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{\sqrt{a}fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{ae}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{\sqrt{b}c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} + \frac{\sqrt{b}ex^2}{2\sqrt{\frac{a}{bx^4}+1}} - \frac{bcx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**3,x)
```

```
[Out] -sqrt(a)*c/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*d*gamma(-1/4)*hyper((-1/2,
 -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - sqrt(a)*e*asin
h(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*f*x*gamma(1/4)*hyper((-1/2, 1/4), (5/
4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a*e/(2*sqrt(b)*x**2*sqrt(a/
(b*x**4) + 1)) + sqrt(b)*c*asinh(sqrt(b)*x**2/sqrt(a))/2 + sqrt(b)*e*x**2/(
2*sqrt(a/(b*x**4) + 1)) - b*c*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^3}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^3}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx=\int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^3}\,dx$$

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^3,x)
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^3, x)
```

3.503 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 357

$$\begin{aligned}\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx &= -\frac{2e\sqrt{a+bx^4}}{x}+\frac{2\sqrt{b}ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3}-\frac{(d-fx^2)\sqrt{a+bx^4}}{2x^2}\\
&\quad+\frac{1}{2}\sqrt{b}d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{2}\sqrt{a}f\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\\
&\quad-\frac{2\sqrt[4]{a}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}\\
&\quad+\frac{\sqrt[4]{b}\left(\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}\end{aligned}$$

```
[Out] -1/2*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+1/2*d*arctanh(x^2*b^(1/2)/(
b*x^4+a)^(1/2))*b^(1/2)-2*e*(b*x^4+a)^(1/2)/x-1/3*(-3*e*x^2+c)*(b*x^4+a)^(1
/2)/x^3-1/2*(-f*x^2+d)*(b*x^4+a)^(1/2)/x^2+2*e*x*b^(1/2)*(b*x^4+a)^(1/2)/(a
^(1/2)+x^2*b^(1/2))-2*a^(1/4)*b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2
)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a
^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2)
)^2)^(1/2)/(b*x^4+a)^(1/2)+1/3*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^
(1/4))),1/2*2^(1/2))*(3*e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+
a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1847, 1286, 1296, 1212, 226, 1210, 1266, 827, 858, 223, 212, 272, 65, 214}

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx$$
$$= \frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}$$
$$-\frac{2\sqrt[4]{a}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}$$
$$+\frac{1}{2}\sqrt{b}d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{2}\sqrt{a}f\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$
$$-\frac{\sqrt{a+bx^4}(c-3ex^2)}{3x^3}-\frac{\sqrt{a+bx^4}(d-fx^2)}{2x^2}-\frac{2e\sqrt{a+bx^4}}{x}+\frac{2\sqrt{b}ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^4,x]
```

```
[Out] (-2*e*Sqrt[a + b*x^4])/x + (2*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[
b]*x^2) - ((c - 3*e*x^2)*Sqrt[a + b*x^4])/(3*x^3) - ((d - f*x^2)*Sqrt[a + b
*x^4])/(2*x^2) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (Sq
rt[a]*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*e*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (b^(1/4)*(Sqrt[b]*c + 3*S
qrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a + b*x^4
])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(c + ex^2)\sqrt{a + bx^4}}{x^4} + \frac{(d + fx^2)\sqrt{a + bx^4}}{x^3} \right) dx$$

$$= \int \frac{(c+ex^2)\sqrt{a+bx^4}}{x^4}\,dx + \int \frac{(d+fx^2)\sqrt{a+bx^4}}{x^3}\,dx$$

$$= -\frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3} + \frac{1}{2}\text{Subst}\left(\int \frac{(d+fx)\sqrt{a+bx^2}}{x^2}\,dx, x, x^2\right) - \frac{2}{3}\int \frac{-3ae-bcx^2}{x^2\sqrt{a+bx^4}}\,dx$$

$$= -\frac{2e\sqrt{a+bx^4}}{x} - \frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3} - \frac{(d-fx^2)\sqrt{a+bx^4}}{2x^2}$$
$$- \frac{1}{4}\text{Subst}\left(\int \frac{-2af-2bdx}{x\sqrt{a+bx^2}}\,dx, x, x^2\right) + \frac{2\int \frac{abc+3abex^2}{\sqrt{a+bx^4}}\,dx}{3a}$$

$$= -\frac{2e\sqrt{a+bx^4}}{x} - \frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3} - \frac{(d-fx^2)\sqrt{a+bx^4}}{2x^2}$$
$$+ \frac{1}{2}(bd)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^2\right) - \left(2\sqrt{a}\sqrt{b}e\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx$$
$$+ \frac{1}{3}\left(2\sqrt{b}\left(\sqrt{b}c+3\sqrt{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx + \frac{1}{2}(af)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, x^2\right)$$

$$= -\frac{2e\sqrt{a+bx^4}}{x} + \frac{2\sqrt{b}ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3} - \frac{(d-fx^2)\sqrt{a+bx^4}}{2x^2}$$
$$- \frac{2\sqrt[4]{a}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}$$
$$+ \frac{\sqrt[4]{b}\left(\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}$$
$$+ \frac{1}{2}(bd)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)$$
$$+ \frac{1}{4}(af)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^4\right)$$

$$= -\frac{2e\sqrt{a+bx^4}}{x} + \frac{2\sqrt{b}ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3} - \frac{(d-fx^2)\sqrt{a+bx^4}}{2x^2} + \frac{1}{2}\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{2\sqrt[4]{a}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} + \frac{\sqrt[4]{b}\left(\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}} + \frac{(af)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)}{2b}$$

$$= -\frac{2e\sqrt{a+bx^4}}{x} + \frac{2\sqrt{b}ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3} - \frac{(d-fx^2)\sqrt{a+bx^4}}{2x^2} + \frac{1}{2}\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{1}{2}\sqrt{a}f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{2\sqrt[4]{a}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} + \frac{\sqrt[4]{b}\left(\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.57

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx$$

$$= \frac{-2ac\sqrt{1+\frac{bx^4}{a}}\text{ Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\frac{bx^4}{a}\right)+3x\left(-ad+afx^2-bdx^4+bfx^6+\sqrt{a}\sqrt{b}dx^2\sqrt{1\right.}{6}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^4,x]
```

```
[Out] (-2*a*c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^4)/a)
] + 3*x*(-(a*d) + a*f*x^2 - b*d*x^4 + b*f*x^6 + Sqrt[a]*Sqrt[b]*d*x^2*Sqrt[
```

```
1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - Sqrt[a]*f*x^2*Sqrt[a + b*x^
4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]] - 2*a*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeom
etric2F1[-1/2, -1/4, 3/4, -((b*x^4)/a)]))/(6*x^3*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(6e\,x^2+3dx+2c)}{6x^3}+\frac{2bc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{f\sqrt{b\,x^4+a}}{2}+\frac{2i\sqrt{b}\,e\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4}}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{3x^3}-\frac{\sqrt{b\,x^4+a}\,d}{2x^2}-\frac{e\sqrt{b\,x^4+a}}{x}+\frac{f\sqrt{b\,x^4+a}}{2}+\frac{2bc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{\sqrt{b}\,d\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4}\right.}{2}$
default	$f\left(\frac{\sqrt{b\,x^4+a}}{2}-\frac{\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2}\right)+c\left(-\frac{\sqrt{b\,x^4+a}}{3x^3}+\frac{2b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+e\left(-\frac{\sqrt{b\,x^4}}{x}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(b*x^4+a)^(1/2)*(6*e*x^2+3*d*x+2*c)/x^3+2/3*b*c/(I/a^(1/2)*b^(1/2))^(1
/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+
a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*f*(b*x^4+a)^(1/2)+2*I
*b^(1/2)*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b
^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*b^(1/2)*d*ln
(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/2*a^(1/2)*f*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/
2))/x^2)
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.90 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.66

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}d}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{a}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{af}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{\sqrt{b}d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} + \frac{\sqrt{b}fx^2}{2\sqrt{\frac{a}{bx^4}+1}} - \frac{bdx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**4,x)
```

```
[Out] sqrt(a)*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)
/(4*x**3*gamma(1/4)) - sqrt(a)*d/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*e*ga
mma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(
3/4)) - sqrt(a)*f*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a*f/(2*sqrt(b)*x**2*sqr
t(a/(b*x**4) + 1)) + sqrt(b)*d*asinh(sqrt(b)*x**2/sqrt(a))/2 + sqrt(b)*f*x*
*2/(2*sqrt(a/(b*x**4) + 1)) - b*d*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx=\int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^4}\,dx$$

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^4,x)
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^4, x)
```

3.504 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 329

$$\begin{aligned}
&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx \\
&= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}+\frac{2\sqrt{b}fx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}+\frac{1}{2}\sqrt{b}e\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) \\
&\quad -\frac{bc\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{2\sqrt[4]{a}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} \\
&\quad +\frac{\sqrt[4]{b}\left(\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}
\end{aligned}$$

```
[Out] -1/4*b*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)+1/2*e*arctanh(x^2*b^(1/2)
/(b*x^4+a)^(1/2))*b^(1/2)-1/12*(3*c/x^4+4*d/x^3+6*e/x^2+12*f/x)*(b*x^4+a)^(
1/2)+2*f*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-2*a^(1/4)*b^(1/4)*
f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)
))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(
1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)+1/3*b^(1/4)
*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4))
)*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*f*a^(1/2)+d*b^
(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1
/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {14, 1839, 1846, 272, 65, 214, 1899, 281, 223, 212, 1212, 226, 1210}

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx$$
$$=\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}$$
$$-\frac{2\sqrt[4]{a}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}-\frac{b\text{carctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$
$$+\frac{1}{2}\sqrt{b}\text{earctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)+\frac{2\sqrt{b}fx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^5,x]
```

```
[Out] -1/12*(((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*Sqrt[a + b*x^4]) + (2
*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) + (Sqrt[b]*e*ArcTanh[
(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/
(4*Sqrt[a]) - (2*a^(1/4)*b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)
/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
Sqrt[a + b*x^4] + (b^(1/4)*(Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\text{integral} = -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}-(2b)\int\frac{-\frac{c}{4}-\frac{dx}{3}-\frac{ex^2}{2}-fx^3}{x\sqrt{a+bx^4}}\,dx$$

$$=-\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}-(2b)\int\frac{-\frac{d}{3}-\frac{ex}{2}-fx^2}{\sqrt{a+bx^4}}\,dx+\frac{1}{2}(bc)\int\frac{1}{x\sqrt{a+bx^4}}\,dx$$

$$=-\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}-(2b)\int\left(-\frac{ex}{2\sqrt{a+bx^4}}+\frac{-\frac{d}{3}-fx^2}{\sqrt{a+bx^4}}\right)dx$$
$$+\frac{1}{8}(bc)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)$$

$$=-\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}-(2b)\int\frac{-\frac{d}{3}-fx^2}{\sqrt{a+bx^4}}\,dx$$
$$+\frac{1}{4}c\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)+(be)\int\frac{x}{\sqrt{a+bx^4}}\,dx$$

$$=-\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$
$$+\frac{1}{2}(be)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\left(2\sqrt{a}\sqrt{b}f\right)\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx$$
$$+\frac{1}{3}\left(2b\left(d+\frac{3\sqrt{a}f}{\sqrt{b}}\right)\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}+\frac{2\sqrt{b}fx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$
$$-\frac{2\sqrt[4]{a}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}$$
$$+\frac{\sqrt[4]{b}\left(\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}$$
$$+\frac{1}{2}(be)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)$$
$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}+\frac{2\sqrt{b}fx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}$$
$$+\frac{1}{2}\sqrt{b}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$
$$-\frac{2\sqrt[4]{a}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}$$
$$+\frac{\sqrt[4]{b}\left(\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.53

$$\int\frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx =$$
$$-\frac{\sqrt{1+\frac{bx^4}{a}}\left(3ac\sqrt{1+\frac{bx^4}{a}}+6aex^2\sqrt{1+\frac{bx^4}{a}}-6\sqrt{a}\sqrt{b}ex^4\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+3bcx^4\text{arctanh}\left(\sqrt{1+\frac{bx^4}{a}}\right)+\right.}{12x^4\sqrt{a+}}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^5,x]
```

```
[Out] -1/12*(Sqrt[1 + (b*x^4)/a]*(3*a*c*Sqrt[1 + (b*x^4)/a] + 6*a*e*x^2*Sqrt[1 +
(b*x^4)/a] - 6*Sqrt[a]*Sqrt[b]*e*x^4*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 3*b*c
*x^4*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 4*a*d*x*Hypergeometric2F1[-3/4, -1/2, 1
/4, -((b*x^4)/a)] + 12*a*f*x^3*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^4)
/a)]))/(x^4*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(12 f\,x^{3}+6 e\,x^{2}+4 d x+3 c\right)}{12 x^{4}}+\frac{b\left(\frac{4 d\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{3 e\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{\sqrt{b}}-\frac{3 c\ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{2\sqrt{a}}+ \cdots}{6}$
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{4 x^{4}}-\frac{d\sqrt{b\,x^{4}+a}}{3 x^{3}}-\frac{e\sqrt{b\,x^{4}+a}}{2 x^{2}}-\frac{f\sqrt{b\,x^{4}+a}}{x}+\frac{2 b d\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{\sqrt{b}\,e\ln\left(2 x^{2}\sqrt{b}+2\sqrt{b\,x^{4}+ \cdots}{2}$
default	$d\left(-\frac{\sqrt{b\,x^{4}+a}}{3 x^{3}}+\frac{2 b\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+f\left(-\frac{\sqrt{b\,x^{4}+a}}{x}+\frac{2 i\sqrt{b}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\cdots}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(b*x^4+a)^(1/2)*(12*f*x^3+6*e*x^2+4*d*x+3*c)/x^4+1/6*b*(4*d/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^
(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3*e*ln(x^2*b
^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)-3/2*c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(
1/2))/x^2)+12*I*f*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,\sqrt{a+bx^4}}{x^5}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.01 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.64

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = \frac{\sqrt{a}d\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}e}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}f\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{\sqrt{b}e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} - \frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}} - \frac{bex^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**5,x)
```

```
[Out] sqrt(a)*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)
/(4*x**3*gamma(1/4)) - sqrt(a)*e/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*f*ga
mma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(
3/4)) - sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*x**2) + sqrt(b)*e*asinh(sqrt(b)*x
**2/sqrt(a))/2 - b*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a)) - b*e*x**2/(
2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(b*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/sqrt(a)
 - 2*sqrt(b*x^4 + a)/x^4)*c + integrate(sqrt(b*x^4 + a)*(f*x^2 + e*x + d)/x
^4, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="giac")
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^5}\,dx$$

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^5,x)
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^5, x)
```

3.505 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 360

$$\begin{aligned}
&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx \\
&= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{5ax} \\
&\quad+\frac{2b^{3/2}cx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{2}\sqrt{b}f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{bd\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}} \\
&\quad-\frac{2b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} \\
&\quad+\frac{b^{3/4}\left(3\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}
\end{aligned}$$

```
[Out] -1/4*b*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)+1/2*f*arctanh(x^2*b^(1/2)
/(b*x^4+a)^(1/2))*b^(1/2)-1/60*(12*c/x^5+15*d/x^4+20*e/x^3+30*f/x^2)*(b*x^4
+a)^(1/2)-2/5*b*c*(b*x^4+a)^(1/2)/a/x+2/5*b^(3/2)*c*x*(b*x^4+a)^(1/2)/a/(a^
(1/2)+x^2*b^(1/2))-2/5*b^(5/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1
/2)/a^(3/4)/(b*x^4+a)^(1/2)+1/15*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/
a^(1/4))),1/2*2^(1/2))*(5*e*a^(1/2)+3*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*
x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {14, 1839, 1847, 1296, 1212, 226, 1210, 1266, 858, 223, 212, 272, 65, 214}

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx$$
$$= \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+3\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}$$
$$-\frac{2b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}-\frac{bd\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$
$$+\frac{1}{2}\sqrt{b}f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\frac{2b^{3/2}cx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)-\frac{2bc\sqrt{a+bx^4}}{5ax}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^6,x]
```

```
[Out] -1/60*(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*Sqrt[a + b*x^4])
 - (2*b*c*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*a*(
Sqrt[a] + Sqrt[b]*x^2)) + (Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]
)/2 - (b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*c*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)
*(3*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^
(3/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

```
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{c}{5}-\frac{dx}{4}-\frac{ex^2}{3}-\frac{fx^3}{2}}{x^2\sqrt{a+bx^4}}\,dx\\
&= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-(2b)\int \left(\frac{-\frac{c}{5}-\frac{ex^2}{3}}{x^2\sqrt{a+bx^4}}+\frac{-\frac{d}{4}-\frac{fx^2}{2}}{x\sqrt{a+bx^4}}\right)dx\\
&= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{c}{5}-\frac{ex^2}{3}}{x^2\sqrt{a+bx^4}}\,dx-(2b)\int \frac{-\frac{d}{4}-\frac{fx^2}{2}}{x\sqrt{a+bx^4}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&=-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{5ax}\\
&\quad-b\text{Subst}\left(\int\frac{-\frac{d}{4}-\frac{fx}{2}}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)+\frac{(2b)\int\frac{\frac{ae}{3}+\frac{1}{5}bcx^2}{\sqrt{a+bx^4}}\,dx}{a}
\end{aligned}
$$

$$
\begin{aligned}
&=-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{5ax}-\frac{\left(2b^{3/2}c\right)\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{5\sqrt{a}}\\
&\quad+\frac{1}{4}(bd)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)+\frac{1}{15}\left(2b\left(\frac{3\sqrt{b}c}{\sqrt{a}}+5e\right)\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx\\
&\quad+\frac{1}{2}(bf)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)
\end{aligned}
$$

$$
\begin{aligned}
&=-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}cx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}\\
&\quad-\frac{2b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}\\
&\quad+\frac{b^{3/4}\left(3\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}\\
&\quad+\frac{1}{8}(bd)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)+\frac{1}{2}(bf)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&=-\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{5ax}\\
&\quad+\frac{2b^{3/2}cx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{2}\sqrt{b}f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)\\
&\quad-\frac{2b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}\\
&\quad+\frac{b^{3/4}\left(3\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}\\
&\quad+\frac{1}{4}d\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)
\end{aligned}
$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{5ax}$$
$$+\frac{2b^{3/2}cx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{2}\sqrt{b}f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$
$$-\frac{2b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{b^{3/4}\left(3\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.50

$$\int\frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx =$$
$$-\frac{\sqrt{a+bx^4}\left(12ac\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{bx^4}{a}\right)+5x\left(3ad\sqrt{1+\frac{bx^4}{a}}+6afx^2\sqrt{1+\frac{bx^4}{a}}-6\sqrt{a}\right.\right.}{60ax^5\sqrt{1}}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^6,x]
```

```
[Out] -1/60*(Sqrt[a + b*x^4]*(12*a*c*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^4
)/a)] + 5*x*(3*a*d*Sqrt[1 + (b*x^4)/a] + 6*a*f*x^2*Sqrt[1 + (b*x^4)/a] - 6*
Sqrt[a]*Sqrt[b]*f*x^4*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 3*b*d*x^4*ArcTanh[Sq
rt[1 + (b*x^4)/a]] + 4*a*e*x*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^4)/a
)])))/(a*x^5*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(24bc\,x^4+30af\,x^3+20ae\,x^2+15adx+12ac)}{60x^5a}+\frac{b\left(\frac{20ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{12i\sqrt{b}\,c\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+i}}{\sqrt{}}\right.}{}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{5x^5}-\frac{d\sqrt{b\,x^4+a}}{4x^4}-\frac{e\sqrt{b\,x^4+a}}{3x^3}-\frac{f\sqrt{b\,x^4+a}}{2x^2}-\frac{2bc\sqrt{b\,x^4+a}}{5ax}+\frac{2be\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{\sqrt{b}\,f\,\mathrm{l}}{}$
default	$e\left(-\frac{\sqrt{b\,x^4+a}}{3x^3}+\frac{2b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+f\left(-\frac{(b\,x^4+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\,x^2\sqrt{b\,x^4+a}}{2a}+\frac{\sqrt{b}\,\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+}\right.}{2}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(b*x^4+a)^(1/2)*(24*b*c*x^4+30*a*f*x^3+20*a*e*x^2+15*a*d*x+12*a*c)/x^
5/a+1/30*b/a*(20*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)+12*I*b^(1/2)*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(
1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(El
lipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2
),I))+15*a*f*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)-15/2*a^(1/2)*d*ln((2*a
+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.05 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}f}{2x^2\sqrt{1+\frac{bx^4}{a}}} - \frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{\sqrt{b}f\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} - \frac{bd\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}} - \frac{bfx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**6,x)
```

```
[Out] sqrt(a)*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**5*gamma(-1/4)) + sqrt(a)*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,),
b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*f/(2*x**2*sqrt(1 +
b*x**4/a)) - sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*x**2) + sqrt(b)*f*asinh(sqrt
(b)*x**2/sqrt(a))/2 - b*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a)) - b*f*x
**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="giac")
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx=\int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^6}\,dx$$

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^6,x)
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^6, x)
```

3.506 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 352

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx$$
$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{6ax^2}$$
$$-\frac{2bd\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}dx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{b e\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$
$$-\frac{2b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{b^{3/4}\left(3\sqrt{b}d+5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -1/4*b*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/60*(10*c/x^6+12*d/x^5+1
5*e/x^4+20*f/x^3)*(b*x^4+a)^(1/2)-1/6*b*c*(b*x^4+a)^(1/2)/a/x^2-2/5*b*d*(b*
x^4+a)^(1/2)/a/x+2/5*b^(3/2)*d*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-2/
5*b^(5/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)
*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/
2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)
^(1/2)+1/15*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)*(5*f*a^(1/2)+3*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b
^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {14, 1839, 1847, 1266, 821, 272, 65, 214, 1296, 1212, 226, 1210}

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx$$
$$= \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+3\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}$$
$$-\frac{2b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}$$
$$-\frac{be\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{2b^{3/2}dx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)-\frac{bc\sqrt{a+bx^4}}{6ax^2}-\frac{2bd\sqrt{a+bx^4}}{5ax}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^7,x]
```

```
[Out] -1/60*(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*Sqrt[a + b*x^4])
 - (b*c*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*d*Sqrt[a + b*x^4])/(5*a*x) + (2*b
^(3/2)*d*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) - (b*e*ArcTanh[Sq
rt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqrt[b]*d + 5*Sqr
t[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4]
)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-(2b)\int\frac{-\frac{c}{6}-\frac{dx}{5}-\frac{ex^2}{4}-\frac{fx^3}{3}}{x^3\sqrt{a+bx^4}}\,dx\\
&= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-(2b)\int\left(\frac{-\frac{c}{6}-\frac{ex^2}{4}}{x^3\sqrt{a+bx^4}}+\frac{-\frac{d}{5}-\frac{fx^2}{3}}{x^2\sqrt{a+bx^4}}\right)dx\\
&= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-(2b)\int\frac{-\frac{c}{6}-\frac{ex^2}{4}}{x^3\sqrt{a+bx^4}}\,dx-(2b)\int\frac{-\frac{d}{5}-\frac{fx^2}{3}}{x^2\sqrt{a+bx^4}}\,dx\\
&= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-\frac{2bd\sqrt{a+bx^4}}{5ax}\\
&\quad -b\text{Subst}\left(\int\frac{-\frac{c}{6}-\frac{ex}{4}}{x^2\sqrt{a+bx^2}}\,dx,x,x^2\right)+\frac{(2b)\int\frac{\frac{af}{3}+\frac{1}{5}bdx^2}{\sqrt{a+bx^4}}\,dx}{a}
\end{aligned}
$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4} - \frac{bc\sqrt{a+bx^4}}{6ax^2} - \frac{2bd\sqrt{a+bx^4}}{5ax} - \frac{\left(2b^{3/2}d\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{5\sqrt{a}} + \frac{1}{4}(be)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, x^2\right) + \frac{1}{15}\left(2b\left(\frac{3\sqrt{b}d}{\sqrt{a}}+5f\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4} - \frac{bc\sqrt{a+bx^4}}{6ax^2} - \frac{2bd\sqrt{a+bx^4}}{5ax} + \frac{2b^{3/2}dx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} + \frac{b^{3/4}\left(3\sqrt{b}d+5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} + \frac{1}{8}(be)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^4\right)$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4} - \frac{bc\sqrt{a+bx^4}}{6ax^2} - \frac{2bd\sqrt{a+bx^4}}{5ax} + \frac{2b^{3/2}dx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} + \frac{b^{3/4}\left(3\sqrt{b}d+5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} + \frac{1}{4}e\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{6ax^2} - \frac{2bd\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}dx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{be\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{2b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} + \frac{b^{3/4}\left(3\sqrt{b}d+5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.41

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx = -\frac{\sqrt{a+bx^4}\left(12adx\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{bx^4}{a}\right)+5\left(\sqrt{1+\frac{bx^4}{a}}(2ac+3aex^2+2bcx^4)+3be\right.\right.}{60ax^6\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^7,x]
```

```
[Out] -1/60*(Sqrt[a + b*x^4]*(12*a*d*x*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x
^4)/a)] + 5*(Sqrt[1 + (b*x^4)/a]*(2*a*c + 3*a*e*x^2 + 2*b*c*x^4) + 3*b*e*x^
6*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 4*a*f*x^3*Hypergeometric2F1[-3/4, -1/2, 1/
4, -((b*x^4)/a)])))/(a*x^6*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(24bd\,x^5+10bc\,x^4+20af\,x^3+15ae\,x^2+12adx+10ac)}{60x^6a}+\frac{b\left(\frac{20af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{12i\sqrt{b}\,d\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x}{\sqrt{a}}}}{}\right.}{}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{6x^6}-\frac{d\sqrt{b\,x^4+a}}{5x^5}-\frac{e\sqrt{b\,x^4+a}}{4x^4}-\frac{f\sqrt{b\,x^4+a}}{3x^3}-\frac{bc\sqrt{b\,x^4+a}}{6a\,x^2}-\frac{2bd\sqrt{b\,x^4+a}}{5ax}+\frac{2bf\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$
default	$-\frac{c(b\,x^4+a)^{\frac{3}{2}}}{6a\,x^6}+f\left(-\frac{\sqrt{b\,x^4+a}}{3x^3}+\frac{2b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+e\left(-\frac{(b\,x^4+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right.}{4\sqrt{a}}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(b*x^4+a)^(1/2)*(24*b*d*x^5+10*b*c*x^4+20*a*f*x^3+15*a*e*x^2+12*a*d*x
+10*a*c)/x^6/a+1/30*b/a*(20*a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*b^(1/2))^(1/2),I)+12*I*b^(1/2)*d*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a
)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^
(1/2))^(1/2),I))-15/2*a^(1/2)*e*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.47

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx = -\frac{48\,\sqrt{a}bdx^6\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)-15\,\sqrt{a}bex^6\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2\,a}{x^4}\right)-16\,(3\,bd-5\,af)\sqrt{}}{12}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/120*(48*sqrt(a)*b*d*x^6*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)),
-1) - 15*sqrt(a)*b*e*x^6*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4
) - 16*(3*b*d - 5*a*f)*sqrt(a)*x^6*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^
(1/4)), -1) + 2*(24*b*d*x^5 + 10*b*c*x^4 + 20*a*f*x^3 + 15*a*e*x^2 + 12*a*d
*x + 10*a*c)*sqrt(b*x^4 + a))/(a*x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.72 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.54

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx = \frac{\sqrt{a}d\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{6a} - \frac{be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**7,x)
```

```
[Out] sqrt(a)*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**5*gamma(-1/4)) + sqrt(a)*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,),
b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(b)*c*sqrt(a/(b*x**4) +
 1)/(6*x**4) - sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*c*sqrt(a/
(b*x**4) + 1)/(6*a) - b*e*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^7}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(b*x^4 + a)^(3/2)*c/(a*x^6) + integrate(sqrt(b*x^4 + a)*(f*x^2 + e*x +
 d)/x^6, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^7}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="giac")
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^7}\,dx$$

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^7,x)
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^7, x)
```

3.507 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8} dx$

Optimal result

Integrand size = 30, antiderivative size = 375

$$\begin{aligned}
&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx \\
&= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{21ax^3} \\
&\quad -\frac{bd\sqrt{a+bx^4}}{6ax^2}-\frac{2be\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}ex\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{bf\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}} \\
&\quad -\frac{2b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} \\
&\quad -\frac{b^{5/4}\left(5\sqrt{b}c-21\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}
\end{aligned}$$

```
[Out] -1/4*b*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/420*(60*c/x^7+70*d/x^6+
84*e/x^5+105*f/x^4)*(b*x^4+a)^(1/2)-2/21*b*c*(b*x^4+a)^(1/2)/a/x^3-1/6*b*d*
(b*x^4+a)^(1/2)/a/x^2-2/5*b*e*(b*x^4+a)^(1/2)/a/x+2/5*b^(3/2)*e*x*(b*x^4+a)
^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-2/5*b^(5/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4
)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4
)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^
(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)-1/105*b^(5/4)*(cos(2*arctan(b^(1/4)
*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arct
an(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-21*e*a^(1/2)+5*c*b^(1/2))*(a^(1/2)+x^
2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^4+a)^(1/2
)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {14, 1839, 1847, 1296, 1212, 226, 1210, 1266, 821, 272, 65, 214}

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx$$
$$= -\frac{b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}c-21\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}$$
$$-\frac{2b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}$$
$$-\frac{bf\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{2b^{3/2}ex\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$-\frac{1}{420}\sqrt{a+bx^4}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)-\frac{2bc\sqrt{a+bx^4}}{21ax^3}-\frac{bd\sqrt{a+bx^4}}{6ax^2}-\frac{2be\sqrt{a+bx^4}}{5ax}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^8,x]
```

```
[Out] -1/420*(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*Sqrt[a + b*x^4
]) - (2*b*c*Sqrt[a + b*x^4])/(21*a*x^3) - (b*d*Sqrt[a + b*x^4])/(6*a*x^2) -
 (2*b*e*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*e*x*Sqrt[a + b*x^4])/(5*a*(Sq
rt[a] + Sqrt[b]*x^2)) - (b*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a])
- (2*b^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a +
b*x^4]) - (b^(5/4)*(5*Sqrt[b]*c - 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(105*a^(5/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\text{integral} = -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-(2b)\int\frac{-\frac{c}{7}-\frac{dx}{6}-\frac{ex^2}{5}-\frac{fx^3}{4}}{x^4\sqrt{a+bx^4}}\,dx$$

$$=-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-(2b)\int\left(\frac{-\frac{c}{7}-\frac{ex^2}{5}}{x^4\sqrt{a+bx^4}}+\frac{-\frac{d}{6}-\frac{fx^2}{4}}{x^3\sqrt{a+bx^4}}\right)dx$$

$$=-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}$$
$$-(2b)\int\frac{-\frac{c}{7}-\frac{ex^2}{5}}{x^4\sqrt{a+bx^4}}\,dx-(2b)\int\frac{-\frac{d}{6}-\frac{fx^2}{4}}{x^3\sqrt{a+bx^4}}\,dx$$

$$=-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{21ax^3}$$
$$-b\text{Subst}\left(\int\frac{-\frac{d}{6}-\frac{fx}{4}}{x^2\sqrt{a+bx^2}}\,dx,x,x^2\right)+\frac{(2b)\int\frac{\frac{3ae}{5}-\frac{1}{7}bcx^2}{x^2\sqrt{a+bx^4}}\,dx}{3a}$$

$$=-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{21ax^3}-\frac{bd\sqrt{a+bx^4}}{6ax^2}-\frac{2be\sqrt{a+bx^4}}{5ax}-\frac{(2b)\int\frac{\frac{abc}{7}-\frac{3}{5}abex^2}{\sqrt{a+bx^4}}\,dx}{3a^2}+\frac{1}{4}(bf)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)$$

$$=-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{21ax^3}-\frac{bd\sqrt{a+bx^4}}{6ax^2}-\frac{2be\sqrt{a+bx^4}}{5ax}-\frac{\left(2b^{3/2}e\right)\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{5\sqrt{a}}-\frac{\left(2b^{3/2}\left(5\sqrt{b}c-21\sqrt{a}e\right)\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx}{105a}+\frac{1}{8}(bf)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)$$

$$=-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{21ax^3}-\frac{bd\sqrt{a+bx^4}}{6ax^2}-\frac{2be\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}ex\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{2b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}-\frac{b^{5/4}\left(5\sqrt{b}c-21\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}+\frac{1}{4}f\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)$$

$$=-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{21ax^3}-\frac{bd\sqrt{a+bx^4}}{6ax^2}-\frac{2be\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}ex\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{bf\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{2b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}-\frac{b^{5/4}\left(5\sqrt{b}c-21\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.39

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = -\frac{\sqrt{a+bx^4}\left(35x\left(\sqrt{1+\frac{bx^4}{a}}(2ad+3afx^2+2bdx^4)+3bfx^6\operatorname{arctanh}\left(\sqrt{1+\frac{bx^4}{a}}\right)\right)+60ac\,\text{Hypergeometri}\right.}{420ax^7\sqrt{1+\frac{bx^4}{a}}}$$

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^8,x]

[Out] -1/420*(Sqrt[a + b*x^4]*(35*x*(Sqrt[1 + (b*x^4)/a]*(2*a*d + 3*a*f*x^2 + 2*b*d*x^4) + 3*b*f*x^6*ArcTanh[Sqrt[1 + (b*x^4)/a]]) + 60*a*c*Hypergeometric2F1[-7/4, -1/2, -3/4, -((b*x^4)/a)] + 84*a*e*x^2*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^4)/a)]))/(a*x^7*Sqrt[1 + (b*x^4)/a])

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(168be\,x^6+70bd\,x^5+40bc\,x^4+105af\,x^3+84ae\,x^2+70adx+60ac)}{420x^7a}+\frac{b\left(-\frac{20bc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{84i\sqrt{b}\,e}{}\right.}{}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{7x^7}-\frac{d\sqrt{b\,x^4+a}}{6x^6}-\frac{e\sqrt{b\,x^4+a}}{5x^5}-\frac{f\sqrt{b\,x^4+a}}{4x^4}-\frac{2bc\sqrt{b\,x^4+a}}{21a\,x^3}-\frac{bd\sqrt{b\,x^4+a}}{6a\,x^2}-\frac{2be\sqrt{b\,x^4+a}}{5ax}-\frac{2b^2c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+}}{21a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}$
default	$-\frac{d(b\,x^4+a)^{\frac{3}{2}}}{6a\,x^6}+c\left(-\frac{\sqrt{b\,x^4+a}}{7x^7}-\frac{2b\sqrt{b\,x^4+a}}{21a\,x^3}-\frac{2b^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+f\left(-\frac{(b\,x^4+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\ln\left(\right.}{}\right.$

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)

[Out] -1/420*(b*x^4+a)^(1/2)*(168*b*e*x^6+70*b*d*x^5+40*b*c*x^4+105*a*f*x^3+84*a*e*x^2+70*a*d*x+60*a*c)/x^7/a+1/210*b/a*(-20*b*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+84*I*b^(1/2)*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE

```
(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-105/2*a^(1/2)*f*ln((2*a+2*a^(1/2)*(b*x^4+a
)^(1/2))/x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.46

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = -\frac{336\sqrt{a}bex^7\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1) - 105\sqrt{a}bfx^7\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right) - 16(5bc+21b}{}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/840*(336*sqrt(a)*b*e*x^7*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)),
 -1) - 105*sqrt(a)*b*f*x^7*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x
^4) - 16*(5*b*c + 21*b*e)*sqrt(a)*x^7*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/
a)^(1/4)), -1) + 2*(168*b*e*x^6 + 70*b*d*x^5 + 40*b*c*x^4 + 105*a*f*x^3 + 8
4*a*e*x^2 + 70*a*d*x + 60*a*c)*sqrt(b*x^4 + a))/(a*x^7)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.85 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.51

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} - \frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{6a} - \frac{bf\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**8,x)
```

```
[Out] sqrt(a)*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**7*gamma(-3/4)) + sqrt(a)*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - sqrt(b)*d*sqrt(a/(b*x**4)
 + 1)/(6*x**4) - sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*d*sqrt(
a/(b*x**4) + 1)/(6*a) - b*f*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="maxima")
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="giac")
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^8}\,dx$$

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^8,x)
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^8, x)
```

3.508 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 400

$$\begin{aligned}
&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx \\
&= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{16ax^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3} \\
&\quad -\frac{be\sqrt{a+bx^4}}{6ax^2}-\frac{2bf\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}fx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{b^2c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}} \\
&\quad -\frac{2b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} \\
&\quad -\frac{b^{5/4}\left(5\sqrt{b}d-21\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}
\end{aligned}$$

```
[Out] 1/16*b^2*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/840*(105*c/x^8+120*d/
x^7+140*e/x^6+168*f/x^5)*(b*x^4+a)^(1/2)-1/16*b*c*(b*x^4+a)^(1/2)/a/x^4-2/2
1*b*d*(b*x^4+a)^(1/2)/a/x^3-1/6*b*e*(b*x^4+a)^(1/2)/a/x^2-2/5*b*f*(b*x^4+a)
^(1/2)/a/x+2/5*b^(3/2)*f*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-2/5*b^(5
/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(
1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2
*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)
-1/105*b^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-2
1*f*a^(1/2)+5*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1
/2))^2)^(1/2)/a^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {14, 1839, 1847, 1266, 849, 821, 272, 65, 214, 1296, 1212, 226, 1210}

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx$$
$$= -\frac{b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}d-21\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}$$
$$-\frac{2b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{b^2c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}+\frac{2b^{3/2}fx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$-\frac{1}{840}\sqrt{a+bx^4}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)-\frac{bc\sqrt{a+bx^4}}{16ax^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3}-\frac{be\sqrt{a+bx^4}}{6ax^2}-\frac{2bf\sqrt{a+bx^4}}{5ax}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^9,x]
```

```
[Out] -1/840*(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*Sqrt[a + b*
x^4]) - (b*c*Sqrt[a + b*x^4])/(16*a*x^4) - (2*b*d*Sqrt[a + b*x^4])/(21*a*x^
3) - (b*e*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*f*Sqrt[a + b*x^4])/(5*a*x) + (2
*b^(3/2)*f*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) + (b^2*c*ArcTan
h[Sqrt[a + b*x^4]/Sqrt[a]])/(16*a^(3/2)) - (2*b^(5/4)*f*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) - (b^(5/4)*(5*Sqrt[b]*d -
21*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(5/4)*Sqrt[a +
 b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-(2b)\int\frac{-\frac{c}{8}-\frac{dx}{7}-\frac{ex^2}{6}-\frac{fx^3}{5}}{x^5\sqrt{a+bx^4}}\,dx\\ &= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-(2b)\int\left(\frac{-\frac{c}{8}-\frac{ex^2}{6}}{x^5\sqrt{a+bx^4}}+\frac{-\frac{d}{7}-\frac{fx^2}{5}}{x^4\sqrt{a+bx^4}}\right)dx\end{aligned}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}$$

$$-(2b)\int\frac{-\frac{c}{8}-\frac{ex^2}{6}}{x^5\sqrt{a+bx^4}}\,dx-(2b)\int\frac{-\frac{d}{7}-\frac{fx^2}{5}}{x^4\sqrt{a+bx^4}}\,dx$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3}$$

$$-b\text{Subst}\left(\int\frac{-\frac{c}{8}-\frac{ex}{6}}{x^3\sqrt{a+bx^2}}\,dx,x,x^2\right)+\frac{(2b)\int\frac{\frac{3af}{5}-\frac{1}{7}bdx^2}{x^2\sqrt{a+bx^4}}\,dx}{3a}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{16ax^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3}$$

$$-\frac{2bf\sqrt{a+bx^4}}{5ax}-\frac{(2b)\int\frac{\frac{abd}{7}-\frac{3}{5}abfx^2}{\sqrt{a+bx^4}}\,dx}{3a^2}+\frac{b\text{Subst}\left(\int\frac{\frac{ae}{3}-\frac{bcx}{8}}{x^2\sqrt{a+bx^2}}\,dx,x,x^2\right)}{2a}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{16ax^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3}$$

$$-\frac{be\sqrt{a+bx^4}}{6ax^2}-\frac{2bf\sqrt{a+bx^4}}{5ax}-\frac{(b^2c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)}{16a}$$

$$-\frac{\left(2b^{3/2}f\right)\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{5\sqrt{a}}-\frac{\left(2b^{3/2}\left(5\sqrt{b}d-21\sqrt{a}f\right)\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx}{105a}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{16ax^4}$$

$$-\frac{2bd\sqrt{a+bx^4}}{21ax^3}-\frac{be\sqrt{a+bx^4}}{6ax^2}-\frac{2bf\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}fx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

$$-\frac{2b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}$$

$$-\frac{b^{5/4}\left(5\sqrt{b}d-21\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}$$

$$-\frac{(b^2c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)}{32a}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{16ax^4} - \frac{2bd\sqrt{a+bx^4}}{21ax^3}-\frac{be\sqrt{a+bx^4}}{6ax^2}-\frac{2bf\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}fx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} - \frac{b^{5/4}\left(5\sqrt{b}d-21\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}} - \frac{(bc)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)}{16a}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{16ax^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3} - \frac{be\sqrt{a+bx^4}}{6ax^2}-\frac{2bf\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}fx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{b^2c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}} - \frac{2b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} - \frac{b^{5/4}\left(5\sqrt{b}d-21\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.36

$$\int\frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx = -\frac{\sqrt{a+bx^4}\left(30a^3d\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{2},-\frac{3}{4},-\frac{bx^4}{a}\right)+7x\left(6a^3fx\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\right.\right.\right.}{210a^3x^7\sqrt{1+\frac{bx}{a}}}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^9,x]
[Out] -1/210*(Sqrt[a + b*x^4]*(30*a^3*d*Hypergeometric2F1[-7/4, -1/2, -3/4, -((b*
x^4)/a)] + 7*x*(6*a^3*f*x*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^4)/a)]
```

```
 + 5*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*(a^2*e + b^2*c*x^6*Hypergeometric2F1[3
/2, 3, 5/2, 1 + (b*x^4)/a]))))/(a^3*x^7*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.33 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(672 b f\,x^7+280 b e\,x^6+160 b d\,x^5+105 b c\,x^4+336 a f\,x^3+280 a e\,x^2+240 a d x+210 a c)}{1680 x^8 a}-\frac{b^2\left(\frac{80 d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.}{}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{8x^8}-\frac{d\sqrt{b\,x^4+a}}{7x^7}-\frac{e\sqrt{b\,x^4+a}}{6x^6}-\frac{f\sqrt{b\,x^4+a}}{5x^5}-\frac{bc\sqrt{b\,x^4+a}}{16a\,x^4}-\frac{2bd\sqrt{b\,x^4+a}}{21a\,x^3}-\frac{be\sqrt{b\,x^4+a}}{6a\,x^2}-\frac{2bf\sqrt{b\,x^4+a}}{5ax}-\frac{2d\,b^2}{}$
default	$-\frac{e\left(b\,x^4+a\right)^{\frac{3}{2}}}{6a\,x^6}+d\left(-\frac{\sqrt{b\,x^4+a}}{7x^7}-\frac{2b\sqrt{b\,x^4+a}}{21a\,x^3}-\frac{2b^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+c\left(-\frac{\left(b\,x^4+a\right)^{\frac{3}{2}}}{8a\,x^8}+\frac{b(b}{1}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1680*(b*x^4+a)^(1/2)*(672*b*f*x^7+280*b*e*x^6+160*b*d*x^5+105*b*c*x^4+33
6*a*f*x^3+280*a*e*x^2+240*a*d*x+210*a*c)/x^8/a-1/840/a*b^2*(80*d/(I/a^(1/2)
*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-336*I*f*a^(1/
2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^
(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-105/2*c/a^(1/2)*ln((2*a+
2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.49

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx =$$

$$-\frac{1344\,a^{\frac{3}{2}}bfx^8\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)-105\,\sqrt{a}b^2cx^8\log\left(-\frac{bx^4+2\,\sqrt{bx^4+a}\sqrt{a}+2\,a}{x^4}\right)-64\,(5\,abd+2}{}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/3360*(1344*a^(3/2)*b*f*x^8*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)
), -1) - 105*sqrt(a)*b^2*c*x^8*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) + 2*
a)/x^4) - 64*(5*a*b*d + 21*a*b*f)*sqrt(a)*x^8*(-b/a)^(3/4)*elliptic_f(arcsi
n(x*(-b/a)^(1/4)), -1) + 2*(672*a*b*f*x^7 + 280*a*b*e*x^6 + 160*a*b*d*x^5 +
 105*a*b*c*x^4 + 336*a^2*f*x^3 + 280*a^2*e*x^2 + 240*a^2*d*x + 210*a^2*c)*s
qrt(b*x^4 + a))/(a^2*x^8)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.04 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.62

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx = \frac{\sqrt{a}d\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} - \frac{ac}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{3\sqrt{b}c}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{b^{\frac{3}{2}}c}{16ax^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^4}+1}}{6a} + \frac{b^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16a^{\frac{3}{2}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**9,x)
```

```
[Out] sqrt(a)*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**7*gamma(-3/4)) + sqrt(a)*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - a*c/(8*sqrt(b)*x**10*sqrt
(a/(b*x**4) + 1)) - 3*sqrt(b)*c/(16*x**6*sqrt(a/(b*x**4) + 1)) - sqrt(b)*e*
sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*c/(16*a*x**2*sqrt(a/(b*x**4) + 1))
 - b**(3/2)*e*sqrt(a/(b*x**4) + 1)/(6*a) + b**2*c*asinh(sqrt(a)/(sqrt(b)*x*
*2))/(16*a**(3/2))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^9}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/32*(b^2*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/a^(
3/2) + 2*((b*x^4 + a)^(3/2)*b^2 + sqrt(b*x^4 + a)*a*b^2)/((b*x^4 + a)^2*a -
 2*(b*x^4 + a)*a^2 + a^3))*c + integrate(sqrt(b*x^4 + a)*(f*x^2 + e*x + d)/
x^8, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^9}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx=\int \frac{\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^9}\,dx$$

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^9,x)
```

```
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^9, x)
```

3.509 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 425

$$\begin{aligned}\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx &= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4}\\ &\quad-\frac{2be\sqrt{a+bx^4}}{21ax^3}-\frac{bf\sqrt{a+bx^4}}{6ax^2}+\frac{2b^2c\sqrt{a+bx^4}}{15a^2x}-\frac{2b^{5/2}cx\sqrt{a+bx^4}}{15a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}\\ &\quad+\frac{b^2d\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}+\frac{2b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^4}}\\ &\quad-\frac{b^{7/4}\left(7\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

```
[Out] 1/16*b^2*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/504*(56*c/x^9+63*d/x^
8+72*e/x^7+84*f/x^6)*(b*x^4+a)^(1/2)-2/45*b*c*(b*x^4+a)^(1/2)/a/x^5-1/16*b*
d*(b*x^4+a)^(1/2)/a/x^4-2/21*b*e*(b*x^4+a)^(1/2)/a/x^3-1/6*b*f*(b*x^4+a)^(1
/2)/a/x^2+2/15*b^2*c*(b*x^4+a)^(1/2)/a^2/x-2/15*b^(5/2)*c*x*(b*x^4+a)^(1/2)
/a^2/(a^(1/2)+x^2*b^(1/2))+2/15*b^(9/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/
2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)-1/105*b^(7/4)*(cos(2*arctan(b^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(
b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(5*e*a^(1/2)+7*c*b^(1/2))*(a^(1/2)+x^2*b^(
1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {14, 1839, 1847, 1296, 1212, 226, 1210, 1266, 849, 821, 272, 65, 214}

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx$$
$$= -\frac{b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+7\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{7/4}\sqrt{a+bx^4}}$$
$$+\frac{2b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^4}}$$
$$+\frac{b^2d\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}-\frac{2b^{5/2}cx\sqrt{a+bx^4}}{15a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2b^2c\sqrt{a+bx^4}}{15a^2x}$$
$$-\frac{1}{504}\sqrt{a+bx^4}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4}-\frac{2be\sqrt{a+bx^4}}{21ax^3}-\frac{bf\sqrt{a+bx^4}}{6ax^2}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^10,x]
```

```
[Out] -1/504*(((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^7 + (84*f)/x^6)*Sqrt[a + b*x^4]
) - (2*b*c*Sqrt[a + b*x^4])/(45*a*x^5) - (b*d*Sqrt[a + b*x^4])/(16*a*x^4) -
 (2*b*e*Sqrt[a + b*x^4])/(21*a*x^3) - (b*f*Sqrt[a + b*x^4])/(6*a*x^2) + (2*
b^2*c*Sqrt[a + b*x^4])/(15*a^2*x) - (2*b^(5/2)*c*x*Sqrt[a + b*x^4])/(15*a^2
*(Sqrt[a] + Sqrt[b]*x^2)) + (b^2*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*a^
(3/2)) + (2*b^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(7/4)*S
qrt[a + b*x^4]) - (b^(7/4)*(7*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(105*a^(7/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\text{integral} = -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-(2b)\int\frac{-\frac{c}{9}-\frac{dx}{8}-\frac{ex^2}{7}-\frac{fx^3}{6}}{x^6\sqrt{a+bx^4}}\,dx$$

$$=-\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-(2b)\int\left(\frac{-\frac{c}{9}-\frac{ex^2}{7}}{x^6\sqrt{a+bx^4}}+\frac{-\frac{d}{8}-\frac{fx^2}{6}}{x^5\sqrt{a+bx^4}}\right)dx$$

$$=-\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-(2b)\int\frac{-\frac{c}{9}-\frac{ex^2}{7}}{x^6\sqrt{a+bx^4}}\,dx-(2b)\int\frac{-\frac{d}{8}-\frac{fx^2}{6}}{x^5\sqrt{a+bx^4}}\,dx$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5} - b\text{Subst}\left(\int\frac{-\frac{d}{8}-\frac{fx}{6}}{x^3\sqrt{a+bx^2}}\,dx,x,x^2\right)+\frac{(2b)\int\frac{\frac{5ae}{7}-\frac{1}{3}bcx^2}{x^4\sqrt{a+bx^4}}\,dx}{5a}$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4} - \frac{2be\sqrt{a+bx^4}}{21ax^3}-\frac{(2b)\int\frac{abc+\frac{5}{7}abex^2}{x^2\sqrt{a+bx^4}}\,dx}{15a^2}+\frac{b\text{Subst}\left(\int\frac{\frac{af}{3}-\frac{bdx}{8}}{x^2\sqrt{a+bx^2}}\,dx,x,x^2\right)}{2a}$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4}-\frac{2be\sqrt{a+bx^4}}{21ax^3} - \frac{bf\sqrt{a+bx^4}}{6ax^2}+\frac{2b^2c\sqrt{a+bx^4}}{15a^2x}+\frac{(2b)\int\frac{-\frac{5}{7}a^2be-ab^2cx^2}{\sqrt{a+bx^4}}\,dx}{15a^3}-\frac{(b^2d)\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)}{16a}$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4} - \frac{2be\sqrt{a+bx^4}}{21ax^3}-\frac{bf\sqrt{a+bx^4}}{6ax^2}+\frac{2b^2c\sqrt{a+bx^4}}{15a^2x}+\frac{(2b^{5/2}c)\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{15a^{3/2}} - \frac{(b^2d)\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)}{32a}-\frac{\left(2b^2\left(7\sqrt{b}c+5\sqrt{a}e\right)\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx}{105a^{3/2}}$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4} - \frac{2be\sqrt{a+bx^4}}{21ax^3}-\frac{bf\sqrt{a+bx^4}}{6ax^2}+\frac{2b^2c\sqrt{a+bx^4}}{15a^2x}-\frac{2b^{5/2}cx\sqrt{a+bx^4}}{15a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^4}} - \frac{b^{7/4}\left(7\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{7/4}\sqrt{a+bx^4}} - \frac{(bd)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)}{16a}$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}$$
$$-\frac{bd\sqrt{a+bx^4}}{16ax^4}-\frac{2be\sqrt{a+bx^4}}{21ax^3}-\frac{bf\sqrt{a+bx^4}}{6ax^2}+\frac{2b^2c\sqrt{a+bx^4}}{15a^2x}$$
$$-\frac{2b^{5/2}cx\sqrt{a+bx^4}}{15a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{b^2d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}$$
$$+\frac{2b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^4}}$$
$$-\frac{b^{7/4}\left(7\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{7/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.35

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx =$$
$$-\frac{\sqrt{a+bx^4}\left(14a^3c\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{1}{2},-\frac{5}{4},-\frac{bx^4}{a}\right)+3x^2\left(6a^3e\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{2},\right.\right.\right.}{126a^3x^9\sqrt{1+}}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^10,x]
```

```
[Out] -1/126*(Sqrt[a + b*x^4]*(14*a^3*c*Hypergeometric2F1[-9/4, -1/2, -5/4, -((b*
x^4)/a)] + 3*x^2*(6*a^3*e*Hypergeometric2F1[-7/4, -1/2, -3/4, -((b*x^4)/a)]
 + 7*x*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*(a^2*f + b^2*d*x^6*Hypergeometric2F1
[3/2, 3, 5/2, 1 + (b*x^4)/a]))))/(a^3*x^9*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.62 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(-672 b^{2} c\,x^{8}+840 a b f\,x^{7}+480 a e b\,x^{6}+315 x^{5} d b a+224 a b c\,x^{4}+840 a^{2} f\,x^{3}+720 a^{2} e\,x^{2}+630 a^{2} d x+560 a^{2} c\right)}{5040 x^{9} a^{2}}-\frac{b^{2}\left(\frac{80 a e\sqrt{1-\frac{i}{}}}{}\right.}{}$
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{9 x^{9}}-\frac{d\sqrt{b\,x^{4}+a}}{8 x^{8}}-\frac{e\sqrt{b\,x^{4}+a}}{7 x^{7}}-\frac{f\sqrt{b\,x^{4}+a}}{6 x^{6}}-\frac{2 b c\sqrt{b\,x^{4}+a}}{45 a\,x^{5}}-\frac{b d\sqrt{b\,x^{4}+a}}{16 a\,x^{4}}-\frac{2 b e\sqrt{b\,x^{4}+a}}{21 a\,x^{3}}-\frac{b f\sqrt{b\,x^{4}+a}}{6 a\,x^{2}}+\frac{2 b^{2} c\sqrt{}}{15 a}$
default	$-\frac{f\left(b\,x^{4}+a\right)^{\frac{3}{2}}}{6 a\,x^{6}}+e\left(-\frac{\sqrt{b\,x^{4}+a}}{7 x^{7}}-\frac{2 b\sqrt{b\,x^{4}+a}}{21 a\,x^{3}}-\frac{2 b^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21 a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+d\left(-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{2}}}{8 a\,x^{8}}+\frac{b(b\,x^{}}{16 a}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5040*(b*x^4+a)^(1/2)*(-672*b^2*c*x^8+840*a*b*f*x^7+480*a*b*e*x^6+315*a*b
*d*x^5+224*a*b*c*x^4+840*a^2*f*x^3+720*a^2*e*x^2+630*a^2*d*x+560*a^2*c)/x^9
/a^2-1/840*b^2/a^2*(80*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^
(1/2)*b^(1/2))^(1/2),I)+112*I*b^(1/2)*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2
))^(1/2),I))-105/2*a^(1/2)*d*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.49

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx = \frac{1344\sqrt{a}b^2cx^9\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right)+315\sqrt{a}b^2dx^9\log\left(-\frac{bx^4+2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-192\left(7b^2c-5a\right.}{}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/10080*(1344*sqrt(a)*b^2*c*x^9*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/
4)), -1) + 315*sqrt(a)*b^2*d*x^9*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) +
2*a)/x^4) - 192*(7*b^2*c - 5*a*b*e)*sqrt(a)*x^9*(-b/a)^(3/4)*elliptic_f(arc
sin(x*(-b/a)^(1/4)), -1) + 2*(672*b^2*c*x^8 - 840*a*b*f*x^7 - 480*a*b*e*x^6
 - 315*a*b*d*x^5 - 224*a*b*c*x^4 - 840*a^2*f*x^3 - 720*a^2*e*x^2 - 630*a^2*
d*x - 560*a^2*c)*sqrt(b*x^4 + a))/(a^2*x^9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.20 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.58

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} - \frac{ad}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{3\sqrt{b}d}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{b^{\frac{3}{2}}d}{16ax^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{6a} + \frac{b^2d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16a^{\frac{3}{2}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**10,x)
```

```
[Out] sqrt(a)*c*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**9*gamma(-5/4)) + sqrt(a)*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) - a*d/(8*sqrt(b)*x**10*sqrt
(a/(b*x**4) + 1)) - 3*sqrt(b)*d/(16*x**6*sqrt(a/(b*x**4) + 1)) - sqrt(b)*f*
sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*d/(16*a*x**2*sqrt(a/(b*x**4) + 1))
 - b**(3/2)*f*sqrt(a/(b*x**4) + 1)/(6*a) + b**2*d*asinh(sqrt(a)/(sqrt(b)*x*
*2))/(16*a**(3/2))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^{10}}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^{10}}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x, algorithm="giac")
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^{10}}\,dx$$

```
[In] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^10,x)
[Out] int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^10, x)
```

3.510 $\int x^4(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 476

$$\int x^4\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{4a^2cx\sqrt{a+bx^4}}{77b}-\frac{a^2dx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}-\frac{4a^3ex\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2ax^5(117c+77ex^2)\sqrt{a+bx^4}}{3003}-\frac{adx^2(a+bx^4)^{3/2}}{48b}+\frac{1}{143}x^5\left(13c+11ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{fx^4(a+bx^4)^{5/2}}{14b}-\frac{(12af-35bdx^2)\left(a+bx^4\right)^{5/2}}{420b^2}-\frac{a^3d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}+$$

```
[Out] -1/48*a*d*x^2*(b*x^4+a)^(3/2)/b+1/143*x^5*(11*e*x^2+13*c)*(b*x^4+a)^(3/2)+1
/14*f*x^4*(b*x^4+a)^(5/2)/b-1/420*(-35*b*d*x^2+12*a*f)*(b*x^4+a)^(5/2)/b^2-
1/32*a^3*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+4/77*a^2*c*x*(b*x^4
+a)^(1/2)/b-1/32*a^2*d*x^2*(b*x^4+a)^(1/2)/b+4/195*a^2*e*x^3*(b*x^4+a)^(1/2
)/b+2/3003*a*x^5*(77*e*x^2+117*c)*(b*x^4+a)^(1/2)-4/65*a^3*e*x*(b*x^4+a)^(1
/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))+4/65*a^(13/4)*e*(cos(2*arctan(b^(1/4)*x/a
^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b
^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x
^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)-2/5005*a^(11/4)*(cos(2*arctan(
b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin
(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(77*e*a^(1/2)+65*c*b^(1/2))*(a^(
1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+
a)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1847, 1288, 1294, 1212, 226, 1210, 1266, 847, 794, 201, 223, 212}

$$\int x^4\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx = -\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{a}e+65\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}} + \frac{4a^{13/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^4}} - \frac{a^3d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}} - \frac{4a^3ex\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{4a^2cx\sqrt{a+bx^4}}{77b} - \frac{a^2dx^2\sqrt{a+bx^4}}{32b} + \frac{4a^2ex^3\sqrt{a+bx^4}}{195b} - \frac{\left(a+bx^4\right)^{5/2}\left(12af-35bdx^2\right)}{420b^2} + \frac{1}{143}x^5\left(a+bx^4\right)^{3/2}\left(13c+11ex^2\right) + \frac{2ax^5\sqrt{a+bx^4}\left(117c+77ex^2\right)}{3003} - \frac{adx^2\left(a+bx^4\right)^{3/2}}{48b} + \frac{fx^4\left(a+bx^4\right)^{5/2}}{14b}$$

```
[In] Int[x^4*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (4*a^2*c*x*Sqrt[a + b*x^4])/(77*b) - (a^2*d*x^2*Sqrt[a + b*x^4])/(32*b) + (
4*a^2*e*x^3*Sqrt[a + b*x^4])/(195*b) - (4*a^3*e*x*Sqrt[a + b*x^4])/(65*b^(3
/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^5*(117*c + 77*e*x^2)*Sqrt[a + b*x^4])
/3003 - (a*d*x^2*(a + b*x^4)^(3/2))/(48*b) + (x^5*(13*c + 11*e*x^2)*(a + b*
x^4)^(3/2))/143 + (f*x^4*(a + b*x^4)^(5/2))/(14*b) - ((12*a*f - 35*b*d*x^2)
*(a + b*x^4)^(5/2))/(420*b^2) - (a^3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4
]])/(32*b^(3/2)) + (4*a^(13/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6
5*b^(7/4)*Sqrt[a + b*x^4]) - (2*a^(11/4)*(65*Sqrt[b]*c + 77*Sqrt[a]*e)*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

```
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(x^4(c+ex^2)\left(a+bx^4\right)^{3/2}+x^5(d+fx^2)\left(a+bx^4\right)^{3/2}\right)\,dx\\
&= \int x^4(c+ex^2)\left(a+bx^4\right)^{3/2}\,dx+\int x^5(d+fx^2)\left(a+bx^4\right)^{3/2}\,dx\\
&= \frac{1}{143}x^5(13c+11ex^2)\left(a+bx^4\right)^{3/2}+\frac{1}{2}\text{Subst}\left(\int x^2(d+fx)\left(a+bx^2\right)^{3/2}\,dx,x,x^2\right)\\
&\quad+\frac{1}{143}(6a)\int x^4(13c+11ex^2)\sqrt{a+bx^4}\,dx
\end{aligned}
$$

$$= \frac{2ax^5(117c+77ex^2)\sqrt{a+bx^4}}{3003} + \frac{1}{143}x^5\left(13c+11ex^2\right)\left(a+bx^4\right)^{3/2} + \frac{fx^4(a+bx^4)^{5/2}}{14b} + \frac{(4a^2)\int \frac{x^4(117c+77ex^2)}{\sqrt{a+bx^4}}\,dx}{3003} + \frac{\text{Subst}\left(\int x(-2af+7bdx)\left(a+bx^2\right)^{3/2}\,dx,x,x^2\right)}{14b}$$

$$= \frac{4a^2ex^3\sqrt{a+bx^4}}{195b} + \frac{2ax^5(117c+77ex^2)\sqrt{a+bx^4}}{3003} + \frac{1}{143}x^5\left(13c+11ex^2\right)\left(a+bx^4\right)^{3/2} + \frac{fx^4(a+bx^4)^{5/2}}{14b} - \frac{(12af-35bdx^2)\left(a+bx^4\right)^{5/2}}{420b^2} - \frac{(4a^2)\int \frac{x^2(231ae-585bcx^2)}{\sqrt{a+bx^4}}\,dx}{15015b} - \frac{(ad)\text{Subst}\left(\int\left(a+bx^2\right)^{3/2}\,dx,x,x^2\right)}{12b}$$

$$= \frac{4a^2cx\sqrt{a+bx^4}}{77b} + \frac{4a^2ex^3\sqrt{a+bx^4}}{195b} + \frac{2ax^5(117c+77ex^2)\sqrt{a+bx^4}}{3003} - \frac{adx^2(a+bx^4)^{3/2}}{48b} + \frac{1}{143}x^5\left(13c+11ex^2\right)\left(a+bx^4\right)^{3/2} + \frac{fx^4(a+bx^4)^{5/2}}{14b} - \frac{(12af-35bdx^2)\left(a+bx^4\right)^{5/2}}{420b^2} + \frac{(4a^2)\int \frac{-585abc-693abex^2}{\sqrt{a+bx^4}}\,dx}{45045b^2} - \frac{(a^2d)\,\text{Subst}\left(\int\sqrt{a+bx^2}\,dx,x,x^2\right)}{16b}$$

$$= \frac{4a^2cx\sqrt{a+bx^4}}{77b} - \frac{a^2dx^2\sqrt{a+bx^4}}{32b} + \frac{4a^2ex^3\sqrt{a+bx^4}}{195b} + \frac{2ax^5(117c+77ex^2)\sqrt{a+bx^4}}{3003} - \frac{adx^2(a+bx^4)^{3/2}}{48b} + \frac{1}{143}x^5\left(13c+11ex^2\right)\left(a+bx^4\right)^{3/2} + \frac{fx^4(a+bx^4)^{5/2}}{14b} - \frac{(12af-35bdx^2)\left(a+bx^4\right)^{5/2}}{420b^2} - \frac{(a^3d)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)}{32b} + \frac{(4a^{7/2}e)\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{65b^{3/2}} - \frac{\left(4a^3\left(65\sqrt{b}c+77\sqrt{a}e\right)\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx}{5005b^{3/2}}$$

$$= \frac{4a^2cx\sqrt{a+bx^4}}{77b} - \frac{a^2dx^2\sqrt{a+bx^4}}{32b} + \frac{4a^2ex^3\sqrt{a+bx^4}}{195b} - \frac{4a^3ex\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2ax^5(117c+77ex^2)\sqrt{a+bx^4}}{3003} - \frac{adx^2(a+bx^4)^{3/2}}{48b} + \frac{1}{143}x^5\left(13c+11ex^2\right)\left(a+bx^4\right)^{3/2} + \frac{fx^4(a+bx^4)^{5/2}}{14b} - \frac{(12af-35bdx^2)(a+bx^4)^{5/2}}{420b^2} + \frac{4a^{13/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^4}} - \frac{2a^{11/4}\left(65\sqrt{b}c+77\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}} - \frac{(a^3d)\,\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)}{32b}$$

$$= \frac{4a^2cx\sqrt{a+bx^4}}{77b} - \frac{a^2dx^2\sqrt{a+bx^4}}{32b} + \frac{4a^2ex^3\sqrt{a+bx^4}}{195b} - \frac{4a^3ex\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2ax^5(117c+77ex^2)\sqrt{a+bx^4}}{3003} - \frac{adx^2(a+bx^4)^{3/2}}{48b} + \frac{1}{143}x^5\left(13c+11ex^2\right)\left(a+bx^4\right)^{3/2} + \frac{fx^4(a+bx^4)^{5/2}}{14b} - \frac{(12af-35bdx^2)(a+bx^4)^{5/2}}{420b^2} - \frac{a^3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}} + \frac{4a^{13/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^4}} - \frac{2a^{11/4}\left(65\sqrt{b}c+77\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.88 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.47

$$\int x^4(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx = \frac{\sqrt{a+bx^4}\left(43680bcx(a+bx^4)^2+36960bex^3(a+bx^4)^2+6864f(a+bx^4)^2\left(-2a+5bx^4\right)+50\right.}{}$$

```
[In] Integrate[x^4*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^4]*(43680*b*c*x*(a + b*x^4)^2 + 36960*b*e*x^3*(a + b*x^4)^2 +
 6864*f*(a + b*x^4)^2*(-2*a + 5*b*x^4) + 5005*b*d*x^2*(3*a^2 + 14*a*b*x^4 +
 8*b^2*x^8) - (15015*a^(5/2)*Sqrt[b]*d*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt
[1 + (b*x^4)/a] - (43680*a^2*b*c*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x
^4)/a)])/Sqrt[1 + (b*x^4)/a] - (36960*a^2*b*e*x^3*Hypergeometric2F1[-3/2, 3
/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/(480480*b^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{\left(-34320b^3fx^{12}-36960b^3ex^{11}-40040b^3dx^{10}-43680b^3cx^9-54912x^8ab^2f-61600ab^2ex^7-70070ab^2dx^6-81120ab^2cx^5-6864a\right.}{480480b^2}$
default	$-\frac{f\sqrt{bx^4+a}\,(-5bx^4+2a)(b^2x^8+2abx^4+a^2)}{70b^2}+e\left(\frac{bx^{11}\sqrt{bx^4+a}}{13}+\frac{5ax^7\sqrt{bx^4+a}}{39}+\frac{4a^2x^3\sqrt{bx^4+a}}{195b}-\frac{4ia^{\frac{7}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{}}{}\right.$
elliptic	$\frac{bfx^{12}\sqrt{bx^4+a}}{14}+\frac{bex^{11}\sqrt{bx^4+a}}{13}+\frac{bdx^{10}\sqrt{bx^4+a}}{12}+\frac{bcx^9\sqrt{bx^4+a}}{11}+\frac{4afx^8\sqrt{bx^4+a}}{35}+\frac{5aex^7\sqrt{bx^4+a}}{39}+\frac{7adx^6\sqrt{bx^4+}}{48}$

```
[In] int(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/480480*(-34320*b^3*f*x^12-36960*b^3*e*x^11-40040*b^3*d*x^10-43680*b^3*c*
x^9-54912*a*b^2*f*x^8-61600*a*b^2*e*x^7-70070*a*b^2*d*x^6-81120*a*b^2*c*x^5
-6864*a^2*b*f*x^4-9856*a^2*b*e*x^3-15015*a^2*b*d*x^2-24960*a^2*b*c*x+13728*
a^3*f)/b^2*(b*x^4+a)^(1/2)-1/80080/b*a^3*(4160*c/(I/a^(1/2)*b^(1/2))^(1/2)*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(
1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+4928*I*e*a^(1/2)/(I/a^(1/2)*b
```

```
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/
2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ellipt
icE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+5005/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)
)/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.55

$$\int x^4\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx = -\frac{59136\,a^3\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1) - 15015\,a^3\sqrt{b}dx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+768}{}$$

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/960960*(59136*a^3*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4
)/x), -1) - 15015*a^3*sqrt(b)*d*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*
x^2 - a) + 768*(65*a^2*b*c - 77*a^3*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(ar
csin((-a/b)^(1/4)/x), -1) - 2*(34320*b^3*f*x^13 + 36960*b^3*e*x^12 + 40040*
b^3*d*x^11 + 43680*b^3*c*x^10 + 54912*a*b^2*f*x^9 + 61600*a*b^2*e*x^8 + 700
70*a*b^2*d*x^7 + 81120*a*b^2*c*x^6 + 6864*a^2*b*f*x^5 + 9856*a^2*b*e*x^4 +
15015*a^2*b*d*x^3 + 24960*a^2*b*c*x^2 - 13728*a^3*f*x - 29568*a^3*e)*sqrt(b
*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 462, normalized size of antiderivative = 0.97

$$\int x^4(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\frac{a^{\frac{5}{2}}dx^2}{32b\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{17a^{\frac{3}{2}}dx^6}{96\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}bcx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{11\sqrt{a}bdx^{10}}{48\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bex^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{15}{4}\right)}-\frac{a^3d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}}+af\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right)+bf\left(\begin{cases}\frac{4a^3\sqrt{a+bx^4}}{105b^3}-\frac{2a^2x^4\sqrt{a+bx^4}}{105b^2}+\frac{ax^8\sqrt{a+bx^4}}{70b}+\frac{x^{12}\sqrt{a+bx^4}}{14} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^{12}}{12} & \text{otherwise}\end{cases}\right)+\frac{b^2dx^{14}}{12\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(5/2)*d*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*c*x**5*gamma(5/4)*hype
r((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 17*a**(3/
2)*d*x**6/(96*sqrt(1 + b*x**4/a)) + a**(3/2)*e*x**7*gamma(7/4)*hyper((-1/2,
 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*c*x**
9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma
(13/4)) + 11*sqrt(a)*b*d*x**10/(48*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*x**11*
gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma
(15/4)) - a**3*d*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*f*Piecewise(
(-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*s
qrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b*f*Piecewise((4*a
**3*sqrt(a + b*x**4)/(105*b**3) - 2*a**2*x**4*sqrt(a + b*x**4)/(105*b**2) +
 a*x**8*sqrt(a + b*x**4)/(70*b) + x**12*sqrt(a + b*x**4)/14, Ne(b, 0)), (sq
rt(a)*x**12/12, True)) + b**2*d*x**14/(12*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int x^4(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int \left(bx^4+a\right)^{\frac{3}{2}}(fx^3+ex^2+dx+c)x^4\,dx$$

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

Giac [F]

$$\int x^4(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int \left(bx^4+a\right)^{\frac{3}{2}}(fx^3+ex^2+dx+c)x^4\,dx$$

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int x^4\,\left(b\,x^4+a\right)^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

```
[In] int(x^4*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
[Out] int(x^4*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.511 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 452

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{4a^2dx\sqrt{a+bx^4}}{77b}-\frac{a^2ex^2\sqrt{a+bx^4}}{32b}+\frac{4a^2fx^3\sqrt{a+bx^4}}{195b}-\frac{4a^3fx\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2ax^5(117d+77fx^2)\sqrt{a+bx^4}}{3003}-\frac{aex^2(a+bx^4)^{3/2}}{48b}+\frac{1}{143}x^5\left(13d+11fx^2\right)\left(a+bx^4\right)^{3/2}+\frac{(6c+5ex^2)\left(a+bx^4\right)^{5/2}}{60b}-\frac{a^3e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}+\frac{4a^{13/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)}{}$$

```
[Out] -1/48*a*e*x^2*(b*x^4+a)^(3/2)/b+1/143*x^5*(11*f*x^2+13*d)*(b*x^4+a)^(3/2)+1
/60*(5*e*x^2+6*c)*(b*x^4+a)^(5/2)/b-1/32*a^3*e*arctanh(x^2*b^(1/2)/(b*x^4+a
)^(1/2))/b^(3/2)+4/77*a^2*d*x*(b*x^4+a)^(1/2)/b-1/32*a^2*e*x^2*(b*x^4+a)^(1
/2)/b+4/195*a^2*f*x^3*(b*x^4+a)^(1/2)/b+2/3003*a*x^5*(77*f*x^2+117*d)*(b*x^
4+a)^(1/2)-4/65*a^3*f*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))+4/65*
a^(13/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2
)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^
(1/2)-2/5005*a^(11/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/
2))*(77*f*a^(1/2)+65*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x
^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1847, 1266, 794, 201, 223, 212, 1288, 1294, 1212, 226, 1210}

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx = -\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{a}f+65\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}} + \frac{4a^{13/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^4}} - \frac{a^3e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}} - \frac{4a^3fx\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{4a^2dx\sqrt{a+bx^4}}{77b} - \frac{a^2ex^2\sqrt{a+bx^4}}{32b} + \frac{4a^2fx^3\sqrt{a+bx^4}}{195b} + \frac{\left(a+bx^4\right)^{5/2}(6c+5ex^2)}{60b} + \frac{1}{143}x^5\left(a+bx^4\right)^{3/2}(13d+11fx^2) + \frac{2ax^5\sqrt{a+bx^4}(117d+77fx^2)}{3003} - \frac{aex^2\left(a+bx^4\right)^{3/2}}{48b}$$

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (4*a^2*d*x*Sqrt[a + b*x^4])/(77*b) - (a^2*e*x^2*Sqrt[a + b*x^4])/(32*b) + (
4*a^2*f*x^3*Sqrt[a + b*x^4])/(195*b) - (4*a^3*f*x*Sqrt[a + b*x^4])/(65*b^(3
/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^5*(117*d + 77*f*x^2)*Sqrt[a + b*x^4])
/3003 - (a*e*x^2*(a + b*x^4)^(3/2))/(48*b) + (x^5*(13*d + 11*f*x^2)*(a + b*
x^4)^(3/2))/143 + ((6*c + 5*e*x^2)*(a + b*x^4)^(5/2))/(60*b) - (a^3*e*ArcTa
nh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(32*b^(3/2)) + (4*a^(13/4)*f*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a + b*x^4]) - (2*a^(11/4)*(65
*Sqrt[b]*d + 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*b^
(7/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(x^3(c+ex^2)\left(a+bx^4\right)^{3/2}+x^4(d+fx^2)\left(a+bx^4\right)^{3/2}\right)\,dx\\
&= \int x^3(c+ex^2)\left(a+bx^4\right)^{3/2}\,dx+\int x^4(d+fx^2)\left(a+bx^4\right)^{3/2}\,dx\\
&= \frac{1}{143}x^5(13d+11fx^2)\left(a+bx^4\right)^{3/2}+\frac{1}{2}\text{Subst}\left(\int x(c+ex)\left(a+bx^2\right)^{3/2}\,dx,x,x^2\right)\\
&\quad+\frac{1}{143}(6a)\int x^4(13d+11fx^2)\sqrt{a+bx^4}\,dx\\
&= \frac{2ax^5(117d+77fx^2)\sqrt{a+bx^4}}{3003}+\frac{1}{143}x^5(13d+11fx^2)\left(a+bx^4\right)^{3/2}\\
&\quad+\frac{(6c+5ex^2)\left(a+bx^4\right)^{5/2}}{60b}+\frac{(4a^2)\int\frac{x^4(117d+77fx^2)}{\sqrt{a+bx^4}}\,dx}{3003}\\
&\quad-\frac{(ae)\text{Subst}\left(\int\left(a+bx^2\right)^{3/2}\,dx,x,x^2\right)}{12b}
\end{aligned}$$

$$= \frac{4a^2fx^3\sqrt{a+bx^4}}{195b} + \frac{2ax^5(117d+77fx^2)\sqrt{a+bx^4}}{3003} - \frac{aex^2(a+bx^4)^{3/2}}{48b} + \frac{1}{143}x^5(13d+11fx^2)(a+bx^4)^{3/2} + \frac{(6c+5ex^2)(a+bx^4)^{5/2}}{60b} - \frac{(4a^2)\int \frac{x^2(231af-585bdx^2)}{\sqrt{a+bx^4}}\,dx}{15015b} - \frac{(a^2e)\,\mathrm{Subst}\left(\int \sqrt{a+bx^2}\,dx,x,x^2\right)}{16b}$$

$$= \frac{4a^2dx\sqrt{a+bx^4}}{77b} - \frac{a^2ex^2\sqrt{a+bx^4}}{32b} + \frac{4a^2fx^3\sqrt{a+bx^4}}{195b} + \frac{2ax^5(117d+77fx^2)\sqrt{a+bx^4}}{3003} - \frac{aex^2(a+bx^4)^{3/2}}{48b} + \frac{1}{143}x^5(13d+11fx^2)(a+bx^4)^{3/2} + \frac{(6c+5ex^2)(a+bx^4)^{5/2}}{60b} + \frac{(4a^2)\int \frac{-585abd-693abfx^2}{\sqrt{a+bx^4}}\,dx}{45045b^2} - \frac{(a^3e)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)}{32b}$$

$$= \frac{4a^2dx\sqrt{a+bx^4}}{77b} - \frac{a^2ex^2\sqrt{a+bx^4}}{32b} + \frac{4a^2fx^3\sqrt{a+bx^4}}{195b} + \frac{2ax^5(117d+77fx^2)\sqrt{a+bx^4}}{3003} - \frac{aex^2(a+bx^4)^{3/2}}{48b} + \frac{1}{143}x^5(13d+11fx^2)(a+bx^4)^{3/2} + \frac{(6c+5ex^2)(a+bx^4)^{5/2}}{60b} - \frac{(a^3e)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)}{32b} + \frac{(4a^{7/2}f)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{65b^{3/2}} - \frac{\left(4a^3\left(65\sqrt{b}d+77\sqrt{a}f\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{5005b^{3/2}}$$

$$= \frac{4a^2dx\sqrt{a+bx^4}}{77b} - \frac{a^2ex^2\sqrt{a+bx^4}}{32b} + \frac{4a^2fx^3\sqrt{a+bx^4}}{195b} - \frac{4a^3fx\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2ax^5(117d+77fx^2)\sqrt{a+bx^4}}{3003} - \frac{aex^2(a+bx^4)^{3/2}}{48b} + \frac{1}{143}x^5(13d+11fx^2)(a+bx^4)^{3/2} + \frac{(6c+5ex^2)(a+bx^4)^{5/2}}{60b} - \frac{a^3e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}} + \frac{4a^{13/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^4}} - \frac{2a^{11/4}\left(65\sqrt{b}d+77\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.75 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.53

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\frac{\sqrt{a+bx^4}\left(6864\sqrt{b}c(a+bx^4)^2+6240\sqrt{b}dx(a+bx^4)^2+5280\sqrt{b}fx^3(a+bx^4)^2+715e\left(\sqrt{b}x^2(3\right.\right.}{}$$

[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]

[Out] (Sqrt[a + b*x^4]*(6864*Sqrt[b]*c*(a + b*x^4)^2 + 6240*Sqrt[b]*d*x*(a + b*x^4)^2 + 5280*Sqrt[b]*f*x^3*(a + b*x^4)^2 + 715*e*(Sqrt[b]*x^2*(3*a^2 + 14*a*b*x^4 + 8*b^2*x^8) - (3*a^(5/2)*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a]) - (6240*a^2*Sqrt[b]*d*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a] - (5280*a^2*Sqrt[b]*f*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/(68640*b^(3/2))

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.70

method	result
risch	$\frac{(36960b^2fx^{11}+40040b^2ex^{10}+43680b^2dx^9+48048b^2cx^8+61600abfx^7+70070aebx^6+81120x^5dba+96096abcx^4+9856a^2fx^3+15015a}{480480b}$
default	$f\left(\frac{bx^{11}\sqrt{bx^4+a}}{13}+\frac{5ax^7\sqrt{bx^4+a}}{39}+\frac{4a^2x^3\sqrt{bx^4+a}}{195b}-\frac{4ia^{\frac{7}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{65b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+a}}\right)+e$
elliptic	$\frac{bfx^{11}\sqrt{bx^4+a}}{13}+\frac{bex^{10}\sqrt{bx^4+a}}{12}+\frac{bdx^9\sqrt{bx^4+a}}{11}+\frac{bcx^8\sqrt{bx^4+a}}{10}+\frac{5afx^7\sqrt{bx^4+a}}{39}+\frac{7aex^6\sqrt{bx^4+a}}{48}+\frac{13adx^5\sqrt{bx^4+a}}{77}$

[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/480480*(36960*b^2*f*x^11+40040*b^2*e*x^10+43680*b^2*d*x^9+48048*b^2*c*x^8+61600*a*b*f*x^7+70070*a*b*e*x^6+81120*a*b*d*x^5+96096*a*b*c*x^4+9856*a^2*f*x^3+15015*a^2*e*x^2+24960*a^2*d*x+48048*a^2*c)/b*(b*x^4+a)^(1/2)-1/80080/b*a^3*(4160*d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+4928*I*f*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x

```
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Elliptic
F(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+
5005/2*e*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.16 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.56

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx = -\frac{59136\,a^3\sqrt{b}fx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-15015\,a^3\sqrt{b}ex\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+76}{}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/960960*(59136*a^3*sqrt(b)*f*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4
)/x), -1) - 15015*a^3*sqrt(b)*e*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*
x^2 - a) + 768*(65*a^2*b*d - 77*a^3*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(ar
csin((-a/b)^(1/4)/x), -1) - 2*(36960*b^3*f*x^12 + 40040*b^3*e*x^11 + 43680*
b^3*d*x^10 + 48048*b^3*c*x^9 + 61600*a*b^2*f*x^8 + 70070*a*b^2*e*x^7 + 8112
0*a*b^2*d*x^6 + 96096*a*b^2*c*x^5 + 9856*a^2*b*f*x^4 + 15015*a^2*b*e*x^3 +
24960*a^2*b*d*x^2 + 48048*a^2*b*c*x - 29568*a^3*f)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.88

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\frac{a^{\frac{5}{2}}ex^2}{32b\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{17a^{\frac{3}{2}}ex^6}{96\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}bdx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{11\sqrt{a}bex^{10}}{48\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bfx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{15}{4}\right)}-\frac{a^3e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}}+ac\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for}\ b=0\\ \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)+bc\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right)+\frac{b^2ex^{14}}{12\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(5/2)*e*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*d*x**5*gamma(5/4)*hype
r((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 17*a**(3/
2)*e*x**6/(96*sqrt(1 + b*x**4/a)) + a**(3/2)*f*x**7*gamma(7/4)*hyper((-1/2,
 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*d*x**
9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma
(13/4)) + 11*sqrt(a)*b*e*x**10/(48*sqrt(1 + b*x**4/a)) + sqrt(a)*b*f*x**11*
gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma
(15/4)) - a**3*e*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*c*Piecewise(
(sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*c*Piecew
ise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x*
*8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*e*x**14/(
12*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int (bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)x^3\,dx$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] 1/10*(b*x^4 + a)^(5/2)*c/b + integrate((b*f*x^10 + b*e*x^9 + b*d*x^8 + a*f*
x^6 + a*e*x^5 + a*d*x^4)*sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int (bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)x^3\,dx$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int x^3\,\left(b\,x^4+a\right)^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

```
[In] int(x^3*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
[Out] int(x^3*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.512 $\int x^2(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 427

$$\int x^2\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{4a^2ex\sqrt{a+bx^4}}{77b}-\frac{a^2fx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2ax^3(77c+45ex^2)\sqrt{a+bx^4}}{1155}-\frac{afx^2(a+bx^4)^{3/2}}{48b}+\frac{1}{99}x^3\left(11c+9ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{(6d+5fx^2)\left(a+bx^4\right)^{5/2}}{60b}-\frac{a^3f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}+\frac{2a^{9/4}\left(77\sqrt{b}c-15\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}$$

```
[Out] -1/48*a*f*x^2*(b*x^4+a)^(3/2)/b+1/99*x^3*(9*e*x^2+11*c)*(b*x^4+a)^(3/2)+1/6
0*(5*f*x^2+6*d)*(b*x^4+a)^(5/2)/b-1/32*a^3*f*arctanh(x^2*b^(1/2)/(b*x^4+a)^
(1/2))/b^(3/2)+4/77*a^2*e*x*(b*x^4+a)^(1/2)/b-1/32*a^2*f*x^2*(b*x^4+a)^(1/2
)/b+2/1155*a*x^3*(45*e*x^2+77*c)*(b*x^4+a)^(1/2)+4/15*a^2*c*x*(b*x^4+a)^(1/
2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))-4/15*a^(9/4)*c*(cos(2*arctan(b^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(
1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2
*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+2/1155*a^(9/4)*(cos(2*arctan(b^(
1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*
arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-15*e*a^(1/2)+77*c*b^(1/2))*(a^(1/
2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)
^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1847, 1288, 1294, 1212, 226, 1210, 1266, 794, 201, 223, 212}

$$\int x^2\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{b}c-15\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{a^3f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}+\frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4a^2ex\sqrt{a+bx^4}}{77b}-\frac{a^2fx^2\sqrt{a+bx^4}}{32b}+\frac{2ax^3\sqrt{a+bx^4}(77c+45ex^2)}{1155}+\frac{1}{99}x^3\left(a+bx^4\right)^{3/2}\left(11c+9ex^2\right)+\frac{\left(a+bx^4\right)^{5/2}\left(6d+5fx^2\right)}{60b}-\frac{afx^2(a+bx^4)^{3/2}}{48b}$$

```
[In] Int[x^2*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (4*a^2*e*x*Sqrt[a + b*x^4])/(77*b) - (a^2*f*x^2*Sqrt[a + b*x^4])/(32*b) + (
4*a^2*c*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^3*
(77*c + 45*e*x^2)*Sqrt[a + b*x^4])/1155 - (a*f*x^2*(a + b*x^4)^(3/2))/(48*b
) + (x^3*(11*c + 9*e*x^2)*(a + b*x^4)^(3/2))/99 + ((6*d + 5*f*x^2)*(a + b*x
^4)^(5/2))/(60*b) - (a^3*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(32*b^(3
/2)) - (4*a^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqr
t[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqr
t[a + b*x^4]) + (2*a^(9/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(1155*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
```

```
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(x^2\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}+x^3\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\right)\,dx\\
&= \int x^2\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}\,dx+\int x^3\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\,dx\\
&= \frac{1}{99}x^3\left(11c+9ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{1}{2}\text{Subst}\left(\int x(d+fx)\left(a+bx^2\right)^{3/2}\,dx,x,x^2\right)\\
&\quad+\frac{1}{33}(2a)\int x^2\left(11c+9ex^2\right)\sqrt{a+bx^4}\,dx\\
&= \frac{2ax^3(77c+45ex^2)\sqrt{a+bx^4}}{1155}+\frac{1}{99}x^3\left(11c+9ex^2\right)\left(a+bx^4\right)^{3/2}\\
&\quad+\frac{(6d+5fx^2)\left(a+bx^4\right)^{5/2}}{60b}+\frac{(4a^2)\int\frac{x^2(77c+45ex^2)}{\sqrt{a+bx^4}}\,dx}{1155}\\
&\quad-\frac{(af)\text{Subst}\left(\int\left(a+bx^2\right)^{3/2}\,dx,x,x^2\right)}{12b}
\end{aligned}$$

$$= \frac{4a^2ex\sqrt{a+bx^4}}{77b} + \frac{2ax^3(77c+45ex^2)\sqrt{a+bx^4}}{1155} - \frac{afx^2(a+bx^4)^{3/2}}{48b} + \frac{1}{99}x^3\left(11c+9ex^2\right)\left(a+bx^4\right)^{3/2} + \frac{(6d+5fx^2)\left(a+bx^4\right)^{5/2}}{60b} - \frac{(4a^2)\int \frac{45ae-231bcx^2}{\sqrt{a+bx^4}}\,dx}{3465b} - \frac{(a^2f)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, x^2\right)}{16b}$$

$$= \frac{4a^2ex\sqrt{a+bx^4}}{77b} - \frac{a^2fx^2\sqrt{a+bx^4}}{32b} + \frac{2ax^3(77c+45ex^2)\sqrt{a+bx^4}}{1155} - \frac{afx^2(a+bx^4)^{3/2}}{48b} + \frac{1}{99}x^3\left(11c+9ex^2\right)\left(a+bx^4\right)^{3/2} + \frac{(6d+5fx^2)\left(a+bx^4\right)^{5/2}}{60b} - \frac{\left(4a^{5/2}c\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{15\sqrt{b}} + \frac{\left(4a^{5/2}\left(77\sqrt{b}c-15\sqrt{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{1155b} - \frac{(a^3f)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{32b}$$

$$= \frac{4a^2ex\sqrt{a+bx^4}}{77b} - \frac{a^2fx^2\sqrt{a+bx^4}}{32b} + \frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2ax^3(77c+45ex^2)\sqrt{a+bx^4}}{1155} - \frac{afx^2(a+bx^4)^{3/2}}{48b} + \frac{1}{99}x^3\left(11c+9ex^2\right)\left(a+bx^4\right)^{3/2} + \frac{(6d+5fx^2)\left(a+bx^4\right)^{5/2}}{60b} - \frac{4a^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} + \frac{2a^{9/4}\left(77\sqrt{b}c-15\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}} - \frac{(a^3f)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)}{32b}$$

$$= \frac{4a^2ex\sqrt{a+bx^4}}{77b} - \frac{a^2fx^2\sqrt{a+bx^4}}{32b} + \frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2ax^3(77c+45ex^2)\sqrt{a+bx^4}}{1155}$$
$$- \frac{afx^2(a+bx^4)^{3/2}}{48b} + \frac{1}{99}x^3(11c+9ex^2)\left(a+bx^4\right)^{3/2} + \frac{(6d+5fx^2)\left(a+bx^4\right)^{5/2}}{60b}$$
$$- \frac{a^3f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}} - \frac{4a^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}$$
$$+ \frac{2a^{9/4}\left(77\sqrt{b}c-15\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.48

$$\int x^2(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx = \frac{\sqrt{a+bx^4}\left(\frac{528d(a+bx^4)^2}{b} + \frac{480ex(a+bx^4)^2}{b} + \frac{55f\left(\sqrt{b}x^2(3a^2+14abx^4+8b^2x^8) - \frac{3a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)}{b^{3/2}} - 480a^2 \cdots\right.}{5280}$$

```
[In] Integrate[x^2*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^4]*((528*d*(a + b*x^4)^2)/b + (480*e*x*(a + b*x^4)^2)/b + (55
*f*(Sqrt[b]*x^2*(3*a^2 + 14*a*b*x^4 + 8*b^2*x^8) - (3*a^(5/2)*ArcSinh[(Sqrt
[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a]))/b^(3/2) - (480*a^2*e*x*Hypergeomet
ric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (1760*a*c*x
^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/5
280
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.09 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.73

method	result
risch	$\frac{(9240b^2 f x^{10}+10080b^2 e x^9+11088b^2 d x^8+12320b^2 c x^7+16170abf x^6+18720abe x^5+22176abd x^4+27104abc x^3+3465a^2 f x^2+5760a^2 e}{110880b}$
default	$f\left(\frac{b x^{10}\sqrt{b x^4+a}}{12}+\frac{7a x^6\sqrt{b x^4+a}}{48}+\frac{a^2x^2\sqrt{b x^4+a}}{32b}-\frac{a^3\ln\left(x^2\sqrt{b}+\sqrt{b x^4+a}\right)}{32b^{\frac{3}{2}}}\right)+e\left(\frac{b x^9\sqrt{b x^4+a}}{11}+\frac{13a x^5\sqrt{b x^4+a}}{77}+\right.$
elliptic	$\frac{bf x^{10}\sqrt{b x^4+a}}{12}+\frac{be x^9\sqrt{b x^4+a}}{11}+\frac{bd x^8\sqrt{b x^4+a}}{10}+\frac{bc x^7\sqrt{b x^4+a}}{9}+\frac{7af x^6\sqrt{b x^4+a}}{48}+\frac{13ae x^5\sqrt{b x^4+a}}{77}+\frac{ad x^4\sqrt{b x^4+a}}{5}+$

```
[In] int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/110880*(9240*b^2*f*x^10+10080*b^2*e*x^9+11088*b^2*d*x^8+12320*b^2*c*x^7+1
6170*a*b*f*x^6+18720*a*b*e*x^5+22176*a*b*d*x^4+27104*a*b*c*x^3+3465*a^2*f*x
^2+5760*a^2*e*x+11088*a^2*d)/b*(b*x^4+a)^(1/2)-1/18480*a^2/b*(960*a*e/(I/a^
(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x
^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-4928*I*b
^(1/2)*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*
(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1155/2*a*f*ln(x^2*
b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.15 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.58

$$\int x^2\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{59136\,a^2b^{\frac{3}{2}}cx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1\right)+3465\,a^3\sqrt{b}fx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-\right.}{}$$

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/221760*(59136*a^2*b^(3/2)*c*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)
/x), -1) + 3465*a^3*sqrt(b)*f*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^
2 - a) - 768*(77*a^2*b*c + 15*a^2*b*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(ar
csin((-a/b)^(1/4)/x), -1) + 2*(9240*b^3*f*x^11 + 10080*b^3*e*x^10 + 11088*b
^3*d*x^9 + 12320*b^3*c*x^8 + 16170*a*b^2*f*x^7 + 18720*a*b^2*e*x^6 + 22176*
a*b^2*d*x^5 + 27104*a*b^2*c*x^4 + 3465*a^2*b*f*x^3 + 5760*a^2*b*e*x^2 + 110
88*a^2*b*d*x + 29568*a^2*b*c)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.93

$$\int x^2\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{a^{\frac{5}{2}}fx^2}{32b\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{a^{\frac{3}{2}}ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{17a^{\frac{3}{2}}fx^6}{96\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bcx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}bex^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{11\sqrt{a}bfx^{10}}{48\sqrt{1+\frac{bx^4}{a}}}-\frac{a^3f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}}+ad\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)+bd\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right)+\frac{b^2fx^{14}}{12\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(5/2)*f*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*c*x**3*gamma(3/4)*hype
r((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a**(3/2)*
e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*g
amma(9/4)) + 17*a**(3/2)*f*x**6/(96*sqrt(1 + b*x**4/a)) + sqrt(a)*b*c*x**7*
gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(1
1/4)) + sqrt(a)*b*e*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_
polar(I*pi)/a)/(4*gamma(13/4)) + 11*sqrt(a)*b*f*x**10/(48*sqrt(1 + b*x**4/a
)) - a**3*f*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*d*Piecewise((sqrt
(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*d*Piecewise((
-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sq
rt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*f*x**14/(12*sq
rt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int x^2(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int (bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)x^2\,dx$$

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

Giac [F]

$$\int x^2(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int (bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)x^2\,dx$$

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int x^2\,\left(b\,x^4+a\right)^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

```
[In] int(x^2*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
[Out] int(x^2*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.513 $\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 409

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{3}{16}acx^2\sqrt{a+bx^4}+\frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2ax^3(77d+45fx^2)\sqrt{a+bx^4}}{1155}+\frac{1}{8}cx^2\left(a+bx^4\right)^{3/2}+\frac{1}{99}x^3(11d+9fx^2)\left(a+bx^4\right)^{3/2}+\frac{e\left(a+bx^4\right)^{5/2}}{10b}+\frac{3a^2c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}-\frac{4a^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}+\frac{2a^{9/4}\left(77\sqrt{b}d-15\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}$$

```
[Out] 1/8*c*x^2*(b*x^4+a)^(3/2)+1/99*x^3*(9*f*x^2+11*d)*(b*x^4+a)^(3/2)+1/10*e*(b
*x^4+a)^(5/2)/b+3/16*a^2*c*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+4/7
7*a^2*f*x*(b*x^4+a)^(1/2)/b+3/16*a*c*x^2*(b*x^4+a)^(1/2)+2/1155*a*x^3*(45*f
*x^2+77*d)*(b*x^4+a)^(1/2)+4/15*a^2*d*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^
2*b^(1/2))-4/15*a^(9/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2
^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(
3/4)/(b*x^4+a)^(1/2)+2/1155*a^(9/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/
4))),1/2*2^(1/2))*(-15*f*a^(1/2)+77*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^
4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1847, 1262, 655, 201, 223, 212, 1288, 1294, 1212, 226, 1210}

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}dx=\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{b}d-15\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}+\frac{3a^2c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{1}{8}cx^2\left(a+bx^4\right)^{3/2}+\frac{3}{16}acx^2\sqrt{a+bx^4}+\frac{1}{99}x^3\left(a+bx^4\right)^{3/2}\left(11d+9fx^2\right)+\frac{2ax^3\sqrt{a+bx^4}(77d+45fx^2)}{1155}+\frac{e(a+b}{10}$$

```
[In] Int[x*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (4*a^2*f*x*Sqrt[a + b*x^4])/(77*b) + (3*a*c*x^2*Sqrt[a + b*x^4])/16 + (4*a^
2*d*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^3*(77*
d + 45*f*x^2)*Sqrt[a + b*x^4])/1155 + (c*x^2*(a + b*x^4)^(3/2))/8 + (x^3*(1
1*d + 9*f*x^2)*(a + b*x^4)^(3/2))/99 + (e*(a + b*x^4)^(5/2))/(10*b) + (3*a^
2*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (4*a^(9/4)*d*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(9
/4)*(77*Sqrt[b]*d - 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
1155*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
```

```
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(x\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}+x^2\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\right)\,dx\\
&= \int x\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}\,dx+\int x^2\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\,dx\\
&= \frac{1}{99}x^3\left(11d+9fx^2\right)\left(a+bx^4\right)^{3/2}+\frac{1}{2}\text{Subst}\left(\int (c+ex)\left(a+bx^2\right)^{3/2}\,dx,x,x^2\right)\\
&\quad+\frac{1}{33}(2a)\int x^2\left(11d+9fx^2\right)\sqrt{a+bx^4}\,dx\\
&= \frac{2ax^3(77d+45fx^2)\sqrt{a+bx^4}}{1155}+\frac{1}{99}x^3\left(11d+9fx^2\right)\left(a+bx^4\right)^{3/2}+\frac{e(a+bx^4)^{5/2}}{10b}\\
&\quad+\frac{(4a^2)\int\frac{x^2(77d+45fx^2)}{\sqrt{a+bx^4}}\,dx}{1155}+\frac{1}{2}c\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{2ax^3(77d+45fx^2)\sqrt{a+bx^4}}{1155}+\frac{1}{8}cx^2\left(a+bx^4\right)^{3/2}\\
&\quad+\frac{1}{99}x^3\left(11d+9fx^2\right)\left(a+bx^4\right)^{3/2}+\frac{e(a+bx^4)^{5/2}}{10b}\\
&\quad-\frac{(4a^2)\int\frac{45af-231bdx^2}{\sqrt{a+bx^4}}\,dx}{3465b}+\frac{1}{8}(3ac)\text{Subst}\left(\int\sqrt{a+bx^2}\,dx,x,x^2\right)
\end{aligned}
$$

$$= \frac{4a^2fx\sqrt{a+bx^4}}{77b} + \frac{3}{16}acx^2\sqrt{a+bx^4} + \frac{2ax^3(77d+45fx^2)\sqrt{a+bx^4}}{1155} + \frac{1}{8}cx^2\left(a+bx^4\right)^{3/2} + \frac{1}{99}x^3\left(11d+9fx^2\right)\left(a+bx^4\right)^{3/2} + \frac{e(a+bx^4)^{5/2}}{10b} + \frac{1}{16}\left(3a^2c\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right) - \frac{\left(4a^{5/2}d\right)\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{15\sqrt{b}} + \frac{\left(4a^{5/2}\left(77\sqrt{b}d-15\sqrt{a}f\right)\right)\int}{1155b}$$

$$= \frac{4a^2fx\sqrt{a+bx^4}}{77b} + \frac{3}{16}acx^2\sqrt{a+bx^4} + \frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2ax^3(77d+45fx^2)\sqrt{a+bx^4}}{1155} + \frac{1}{8}cx^2\left(a+bx^4\right)^{3/2} + \frac{1}{99}x^3\left(11d+9fx^2\right)\left(a+bx^4\right)^{3/2} + \frac{e(a+bx^4)^{5/2}}{10b} - \frac{4a^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} + \frac{2a^{9/4}\left(77\sqrt{b}d-15\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}} + \frac{1}{16}\left(3a^2c\right)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)$$

$$= \frac{4a^2fx\sqrt{a+bx^4}}{77b} + \frac{3}{16}acx^2\sqrt{a+bx^4} + \frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2ax^3(77d+45fx^2)\sqrt{a+bx^4}}{1155} + \frac{1}{8}cx^2\left(a+bx^4\right)^{3/2} + \frac{1}{99}x^3\left(11d+9fx^2\right)\left(a+bx^4\right)^{3/2} + \frac{e(a+bx^4)^{5/2}}{10b} + \frac{3a^2c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} - \frac{4a^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} + \frac{2a^{9/4}\left(77\sqrt{b}d-15\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.71 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.48

$$\int x(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}\,dx=\frac{\sqrt{a+bx^4}\left(\frac{264e(a+bx^4)^2}{b}+\frac{240fx(a+bx^4)^2}{b}+165c\left(5ax^2+2bx^6+\frac{3a^{5/2}\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}(a+bx^4)}\right)-\frac{240\cdots}{\cdots}\right.}{2640}$$

```
[In] Integrate[x*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^4]*((264*e*(a + b*x^4)^2)/b + (240*f*x*(a + b*x^4)^2)/b + 165
*c*(5*a*x^2 + 2*b*x^6 + (3*a^(5/2)*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2
)/Sqrt[a]])/(Sqrt[b]*(a + b*x^4))) - (240*a^2*f*x*Hypergeometric2F1[-3/2, 1
/4, 5/4, -((b*x^4)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (880*a*d*x^3*Hypergeometr
ic2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/2640
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.06 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.73

method	result
risch	$\frac{(5040b^2f\,x^9+5544b^2e\,x^8+6160b^2d\,x^7+6930b^2c\,x^6+9360abf\,x^5+11088abe\,x^4+13552x^3abd+17325abc\,x^2+2880a^2fx+5544a^2e)\sqrt{b\,x^4\cdots}}{55440b}$
default	$f\left(\frac{b\,x^9\sqrt{b\,x^4+a}}{11}+\frac{13a\,x^5\sqrt{b\,x^4+a}}{77}+\frac{4a^2x\sqrt{b\,x^4+a}}{77b}-\frac{4a^3\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{77b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+\frac{e(b\,x^4+a)^{\frac{5}{2}}}{10b}+d\left(\frac{b\cdots}{\cdots}\right.$
elliptic	$\frac{bf\,x^9\sqrt{b\,x^4+a}}{11}+\frac{be\,x^8\sqrt{b\,x^4+a}}{10}+\frac{bd\,x^7\sqrt{b\,x^4+a}}{9}+\frac{bc\,x^6\sqrt{b\,x^4+a}}{8}+\frac{13af\,x^5\sqrt{b\,x^4+a}}{77}+\frac{ae\,x^4\sqrt{b\,x^4+a}}{5}+\frac{11ad\,x^3\sqrt{b\,x^4+a}}{45}+\cdots$

```
[In] int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/55440*(5040*b^2*f*x^9+5544*b^2*e*x^8+6160*b^2*d*x^7+6930*b^2*c*x^6+9360*a
*b*f*x^5+11088*a*b*e*x^4+13552*a*b*d*x^3+17325*a*b*c*x^2+2880*a^2*f*x+5544*
a^2*e)/b*(b*x^4+a)^(1/2)-1/9240*a^2/b*(480*a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1
-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2464*I*b^(1/2)*d*a^(1/2)/(I/a^(
1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^
```

```
2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Elliptic
E(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3465/2*b^(1/2)*c*ln(x^2*b^(1/2)+(b*x^4+a)
^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.55

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\frac{29568\,a^2\sqrt{b}dx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+10395\,a^2\sqrt{b}cx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x\right.}{}$$

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/110880*(29568*a^2*sqrt(b)*d*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)
/x), -1) + 10395*a^2*sqrt(b)*c*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x
^2 - a) - 384*(77*a^2*d + 15*a^2*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsi
n((-a/b)^(1/4)/x), -1) + 2*(5040*b^2*f*x^10 + 5544*b^2*e*x^9 + 6160*b^2*d*x
^8 + 6930*b^2*c*x^7 + 9360*a*b*f*x^6 + 11088*a*b*e*x^5 + 13552*a*b*d*x^4 +
17325*a*b*c*x^3 + 2880*a^2*f*x^2 + 5544*a^2*e*x + 14784*a^2*d)*sqrt(b*x^4 +
 a))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.97

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\frac{a^{\frac{3}{2}}cx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{a^{\frac{3}{2}}cx^2}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{a^{\frac{3}{2}}fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}bcx^6}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bdx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}bfx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{3a^2c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}}+ae\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for}\ b=0\\ \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)+be\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right)+\frac{b^2cx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(3/2)*c*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*c*x**2/(16*sqrt(1 + b*x**4/
a)) + a**(3/2)*d*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*gamma(7/4)) + a**(3/2)*f*x**5*gamma(5/4)*hyper((-1/2, 5/4), (
9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*c*x**6/(16*sq
rt(1 + b*x**4/a)) + sqrt(a)*b*d*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*f*x**9*gamma(9/4)*hy
per((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4)) + 3*a**
2*c*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*e*Piecewise((sqrt(a)*x**4/
4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*e*Piecewise((-a**2*sqr
t(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*
x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*c*x**10/(8*sqrt(a)*sqrt
(1 + b*x**4/a))
```

Maxima [F]

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int (bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)x\,dx$$

[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")

[Out] -1/32*(3*a^2*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 + a)/x^2))/sqrt(b) + 2*(3*sqrt(b*x^4 + a)*a^2*b/x^2 - 5*(b*x^4 + a)^(3/2)*a^2/x^6)/(b^2 - 2*(b*x^4 + a)*b/x^4 + (b*x^4 + a)^2/x^8))*c + integrate((b*f*x^8 + b*e*x^7 + b*d*x^6 + a*f*x^4 + a*e*x^3 + a*d*x^2)*sqrt(b*x^4 + a), x)

Giac [F]

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int (bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)x\,dx$$

[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x, x)

Mupad [F(-1)]

Timed out.

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int x\left(b\,x^4+a\right)^{3/2}\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

[In] int(x*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)

[Out] int(x*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)

3.514 $\int (c+dx+ex^2+fx^3)(a+bx^4)^{3/2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 382

$$\begin{aligned}\int (c+dx+ex^2+fx^3)(a+bx^4)^{3/2}\,dx &= \frac{3}{16}adx^2\sqrt{a+bx^4} \\ &+\frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2}{105}ax(15c+7ex^2)\sqrt{a+bx^4}+\frac{1}{8}dx^2(a+bx^4)^{3/2} \\ &+\frac{1}{63}x(9c+7ex^2)(a+bx^4)^{3/2}+\frac{f(a+bx^4)^{5/2}}{10b}+\frac{3a^2d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} \\ &-\frac{4a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} \\ &+\frac{2a^{7/4}\left(15\sqrt{b}c+7\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}\end{aligned}$$

```
[Out] 1/8*d*x^2*(b*x^4+a)^(3/2)+1/63*x*(7*e*x^2+9*c)*(b*x^4+a)^(3/2)+1/10*f*(b*x^
4+a)^(5/2)/b+3/16*a^2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+3/16*a
*d*x^2*(b*x^4+a)^(1/2)+2/105*a*x*(7*e*x^2+15*c)*(b*x^4+a)^(1/2)+4/15*a^2*e*
x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))-4/15*a^(9/4)*e*(cos(2*arcta
n(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(s
in(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+
a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+2/105*a^(7/4)*(co
s(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*El
lipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(7*e*a^(1/2)+15*c*b^(
1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/
4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {1899, 1191, 1212, 226, 1210, 1262, 655, 201, 223, 212}

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}e+15\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}+\frac{3a^2d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{63}x\left(a+bx^4\right)^{3/2}\left(9c+7ex^2\right)+\frac{2}{105}ax\sqrt{a+bx^4}\left(15c+7ex^2\right)+\frac{1}{8}dx^2\left(a+bx^4\right)^{3/2}+\frac{3}{16}adx^2\sqrt{a+bx^4}+\frac{f(a+bx}{10b}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (3*a*d*x^2*Sqrt[a + b*x^4])/16 + (4*a^2*e*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(S
qrt[a] + Sqrt[b]*x^2)) + (2*a*x*(15*c + 7*e*x^2)*Sqrt[a + b*x^4])/105 + (d*
x^2*(a + b*x^4)^(3/2))/8 + (x*(9*c + 7*e*x^2)*(a + b*x^4)^(3/2))/63 + (f*(a
 + b*x^4)^(5/2))/(10*b) + (3*a^2*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/
(16*Sqrt[b]) - (4*a^(9/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(
3/4)*Sqrt[a + b*x^4]) + (2*a^(7/4)*(15*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(3/4)*Sqrt[a + b*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\text{integral} = \int \left(\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}+x\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\right)\,dx$$

$$= \int \left(c+ex^2\right)\left(a+bx^4\right)^{3/2}\,dx+\int x\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\,dx$$

$$= \frac{1}{63}x\left(9c+7ex^2\right)\left(a+bx^4\right)^{3/2} + \frac{1}{21}\int \left(18ac+14aex^2\right)\sqrt{a+bx^4}\,dx+\frac{1}{2}\text{Subst}\left(\int (d+fx)\left(a+bx^2\right)^{3/2}\,dx,x,x^2\right)$$

$$= \frac{2}{105}ax\left(15c+7ex^2\right)\sqrt{a+bx^4}+\frac{1}{63}x\left(9c+7ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{f\left(a+bx^4\right)^{5/2}}{10b} + \frac{1}{315}\int \frac{180a^2c+84a^2ex^2}{\sqrt{a+bx^4}}\,dx+\frac{1}{2}d\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx,x,x^2\right)$$

$$= \frac{2}{105}ax\left(15c+7ex^2\right)\sqrt{a+bx^4}+\frac{1}{8}dx^2\left(a+bx^4\right)^{3/2} + \frac{1}{63}x\left(9c+7ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{f\left(a+bx^4\right)^{5/2}}{10b} + \frac{1}{8}(3ad)\text{Subst}\left(\int \sqrt{a+bx^2}\,dx,x,x^2\right)-\frac{\left(4a^{5/2}e\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{15\sqrt{b}}+\frac{1}{105}\left(4a^2\left(15c+\frac{7\sqrt{a}e}{\sqrt{b}}\right)\right)\int \frac{}{\sqrt{a}}$$

$$= \frac{3}{16}adx^2\sqrt{a+bx^4}+\frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2}{105}ax\left(15c+7ex^2\right)\sqrt{a+bx^4} + \frac{1}{8}dx^2\left(a+bx^4\right)^{3/2}+\frac{1}{63}x\left(9c+7ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{f\left(a+bx^4\right)^{5/2}}{10b} - \frac{4a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} + \frac{2a^{7/4}\left(15\sqrt{b}c+7\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}} + \frac{1}{16}\left(3a^2d\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)$$

$$= \frac{3}{16}adx^2\sqrt{a+bx^4} + \frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2}{105}ax(15c+7ex^2)\sqrt{a+bx^4} + \frac{1}{8}dx^2(a+bx^4)^{3/2} + \frac{1}{63}x(9c+7ex^2)(a+bx^4)^{3/2} + \frac{f(a+bx^4)^{5/2}}{10b} - \frac{4a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} + \frac{2a^{7/4}\left(15\sqrt{b}c+7\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}} + \frac{1}{16}(3a^2d)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)$$

$$= \frac{3}{16}adx^2\sqrt{a+bx^4} + \frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2}{105}ax(15c+7ex^2)\sqrt{a+bx^4} + \frac{1}{8}dx^2(a+bx^4)^{3/2} + \frac{1}{63}x(9c+7ex^2)(a+bx^4)^{3/2} + \frac{f(a+bx^4)^{5/2}}{10b} + \frac{3a^2d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} - \frac{4a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} + \frac{2a^{7/4}\left(15\sqrt{b}c+7\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.51 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.46

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^{3/2}\,dx = \frac{1}{240}\sqrt{a+bx^4}\left(\frac{24f(a+bx^4)^2}{b} + 15d\left(5ax^2+2bx^6+\frac{3a^{5/2}\sqrt{1+\frac{bx^4}{a}}\,\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}(a+bx^4)}\right) + \frac{240acx\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}} + \frac{80aex^3}{\quad}\right.$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^4]*((24*f*(a + b*x^4)^2)/b + 15*d*(5*a*x^2 + 2*b*x^6 + (3*a^(
5/2)*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*(a + b*x^
4))) + (240*a*c*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 +
 (b*x^4)/a] + (80*a*e*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/
Sqrt[1 + (b*x^4)/a]))/240
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.75

method	result
risch	$\frac{\left(504 b^{2} f \,x^{8}+560 b^{2} e \,x^{7}+630 b^{2} d \,x^{6}+720 b^{2} c \,x^{5}+1008 a b f \,x^{4}+1232 a b e \,x^{3}+1575 x^{2} a b d+2160 a b c x+504 a^{2} f \right) \sqrt{b \,x^{4}+a}}{5040 b}+\frac{a^{2} \left(\frac{480 c \sqrt{1-}}{}\right.}{}$
default	$c \left(\frac{b \,x^{5} \sqrt{b \,x^{4}+a}}{7}+\frac{3 a x \sqrt{b \,x^{4}+a}}{7}+\frac{4 a^{2} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{7 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}\right)+\frac{f \left(b \,x^{4}+a \right)^{\frac{5}{2}}}{10 b}+e \left(\frac{b \,x^{7} \sqrt{b \,x^{4}+a}}{9}+\frac{11 a}{}\right.$
elliptic	$\frac{b f \,x^{8} \sqrt{b \,x^{4}+a}}{10}+\frac{b e \,x^{7} \sqrt{b \,x^{4}+a}}{9}+\frac{b d \,x^{6} \sqrt{b \,x^{4}+a}}{8}+\frac{b c \,x^{5} \sqrt{b \,x^{4}+a}}{7}+\frac{a f \,x^{4} \sqrt{b \,x^{4}+a}}{5}+\frac{11 a e \,x^{3} \sqrt{b \,x^{4}+a}}{45}+\frac{5 a d \,x^{2} \sqrt{b \,x^{4}+a}}{16}$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5040*(504*b^2*f*x^8+560*b^2*e*x^7+630*b^2*d*x^6+720*b^2*c*x^5+1008*a*b*f*
x^4+1232*a*b*e*x^3+1575*a*b*d*x^2+2160*a*b*c*x+504*a^2*f)/b*(b*x^4+a)^(1/2)
+1/840*a^2*(480*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)
*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)+224*I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Elli
pticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),
I))+315/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.56

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{2688\,a^2\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+945\,a^2\sqrt{b}dx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2-\right.}{}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10080*(2688*a^2*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x
), -1) + 945*a^2*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 -
 a) + 384*(15*a*b*c - 7*a^2*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a
/b)^(1/4)/x), -1) + 2*(504*b^2*f*x^9 + 560*b^2*e*x^8 + 630*b^2*d*x^7 + 720*
b^2*c*x^6 + 1008*a*b*f*x^5 + 1232*a*b*e*x^4 + 1575*a*b*d*x^3 + 2160*a*b*c*x
^2 + 504*a^2*f*x + 1344*a^2*e)*sqrt(b*x^4 + a))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.03

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx = \frac{a^{\frac{3}{2}}cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}dx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}dx^2}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}bcx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}bdx^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{3a^2d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}} + af\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + bf\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right) + \frac{b^2dx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(3/2)*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a
)/(4*gamma(5/4)) + a**(3/2)*d*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*d*x**2/(
16*sqrt(1 + b*x**4/a)) + a**(3/2)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4
,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*b*c*x**5*gamma(5/4)*
hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqr
t(a)*b*d*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*x**7*gamma(7/4)*hyper((
-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + 3*a**2*d*a
sinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*f*Piecewise((sqrt(a)*x**4/4, Eq
```

```
(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*f*Piecewise((-a**2*sqrt(a +
 b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)
/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*d*x**10/(8*sqrt(a)*sqrt(1 +
b*x**4/a))
```

Maxima [F]

$$\int \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^{3/2} \, dx = \int \left(bx^4+a\right)^{\frac{3}{2}} \left(fx^3+ex^2+dx+c\right) dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c), x)
```

Giac [F]

$$\int \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^{3/2} \, dx = \int \left(bx^4+a\right)^{\frac{3}{2}} \left(fx^3+ex^2+dx+c\right) dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^{3/2} \, dx = \int \left(b\,x^4+a\right)^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right) \,dx$$

```
[In] int((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
[Out] int((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.515 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 403

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x}\,dx = \frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

$$+\frac{1}{16}a(8c+3ex^2)\sqrt{a+bx^4}+\frac{2}{105}ax(15d+7fx^2)\sqrt{a+bx^4}$$

$$+\frac{1}{24}(4c+3ex^2)\left(a+bx^4\right)^{3/2}+\frac{1}{63}x(9d+7fx^2)\left(a+bx^4\right)^{3/2}+\frac{3a^2e\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}$$

$$-\frac{1}{2}a^{3/2}c\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{4a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}+\frac{2a^{7/4}\left(15\sqrt{b}d-\right.}{}$$

```
[Out] 1/24*(3*e*x^2+4*c)*(b*x^4+a)^(3/2)+1/63*x*(7*f*x^2+9*d)*(b*x^4+a)^(3/2)-1/2
*a^(3/2)*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))+3/16*a^2*e*arctanh(x^2*b^(1/2)/
(b*x^4+a)^(1/2))/b^(1/2)+1/16*a*(3*e*x^2+8*c)*(b*x^4+a)^(1/2)+2/105*a*x*(7*
f*x^2+15*d)*(b*x^4+a)^(1/2)+4/15*a^2*f*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x
^2*b^(1/2))-4/15*a^(9/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*
2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^
(3/4)/(b*x^4+a)^(1/2)+2/105*a^(7/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/
4))),1/2*2^(1/2))*(7*f*a^(1/2)+15*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+
a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {1847, 1266, 829, 858, 223, 212, 272, 65, 214, 1191, 1212, 226, 1210}

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x}\,dx = \frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}f+15\sqrt{b}d\right)\text{EllipticF}\left(2\right.}{105b^{3/4}\sqrt{a+bx^4}}$$
$$-\frac{4a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}$$
$$-\frac{1}{2}a^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3a^2e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{16}a\sqrt{a+bx^4}(8c+3ex^2)+\frac{1}{2}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x,x]
```

```
[Out] (4*a^2*f*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (a*(8*c
+ 3*e*x^2)*Sqrt[a + b*x^4])/16 + (2*a*x*(15*d + 7*f*x^2)*Sqrt[a + b*x^4])/1
05 + ((4*c + 3*e*x^2)*(a + b*x^4)^(3/2))/24 + (x*(9*d + 7*f*x^2)*(a + b*x^4
)^(3/2))/63 + (3*a^2*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b])
 - (a^(3/2)*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (4*a^(9/4)*f*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(7/4)*(15
*Sqrt[b]*d + 7*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(3
/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(c+ex^2)\,(a+bx^4)^{3/2}}{x}+\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\right)\,dx\\
&= \int \frac{(c+ex^2)\,(a+bx^4)^{3/2}}{x}\,dx+\int \left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\,dx\\
&= \frac{1}{63}x\left(9d+7fx^2\right)\left(a+bx^4\right)^{3/2}+\frac{1}{21}\int \left(18ad+14afx^2\right)\sqrt{a+bx^4}\,dx\\
&\quad +\frac{1}{2}\text{Subst}\left(\int \frac{(c+ex)\,(a+bx^2)^{3/2}}{x}\,dx,x,x^2\right)
\end{aligned}$$

$$= \frac{2}{105} a x\left(15 d+7 f x^{2}\right) \sqrt{a+b x^{4}}+\frac{1}{24}\left(4 c+3 e x^{2}\right)\left(a+b x^{4}\right)^{3/2}+\frac{1}{63} x\left(9 d+7 f x^{2}\right)\left(a+b x^{4}\right)^{3/2}+\frac{1}{315} \int \frac{180 a^{2} d+84 a^{2} f x^{2}}{\sqrt{a+b x^{4}}} d x+\frac{\operatorname{Subst}\left(\int \frac{(4 a b c+3 a b e x) \sqrt{a+b x^{2}}}{x} d x, x, x^{2}\right)}{8 b}$$

$$= \frac{1}{16} a\left(8 c+3 e x^{2}\right) \sqrt{a+b x^{4}}+\frac{2}{105} a x\left(15 d+7 f x^{2}\right) \sqrt{a+b x^{4}}+\frac{1}{24}\left(4 c+3 e x^{2}\right)\left(a+b x^{4}\right)^{3/2}+\frac{1}{63} x\left(9 d+7 f x^{2}\right)\left(a+b x^{4}\right)^{3/2}+\frac{\operatorname{Subst}\left(\int \frac{8 a^{2} b^{2} c+3 a^{2} b^{2} e x}{x \sqrt{a+b x^{2}}} d x, x, x^{2}\right)}{16 b^{2}}-\frac{\left(4 a^{5/2} f\right) \int \frac{1-\frac{\sqrt{b} x^{2}}{\sqrt{a}}}{\sqrt{a+b x^{4}}} d x}{15 \sqrt{b}}+\frac{1}{105}\left(4 a^{2}\left(15 d+\frac{7 \sqrt{a} f}{\sqrt{b}}\right)\right) \int \frac{1}{\sqrt{a+b x^{4}}} d x$$

$$= \frac{4 a^{2} f x \sqrt{a+b x^{4}}}{15 \sqrt{b}\left(\sqrt{a}+\sqrt{b} x^{2}\right)}+\frac{1}{16} a\left(8 c+3 e x^{2}\right) \sqrt{a+b x^{4}}+\frac{2}{105} a x\left(15 d+7 f x^{2}\right) \sqrt{a+b x^{4}}+\frac{1}{24}\left(4 c+3 e x^{2}\right)\left(a+b x^{4}\right)^{3/2}+\frac{1}{63} x\left(9 d+7 f x^{2}\right)\left(a+b x^{4}\right)^{3/2}-\frac{4 a^{9/4} f\left(\sqrt{a}+\sqrt{b} x^{2}\right) \sqrt{\frac{a+b x^{4}}{\left(\sqrt{a}+\sqrt{b} x^{2}\right)^{2}}} E\left(2 \tan ^{-1}\left(\frac{\sqrt[4]{b} x}{\sqrt[4]{a}}\right) | \frac{1}{2}\right)}{15 b^{3/4} \sqrt{a+b x^{4}}}+\frac{2 a^{7/4}\left(15 \sqrt{b} d+7 \sqrt{a} f\right)\left(\sqrt{a}+\sqrt{b} x^{2}\right) \sqrt{\frac{a+b x^{4}}{\left(\sqrt{a}+\sqrt{b} x^{2}\right)^{2}}} F\left(2 \tan ^{-1}\left(\frac{\sqrt[4]{b} x}{\sqrt[4]{a}}\right) | \frac{1}{2}\right)}{105 b^{3/4} \sqrt{a+b x^{4}}}+\frac{1}{2}\left(a^{2} c\right) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a+b x^{2}}} d x, x, x^{2}\right)+\frac{1}{16}\left(3 a^{2} e\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a+b x^{2}}} d x, x, x^{2}\right)$$

$$= \frac{4 a^{2} f x \sqrt{a+b x^{4}}}{15 \sqrt{b}\left(\sqrt{a}+\sqrt{b} x^{2}\right)}+\frac{1}{16} a\left(8 c+3 e x^{2}\right) \sqrt{a+b x^{4}}+\frac{2}{105} a x\left(15 d+7 f x^{2}\right) \sqrt{a+b x^{4}}+\frac{1}{24}\left(4 c+3 e x^{2}\right)\left(a+b x^{4}\right)^{3/2}+\frac{1}{63} x\left(9 d+7 f x^{2}\right)\left(a+b x^{4}\right)^{3/2}-\frac{4 a^{9/4} f\left(\sqrt{a}+\sqrt{b} x^{2}\right) \sqrt{\frac{a+b x^{4}}{\left(\sqrt{a}+\sqrt{b} x^{2}\right)^{2}}} E\left(2 \tan ^{-1}\left(\frac{\sqrt[4]{b} x}{\sqrt[4]{a}}\right) | \frac{1}{2}\right)}{15 b^{3/4} \sqrt{a+b x^{4}}}+\frac{2 a^{7/4}\left(15 \sqrt{b} d+7 \sqrt{a} f\right)\left(\sqrt{a}+\sqrt{b} x^{2}\right) \sqrt{\frac{a+b x^{4}}{\left(\sqrt{a}+\sqrt{b} x^{2}\right)^{2}}} F\left(2 \tan ^{-1}\left(\frac{\sqrt[4]{b} x}{\sqrt[4]{a}}\right) | \frac{1}{2}\right)}{105 b^{3/4} \sqrt{a+b x^{4}}}+\frac{1}{4}\left(a^{2} c\right) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a+b x}} d x, x, x^{4}\right)+\frac{1}{16}\left(3 a^{2} e\right) \operatorname{Subst}\left(\int \frac{1}{1-b x^{2}} d x, x, \frac{x^{2}}{\sqrt{a+b x^{4}}}\right)$$

$$= \frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{16}a\left(8c+3ex^2\right)\sqrt{a+bx^4} + \frac{2}{105}ax\left(15d+7fx^2\right)\sqrt{a+bx^4}$$

$$+\frac{1}{24}\left(4c+3ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{1}{63}x\left(9d+7fx^2\right)\left(a+bx^4\right)^{3/2}+\frac{3a^2e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}$$

$$-\frac{4a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}$$

$$+\frac{2a^{7/4}\left(15\sqrt{b}d+7\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}$$

$$+\frac{(a^2c)\,\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)}{2b}$$

$$= \frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{16}a\left(8c+3ex^2\right)\sqrt{a+bx^4} + \frac{2}{105}ax\left(15d+7fx^2\right)\sqrt{a+bx^4}$$

$$+\frac{1}{24}\left(4c+3ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{1}{63}x\left(9d+7fx^2\right)\left(a+bx^4\right)^{3/2}+\frac{3a^2e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}$$

$$-\frac{1}{2}a^{3/2}c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{4a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}+\frac{2a^{7/4}}{}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.52 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.56

$$\int\frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x}\,dx=\frac{1}{16}e\sqrt{a+bx^4}\left(5ax^2+2bx^6+\frac{3a^{3/2}\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{1+\frac{bx^4}{a}}}\right)$$

$$+\frac{1}{6}c\left(\sqrt{a+bx^4}\left(4a+bx^4\right)-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+\frac{adx\sqrt{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x,x]
```

```
[Out] (e*Sqrt[a + b*x^4]*(5*a*x^2 + 2*b*x^6 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*x^2)/Sq
rt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^4)/a])))/16 + (c*(Sqrt[a + b*x^4]*(4*a + b*x
^4) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]))/6 + (a*d*x*Sqrt[a + b*x^
```

```
4]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a] + (
a*f*x^3*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/(3
*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.86

method	result
elliptic	$\frac{bf\,x^7\sqrt{b\,x^4+a}}{9}+\frac{be\,x^6\sqrt{b\,x^4+a}}{8}+\frac{bd\,x^5\sqrt{b\,x^4+a}}{7}+\frac{bc\,x^4\sqrt{b\,x^4+a}}{6}+\frac{11af\,x^3\sqrt{b\,x^4+a}}{45}+\frac{5ae\,x^2\sqrt{b\,x^4+a}}{16}+\frac{3adx\sqrt{b\,x^4+a}}{7}+$
default	$d\left(\frac{b\,x^5\sqrt{b\,x^4+a}}{7}+\frac{3ax\sqrt{b\,x^4+a}}{7}+\frac{4a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+f\left(\frac{b\,x^7\sqrt{b\,x^4+a}}{9}+\frac{11a\,x^3\sqrt{b\,x^4+a}}{45}+\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b*f*x^7*(b*x^4+a)^(1/2)+1/8*b*e*x^6*(b*x^4+a)^(1/2)+1/7*b*d*x^5*(b*x^4+
a)^(1/2)+1/6*b*c*x^4*(b*x^4+a)^(1/2)+11/45*a*f*x^3*(b*x^4+a)^(1/2)+5/16*a*e
*x^2*(b*x^4+a)^(1/2)+3/7*a*d*x*(b*x^4+a)^(1/2)+2/3*a*c*(b*x^4+a)^(1/2)+4/7*
a^2*d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)+3/16*a^2*e*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))/b^(1/2)+4/15*I*a^(5/2)*f/
(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1
/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/
2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*a^(3/2)*c*arctanh(a^(1/
2)/(b*x^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x, x)
```

Sympy [A] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x}\,dx = -\frac{a^{\frac{3}{2}}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{a^{\frac{3}{2}}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}ex^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}ex^2}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}bdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}bex^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bfx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{a^2c}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a^2e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}} + \frac{a\sqrt{b}cx^2}{2\sqrt{\frac{a}{bx^4}+1}} + bc\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + \frac{b^2ex^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x,x)
```

```
[Out] -a**(3/2)*c*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*d*x*gamma(1/4)*hyper
((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a**(3/2)*e
*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*e*x**2/(16*sqrt(1 + b*x**4/a)) + a**(
3/2)*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)
/(4*gamma(7/4)) + sqrt(a)*b*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*
x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*e*x**6/(16*sqrt(1 + b*
x**4/a)) + sqrt(a)*b*f*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*e
xp_polar(I*pi)/a)/(4*gamma(11/4)) + a**2*c/(2*sqrt(b)*x**2*sqrt(a/(b*x**4)
+ 1)) + 3*a**2*e*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*sqrt(b)*c*x**
2/(2*sqrt(a/(b*x**4) + 1)) + b*c*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a
+ b*x**4)**(3/2)/(6*b), True)) + b**2*e*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a)
)
```

Maxima [F]

$$\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x}\, dx = \int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x}\, dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Giac [F]

$$\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x}\, dx = \int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x}\, dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x}\, dx = \int \frac{(b\,x^4 + a)^{3/2}\,(f\,x^3 + e\,x^2 + d\,x + c)}{x}\, dx$$

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x, x)
```

3.516 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 404

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx = \frac{12a\sqrt{b}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2}{35}x(5ae+21bcx^2)\sqrt{a+bx^4} + \frac{1}{16}a(8d+3fx^2)\sqrt{a+bx^4} - \frac{(7c-ex^2)(a+bx^4)^{3/2}}{7x} + \frac{1}{24}(4d+3fx^2)(a+bx^4)^{3/2} + \frac{3a^2f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} - \frac{1}{2}a^{3/2}d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12a^{5/4}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}} + \frac{2a^{5/4}(21}{}$$

```
[Out] -1/7*(-e*x^2+7*c)*(b*x^4+a)^(3/2)/x+1/24*(3*f*x^2+4*d)*(b*x^4+a)^(3/2)-1/2*
a^(3/2)*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))+3/16*a^2*f*arctanh(x^2*b^(1/2)/(
b*x^4+a)^(1/2))/b^(1/2)+2/35*x*(21*b*c*x^2+5*a*e)*(b*x^4+a)^(1/2)+1/16*a*(3
*f*x^2+8*d)*(b*x^4+a)^(1/2)+12/5*a*c*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2
*b^(1/2))-12/5*a^(5/4)*b^(1/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1
/2)/(b*x^4+a)^(1/2)+2/35*a^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(5*e*a^(1/2)+21*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/
(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1847, 1286, 1191, 1212, 226, 1210, 1266, 829, 858, 223, 212, 272, 65, 214}

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx = \frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+21\sqrt{b}c\right)\text{EllipticF}\left(2\,\text{ar}\right.}{35\sqrt[4]{b}\sqrt{a+bx^4}}$$
$$-\frac{12a^{5/4}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$-\frac{1}{2}a^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3a^2f\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}-\frac{(a+bx^4)^{3/2}(7c-ex^2)}{7x}+\frac{2}{35}x\sqrt{a+bx^4}(5ae+21bcx^{2}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^2,x]
```

```
[Out] (12*a*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + (2*x*(5*a*
e + 21*b*c*x^2)*Sqrt[a + b*x^4])/35 + (a*(8*d + 3*f*x^2)*Sqrt[a + b*x^4])/1
6 - ((7*c - e*x^2)*(a + b*x^4)^(3/2))/(7*x) + ((4*d + 3*f*x^2)*(a + b*x^4)^
(3/2))/24 + (3*a^2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) -
 (a^(3/2)*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*c*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)*(21*S
qrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*b^(1/4)
*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(c+ex^2)\left(a+bx^4\right)^{3/2}}{x^2} + \frac{(d+fx^2)\left(a+bx^4\right)^{3/2}}{x} \right) dx$$

$$= \int \frac{(c+ex^2)\left(a+bx^4\right)^{3/2}}{x^2}\, dx + \int \frac{(d+fx^2)\left(a+bx^4\right)^{3/2}}{x}\, dx$$

$$=-\frac{(7c-ex^2)(a+bx^4)^{3/2}}{7x}+\frac{1}{2}\text{Subst}\left(\int\frac{(d+fx)(a+bx^2)^{3/2}}{x}\,dx,x,x^2\right)-\frac{6}{7}\int(-ae-7bcx^2)\sqrt{a+bx^4}\,dx$$

$$=\frac{2}{35}x(5ae+21bcx^2)\sqrt{a+bx^4}-\frac{(7c-ex^2)(a+bx^4)^{3/2}}{7x}+\frac{1}{24}(4d+3fx^2)(a+bx^4)^{3/2}-\frac{2}{35}\int\frac{-10a^2e-42abcx^2}{\sqrt{a+bx^4}}\,dx+\frac{\text{Subst}\left(\int\frac{(4abd+3abfx)\sqrt{a+bx^2}}{x}\,dx,x,x^2\right)}{8b}$$

$$=\frac{2}{35}x(5ae+21bcx^2)\sqrt{a+bx^4}+\frac{1}{16}a(8d+3fx^2)\sqrt{a+bx^4}-\frac{(7c-ex^2)(a+bx^4)^{3/2}}{7x}+\frac{1}{24}(4d+3fx^2)(a+bx^4)^{3/2}+\frac{\text{Subst}\left(\int\frac{8a^2b^2d+3a^2b^2fx}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)}{16b^2}-\frac{1}{5}\left(12a^{3/2}\sqrt{b}c\right)\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx+\frac{1}{35}\left(4a^{3/2}\left(21\sqrt{b}c+5\sqrt{a}e\right)\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx$$

$$=\frac{12a\sqrt{b}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2}{35}x(5ae+21bcx^2)\sqrt{a+bx^4}+\frac{1}{16}a(8d+3fx^2)\sqrt{a+bx^4}-\frac{(7c-ex^2)(a+bx^4)^{3/2}}{7x}+\frac{1}{24}(4d+3fx^2)(a+bx^4)^{3/2}-\frac{12a^{5/4}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}+\frac{2a^{5/4}\left(21\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{1}{2}(a^2d)\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)+\frac{1}{16}(3a^2f)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)$$

$$=\frac{12a\sqrt{b}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2}{35}x\left(5ae+21bcx^2\right)\sqrt{a+bx^4}+\frac{1}{16}a\left(8d+3fx^2\right)\sqrt{a+bx^4}$$
$$-\frac{\left(7c-ex^2\right)\left(a+bx^4\right)^{3/2}}{7x}+\frac{1}{24}\left(4d+3fx^2\right)\left(a+bx^4\right)^{3/2}$$
$$-\frac{12a^{5/4}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$+\frac{2a^{5/4}\left(21\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}$$
$$+\frac{1}{4}\left(a^2d\right)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)+\frac{1}{16}\left(3a^2f\right)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)$$

$$=\frac{12a\sqrt{b}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2}{35}x\left(5ae+21bcx^2\right)\sqrt{a+bx^4}+\frac{1}{16}a\left(8d+3fx^2\right)\sqrt{a+bx^4}$$
$$-\frac{\left(7c-ex^2\right)\left(a+bx^4\right)^{3/2}}{7x}+\frac{1}{24}\left(4d+3fx^2\right)\left(a+bx^4\right)^{3/2}+\frac{3a^2f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}$$
$$-\frac{12a^{5/4}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$+\frac{2a^{5/4}\left(21\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}$$
$$+\frac{\left(a^2d\right)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)}{2b}$$

$$=\frac{12a\sqrt{b}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2}{35}x\left(5ae+21bcx^2\right)\sqrt{a+bx^4}+\frac{1}{16}a\left(8d+3fx^2\right)\sqrt{a+bx^4}$$
$$-\frac{\left(7c-ex^2\right)\left(a+bx^4\right)^{3/2}}{7x}+\frac{1}{24}\left(4d+3fx^2\right)\left(a+bx^4\right)^{3/2}+\frac{3a^2f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}$$
$$-\frac{1}{2}a^{3/2}d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{12a^{5/4}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}+\cdots$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.46 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.55

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx = \frac{1}{16}f\sqrt{a+bx^4}\left(5ax^2 + 2bx^6 + \frac{3a^{3/2}\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{1}{6}d\left(\sqrt{a+bx^4}(4a+bx^4)-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right) - \frac{ac\sqrt{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^2,x]
```

```
[Out] (f*Sqrt[a + b*x^4]*(5*a*x^2 + 2*b*x^6 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*x^2)/Sq
rt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^4)/a])))/16 + (d*(Sqrt[a + b*x^4]*(4*a + b*x
^4) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]))/6 - (a*c*Sqrt[a + b*x^4]
*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^4)/a)])/(x*Sqrt[1 + (b*x^4)/a])
+ (a*e*x*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/S
qrt[1 + (b*x^4)/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.40 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.86

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{x}+\frac{bf\,x^6\sqrt{b\,x^4+a}}{8}+\frac{be\,x^5\sqrt{b\,x^4+a}}{7}+\frac{bd\,x^4\sqrt{b\,x^4+a}}{6}+\frac{bc\,x^3\sqrt{b\,x^4+a}}{5}+\frac{5af\,x^2\sqrt{b\,x^4+a}}{16}+\frac{3aex\sqrt{b\,x^4+a}}{7}+$
default	$e\left(\frac{b\,x^5\sqrt{b\,x^4+a}}{7}+\frac{3ax\sqrt{b\,x^4+a}}{7}+\frac{4a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+f\left(\frac{3a^2\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{16\sqrt{b}}+\frac{b\,x^6\sqrt{b\,x}}{8}\right.$
risch	$-\frac{ac\sqrt{b\,x^4+a}}{x}+\frac{4a^2e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{bf\,x^6\sqrt{b\,x^4+a}}{8}+\frac{5af\,x^2\sqrt{b\,x^4+a}}{16}+\frac{3a^2f\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{16\sqrt{b}}$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a*c*(b*x^4+a)^(1/2)/x+1/8*b*f*x^6*(b*x^4+a)^(1/2)+1/7*b*e*x^5*(b*x^4+a)^(1
/2)+1/6*b*d*x^4*(b*x^4+a)^(1/2)+1/5*b*c*x^3*(b*x^4+a)^(1/2)+5/16*a*f*x^2*(b
*x^4+a)^(1/2)+3/7*a*e*x*(b*x^4+a)^(1/2)+2/3*a*d*(b*x^4+a)^(1/2)+4/7*a^2*e/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/16
*a^2*f*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))/b^(1/2)+12/5*I*a^(3/2)*b^(1/2)*c
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-E
llipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*a^(3/2)*d*arctanh(a^(1/2)/(b*x
^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^2, x)
```

Sympy [A] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^2}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^{\frac{3}{2}}d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{a^{\frac{3}{2}}ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}fx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}fx^2}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bcx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}bex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}bfx^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{a^2d}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a^2f\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}} + \frac{a\sqrt{b}dx^2}{2\sqrt{\frac{a}{bx^4}+1}} + bd\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + \frac{b^2fx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**2,x)
```

```
[Out] a**(3/2)*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x*gamma(3/4)) - a**(3/2)*d*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*
e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamm
a(5/4)) + a**(3/2)*f*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*f*x**2/(16*sqrt(1
 + b*x**4/a)) + sqrt(a)*b*c*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*b*e*x**5*gamma(5/4)*hyper((-
1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*f
*x**6/(16*sqrt(1 + b*x**4/a)) + a**2*d/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1)
) + 3*a**2*f*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*sqrt(b)*d*x**2/(2
*sqrt(a/(b*x**4) + 1)) + b*d*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*
x**4)**(3/2)/(6*b), True)) + b**2*f*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^2}\,dx$$

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^2,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^2, x)
```

3.517 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 406

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^3} dx = \frac{12a\sqrt{b}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{4}(2ae+3bcx^2)\sqrt{a+bx^4} + \frac{2}{35}x(5af+21bdx^2)\sqrt{a+bx^4} - \frac{(3c-ex^2)(a+bx^4)^{3/2}}{6x^2} - \frac{(7d-fx^2)(a+bx^4)^{3/2}}{7x} + \frac{3}{4}a\sqrt{b}c\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{1}{2}a^{3/2}e\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\Big(2 \ldots}{5\sqrt{a+bx^4}}$$

```
[Out] -1/6*(-e*x^2+3*c)*(b*x^4+a)^(3/2)/x^2-1/7*(-f*x^2+7*d)*(b*x^4+a)^(3/2)/x-1/
2*a^(3/2)*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))+3/4*a*c*arctanh(x^2*b^(1/2)/(b
*x^4+a)^(1/2))*b^(1/2)+1/4*(3*b*c*x^2+2*a*e)*(b*x^4+a)^(1/2)+2/35*x*(21*b*d
*x^2+5*a*f)*(b*x^4+a)^(1/2)+12/5*a*d*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2
*b^(1/2))-12/5*a^(5/4)*b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1
/2)/(b*x^4+a)^(1/2)+2/35*a^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(5*f*a^(1/2)+21*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/
(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1847, 1266, 827, 829, 858, 223, 212, 272, 65, 214, 1286, 1191, 1212, 226, 1210}

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^3}\,dx = \frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+21\sqrt{b}d\right)\text{EllipticF}\left(2\,\text{ar}\right.}{35\sqrt[4]{b}\sqrt{a+bx^4}}$$

$$-\frac{12a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$

$$-\frac{1}{2}a^{3/2}e\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3}{4}a\sqrt{b}c\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{(a+bx^4)^{3/2}(3c-ex^2)}{6x^2}+\frac{1}{4}\sqrt{a+bx^4}(2ae+3bc$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^3,x]
```

```
[Out] (12*a*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*a*e +
3*b*c*x^2)*Sqrt[a + b*x^4])/4 + (2*x*(5*a*f + 21*b*d*x^2)*Sqrt[a + b*x^4])/
35 - ((3*c - e*x^2)*(a + b*x^4)^(3/2))/(6*x^2) - ((7*d - f*x^2)*(a + b*x^4)
^(3/2))/(7*x) + (3*a*Sqrt[b]*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 -
(a^(3/2)*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*d*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)*(21*Sq
rt[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*b^(1/4)*
Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
```

```
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{(c+ex^2)\,(a+bx^4)^{3/2}}{x^3}+\frac{(d+fx^2)\,(a+bx^4)^{3/2}}{x^2}\right) dx \\
&= \int \frac{(c+ex^2)\,(a+bx^4)^{3/2}}{x^3}\,dx+\int \frac{(d+fx^2)\,(a+bx^4)^{3/2}}{x^2}\,dx \\
&= -\frac{(7d-fx^2)\,(a+bx^4)^{3/2}}{7x} \\
&\quad +\frac{1}{2}\text{Subst}\left(\int \frac{(c+ex)\,(a+bx^2)^{3/2}}{x^2}\,dx,x,x^2\right)-\frac{6}{7}\int \left(-af-7bdx^2\right)\sqrt{a+bx^4}\,dx \\
&= \frac{2}{35}x\left(5af+21bdx^2\right)\sqrt{a+bx^4}-\frac{(3c-ex^2)\,(a+bx^4)^{3/2}}{6x^2}-\frac{(7d-fx^2)\,(a+bx^4)^{3/2}}{7x} \\
&\quad -\frac{2}{35}\int \frac{-10a^2f-42abdx^2}{\sqrt{a+bx^4}}\,dx-\frac{1}{4}\text{Subst}\left(\int \frac{(-2ae-6bcx)\sqrt{a+bx^2}}{x}\,dx,x,x^2\right) \\
&= \frac{1}{4}\left(2ae+3bcx^2\right)\sqrt{a+bx^4}+\frac{2}{35}x\left(5af+21bdx^2\right)\sqrt{a+bx^4}-\frac{(3c-ex^2)\,(a+bx^4)^{3/2}}{6x^2} \\
&\quad -\frac{(7d-fx^2)\,(a+bx^4)^{3/2}}{7x}-\frac{\text{Subst}\left(\int \frac{-4a^2be-6ab^2cx}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)}{8b} \\
&\quad -\frac{1}{5}\left(12a^{3/2}\sqrt{b}d\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx+\frac{1}{35}\left(4a^{3/2}\left(21\sqrt{b}d+5\sqrt{a}f\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx \\
&= \frac{12a\sqrt{b}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{4}\left(2ae+3bcx^2\right)\sqrt{a+bx^4}+\frac{2}{35}x\left(5af+21bdx^2\right)\sqrt{a+bx^4} \\
&\quad -\frac{(3c-ex^2)\,(a+bx^4)^{3/2}}{6x^2}-\frac{(7d-fx^2)\,(a+bx^4)^{3/2}}{7x} \\
&\quad -\frac{12a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}} \\
&\quad +\frac{2a^{5/4}\left(21\sqrt{b}d+5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}} \\
&\quad +\frac{1}{4}(3abc)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)+\frac{1}{2}\left(a^2e\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)
\end{aligned}
$$

$$=\frac{12a\sqrt{b}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{4}\left(2ae+3bcx^2\right)\sqrt{a+bx^4}+\frac{2}{35}x\left(5af+21bdx^2\right)\sqrt{a+bx^4}-\frac{\left(3c-ex^2\right)\left(a+bx^4\right)^{3/2}}{6x^2}-\frac{\left(7d-fx^2\right)\left(a+bx^4\right)^{3/2}}{7x}-\frac{12a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}+\frac{2a^{5/4}\left(21\sqrt{b}d+5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{1}{4}(3abc)\text{Subst}\left(\int\frac{1}{1-bx^2}dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)+\frac{1}{4}\left(a^2e\right)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}dx,x,x^4\right)$$

$$=\frac{12a\sqrt{b}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{4}\left(2ae+3bcx^2\right)\sqrt{a+bx^4}+\frac{2}{35}x\left(5af+21bdx^2\right)\sqrt{a+bx^4}-\frac{\left(3c-ex^2\right)\left(a+bx^4\right)^{3/2}}{6x^2}-\frac{\left(7d-fx^2\right)\left(a+bx^4\right)^{3/2}}{7x}+\frac{3}{4}a\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{12a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}+\frac{2a^{5/4}\left(21\sqrt{b}d+5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{\left(a^2e\right)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}dx,x,\sqrt{a+bx^4}\right)}{2b}$$

$$=\frac{12a\sqrt{b}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{4}\left(2ae+3bcx^2\right)\sqrt{a+bx^4}+\frac{2}{35}x\left(5af+21bdx^2\right)\sqrt{a+bx^4}-\frac{\left(3c-ex^2\right)\left(a+bx^4\right)^{3/2}}{6x^2}-\frac{\left(7d-fx^2\right)\left(a+bx^4\right)^{3/2}}{7x}+\frac{3}{4}a\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{2}a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{12a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}+$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.34 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.48

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^3}\,dx = \frac{-3ac\sqrt{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2},-\frac{bx^4}{a}\right)+x\left(ex\sqrt{\ldots}\right.}{\ldots}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^3,x]
```

```
[Out] (-3*a*c*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] +
x*(e*x*Sqrt[1 + (b*x^4)/a]*(Sqrt[a + b*x^4]*(4*a + b*x^4) - 3*a^(3/2)*ArcTa
nh[Sqrt[a + b*x^4]/Sqrt[a]]) - 6*a*d*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2
, -1/4, 3/4, -((b*x^4)/a)] + 6*a*f*x^2*Sqrt[a + b*x^4]*Hypergeometric2F1[-3
/2, 1/4, 5/4, -((b*x^4)/a)]))/(6*x^2*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.31 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.85

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{2x^2}-\frac{ad\sqrt{b\,x^4+a}}{x}+\frac{bf\,x^5\sqrt{b\,x^4+a}}{7}+\frac{be\,x^4\sqrt{b\,x^4+a}}{6}+\frac{bd\,x^3\sqrt{b\,x^4+a}}{5}+\frac{bc\,x^2\sqrt{b\,x^4+a}}{4}+\frac{3afx\sqrt{b\,x^4+a}}{7}+\frac{2ae\ldots}{}$
default	$f\left(\frac{b\,x^5\sqrt{b\,x^4+a}}{7}+\frac{3ax\sqrt{b\,x^4+a}}{7}+\frac{4a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+e\left(\frac{b\,x^4\sqrt{b\,x^4+a}}{6}+\frac{2a\sqrt{b\,x^4+a}}{3}-\frac{a^{\frac{3}{2}}\ldots}{}\right.$
risch	$-\frac{a\sqrt{b\,x^4+a}\,(2dx+c)}{2x^2}+\frac{4a^2f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{bf\,x^5\sqrt{b\,x^4+a}}{7}+\frac{3afx\sqrt{b\,x^4+a}}{7}-\frac{e\sqrt{b\,x^4+a}\,(-b\,x^4+\ldots}{6}$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*c*(b*x^4+a)^(1/2)/x^2-a*d*(b*x^4+a)^(1/2)/x+1/7*b*f*x^5*(b*x^4+a)^(1
/2)+1/6*b*e*x^4*(b*x^4+a)^(1/2)+1/5*b*d*x^3*(b*x^4+a)^(1/2)+1/4*b*c*x^2*(b*
x^4+a)^(1/2)+3/7*a*f*x*(b*x^4+a)^(1/2)+2/3*a*e*(b*x^4+a)^(1/2)+4/7*a^2*f/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2
)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/4*a
*b^(1/2)*c*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))+12/5*I*a^(3/2)*b^(1/2)*d/(I/
a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ellip
```

```
ticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*a^(3/2)*e*arctanh(a^(1/2)/(b*x^4+a
)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^3}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^3}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^3, x)
```

Sympy [A] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^3}\,dx &= -\frac{a^{\frac{3}{2}}c}{2x^2\sqrt{1+\frac{bx^4}{a}}} \\ &+ \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^{\frac{3}{2}}e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} \\ &+ \frac{a^{\frac{3}{2}}fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}bcx^2\sqrt{1+\frac{bx^4}{a}}}{4} - \frac{\sqrt{a}bcx^2}{2\sqrt{1+\frac{bx^4}{a}}} \\ &+ \frac{\sqrt{a}bdx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}bfx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} \\ &+ \frac{a^2e}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a\sqrt{b}c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4} + \frac{a\sqrt{b}ex^2}{2\sqrt{\frac{a}{bx^4}+1}} + be\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)\end{aligned}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**3,x)
```

```
[Out] -a**(3/2)*c/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)*d*gamma(-1/4)*hyper((-1/
2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - a**(3/2)*e*a
sinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*f*x*gamma(1/4)*hyper((-1/2, 1/4),
```

```
(5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*c*x**2*sqrt(1
 + b*x**4/a)/4 - sqrt(a)*b*c*x**2/(2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*d*x**3
*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7
/4)) + sqrt(a)*b*f*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*gamma(9/4)) + a**2*e/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1))
+ 3*a*sqrt(b)*c*asinh(sqrt(b)*x**2/sqrt(a))/4 + a*sqrt(b)*e*x**2/(2*sqrt(a/
(b*x**4) + 1)) + b*e*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(
3/2)/(6*b), True))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^3}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^3}\,dx$$

[In] `integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="maxima")`

[Out] `integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)`

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^3}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^3}\,dx$$

[In] `integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="giac")`

[Out] `integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^3}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^3}\,dx$$

[In] `int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^3,x)`

[Out] `int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^3, x)`

3.518 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 408

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx = \frac{12a\sqrt{b}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2(9ae-5bcx^2)\sqrt{a+bx^4}}{15x} + \frac{1}{4}(2af+3bdx^2)\sqrt{a+bx^4} - \frac{(5c-3ex^2)(a+bx^4)^{3/2}}{15x^3} - \frac{(3d-fx^2)(a+bx^4)^{3/2}}{6x^2} + \frac{3}{4}a\sqrt{b}d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{1}{2}a^{3/2}f\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12a^{5/4}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\,\text{ar}\right.}{5\sqrt{a+bx^4}}$$

```
[Out] -1/15*(-3*e*x^2+5*c)*(b*x^4+a)^(3/2)/x^3-1/6*(-f*x^2+3*d)*(b*x^4+a)^(3/2)/x
^2-1/2*a^(3/2)*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))+3/4*a*d*arctanh(x^2*b^(1/
2)/(b*x^4+a)^(1/2))*b^(1/2)-2/15*(-5*b*c*x^2+9*a*e)*(b*x^4+a)^(1/2)/x+1/4*(
3*b*d*x^2+2*a*f)*(b*x^4+a)^(1/2)+12/5*a*e*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2
)+x^2*b^(1/2))-12/5*a^(5/4)*b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^
2)^(1/2)/(b*x^4+a)^(1/2)+2/15*a^(3/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/
4)*x/a^(1/4))),1/2*2^(1/2))*(9*e*a^(1/2)+5*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))
*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1847, 1286, 1212, 226, 1210, 1266, 827, 829, 858, 223, 212, 272, 65, 214}

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx = \frac{2a^{3/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\right.}{15\sqrt{a+bx^4}}$$
$$-\frac{12a^{5/4}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$-\frac{1}{2}a^{3/2}f\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3}{4}a\sqrt{b}d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{2\sqrt{a+bx^4}(9ae-5bcx^2)}{15x}-\frac{(a+bx^4)^{3/2}(5c-}{15x^3}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^4,x]
```

```
[Out] (12*a*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2*(9*a*e
- 5*b*c*x^2)*Sqrt[a + b*x^4])/(15*x) + ((2*a*f + 3*b*d*x^2)*Sqrt[a + b*x^4]
)/4 - ((5*c - 3*e*x^2)*(a + b*x^4)^(3/2))/(15*x^3) - ((3*d - f*x^2)*(a + b*
x^4)^(3/2))/(6*x^2) + (3*a*Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]
)/4 - (a^(3/2)*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*
e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(3/4)*
b^(1/4)*(5*Sqrt[b]*c + 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4
)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])
/(15*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{(c+ex^2)\left(a+bx^4\right)^{3/2}}{x^4}+\frac{(d+fx^2)\left(a+bx^4\right)^{3/2}}{x^3}\right) dx\\
&= \int \frac{(c+ex^2)\left(a+bx^4\right)^{3/2}}{x^4}\,dx+\int \frac{(d+fx^2)\left(a+bx^4\right)^{3/2}}{x^3}\,dx\\
&= -\frac{(5c-3ex^2)\left(a+bx^4\right)^{3/2}}{15x^3}-\frac{2}{5}\int \frac{(-3ae-5bcx^2)\sqrt{a+bx^4}}{x^2}\,dx\\
&\quad+\frac{1}{2}\text{Subst}\left(\int \frac{(d+fx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{2(9ae-5bcx^2)\sqrt{a+bx^4}}{15x}-\frac{(5c-3ex^2)\left(a+bx^4\right)^{3/2}}{15x^3}-\frac{(3d-fx^2)\left(a+bx^4\right)^{3/2}}{6x^2}\\
&\quad-\frac{1}{4}\text{Subst}\left(\int \frac{(-2af-6bdx)\sqrt{a+bx^2}}{x}\,dx,x,x^2\right)+\frac{4}{15}\int \frac{5abc+9abex^2}{\sqrt{a+bx^4}}\,dx\\
&= -\frac{2(9ae-5bcx^2)\sqrt{a+bx^4}}{15x}+\frac{1}{4}(2af+3bdx^2)\sqrt{a+bx^4}-\frac{(5c-3ex^2)\left(a+bx^4\right)^{3/2}}{15x^3}\\
&\quad-\frac{(3d-fx^2)\left(a+bx^4\right)^{3/2}}{6x^2}-\frac{\text{Subst}\left(\int \frac{-4a^2bf-6ab^2dx}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)}{8b}\\
&\quad-\frac{1}{5}\left(12a^{3/2}\sqrt{b}e\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx+\frac{1}{15}\left(4a\sqrt{b}\left(5\sqrt{b}c+9\sqrt{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx\\
&= \frac{12a\sqrt{b}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{2(9ae-5bcx^2)\sqrt{a+bx^4}}{15x}+\frac{1}{4}(2af+3bdx^2)\sqrt{a+bx^4}\\
&\quad-\frac{(5c-3ex^2)\left(a+bx^4\right)^{3/2}}{15x^3}-\frac{(3d-fx^2)\left(a+bx^4\right)^{3/2}}{6x^2}\\
&\quad-\frac{12a^{5/4}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}\\
&\quad+\frac{2a^{3/4}\sqrt[4]{b}\left(5\sqrt{b}c+9\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}\\
&\quad+\frac{1}{4}(3abd)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)+\frac{1}{2}\left(a^2f\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)
\end{aligned}
$$

$$= \frac{12a\sqrt{b}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2(9ae-5bcx^2)\sqrt{a+bx^4}}{15x} + \frac{1}{4}(2af+3bdx^2)\sqrt{a+bx^4}$$
$$- \frac{(5c-3ex^2)(a+bx^4)^{3/2}}{15x^3} - \frac{(3d-fx^2)(a+bx^4)^{3/2}}{6x^2}$$
$$- \frac{12a^{5/4}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$+ \frac{2a^{3/4}\sqrt[4]{b}\left(5\sqrt{b}c+9\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}$$
$$+ \frac{1}{4}(3abd)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right) + \frac{1}{4}\left(a^2f\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^4\right)$$

$$= \frac{12a\sqrt{b}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2(9ae-5bcx^2)\sqrt{a+bx^4}}{15x}$$
$$+ \frac{1}{4}(2af+3bdx^2)\sqrt{a+bx^4} - \frac{(5c-3ex^2)(a+bx^4)^{3/2}}{15x^3}$$
$$- \frac{(3d-fx^2)(a+bx^4)^{3/2}}{6x^2} + \frac{3}{4}a\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)$$
$$- \frac{12a^{5/4}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$+ \frac{2a^{3/4}\sqrt[4]{b}\left(5\sqrt{b}c+9\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}$$
$$+ \frac{\left(a^2f\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)}{2b}$$

$$= \frac{12a\sqrt{b}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2(9ae-5bcx^2)\sqrt{a+bx^4}}{15x} + \frac{1}{4}(2af+3bdx^2)\sqrt{a+bx^4}$$
$$- \frac{(5c-3ex^2)(a+bx^4)^{3/2}}{15x^3} - \frac{(3d-fx^2)(a+bx^4)^{3/2}}{6x^2} + \frac{3}{4}a\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)$$
$$- \frac{1}{2}a^{3/2}f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12a^{5/4}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}} +$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.34 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.48

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx = \frac{-2ac\sqrt{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{bx^4}{a}\right)-3adx\sqrt{a \ldots}}{\ldots}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^4,x]
```

```
[Out] (-2*a*c*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] -
3*a*d*x*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] +
x^2*(f*x*Sqrt[1 + (b*x^4)/a]*(Sqrt[a + b*x^4]*(4*a + b*x^4) - 3*a^(3/2)*Arc
Tanh[Sqrt[a + b*x^4]/Sqrt[a]]) - 6*a*e*Sqrt[a + b*x^4]*Hypergeometric2F1[-3
/2, -1/4, 3/4, -((b*x^4)/a)]))/(6*x^3*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.96 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.84

method	result
elliptic	$-\frac{ac\sqrt{bx^4+a}}{3x^3}-\frac{ad\sqrt{bx^4+a}}{2x^2}-\frac{ae\sqrt{bx^4+a}}{x}+\frac{bfx^4\sqrt{bx^4+a}}{6}+\frac{bex^3\sqrt{bx^4+a}}{5}+\frac{bdx^2\sqrt{bx^4+a}}{4}+\frac{bcx\sqrt{bx^4+a}}{3}+\frac{2af\sqrt{bx^4 \ldots}}{3}$
default	$f\left(\frac{bx^4\sqrt{bx^4+a}}{6}+\frac{2a\sqrt{bx^4+a}}{3}-\frac{a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^4+a}}{x^2}\right)}{2}\right)+c\left(-\frac{a\sqrt{bx^4+a}}{3x^3}+\frac{bx\sqrt{bx^4+a}}{3}+\frac{4ab\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\ldots}{\ldots}}}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx \ldots}} \ldots\right.$
risch	$-\frac{a\sqrt{bx^4+a}\,(6ex^2+3dx+2c)}{6x^3}+\frac{4abc\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+a}}-\frac{f\sqrt{bx^4+a}\,(-bx^4+2a)}{6}+\frac{bex^3\sqrt{bx^4+a}}{5}-\frac{3i\sqrt{b}\ldots}{\ldots}$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a*c*(b*x^4+a)^(1/2)/x^3-1/2*a*d*(b*x^4+a)^(1/2)/x^2-a*e*(b*x^4+a)^(1/2
)/x+1/6*b*f*x^4*(b*x^4+a)^(1/2)+1/5*b*e*x^3*(b*x^4+a)^(1/2)+1/4*b*d*x^2*(b*
x^4+a)^(1/2)+1/3*b*c*x*(b*x^4+a)^(1/2)+2/3*a*f*(b*x^4+a)^(1/2)+4/3*a*b*c/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2
)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/4*a
*b^(1/2)*d*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))+12/5*I*a^(3/2)*e*b^(1/2)/(I/
a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ellip
```

```
ticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*a^(3/2)*f*arctanh(a^(1/2)/(b*x^4+a
)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^4}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="fricas")
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^4, x)
```

Sympy [A] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^4}\,dx &= \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} \\ &- \frac{a^{\frac{3}{2}}d}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^{\frac{3}{2}}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} \\ &+ \frac{\sqrt{a}bcx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}bdx^2\sqrt{1+\frac{bx^4}{a}}}{4} - \frac{\sqrt{a}bdx^2}{2\sqrt{1+\frac{bx^4}{a}}} \\ &+ \frac{\sqrt{a}bex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{a^2f}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} \\ &+ \frac{3a\sqrt{b}d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4} + \frac{a\sqrt{b}fx^2}{2\sqrt{\frac{a}{bx^4}+1}} + bf\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for}\ b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)\end{aligned}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**4,x)
[Out] a**(3/2)*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**3*gamma(1/4)) - a**(3/2)*d/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)*e
```

```
*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gam
ma(3/4)) - a**(3/2)*f*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*b*c*x*gamma
(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) +
 sqrt(a)*b*d*x**2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*b*d*x**2/(2*sqrt(1 + b*x**
4/a)) + sqrt(a)*b*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*gamma(7/4)) + a**2*f/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1))
 + 3*a*sqrt(b)*d*asinh(sqrt(b)*x**2/sqrt(a))/4 + a*sqrt(b)*f*x**2/(2*sqrt(a
/(b*x**4) + 1)) + b*f*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**
(3/2)/(6*b), True))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx=\int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^4}\,dx$$

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^4,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^4, x)
```

3.519 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 386

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5}\,dx = \frac{12a\sqrt{b}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{3}{4}b(c+ex^2)\sqrt{a+bx^4} + \frac{2}{15}bx(5d+9fx^2)\sqrt{a+bx^4} - \frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)(a+bx^4)^{3/2} + \frac{3}{4}a\sqrt{b}e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3}{4}\sqrt{a}bc\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12a^{5/4}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}} + \frac{2a^{3/4}\sqrt[4]{b}\left(5\sqrt{b}d+9\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15\sqrt{a+bx^4}}$$

```
[Out] -1/12*(3*c/x^4+4*d/x^3+6*e/x^2+12*f/x)*(b*x^4+a)^(3/2)-3/4*b*c*arctanh((b*x
^4+a)^(1/2)/a^(1/2))*a^(1/2)+3/4*a*e*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))*b
^(1/2)+3/4*b*(e*x^2+c)*(b*x^4+a)^(1/2)+2/15*b*x*(9*f*x^2+5*d)*(b*x^4+a)^(1/
2)+12/5*a*f*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-12/5*a^(5/4)*b^
(1/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a
^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x
^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)+2/15*
a^(3/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(9
*f*a^(1/2)+5*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/
2))^2)^(1/2)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {14, 1839, 1847, 1266, 829, 858, 223, 212, 272, 65, 214, 1191, 1212, 226, 1210}

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5}\,dx = \frac{2a^{3/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\right.}{15\sqrt{a+bx^4}}$$
$$-\frac{12a^{5/4}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$-\frac{3}{4}\sqrt{a}bc\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3}{4}a\sqrt{b}e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{12}(a+bx^4)^{3/2}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)+\frac{3}{4}b\sqrt{a}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^5,x]
```

```
[Out] (12*a*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + (3*b*(c +
e*x^2)*Sqrt[a + b*x^4])/4 + (2*b*x*(5*d + 9*f*x^2)*Sqrt[a + b*x^4])/15 - ((
(3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*(a + b*x^4)^(3/2))/12 + (3*a*
Sqrt[b]*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 - (3*Sqrt[a]*b*c*ArcTan
h[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(5/4)*b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(3/4)*b^(1/4)*(5*Sqrt[b]*d +
9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
```

```
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\text{integral} = -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\left(a+bx^4\right)^{3/2} - (6b)\int \frac{\left(-\frac{c}{4}-\frac{dx}{3}-\frac{ex^2}{2}-fx^3\right)\sqrt{a+bx^4}}{x}\,dx$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\left(a+bx^4\right)^{3/2} - (6b)\int \left(\frac{\left(-\frac{c}{4}-\frac{ex^2}{2}\right)\sqrt{a+bx^4}}{x}+\left(-\frac{d}{3}-fx^2\right)\sqrt{a+bx^4}\right)dx$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\left(a+bx^4\right)^{3/2} - (6b)\int \frac{\left(-\frac{c}{4}-\frac{ex^2}{2}\right)\sqrt{a+bx^4}}{x}\,dx - (6b)\int\left(-\frac{d}{3}-fx^2\right)\sqrt{a+bx^4}\,dx$$

$$= \frac{2}{15}bx\left(5d+9fx^2\right)\sqrt{a+bx^4} - \frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\left(a+bx^4\right)^{3/2} - \frac{1}{5}(2b)\int \frac{-\frac{10ad}{3}-6afx^2}{\sqrt{a+bx^4}}\,dx - (3b)\text{Subst}\left(\int \frac{\left(-\frac{c}{4}-\frac{ex}{2}\right)\sqrt{a+bx^2}}{x}\,dx, x, x^2\right)$$

$$= \frac{3}{4}b\left(c+ex^2\right)\sqrt{a+bx^4} + \frac{2}{15}bx\left(5d+9fx^2\right)\sqrt{a+bx^4} - \frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\left(a+bx^4\right)^{3/2} - \frac{3}{2}\text{Subst}\left(\int \frac{-\frac{1}{2}abc-\frac{1}{2}abex}{x\sqrt{a+bx^2}}\,dx, x, x^2\right) - \frac{1}{5}\left(12a^{3/2}\sqrt{b}f\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx + \frac{1}{15}\left(4ab\left(5d+\frac{9\sqrt{a}f}{\sqrt{b}}\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{12a\sqrt{b}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{3}{4}b\left(c+ex^2\right)\sqrt{a+bx^4} + \frac{2}{15}bx\left(5d+9fx^2\right)\sqrt{a+bx^4} - \frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\left(a+bx^4\right)^{3/2} - \frac{12a^{5/4}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}} + \frac{2a^{3/4}\sqrt[4]{b}\left(5\sqrt{b}d+9\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}} + \frac{1}{4}(3abc)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, x^2\right) + \frac{1}{4}(3abe)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^2\right)$$

$$=\frac{12a\sqrt{b}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3}{4}b\left(c+ex^2\right)\sqrt{a+bx^4}$$
$$+\frac{2}{15}bx\left(5d+9fx^2\right)\sqrt{a+bx^4}-\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\left(a+bx^4\right)^{3/2}$$
$$-\frac{12a^{5/4}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$+\frac{2a^{3/4}\sqrt[4]{b}\left(5\sqrt{b}d+9\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}$$
$$+\frac{1}{8}(3abc)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)+\frac{1}{4}(3abe)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)$$
$$=\frac{12a\sqrt{b}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3}{4}b\left(c+ex^2\right)\sqrt{a+bx^4}+\frac{2}{15}bx\left(5d+9fx^2\right)\sqrt{a+bx^4}$$
$$-\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\left(a+bx^4\right)^{3/2}+\frac{3}{4}a\sqrt{b}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)$$
$$-\frac{12a^{5/4}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$+\frac{2a^{3/4}\sqrt[4]{b}\left(5\sqrt{b}d+9\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}$$
$$+\frac{1}{4}(3ac)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)$$
$$=\frac{12a\sqrt{b}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3}{4}b\left(c+ex^2\right)\sqrt{a+bx^4}$$
$$+\frac{2}{15}bx\left(5d+9fx^2\right)\sqrt{a+bx^4}-\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\left(a+bx^4\right)^{3/2}$$
$$+\frac{3}{4}a\sqrt{b}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{3}{4}\sqrt{a}bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$
$$-\frac{12a^{5/4}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$+\frac{2a^{3/4}\sqrt[4]{b}\left(5\sqrt{b}d+9\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.42

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5}\,dx = \frac{\sqrt{a+bx^4}\left(-10a^3d\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{bx^4}{a}\right)+3x\left(\right.\right.}{}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[a + b*x^4]*(-10*a^3*d*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)
] + 3*x*(-5*a^3*e*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] - 10*a^3
*f*x*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^4)/a)] + b*c*x^2*(a + b*x^4)
^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a])))/(30
*a^2*x^3*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.84 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.89

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{4x^4}-\frac{ad\sqrt{b\,x^4+a}}{3x^3}-\frac{ae\sqrt{b\,x^4+a}}{2x^2}-\frac{af\sqrt{b\,x^4+a}}{x}+\frac{bf\,x^3\sqrt{b\,x^4+a}}{5}+\frac{be\,x^2\sqrt{b\,x^4+a}}{4}+\frac{bdx\sqrt{b\,x^4+a}}{3}+\frac{bc\sqrt{b\,x^4+a}}{2}$
default	$d\left(-\frac{a\sqrt{b\,x^4+a}}{3x^3}+\frac{bx\sqrt{b\,x^4+a}}{3}+\frac{4ab\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+f\left(-\frac{a\sqrt{b\,x^4+a}}{x}+\frac{\sqrt{b\,x^4+a}\,b\,x^3}{5}+\frac{12i}{}\right.$
risch	$-\frac{a\sqrt{b\,x^4+a}\,(12f\,x^3+6e\,x^2+4dx+3c)}{12x^4}+\frac{4bda\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{bf\,x^3\sqrt{b\,x^4+a}}{5}+\frac{12i\sqrt{b}\,f\,a^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{5\sqrt{\frac{i}{}}}$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a*c*(b*x^4+a)^(1/2)/x^4-1/3*a*d*(b*x^4+a)^(1/2)/x^3-1/2*a*e*(b*x^4+a)^
(1/2)/x^2-a*f*(b*x^4+a)^(1/2)/x+1/5*b*f*x^3*(b*x^4+a)^(1/2)+1/4*b*e*x^2*(b*
x^4+a)^(1/2)+1/3*b*d*x*(b*x^4+a)^(1/2)+1/2*b*c*(b*x^4+a)^(1/2)+4/3*b*d*a/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2
)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/4*a
*e*b^(1/2)*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))+12/5*I*a^(3/2)*f*b^(1/2)/(I/
a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ellip
```

```
ticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3/4*a^(1/2)*b*c*arctanh(a^(1/2)/(b*x^4
+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^5}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^5, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.48 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^5}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{a^{\frac{3}{2}}e}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} + \frac{\sqrt{a}bdx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}bex^2\sqrt{1+\frac{bx^4}{a}}}{4} - \frac{\sqrt{a}bex^2}{2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bfx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{a\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{a\sqrt{b}c}{2x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a\sqrt{b}e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4} + \frac{b^{\frac{3}{2}}cx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**5,x)
```

```
[Out] a**(3/2)*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**3*gamma(1/4)) - a**(3/2)*e/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)*f
*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gam
ma(3/4)) - 3*sqrt(a)*b*c*asinh(sqrt(a)/(sqrt(b)*x**2))/4 + sqrt(a)*b*d*x*ga
mma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)
) + sqrt(a)*b*e*x**2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*b*e*x**2/(2*sqrt(1 + b*
x**4/a)) + sqrt(a)*b*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*gamma(7/4)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*x**2)
 + a*sqrt(b)*c/(2*x**2*sqrt(a/(b*x**4) + 1)) + 3*a*sqrt(b)*e*asinh(sqrt(b)*
x**2/sqrt(a))/4 + b**(3/2)*c*x**2/(2*sqrt(a/(b*x**4) + 1))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^5}\,dx=\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(3*sqrt(a)*b*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)
)) + 4*sqrt(b*x^4 + a)*b - 2*sqrt(b*x^4 + a)*a/x^4)*c + integrate((b*f*x^6
+ b*e*x^5 + b*d*x^4 + a*f*x^2 + a*e*x + a*d)*sqrt(b*x^4 + a)/x^4, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^5}\,dx=\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^5}\,dx=\int \frac{{\left(b\,x^4+a\right)}^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^5}\,dx$$

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^5,x)
```

```
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^5, x)
```

3.520 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 387

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx = \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b(9c-5ex^2)\sqrt{a+bx^4}}{15x} + \frac{3}{4}b(d+fx^2)\sqrt{a+bx^4} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)(a+bx^4)^{3/2} + \frac{3}{4}a\sqrt{b}f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3}{4}\sqrt{a}bd\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12\sqrt[4]{a}b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\,\text{ar}\right.}{5\sqrt{a+bx^4}}$$

```
[Out] -1/60*(12*c/x^5+15*d/x^4+20*e/x^3+30*f/x^2)*(b*x^4+a)^(3/2)-3/4*b*d*arctanh
((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+3/4*a*f*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/
2))*b^(1/2)-2/15*b*(-5*e*x^2+9*c)*(b*x^4+a)^(1/2)/x+3/4*b*(f*x^2+d)*(b*x^4+
a)^(1/2)+12/5*b^(3/2)*c*x*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-12/5*a^(1/4
)*b^(5/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)
*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/
2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)+2
/15*a^(1/4)*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)*(5*e*a^(1/2)+9*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b
^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {14, 1839, 1847, 1286, 1212, 226, 1210, 1266, 829, 858, 223, 212, 272, 65, 214}

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx = \frac{2\sqrt[4]{a}b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+9\sqrt{b}c\right)\text{EllipticF}\left(\right.}{15\sqrt{a+bx^4}}$$
$$-\frac{12\sqrt[4]{a}b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$-\frac{3}{4}\sqrt{a}bd\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3}{4}a\sqrt{b}f\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{1}{60}\left(a+bx^4\right)^{3/2}\left(\frac{12c}{x^5}+\right.$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^6,x]
```

```
[Out] (12*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2*b*(9*c -
5*e*x^2)*Sqrt[a + b*x^4])/(15*x) + (3*b*(d + f*x^2)*Sqrt[a + b*x^4])/4 - ((
(12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*(a + b*x^4)^(3/2))/60 +
(3*a*Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 - (3*Sqrt[a]*b*d*A
rcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(1/4)*b^(5/4)*c*(Sqrt[a] + Sqrt[
b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(
1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(1/4)*b^(3/4)*(9*Sqrt[b]
*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[
b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a + b*x^
4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\text{integral} = -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\left(a+bx^4\right)^{3/2} - (6b)\int \frac{\left(-\frac{c}{5}-\frac{dx}{4}-\frac{ex^2}{3}-\frac{fx^3}{2}\right)\sqrt{a+bx^4}}{x^2}\,dx$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\left(a+bx^4\right)^{3/2} - (6b)\int \left(\frac{\left(-\frac{c}{5}-\frac{ex^2}{3}\right)\sqrt{a+bx^4}}{x^2}+\frac{\left(-\frac{d}{4}-\frac{fx^2}{2}\right)\sqrt{a+bx^4}}{x}\right)dx$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\left(a+bx^4\right)^{3/2} - (6b)\int \frac{\left(-\frac{c}{5}-\frac{ex^2}{3}\right)\sqrt{a+bx^4}}{x^2}\,dx - (6b)\int \frac{\left(-\frac{d}{4}-\frac{fx^2}{2}\right)\sqrt{a+bx^4}}{x}\,dx$$

$$= -\frac{2b(9c-5ex^2)\sqrt{a+bx^4}}{15x} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\left(a+bx^4\right)^{3/2} - (3b)\text{Subst}\left(\int \frac{\left(-\frac{d}{4}-\frac{fx}{2}\right)\sqrt{a+bx^2}}{x}\,dx, x, x^2\right) + (4b)\int \frac{\frac{ae}{3}+\frac{3}{5}bcx^2}{\sqrt{a+bx^4}}\,dx$$

$$= -\frac{2b(9c-5ex^2)\sqrt{a+bx^4}}{15x} + \frac{3}{4}b\left(d+fx^2\right)\sqrt{a+bx^4} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\left(a+bx^4\right)^{3/2} - \frac{3}{2}\text{Subst}\left(\int \frac{-\frac{1}{2}abd-\frac{1}{2}abfx}{x\sqrt{a+bx^2}}\,dx, x, x^2\right) - \frac{1}{5}\left(12\sqrt{a}b^{3/2}c\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx + \frac{1}{15}\left(4\sqrt{a}b\left(9\sqrt{b}c+5\sqrt{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx$$

$$
= \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b(9c-5ex^2)\sqrt{a+bx^4}}{15x}
$$
$$
+ \frac{3}{4}b(d+fx^2)\sqrt{a+bx^4} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\left(a+bx^4\right)^{3/2}
$$
$$
- \frac{12\sqrt[4]{a}b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}
$$
$$
+ \frac{2\sqrt[4]{a}b^{3/4}\left(9\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}
$$
$$
+\frac{1}{4}(3abd)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)+\frac{1}{4}(3abf)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)
$$
$$
= \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b(9c-5ex^2)\sqrt{a+bx^4}}{15x}
$$
$$
+ \frac{3}{4}b(d+fx^2)\sqrt{a+bx^4} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\left(a+bx^4\right)^{3/2}
$$
$$
- \frac{12\sqrt[4]{a}b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}
$$
$$
+ \frac{2\sqrt[4]{a}b^{3/4}\left(9\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}
$$
$$
+\frac{1}{8}(3abd)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)+\frac{1}{4}(3abf)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)
$$
$$
= \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b(9c-5ex^2)\sqrt{a+bx^4}}{15x} + \frac{3}{4}b(d+fx^2)\sqrt{a+bx^4}
$$
$$
- \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\left(a+bx^4\right)^{3/2} + \frac{3}{4}a\sqrt{b}f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)
$$
$$
- \frac{12\sqrt[4]{a}b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}
$$
$$
+ \frac{2\sqrt[4]{a}b^{3/4}\left(9\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}
$$
$$
+\frac{1}{4}(3ad)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)
$$

$$= \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b(9c-5ex^2)\sqrt{a+bx^4}}{15x} + \frac{3}{4}b(d+fx^2)\sqrt{a+bx^4} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)(a+bx^4)^{3/2} + \frac{3}{4}a\sqrt{b}f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3}{4}\sqrt{a}bd\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12\sqrt[4]{a}b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}} + \frac{2\sqrt[4]{a}b^{3/4}\left(9\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.43

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx = \frac{\sqrt{a+bx^4}\left(-6a^3c\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{4},-\frac{bx^4}{a}\right)-10a^3\right.}{}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^4]*(-6*a^3*c*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a)
] - 10*a^3*e*x^2*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] - 15*a^3*
f*x^3*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] + 3*b*d*x^5*(a + b*x
^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a]))/(
30*a^2*x^5*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.50 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.89

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{5x^5}-\frac{ad\sqrt{b\,x^4+a}}{4x^4}-\frac{ae\sqrt{b\,x^4+a}}{3x^3}-\frac{af\sqrt{b\,x^4+a}}{2x^2}-\frac{7bc\sqrt{b\,x^4+a}}{5x}+\frac{bf\,x^2\sqrt{b\,x^4+a}}{4}+\frac{bex\sqrt{b\,x^4+a}}{3}+\frac{bd\sqrt{b\,x^4+a}}{2}$
default	$e\left(-\frac{a\sqrt{b\,x^4+a}}{3x^3}+\frac{bx\sqrt{b\,x^4+a}}{3}+\frac{4ab\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+f\left(\frac{b\,x^2\sqrt{b\,x^4+a}}{4}+\frac{3a\sqrt{b}\,\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4}\right.}{4}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,(84bc\,x^4+30af\,x^3+20ae\,x^2+15adx+12ac)}{60x^5}+\frac{4bea\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{bf\,x^2\sqrt{b\,x^4+a}}{4}+\frac{3\sqrt{b}\,c}{}$

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x,method=_RETURNVERBOSE)

[Out] -1/5*a*c*(b*x^4+a)^(1/2)/x^5-1/4*a*d*(b*x^4+a)^(1/2)/x^4-1/3*a*e*(b*x^4+a)^(1/2)/x^3-1/2*a*f*(b*x^4+a)^(1/2)/x^2-7/5*b*c*(b*x^4+a)^(1/2)/x+1/4*b*f*x^2*(b*x^4+a)^(1/2)+1/3*b*e*x*(b*x^4+a)^(1/2)+1/2*b*d*(b*x^4+a)^(1/2)+4/3*b*e*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/4*a*f*b^(1/2)*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))+12/5*I*b^(3/2)*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3/4*a^(1/2)*b*d*arctanh(a^(1/2)/(b*x^4+a)^(1/2))

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^6}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^6, x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.65 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{a^{\frac{3}{2}}f}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bc\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} + \frac{\sqrt{a}bex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}bfx^2\sqrt{1+\frac{bx^4}{a}}}{4} - \frac{\sqrt{a}bfx^2}{2\sqrt{1+\frac{bx^4}{a}}} - \frac{a\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{a\sqrt{b}d}{2x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a\sqrt{b}f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4} + \frac{b^{\frac{3}{2}}dx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**6,x)
```

```
[Out] a**(3/2)*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**5*gamma(-1/4)) + a**(3/2)*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,)
, b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - a**(3/2)*f/(2*x**2*sqrt(1
 + b*x**4/a)) + sqrt(a)*b*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*
exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*d*asinh(sqrt(a)/(sqrt(b)*
x**2))/4 + sqrt(a)*b*e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*f*x**2*sqrt(1 + b*x**4/a)/4 - sqrt
(a)*b*f*x**2/(2*sqrt(1 + b*x**4/a)) - a*sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*x
**2) + a*sqrt(b)*d/(2*x**2*sqrt(a/(b*x**4) + 1)) + 3*a*sqrt(b)*f*asinh(sqrt
(b)*x**2/sqrt(a))/4 + b**(3/2)*d*x**2/(2*sqrt(a/(b*x**4) + 1))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^6}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^6}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^6}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^6}\,dx$$

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^6,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^6, x)
```

3.521 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 30, antiderivative size = 392

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7} dx = \frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{b(2c-3ex^2)\sqrt{a+bx^4}}{4x^2} - \frac{2b(9d-5fx^2)\sqrt{a+bx^4}}{15x} - \frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)(a+bx^4)^{3/2} + \frac{1}{2}b^{3/2}c\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3}{4}\sqrt{a}be\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12\sqrt[4]{a}b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\text{ar}\right.}{5\sqrt{a+bx^4}}$$

```
[Out] -1/60*(10*c/x^6+12*d/x^5+15*e/x^4+20*f/x^3)*(b*x^4+a)^(3/2)+1/2*b^(3/2)*c*a
rctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))-3/4*b*e*arctanh((b*x^4+a)^(1/2)/a^(1/2)
)*a^(1/2)-1/4*b*(-3*e*x^2+2*c)*(b*x^4+a)^(1/2)/x^2-2/15*b*(-5*f*x^2+9*d)*(b
*x^4+a)^(1/2)/x+12/5*b^(3/2)*d*x*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-12/5
*a^(1/4)*b^(5/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(
b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))
*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^
(1/2)+2/15*a^(1/4)*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*
2^(1/2))*(5*f*a^(1/2)+9*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2
)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {14, 1839, 1847, 1266, 827, 858, 223, 212, 272, 65, 214, 1286, 1212, 226, 1210}

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx = \frac{2\sqrt[4]{a}b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+9\sqrt{b}d\right)\text{EllipticF}\left(\right.}{15\sqrt{a+bx^4}}$$
$$-\frac{12\sqrt[4]{a}b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$+\frac{1}{2}b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{3}{4}\sqrt{a}be\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{1}{60}(a+bx^4)^{3/2}\left(\frac{10c}{x^6}+\frac{12}{x}\right.$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^7,x]
```

```
[Out] (12*b^(3/2)*d*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (b*(2*c - 3*
e*x^2)*Sqrt[a + b*x^4])/(4*x^2) - (2*b*(9*d - 5*f*x^2)*Sqrt[a + b*x^4])/(15
*x) - (((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*(a + b*x^4)^(3/2
))/60 + (b^(3/2)*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (3*Sqrt[a]*b
*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(1/4)*b^(5/4)*d*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(1/4)*b^(3/4)*(9*Sqr
t[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a +
b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\left(a+bx^4\right)^{3/2} \\
&\quad -(6b)\int \frac{\left(-\frac{c}{6}-\frac{dx}{5}-\frac{ex^2}{4}-\frac{fx^3}{3}\right)\sqrt{a+bx^4}}{x^3}\,dx \\
&= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\left(a+bx^4\right)^{3/2} \\
&\quad -(6b)\int\left(\frac{\left(-\frac{c}{6}-\frac{ex^2}{4}\right)\sqrt{a+bx^4}}{x^3}+\frac{\left(-\frac{d}{5}-\frac{fx^2}{3}\right)\sqrt{a+bx^4}}{x^2}\right)dx \\
&= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\left(a+bx^4\right)^{3/2} \\
&\quad -(6b)\int \frac{\left(-\frac{c}{6}-\frac{ex^2}{4}\right)\sqrt{a+bx^4}}{x^3}\,dx-(6b)\int \frac{\left(-\frac{d}{5}-\frac{fx^2}{3}\right)\sqrt{a+bx^4}}{x^2}\,dx \\
&= -\frac{2b(9d-5fx^2)\sqrt{a+bx^4}}{15x}-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\left(a+bx^4\right)^{3/2} \\
&\quad -(3b)\text{Subst}\left(\int \frac{\left(-\frac{c}{6}-\frac{ex}{4}\right)\sqrt{a+bx^2}}{x^2}\,dx,x,x^2\right)+(4b)\int \frac{\frac{af}{3}+\frac{3}{5}bdx^2}{\sqrt{a+bx^4}}\,dx \\
&= -\frac{b(2c-3ex^2)\sqrt{a+bx^4}}{4x^2}-\frac{2b(9d-5fx^2)\sqrt{a+bx^4}}{15x} \\
&\quad -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\left(a+bx^4\right)^{3/2} \\
&\quad +\frac{1}{2}(3b)\text{Subst}\left(\int \frac{\frac{ae}{2}+\frac{bcx}{3}}{x\sqrt{a+bx^2}}\,dx,x,x^2\right) \\
&\quad -\frac{1}{5}\left(12\sqrt{a}b^{3/2}d\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx+\frac{1}{15}\left(4\sqrt{a}b\left(9\sqrt{b}d+5\sqrt{a}f\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx
\end{aligned}
$$

$$=\frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{b(2c-3ex^2)\sqrt{a+bx^4}}{4x^2}-\frac{2b(9d-5fx^2)\sqrt{a+bx^4}}{15x}$$
$$-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)(a+bx^4)^{3/2}$$
$$-\frac{12\sqrt[4]{a}b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$+\frac{2\sqrt[4]{a}b^{3/4}\left(9\sqrt{b}d+5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}$$
$$+\frac{1}{2}(b^2c)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)+\frac{1}{4}(3abe)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)$$

$$=\frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{b(2c-3ex^2)\sqrt{a+bx^4}}{4x^2}-\frac{2b(9d-5fx^2)\sqrt{a+bx^4}}{15x}$$
$$-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)(a+bx^4)^{3/2}$$
$$-\frac{12\sqrt[4]{a}b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$+\frac{2\sqrt[4]{a}b^{3/4}\left(9\sqrt{b}d+5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}$$
$$+\frac{1}{2}(b^2c)\,\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)+\frac{1}{8}(3abe)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)$$

$$=\frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{b(2c-3ex^2)\sqrt{a+bx^4}}{4x^2}-\frac{2b(9d-5fx^2)\sqrt{a+bx^4}}{15x}$$
$$-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)(a+bx^4)^{3/2}$$
$$+\frac{1}{2}b^{3/2}c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{12\sqrt[4]{a}b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}+$$

$$= \frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{b(2c-3ex^2)\sqrt{a+bx^4}}{4x^2} - \frac{2b(9d-5fx^2)\sqrt{a+bx^4}}{15x}$$
$$-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\left(a+bx^4\right)^{3/2}$$
$$+\frac{1}{2}b^{3/2}c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{3}{4}\sqrt{a}be\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{12\sqrt[4]{a}b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x\right.}}}{5\sqrt{a+bx}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.42

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^7}\,dx = \frac{\sqrt{a+bx^4}\left(-5a^3c\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-\frac{bx^4}{a}\right)-6a^3c\right.}{}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^7,x]
[Out] (Sqrt[a + b*x^4]*(-5*a^3*c*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x^4)/a)
] - 6*a^3*d*x*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a)] - 10*a^3*f*
x^3*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] + 3*b*e*x^6*(a + b*x^4
)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a]))/(30
*a^2*x^6*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.88

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{6x^6}-\frac{ad\sqrt{b\,x^4+a}}{5x^5}-\frac{ae\sqrt{b\,x^4+a}}{4x^4}-\frac{af\sqrt{b\,x^4+a}}{3x^3}-\frac{2bc\sqrt{b\,x^4+a}}{3x^2}-\frac{7bd\sqrt{b\,x^4+a}}{5x}+\frac{bfx\sqrt{b\,x^4+a}}{3}+\frac{be\sqrt{b\,x^4+a}}{2}+$
default	$c\left(\frac{b^{\frac{3}{2}}\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{2}-\frac{a\sqrt{b\,x^4+a}}{6x^6}-\frac{2b\sqrt{b\,x^4+a}}{3x^2}\right)+f\left(-\frac{a\sqrt{b\,x^4+a}}{3x^3}+\frac{bx\sqrt{b\,x^4+a}}{3}+\frac{4ab\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,(84bd\,x^5+40bc\,x^4+20af\,x^3+15ae\,x^2+12adx+10ac)}{60x^6}+\frac{4bfa\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{bfx\sqrt{b\,x^4+a}}{3}+$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a*c*(b*x^4+a)^(1/2)/x^6-1/5*a*d*(b*x^4+a)^(1/2)/x^5-1/4*a*e*(b*x^4+a)^
(1/2)/x^4-1/3*a*f*(b*x^4+a)^(1/2)/x^3-2/3*b*c*(b*x^4+a)^(1/2)/x^2-7/5*b*d*(
b*x^4+a)^(1/2)/x+1/3*b*f*x*(b*x^4+a)^(1/2)+1/2*b*e*(b*x^4+a)^(1/2)+4/3*b*f*
a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1
/2*b^(3/2)*c*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))+12/5*I*b^(3/2)*d*a^(1/2)/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ell
ipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3/4*a^(1/2)*e*b*arctanh(a^(1/2)/(b*x
^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^7}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^7}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^7, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.25 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\ -\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}bc}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bd\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} + \frac{\sqrt{a}bfx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{a\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{a\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{a\sqrt{b}e}{2x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{6} + \frac{b^{\frac{3}{2}}c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} + \frac{b^{\frac{3}{2}}ex^2}{2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^2cx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**7,x)
```

```
[Out] a**(3/2)*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**5*gamma(-1/4)) + a**(3/2)*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,)
, b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*c/(2*x**2*sqrt(
1 + b*x**4/a)) + sqrt(a)*b*d*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4
*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*e*asinh(sqrt(a)/(sqrt(b)
*x**2))/4 + sqrt(a)*b*f*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_
polar(I*pi)/a)/(4*gamma(5/4)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(6*x**4) -
 a*sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b)*e/(2*x**2*sqrt(a/(b*
x**4) + 1)) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*c*asinh(sqrt(b)*
x**2/sqrt(a))/2 + b**(3/2)*e*x**2/(2*sqrt(a/(b*x**4) + 1)) - b**2*c*x**2/(2
*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^7}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^(3/2)*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4
 + a)/x^2)) + 6*sqrt(b*x^4 + a)*b/x^2 + 2*(b*x^4 + a)^(3/2)/x^6)*c + integr
ate((b*f*x^6 + b*e*x^5 + b*d*x^4 + a*f*x^2 + a*e*x + a*d)*sqrt(b*x^4 + a)/x
^6, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^7}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^7}\,dx$$

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^7,x)
```

```
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^7, x)
```

3.522 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 412

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx = -\frac{12be\sqrt{a+bx^4}}{5x} + \frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b(5c-21ex^2)\sqrt{a+bx^4}}{35x^3} - \frac{b(2d-3fx^2)\sqrt{a+bx^4}}{4x^2} - \frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)(a+bx^4)^{3/2} + \frac{1}{2}b^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3}{4}\sqrt{a}bf\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12\sqrt[4]{a}b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\,\text{ar}\right.}{5\sqrt{a+bx^4}}$$

```
[Out] -1/420*(60*c/x^7+70*d/x^6+84*e/x^5+105*f/x^4)*(b*x^4+a)^(3/2)+1/2*b^(3/2)*d
*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))-3/4*b*f*arctanh((b*x^4+a)^(1/2)/a^(1/
2))*a^(1/2)-12/5*b*e*(b*x^4+a)^(1/2)/x-2/35*b*(-21*e*x^2+5*c)*(b*x^4+a)^(1/
2)/x^3-1/4*b*(-3*f*x^2+2*d)*(b*x^4+a)^(1/2)/x^2+12/5*b^(3/2)*e*x*(b*x^4+a)^
(1/2)/(a^(1/2)+x^2*b^(1/2))-12/5*a^(1/4)*b^(5/4)*e*(cos(2*arctan(b^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(
b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+
x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)+2/35*b^(5/4)*(cos(2*arctan(b^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(
b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(21*e*a^(1/2)+5*c*b^(1/2))*(a^(1/2)+x^2*b^
(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {14, 1839, 1847, 1286, 1296, 1212, 226, 1210, 1266, 827, 858, 223, 212, 272, 65, 214}

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}dx = \frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\,\text{a}\right.}{35\sqrt[4]{a}\sqrt{a+bx^4}}$$
$$-\frac{12\sqrt[4]{a}b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$+\frac{1}{2}b^{3/2}d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{3}{4}\sqrt{a}bf\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{1}{420}(a+bx^4)^{3/2}\left(\frac{60c}{x^7}+\right.$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^8,x]
```

```
[Out] (-12*b*e*Sqrt[a + b*x^4])/(5*x) + (12*b^(3/2)*e*x*Sqrt[a + b*x^4])/(5*(Sqrt
[a] + Sqrt[b]*x^2)) - (2*b*(5*c - 21*e*x^2)*Sqrt[a + b*x^4])/(35*x^3) - (b*
(2*d - 3*f*x^2)*Sqrt[a + b*x^4])/(4*x^2) - (((60*c)/x^7 + (70*d)/x^6 + (84*
e)/x^5 + (105*f)/x^4)*(a + b*x^4)^(3/2))/420 + (b^(3/2)*d*ArcTanh[(Sqrt[b]*
x^2)/Sqrt[a + b*x^4]])/2 - (3*Sqrt[a]*b*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])
/4 - (12*a^(1/4)*b^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[
a + b*x^4]) + (2*b^(5/4)*(5*Sqrt[b]*c + 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(35*a^(1/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
```

```
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2} \\
&\quad -(6b)\int\frac{\left(-\frac{c}{7}-\frac{dx}{6}-\frac{ex^2}{5}-\frac{fx^3}{4}\right)\sqrt{a+bx^4}}{x^4}\,dx \\
&= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2} \\
&\quad -(6b)\int\left(\frac{\left(-\frac{c}{7}-\frac{ex^2}{5}\right)\sqrt{a+bx^4}}{x^4}+\frac{\left(-\frac{d}{6}-\frac{fx^2}{4}\right)\sqrt{a+bx^4}}{x^3}\right)dx \\
&= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2} \\
&\quad -(6b)\int\frac{\left(-\frac{c}{7}-\frac{ex^2}{5}\right)\sqrt{a+bx^4}}{x^4}\,dx-(6b)\int\frac{\left(-\frac{d}{6}-\frac{fx^2}{4}\right)\sqrt{a+bx^4}}{x^3}\,dx \\
&= -\frac{2b(5c-21ex^2)\sqrt{a+bx^4}}{35x^3}-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2} \\
&\quad -(3b)\text{Subst}\left(\int\frac{\left(-\frac{d}{6}-\frac{fx}{4}\right)\sqrt{a+bx^2}}{x^2}\,dx,x,x^2\right)+(4b)\int\frac{\frac{3ae}{5}+\frac{1}{7}bcx^2}{x^2\sqrt{a+bx^4}}\,dx \\
&= -\frac{12be\sqrt{a+bx^4}}{5x}-\frac{2b(5c-21ex^2)\sqrt{a+bx^4}}{35x^3}-\frac{b(2d-3fx^2)\sqrt{a+bx^4}}{4x^2} \\
&\quad -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2} \\
&\quad +\frac{1}{2}(3b)\text{Subst}\left(\int\frac{\frac{af}{2}+\frac{bdx}{3}}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{(4b)\int\frac{-\frac{1}{7}abc-\frac{3}{5}abex^2}{\sqrt{a+bx^4}}\,dx}{a} \\
&= -\frac{12be\sqrt{a+bx^4}}{5x}-\frac{2b(5c-21ex^2)\sqrt{a+bx^4}}{35x^3}-\frac{b(2d-3fx^2)\sqrt{a+bx^4}}{4x^2} \\
&\quad -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2} \\
&\quad +\frac{1}{2}\left(b^2d\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{1}{5}\left(12\sqrt{a}b^{3/2}e\right)\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx \\
&\quad +\frac{1}{35}\left(4b^{3/2}\left(5\sqrt{b}c+21\sqrt{a}e\right)\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx+\frac{1}{4}(3abf)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)
\end{aligned}$$

$$= -\frac{12be\sqrt{a+bx^4}}{5x} + \frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b(5c-21ex^2)\sqrt{a+bx^4}}{35x^3} - \frac{b(2d-3fx^2)\sqrt{a+bx^4}}{4x^2} - \frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2} - \frac{12\sqrt[4]{a}b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}} + \frac{2b^{5/4}\left(5\sqrt{b}c+21\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}} + \frac{1}{2}(b^2d)\,\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right) + \frac{1}{8}(3abf)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)$$

$$= -\frac{12be\sqrt{a+bx^4}}{5x} + \frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b(5c-21ex^2)\sqrt{a+bx^4}}{35x^3} - \frac{b(2d-3fx^2)\sqrt{a+bx^4}}{4x^2} - \frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2} + \frac{1}{2}b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{12\sqrt[4]{a}b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}} +$$

$$= -\frac{12be\sqrt{a+bx^4}}{5x} + \frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b(5c-21ex^2)\sqrt{a+bx^4}}{35x^3} - \frac{b(2d-3fx^2)\sqrt{a+bx^4}}{4x^2} - \frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2} + \frac{1}{2}b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3}{4}\sqrt{a}bf\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12\sqrt[4]{a}b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+b}{(\sqrt{a}+\sqrt{}}}}{5\sqrt{a+b}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.40

$$\int\frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^8}\,dx = \frac{\sqrt{a+bx^4}\left(-30a^3c\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},-\frac{3}{4},-\frac{bx^4}{a}\right)+7\right.}{}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^8,x]
```

```
[Out] (Sqrt[a + b*x^4]*(-30*a^3*c*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^4)/a
)] + 7*x*(-5*a^3*d*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x^4)/a)] - 6*a^
3*e*x*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a)] + 3*b*f*x^6*(a + b*
x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a]))
/(210*a^2*x^7*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(588be\,x^6+280bd\,x^5+180bc\,x^4+105af\,x^3+84ae\,x^2+70adx+60ac)}{420x^7}+\frac{b\left(\frac{40bc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+35f\sqrt{b}\right.}{}$
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{7x^7}-\frac{ad\sqrt{b\,x^4+a}}{6x^6}-\frac{ae\sqrt{b\,x^4+a}}{5x^5}-\frac{af\sqrt{b\,x^4+a}}{4x^4}-\frac{3bc\sqrt{b\,x^4+a}}{7x^3}-\frac{2bd\sqrt{b\,x^4+a}}{3x^2}-\frac{7be\sqrt{b\,x^4+a}}{5x}+\frac{bf\sqrt{b\,x^4+a}}{2}+$
default	$d\left(\frac{b^{\frac{3}{2}}\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{2}-\frac{a\sqrt{b\,x^4+a}}{6x^6}-\frac{2b\sqrt{b\,x^4+a}}{3x^2}\right)+c\left(-\frac{a\sqrt{b\,x^4+a}}{7x^7}-\frac{3b\sqrt{b\,x^4+a}}{7x^3}+\frac{4b^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/420*(b*x^4+a)^(1/2)*(588*b*e*x^6+280*b*d*x^5+180*b*c*x^4+105*a*f*x^3+84*
a*e*x^2+70*a*d*x+60*a*c)/x^7+1/70*b*(40*b*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/
a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*
EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+35*f*(b*x^4+a)^(1/2)+35*b^(1/2)*d*
ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+168*I*b^(1/2)*e*a^(1/2)/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x
^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2
)*b^(1/2))^(1/2),I))-105/2*a^(1/2)*f*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2
))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^8}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.32 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, -\frac{1}{2} \\ -\frac{3}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{2} \\ -\frac{1}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}bd}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}be\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}bf\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} - \frac{a\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{a\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{a\sqrt{b}f}{2x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{6} + \frac{b^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} + \frac{b^{\frac{3}{2}}fx^2}{2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^2dx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**8,x)
```

```
[Out] a**(3/2)*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**7*gamma(-3/4)) + a**(3/2)*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*b*c*gamma(-3/4)
*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4))
- sqrt(a)*b*d/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*gamma(-1/4)*hyper((
-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)
*b*f*asinh(sqrt(a)/(sqrt(b)*x**2))/4 - a*sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(6*
x**4) - a*sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b)*f/(2*x**2*sqr
t(a/(b*x**4) + 1)) - b**(3/2)*d*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*d*asinh(s
qrt(b)*x**2/sqrt(a))/2 + b**(3/2)*f*x**2/(2*sqrt(a/(b*x**4) + 1)) - b**2*d*
x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^8}\,dx$$

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^8,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^8, x)
```

3.523 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9} dx$

Optimal result

Integrand size = 30, antiderivative size = 377

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9} dx = -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}$$
$$+\frac{12b^{3/2}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)(a+bx^4)^{3/2}$$
$$+\frac{1}{2}b^{3/2}e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{3b^2c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{12\sqrt[4]{a}b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\Big(2\arctan}{5\sqrt{a+bx^4}}$$

```
[Out] -1/840*(105*c/x^8+120*d/x^7+140*e/x^6+168*f/x^5)*(b*x^4+a)^(3/2)+1/2*b^(3/2
)*e*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))-3/16*b^2*c*arctanh((b*x^4+a)^(1/2)
/a^(1/2))/a^(1/2)-1/560*b*(105*c/x^4+160*d/x^3+280*e/x^2+672*f/x)*(b*x^4+a)
^(1/2)+12/5*b^(3/2)*f*x*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-12/5*a^(1/4)*
b^(5/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x
/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)
+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)+2/3
5*b^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x
/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(21*f*a^
(1/2)+5*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2
)^(1/2)/a^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {14, 1839, 1846, 272, 65, 214, 1899, 281, 223, 212, 1212, 226, 1210}

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = \frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\,\text{ar}\right.}{35\sqrt[4]{a}\sqrt{a+bx^4}}$$

$$-\frac{12\sqrt[4]{a}b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$

$$+\frac{1}{2}b^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{3b^2c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}+\frac{12b^{3/2}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{1}{560}b\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{2}{}\right.$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^9,x]
```

```
[Out] -1/560*(b*((105*c)/x^4 + (160*d)/x^3 + (280*e)/x^2 + (672*f)/x)*Sqrt[a + b*
x^4]) + (12*b^(3/2)*f*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (((1
05*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*(a + b*x^4)^(3/2))/840
 + (b^(3/2)*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (3*b^2*c*ArcTanh[
Sqrt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (12*a^(1/4)*b^(5/4)*f*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*b^(5/4)*(5*Sqrt[b]*d +
 21*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*a^(1/4)*Sqrt[a +
 b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\left(a+bx^4\right)^{3/2} \\
&\quad -(6b)\int\frac{\left(-\frac{c}{8}-\frac{dx}{7}-\frac{ex^2}{6}-\frac{fx^3}{5}\right)\sqrt{a+bx^4}}{x^5}\,dx \\
&= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4} \\
&\quad -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\left(a+bx^4\right)^{3/2} \\
&\quad +\left(12b^2\right)\int\frac{\frac{c}{32}+\frac{dx}{21}+\frac{ex^2}{12}+\frac{fx^3}{5}}{x\sqrt{a+bx^4}}\,dx \\
&= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4} \\
&\quad -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\left(a+bx^4\right)^{3/2} \\
&\quad +\left(12b^2\right)\int\frac{\frac{d}{21}+\frac{ex}{12}+\frac{fx^2}{5}}{\sqrt{a+bx^4}}\,dx+\frac{1}{8}\left(3b^2c\right)\int\frac{1}{x\sqrt{a+bx^4}}\,dx
\end{aligned}$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}$$
$$-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\left(a+bx^4\right)^{3/2}$$
$$+\left(12b^2\right)\int\left(\frac{ex}{12\sqrt{a+bx^4}}+\frac{\frac{d}{21}+\frac{fx^2}{5}}{\sqrt{a+bx^4}}\right)dx$$
$$+\frac{1}{32}\left(3b^2c\right)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}$$
$$-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\left(a+bx^4\right)^{3/2}+\left(12b^2\right)\int\frac{\frac{d}{21}+\frac{fx^2}{5}}{\sqrt{a+bx^4}}\,dx$$
$$+\frac{1}{16}(3bc)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)+\left(b^2e\right)\int\frac{x}{\sqrt{a+bx^4}}\,dx$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}$$
$$-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\left(a+bx^4\right)^{3/2}$$
$$-\frac{3b^2c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}+\frac{1}{2}\left(b^2e\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)$$
$$-\frac{1}{5}\left(12\sqrt{a}b^{3/2}f\right)\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx+\frac{1}{35}\left(4b^2\left(5d+\frac{21\sqrt{a}f}{\sqrt{b}}\right)\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}+\frac{12b^{3/2}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\left(a+bx^4\right)^{3/2}-\frac{3b^2c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}$$
$$-\frac{12\sqrt[4]{a}b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$+\frac{2b^{5/4}\left(5\sqrt{b}d+21\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}}$$
$$+\frac{1}{2}\left(b^2e\right)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}+\frac{12b^{3/2}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\left(a+bx^4\right)^{3/2}$$
$$+\frac{1}{2}b^{3/2}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{3b^2c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{12\sqrt[4]{a}b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\Big(}{5\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.46

$$\int\frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^9}\,dx=$$
$$-\frac{\sqrt{a+bx^4}\Big(240a^2dx\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},-\frac{3}{4},-\frac{bx^4}{a}\right)+7\Big(15c\Big(a(2a+5bx^4)\sqrt{1+\frac{bx^4}{a}}+3b^2x^8\text{arct}}{}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^9,x]
```

```
[Out] -1/1680*(Sqrt[a + b*x^4]*(240*a^2*d*x*Hypergeometric2F1[-7/4, -3/2, -3/4, -
((b*x^4)/a)] + 7*(15*c*(a*(2*a + 5*b*x^4)*Sqrt[1 + (b*x^4)/a] + 3*b^2*x^8*A
rcTanh[Sqrt[1 + (b*x^4)/a]]) + 40*a^2*e*x^2*Hypergeometric2F1[-3/2, -3/2, -
1/2, -((b*x^4)/a)] + 48*a^2*f*x^3*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*
x^4)/a)])))/(a*x^8*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(2352bf\,x^{7}+1120be\,x^{6}+720bd\,x^{5}+525bc\,x^{4}+336af\,x^{3}+280ae\,x^{2}+240adx+210ac)}{1680x^{8}}+\frac{b^{2}\left(\frac{160d\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right.}{}$
elliptic	$-\frac{ac\sqrt{b\,x^{4}+a}}{8x^{8}}-\frac{ad\sqrt{b\,x^{4}+a}}{7x^{7}}-\frac{ae\sqrt{b\,x^{4}+a}}{6x^{6}}-\frac{af\sqrt{b\,x^{4}+a}}{5x^{5}}-\frac{5bc\sqrt{b\,x^{4}+a}}{16x^{4}}-\frac{3bd\sqrt{b\,x^{4}+a}}{7x^{3}}-\frac{2be\sqrt{b\,x^{4}+a}}{3x^{2}}-\frac{7bf\sqrt{b\,x^{4}+a}}{5x}$
default	$e\left(\frac{b^{\frac{3}{2}}\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{2}-\frac{a\sqrt{b\,x^{4}+a}}{6x^{6}}-\frac{2b\sqrt{b\,x^{4}+a}}{3x^{2}}\right)+d\left(-\frac{a\sqrt{b\,x^{4}+a}}{7x^{7}}-\frac{3b\sqrt{b\,x^{4}+a}}{7x^{3}}+\frac{4b^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+}}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1680*(b*x^4+a)^(1/2)*(2352*b*f*x^7+1120*b*e*x^6+720*b*d*x^5+525*b*c*x^4+
336*a*f*x^3+280*a*e*x^2+240*a*d*x+210*a*c)/x^8+1/280*b^2*(160*d/(I/a^(1/2)*
b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1
/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+672*I*f*a^(1/2
)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+140*e*ln(x^2*b^(1/2)+(b*x
^4+a)^(1/2))/b^(1/2)-105/2*c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2
))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^9}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^9}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^9, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.50 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.18

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}be}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bf\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^2c}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{3a\sqrt{b}c}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}c}{16x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^4}+1}}{6} + \frac{b^{\frac{3}{2}}e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} - \frac{3b^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}} - \frac{b^2ex^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**9,x)
```

```
[Out] a**(3/2)*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**7*gamma(-3/4)) + a**(3/2)*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*b*d*gamma(-3/4)
*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4))
- sqrt(a)*b*e/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*f*gamma(-1/4)*hyper((
-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - a**2*c/(8
*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*a*sqrt(b)*c/(16*x**6*sqrt(a/(b*x**
4) + 1)) - a*sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*c*sqrt(a/(b
*x**4) + 1)/(4*x**2) - b**(3/2)*c/(16*x**2*sqrt(a/(b*x**4) + 1)) - b**(3/2)
*e*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*e*asinh(sqrt(b)*x**2/sqrt(a))/2 - 3*b*
*2*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a)) - b**2*e*x**2/(2*sqrt(a)*sq
rt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^9}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/32*(3*b^2*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/sq
rt(a) - 2*(5*(b*x^4 + a)^(3/2)*b^2 - 3*sqrt(b*x^4 + a)*a*b^2)/((b*x^4 + a)^
2 - 2*(b*x^4 + a)*a + a^2))*c + integrate((b*f*x^6 + b*e*x^5 + b*d*x^4 + a*
f*x^2 + a*e*x + a*d)*sqrt(b*x^4 + a)/x^8, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^9}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^9}\,dx$$

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^9,x)
```

```
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^9, x)
```

3.524 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}} dx$

Optimal result

Integrand size = 30, antiderivative size = 405

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}} dx = -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}$$
$$-\frac{4b^2c\sqrt{a+bx^4}}{15ax}+\frac{4b^{5/2}cx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)(a+bx^4)^{3/2}$$
$$+\frac{1}{2}b^{3/2}f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{3b^2d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{4b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\ldots\right.}{15a^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -1/504*(56*c/x^9+63*d/x^8+72*e/x^7+84*f/x^6)*(b*x^4+a)^(3/2)+1/2*b^(3/2)*f*
arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))-3/16*b^2*d*arctanh((b*x^4+a)^(1/2)/a^(
1/2))/a^(1/2)-1/1680*b*(224*c/x^5+315*d/x^4+480*e/x^3+840*f/x^2)*(b*x^4+a)^
(1/2)-4/15*b^2*c*(b*x^4+a)^(1/2)/a/x+4/15*b^(5/2)*c*x*(b*x^4+a)^(1/2)/a/(a^
(1/2)+x^2*b^(1/2))-4/15*b^(9/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4)
)),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(
1/2)/a^(3/4)/(b*x^4+a)^(1/2)+2/105*b^(7/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*
x/a^(1/4))),1/2*2^(1/2))*(15*e*a^(1/2)+7*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*(
(b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {14, 1839, 1847, 1296, 1212, 226, 1210, 1266, 858, 223, 212, 272, 65, 214}

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx = \frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{a}e+7\sqrt{b}c\right)\text{EllipticF}\left(2\,\text{a}\right.}{105a^{3/4}\sqrt{a+bx^4}}$$
$$-\frac{4b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{1}{2}b^{3/2}f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{3b^2d\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}+\frac{4b^{5/2}cx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{4b^2c\sqrt{a+bx^4}}{15ax}-\frac{b\sqrt{a+bx^4}\left(\frac{224}{x}\right.}{}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^10,x]
```

```
[Out] -1/1680*(b*((224*c)/x^5 + (315*d)/x^4 + (480*e)/x^3 + (840*f)/x^2)*Sqrt[a +
 b*x^4]) - (4*b^2*c*Sqrt[a + b*x^4])/(15*a*x) + (4*b^(5/2)*c*x*Sqrt[a + b*x
^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) - (((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^
7 + (84*f)/x^6)*(a + b*x^4)^(3/2))/504 + (b^(3/2)*f*ArcTanh[(Sqrt[b]*x^2)/S
qrt[a + b*x^4]])/2 - (3*b^2*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]
) - (4*b^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b
]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a
 + b*x^4]) + (2*b^(7/4)*(7*Sqrt[b]*c + 15*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(105*a^(3/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

```
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\left(a+bx^4\right)^{3/2} \\
&\quad -(6b)\int\frac{\left(-\frac{c}{9}-\frac{dx}{8}-\frac{ex^2}{7}-\frac{fx^3}{6}\right)\sqrt{a+bx^4}}{x^6}\,dx \\
&= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680} \\
&\quad -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\left(a+bx^4\right)^{3/2}+\left(12b^2\right)\int\frac{\frac{c}{45}+\frac{dx}{32}+\frac{ex^2}{21}+\frac{fx^3}{12}}{x^2\sqrt{a+bx^4}}\,dx
\end{aligned}
$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680} - \frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\left(a+bx^4\right)^{3/2} + \left(12b^2\right)\int\left(\frac{\frac{c}{45}+\frac{ex^2}{21}}{x^2\sqrt{a+bx^4}}+\frac{\frac{d}{32}+\frac{fx^2}{12}}{x\sqrt{a+bx^4}}\right)dx$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680} - \frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\left(a+bx^4\right)^{3/2} + \left(12b^2\right)\int\frac{\frac{c}{45}+\frac{ex^2}{21}}{x^2\sqrt{a+bx^4}}dx + \left(12b^2\right)\int\frac{\frac{d}{32}+\frac{fx^2}{12}}{x\sqrt{a+bx^4}}dx$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680} - \frac{4b^2c\sqrt{a+bx^4}}{15ax} - \frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\left(a+bx^4\right)^{3/2} + \left(6b^2\right)\text{Subst}\left(\int\frac{\frac{d}{32}+\frac{fx}{12}}{x\sqrt{a+bx^2}}dx,x,x^2\right) - \frac{\left(12b^2\right)\int\frac{-\frac{ae}{21}-\frac{1}{45}bcx^2}{\sqrt{a+bx^4}}dx}{a}$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680} - \frac{4b^2c\sqrt{a+bx^4}}{15ax} - \frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\left(a+bx^4\right)^{3/2} - \frac{\left(4b^{5/2}c\right)\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}dx}{15\sqrt{a}} + \frac{1}{16}\left(3b^2d\right)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx^2}}dx,x,x^2\right) + \frac{1}{105}\left(4b^2\left(\frac{7\sqrt{b}c}{\sqrt{a}}+15e\right)\right)\int\frac{1}{\sqrt{a+bx^4}}dx + \frac{1}{2}\left(b^2f\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}dx,x,x^2\right)$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680} - \frac{4b^2c\sqrt{a+bx^4}}{15ax} + \frac{4b^{5/2}cx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\left(a+bx^4\right)^{3/2} - \frac{4b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} + \frac{2b^{7/4}\left(7\sqrt{b}c+15\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}} + \frac{1}{32}\left(3b^2d\right)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}dx,x,x^4\right) + \frac{1}{2}\left(b^2f\right)\text{Subst}\left(\int\frac{1}{1-bx^2}dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)$$

$$= -\frac{b\left(\frac{224c}{x^5} + \frac{315d}{x^4} + \frac{480e}{x^3} + \frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680} - \frac{4b^2c\sqrt{a+bx^4}}{15ax} + \frac{4b^{5/2}cx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{1}{504}\left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6}\right)\left(a+bx^4\right)^{3/2} + \frac{1}{2}b^{3/2}f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{4b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} + \frac{2b^{7/}}{}$$

$$= -\frac{b\left(\frac{224c}{x^5} + \frac{315d}{x^4} + \frac{480e}{x^3} + \frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680} - \frac{4b^2c\sqrt{a+bx^4}}{15ax} + \frac{4b^{5/2}cx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{1}{504}\left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6}\right)\left(a+bx^4\right)^{3/2} + \frac{1}{2}b^{3/2}f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3b^2d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}} - \frac{4b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\,\mathrm{t}\right.}{15a^{3/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.43

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}}{x^{10}}\,dx =$$

$$-\frac{\sqrt{a+bx^4}\left(112a^2c\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{3}{2},-\frac{5}{4},-\frac{bx^4}{a}\right)+3x\left(48a^2ex\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},\right.\right.\right.}{}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^10,x]
```

```
[Out] -1/1008*(Sqrt[a + b*x^4]*(112*a^2*c*Hypergeometric2F1[-9/4, -3/2, -5/4, -((
b*x^4)/a)] + 3*x*(48*a^2*e*x*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^4)/
a)] + 7*(3*a*d*(2*a + 5*b*x^4)*Sqrt[1 + (b*x^4)/a] + 9*b^2*d*x^8*ArcTanh[Sq
rt[1 + (b*x^4)/a]] + 8*a^2*f*x^2*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x
^4)/a)]))))/(a*x^9*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(1344 b^{2} c\,x^{8}+3360 a b f\,x^{7}+2160 a e b\,x^{6}+1575 x^{5} d b a+1232 a b c\,x^{4}+840 a^{2} f\,x^{3}+720 a^{2} e\,x^{2}+630 a^{2} d x+560 a^{2} c\right)}{5040 x^{9} a}+\frac{b^{2}\left(\frac{480 a e}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{a c\sqrt{b\,x^{4}+a}}{9 x^{9}}-\frac{a d\sqrt{b\,x^{4}+a}}{8 x^{8}}-\frac{a e\sqrt{b\,x^{4}+a}}{7 x^{7}}-\frac{a f\sqrt{b\,x^{4}+a}}{6 x^{6}}-\frac{11 b c\sqrt{b\,x^{4}+a}}{45 x^{5}}-\frac{5 b d\sqrt{b\,x^{4}+a}}{16 x^{4}}-\frac{3 b e\sqrt{b\,x^{4}+a}}{7 x^{3}}-\frac{2 b f\sqrt{b\,x^{4}+a}}{3 x^{2}}-$
default	$f\left(\frac{b^{\frac{3}{2}}\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{2}-\frac{a\sqrt{b\,x^{4}+a}}{6 x^{6}}-\frac{2 b\sqrt{b\,x^{4}+a}}{3 x^{2}}\right)+e\left(-\frac{a\sqrt{b\,x^{4}+a}}{7 x^{7}}-\frac{3 b\sqrt{b\,x^{4}+a}}{7 x^{3}}+\frac{4 b^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5040*(b*x^4+a)^(1/2)*(1344*b^2*c*x^8+3360*a*b*f*x^7+2160*a*b*e*x^6+1575*
a*b*d*x^5+1232*a*b*c*x^4+840*a^2*f*x^3+720*a^2*e*x^2+630*a^2*d*x+560*a^2*c)
/x^9/a+1/840/a*b^2*(480*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a
^(1/2)*b^(1/2))^(1/2),I)+224*I*b^(1/2)*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(
1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/
2))^(1/2),I))+420*a*f*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)-315/2*a^(1/2)
*d*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{10}}\,dx=\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^{10}}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^10, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.79 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{10}}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}bf}{2x^2\sqrt{1+\frac{bx^4}{a}}} - \frac{a^2d}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{3a\sqrt{b}d}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}d}{16x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{6} + \frac{b^{\frac{3}{2}}f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} - \frac{3b^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}} - \frac{b^2fx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**10,x)
```

```
[Out] a**(3/2)*c*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**9*gamma(-5/4)) + a**(3/2)*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*c*gamma(-5/4)
*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)
) + sqrt(a)*b*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*
pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*f/(2*x**2*sqrt(1 + b*x**4/a)) - a**2
*d/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*a*sqrt(b)*d/(16*x**6*sqrt(a/(
b*x**4) + 1)) - a*sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*d*sqrt
(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*d/(16*x**2*sqrt(a/(b*x**4) + 1)) - b**
(3/2)*f*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*f*asinh(sqrt(b)*x**2/sqrt(a))/2 -
 3*b**2*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a)) - b**2*f*x**2/(2*sqrt(
a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^{10}}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

Giac [F]

$$\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^{10}}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(b\,x^4 + a)^{3/2}\,(f\,x^3 + e\,x^2 + d\,x + c)}{x^{10}}\,dx$$

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^10,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^10, x)
```

3.525 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 399

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx = -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{b^2c\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2d\sqrt{a+bx^4}}{15ax} + \frac{4b^{5/2}dx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)(a+bx^4)^{3/2}}{2520} - \frac{3b^2e\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}} - \frac{4b^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} + \frac{2b^{7/4}\left(7\sqrt{b}d+15\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -1/2520*(252*c/x^10+280*d/x^9+315*e/x^8+360*f/x^7)*(b*x^4+a)^(3/2)-3/16*b^2
*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/1680*b*(168*c/x^6+224*d/x^5+3
15*e/x^4+480*f/x^3)*(b*x^4+a)^(1/2)-1/10*b^2*c*(b*x^4+a)^(1/2)/a/x^2-4/15*b
^2*d*(b*x^4+a)^(1/2)/a/x+4/15*b^(5/2)*d*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^
(1/2))-4/15*b^(9/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)
/(b*x^4+a)^(1/2)+2/105*b^(7/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),
1/2*2^(1/2))*(15*f*a^(1/2)+7*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a
^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {14, 1839, 1847, 1266, 821, 272, 65, 214, 1296, 1212, 226, 1210}

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{11}}\,dx = \frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{a}f+7\sqrt{b}d\right)\text{EllipticF}\left(2\,\text{ar}\right.}{105a^{3/4}\sqrt{a+bx^4}}$$
$$-\frac{4b^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}$$
$$-\frac{3b^2e\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}+\frac{4b^{5/2}dx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{b^2c\sqrt{a+bx^4}}{10ax^2}-\frac{4b^2d\sqrt{a+bx^4}}{15ax}$$
$$-\frac{b\sqrt{a+bx^4}\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)}{1680}-\frac{(a+bx^4)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^11,x]
```

```
[Out] -1/1680*(b*((168*c)/x^6 + (224*d)/x^5 + (315*e)/x^4 + (480*f)/x^3)*Sqrt[a +
 b*x^4]) - (b^2*c*Sqrt[a + b*x^4])/(10*a*x^2) - (4*b^2*d*Sqrt[a + b*x^4])/(
15*a*x) + (4*b^(5/2)*d*x*Sqrt[a + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) -
(((252*c)/x^10 + (280*d)/x^9 + (315*e)/x^8 + (360*f)/x^7)*(a + b*x^4)^(3/2)
)/2520 - (3*b^2*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (4*b^(9/
4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4]) +
 (2*b^(7/4)*(7*Sqrt[b]*d + 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
1/2])/(105*a^(3/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)\left(a+bx^4\right)^{3/2}}{2520} \\
&\quad -(6b)\int \frac{\left(-\frac{c}{10}-\frac{dx}{9}-\frac{ex^2}{8}-\frac{fx^3}{7}\right)\sqrt{a+bx^4}}{x^7}\,dx \\
&= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} \\
&\quad -\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)\left(a+bx^4\right)^{3/2}}{2520}+\left(12b^2\right)\int \frac{\frac{c}{60}+\frac{dx}{45}+\frac{ex^2}{32}+\frac{fx^3}{21}}{x^3\sqrt{a+bx^4}}\,dx \\
&= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680}-\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)\left(a+bx^4\right)^{3/2}}{2520} \\
&\quad +\left(12b^2\right)\int \left(\frac{\frac{c}{60}+\frac{ex^2}{32}}{x^3\sqrt{a+bx^4}}+\frac{\frac{d}{45}+\frac{fx^2}{21}}{x^2\sqrt{a+bx^4}}\right)dx \\
&= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680}-\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)\left(a+bx^4\right)^{3/2}}{2520} \\
&\quad +\left(12b^2\right)\int \frac{\frac{c}{60}+\frac{ex^2}{32}}{x^3\sqrt{a+bx^4}}\,dx+\left(12b^2\right)\int \frac{\frac{d}{45}+\frac{fx^2}{21}}{x^2\sqrt{a+bx^4}}\,dx
\end{aligned}$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680}-\frac{4b^2d\sqrt{a+bx^4}}{15ax}$$
$$-\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)\left(a+bx^4\right)^{3/2}}{2520}$$
$$+\left(6b^2\right)\text{Subst}\left(\int\frac{\frac{c}{60}+\frac{ex}{32}}{x^2\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\left(12b^2\right)\int\frac{-\frac{af}{21}-\frac{1}{45}bdx^2}{\sqrt{a+bx^4}}\,dx}{a}$$
$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680}-\frac{b^2c\sqrt{a+bx^4}}{10ax^2}$$
$$-\frac{4b^2d\sqrt{a+bx^4}}{15ax}-\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)\left(a+bx^4\right)^{3/2}}{2520}$$
$$-\frac{\left(4b^{5/2}d\right)\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{15\sqrt{a}}+\frac{1}{16}\left(3b^2e\right)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)$$
$$+\frac{1}{105}\left(4b^2\left(\frac{7\sqrt{b}d}{\sqrt{a}}+15f\right)\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx$$
$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680}-\frac{b^2c\sqrt{a+bx^4}}{10ax^2}-\frac{4b^2d\sqrt{a+bx^4}}{15ax}$$
$$+\frac{4b^{5/2}dx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)\left(a+bx^4\right)^{3/2}}{2520}$$
$$-\frac{4b^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{2b^{7/4}\left(7\sqrt{b}d+15\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{1}{32}\left(3b^2e\right)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{b^2c\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2d\sqrt{a+bx^4}}{15ax} + \frac{4b^{5/2}dx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)(a+bx^4)^{3/2}}{2520} - \frac{4b^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} + \frac{2b^{7/4}\left(7\sqrt{b}d+15\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}} + \frac{1}{16}(3be)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{b^2c\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2d\sqrt{a+bx^4}}{15ax} + \frac{4b^{5/2}dx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)(a+bx^4)^{3/2}}{2520} - \frac{3b^2e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}} - \frac{4b^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} + \frac{2b^{7/4}\left(7\sqrt{b}d+15\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.34 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.43

$$\int\frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx =$$

$$-\frac{\sqrt{a+bx^4}\left(63\sqrt{1+\frac{bx^4}{a}}(8b^2cx^8+2a^2(4c+5ex^2)+abx^4(16c+25ex^2))+945b^2ex^{10}\text{arctanh}\left(\sqrt{1+\frac{bx^4}{a}}\right)+\right.}{5040ax^{10}}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^11,x]
```

```
[Out] -1/5040*(Sqrt[a + b*x^4]*(63*Sqrt[1 + (b*x^4)/a]*(8*b^2*c*x^8 + 2*a^2*(4*c
+ 5*e*x^2) + a*b*x^4*(16*c + 25*e*x^2)) + 945*b^2*e*x^10*ArcTanh[Sqrt[1 + (
b*x^4)/a]] + 560*a^2*d*x*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^4)/a)]
+ 720*a^2*f*x^3*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^4)/a)]))/(a*x^10
*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(1344b^2d\,x^9+504b^2c\,x^8+2160abf\,x^7+1575aeb\,x^6+1232x^5dba+1008abc\,x^4+720a^2f\,x^3+630a^2e\,x^2+560a^2dx+504a^2c)}{5040x^{10}a}$
default	$f\left(-\frac{a\sqrt{b\,x^4+a}}{7x^7}-\frac{3b\sqrt{b\,x^4+a}}{7x^3}+\frac{4b^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+e\left(-\frac{3b^2\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{16\sqrt{a}}-\frac{a\sqrt{b\,x^4}}{8x^8}\right.$
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{10x^{10}}-\frac{ad\sqrt{b\,x^4+a}}{9x^9}-\frac{ae\sqrt{b\,x^4+a}}{8x^8}-\frac{af\sqrt{b\,x^4+a}}{7x^7}-\frac{bc\sqrt{b\,x^4+a}}{5x^6}-\frac{11bd\sqrt{b\,x^4+a}}{45x^5}-\frac{5be\sqrt{b\,x^4+a}}{16x^4}-\frac{3bf\sqrt{b\,x^4+a}}{7x^3}$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5040*(b*x^4+a)^(1/2)*(1344*b^2*d*x^9+504*b^2*c*x^8+2160*a*b*f*x^7+1575*a
*b*e*x^6+1232*a*b*d*x^5+1008*a*b*c*x^4+720*a^2*f*x^3+630*a^2*e*x^2+560*a^2*
d*x+504*a^2*c)/x^10/a+1/840/a*b^2*(480*a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a
^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*E
llipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+224*I*b^(1/2)*d*a^(1/2)/(I/a^(1/2)*
b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1
/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(
I/a^(1/2)*b^(1/2))^(1/2),I))-315/2*a^(1/2)*e*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1
/2))/x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.54

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{11}}\,dx =$$

$$-\frac{2688\,\sqrt{a}b^2dx^{10}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)-945\,\sqrt{a}b^2ex^{10}\log\left(-\frac{bx^4-2\,\sqrt{bx^4+a}\sqrt{a}+2\,a}{x^4}\right)-384\,(7\,b^2d-}{}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/10080*(2688*sqrt(a)*b^2*d*x^10*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(
1/4)), -1) - 945*sqrt(a)*b^2*e*x^10*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a)
 + 2*a)/x^4) - 384*(7*b^2*d - 15*a*b*f)*sqrt(a)*x^10*(-b/a)^(3/4)*elliptic_
f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(1344*b^2*d*x^9 + 504*b^2*c*x^8 + 2160*a*
b*f*x^7 + 1575*a*b*e*x^6 + 1232*a*b*d*x^5 + 1008*a*b*c*x^4 + 720*a^2*f*x^3
+ 630*a^2*e*x^2 + 560*a^2*d*x + 504*a^2*c)*sqrt(b*x^4 + a))/(a*x^10)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.67 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}bf\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{a^2e}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{10x^8} - \frac{3a\sqrt{b}e}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{5x^4} - \frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}e}{16x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{5}{2}}c\sqrt{\frac{a}{bx^4}+1}}{10a} - \frac{3b^2e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**11,x)
```

```
[Out] a**(3/2)*d*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**9*gamma(-5/4)) + a**(3/2)*f*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*d*gamma(-5/4)
*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)
) + sqrt(a)*b*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*
pi)/a)/(4*x**3*gamma(1/4)) - a**2*e/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1))
- a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(10*x**8) - 3*a*sqrt(b)*e/(16*x**6*sqrt(
a/(b*x**4) + 1)) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(3/2)*e*sq
rt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*e/(16*x**2*sqrt(a/(b*x**4) + 1)) - b
**(5/2)*c*sqrt(a/(b*x**4) + 1)/(10*a) - 3*b**2*e*asinh(sqrt(a)/(sqrt(b)*x**
2))/(16*sqrt(a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{11}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{11}}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="maxima")
[Out] -1/10*(b*x^4 + a)^(5/2)*c/(a*x^10) + integrate((b*f*x^6 + b*e*x^5 + b*d*x^4
 + a*f*x^2 + a*e*x + a*d)*sqrt(b*x^4 + a)/x^10, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{11}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{11}}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{11}}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^{11}}\,dx$$

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^11,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^11, x)
```

3.526 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}} dx$

Optimal result

Integrand size = 30, antiderivative size = 424

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}} dx = -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \frac{b^2d\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2e\sqrt{a+bx^4}}{15ax} + \frac{4b^{5/2}ex\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)(a+bx^4)^{3/2}}{3960} - \frac{3b^2f\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}} - \frac{4b^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} - \frac{2b^{9/4}\left(15\sqrt{b}c-77\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}$$

```
[Out] -1/3960*(360*c/x^11+396*d/x^10+440*e/x^9+495*f/x^8)*(b*x^4+a)^(3/2)-3/16*b^
2*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/18480*b*(1440*c/x^7+1848*d/x
^6+2464*e/x^5+3465*f/x^4)*(b*x^4+a)^(1/2)-4/77*b^2*c*(b*x^4+a)^(1/2)/a/x^3-
1/10*b^2*d*(b*x^4+a)^(1/2)/a/x^2-4/15*b^2*e*(b*x^4+a)^(1/2)/a/x+4/15*b^(5/2
)*e*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-4/15*b^(9/4)*e*(cos(2*arctan(
b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin
(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)
/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)-2/1155*b^(9/4)*(cos
(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ell
ipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-77*e*a^(1/2)+15*c*b^
```

```
(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5
/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {14, 1839, 1847, 1296, 1212, 226, 1210, 1266, 821, 272, 65, 214}

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx = -\frac{2b^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}c-77\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}} - \frac{4b^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} - \frac{3b^2f\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}} + \frac{4b^{5/2}ex\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \frac{b^2d\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2e\sqrt{a+bx^4}}{15ax} - \frac{b\sqrt{a+bx^4}\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)}{18480} - \frac{(a+bx^4)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^12,x]
```

```
[Out] -1/18480*(b*((1440*c)/x^7 + (1848*d)/x^6 + (2464*e)/x^5 + (3465*f)/x^4)*Sqr
t[a + b*x^4]) - (4*b^2*c*Sqrt[a + b*x^4])/(77*a*x^3) - (b^2*d*Sqrt[a + b*x^
4])/(10*a*x^2) - (4*b^2*e*Sqrt[a + b*x^4])/(15*a*x) + (4*b^(5/2)*e*x*Sqrt[a
 + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) - (((360*c)/x^11 + (396*d)/x^10 +
 (440*e)/x^9 + (495*f)/x^8)*(a + b*x^4)^(3/2))/3960 - (3*b^2*f*ArcTanh[Sqrt
[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (4*b^(9/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4]) - (2*b^(9/4)*(15*Sqrt[b]*c - 77
*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(1155*a^(5/4)*Sqrt[a +
b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)\left(a+bx^4\right)^{3/2}}{3960} - (6b)\int \frac{\left(-\frac{c}{11}-\frac{dx}{10}-\frac{ex^2}{9}-\frac{fx^3}{8}\right)\sqrt{a+bx^4}}{x^8}\,dx \\
&= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)\left(a+bx^4\right)^{3/2}}{3960} + \left(12b^2\right)\int \frac{\frac{c}{77}+\frac{dx}{60}+\frac{ex^2}{45}+\frac{fx^3}{32}}{x^4\sqrt{a+bx^4}}\,dx
\end{aligned}$$

$$= -\frac{b\left(\frac{1440c}{x^7} + \frac{1848d}{x^6} + \frac{2464e}{x^5} + \frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{360c}{x^{11}} + \frac{396d}{x^{10}} + \frac{440e}{x^9} + \frac{495f}{x^8}\right)(a+bx^4)^{3/2}}{3960} + (12b^2)\int \left(\frac{\frac{c}{77} + \frac{ex^2}{45}}{x^4\sqrt{a+bx^4}} + \frac{\frac{d}{60} + \frac{fx^2}{32}}{x^3\sqrt{a+bx^4}}\right) dx$$

$$= -\frac{b\left(\frac{1440c}{x^7} + \frac{1848d}{x^6} + \frac{2464e}{x^5} + \frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{360c}{x^{11}} + \frac{396d}{x^{10}} + \frac{440e}{x^9} + \frac{495f}{x^8}\right)(a+bx^4)^{3/2}}{3960} + (12b^2)\int \frac{\frac{c}{77} + \frac{ex^2}{45}}{x^4\sqrt{a+bx^4}}\,dx + (12b^2)\int \frac{\frac{d}{60} + \frac{fx^2}{32}}{x^3\sqrt{a+bx^4}}\,dx$$

$$= -\frac{b\left(\frac{1440c}{x^7} + \frac{1848d}{x^6} + \frac{2464e}{x^5} + \frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \frac{\left(\frac{360c}{x^{11}} + \frac{396d}{x^{10}} + \frac{440e}{x^9} + \frac{495f}{x^8}\right)(a+bx^4)^{3/2}}{3960} + (6b^2)\,\text{Subst}\left(\int \frac{\frac{d}{60} + \frac{fx}{32}}{x^2\sqrt{a+bx^2}}\,dx, x, x^2\right) - \frac{(4b^2)\int \frac{-\frac{ae}{15} + \frac{1}{77}bcx^2}{x^2\sqrt{a+bx^4}}\,dx}{a}$$

$$= -\frac{b\left(\frac{1440c}{x^7} + \frac{1848d}{x^6} + \frac{2464e}{x^5} + \frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \frac{b^2d\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2e\sqrt{a+bx^4}}{15ax} - \frac{\left(\frac{360c}{x^{11}} + \frac{396d}{x^{10}} + \frac{440e}{x^9} + \frac{495f}{x^8}\right)(a+bx^4)^{3/2}}{3960} + \frac{(4b^2)\int \frac{-\frac{1}{77}abc + \frac{1}{15}abex^2}{\sqrt{a+bx^4}}\,dx}{a^2} + \frac{1}{16}(3b^2f)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, x^2\right)$$

$$= -\frac{b\left(\frac{1440c}{x^7} + \frac{1848d}{x^6} + \frac{2464e}{x^5} + \frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \frac{b^2d\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2e\sqrt{a+bx^4}}{15ax} - \frac{\left(\frac{360c}{x^{11}} + \frac{396d}{x^{10}} + \frac{440e}{x^9} + \frac{495f}{x^8}\right)(a+bx^4)^{3/2}}{3960} - \frac{(4b^{5/2}e)\int \frac{1 - \frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{15\sqrt{a}} - \frac{\left(4b^{5/2}\left(15\sqrt{b}c - 77\sqrt{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{1155a} + \frac{1}{32}(3b^2f)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^4\right)$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \frac{b^2d\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2e\sqrt{a+bx^4}}{15ax} + \frac{4b^{5/2}ex\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)(a+bx^4)^{3/2}}{3960} - \frac{4b^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} - \frac{2b^{9/4}\left(15\sqrt{b}c-77\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}} + \frac{1}{16}(3bf)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \frac{b^2d\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2e\sqrt{a+bx^4}}{15ax} + \frac{4b^{5/2}ex\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)(a+bx^4)^{3/2}}{3960} - \frac{3b^2f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}} - \frac{4b^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} - \frac{2b^{9/4}\left(15\sqrt{b}c-77\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.36 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.41

$$\int\frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx = -\frac{\sqrt{a+bx^4}\left(720a^2c\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{3}{2},-\frac{7}{4},-\frac{bx^4}{a}\right)+11x\left(9\sqrt{1+\frac{bx^4}{a}}(8b^2dx^8+2a^2(4d+5fx^2)\right.\right.}{7920a}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^12,x]
```

[Out] -1/7920*(Sqrt[a + b*x^4]*(720*a^2*c*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*x^4)/a)] + 11*x*(9*Sqrt[1 + (b*x^4)/a]*(8*b^2*d*x^8 + 2*a^2*(4*d + 5*f*x^2) + a*b*x^4*(16*d + 25*f*x^2)) + 135*b^2*f*x^10*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 80*a^2*e*x*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^4)/a)])))/(a*x^11*Sqrt[1 + (b*x^4)/a])

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.20 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(14784 b^2 e\,x^{10}+5544 b^2 d\,x^9+2880 b^2 c\,x^8+17325 a b f\,x^7+13552 a e b\,x^6+11088 x^5 d b a+9360 a b c\,x^4+6930 a^2 f\,x^3+6160 a^2 e\,x^2+}{55440 x^{11} a}$
default	$f\left(-\frac{3 b^2 \ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{16\sqrt{a}}-\frac{a\sqrt{b\,x^4+a}}{8x^8}-\frac{5b\sqrt{b\,x^4+a}}{16x^4}\right)-\frac{d\left(b^2 x^8+2ab\,x^4+a^2\right)\sqrt{b\,x^4+a}}{10x^{10}a}+c\left(-\frac{a\sqrt{b\,x^4+a}}{11x^{11}}-\frac{13b\sqrt{b}}{77x}\right.$
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{11x^{11}}-\frac{ad\sqrt{b\,x^4+a}}{10x^{10}}-\frac{ae\sqrt{b\,x^4+a}}{9x^9}-\frac{af\sqrt{b\,x^4+a}}{8x^8}-\frac{13bc\sqrt{b\,x^4+a}}{77x^7}-\frac{bd\sqrt{b\,x^4+a}}{5x^6}-\frac{11be\sqrt{b\,x^4+a}}{45x^5}-\frac{5bf\sqrt{b\,x^4+a}}{16x^4}-$

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x,method=_RETURNVERBOSE)

[Out] -1/55440*(b*x^4+a)^(1/2)*(14784*b^2*e*x^10+5544*b^2*d*x^9+2880*b^2*c*x^8+17325*a*b*f*x^7+13552*a*b*e*x^6+11088*a*b*d*x^5+9360*a*b*c*x^4+6930*a^2*f*x^3+6160*a^2*e*x^2+5544*a^2*d*x+5040*a^2*c)/x^11/a+1/9240/a*b^2*(-480*b*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+2464*I*b^(1/2)*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3465/2*a^(1/2)*f*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.54

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{12}}\,dx =$$

$$-\frac{29568\,\sqrt{a}b^2ex^{11}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)-10395\,\sqrt{a}b^2fx^{11}\log\left(-\frac{bx^4-2\,\sqrt{bx^4+a}\sqrt{a}+2\,a}{x^4}\right)-384\,(15\,b^2}{}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/110880*(29568*sqrt(a)*b^2*e*x^11*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)
^(1/4)), -1) - 10395*sqrt(a)*b^2*f*x^11*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqr
t(a) + 2*a)/x^4) - 384*(15*b^2*c + 77*b^2*e)*sqrt(a)*x^11*(-b/a)^(3/4)*elli
ptic_f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(14784*b^2*e*x^10 + 5544*b^2*d*x^9 +
 2880*b^2*c*x^8 + 17325*a*b*f*x^7 + 13552*a*b*e*x^6 + 11088*a*b*d*x^5 + 936
0*a*b*c*x^4 + 6930*a^2*f*x^3 + 6160*a^2*e*x^2 + 5544*a^2*d*x + 5040*a^2*c)*
sqrt(b*x^4 + a))/(a*x^11)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.99 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{9}\Gamma\left(-\frac{5}{4}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{7}\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{5}\Gamma\left(-\frac{1}{4}\right)} - \frac{a^2f}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{10x^8} - \frac{3a\sqrt{b}f}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{5x^4} - \frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}f}{16x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{5}{2}}d\sqrt{\frac{a}{bx^4}+1}}{10a} - \frac{3b^2f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**12,x)
```

```
[Out] a**(3/2)*c*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**4*exp_polar(I*pi
)/a)/(4*x**11*gamma(-7/4)) + a**(3/2)*e*gamma(-9/4)*hyper((-9/4, -1/2), (-5
/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*b*c*gamma(-7
/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3
/4)) + sqrt(a)*b*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*x**5*gamma(-1/4)) - a**2*f/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) +
 1)) - a*sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(10*x**8) - 3*a*sqrt(b)*f/(16*x**6*
sqrt(a/(b*x**4) + 1)) - b**(3/2)*d*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(3/2)
```

```
*f*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*f/(16*x**2*sqrt(a/(b*x**4) + 1)
) - b**(5/2)*d*sqrt(a/(b*x**4) + 1)/(10*a) - 3*b**2*f*asinh(sqrt(a)/(sqrt(b
)*x**2))/(16*sqrt(a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{12}}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^12, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{12}}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^12, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^{12}}\,dx$$

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^12,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^12, x)
```

3.527 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 449

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx = -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a+bx^4}}{77ax^3} - \frac{b^2e\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2f\sqrt{a+bx^4}}{15ax} + \frac{4b^{5/2}fx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)(a+bx^4)^{3/2}}{1980} + \frac{b^3c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}} - \frac{4b^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} - \frac{2b^{9/4}\left(15\sqrt{b}d-77\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}$$

```
[Out] -1/1980*(165*c/x^12+180*d/x^11+198*e/x^10+220*f/x^9)*(b*x^4+a)^(3/2)+1/32*b
^3*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/18480*b*(1155*c/x^8+1440*d/
x^7+1848*e/x^6+2464*f/x^5)*(b*x^4+a)^(1/2)-1/32*b^2*c*(b*x^4+a)^(1/2)/a/x^4
-4/77*b^2*d*(b*x^4+a)^(1/2)/a/x^3-1/10*b^2*e*(b*x^4+a)^(1/2)/a/x^2-4/15*b^2
*f*(b*x^4+a)^(1/2)/a/x+4/15*b^(5/2)*f*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1
/2))-4/15*b^(9/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(
b*x^4+a)^(1/2)-2/1155*b^(9/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1
/2*2^(1/2))*(-77*f*a^(1/2)+15*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(
a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {14, 1839, 1847, 1266, 849, 821, 272, 65, 214, 1296, 1212, 226, 1210}

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx = -\frac{2b^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}d-77\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}} - \frac{4b^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} + \frac{b^3c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}} + \frac{4b^{5/2}fx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a+bx^4}}{77ax^3} - \frac{b^2e\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2f\sqrt{a+bx^4}}{15ax} - \frac{(a+bx^4)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980} - \frac{b\sqrt{a+bx^4}\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)}{18480}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^13,x]
```

```
[Out] -1/18480*(b*((1155*c)/x^8 + (1440*d)/x^7 + (1848*e)/x^6 + (2464*f)/x^5)*Sqr
t[a + b*x^4]) - (b^2*c*Sqrt[a + b*x^4])/(32*a*x^4) - (4*b^2*d*Sqrt[a + b*x^
4])/(77*a*x^3) - (b^2*e*Sqrt[a + b*x^4])/(10*a*x^2) - (4*b^2*f*Sqrt[a + b*x
^4])/(15*a*x) + (4*b^(5/2)*f*x*Sqrt[a + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^
2)) - (((165*c)/x^12 + (180*d)/x^11 + (198*e)/x^10 + (220*f)/x^9)*(a + b*x^
4)^(3/2))/1980 + (b^3*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(32*a^(3/2)) - (4
*b^(9/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x
^4]) - (2*b^(9/4)*(15*Sqrt[b]*d - 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(1155*a^(5/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\text{integral} = -\frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)\left(a+bx^4\right)^{3/2}}{1980} - (6b)\int\frac{\left(-\frac{c}{12}-\frac{dx}{11}-\frac{ex^2}{10}-\frac{fx^3}{9}\right)\sqrt{a+bx^4}}{x^9}\,dx$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)\left(a+bx^4\right)^{3/2}}{1980} + \left(12b^2\right)\int \frac{\frac{c}{96}+\frac{dx}{77}+\frac{ex^2}{60}+\frac{fx^3}{45}}{x^5\sqrt{a+bx^4}}\,dx$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)\left(a+bx^4\right)^{3/2}}{1980} + \left(12b^2\right)\int \left(\frac{\frac{c}{96}+\frac{ex^2}{60}}{x^5\sqrt{a+bx^4}}+\frac{\frac{d}{77}+\frac{fx^2}{45}}{x^4\sqrt{a+bx^4}}\right)dx$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)\left(a+bx^4\right)^{3/2}}{1980} + \left(12b^2\right)\int \frac{\frac{c}{96}+\frac{ex^2}{60}}{x^5\sqrt{a+bx^4}}\,dx + \left(12b^2\right)\int \frac{\frac{d}{77}+\frac{fx^2}{45}}{x^4\sqrt{a+bx^4}}\,dx$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2d\sqrt{a+bx^4}}{77ax^3} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)\left(a+bx^4\right)^{3/2}}{1980} + \left(6b^2\right)\text{Subst}\left(\int \frac{\frac{c}{96}+\frac{ex}{60}}{x^3\sqrt{a+bx^2}}\,dx, x, x^2\right) - \frac{\left(4b^2\right)\int \frac{-\frac{af}{15}+\frac{1}{77}bdx^2}{x^2\sqrt{a+bx^4}}\,dx}{a}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a+bx^4}}{77ax^3} - \frac{4b^2f\sqrt{a+bx^4}}{15ax} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)\left(a+bx^4\right)^{3/2}}{1980} + \frac{\left(4b^2\right)\int \frac{-\frac{1}{77}abd+\frac{1}{15}abfx^2}{\sqrt{a+bx^4}}\,dx}{a^2} - \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{-\frac{ae}{30}+\frac{bcx}{96}}{x^2\sqrt{a+bx^2}}\,dx, x, x^2\right)}{a}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a+bx^4}}{77ax^3} - \frac{b^2e\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2f\sqrt{a+bx^4}}{15ax} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)\left(a+bx^4\right)^{3/2}}{1980} - \frac{\left(b^3c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, x^2\right)}{32a} - \frac{\left(4b^{5/2}f\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{15\sqrt{a}} - \frac{\left(4b^{5/2}\left(15\sqrt{b}d-77\sqrt{a}f\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{1155a}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a+bx^4}}{77ax^3} - \frac{b^2e\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2f\sqrt{a+bx^4}}{15ax} + \frac{4b^{5/2}fx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)\left(a+bx^4\right)^{3/2}}{1980} - \frac{4b^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} - \frac{2b^{9/4}\left(15\sqrt{b}d-77\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}} - \frac{\left(b^3c\right)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)}{64a}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a+bx^4}}{77ax^3} - \frac{b^2e\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2f\sqrt{a+bx^4}}{15ax} + \frac{4b^{5/2}fx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)\left(a+bx^4\right)^{3/2}}{1980} - \frac{4b^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} - \frac{2b^{9/4}\left(15\sqrt{b}d-77\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}} - \frac{\left(b^2c\right)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)}{32a}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480}-\frac{b^2c\sqrt{a+bx^4}}{32ax^4}-\frac{4b^2d\sqrt{a+bx^4}}{77ax^3}-\frac{b^2e\sqrt{a+bx^4}}{10ax^2}-\frac{4b^2f\sqrt{a+bx^4}}{15ax}+\frac{4b^{5/2}fx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)(a+bx^4)^{3/2}}{1980}+\frac{b^3c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}}-\frac{4b^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}-\frac{2b^{9/4}\left(15\sqrt{b}d-77\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.33

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx = -\frac{\sqrt{a+bx^4}\left(90a^5d\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{3}{2},-\frac{7}{4},-\frac{bx^4}{a}\right)+11x\left(10a^5fx\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{3}{2}\right.\right.\right.}{990a^4x^{11}\sqrt{1+}}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^13,x]
```

```
[Out] -1/990*(Sqrt[a + b*x^4]*(90*a^5*d*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b
*x^4)/a)] + 11*x*(10*a^5*f*x*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^4)/
a)] + 9*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*(a^3*e - b^3*c*x^10*Hypergeometri
c2F1[5/2, 4, 7/2, 1 + (b*x^4)/a]))))/(a^4*x^11*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.37 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(29568 b^{2} f\,x^{11}+11088 b^{2} e\,x^{10}+5760 b^{2} d\,x^{9}+3465 b^{2} c\,x^{8}+27104 a b f\,x^{7}+22176 a e b\,x^{6}+18720 x^{5} d b a+16170 a b c\,x^{4}+12320 a^{2} f}{110880 x^{12} a}$
default	$c\left(-\frac{a\sqrt{b\,x^{4}+a}}{12 x^{12}}-\frac{7 b\sqrt{b\,x^{4}+a}}{48 x^{8}}-\frac{b^{2}\sqrt{b\,x^{4}+a}}{32 a\,x^{4}}+\frac{b^{3}\ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{32 a^{\frac{3}{2}}}\right)-\frac{e\left(b^{2} x^{8}+2 a b\,x^{4}+a^{2}\right)\sqrt{b\,x^{4}+a}}{10 x^{10} a}+d\left(-\frac{a\sqrt{b\,x^{4}}}{11 x^{11}}\right.$
elliptic	$-\frac{a c\sqrt{b\,x^{4}+a}}{12 x^{12}}-\frac{a d\sqrt{b\,x^{4}+a}}{11 x^{11}}-\frac{a e\sqrt{b\,x^{4}+a}}{10 x^{10}}-\frac{a f\sqrt{b\,x^{4}+a}}{9 x^{9}}-\frac{7 b c\sqrt{b\,x^{4}+a}}{48 x^{8}}-\frac{13 b d\sqrt{b\,x^{4}+a}}{77 x^{7}}-\frac{b e\sqrt{b\,x^{4}+a}}{5 x^{6}}-\frac{11 b f\sqrt{b\,x^{4}+a}}{45 x^{5}}-$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/110880*(b*x^4+a)^(1/2)*(29568*b^2*f*x^11+11088*b^2*e*x^10+5760*b^2*d*x^9
+3465*b^2*c*x^8+27104*a*b*f*x^7+22176*a*b*e*x^6+18720*a*b*d*x^5+16170*a*b*c
*x^4+12320*a^2*f*x^3+11088*a^2*e*x^2+10080*a^2*d*x+9240*a^2*c)/x^12/a-1/184
80*b^3/a*(960*d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(
1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/
2))^(1/2),I)-4928*I*f*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ellip
ticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I
))-1155/2*c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.56

$$\int\frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{13}}\,dx = -\frac{59136\,a^{\frac{3}{2}}b^2fx^{12}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)-3465\,\sqrt{a}b^3cx^{12}\log\left(-\frac{bx^4+2\,\sqrt{bx^4+a}\sqrt{a}+2\,a}{x^4}\right)-768\,(15\,ab^2d}{}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/221760*(59136*a^(3/2)*b^2*f*x^12*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)
^(1/4)), -1) - 3465*sqrt(a)*b^3*c*x^12*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt
(a) + 2*a)/x^4) - 768*(15*a*b^2*d + 77*a*b^2*f)*sqrt(a)*x^12*(-b/a)^(3/4)*e
lliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(29568*a*b^2*f*x^11 + 11088*a*b^2
*e*x^10 + 5760*a*b^2*d*x^9 + 3465*a*b^2*c*x^8 + 27104*a^2*b*f*x^7 + 22176*a
^2*b*e*x^6 + 18720*a^2*b*d*x^5 + 16170*a^2*b*c*x^4 + 12320*a^3*f*x^3 + 1108
8*a^3*e*x^2 + 10080*a^3*d*x + 9240*a^3*c)*sqrt(b*x^4 + a))/(a^2*x^12)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.77 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\ -\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\ -\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\ -\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}bf\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\ -\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} - \frac{a^2c}{12\sqrt{b}x^{14}\sqrt{\frac{a}{bx^4}+1}} - \frac{11a\sqrt{b}c}{48x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{10x^8} - \frac{17b^{\frac{3}{2}}c}{96x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^4}+1}}{5x^4} - \frac{b^{\frac{5}{2}}c}{32ax^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{5}{2}}e\sqrt{\frac{a}{bx^4}+1}}{10a} + \frac{b^3c\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{32a^{\frac{3}{2}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**13,x)
```

```
[Out] a**(3/2)*d*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**4*exp_polar(I*pi
)/a)/(4*x**11*gamma(-7/4)) + a**(3/2)*f*gamma(-9/4)*hyper((-9/4, -1/2), (-5
/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*b*d*gamma(-7
/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3
/4)) + sqrt(a)*b*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*x**5*gamma(-1/4)) - a**2*c/(12*sqrt(b)*x**14*sqrt(a/(b*x**4)
+ 1)) - 11*a*sqrt(b)*c/(48*x**10*sqrt(a/(b*x**4) + 1)) - a*sqrt(b)*e*sqrt(a
/(b*x**4) + 1)/(10*x**8) - 17*b**(3/2)*c/(96*x**6*sqrt(a/(b*x**4) + 1)) - b
**(3/2)*e*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(5/2)*c/(32*a*x**2*sqrt(a/(b*x
**4) + 1)) - b**(5/2)*e*sqrt(a/(b*x**4) + 1)/(10*a) + b**3*c*asinh(sqrt(a)/
(sqrt(b)*x**2))/(32*a**(3/2))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{13}}\,dx = \int \frac{\left(bx^4+a\right)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{13}}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] -1/192*(3*b^3*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/
a^(3/2) + 2*(3*(b*x^4 + a)^(5/2)*b^3 + 8*(b*x^4 + a)^(3/2)*a*b^3 - 3*sqrt(b
*x^4 + a)*a^2*b^3)/((b*x^4 + a)^3*a - 3*(b*x^4 + a)^2*a^2 + 3*(b*x^4 + a)*a
^3 - a^4))*c + integrate((b*f*x^6 + b*e*x^5 + b*d*x^4 + a*f*x^2 + a*e*x + a
*d)*sqrt(b*x^4 + a)/x^12, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{13}}\,dx = \int \frac{\left(bx^4+a\right)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{13}}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^13, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{13}}\,dx = \int \frac{\left(b\,x^4+a\right)^{3/2}\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^{13}}\,dx$$

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^13,x)
```

```
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^13, x)
```

3.528 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 474

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx = -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5} - \frac{b^2d\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2e\sqrt{a+bx^4}}{77ax^3} - \frac{b^2f\sqrt{a+bx^4}}{10ax^2} + \frac{4b^3c\sqrt{a+bx^4}}{65a^2x} - \frac{4b^{7/2}cx\sqrt{a+bx^4}}{65a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)(a+bx^4)^{3/2}}{8580} + \frac{b^3d\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}} + \frac{4b^{13/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{65a^{7/4}\sqrt{a+bx^4}} - \frac{2b^{11/4}\left(77\sqrt{b}c+65\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5005a^{7/4}\sqrt{a+bx^4}}$$

```
[Out] -1/8580*(660*c/x^13+715*d/x^12+780*e/x^11+858*f/x^10)*(b*x^4+a)^(3/2)+1/32*
b^3*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/240240*b*(12320*c/x^9+1501
5*d/x^8+18720*e/x^7+24024*f/x^6)*(b*x^4+a)^(1/2)-4/195*b^2*c*(b*x^4+a)^(1/2
)/a/x^5-1/32*b^2*d*(b*x^4+a)^(1/2)/a/x^4-4/77*b^2*e*(b*x^4+a)^(1/2)/a/x^3-1
/10*b^2*f*(b*x^4+a)^(1/2)/a/x^2+4/65*b^3*c*(b*x^4+a)^(1/2)/a^2/x-4/65*b^(7/
2)*c*x*(b*x^4+a)^(1/2)/a^2/(a^(1/2)+x^2*b^(1/2))+4/65*b^(13/4)*c*(cos(2*arc
tan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE
(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^
4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)-2/5005*b^(11/4)
*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4))
)*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(65*e*a^(1/2)+77*
```

```
c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/
a^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {14, 1839, 1847, 1296, 1212, 226, 1210, 1266, 849, 821, 272, 65, 214}

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx = -\frac{2b^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(65\sqrt{a}e+77\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5005a^{7/4}\sqrt{a+bx^4}} + \frac{4b^{13/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65a^{7/4}\sqrt{a+bx^4}} + \frac{b^3d\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}} - \frac{4b^{7/2}cx\sqrt{a+bx^4}}{65a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{4b^3c\sqrt{a+bx^4}}{65a^2x} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5} - \frac{b^2d\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2e\sqrt{a+bx^4}}{77ax^3} - \frac{b^2f\sqrt{a+bx^4}}{10ax^2} - \frac{(a+bx^4)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580} - \frac{b\sqrt{a+bx^4}\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)}{240240}$$

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^14,x]
```

```
[Out] -1/240240*(b*((12320*c)/x^9 + (15015*d)/x^8 + (18720*e)/x^7 + (24024*f)/x^6
)*Sqrt[a + b*x^4]) - (4*b^2*c*Sqrt[a + b*x^4])/(195*a*x^5) - (b^2*d*Sqrt[a
+ b*x^4])/(32*a*x^4) - (4*b^2*e*Sqrt[a + b*x^4])/(77*a*x^3) - (b^2*f*Sqrt[a
 + b*x^4])/(10*a*x^2) + (4*b^3*c*Sqrt[a + b*x^4])/(65*a^2*x) - (4*b^(7/2)*c
*x*Sqrt[a + b*x^4])/(65*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (((660*c)/x^13 + (71
5*d)/x^12 + (780*e)/x^11 + (858*f)/x^10)*(a + b*x^4)^(3/2))/8580 + (b^3*d*A
rcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(32*a^(3/2)) + (4*b^(13/4)*c*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(65*a^(7/4)*Sqrt[a + b*x^4]) - (2*b^(11/4)*(77*S
qrt[b]*c + 65*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
+ Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*a^(7
/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1839

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)(a+bx^4)^{3/2}}{8580} \\
&\quad -(6b)\int \frac{\left(-\frac{c}{13}-\frac{dx}{12}-\frac{ex^2}{11}-\frac{fx^3}{10}\right)\sqrt{a+bx^4}}{x^{10}}\,dx \\
&= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} \\
&\quad -\frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)(a+bx^4)^{3/2}}{8580}+\left(12b^2\right)\int \frac{\frac{c}{117}+\frac{dx}{96}+\frac{ex^2}{77}+\frac{fx^3}{60}}{x^6\sqrt{a+bx^4}}\,dx \\
&= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} \\
&\quad -\frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)(a+bx^4)^{3/2}}{8580} \\
&\quad +\left(12b^2\right)\int \left(\frac{\frac{c}{117}+\frac{ex^2}{77}}{x^6\sqrt{a+bx^4}}+\frac{\frac{d}{96}+\frac{fx^2}{60}}{x^5\sqrt{a+bx^4}}\right)dx \\
&= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} \\
&\quad -\frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)(a+bx^4)^{3/2}}{8580} \\
&\quad +\left(12b^2\right)\int \frac{\frac{c}{117}+\frac{ex^2}{77}}{x^6\sqrt{a+bx^4}}\,dx+\left(12b^2\right)\int \frac{\frac{d}{96}+\frac{fx^2}{60}}{x^5\sqrt{a+bx^4}}\,dx \\
&= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} \\
&\quad -\frac{4b^2c\sqrt{a+bx^4}}{195ax^5}-\frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)(a+bx^4)^{3/2}}{8580} \\
&\quad +\left(6b^2\right)\mathrm{Subst}\left(\int \frac{\frac{d}{96}+\frac{fx}{60}}{x^3\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\left(12b^2\right)\int \frac{-\frac{5ae}{77}+\frac{1}{39}bcx^2}{x^4\sqrt{a+bx^4}}\,dx}{5a} \\
&= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240}-\frac{4b^2c\sqrt{a+bx^4}}{195ax^5} \\
&\quad -\frac{b^2d\sqrt{a+bx^4}}{32ax^4}-\frac{4b^2e\sqrt{a+bx^4}}{77ax^3}-\frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)(a+bx^4)^{3/2}}{8580} \\
&\quad +\frac{\left(4b^2\right)\int \frac{-\frac{1}{13}abc-\frac{5}{77}abex^2}{x^2\sqrt{a+bx^4}}\,dx}{5a^2}-\frac{\left(3b^2\right)\mathrm{Subst}\left(\int \frac{-\frac{af}{30}+\frac{bdx}{96}}{x^2\sqrt{a+bx^2}}\,dx,x,x^2\right)}{a}
\end{aligned}
$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240}-\frac{4b^2c\sqrt{a+bx^4}}{195ax^5}$$
$$-\frac{b^2d\sqrt{a+bx^4}}{32ax^4}-\frac{4b^2e\sqrt{a+bx^4}}{77ax^3}-\frac{b^2f\sqrt{a+bx^4}}{10ax^2}$$
$$+\frac{4b^3c\sqrt{a+bx^4}}{65a^2x}-\frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)(a+bx^4)^{3/2}}{8580}$$
$$-\frac{(4b^2)\int\frac{\frac{5}{77}a^2be+\frac{1}{13}ab^2cx^2}{\sqrt{a+bx^4}}\,dx}{5a^3}-\frac{(b^3d)\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)}{32a}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240}-\frac{4b^2c\sqrt{a+bx^4}}{195ax^5}$$
$$-\frac{b^2d\sqrt{a+bx^4}}{32ax^4}-\frac{4b^2e\sqrt{a+bx^4}}{77ax^3}-\frac{b^2f\sqrt{a+bx^4}}{10ax^2}+\frac{4b^3c\sqrt{a+bx^4}}{65a^2x}$$
$$-\frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)(a+bx^4)^{3/2}}{8580}+\frac{(4b^{7/2}c)\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{65a^{3/2}}$$
$$-\frac{(b^3d)\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)}{64a}-\frac{\left(4b^3\left(77\sqrt{b}c+65\sqrt{a}e\right)\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx}{5005a^{3/2}}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240}-\frac{4b^2c\sqrt{a+bx^4}}{195ax^5}$$
$$-\frac{b^2d\sqrt{a+bx^4}}{32ax^4}-\frac{4b^2e\sqrt{a+bx^4}}{77ax^3}-\frac{b^2f\sqrt{a+bx^4}}{10ax^2}+\frac{4b^3c\sqrt{a+bx^4}}{65a^2x}$$
$$-\frac{4b^{7/2}cx\sqrt{a+bx^4}}{65a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)(a+bx^4)^{3/2}}{8580}$$
$$+\frac{4b^{13/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65a^{7/4}\sqrt{a+bx^4}}$$
$$-\frac{2b^{11/4}\left(77\sqrt{b}c+65\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005a^{7/4}\sqrt{a+bx^4}}$$
$$-\frac{(b^2d)\,\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)}{32a}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5} - \frac{b^2d\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2e\sqrt{a+bx^4}}{77ax^3} - \frac{b^2f\sqrt{a+bx^4}}{10ax^2} + \frac{4b^3c\sqrt{a+bx^4}}{65a^2x} - \frac{4b^{7/2}cx\sqrt{a+bx^4}}{65a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)(a+bx^4)^{3/2}}{8580} + \frac{b^3d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}} + \frac{4b^{13/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65a^{7/4}\sqrt{a+bx^4}} - \frac{2b^{11/4}\left(77\sqrt{b}c+65\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005a^{7/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.32

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx =$$

$$-\frac{\sqrt{a+bx^4}\left(110a^5c\,\text{Hypergeometric2F1}\left(-\frac{13}{4},-\frac{3}{2},-\frac{9}{4},-\frac{bx^4}{a}\right)+13x^2\left(10a^5e\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\right.\right.\right.}{1430a^4x^{13}\sqrt{1}}$$

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^14,x]
```

```
[Out] -1/1430*(Sqrt[a + b*x^4]*(110*a^5*c*Hypergeometric2F1[-13/4, -3/2, -9/4, -(
(b*x^4)/a)] + 13*x^2*(10*a^5*e*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*x^
4)/a)] + 11*x*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*(a^3*f - b^3*d*x^10*Hyperge
ometric2F1[5/2, 4, 7/2, 1 + (b*x^4)/a]))))/(a^4*x^13*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.65 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(-29568 b^{3} c\,x^{12}+48048 x^{11} a\,b^{2} f+24960 a\,b^{2} e\,x^{10}+15015 x^{9} a\,b^{2} d+9856 a\,b^{2} c\,x^{8}+96096 a^{2} b f\,x^{7}+81120 a^{2} b e\,x^{6}+70070 x^{5} b d}{480480 x^{13} a^{2}}$
default	$d\left(-\frac{a\sqrt{b\,x^{4}+a}}{12 x^{12}}-\frac{7 b\sqrt{b\,x^{4}+a}}{48 x^{8}}-\frac{b^{2}\sqrt{b\,x^{4}+a}}{32 a\,x^{4}}+\frac{b^{3}\ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{32 a^{\frac{3}{2}}}\right)-\frac{f\,(b^{2} x^{8}+2 a b\,x^{4}+a^{2})\sqrt{b\,x^{4}+a}}{10 x^{10} a}+e\left(-\frac{a\sqrt{b\,x^{4}}}{11 x^{11}}\right.$
elliptic	$-\frac{a c\sqrt{b\,x^{4}+a}}{13 x^{13}}-\frac{a d\sqrt{b\,x^{4}+a}}{12 x^{12}}-\frac{a e\sqrt{b\,x^{4}+a}}{11 x^{11}}-\frac{a f\sqrt{b\,x^{4}+a}}{10 x^{10}}-\frac{5 b c\sqrt{b\,x^{4}+a}}{39 x^{9}}-\frac{7 b d\sqrt{b\,x^{4}+a}}{48 x^{8}}-\frac{13 b e\sqrt{b\,x^{4}+a}}{77 x^{7}}-\frac{b f\sqrt{b\,x^{4}+a}}{5 x^{6}}-$

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/480480*(b*x^4+a)^(1/2)*(-29568*b^3*c*x^12+48048*a*b^2*f*x^11+24960*a*b^2
*e*x^10+15015*a*b^2*d*x^9+9856*a*b^2*c*x^8+96096*a^2*b*f*x^7+81120*a^2*b*e*
x^6+70070*a^2*b*d*x^5+61600*a^2*b*c*x^4+48048*a^3*f*x^3+43680*a^3*e*x^2+400
40*a^3*d*x+36960*a^3*c)/x^13/a^2-1/80080/a^2*b^3*(4160*a*e/(I/a^(1/2)*b^(1/
2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(
b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+4928*I*b^(1/2)*c*a^
(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2)
,I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-5005/2*a^(1/2)*d*ln((2*a+2*a^
(1/2)*(b*x^4+a)^(1/2))/x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.54

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{14}}\,dx = \frac{59136\,\sqrt{a}b^3cx^{13}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)+15015\,\sqrt{a}b^3dx}{}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] 1/960960*(59136*sqrt(a)*b^3*c*x^13*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^
(1/4)), -1) + 15015*sqrt(a)*b^3*d*x^13*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt
(a) + 2*a)/x^4) - 768*(77*b^3*c - 65*a*b^2*e)*sqrt(a)*x^13*(-b/a)^(3/4)*ell
iptic_f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(29568*b^3*c*x^12 - 48048*a*b^2*f*x
^11 - 24960*a*b^2*e*x^10 - 15015*a*b^2*d*x^9 - 9856*a*b^2*c*x^8 - 96096*a^2
*b*f*x^7 - 81120*a^2*b*e*x^6 - 70070*a^2*b*d*x^5 - 61600*a^2*b*c*x^4 - 4804
8*a^3*f*x^3 - 43680*a^3*e*x^2 - 40040*a^3*d*x - 36960*a^3*c)*sqrt(b*x^4 + a
))/(a^2*x^13)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.57 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{13}{4},-\frac{1}{2}\\-\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{13}\Gamma\left(-\frac{9}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{9}\Gamma\left(-\frac{5}{4}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{7}\Gamma\left(-\frac{3}{4}\right)} - \frac{a^2d}{12\sqrt{b}x^{14}\sqrt{\frac{a}{bx^4}+1}} - \frac{11a\sqrt{b}d}{48x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{10x^8} - \frac{17b^{\frac{3}{2}}d}{96x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{5x^4} - \frac{b^{\frac{5}{2}}d}{32ax^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{5}{2}}f\sqrt{\frac{a}{bx^4}+1}}{10a} + \frac{b^3d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{32a^{\frac{3}{2}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**14,x)
```

```
[Out] a**(3/2)*c*gamma(-13/4)*hyper((-13/4, -1/2), (-9/4,), b*x**4*exp_polar(I*pi
)/a)/(4*x**13*gamma(-9/4)) + a**(3/2)*e*gamma(-11/4)*hyper((-11/4, -1/2), (
-7/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**11*gamma(-7/4)) + sqrt(a)*b*c*gamma
(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma
(-5/4)) + sqrt(a)*b*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*x**7*gamma(-3/4)) - a**2*d/(12*sqrt(b)*x**14*sqrt(a/(b*x**
4) + 1)) - 11*a*sqrt(b)*d/(48*x**10*sqrt(a/(b*x**4) + 1)) - a*sqrt(b)*f*sqr
t(a/(b*x**4) + 1)/(10*x**8) - 17*b**(3/2)*d/(96*x**6*sqrt(a/(b*x**4) + 1))
- b**(3/2)*f*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(5/2)*d/(32*a*x**2*sqrt(a/(
b*x**4) + 1)) - b**(5/2)*f*sqrt(a/(b*x**4) + 1)/(10*a) + b**3*d*asinh(sqrt(
a)/(sqrt(b)*x**2))/(32*a**(3/2))
```

Maxima [F]

$$\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^{14}}\, dx = \int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^{14}}\, dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^14, x)
```

Giac [F]

$$\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^{14}}\, dx = \int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^{14}}\, dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^14, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^{14}}\, dx = \int \frac{(b\,x^4 + a)^{3/2}\,(f\,x^3 + e\,x^2 + d\,x + c)}{x^{14}}\, dx$$

```
[In] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^14,x)
[Out] int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^14, x)
```

3.529 $\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 361

$$\begin{aligned}
&\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx\\
&=\frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}-\frac{3aex\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}\\
&-\frac{(4af-3bdx^2)\sqrt{a+bx^4}}{12b^2}-\frac{ad\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}\\
&+\frac{3a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}\\
&-\frac{a^{3/4}\left(5\sqrt{b}c+9\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}
\end{aligned}$$

```
[Out] -1/4*a*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/3*c*x*(b*x^4+a)^(1/
2)/b+1/5*e*x^3*(b*x^4+a)^(1/2)/b+1/6*f*x^4*(b*x^4+a)^(1/2)/b-1/12*(-3*b*d*x
^2+4*a*f)*(b*x^4+a)^(1/2)/b^2-3/5*a*e*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^
2*b^(1/2))+3/5*a^(5/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^
(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7
/4)/(b*x^4+a)^(1/2)-1/30*a^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(9*e*a^(1/2)+5*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(
a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1847, 1294, 1212, 226, 1210, 1266, 847, 794, 223, 212}

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$= -\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}$$
$$+\frac{3a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}$$
$$-\frac{ad\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}-\frac{3aex\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\sqrt{a+bx^4}(4af-3bdx^2)}{12b^2}$$
$$+\frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}$$

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (c*x*Sqrt[a + b*x^4])/(3*b) + (e*x^3*Sqrt[a + b*x^4])/(5*b) + (f*x^4*Sqrt[a
 + b*x^4])/(6*b) - (3*a*e*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*
x^2)) - ((4*a*f - 3*b*d*x^2)*Sqrt[a + b*x^4])/(12*b^2) - (a*d*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) + (3*a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(3/4)*(5*Sqrt[b]*c + 9*
Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x
^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
```

```
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{x^4(c+ex^2)}{\sqrt{a+bx^4}}+\frac{x^5(d+fx^2)}{\sqrt{a+bx^4}}\right)dx\\
&= \int \frac{x^4(c+ex^2)}{\sqrt{a+bx^4}}\,dx+\int \frac{x^5(d+fx^2)}{\sqrt{a+bx^4}}\,dx\\
&= \frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{1}{2}\text{Subst}\left(\int \frac{x^2(d+fx)}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{x^2(3ae-5bcx^2)}{\sqrt{a+bx^4}}\,dx}{5b}\\
&= \frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}\\
&\quad+\frac{\int \frac{-5abc-9abex^2}{\sqrt{a+bx^4}}\,dx}{15b^2}+\frac{\text{Subst}\left(\int \frac{x(-2af+3bdx)}{\sqrt{a+bx^2}}\,dx,x,x^2\right)}{6b}\\
&= \frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}\\
&\quad-\frac{(4af-3bdx^2)\sqrt{a+bx^4}}{12b^2}-\frac{(ad)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)}{4b}\\
&\quad+\frac{(3a^{3/2}e)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{5b^{3/2}}-\frac{\left(a\left(5\sqrt{b}c+9\sqrt{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{15b^{3/2}}
\end{aligned}
$$

$$= \frac{cx\sqrt{a+bx^4}}{3b} + \frac{ex^3\sqrt{a+bx^4}}{5b} + \frac{fx^4\sqrt{a+bx^4}}{6b} - \frac{3aex\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{(4af-3bdx^2)\sqrt{a+bx^4}}{12b^2} + \frac{3a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}} - \frac{a^{3/4}\left(5\sqrt{b}c+9\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}} - \frac{(ad)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)}{4b}$$

$$= \frac{cx\sqrt{a+bx^4}}{3b} + \frac{ex^3\sqrt{a+bx^4}}{5b} + \frac{fx^4\sqrt{a+bx^4}}{6b} - \frac{3aex\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{(4af-3bdx^2)\sqrt{a+bx^4}}{12b^2} - \frac{ad\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}} + \frac{3a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}} - \frac{a^{3/4}\left(5\sqrt{b}c+9\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.59

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{-20a^2f+20abcx+15abdx^2+12abex^3-10abfx^4+20b^2cx^5+15b^2dx^6+12b^2ex^7+10b^2fx^8-15a\sqrt{b}d}{}$$

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (-20*a^2*f + 20*a*b*c*x + 15*a*b*d*x^2 + 12*a*b*e*x^3 - 10*a*b*f*x^4 + 20*b
^2*c*x^5 + 15*b^2*d*x^6 + 12*b^2*e*x^7 + 10*b^2*f*x^8 - 15*a*Sqrt[b]*d*Sqrt
[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*a*b*c*x*Sqrt[1 + (b
```

```
*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*a*b*e*x^3*Sqrt
[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^2*Sqr
t[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.18 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{\left(-10bf\,x^{4}-12be\,x^{3}-15bd\,x^{2}-20bcx+20af\right)\sqrt{b\,x^{4}+a}}{60b^{2}}-\frac{a\left(\frac{10c\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{18ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right.}{30b}$
default	$-\frac{f\sqrt{b\,x^{4}+a}\,\left(-b\,x^{4}+2a\right)}{6b^{2}}+e\left(\frac{x^{3}\sqrt{b\,x^{4}+a}}{5b}-\frac{3ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+d\left(\frac{x^{2}\sqrt{b\,x^{4}+a}}{4b}\right.$
elliptic	$\frac{f\,x^{4}\sqrt{b\,x^{4}+a}}{6b}+\frac{e\,x^{3}\sqrt{b\,x^{4}+a}}{5b}+\frac{d\,x^{2}\sqrt{b\,x^{4}+a}}{4b}+\frac{cx\sqrt{b\,x^{4}+a}}{3b}-\frac{af\sqrt{b\,x^{4}+a}}{3b^{2}}-\frac{ac\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{ad}{}$

```
[In] int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(-10*b*f*x^4-12*b*e*x^3-15*b*d*x^2-20*b*c*x+20*a*f)/b^2*(b*x^4+a)^(1/
2)-1/30*a/b*(10*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)
*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)+18*I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ellip
ticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I
))+15/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.45

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx =$$

$$-\frac{72\,a\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-15\,a\sqrt{b}dx\log\left(-2\,bx^{4}+2\,\sqrt{bx^{4}+a}\sqrt{b}x^{2}-a\right)+8\,(5\,bc-9}{}$$

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(72*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1
) - 15*a*sqrt(b)*d*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 8*
(5*b*c - 9*a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -
1) - 2*(10*b*f*x^5 + 12*b*e*x^4 + 15*b*d*x^3 + 20*b*c*x^2 - 20*a*f*x - 36*a
*e)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.49

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a}dx^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{ad\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + f\left(\begin{cases} -\frac{a\sqrt{a+bx^4}}{3b^2} + \frac{x^4\sqrt{a+bx^4}}{6b} & \text{for } b \neq 0 \\ \frac{x^8}{8\sqrt{a}} & \text{otherwise} \end{cases}\right) + \frac{cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

```
[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*d*asinh(sqrt(b)*x**2/sqrt(a))/(
4*b**(3/2)) + f*Piecewise((-a*sqrt(a + b*x**4)/(3*b**2) + x**4*sqrt(a + b*x
**4)/(6*b), Ne(b, 0)), (x**8/(8*sqrt(a)), True)) + c*x**5*gamma(5/4)*hyper(
(1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + e*x*
*7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^4}{\sqrt{bx^4+a}}\,dx$$

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^4(c + dx + ex^2 + fx^3)}{\sqrt{a + bx^4}}\,dx = \int \frac{(fx^3 + ex^2 + dx + c)x^4}{\sqrt{bx^4 + a}}\,dx$$

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c + dx + ex^2 + fx^3)}{\sqrt{a + bx^4}}\,dx = \int \frac{x^4\,(f\,x^3 + e\,x^2 + d\,x + c)}{\sqrt{b\,x^4 + a}}\,dx$$

```
[In] int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
[Out] int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

3.530 $\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 336

$$\begin{aligned}
&\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx\\
&=\frac{dx\sqrt{a+bx^4}}{3b}+\frac{fx^3\sqrt{a+bx^4}}{5b}-\frac{3afx\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{(2c+ex^2)\sqrt{a+bx^4}}{4b}\\
&\quad-\frac{ae\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}+\frac{3a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}\\
&\quad-\frac{a^{3/4}\left(5\sqrt{b}d+9\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}
\end{aligned}$$

```
[Out] -1/4*a*e*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/3*d*x*(b*x^4+a)^(1/
2)/b+1/5*f*x^3*(b*x^4+a)^(1/2)/b+1/4*(e*x^2+2*c)*(b*x^4+a)^(1/2)/b-3/5*a*f*
x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))+3/5*a^(5/4)*f*(cos(2*arctan
(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(si
n(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a
)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)-1/30*a^(3/4)*(cos(
2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Elli
pticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(9*f*a^(1/2)+5*d*b^(1/2
))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/
(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1847, 1266, 794, 223, 212, 1294, 1212, 226, 1210}

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$= -\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}$$
$$+\frac{3a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}-\frac{ae\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}$$
$$-\frac{3afx\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\sqrt{a+bx^4}(2c+ex^2)}{4b}+\frac{dx\sqrt{a+bx^4}}{3b}+\frac{fx^3\sqrt{a+bx^4}}{5b}$$

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (d*x*Sqrt[a + b*x^4])/(3*b) + (f*x^3*Sqrt[a + b*x^4])/(5*b) - (3*a*f*x*Sqrt
[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*c + e*x^2)*Sqrt[a +
b*x^4])/(4*b) - (a*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) +
(3*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*
x^4]) - (a^(3/4)*(5*Sqrt[b]*d + 9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(
a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x^3(c+ex^2)}{\sqrt{a+bx^4}}+\frac{x^4(d+fx^2)}{\sqrt{a+bx^4}}\right)dx\\
&= \int \frac{x^3(c+ex^2)}{\sqrt{a+bx^4}}\,dx+\int \frac{x^4(d+fx^2)}{\sqrt{a+bx^4}}\,dx\\
&= \frac{fx^3\sqrt{a+bx^4}}{5b}+\frac{1}{2}\text{Subst}\left(\int \frac{x(c+ex)}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{x^2(3af-5bdx^2)}{\sqrt{a+bx^4}}\,dx}{5b}\\
&= \frac{dx\sqrt{a+bx^4}}{3b}+\frac{fx^3\sqrt{a+bx^4}}{5b}+\frac{(2c+ex^2)\sqrt{a+bx^4}}{4b}\\
&\quad+\frac{\int \frac{-5abd-9abfx^2}{\sqrt{a+bx^4}}\,dx}{15b^2}-\frac{(ae)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)}{4b}\\
&= \frac{dx\sqrt{a+bx^4}}{3b}+\frac{fx^3\sqrt{a+bx^4}}{5b}+\frac{(2c+ex^2)\sqrt{a+bx^4}}{4b}\\
&\quad-\frac{(ae)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)}{4b}+\frac{\left(3a^{3/2}f\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{5b^{3/2}}\\
&\quad-\frac{\left(a\left(5\sqrt{b}d+9\sqrt{a}f\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{15b^{3/2}}\\
&= \frac{dx\sqrt{a+bx^4}}{3b}+\frac{fx^3\sqrt{a+bx^4}}{5b}-\frac{3afx\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{(2c+ex^2)\sqrt{a+bx^4}}{4b}\\
&\quad-\frac{ae\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}+\frac{3a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}\\
&\quad-\frac{a^{3/4}\left(5\sqrt{b}d+9\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.63

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{30\sqrt{b}c(a+bx^4)+20\sqrt{b}dx(a+bx^4)+15\sqrt{b}ex^2(a+bx^4)+12\sqrt{b}fx^3(a+bx^4)-15ae\sqrt{a+bx^4}\text{arctanh}\left(\frac{\sqrt{}}{\sqrt{a}}\right.}{}$$

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (30*Sqrt[b]*c*(a + b*x^4) + 20*Sqrt[b]*d*x*(a + b*x^4) + 15*Sqrt[b]*e*x^2*(
a + b*x^4) + 12*Sqrt[b]*f*x^3*(a + b*x^4) - 15*a*e*Sqrt[a + b*x^4]*ArcTanh[
(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*a*Sqrt[b]*d*x*Sqrt[1 + (b*x^4)/a]*Hyper
geometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*a*Sqrt[b]*f*x^3*Sqrt[1 + (b*
x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^(3/2)*Sqrt[a
+ b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.70

method	result
risch	$\frac{(12f\,x^3+15e\,x^2+20dx+30c)\sqrt{b\,x^4+a}}{60b}-\frac{a\left(\frac{10d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{18if\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}\right.}{30b}$
default	$f\left(\frac{x^3\sqrt{b\,x^4+a}}{5b}-\frac{3ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+e\left(\frac{x^2\sqrt{b\,x^4+a}}{4b}-\frac{a\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+}\right.}{4b^{\frac{3}{2}}}\right.$
elliptic	$\frac{f\,x^3\sqrt{b\,x^4+a}}{5b}+\frac{e\,x^2\sqrt{b\,x^4+a}}{4b}+\frac{dx\sqrt{b\,x^4+a}}{3b}+\frac{c\sqrt{b\,x^4+a}}{2b}-\frac{ad\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{ae\ln\left(2x^2\sqrt{b}+2\sqrt{b}\right.}{4b^{\frac{3}{2}}}$

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(12*f*x^3+15*e*x^2+20*d*x+30*c)/b*(b*x^4+a)^(1/2)-1/30*a/b*(10*d/(I/a^
(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x
^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+18*I*f*a
^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/
2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/
2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+15/2*e*ln(x^2*b^(1/2
)+(b*x^4+a)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.46

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx =$$

$$-\frac{72\,a\sqrt{b}fx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-15\,a\sqrt{b}ex\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+8\,(5\,bd-}{120}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(72*a*sqrt(b)*f*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1
) - 15*a*sqrt(b)*e*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 8*
(5*b*d - 9*a*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -
1) - 2*(12*b*f*x^4 + 15*b*e*x^3 + 20*b*d*x^2 + 30*b*c*x - 36*a*f)*sqrt(b*x^
4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.46

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a}ex^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{ae\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + c\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right) + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*e*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*e*asinh(sqrt(b)*x**2/sqrt(a))/(
4*b**(3/2)) + c*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(
2*b), True)) + d*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar
(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + f*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/
4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^3}{\sqrt{bx^4+a}}\,dx$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*c/b + integrate((f*x^6 + e*x^5 + d*x^4)/sqrt(b*x^4 + a)
, x)
```

Giac [F]

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^3}{\sqrt{bx^4+a}}\,dx$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{x^3\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{\sqrt{b\,x^4+a}}\,dx$$

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
[Out] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

3.531 $\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 308

$$\begin{aligned}
&\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx \\
&= \frac{ex\sqrt{a+bx^4}}{3b}+\frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{(2d+fx^2)\sqrt{a+bx^4}}{4b}-\frac{af\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}} \\
&-\frac{\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} \\
&+\frac{\sqrt[4]{a}\left(3\sqrt{b}c-\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}
\end{aligned}$$

```
[Out] -1/4*a*f*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/3*e*x*(b*x^4+a)^(1/
2)/b+1/4*(f*x^2+2*d)*(b*x^4+a)^(1/2)/b+c*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)
+x^2*b^(1/2))-a^(1/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(
1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/
4)/(b*x^4+a)^(1/2)+1/6*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),
1/2*2^(1/2))*(-e*a^(1/2)+3*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(
1/2)+x^2*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1847, 1294, 1212, 226, 1210, 1266, 794, 223, 212}

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{b}c-\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} - \frac{af\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}} + \frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{\sqrt{a+bx^4}(2d+fx^2)}{4b} + \frac{ex\sqrt{a+bx^4}}{3b}$$

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (e*x*Sqrt[a + b*x^4])/(3*b) + (c*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqr
t[b]*x^2)) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/(4*b) - (a*f*ArcTanh[(Sqrt[b]*
x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*c - Sqrt[a]*e)
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{x^2(c+ex^2)}{\sqrt{a+bx^4}}+\frac{x^3(d+fx^2)}{\sqrt{a+bx^4}}\right)\,dx\\
&= \int \frac{x^2(c+ex^2)}{\sqrt{a+bx^4}}\,dx+\int \frac{x^3(d+fx^2)}{\sqrt{a+bx^4}}\,dx\\
&= \frac{ex\sqrt{a+bx^4}}{3b}+\frac{1}{2}\text{Subst}\left(\int \frac{x(d+fx)}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{ae-3bcx^2}{\sqrt{a+bx^4}}\,dx}{3b}\\
&= \frac{ex\sqrt{a+bx^4}}{3b}+\frac{(2d+fx^2)\sqrt{a+bx^4}}{4b}-\frac{\left(\sqrt{a}c\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{b}}\\
&\quad+\frac{\left(\sqrt{a}\left(3\sqrt{b}c-\sqrt{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{3b}-\frac{(af)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)}{4b}\\
&= \frac{ex\sqrt{a+bx^4}}{3b}+\frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{(2d+fx^2)\sqrt{a+bx^4}}{4b}\\
&\quad-\frac{\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}\\
&\quad+\frac{\sqrt[4]{a}\left(3\sqrt{b}c-\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}\\
&\quad-\frac{(af)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)}{4b}\\
&= \frac{ex\sqrt{a+bx^4}}{3b}+\frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{(2d+fx^2)\sqrt{a+bx^4}}{4b}-\frac{af\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}\\
&\quad-\frac{\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}\\
&\quad+\frac{\sqrt[4]{a}\left(3\sqrt{b}c-\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.63

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$=\frac{6\sqrt{b}d(a+bx^4)+4\sqrt{b}ex(a+bx^4)+3\sqrt{b}fx^2(a+bx^4)-3af\sqrt{a+bx^4}\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-4a\sqrt{b}ex\sqrt{1+\frac{bx}{a}}}{12b^{3/2}\sqrt{a+bx}}$$

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (6*Sqrt[b]*d*(a + b*x^4) + 4*Sqrt[b]*e*x*(a + b*x^4) + 3*Sqrt[b]*f*x^2*(a +
 b*x^4) - 3*a*f*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 4*
a*Sqrt[b]*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4
)/a)] + 4*b^(3/2)*c*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4
, -((b*x^4)/a)])/(12*b^(3/2)*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.75

method	result
risch	$\frac{(3f\,x^2+4ex+6d)\sqrt{b\,x^4+a}}{12b}-\frac{\frac{2ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{6i\sqrt{b}\,c\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}}{6b}$
default	$f\left(\frac{x^2\sqrt{b\,x^4+a}}{4b}-\frac{a\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{4b^{\frac{3}{2}}}\right)+e\left(\frac{x\sqrt{b\,x^4+a}}{3b}-\frac{a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+\frac{d\sqrt{b\,x^4+a}}{2b}+$
elliptic	$\frac{f\,x^2\sqrt{b\,x^4+a}}{4b}+\frac{ex\sqrt{b\,x^4+a}}{3b}+\frac{d\sqrt{b\,x^4+a}}{2b}-\frac{ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{af\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{4b^{\frac{3}{2}}}+$

```
[In] int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(3*f*x^2+4*e*x+6*d)/b*(b*x^4+a)^(1/2)-1/6/b*(2*a*e/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x
^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-6*I*b^(1/2)*c*a^(1/2)/
(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1
/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-El
lipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+3/2*a*f*ln(x^2*b^(1/2)+(b*x^4+a)^(1
/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.48

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$=\frac{24\,b^{\frac{3}{2}}cx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+3\,a\sqrt{b}fx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)-8\,(3\,bc+be)\sqrt{\ }}{24\,b^2x}$$

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^(3/2)*c*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) +
3*a*sqrt(b)*f*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) - 8*(3*b*
c + b*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 2*
(3*b*f*x^3 + 4*b*e*x^2 + 6*b*d*x + 12*b*c)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.51

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx=\frac{\sqrt{a}fx^2\sqrt{1+\frac{bx^4}{a}}}{4b}-\frac{af\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+d\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right)+\frac{cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}+\frac{ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*f*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*f*asinh(sqrt(b)*x**2/sqrt(a))/(
4*b**(3/2)) + d*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(
2*b), True)) + c*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar
(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + e*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4
,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Maxima [F]

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^2}{\sqrt{bx^4+a}}\,dx$$

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^2/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^2}{\sqrt{bx^4+a}}\,dx$$

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^2/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{x^2\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{\sqrt{b\,x^4+a}}\,dx$$

```
[In] int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
[Out] int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

3.532 $\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 299

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{e\sqrt{a+bx^4}}{2b}+\frac{fx\sqrt{a+bx^4}}{3b}+\frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \frac{\sqrt[4]{a}\left(3\sqrt{b}d-\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}$$

```
[Out] 1/2*c*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+1/2*e*(b*x^4+a)^(1/2)/b+
1/3*f*x*(b*x^4+a)^(1/2)/b+d*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))
-a^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2
)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^
(1/2)+1/6*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b
^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*
(-f*a^(1/2)+3*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1
/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1847, 1262, 655, 223, 212, 1294, 1212, 226, 1210}

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{b}d-\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{e\sqrt{a+bx^4}}{2b} + \frac{fx\sqrt{a+bx^4}}{3b}$$

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (e*Sqrt[a + b*x^4])/(2*b) + (f*x*Sqrt[a + b*x^4])/(3*b) + (d*x*Sqrt[a + b*x
^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\text{integral} = \int \left(\frac{x(c+ex^2)}{\sqrt{a+bx^4}}+\frac{x^2(d+fx^2)}{\sqrt{a+bx^4}}\right)dx$$

$$= \int \frac{x(c+ex^2)}{\sqrt{a+bx^4}}\,dx+\int \frac{x^2(d+fx^2)}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{fx\sqrt{a+bx^4}}{3b}+\frac{1}{2}\text{Subst}\left(\int \frac{c+ex}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{af-3bdx^2}{\sqrt{a+bx^4}}\,dx}{3b}$$

$$= \frac{e\sqrt{a+bx^4}}{2b}+\frac{fx\sqrt{a+bx^4}}{3b}+\frac{1}{2}c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right) - \frac{\left(\sqrt{a}d\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{b}}+\frac{\left(\sqrt{a}\left(3\sqrt{b}d-\sqrt{a}f\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{3b}$$

$$= \frac{e\sqrt{a+bx^4}}{2b}+\frac{fx\sqrt{a+bx^4}}{3b}+\frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \frac{\sqrt[4]{a}\left(3\sqrt{b}d-\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}} + \frac{1}{2}c\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)$$

$$= \frac{e\sqrt{a+bx^4}}{2b}+\frac{fx\sqrt{a+bx^4}}{3b}+\frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \frac{\sqrt[4]{a}\left(3\sqrt{b}d-\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.54

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{3ae+2afx+3bex^4+2bfx^5+3\sqrt{b}c\sqrt{a+bx^4}\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-2afx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4}\right.}{6b\sqrt{a+bx^4}}$$

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (3*a*e + 2*a*f*x + 3*b*e*x^4 + 2*b*f*x^5 + 3*Sqrt[b]*c*Sqrt[a + b*x^4]*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 2*a*f*x*Sqrt[1 + (b*x^4)/a]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*d*x^3*Sqrt[1 + (b*x^4)/a]*Hyperg
eometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(6*b*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.74

method	result
risch	$\frac{(2fx+3e)\sqrt{b\,x^4+a}}{6b}-\frac{\frac{af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{3i\sqrt{b}\,d\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-3\sqrt{b}}{3b}$
default	$f\left(\frac{x\sqrt{b\,x^4+a}}{3b}-\frac{a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+\frac{e\sqrt{b\,x^4+a}}{2b}+\frac{id\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
elliptic	$\frac{fx\sqrt{b\,x^4+a}}{3b}+\frac{e\sqrt{b\,x^4+a}}{2b}-\frac{af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{c\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{id\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+}}{\sqrt{}}$

```
[In] int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*f*x+3*e)/b*(b*x^4+a)^(1/2)-1/3/b*(a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I
/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)
*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-3*I*b^(1/2)*d*a^(1/2)/(I/a^(1/2)*
b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1
/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(
I/a^(1/2)*b^(1/2))^(1/2),I))-3/2*b^(1/2)*c*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.44

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{12\sqrt{b}dx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-4\sqrt{b}(3\,d+f)x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+3\sqrt{b}cx\log\left(-\right.}{12\,bx}$$

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(b)*d*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) -
4*sqrt(b)*(3*d + f)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) +
 3*sqrt(b)*c*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*sqrt(b
*x^4 + a)*(2*f*x^2 + 3*e*x + 6*d))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.43

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = e\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right)+\frac{c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}+\frac{fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] e*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b), True)) +
 c*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + d*x**3*gamma(3/4)*hyper((1/2,
3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + f*x**5*gam
ma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamm
a(9/4))
```

Maxima [F]

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x}{\sqrt{bx^4+a}}\,dx$$

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] -1/4*c*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 + a)/x^2)
)/sqrt(b) + integrate((f*x^4 + e*x^3 + d*x^2)/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x}{\sqrt{bx^4+a}}\,dx$$

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{x\,(f\,x^3+e\,x^2+d\,x+c)}{\sqrt{b\,x^4+a}}\,dx$$

```
[In] int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
[Out] int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

3.533 $\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 276

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx &= \frac{f\sqrt{a+bx^4}}{2b}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}\\&\quad-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}\\&\quad+\frac{\sqrt[4]{a}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}\end{aligned}$$

```
[Out] 1/2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+1/2*f*(b*x^4+a)^(1/2)/b+
e*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))-a^(1/4)*e*(cos(2*arctan(b
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(
2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/
(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*a
rctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ellipti
cF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(e+c
*b^(1/2)/a^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+
a)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1899, 1212, 226, 1210, 1262, 655, 223, 212}

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{f\sqrt{a+bx^4}}{2b}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x^4],x]
```

```
[Out] (f*Sqrt[a + b*x^4])/(2*b) + (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[
b]*x^2)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4
)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^
(1/4)*((Sqrt[b]*c)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(S
qrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*
b^(3/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{c+ex^2}{\sqrt{a+bx^4}}+\frac{x(d+fx^2)}{\sqrt{a+bx^4}}\right) dx\\
&= \int \frac{c+ex^2}{\sqrt{a+bx^4}}\,dx+\int \frac{x(d+fx^2)}{\sqrt{a+bx^4}}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\left(\sqrt{a}e\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{b}}+\left(c+\frac{\sqrt{a}e}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx
\end{aligned}
$$

$$=\frac{f\sqrt{a+bx^4}}{2b}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{1}{2}d\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)$$

$$=\frac{f\sqrt{a+bx^4}}{2b}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{1}{2}d\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)$$

$$=\frac{f\sqrt{a+bx^4}}{2b}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$
$$-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.54

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = \frac{f\sqrt{a+bx^4}}{2b}+\frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{cx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}} + \frac{ex^3\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x^4],x]
```

```
[Out] (f*Sqrt[a + b*x^4])/(2*b) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*S
qrt[b]) + (c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^
4)/a)])/Sqrt[a + b*x^4] + (e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2,
 3/4, 7/4, -((b*x^4)/a)])/(3*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.75

method	result
default	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{f\sqrt{b\,x^4+a}}{2b}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}+ d\ln($
risch	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{f\sqrt{b\,x^4+a}}{2b}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}+ d\ln($
elliptic	$\frac{f\sqrt{b\,x^4+a}}{2b}+\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1
/2*f*(b*x^4+a)^(1/2)/b+I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(
```

```
EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1
/2),I))+1/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.49

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = \frac{4\,a\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+a\sqrt{b}dx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+4\,(bc-ae)\sqrt{b}x}{4\,abx}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) +
a*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 4*(b*c -
a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 2*sqrt
(b*x^4 + a)*(a*f*x + 2*a*e))/(a*b*x)
```

Sympy [A] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.46

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = f\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for}\ b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right)+\frac{d\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] f*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b), True)) +
 d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2
), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma(
3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7
/4))
```

Maxima [F]

$$\int \frac{c + dx + ex^2 + fx^3}{\sqrt{a + bx^4}}\, dx = \int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}}\, dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{c + dx + ex^2 + fx^3}{\sqrt{a + bx^4}}\, dx = \int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}}\, dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3}{\sqrt{a + bx^4}}\, dx = \int \frac{f\, x^3 + e\, x^2 + d\, x + c}{\sqrt{b\, x^4 + a}}\, dx$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(1/2),x)
[Out] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(1/2), x)
```

3.534 $\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 285

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx &= \frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\\ &\quad-\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}\\ &\quad+\frac{\sqrt[4]{a}\left(\frac{\sqrt{b}d}{\sqrt{a}}+f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}\end{aligned}$$

```
[Out] -1/2*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)+1/2*e*arctanh(x^2*b^(1/2)/(
b*x^4+a)^(1/2))/b^(1/2)+f*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))-a
^(1/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/
a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1
/2)+1/2*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*b^(1/2))*(f+d*b^(1/2)/a^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^
2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1846, 272, 65, 214, 1899, 281, 223, 212, 1212, 226, 1210}

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx$$
$$= \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}d}{\sqrt{a}}+f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}$$
$$-\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}$$
$$-\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x*Sqrt[a + b*x^4]),x]
```

```
[Out] (f*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]
)/(2*Sqrt[a]) - (a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)
*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*d)/Sqrt[a] + f)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= c\int \frac{1}{x\sqrt{a+bx^4}}\,dx+\int \frac{d+ex+fx^2}{\sqrt{a+bx^4}}\,dx\\
&= \frac{1}{4}c\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)+\int \left(\frac{ex}{\sqrt{a+bx^4}}+\frac{d+fx^2}{\sqrt{a+bx^4}}\right)dx\\
&= \frac{c\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)}{2b}+e\int \frac{x}{\sqrt{a+bx^4}}\,dx+\int \frac{d+fx^2}{\sqrt{a+bx^4}}\,dx\\
&= -\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{1}{2}e\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)\\
&\quad -\frac{\left(\sqrt{a}f\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{b}}+\left(d+\frac{\sqrt{a}f}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx\\
&= \frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\\
&\quad -\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}\\
&\quad +\frac{\left(\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}\\
&\quad +\frac{1}{2}e\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)\\
&= \frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\\
&\quad -\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}\\
&\quad +\frac{\left(\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.56

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx = \frac{e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{dx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}} + \frac{fx^3\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x*Sqrt[a + b*x^4]),x]
```

```
[Out] (e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (c*ArcTanh[Sqrt[a
+ b*x^4]/Sqrt[a]])/(2*Sqrt[a]) + (d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1
[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4] + (f*x^3*Sqrt[1 + (b*x^4)/a]
*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(3*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.76

method	result
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{e\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{if\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\cdots}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
default	$\frac{d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{if\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}+\frac{e\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4\cdots}\right.}{2\sqrt{b}}$

```
[In] int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1
/2*e*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))/b^(1/2)+I*f*a^(1/2)/(I/a^(1/2)*b^(
1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)
/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Elliptic
E(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*c/a^(1/2)*arctanh(a^(1/2)/(b*x^4+a)^(
1/2))
```

Fricas [F]

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^5 + a*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.44

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx=\frac{e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}}-\frac{c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}}+\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(1/2),x)
[Out] e*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) - c*asinh(sqrt(a)/(sqrt(b)*x**2))
/(2*sqrt(a)) + d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*
pi)/a)/(4*sqrt(a)*gamma(5/4)) + f*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx=\int \frac{f\,x^3+e\,x^2+d\,x+c}{x\,\sqrt{b\,x^4+a}}\,dx$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(1/2)),x)
[Out] int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(1/2)), x)
```

3.535 $\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}} dx$

Optimal result

Integrand size = 30, antiderivative size = 309

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}} dx = -\frac{c\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{d\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$
$$-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

```
[Out] -1/2*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)+1/2*f*arctanh(x^2*b^(1/2)/(
b*x^4+a)^(1/2))/b^(1/2)-c*(b*x^4+a)^(1/2)/a/x+c*x*b^(1/2)*(b*x^4+a)^(1/2)/a
/(a^(1/2)+x^2*b^(1/2))-b^(1/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1
/2)/a^(3/4)/(b*x^4+a)^(1/2)+1/2*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/
cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4)))
,1/2*2^(1/2))*(e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/
2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1847, 1296, 1212, 226, 1210, 1266, 858, 223, 212, 272, 65, 214}

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}} - \frac{d\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{f\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{c\sqrt{a+bx^4}}{ax} + \frac{\sqrt{b}cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^2*Sqrt[a + b*x^4]),x]
```

```
[Out] -((c*Sqrt[a + b*x^4])/(a*x)) + (Sqrt[b]*c*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c+ex^2}{x^2\sqrt{a+bx^4}}+\frac{d+fx^2}{x\sqrt{a+bx^4}}\right) dx\\
&= \int \frac{c+ex^2}{x^2\sqrt{a+bx^4}}\,dx+\int \frac{d+fx^2}{x\sqrt{a+bx^4}}\,dx\\
&= -\frac{c\sqrt{a+bx^4}}{ax}+\frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{-ae-bcx^2}{\sqrt{a+bx^4}}\,dx}{a}\\
&= -\frac{c\sqrt{a+bx^4}}{ax}-\frac{\left(\sqrt{b}c\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{a}}+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)\\
&\quad+\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx+\frac{1}{2}f\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)\\
&= -\frac{c\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}\\
&\quad-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}\\
&\quad+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}\\
&\quad+\frac{1}{4}d\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)+\frac{1}{2}f\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)
\end{aligned}$$

$$= -\frac{c\sqrt{a+bx^4}}{ax} + \frac{\sqrt{b}cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$
$$- \frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}$$
$$+ \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$
$$+ \frac{d\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)}{2b}$$

$$= -\frac{c\sqrt{a+bx^4}}{ax} + \frac{\sqrt{b}cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$
$$- \frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}$$
$$+ \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.51

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx = \frac{f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{d\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$
$$- \frac{c\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\sqrt{a+bx^4}}$$
$$+ \frac{ex\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^2*Sqrt[a + b*x^4]),x]
```

```
[Out] (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (d*ArcTanh[Sqrt[a
+ b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-
1/4, 1/2, 3/4, -((b*x^4)/a)])/(x*Sqrt[a + b*x^4]) + (e*x*Sqrt[1 + (b*x^4)/a
]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.03 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.76

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{ax}+\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{f\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{ic\sqrt{b}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\ldots}\right.\right.}{\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+\ldots}}$
default	$\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{f\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}-\frac{d\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2\sqrt{a}}+c\left(-\frac{\sqrt{b\,x^4+a}}{ax}+\frac{i\sqrt{b}\,\sqrt{1-\ldots}}{\ldots}\right.$
risch	$-\frac{c\sqrt{b\,x^4+a}}{ax}+\frac{\frac{ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{i\sqrt{b}\,c\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{af\ln\left(x^2\sqrt{b}-\ldots\right.}{2\sqrt{\ldots}}}{a}$

```
[In] int((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -c*(b*x^4+a)^(1/2)/a/x+e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2
)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1
/2)*b^(1/2))^(1/2),I)+1/2*f*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))/b^(1/2)+I*c
/a^(1/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*
(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*d/a^(1/2)*arct
anh(a^(1/2)/(b*x^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^2}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^6 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.41

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx = \frac{f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} - \frac{d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}} + \frac{ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(1/2),x)
```

```
[Out] f*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*gamma(-1/4)*hyper((-1/4, 1/2)
, (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) - d*asinh(sqrt
(a)/(sqrt(b)*x**2))/(2*sqrt(a)) + e*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,),
b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^2}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^2}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx=\int \frac{f\,x^3+e\,x^2+d\,x+c}{x^2\,\sqrt{b\,x^4+a}}\,dx$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(1/2)),x)
[Out] int((c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(1/2)), x)
```

3.536 $\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 300

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = -\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{e\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

```
[Out] -1/2*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/2*c*(b*x^4+a)^(1/2)/a/x^2
-d*(b*x^4+a)^(1/2)/a/x+d*x*b^(1/2)*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-
b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x
/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)
+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(
1/2)+1/2*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/
a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(f*a^(1/2
)+d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2
)/a^(3/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1847, 1266, 821, 272, 65, 214, 1296, 1212, 226, 1210}

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}} - \frac{e\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{c\sqrt{a+bx^4}}{2ax^2} - \frac{d\sqrt{a+bx^4}}{ax} + \frac{\sqrt{b}dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^3*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/2*(c*Sqrt[a + b*x^4])/(a*x^2) - (d*Sqrt[a + b*x^4])/(a*x) + (Sqrt[b]*d*x
*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (e*ArcTanh[Sqrt[a + b*x^4]/
Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)
/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(a^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c+ex^2}{x^3\sqrt{a+bx^4}}+\frac{d+fx^2}{x^2\sqrt{a+bx^4}}\right)dx\\
&= \int \frac{c+ex^2}{x^3\sqrt{a+bx^4}}\,dx+\int \frac{d+fx^2}{x^2\sqrt{a+bx^4}}\,dx\\
&= -\frac{d\sqrt{a+bx^4}}{ax}+\frac{1}{2}\text{Subst}\left(\int \frac{c+ex}{x^2\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{-af-bdx^2}{\sqrt{a+bx^4}}\,dx}{a}\\
&= -\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{ax}-\frac{\left(\sqrt{b}d\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{a}}\\
&\quad+\frac{1}{2}e\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)+\left(\frac{\sqrt{b}d}{\sqrt{a}}+f\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx\\
&= -\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}\\
&\quad-\frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}\\
&\quad+\frac{\left(\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}\\
&\quad+\frac{1}{4}e\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)
\end{aligned}$$

$$= -\frac{c\sqrt{a+bx^4}}{2ax^2} - \frac{d\sqrt{a+bx^4}}{ax} + \frac{\sqrt{b}dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}} + \frac{e\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)}{2b}$$

$$= -\frac{c\sqrt{a+bx^4}}{2ax^2} - \frac{d\sqrt{a+bx^4}}{ax} + \frac{\sqrt{b}dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.49

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = -\frac{c\sqrt{a+bx^4}}{2ax^2} - \frac{\text{earctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{d\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\sqrt{a+bx^4}} + \frac{fx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^3*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/2*(c*Sqrt[a + b*x^4])/(a*x^2) - (e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*
Sqrt[a]) - (d*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^
4)/a)])/(x*Sqrt[a + b*x^4]) + (f*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/
4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.75

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{2a\,x^2}-\frac{d\sqrt{b\,x^4+a}}{ax}+\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{i\sqrt{b}\,d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\right.\right.}{\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
risch	$-\frac{\sqrt{b\,x^4+a}\,(2dx+c)}{2a\,x^2}+\frac{\frac{af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{i\sqrt{b}\,d\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sqrt{a}}{}}{a}$
default	$\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2\sqrt{a}}+d\left(-\frac{\sqrt{b\,x^4+a}}{ax}+\frac{i\sqrt{b}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\right.\right.}{\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4}}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c*(b*x^4+a)^(1/2)/a/x^2-d*(b*x^4+a)^(1/2)/a/x+f/(I/a^(1/2)*b^(1/2))^(1
/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+
a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+I*b^(1/2)/a^(1/2)*d/(I/a^
(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x
^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ellipti
cE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*e/a^(1/2)*arctanh(a^(1/2)/(b*x^4+a)^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.46

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx =$$
$$-\frac{4\sqrt{a}bdx^2\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-\sqrt{a}bex^2\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-4\left(bd-af\right)\sqrt{a}x^2\left(-\right.}{4abx^2}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(a)*b*d*x^2*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1)
 - sqrt(a)*b*e*x^2*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) - 4*
(b*d - a*f)*sqrt(a)*x^2*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1)
 + 2*sqrt(b*x^4 + a)*(2*b*d*x + b*c))/(a*b*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.42

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} - \frac{e\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}} + \frac{fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(2*a) + d*gamma(-1/4)*hyper((-1/4, 1/2), (3
/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) - e*asinh(sqrt(a)/
(sqrt(b)*x**2))/(2*sqrt(a)) + f*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^3}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^3}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.39

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = \frac{f\,x\,\sqrt{\frac{b\,x^4}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\sqrt{b\,x^4+a}} - \frac{c\,\sqrt{b\,x^4+a}}{2\,a\,x^2} - \frac{d\,\sqrt{\frac{a}{b\,x^4}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{a}{b\,x^4}\right)}{3\,x\,\sqrt{b\,x^4+a}} - \frac{e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(1/2)),x)
```

```
[Out] (f*x*((b*x^4)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(b*x^4)/a))/(a + b*x
^4)^(1/2) - (c*(a + b*x^4)^(1/2))/(2*a*x^2) - (d*(a/(b*x^4) + 1)^(1/2)*hype
rgeom([1/2, 3/4], 7/4, -a/(b*x^4)))/(3*x*(a + b*x^4)^(1/2)) - (e*atanh((a +
 b*x^4)^(1/2)/a^(1/2)))/(2*a^(1/2))
```

3.537 $\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 323

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx = -\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{f\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}\left(\sqrt{b}c-3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}$$

```
[Out] -1/2*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/3*c*(b*x^4+a)^(1/2)/a/x^3
-1/2*d*(b*x^4+a)^(1/2)/a/x^2-e*(b*x^4+a)^(1/2)/a/x+e*x*b^(1/2)*(b*x^4+a)^(1
/2)/a/(a^(1/2)+x^2*b^(1/2))-b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^
2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)-1/6*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4
)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4
)*x/a^(1/4))),1/2*2^(1/2))*(-3*e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*(
(b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1847, 1296, 1212, 226, 1210, 1266, 821, 272, 65, 214}

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx$$
$$= -\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-3\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}$$
$$-\frac{\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{f\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$
$$-\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^4*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/3*(c*Sqrt[a + b*x^4])/(a*x^3) - (d*Sqrt[a + b*x^4])/(2*a*x^2) - (e*Sqrt[
a + b*x^4])/(a*x) + (Sqrt[b]*e*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2
)) - (f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*e*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[
b]*c - 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqr
t[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt
[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{c+ex^2}{x^4\sqrt{a+bx^4}}+\frac{d+fx^2}{x^3\sqrt{a+bx^4}}\right)dx \\
&= \int \frac{c+ex^2}{x^4\sqrt{a+bx^4}}\,dx+\int \frac{d+fx^2}{x^3\sqrt{a+bx^4}}\,dx \\
&= -\frac{c\sqrt{a+bx^4}}{3ax^3}+\frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{x^2\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{-3ae+bcx^2}{x^2\sqrt{a+bx^4}}\,dx}{3a} \\
&= -\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax} \\
&\quad +\frac{\int \frac{-abc+3abex^2}{\sqrt{a+bx^4}}\,dx}{3a^2}+\frac{1}{2}f\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right) \\
&= -\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}-\frac{\left(\sqrt{b}e\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{a}} \\
&\quad -\frac{\left(\sqrt{b}\left(\sqrt{b}c-3\sqrt{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{3a}+\frac{1}{4}f\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right) \\
&= -\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)} \\
&\quad -\frac{\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}} \\
&\quad -\frac{\sqrt[4]{b}\left(\sqrt{b}c-3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}} \\
&\quad +\frac{f\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)}{2b}
\end{aligned}
$$

$$= -\frac{c\sqrt{a+bx^4}}{3ax^3} - \frac{d\sqrt{a+bx^4}}{2ax^2} - \frac{e\sqrt{a+bx^4}}{ax} + \frac{\sqrt{b}ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$- \frac{f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}$$
$$- \frac{\sqrt[4]{b}\left(\sqrt{b}c-3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.46

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx$$
$$= \frac{-2ac\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\frac{bx^4}{a}\right) - 3x\left(ad+bdx^4+\sqrt{a}fx^2\sqrt{a+bx^4}\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right.}{6ax^3\sqrt{a+bx^4}}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^4*Sqrt[a + b*x^4]),x]
```

```
[Out] (-2*a*c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^4)/a)]
 - 3*x*(a*d + b*d*x^4 + Sqrt[a]*f*x^2*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^
4]/Sqrt[a]] + 2*a*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4,
 -((b*x^4)/a)]))/(6*a*x^3*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(6e\,x^{2}+3dx+2c)}{6a\,x^{3}}+\frac{-\frac{bc\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{3i\sqrt{b}\,e\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}}{3a}$
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{3a\,x^{3}}-\frac{d\sqrt{b\,x^{4}+a}}{2a\,x^{2}}-\frac{e\sqrt{b\,x^{4}+a}}{ax}-\frac{cb\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{i\sqrt{b}\,e\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\right.\right.}{\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}}}$
default	$-\frac{f\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{2\sqrt{a}}+c\left(-\frac{\sqrt{b\,x^{4}+a}}{3a\,x^{3}}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+e\left(-\frac{\sqrt{b\,x^{4}+a}}{ax}+\frac{i\sqrt{b}\,\sqrt{1-\frac{i\sqrt{}}{}}}{}\right.$

[In] int((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/6*(b*x^4+a)^(1/2)*(6*e*x^2+3*d*x+2*c)/a/x^3+1/3/a*(-b*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3*I*b^(1/2)*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3/2*a^(1/2)*f*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.42

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx = -\frac{12\,\sqrt{a}ex^3\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)-4\,\sqrt{a}(c+3\,e)x^3\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)-3\,\sqrt{a}}{12\,ax^3}$$

[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] -1/12*(12*sqrt(a)*e*x^3*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1) - 4*sqrt(a)*(c + 3*e)*x^3*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - 3*sqrt(a)*f*x^3*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*sqrt(b*x^4 + a)*(6*e*x^2 + 3*d*x + 2*c))/(a*x^3)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.41

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} - \frac{f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**4/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(2*a) + c*gamma(-3/4)*hyper((-3/4, 1/2), (1
/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4)) + e*gamma(-1/4)
*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/
4)) - f*asinh(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^4}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^4}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx=\int \frac{f\,x^3+e\,x^2+d\,x+c}{x^4\,\sqrt{b\,x^4+a}}\,dx$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(1/2)),x)
[Out] int((c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(1/2)), x)
```

3.538 $\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 346

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx = -\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{b\text{carctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}\left(\sqrt{b}d-3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}$$

```
[Out] 1/4*b*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/4*c*(b*x^4+a)^(1/2)/a/x^
4-1/3*d*(b*x^4+a)^(1/2)/a/x^3-1/2*e*(b*x^4+a)^(1/2)/a/x^2-f*(b*x^4+a)^(1/2)
/a/x+f*x*b^(1/2)*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-b^(1/4)*f*(cos(2*a
rctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ellipti
cE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*
x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)-1/6*b^(1/4)*(
cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*
EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-3*f*a^(1/2)+d*b^(
1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/
4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1847, 1266, 849, 821, 272, 65, 214, 1296, 1212, 226, 1210}

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx$$
$$= -\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-3\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}$$
$$-\frac{\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}+\frac{b\text{carctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}$$
$$-\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^5*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/4*(c*Sqrt[a + b*x^4])/(a*x^4) - (d*Sqrt[a + b*x^4])/(3*a*x^3) - (e*Sqrt[
a + b*x^4])/(2*a*x^2) - (f*Sqrt[a + b*x^4])/(a*x) + (Sqrt[b]*f*x*Sqrt[a + b
*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) + (b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])
/(4*a^(3/2)) - (b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*
Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b]*d - 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c+ex^2}{x^5\sqrt{a+bx^4}}+\frac{d+fx^2}{x^4\sqrt{a+bx^4}}\right)dx\\
&= \int \frac{c+ex^2}{x^5\sqrt{a+bx^4}}\,dx+\int \frac{d+fx^2}{x^4\sqrt{a+bx^4}}\,dx\\
&= -\frac{d\sqrt{a+bx^4}}{3ax^3}+\frac{1}{2}\text{Subst}\left(\int \frac{c+ex}{x^3\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{-3af+bdx^2}{x^2\sqrt{a+bx^4}}\,dx}{3a}\\
&= -\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\int \frac{-abd+3abfx^2}{\sqrt{a+bx^4}}\,dx}{3a^2}-\frac{\text{Subst}\left(\int \frac{-2ae+bcx}{x^2\sqrt{a+bx^2}}\,dx,x,x^2\right)}{4a}\\
&= -\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}\\
&\quad-\frac{f\sqrt{a+bx^4}}{ax}-\frac{(bc)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)}{4a}\\
&\quad-\frac{\left(\sqrt{b}f\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{a}}-\frac{\left(\sqrt{b}\left(\sqrt{b}d-3\sqrt{a}f\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{3a}
\end{aligned}$$

$$= -\frac{c\sqrt{a+bx^4}}{4ax^4} - \frac{d\sqrt{a+bx^4}}{3ax^3} - \frac{e\sqrt{a+bx^4}}{2ax^2} - \frac{f\sqrt{a+bx^4}}{ax} + \frac{\sqrt{b}fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}\left(\sqrt{b}d-3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}} - \frac{(bc)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}dx, x, x^4\right)}{8a}$$

$$= -\frac{c\sqrt{a+bx^4}}{4ax^4} - \frac{d\sqrt{a+bx^4}}{3ax^3} - \frac{e\sqrt{a+bx^4}}{2ax^2} - \frac{f\sqrt{a+bx^4}}{ax} + \frac{\sqrt{b}fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}\left(\sqrt{b}d-3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}} - \frac{c\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}dx, x, \sqrt{a+bx^4}\right)}{4a}$$

$$= -\frac{c\sqrt{a+bx^4}}{4ax^4} - \frac{d\sqrt{a+bx^4}}{3ax^3} - \frac{e\sqrt{a+bx^4}}{2ax^2} - \frac{f\sqrt{a+bx^4}}{ax} + \frac{\sqrt{b}fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}\left(\sqrt{b}d-3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.42

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}\left(3ac\sqrt{1+\frac{bx^4}{a}}+6aex^2\sqrt{1+\frac{bx^4}{a}}-3bcx^4\text{arctanh}\left(\sqrt{1+\frac{bx^4}{a}}\right)+4adx\,\text{Hypergeometric2F1}\left(\right.\right.}{12a^2x^4\sqrt{1+\frac{bx^4}{a}}}$$

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^5*Sqrt[a + b*x^4]),x]

[Out] -1/12*(Sqrt[a + b*x^4]*(3*a*c*Sqrt[1 + (b*x^4)/a] + 6*a*e*x^2*Sqrt[1 + (b*x^4)/a] - 3*b*c*x^4*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 4*a*d*x*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^4)/a)] + 12*a*f*x^3*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^4)/a)]))/(a^2*x^4*Sqrt[1 + (b*x^4)/a])

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(12f\,x^3+6e\,x^2+4dx+3c)}{12a\,x^4}-\frac{b\left(\frac{2d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}-\frac{6if\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}\sqrt{b}}\right.}{6a}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{4a\,x^4}-\frac{d\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{e\sqrt{b\,x^4+a}}{2a\,x^2}-\frac{f\sqrt{b\,x^4+a}}{ax}-\frac{bd\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{i\sqrt{b}\,f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\right.}{\sqrt{}}$
default	$d\left(-\frac{\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+f\left(-\frac{\sqrt{b\,x^4+a}}{ax}+\frac{i\sqrt{b}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.}{\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right.$

[In] int((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/12*(b*x^4+a)^(1/2)*(12*f*x^3+6*e*x^2+4*d*x+3*c)/a/x^4-1/6*b/a*(2*d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-6*I*f*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3/2*c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.43

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx = -\frac{24\,a^{\frac{3}{2}}fx^4\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1) - 3\sqrt{a}bcx^4\log\left(-\frac{bx^4+2\sqrt{bx^4+a}\sqrt{a}+2\,a}{x^4}\right) - 8\,(ad+3\,af)\sqrt{a}x^4(-}{24\,a^2x^4}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(24*a^(3/2)*f*x^4*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1)
 - 3*sqrt(a)*b*c*x^4*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) -
8*(a*d + 3*a*f)*sqrt(a)*x^4*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)),
 -1) + 2*(12*a*f*x^3 + 6*a*e*x^2 + 4*a*d*x + 3*a*c)*sqrt(b*x^4 + a))/(a^2*x
^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.41 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.46

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{4ax^2} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{d\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{f\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} + \frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{3}{2}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**5/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*a*x**2) - sqrt(b)*e*sqrt(a/(b*x**4) + 1)
/(2*a) + d*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)
/(4*sqrt(a)*x**3*gamma(1/4)) + f*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x
**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) + b*c*asinh(sqrt(a)/(sqrt(b
)*x**2))/(4*a**(3/2))
```

Maxima [F]

$$\int \frac{c + dx + ex^2 + fx^3}{x^5\sqrt{a + bx^4}}\,dx = \int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^5}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] -1/8*c*(2*sqrt(b*x^4 + a)*b/((b*x^4 + a)*a - a^2) + b*log((sqrt(b*x^4 + a)
- sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/a^(3/2)) + integrate((f*x^2 + e*x +
 d)/(sqrt(b*x^4 + a)*x^4), x)
```

Giac [F]

$$\int \frac{c + dx + ex^2 + fx^3}{x^5\sqrt{a + bx^4}}\,dx = \int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^5}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3}{x^5\sqrt{a + bx^4}}\,dx = \int \frac{f\,x^3 + e\,x^2 + d\,x + c}{x^5\,\sqrt{b\,x^4 + a}}\,dx$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^5*(a + b*x^4)^(1/2)),x)
[Out] int((c + d*x + e*x^2 + f*x^3)/(x^5*(a + b*x^4)^(1/2)), x)
```

3.539 $\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 377

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = -\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2}+\frac{3bc\sqrt{a+bx^4}}{5a^2x}-\frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{bd\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}+\frac{3b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{a+bx^4}}-\frac{b^{3/4}\left(9\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30a^{7/4}\sqrt{a+bx^4}}$$

```
[Out] 1/4*b*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/5*c*(b*x^4+a)^(1/2)/a/x^
5-1/4*d*(b*x^4+a)^(1/2)/a/x^4-1/3*e*(b*x^4+a)^(1/2)/a/x^3-1/2*f*(b*x^4+a)^(
1/2)/a/x^2+3/5*b*c*(b*x^4+a)^(1/2)/a^2/x-3/5*b^(3/2)*c*x*(b*x^4+a)^(1/2)/a^
2/(a^(1/2)+x^2*b^(1/2))+3/5*b^(5/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^
2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)-1/30*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/
4)*x/a^(1/4))),1/2*2^(1/2))*(5*e*a^(1/2)+9*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))
*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1847, 1296, 1212, 226, 1210, 1266, 849, 821, 272, 65, 214}

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx$$
$$= -\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+9\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30a^{7/4}\sqrt{a+bx^4}}$$
$$+\frac{3b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{a+bx^4}}$$
$$+\frac{bd\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3bc\sqrt{a+bx^4}}{5a^2x}$$
$$-\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^6*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/5*(c*Sqrt[a + b*x^4])/(a*x^5) - (d*Sqrt[a + b*x^4])/(4*a*x^4) - (e*Sqrt[
a + b*x^4])/(3*a*x^3) - (f*Sqrt[a + b*x^4])/(2*a*x^2) + (3*b*c*Sqrt[a + b*x
^4])/(5*a^2*x) - (3*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*a^2*(Sqrt[a] + Sqrt[b]*
x^2)) + (b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(3/2)) + (3*b^(5/4)*c*(
Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elliptic
E[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[a + b*x^4]) - (b^(3/
4)*(9*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sq
rt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*
a^(7/4)*Sqrt[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{c+ex^2}{x^6\sqrt{a+bx^4}}+\frac{d+fx^2}{x^5\sqrt{a+bx^4}}\right)dx \\
&= \int \frac{c+ex^2}{x^6\sqrt{a+bx^4}}\,dx+\int \frac{d+fx^2}{x^5\sqrt{a+bx^4}}\,dx \\
&= -\frac{c\sqrt{a+bx^4}}{5ax^5}+\frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{x^3\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{-5ae+3bcx^2}{x^4\sqrt{a+bx^4}}\,dx}{5a} \\
&= -\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}+\frac{\int \frac{-9abc-5abex^2}{x^2\sqrt{a+bx^4}}\,dx}{15a^2}-\frac{\text{Subst}\left(\int \frac{-2af+bdx}{x^2\sqrt{a+bx^2}}\,dx,x,x^2\right)}{4a} \\
&= -\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2} \\
&\quad +\frac{3bc\sqrt{a+bx^4}}{5a^2x}-\frac{\int \frac{5a^2be+9ab^2cx^2}{\sqrt{a+bx^4}}\,dx}{15a^3}-\frac{(bd)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)}{4a} \\
&= -\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3} \\
&\quad -\frac{f\sqrt{a+bx^4}}{2ax^2}+\frac{3bc\sqrt{a+bx^4}}{5a^2x}+\frac{\left(3b^{3/2}c\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{5a^{3/2}} \\
&\quad -\frac{(bd)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)}{8a}-\frac{\left(b\left(9\sqrt{b}c+5\sqrt{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{15a^{3/2}}
\end{aligned}
$$

$$= -\frac{c\sqrt{a+bx^4}}{5ax^5} - \frac{d\sqrt{a+bx^4}}{4ax^4} - \frac{e\sqrt{a+bx^4}}{3ax^3} - \frac{f\sqrt{a+bx^4}}{2ax^2} + \frac{3bc\sqrt{a+bx^4}}{5a^2x} - \frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{3b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{a+bx^4}} - \frac{b^{3/4}\left(9\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30a^{7/4}\sqrt{a+bx^4}} - \frac{d\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)}{4a}$$

$$= -\frac{c\sqrt{a+bx^4}}{5ax^5} - \frac{d\sqrt{a+bx^4}}{4ax^4} - \frac{e\sqrt{a+bx^4}}{3ax^3} - \frac{f\sqrt{a+bx^4}}{2ax^2} + \frac{3bc\sqrt{a+bx^4}}{5a^2x} - \frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}} + \frac{3b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{a+bx^4}} - \frac{b^{3/4}\left(9\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30a^{7/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.36

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}\left(12ac\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},-\frac{bx^4}{a}\right)+5x\left(3a(d+2fx^2)\sqrt{1+\frac{bx^4}{a}}-3bdx^4\text{arctanh}\left(\right.\right.\right.}{60a^2x^5\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^6*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/60*(Sqrt[a + b*x^4]*(12*a*c*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^4)
/a)] + 5*x*(3*a*(d + 2*f*x^2)*Sqrt[1 + (b*x^4)/a] - 3*b*d*x^4*ArcTanh[Sqrt[
1 + (b*x^4)/a]] + 4*a*e*x*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^4)/a)]))
)/(a^2*x^5*Sqrt[1 + (b*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(-36bc\,x^4+30af\,x^3+20ae\,x^2+15adx+12ac)}{60a^2x^5}-\frac{b\left(\frac{10ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{18i\sqrt{b}\,c\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1-}}{}\right.}{30}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{5a\,x^5}-\frac{d\sqrt{b\,x^4+a}}{4a\,x^4}-\frac{e\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{f\sqrt{b\,x^4+a}}{2a\,x^2}+\frac{3bc\sqrt{b\,x^4+a}}{5a^2x}-\frac{be\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}-\frac{3ib^{\frac{3}{2}}c\sqrt{}}{}$
default	$e\left(-\frac{\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)-\frac{f\sqrt{b\,x^4+a}}{2a\,x^2}+d\left(-\frac{\sqrt{b\,x^4+a}}{4a\,x^4}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\sqrt{b\,x^4+a}}{x^2}\right)}{4a^{\frac{3}{2}}}\right)-$

```
[In] int((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(b*x^4+a)^(1/2)*(-36*b*c*x^4+30*a*f*x^3+20*a*e*x^2+15*a*d*x+12*a*c)/a
^2/x^5-1/30*b/a^2*(10*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(
1/2)*b^(1/2))^(1/2),I)+18*I*b^(1/2)*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-
I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2
)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))
^(1/2),I))-15/2*a^(1/2)*d*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.42

$$\int\frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx$$
$$=\frac{72\sqrt{a}bcx^5\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right)+15\sqrt{a}bdx^5\log\left(-\frac{bx^4+2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-8\,(9\,bc-5\,ae)\sqrt{a}x^5}{120\,a^2x^5}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/120*(72*sqrt(a)*b*c*x^5*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -
1) + 15*sqrt(a)*b*d*x^5*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4)
 - 8*(9*b*c - 5*a*e)*sqrt(a)*x^5*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1
/4)), -1) + 2*(36*b*c*x^4 - 30*a*f*x^3 - 20*a*e*x^2 - 15*a*d*x - 12*a*c)*sq
rt(b*x^4 + a))/(a^2*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.43

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{4ax^2} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{c\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{e\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{3}{2}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**6/(b*x**4+a)**(1/2),x)
[Out] -sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*a*x**2) - sqrt(b)*f*sqrt(a/(b*x**4) + 1)
/(2*a) + c*gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*sqrt(a)*x**5*gamma(-1/4)) + e*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b
*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4)) + b*d*asinh(sqrt(a)/(s
qrt(b)*x**2))/(4*a**(3/2))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^6}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^6}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx=\int \frac{f\,x^3+e\,x^2+d\,x+c}{x^6\,\sqrt{b\,x^4+a}}\,dx$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^6*(a + b*x^4)^(1/2)),x)
[Out] int((c + d*x + e*x^2 + f*x^3)/(x^6*(a + b*x^4)^(1/2)), x)
```

3.540 $\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 365

$$\begin{aligned}\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx &= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}}+\frac{d\sqrt{a+bx^4}}{b^2} \\ &+\frac{ex\sqrt{a+bx^4}}{3b^2}+\frac{fx^2\sqrt{a+bx^4}}{4b^2}+\frac{3cx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{3af\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{5/2}} \\ &-\frac{3\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} \\ &+\frac{\sqrt[4]{a}\left(9\sqrt{b}c-5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}\end{aligned}$$

```
[Out] -3/4*a*f*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(5/2)+1/2*x*(-b*d*x^3-b*c*x
^2+a*f*x+a*e)/b^2/(b*x^4+a)^(1/2)+d*(b*x^4+a)^(1/2)/b^2+1/3*e*x*(b*x^4+a)^(
1/2)/b^2+1/4*f*x^2*(b*x^4+a)^(1/2)/b^2+3/2*c*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(
1/2)+x^2*b^(1/2))-3/2*a^(1/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/
cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4)))
,1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/
2)/b^(7/4)/(b*x^4+a)^(1/2)+1/12*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2
)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a
^(1/4))),1/2*2^(1/2))*(-5*e*a^(1/2)+9*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*
x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1842, 1899, 1902, 1212, 226, 1210, 1833, 1829, 655, 223, 212}

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{b}c-5\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{12b^{9/4}\sqrt{a+bx^4}}$$
$$-\frac{3\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}$$
$$-\frac{3af\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{5/2}}+\frac{3cx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$+\frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}}+\frac{d\sqrt{a+bx^4}}{b^2}+\frac{ex\sqrt{a+bx^4}}{3b^2}+\frac{fx^2\sqrt{a+bx^4}}{4b^2}$$

```
[In] Int[(x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (d*Sqrt[a +
 b*x^4])/b^2 + (e*x*Sqrt[a + b*x^4])/(3*b^2) + (f*x^2*Sqrt[a + b*x^4])/(4*b
^2) + (3*c*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) - (3*a*f*
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(5/2)) - (3*a^(1/4)*c*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(9*S
qrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(12*b^(9/4)
*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
```

```
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}} - \frac{\int \frac{a^2be+2a^2bfx-3ab^2cx^2-4ab^2dx^3-2ab^2ex^4-2ab^2fx^5}{\sqrt{a+bx^4}}\,dx}{2ab^3}\\
&= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}} - \frac{\int \left(\frac{a^2be-3ab^2cx^2-2ab^2ex^4}{\sqrt{a+bx^4}} + \frac{x(2a^2bf-4ab^2dx^2-2ab^2fx^4)}{\sqrt{a+bx^4}}\right)dx}{2ab^3}\\
&= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}} - \frac{\int \frac{a^2be-3ab^2cx^2-2ab^2ex^4}{\sqrt{a+bx^4}}\,dx}{2ab^3} - \frac{\int \frac{x(2a^2bf-4ab^2dx^2-2ab^2fx^4)}{\sqrt{a+bx^4}}\,dx}{2ab^3}\\
&= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}} + \frac{ex\sqrt{a+bx^4}}{3b^2}\\
&\quad - \frac{\int \frac{5a^2b^2e-9ab^3cx^2}{\sqrt{a+bx^4}}\,dx}{6ab^4} - \frac{\text{Subst}\left(\int \frac{2a^2bf-4ab^2dx-2ab^2fx^2}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{4ab^3}\\
&= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}} + \frac{ex\sqrt{a+bx^4}}{3b^2}\\
&\quad + \frac{fx^2\sqrt{a+bx^4}}{4b^2} - \frac{\text{Subst}\left(\int \frac{6a^2b^2f-8ab^3dx}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{8ab^4}\\
&\quad - \frac{(3\sqrt{a}c)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2b^{3/2}} + \frac{\left(\sqrt{a}\left(9\sqrt{b}c-5\sqrt{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{6b^2}
\end{aligned}
$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}}+\frac{d\sqrt{a+bx^4}}{b^2}+\frac{ex\sqrt{a+bx^4}}{3b^2}+\frac{fx^2\sqrt{a+bx^4}}{4b^2}+\frac{3cx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{3\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}+\frac{\sqrt[4]{a}\left(9\sqrt{b}c-5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}-\frac{(3af)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)}{4b^2}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}}+\frac{d\sqrt{a+bx^4}}{b^2}+\frac{ex\sqrt{a+bx^4}}{3b^2}+\frac{fx^2\sqrt{a+bx^4}}{4b^2}+\frac{3cx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{3\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}+\frac{\sqrt[4]{a}\left(9\sqrt{b}c-5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}-\frac{(3af)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)}{4b^2}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}}+\frac{d\sqrt{a+bx^4}}{b^2}+\frac{ex\sqrt{a+bx^4}}{3b^2}+\frac{fx^2\sqrt{a+bx^4}}{4b^2}+\frac{3cx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{3af\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{5/2}}-\frac{3\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}+\frac{\sqrt[4]{a}\left(9\sqrt{b}c-5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.60

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{12a\sqrt{b}d+10a\sqrt{b}ex+9a\sqrt{b}fx^2+12b^{3/2}cx^3+6b^{3/2}dx^4+4b^{3/2}ex^5+3b^{3/2}}{}$$

```
[In] Integrate[(x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (12*a*Sqrt[b]*d + 10*a*Sqrt[b]*e*x + 9*a*Sqrt[b]*f*x^2 + 12*b^(3/2)*c*x^3 +
 6*b^(3/2)*d*x^4 + 4*b^(3/2)*e*x^5 + 3*b^(3/2)*f*x^6 - 9*a^(3/2)*f*Sqrt[1 +
 (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - 10*a*Sqrt[b]*e*x*Sqrt[1 + (b*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*b^(3/2)*c*x^3*Sq
rt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(12*b^(5/
2)*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.07 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.83

method	result
elliptic	$-\frac{2b\left(\frac{c\,x^3}{4b^2}-\frac{a f\,x^2}{4b^3}-\frac{a e x}{4b^3}-\frac{a d}{4b^3}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{f\,x^2\sqrt{b\,x^4+a}}{4b^2}+\frac{e x\sqrt{b\,x^4+a}}{3b^2}+\frac{d\sqrt{b\,x^4+a}}{2b^2}-\frac{5ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-$
default	$f\left(\frac{x^6}{4b\sqrt{b\,x^4+a}}+\frac{3a\,x^2}{4b^2\sqrt{b\,x^4+a}}-\frac{3a\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{4b^{\frac{5}{2}}}\right)+e\left(\frac{ax}{2b^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{x\sqrt{b\,x^4+a}}{3b^2}-\frac{5a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b}}\right.$
risch	$\frac{\left(3f\,x^2+4ex+6d\right)\sqrt{b\,x^4+a}}{12b^2}+\frac{aex}{2b^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{5ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{c\,x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3ic\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}}{\sqrt{}}}}{2b}$

```
[In] int(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(1/4*c/b^2*x^3-1/4*a*f/b^3*x^2-1/4/b^3*a*e*x-1/4*a*d/b^3)/((x^4+a/b)*b
)^(1/2)+1/4*f*x^2*(b*x^4+a)^(1/2)/b^2+1/3*e*x*(b*x^4+a)^(1/2)/b^2+1/2*d*(b*
x^4+a)^(1/2)/b^2-5/6/b^2*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)
*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/
a^(1/2)*b^(1/2))^(1/2),I)-3/4*a*f/b^(5/2)*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2
))+3/2*I*c/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a
^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.66

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{36\,(b^2cx^5+abcx)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-4\,((9\,b^2c+5\,b^2e)x^5+$$

```
[In] integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(36*(b^2*c*x^5 + a*b*c*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b
)^(1/4)/x), -1) - 4*((9*b^2*c + 5*b^2*e)*x^5 + (9*a*b*c + 5*a*b*e)*x)*sqrt(
b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 9*(a*b*f*x^5 + a^2
*f*x)*sqrt(b)*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(3*b^2*
f*x^7 + 4*b^2*e*x^6 + 6*b^2*d*x^5 + 12*b^2*c*x^4 + 9*a*b*f*x^3 + 10*a*b*e*x
^2 + 12*a*b*d*x + 18*a*b*c)*sqrt(b*x^4 + a))/(b^4*x^5 + a*b^3*x)
```

Sympy [A] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.55

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = d\left(\begin{cases}\frac{a}{b^2\sqrt{a+bx^4}}+\frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0\\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + f\left(\frac{3\sqrt{a}x^2}{4b^2\sqrt{1+\frac{bx^4}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}+\frac{x^6}{4\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)} + \frac{ex^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

```
[In] integrate(x**6*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] d*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x**4)), Ne(b,
 0)), (x**8/(8*a**(3/2)), True)) + f*(3*sqrt(a)*x**2/(4*b**2*sqrt(1 + b*x**
4/a)) - 3*a*asinh(sqrt(b)*x**2/sqrt(a))/(4*b**(5/2)) + x**6/(4*sqrt(a)*b*sq
rt(1 + b*x**4/a))) + c*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4)) + e*x**9*gamma(9/4)*hyper((3/2, 9
/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^6}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^6/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^6}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^6/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^6\,(f\,x^3+e\,x^2+d\,x+c)}{(b\,x^4+a)^{3/2}}\,dx$$

```
[In] int((x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
[Out] int((x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.541 $\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} dx$

Optimal result

Integrand size = 30, antiderivative size = 343

$$\begin{aligned} \int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} dx &= \frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} \\ &+ \frac{e\sqrt{a+bx^4}}{b^2} + \frac{fx\sqrt{a+bx^4}}{3b^2} + \frac{3dx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{c\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} \\ &- \frac{3\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} \\ &+ \frac{\sqrt[4]{a}\left(9\sqrt{b}d-5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}} \end{aligned}$$

```
[Out] 1/2*c*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/2*x*(-b*e*x^3-b*d*x^2-
b*c*x+a*f)/b^2/(b*x^4+a)^(1/2)+e*(b*x^4+a)^(1/2)/b^2+1/3*f*x*(b*x^4+a)^(1/2
)/b^2+3/2*d*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))-3/2*a^(1/4)*d*(
cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*
EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2
))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)+1/12*a
^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^
(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-5*f*a^(1/
2)+9*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(
1/2)/b^(9/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1842, 1899, 1262, 655, 223, 212, 1902, 1212, 226, 1210}

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{b}d-5\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{12b^{9/4}\sqrt{a+bx^4}}$$
$$-\frac{3\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}+\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}$$
$$+\frac{3dx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}}+\frac{e\sqrt{a+bx^4}}{b^2}+\frac{fx\sqrt{a+bx^4}}{3b^2}$$

```
[In] Int[(x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (e*Sqrt[a +
 b*x^4])/b^2 + (f*x*Sqrt[a + b*x^4])/(3*b^2) + (3*d*x*Sqrt[a + b*x^4])/(2*b
^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]
)/(2*b^(3/2)) - (3*a^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(
7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(9*Sqrt[b]*d - 5*Sqrt[a]*f)*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(12*b^(9/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
```

```
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} - \frac{\int \frac{a^2f-2abcx-3abdx^2-4abex^3-2abfx^4}{\sqrt{a+bx^4}}\,dx}{2ab^2} \\
&= \frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} - \frac{\int \left(\frac{x(-2abc-4abex^2)}{\sqrt{a+bx^4}} + \frac{a^2f-3abdx^2-2abfx^4}{\sqrt{a+bx^4}}\right)dx}{2ab^2} \\
&= \frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} - \frac{\int \frac{x(-2abc-4abex^2)}{\sqrt{a+bx^4}}\,dx}{2ab^2} - \frac{\int \frac{a^2f-3abdx^2-2abfx^4}{\sqrt{a+bx^4}}\,dx}{2ab^2} \\
&= \frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} + \frac{fx\sqrt{a+bx^4}}{3b^2} \\
&\quad - \frac{\int \frac{5a^2bf-9ab^2dx^2}{\sqrt{a+bx^4}}\,dx}{6ab^3} - \frac{\text{Subst}\left(\int \frac{-2abc-4abex}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{4ab^2} \\
&= \frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{b^2} \\
&\quad + \frac{fx\sqrt{a+bx^4}}{3b^2} + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{2b} \\
&\quad - \frac{\left(3\sqrt{a}d\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2b^{3/2}} + \frac{\left(\sqrt{a}\left(9\sqrt{b}d-5\sqrt{a}f\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{6b^2} \\
&= \frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{b^2} + \frac{fx\sqrt{a+bx^4}}{3b^2} + \frac{3dx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} \\
&\quad - \frac{3\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} \\
&\quad + \frac{\sqrt[4]{a}\left(9\sqrt{b}d-5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}} \\
&\quad + \frac{c\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)}{2b}
\end{aligned}$$

$$= \frac{x(af - bcx - bdx^2 - bex^3)}{2b^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{b^2} + \frac{fx\sqrt{a+bx^4}}{3b^2} + \frac{3dx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} - \frac{3\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} + \frac{\sqrt[4]{a}\left(9\sqrt{b}d-5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.51

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{6ae+5afx-3bcx^2+6bdx^3+3bex^4+2bfx^5+3\sqrt{a}\sqrt{b}c\sqrt{1+\frac{bx^4}{a}}\text{arcsinh}\left(\right.}{}$$

```
[In] Integrate[(x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (6*a*e + 5*a*f*x - 3*b*c*x^2 + 6*b*d*x^3 + 3*b*e*x^4 + 2*b*f*x^5 + 3*Sqrt[a
]*Sqrt[b]*c*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - 5*a*f*x*Sq
rt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 6*b*d*x^
3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*b^
2*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.82

method	result
elliptic	$-\frac{2b\left(\frac{dx^3}{4b^2}+\frac{cx^2}{4b^2}-\frac{afx}{4b^3}-\frac{ae}{4b^3}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{fx\sqrt{bx^4+a}}{3b^2}+\frac{e\sqrt{bx^4+a}}{2b^2}-\frac{5af\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}x^2}{\sqrt{a}}}F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+a}}+\frac{c\ln\left(2x^2\sqrt{b}+2\sqrt{bx^4}\right.}{2b^{\frac{3}{2}}}$
default	$f\left(\frac{ax}{2b^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{x\sqrt{bx^4+a}}{3b^2}-\frac{5a\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}x^2}{\sqrt{a}}}F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+a}}\right)+\frac{e(bx^4+2a)}{2\sqrt{bx^4+a}\,b^2}+d\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i}{}\right.$
risch	$\frac{(2fx+3e)\sqrt{bx^4+a}}{6b^2}+\frac{afx}{2b^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{5af\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}x^2}{\sqrt{a}}}F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+a}}-\frac{dx^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3id\sqrt{a}\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\sqrt{1+}}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}$

```
[In] int(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(1/4*d*x^3/b^2+1/4*c/b^2*x^2-1/4*a*f/b^3*x-1/4*a*e/b^3)/((x^4+a/b)*b)^
(1/2)+1/3*f*x*(b*x^4+a)^(1/2)/b^2+1/2*e*(b*x^4+a)^(1/2)/b^2-5/6/b^2*a*f/(I/
a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*c/
b^(3/2)*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))+3/2*I*d/b^(3/2)*a^(1/2)/(I/a^(1
/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE
(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.62

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{18\,(bdx^5+adx)\sqrt{b}(-\frac{a}{b})^{\frac{3}{4}}\,E(\arcsin\left(\frac{(-\frac{a}{b})^{\frac{1}{4}}}{x}\right)\,|-1)-2\,((9\,bd+5\,bf)x^5+(}{}$$

```
[In] integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(18*(b*d*x^5 + a*d*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1
/4)/x), -1) - 2*((9*b*d + 5*b*f)*x^5 + (9*a*d + 5*a*f)*x)*sqrt(b)*(-a/b)^(3
/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 3*(b*c*x^5 + a*c*x)*sqrt(b)*lo
g(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(2*b*f*x^6 + 3*b*e*x^5
+ 6*b*d*x^4 - 3*b*c*x^3 + 5*a*f*x^2 + 6*a*e*x + 9*a*d)*sqrt(b*x^4 + a))/(b^
3*x^5 + a*b^2*x)
```

Sympy [A] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.50

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = c\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + e\left(\begin{cases}\frac{a}{b^2\sqrt{a+bx^4}}+\frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0\\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + \frac{dx^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)} + \frac{fx^9\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

```
[In] integrate(x**5*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] c*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b*
x**4/a))) + e*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x
**4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True)) + d*x**7*gamma(7/4)*hyper((3/2
, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4)) + f*x**
9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/
2)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^5}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*c*(2*x^2/(sqrt(b*x^4 + a)*b) + log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(s
qrt(b) + sqrt(b*x^4 + a)/x^2))/b^(3/2)) + integrate((f*x^8 + e*x^7 + d*x^6)
/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^5}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^5/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^5\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{{\left(b\,x^4+a\right)}^{3/2}}\,dx$$

```
[In] int((x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

```
[Out] int((x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.542 $\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 314

$$\begin{aligned}\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = & -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} \\ & +\frac{f\sqrt{a+bx^4}}{b^2}+\frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} \\ & -\frac{3\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} \\ & +\frac{\left(\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

```
[Out] 1/2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)-1/2*x*(f*x^3+e*x^2+d*x+c
)/b/(b*x^4+a)^(1/2)+f*(b*x^4+a)^(1/2)/b^2+3/2*e*x*(b*x^4+a)^(1/2)/b^(3/2)/(
a^(1/2)+x^2*b^(1/2))-3/2*a^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/
2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4
))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^
(1/2)/b^(7/4)/(b*x^4+a)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4)
)),1/2*2^(1/2))*(3*e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a
^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1842, 1899, 1212, 226, 1210, 1262, 655, 223, 212}

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}} - \frac{3\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} + \frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} + \frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{f\sqrt{a+bx^4}}{b^2} - \frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}}$$

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] -1/2*(x*(c + d*x + e*x^2 + f*x^3))/(b*Sqrt[a + b*x^4]) + (f*Sqrt[a + b*x^4]
)/b^2 + (3*e*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (d*Ar
cTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2)) - (3*a^(1/4)*e*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + 3
*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(7/4)*Sqrt
[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\text{integral} = -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} - \frac{\int \frac{-abc-2abdx-3abex^2-4abfx^3}{\sqrt{a+bx^4}}\,dx}{2ab^2}$$

$$= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} - \frac{\int \left(\frac{-abc-3abex^2}{\sqrt{a+bx^4}} + \frac{x(-2abd-4abfx^2)}{\sqrt{a+bx^4}}\right)dx}{2ab^2}$$

$$= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} - \frac{\int \frac{-abc-3abex^2}{\sqrt{a+bx^4}}\,dx}{2ab^2} - \frac{\int \frac{x(-2abd-4abfx^2)}{\sqrt{a+bx^4}}\,dx}{2ab^2}$$

$$= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} - \frac{\text{Subst}\left(\int \frac{-2abd-4abfx}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{4ab^2} - \frac{\left(3\sqrt{a}e\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2b^{3/2}} + \frac{\left(\sqrt{b}c+3\sqrt{a}e\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2b^{3/2}}$$

$$= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{b^2} + \frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{3\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}} + \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{2b}$$

$$= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{b^2} + \frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{3\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}} + \frac{d\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)}{2b}$$

$$= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{b^2} + \frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} - \frac{3\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.53

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{2af-bcx-bdx^2+2bex^3+bfx^4+\sqrt{a}\sqrt{b}d\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+bcx}{}$$

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (2*a*f - b*c*x - b*d*x^2 + 2*b*e*x^3 + b*f*x^4 + Sqrt[a]*Sqrt[b]*d*Sqrt[1 +
 (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + b*c*x*Sqrt[1 + (b*x^4)/a]*Hype
rgeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 2*b*e*x^3*Sqrt[1 + (b*x^4)/a]*
Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(2*b^2*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.84

method	result
elliptic	$-\frac{2b\left(\frac{e\,x^3}{4b^2}+\frac{d\,x^2}{4b^2}+\frac{cx}{4b^2}-\frac{af}{4b^3}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{f\sqrt{b\,x^4+a}}{2b^2}+\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{d\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2b^{\frac{3}{2}}}+\frac{3ie\sqrt{a}\,\sqrt{}}{}$
default	$\frac{f\left(b\,x^4+2a\right)}{2\sqrt{b\,x^4+a}\,b^2}+e\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+d\left(-\frac{x^2}{2b\sqrt{b\,x^4+}}\right.$
risch	$\frac{f\sqrt{b\,x^4+a}}{2b^2}+\frac{be\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+bd\left(-\frac{x^2}{2b\sqrt{b\,x^4+a}}+\frac{\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4}\right.}{2b^{\frac{3}{2}}}\right.}{b}$

```
[In] int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(1/4*e/b^2*x^3+1/4*d*x^2/b^2+1/4*c/b^2*x-1/4*a*f/b^3)/((x^4+a/b)*b)^(1
/2)+1/2*f*(b*x^4+a)^(1/2)/b^2+1/2*c/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2
)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellipt
icF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*d/b^(3/2)*ln(2*x^2*b^(1/2)+2*(b*x^4+
a)^(1/2))+3/2*I/b^(3/2)*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.71

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{6\,(abex^5+a^2ex)\sqrt{b}(-\frac{a}{b})^{\frac{3}{4}}\,E(\arcsin\left(\frac{(-\frac{a}{b})^{\frac{1}{4}}}{x}\right)\,|-1)+2\,((b^2c-3\,abe)x^5+(a$$

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(6*(a*b*e*x^5 + a^2*e*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^
(1/4)/x), -1) + 2*((b^2*c - 3*a*b*e)*x^5 + (a*b*c - 3*a^2*e)*x)*sqrt(b)*(-a
/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + (a*b*d*x^5 + a^2*d*x)*sq
rt(b)*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(a*b*f*x^5 + 2*
a*b*e*x^4 - a*b*d*x^3 - a*b*c*x^2 + 2*a^2*f*x + 3*a^2*e)*sqrt(b*x^4 + a))/(
a*b^3*x^5 + a^2*b^2*x)
```

Sympy [A] (verification not implemented)

Time = 7.20 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.55

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = d\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + f\left(\begin{cases}\frac{a}{b^2\sqrt{a+bx^4}}+\frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0\\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + \frac{cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

```
[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

[Out] d*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b*x**4/a))) + f*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True)) + c*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4))

Maxima [F]

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^4}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^4 + a)^(3/2), x)

Giac [F]

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^4}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^4 + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^4\,(f\,x^3+e\,x^2+d\,x+c)}{(b\,x^4+a)^{3/2}}\,dx$$

[In] int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)

[Out] int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)

3.543 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 302

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{-c-dx-ex^2-fx^3}{2b\sqrt{a+bx^4}} + \frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} - \frac{3\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}$$

```
[Out] 1/2*e*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/2*(-f*x^3-e*x^2-d*x-c)
/b/(b*x^4+a)^(1/2)+3/2*f*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))-3/
2*a^(1/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)
*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/
2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)
^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*f*a^
(1/2)+d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^
(1/2)/a^(1/4)/b^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1837, 1899, 281, 223, 212, 1212, 226, 1210}

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}$$
$$-\frac{3\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}$$
$$+\frac{e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}$$

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] -1/2*(c + d*x + e*x^2 + f*x^3)/(b*Sqrt[a + b*x^4]) + (3*f*x*Sqrt[a + b*x^4]
)/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b
*x^4]])/(2*b^(3/2)) - (3*a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)
/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(2*b^(7/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1837

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{\int \frac{d+2ex+3fx^2}{\sqrt{a+bx^4}}\,dx}{2b}\\
&= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{\int \left(\frac{2ex}{\sqrt{a+bx^4}}+\frac{d+3fx^2}{\sqrt{a+bx^4}}\right)dx}{2b}\\
&= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{\int \frac{d+3fx^2}{\sqrt{a+bx^4}}\,dx}{2b}+\frac{e\int \frac{x}{\sqrt{a+bx^4}}\,dx}{b}
\end{aligned}$$

$$= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}} + \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{2b} - \frac{\left(3\sqrt{a}f\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2b^{3/2}} + \frac{\left(d+\frac{3\sqrt{a}f}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2b}$$

$$= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}} + \frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{3\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}} + \frac{e\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)}{2b}$$

$$= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}} + \frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} - \frac{3\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.60

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{-\sqrt{b}c-\sqrt{b}dx-\sqrt{b}ex^2+2\sqrt{b}fx^3+\sqrt{a}e\sqrt{1+\frac{bx^4}{a}}\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+\sqrt{b}dx$$

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[b]*c) - Sqrt[b]*d*x - Sqrt[b]*e*x^2 + 2*Sqrt[b]*f*x^3 + Sqrt[a]*e*S
qrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + Sqrt[b]*d*x*Sqrt[1 + (b
*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 2*Sqrt[b]*f*x^3*S
qrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(2*b^(3/
2)*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.82

method	result
elliptic	$-\frac{2b\left(\frac{f\,x^3}{4b^2}+\frac{e\,x^2}{4b^2}+\frac{dx}{4b^2}+\frac{c}{4b^2}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{e\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2b^{\frac{3}{2}}}+\frac{3if\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+}}{2}$
default	$f\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+e\left(-\frac{x^2}{2b\sqrt{b\,x^4+a}}+\frac{\ln\left(x^2\sqrt{b}+\sqrt{b}\right.}{2b^{\frac{3}{2}}}\right.$

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(1/4*f*x^3/b^2+1/4*e/b^2*x^2+1/4*d*x/b^2+1/4*c/b^2)/((x^4+a/b)*b)^(1/2
)+1/2*d/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1
/2),I)+1/2/b^(3/2)*e*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))+3/2*I*f/b^(3/2)*a^
(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2)
,I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.71

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx=\frac{6\,(abfx^5+a^2fx)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+2\,((b^2d-3\,abf)x^5+}{}$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(6*(a*b*f*x^5 + a^2*f*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^
(1/4)/x), -1) + 2*((b^2*d - 3*a*b*f)*x^5 + (a*b*d - 3*a^2*f)*x)*sqrt(b)*(-a
/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + (a*b*e*x^5 + a^2*e*x)*sq
rt(b)*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(2*a*b*f*x^4 -
a*b*e*x^3 - a*b*d*x^2 - a*b*c*x + 3*a^2*f)*sqrt(b*x^4 + a))/(a*b^3*x^5 + a^
2*b^2*x)
```

Sympy [A] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.52

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = c\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + e\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] c*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + e*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1
 + b*x**4/a))) + d*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + f*x**7*gamma(7/4)*hyper((3/2, 7/4), (
11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^3}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*c/(sqrt(b*x^4 + a)*b) + integrate((f*x^6 + e*x^5 + d*x^4)/(b*x^4 + a)^
(3/2), x)
```

Giac [F]

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^3}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^3\,(f\,x^3+e\,x^2+d\,x+c)}{(b\,x^4+a)^{3/2}}\,dx$$

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
[Out] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.544 $\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 333

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{d\sqrt{a+bx^4}}{2ab} - \frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} + \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}$$

```
[Out] 1/2*f*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)-1/2*x*(-b*d*x^3-b*c*x^2+
a*f*x+a*e)/a/b/(b*x^4+a)^(1/2)-1/2*d*(b*x^4+a)^(1/2)/a/b-1/2*c*x*(b*x^4+a)^
(1/2)/a/b^(1/2)/(a^(1/2)+x^2*b^(1/2))+1/2*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)
*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(
1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)-1/4*(cos(2*arctan(b^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(
b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2
))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/b^(5/4)/(b*x^4+a)^(1/2
)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1842, 1899, 1212, 226, 1210, 1262, 655, 223, 212}

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = -\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}} + \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} + \frac{f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} - \frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{d\sqrt{a+bx^4}}{2ab}$$

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] -1/2*(x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(a*b*Sqrt[a + b*x^4]) - (d*Sqrt[
a + b*x^4])/(2*a*b) - (c*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]
*x^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2)) + (c*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*A
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) - ((S
qrt[b]*c - Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(3/4)*b^
(5/4)*Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\text{integral} = -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{\int \frac{-abe-2abfx+b^2cx^2+2b^2dx^3}{\sqrt{a+bx^4}}\,dx}{2ab^2}$$

$$= -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{\int \left(\frac{-abe+b^2cx^2}{\sqrt{a+bx^4}} + \frac{x(-2abf+2b^2dx^2)}{\sqrt{a+bx^4}}\right)dx}{2ab^2}$$

$$= -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{\int \frac{-abe+b^2cx^2}{\sqrt{a+bx^4}}\,dx}{2ab^2} - \frac{\int \frac{x(-2abf+2b^2dx^2)}{\sqrt{a+bx^4}}\,dx}{2ab^2}$$

$$= -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{\text{Subst}\left(\int \frac{-2abf+2b^2dx}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{4ab^2} + \frac{c\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2\sqrt{a}\sqrt{b}} - \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-e\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2b}$$

$$= -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{d\sqrt{a+bx^4}}{2ab} - \frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}} + \frac{f\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{2b}$$

$$= -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{d\sqrt{a+bx^4}}{2ab} - \frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}} + \frac{f\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)}{2b}$$

$$= -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{d\sqrt{a+bx^4}}{2ab} - \frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$+ \frac{f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} + \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$
$$- \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.50

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{-3a\sqrt{b}(d+x(e+fx))+3a^{3/2}f\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+3a\sqrt{b}ex\sqrt{1+\frac{b}{}}$$

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (-3*a*Sqrt[b]*(d + x*(e + f*x)) + 3*a^(3/2)*f*Sqrt[1 + (b*x^4)/a]*ArcSinh[(
Sqrt[b]*x^2)/Sqrt[a]] + 3*a*Sqrt[b]*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b^(3/2)*c*x^3*Sqrt[1 + (b*x^4)/a]*Hyper
geometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b^(3/2)*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.75

method	result
elliptic	$-\frac{2b\left(-\frac{c\,x^3}{4ba}+\frac{f\,x^2}{4b^2}+\frac{ex}{4b^2}+\frac{d}{4b^2}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{f\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2b^{\frac{3}{2}}}-\frac{ic\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+}}{2}$
default	$f\left(-\frac{x^2}{2b\sqrt{b\,x^4+a}}+\frac{\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{2b^{\frac{3}{2}}}\right)+e\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)-\frac{d}{2b\sqrt{b\,x^4}}$

```
[In] int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(-1/4/b/a*c*x^3+1/4*f*x^2/b^2+1/4*e/b^2*x+1/4*d/b^2)/((x^4+a/b)*b)^(1/
2)+1/2/b*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
```

```
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)+1/2*f/b^(3/2)*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))-1/2*I*c/a^(1/2)/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.60

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = -\frac{2\left(b^2cx^5+abcx\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-2\left(\left(b^2c+b^2e\right)x^5+\left(abc+abe\right)x\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\right.\right.}{4\,(a}$$

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2*c*x^5 + a*b*c*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)
^(1/4)/x), -1) - 2*((b^2*c + b^2*e)*x^5 + (a*b*c + a*b*e)*x)*sqrt(b)*(-a/b)
^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) - (a*b*f*x^5 + a^2*f*x)*sqrt(
b)*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(a*b*f*x^3 + a*b*e
*x^2 + a*b*d*x + a*b*c)*sqrt(b*x^4 + a))/(a*b^3*x^5 + a^2*b^2*x)
```

Sympy [A] (verification not implemented)

Time = 5.90 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.47

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = d\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for}\ b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + f\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] d*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + f*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1
 + b*x**4/a))) + c*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + e*x**5*gamma(5/4)*hyper((5/4, 3/2), (
9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^2}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^2}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^2\,(f\,x^3+e\,x^2+d\,x+c)}{(b\,x^4+a)^{3/2}}\,dx$$

```
[In] int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
[Out] int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.545 $\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 303

$$\begin{aligned}\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx &= -\frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}}-\frac{e\sqrt{a+bx^4}}{2ab}\\ &\quad -\frac{dx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}\\ &\quad -\frac{\left(\sqrt{b}d-\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}\end{aligned}$$

```
[Out] -1/2*x*(-b*e*x^3-b*d*x^2-b*c*x+a*f)/a/b/(b*x^4+a)^(1/2)-1/2*e*(b*x^4+a)^(1/
2)/a/b-1/2*d*x*(b*x^4+a)^(1/2)/a/b^(1/2)/(a^(1/2)+x^2*b^(1/2))+1/2*d*(cos(2
*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ellip
ticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((
b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)-1/4
*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4))
)*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-f*a^(1/2)+d*b^(
1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/
4)/b^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1842, 1899, 267, 1212, 226, 1210}

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = -\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}} + \frac{d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} - \frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}} - \frac{dx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{e\sqrt{a+bx^4}}{2ab}$$

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] -1/2*(x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(a*b*Sqrt[a + b*x^4]) - (e*Sqrt[
a + b*x^4])/(2*a*b) - (d*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]
*x^2)) + (d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt
[a + b*x^4]) - ((Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1
/2])/(4*a^(3/4)*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1842

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] + Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n]
+ 1))), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
 && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}}-\frac{\int \frac{-af+bdx^2+2bex^3}{\sqrt{a+bx^4}}\,dx}{2ab}\\
&= -\frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}}-\frac{\int \left(\frac{2bex^3}{\sqrt{a+bx^4}}+\frac{-af+bdx^2}{\sqrt{a+bx^4}}\right)dx}{2ab}\\
&= -\frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}}-\frac{\int \frac{-af+bdx^2}{\sqrt{a+bx^4}}\,dx}{2ab}-\frac{e\int \frac{x^3}{\sqrt{a+bx^4}}\,dx}{a}\\
&= -\frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}}-\frac{e\sqrt{a+bx^4}}{2ab}+\frac{d\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2\sqrt{a}\sqrt{b}}-\frac{\left(\frac{\sqrt{b}d}{\sqrt{a}}-f\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2b}
\end{aligned}$$

$$= -\frac{x(af - bcx - bdx^2 - bex^3)}{2ab\sqrt{a+bx^4}} - \frac{e\sqrt{a+bx^4}}{2ab} - \frac{dx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$+ \frac{d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$
$$- \frac{\left(\sqrt{b}d - \sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.38

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{-3ae - 3afx + 3bcx^2 + 3afx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{6ab\sqrt{a+bx^4}}$$

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (-3*a*e - 3*a*f*x + 3*b*c*x^2 + 3*a*f*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*d*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeomet
ric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.75

method	result
elliptic	$-\frac{2b\left(-\frac{d\,x^3}{4ab}-\frac{c\,x^2}{4ba}+\frac{fx}{4b^2}+\frac{e}{4b^2}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}-\frac{id\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\right.\right.}{2\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}\sqrt{b}}$
default	$f\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)-\frac{e}{2b\sqrt{b\,x^4+a}}+d\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+}}{2}\right.$

```
[In] int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(-1/4/a/b*d*x^3-1/4/b/a*c*x^2+1/4*f*x/b^2+1/4*e/b^2)/((x^4+a/b)*b)^(1/
2)+1/2*f/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(
```

```
1/2),I)-1/2*I*d/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.49

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{(b^2dx^4+abd)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right)-((b^2d+abf)x^4+abd+}{2\,(ab}$$

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((b^2*d*x^4 + a*b*d)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1
/4)), -1) - ((b^2*d + a*b*f)*x^4 + a*b*d + a^2*f)*sqrt(a)*(-b/a)^(3/4)*elli
ptic_f(arcsin(x*(-b/a)^(1/4)), -1) + (b^2*d*x^3 + b^2*c*x^2 - a*b*f*x - a*b
*e)*sqrt(b*x^4 + a))/(a*b^3*x^4 + a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.44

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = e\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}} + \frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] e*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + c*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a)) + d*x**3*gamma(3/4)*hyper((3/4,
 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + f*x**5*g
amma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*g
amma(9/4))
```

Maxima [F]

$$\int \frac{x(c + dx + ex^2 + fx^3)}{(a + bx^4)^{3/2}} \, dx = \int \frac{(fx^3 + ex^2 + dx + c)x}{(bx^4 + a)^{\frac{3}{2}}} \, dx$$

[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")

[Out] 1/2*c*x^2/(sqrt(b*x^4 + a)*a) + integrate((f*x^4 + e*x^3 + d*x^2)/(b*x^4 + a)^(3/2), x)

Giac [F]

$$\int \frac{x(c + dx + ex^2 + fx^3)}{(a + bx^4)^{3/2}} \, dx = \int \frac{(fx^3 + ex^2 + dx + c)x}{(bx^4 + a)^{\frac{3}{2}}} \, dx$$

[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x/(b*x^4 + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c + dx + ex^2 + fx^3)}{(a + bx^4)^{3/2}} \, dx = \int \frac{x \left(f\,x^3 + e\,x^2 + d\,x + c\right)}{\left(b\,x^4 + a\right)^{3/2}} \, dx$$

[In] int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)

[Out] int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)

3.546 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 275

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = -\frac{ex\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}} + \frac{e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] 1/2*(-a*f+b*x*(e*x^2+d*x+c))/a/b/(b*x^4+a)^(1/2)-1/2*e*x*(b*x^4+a)^(1/2)/a/
b^(1/2)/(a^(1/2)+x^2*b^(1/2))+1/2*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)
^(1/2)/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*
x/a^(1/4))),1/2*2^(1/2))*(-e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x
^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/b^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1868, 1212, 226, 1210}

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = \frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$
$$-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}-\frac{ex\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x]
```

```
[Out] -1/2*(e*x*Sqrt[a + b*x^4])/(a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) - (a*f - b*x
*(c + d*x + e*x^2))/(2*a*b*Sqrt[a + b*x^4]) + (e*(Sqrt[a] + Sqrt[b]*x^2)*Sq
rt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^
(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c - Sqrt[a]*e
)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(3/4)*Sqrt[a + b*x^4
])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

```
d, e}, x] && PosQ[c/a]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}-\frac{\int \frac{-c+ex^2}{\sqrt{a+bx^4}}\,dx}{2a}\\
&= -\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}+\frac{e\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2\sqrt{a}\sqrt{b}}+\frac{\left(c-\frac{\sqrt{a}e}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2a}\\
&= -\frac{ex\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}\\
&\quad+\frac{e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}\\
&\quad+\frac{\left(c-\frac{\sqrt{a}e}{\sqrt{b}}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+bx^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.42

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx=\frac{-3af+3bcx+3bdx^2+3bcx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)+2be}{6ab\sqrt{a+bx^4}}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x]
```

```
[Out] (-3*a*f + 3*b*c*x + 3*b*d*x^2 + 3*b*c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeomet
ric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.84

method	result
elliptic	$-\frac{2b\left(-\frac{e\,x^{3}}{4ba}-\frac{d\,x^{2}}{4ab}-\frac{cx}{4ba}+\frac{f}{4b^{2}}\right)}{\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{ie\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\right.\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}$
default	$c\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)-\frac{f}{2b\sqrt{b\,x^{4}+a}}+e\left(\frac{x^{3}}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}}{2\sqrt{a}}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(-1/4/b/a*e*x^3-1/4/a/b*d*x^2-1/4/b/a*c*x+1/4*f/b^2)/((x^4+a/b)*b)^(1/
2)+1/2*c/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)-1/2*I/a^(1/2)*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.47

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = \frac{(bex^4+ae)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-\left((bc+be)x^4+ac+ae\right)\sqrt{a}(}{2\left(ab^2x^4+a\right.}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((b*e*x^4 + a*e)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4))
, -1) - ((b*c + b*e)*x^4 + a*c + a*e)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsi
n(x*(-b/a)^(1/4)), -1) + (b*e*x^3 + b*d*x^2 + b*c*x - a*f)*sqrt(b*x^4 + a))
/(a*b^2*x^4 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.48

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = f\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + c*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(3/2)*gamma(5/4)) + d*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a)) + e*x**3*gamm
a(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamm
a(7/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = \int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = \int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = \int \frac{f\,x^3+e\,x^2+d\,x+c}{(b\,x^4+a)^{3/2}}\,dx$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x)
[Out] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2), x)
```

3.547 $\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 323

$$\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}}\,dx = \frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}} + \frac{c\sqrt{a+bx^4}}{2a^2} - \frac{fx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}d-\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -1/2*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)+1/2*x*(-b*c*x^3+a*f*x^2+a*e
*x+a*d)/a^2/(b*x^4+a)^(1/2)+1/2*c*(b*x^4+a)^(1/2)/a^2-1/2*f*x*(b*x^4+a)^(1/
2)/a/b^(1/2)/(a^(1/2)+x^2*b^(1/2))+1/2*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2
))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(
1/4)*x/a^(1/4))),1/2*2^(1/2))*(-f*a^(1/2)+d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*
((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/b^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1843, 1846, 272, 65, 214, 1899, 267, 1212, 226, 1210}

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx=\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}+\frac{f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}}+\frac{c\sqrt{a+bx^4}}{2a^2}-\frac{fx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x]
```

```
[Out] (x*(a*d + a*e*x + a*f*x^2 - b*c*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (c*Sqrt[a +
 b*x^4])/(2*a^2) - (f*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^
2)) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) + (f*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d
- Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(3/4)*Sqr
t[a + b*x^4])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\text{integral} = \frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-bdx+bfx^3-\frac{2b^2cx^4}{a}}{x\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= \frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-bd+bfx^2-\frac{2b^2cx^3}{a}}{\sqrt{a+bx^4}}\,dx}{2ab} + \frac{c\int \frac{1}{x\sqrt{a+bx^4}}\,dx}{a}$$

$$= \frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \left(-\frac{2b^2cx^3}{a\sqrt{a+bx^4}} + \frac{-bd+bfx^2}{\sqrt{a+bx^4}}\right)dx}{2ab} + \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^4\right)}{4a}$$

$$= \frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-bd+bfx^2}{\sqrt{a+bx^4}}\,dx}{2ab} + \frac{c\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)}{2ab} + \frac{(bc)\int \frac{x^3}{\sqrt{a+bx^4}}\,dx}{a^2}$$

$$= \frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}} + \frac{c\sqrt{a+bx^4}}{2a^2} - \frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{f\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2\sqrt{a}\sqrt{b}} + \frac{\left(d-\frac{\sqrt{a}f}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2a}$$

$$= \frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}} + \frac{c\sqrt{a+bx^4}}{2a^2} - \frac{fx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} + \frac{\left(d-\frac{\sqrt{a}f}{\sqrt{b}}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.39

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx = \frac{3c\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{bx^4}{a}\right)+x\left(3d+3ex+3d\sqrt{1+\frac{bx^4}{a}}\,\text{Hyperge}\right.}{6a\sqrt{c}}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x]
```

```
[Out] (3*c*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^4)/a] + x*(3*d + 3*e*x + 3*d*
Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*f*x^
2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)]))/(6*a
*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.78

method	result
elliptic	$-\frac{2b\left(-\frac{f\,x^3}{4ab}-\frac{x^2 e}{4ab}-\frac{dx}{4ab}-\frac{c}{4ba}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{if\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\right.\right.}}{2\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
default	$d\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+f\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right.\right.}{2\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{}}\right.$

```
[In] int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(-1/4/a/b*f*x^3-1/4/a/b*x^2*e-1/4/a/b*d*x-1/4/b/a*c)/((x^4+a/b)*b)^(1/
2)+1/2*d/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)-1/2*I*f/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2
*c/a^(3/2)*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.59

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx = \frac{2\left(abfx^4+a^2f\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)-2\left((abd+abf)x^4+a^2d\right.}{}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*b*f*x^4 + a^2*f)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^
(1/4)), -1) - 2*((a*b*d + a*b*f)*x^4 + a^2*d + a^2*f)*sqrt(a)*(-b/a)^(3/4)*
elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + (b^2*c*x^4 + a*b*c)*sqrt(a)*log(-(
b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(a*b*f*x^3 + a*b*e*x^2 +
a*b*d*x + a*b*c)*sqrt(b*x^4 + a))/(a^2*b^2*x^4 + a^3*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.64 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.89

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx = c\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right)+\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}+\frac{fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(3/2),x)
```

```
[Out] c*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**3*log(b*
x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a) +
1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9/2)
 + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/
2) + 4*a**(7/2)*b*x**4)) + d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*
x**4/a)) + f*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*p
i)/a)/(4*a**(3/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\left(bx^4+a\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\left(bx^4+a\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx = \int \frac{f\,x^3+e\,x^2+d\,x+c}{x\,\left(b\,x^4+a\right)^{3/2}}\,dx$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x)
[Out] int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)), x)
```

3.548 $\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 344

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx=\frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}}+\frac{d\sqrt{a+bx^4}}{2a^2}-\frac{c\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}cx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{3\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}}+\frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

```
[Out] -1/2*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)+1/2*x*(-b*d*x^3-b*c*x^2+a*f
*x+a*e)/a^2/(b*x^4+a)^(1/2)+1/2*d*(b*x^4+a)^(1/2)/a^2-c*(b*x^4+a)^(1/2)/a^2
/x+3/2*c*x*b^(1/2)*(b*x^4+a)^(1/2)/a^2/(a^(1/2)+x^2*b^(1/2))-3/2*b^(1/4)*c*
(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))
*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/
2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)+1/4*(
cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*
EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(e*a^(1/2)+3*c*b^(1
/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4
)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1843, 1847, 1849, 1598, 1212, 226, 1210, 21, 272, 52, 65, 214}

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx = \frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+3\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{3\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} - \frac{d\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}cx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{d\sqrt{a+bx^4}}{2a^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x]
```

```
[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (d*Sqrt[a +
 b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqrt[b]*c*x*Sqrt[a + b*
x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]
)/(2*a^(3/2)) - (3*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(
7/4)*Sqrt[a + b*x^4]) + ((3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(4*a^(7/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
```

```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}}-\frac{\int \frac{-2bc-2bdx-bex^2-\frac{b^2cx^4}{a}-\frac{2b^2dx^5}{a}}{x^2\sqrt{a+bx^4}}\,dx}{2ab}\\
&= \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}}-\frac{\int \left(\frac{-2bc-bex^2-\frac{b^2cx^4}{a}}{x^2\sqrt{a+bx^4}}+\frac{-2bd-\frac{2b^2dx^4}{a}}{x\sqrt{a+bx^4}}\right)dx}{2ab}\\
&= \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}}-\frac{\int \frac{-2bc-bex^2-\frac{b^2cx^4}{a}}{x^2\sqrt{a+bx^4}}\,dx}{2ab}-\frac{\int \frac{-2bd-\frac{2b^2dx^4}{a}}{x\sqrt{a+bx^4}}\,dx}{2ab}\\
&= \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{a^2x}+\frac{\int \frac{2abex+6b^2cx^3}{x\sqrt{a+bx^4}}\,dx}{4a^2b}+\frac{d\int \frac{\sqrt{a+bx^4}}{x}\,dx}{a^2}\\
&= \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{a^2x}+\frac{\int \frac{2abe+6b^2cx^2}{\sqrt{a+bx^4}}\,dx}{4a^2b}+\frac{d\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^4\right)}{4a^2}
\end{aligned}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}} + \frac{d\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{a^2x} - \frac{\left(3\sqrt{b}c\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2a^{3/2}} + \frac{d\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^4\right)}{4a} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2a^{3/2}}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}} + \frac{d\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}cx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{3\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}} + \frac{d\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)}{2ab}$$

$$= \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}} + \frac{d\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}cx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{3\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.36

$$\int \frac{c+dx+ex^2+fx^3}{x^2\left(a+bx^4\right)^{3/2}}\,dx = \frac{dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{bx^4}{a}\right) - 2c\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\right.}{2ax\sqrt{}}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x]
[Out] (d*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^4)/a] - 2*c*Sqrt[1 + (b*x^4)/
a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^4)/a)] + x^2*(e + f*x + e*Sqrt[
1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)]))/(2*a*x*Sqrt
[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.85 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.78

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{a^2 x}-\frac{2b\left(\frac{c\,x^3}{4a^2}-\frac{x^2 f}{4ab}-\frac{xe}{4ab}-\frac{d}{4ab}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{3i\sqrt{b}\,c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\cdots}\right.\right.}{2a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4\cdots}}$
default	$e\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+\frac{f\,x^2}{2a\sqrt{b\,x^4+a}}+d\left(\frac{1}{2a\sqrt{b\,x^4+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2a^{\frac{3}{2}}}\right)+\cdots$
risch	$-\frac{c\sqrt{b\,x^4+a}}{a^2 x}+\frac{a^2 e\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+b^2 c\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(\cdots\right.\right.}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\cdots}}\right.}{a^2}$

```
[In] int((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -c*(b*x^4+a)^(1/2)/a^2/x-2*b*(1/4/a^2*c*x^3-1/4/a/b*x^2*f-1/4/a/b*x*e-1/4/a
/b*d)/((x^4+a/b)*b)^(1/2)+1/2/a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/2*I*b^(1/2)*c/a^(3/2)/(I/a^(1/2)*b^(1/2))^
(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^
4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)
*b^(1/2))^(1/2),I))-1/2*d/a^(3/2)*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.62

$$\int \frac{c+dx+ex^2+fx^3}{x^2\left(a+bx^4\right)^{3/2}}\,dx =$$
$$-\frac{6\left(b^2cx^5+abcx\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-2\left(\left(3\,b^2c-abe\right)x^5+\left(3\,abc-a^2e\right)x\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\mathrm{ar}\cdots}{\cdots}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(6*(b^2*c*x^5 + a*b*c*x)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/
a)^(1/4)), -1) - 2*((3*b^2*c - a*b*e)*x^5 + (3*a*b*c - a^2*e)*x)*sqrt(a)*(-
b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - (b^2*d*x^5 + a*b*d*x)*s
qrt(a)*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(3*b^2*c*x^4
 - a*b*f*x^3 - a*b*e*x^2 - a*b*d*x + 2*a*b*c)*sqrt(b*x^4 + a))/(a^2*b^2*x^5
 + a^3*b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.08 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.85

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx = d\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right) + \frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)}+\frac{ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{fx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(3/2),x)
```

```
[Out] d*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**3*log(b*
x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a) +
1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9/2)
 + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/
2) + 4*a**(7/2)*b*x**4)) + c*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3/4)) + e*x*gamma(1/4)*hyper((1/4, 3
/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + f*x**2/(2*
a**(3/2)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx = \int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx = \int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.97 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.39

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx = \frac{d}{2\,a\,\sqrt{b\,x^4+a}} - \frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} + \frac{f\,x^2}{2\,a\,\sqrt{b\,x^4+a}} - \frac{c\,\left(\frac{a}{b\,x^4}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{a}{b\,x^4}\right)}{7\,x\,(b\,x^4+a)^{3/2}} + \frac{e\,x\,\left(\frac{b\,x^4}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{(b\,x^4+a)^{3/2}}$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x)
[Out] d/(2*a*(a + b*x^4)^(1/2)) - (d*atanh((a + b*x^4)^(1/2)/a^(1/2)))/(2*a^(3/2)
) + (f*x^2)/(2*a*(a + b*x^4)^(1/2)) - (c*(a/(b*x^4) + 1)^(3/2)*hypergeom([3
/2, 7/4], 11/4, -a/(b*x^4)))/(7*x*(a + b*x^4)^(3/2)) + (e*x*((b*x^4)/a + 1)
^(3/2)*hypergeom([1/4, 3/2], 5/4, -(b*x^4)/a))/(a + b*x^4)^(3/2)
```

3.549 $\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 367

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{x^3\left(a+bx^4\right)^{3/2}}\,dx &= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}}+\frac{e\sqrt{a+bx^4}}{2a^2}\\ &-\frac{c\sqrt{a+bx^4}}{2a^2x^2}-\frac{d\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{e\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}\\ &-\frac{3\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}}\\ &+\frac{\left(3\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}}\end{aligned}$$

```
[Out] -1/2*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)+1/2*x*(-b*e*x^3-b*d*x^2-b*c
*x+a*f)/a^2/(b*x^4+a)^(1/2)+1/2*e*(b*x^4+a)^(1/2)/a^2-1/2*c*(b*x^4+a)^(1/2)
/a^2/x^2-d*(b*x^4+a)^(1/2)/a^2/x+3/2*d*x*b^(1/2)*(b*x^4+a)^(1/2)/a^2/(a^(1/
2)+x^2*b^(1/2))-3/2*b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1
/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)
/a^(7/4)/(b*x^4+a)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos
(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/
2*2^(1/2))*(f*a^(1/2)+3*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2
)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1843, 1847, 1849, 1598, 1212, 226, 1210, 21, 272, 52, 65, 214}

$$\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx = \frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+3\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{3\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} - \frac{e\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{e\sqrt{a+bx^4}}{2a^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x]
```

```
[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (e*Sqrt[a +
 b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(2*a^2*x^2) - (d*Sqrt[a + b*x^4])/(a
^2*x) + (3*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (
e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) - (3*b^(1/4)*d*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a + b*x^4]) + ((3*Sqrt[b]*d + S
qrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(7/4)*b^(1/4)*Sqrt[a
 + b*x^4])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\text{integral} = \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-2bdx-2bex^2-bfx^3-\frac{b^2dx^5}{a}-\frac{2b^2ex^6}{a}}{x^3\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \left(\frac{-2bd-bfx^2-\frac{b^2dx^4}{a}}{x^2\sqrt{a+bx^4}} + \frac{-2bc-2bex^2-\frac{2b^2ex^6}{a}}{x^3\sqrt{a+bx^4}}\right)dx}{2ab}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bd-bfx^2-\frac{b^2dx^4}{a}}{x^2\sqrt{a+bx^4}}\,dx}{2ab} - \frac{\int \frac{-2bc-2bex^2-\frac{2b^2ex^6}{a}}{x^3\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{\int \frac{8abex+8b^2ex^5}{x^2\sqrt{a+bx^4}}\,dx}{8a^2b} + \frac{\int \frac{2abfx+6b^2dx^3}{x\sqrt{a+bx^4}}\,dx}{4a^2b}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{\int \frac{8abe+8b^2ex^4}{x\sqrt{a+bx^4}}\,dx}{8a^2b} + \frac{\int \frac{2abf+6b^2dx^2}{\sqrt{a+bx^4}}\,dx}{4a^2b}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} - \frac{\left(3\sqrt{b}d\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2a^{3/2}} + \frac{e\int \frac{\sqrt{a+bx^4}}{x}\,dx}{a^2} + \frac{\left(3\sqrt{b}d+\sqrt{a}f\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2a^{3/2}}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{3\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} + \frac{\left(3\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}} + \frac{e\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^4\right)}{4a^2}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{3\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} + \frac{\left(3\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}} + \frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)}{4a}$$

$$= \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{3\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} + \frac{\left(3\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}} + \frac{e\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)}{2ab}$$

$$= \frac{x(af - bcx - bdx^2 - bex^3)}{2a^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{3\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} + \frac{\left(3\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.38

$$\int \frac{c+dx+ex^2+fx^3}{x^3\left(a+bx^4\right)^{3/2}}\,dx = \frac{-ac+afx^3-2bcx^4+aex^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{bx^4}{a}\right)-2adx\sqrt{1+}}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x]
```

```
[Out] (-(a*c) + a*f*x^3 - 2*b*c*x^4 + a*e*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 +
 (b*x^4)/a] - 2*a*d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4,
 -((b*x^4)/a)] + a*f*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/
4, -((b*x^4)/a)])/(2*a^2*x^2*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.84 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.77

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{2a^2x^2}-\frac{d\sqrt{b\,x^4+a}}{a^2x}-\frac{2b\left(\frac{d\,x^3}{4a^2}+\frac{c\,x^2}{4a^2}-\frac{xf}{4ab}-\frac{e}{4ba}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{3i\sqrt{b}\,d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1-}}{}$
default	$f\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+e\left(\frac{1}{2a\sqrt{b\,x^4+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2a^{\frac{3}{2}}}\right)+d\left(-\frac{}{2a^2\sqrt{}}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,(2dx+c)}{2a^2x^2}+\frac{a^2f\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+b^2d\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+}}{2b^{\frac{3}{2}}}\right.}{a^2}$

```
[In] int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c*(b*x^4+a)^(1/2)/a^2/x^2-d*(b*x^4+a)^(1/2)/a^2/x-2*b*(1/4*d/a^2*x^3+1
/4/a^2*c*x^2-1/4/a/b*x*f-1/4/b/a*e)/((x^4+a/b)*b)^(1/2)+1/2*f/a/(I/a^(1/2)*
b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1
/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/2*I*b^(1/2)*
d/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2/a^(3/2)*e*arctanh(a^(
1/2)/(b*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.63

$$\int \frac{c+dx+ex^2+fx^3}{x^3\left(a+bx^4\right)^{3/2}}\,dx =$$

$$-\frac{6\left(b^2dx^6+abdx^2\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)-2\left(\left(3\,b^2d-abf\right)x^6+\left(3\,abd-a^2f\right)x^2\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}}{}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(6*(b^2*d*x^6 + a*b*d*x^2)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-
b/a)^(1/4)), -1) - 2*((3*b^2*d - a*b*f)*x^6 + (3*a*b*d - a^2*f)*x^2)*sqrt(a
)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - (b^2*e*x^6 + a*b*e*
x^2)*sqrt(a)*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(3*b^2
```

```
*d*x^5 + 2*b^2*c*x^4 - a*b*f*x^3 - a*b*e*x^2 + 2*a*b*d*x + a*b*c)*sqrt(b*x^
4 + a))/(a^2*b^2*x^6 + a^3*b*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.47 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.86

$$\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx = c\left(-\frac{1}{2a\sqrt{b}x^4\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}+1}}\right) + e\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} + \frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right) + \frac{d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)}+\frac{fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(3/2),x)
```

```
[Out] c*(-1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4) + 1)) - sqrt(b)/(a**2*sqrt(a/(b*x**
4) + 1))) + e*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) +
 a**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 +
b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)
/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) +
1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4)) + d*gamma(-1/4)*hyper((-1/4, 3/2), (3/
4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3/4)) + f*x*gamma(1/4)*h
yper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx=\int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx=\int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 10.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.40

$$\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx=\frac{e}{2\,a\,\sqrt{b\,x^4+a}}-\frac{e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{2\,c\,(b\,x^4+a)-a\,c}{2\,a^2\,x^2\,\sqrt{b\,x^4+a}}-\frac{d\left(\frac{a}{b\,x^4}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{a}{b\,x^4}\right)}{7\,x\,(b\,x^4+a)^{3/2}}+\frac{f\,x\left(\frac{b\,x^4}{a}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{(b\,x^4+a)^{3/2}}$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x)
[Out] e/(2*a*(a + b*x^4)^(1/2)) - (e*atanh((a + b*x^4)^(1/2)/a^(1/2)))/(2*a^(3/2)
) - (2*c*(a + b*x^4) - a*c)/(2*a^2*x^2*(a + b*x^4)^(1/2)) - (d*(a/(b*x^4) +
 1)^(3/2)*hypergeom([3/2, 7/4], 11/4, -a/(b*x^4)))/(7*x*(a + b*x^4)^(3/2))
+ (f*x*((b*x^4)/a + 1)^(3/2)*hypergeom([1/4, 3/2], 5/4, -(b*x^4)/a))/(a + b
*x^4)^(3/2)
```

3.550 $\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 387

$$\int \frac{c+dx+ex^2+fx^3}{x^4\,(a+bx^4)^{3/2}}\,dx = -\frac{x(bc+bdx+bex^2+bfx^3)}{2a^2\sqrt{a+bx^4}}+\frac{f\sqrt{a+bx^4}}{2a^2} -\frac{c\sqrt{a+bx^4}}{3a^2x^3}-\frac{d\sqrt{a+bx^4}}{2a^2x^2}-\frac{e\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}ex\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} -\frac{f\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{3\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} -\frac{\sqrt[4]{b}\left(5\sqrt{b}c-9\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}}$$

```
[Out] -1/2*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*x*(b*f*x^3+b*e*x^2+b*d*
x+b*c)/a^2/(b*x^4+a)^(1/2)+1/2*f*(b*x^4+a)^(1/2)/a^2-1/3*c*(b*x^4+a)^(1/2)/
a^2/x^3-1/2*d*(b*x^4+a)^(1/2)/a^2/x^2-e*(b*x^4+a)^(1/2)/a^2/x+3/2*e*x*b^(1/
2)*(b*x^4+a)^(1/2)/a^2/(a^(1/2)+x^2*b^(1/2))-3/2*b^(1/4)*e*(cos(2*arctan(b^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2
*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(
a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)-1/12*b^(1/4)*(cos(2*a
rctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ellipti
cF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-9*e*a^(1/2)+5*c*b^(1/2))
*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(9/4)/(b
*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {1843, 1847, 1849, 1599, 1598, 1212, 226, 1210, 21, 272, 52, 65, 214}

$$\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx^4)^{3/2}}\,dx = -\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}c-9\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}} - \frac{3\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} - \frac{f\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{x(bc+bdx+bex^2+bfx^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - \frac{e\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}ex\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{f\sqrt{a+bx^4}}{2a^2}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)),x]
```

```
[Out] -1/2*(x*(b*c + b*d*x + b*e*x^2 + b*f*x^3))/(a^2*Sqrt[a + b*x^4]) + (f*Sqrt[
a + b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(3*a^2*x^3) - (d*Sqrt[a + b*x^4])
/(2*a^2*x^2) - (e*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqrt[b]*e*x*Sqrt[a + b*x^4]
)/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2
*a^(3/2)) - (3*b^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
+ Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)
*Sqrt[a + b*x^4]) - (b^(1/4)*(5*Sqrt[b]*c - 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(12*a^(9/4)*Sqrt[a + b*x^4])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeqQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1849

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[2*a*
(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\text{integral} = -\frac{x(bc+bdx+bex^2+bfx^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-2bdx-2bex^2-2bfx^3+\frac{b^2cx^4}{a}-\frac{b^2ex^6}{a}-\frac{2b^2fx^7}{a}}{x^4\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= -\frac{x(bc+bdx+bex^2+bfx^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \left(\frac{-2bc-2bex^2+\frac{b^2cx^4}{a}-\frac{b^2ex^6}{a}}{x^4\sqrt{a+bx^4}}+\frac{-2bd-2bfx^2-\frac{2b^2fx^6}{a}}{x^3\sqrt{a+bx^4}}\right)dx}{2ab}$$

$$= -\frac{x(bc+bdx+bex^2+bfx^3)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-2bex^2+\frac{b^2cx^4}{a}-\frac{b^2ex^6}{a}}{x^4\sqrt{a+bx^4}}\,dx}{2ab} - \frac{\int \frac{-2bd-2bfx^2-\frac{2b^2fx^6}{a}}{x^3\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= -\frac{x(bc+bdx+bex^2+bfx^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} + \frac{\int \frac{12abex-10b^2cx^3+6b^2ex^5}{x^3\sqrt{a+bx^4}}\,dx}{12a^2b} + \frac{\int \frac{8abfx+8b^2fx^5}{x^2\sqrt{a+bx^4}}\,dx}{8a^2b}$$

$$= -\frac{x(bc+bdx+bex^2+bfx^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} + \frac{\int \frac{12abe-10b^2cx^2+6b^2ex^4}{x^2\sqrt{a+bx^4}}\,dx}{12a^2b} + \frac{\int \frac{8abf+8b^2fx^4}{x\sqrt{a+bx^4}}\,dx}{8a^2b}$$

$$= -\frac{x(bc+bdx+bex^2+bfx^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - \frac{e\sqrt{a+bx^4}}{a^2x} - \frac{\int \frac{20ab^2cx-36ab^2ex^3}{x\sqrt{a+bx^4}}\,dx}{24a^3b} + \frac{f\int \frac{\sqrt{a+bx^4}}{x}\,dx}{a^2}$$

$$= -\frac{x(bc+bdx+bex^2+bfx^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - \frac{e\sqrt{a+bx^4}}{a^2x} - \frac{\int \frac{20ab^2c-36ab^2ex^2}{\sqrt{a+bx^4}}\,dx}{24a^3b} + \frac{f\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^4\right)}{4a^2}$$

$$= -\frac{x(bc+bdx+bex^2+bfx^3)}{2a^2\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - \frac{e\sqrt{a+bx^4}}{a^2x} - \frac{\left(3\sqrt{b}e\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2a^{3/2}} - \frac{\left(\sqrt{b}\left(5\sqrt{b}c-9\sqrt{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{6a^2} + \frac{f\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^4\right)}{4a}$$

$$= -\frac{x(bc+bdx+bex^2+bfx^3)}{2a^2\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - \frac{e\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}ex\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{3\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}\left(5\sqrt{b}c-9\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}} + \frac{f\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)}{2ab}$$

$$= -\frac{x(bc+bdx+bex^2+bfx^3)}{2a^2\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - \frac{e\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}ex\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{3\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}\left(5\sqrt{b}c-9\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.35

$$\int \frac{c+dx+ex^2+fx^3}{x^4\left(a+bx^4\right)^{3/2}}\,dx = \frac{-2ac\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{bx^4}{a}\right) - 3x\left(ad+2bdx^4-afx^2\right.}{}$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)),x]
```

```
[Out] (-2*a*c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^4)/a)]
 - 3*x*(a*d + 2*b*d*x^4 - a*f*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^
4)/a] + 2*a*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*
x^4)/a)]))/(6*a^2*x^3*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.80 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.77

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{3a^2x^3}-\frac{d\sqrt{b\,x^4+a}}{2a^2x^2}-\frac{e\sqrt{b\,x^4+a}}{a^2x}-\frac{2b\left(\frac{e\,x^3}{4a^2}+\frac{x^2d}{4a^2}+\frac{cx}{4a^2}-\frac{f}{4ab}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{5bc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{3i\sqrt{b}\,e\sqrt{\cdots}}{\cdots}$
default	$f\left(\frac{1}{2a\sqrt{b\,x^4+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2a^{\frac{3}{2}}}\right)+c\left(-\frac{bx}{2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{\sqrt{b\,x^4+a}}{3a^2x^3}-\frac{5b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)\cdots$
risch	$-\frac{\sqrt{b\,x^4+a}\,(6e\,x^2+3dx+2c)}{6a^2x^3}-\frac{bcx}{2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{5bc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{be\,x^3}{2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{b}\,e\sqrt{1-\cdots}}{2\cdots}$

```
[In] int((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*c*(b*x^4+a)^(1/2)/a^2/x^3-1/2*d*(b*x^4+a)^(1/2)/a^2/x^2-e*(b*x^4+a)^(1
/2)/a^2/x-2*b*(1/4/a^2*e*x^3+1/4/a^2*x^2*d+1/4/a^2*c*x-1/4/a/b*f)/((x^4+a/b
)*b)^(1/2)-5/6/a^2*b*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^
(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2
)*b^(1/2))^(1/2),I)+3/2*I*b^(1/2)*e/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/
a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*
(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(
1/2),I))-1/2*f/a^(3/2)*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.56

$$\int \frac{c+dx+ex^2+fx^3}{x^4\left(a+bx^4\right)^{3/2}}\,dx =$$

$$-\frac{18\left(bex^7+aex^3\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)-2\left((5\,bc+9\,be)x^7+(5\,ac+9\,ae)x^3\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\mathrm{ar}\cdots}{\cdots}$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(18*(b*e*x^7 + a*e*x^3)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/
a)^(1/4)), -1) - 2*((5*b*c + 9*b*e)*x^7 + (5*a*c + 9*a*e)*x^3)*sqrt(a)*(-b/
a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - 3*(b*f*x^7 + a*f*x^3)*sqr
t(a)*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(9*b*e*x^6 + 6
*b*d*x^5 + 5*b*c*x^4 - 3*a*f*x^3 + 6*a*e*x^2 + 3*a*d*x + 2*a*c)*sqrt(b*x^4
+ a))/(a^2*b*x^7 + a^3*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.87 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.83

$$\int \frac{c+dx+ex^2+fx^3}{x^4\left(a+bx^4\right)^{3/2}}\,dx = d\left(-\frac{1}{2a\sqrt{b}x^4\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}+1}}\right) + f\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} + \frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right) + \frac{c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)}$$

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**4/(b*x**4+a)**(3/2),x)
```

```
[Out] d*(-1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4) + 1)) - sqrt(b)/(a**2*sqrt(a/(b*x**
4) + 1))) + f*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) +
 a**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 +
b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)
/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) +
1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4)) + c*gamma(-3/4)*hyper((-3/4, 3/2), (1/
4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x**3*gamma(1/4)) + e*gamma(-1/4)
*hyper((-1/4, 3/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3
/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^4\left(a+bx^4\right)^{3/2}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\left(bx^4+a\right)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^4\left(a+bx^4\right)^{3/2}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\left(bx^4+a\right)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x^4\left(a+bx^4\right)^{3/2}}\,dx=\int \frac{f\,x^3+e\,x^2+d\,x+c}{x^4\,{\left(b\,x^4+a\right)}^{3/2}}\,dx$$

```
[In] int((c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)),x)
[Out] int((c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)), x)
```

3.551 $\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \, dx$

Optimal result

Integrand size = 30, antiderivative size = 269

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \, dx = \frac{c(gx)^{1+m} \left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)}{g(1+m)} + \frac{d(gx)^{2+m} \left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{2+m}{4},-p,\frac{6+m}{4},-\frac{bx^4}{a}\right)}{g^2(2+m)} + \frac{e(gx)^{3+m} \left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3+m}{4},-p,\frac{7+m}{4},-\frac{bx^4}{a}\right)}{g^3(3+m)} + \frac{f(gx)^{4+m} \left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{4+m}{4},-p,\frac{8+m}{4},-\frac{bx^4}{a}\right)}{g^4(4+m)}$$

```
[Out] c*(g*x)^(1+m)*(b*x^4+a)^p*hypergeom([-p, 1/4+1/4*m],[5/4+1/4*m],-b*x^4/a)/g
/(1+m)/((1+b*x^4/a)^p)+d*(g*x)^(2+m)*(b*x^4+a)^p*hypergeom([-p, 1/2+1/4*m],
[3/2+1/4*m],-b*x^4/a)/g^2/(2+m)/((1+b*x^4/a)^p)+e*(g*x)^(3+m)*(b*x^4+a)^p*h
ypergeom([-p, 3/4+1/4*m],[7/4+1/4*m],-b*x^4/a)/g^3/(3+m)/((1+b*x^4/a)^p)+f*
(g*x)^(4+m)*(b*x^4+a)^p*hypergeom([-p, 1+1/4*m],[2+1/4*m],-b*x^4/a)/g^4/(4+
m)/((1+b*x^4/a)^p)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1847, 1350, 372, 371}

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx$$
$$=\frac{c(gx)^{m+1}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{4},-p,\frac{m+5}{4},-\frac{bx^4}{a}\right)}{g(m+1)}$$
$$+\frac{d(gx)^{m+2}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{4},-p,\frac{m+6}{4},-\frac{bx^4}{a}\right)}{g^2(m+2)}$$
$$+\frac{e(gx)^{m+3}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+3}{4},-p,\frac{m+7}{4},-\frac{bx^4}{a}\right)}{g^3(m+3)}$$
$$+\frac{f(gx)^{m+4}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+4}{4},-p,\frac{m+8}{4},-\frac{bx^4}{a}\right)}{g^4(m+4)}$$

```
[In] Int[(g*x)^m*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] (c*(g*x)^(1 + m)*(a + b*x^4)^p*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4,
-((b*x^4)/a)])/(g*(1 + m)*(1 + (b*x^4)/a)^p) + (d*(g*x)^(2 + m)*(a + b*x^4)
^p*Hypergeometric2F1[(2 + m)/4, -p, (6 + m)/4, -((b*x^4)/a)])/(g^2*(2 + m)*
(1 + (b*x^4)/a)^p) + (e*(g*x)^(3 + m)*(a + b*x^4)^p*Hypergeometric2F1[(3 +
m)/4, -p, (7 + m)/4, -((b*x^4)/a)])/(g^3*(3 + m)*(1 + (b*x^4)/a)^p) + (f*(g
*x)^(4 + m)*(a + b*x^4)^p*Hypergeometric2F1[(4 + m)/4, -p, (8 + m)/4, -((b*
x^4)/a)])/(g^4*(4 + m)*(1 + (b*x^4)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1350

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left((gx)^m \left(c+ex^2\right) \left(a+bx^4\right)^p+\frac{(gx)^{1+m} \left(d+fx^2\right) \left(a+bx^4\right)^p}{g}\right) dx \\
&= \frac{\int (gx)^{1+m} \left(d+fx^2\right) \left(a+bx^4\right)^p \, dx}{g}+\int (gx)^m \left(c+ex^2\right) \left(a+bx^4\right)^p \, dx \\
&= \frac{\int \left(d(gx)^{1+m} \left(a+bx^4\right)^p+\frac{f(gx)^{3+m} \left(a+bx^4\right)^p}{g^2}\right) dx}{g} \\
&\quad +\int \left(c(gx)^m \left(a+bx^4\right)^p+\frac{e(gx)^{2+m} \left(a+bx^4\right)^p}{g^2}\right) dx \\
&= c\int (gx)^m \left(a+bx^4\right)^p \, dx+\frac{f\int (gx)^{3+m} \left(a+bx^4\right)^p \, dx}{g^3} \\
&\quad +\frac{e\int (gx)^{2+m} \left(a+bx^4\right)^p \, dx}{g^2}+\frac{d\int (gx)^{1+m} \left(a+bx^4\right)^p \, dx}{g} \\
&= \left(c\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p}\right) \int (gx)^m \left(1+\frac{bx^4}{a}\right)^p dx \\
&\quad +\frac{\left(f\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p}\right) \int (gx)^{3+m} \left(1+\frac{bx^4}{a}\right)^p dx}{g^3} \\
&\quad +\frac{\left(e\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p}\right) \int (gx)^{2+m} \left(1+\frac{bx^4}{a}\right)^p dx}{g^2} \\
&\quad +\frac{\left(d\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p}\right) \int (gx)^{1+m} \left(1+\frac{bx^4}{a}\right)^p dx}{g}
\end{aligned}$$

$$= \frac{c(gx)^{1+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{4},-p;\frac{5+m}{4};-\frac{bx^4}{a}\right)}{g(1+m)} + \frac{d(gx)^{2+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{2+m}{4},-p;\frac{6+m}{4};-\frac{bx^4}{a}\right)}{g^2(2+m)} + \frac{e(gx)^{3+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{3+m}{4},-p;\frac{7+m}{4};-\frac{bx^4}{a}\right)}{g^3(3+m)} + \frac{f(gx)^{4+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{4+m}{4},-p;\frac{8+m}{4};-\frac{bx^4}{a}\right)}{g^4(4+m)}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.65

$$\int (gx)^m\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx$$

$$= x(gx)^m\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(\frac{c\,\text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)}{1+m} + x\left(\frac{d\,\text{Hypergeometric2F1}\left(\frac{2+m}{4},-p,\frac{6+m}{4},-\frac{bx^4}{a}\right)}{2+m} + x\left(\frac{e\,\text{Hypergeometric2F1}\left(\frac{3+m}{4},-p,\frac{7+m}{4},-\frac{bx^4}{a}\right)}{3+m} + \frac{fx\,\text{Hypergeometric2F1}\left(\frac{4+m}{4},-p,\frac{8+m}{4},-\frac{bx^4}{a}\right)}{4+m}\right)\right)\right)$$

```
[In] Integrate[(g*x)^m*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] (x*(g*x)^m*(a + b*x^4)^p*((c*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4, -(
(b*x^4)/a)])/(1 + m) + x*((d*Hypergeometric2F1[(2 + m)/4, -p, (6 + m)/4, -(
(b*x^4)/a)])/(2 + m) + x*((e*Hypergeometric2F1[(3 + m)/4, -p, (7 + m)/4, -(
(b*x^4)/a)])/(3 + m) + (f*x*Hypergeometric2F1[(4 + m)/4, -p, (8 + m)/4, -((
b*x^4)/a)])/(4 + m)))))/(1 + (b*x^4)/a)^p
```

Maple [F]

$$\int (gx)^m \left(f\,x^3+e\,x^2+dx+c\right) \left(b\,x^4+a\right)^p dx$$

```
[In] int((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
[Out] int((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \,dx = \int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p (gx)^m \,dx$$

```
[In] integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")
[Out] integral((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \,dx = \text{Timed out}$$

```
[In] integrate((g*x)**m*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \,dx = \int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p (gx)^m \,dx$$

```
[In] integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \, dx = \int \left(fx^3+ex^2+dx+c\right) \left(bx^4+a\right)^p (gx)^m \, dx$$

```
[In] integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \, dx = \int (g\,x)^m \left(b\,x^4+a\right)^p \left(f\,x^3+e\,x^2+d\,x+c\right) \, dx$$

```
[In] int((g*x)^m*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3),x)
[Out] int((g*x)^m*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3), x)
```

3.552 $\int (c+dx+ex^2+fx^3) (a+bx^4)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\begin{aligned}
&\int (c+dx+ex^2+fx^3) (a+bx^4)^p \, dx \\
&= \frac{f(a+bx^4)^{1+p}}{4b(1+p)}+\frac{cx(a+bx^4)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{5}{4}+p,\frac{5}{4},-\frac{bx^4}{a}\right)}{a} \\
&+\frac{dx^2(a+bx^4)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2}+p,\frac{3}{2},-\frac{bx^4}{a}\right)}{2a} \\
&+\frac{ex^3(a+bx^4)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{7}{4}+p,\frac{7}{4},-\frac{bx^4}{a}\right)}{3a}
\end{aligned}$$

```
[Out] 1/4*f*(b*x^4+a)^(p+1)/b/(p+1)+c*x*(b*x^4+a)^(p+1)*hypergeom([1, 5/4+p],[5/4
],-b*x^4/a)/a+1/2*d*x^2*(b*x^4+a)^(p+1)*hypergeom([1, 3/2+p],[3/2],-b*x^4/a
)/a+1/3*e*x^3*(b*x^4+a)^(p+1)*hypergeom([1, 7/4+p],[7/4],-b*x^4/a)/a
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used

$= \{1899, 1218, 252, 251, 372, 371, 1262, 655\}$

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^p\,dx = cx(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{1}{2}dx^2(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)+\frac{f(a+bx^4)^{p+1}}{4b(p+1)}$$

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] (f*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (c*x*(a + b*x^4)^p*Hypergeometric2F
1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (d*x^2*(a + b*x^4)^p*Hyp
ergeometric2F1[1/2, -p, 3/2, -((b*x^4)/a)])/(2*(1 + (b*x^4)/a)^p) + (e*x^3*
(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)])/(3*(1 + (b*x^4
)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1218

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left((c+ex^2)\left(a+bx^4\right)^p + x\left(d+fx^2\right)\left(a+bx^4\right)^p\right)\,dx\\
&= \int (c+ex^2)\left(a+bx^4\right)^p\,dx + \int x\left(d+fx^2\right)\left(a+bx^4\right)^p\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int (d+fx)\left(a+bx^2\right)^p\,dx, x, x^2\right) + \int \left(c\left(a+bx^4\right)^p + ex^2\left(a+bx^4\right)^p\right)\,dx\\
&= \frac{f(a+bx^4)^{1+p}}{4b(1+p)} + c\int \left(a+bx^4\right)^p\,dx\\
&\quad + \frac{1}{2}d\text{Subst}\left(\int \left(a+bx^2\right)^p\,dx, x, x^2\right) + e\int x^2\left(a+bx^4\right)^p\,dx
\end{aligned}
$$

$$= \frac{f(a+bx^4)^{1+p}}{4b(1+p)} + \left(c(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p}\right) \int \left(1+\frac{bx^4}{a}\right)^p dx + \frac{1}{2}\left(d(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p}\right) \text{Subst}\left(\int \left(1+\frac{bx^2}{a}\right)^p dx, x, x^2\right) + \left(e(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p}\right) \int x^2\left(1+\frac{bx^4}{a}\right)^p dx$$

$$= \frac{f(a+bx^4)^{1+p}}{4b(1+p)} + cx(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a}\right) + \frac{1}{2}dx^2(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^4}{a}\right) + \frac{1}{3}ex^3(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -\frac{bx^4}{a}\right)$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^p\,dx = \frac{1}{12}(a+bx^4)^p \left(\frac{3f(a+bx^4)}{b(1+p)} + 12cx\left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4}, -p, \frac{5}{4}, -\frac{bx^4}{a}\right) + 6dx^2\left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, -\frac{bx^4}{a}\right) + 4ex^3\left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4}, -p, \frac{7}{4}, -\frac{bx^4}{a}\right)\right)$$

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] ((a + b*x^4)^p*((3*f*(a + b*x^4))/(b*(1 + p)) + (12*c*x*Hypergeometric2F1[1
/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (6*d*x^2*Hypergeometric2F1[
1/2, -p, 3/2, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (4*e*x^3*Hypergeometric2F1
[3/4, -p, 7/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p))/12
```

Maple [F]

$$\int \left(f\,x^{3}+e\,x^{2}+dx+c\right)\left(b\,x^{4}+a\right)^{p}dx$$

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)

[Out] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)

Fricas [F]

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx=\int \left(fx^{3}+ex^{2}+dx+c\right)\left(bx^{4}+a\right)^{p}dx$$

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")

[Out] integral((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)

Sympy [A] (verification not implemented)

Time = 20.54 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx=\frac{a^{p}cx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^{p}dx^{2}{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{2}+\frac{a^{p}ex^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+f\left(\begin{cases}\frac{a^{p}x^{4}}{4} & \text{for}\ b=0\\ \frac{\begin{cases}\frac{(a+bx^{4})^{p+1}}{p+1} & \text{for}\ p\neq -1\\ \log\left(a+bx^{4}\right) & \text{otherwise}\end{cases}}{4b} & \text{otherwise}\end{cases}\right)$$

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)

[Out] a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a**p*d*x**2*hyper((1/2, -p), (3/2,), b*x**4*exp_polar(I*pi)/a)

```
/2 + a**p*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(7/4)) + f*Piecewise((a**p*x**4/4, Eq(b, 0)), (Piecewise(((a +
b*x**4)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**4), True))/(4*b), True)
)
```

Maxima [F]

$$\int (c + dx + ex^2 + fx^3) (a + bx^4)^p \, dx = \int (fx^3 + ex^2 + dx + c) (bx^4 + a)^p \, dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int (c + dx + ex^2 + fx^3) (a + bx^4)^p \, dx = \int (fx^3 + ex^2 + dx + c) (bx^4 + a)^p \, dx$$

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c + dx + ex^2 + fx^3) (a + bx^4)^p \, dx = \int (b\,x^4 + a)^p \left(f\,x^3 + e\,x^2 + d\,x + c\right) \, dx$$

```
[In] int((a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3),x)
[Out] int((a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3), x)
```

3.553 $\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^p\,dx$

Optimal result

Integrand size = 28, antiderivative size = 175

$$\begin{aligned}&\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^p\,dx\\&=\frac{c(a+bx^4)^{1+p}}{4b(1+p)}+\frac{1}{5}dx^5(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)\\&\quad+\frac{1}{6}ex^6(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^4}{a}\right)\\&\quad+\frac{1}{7}fx^7(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-\frac{bx^4}{a}\right)\end{aligned}$$

```
[Out] 1/4*c*(b*x^4+a)^(p+1)/b/(p+1)+1/5*d*x^5*(b*x^4+a)^p*hypergeom([5/4, -p],[9/
4],-b*x^4/a)/((1+b*x^4/a)^p)+1/6*e*x^6*(b*x^4+a)^p*hypergeom([3/2, -p],[5/2
],-b*x^4/a)/((1+b*x^4/a)^p)+1/7*f*x^7*(b*x^4+a)^p*hypergeom([7/4, -p],[11/4
],-b*x^4/a)/((1+b*x^4/a)^p)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{1847, 1266, 778, 267, 372, 371, 1350\}$

$$\int x^3 (c + dx + ex^2 + fx^3) (a + bx^4)^p \, dx = \frac{c(a+bx^4)^{p+1}}{4b(p+1)} + \frac{1}{5} dx^5 (a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{4}, -p, \frac{9}{4}, -\frac{bx^4}{a}\right) + \frac{1}{6} ex^6 (a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2}, -p, \frac{5}{2}, -\frac{bx^4}{a}\right) + \frac{1}{7} fx^7 (a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{7}{4}, -p, \frac{11}{4}, -\frac{bx^4}{a}\right)$$

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] (c*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (d*x^5*(a + b*x^4)^p*Hypergeometric
2F1[5/4, -p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p) + (e*x^6*(a + b*x^4)
^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^4)/a)])/(6*(1 + (b*x^4)/a)^p) + (
f*x^7*(a + b*x^4)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^4)/a)])/(7*(1 +
 (b*x^4)/a)^p)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1350

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(x^3(c+ex^2)\left(a+bx^4\right)^p + x^4(d+fx^2)\left(a+bx^4\right)^p\right) dx \\
&= \int x^3(c+ex^2)\left(a+bx^4\right)^p dx + \int x^4(d+fx^2)\left(a+bx^4\right)^p dx \\
&= \frac{1}{2}\text{Subst}\left(\int x(c+ex)\left(a+bx^2\right)^p dx, x, x^2\right) + \int \left(dx^4\left(a+bx^4\right)^p + fx^6\left(a+bx^4\right)^p\right) dx \\
&= \frac{1}{2}c\text{Subst}\left(\int x\left(a+bx^2\right)^p dx, x, x^2\right) + d\int x^4\left(a+bx^4\right)^p dx \\
&\quad + \frac{1}{2}e\text{Subst}\left(\int x^2\left(a+bx^2\right)^p dx, x, x^2\right) + f\int x^6\left(a+bx^4\right)^p dx
\end{aligned}$$

$$= \frac{c(a+bx^4)^{1+p}}{4b(1+p)} + \left(d(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^4}{a}\right)^p dx$$
$$+ \frac{1}{2}\left(e(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\text{Subst}\left(\int x^2\left(1+\frac{bx^2}{a}\right)^p dx, x, x^2\right)$$
$$+ \left(f(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int x^6\left(1+\frac{bx^4}{a}\right)^p dx$$
$$= \frac{c(a+bx^4)^{1+p}}{4b(1+p)} + \frac{1}{5}dx^5(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)$$
$$+ \frac{1}{6}ex^6(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^4}{a}\right)$$
$$+ \frac{1}{7}fx^7(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-\frac{bx^4}{a}\right)$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.83

$$\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^p\,dx$$
$$= \frac{(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(105c(a+bx^4)\left(1+\frac{bx^4}{a}\right)^p + 84bd(1+p)x^5\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right) + \right.}{420b}$$

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] ((a + b*x^4)^p*(105*c*(a + b*x^4)*(1 + (b*x^4)/a)^p + 84*b*d*(1 + p)*x^5*Hy
pergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 70*b*e*(1 + p)*x^6*Hypergeome
tric2F1[3/2, -p, 5/2, -((b*x^4)/a)] + 60*b*f*(1 + p)*x^7*Hypergeometric2F1[
7/4, -p, 11/4, -((b*x^4)/a)]))/(420*b*(1 + p)*(1 + (b*x^4)/a)^p)
```

Maple [F]

$$\int x^3\left(f\,x^3+e\,x^2+dx+c\right)\left(b\,x^4+a\right)^p dx$$

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

```
[Out] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^p\,dx=\int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p x^3\,dx$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((f*x^6 + e*x^5 + d*x^4 + c*x^3)*(b*x^4 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 47.59 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.82

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^p\,dx$$
$$=\frac{a^pdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},-p\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{a^pex^6{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{6}$$
$$+\frac{a^pfx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},-p\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+c\left(\begin{cases}\frac{a^px^4}{4} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^4)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+bx^4\right) & \text{otherwise}\end{cases}}{4b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

```
[Out] a**p*d*x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi)/a)/(
4*gamma(9/4)) + a**p*e*x**6*hyper((3/2, -p), (5/2,), b*x**4*exp_polar(I*pi)
/a)/6 + a**p*f*x**7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_polar(I
*pi)/a)/(4*gamma(11/4)) + c*Piecewise((a**p*x**4/4, Eq(b, 0)), (Piecewise((
(a + b*x**4)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**4), True))/(4*b),
True))
```

Maxima [F]

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^p\,dx=\int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p x^3\,dx$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^4 + a)^(p + 1)*c/(b*(p + 1)) + integrate((f*x^6 + e*x^5 + d*x^4)*(
b*x^4 + a)^p, x)
```

Giac [F]

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^p\,dx=\int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^px^3\,dx$$

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^p\,dx=\int x^3\,\left(b\,x^4+a\right)^p\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

```
[In] int(x^3*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3),x)
[Out] int(x^3*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3), x)
```

3.554 $\int \frac{1+x+x^2+x^3+x^4}{1-x^5} dx$

Optimal result

Integrand size = 22, antiderivative size = 8

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5} dx = -\log(1-x)$$

```
[Out] -ln(1-x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1600, 31}

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5} dx = -\log(1-x)$$

```
[In] Int[(1 + x + x^2 + x^3 + x^4)/(1 - x^5),x]
```

```
[Out] -Log[1 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{1-x}\,dx \\ &= -\log(1-x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx = -\log(1-x)$$

```
[In] Integrate[(1 + x + x^2 + x^3 + x^4)/(1 - x^5),x]
[Out] -Log[1 - x]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$-\ln(-1+x)$	7
norman	$-\ln(-1+x)$	7
risch	$-\ln(-1+x)$	7
parallelrisch	$-\ln(-1+x)$	7
meijerg	Expression too large to display	542

```
[In] int((x^4+x^3+x^2+x+1)/(-x^5+1),x,method=_RETURNVERBOSE)
[Out] -ln(-1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx = -\log(x-1)$$

```
[In] integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="fricas")
[Out] -log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx = -\log(x-1)$$

```
[In] integrate((x**4+x**3+x**2+x+1)/(-x**5+1),x)
[Out] -log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx = -\log(x-1)$$

```
[In] integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="maxima")
[Out] -log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx = -\log(|x-1|)$$

```
[In] integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="giac")
[Out] -log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx = -\ln(x-1)$$

```
[In] int(-(x + x^2 + x^3 + x^4 + 1)/(x^5 - 1),x)
[Out] -log(x - 1)
```

3.555 $\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 10

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx=\frac{1}{2}\log(3+2x)$$

```
[Out] 1/2*ln(3+2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1600, 31}

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx=\frac{1}{2}\log(2x+3)$$

```
[In] Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6),x]
```

```
[Out] Log[3 + 2*x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{3+2x}\,dx \\ &= \frac{1}{2}\log(3+2x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx = \frac{1}{2}\log(3+2x)$$

```
[In] Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)
,x]
```

```
[Out] Log[3 + 2*x]/2
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{\ln\left(x+\frac{3}{2}\right)}{2}$
default	$\frac{\ln(2x+3)}{2}$
norman	$\frac{\ln(2x+3)}{2}$
risch	$\frac{\ln(2x+3)}{2}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\right.}{12(x^6)^{\frac{1}{6}}}$

```
[In] int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x,method=_RETUR
NVERBOSE)
```

```
[Out] 1/2*ln(x+3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx=\frac{1}{2}\,\log\left(2\,x+3\right)$$

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algori
thm="fricas")
```

```
[Out] 1/2*log(2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx=\frac{\log\left(2x+3\right)}{2}$$

```
[In] integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729),x)
```

```
[Out] log(2*x + 3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx=\frac{1}{2}\,\log\left(2\,x+3\right)$$

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algori
thm="maxima")
```

```
[Out] 1/2*log(2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx = \frac{1}{2}\,\log\left(|2\,x+3|\right)$$

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algori
thm="giac")
```

```
[Out] 1/2*log(abs(2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{2}$$

```
[In] int((162*x - 108*x^2 + 72*x^3 - 48*x^4 + 32*x^5 - 243)/(64*x^6 - 729),x)
```

```
[Out] log(x + 3/2)/2
```

3.556 $\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 10

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6} \, dx = -\frac{1}{2}\log(3-2x)$$

```
[Out] -1/2*ln(3-2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1600, 31}

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6} \, dx = -\frac{1}{2}\log(3-2x)$$

```
[In] Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6),x]
```

```
[Out] -1/2*Log[3 - 2*x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{3-2x}\,dx \\ &= -\frac{1}{2}\log(3-2x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx = -\frac{1}{2}\log(3-2x)$$

```
[In] Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)
,x]
```

```
[Out] -1/2*Log[3 - 2*x]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result
parallelrisch	$-\frac{\ln\left(x-\frac{3}{2}\right)}{2}$
default	$-\frac{\ln(-3+2x)}{2}$
norman	$-\frac{\ln(-3+2x)}{2}$
risch	$-\frac{\ln(-3+2x)}{2}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\right.}{12(x^6)^{\frac{1}{6}}}$

```
[In] int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/2*ln(x-3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx=-\frac{1}{2}\,\log\left(2\,x-3\right)$$

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algorit
hm="fricas")
```

```
[Out] -1/2*log(2*x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx=-\frac{\log\left(2x-3\right)}{2}$$

```
[In] integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729),x)
```

```
[Out] -log(2*x - 3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx=-\frac{1}{2}\,\log\left(2\,x-3\right)$$

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algorit
hm="maxima")
```

```
[Out] -1/2*log(2*x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx=-\frac{1}{2}\,\log\left(|2\,x-3|\right)$$

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algorit
hm="giac")
```

```
[Out] -1/2*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx=-\frac{\ln\left(x-\frac{3}{2}\right)}{2}$$

```
[In] int(-(162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5 + 243)/(64*x^6 - 729),x)
```

```
[Out] -log(x - 3/2)/2
```

3.557 $\int \frac{81+36x^2+16x^4}{729-64x^6} dx$

Optimal result

Integrand size = 22, antiderivative size = 10

$$\int \frac{81 + 36x^2 + 16x^4}{729 - 64x^6} dx = \frac{1}{6}\text{arctanh}\left(\frac{2x}{3}\right)$$

```
[Out] 1/6*arctanh(2/3*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1600, 212}

$$\int \frac{81 + 36x^2 + 16x^4}{729 - 64x^6} dx = \frac{1}{6}\text{arctanh}\left(\frac{2x}{3}\right)$$

```
[In] Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6),x]
```

```
[Out] ArcTanh[(2*x)/3]/6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{9-4x^2}\,dx \\ &= \frac{1}{6}\tanh^{-1}\left(\frac{2x}{3}\right) \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx = -\frac{1}{12}\log(3-2x)+\frac{1}{12}\log(3+2x)$$

```
[In] Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6),x]
[Out] -1/12*Log[3 - 2*x] + Log[3 + 2*x]/12
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(6) = 12.

Time = 1.53 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

method	result
parallelrisch	$-\frac{\ln\left(x-\frac{3}{2}\right)}{12}+\frac{\ln\left(x+\frac{3}{2}\right)}{12}$
default	$-\frac{\ln(-3+2x)}{12}+\frac{\ln(2x+3)}{12}$
norman	$-\frac{\ln(-3+2x)}{12}+\frac{\ln(2x+3)}{12}$
risch	$-\frac{\ln(-3+2x)}{12}+\frac{\ln(2x+3)}{12}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^6\right)^{\frac{1}{6}}}{3-\left(x^6\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2} - \right.}{36\left(x^6\right)^{\frac{1}{6}}}$

```
[In] int((16*x^4+36*x^2+81)/(-64*x^6+729),x,method=_RETURNVERBOSE)
[Out] -1/12*ln(x-3/2)+1/12*ln(x+3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{81 + 36x^2 + 16x^4}{729 - 64x^6}\,dx = \frac{1}{12}\,\log\left(2\,x + 3\right) - \frac{1}{12}\,\log\left(2\,x - 3\right)$$

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="fricas")
[Out] 1/12*log(2*x + 3) - 1/12*log(2*x - 3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{81 + 36x^2 + 16x^4}{729 - 64x^6}\,dx = -\frac{\log\left(x - \frac{3}{2}\right)}{12} + \frac{\log\left(x + \frac{3}{2}\right)}{12}$$

```
[In] integrate((16*x**4+36*x**2+81)/(-64*x**6+729),x)
[Out] -log(x - 3/2)/12 + log(x + 3/2)/12
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{81 + 36x^2 + 16x^4}{729 - 64x^6}\,dx = \frac{1}{12}\,\log\left(2\,x + 3\right) - \frac{1}{12}\,\log\left(2\,x - 3\right)$$

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="maxima")
[Out] 1/12*log(2*x + 3) - 1/12*log(2*x - 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{81 + 36x^2 + 16x^4}{729 - 64x^6} \, dx = \frac{1}{12} \log \left(\left| x + \frac{3}{2} \right| \right) - \frac{1}{12} \log \left(\left| x - \frac{3}{2} \right| \right)$$

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="giac")
[Out] 1/12*log(abs(x + 3/2)) - 1/12*log(abs(x - 3/2))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{81 + 36x^2 + 16x^4}{729 - 64x^6} \, dx = \frac{\operatorname{atanh}\left(\frac{2x}{3}\right)}{6}$$

```
[In] int(-(36*x^2 + 16*x^4 + 81)/(64*x^6 - 729),x)
[Out] atanh((2*x)/3)/6
```

3.558 $\int \frac{81+54x-24x^3-16x^4}{729-64x^6} dx$

Optimal result

Integrand size = 25, antiderivative size = 24

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6} dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -1/9*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1600, 632, 210}

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6} dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[In] Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6),x]
```

```
[Out] -1/3*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{9-6x+4x^2}\,dx\\ &= -\left(2\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, -6+8x\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx = \frac{\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[In] Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6),x]
[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(3*Sqrt[3])
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{9}$
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{(4x-3)\sqrt{3}}{9}\right)}{9}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\mathrm{ar}\right.}{36(x^6)^{\frac{1}{6}}}$

```
[In] int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x,method=_RETURNVERBOSE)
[Out] 1/9*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx = \frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right)$$

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="fricas")
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729),x)
[Out] sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx = \frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right)$$

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="maxima")
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx = \frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right)$$

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="giac")
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(4\,x-3)}{9}\right)}{9}$$

```
[In] int(-(54*x - 24*x^3 - 16*x^4 + 81)/(64*x^6 - 729),x)
[Out] (3^(1/2)*atan((3^(1/2)*(4*x - 3))/9))/9
```

3.559 $\int \frac{3-2x}{729-64x^6} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{3-2x}{729-64x^6} \, dx = \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{162\sqrt{3}} + \frac{1}{486}\log(3+2x) - \frac{1}{972}\log\left(9-6x+4x^2\right)$$

```
[Out] 1/486*ln(3+2*x)-1/972*ln(4*x^2-6*x+9)+1/486*arctan(1/9*(3+4*x)*3^(1/2))*3^(
1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1600, 2083, 642, 632, 210}

$$\int \frac{3-2x}{729-64x^6} \, dx = \frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{162\sqrt{3}} - \frac{1}{972}\log\left(4x^2-6x+9\right) + \frac{1}{486}\log(2x+3)$$

```
[In] Int[(3 - 2*x)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6*
x + 4*x^2]/972
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{243+162x+108x^2+72x^3+48x^4+32x^5}\,dx\\
&= \int \left(\frac{1}{243(3+2x)}+\frac{3-4x}{486\,(9-6x+4x^2)}+\frac{1}{54\,(9+6x+4x^2)}\right)\,dx\\
&= \frac{1}{486}\log(3+2x)+\frac{1}{486}\int \frac{3-4x}{9-6x+4x^2}\,dx+\frac{1}{54}\int \frac{1}{9+6x+4x^2}\,dx\\
&= \frac{1}{486}\log(3+2x)-\frac{1}{972}\log\left(9-6x+4x^2\right)-\frac{1}{27}\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,6+8x\right)\\
&= \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{162\sqrt{3}}+\frac{1}{486}\log(3+2x)-\frac{1}{972}\log\left(9-6x+4x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{162\sqrt{3}} + \frac{1}{486}\log(3+2x) - \frac{1}{972}\log\left(9-6x+4x^2\right)$$

[In] Integrate[(3 - 2*x)/(729 - 64*x^6),x]

[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6*x + 4*x^2]/972

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\ln(4x^2-6x+9)}{972}+\frac{\ln(2x+3)}{486}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{486}$
risch	$\frac{\ln(2x+3)}{486}-\frac{\ln(4x^2-6x+9)}{972}+\frac{\arctan\left(\frac{(4x+3)\sqrt{3}}{9}\right)\sqrt{3}}{486}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\mathrm{arct}\right.}{972(x^6)^{\frac{1}{6}}}$

[In] int((3-2*x)/(-64*x^6+729),x,method=_RETURNVERBOSE)

[Out] -1/972*ln(4*x^2-6*x+9)+1/486*ln(2*x+3)+1/486*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) - \frac{1}{972}\log\left(4x^2-6x+9\right) + \frac{1}{486}\log\left(2x+3\right)$$

[In] integrate((3-2*x)/(-64*x^6+729),x, algorithm="fricas")

[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) + 1/486*log(2*x + 3)

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{\log\left(x+\frac{3}{2}\right)}{486} - \frac{\log\left(4x^2-6x+9\right)}{972} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{486}$$

```
[In] integrate((3-2*x)/(-64*x**6+729),x)
```

```
[Out] log(x + 3/2)/486 - log(4*x**2 - 6*x + 9)/972 + sqrt(3)*atan(4*sqrt(3)*x/9 +
 sqrt(3)/3)/486
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{1}{486}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) - \frac{1}{972}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{486}\,\log\left(2\,x+3\right)$$

```
[In] integrate((3-2*x)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
1/486*log(2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{1}{486}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) - \frac{1}{972}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{486}\,\log\left(|2\,x+3|\right)$$

```
[In] integrate((3-2*x)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
1/486*log(abs(2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{486} - \frac{\ln\left(x^2-\frac{3\,x}{2}+\frac{9}{4}\right)}{972} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{1327104\,\left(\frac{x}{884736}+\frac{1}{884736}\right)}-\frac{\sqrt{3}\,x}{7962624\,\left(\frac{x}{884736}+\frac{1}{884736}\right)}\right)}{486}$$

```
[In] int((2*x - 3)/(64*x^6 - 729),x)
```

```
[Out] log(x + 3/2)/486 - log(x^2 - (3*x)/2 + 9/4)/972 - (3^(1/2)*atan(3^(1/2)/(13
27104*(x/884736 + 1/884736)) - (3^(1/2)*x)/(7962624*(x/884736 + 1/884736)))
)/486
```

3.560 $\int \frac{3+2x}{729-64x^6} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{3+2x}{729-64x^6} \, dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}} - \frac{1}{486}\log(3-2x) + \frac{1}{972}\log\left(9+6x+4x^2\right)$$

```
[Out] -1/486*ln(3-2*x)+1/972*ln(4*x^2+6*x+9)-1/486*arctan(1/9*(3-4*x)*3^(1/2))*3^
(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1600, 2083, 632, 210, 642}

$$\int \frac{3+2x}{729-64x^6} \, dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}} + \frac{1}{972}\log\left(4x^2+6x+9\right) - \frac{1}{486}\log(3-2x)$$

```
[In] Int[(3 + 2*x)/(729 - 64*x^6),x]
```

```
[Out] -1/162*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3] - Log[3 - 2*x]/486 + Log[9 + 6
*x + 4*x^2]/972
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{243-162x+108x^2-72x^3+48x^4-32x^5}\,dx\\
&= \int \left(-\frac{1}{243(-3+2x)}+\frac{1}{54\,(9-6x+4x^2)}+\frac{3+4x}{486\,(9+6x+4x^2)}\right)dx\\
&= -\frac{1}{486}\log(3-2x)+\frac{1}{486}\int \frac{3+4x}{9+6x+4x^2}\,dx+\frac{1}{54}\int \frac{1}{9-6x+4x^2}\,dx\\
&= -\frac{1}{486}\log(3-2x)+\frac{1}{972}\log\left(9+6x+4x^2\right)-\frac{1}{27}\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right)\\
&= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}-\frac{1}{486}\log(3-2x)+\frac{1}{972}\log\left(9+6x+4x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{3+2x}{729-64x^6}\,dx=\frac{1}{972}\left(2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)-2\log(3-2x)+\log\left(9+6x+4x^2\right)\right)$$

```
[In] Integrate[(3 + 2*x)/(729 - 64*x^6),x]
[Out] (2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 2*Log[3 - 2*x] + Log[9 + 6*x + 4*x^2])/972
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\ln(-3+2x)}{486}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{486}+\frac{\ln(4x^2+6x+9)}{972}$
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{(4x-3)\sqrt{3}}{9}\right)}{486}-\frac{\ln(-3+2x)}{486}+\frac{\ln(4x^2+6x+9)}{972}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\mathrm{ar}\ldots\right.}{972(x^6)^{\frac{1}{6}}}$

```
[In] int((2*x+3)/(-64*x^6+729),x,method=_RETURNVERBOSE)
[Out] -1/486*ln(-3+2*x)+1/486*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/972*ln(4*x^2+6*x+9)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{3+2x}{729-64x^6}\,dx=\frac{1}{486}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right)+\frac{1}{972}\,\log\left(4\,x^2+6\,x+9\right)-\frac{1}{486}\,\log\left(2\,x-3\right)$$

```
[In] integrate((3+2*x)/(-64*x^6+729),x, algorithm="fricas")
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) - 1/486*log(2*x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{3+2x}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{486} + \frac{\log\left(4x^2+6x+9\right)}{972} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{486}$$

```
[In] integrate((3+2*x)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/486 + log(4*x**2 + 6*x + 9)/972 + sqrt(3)*atan(4*sqrt(3)*x/9
- sqrt(3)/3)/486
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{3+2x}{729-64x^6}\,dx = \frac{1}{486}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) + \frac{1}{972}\,\log\left(4\,x^2+6\,x+9\right) - \frac{1}{486}\,\log\left(2\,x-3\right)$$

```
[In] integrate((3+2*x)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
1/486*log(2*x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{3+2x}{729-64x^6}\,dx = \frac{1}{486}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) + \frac{1}{972}\,\log\left(4\,x^2+6\,x+9\right) - \frac{1}{486}\,\log\left(|2\,x-3|\right)$$

```
[In] integrate((3+2*x)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
1/486*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{3+2x}{729-64x^6}\,dx = \frac{\ln\left(x^2+\frac{3\,x}{2}+\frac{9}{4}\right)}{972} - \frac{\ln\left(x-\frac{3}{2}\right)}{486} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{1327104\,\left(\frac{x}{884736}-\frac{1}{884736}\right)}+\frac{\sqrt{3}\,x}{7962624\,\left(\frac{x}{884736}-\frac{1}{884736}\right)}\right)}{486}$$

```
[In] int(-(2*x + 3)/(64*x^6 - 729),x)
```

```
[Out] log((3*x)/2 + x^2 + 9/4)/972 - log(x - 3/2)/486 - (3^(1/2)*atan(3^(1/2)/(13
27104*(x/884736 - 1/884736)) + (3^(1/2)*x)/(7962624*(x/884736 - 1/884736)))
)/486
```

3.561 $\int \frac{9-6x+4x^2}{729-64x^6} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{9-6x+4x^2}{729-64x^6} \, dx = \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{54\sqrt{3}} - \frac{1}{324}\log(3-2x) + \frac{1}{108}\log(3+2x) - \frac{1}{324}\log\left(9+6x+4x^2\right)$$

```
[Out] -1/324*ln(3-2*x)+1/108*ln(3+2*x)-1/324*ln(4*x^2+6*x+9)+1/162*arctan(1/9*(3+
4*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1600, 2083, 648, 632, 210, 642}

$$\int \frac{9-6x+4x^2}{729-64x^6} \, dx = \frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{54\sqrt{3}} - \frac{1}{324}\log\left(4x^2+6x+9\right) - \frac{1}{324}\log(3-2x) + \frac{1}{108}\log(2x+3)$$

```
[In] Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/324 + Log[3 + 2*x
]/108 - Log[9 + 6*x + 4*x^2]/324
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{81+54x-24x^3-16x^4}\,dx \\
&= \int \left(-\frac{1}{162(-3+2x)}+\frac{1}{54(3+2x)}+\frac{3-2x}{81\,(9+6x+4x^2)}\right)dx \\
&= -\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(3+2x)+\frac{1}{81}\int \frac{3-2x}{9+6x+4x^2}\,dx \\
&= -\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(3+2x)-\frac{1}{324}\int \frac{6+8x}{9+6x+4x^2}\,dx+\frac{1}{18}\int \frac{1}{9+6x+4x^2}\,dx
\end{aligned}$$

$$= -\frac{1}{324}\log(3-2x) + \frac{1}{108}\log(3+2x) - \frac{1}{324}\log\left(9+6x+4x^2\right) - \frac{1}{9}\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, 6+8x\right)$$

$$= \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{54\sqrt{3}} - \frac{1}{324}\log(3-2x) + \frac{1}{108}\log(3+2x) - \frac{1}{324}\log\left(9+6x+4x^2\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = \frac{1}{324}\left(2\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right) - \log(3-2x) + 3\log(3+2x) - \log\left(9+6x+4x^2\right)\right)$$

```
[In] Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - Log[3 - 2*x] + 3*Log[3 + 2*x] -
Log[9 + 6*x + 4*x^2])/324
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\ln(-3+2x)}{324}+\frac{\ln(2x+3)}{108}-\frac{\ln(4x^2+6x+9)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{162}$
risch	$-\frac{\ln(-3+2x)}{324}-\frac{\ln(4x^2+6x+9)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{162}+\frac{\ln(2x+3)}{108}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^6\right)^{\frac{1}{6}}}{3-\left(x^6\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\text{arct}\right.}{324\left(x^6\right)^{\frac{1}{6}}}$

```
[In] int((4*x^2-6*x+9)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

```
[Out] -1/324*ln(-3+2*x)+1/108*ln(2*x+3)-1/324*ln(4*x^2+6*x+9)+1/162*3^(1/2)*arcta
n(1/18*(8*x+6)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) - \frac{1}{324}\,\log\left(4\,x^2+6\,x+9\right) + \frac{1}{108}\,\log\left(2\,x+3\right) - \frac{1}{324}\,\log\left(2\,x-3\right)$$

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="fricas")
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
1/108*log(2*x + 3) - 1/324*log(2*x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{324} + \frac{\log\left(x+\frac{3}{2}\right)}{108} - \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{324} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{162}$$

```
[In] integrate((4*x**2-6*x+9)/(-64*x**6+729),x)
[Out] -log(x - 3/2)/324 + log(x + 3/2)/108 - log(x**2 + 3*x/2 + 9/4)/324 + sqrt(3
)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/162
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) - \frac{1}{324}\,\log\left(4\,x^2+6\,x+9\right) + \frac{1}{108}\,\log\left(2\,x+3\right) - \frac{1}{324}\,\log\left(2\,x-3\right)$$

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="maxima")
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
1/108*log(2*x + 3) - 1/324*log(2*x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) - \frac{1}{324}\,\log\left(4\,x^2+6\,x+9\right) + \frac{1}{108}\,\log\left(|2\,x+3|\right) - \frac{1}{324}\,\log\left(|2\,x-3|\right)$$

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
1/108*log(abs(2*x + 3)) - 1/324*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{108} - \frac{\ln\left(x-\frac{3}{2}\right)}{324} - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)$$

```
[In] int(-(4*x^2 - 6*x + 9)/(64*x^6 - 729),x)
```

```
[Out] log(x + 3/2)/108 - log(x - 3/2)/324 - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/
2)*1i)/324 + 1/324) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/324 - 1/3
24)
```

3.562 $\int \frac{9+6x+4x^2}{729-64x^6} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{9+6x+4x^2}{729-64x^6} \, dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}} - \frac{1}{108}\log(3-2x) + \frac{1}{324}\log(3+2x) + \frac{1}{324}\log\left(9-6x+4x^2\right)$$

```
[Out] -1/108*ln(3-2*x)+1/324*ln(3+2*x)+1/324*ln(4*x^2-6*x+9)-1/162*arctan(1/9*(3-
4*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1600, 2083, 648, 632, 210, 642}

$$\int \frac{9+6x+4x^2}{729-64x^6} \, dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}} + \frac{1}{324}\log\left(4x^2-6x+9\right) - \frac{1}{108}\log(3-2x) + \frac{1}{324}\log(2x+3)$$

```
[In] Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] -1/54*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3] - Log[3 - 2*x]/108 + Log[3 + 2*
x]/324 + Log[9 - 6*x + 4*x^2]/324
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{81-54x+24x^3-16x^4}\,dx \\
&= \int \left(-\frac{1}{54(-3+2x)}+\frac{1}{162(3+2x)}+\frac{3+2x}{81\,(9-6x+4x^2)}\right)dx \\
&= -\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(3+2x)+\frac{1}{81}\int \frac{3+2x}{9-6x+4x^2}\,dx \\
&= -\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(3+2x)+\frac{1}{324}\int \frac{-6+8x}{9-6x+4x^2}\,dx+\frac{1}{18}\int \frac{1}{9-6x+4x^2}\,dx
\end{aligned}
$$

$$= -\frac{1}{108}\log(3-2x) + \frac{1}{324}\log(3+2x) + \frac{1}{324}\log\left(9-6x+4x^2\right) - \frac{1}{9}\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, -6+8x\right)$$

$$= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}} - \frac{1}{108}\log(3-2x) + \frac{1}{324}\log(3+2x) + \frac{1}{324}\log\left(9-6x+4x^2\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = \frac{1}{324}\left(2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) - 3\log(3-2x) + \log(3+2x) + \log\left(9-6x+4x^2\right)\right)$$

```
[In] Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]
[Out] (2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + Log[3 + 2*x] +
 Log[9 - 6*x + 4*x^2])/324
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\ln(-3+2x)}{108}+\frac{\ln(4x^2-6x+9)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{162}+\frac{\ln(2x+3)}{324}$
risch	$\frac{\ln(16x^2-24x+36)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{(4x-3)\sqrt{3}}{9}\right)}{162}-\frac{\ln(-3+2x)}{108}+\frac{\ln(2x+3)}{324}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\text{ar}\right.}{324(x^6)^{\frac{1}{6}}}$

```
[In] int((4*x^2+6*x+9)/(-64*x^6+729),x,method=_RETURNVERBOSE)
[Out] -1/108*ln(-3+2*x)+1/324*ln(4*x^2-6*x+9)+1/162*3^(1/2)*arctan(1/18*(8*x-6)*3
^(1/2))+1/324*ln(2*x+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right)+\frac{1}{324}\,\log\left(4\,x^2-6\,x+9\right)+\frac{1}{324}\,\log\left(2\,x+3\right)-\frac{1}{108}\,\log\left(2\,x-3\right)$$

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="fricas")
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
1/324*log(2*x + 3) - 1/108*log(2*x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{108}+\frac{\log\left(x+\frac{3}{2}\right)}{324}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{324}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{162}$$

```
[In] integrate((4*x**2+6*x+9)/(-64*x**6+729),x)
[Out] -log(x - 3/2)/108 + log(x + 3/2)/324 + log(x**2 - 3*x/2 + 9/4)/324 + sqrt(3
)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/162
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right)+\frac{1}{324}\,\log\left(4\,x^2-6\,x+9\right)+\frac{1}{324}\,\log\left(2\,x+3\right)-\frac{1}{108}\,\log\left(2\,x-3\right)$$

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="maxima")
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
1/324*log(2*x + 3) - 1/108*log(2*x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{324}\log\left(4x^2-6x+9\right)+\frac{1}{324}\log\left(|2x+3|\right)-\frac{1}{108}\log\left(|2x-3|\right)$$

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
1/324*log(abs(2*x + 3)) - 1/108*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{324}-\frac{\ln\left(x-\frac{3}{2}\right)}{108}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)$$

```
[In] int(-(6*x + 4*x^2 + 9)/(64*x^6 - 729),x)
```

```
[Out] log(x + 3/2)/324 - log(x - 3/2)/108 - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/
2)*1i)/324 - 1/324) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/324 + 1/3
24)
```

3.563 $\int \frac{27-8x^3}{729-64x^6} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int \frac{27-8x^3}{729-64x^6} \, dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}} + \frac{1}{54}\log(3+2x) - \frac{1}{108}\log\left(9-6x+4x^2\right)$$

```
[Out] 1/54*ln(3+2*x)-1/108*ln(4*x^2-6*x+9)-1/54*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {26, 206, 31, 648, 632, 210, 642}

$$\int \frac{27-8x^3}{729-64x^6} \, dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}} - \frac{1}{108}\log\left(4x^2-6x+9\right) + \frac{1}{54}\log(2x+3)$$

```
[In] Int[(27 - 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] -1/18*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3] + Log[3 + 2*x]/54 - Log[9 - 6*x
 + 4*x^2]/108
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{27+8x^3}\,dx \\
&= \frac{1}{27}\int \frac{1}{3+2x}\,dx + \frac{1}{27}\int \frac{6-2x}{9-6x+4x^2}\,dx \\
&= \frac{1}{54}\log(3+2x) - \frac{1}{108}\int \frac{-6+8x}{9-6x+4x^2}\,dx + \frac{1}{6}\int \frac{1}{9-6x+4x^2}\,dx
\end{aligned}$$

$$= \frac{1}{54}\log(3+2x) - \frac{1}{108}\log\left(9-6x+4x^2\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, -6+8x\right)$$

$$= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}} + \frac{1}{54}\log(3+2x) - \frac{1}{108}\log\left(9-6x+4x^2\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)}{18\sqrt{3}} + \frac{1}{54}\log(3+2x) - \frac{1}{108}\log\left(9-6x+4x^2\right)$$

```
[In] Integrate[(27 - 8*x^3)/(729 - 64*x^6),x]
[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) + Log[3 + 2*x]/54 - Log[9 - 6*x
 + 4*x^2]/108
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\ln(4x^2-6x+9)}{108}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{54}+\frac{\ln(2x+3)}{54}$
risch	$\frac{\ln(2x+3)}{54}-\frac{\ln(4x^2-6x+9)}{108}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(-\frac{3}{2}+2x\right)\sqrt{3}}{9}\right)}{54}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\text{arct}\right.}{108(x^6)^{\frac{1}{6}}}$

```
[In] int((-8*x^3+27)/(-64*x^6+729),x,method=_RETURNVERBOSE)
[Out] -1/108*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/54*ln(2*
x+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{1}{54}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{1}{108}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{54}\,\log\left(2\,x+3\right)$$

```
[In] integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="fricas")
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(4*x^2 - 6*x + 9) + 1
/54*log(2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{\log\left(x+\frac{3}{2}\right)}{54} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{108} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{54}$$

```
[In] integrate((-8*x**3+27)/(-64*x**6+729),x)
[Out] log(x + 3/2)/54 - log(x**2 - 3*x/2 + 9/4)/108 + sqrt(3)*atan(4*sqrt(3)*x/9
- sqrt(3)/3)/54
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{1}{54}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{1}{108}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{54}\,\log\left(2\,x+3\right)$$

```
[In] integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="maxima")
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(4*x^2 - 6*x + 9) + 1
/54*log(2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{1}{54}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{1}{108}\,\log\left(x^2-\frac{3}{2}\,x+\frac{9}{4}\right)+\frac{1}{54}\,\log\left(\left|x+\frac{3}{2}\right|\right)$$

[In] integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="giac")

[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(x^2 - 3/2*x + 9/4) + 1/54*log(abs(x + 3/2))

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{54} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)$$

[In] int((8*x^3 - 27)/(64*x^6 - 729),x)

[Out] log(x + 3/2)/54 - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/108 + 1/108) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/108 - 1/108)

3.564 $\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 50

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx=-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}-\frac{1}{18}\log(3-2x)+\frac{1}{36}\log\left(9-6x+4x^2\right)$$

```
[Out] -1/18*ln(3-2*x)+1/36*ln(4*x^2-6*x+9)-1/54*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1600, 2083, 648, 632, 210, 642}

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx=-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}+\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(3-2x)$$

```
[In] Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] -1/18*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3] - Log[3 - 2*x]/18 + Log[9 - 6*x
 + 4*x^2]/36
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{27-36x+24x^2-8x^3}\,dx \\
&= \int \left(-\frac{1}{9(-3+2x)}+\frac{2x}{9\left(9-6x+4x^2\right)}\right)\,dx \\
&= -\frac{1}{18}\log(3-2x)+\frac{2}{9}\int \frac{x}{9-6x+4x^2}\,dx \\
&= -\frac{1}{18}\log(3-2x)+\frac{1}{36}\int \frac{-6+8x}{9-6x+4x^2}\,dx+\frac{1}{6}\int \frac{1}{9-6x+4x^2}\,dx \\
&= -\frac{1}{18}\log(3-2x)+\frac{1}{36}\log\left(9-6x+4x^2\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right) \\
&= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}-\frac{1}{18}\log(3-2x)+\frac{1}{36}\log\left(9-6x+4x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx=\frac{\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)}{18\sqrt{3}}-\frac{1}{18}\log(3-2x)+\frac{1}{36}\log\left(9-6x+4x^2\right)$$

```
[In] Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6),x]
[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) - Log[3 - 2*x]/18 + Log[9 - 6*x
 + 4*x^2]/36
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\ln(-3+2x)}{18}+\frac{\ln(4x^2-6x+9)}{36}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{54}$
risch	$\frac{\ln(16x^2-24x+36)}{36}+\frac{\sqrt{3}\,\arctan\left(\frac{(4x-3)\sqrt{3}}{9}\right)}{54}-\frac{\ln(-3+2x)}{18}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\mathrm{ar}\right.}{108(x^6)^{\frac{1}{6}}}$

```
[In] int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x,method=_RETURNVERBOSE)
[Out] -1/18*ln(-3+2*x)+1/36*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8*x-6)*3^(1
/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx=\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(2x-3)$$

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="fricas")
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1/
18*log(2*x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{18}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{36}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{54}$$

```
[In] integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729),x)
[Out] -log(x - 3/2)/18 + log(x**2 - 3*x/2 + 9/4)/36 + sqrt(3)*atan(4*sqrt(3)*x/9
- sqrt(3)/3)/54
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx = \frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log\left(2x-3\right)$$

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="maxima")
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1/
18*log(2*x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx = \frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log\left(|2x-3|\right)$$

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="giac")
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1/
18*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx = -\frac{\ln\left(x-\frac{3}{2}\right)}{18} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)$$

```
[In] int(-(36*x + 24*x^2 + 8*x^3 + 27)/(64*x^6 - 729),x)
[Out] log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/108 + 1/36) - log(x - (3^(1/2)*
3i)/4 - 3/4)*((3^(1/2)*1i)/108 - 1/36) - log(x - 3/2)/18
```

3.565 $\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 110

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2} \, dx = -\frac{1}{2916(3+2x)}-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}-\frac{\log(3-2x)}{17496}+\frac{5\log(3+2x)}{17496}-\frac{\log(9-6x+4x^2)}{17496}-\frac{\log(9+6x+4x^2)}{17496}$$

```
[Out] -1/2916/(3+2*x)-1/17496*ln(3-2*x)+5/17496*ln(3+2*x)-1/17496*ln(4*x^2-6*x+9)
-1/17496*ln(4*x^2+6*x+9)-1/26244*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/8748
*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1600, 2099, 648, 632, 210, 642}

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2} \, dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{2916\sqrt{3}}-\frac{\log(4x^2-6x+9)}{17496}-\frac{\log(4x^2+6x+9)}{17496}-\frac{1}{2916(2x+3)}-\frac{\log(3-2x)}{17496}+\frac{5\log(2x+3)}{17496}$$

```
[In] Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)^2,x]
```

```
[Out] -1/2916*1/(3 + 2*x) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) + ArcTan
[(3 + 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) - Log[3 - 2*x]/17496 + (5*Log[3 + 2*
x])/17496 - Log[9 - 6*x + 4*x^2]/17496 - Log[9 + 6*x + 4*x^2]/17496
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(3+2x)^2\,(243-162x+108x^2-72x^3+48x^4-32x^5)}\,dx$$

$$= \int \left(-\frac{1}{8748(-3+2x)} + \frac{1}{1458(3+2x)^2} + \frac{5}{8748(3+2x)} + \frac{3-2x}{4374\,(9-6x+4x^2)} + \frac{3-2x}{4374\,(9+6x+4x^2)}\right) dx$$

$$= -\frac{1}{2916(3+2x)} - \frac{\log(3-2x)}{17496} + \frac{5\log(3+2x)}{17496} + \frac{\int \frac{3-2x}{9-6x+4x^2}\,dx}{4374} + \frac{\int \frac{3-2x}{9+6x+4x^2}\,dx}{4374}$$

$$= -\frac{1}{2916(3+2x)} - \frac{\log(3-2x)}{17496} + \frac{5\log(3+2x)}{17496} - \frac{\int \frac{-6+8x}{9-6x+4x^2}\,dx}{17496} - \frac{\int \frac{6+8x}{9+6x+4x^2}\,dx}{17496} + \frac{\int \frac{1}{9-6x+4x^2}\,dx}{2916} + \frac{1}{972}\int \frac{1}{9+6x+4x^2}\,dx$$

$$= -\frac{1}{2916(3+2x)} - \frac{\log(3-2x)}{17496} + \frac{5\log(3+2x)}{17496} - \frac{\log(9-6x+4x^2)}{17496} - \frac{\log(9+6x+4x^2)}{17496} - \frac{\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, -6+8x\right)}{1458} - \frac{1}{486}\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, 6+8x\right)$$

$$= -\frac{1}{2916(3+2x)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{2916\sqrt{3}} - \frac{\log(3-2x)}{17496} + \frac{5\log(3+2x)}{17496} - \frac{\log(9-6x+4x^2)}{17496} - \frac{\log(9+6x+4x^2)}{17496}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$$
$$= \frac{-\frac{18}{3+2x} + 2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) + 6\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right) - 3\log(3-2x) + 15\log(3+2x) - 3\log(9-6x+4x^2}{52488}$$

```
[In] Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)
^2,x]
```

```
[Out] (-18/(3 + 2*x) + 2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 6*Sqrt[3]*ArcTa
n[(3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + 15*Log[3 + 2*x] - 3*Log[9 - 6*x
 + 4*x^2] - 3*Log[9 + 6*x + 4*x^2])/52488
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{1}{5832\left(x+\frac{3}{2}\right)}+\frac{5\ln(2x+3)}{17496}-\frac{\ln(4x^2+6x+9)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{8748}-\frac{\ln(-3+2x)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(-\frac{3}{2}+2x\right)\sqrt{3}}{9}\right)}{26244}-$
default	$-\frac{\ln(-3+2x)}{17496}-\frac{\ln(4x^2-6x+9)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{26244}-\frac{1}{2916(2x+3)}+\frac{5\ln(2x+3)}{17496}-\frac{\ln(4x^2+6x+9)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{(8}{8748}\right.}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right.}{}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{8748}$

```
[In] int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x,method=_RET
URNVERBOSE)
```

```
[Out] -1/5832/(x+3/2)+5/17496*ln(2*x+3)-1/17496*ln(4*x^2+6*x+9)+1/8748*3^(1/2)*ar
ctan(2/9*(2*x+3/2)*3^(1/2))-1/17496*ln(-3+2*x)+1/26244*3^(1/2)*arctan(2/9*(
-3/2+2*x)*3^(1/2))-1/17496*ln(4*x^2-6*x+9)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx = \frac{6\sqrt{3}(2x+3)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+2\sqrt{3}(2x+3)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-3(2x+3)\log\left(4x^2+6x\right.}{52488\,(2x+}$$

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, algo
rithm="fricas")
```

```
[Out] 1/52488*(6*sqrt(3)*(2*x + 3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 2*sqrt(3)*(2*x
 + 3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 3*(2*x + 3)*log(4*x^2 + 6*x + 9) - 3*
(2*x + 3)*log(4*x^2 - 6*x + 9) + 15*(2*x + 3)*log(2*x + 3) - 3*(2*x + 3)*lo
g(2*x - 3) - 18)/(2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{17496}+\frac{5\log\left(x+\frac{3}{2}\right)}{17496}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{17496}-\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{26244}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{8748}-\frac{1}{5832x+8748}$$

```
[In] integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729)**2,x
)
```

```
[Out] -log(x - 3/2)/17496 + 5*log(x + 3/2)/17496 - log(x**2 - 3*x/2 + 9/4)/17496
- log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/2
6244 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/8748 - 1/(5832*x + 8748)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx = \frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{2916(2x+3)}-\frac{1}{17496}\log\left(4x^2+6x+9\right)-\frac{1}{17496}\log\left(4x^2-6x+9\right)+\frac{5}{17496}\log\left(2x+3\right)-\frac{1}{17496}\log\left(2x-3\right)$$

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, algo
rithm="maxima")
```

```
[Out] 1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*log(4*x^2 + 6*x + 9) - 1/174
96*log(4*x^2 - 6*x + 9) + 5/17496*log(2*x + 3) - 1/17496*log(2*x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx = \frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{1}{2916(2x+3)}-\frac{1}{17496}\log\left(4x^2+6x+9\right)-\frac{1}{17496}\log\left(4x^2-6x+9\right) + \frac{5}{17496}\log\left(|2x+3|\right)-\frac{1}{17496}\log\left(|2x-3|\right)$$

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, algo
rithm="giac")
```

```
[Out] 1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*log(4*x^2 + 6*x + 9) - 1/174
96*log(4*x^2 - 6*x + 9) + 5/17496*log(abs(2*x + 3)) - 1/17496*log(abs(2*x -
 3))
```

Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx = \frac{5\ln\left(x+\frac{3}{2}\right)}{17496}-\frac{\ln\left(x-\frac{3}{2}\right)}{17496}-\frac{1}{5832\left(x+\frac{3}{2}\right)}-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{17496}\right) +\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{17496}\right)-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{52488}\right) +\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{52488}\right)$$

```
[In] int(-(162*x - 108*x^2 + 72*x^3 - 48*x^4 + 32*x^5 - 243)/(64*x^6 - 729)^2,x)
```

```
[Out] (5*log(x + 3/2))/17496 - log(x - 3/2)/17496 - 1/(5832*(x + 3/2)) - log(x -
(3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/17496 + 1/17496) + log(x + (3^(1/2)*3i)
/4 + 3/4)*((3^(1/2)*1i)/17496 - 1/17496) - log(x - (3^(1/2)*3i)/4 - 3/4)*((
3^(1/2)*1i)/52488 + 1/17496) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/
52488 - 1/17496)
```

3.566 $\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 110

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2} \, dx = \frac{1}{2916(3-2x)} - \frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}} + \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{8748\sqrt{3}} - \frac{5\log(3-2x)}{17496} + \frac{\log(3+2x)}{17496} + \frac{\log\left(9-6x+4x^2\right)}{17496} + \frac{\log\left(9+6x+4x^2\right)}{17496}$$

```
[Out] 1/2916/(3-2*x)-5/17496*ln(3-2*x)+1/17496*ln(3+2*x)+1/17496*ln(4*x^2-6*x+9)+
1/17496*ln(4*x^2+6*x+9)-1/8748*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/26244*
arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1600, 2099, 648, 632, 210, 642}

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2} \, dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}} + \frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{8748\sqrt{3}} + \frac{\log\left(4x^2-6x+9\right)}{17496} + \frac{\log\left(4x^2+6x+9\right)}{17496} + \frac{1}{2916(3-2x)} - \frac{5\log(3-2x)}{17496} + \frac{\log(2x+3)}{17496}$$

```
[In] Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(2916*(3 - 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) + ArcTan[
(3 + 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) - (5*Log[3 - 2*x])/17496 + Log[3 + 2*
x]/17496 + Log[9 - 6*x + 4*x^2]/17496 + Log[9 + 6*x + 4*x^2]/17496
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(3-2x)^2\left(243+162x+108x^2+72x^3+48x^4+32x^5\right)}\,dx$$

$$= \int \left(\frac{1}{1458(-3+2x)^2}-\frac{5}{8748(-3+2x)}+\frac{1}{8748(3+2x)}+\frac{3+2x}{4374\,(9-6x+4x^2)}+\frac{3+2x}{4374\,(9+6x+4x^2)}\right)dx$$

$$=\frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(3+2x)}{17496}+\frac{\int\frac{3+2x}{9-6x+4x^2}\,dx}{4374}+\frac{\int\frac{3+2x}{9+6x+4x^2}\,dx}{4374}$$

$$=\frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(3+2x)}{17496}+\frac{\int\frac{-6+8x}{9-6x+4x^2}\,dx}{17496}+\frac{\int\frac{6+8x}{9+6x+4x^2}\,dx}{17496}+\frac{\int\frac{1}{9+6x+4x^2}\,dx}{2916}+\frac{1}{972}\int\frac{1}{9-6x+4x^2}\,dx$$

$$=\frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(3+2x)}{17496}+\frac{\log(9-6x+4x^2)}{17496}+\frac{\log(9+6x+4x^2)}{17496}-\frac{\text{Subst}\left(\int\frac{1}{-108-x^2}\,dx,x,6+8x\right)}{1458}-\frac{1}{486}\text{Subst}\left(\int\frac{1}{-108-x^2}\,dx,x,-6+8x\right)$$

$$=\frac{1}{2916(3-2x)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}-\frac{5\log(3-2x)}{17496}+\frac{\log(3+2x)}{17496}+\frac{\log(9-6x+4x^2)}{17496}+\frac{\log(9+6x+4x^2)}{17496}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int\frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx$$
$$=\frac{6\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)+2\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)+3\left(\frac{6}{3-2x}-5\log(3-2x)+\log(3+2x)+\log(9-6x+4x^2)+\right.}{52488}$$

```
[In] Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)
^2,x]
```

```
[Out] (6*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*S
qrt[3])] + 3*(6/(3 - 2*x) - 5*Log[3 - 2*x] + Log[3 + 2*x] + Log[9 - 6*x + 4
*x^2] + Log[9 + 6*x + 4*x^2]))/52488
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{1}{5832\left(x-\frac{3}{2}\right)}-\frac{5\ln\left(-3+2x\right)}{17496}+\frac{\ln\left(2x+3\right)}{17496}+\frac{\ln\left(4x^{2}+6x+9\right)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{26244}+\frac{\ln\left(4x^{2}-6x+9\right)}{17496}+\frac{\sqrt{3}\,\arctan}{}$
default	$-\frac{1}{2916\left(-3+2x\right)}-\frac{5\ln\left(-3+2x\right)}{17496}+\frac{\ln\left(4x^{2}-6x+9\right)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(8x-6\right)\sqrt{3}}{18}\right)}{8748}+\frac{\ln\left(2x+3\right)}{17496}+\frac{\ln\left(4x^{2}+6x+9\right)}{17496}+\frac{\sqrt{3}\,\arctan\left(\right.}{262}$
meijerg	$-\frac{\left(-1\right)^{\frac{5}{6}}\left(\frac{4x\left(-1\right)^{\frac{1}{6}}}{6-\frac{128x^{6}}{243}}-\frac{5x\left(-1\right)^{\frac{1}{6}}\left(\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{6}\right)^{\frac{1}{6}}}{3-\left(x^{6}\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right.}{}\right.}{6\left(x^{6}\right)^{\frac{1}{6}}}\right.}{8748}$

[In] int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x,method=_RETU
RNVERBOSE)

[Out] -1/5832/(x-3/2)-5/17496*ln(-3+2*x)+1/17496*ln(2*x+3)+1/17496*ln(4*x^2+6*x+9
)+1/26244*3^(1/2)*arctan(2/9*(2*x+3/2)*3^(1/2))+1/17496*ln(4*x^2-6*x+9)+1/8
748*3^(1/2)*arctan(2/9*(-3/2+2*x)*3^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{\left(729-64x^6\right)^2}\,dx$$
$$=\frac{2\sqrt{3}(2x-3)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+6\sqrt{3}(2x-3)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+3(2x-3)\log\left(4x^2+6x\right.}{52488\,(2x-}$$

[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algor
ithm="fricas")

[Out] 1/52488*(2*sqrt(3)*(2*x - 3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 6*sqrt(3)*(2*x
 - 3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(2*x - 3)*log(4*x^2 + 6*x + 9) + 3*
(2*x - 3)*log(4*x^2 - 6*x + 9) + 3*(2*x - 3)*log(2*x + 3) - 15*(2*x - 3)*lo
g(2*x - 3) - 18)/(2*x - 3)

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx = -\frac{5\log\left(x-\frac{3}{2}\right)}{17496}+\frac{\log\left(x+\frac{3}{2}\right)}{17496}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{17496} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{8748}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{26244}-\frac{1}{5832x-8748}$$

```
[In] integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729)**2,x)
```

```
[Out] -5*log(x - 3/2)/17496 + log(x + 3/2)/17496 + log(x**2 - 3*x/2 + 9/4)/17496
+ log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/8
748 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/26244 - 1/(5832*x - 8748)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx = \frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{1}{2916\,(2x-3)}+\frac{1}{17496}\log\left(4x^2+6x+9\right) + \frac{1}{17496}\log\left(4x^2-6x+9\right)+\frac{1}{17496}\log\left(2x+3\right)-\frac{5}{17496}\log\left(2x-3\right)$$

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algor
ithm="maxima")
```

```
[Out] 1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*log(4*x^2 + 6*x + 9) + 1/174
96*log(4*x^2 - 6*x + 9) + 1/17496*log(2*x + 3) - 5/17496*log(2*x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx$$
$$=\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{2916(2x-3)}+\frac{1}{17496}\log\left(4x^2+6x+9\right)+\frac{1}{17496}\log\left(4x^2-6x+9\right)+\frac{1}{17496}\log\left(|2x+3|\right)-\frac{5}{17496}\log\left(|2x-3|\right)$$

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algor
ithm="giac")
```

```
[Out] 1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*log(4*x^2 + 6*x + 9) + 1/174
96*log(4*x^2 - 6*x + 9) + 1/17496*log(abs(2*x + 3)) - 5/17496*log(abs(2*x -
 3))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx$$
$$=\frac{\ln\left(x+\frac{3}{2}\right)}{17496}-\frac{5\,\ln\left(x-\frac{3}{2}\right)}{17496}-\frac{1}{5832\,\left(x-\frac{3}{2}\right)}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{17496}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{17496}\right)-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{52488}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{52488}\right)$$

```
[In] int((162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5 + 243)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/17496 - (5*log(x - 3/2))/17496 - 1/(5832*(x - 3/2)) - log(x -
(3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/17496 - 1/17496) + log(x + (3^(1/2)*3i)
/4 - 3/4)*((3^(1/2)*1i)/17496 + 1/17496) - log(x - (3^(1/2)*3i)/4 + 3/4)*((
3^(1/2)*1i)/52488 - 1/17496) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/
52488 + 1/17496)
```

3.567 $\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = \frac{1}{17496(3-2x)}-\frac{1}{17496(3+2x)}-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{2x}{3}\right)}{8748}$$

```
[Out] 1/17496/(3-2*x)-1/17496/(3+2*x)+1/8748*arctanh(2/3*x)-1/39366*arctan(1/9*(3
-4*x)*3^(1/2))*3^(1/2)+1/39366*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1600, 1184, 213, 632, 210}

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{2x}{3}\right)}{8748}+\frac{1}{17496(3-2x)}-\frac{1}{17496(2x+3)}$$

```
[In] Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(17496*(3 - 2*x)) - 1/(17496*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(
13122*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(13122*Sqrt[3]) + ArcTanh[(2
*x)/3]/8748
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{(9-4x^2)^2(81+36x^2+16x^4)}\,dx \\
&= \int \left(\frac{1}{8748(-3+2x)^2}+\frac{1}{8748(3+2x)^2}-\frac{1}{1458(-9+4x^2)}+\frac{1}{4374(9-6x+4x^2)}\right. \\
&\qquad \left.+\frac{1}{4374(9+6x+4x^2)}\right)dx \\
&= \frac{1}{17496(3-2x)}-\frac{1}{17496(3+2x)}+\frac{\int \frac{1}{9-6x+4x^2}\,dx}{4374}+\frac{\int \frac{1}{9+6x+4x^2}\,dx}{4374}-\frac{\int \frac{1}{-9+4x^2}\,dx}{1458} \\
&= \frac{1}{17496(3-2x)}-\frac{1}{17496(3+2x)}+\frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748} \\
&\quad -\frac{\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right)}{2187}-\frac{\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,6+8x\right)}{2187}
\end{aligned}
$$

$$= \frac{1}{17496(3-2x)} - \frac{1}{17496(3+2x)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{13122\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.51

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx$$

$$= \frac{\frac{36x}{9-4x^2} + 3\sqrt{3}\arctan\left(\frac{1}{3}\left(-i+\sqrt{3}\right)x\right) + 4i\sqrt{3}\operatorname{arctanh}\left(\frac{1}{3}\left(1-i\sqrt{3}\right)x\right) + \left(-3+\frac{2}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}\right)\operatorname{arctanh}\left(\frac{1}{3}(x+\right.}{157464}$$

```
[In] Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] ((36*x)/(9 - 4*x^2) + 3*Sqrt[3]*ArcTan[((-I + Sqrt[3])*x)/3] + (4*I)*Sqrt[3
]*ArcTanh[((1 - I*Sqrt[3])*x)/3] + (-3 + 2/Sqrt[(1 + I*Sqrt[3])/6])*ArcTanh
[(x + I*Sqrt[3]*x)/3] - 9*Log[3 - 2*x] + 9*Log[3 + 2*x])/157464
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{x}{17496\left(x^2-\frac{9}{4}\right)} - \frac{\ln(-3+2x)}{17496} + \frac{\ln(2x+3)}{17496} + \frac{\sqrt{3}\arctan\left(\frac{2\sqrt{3}\,x}{9}\right)}{39366} + \frac{\sqrt{3}\arctan\left(\frac{8x^3\sqrt{3}}{81}+\frac{4\sqrt{3}\,x}{9}\right)}{39366}$
default	$-\frac{1}{17496(-3+2x)} - \frac{\ln(-3+2x)}{17496} + \frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{39366} - \frac{1}{17496(2x+3)} + \frac{\ln(2x+3)}{17496} + \frac{\sqrt{3}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{39366}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}} - \frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right) - \ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right) + \frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2} - \sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right) - \frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right.}{2}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{26244}$

```
[In] int((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/17496*x/(x^2-9/4)-1/17496*ln(-3+2*x)+1/17496*ln(2*x+3)+1/39366*3^(1/2)*a
rctan(2/9*3^(1/2)*x)+1/39366*3^(1/2)*arctan(8/81*x^3*3^(1/2)+4/9*3^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = \frac{4\sqrt{3}(4x^2-9)\arctan\left(\frac{4}{81}\sqrt{3}(2x^3+9x)\right)+4\sqrt{3}(4x^2-9)\arctan\left(\frac{2}{9}\sqrt{3}x\right)+9(4x^2-9)\log(2x+3)-}{157464(4x^2-9)}$$

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/157464*(4*sqrt(3)*(4*x^2 - 9)*arctan(4/81*sqrt(3)*(2*x^3 + 9*x)) + 4*sqrt
(3)*(4*x^2 - 9)*arctan(2/9*sqrt(3)*x) + 9*(4*x^2 - 9)*log(2*x + 3) - 9*(4*x
^2 - 9)*log(2*x - 3) - 36*x)/(4*x^2 - 9)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = -\frac{x}{17496x^2-39366} + \frac{\sqrt{3}\cdot\left(2\operatorname{atan}\left(\frac{2\sqrt{3}x}{9}\right)+2\operatorname{atan}\left(\frac{8\sqrt{3}x^3}{81}+\frac{4\sqrt{3}x}{9}\right)\right)}{78732} - \frac{\log\left(x-\frac{3}{2}\right)}{17496} + \frac{\log\left(x+\frac{3}{2}\right)}{17496}$$

```
[In] integrate((16*x**4+36*x**2+81)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(17496*x**2 - 39366) + sqrt(3)*(2*atan(2*sqrt(3)*x/9) + 2*atan(8*sqrt(3)
*x**3/81 + 4*sqrt(3)*x/9))/78732 - log(x - 3/2)/17496 + log(x + 3/2)/17496
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = \frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{x}{4374(4x^2-9)} + \frac{1}{17496}\log(2x+3) - \frac{1}{17496}\log(2x-3)$$

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*log(2*x + 3) - 1/17496*
log(2*x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.78

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = \frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{x}{4374\,(4x^2-9)} + \frac{1}{17496}\log\left(|2x+3|\right) - \frac{1}{17496}\log\left(|2x-3|\right)$$

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*log(abs(2*x + 3)) - 1/1
7496*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{2x}{3}\right)}{8748} + \frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{8\sqrt{3}\,x^3}{81}+\frac{4\sqrt{3}\,x}{9}\right)+2\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{9}\right)\right)}{78732} - \frac{x}{17496\,\left(x^2-\frac{9}{4}\right)}$$

```
[In] int((36*x^2 + 16*x^4 + 81)/(64*x^6 - 729)^2,x)
```

```
[Out] atanh((2*x)/3)/8748 + (3^(1/2)*(2*atan((4*3^(1/2)*x)/9 + (8*3^(1/2)*x^3)/81
) + 2*atan((2*3^(1/2)*x)/9)))/78732 - x/(17496*(x^2 - 9/4))
```

3.568 $\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 92

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2} dx = \frac{x}{4374(9-6x+4x^2)} - \frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}} - \frac{\log(3-2x)}{26244} + \frac{\log(3+2x)}{78732} - \frac{\log(9-6x+4x^2)}{157464} + \frac{\log(9+6x+4x^2)}{52488}$$

```
[Out] 1/4374*x/(4*x^2-6*x+9)-1/26244*ln(3-2*x)+1/78732*ln(3+2*x)-1/157464*ln(4*x^
2-6*x+9)+1/52488*ln(4*x^2+6*x+9)-1/13122*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2
)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1600, 2099, 652, 632, 210, 648, 642}

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2} dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}} + \frac{x}{4374(4x^2-6x+9)} - \frac{\log(4x^2-6x+9)}{157464} + \frac{\log(4x^2+6x+9)}{52488} - \frac{\log(3-2x)}{26244} + \frac{\log(2x+3)}{78732}$$

```
[In] Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] x/(4374*(9 - 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(4374*Sqrt[3]) -
 Log[3 - 2*x]/26244 + Log[3 + 2*x]/78732 - Log[9 - 6*x + 4*x^2]/157464 + Lo
g[9 + 6*x + 4*x^2]/52488
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
```

```
PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(9-6x+4x^2)^2\,(81+54x-24x^3-16x^4)}\,dx$$

$$= \int \left(-\frac{1}{13122(-3+2x)}+\frac{1}{39366(3+2x)}+\frac{3-x}{729\,(9-6x+4x^2)^2}+\frac{39-4x}{78732\,(9-6x+4x^2)}+\frac{3+4x}{26244\,(9+6x+4x^2)}\right)dx$$

$$= -\frac{\log(3-2x)}{26244}+\frac{\log(3+2x)}{78732}+\frac{\int \frac{39-4x}{9-6x+4x^2}\,dx}{78732}+\frac{\int \frac{3+4x}{9+6x+4x^2}\,dx}{26244}+\frac{1}{729}\int \frac{3-x}{(9-6x+4x^2)^2}\,dx$$

$$= \frac{x}{4374\,(9-6x+4x^2)}-\frac{\log(3-2x)}{26244}+\frac{\log(3+2x)}{78732}+\frac{\log(9+6x+4x^2)}{52488}-\frac{\int \frac{-6+8x}{9-6x+4x^2}\,dx}{157464}+\frac{\int \frac{1}{9-6x+4x^2}\,dx}{4374}+\frac{\int \frac{1}{9-6x+4x^2}\,dx}{2187}$$

$$= \frac{x}{4374\,(9-6x+4x^2)}-\frac{\log(3-2x)}{26244}+\frac{\log(3+2x)}{78732}-\frac{\log(9-6x+4x^2)}{157464}+\frac{\log(9+6x+4x^2)}{52488}-\frac{\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right)}{2187}-\frac{2\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right)}{2187}$$

$$= \frac{x}{4374\,(9-6x+4x^2)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}}-\frac{\log(3-2x)}{26244}+\frac{\log(3+2x)}{78732}-\frac{\log(9-6x+4x^2)}{157464}+\frac{\log(9+6x+4x^2)}{52488}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx$$

$$= \frac{\frac{36x}{9-6x+4x^2}+12\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)-6\log(3-2x)+2\log(3+2x)-\log(9-6x+4x^2)+3\log(9+6x+}{157464}$$

```
[In] Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] ((36*x)/(9 - 6*x + 4*x^2) + 12*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 6*L
og[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2
])/157464
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

method	result
default	$-\frac{\ln(-3+2x)}{26244}+\frac{x}{17496x^2-26244x+39366}-\frac{\ln(4x^2-6x+9)}{157464}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{13122}+\frac{\ln(2x+3)}{78732}+\frac{\ln(4x^2+6x+9)}{52488}$
risch	$\frac{x}{17496x^2-26244x+39366}-\frac{\ln(-3+2x)}{26244}-\frac{\ln(64x^2-96x+144)}{157464}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{13122}+\frac{\ln(2x+3)}{78732}+\frac{\ln(4x^2+6x+9)}{52488}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right.}{2}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{26244}$

[In] int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)

[Out] -1/26244*ln(-3+2*x)+1/17496*x/(x^2-3/2*x+9/4)-1/157464*ln(4*x^2-6*x+9)+1/13 122*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/78732*ln(2*x+3)+1/52488*ln(4*x^2 +6*x+9)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.37

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx$$
$$=\frac{12\sqrt{3}(4x^2-6x+9)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+3\,(4x^2-6x+9)\log\left(4x^2+6x+9\right)-(4x^2-6x+9)\log}{157464\,(4x^2-6x+}$$

[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="fricas")

[Out] 1/157464*(12*sqrt(3)*(4*x^2 - 6*x + 9)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(4 *x^2 - 6*x + 9)*log(4*x^2 + 6*x + 9) - (4*x^2 - 6*x + 9)*log(4*x^2 - 6*x + 9) + 2*(4*x^2 - 6*x + 9)*log(2*x + 3) - 6*(4*x^2 - 6*x + 9)*log(2*x - 3) + 36*x)/(4*x^2 - 6*x + 9)

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \frac{x}{17496x^2-26244x+39366} - \frac{\log\left(x-\frac{3}{2}\right)}{26244} + \frac{\log\left(x+\frac{3}{2}\right)}{78732} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{157464} + \frac{\log\left(4x^2+6x+9\right)}{52488} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{13122}$$

```
[In] integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729)**2,x)
[Out] x/(17496*x**2 - 26244*x + 39366) - log(x - 3/2)/26244 + log(x + 3/2)/78732
- log(x**2 - 3*x/2 + 9/4)/157464 + log(4*x**2 + 6*x + 9)/52488 + sqrt(3)*at
an(4*sqrt(3)*x/9 - sqrt(3)/3)/13122
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \frac{1}{13122}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{x}{4374(4x^2-6x+9)} + \frac{1}{52488}\log\left(4x^2+6x+9\right) - \frac{1}{157464}\log\left(4x^2-6x+9\right) + \frac{1}{78732}\log\left(2x+3\right) - \frac{1}{26244}\log\left(2x-3\right)$$

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="maxima")
[Out] 1/13122*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(4*x^2 - 6*x + 9)
+ 1/52488*log(4*x^2 + 6*x + 9) - 1/157464*log(4*x^2 - 6*x + 9) + 1/78732*lo
g(2*x + 3) - 1/26244*log(2*x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \frac{1}{13122}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{x}{4374(4x^2-6x+9)} + \frac{1}{52488}\log\left(4x^2+6x+9\right) - \frac{1}{157464}\log\left(4x^2-6x+9\right) + \frac{1}{78732}\log\left(|2x+3|\right) - \frac{1}{26244}\log\left(|2x-3|\right)$$

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/13122*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(4*x^2 - 6*x + 9)
+ 1/52488*log(4*x^2 + 6*x + 9) - 1/157464*log(4*x^2 - 6*x + 9) + 1/78732*lo
g(abs(2*x + 3)) - 1/26244*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{78732} - \frac{\ln\left(x-\frac{3}{2}\right)}{26244} + \frac{\ln\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{52488} + \frac{x}{17496\,\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{157464}+\frac{\sqrt{3}\,1\mathrm{i}}{26244}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{157464}+\frac{\sqrt{3}\,1\mathrm{i}}{26244}\right)$$

```
[In] int((54*x - 24*x^3 - 16*x^4 + 81)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/78732 - log(x - 3/2)/26244 + log((3*x)/2 + x^2 + 9/4)/52488 +
x/(17496*(x^2 - (3*x)/2 + 9/4)) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1
i)/26244 + 1/157464) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/26244 -
1/157464)
```

3.569 $\int \frac{3-2x}{(729-64x^6)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 148

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = -\frac{1}{708588(3+2x)}+\frac{3-x}{708588(9-6x+4x^2)}+\frac{x}{236196(9+6x+4x^2)} -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}+\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{\log(3-2x)}{4251528} +\frac{\log(3+2x)}{472392}-\frac{\log(9-6x+4x^2)}{944784}+\frac{\log(9+6x+4x^2)}{8503056}$$

```
[Out] -1/708588/(3+2*x)+1/708588*(3-x)/(4*x^2-6*x+9)+1/236196*x/(4*x^2+6*x+9)-1/4
251528*ln(3-2*x)+1/472392*ln(3+2*x)-1/944784*ln(4*x^2-6*x+9)+1/8503056*ln(4
*x^2+6*x+9)-1/4251528*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/472392*arctan(1
/9*(3+4*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1600, 2099, 652, 632, 210, 648, 642}

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{3-x}{708588(4x^2-6x+9)} +\frac{x}{236196(4x^2+6x+9)}-\frac{\log(4x^2-6x+9)}{944784}+\frac{\log(4x^2+6x+9)}{8503056} -\frac{1}{708588(2x+3)}-\frac{\log(3-2x)}{4251528}+\frac{\log(2x+3)}{472392}$$

```
[In] Int[(3 - 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] -1/708588*1/(3 + 2*x) + (3 - x)/(708588*(9 - 6*x + 4*x^2)) + x/(236196*(9 +
 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) + ArcTan[(
3 + 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) - Log[3 - 2*x]/4251528 + Log[3 + 2*x
]/472392 - Log[9 - 6*x + 4*x^2]/944784 + Log[9 + 6*x + 4*x^2]/8503056
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(3-2x)\left(243+162x+108x^2+72x^3+48x^4+32x^5\right)^2}\,dx \\
&= \int \left(-\frac{1}{2125764(-3+2x)}+\frac{1}{354294(3+2x)^2}+\frac{1}{236196(3+2x)}-\frac{x}{39366\left(9-6x+4x^2\right)^2}\right. \\
&\quad \left.+\frac{7-6x}{708588\left(9-6x+4x^2\right)}+\frac{3+x}{39366\left(9+6x+4x^2\right)^2}+\frac{33+2x}{2125764\left(9+6x+4x^2\right)}\right)dx \\
&= -\frac{1}{708588(3+2x)}-\frac{\log(3-2x)}{4251528}+\frac{\log(3+2x)}{472392}+\frac{\int \frac{33+2x}{9+6x+4x^2}\,dx}{2125764} \\
&\quad +\frac{\int \frac{7-6x}{9-6x+4x^2}\,dx}{708588}-\frac{\int \frac{x}{\left(9-6x+4x^2\right)^2}\,dx}{39366}+\frac{\int \frac{3+x}{\left(9+6x+4x^2\right)^2}\,dx}{39366} \\
&= -\frac{1}{708588(3+2x)}+\frac{3-x}{708588\left(9-6x+4x^2\right)}+\frac{x}{236196\left(9+6x+4x^2\right)} \\
&\quad -\frac{\log(3-2x)}{4251528}+\frac{\log(3+2x)}{472392}+\frac{\int \frac{6+8x}{9+6x+4x^2}\,dx}{8503056}-\frac{\int \frac{-6+8x}{9-6x+4x^2}\,dx}{944784} \\
&\quad -\frac{\int \frac{1}{9-6x+4x^2}\,dx}{708588}+\frac{5\int \frac{1}{9-6x+4x^2}\,dx}{1417176}+\frac{\int \frac{1}{9+6x+4x^2}\,dx}{236196}+\frac{7\int \frac{1}{9+6x+4x^2}\,dx}{472392} \\
&= -\frac{1}{708588(3+2x)}+\frac{3-x}{708588\left(9-6x+4x^2\right)}+\frac{x}{236196\left(9+6x+4x^2\right)} \\
&\quad -\frac{\log(3-2x)}{4251528}+\frac{\log(3+2x)}{472392}-\frac{\log\left(9-6x+4x^2\right)}{944784}+\frac{\log\left(9+6x+4x^2\right)}{8503056} \\
&\quad +\frac{\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right)}{354294}-\frac{5\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right)}{708588} \\
&\quad -\frac{\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,6+8x\right)}{118098}-\frac{7\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,6+8x\right)}{236196} \\
&= -\frac{1}{708588(3+2x)}+\frac{3-x}{708588\left(9-6x+4x^2\right)}+\frac{x}{236196\left(9+6x+4x^2\right)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}} \\
&\quad +\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{\log(3-2x)}{4251528}+\frac{\log(3+2x)}{472392}-\frac{\log\left(9-6x+4x^2\right)}{944784}+\frac{\log\left(9+6x+4x^2\right)}{8503056}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx$$
$$= \frac{\frac{1944x}{243+162x+108x^2+72x^3+48x^4+32x^5}+2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)+18\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-2\log(3-2x)+18\log(3+2x}{8503056}$$

```
[In] Integrate[(3 - 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] ((1944*x)/(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5) + 2*Sqrt[3]*Ar
cTan[(-3 + 4*x)/(3*Sqrt[3])] + 18*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 2
*Log[3 - 2*x] + 18*Log[3 + 2*x] - 9*Log[9 - 6*x + 4*x^2] + Log[9 + 6*x + 4*
x^2])/8503056
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.70

method	result
risch	$\frac{x}{139968x^5+209952x^4+314928x^3+472392x^2+708588x+1062882}+\frac{\ln(16x^2+24x+36)}{8503056}+\frac{\arctan\left(\frac{(4x+3)\sqrt{3}}{9}\right)\sqrt{3}}{472392}-\frac{\ln(16x^2-24x+}{944784}$
default	$-\frac{\ln(-3+2x)}{4251528}-\frac{\frac{x}{4}-\frac{3}{4}}{708588\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)}-\frac{\ln(4x^2-6x+9)}{944784}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{4251528}-\frac{1}{708588(2x+3)}+\frac{\ln(2x+3)}{472392}+\frac{}{944784x^2}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right.}{2}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{708588}$

```
[In] int((3-2*x)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/139968*x/(x^5+3/2*x^4+9/4*x^3+27/8*x^2+81/16*x+243/32)+1/8503056*ln(16*x^
2+24*x+36)+1/472392*arctan(1/9*(4*x+3)*3^(1/2))*3^(1/2)-1/944784*ln(16*x^2-
24*x+36)+1/4251528*3^(1/2)*arctan(1/9*(4*x-3)*3^(1/2))+1/472392*ln(2*x+3)-1
/4251528*ln(-3+2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(116) = 232.

Time = 0.28 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.73

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx$$
$$= \frac{18\sqrt{3}(32x^5+48x^4+72x^3+108x^2+162x+243)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+2\sqrt{3}(32x^5+48x^4+72x^3}{}$$

```
[In] integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/8503056*(18*sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)*ar
ctan(1/9*sqrt(3)*(4*x + 3)) + 2*sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2
 + 162*x + 243)*arctan(1/9*sqrt(3)*(4*x - 3)) + (32*x^5 + 48*x^4 + 72*x^3 +
 108*x^2 + 162*x + 243)*log(4*x^2 + 6*x + 9) - 9*(32*x^5 + 48*x^4 + 72*x^3
+ 108*x^2 + 162*x + 243)*log(4*x^2 - 6*x + 9) + 18*(32*x^5 + 48*x^4 + 72*x^
3 + 108*x^2 + 162*x + 243)*log(2*x + 3) - 2*(32*x^5 + 48*x^4 + 72*x^3 + 108
*x^2 + 162*x + 243)*log(2*x - 3) + 1944*x)/(32*x^5 + 48*x^4 + 72*x^3 + 108*
x^2 + 162*x + 243)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = \frac{x}{139968x^5+209952x^4+314928x^3+472392x^2+708588x+1062882} - \frac{\log\left(x-\frac{3}{2}\right)}{4251528} + \frac{\log\left(x+\frac{3}{2}\right)}{472392} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{8503056} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{4251528} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{472392}$$

```
[In] integrate((3-2*x)/(-64*x**6+729)**2,x)
```

```
[Out] x/(139968*x**5 + 209952*x**4 + 314928*x**3 + 472392*x**2 + 708588*x + 10628
82) - log(x - 3/2)/4251528 + log(x + 3/2)/472392 - log(x**2 - 3*x/2 + 9/4)/
944784 + log(x**2 + 3*x/2 + 9/4)/8503056 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqr
t(3)/3)/4251528 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/472392
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.71

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = \frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right) + \frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right) + \frac{x}{4374\,(32\,x^5+48\,x^4+72\,x^3+108\,x^2+162\,x+243)} + \frac{1}{8503056}\log\left(4\,x^2+6\,x+9\right) - \frac{1}{944784}\log\left(4\,x^2-6\,x+9\right) + \frac{1}{472392}\log\left(2\,x+3\right) - \frac{1}{4251528}\log\left(2\,x-3\right)$$

```
[In] integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*
x + 243) + 1/8503056*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) +
 1/472392*log(2*x + 3) - 1/4251528*log(2*x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.75

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = \frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right) + \frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right) + \frac{x}{4374\,(4\,x^2+6\,x+9)(4\,x^2-6\,x+9)(2\,x+3)} + \frac{1}{8503056}\log\left(4\,x^2+6\,x+9\right) - \frac{1}{944784}\log\left(4\,x^2-6\,x+9\right) + \frac{1}{472392}\log\left(|2\,x+3|\right) - \frac{1}{4251528}\log\left(|2\,x-3|\right)$$

```
[In] integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) + 1/4374*x/((4*x^2 + 6*x + 9)*(4*x^2 - 6*x + 9)*(2*x
+ 3)) + 1/8503056*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/
472392*log(abs(2*x + 3)) - 1/4251528*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{472392} - \frac{\ln\left(x-\frac{3}{2}\right)}{4251528} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right) - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) + \frac{x}{139968\left(x^5+\frac{3x^4}{2}+\frac{9x^3}{4}+\frac{27x^2}{8}+\frac{81x}{16}+\frac{243}{32}\right)}$$

```
[In] int(-(2*x - 3)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/472392 - log(x - 3/2)/4251528 - log(x - (3^(1/2)*3i)/4 - 3/4)*
((3^(1/2)*1i)/8503056 + 1/944784) - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)
*1i)/944784 - 1/8503056) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/8503
056 - 1/944784) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/944784 + 1/85
03056) + x/(139968*((81*x)/16 + (27*x^2)/8 + (9*x^3)/4 + (3*x^4)/2 + x^5 +
243/32))
```

3.570 $\int \frac{3+2x}{(729-64x^6)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 146

$$\int \frac{3+2x}{(729-64x^6)^2} \, dx = \frac{1}{708588(3-2x)} + \frac{x}{236196(9-6x+4x^2)} - \frac{3+x}{708588(9+6x+4x^2)} - \frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} + \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}} - \frac{\log(3-2x)}{472392} + \frac{\log(3+2x)}{4251528} - \frac{\log(9-6x+4x^2)}{8503056} + \frac{\log(9+6x+4x^2)}{944784}$$

```
[Out] 1/708588/(3-2*x)+1/236196*x/(4*x^2-6*x+9)+1/708588*(-3-x)/(4*x^2+6*x+9)-1/4
72392*ln(3-2*x)+1/4251528*ln(3+2*x)-1/8503056*ln(4*x^2-6*x+9)+1/944784*ln(4
*x^2+6*x+9)-1/472392*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/4251528*arctan(1
/9*(3+4*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1600, 2099, 652, 632, 210, 648, 642}

$$\int \frac{3+2x}{(729-64x^6)^2} \, dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} + \frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{1417176\sqrt{3}} + \frac{x}{236196(4x^2-6x+9)} - \frac{x+3}{708588(4x^2+6x+9)} - \frac{\log(4x^2-6x+9)}{8503056} + \frac{\log(4x^2+6x+9)}{944784} + \frac{1}{708588(3-2x)} - \frac{\log(3-2x)}{472392} + \frac{\log(2x+3)}{4251528}$$

```
[In] Int[(3 + 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(708588*(3 - 2*x)) + x/(236196*(9 - 6*x + 4*x^2)) - (3 + x)/(708588*(9 +
6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) + ArcTan[(3
+ 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) - Log[3 - 2*x]/472392 + Log[3 + 2*x]/
4251528 - Log[9 - 6*x + 4*x^2]/8503056 + Log[9 + 6*x + 4*x^2]/944784
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(3+2x)\left(243-162x+108x^2-72x^3+48x^4-32x^5\right)^2}\,dx \\
&= \int \left(\frac{1}{354294(-3+2x)^2}-\frac{1}{236196(-3+2x)}+\frac{1}{2125764(3+2x)}+\frac{3-x}{39366\left(9-6x+4x^2\right)^2}\right. \\
&\quad \left.+\frac{33-2x}{2125764\left(9-6x+4x^2\right)}+\frac{x}{39366\left(9+6x+4x^2\right)^2}+\frac{7+6x}{708588\left(9+6x+4x^2\right)}\right)dx \\
&= \frac{1}{708588(3-2x)}-\frac{\log(3-2x)}{472392}+\frac{\log(3+2x)}{4251528}+\frac{\int \frac{33-2x}{9-6x+4x^2}\,dx}{2125764} \\
&\quad +\frac{\int \frac{7+6x}{9+6x+4x^2}\,dx}{708588}+\frac{\int \frac{3-x}{\left(9-6x+4x^2\right)^2}\,dx}{39366}+\frac{\int \frac{x}{\left(9+6x+4x^2\right)^2}\,dx}{39366} \\
&= \frac{1}{708588(3-2x)}+\frac{x}{236196\left(9-6x+4x^2\right)}-\frac{3+x}{708588\left(9+6x+4x^2\right)} \\
&\quad -\frac{\log(3-2x)}{472392}+\frac{\log(3+2x)}{4251528}-\frac{\int \frac{-6+8x}{9-6x+4x^2}\,dx}{8503056}+\frac{\int \frac{6+8x}{9+6x+4x^2}\,dx}{944784} \\
&\quad -\frac{\int \frac{1}{9+6x+4x^2}\,dx}{708588}+\frac{5\int \frac{1}{9+6x+4x^2}\,dx}{1417176}+\frac{\int \frac{1}{9-6x+4x^2}\,dx}{236196}+\frac{7\int \frac{1}{9-6x+4x^2}\,dx}{472392} \\
&= \frac{1}{708588(3-2x)}+\frac{x}{236196\left(9-6x+4x^2\right)}-\frac{3+x}{708588\left(9+6x+4x^2\right)} \\
&\quad -\frac{\log(3-2x)}{472392}+\frac{\log(3+2x)}{4251528}-\frac{\log\left(9-6x+4x^2\right)}{8503056}+\frac{\log\left(9+6x+4x^2\right)}{944784} \\
&\quad +\frac{\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,6+8x\right)}{354294}-\frac{5\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,6+8x\right)}{708588} \\
&\quad -\frac{\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right)}{118098}-\frac{7\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right)}{236196} \\
&= \frac{1}{708588(3-2x)}+\frac{x}{236196\left(9-6x+4x^2\right)}-\frac{3+x}{708588\left(9+6x+4x^2\right)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} \\
&\quad +\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}-\frac{\log(3-2x)}{472392}+\frac{\log(3+2x)}{4251528}-\frac{\log\left(9-6x+4x^2\right)}{8503056}+\frac{\log\left(9+6x+4x^2\right)}{944784}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.83

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx$$
$$= \frac{\frac{1944x}{243-162x+108x^2-72x^3+48x^4-32x^5} + 18\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) + 2\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right) - 18\log(3-2x) + 2\log(3+2}{8503056}$$

```
[In] Integrate[(3 + 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] ((1944*x)/(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5) + 18*Sqrt[3]*A
rcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 1
8*Log[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x + 4*x^2] + 9*Log[9 + 6*x + 4*
x^2])/8503056
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{x}{139968\left(x^5-\frac{3}{2}x^4+\frac{9}{4}x^3-\frac{27}{8}x^2+\frac{81}{16}x-\frac{243}{32}\right)}-\frac{\ln(-3+2x)}{472392}+\frac{\ln(4x^2+6x+9)}{944784}+\frac{\sqrt{3}\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{4251528}+\frac{\ln(2x+3)}{4251528}-\frac{\ln}{}$
default	$-\frac{1}{708588(-3+2x)}-\frac{\ln(-3+2x)}{472392}+\frac{x}{944784x^2-1417176x+2125764}-\frac{\ln(4x^2-6x+9)}{8503056}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{472392}+\frac{\ln(2x+3)}{4251528}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)}{3}\right.}{}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{708588}$

```
[In] int((2*x+3)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/139968*x/(x^5-3/2*x^4+9/4*x^3-27/8*x^2+81/16*x-243/32)-1/472392*ln(-3+2*
x)+1/944784*ln(4*x^2+6*x+9)+1/4251528*3^(1/2)*arctan(2/9*(2*x+3/2)*3^(1/2))
+1/4251528*ln(2*x+3)-1/8503056*ln(36*x^2-54*x+81)+1/472392*3^(1/2)*arctan(2
/27*(6*x-9/2)*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(116) = 232.

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.76

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx = \frac{2\sqrt{3}(32x^5-48x^4+72x^3-108x^2+162x-243)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+18\sqrt{3}(32x^5-48x^4+72x^3 \dots}{}$$

```
[In] integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/8503056*(2*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)*arc
tan(1/9*sqrt(3)*(4*x + 3)) + 18*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2
 + 162*x - 243)*arctan(1/9*sqrt(3)*(4*x - 3)) + 9*(32*x^5 - 48*x^4 + 72*x^3
 - 108*x^2 + 162*x - 243)*log(4*x^2 + 6*x + 9) - (32*x^5 - 48*x^4 + 72*x^3
- 108*x^2 + 162*x - 243)*log(4*x^2 - 6*x + 9) + 2*(32*x^5 - 48*x^4 + 72*x^3
 - 108*x^2 + 162*x - 243)*log(2*x + 3) - 18*(32*x^5 - 48*x^4 + 72*x^3 - 108
*x^2 + 162*x - 243)*log(2*x - 3) - 1944*x)/(32*x^5 - 48*x^4 + 72*x^3 - 108*
x^2 + 162*x - 243)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.85

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx = -\frac{x}{139968x^5-209952x^4+314928x^3-472392x^2+708588x-1062882} - \frac{\log\left(x-\frac{3}{2}\right)}{472392} + \frac{\log\left(x+\frac{3}{2}\right)}{4251528} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{8503056} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{944784} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{4251528}$$

```
[In] integrate((3+2*x)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(139968*x**5 - 209952*x**4 + 314928*x**3 - 472392*x**2 + 708588*x - 1062
882) - log(x - 3/2)/472392 + log(x + 3/2)/4251528 - log(x**2 - 3*x/2 + 9/4)
/8503056 + log(x**2 + 3*x/2 + 9/4)/944784 + sqrt(3)*atan(4*sqrt(3)*x/9 - sq
rt(3)/3)/472392 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/4251528
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.72

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx = \frac{1}{4251528}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{1}{472392}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{x}{4374\,(32\,x^5-48\,x^4+72\,x^3-108\,x^2+162\,x-243)} + \frac{1}{944784}\,\log\left(4\,x^2+6\,x+9\right) - \frac{1}{8503056}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{4251528}\,\log\left(2\,x+3\right) - \frac{1}{472392}\,\log\left(2\,x-3\right)$$

```
[In] integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*
x - 243) + 1/944784*log(4*x^2 + 6*x + 9) - 1/8503056*log(4*x^2 - 6*x + 9) +
 1/4251528*log(2*x + 3) - 1/472392*log(2*x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.76

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx = \frac{1}{4251528}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{1}{472392}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{x}{4374\,(4\,x^2+6\,x+9)(4\,x^2-6\,x+9)(2\,x-3)} + \frac{1}{944784}\,\log\left(4\,x^2+6\,x+9\right) - \frac{1}{8503056}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{4251528}\,\log\left(|2\,x+3|\right) - \frac{1}{472392}\,\log\left(|2\,x-3|\right)$$

```
[In] integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(4*x^2 - 6*x + 9)*(2*x
- 3)) + 1/944784*log(4*x^2 + 6*x + 9) - 1/8503056*log(4*x^2 - 6*x + 9) + 1/
4251528*log(abs(2*x + 3)) - 1/472392*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 9.93 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.83

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{4251528} - \frac{\ln\left(x-\frac{3}{2}\right)}{472392} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right) - \frac{x}{139968\left(x^5-\frac{3x^4}{2}+\frac{9x^3}{4}-\frac{27x^2}{8}+\frac{81x}{16}-\frac{243}{32}\right)}$$

```
[In] int((2*x + 3)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/4251528 - log(x - 3/2)/472392 - log(x - (3^(1/2)*3i)/4 - 3/4)*
((3^(1/2)*1i)/944784 + 1/8503056) - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)
*1i)/8503056 - 1/944784) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/9447
84 - 1/8503056) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/8503056 + 1/9
44784) - x/(139968*((81*x)/16 - (27*x^2)/8 + (9*x^3)/4 - (3*x^4)/2 + x^5 -
243/32))
```

3.571 $\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = \frac{1}{472392(3-2x)}-\frac{1}{157464(3+2x)}+\frac{3+4x}{236196(9+6x+4x^2)} -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}}+\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}-\frac{\log(3-2x)}{354294} +\frac{\log(3+2x)}{118098}-\frac{\log(9-6x+4x^2)}{944784}-\frac{5\log(9+6x+4x^2)}{2834352}$$

```
[Out] 1/472392/(3-2*x)-1/157464/(3+2*x)+1/236196*(3+4*x)/(4*x^2+6*x+9)-1/354294*l
n(3-2*x)+1/118098*ln(3+2*x)-1/944784*ln(4*x^2-6*x+9)-5/2834352*ln(4*x^2+6*x
+9)-1/1417176*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/157464*arctan(1/9*(3+4*
x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1600, 2099, 648, 632, 210, 642, 628}

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{4x+3}{236196(4x^2+6x+9)} -\frac{\log(4x^2-6x+9)}{944784}-\frac{5\log(4x^2+6x+9)}{2834352}+\frac{1}{472392(3-2x)} -\frac{1}{157464(2x+3)}-\frac{\log(3-2x)}{354294}+\frac{\log(2x+3)}{118098}$$

```
[In] Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(472392*(3 - 2*x)) - 1/(157464*(3 + 2*x)) + (3 + 4*x)/(236196*(9 + 6*x +
4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(472392*Sqrt[3]) + ArcTan[(3 + 4*x)
/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/354294 + Log[3 + 2*x]/118098 -
 Log[9 - 6*x + 4*x^2]/944784 - (5*Log[9 + 6*x + 4*x^2])/2834352
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(9-6x+4x^2)(81+54x-24x^3-16x^4)^2}\,dx$$

$$= \int \left(\frac{1}{236196(-3+2x)^2} - \frac{1}{177147(-3+2x)} + \frac{1}{78732(3+2x)^2} + \frac{1}{59049(3+2x)} + \frac{3-2x}{236196(9-6x+4x^2)} + \frac{1}{4374(9+6x+4x^2)^2} + \frac{21-10x}{708588(9+6x+4x^2)}\right) dx$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} - \frac{\log(3-2x)}{354294} + \frac{\log(3+2x)}{118098} + \frac{\int \frac{21-10x}{9+6x+4x^2}\,dx}{708588} + \frac{\int \frac{3-2x}{9-6x+4x^2}\,dx}{236196} + \frac{\int \frac{1}{(9+6x+4x^2)^2}\,dx}{4374}$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} + \frac{3+4x}{236196(9+6x+4x^2)} - \frac{\log(3-2x)}{354294} + \frac{\log(3+2x)}{118098} - \frac{\int \frac{-6+8x}{9-6x+4x^2}\,dx}{944784} - \frac{5\int \frac{6+8x}{9+6x+4x^2}\,dx}{2834352} + \frac{\int \frac{1}{9-6x+4x^2}\,dx}{157464} + \frac{\int \frac{1}{9+6x+4x^2}\,dx}{59049} + \frac{19\int \frac{1}{9+6x+4x^2}\,dx}{472392}$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} + \frac{3+4x}{236196(9+6x+4x^2)} - \frac{\log(3-2x)}{354294} + \frac{\log(3+2x)}{118098} - \frac{\log(9-6x+4x^2)}{944784} - \frac{5\log(9+6x+4x^2)}{2834352} - \frac{\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, -6+8x\right)}{78732} - \frac{2\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, 6+8x\right)}{59049} - \frac{19\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, 6+8x\right)}{236196}$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} + \frac{3+4x}{236196(9+6x+4x^2)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} - \frac{\log(3-2x)}{354294} + \frac{\log(3+2x)}{118098} - \frac{\log(9-6x+4x^2)}{944784} - \frac{5\log(9+6x+4x^2)}{2834352}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.78

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = \frac{\frac{648x}{81+54x-24x^3-16x^4}+2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)+18\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-8\log(3-2x)+24\log(3+2x)-3\log(9\ldots}{2834352}$$

```
[In] Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] ((648*x)/(81 + 54*x - 24*x^3 - 16*x^4) + 2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqr
t[3])] + 18*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 8*Log[3 - 2*x] + 24*Log
[3 + 2*x] - 3*Log[9 - 6*x + 4*x^2] - 5*Log[9 + 6*x + 4*x^2])/2834352
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{x}{69984\left(x^4+\frac{3}{2}x^3-\frac{27}{8}x-\frac{81}{16}\right)}-\frac{5\ln(16x^2+24x+36)}{2834352}+\frac{\arctan\left(\frac{(4x+3)\sqrt{3}}{9}\right)\sqrt{3}}{157464}-\frac{\ln(-3+2x)}{354294}+\frac{\ln(2x+3)}{118098}+\frac{\sqrt{3}\arctan\left(\frac{2\left(-\frac{3}{2}\ldots\right.}{}\right)}{1417176}$
default	$-\frac{1}{472392(-3+2x)}-\frac{\ln(-3+2x)}{354294}-\frac{\ln(4x^2-6x+9)}{944784}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{1417176}-\frac{1}{157464(2x+3)}+\frac{\ln(2x+3)}{118098}-\frac{-3x-\frac{9}{4}\ldots}{708588(x^2+\frac{3}{2}\ldots}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\ldots\right)}{2}\ldots\right.}{6(x^6)^{\frac{1}{6}}}\ldots\right)}{236196}$

```
[In] int((4*x^2-6*x+9)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/69984*x/(x^4+3/2*x^3-27/8*x-81/16)-5/2834352*ln(16*x^2+24*x+36)+1/157464
*arctan(1/9*(4*x+3)*3^(1/2))*3^(1/2)-1/354294*ln(-3+2*x)+1/118098*ln(2*x+3)
+1/1417176*3^(1/2)*arctan(2/9*(-3/2+2*x)*3^(1/2))-1/944784*ln(4*x^2-6*x+9)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.32

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = \frac{18\sqrt{3}(16\,x^4+24\,x^3-54\,x-81)\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+2\sqrt{3}(16\,x^4+24\,x^3-54\,x-81)\arctan\left(\frac{1}{9}\sqrt{3}\right.}{}$$

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/2834352*(18*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
 + 3)) + 2*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*x -
3)) - 5*(16*x^4 + 24*x^3 - 54*x - 81)*log(4*x^2 + 6*x + 9) - 3*(16*x^4 + 24
*x^3 - 54*x - 81)*log(4*x^2 - 6*x + 9) + 24*(16*x^4 + 24*x^3 - 54*x - 81)*l
og(2*x + 3) - 8*(16*x^4 + 24*x^3 - 54*x - 81)*log(2*x - 3) - 648*x)/(16*x^4
 + 24*x^3 - 54*x - 81)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\begin{aligned}\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = &-\frac{x}{69984x^4+104976x^3-236196x-354294}-\frac{\log\left(x-\frac{3}{2}\right)}{354294}\\&+\frac{\log\left(x+\frac{3}{2}\right)}{118098}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784}-\frac{5\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{2834352}\\&+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{1417176}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{157464}\end{aligned}$$

```
[In] integrate((4*x**2-6*x+9)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(69984*x**4 + 104976*x**3 - 236196*x - 354294) - log(x - 3/2)/354294 + l
og(x + 3/2)/118098 - log(x**2 - 3*x/2 + 9/4)/944784 - 5*log(x**2 + 3*x/2 +
9/4)/2834352 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/1417176 + sqrt(3)*at
an(4*sqrt(3)*x/9 + sqrt(3)/3)/157464
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = \frac{1}{157464}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{1}{1417176}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{x}{4374\,(16\,x^4+24\,x^3-54\,x-81)} - \frac{5}{2834352}\,\log\left(4\,x^2+6\,x+9\right) - \frac{1}{944784}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{118098}\,\log\left(2\,x+3\right) - \frac{1}{354294}\,\log\left(2\,x-3\right)$$

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 + 24*x^3 - 54*x - 81) - 5/2834352*
log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/118098*log(2*x + 3
) - 1/354294*log(2*x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = \frac{1}{157464}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{1}{1417176}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{x}{4374\,(4\,x^2+6\,x+9)(2\,x+3)(2\,x-3)} - \frac{5}{2834352}\,\log\left(4\,x^2+6\,x+9\right) - \frac{1}{944784}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{118098}\,\log\left(|2\,x+3|\right) - \frac{1}{354294}\,\log\left(|2\,x-3|\right)$$

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(2*x + 3)*(2*x - 3)) -
5/2834352*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/118098*l
og(abs(2*x + 3)) - 1/354294*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 11.76 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{118098} - \frac{\ln\left(x-\frac{3}{2}\right)}{354294} - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right) - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right) + \frac{x}{69984\left(-x^4-\frac{3x^3}{2}+\frac{27x}{8}+\frac{81}{16}\right)}$$

```
[In] int((4*x^2 - 6*x + 9)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/118098 - log(x - 3/2)/354294 - log(x - (3^(1/2)*3i)/4 + 3/4)*(
(3^(1/2)*1i)/314928 + 5/2834352) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*
1i)/314928 - 5/2834352) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/28343
52 + 1/944784) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/2834352 - 1/94
4784) + x/(69984*((27*x)/8 - (3*x^3)/2 - x^4 + 81/16))
```

3.572 $\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx=\frac{1}{157464(3-2x)}-\frac{1}{472392(3+2x)}-\frac{3-4x}{236196(9-6x+4x^2)}-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{472392\sqrt{3}}-\frac{\log(3-2x)}{118098}+\frac{\log(3+2x)}{354294}+\frac{5\log(9-6x+4x^2)}{2834352}+\frac{\log(9+6x+4x^2)}{944784}$$

```
[Out] 1/157464/(3-2*x)-1/472392/(3+2*x)+1/236196*(-3+4*x)/(4*x^2-6*x+9)-1/118098*
ln(3-2*x)+1/354294*ln(3+2*x)+5/2834352*ln(4*x^2-6*x+9)+1/944784*ln(4*x^2+6*
x+9)-1/157464*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/1417176*arctan(1/9*(3+4
*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1600, 2099, 628, 632, 210, 648, 642}

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx=-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{472392\sqrt{3}}-\frac{3-4x}{236196(4x^2-6x+9)}+\frac{5\log(4x^2-6x+9)}{2834352}+\frac{\log(4x^2+6x+9)}{944784}+\frac{1}{157464(3-2x)}-\frac{1}{472392(2x+3)}-\frac{\log(3-2x)}{118098}+\frac{\log(2x+3)}{354294}$$

```
[In] Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(157464*(3 - 2*x)) - 1/(472392*(3 + 2*x)) - (3 - 4*x)/(236196*(9 - 6*x +
4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) + ArcTan[(3 + 4*x)/
(3*Sqrt[3])]/(472392*Sqrt[3]) - Log[3 - 2*x]/118098 + Log[3 + 2*x]/354294 +
 (5*Log[9 - 6*x + 4*x^2])/2834352 + Log[9 + 6*x + 4*x^2]/944784
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(9+6x+4x^2)\,(81-54x+24x^3-16x^4)^2}\,dx \\
&= \int \left(\frac{1}{78732(-3+2x)^2}-\frac{1}{59049(-3+2x)}+\frac{1}{236196(3+2x)^2}+\frac{1}{177147(3+2x)}\right. \\
&\quad \left.+\frac{1}{4374\,(9-6x+4x^2)^2}+\frac{21+10x}{708588\,(9-6x+4x^2)}+\frac{3+2x}{236196\,(9+6x+4x^2)}\right) dx \\
&= \frac{1}{157464(3-2x)}-\frac{1}{472392(3+2x)}-\frac{\log(3-2x)}{118098}+\frac{\log(3+2x)}{354294} \\
&\quad +\frac{\int \frac{21+10x}{9-6x+4x^2}\,dx}{708588}+\frac{\int \frac{3+2x}{9+6x+4x^2}\,dx}{236196}+\frac{\int \frac{1}{(9-6x+4x^2)^2}\,dx}{4374} \\
&= \frac{1}{157464(3-2x)}-\frac{1}{472392(3+2x)}-\frac{3-4x}{236196\,(9-6x+4x^2)}-\frac{\log(3-2x)}{118098}+\frac{\log(3+2x)}{354294} \\
&\quad +\frac{\int \frac{6+8x}{9+6x+4x^2}\,dx}{944784}+\frac{5\int \frac{-6+8x}{9-6x+4x^2}\,dx}{2834352}+\frac{\int \frac{1}{9+6x+4x^2}\,dx}{157464}+\frac{\int \frac{1}{9-6x+4x^2}\,dx}{59049}+\frac{19\int \frac{1}{9-6x+4x^2}\,dx}{472392} \\
&= \frac{1}{157464(3-2x)}-\frac{1}{472392(3+2x)}-\frac{3-4x}{236196\,(9-6x+4x^2)} \\
&\quad -\frac{\log(3-2x)}{118098}+\frac{\log(3+2x)}{354294}+\frac{5\log(9-6x+4x^2)}{2834352} \\
&\quad +\frac{\log(9+6x+4x^2)}{944784}-\frac{\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,6+8x\right)}{78732} \\
&\quad -\frac{2\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right)}{59049}-\frac{19\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right)}{236196} \\
&= \frac{1}{157464(3-2x)}-\frac{1}{472392(3+2x)}-\frac{3-4x}{236196\,(9-6x+4x^2)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} \\
&\quad +\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{472392\sqrt{3}}-\frac{\log(3-2x)}{118098}+\frac{\log(3+2x)}{354294}+\frac{5\log(9-6x+4x^2)}{2834352} \\
&\quad +\frac{\log(9+6x+4x^2)}{944784}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.78

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx$$
$$= \frac{\frac{648x}{81-54x+24x^3-16x^4}+18\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)+2\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-24\log(3-2x)+8\log(3+2x)+5\log(9}{2834352}$$

```
[In] Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] ((648*x)/(81 - 54*x + 24*x^3 - 16*x^4) + 18*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sq
rt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 24*Log[3 - 2*x] + 8*Log
[3 + 2*x] + 5*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/2834352
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{x}{69984\left(x^4-\frac{3}{2}x^3+\frac{27}{8}x-\frac{81}{16}\right)}+\frac{\ln\left(16x^2+24x+36\right)}{944784}+\frac{\arctan\left(\frac{(4x+3)\sqrt{3}}{9}\right)\sqrt{3}}{1417176}-\frac{\ln(-3+2x)}{118098}+\frac{5\ln\left(36x^2-54x+81\right)}{2834352}+\frac{\sqrt{3}\,\mathrm{arc}}{}$
default	$-\frac{1}{157464(-3+2x)}-\frac{\ln(-3+2x)}{118098}+\frac{3x-\frac{9}{4}}{708588x^2-1062882x+1594323}+\frac{5\ln\left(4x^2-6x+9\right)}{2834352}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{157464}-\frac{1}{472392(}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right.}{}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{236196}$

```
[In] int((4*x^2+6*x+9)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/69984*x/(x^4-3/2*x^3+27/8*x-81/16)+1/944784*ln(16*x^2+24*x+36)+1/1417176
*arctan(1/9*(4*x+3)*3^(1/2))*3^(1/2)-1/118098*ln(-3+2*x)+5/2834352*ln(36*x^
2-54*x+81)+1/157464*3^(1/2)*arctan(2/27*(6*x-9/2)*3^(1/2))+1/354294*ln(2*x+
3)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.32

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = \frac{2\sqrt{3}(16\,x^4-24\,x^3+54\,x-81)\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+18\sqrt{3}(16\,x^4-24\,x^3+54\,x-81)\arctan\left(\frac{1}{9}\sqrt{3}(4\right.}{}$$

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/2834352*(2*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
+ 3)) + 18*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x -
3)) + 3*(16*x^4 - 24*x^3 + 54*x - 81)*log(4*x^2 + 6*x + 9) + 5*(16*x^4 - 24
*x^3 + 54*x - 81)*log(4*x^2 - 6*x + 9) + 8*(16*x^4 - 24*x^3 + 54*x - 81)*lo
g(2*x + 3) - 24*(16*x^4 - 24*x^3 + 54*x - 81)*log(2*x - 3) - 648*x)/(16*x^4
 - 24*x^3 + 54*x - 81)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = -\frac{x}{69984x^4-104976x^3+236196x-354294} - \frac{\log\left(x-\frac{3}{2}\right)}{118098} + \frac{\log\left(x+\frac{3}{2}\right)}{354294} + \frac{5\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{2834352} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{944784} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{157464} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{1417176}$$

```
[In] integrate((4*x**2+6*x+9)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(69984*x**4 - 104976*x**3 + 236196*x - 354294) - log(x - 3/2)/118098 + l
og(x + 3/2)/354294 + 5*log(x**2 - 3*x/2 + 9/4)/2834352 + log(x**2 + 3*x/2 +
 9/4)/944784 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/157464 + sqrt(3)*ata
n(4*sqrt(3)*x/9 + sqrt(3)/3)/1417176
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\begin{aligned}\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = {} & \frac{1}{1417176}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) \\ & + \frac{1}{157464}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) \\ & - \frac{x}{4374\,(16\,x^4-24\,x^3+54\,x-81)} \\ & + \frac{1}{944784}\,\log\left(4\,x^2+6\,x+9\right) + \frac{5}{2834352}\,\log\left(4\,x^2-6\,x+9\right) \\ & + \frac{1}{354294}\,\log\left(2\,x+3\right) - \frac{1}{118098}\,\log\left(2\,x-3\right)\end{aligned}$$

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 - 24*x^3 + 54*x - 81) + 1/944784*l
og(4*x^2 + 6*x + 9) + 5/2834352*log(4*x^2 - 6*x + 9) + 1/354294*log(2*x + 3
) - 1/118098*log(2*x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = {} & \frac{1}{1417176}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) \\ & + \frac{1}{157464}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) \\ & - \frac{x}{4374\,(4\,x^2-6\,x+9)(2\,x+3)(2\,x-3)} \\ & + \frac{1}{944784}\,\log\left(4\,x^2+6\,x+9\right) + \frac{5}{2834352}\,\log\left(4\,x^2-6\,x+9\right) \\ & + \frac{1}{354294}\,\log\left(|2\,x+3|\right) - \frac{1}{118098}\,\log\left(|2\,x-3|\right)\end{aligned}$$

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2*x + 3)*(2*x - 3)) +
1/944784*log(4*x^2 + 6*x + 9) + 5/2834352*log(4*x^2 - 6*x + 9) + 1/354294*l
og(abs(2*x + 3)) - 1/118098*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.78

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{354294} - \frac{\ln\left(x-\frac{3}{2}\right)}{118098} - \frac{x}{69984\left(x^4-\frac{3\,x^3}{2}+\frac{27\,x}{8}-\frac{81}{16}\right)} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right) - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right)$$

```
[In] int((6*x + 4*x^2 + 9)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/354294 - log(x - 3/2)/118098 - x/(69984*((27*x)/8 - (3*x^3)/2
+ x^4 - 81/16)) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/314928 - 5/28
34352) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/314928 + 5/2834352) -
log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/2834352 - 1/944784) + log(x + (
3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/2834352 + 1/944784)
```

3.573 $\int \frac{27-8x^3}{(729-64x^6)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 113

$$\int \frac{27-8x^3}{(729-64x^6)^2} \, dx = \frac{x}{4374(27+8x^3)} - \frac{7\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} + \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} - \frac{\log(3-2x)}{157464} + \frac{7\log(3+2x)}{472392} - \frac{7\log(9-6x+4x^2)}{944784} + \frac{\log(9+6x+4x^2)}{314928}$$

```
[Out] 1/4374*x/(8*x^3+27)-1/157464*ln(3-2*x)+7/472392*ln(3+2*x)-7/944784*ln(4*x^2
-6*x+9)+1/314928*ln(4*x^2+6*x+9)-7/472392*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/
2)+1/157464*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1418, 425, 536, 206, 31, 648, 632, 210, 642}

$$\int \frac{27-8x^3}{(729-64x^6)^2} \, dx = -\frac{7\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} + \frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488\sqrt{3}} + \frac{x}{4374(8x^3+27)} - \frac{7\log(4x^2-6x+9)}{944784} + \frac{\log(4x^2+6x+9)}{314928} - \frac{\log(3-2x)}{157464} + \frac{7\log(2x+3)}{472392}$$

```
[In] Int[(27 - 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] x/(4374*(27 + 8*x^3)) - (7*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3])
+ ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/157464 + (7*
```

```
Log[3 + 2*x])/472392 - (7*Log[9 - 6*x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^
2]/314928
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1418

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(27-8x^3)(27+8x^3)^2}\,dx$$

$$= \frac{x}{4374(27+8x^3)} - \frac{\int \frac{-1080+128x^3}{(27-8x^3)(27+8x^3)}\,dx}{34992}$$

$$= \frac{x}{4374(27+8x^3)} + \frac{\int \frac{1}{27-8x^3}\,dx}{2916} + \frac{7\int \frac{1}{27+8x^3}\,dx}{8748}$$

$$= \frac{x}{4374(27+8x^3)} + \frac{\int \frac{1}{3-2x}\,dx}{78732} + \frac{\int \frac{6+2x}{9+6x+4x^2}\,dx}{78732} + \frac{7\int \frac{1}{3+2x}\,dx}{236196} + \frac{7\int \frac{6-2x}{9-6x+4x^2}\,dx}{236196}$$

$$= \frac{x}{4374(27+8x^3)} - \frac{\log(3-2x)}{157464} + \frac{7\log(3+2x)}{472392} + \frac{\int \frac{6+8x}{9+6x+4x^2}\,dx}{314928} - \frac{7\int \frac{-6+8x}{9-6x+4x^2}\,dx}{944784} + \frac{\int \frac{1}{9+6x+4x^2}\,dx}{17496} + \frac{7\int \frac{1}{9-6x+4x^2}\,dx}{52488}$$

$$= \frac{x}{4374(27+8x^3)} - \frac{\log(3-2x)}{157464} + \frac{7\log(3+2x)}{472392} - \frac{7\log(9-6x+4x^2)}{944784} + \frac{\log(9+6x+4x^2)}{314928} - \frac{\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, 6+8x\right)}{8748} - \frac{7\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, -6+8x\right)}{26244}$$

$$= \frac{x}{4374(27+8x^3)} - \frac{7\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} - \frac{\log(3-2x)}{157464} + \frac{7\log(3+2x)}{472392} - \frac{7\log(9-6x+4x^2)}{944784} + \frac{\log(9+6x+4x^2)}{314928}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx$$
$$= \frac{\frac{216x}{27+8x^3}+14\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)+6\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-6\log(3-2x)+14\log(3+2x)-7\log\left(9-6x+4x\right.}{944784}$$

```
[In] Integrate[(27 - 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] ((216*x)/(27 + 8*x^3) + 14*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 6*Sqrt[
3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 6*Log[3 - 2*x] + 14*Log[3 + 2*x] - 7*Log
[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/944784
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

method	result
risch	$\frac{x}{34992x^3+118098}-\frac{\ln(-3+2x)}{157464}+\frac{7\ln(2x+3)}{472392}-\frac{7\ln(4x^2-6x+9)}{944784}+\frac{7\sqrt{3}\arctan\left(\frac{2\left(-\frac{3}{2}+2x\right)\sqrt{3}}{9}\right)}{472392}+\frac{\ln(4x^2+6x+9)}{314928}+\frac{\sqrt{3}\,\text{ar}}{}$
default	$-\frac{\ln(-3+2x)}{157464}-\frac{-\frac{3x}{4}-\frac{9}{8}}{118098\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)}-\frac{7\ln(4x^2-6x+9)}{944784}+\frac{7\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{472392}-\frac{1}{78732(2x+3)}+\frac{7\ln(2x+3)}{472392}+\frac{\ln(4x^2}{31}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right.}{2}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{78732}$

```
[In] int((-8*x^3+27)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/34992*x/(x^3+27/8)-1/157464*ln(-3+2*x)+7/472392*ln(2*x+3)-7/944784*ln(4*x
^2-6*x+9)+7/472392*3^(1/2)*arctan(2/9*(-3/2+2*x)*3^(1/2))+1/314928*ln(4*x^2
+6*x+9)+1/157464*3^(1/2)*arctan(2/9*(2*x+3/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.16

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx$$
$$= \frac{6\sqrt{3}(8\,x^3+27)\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+14\sqrt{3}(8\,x^3+27)\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right)+3\,(8\,x^3+27)\log\left(4\,x\right.}{94478}$$

```
[In] integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/944784*(6*sqrt(3)*(8*x^3 + 27)*arctan(1/9*sqrt(3)*(4*x + 3)) + 14*sqrt(3)
*(8*x^3 + 27)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(8*x^3 + 27)*log(4*x^2 + 6*
x + 9) - 7*(8*x^3 + 27)*log(4*x^2 - 6*x + 9) + 14*(8*x^3 + 27)*log(2*x + 3)
 - 6*(8*x^3 + 27)*log(2*x - 3) + 216*x)/(8*x^3 + 27)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.97

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx = \frac{x}{34992x^3+118098} - \frac{\log\left(x-\frac{3}{2}\right)}{157464} + \frac{7\log\left(x+\frac{3}{2}\right)}{472392} - \frac{7\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{314928} + \frac{7\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{157464}$$

```
[In] integrate((-8*x**3+27)/(-64*x**6+729)**2,x)
```

```
[Out] x/(34992*x**3 + 118098) - log(x - 3/2)/157464 + 7*log(x + 3/2)/472392 - 7*l
og(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x/2 + 9/4)/314928 + 7*sqrt(3)*
atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(
3)/3)/157464
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.77

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx = \frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right) + \frac{7}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right) + \frac{x}{4374\,(8\,x^3+27)} + \frac{1}{314928}\log\left(4\,x^2+6\,x+9\right) - \frac{7}{944784}\log\left(4\,x^2-6\,x+9\right) + \frac{7}{472392}\log\left(2\,x+3\right) - \frac{1}{157464}\log\left(2\,x-3\right)$$

```
[In] integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)*arctan(1/
9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928*log(4*x^2 + 6*x + 9
) - 7/944784*log(4*x^2 - 6*x + 9) + 7/472392*log(2*x + 3) - 1/157464*log(2*
x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

$$\int \frac{27-8x^3}{\left(729-64x^6\right)^2}\,dx = \frac{1}{157464}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{7}{472392}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) + \frac{x}{4374\,(8\,x^3+27)} + \frac{1}{314928}\,\log\left(4\,x^2+6\,x+9\right) - \frac{7}{944784}\,\log\left(4\,x^2-6\,x+9\right) + \frac{7}{472392}\,\log\left(|2\,x+3|\right) - \frac{1}{157464}\,\log\left(|2\,x-3|\right)$$

```
[In] integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)*arctan(1/
9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928*log(4*x^2 + 6*x + 9
) - 7/944784*log(4*x^2 - 6*x + 9) + 7/472392*log(abs(2*x + 3)) - 1/157464*l
og(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int \frac{27-8x^3}{\left(729-64x^6\right)^2}\,dx = \frac{7\,\ln\left(x+\frac{3}{2}\right)}{472392} - \frac{\ln\left(x-\frac{3}{2}\right)}{157464} + \frac{x}{34992\,\left(x^3+\frac{27}{8}\right)} - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\,\left(-\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\,\left(\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right) - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\,\left(\frac{7}{944784}+\frac{\sqrt{3}\,7\mathrm{i}}{944784}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\,\left(-\frac{7}{944784}+\frac{\sqrt{3}\,7\mathrm{i}}{944784}\right)$$

```
[In] int(-(8*x^3 - 27)/(64*x^6 - 729)^2,x)
```

```
[Out] (7*log(x + 3/2))/472392 - log(x - 3/2)/157464 + x/(34992*(x^3 + 27/8)) - lo
g(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/314928 - 1/314928) + log(x + (3^(
1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/314928 + 1/314928) - log(x - (3^(1/2)*3i)/4
 - 3/4)*((3^(1/2)*7i)/944784 + 7/944784) + log(x + (3^(1/2)*3i)/4 - 3/4)*((
3^(1/2)*7i)/944784 - 7/944784)
```

3.574 $\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2} dx = \frac{1}{26244(3-2x)} - \frac{3-2x}{26244(9-6x+4x^2)} - \frac{11\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} - \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} - \frac{7\log(3-2x)}{157464} + \frac{\log(3+2x)}{472392} + \frac{17\log(9-6x+4x^2)}{944784} + \frac{\log(9+6x+4x^2)}{314928}$$

```
[Out] 1/26244/(3-2*x)+1/26244*(-3+2*x)/(4*x^2-6*x+9)-7/157464*ln(3-2*x)+1/472392*
ln(3+2*x)+17/944784*ln(4*x^2-6*x+9)+1/314928*ln(4*x^2+6*x+9)-11/472392*arct
an(1/9*(3-4*x)*3^(1/2))*3^(1/2)-1/472392*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2
)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1600, 2099, 652, 632, 210, 648, 642}

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2} dx = -\frac{11\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} - \frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{157464\sqrt{3}} - \frac{3-2x}{26244(4x^2-6x+9)} + \frac{17\log(4x^2-6x+9)}{944784} + \frac{\log(4x^2+6x+9)}{314928} + \frac{1}{26244(3-2x)} - \frac{7\log(3-2x)}{157464} + \frac{\log(2x+3)}{472392}$$

```
[In] Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(26244*(3 - 2*x)) - (3 - 2*x)/(26244*(9 - 6*x + 4*x^2)) - (11*ArcTan[(3 -
 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3]) - ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(15746
4*Sqrt[3]) - (7*Log[3 - 2*x])/157464 + Log[3 + 2*x]/472392 + (17*Log[9 - 6*
x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^2]/314928
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{(27-36x+24x^2-8x^3)^2\,(27+36x+24x^2+8x^3)}\,dx \\
&= \int \left(\frac{1}{13122(-3+2x)^2}-\frac{7}{78732(-3+2x)}+\frac{1}{236196(3+2x)}+\frac{3+2x}{4374\,(9-6x+4x^2)^2}+\frac{3+17x}{118098\,(9-6x+4x^2)}+\frac{x}{39366\,(9+6x+4x^2)}\right)dx \\
&= \frac{1}{26244(3-2x)}-\frac{7\log(3-2x)}{157464}+\frac{\log(3+2x)}{472392}+\frac{\int\frac{3+17x}{9-6x+4x^2}\,dx}{118098}+\frac{\int\frac{x}{9+6x+4x^2}\,dx}{39366}+\frac{\int\frac{3+2x}{(9-6x+4x^2)^2}\,dx}{4374} \\
&= \frac{1}{26244(3-2x)}-\frac{3-2x}{26244\,(9-6x+4x^2)}-\frac{7\log(3-2x)}{157464}+\frac{\log(3+2x)}{472392}+\frac{\int\frac{6+8x}{9+6x+4x^2}\,dx}{314928}+\frac{17\int\frac{-6+8x}{9-6x+4x^2}\,dx}{944784}-\frac{\int\frac{1}{9+6x+4x^2}\,dx}{52488}+\frac{\int\frac{1}{9-6x+4x^2}\,dx}{13122}+\frac{7\int\frac{1}{9-6x+4x^2}\,dx}{52488} \\
&= \frac{1}{26244(3-2x)}-\frac{3-2x}{26244\,(9-6x+4x^2)}-\frac{7\log(3-2x)}{157464}+\frac{\log(3+2x)}{472392}+\frac{17\log(9-6x+4x^2)}{944784}+\frac{\log(9+6x+4x^2)}{314928}+\frac{\text{Subst}\left(\int\frac{1}{-108-x^2}\,dx,x,6+8x\right)}{26244}-\frac{\text{Subst}\left(\int\frac{1}{-108-x^2}\,dx,x,-6+8x\right)}{6561}-\frac{7\text{Subst}\left(\int\frac{1}{-108-x^2}\,dx,x,-6+8x\right)}{26244} \\
&= \frac{1}{26244(3-2x)}-\frac{3-2x}{26244\,(9-6x+4x^2)}-\frac{11\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{7\log(3-2x)}{157464}+\frac{\log(3+2x)}{472392}+\frac{17\log(9-6x+4x^2)}{944784}+\frac{\log(9+6x+4x^2)}{314928}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx$$
$$= \frac{\frac{216x}{27-36x+24x^2-8x^3}+22\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-42\log(3-2x)+2\log(3+2x)+17\log(\cdots}{944784}$$

```
[In] Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] ((216*x)/(27 - 36*x + 24*x^2 - 8*x^3) + 22*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqr
t[3])] - 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 42*Log[3 - 2*x] + 2*Log[
3 + 2*x] + 17*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/944784
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{x}{34992\left(x^3-3x^2+\frac{9}{2}x-\frac{27}{8}\right)}-\frac{7\ln(-3+2x)}{157464}+\frac{\ln(4x^2+6x+9)}{314928}-\frac{\sqrt{3}\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{472392}+\frac{17\ln(484x^2-726x+1089)}{944784}+\cdots$
default	$-\frac{1}{26244(-3+2x)}-\frac{7\ln(-3+2x)}{157464}+\frac{\frac{9x}{4}-\frac{27}{8}}{118098x^2-177147x+\frac{531441}{2}}+\frac{17\ln(4x^2-6x+9)}{944784}+\frac{11\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{472392}+\frac{\ln(2x+\cdots}{\cdots}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\cdots\right)}{\cdots}\cdots\right)}{6(x^6)^{\frac{1}{6}}}\cdots\right)}{78732}$

```
[In] int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/34992*x/(x^3-3*x^2+9/2*x-27/8)-7/157464*ln(-3+2*x)+1/314928*ln(4*x^2+6*x
+9)-1/472392*3^(1/2)*arctan(2/9*(2*x+3/2)*3^(1/2))+17/944784*ln(484*x^2-726
*x+1089)+11/472392*3^(1/2)*arctan(2/99*(22*x-33/2)*3^(1/2))+1/472392*ln(2*x
+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.43

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = -\frac{2\sqrt{3}(8x^3-24x^2+36x-27)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-22\sqrt{3}(8x^3-24x^2+36x-27)\arctan\left(\frac{1}{9}\sqrt{3}(4\right.}{}$$

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] -1/944784*(2*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x +
 3)) - 22*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x - 3)
) - 3*(8*x^3 - 24*x^2 + 36*x - 27)*log(4*x^2 + 6*x + 9) - 17*(8*x^3 - 24*x^
2 + 36*x - 27)*log(4*x^2 - 6*x + 9) - 2*(8*x^3 - 24*x^2 + 36*x - 27)*log(2*
x + 3) + 42*(8*x^3 - 24*x^2 + 36*x - 27)*log(2*x - 3) + 216*x)/(8*x^3 - 24*
x^2 + 36*x - 27)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = -\frac{x}{34992x^3-104976x^2+157464x-118098}-\frac{7\log\left(x-\frac{3}{2}\right)}{157464}+\frac{\log\left(x+\frac{3}{2}\right)}{472392}+\frac{17\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{314928}+\frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{472392}$$

```
[In] integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(34992*x**3 - 104976*x**2 + 157464*x - 118098) - 7*log(x - 3/2)/157464 +
 log(x + 3/2)/472392 + 17*log(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x/2
 + 9/4)/314928 + 11*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 - sqrt(3
)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/472392
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = -\frac{1}{472392}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{11}{472392}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{x}{4374\,(8\,x^3-24\,x^2+36\,x-27)} + \frac{1}{314928}\,\log\left(4\,x^2+6\,x+9\right) + \frac{17}{944784}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{472392}\,\log\left(2\,x+3\right) - \frac{7}{157464}\,\log\left(2\,x-3\right)$$

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] -1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(8*x^3 - 24*x^2 + 36*x - 27) + 1/314928*l
og(4*x^2 + 6*x + 9) + 17/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(2*x + 3
) - 7/157464*log(2*x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = -\frac{1}{472392}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{11}{472392}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{x}{4374\,(4\,x^2-6\,x+9)(2\,x-3)} + \frac{1}{314928}\,\log\left(4\,x^2+6\,x+9\right) + \frac{17}{944784}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{472392}\,\log\left(|2\,x+3|\right) - \frac{7}{157464}\,\log\left(|2\,x-3|\right)$$

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] -1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2*x - 3)) + 1/314928*
log(4*x^2 + 6*x + 9) + 17/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(abs(2*
x + 3)) - 7/157464*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{472392} - \frac{7\ \ln\left(x-\frac{3}{2}\right)}{157464} - \frac{x}{34992\ \left(x^3-3\,x^2+\frac{9\,x}{2}-\frac{27}{8}\right)} + \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) - \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{17}{944784}+\frac{\sqrt{3}\,11\mathrm{i}}{944784}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{17}{944784}+\frac{\sqrt{3}\,11\mathrm{i}}{944784}\right)$$

```
[In] int((36*x + 24*x^2 + 8*x^3 + 27)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/472392 - (7*log(x - 3/2))/157464 - x/(34992*((9*x)/2 - 3*x^2 +
 x^3 - 27/8)) + log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/944784 + 1/3149
28) - log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/944784 - 1/314928) - log(
x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*11i)/944784 - 17/944784) + log(x + (3^(
1/2)*3i)/4 - 3/4)*((3^(1/2)*11i)/944784 + 17/944784)
```

3.575 $\int \frac{x(27-2x^3)}{729-64x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 99

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{5\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}} - \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{32\sqrt{3}} - \frac{1}{96}\log(3-2x) - \frac{5}{288}\log(3+2x) + \frac{5}{576}\log\left(9-6x+4x^2\right) + \frac{1}{192}\log\left(9+6x+4x^2\right)$$

```
[Out] -1/96*ln(3-2*x)-5/288*ln(3+2*x)+5/576*ln(4*x^2-6*x+9)+1/192*ln(4*x^2+6*x+9)
-5/288*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)-1/96*arctan(1/9*(3+4*x)*3^(1/2))
*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1525, 298, 31, 648, 632, 210, 642}

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{5\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}} - \frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{32\sqrt{3}} + \frac{5}{576}\log\left(4x^2-6x+9\right) + \frac{1}{192}\log\left(4x^2+6x+9\right) - \frac{1}{96}\log(3-2x) - \frac{5}{288}\log(2x+3)$$

```
[In] Int[(x*(27 - 2*x^3))/(729 - 64*x^6),x]
```

```
[Out] (-5*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(96*Sqrt[3]) - ArcTan[(3 + 4*x)/(3*Sqrt[
3])]/(32*Sqrt[3]) - Log[3 - 2*x]/96 - (5*Log[3 + 2*x])/288 + (5*Log[9 - 6*x
 + 4*x^2])/576 + Log[9 + 6*x + 4*x^2]/192
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1525

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[-(e/2 + c*(d/(2*q))), Int[(f*
x)^m/(q - c*x^n), x], x] + Dist[e/2 - c*(d/(2*q)), Int[(f*x)^m/(q + c*x^n),
 x], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = 3\int \frac{x}{216-64x^3}\,dx + 5\int \frac{x}{216+64x^3}\,dx$$

$$
\begin{aligned}
&= \frac{1}{24}\int \frac{1}{6-4x}\,dx - \frac{1}{24}\int \frac{6-4x}{36+24x+16x^2}\,dx - \frac{5}{72}\int \frac{1}{6+4x}\,dx + \frac{5}{72}\int \frac{6+4x}{36-24x+16x^2}\,dx \\
&= -\frac{1}{96}\log(3-2x) - \frac{5}{288}\log(3+2x) \\
&\quad + \frac{1}{192}\int \frac{24+32x}{36+24x+16x^2}\,dx + \frac{5}{576}\int \frac{-24+32x}{36-24x+16x^2}\,dx \\
&\quad - \frac{3}{8}\int \frac{1}{36+24x+16x^2}\,dx + \frac{5}{8}\int \frac{1}{36-24x+16x^2}\,dx \\
&= -\frac{1}{96}\log(3-2x) - \frac{5}{288}\log(3+2x) + \frac{5}{576}\log\left(9-6x+4x^2\right) + \frac{1}{192}\log\left(9+6x+4x^2\right) \\
&\quad + \frac{3}{4}\text{Subst}\left(\int \frac{1}{-1728-x^2}\,dx, x, 24+32x\right) - \frac{5}{4}\text{Subst}\left(\int \frac{1}{-1728-x^2}\,dx, x, -24+32x\right) \\
&= -\frac{5\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}} - \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{32\sqrt{3}} - \frac{1}{96}\log(3-2x) - \frac{5}{288}\log(3+2x) \\
&\quad + \frac{5}{576}\log\left(9-6x+4x^2\right) + \frac{1}{192}\log\left(9+6x+4x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$
\begin{aligned}
\int \frac{x(27-2x^3)}{729-64x^6}\,dx = \frac{1}{576}\Bigg(&10\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) - 6\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right) - 6\log(3-2x) \\
&- 10\log(3+2x) + 5\log\left(9-6x+4x^2\right) + 3\log\left(9+6x+4x^2\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[(x*(27 - 2*x^3))/(729 - 64*x^6),x]
```

```
[Out] (10*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 6*Sqrt[3]*ArcTan[(3 + 4*x)/(3*
Sqrt[3])] - 6*Log[3 - 2*x] - 10*Log[3 + 2*x] + 5*Log[9 - 6*x + 4*x^2] + 3*L
og[9 + 6*x + 4*x^2])/576
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.77

method	result
default	$-\frac{\ln(-3+2x)}{96}+\frac{5\ln(4x^2-6x+9)}{576}+\frac{5\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{288}-\frac{5\ln(2x+3)}{288}+\frac{\ln(4x^2+6x+9)}{192}-\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{96}$
risch	$-\frac{\ln(-3+2x)}{96}-\frac{5\ln(2x+3)}{288}+\frac{5\ln(4x^2-6x+9)}{576}+\frac{5\sqrt{3}\,\arctan\left(\frac{2\left(-\frac{3}{2}+2x\right)\sqrt{3}}{9}\right)}{288}+\frac{\ln(4x^2+6x+9)}{192}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{96}$
meijerg	$\frac{x^5\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}+\sqrt{3}\,\text{arcta}\ldots\right.}{288(x^6)^{\frac{5}{6}}}$

[In] int(x*(-2*x^3+27)/(-64*x^6+729),x,method=_RETURNVERBOSE)

[Out] -1/96*ln(-3+2*x)+5/576*ln(4*x^2-6*x+9)+5/288*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))-5/288*ln(2*x+3)+1/192*ln(4*x^2+6*x+9)-1/96*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.76

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{192}\log\left(4x^2+6x+9\right)+\frac{5}{576}\log\left(4x^2-6x+9\right)-\frac{5}{288}\log\left(2x+3\right)-\frac{1}{96}\log\left(2x-3\right)$$

[In] integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="fricas")

[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/192*log(4*x^2 + 6*x + 9) + 5/576*log(4*x^2 - 6*x + 9) - 5/288*log(2*x + 3) - 1/96*log(2*x - 3)

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{96} - \frac{5\log\left(x+\frac{3}{2}\right)}{288} + \frac{5\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{576} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{192} + \frac{5\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{288} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{96}$$

```
[In] integrate(x*(-2*x**3+27)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/96 - 5*log(x + 3/2)/288 + 5*log(x**2 - 3*x/2 + 9/4)/576 + log
(x**2 + 3*x/2 + 9/4)/192 + 5*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/288 -
sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/96
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.76

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{1}{192}\log\left(4x^2+6x+9\right) + \frac{5}{576}\log\left(4x^2-6x+9\right) - \frac{5}{288}\log\left(2x+3\right) - \frac{1}{96}\log\left(2x-3\right)$$

```
[In] integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqrt
(3)*(4*x - 3)) + 1/192*log(4*x^2 + 6*x + 9) + 5/576*log(4*x^2 - 6*x + 9) -
5/288*log(2*x + 3) - 1/96*log(2*x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{1}{192}\log\left(x^2+\frac{3}{2}x+\frac{9}{4}\right) + \frac{5}{576}\log\left(x^2-\frac{3}{2}x+\frac{9}{4}\right) - \frac{5}{288}\log\left(\left|x+\frac{3}{2}\right|\right) - \frac{1}{96}\log\left(\left|x-\frac{3}{2}\right|\right)$$

```
[In] integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="giac")

[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqrt
(3)*(4*x - 3)) + 1/192*log(x^2 + 3/2*x + 9/4) + 5/576*log(x^2 - 3/2*x + 9/4
) - 5/288*log(abs(x + 3/2)) - 1/96*log(abs(x - 3/2))
```

Mupad [B] (verification not implemented)

Time = 9.68 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{\ln\left(x-\frac{3}{2}\right)}{96} - \frac{5\,\ln\left(x+\frac{3}{2}\right)}{288} + \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{192}+\frac{\sqrt{3}\,1\mathrm{i}}{192}\right) - \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{192}+\frac{\sqrt{3}\,1\mathrm{i}}{192}\right) - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{5}{576}+\frac{\sqrt{3}\,5\mathrm{i}}{576}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{5}{576}+\frac{\sqrt{3}\,5\mathrm{i}}{576}\right)$$

```
[In] int((x*(2*x^3 - 27))/(64*x^6 - 729),x)

[Out] log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/192 + 1/192) - (5*log(x + 3/2))
/288 - log(x - 3/2)/96 - log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/192 -
1/192) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*5i)/576 - 5/576) + log(x +
 (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*5i)/576 + 5/576)
```

3.576 $\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{a+bx^n}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 162

$$\int \frac{(cx)^m\left(d+ex^n+fx^{2n}+gx^{3n}\right)}{a+bx^n}\,dx = \frac{(bf-ag)x^{1+n}(cx)^m}{b^2(1+m+n)}+\frac{gx^{1+2n}(cx)^m}{b(1+m+2n)}+\frac{\left(b^2e-abf+a^2g\right)(cx)^{1+m}}{b^3c(1+m)} + \frac{\left(b^3d-ab^2e+a^2bf-a^3g\right)(cx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{ab^3c(1+m)}$$

```
[Out] (-a*g+b*f)*x^(1+n)*(c*x)^m/b^2/(1+m+n)+g*x^(1+2*n)*(c*x)^m/b/(1+m+2*n)+(a^2
*g-a*b*f+b^2*e)*(c*x)^(1+m)/b^3/c/(1+m)+(-a^3*g+a^2*b*f-a*b^2*e+b^3*d)*(c*x
)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/b^3/c/(1+m)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1858, 20, 30, 371}

$$\int \frac{(cx)^m\left(d+ex^n+fx^{2n}+gx^{3n}\right)}{a+bx^n}\,dx = \frac{(cx)^{m+1}\left(a^2g-abf+b^2e\right)}{b^3c(m+1)} + \frac{(cx)^{m+1}\left(a^3(-g)+a^2bf-ab^2e+b^3d\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{ab^3c(m+1)} + \frac{x^{n+1}(cx)^m(bf-ag)}{b^2(m+n+1)}+\frac{gx^{2n+1}(cx)^m}{b(m+2n+1)}$$

```
[In] Int[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n),x]
```

```
[Out] ((b*f - a*g)*x^(1 + n)*(c*x)^m)/(b^2*(1 + m + n)) + (g*x^(1 + 2*n)*(c*x)^m)
/(b*(1 + m + 2*n)) + ((b^2*e - a*b*f + a^2*g)*(c*x)^(1 + m))/(b^3*c*(1 + m)
) + ((b^3*d - a*b^2*e + a^2*b*f - a^3*g)*(c*x)^(1 + m)*Hypergeometric2F1[1,
 (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*b^3*c*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1858

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(b^2e - abf + a^2g)\,(cx)^m}{b^3} + \frac{(bf-ag)x^n(cx)^m}{b^2} + \frac{gx^{2n}(cx)^m}{b} + \frac{(b^3d - ab^2e + a^2bf - a^3g)\,(cx)^m}{b^3\,(a+bx^n)} \right) dx$$

$$= \frac{(b^2e - abf + a^2g)\,(cx)^{1+m}}{b^3c(1+m)} + \frac{g\int x^{2n}(cx)^m\,dx}{b} + \frac{(bf-ag)\int x^n(cx)^m\,dx}{b^2} + \frac{(b^3d - ab^2e + a^2bf - a^3g)\int \frac{(cx)^m}{a+bx^n}\,dx}{b^3}$$

$$
\begin{aligned}
&= \frac{(b^2e-abf+a^2g)\,(cx)^{1+m}}{b^3c(1+m)} \\
&\quad+ \frac{(b^3d-ab^2e+a^2bf-a^3g)\,(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{ab^3c(1+m)} \\
&\quad+ \frac{(gx^{-m}(cx)^m)\int x^{m+2n}\,dx}{b}+\frac{((bf-ag)x^{-m}(cx)^m)\int x^{m+n}\,dx}{b^2} \\
&= \frac{(bf-ag)x^{1+n}(cx)^m}{b^2(1+m+n)}+\frac{gx^{1+2n}(cx)^m}{b(1+m+2n)}+\frac{(b^2e-abf+a^2g)\,(cx)^{1+m}}{b^3c(1+m)} \\
&\quad+ \frac{(b^3d-ab^2e+a^2bf-a^3g)\,(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{ab^3c(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.80

$$
\int \frac{(cx)^m\,(d+ex^n+fx^{2n}+gx^{3n})}{a+bx^n}\,dx
= \frac{x(cx)^m\left(\frac{b^2e-abf+a^2g}{1+m}+\frac{b(bf-ag)x^n}{1+m+n}+\frac{b^2gx^{2n}}{1+m+2n}+\frac{(b^3d-ab^2e+a^2bf-a^3g)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a(1+m)}\right)}{b^3}
$$

```
[In] Integrate[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n),x]
```

```
[Out] (x*(c*x)^m*((b^2*e - a*b*f + a^2*g)/(1 + m) + (b*(b*f - a*g)*x^n)/(1 + m +
n) + (b^2*g*x^(2*n))/(1 + m + 2*n) + ((b^3*d - a*b^2*e + a^2*b*f - a^3*g)*H
ypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*(1 + m))))/
b^3
```

Maple [F]

$$
\int \frac{(cx)^m\,(d+e\,x^n+f\,x^{2n}+g\,x^{3n})}{a+b\,x^n}dx
$$

```
[In] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x)
```

```
[Out] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(cx)^m (d + ex^n + fx^{2n} + gx^{3n})}{a + bx^n} \, dx = \int \frac{(gx^{3n} + fx^{2n} + ex^n + d)(cx)^m}{bx^n + a} \, dx$$

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x, algorithm="fri
cas")
```

```
[Out] integral((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.50 (sec) , antiderivative size = 860, normalized size of antiderivative = 5.31

$$\int \frac{(cx)^m (d + ex^n + fx^{2n} + gx^{3n})}{a + bx^n} \, dx = \text{Too large to display}$$

```
[In] integrate((c*x)**m*(d+e*x**n+f*x**(2*n)+g*x**(3*n))/(a+b*x**n),x)
```

```
[Out] a**(m/n + 1/n)*a**(-m/n - 1 - 1/n)*c**m*d*m*x**(m + 1)*lerchphi(b*x**n*exp_
polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 + 1/n)) +
 a**(m/n + 1/n)*a**(-m/n - 1 - 1/n)*c**m*d*x**(m + 1)*lerchphi(b*x**n*exp_p
olar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 + 1/n)) +
a**(-m/n - 4 - 1/n)*a**(m/n + 3 + 1/n)*c**m*g*m*x**(m + 3*n + 1)*lerchphi(b
*x**n*exp_polar(I*pi)/a, 1, m/n + 3 + 1/n)*gamma(m/n + 3 + 1/n)/(n**2*gamma
(m/n + 4 + 1/n)) + 3*a**(-m/n - 4 - 1/n)*a**(m/n + 3 + 1/n)*c**m*g*x**(m +
3*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 3 + 1/n)*gamma(m/n + 3
 + 1/n)/(n*gamma(m/n + 4 + 1/n)) + a**(-m/n - 4 - 1/n)*a**(m/n + 3 + 1/n)*c
**m*g*x**(m + 3*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 3 + 1/n)
*gamma(m/n + 3 + 1/n)/(n**2*gamma(m/n + 4 + 1/n)) + a**(-m/n - 3 - 1/n)*a**
(m/n + 2 + 1/n)*c**m*f*m*x**(m + 2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a
, 1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/(n**2*gamma(m/n + 3 + 1/n)) + 2*a*
*(-m/n - 3 - 1/n)*a**(m/n + 2 + 1/n)*c**m*f*x**(m + 2*n + 1)*lerchphi(b*x**
n*exp_polar(I*pi)/a, 1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/(n*gamma(m/n +
3 + 1/n)) + a**(-m/n - 3 - 1/n)*a**(m/n + 2 + 1/n)*c**m*f*x**(m + 2*n + 1)*
lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/(
n**2*gamma(m/n + 3 + 1/n)) + a**(-m/n - 2 - 1/n)*a**(m/n + 1 + 1/n)*c**m*e*
m*x**(m + n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1 + 1/n)*gamma
(m/n + 1 + 1/n)/(n**2*gamma(m/n + 2 + 1/n)) + a**(-m/n - 2 - 1/n)*a**(m/n +
 1 + 1/n)*c**m*e*x**(m + n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n +
 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n*gamma(m/n + 2 + 1/n)) + a**(-m/n - 2 - 1/
n)*a**(m/n + 1 + 1/n)*c**m*e*x**(m + n + 1)*lerchphi(b*x**n*exp_polar(I*pi)
/a, 1, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n**2*gamma(m/n + 2 + 1/n))
```

Maxima [F]

$$\int \frac{(cx)^m (d+ex^n+fx^{2n}+gx^{3n})}{a+bx^n}\,dx = \int \frac{(gx^{3n}+fx^{2n}+ex^n+d)(cx)^m}{bx^n+a}\,dx$$

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x, algorithm="max
ima")
```

```
[Out] (b^3*c^m*d - a*b^2*c^m*e + a^2*b*c^m*f - a^3*c^m*g)*integrate(x^m/(b^4*x^n
+ a*b^3), x) + ((m^2 + m*(n + 2) + n + 1)*b^2*c^m*g*x*e^(m*log(x) + 2*n*log
(x)) + ((m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*b^2*c^m*e - (m^2 + m*(3*n + 2
) + 2*n^2 + 3*n + 1)*a*b*c^m*f + (m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*a^2*
c^m*g)*x*x^m + ((m^2 + 2*m*(n + 1) + 2*n + 1)*b^2*c^m*f - (m^2 + 2*m*(n + 1
) + 2*n + 1)*a*b*c^m*g)*x*e^(m*log(x) + n*log(x)))/((m^3 + 3*m^2*(n + 1) +
(2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n + 1)*b^3)
```

Giac [F]

$$\int \frac{(cx)^m (d+ex^n+fx^{2n}+gx^{3n})}{a+bx^n}\,dx = \int \frac{(gx^{3n}+fx^{2n}+ex^n+d)(cx)^m}{bx^n+a}\,dx$$

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x, algorithm="gia
c")
```

```
[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (d+ex^n+fx^{2n}+gx^{3n})}{a+bx^n}\,dx = \int \frac{(c\,x)^m\,(d+e\,x^n+f\,x^{2\,n}+g\,x^{3\,n})}{a+b\,x^n}\,dx$$

```
[In] int(((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n),x)
```

```
[Out] int(((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n), x)
```

3.577 $\int (c+dx^{-1+n}) (a+bx^n)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int (c+dx^{-1+n}) (a+bx^n)^3 \, dx = a^3cx+\frac{3a^2bcx^{1+n}}{1+n}+\frac{3ab^2cx^{1+2n}}{1+2n}+\frac{b^3cx^{1+3n}}{1+3n}+\frac{d(a+bx^n)^4}{4bn}$$

```
[Out] a^3*c*x+3*a^2*b*c*x^(1+n)/(1+n)+3*a*b^2*c*x^(1+2*n)/(1+2*n)+b^3*c*x^(1+3*n)
/(1+3*n)+1/4*d*(a+b*x^n)^4/b/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1905, 250, 267}

$$\int (c+dx^{-1+n}) (a+bx^n)^3 \, dx = a^3cx+\frac{3a^2bcx^{n+1}}{n+1}+\frac{3ab^2cx^{2n+1}}{2n+1}+\frac{d(a+bx^n)^4}{4bn}+\frac{b^3cx^{3n+1}}{3n+1}$$

```
[In] Int[(c + d*x^(-1 + n))*(a + b*x^n)^3,x]
```

```
[Out] a^3*c*x + (3*a^2*b*c*x^(1 + n))/(1 + n) + (3*a*b^2*c*x^(1 + 2*n))/(1 + 2*n)
 + (b^3*c*x^(1 + 3*n))/(1 + 3*n) + (d*(a + b*x^n)^4)/(4*b*n)
```

Rule 250

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 1905

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= c\int (a+bx^n)^3\,dx + d\int x^{-1+n}(a+bx^n)^3\,dx \\
&= \frac{d(a+bx^n)^4}{4bn} + c\int \left(a^3+3a^2bx^n+3ab^2x^{2n}+b^3x^{3n}\right)dx \\
&= a^3cx + \frac{3a^2bcx^{1+n}}{1+n} + \frac{3ab^2cx^{1+2n}}{1+2n} + \frac{b^3cx^{1+3n}}{1+3n} + \frac{d(a+bx^n)^4}{4bn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$
\int \left(c+dx^{-1+n}\right)(a+bx^n)^3\,dx = \frac{x\left(c+dx^{-1+n}\right)\left(4a^3cx+\frac{12a^2bcx^{1+n}}{1+n}+\frac{12ab^2cx^{1+2n}}{1+2n}+\frac{4b^3cx^{1+3n}}{1+3n}+\frac{d(a+bx^n)^4}{bn}\right)}{4\,(cx+dx^n)}
$$

```
[In] Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^3,x]
[Out] (x*(c + d*x^(-1 + n))*(4*a^3*c*x + (12*a^2*b*c*x^(1 + n))/(1 + n) + (12*a*b
^2*c*x^(1 + 2*n))/(1 + 2*n) + (4*b^3*c*x^(1 + 3*n))/(1 + 3*n) + (d*(a + b*x
^n)^4)/(b*n)))/(4*(c*x + d*x^n))
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

method	result
risch	$a^3cx+\frac{b^3d\,x^{4n}}{4n}+\frac{b^2(nbcx+3adn+ad)x^{3n}}{n(1+3n)}+\frac{3ab(2nbcx+2adn+ad)x^{2n}}{2n(1+2n)}+\frac{a^2(3nbcx+adn+ad)x^n}{n(1+n)}$
norman	$a^3cx+\frac{a^3d\,\mathrm{e}^{n\ln(x)}}{n}+\frac{a\,b^2d\,\mathrm{e}^{3n\ln(x)}}{n}+\frac{b^3cx\,\mathrm{e}^{3n\ln(x)}}{1+3n}+\frac{b^3d\,\mathrm{e}^{4n\ln(x)}}{4n}+\frac{3d\,a^2b\,\mathrm{e}^{2n\ln(x)}}{2n}+\frac{3a\,b^2cx\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{3a^2bc}{1}$
parallelrisch	$\frac{6x\,x^nx^{-1+n}a^2bd+4a^3cxn+66x\,x^nx^{-1+n}a^2bd\,n^2+36x\,x^nx^{-1+n}a^2bdn+36x^{2n}a\,b^2c\,n^3x+6x\,x^{3n}x^{-1+n}b^3d\,n^3+11x\,x^{3n}x^{-1+}}{}$

```
[In] int((c+d*x^(-1+n))*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*c*x+1/4*b^3*d/n*(x^n)^4+b^2*(b*c*n*x+3*a*d*n+a*d)/n/(1+3*n)*(x^n)^3+3/2
*a*b*(2*b*c*n*x+2*a*d*n+a*d)/n/(1+2*n)*(x^n)^2+a^2*(3*b*c*n*x+a*d*n+a*d)/n/
(1+n)*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(82) = 164$.

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.63

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)^3\,dx$$
$$= \frac{4\,(6\,a^3cn^4+11\,a^3cn^3+6\,a^3cn^2+a^3cn)x+(6\,b^3dn^3+11\,b^3dn^2+6\,b^3dn+b^3d)x^{4\,n}+4\,(6\,ab^2dn^3+11\,ab^2d}{}$$

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(6*a^3*c*n^4 + 11*a^3*c*n^3 + 6*a^3*c*n^2 + a^3*c*n)*x + (6*b^3*d*n^
3 + 11*b^3*d*n^2 + 6*b^3*d*n + b^3*d)*x^(4*n) + 4*(6*a*b^2*d*n^3 + 11*a*b^2
*d*n^2 + 6*a*b^2*d*n + a*b^2*d + (2*b^3*c*n^3 + 3*b^3*c*n^2 + b^3*c*n)*x)*x
^(3*n) + 6*(6*a^2*b*d*n^3 + 11*a^2*b*d*n^2 + 6*a^2*b*d*n + a^2*b*d + 2*(3*a
*b^2*c*n^3 + 4*a*b^2*c*n^2 + a*b^2*c*n)*x)*x^(2*n) + 4*(6*a^3*d*n^3 + 11*a^
3*d*n^2 + 6*a^3*d*n + a^3*d + 3*(6*a^2*b*c*n^3 + 5*a^2*b*c*n^2 + a^2*b*c*n)
*x)*x^n)/(6*n^4 + 11*n^3 + 6*n^2 + n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1340 vs. $2(75) = 150$.

Time = 0.78 (sec) , antiderivative size = 1340, normalized size of antiderivative = 15.95

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)^3\,dx = \text{Too large to display}$$

```
[In] integrate((c+d*x**(-1+n))*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((a**3*c*x - a**3*d/x + 3*a**2*b*c*log(x) - 3*a**2*b*d/(2*x**2) -
3*a*b**2*c/x - a*b**2*d/x**3 - b**3*c/(2*x**2) - b**3*d/(4*x**4), Eq(n, -1)
), (a**3*c*x - 2*a**3*d/sqrt(x) + 6*a**2*b*c*sqrt(x) - 3*a**2*b*d/x + 3*a*b
**2*c*log(x) - 2*a*b**2*d/x**(3/2) - 2*b**3*c/sqrt(x) - b**3*d/(2*x**2), Eq
(n, -1/2)), (a**3*c*x - 3*a**3*d/x**(1/3) + 9*a**2*b*c*x**(2/3)/2 - 9*a**2*
b*d/(2*x**(2/3)) + 9*a*b**2*c*x**(1/3) - 3*a*b**2*d/x + b**3*c*log(x) - 3*b
**3*d/(4*x**(4/3)), Eq(n, -1/3)), ((a + b)**3*(c*x + d*log(x)), Eq(n, 0)),
(24*a**3*c*n**4*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 44*a**3*c*n**3*x/(2
4*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*c*n**2*x/(24*n**4 + 44*n**3 + 2
```

```
4*n**2 + 4*n) + 4*a**3*c*n*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*
d*n**3*x*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 44*a**3*d*n**2*x*
x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*d*n*x*x**(n - 1)/(
24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 4*a**3*d*x*x**(n - 1)/(24*n**4 + 44*n*
*3 + 24*n**2 + 4*n) + 72*a**2*b*c*n**3*x*x**n/(24*n**4 + 44*n**3 + 24*n**2
+ 4*n) + 60*a**2*b*c*n**2*x*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 12*a
**2*b*c*n*x*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 36*a**2*b*d*n**3*x*x
**n*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 66*a**2*b*d*n**2*x*x**
n*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 36*a**2*b*d*n*x*x**n*x**
(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6*a**2*b*d*x*x**n*x**(n - 1)/
(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 36*a*b**2*c*n**3*x*x**(2*n)/(24*n**4
+ 44*n**3 + 24*n**2 + 4*n) + 48*a*b**2*c*n**2*x*x**(2*n)/(24*n**4 + 44*n**3
 + 24*n**2 + 4*n) + 12*a*b**2*c*n*x*x**(2*n)/(24*n**4 + 44*n**3 + 24*n**2 +
 4*n) + 24*a*b**2*d*n**3*x*x**(2*n)*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2
 + 4*n) + 44*a*b**2*d*n**2*x*x**(2*n)*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n*
*2 + 4*n) + 24*a*b**2*d*n*x*x**(2*n)*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**
2 + 4*n) + 4*a*b**2*d*x*x**(2*n)*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 +
4*n) + 8*b**3*c*n**3*x*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 12*b*
*3*c*n**2*x*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 4*b**3*c*n*x*x**
(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6*b**3*d*n**3*x*x**(3*n)*x**(n
- 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 11*b**3*d*n**2*x*x**(3*n)*x**(n
- 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6*b**3*d*n*x*x**(3*n)*x**(n - 1)
/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + b**3*d*x*x**(3*n)*x**(n - 1)/(24*n**
4 + 44*n**3 + 24*n**2 + 4*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

$$\int (c+dx^{-1+n})(a+bx^n)^3\,dx = a^3cx + \frac{b^3dx^{4n}}{4n} + \frac{ab^2dx^{3n}}{n} + \frac{3a^2bdx^{2n}}{2n} + \frac{b^3cx^{3n+1}}{3n+1} + \frac{3ab^2cx^{2n+1}}{2n+1} + \frac{3a^2bcx^{n+1}}{n+1} + \frac{a^3dx^n}{n}$$

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] a^3*c*x + 1/4*b^3*d*x^(4*n)/n + a*b^2*d*x^(3*n)/n + 3/2*a^2*b*d*x^(2*n)/n +
 b^3*c*x^(3*n + 1)/(3*n + 1) + 3*a*b^2*c*x^(2*n + 1)/(2*n + 1) + 3*a^2*b*c*
x^(n + 1)/(n + 1) + a^3*d*x^n/n
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(82) = 164.

Time = 0.29 (sec) , antiderivative size = 392, normalized size of antiderivative = 4.67

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)^3\,dx = \frac{24\,a^3cn^4x+8\,b^3cn^3xx^{3\,n}+36\,ab^2cn^3xx^{2\,n}+72\,a^2bcn^3xx^n+44\,a^3cn^3x+6\,b^3dn^3x^{4\,n}+24\,ab^2dn^3x^{3\,n}+12\,b}{}$$

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] 1/4*(24*a^3*c*n^4*x + 8*b^3*c*n^3*x*x^(3*n) + 36*a*b^2*c*n^3*x*x^(2*n) + 72
*a^2*b*c*n^3*x*x^n + 44*a^3*c*n^3*x + 6*b^3*d*n^3*x^(4*n) + 24*a*b^2*d*n^3*
x^(3*n) + 12*b^3*c*n^2*x*x^(3*n) + 36*a^2*b*d*n^3*x^(2*n) + 48*a*b^2*c*n^2*
x*x^(2*n) + 24*a^3*d*n^3*x^n + 60*a^2*b*c*n^2*x*x^n + 24*a^3*c*n^2*x + 11*b
^3*d*n^2*x^(4*n) + 44*a*b^2*d*n^2*x^(3*n) + 4*b^3*c*n*x*x^(3*n) + 66*a^2*b*
d*n^2*x^(2*n) + 12*a*b^2*c*n*x*x^(2*n) + 44*a^3*d*n^2*x^n + 12*a^2*b*c*n*x*
x^n + 4*a^3*c*n*x + 6*b^3*d*n*x^(4*n) + 24*a*b^2*d*n*x^(3*n) + 36*a^2*b*d*n
*x^(2*n) + 24*a^3*d*n*x^n + b^3*d*x^(4*n) + 4*a*b^2*d*x^(3*n) + 6*a^2*b*d*x
^(2*n) + 4*a^3*d*x^n)/(6*n^4 + 11*n^3 + 6*n^2 + n)
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.37

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)^3\,dx = a^3\,c\,x+\frac{a^3\,d\,x^n}{n}+\frac{b^3\,d\,x^{4\,n}}{4\,n}+\frac{b^3\,c\,x\,x^{3\,n}}{3\,n+1}+\frac{3\,a^2\,b\,d\,x^{2\,n}}{2\,n}+\frac{a\,b^2\,d\,x^{3\,n}}{n}+\frac{3\,a\,b^2\,c\,x\,x^{2\,n}}{2\,n+1}+\frac{3\,a^2\,b\,c\,x\,x^n}{n+1}$$

```
[In] int((c + d*x^(n - 1))*(a + b*x^n)^3,x)
```

```
[Out] a^3*c*x + (a^3*d*x^n)/n + (b^3*d*x^(4*n))/(4*n) + (b^3*c*x*x^(3*n))/(3*n +
1) + (3*a^2*b*d*x^(2*n))/(2*n) + (a*b^2*d*x^(3*n))/n + (3*a*b^2*c*x*x^(2*n)
)/(2*n + 1) + (3*a^2*b*c*x*x^n)/(n + 1)
```

3.578 $\int (c+dx^{-1+n}) (a+bx^n)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int (c+dx^{-1+n}) (a+bx^n)^2 \, dx = a^2cx + \frac{2abcx^{1+n}}{1+n} + \frac{b^2cx^{1+2n}}{1+2n} + \frac{d(a+bx^n)^3}{3bn}$$

```
[Out] a^2*c*x+2*a*b*c*x^(1+n)/(1+n)+b^2*c*x^(1+2*n)/(1+2*n)+1/3*d*(a+b*x^n)^3/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1905, 250, 267}

$$\int (c+dx^{-1+n}) (a+bx^n)^2 \, dx = a^2cx + \frac{2abcx^{n+1}}{n+1} + \frac{d(a+bx^n)^3}{3bn} + \frac{b^2cx^{2n+1}}{2n+1}$$

```
[In] Int[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]
```

```
[Out] a^2*c*x + (2*a*b*c*x^(1 + n))/(1 + n) + (b^2*c*x^(1 + 2*n))/(1 + 2*n) + (d*
(a + b*x^n)^3)/(3*b*n)
```

Rule 250

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 1905

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= c\int \left(a+bx^n\right)^2\,dx + d\int x^{-1+n}(a+bx^n)^2\,dx\\ &= \frac{d(a+bx^n)^3}{3bn} + c\int \left(a^2+2abx^n+b^2x^{2n}\right)\,dx\\ &= a^2cx + \frac{2abcx^{1+n}}{1+n} + \frac{b^2cx^{1+2n}}{1+2n} + \frac{d(a+bx^n)^3}{3bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.97

$$\int \left(c+dx^{-1+n}\right)\left(a+bx^n\right)^2\,dx$$
$$= \frac{a^3d(1+3n+2n^2)+3a^2b(1+3n+2n^2)\left(cnx+dx^n\right)+3ab^2(1+2n)x^n(2cnx+d(1+n)x^n)+b^3(1+n)x^{2}}{3bn(1+n)(1+2n)}$$

```
[In] Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]
[Out] (a^3*d*(1 + 3*n + 2*n^2) + 3*a^2*b*(1 + 3*n + 2*n^2)*(c*n*x + d*x^n) + 3*a*
b^2*(1 + 2*n)*x^n*(2*c*n*x + d*(1 + n)*x^n) + b^3*(1 + n)*x^(2*n)*(3*c*n*x
+ d*(1 + 2*n)*x^n))/(3*b*n*(1 + n)*(1 + 2*n))
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

method	result
risch	$a^2cx+\frac{b^2d\,x^{3n}}{3n}+\frac{b(nbcx+2adn+ad)x^{2n}}{n(1+2n)}+\frac{a(2nbcx+adn+ad)x^n}{n(1+n)}$
norman	$a^2cx+\frac{a^2d\,e^{n\ln(x)}}{n}+\frac{abd\,e^{2n\ln(x)}}{n}+\frac{b^2cx\,e^{2n\ln(x)}}{1+2n}+\frac{b^2d\,e^{3n\ln(x)}}{3n}+\frac{2abcx\,e^{n\ln(x)}}{1+n}$
parallelrisch	$\frac{2x\,x^{2n}x^{-1+n}b^2d\,n^2+3x\,x^{2n}x^{-1+n}b^2dn+3x^{2n}b^2c\,n^2x+6x\,x^nx^{-1+n}abd\,n^2+x\,x^{2n}x^{-1+n}b^2d+3x^{2n}b^2cnx+9x\,x^nx^{-1+n}abdn+1}{3n(1+2n)(}$

```
[In] int((c+d*x^(-1+n))*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*c*x+1/3*b^2*d/n*(x^n)^3+b*(b*c*n*x+2*a*d*n+a*d)/n/(1+2*n)*(x^n)^2+a*(2*
b*c*n*x+a*d*n+a*d)/n/(1+n)*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(59) = 118.

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.62

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)^2\,dx = \frac{3\,(2\,a^2cn^3+3\,a^2cn^2+a^2cn)x+(2\,b^2dn^2+3\,b^2dn+b^2d)x^{3\,n}+3\,(2\,abdn^2+3\,abdn+abd+(b^2cn^2+b^2cn}{3\,(2\,n^3+3\,n^2+n)}$$

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(2*a^2*c*n^3 + 3*a^2*c*n^2 + a^2*c*n)*x + (2*b^2*d*n^2 + 3*b^2*d*n +
 b^2*d)*x^(3*n) + 3*(2*a*b*d*n^2 + 3*a*b*d*n + a*b*d + (b^2*c*n^2 + b^2*c*n
)*x)*x^(2*n) + 3*(2*a^2*d*n^2 + 3*a^2*d*n + a^2*d + 2*(2*a*b*c*n^2 + a*b*c*
n)*x)*x^n)/(2*n^3 + 3*n^2 + n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. 2(53) = 106.

Time = 0.48 (sec) , antiderivative size = 598, normalized size of antiderivative = 9.80

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)^2\,dx = \begin{cases} a^2cx-\frac{a^2d}{x}+2abc\log(x)-\frac{abd}{x^2}-\frac{b^2c}{x}-\frac{b^2d}{3x^3} \\ a^2cx-\frac{2a^2d}{\sqrt{x}}+4abc\sqrt{x}-\frac{2abd}{x}+b^2c\log(x)-\frac{2b^2d}{3x^{\frac{3}{2}}} \\ (a+b)^2(cx+d\log(x)) \\ \frac{6a^2cn^3x}{6n^3+9n^2+3n}+\frac{9a^2cn^2x}{6n^3+9n^2+3n}+\frac{3a^2cnx}{6n^3+9n^2+3n}+\frac{6a^2dn^2xx^{n-1}}{6n^3+9n^2+3n}+\frac{9a^2dnxx^{n-1}}{6n^3+9n^2+3n}+\frac{3a^2dxx^{n-1}}{6n^3+9n^2+3n}+\frac{12abcn^2xx^n}{6n^3+9n^2+3n}+\frac{6abcnxx^n}{6n^3+9n^2+3} \end{cases}$$

```
[In] integrate((c+d*x**(-1+n))*(a+b*x**n)**2,x)
```

```
[Out] Piecewise((a**2*c*x - a**2*d/x + 2*a*b*c*log(x) - a*b*d/x**2 - b**2*c/x - b
**2*d/(3*x**3), Eq(n, -1)), (a**2*c*x - 2*a**2*d/sqrt(x) + 4*a*b*c*sqrt(x)
- 2*a*b*d/x + b**2*c*log(x) - 2*b**2*d/(3*x**(3/2)), Eq(n, -1/2)), ((a + b)
**2*(c*x + d*log(x)), Eq(n, 0)), (6*a**2*c*n**3*x/(6*n**3 + 9*n**2 + 3*n) +
 9*a**2*c*n**2*x/(6*n**3 + 9*n**2 + 3*n) + 3*a**2*c*n*x/(6*n**3 + 9*n**2 +
3*n) + 6*a**2*d*n**2*x*x**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + 9*a**2*d*n*x*x*
*(n - 1)/(6*n**3 + 9*n**2 + 3*n) + 3*a**2*d*x*x**(n - 1)/(6*n**3 + 9*n**2 +
 3*n) + 12*a*b*c*n**2*x*x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*c*n*x*x**n/(6*
```

```
n**3 + 9*n**2 + 3*n) + 6*a*b*d*n**2*x*x**n*x**(n - 1)/(6*n**3 + 9*n**2 + 3*
n) + 9*a*b*d*n*x*x**n*x**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + 3*a*b*d*x*x**n*x
**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*c*n**2*x*x**(2*n)/(6*n**3 + 9*n*
*2 + 3*n) + 3*b**2*c*n*x*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 2*b**2*d*n**2*x
*x**(2*n)*x**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*d*n*x*x**(2*n)*x**(n
- 1)/(6*n**3 + 9*n**2 + 3*n) + b**2*d*x*x**(2*n)*x**(n - 1)/(6*n**3 + 9*n**
2 + 3*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \left(c+dx^{-1+n}\right)\left(a+bx^n\right)^2\,dx=a^2cx+\frac{b^2dx^{3n}}{3n}+\frac{abdx^{2n}}{n}+\frac{b^2cx^{2n+1}}{2n+1}+\frac{2\,abcx^{n+1}}{n+1}+\frac{a^2dx^n}{n}$$

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] a^2*c*x + 1/3*b^2*d*x^(3*n)/n + a*b*d*x^(2*n)/n + b^2*c*x^(2*n + 1)/(2*n +
1) + 2*a*b*c*x^(n + 1)/(n + 1) + a^2*d*x^n/n
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(59) = 118.

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.21

$$\int \left(c+dx^{-1+n}\right)\left(a+bx^n\right)^2\,dx$$
$$=\frac{6\,a^2cn^3x+3\,b^2cn^2xx^{2n}+12\,abcn^2xx^n+9\,a^2cn^2x+2\,b^2dn^2x^{3n}+6\,abdn^2x^{2n}+3\,b^2cnxx^{2n}+6\,a^2dn^2x^n+}{3\left(2\,n^3+3\,n^2+n\right.}$$

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*a^2*c*n^3*x + 3*b^2*c*n^2*x*x^(2*n) + 12*a*b*c*n^2*x*x^n + 9*a^2*c*n
^2*x + 2*b^2*d*n^2*x^(3*n) + 6*a*b*d*n^2*x^(2*n) + 3*b^2*c*n*x*x^(2*n) + 6*
a^2*d*n^2*x^n + 6*a*b*c*n*x*x^n + 3*a^2*c*n*x + 3*b^2*d*n*x^(3*n) + 9*a*b*d
*n*x^(2*n) + 9*a^2*d*n*x^n + b^2*d*x^(3*n) + 3*a*b*d*x^(2*n) + 3*a^2*d*x^n)
/(2*n^3 + 3*n^2 + n)
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \left(c+dx^{-1+n}\right) (a+bx^n)^2 \, dx = a^2 \, c \, x + \frac{a^2 \, d \, x^n}{n} + \frac{b^2 \, d \, x^{3\,n}}{3\,n} + \frac{b^2 \, c \, x \, x^{2\,n}}{2\,n+1} + \frac{a \, b \, d \, x^{2\,n}}{n} + \frac{2 \, a \, b \, c \, x \, x^n}{n+1}$$

```
[In] int((c + d*x^(n - 1))*(a + b*x^n)^2,x)
[Out] a^2*c*x + (a^2*d*x^n)/n + (b^2*d*x^(3*n))/(3*n) + (b^2*c*x*x^(2*n))/(2*n +
1) + (a*b*d*x^(2*n))/n + (2*a*b*c*x*x^n)/(n + 1)
```

3.579 $\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx=acx+\frac{adx^n}{n}+\frac{bdx^{2n}}{2n}+\frac{bcx^{1+n}}{1+n}$$

```
[Out] a*c*x+a*d*x^n/n+1/2*b*d*x^(2*n)/n+b*c*x^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1905, 14}

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx=acx+\frac{adx^n}{n}+\frac{bcx^{n+1}}{n+1}+\frac{bdx^{2n}}{2n}$$

```
[In] Int[(c + d*x^(-1 + n))*(a + b*x^n),x]
```

```
[Out] a*c*x + (a*d*x^n)/n + (b*d*x^(2*n))/(2*n) + (b*c*x^(1 + n))/(1 + n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1905

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= c\int (a+bx^n)\ dx + d\int x^{-1+n}(a+bx^n)\ dx\\
&= acx + \frac{bcx^{1+n}}{1+n} + d\int \left(ax^{-1+n}+bx^{-1+2n}\right)\ dx\\
&= acx + \frac{adx^n}{n} + \frac{bdx^{2n}}{2n} + \frac{bcx^{1+n}}{1+n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$
\int \left(c+dx^{-1+n}\right)(a+bx^n)\ dx = \frac{2a(cnx+dx^n)+bx^n\left(\frac{2cnx}{1+n}+dx^n\right)}{2n}
$$

```
[In] Integrate[(c + d*x^(-1 + n))*(a + b*x^n),x]
[Out] (2*a*(c*n*x + d*x^n) + b*x^n*((2*c*n*x)/(1 + n) + d*x^n))/(2*n)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

method	result	size
risch	$acx+\frac{bd\,x^{2n}}{2n}+\frac{(nbcx+adn+ad)x^n}{n(1+n)}$	43
norman	$acx+\frac{ad\,e^{n\ln(x)}}{n}+\frac{bcx\,e^{n\ln(x)}}{1+n}+\frac{bd\,e^{2n\ln(x)}}{2n}$	45
parallelrisch	$\frac{x\,x^n x^{-1+n} bdn+x\,x^n x^{-1+n} bd+2x^n bcnx+2x\,x^{-1+n} adn+2acx\,n^2+2x\,x^{-1+n} ad+2acxn}{2n(1+n)}$	81

```
[In] int((c+d*x^(-1+n))*(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] a*c*x+1/2*b*d/n*(x^n)^2+(b*c*n*x+a*d*n+a*d)/n/(1+n)*x^n
```

Fricas [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx = \frac{2\,(acn^2+acn)x+(bdn+bd)x^{2\,n}+2\,(bcnx+adn+ad)x^n}{2\,(n^2+n)}$$

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*c*n^2 + a*c*n)*x + (b*d*n + b*d)*x^(2*n) + 2*(b*c*n*x + a*d*n + a
*d)*x^n)/(n^2 + n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(36) = 72.

Time = 0.30 (sec) , antiderivative size = 180, normalized size of antiderivative = 4.39

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx = \begin{cases} acx-\frac{ad}{x}+bc\log(x)-\frac{bd}{2x^2} & \text{for } n=-1 \\ (a+b)\,(cx+d\log(x)) & \text{for } n=0 \\ \frac{2acn^2x}{2n^2+2n}+\frac{2acnx}{2n^2+2n}+\frac{2adnxx^{n-1}}{2n^2+2n}+\frac{2adxx^{n-1}}{2n^2+2n}+\frac{2bcnxx^n}{2n^2+2n}+\frac{bdnxx^nx^{n-1}}{2n^2+2n}+\frac{bdxx^nx^{n-1}}{2n^2+2n} & \text{otherwise} \end{cases}$$

```
[In] integrate((c+d*x**(-1+n))*(a+b*x**n),x)
```

```
[Out] Piecewise((a*c*x - a*d/x + b*c*log(x) - b*d/(2*x**2), Eq(n, -1)), ((a + b)*
(c*x + d*log(x)), Eq(n, 0)), (2*a*c*n**2*x/(2*n**2 + 2*n) + 2*a*c*n*x/(2*n*
*2 + 2*n) + 2*a*d*n*x*x**(n - 1)/(2*n**2 + 2*n) + 2*a*d*x*x**(n - 1)/(2*n**
2 + 2*n) + 2*b*c*n*x*x**n/(2*n**2 + 2*n) + b*d*n*x*x**n*x**(n - 1)/(2*n**2
+ 2*n) + b*d*x*x**n*x**(n - 1)/(2*n**2 + 2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx = acx+\frac{bdx^{2\,n}}{2\,n}+\frac{bcx^{n+1}}{n+1}+\frac{adx^n}{n}$$

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="maxima")
```

```
[Out] a*c*x + 1/2*b*d*x^(2*n)/n + b*c*x^(n + 1)/(n + 1) + a*d*x^n/n
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int (c+dx^{-1+n}) (a+bx^n)\ dx = \frac{2\,acn^2x + 2\,bcnxx^n + 2\,acnx + bdnx^{2\,n} + 2\,adnx^n + bdx^{2\,n} + 2\,adx^n}{2\,(n^2+n)}$$

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="giac")
[Out] 1/2*(2*a*c*n^2*x + 2*b*c*n*x*x^n + 2*a*c*n*x + b*d*n*x^(2*n) + 2*a*d*n*x^n
+ b*d*x^(2*n) + 2*a*d*x^n)/(n^2 + n)
```

Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int (c+dx^{-1+n}) (a+bx^n)\ dx = a\,c\,x + \frac{a\,d\,x^n}{n} + \frac{b\,d\,x^{2\,n}}{2\,n} + \frac{b\,c\,x\,x^n}{n+1}$$

```
[In] int((c + d*x^(n - 1))*(a + b*x^n),x)
[Out] a*c*x + (a*d*x^n)/n + (b*d*x^(2*n))/(2*n) + (b*c*x*x^n)/(n + 1)
```

3.580 $\int (c + dx^{-1+n})\, dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int (c + dx^{-1+n})\, dx = cx + \frac{dx^n}{n}$$

```
[Out] c*x+d*x^n/n
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c + dx^{-1+n})\, dx = cx + \frac{dx^n}{n}$$

```
[In] Int[c + d*x^(-1 + n),x]
```

```
[Out] c*x + (d*x^n)/n
```

Rubi steps

$$\text{integral} = cx + \frac{dx^n}{n}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(c+dx^{-1+n}\right)\,dx=cx+\frac{dx^n}{n}$$

```
[In] Integrate[c + d*x^(-1 + n),x]
[Out] c*x + (d*x^n)/n
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$cx+\frac{d\,x^{n}}{n}$	13
parts	$cx+\frac{d\,x^{n}}{n}$	13
risch	$cx+\frac{dx\,x^{-1+n}}{n}$	16
parallelrisch	$cx+\frac{dx\,x^{-1+n}}{n}$	16
norman	$cx+\frac{dx\,\mathrm{e}^{(-1+n)\ln(x)}}{n}$	18

```
[In] int(c+d*x^(-1+n),x,method=_RETURNVERBOSE)
[Out] c*x+d*x^n/n
```

Fricas [A] (verification not implemented)

none

Time = 0.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \left(c+dx^{-1+n}\right)\,dx=\frac{cnx+dxx^{n-1}}{n}$$

```
[In] integrate(c+d*x^(-1+n),x, algorithm="fricas")
[Out] (c*n*x + d*x*x^(n - 1))/n
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(c+dx^{-1+n}\right) dx = cx + d\left(\begin{cases} \frac{x^n}{n} & \text{for } n \neq 0 \\ \log\left(x\right) & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(c+d*x**(-1+n),x)
[Out] c*x + d*Piecewise((x**n/n, Ne(n, 0)), (log(x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(c+dx^{-1+n}\right) dx = cx + \frac{dx^n}{n}$$

```
[In] integrate(c+d*x^(-1+n),x, algorithm="maxima")
[Out] c*x + d*x^n/n
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(c+dx^{-1+n}\right) dx = cx + \frac{dx^n}{n}$$

```
[In] integrate(c+d*x^(-1+n),x, algorithm="giac")
[Out] c*x + d*x^n/n
```

Mupad [B] (verification not implemented)

Time = 11.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(c+dx^{-1+n}\right) dx = c\,x + \frac{d\,x^n}{n}$$

```
[In] int(c + d*x^(n - 1),x)
[Out] c*x + (d*x^n)/n
```

3.581 $\int \frac{c+dx^{-1+n}}{a+bx^n} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 42

$$\int \frac{c+dx^{-1+n}}{a+bx^n} \, dx = \frac{cx \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}$$

```
[Out] c*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a+d*ln(a+b*x^n)/b/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1905, 251, 266}

$$\int \frac{c+dx^{-1+n}}{a+bx^n} \, dx = \frac{cx \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}$$

```
[In] Int[(c + d*x^(-1 + n))/(a + b*x^n),x]
```

```
[Out] (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a + (d*Log[a + b*x^n])/(b*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1905

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= c\int \frac{1}{a+bx^n}\,dx + d\int \frac{x^{-1+n}}{a+bx^n}\,dx\\ &= \frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a} + \frac{d\log\left(a+bx^n\right)}{bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx = \frac{cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a} + \frac{d\log\left(a+bx^n\right)}{bn}$$

```
[In] Integrate[(c + d*x^(-1 + n))/(a + b*x^n),x]
[Out] (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a + (d*Log[a +
 b*x^n])/(b*n)
```

Maple [F]

$$\int \frac{c+d\,x^{-1+n}}{a+b\,x^n}dx$$

```
[In] int((c+d*x^(-1+n))/(a+b*x^n),x)
[Out] int((c+d*x^(-1+n))/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx = \int \frac{dx^{n-1}+c}{bx^n+a}\,dx$$

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="fricas")
[Out] integral((d*x^(n - 1) + c)/(b*x^n + a), x)
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx = \frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}cx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} + d\left(\begin{cases}\frac{x^n}{an} & \text{for } b=0\\ \tilde{\infty}x^n & \text{for } n=0\\ \frac{\log(an+bnx^n)}{bn} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((c+d*x**(-1+n))/(a+b*x**n),x)
[Out] a**(1/n)*a**(-1 - 1/n)*c*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma
(1/n)/(n**2*gamma(1 + 1/n)) + d*Piecewise((x**n/(a*n), Eq(b, 0)), (zoo*x**n
, Eq(n, 0)), (log(a*n + b*n*x**n)/(b*n), True))
```

Maxima [F]

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx = \int \frac{dx^{n-1}+c}{bx^n+a}\,dx$$

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="maxima")
[Out] d*log(x)/b + integrate((b*c*x - a*d)/(b^2*x*x^n + a*b*x), x)
```

Giac [F]

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx = \int \frac{dx^{n-1}+c}{bx^n+a}\,dx$$

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="giac")
[Out] integrate((d*x^(n - 1) + c)/(b*x^n + a), x)
```

Mupad [B] (verification not implemented)

Time = 11.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx = \frac{c\,x\,{}_2\mathrm{F}_1\left(1,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{a}+\frac{d\,\ln\left(a+b\,x^n\right)}{b\,n}$$

```
[In] int((c + d*x^(n - 1))/(a + b*x^n),x)
[Out] (c*x*hypergeom([1, 1/n], 1/n + 1, -(b*x^n)/a))/a + (d*log(a + b*x^n))/(b*n)
```

3.582 $\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx=-\frac{d}{bn(a+bx^n)}+\frac{cx\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}$$

```
[Out] -d/b/n/(a+b*x^n)+c*x*hypergeom([2, 1/n],[1+1/n],-b*x^n/a)/a^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1905, 251, 267}

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx=\frac{cx\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}-\frac{d}{bn(a+bx^n)}$$

```
[In] Int[(c + d*x^(-1 + n))/(a + b*x^n)^2,x]
```

```
[Out] -(d/(b*n*(a + b*x^n))) + (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b
*x^n)/a)])/a^2
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1905

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= c\int \frac{1}{(a+bx^n)^2}\,dx + d\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx \\ &= -\frac{d}{bn\,(a+bx^n)} + \frac{cx\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx = -\frac{d}{abn+b^2nx^n} + \frac{cx\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}$$

```
[In] Integrate[(c + d*x^(-1 + n))/(a + b*x^n)^2,x]
[Out] -(d/(a*b*n + b^2*n*x^n)) + (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(
(b*x^n)/a)])/a^2
```

Maple [F]

$$\int \frac{c+d\,x^{-1+n}}{(a+b\,x^n)^2}dx$$

```
[In] int((c+d*x^(-1+n))/(a+b*x^n)^2,x)
[Out] int((c+d*x^(-1+n))/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx=\int \frac{dx^{n-1}+c}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^(n - 1) + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.87 (sec) , antiderivative size = 388, normalized size of antiderivative = 8.82

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx = c\left(\frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}nx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)}+\frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}nx\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} - \frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{\frac{1}{n}}a^{-2-\frac{1}{n}}bnxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} - \frac{a^{\frac{1}{n}}a^{-2-\frac{1}{n}}bxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)}\right) + d\left(\begin{cases} \tilde{\infty}\log(x) & \text{for}\ a=0\wedge b=0\wedge n=0 \\ -\frac{xx^{-2n}x^{n-1}}{b^2n} & \text{for}\ a=0 \\ \frac{\tilde{\infty}xx^{n-1}}{n} & \text{for}\ b=-ax^{-n} \\ \frac{\log(x)}{(a+b)^2} & \text{for}\ n=0 \\ \frac{xx^{n-1}}{a^2n+abnx^n} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((c+d*x**(-1+n))/(a+b*x**n)**2,x)
```

```
[Out] c*(a*a**(1/n)*a**(-2 - 1/n)*n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*
gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) + a*a**(1/n
)*a**(-2 - 1/n)*n*x*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1
 + 1/n)) - a*a**(1/n)*a**(-2 - 1/n)*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
 1/n)*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) + a**
(1/n)*a**(-2 - 1/n)*b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*g
```

```
amma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) - a**(1/n)*a
**(-2 - 1/n)*b*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)
/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) + d*Piecewise((zoo*l
og(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(n - 1)/(b**2*n*x**(2*n)), E
q(a, 0)), (zoo*x*x**(n - 1)/n, Eq(b, -a/x**n)), (log(x)/(a + b)**2, Eq(n, 0
)), (x*x**(n - 1)/(a**2*n + a*b*n*x**n), True))
```

Maxima [F]

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx = \int \frac{dx^{n-1}+c}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="maxima")
[Out] c*(n - 1)*integrate(1/(a*b*n*x^n + a^2*n), x) + (b*c*x - a*d)/(a*b^2*n*x^n
+ a^2*b*n)
```

Giac [F]

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx = \int \frac{dx^{n-1}+c}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate((d*x^(n - 1) + c)/(b*x^n + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 10.89 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx = \frac{c\,x\,{}_2\mathrm{F}_1\left(2,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{a^2} - \frac{a\,d}{b\,\left(a^2\,n+a\,b\,n\,x^n\right)}$$

```
[In] int((c + d*x^(n - 1))/(a + b*x^n)^2,x)
[Out] (c*x*hypergeom([2, 1/n], 1/n + 1, -(b*x^n)/a))/a^2 - (a*d)/(b*(a^2*n + a*b*
n*x^n))
```

3.583 $\int \frac{c+dx^{-1+n}}{(a+bx^n)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3} dx = -\frac{d}{2bn(a+bx^n)^2} + \frac{cx\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3}$$

```
[Out] -1/2*d/b/n/(a+b*x^n)^2+c*x*hypergeom([3, 1/n],[1+1/n],-b*x^n/a)/a^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1905, 251, 267}

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3} dx = \frac{cx\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3} - \frac{d}{2bn(a+bx^n)^2}$$

```
[In] Int[(c + d*x^(-1 + n))/(a + b*x^n)^3,x]
```

```
[Out] -1/2*d/(b*n*(a + b*x^n)^2) + (c*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1),
-((b*x^n)/a)])/a^3
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1905

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= c\int \frac{1}{(a+bx^n)^3}\,dx + d\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx\\ &= -\frac{d}{2bn\,(a+bx^n)^2} + \frac{cx\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx = \frac{-a^3d + 2bcnx(a+bx^n)^2\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{2a^3bn\,(a+bx^n)^2}$$

```
[In] Integrate[(c + d*x^(-1 + n))/(a + b*x^n)^3,x]
[Out] (-(a^3*d) + 2*b*c*n*x*(a + b*x^n)^2*Hypergeometric2F1[3, n^(-1), 1 + n^(-1)
, -((b*x^n)/a)])/(2*a^3*b*n*(a + b*x^n)^2)
```

Maple [F]

$$\int \frac{c+d\,x^{-1+n}}{(a+b\,x^n)^3}dx$$

```
[In] int((c+d*x^(-1+n))/(a+b*x^n)^3,x)
[Out] int((c+d*x^(-1+n))/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{c + dx^{-1+n}}{(a + bx^n)^3} \, dx = \int \frac{dx^{n-1} + c}{(bx^n + a)^3} \, dx$$

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="fricas")
[Out] integral((d*x^(n - 1) + c)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a
^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^{-1+n}}{(a + bx^n)^3} \, dx = \text{Timed out}$$

```
[In] integrate((c+d*x**(-1+n))/(a+b*x**n)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{c + dx^{-1+n}}{(a + bx^n)^3} \, dx = \int \frac{dx^{n-1} + c}{(bx^n + a)^3} \, dx$$

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="maxima")
[Out] (2*n^2 - 3*n + 1)*c*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b^2*
c*(2*n - 1)*x*x^n + a*b*c*(3*n - 1)*x - a^2*d*n)/(a^2*b^3*n^2*x^(2*n) + 2*a
^3*b^2*n^2*x^n + a^4*b*n^2)
```

Giac [F]

$$\int \frac{c + dx^{-1+n}}{(a + bx^n)^3} \, dx = \int \frac{dx^{n-1} + c}{(bx^n + a)^3} \, dx$$

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate((d*x^(n - 1) + c)/(b*x^n + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 10.81 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx=\frac{c\,x\,{}_2\mathrm{F}_1\left(3,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{a^3}-\frac{d}{2\,b\,\left(a^2\,n+b^2\,n\,x^{2\,n}+2\,a\,b\,n\,x^n\right)}$$

```
[In] int((c + d*x^(n - 1))/(a + b*x^n)^3,x)
```

```
[Out] (c*x*hypergeom([3, 1/n], 1/n + 1, -(b*x^n)/a))/a^3 - d/(2*b*(a^2*n + b^2*n*
x^(2*n) + 2*a*b*n*x^n))
```

3.584 $\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 305

$$\begin{aligned}\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx &= \frac{d(cx)^{1+m}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{c(1+m)\sqrt{a+bx^n}}\\ &+\frac{ex^{1+n}(cx)^m\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m+n}{n},\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{(1+m+n)\sqrt{a+bx^n}}\\ &+\frac{fx^{1+2n}(cx)^m\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m+2n}{n},\frac{1+m+3n}{n},-\frac{bx^n}{a}\right)}{(1+m+2n)\sqrt{a+bx^n}}\\ &+\frac{gx^{1+3n}(cx)^m\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m+3n}{n},\frac{1+m+4n}{n},-\frac{bx^n}{a}\right)}{(1+m+3n)\sqrt{a+bx^n}}\end{aligned}$$

```
[Out] d*(c*x)^(1+m)*hypergeom([1/2, (1+m)/n],[(1+m+n)/n],-b*x^n/a)*(1+b*x^n/a)^(1
/2)/c/(1+m)/(a+b*x^n)^(1/2)+e*x^(1+n)*(c*x)^m*hypergeom([1/2, (1+m+n)/n],[(
1+m+2*n)/n],-b*x^n/a)*(1+b*x^n/a)^(1/2)/(1+m+n)/(a+b*x^n)^(1/2)+f*x^(1+2*n)
*(c*x)^m*hypergeom([1/2, (1+m+2*n)/n],[(1+m+3*n)/n],-b*x^n/a)*(1+b*x^n/a)^(
1/2)/(1+m+2*n)/(a+b*x^n)^(1/2)+g*x^(1+3*n)*(c*x)^m*hypergeom([1/2, (1+m+3*n
)/n],[(1+m+4*n)/n],-b*x^n/a)*(1+b*x^n/a)^(1/2)/(1+m+3*n)/(a+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1858, 372, 371, 20}

$$\int \frac{(cx)^m (d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx$$
$$= \frac{d(cx)^{m+1}\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{c(m+1)\sqrt{a+bx^n}}$$
$$+\frac{ex^{n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+n+1}{n},\frac{m+2n+1}{n},-\frac{bx^n}{a}\right)}{(m+n+1)\sqrt{a+bx^n}}$$
$$+\frac{fx^{2n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2n+1}{n},\frac{m+3n+1}{n},-\frac{bx^n}{a}\right)}{(m+2n+1)\sqrt{a+bx^n}}$$
$$+\frac{gx^{3n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3n+1}{n},\frac{m+4n+1}{n},-\frac{bx^n}{a}\right)}{(m+3n+1)\sqrt{a+bx^n}}$$

```
[In] Int[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/Sqrt[a + b*x^n],x]
```

```
[Out] (d*(c*x)^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m)/n, (1 +
 m + n)/n, -((b*x^n)/a)])/(c*(1 + m)*Sqrt[a + b*x^n]) + (e*x^(1 + n)*(c*x)^
m*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + n)/n, (1 + m + 2*n)/n
, -((b*x^n)/a)])/((1 + m + n)*Sqrt[a + b*x^n]) + (f*x^(1 + 2*n)*(c*x)^m*Sqr
t[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 2*n)/n, (1 + m + 3*n)/n, -
((b*x^n)/a)])/((1 + m + 2*n)*Sqrt[a + b*x^n]) + (g*x^(1 + 3*n)*(c*x)^m*Sqrt
[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 3*n)/n, (1 + m + 4*n)/n, -(
(b*x^n)/a)])/((1 + m + 3*n)*Sqrt[a + b*x^n])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1858

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d(cx)^m}{\sqrt{a+bx^n}}+\frac{ex^n(cx)^m}{\sqrt{a+bx^n}}+\frac{fx^{2n}(cx)^m}{\sqrt{a+bx^n}}+\frac{gx^{3n}(cx)^m}{\sqrt{a+bx^n}}\right)dx\\
&= d\int \frac{(cx)^m}{\sqrt{a+bx^n}}\,dx+e\int \frac{x^n(cx)^m}{\sqrt{a+bx^n}}\,dx+f\int \frac{x^{2n}(cx)^m}{\sqrt{a+bx^n}}\,dx+g\int \frac{x^{3n}(cx)^m}{\sqrt{a+bx^n}}\,dx\\
&= \left(ex^{-m}(cx)^m\right)\int \frac{x^{m+n}}{\sqrt{a+bx^n}}\,dx+\left(fx^{-m}(cx)^m\right)\int \frac{x^{m+2n}}{\sqrt{a+bx^n}}\,dx\\
&\quad+\left(gx^{-m}(cx)^m\right)\int \frac{x^{m+3n}}{\sqrt{a+bx^n}}\,dx+\frac{\left(d\sqrt{1+\frac{bx^n}{a}}\right)\int \frac{(cx)^m}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\
&= \frac{d(cx)^{1+m}\sqrt{1+\frac{bx^n}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)\sqrt{a+bx^n}}+\frac{\left(ex^{-m}(cx)^m\sqrt{1+\frac{bx^n}{a}}\right)\int \frac{x^{m+n}}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\
&\quad+\frac{\left(fx^{-m}(cx)^m\sqrt{1+\frac{bx^n}{a}}\right)\int \frac{x^{m+2n}}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}+\frac{\left(gx^{-m}(cx)^m\sqrt{1+\frac{bx^n}{a}}\right)\int \frac{x^{m+3n}}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\
&= \frac{d(cx)^{1+m}\sqrt{1+\frac{bx^n}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)\sqrt{a+bx^n}}\\
&\quad+\frac{ex^{1+n}(cx)^m\sqrt{1+\frac{bx^n}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m+n}{n};\frac{1+m+2n}{n};-\frac{bx^n}{a}\right)}{(1+m+n)\sqrt{a+bx^n}}\\
&\quad+\frac{fx^{1+2n}(cx)^m\sqrt{1+\frac{bx^n}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m+2n}{n};\frac{1+m+3n}{n};-\frac{bx^n}{a}\right)}{(1+m+2n)\sqrt{a+bx^n}}\\
&\quad+\frac{gx^{1+3n}(cx)^m\sqrt{1+\frac{bx^n}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m+3n}{n};\frac{1+m+4n}{n};-\frac{bx^n}{a}\right)}{(1+m+3n)\sqrt{a+bx^n}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.68

$$\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx$$
$$= \frac{x(cx)^m\sqrt{1+\frac{bx^n}{a}}\left(\frac{d\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{1+m}+x^n\left(\frac{e\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m+n}{n},\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{1+m+n}+x^n\left(\frac{f}{\right.\right.\right.}}{\sqrt{a+bx^n}}$$

```
[In] Integrate[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/Sqrt[a + b*x^n],x]
```

```
[Out] (x*(c*x)^m*Sqrt[1 + (b*x^n)/a]*((d*Hypergeometric2F1[1/2, (1 + m)/n, (1 + m
 + n)/n, -((b*x^n)/a)])/(1 + m) + x^n*((e*Hypergeometric2F1[1/2, (1 + m + n
)/n, (1 + m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n) + x^n*((f*Hypergeometric2F
1[1/2, (1 + m + 2*n)/n, (1 + m + 3*n)/n, -((b*x^n)/a)])/(1 + m + 2*n) + (g*
x^n*Hypergeometric2F1[1/2, (1 + m + 3*n)/n, (1 + m + 4*n)/n, -((b*x^n)/a)])
/(1 + m + 3*n)))))/Sqrt[a + b*x^n]
```

Maple [F]

$$\int \frac{(cx)^m\left(d+e\,x^n+f\,x^{2n}+g\,x^{3n}\right)}{\sqrt{a+b\,x^n}}dx$$

```
[In] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x, algorith
m="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.81 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.12

$$\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx$$
$$= \frac{a^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}-\frac{1}{2}-\frac{1}{n}}c^m dx^{m+1}\Gamma\left(\frac{m}{n}+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$
$$+\frac{a^{-\frac{m}{n}-\frac{7}{2}-\frac{1}{n}}a^{\frac{m}{n}+3+\frac{1}{n}}c^m gx^{m+3n+1}\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{n}+3+\frac{1}{n}\\ \frac{m}{n}+4+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+4+\frac{1}{n}\right)}$$
$$+\frac{a^{-\frac{m}{n}-\frac{5}{2}-\frac{1}{n}}a^{\frac{m}{n}+2+\frac{1}{n}}c^m fx^{m+2n+1}\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{n}+2+\frac{1}{n}\\ \frac{m}{n}+3+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right)}$$
$$+\frac{a^{-\frac{m}{n}-\frac{3}{2}-\frac{1}{n}}a^{\frac{m}{n}+1+\frac{1}{n}}c^m ex^{m+n+1}\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{n}+1+\frac{1}{n}\\ \frac{m}{n}+2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}$$

```
[In] integrate((c*x)**m*(d+e*x**n+f*x**(2*n)+g*x**(3*n))/(a+b*x**n)**(1/2),x)
```

```
[Out] a**(m/n + 1/n)*a**(-m/n - 1/2 - 1/n)*c**m*d*x**(m + 1)*gamma(m/n + 1/n)*hyp
er((1/2, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m
/n + 1 + 1/n)) + a**(-m/n - 7/2 - 1/n)*a**(m/n + 3 + 1/n)*c**m*g*x**(m + 3*
n + 1)*gamma(m/n + 3 + 1/n)*hyper((1/2, m/n + 3 + 1/n), (m/n + 4 + 1/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 4 + 1/n)) + a**(-m/n - 5/2 - 1/n)*a
**(m/n + 2 + 1/n)*c**m*f*x**(m + 2*n + 1)*gamma(m/n + 2 + 1/n)*hyper((1/2,
m/n + 2 + 1/n), (m/n + 3 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n +
3 + 1/n)) + a**(-m/n - 3/2 - 1/n)*a**(m/n + 1 + 1/n)*c**m*e*x**(m + n + 1)*
gamma(m/n + 1 + 1/n)*hyper((1/2, m/n + 1 + 1/n), (m/n + 2 + 1/n,), b*x**n*e
xp_polar(I*pi)/a)/(n*gamma(m/n + 2 + 1/n))
```

Maxima [F]

$$\int \frac{(cx)^m (d + ex^n + fx^{2n} + gx^{3n})}{\sqrt{a + bx^n}} \, dx = \int \frac{(gx^{3n} + fx^{2n} + ex^n + d)(cx)^m}{\sqrt{bx^n + a}} \, dx$$

[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/sqrt(b*x^n + a), x)

Giac [F]

$$\int \frac{(cx)^m (d + ex^n + fx^{2n} + gx^{3n})}{\sqrt{a + bx^n}} \, dx = \int \frac{(gx^{3n} + fx^{2n} + ex^n + d)(cx)^m}{\sqrt{bx^n + a}} \, dx$$

[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x, algorithm="giac")

[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/sqrt(b*x^n + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (d + ex^n + fx^{2n} + gx^{3n})}{\sqrt{a + bx^n}} \, dx = \int \frac{(c\,x)^m \left(d + e\,x^n + f\,x^{2\,n} + g\,x^{3\,n}\right)}{\sqrt{a + b\,x^n}} \, dx$$

[In] int(((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n)^(1/2),x)

[Out] int(((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n)^(1/2), x)

3.585 $\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}} \, dx$

Optimal result

Integrand size = 58, antiderivative size = 45

$$\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}} \, dx = -\frac{2(ag+2ahx^{n/4}-bfx^{n/2})}{an\sqrt{a+bx^n}}$$

```
[Out] -2*(a*g+2*a*h*x^(1/4*n)-b*f*x^(1/2*n))/a/n/(a+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {6873, 1830}

$$\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}} \, dx = -\frac{2(ag+2ahx^{n/4}-bfx^{n/2})}{an\sqrt{a+bx^n}}$$

```
[In] Int[(-(a*h*x^(-1 + n/4)) + b*f*x^(-1 + n/2) + b*g*x^(-1 + n) + b*h*x^(-1 + (5*n)/4))/(a + b*x^n)^(3/2),x]
```

```
[Out] (-2*(a*g + 2*a*h*x^(n/4) - b*f*x^(n/2)))/(a*n*Sqrt[a + b*x^n])
```

Rule 1830

```
Int[((x_)^(m_.)*((e_) + (h_.)*(x_)^(n_.) + (f_.)*(x_)^(q_.) + (g_.)*(x_)^(r_.)))/((a_) + (c_.)*(x_)^(n_.))^(3/2), x_Symbol] :> Simp[-(2*a*g + 4*a*h*x^(n/4) - 2*c*f*x^(n/2))/(a*c*n*Sqrt[a + c*x^n]), x] /; FreeQ[{a, c, e, f, g, h, m, n}, x] && EqQ[q, n/4] && EqQ[r, 3*(n/4)] && EqQ[4*m - n + 4, 0] && EqQ[c*e + a*h, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = \int \frac{x^{-1+\frac{n}{4}}\left(-ah+bfx^{n/4}+bgx^{3n/4}+bhx^n\right)}{(a+bx^n)^{3/2}}\,dx$$
$$= -\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx = \frac{2bfx^{n/2}-2a\left(g+2hx^{n/4}\right)}{an\sqrt{a+bx^n}}$$

```
[In] Integrate[(-(a*h*x^(-1 + n/4)) + b*f*x^(-1 + n/2) + b*g*x^(-1 + n) + b*h*x^
(-1 + (5*n)/4))/(a + b*x^n)^(3/2),x]
[Out] (2*b*f*x^(n/2) - 2*a*(g + 2*h*x^(n/4)))/(a*n*Sqrt[a + b*x^n])
```

Maple [F]

$$\int \frac{-ah\,x^{-1+\frac{n}{4}}+bf\,x^{-1+\frac{n}{2}}+bg\,x^{-1+n}+bh\,x^{-1+\frac{5n}{4}}}{(a+b\,x^n)^{\frac{3}{2}}}dx$$

```
[In] int((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n))/(a+b
*x^n)^(3/2),x)
[Out] int((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n))/(a+b
*x^n)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx = \frac{2\sqrt{bx^4x^{n-4}+a}\left(bfx^2x^{\frac{1}{2}\,n-2}-2\,ahxx^{\frac{1}{4}\,n-1}-ag\right)}{abnx^4x^{n-4}+a^2n}$$

```
[In] integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n)
)/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x^4*x^(n - 4) + a)*(b*f*x^2*x^(1/2*n - 2) - 2*a*h*x*x^(1/4*n - 1)
- a*g)/(a*b*n*x^4*x^(n - 4) + a^2*n)
```

Sympy [A] (verification not implemented)

Time = 137.42 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.60

$$\int \frac{-ahx^{-1+\frac{n}{4}} + bfx^{-1+\frac{n}{2}} + bgx^{-1+n} + bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx = bg\left(\begin{cases} \frac{\log(x)}{a^{\frac{3}{2}}} & \text{for}\ b=0\wedge n=0 \\ \frac{xx^{n-1}}{a^{\frac{3}{2}}n} & \text{for}\ b=0 \\ \frac{\log(x)}{(a+b)^{\frac{3}{2}}} & \text{for}\ n=0 \\ -\frac{2}{bn\sqrt{a+bx^n}} & \text{otherwise} \end{cases}\right) + \frac{2\sqrt{b}f}{an\sqrt{\frac{ax^{-n}}{b}+1}} - \frac{hx^{\frac{n}{4}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{a}n\Gamma\left(\frac{5}{4}\right)} + \frac{bhx^{\frac{5n}{4}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{a^{\frac{3}{2}}n\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate((-a*h*x**(-1+1/4*n)+b*f*x**(-1+1/2*n)+b*g*x**(-1+n)+b*h*x**(-1+5/
4*n))/(a+b*x**n)**(3/2),x)
```

```
[Out] b*g*Piecewise((log(x)/a**(3/2), Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)/(a**(3/
2)*n), Eq(b, 0)), (log(x)/(a + b)**(3/2), Eq(n, 0)), (-2/(b*n*sqrt(a + b*x*
*n)), True)) + 2*sqrt(b)*f/(a*n*sqrt(a/(b*x**n) + 1)) - h*x**(n/4)*gamma(1/
4)*hyper((1/4, 3/2), (5/4,), b*x**n*exp_polar(I*pi)/a)/(sqrt(a)*n*gamma(5/4
)) + b*h*x**(5*n/4)*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**n*exp_polar(I
*pi)/a)/(a**(3/2)*n*gamma(9/4))
```

Maxima [F]

$$\int \frac{-ahx^{-1+\frac{n}{4}} + bfx^{-1+\frac{n}{2}} + bgx^{-1+n} + bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx = \int \frac{bhx^{\frac{5}{4}n-1} + bgx^{n-1} + bfx^{\frac{1}{2}n-1} - ahx^{\frac{1}{4}n-1}}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n)
)/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1) - a*h*x^(1
/4*n - 1))/(b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{-ahx^{-1+\frac{n}{4}} + bfx^{-1+\frac{n}{2}} + bgx^{-1+n} + bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx = \int \frac{bhx^{\frac{5}{4}\,n-1} + bgx^{n-1} + bfx^{\frac{1}{2}\,n-1} - ahx^{\frac{1}{4}\,n-1}}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n)
)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1) - a*h*x^(1
/4*n - 1))/(b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-ahx^{-1+\frac{n}{4}} + bfx^{-1+\frac{n}{2}} + bgx^{-1+n} + bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx = \int \frac{b\,f\,x^{\frac{n}{2}-1} - a\,h\,x^{\frac{n}{4}-1} + b\,h\,x^{\frac{5\,n}{4}-1} + b\,g\,x^{n-1}}{(a+b\,x^n)^{3/2}}\,dx$$

```
[In] int((b*f*x^(n/2 - 1) - a*h*x^(n/4 - 1) + b*h*x^((5*n)/4 - 1) + b*g*x^(n - 1
))/(a + b*x^n)^(3/2),x)
```

```
[Out] int((b*f*x^(n/2 - 1) - a*h*x^(n/4 - 1) + b*h*x^((5*n)/4 - 1) + b*g*x^(n - 1
))/(a + b*x^n)^(3/2), x)
```

3.586 $\int (cx)^m (d+ex+fx^2+gx^3) (a+bx^n)^p \, dx$

Optimal result

Integrand size = 30, antiderivative size = 273

$$\int (cx)^m \left(d+ex+fx^2+gx^3\right) (a+bx^n)^p \, dx = \frac{d(cx)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{c(1+m)} + \frac{e(cx)^{2+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{2+m}{n},-p,\frac{2+m+n}{n},-\frac{bx^n}{a}\right)}{c^2(2+m)} + \frac{f(cx)^{3+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3+m}{n},-p,\frac{3+m+n}{n},-\frac{bx^n}{a}\right)}{c^3(3+m)} + \frac{g(cx)^{4+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{4+m}{n},-p,\frac{4+m+n}{n},-\frac{bx^n}{a}\right)}{c^4(4+m)}$$

```
[Out] d*(c*x)^(1+m)*(a+b*x^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/c/(
1+m)/((1+b*x^n/a)^p)+e*(c*x)^(2+m)*(a+b*x^n)^p*hypergeom([-p, (2+m)/n],[(2+
m+n)/n],-b*x^n/a)/c^2/(2+m)/((1+b*x^n/a)^p)+f*(c*x)^(3+m)*(a+b*x^n)^p*hyper
geom([-p, (3+m)/n],[(3+m+n)/n],-b*x^n/a)/c^3/(3+m)/((1+b*x^n/a)^p)+g*(c*x)^
(4+m)*(a+b*x^n)^p*hypergeom([-p, (4+m)/n],[(4+m+n)/n],-b*x^n/a)/c^4/(4+m)/(
(1+b*x^n/a)^p)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1858, 372, 371}

$$\int (cx)^m \left(d+ex+fx^2+gx^3\right) (a+bx^n)^p \, dx$$
$$= \frac{g(cx)^{m+4} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+4}{n},-p,\frac{m+n+4}{n},-\frac{bx^n}{a}\right)}{c^4(m+4)}$$
$$+\frac{f(cx)^{m+3} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+3}{n},-p,\frac{m+n+3}{n},-\frac{bx^n}{a}\right)}{c^3(m+3)}$$
$$+\frac{e(cx)^{m+2} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+2}{n},-p,\frac{m+n+2}{n},-\frac{bx^n}{a}\right)}{c^2(m+2)}$$
$$+\frac{d(cx)^{m+1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{n},-p,\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{c(m+1)}$$

```
[In] Int[(c*x)^m*(d + e*x + f*x^2 + g*x^3)*(a + b*x^n)^p,x]
```

```
[Out] (d*(c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)
/n, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p) + (e*(c*x)^(2 + m)*(a + b*
x^n)^p*Hypergeometric2F1[(2 + m)/n, -p, (2 + m + n)/n, -((b*x^n)/a)])/(c^2*
(2 + m)*(1 + (b*x^n)/a)^p) + (f*(c*x)^(3 + m)*(a + b*x^n)^p*Hypergeometric2
F1[(3 + m)/n, -p, (3 + m + n)/n, -((b*x^n)/a)])/(c^3*(3 + m)*(1 + (b*x^n)/a
)^p) + (g*(c*x)^(4 + m)*(a + b*x^n)^p*Hypergeometric2F1[(4 + m)/n, -p, (4 +
 m + n)/n, -((b*x^n)/a)])/(c^4*(4 + m)*(1 + (b*x^n)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1858

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
```

```
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(d(cx)^m (a+bx^n)^p + \frac{e(cx)^{1+m}(a+bx^n)^p}{c} + \frac{f(cx)^{2+m}(a+bx^n)^p}{c^2} + \frac{g(cx)^{3+m}(a+bx^n)^p}{c^3}\right) dx \\
&= d\int (cx)^m (a+bx^n)^p \, dx + \frac{e\int (cx)^{1+m}(a+bx^n)^p \, dx}{c} + \frac{f\int (cx)^{2+m}(a+bx^n)^p \, dx}{c^2} + \frac{g\int (cx)^{3+m}(a+bx^n)^p \, dx}{c^3} \\
&= \left(d(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p}\right)\int (cx)^m \left(1+\frac{bx^n}{a}\right)^p dx + \frac{\left(e(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p}\right)\int (cx)^{1+m}\left(1+\frac{bx^n}{a}\right)^p dx}{c} \\
&\quad + \frac{\left(f(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p}\right)\int (cx)^{2+m}\left(1+\frac{bx^n}{a}\right)^p dx}{c^2} + \frac{\left(g(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p}\right)\int (cx)^{3+m}\left(1+\frac{bx^n}{a}\right)^p dx}{c^3} \\
&= \frac{d(cx)^{1+m}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{n},-p;\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)} \\
&\quad + \frac{e(cx)^{2+m}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{2+m}{n},-p;\frac{2+m+n}{n};-\frac{bx^n}{a}\right)}{c^2(2+m)} \\
&\quad + \frac{f(cx)^{3+m}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{3+m}{n},-p;\frac{3+m+n}{n};-\frac{bx^n}{a}\right)}{c^3(3+m)} \\
&\quad + \frac{g(cx)^{4+m}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{4+m}{n},-p;\frac{4+m+n}{n};-\frac{bx^n}{a}\right)}{c^4(4+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.65

$$\int (cx)^m \left(d+ex+fx^2+gx^3\right) (a+bx^n)^p \, dx$$
$$= x(cx)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(\frac{d\,\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{1+m} + x\left(\frac{e\,\text{Hypergeometric2F1}\left(\frac{2+m}{n},-p,\frac{2+m+n}{n},-\frac{bx^n}{a}\right)}{2+m} + x\left(\frac{f\,\text{Hypergeometric2F1}\left(\frac{3+m}{n},-p,\frac{3+m+n}{n},-\frac{bx^n}{a}\right)}{3+m} + \frac{gx\,\text{Hypergeometric2F1}\left(\frac{4+m}{n},-p,\frac{4+m+n}{n},-\frac{bx^n}{a}\right)}{4+m}\right)\right)\right)$$

```
[In] Integrate[(c*x)^m*(d + e*x + f*x^2 + g*x^3)*(a + b*x^n)^p,x]
```

```
[Out] (x*(c*x)^m*(a + b*x^n)^p*((d*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n
, -((b*x^n)/a)])/(1 + m) + x*((e*Hypergeometric2F1[(2 + m)/n, -p, (2 + m +
n)/n, -((b*x^n)/a)])/(2 + m) + x*((f*Hypergeometric2F1[(3 + m)/n, -p, (3 +
m + n)/n, -((b*x^n)/a)])/(3 + m) + (g*x*Hypergeometric2F1[(4 + m)/n, -p, (4
 + m + n)/n, -((b*x^n)/a)])/(4 + m)))))/(1 + (b*x^n)/a)^p
```

Maple [F]

$$\int (cx)^m \left(g\,x^3+f\,x^2+ex+d\right) (a+b\,x^n)^p \, dx$$

```
[In] int((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x)
```

```
[Out] int((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int (cx)^m \left(d+ex+fx^2+gx^3\right) (a+bx^n)^p \, dx = \int \left(gx^3+fx^2+ex+d\right)(bx^n+a)^p(cx)^m \, dx$$

```
[In] integrate((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 104.02 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.11

$$\int (cx)^m\left(d+ex+fx^2+gx^3\right)(a+bx^n)^p\,dx$$
$$=\frac{a^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}+p-\frac{1}{n}}c^m dx^{m+1}\Gamma\left(\frac{m}{n}+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$
$$+\frac{a^{\frac{m}{n}+\frac{2}{n}}a^{-\frac{m}{n}+p-\frac{2}{n}}c^m ex^{m+2}\Gamma\left(\frac{m}{n}+\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{n}+\frac{2}{n}\\ \frac{m}{n}+1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{2}{n}\right)}$$
$$+\frac{a^{\frac{m}{n}+\frac{3}{n}}a^{-\frac{m}{n}+p-\frac{3}{n}}c^m fx^{m+3}\Gamma\left(\frac{m}{n}+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{n}+\frac{3}{n}\\ \frac{m}{n}+1+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{3}{n}\right)}$$
$$+\frac{a^{\frac{m}{n}+\frac{4}{n}}a^{-\frac{m}{n}+p-\frac{4}{n}}c^m gx^{m+4}\Gamma\left(\frac{m}{n}+\frac{4}{n}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{n}+\frac{4}{n}\\ \frac{m}{n}+1+\frac{4}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{4}{n}\right)}$$

```
[In] integrate((c*x)**m*(g*x**3+f*x**2+e*x+d)*(a+b*x**n)**p,x)
```

```
[Out] a**(m/n + 1/n)*a**(-m/n + p - 1/n)*c**m*d*x**(m + 1)*gamma(m/n + 1/n)*hyper
((-p, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n
+ 1 + 1/n)) + a**(m/n + 2/n)*a**(-m/n + p - 2/n)*c**m*e*x**(m + 2)*gamma(m/
n + 2/n)*hyper((-p, m/n + 2/n), (m/n + 1 + 2/n,), b*x**n*exp_polar(I*pi)/a)
/(n*gamma(m/n + 1 + 2/n)) + a**(m/n + 3/n)*a**(-m/n + p - 3/n)*c**m*f*x**(m
 + 3)*gamma(m/n + 3/n)*hyper((-p, m/n + 3/n), (m/n + 1 + 3/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(m/n + 1 + 3/n)) + a**(m/n + 4/n)*a**(-m/n + p - 4/n
)*c**m*g*x**(m + 4)*gamma(m/n + 4/n)*hyper((-p, m/n + 4/n), (m/n + 1 + 4/n,
), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1 + 4/n))
```

Maxima [F]

$$\int (cx)^m \left(d+ex+fx^2+gx^3\right)(a+bx^n)^p\,dx = \int \left(gx^3+fx^2+ex+d\right)(bx^n+a)^p(cx)^m\,dx$$

```
[In] integrate((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x, algorithm="maxima")
[Out] integrate((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(d+ex+fx^2+gx^3\right)(a+bx^n)^p\,dx = \int \left(gx^3+fx^2+ex+d\right)(bx^n+a)^p(cx)^m\,dx$$

```
[In] integrate((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x, algorithm="giac")
[Out] integrate((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(d+ex+fx^2+gx^3\right)(a+bx^n)^p\,dx = \int (c\,x)^m\,(a+b\,x^n)^p\,\left(g\,x^3+f\,x^2+e\,x+d\right)\,dx$$

```
[In] int((c*x)^m*(a + b*x^n)^p*(d + e*x + f*x^2 + g*x^3),x)
[Out] int((c*x)^m*(a + b*x^n)^p*(d + e*x + f*x^2 + g*x^3), x)
```

3.587 $\int (cx)^m (a+bx^n)^p (d+ex^n+fx^{2n}+gx^{3n}) \, dx$

Optimal result

Integrand size = 36, antiderivative size = 297

$$\int (cx)^m (a+bx^n)^p (d+ex^n+fx^{2n}+gx^{3n}) \, dx = \frac{d(cx)^{1+m}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{c(1+m)} + \frac{ex^{1+n}(cx)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m+n}{n},-p,\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{1+m+n} + \frac{fx^{1+2n}(cx)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m+2n}{n},-p,\frac{1+m+3n}{n},-\frac{bx^n}{a}\right)}{1+m+2n} + \frac{gx^{1+3n}(cx)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m+3n}{n},-p,\frac{1+m+4n}{n},-\frac{bx^n}{a}\right)}{1+m+3n}$$

```
[Out] d*(c*x)^(1+m)*(a+b*x^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/c/(
1+m)/((1+b*x^n/a)^p)+e*x^(1+n)*(c*x)^m*(a+b*x^n)^p*hypergeom([-p, (1+m+n)/n
],[(1+m+2*n)/n],-b*x^n/a)/(1+m+n)/((1+b*x^n/a)^p)+f*x^(1+2*n)*(c*x)^m*(a+b*
x^n)^p*hypergeom([-p, (1+m+2*n)/n],[(1+m+3*n)/n],-b*x^n/a)/(1+m+2*n)/((1+b*
x^n/a)^p)+g*x^(1+3*n)*(c*x)^m*(a+b*x^n)^p*hypergeom([-p, (1+m+3*n)/n],[(1+m
+4*n)/n],-b*x^n/a)/(1+m+3*n)/((1+b*x^n/a)^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1858, 372, 371, 20}

$$\int (cx)^m (a+bx^n)^p \left(d+ex^n+fx^{2n}+gx^{3n}\right) dx$$
$$= \frac{d(cx)^{m+1}(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{n},-p,\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{c(m+1)}$$
$$+\frac{ex^{n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+n+1}{n},-p,\frac{m+2n+1}{n},-\frac{bx^n}{a}\right)}{m+n+1}$$
$$+\frac{fx^{2n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2n+1}{n},-p,\frac{m+3n+1}{n},-\frac{bx^n}{a}\right)}{m+2n+1}$$
$$+\frac{gx^{3n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+3n+1}{n},-p,\frac{m+4n+1}{n},-\frac{bx^n}{a}\right)}{m+3n+1}$$

```
[In] Int[(c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)),x]
```

```
[Out] (d*(c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)
/n, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p) + (e*x^(1 + n)*(c*x)^m*(a
+ b*x^n)^p*Hypergeometric2F1[(1 + m + n)/n, -p, (1 + m + 2*n)/n, -((b*x^n)/
a)])/((1 + m + n)*(1 + (b*x^n)/a)^p) + (f*x^(1 + 2*n)*(c*x)^m*(a + b*x^n)^p
*Hypergeometric2F1[(1 + m + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)])/((1
 + m + 2*n)*(1 + (b*x^n)/a)^p) + (g*x^(1 + 3*n)*(c*x)^m*(a + b*x^n)^p*Hyper
geometric2F1[(1 + m + 3*n)/n, -p, (1 + m + 4*n)/n, -((b*x^n)/a)])/((1 + m +
 3*n)*(1 + (b*x^n)/a)^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
```

```
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1858

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(d(cx)^m (a+bx^n)^p + ex^n(cx)^m (a+bx^n)^p + fx^{2n}(cx)^m (a+bx^n)^p \right. \\
&\qquad \left. + gx^{3n}(cx)^m (a+bx^n)^p\right) dx \\
&= d\int (cx)^m (a+bx^n)^p \, dx + e\int x^n(cx)^m (a+bx^n)^p \, dx \\
&\quad + f\int x^{2n}(cx)^m (a+bx^n)^p \, dx + g\int x^{3n}(cx)^m (a+bx^n)^p \, dx \\
&= \left(ex^{-m}(cx)^m\right)\int x^{m+n}(a+bx^n)^p \, dx + \left(fx^{-m}(cx)^m\right)\int x^{m+2n}(a+bx^n)^p \, dx \\
&\quad + \left(gx^{-m}(cx)^m\right)\int x^{m+3n}(a+bx^n)^p \, dx \\
&\quad + \left(d(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int (cx)^m\left(1+\frac{bx^n}{a}\right)^p dx \\
&= \frac{d(cx)^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{n},-p;\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)} \\
&\quad + \left(ex^{-m}(cx)^m (a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int x^{m+n}\left(1+\frac{bx^n}{a}\right)^p dx \\
&\quad + \left(fx^{-m}(cx)^m (a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int x^{m+2n}\left(1+\frac{bx^n}{a}\right)^p dx \\
&\quad + \left(gx^{-m}(cx)^m (a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int x^{m+3n}\left(1+\frac{bx^n}{a}\right)^p dx
\end{aligned}$$

$$= \frac{d(cx)^{1+m}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{n},-p;\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)} + \frac{ex^{1+n}(cx)^m(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1+m+n}{n},-p;\frac{1+m+2n}{n};-\frac{bx^n}{a}\right)}{1+m+n} + \frac{fx^{1+2n}(cx)^m(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1+m+2n}{n},-p;\frac{1+m+3n}{n};-\frac{bx^n}{a}\right)}{1+m+2n} + \frac{gx^{1+3n}(cx)^m(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1+m+3n}{n},-p;\frac{1+m+4n}{n};-\frac{bx^n}{a}\right)}{1+m+3n}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.69

$$\int (cx)^m (a+bx^n)^p \left(d+ex^n+fx^{2n}+gx^{3n}\right) dx$$
$$= x(cx)^m(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(\frac{d\,\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{1+m} + x^n\left(\frac{e\,\text{Hypergeometric2F1}\left(\frac{1+m+n}{n},-p,\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{1+m+n} + x^n\left(\frac{f\,\text{Hypergeometric2F1}\left(\frac{1+m+2n}{n},-p,\frac{1+m+3n}{n},-\frac{bx^n}{a}\right)}{1+m+2n} + \frac{gx^n\,\text{Hypergeometric2F1}\left(\frac{1+m+3n}{n},-p,\frac{1+m+4n}{n}\right.}{1+m+3n}\right.\right.\right.$$

```
[In] Integrate[(c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)),x]
```

```
[Out] (x*(c*x)^m*(a + b*x^n)^p*((d*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n
, -((b*x^n)/a)])/(1 + m) + x^n*((e*Hypergeometric2F1[(1 + m + n)/n, -p, (1
+ m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n) + x^n*((f*Hypergeometric2F1[(1 + m
 + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)])/(1 + m + 2*n) + (g*x^n*Hyper
geometric2F1[(1 + m + 3*n)/n, -p, (1 + m + 4*n)/n, -((b*x^n)/a)])/(1 + m +
3*n)))))/(1 + (b*x^n)/a)^p
```

Maple [F]

$$\int (cx)^m (a+b\,x^n)^p \left(d+e\,x^n+f\,x^{2n}+g\,x^{3n}\right) dx$$

```
[In] int((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x)
```

```
[Out] int((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x)
```

Fricas [F]

$$\int (cx)^m (a + bx^n)^p \left(d + ex^n + fx^{2n} + gx^{3n}\right) dx$$
$$= \int \left(gx^{3n} + fx^{2n} + ex^n + d\right)(bx^n + a)^p (cx)^m dx$$

```
[In] integrate((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x, algorithm="f
ricas")
[Out] integral((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (cx)^m (a + bx^n)^p \left(d + ex^n + fx^{2n} + gx^{3n}\right) dx = \text{Timed out}$$

```
[In] integrate((c*x)**m*(a+b*x**n)**p*(d+e*x**n+f*x**(2*n)+g*x**(3*n)),x)
[Out] Timed out
```

Maxima [F]

$$\int (cx)^m (a + bx^n)^p \left(d + ex^n + fx^{2n} + gx^{3n}\right) dx$$
$$= \int \left(gx^{3n} + fx^{2n} + ex^n + d\right)(bx^n + a)^p (cx)^m dx$$

```
[In] integrate((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x, algorithm="m
axima")
[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (cx)^m (a+bx^n)^p \left(d+ex^n+fx^{2n}+gx^{3n}\right)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2,0,6,4,0,2,4,4,1,0,0,0]%%%}+%%%{4,[2,0,6,4,0,2,3,4,1,0,0,
0]%%%}
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m (a+bx^n)^p \left(d+ex^n+fx^{2n}+gx^{3n}\right)\,dx = \int (c\,x)^m\,(a+b\,x^n)^p\,\left(d+e\,x^n+f\,x^{2\,n}+g\,x^{3\,n}\right)\,dx$$

```
[In] int((c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)),x)
```

```
[Out] int((c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)), x)
```

3.588 $\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 35, antiderivative size = 162

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2} dx = \frac{x\left(bc-ae+(bd-af)x^{n/2}\right)}{abn\left(a+bx^n\right)} - \frac{(bd(2-n)-af(2+n))x^{\frac{2+n}{2}}\operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{a^2bn(2+n)} + \frac{(ae-bc(1-n))x\operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2bn}$$

```
[Out] x*(b*c-a*e+(-a*f+b*d)*x^(1/2*n))/a/b/n/(a+b*x^n)-(b*d*(2-n)-a*f*(2+n))*x^(1
+1/2*n)*hypergeom([1, 1/2+1/n],[3/2+1/n],-b*x^n/a)/a^2/b/n/(2+n)+(a*e-b*c*(
1-n))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a^2/b/n
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1906, 1432, 251, 371}

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2} dx = \frac{x(ae-bc(1-n))\operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2bn} - \frac{x^{\frac{n+2}{2}}(bd(2-n)-af(n+2))\operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{a^2bn(n+2)} + \frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{abn\left(a+bx^n\right)}$$

```
[In] Int[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^2,x]
```

```
[Out] (x*(b*c - a*e + (b*d - a*f)*x^(n/2)))/(a*b*n*(a + b*x^n)) - ((b*d*(2 - n) -
 a*f*(2 + n))*x^((2 + n)/2)*Hypergeometric2F1[1, (1 + 2/n)/2, (3 + 2/n)/2,
-((b*x^n)/a)])/(a^2*b*n*(2 + n)) + ((a*e - b*c*(1 - n))*x*Hypergeometric2F1
[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1906

```
Int[(P3_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{A = Coeff[P3, x
^(n/2), 0], B = Coeff[P3, x^(n/2), 1], C = Coeff[P3, x^(n/2), 2], D = Coeff
[P3, x^(n/2), 3]}, Simp[-(x*(b*A - a*C + (b*B - a*D)*x^(n/2))*(a + b*x^n)^(
p + 1))/(a*b*n*(p + 1)), x] - Dist[1/(2*a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*Simp[2*a*C - 2*b*A*(n*(p + 1) + 1) + (a*D*(n + 2) - b*B*(n*(2*p + 3) +
 2))*x^(n/2), x], x], x]] /; FreeQ[{a, b, n}, x] && PolyQ[P3, x^(n/2), 3] &
& ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\left(bc-ae+(bd-af)x^{n/2}\right)}{abn\left(a+bx^n\right)}+\frac{\int \frac{2(ae-bc(1-n))-(bd(2-n)-af(2+n))x^{n/2}}{a+bx^n}\,dx}{2abn}\\ &= \frac{x\left(bc-ae+(bd-af)x^{n/2}\right)}{abn\left(a+bx^n\right)}+\frac{(ae-bc(1-n))\int \frac{1}{a+bx^n}\,dx}{abn}\\ &\quad -\frac{(bd(2-n)-af(2+n))\int \frac{x^{n/2}}{a+bx^n}\,dx}{2abn}\end{aligned}$$

$$= \frac{x\left(bc-ae+(bd-af)x^{n/2}\right)}{abn\left(a+bx^n\right)} - \frac{(bd(2-n)-af(2+n))x^{\frac{2+n}{2}}\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-\frac{bx^n}{a}\right)}{a^2bn(2+n)} + \frac{(ae-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx = \frac{x\left(\frac{2afx^{n/2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{1}{n},\frac{3}{2}+\frac{1}{n},-\frac{bx^n}{a}\right)}{2+n} + ae\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},\right.\right.}{}$$

```
[In] Integrate[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^2,x]
[Out] (x*((2*a*f*x^(n/2)*Hypergeometric2F1[1, 1/2 + n^(-1), 3/2 + n^(-1), -((b*x^
n)/a)])/(2 + n) + a*e*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)
] + (2*(b*d - a*f)*x^(n/2)*Hypergeometric2F1[2, 1/2 + n^(-1), 3/2 + n^(-1),
 -((b*x^n)/a)])/(2 + n) + (b*c - a*e)*Hypergeometric2F1[2, n^(-1), 1 + n^(-
1), -((b*x^n)/a)]))/(a^2*b)
```

Maple [F]

$$\int \frac{c+d\,x^{\frac{n}{2}}+e\,x^n+f\,x^{\frac{3n}{2}}}{(a+b\,x^n)^2}dx$$

```
[In] int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x)
[Out] int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx = \int \frac{fx^{\frac{3}{2}n}+dx^{\frac{1}{2}n}+ex^n+c}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="frica
s")
[Out] integral((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b^2*x^(2*n) + 2*a*b*x^n +
 a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx = \text{Timed out}$$

```
[In] integrate((c+d*x**(1/2*n)+e*x**n+f*x**(3/2*n))/(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx = \int \frac{fx^{\frac{3}{2}n}+dx^{\frac{1}{2}n}+ex^n+c}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="maxim
a")
```

```
[Out] ((b*d - a*f)*x*x^(1/2*n) + (b*c - a*e)*x)/(a*b^2*n*x^n + a^2*b*n) + integra
te(1/2*(2*b*c*(n - 1) + 2*a*e + (a*f*(n + 2) + b*d*(n - 2))*x^(1/2*n))/(a*b
^2*n*x^n + a^2*b*n), x)
```

Giac [F]

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx = \int \frac{fx^{\frac{3}{2}n}+dx^{\frac{1}{2}n}+ex^n+c}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="giac"
)
```

```
[Out] integrate((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx = \int \frac{c+e\,x^n+d\,x^{n/2}+f\,x^{\frac{3\,n}{2}}}{(a+b\,x^n)^2}\,dx$$

```
[In] int((c + e*x^n + d*x^(n/2) + f*x^((3*n)/2))/(a + b*x^n)^2,x)
```

```
[Out] int((c + e*x^n + d*x^(n/2) + f*x^((3*n)/2))/(a + b*x^n)^2, x)
```

3.589 $\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 24

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

```
[Out] x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {1604}

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

```
[In] Int[(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
,x]
```

```
[Out] x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Mathematica [A] (verified)

Time = 7.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

```
[In] Integrate[(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b*x^2]*Sqrt[c + d
*x^2]),x]
```

```
[Out] x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
gosper	$x\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}$	21
default	$x\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}$	21
risch	$x\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}$	21
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\,x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$	62

```
[In] int((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \sqrt{bx^2+a}\sqrt{dx^2+c}x$$

```
[In] integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x
, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x
```

Sympy [F]

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{ac+2adx^2+2bcx^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

```
[In] integrate((a*c+2*(a*d+b*c)*x**2+3*b*d*x**4)/(b*x**2+a)**(1/2)/(d*x**2+c)**(
1/2),x)
```

```
[Out] Integral((a*c + 2*a*d*x**2 + 2*b*c*x**2 + 3*b*d*x**4)/(sqrt(a + b*x**2)*sqr
t(c + d*x**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\sqrt{bx^2+a}\sqrt{dx^2+c}x$$

```
[In] integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x
, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x
```

Giac [F]

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{3\,bdx^4+2\,(bc+ad)x^2+ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((3*b*d*x^4 + 2*(b*c + a*d)*x^2 + a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2
 + c)), x)
```

Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = x\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}$$

```
[In] int((a*c + 2*x^2*(a*d + b*c) + 3*b*d*x^4)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1
/2)),x)
```

```
[Out] x*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)
```

3.590 $\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} - \frac{\arctan\left(\frac{\sqrt[4]{1+x^4}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1+x^4}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

```
[Out] 1/4*arctan(2^(1/4)*x/(x^4+1)^(1/4))*2^(3/4)-1/4*arctan(1/2*(x^4+1)^(1/4)*2^
(3/4))*2^(3/4)+1/4*arctanh(2^(1/4)*x/(x^4+1)^(1/4))*2^(3/4)+1/4*arctanh(1/2
*(x^4+1)^(1/4)*2^(3/4))*2^(3/4)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1913, 385, 218, 212, 209, 455, 65, 304}

$$\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}} - \frac{\arctan\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

```
[In] Int[(1 + x^3)/((1 - x^4)*(1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) - ArcTan[(1 + x^4)^(1/4)/2^
(1/4)]/(2*2^(1/4)) + ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) + Arc
Tanh[(1 + x^4)^(1/4)/2^(1/4)]/(2*2^(1/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1913

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)
*(x_)^(n_))^(q_.), x_Symbol] :> Dist[A, Int[(a + b*x^n)^p*(c + d*x^n)^q, x]
, x] + Dist[B, Int[x^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b,
c, d, A, B, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(1-x^4)\sqrt[4]{1+x^4}}\,dx + \int \frac{x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt[4]{1+x}}\,dx, x, x^4\right) + \text{Subst}\left(\int \frac{1}{1-2x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right) \\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right) + \text{Subst}\left(\int \frac{x^2}{2-x^4}\,dx, x, \sqrt[4]{1+x^4}\right) \\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} \\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx, x, \sqrt[4]{1+x^4}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}\,dx, x, \sqrt[4]{1+x^4}\right) \\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{1+x^4}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+x^4}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\begin{aligned}
&\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx \\
&= \frac{1}{4}x^4\,\text{AppellF1}\left(1,\frac{1}{4},1,2,-x^4,x^4\right) \\
&\quad + \frac{2\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right) - \log\left(1-\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right) + \log\left(1+\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{4\sqrt[4]{2}}
\end{aligned}$$

```
[In] Integrate[(1 + x^3)/((1 - x^4)*(1 + x^4)^(1/4)),x]
```

```
[Out] (x^4*AppellF1[1, 1/4, 1, 2, -x^4, x^4])/4 + (2*ArcTan[(2^(1/4)*x)/(1 + x^4)
^(1/4)] - Log[1 - (2^(1/4)*x)/(1 + x^4)^(1/4)] + Log[1 + (2^(1/4)*x)/(1 + x
^4)^(1/4)])/(4*2^(1/4))
```

Maple [F(-1)]

Timed out.

$$\int \frac{x^3+1}{(-x^4+1)(x^4+1)^{\frac{1}{4}}}dx$$

```
[In] int((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x)
```

```
[Out] int((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

Sympy [F]

$$\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx = -\int \left(-\frac{x}{x^3\sqrt[4]{x^4+1}-x^2\sqrt[4]{x^4+1}+x\sqrt[4]{x^4+1}-\sqrt[4]{x^4+1}}\right)dx - \int \frac{x^2}{x^3\sqrt[4]{x^4+1}-x^2\sqrt[4]{x^4+1}+x\sqrt[4]{x^4+1}-\sqrt[4]{x^4+1}}\,dx - \int \frac{1}{x^3\sqrt[4]{x^4+1}-x^2\sqrt[4]{x^4+1}+x\sqrt[4]{x^4+1}-\sqrt[4]{x^4+1}}\,dx$$

```
[In] integrate((x**3+1)/(-x**4+1)/(x**4+1)**(1/4),x)
```

```
[Out] -Integral(-x/(x**3*(x**4 + 1)**(1/4) - x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1
)**(1/4) - (x**4 + 1)**(1/4)), x) - Integral(x**2/(x**3*(x**4 + 1)**(1/4) -
 x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1)**(1/4) - (x**4 + 1)**(1/4)), x) - In
tegral(1/(x**3*(x**4 + 1)**(1/4) - x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1)**(
1/4) - (x**4 + 1)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx = \int -\frac{x^3+1}{(x^4+1)^{\frac{1}{4}}(x^4-1)}\,dx$$

```
[In] integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="maxima")
[Out] -integrate((x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)), x)
```

Giac [F]

$$\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx = \int -\frac{x^3+1}{(x^4+1)^{\frac{1}{4}}(x^4-1)}\,dx$$

```
[In] integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="giac")
[Out] integrate(-(x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx = \int -\frac{x^3+1}{(x^4-1)\,(x^4+1)^{1/4}}\,dx$$

```
[In] int(-(x^3 + 1)/((x^4 - 1)*(x^4 + 1)^(1/4)),x)
[Out] int(-(x^3 + 1)/((x^4 - 1)*(x^4 + 1)^(1/4)), x)
```

3.591 $\int (a+bx^n)^{\frac{-1-n}{n}} (c+dx^n)^{\frac{-1-n}{n}} (ac-bdx^{2n})\,dx$

Optimal result

Integrand size = 48, antiderivative size = 28

$$\int (a+bx^n)^{\frac{-1-n}{n}} (c+dx^n)^{\frac{-1-n}{n}} (ac-bdx^{2n})\,dx = x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

```
[Out] x/((a+b*x^n)^(1/n))/((c+d*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {1912}

$$\int (a+bx^n)^{\frac{-1-n}{n}} (c+dx^n)^{\frac{-1-n}{n}} (ac-bdx^{2n})\,dx = x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

```
[In] Int[(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)*(a*c - b*d*x^(2*n)),x
]
```

```
[Out] x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))
```

Rule 1912

```
Int[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(p_.)*((e_) +
 (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[e*x*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(p + 1)/(a*c)), x] /; FreeQ[{a, b, c, d, e, g, n, p}, x] && EqQ[n2, 2*n] &
& EqQ[n*(p + 1) + 1, 0] && EqQ[a*c*g - b*d*e*(2*n*(p + 1) + 1), 0]
```

Rubi steps

$$\text{integral} = x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^{\frac{-1-n}{n}}(c+dx^n)^{\frac{-1-n}{n}}\left(ac-bdx^{2n}\right)\,dx=x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

```
[In] Integrate[(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)*(a*c - b*d*x^(2
*n)),x]
```

```
[Out] x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))
```

Maple [F]

$$\int (a+b\,x^n)^{\frac{-1-n}{n}}(c+d\,x^n)^{\frac{-1-n}{n}}\left(ac-bd\,x^{2n}\right)dx$$

```
[In] int((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x)
```

```
[Out] int((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

$$\int (a+bx^n)^{\frac{-1-n}{n}}(c+dx^n)^{\frac{-1-n}{n}}\left(ac-bdx^{2n}\right)\,dx=\frac{bdxx^{2\,n}+acx+(bc+ad)xx^n}{(bx^n+a)^{\frac{n+1}{n}}(dx^n+c)^{\frac{n+1}{n}}}$$

```
[In] integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, al
gorithm="fricas")
```

```
[Out] (b*d*x*x^(2*n) + a*c*x + (b*c + a*d)*x*x^n)/((b*x^n + a)^((n + 1)/n)*(d*x^n
 + c)^((n + 1)/n))
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx^n)^{\frac{-1-n}{n}}(c+dx^n)^{\frac{-1-n}{n}}\left(ac-bdx^{2n}\right)\,dx=\text{Timed out}$$

```
[In] integrate((a+b*x**n)**((-1-n)/n)*(c+d*x**n)**((-1-n)/n)*(a*c-b*d*x**(2*n)),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+bx^n)^{\frac{-1-n}{n}}(c+dx^n)^{\frac{-1-n}{n}}\left(ac-bdx^{2n}\right)\,dx=\int -\frac{bdx^{2\,n}-ac}{(bx^n+a)^{\frac{n+1}{n}}(dx^n+c)^{\frac{n+1}{n}}}\,dx$$

```
[In] integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, al
gorithm="maxima")
```

```
[Out] -integrate((b*d*x^(2*n) - a*c)/((b*x^n + a)^((n + 1)/n)*(d*x^n + c)^((n + 1
)/n)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(28) = 56.

Time = 0.33 (sec) , antiderivative size = 228, normalized size of antiderivative = 8.14

$$\begin{aligned}\int (a+bx^n)^{\frac{-1-n}{n}}(c+dx^n)^{\frac{-1-n}{n}}\left(ac-bdx^{2n}\right)\,dx\\ &=bdxx^{2\,n}e^{\left(-\frac{n\log(bx^n+a)+\log(bx^n+a)}{n}-\frac{n\log(dx^n+c)+\log(dx^n+c)}{n}\right)}\\ &\quad+bcxx^ne^{\left(-\frac{n\log(bx^n+a)+\log(bx^n+a)}{n}-\frac{n\log(dx^n+c)+\log(dx^n+c)}{n}\right)}\\ &\quad+adxx^ne^{\left(-\frac{n\log(bx^n+a)+\log(bx^n+a)}{n}-\frac{n\log(dx^n+c)+\log(dx^n+c)}{n}\right)}\\ &\quad+acxe^{\left(-\frac{n\log(bx^n+a)+\log(bx^n+a)}{n}-\frac{n\log(dx^n+c)+\log(dx^n+c)}{n}\right)}\end{aligned}$$

```
[In] integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, al
gorithm="giac")
```

```
[Out] b*d*x*x^(2*n)*e^(-(n*log(b*x^n + a) + log(b*x^n + a))/n - (n*log(d*x^n + c)
 + log(d*x^n + c))/n) + b*c*x*x^n*e^(-(n*log(b*x^n + a) + log(b*x^n + a))/n
 - (n*log(d*x^n + c) + log(d*x^n + c))/n) + a*d*x*x^n*e^(-(n*log(b*x^n + a)
 + log(b*x^n + a))/n - (n*log(d*x^n + c) + log(d*x^n + c))/n) + a*c*x*e^(-(
n*log(b*x^n + a) + log(b*x^n + a))/n - (n*log(d*x^n + c) + log(d*x^n + c))/
n)
```

Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.39

$$\int (a+bx^n)^{\frac{-1-n}{n}}(c+dx^n)^{\frac{-1-n}{n}}\left(ac-bdx^{2n}\right)\,dx=\frac{\frac{a\,c\,x}{(a+b\,x^n)^{\frac{n+1}{n}}}+\frac{x\,x^n\,(a\,d+b\,c)}{(a+b\,x^n)^{\frac{n+1}{n}}}+\frac{b\,d\,x\,x^{2\,n}}{(a+b\,x^n)^{\frac{n+1}{n}}}}{(c+d\,x^n)^{\frac{n+1}{n}}}$$

```
[In] int((a*c - b*d*x^(2*n))/((a + b*x^n)^((n + 1)/n)*(c + d*x^n)^((n + 1)/n)),x
)
```

```
[Out] ((a*c*x)/(a + b*x^n)^((n + 1)/n) + (x*x^n*(a*d + b*c))/(a + b*x^n)^((n + 1)
/n) + (b*d*x*x^(2*n))/(a + b*x^n)^((n + 1)/n))/(c + d*x^n)^((n + 1)/n)
```

3.592 $\int (hx)^{-1-n-np} (a+bx^n)^p (c+dx^n)^p (ac-bdx^{2n}) \, dx$

Optimal result

Integrand size = 45, antiderivative size = 45

$$\int (hx)^{-1-n-np} (a+bx^n)^p (c+dx^n)^p (ac-bdx^{2n}) \, dx = -\frac{(hx)^{-n(1+p)} (a+bx^n)^{1+p} (c+dx^n)^{1+p}}{hn(1+p)}$$

```
[Out] -(a+b*x^n)^(p+1)*(c+d*x^n)^(p+1)/h/n/(p+1)/((h*x)^(n*(p+1)))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {1863}

$$\int (hx)^{-1-n-np} (a+bx^n)^p (c+dx^n)^p (ac-bdx^{2n}) \, dx = -\frac{(hx)^{-n(p+1)} (a+bx^n)^{p+1} (c+dx^n)^{p+1}}{hn(p+1)}$$

```
[In] Int[(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*(a*c - b*d*x^(2*n)),x]
[Out] -(((a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n*(1 + p)*(h*x)^(n*(1 + p)))
)
```

Rule 1863

```
Int[((h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(
n_.))^(p_.)*((e_) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[e*(h*x)^(m + 1)*(
a + b*x^n)^(p + 1)*((c + d*x^n)^(p + 1)/(a*c*h*(m + 1))), x] /; FreeQ[{a, b
, c, d, e, g, h, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[m + n*(p + 1) + 1, 0]
&& EqQ[a*c*g*(m + 1) - b*d*e*(m + 2*n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(hx)^{-n(1+p)} (a+bx^n)^{1+p} (c+dx^n)^{1+p}}{hn(1+p)}$$

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int (hx)^{-1-n-np}(a+bx^n)^p(c+dx^n)^p\left(ac-bdx^{2n}\right)\,dx=-\frac{(hx)^{-n-np}(a+bx^n)^{1+p}(c+dx^n)^{1+p}}{hn+hnp}$$

```
[In] Integrate[(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*(a*c - b*d*x^(2*
n)),x]
```

```
[Out] -(((h*x)^(-n - n*p)*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n + h*n*p))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.07

$$-\frac{(a+b\,x^n)^p\,\mathrm{e}^{-\frac{(np+n+1)\left(-i\pi\,\mathrm{csgn}(ihx)^3+i\pi\,\mathrm{csgn}(ihx)^2\,\mathrm{csgn}(ih)+i\pi\,\mathrm{csgn}(ihx)^2\,\mathrm{csgn}(ix)-i\pi\,\mathrm{csgn}(ihx)\,\mathrm{csgn}(ih)\,\mathrm{csgn}(ix)+2\ln(x)+2\ln(h)\right)}{2}}(bd\,x^{2n}+c}{n\,(1+p)}$$

```
[In] int((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x)
```

```
[Out] -(a+b*x^n)^p*exp(-1/2*(n*p+n+1)*(-I*Pi*csgn(I*h*x)^3+I*Pi*csgn(I*h*x)^2*csg
n(I*h)+I*Pi*csgn(I*h*x)^2*csgn(I*x)-I*Pi*csgn(I*h*x)*csgn(I*h)*csgn(I*x)+2*
ln(x)+2*ln(h)))*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*x/n/(1+p)*(c+d*x^n)^p
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(46) = 92.

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.64

$$\int (hx)^{-1-n-np}(a+bx^n)^p(c+dx^n)^p\left(ac-bdx^{2n}\right)\,dx=$$
$$-\frac{\left(bdxx^{2n}e^{(-(np+n+1)\log(h)-(np+n+1)\log(x))}+acxe^{(-(np+n+1)\log(h)-(np+n+1)\log(x))}+(bc+ad)xx^ne^{(-(np+n+1)}}{np+n}$$

```
[In] integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, alg
orithm="fricas")
```

```
[Out] -(b*d*x*x^(2*n)*e^(-(n*p + n + 1)*log(h) - (n*p + n + 1)*log(x)) + a*c*x*e^
(-(n*p + n + 1)*log(h) - (n*p + n + 1)*log(x)) + (b*c + a*d)*x*x^n*e^(-(n*p
 + n + 1)*log(h) - (n*p + n + 1)*log(x)))*(b*x^n + a)^p*(d*x^n + c)^p/(n*p
+ n)
```

Sympy [F(-2)]

Exception generated.

$$\int (hx)^{-1-n-np} (a+bx^n)^p (c+dx^n)^p (ac-bdx^{2n}) \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((h*x)**(-n*p-n-1)*(a+b*x**n)**p*(c+d*x**n)**p*(a*c-b*d*x**(2*n)),
x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.71

$$\int (hx)^{-1-n-np} (a+bx^n)^p (c+dx^n)^p (ac-bdx^{2n}) \, dx = -\frac{(bdx^{2n} + ac + (bc+ad)x^n)h^{-np-n-1}e^{(-np\log(x)+p\log(bx^n+a)+p\log(dx^n+c)-n\log(x))}}{n(p+1)}$$

```
[In] integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, alg
orithm="maxima")
[Out] -(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n)*h^(-n*p - n - 1)*e^(-n*p*log(x) + p*
log(b*x^n + a) + p*log(d*x^n + c) - n*log(x))/(n*(p + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(46) = 92.

Time = 0.35 (sec) , antiderivative size = 237, normalized size of antiderivative = 5.27

$$\int (hx)^{-1-n-np} (a+bx^n)^p (c+dx^n)^p (ac-bdx^{2n}) \, dx = -\frac{(bx^n+a)^p(dx^n+c)^p bdxx^{2n}e^{(-np\log(h)-np\log(x)-n\log(h)-n\log(x)-\log(h)-\log(x))} + (bx^n+a)^p(dx^n+c)^p bcxx^n e^{(}}{}$$

```
[In] integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, alg
orithm="giac")
[Out] -((b*x^n + a)^p*(d*x^n + c)^p*b*d*x*x^(2*n)*e^(-n*p*log(h) - n*p*log(x) - n
*log(h) - n*log(x) - log(h) - log(x)) + (b*x^n + a)^p*(d*x^n + c)^p*b*c*x*x
^n*e^(-n*p*log(h) - n*p*log(x) - n*log(h) - n*log(x) - log(h) - log(x)) + (
b*x^n + a)^p*(d*x^n + c)^p*a*d*x*x^n*e^(-n*p*log(h) - n*p*log(x) - n*log(h)
 - n*log(x) - log(h) - log(x)) + (b*x^n + a)^p*(d*x^n + c)^p*a*c*x*e^(-n*p*
log(h) - n*p*log(x) - n*log(h) - n*log(x) - log(h) - log(x)))/(n*p + n)
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.76

$$\int (hx)^{-1-n-np}(a+bx^n)^p(c+dx^n)^p\left(ac-bdx^{2n}\right)\,dx$$
$$= -(c+d\,x^n)^p\left(\frac{a\,c\,x\,(a+b\,x^n)^p}{n\,(h\,x)^{n+n\,p+1}\,(p+1)}+\frac{x\,x^n\,(a\,d+b\,c)\,(a+b\,x^n)^p}{n\,(h\,x)^{n+n\,p+1}\,(p+1)}+\frac{b\,d\,x\,x^{2\,n}\,(a+b\,x^n)^p}{n\,(h\,x)^{n+n\,p+1}\,(p+1)}\right)$$

```
[In] int(((a*c - b*d*x^(2*n))*(a + b*x^n)^p*(c + d*x^n)^p)/(h*x)^(n + n*p + 1),x
)

[Out] -(c + d*x^n)^p*((a*c*x*(a + b*x^n)^p)/(n*(h*x)^(n + n*p + 1)*(p + 1)) + (x*
x^n*(a*d + b*c)*(a + b*x^n)^p)/(n*(h*x)^(n + n*p + 1)*(p + 1)) + (b*d*x*x^(
2*n)*(a + b*x^n)^p)/(n*(h*x)^(n + n*p + 1)*(p + 1)))
```

3.593 $\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1-$

Optimal result

Integrand size = 69, antiderivative size = 31

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx = \frac{ex(a+bx^n)^{1+p}(c+dx^n)^{1+p}}{ac}$$

```
[Out] e*x*(a+b*x^n)^(p+1)*(c+d*x^n)^(p+1)/a/c
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.014, Rules used = {1911}

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx = \frac{ex(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ac}$$

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + n + n*p)*x^n)/(a*c
) + (b*d*e*(1 + 2*n + 2*n*p)*x^(2*n))/(a*c)),x]
```

```
[Out] (e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)
```

Rule 1911

```
Int[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(p_.)*((e_) +
 (f_.)*(x_)^(n_.) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[e*x*(a + b*x^n)^(
p + 1)*((c + d*x^n)^(p + 1)/(a*c)), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}
```

```
, x] && EqQ[n2, 2*n] && EqQ[a*c*f - e*(b*c + a*d)*(n*(p + 1) + 1), 0] && Eq
Q[a*c*g - b*d*e*(2*n*(p + 1) + 1), 0]
```

Rubi steps

$$\text{integral} = \frac{ex(a+bx^n)^{1+p}(c+dx^n)^{1+p}}{ac}$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^p(c+dx^n)^p\left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right)dx = \frac{ex(a+bx^n)^{1+p}(c+dx^n)^{1+p}}{ac}$$

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + n + n*p)*x^n
)/(a*c) + (b*d*e*(1 + 2*n + 2*n*p)*x^(2*n))/(a*c)),x]
[Out] (e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.68

$$\frac{(a+b\,x^n)^p\,(bd\,x^{2n}+ad\,x^n+bc\,x^n+ac)\,ex(c+d\,x^n)^p}{ac}$$

```
[In] int((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2*n*p+2
*n+1)*x^(2*n)/a/c),x)
[Out] (a+b*x^n)^p*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*e*x/a/c*(c+d*x^n)^p
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int (a+bx^n)^p(c+dx^n)^p\left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right)dx = \frac{(bdexx^{2n}+acex+(bc+ad)exx^n)(bx^n+a)^p(dx^n+c)^p}{ac}$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2
*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="fricas")
[Out] (b*d*e*x*x^(2*n) + a*c*e*x + (b*c + a*d)*e*x*x^n)*(b*x^n + a)^p*(d*x^n + c)
^p/(a*c)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c)*e*(n*p+n+1)*x**n/a/c+b*d
*e*(2*n*p+2*n+1)*x**(2*n)/a/c),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx$$
$$= \frac{(bdexx^{2n}+acex+(bce+ade)xx^n)e^{(p\log(bx^n+a)+p\log(dx^n+c))}}{ac}$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2
*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="maxima")
```

```
[Out] (b*d*e*x*x^(2*n) + a*c*e*x + (b*c*e + a*d*e)*x*x^n)*e^(p*log(b*x^n + a) + p
*log(d*x^n + c))/(a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(31) = 62.

Time = 0.35 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.58

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx$$
$$= \frac{(bx^n+a)^p(dx^n+c)^pbdexx^{2n}+(bx^n+a)^p(dx^n+c)^pbcexx^n+(bx^n+a)^p(dx^n+c)^padexx^n+(bx^n+a)^p(d}{ac}$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2
*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="giac")
```

```
[Out] ((b*x^n + a)^p*(d*x^n + c)^p*b*d*e*x*x^(2*n) + (b*x^n + a)^p*(d*x^n + c)^p*
b*c*e*x*x^n + (b*x^n + a)^p*(d*x^n + c)^p*a*d*e*x*x^n + (b*x^n + a)^p*(d*x^
n + c)^p*a*c*e*x)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 10.78 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int (a+bx^n)^p\,(c+dx^n)^p\left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right)\,dx$$
$$=(c+d\,x^n)^p\left(e\,x\,(a+b\,x^n)^p+\frac{e\,x\,x^n\,(a\,d+b\,c)\,(a+b\,x^n)^p}{a\,c}+\frac{b\,d\,e\,x\,x^{2\,n}\,(a+b\,x^n)^p}{a\,c}\right)$$

```
[In] int((a + b*x^n)^p*(c + d*x^n)^p*(e + (e*x^n*(a*d + b*c)*(n + n*p + 1))/(a*c
) + (b*d*e*x^(2*n)*(2*n + 2*n*p + 1))/(a*c)),x)
```

```
[Out] (c + d*x^n)^p*(e*x*(a + b*x^n)^p + (e*x*x^n*(a*d + b*c)*(a + b*x^n)^p)/(a*c
) + (b*d*e*x*x^(2*n)*(a + b*x^n)^p)/(a*c))
```

3.594 $\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+m+n+np)x}{ac(1+m)}\right.$

Optimal result

Integrand size = 86, antiderivative size = 45

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)} + \frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx = \frac{e(hx)^{1+m} (a+bx^n)^{1+p} (c+dx^n)^{1+p}}{ach(1+m)}$$

```
[Out] e*(h*x)^(1+m)*(a+b*x^n)^(p+1)*(c+d*x^n)^(p+1)/a/c/h/(1+m)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.012, Rules used = {1862}

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)} + \frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx = \frac{e(hx)^{m+1} (a+bx^n)^{p+1} (c+dx^n)^{p+1}}{ach(m+1)}$$

```
[In] Int[(h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + m + n + n*
p)*x^n)/(a*c*(1 + m)) + (b*d*e*(1 + m + 2*n + 2*n*p)*x^(2*n))/(a*c*(1 + m))
),x]
```

```
[Out] (e*(h*x)^(1 + m)*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c*h*(1 + m))
```

Rule 1862

```
Int[((h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(
n_.))^(p_.)*((e_) + (f_.)*(x_)^(n_.) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Sim
```

```
p[e*(h*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(p + 1)/(a*c*h*(m + 1)))
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[
a*c*f*(m + 1) - e*(b*c + a*d)*(m + n*(p + 1) + 1), 0] && EqQ[a*c*g*(m + 1)
- b*d*e*(m + 2*n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{e(hx)^{1+m}(a+bx^n)^{1+p}(c+dx^n)^{1+p}}{ach(1+m)}$$

Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int (hx)^m(a+bx^n)^p(c+dx^n)^p\left(e+\frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)}+\frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right)dx = \frac{ex(hx)^m(a+bx^n)^{1+p}(c+dx^n)^{1+p}}{ac(1+m)}$$

```
[In] Integrate[(h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + m +
n + n*p)*x^n)/(a*c*(1 + m)) + (b*d*e*(1 + m + 2*n + 2*n*p)*x^(2*n))/(a*c*(1
 + m))),x]
```

```
[Out] (e*x*(h*x)^m*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c*(1 + m))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.29

$$\frac{(a+b\,x^n)^p\,x^m h^m \mathrm{e}^{\frac{i\pi\,\operatorname{csgn}(ihx)m(\operatorname{csgn}(ihx)-\operatorname{csgn}(ix))(-\operatorname{csgn}(ihx)+\operatorname{csgn}(ih))}{2}}\left(bd\,x^{2n}+ad\,x^n+bc\,x^n+ac\right)ex(c+d\,x^n)^p}{ac\,(1+m)}$$

```
[In] int((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/c/(1+m
)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x)
```

```
[Out] (a+b*x^n)^p*x^m*h^m*exp(1/2*I*Pi*csgn(I*h*x)*m*(csgn(I*h*x)-csgn(I*x))*(-cs
gn(I*h*x)+csgn(I*h)))*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*e*x/a/c/(1+m)*(c+d*
x^n)^p
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.96

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e + \frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)} + \frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx = \frac{\left(bdexx^{2n}e^{(m\log(h)+m\log(x))} + acexe^{(m\log(h)+m\log(x))} + (bc+ad)exx^n e^{(m\log(h)+m\log(x))}\right)(bx^n+a)^p(dx^n+c)^p}{acm+ac}$$

```
[In] integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/
c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="fricas")
```

```
[Out] (b*d*e*x*x^(2*n)*e^(m*log(h) + m*log(x)) + a*c*e*x*e^(m*log(h) + m*log(x))
+ (b*c + a*d)*e*x*x^n*e^(m*log(h) + m*log(x)))*(b*x^n + a)^p*(d*x^n + c)^p/
(a*c*m + a*c)
```

Sympy [F(-2)]

Exception generated.

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e + \frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)} + \frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((h*x)**m*(a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x
**n/a/c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x**(2*n)/a/c/(1+m)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(45) = 90$.

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.04

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e + \frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)} + \frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx = \frac{\left(aceh^m xx^m + bdeh^m xe^{(m\log(x)+2n\log(x))} + (bceh^m + adeh^m)xe^{(m\log(x)+n\log(x))}\right)e^{(p\log(bx^n+a)+p\log(dx^n+c))}}{ac(m+1)}$$

```
[In] integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/
c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="maxima")
```

```
[Out] (a*c*e*h^m*x*x^m + b*d*e*h^m*x*e^(m*log(x) + 2*n*log(x)) + (b*c*e*h^m + a*d
*e*h^m)*x*e^(m*log(x) + n*log(x)))*e^(p*log(b*x^n + a) + p*log(d*x^n + c))/
(a*c*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(45) = 90$.

Time = 0.46 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.44

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e + \frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)} + \frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx = \frac{(bx^n+a)^p(dx^n+c)^p bdexx^{2n}e^{(m\log(h)+m\log(x))} + (bx^n+a)^p(dx^n+c)^p bcexx^n e^{(m\log(h)+m\log(x))} + (bx^n+a)}{acm+ac}$$

```
[In] integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/
c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="giac")
```

```
[Out] ((b*x^n + a)^p*(d*x^n + c)^p*b*d*e*x*x^(2*n)*e^(m*log(h) + m*log(x)) + (b*x
^n + a)^p*(d*x^n + c)^p*b*c*e*x*x^n*e^(m*log(h) + m*log(x)) + (b*x^n + a)^p
*(d*x^n + c)^p*a*d*e*x*x^n*e^(m*log(h) + m*log(x)) + (b*x^n + a)^p*(d*x^n +
 c)^p*a*c*e*x*e^(m*log(h) + m*log(x)))/(a*c*m + a*c)
```

Mupad [B] (verification not implemented)

Time = 11.69 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.36

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e + \frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)} + \frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx = (c+d\,x^n)^p \left(\frac{e\,x\,(h\,x)^m\,(a+b\,x^n)^p}{m+1} + \frac{e\,x\,x^n\,(h\,x)^m\,(a\,d+b\,c)\,(a+b\,x^n)^p}{a\,c\,(m+1)} + \frac{b\,d\,e\,x\,x^{2\,n}\,(h\,x)^m\,(a+b\,x^n)^p}{a\,c\,(m+1)}\right)$$

```
[In] int((h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + (e*x^n*(a*d + b*c)*(m + n + n*
p + 1))/(a*c*(m + 1)) + (b*d*e*x^(2*n)*(m + 2*n + 2*n*p + 1))/(a*c*(m + 1))
),x)
```

```
[Out] (c + d*x^n)^p*((e*x*(h*x)^m*(a + b*x^n)^p)/(m + 1) + (e*x*x^n*(h*x)^m*(a*d
 + b*c)*(a + b*x^n)^p)/(a*c*(m + 1)) + (b*d*e*x*x^(2*n)*(h*x)^m*(a + b*x^n)^
p)/(a*c*(m + 1)))
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                   end if
               else #result contains complex but optimal is not
                   if debug then
                           print("result contains complex but optimal is not");
                   fi;
                   return "C","Result contains complex when optimal does not.";
               fi;
           else # result do not contain complex
                # this assumes optimal do not as well. No check is needed here.
               if debug then
                      print("result do not contain complex, this assumes optimal do not as well")
               fi;
               if leaf_count_result<=2*leaf_count_optimal then
                   if debug then
                       print("leaf_count_result<=2*leaf_count_optimal");
                   fi;
                   return "A"," ";
               else
                   if debug then
                       print("leaf_count_result>2*leaf_count_optimal");
                   fi;
                   return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
               fi;
           fi;
       else #ExpnType(result) > ExpnType(optimal)
           if debug then
               print("ExpnType(result) > ExpnType(optimal)");
           fi;
           return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
       fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.a
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```